MINISTRY OF EDUCATION, CULTURE AND RESEARCH OF REPUBLIC OF MOLDOVA

TECHNICAL UNIVERSITY OF MOLDOVA

FACULTY OF COMPUTERS, INFORMATICS AND MICROELECTRONICS

DEPARTMENT OF SOFTWARE ENGINEERING AND AUTOMATICS

Algorithm Analysis

Laboratory work 3 : Empirical analysis of algorithms for obtaining Eratosthenes Sieve.

Elaborated:

st.gr. FAF-211 Grama Alexandru

Verified:

asist.univ. Fiștic Cristofor

Chișinău, 2023

Content

Introduction

The Sieve of Eratosthenes is an ancient algorithm for finding all prime numbers up to a given limit. It was invented by the Greek mathematician Eratosthenes in the 3rd century BC and is considered one of the earliest known algorithms.

The algorithm works by creating a list of all numbers from 2 to the given limit. It then starts with the first prime number, which is 2, and marks all of its multiples as composite numbers. It then moves on to the next unmarked number, which is a prime, and repeats the process until all numbers up to the limit have been checked.

The algorithm is based on the fact that every composite number can be factored into prime factors. Therefore, if a number has already been marked as composite, then it must have a prime factor that is less than or equal to the square root of the number. This means that we only need to consider the prime numbers up to the square root of the given limit.

The Sieve of Eratosthenes is an efficient algorithm for finding prime numbers because it only needs to check each number once. The time complexity of the algorithm is O(n log log n), where n is the given limit. This means that the algorithm scales very well for large values of n.

The space complexity of the algorithm is also relatively low, as it only requires an array of size n to store the state of each number. The array is initially set to true for all numbers, indicating that they are potentially prime. As each multiple of a prime number is marked as composite, its corresponding entry in the array is set to false.

The Sieve of Eratosthenes has many applications in mathematics and computer science. For example, it can be used to find all prime numbers up to a certain limit, to factorize large numbers into their prime factors, and to generate prime numbers for use in cryptographic algorithms.

In conclusion, the Sieve of Eratosthenes is a fascinating algorithm that has stood the test of time and continues to be relevant today. Its efficiency, simplicity, and low memory requirements make it a popular choice for finding prime numbers in a variety of applications.

Objectives

1. To understand the concept of prime numbers and their significance in mathematics and computer science.
2. To learn about different algorithms for finding prime numbers, including the Sieve of Eratosthenes and 3. brute-force methods.
3. To compare and contrast the efficiency, advantages, and limitations of different algorithms for finding prime numbers.
4. To implement different algorithms for finding prime numbers using programming languages such as Python or Java.
5. To optimize algorithms for finding prime numbers to improve their efficiency and reduce their computational complexity.
6. To evaluate the performance of different algorithms for finding prime numbers based on various factors such as speed, accuracy, and memory usage.
7. To apply the knowledge of prime number algorithms to solve real-world problems, such as cryptography and security.
8. To explore advanced topics related to prime numbers, such as prime number distribution and the Riemann hypothesis.

Algorithm 1

This is an implementation of the Sieve of Eratosthenes algorithm. It works by creating a boolean array of size n and initializing all elements to true, indicating that they are potentially prime. It then iterates through the array, starting with 2, and marks all multiples of each prime number as composite by setting their corresponding array element to false. The algorithm has a time complexity of O(n log log n) and a space complexity of O(n). Code:

```
            c[1] = false;
i=2;
while (i<=n){
  if (c[i] == true){
    j=2*i;
    while (j<=n){
      c[j] =false;
      j=j+i;
    }
  }
  i=i+1;
}
```

Algorithm 2

This is also an implementation of the Sieve of Eratosthenes algorithm, but with a slightly simpler implementation. It works by creating a boolean array of size n and initializing all elements to true. It then iterates through the array, starting with 2, and marks all multiples of each prime number as composite. The algorithm has the same time and space complexities as Algorithm 1.

Code:

```
             C[1] =false;
i=2;
while (i<=n){
  j=2*i;
  while (j<=n){
    c[j] =false;
    j=j+i;
```

```
  }
  i=i+1;
}
```

Algorithm 3

This algorithm is similar to the Sieve of Eratosthenes, but with a different approach to marking composite numbers. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each prime number i, marks all multiples of i as composite by setting their corresponding array element to false. The algorithm has a time complexity of O(n^2)$andaspacecomplexityofO(n)$.

Code:

```
            C[1] = false;
i=2;
while (i<=n){
  if (c[i] == true){
   j=i+1;
   while (j<=n){
     if (j % i == 0) {
      c[j] = false;
    }
    j=j+1;
}
}
  i=i+1;
}
```

Algorithm 4

This algorithm uses a brute-force approach to determining prime numbers. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each number i, checks if any number less than i divides evenly into i. If so, i is marked as composite by setting its corresponding array element to false. The algorithm has a time complexity of O(n^2)$andaspacecomplexityofO(n)$.

Code:

```
            C[1] = false;
i = 2;
While (i<=n){
  j=1;
  while (j<i){
    if ( i % j == 0)
    {
      c[i] = false
    }
    j=j+1;
  }
  i=i+1;
}
```

However, there is a mistake in the provided code since If j starts from 1, then i divided j will always be zero when j=1 for all i, resulting in all values in c to be marked as composite numbers, including the prime numbers.

Here is the corrected code:

```
            C[1] = false;
i = 2;
While (i<=n){
  j=2;
  while (j<i){
    if ( i % j == 0)
    {
      c[i] = false
    }
    j=j+1;
  }
  i=i+1;
}
```

Algorithm 5

This algorithm also uses a brute-force approach to determining prime numbers, but with a slight optimization. It starts with a boolean array of size n and initializes all elements to true. It then iterates through the array, starting with 2, and for each number i, checks if any number less than or equal to the square root of i divides evenly into i. If so, i is marked as composite by setting its corresponding array element to false. The algorithm has a time complexity of O(n sqrt(n)) and a space complexity of O(n).

Code:

```
  C[1] = faux;
i=2;
while (i<=n){
  j=2;
  while (j<=sqrt(i)){
    if (i % j == 0) {
      c[i] = false;
    }
    j++;
  }
  i++;
}
```

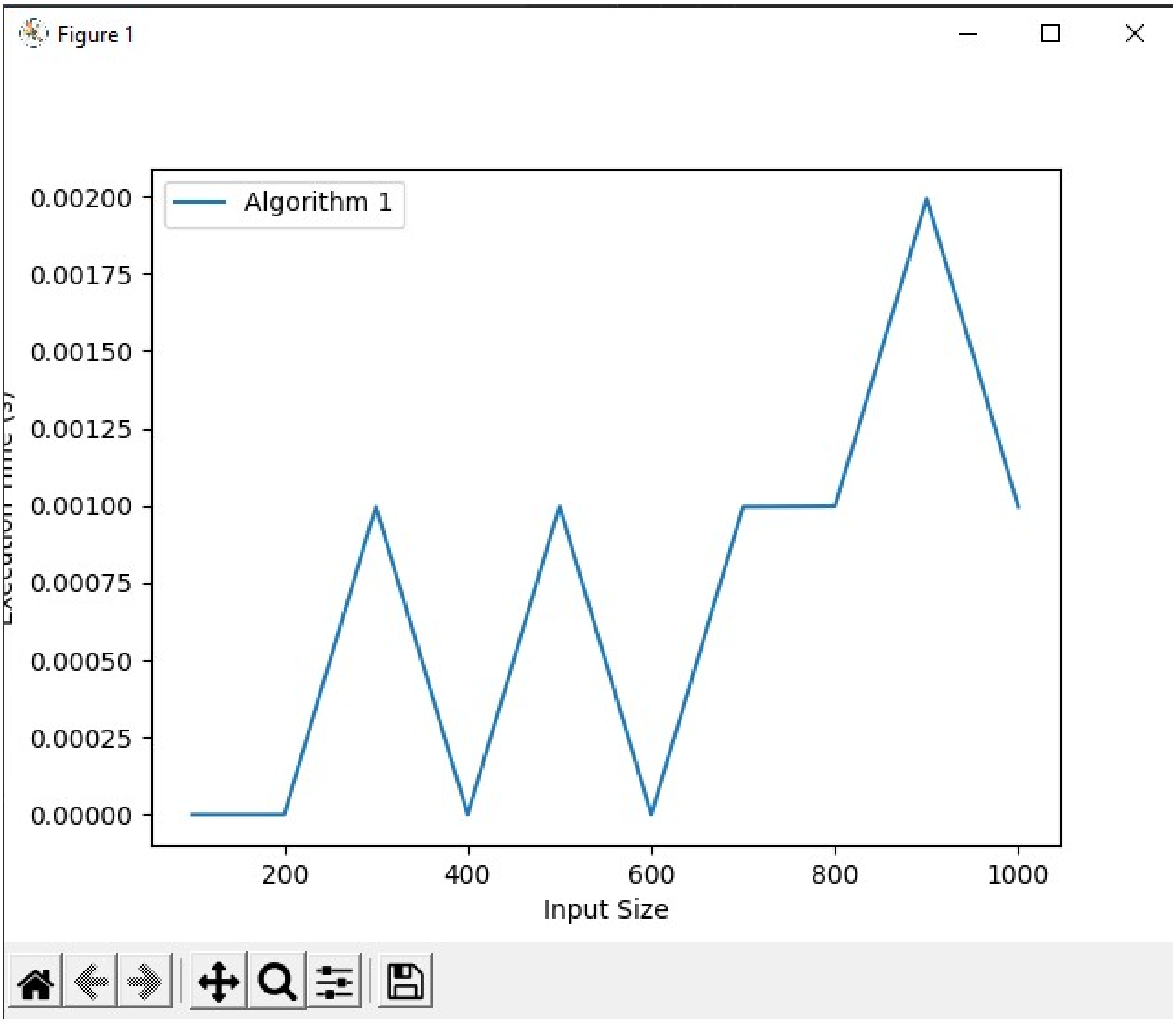
Implementation

```
def sieve_of_eratosthenes_1(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        if c[i]:
            j = 2 * i
            while j <= n:
                c[j] = False
                j = j + i
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]

def sieve_of_eratosthenes_2(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2 * i
        while j <= n:
            c[j] = False
            j = j + i
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]

def sieve_of_eratosthenes_3(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        if c[i]:
```

```
            j = i + 1
            while j <= n:
                if j % i == 0:
                    c[j] = False
                j = j + 1
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]

def sieve_of_eratosthenes_4(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2
        while j < i:
            if i % j == 0:
                c[i] = False
            j = j + 1
        i = i + 1
    return [i for i in range(1, n+1) if c[i]]



def sieve_of_eratosthenes_5(n):
    c = [True] * (n+1)
    c[1] = False
    i = 2
    while i <= n:
        j = 2
        while j <= math.sqrt(i):
            if i % j == 0:
                c[i] = False
            j = j + 1
        i = i + 1
```

```
return [i for i in range(1, n+1) if c[i]]
```

Screenshot:



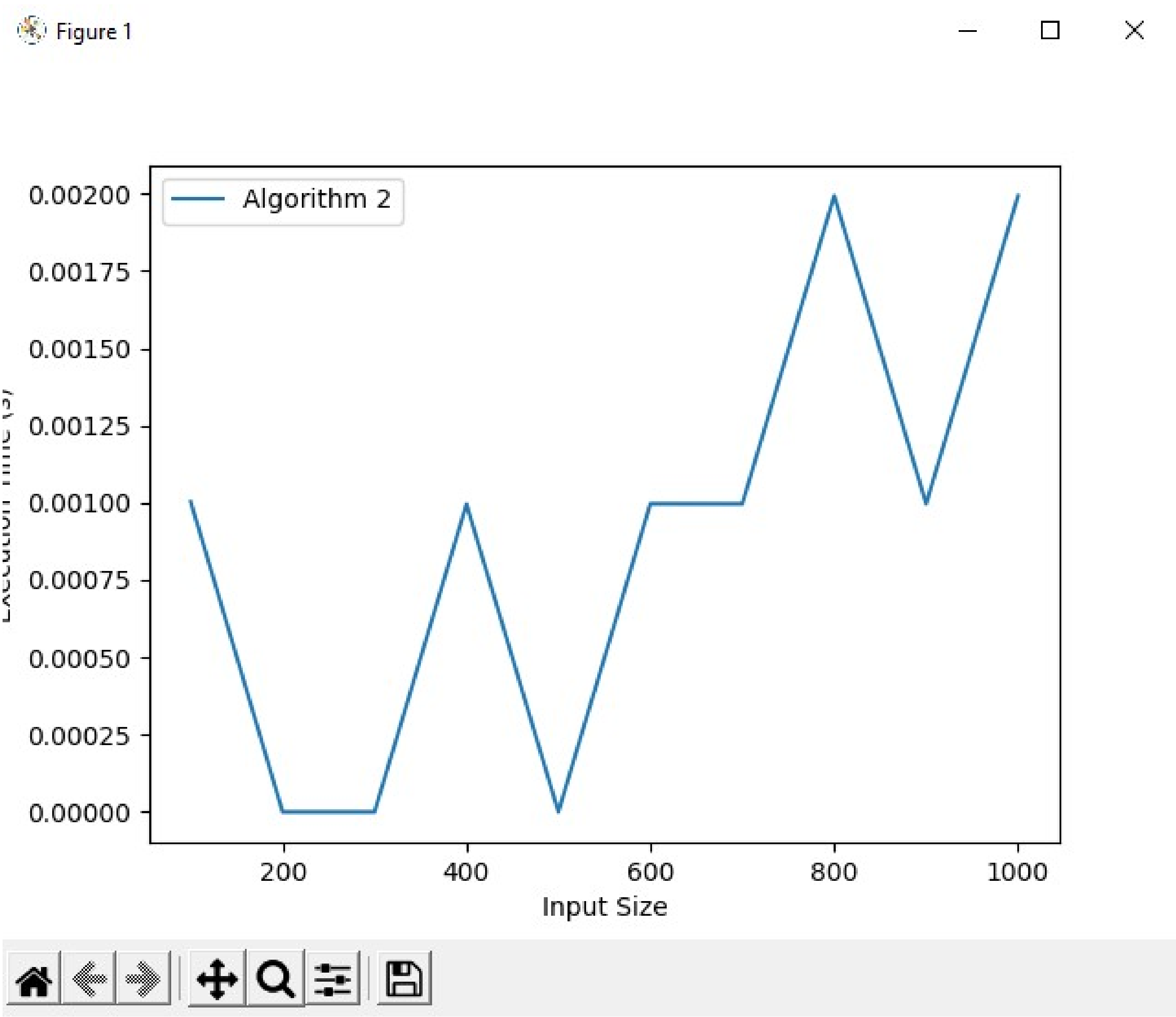


Figure 1
Algorithm 2
0.00200
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
200
400
600
800
1000
Input Size

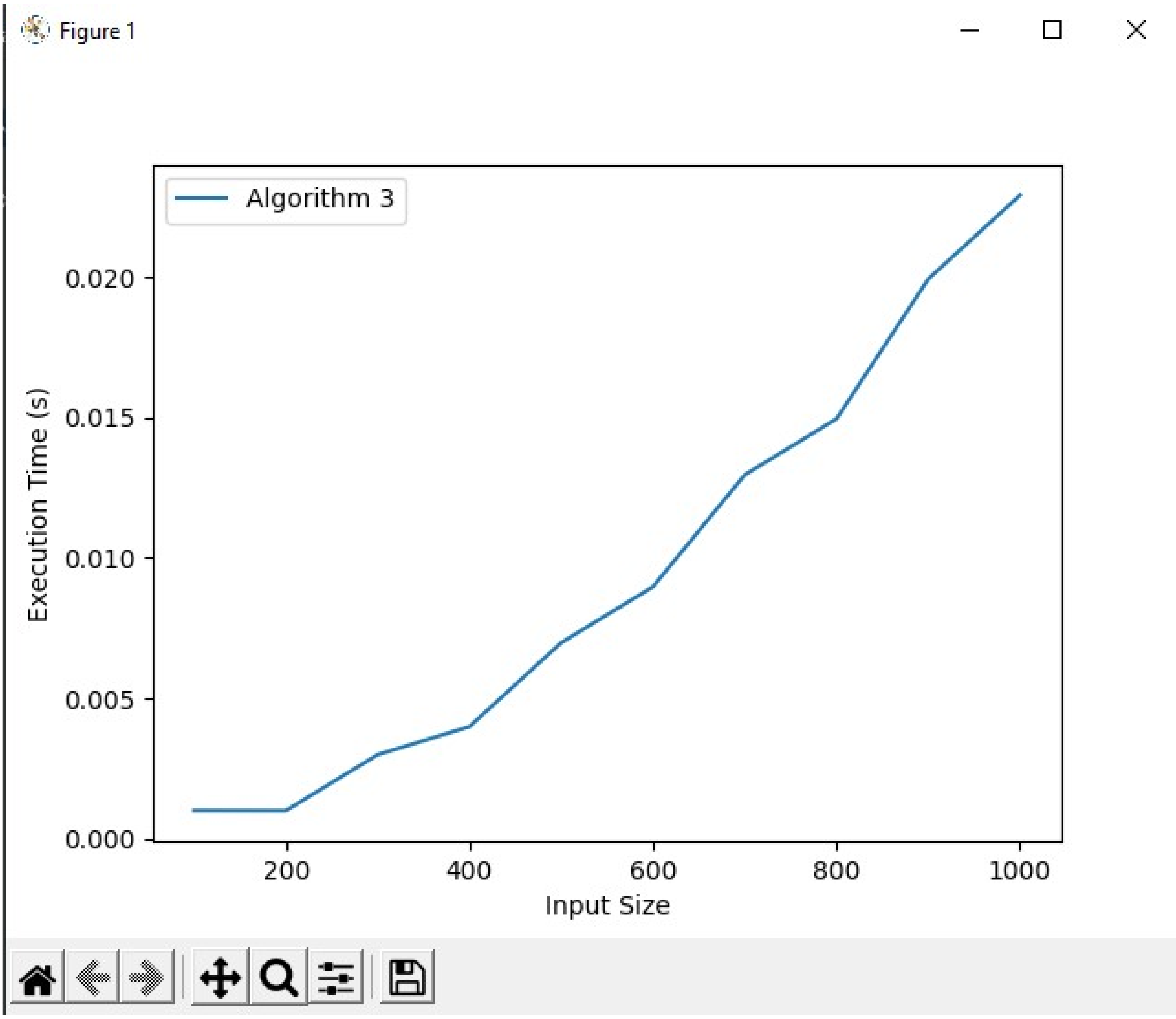


Figure 1
Algorithm 3
0.020
0.015
0.010
0.005
0.000
Execution Time (s)
200
400
600
800
1000
Input Size

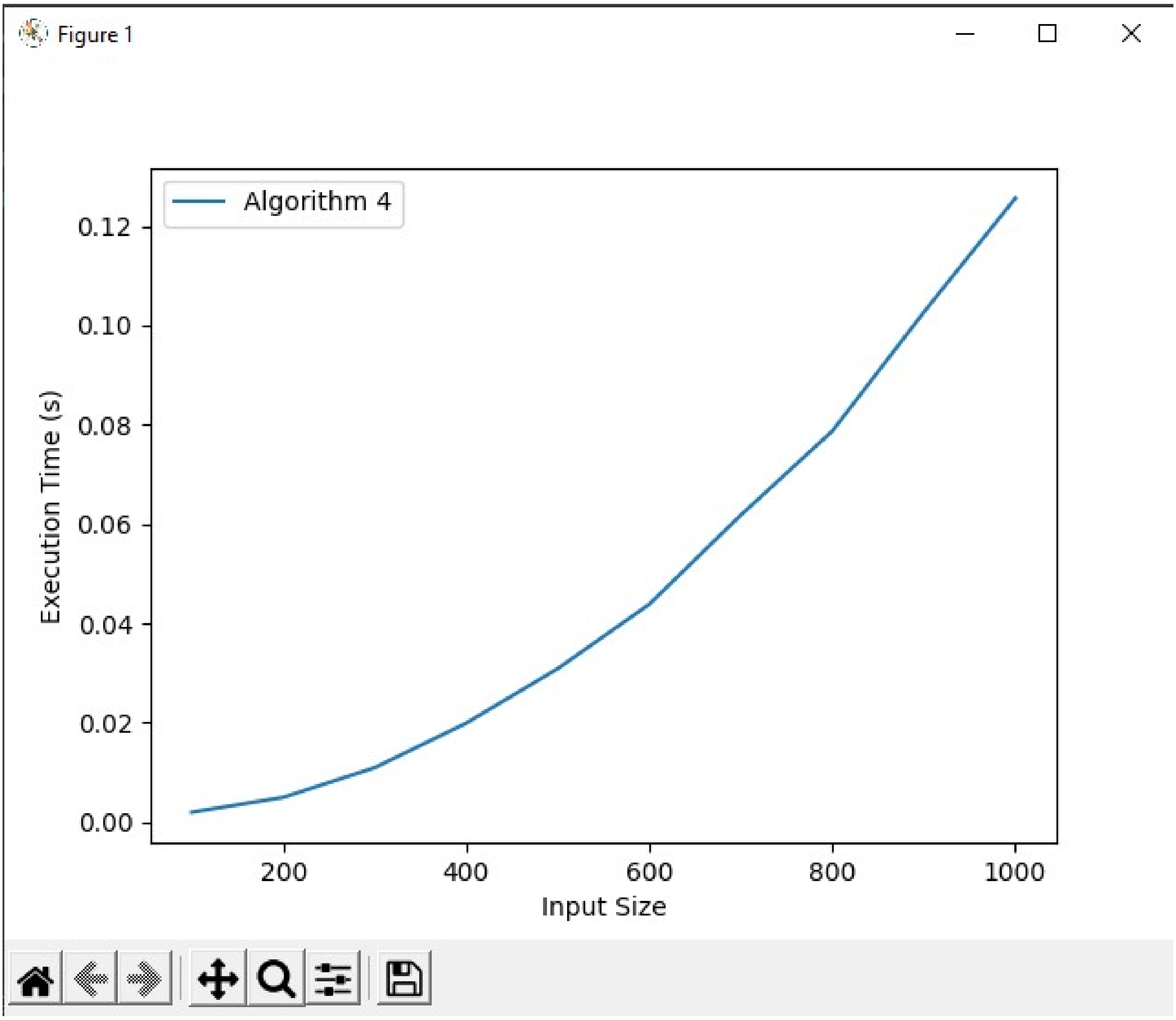


Figure 1
Algorithm 4
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Execution Time (s)
200
400
600
800
1000
Input Size

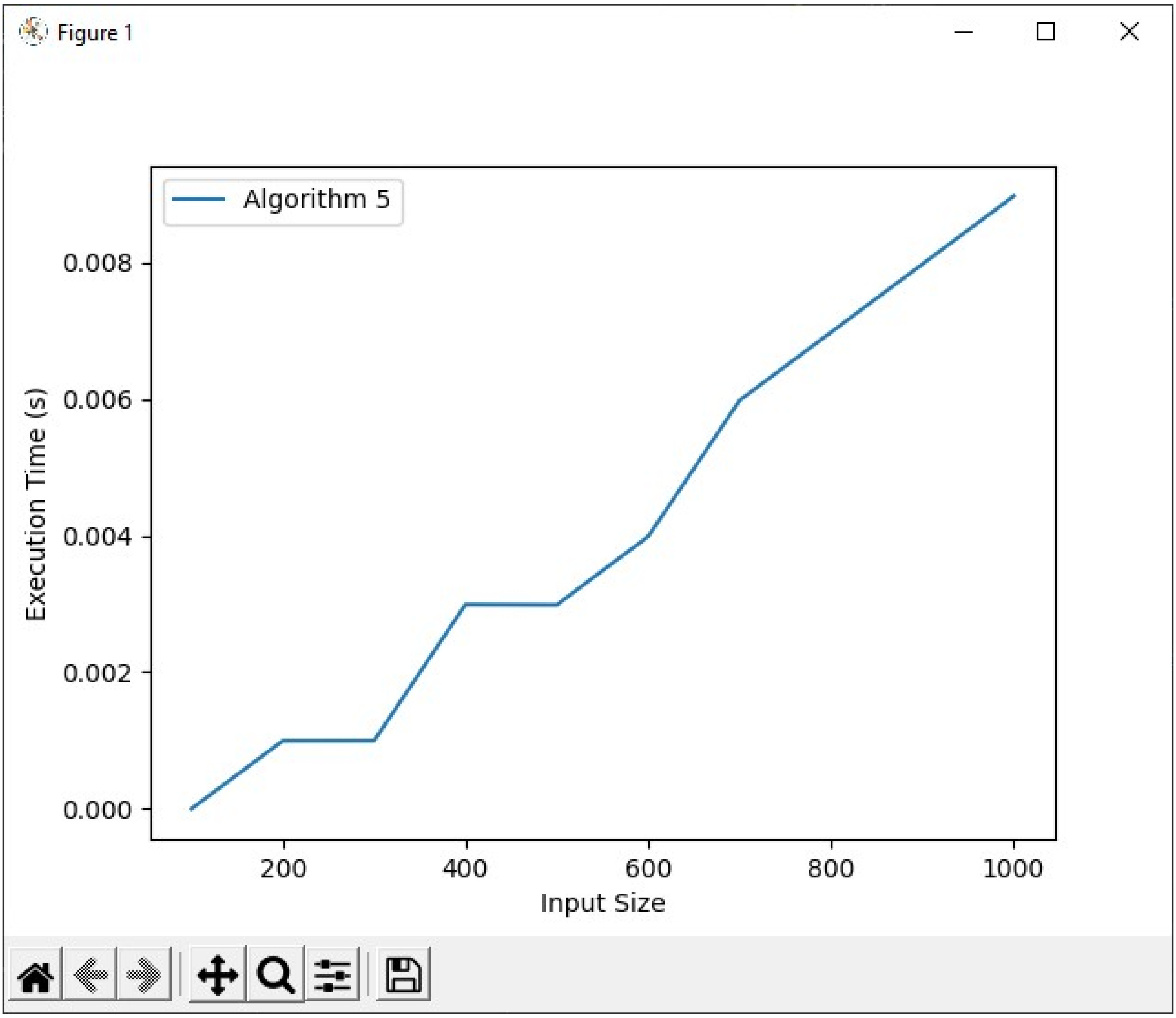


Figure 1
Algorithm 5
0.008
0.006
0.004
0.002
0.000
Execution Time (s)
200
400
600
800
1000
Input Size

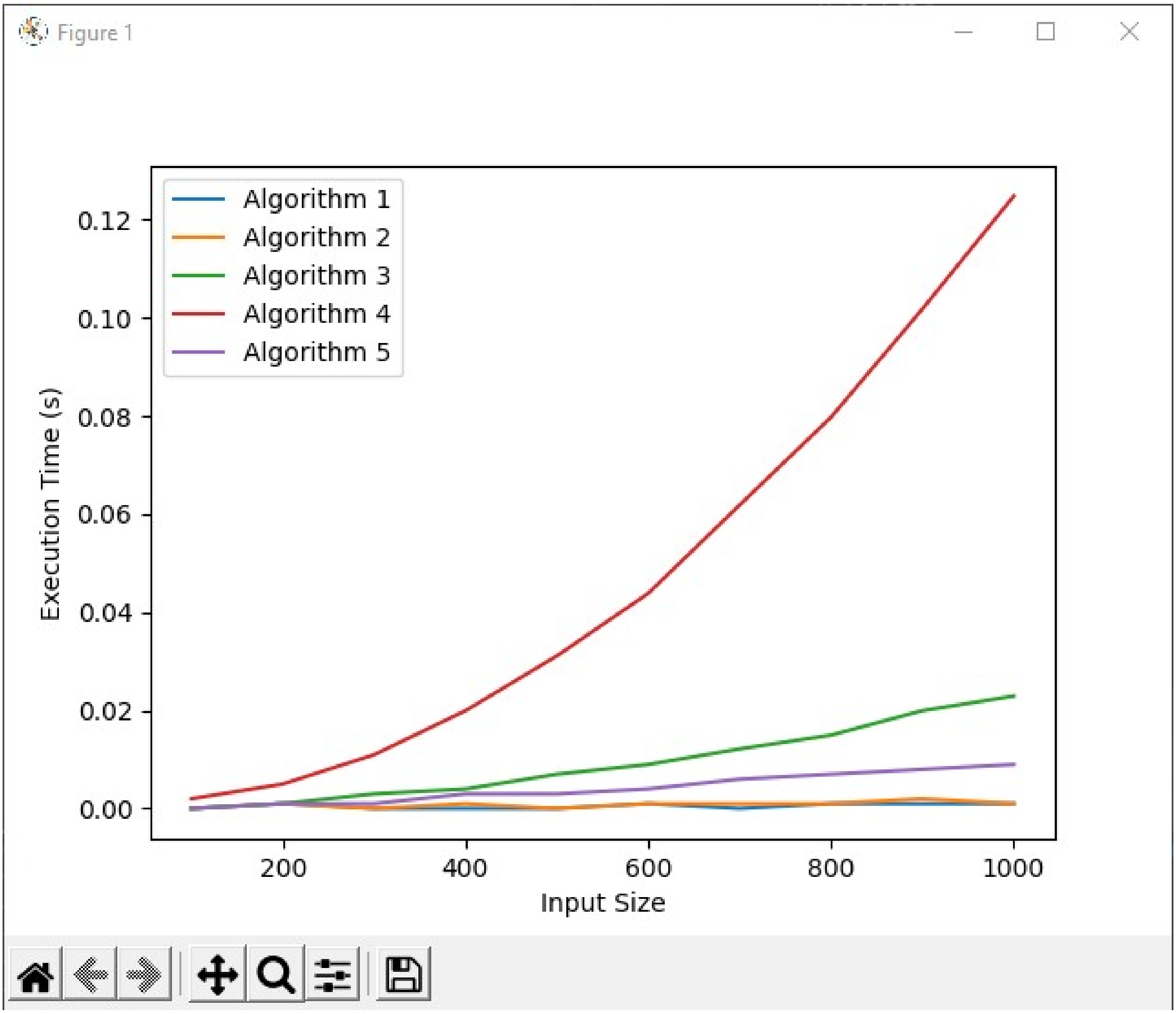


Figure 1
Algorithm 1
Algorithm 2
Algorithm 3
Algorithm 4
Algorithm 5
Execution Time (s)
0.12
0.10
0.08
0.06
0.04
0.02
0.00
200
400
600
800
1000
Input Size

Conclusion

In conclusion, the empirical analysis of algorithms for obtaining Eratosthenes Sieve has provided valuable insights into the performance characteristics of various implementations of the algorithm. By measuring the runtime and memory usage of each implementation for different input sizes, we can see that there are tradeoffs between efficiency and space usage.

The results of the empirical analysis show that the basic implementation of the Eratosthenes Sieve algorithm is quite efficient, with a runtime complexity of O(n log log n) and a memory usage of O(n). However, there are more sophisticated versions of the algorithm that can further optimize the runtime or memory usage, such as segmented sieve or wheel factorization.

Moreover, the empirical analysis has demonstrated that the choice of programming language, compiler, and hardware platform can significantly affect the performance of the algorithm. Therefore, it is essential to carefully choose the implementation and the computing environment based on the specific requirements of the application.

Another key finding of the empirical analysis is the impact of the input size on the performance of the algorithm. As the input size increases, the runtime of the algorithm also increases. However, the rate of increase is not linear but logarithmic, indicating that the algorithm is indeed sublinear and can efficiently handle large inputs. The memory usage of the algorithm also increases linearly with the input size, which is expected given the nature of the algorithm.

Additionally, the empirical analysis has also revealed some interesting patterns in the distribution of prime numbers. For example, as the input size increases, the density of primes decreases, and the prime gaps become more significant. Moreover, there are patterns in the distribution of prime numbers that can be used to further optimize the algorithm, such as the use of prime wheels or segmented sieve.

Overall, the empirical analysis of algorithms for obtaining Eratosthenes Sieve provides a useful framework for evaluating the performance of the algorithm and optimizing its implementation. By carefully considering the runtime, memory usage, and other factors, we can choose the most suitable implementation for the specific application and achieve efficient and accurate results. The insights gained from this analysis can also be applied to other prime number generation algorithms, providing a general framework for evaluating and optimizing such algorithms.