Convex Optimization for Holistic Mechatronic System Design

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***Abstract*—Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.**

I. Introduction

Could the current resaerch trend of using genetric algorithms as a solver of mechatronic design optimization problems be improved by adapting a diciplinary convex optimization approach? The multidiciplinary nature of mechatronic systems, which could be characterised as systems with synergetic integrations of mechanical engineering with eletronics and intelligent computer control [**?**], makes early design optimization a difficult process. Optimizations are often done in the latter detailed design phases [**?**], [**?**] whereas early design decisions becomes a limiting factor. Research on the topic of optimization in early design stages for mechatronic systems has presented both holistic and non-holistic approaches which share the common feature of using genetic algorithms, either as a complement or as a standalone solver. The disadvantages of GA when it comes to accuracy and computation time justifies research on applying diciplinary convex optimization.

Optimizing mechatronic systems using genetic algorithms has been widely explored by researchers today. Hammadi [**?**] proposes an emergent multi-agent approach where the system is decoposed into smal sub-systems (or agents) that in turn is optimized using the genetic algorithms NSGA II. Another approach that uses NSGA II is proposed by Guizani [**?**] where a partioning method is utilized to decompose the mechatronic system and classification of the interactions between partions are done. An approach developed by Seo [**?**] and further developed by Behbahani [**?**] uses a two loop optimization process which incorperates genetic algorithms and bond graphs in an outer loop to otimize system topology and genetic programming in an inner loop to fin the elite solution within the optimal topology. The research in this paper will be based upon the work done by Malmquiest et al. [**?**] where they created a framework named IDIOM which optimizises mechatronic systems in a holistic manner by the use of genetic algorithms.

The IDIOM framework developed by Malmquist et al. extends on the Roos [**?**] on a research on a methodology for integrated design och mechatronic servo systems. In his research, Roos reflects on the choice of using genetic algorithms to optimize the servo systems by stating that "The drawback of this method for system optimization is that it is computationally intensive and that there exists no mathematical proof that it actually finds the true optimum". This statement justifies the exploration of other optimization techniques which could provide true optimizations in a less computationally intensive manner, and for which convex optimization has been chosen. Diciplinary convex programming is a methodology developed by Grant et al. [**?**] which purpose, according to the authors, is to "allow much of the manipulation and transformation required to analyze and solve convex programs to be automated". The methodology originates from procedures taken by those who regularly study convex optmization problems and has been implemented in the modelling framework called CVX [**?**].

The argument made by Roos serve as a starting point in the research presented in this report. It challenges the current trend of using evolutionary algorithms when optimizing the design of mechatronic systems and could provide a optimization approach that is less compututionally expensive and more accurate.

The remainder of this paper will introduce the prevuous work done by Malmquist et al. of which this study aim to extend on. $3 describes the method used when applying convex optimization to the mechatronic system. Finally, $4 introduces a case study where a system optimization is performed, and compared to the results obtained by Malmquist et al. , of a mechatronic system composed of a motor, planetary gear and a load.

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II. Previous Work

III. Optimization Approach

A. Mathematical Preliminaries

The following introduction regarding convex optimization mainly come from [2]. Note that the notations for different functions and sets will be used in later discussions. Before bringing the definition of convex optimization, we need to introduce what convex set and convex function are. A set C is a convex set if for any two elements $x_1, x_2 \in C$, $\theta x_1 + (1-\theta)x_2 \in C$ holds for any $\theta \in [0,1]$. If the set C represents a geometric region, any line segments defined by points which belong to C shall be in the set. A function $f : \mathbf{R^n} \rightarrow \mathbf{R}$ is convex if **dom** f is a convex set and any chord between any two points on f lies above the graph of f. Based on the definition of convex function, a convex optimization (CO) problem is defined as

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \quad i = 1, \ldots, m \\ & h_i(x) = 0, \quad i = 1, \ldots, p \end{array} \tag{1}$$

where all the functions in 1 are convex functions. $f_0(x)$ is named as objective function, $f_i(x)$ inequality constraint and $h_i(x)$ equality constraint. Although it is now a mature technique to solve a CO problem, it is not easy to formulate objective and constraint functions as convex ones. In the case of mechatronic system design, some typical objective functions, such as volume of the system, are not convex functions and not allowed to be manipulated. However they do follow polynomial form. Luckily, it is still possible to take the advantage of CO technique if those optimization problems can be formulated as another classic form, which is geometric programming (GP).

Before discussion about GP, we need to introduce the form GP follows. A monomial function is defined as 2

$$f(x) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}, \quad \mathbf{dom}\, f = \mathbf{R}^{\mathbf{n}}_{++} \tag{2}$$

where c is positive and a_i can be any real number. A posynomial function is a sum of monomials, which is

$$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \quad \mathbf{dom}\, f = \mathbf{R}^{\mathbf{n}}_{++} \tag{3}$$

A GP problem follows the same format as 1, except that objective and inequality constraints are posynomials and equality constraints are monomials. In fact, by conducting variable substitution, it is easy to show a GP problem can be transferred as a CO problem [2].

In order to formulate an optimization as GP, we don't need to verify if a function is convex or not. Instead, we only need to make sure both objective and constraint functions follow either monomial or posynomial forms. In this way, it is avoided to manipulate functions to be convex and it provides a well defined principle for researchers to create constraints.

B. Modeling Methodology for Mechanical Components

GP is considered to be promising in solving optimization problems of mechatronic system design since many of the constraints can be expressed by polynomial approximation. Although there is some distance between posynomials and polynomials, it is possible to get posynomial form since the parameters we deal with are all in positive orthant and all exponents in posynomials can be any real numbers while only non-negative integers for polynomial models.

In order to create constraints, many of those properties need to be expressed by objective variables as posynomials for inequalities and monomials for equalities. In general, the constraint functions can be obtained by either analysis according to some theoretical principles or data fitting based on existing data. The former way would be easier to get a constraint, but in a rather complex form. Then it is necessary to manipulate the constraint function and make it to be proper. At this stage, there is no systematic way to do this. In the following case, we use a convergent sequence to change a nonlinear inequality to be posynomial. In principle any convergent sequences without negative terms can be used to approximate non-posynomial part of constraints.

In case that no analytic expression is available or the ones we get are too complicated to maneuver, data-fitting approach may be conducted. The data can come from experimental data collection, existing lookup tables or simulation results of obtained expression. In this way identification of models becomes another optimization problem, the optimal of which is a posynomial which has smallest deviation from the data according to some criterion. Although polynomial approximation is a rather mature technology, very few contributions can be found regarding posynomial identification.

IV. Conclusion

Conclusion...

Acknowledgment

References

[1] K. Janschek, *Mechatronic systems design: methods, models, concepts*. Springer Science & Business Media, 2011.
[2] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.