Question 1

The fractional quantum Hall effect (FQHE) was discovered by D. C. Tsui and H. Stormer at Bell Labs in 1981. In the experiment electrons were confined in two dimensions on the GaAs side by the interface potential of a GaAs/AlGaAs heterojunction fabricated by A. C. Gossard (here we neglect the thickness of the two-dimensional electron layer). A strong uniform magnetic field B was applied perpendicular to the two-dimensional electron system. As illustrated in Figure 1, when a current I was passing through the sample, the voltage V_{H} across the current path exhibited an unexpected quantized plateau (corresponding to a Hall resistance $R_{\mathrm{H}} = 3h/e^2$) at sufficiently low temperatures. The appearance of the plateau would imply the presence of fractionally charged quasiparticles in the system, which we analyze below. For simplicity, we neglect the scattering of the electrons by random potential, as well as the electron spin.

(a) In a classical model, two-dimensional electrons behave like charged billiard balls on a table. In the GaAs/AlGaAs sample, however, the mass of the electrons is reduced to an effective mass m^* due to their interaction with ions.

(i) (**2 point**) Write down the equation of motion of an electron in perpendicular electric field $\vec{E} = -E_y\hat{y}$ and magnetic field $\vec{B} = B\hat{z}$.

(ii) (**1 point**) Determine the velocity v_{s} of the electrons in the stationary case.

(iii) (**1 point**) Which direction is the velocity pointing at?

(b) (**2 points**) The Hall resistance is defined as $R_{\mathrm{H}} = V_{\mathrm{H}}/I$. In the classical model, find R_{H} as a function of the number of the electrons N and the magnetic flux $\phi = BA = BWL$, where A is the area of the sample, and W and L the effective width and length of the sample, respectively.

(c) (**2 points**) We know that electrons move in circular orbits in the magnetic field. In the quantum mechanical picture, the impinging magnetic field B could be viewed as creating tiny whirlpools, so-called vortices, in the sea of electrons—one whirlpool for each flux quantum h/e of the magnetic field, where h is the Planck's constant and e the elementary charge of an electron. For the case of $R_{\mathrm{H}} = 3h/e^2$, which was discovered by Tsui and Stormer, derive the ratio of the number of the electrons N to the number of the flux quanta N_{ϕ}, known as the filling factor ν.

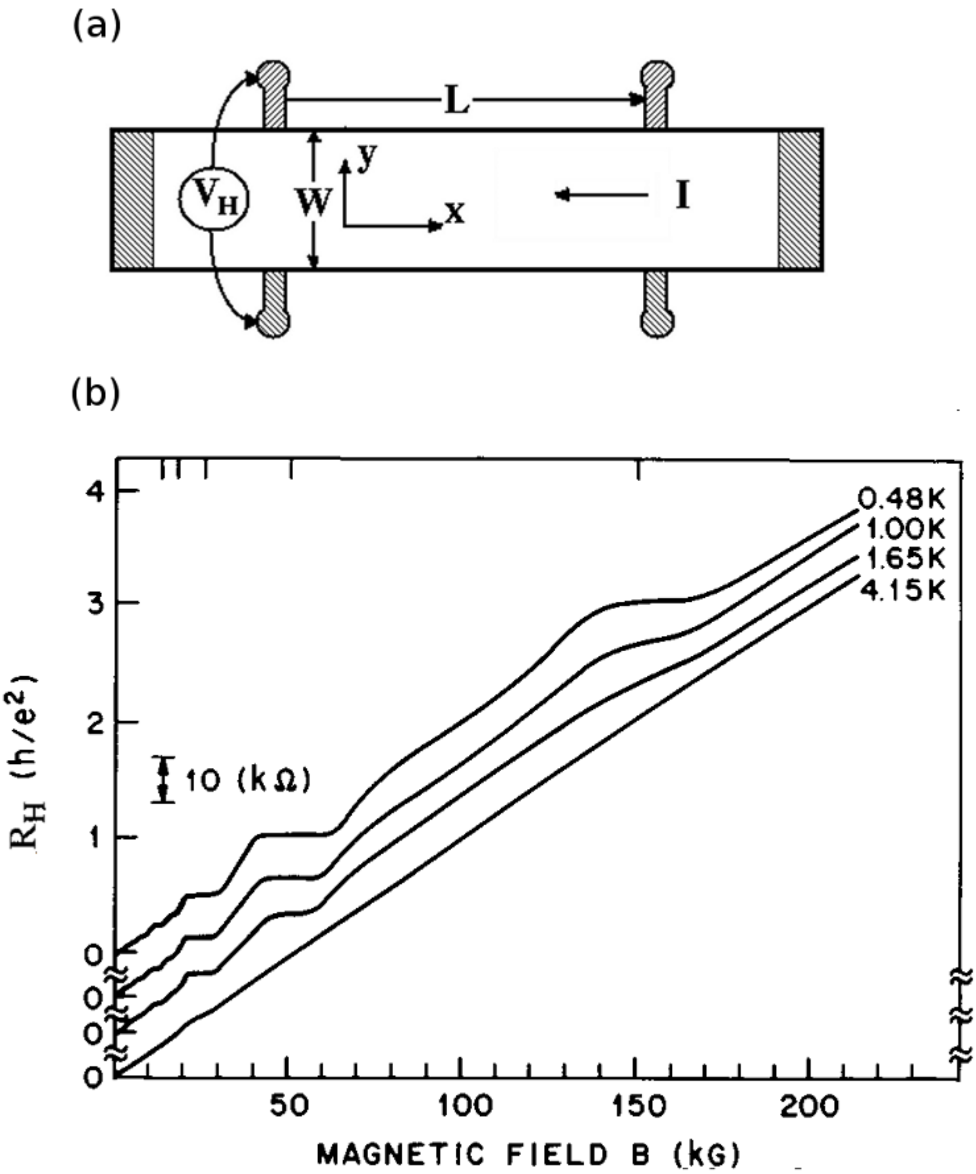


Figure 1: (a) Sketch of the experimental setup for the observation of the FQHE. As indicated, a current I is passing through a two-dimensional electron system in the longitudinal direction with an effective length L. The Hall voltage V_{H} is measured in the transverse direction with an effective width W. In addition, a uniform magnetic field B is applied perpendicular to the plane. The direction of the current is given for illustrative purpose only, which may not be correct. (b) Hall resistance R_{H} versus B at four different temperatures (curves shifted for clarity) in the original publication on the FQHE. The features at $R_{\mathrm{H}} = 3h/e^2$ are due to the FQHE.

(d) (**2 points**) It turns out that binding an integer number of vortices ($n > 1$) with each electron generates a bigger surrounding whirlpool, hence pushes away all other electrons. Therefore, the system can considerably reduce its electrostatic

Coulomb energy at the corresponding filling factor. Determine the scaling exponent α of the amount of energy gain for each electron $\Delta U(B) \propto B^{\alpha}$.

(e) (**2 points**) As the magnetic field deviates from the exact filling $\nu = 1/n$ to a higher field, more vortices (whirlpools in the electron sea) are being created. They are not bound to electrons and behave like particles carrying effectively positive charges, hence known as quasiholes, compared to the negatively charged electrons. The amount of charge deficit in any of these quasiholes amounts to exactly $1/n$ of an electronic charge. An analogous argument can be made for magnetic fields slightly below ν and the creation of quasielectrons of negative charge $e^* = -e/n$. At the quantized Hall plateau of $R_{\mathrm{H}} = 3h/e^2$, calculate the amount of change in B that corresponds to the introduction of exactly one fractionally charged quasihole. (When their density is low, the quasiparticles are confined by the random potential generated by impurities and imperfections, hence the Hall resistance remains quantized for a finite range of B.)

(f) In Tsui *et al.* experiment,

the magnetic field corresponding to the center of the quantized Hall plateau

$R_{\mathrm{H}} = 3h/e^2$, $B_{1/3} = 15$ Tesla,

the effective mass of an electron in GaAs, $m^* = 0.067\, m_e$,
the electron mass, $m_e = 9.1 \times 10^{-31}$ kg,
Coulomb's constant, $k = 9.0 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$,
the vacuum permittivity, $\varepsilon_0 = 1/4\pi k = 8.854 \times 10^{-12}$ F/m,
the relative permittivity (the ratio of the permittivity of a substance to the vacuum permittivity) of GaAs, $\varepsilon_r = 13$,
the elementary charge, $e = 1.6 \times 10^{-19}$ C,
Planck's constant, $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$, and
Boltzmann's constant, $k_{\mathrm{B}} = 1.38 \times 10^{-23}$ J/K.

In our analysis, we have neglected several factors, whose corresponding energy scales, compared to $\Delta U(B)$ discussed in (d), are either too large to excite or too small to be relevant.

(i) (**1 point**) Calculate the thermal energy E_{th} at temperature $T = 1.0$ K.

(ii) (**2 point**) The electrons spatially confined in the whirlpools (or vortices) have a large kinetic energy. Using the uncertainty relation, estimate the order of magnitude of the kinetic energy. (This amount would also be the additional energy penalty if we put two electrons in the same whirlpool, instead of in two separate whirlpools, due to Pauli exclusion principle.)

(g) There are also a series of plateaus at $R_{\mathrm{H}} = h/ie^2$, where $i = 1, 2, 3, \ldots$ in Tsui *et al.* experiment, as shown in Figure 1(b). These plateaus, known as the integer quantum Hall effect (IQHE), were reported previously by K. von Klitzing in 1980. Repeating (c)-(f) for the integer plateaus, one realizes that the novelty of the FQHE lies critically in the existence of fractionally charged quasiparticles. R.

de-Picciotto *et al.* and L. Saminadayar *et al.* independently reported the observation of fractional charges at the $\nu = 1/3$ filling in 1997. In the experiments, they measured the noise in the charge current across a narrow constriction, the so-called quantum point contact (QPC). In a simple statistical model, carriers with discrete charge e^* tunnel across the QPC and generate charge current I_{B} (on top of a trivial background). The number of the carriers n_τ arriving at the electrode during a sufficiently small time interval τ obeys Poisson probability distribution with parameter λ

$$\mathrm{P}(n_\tau = k) = \frac{\lambda^k \mathrm{e}^{-\lambda}}{k!}$$

where $k!$ is the factorial of k. You may need the following summation

$$\mathrm{e}^{\lambda} = \sum_{\mathrm{k}=0}^{\infty} \frac{\lambda^k}{k!}$$

(i) (**2 point**) Determine the charge current I_{B}, which measures total charge per unit of time, in terms of λ and τ.

(ii) (**2 points**) Current noise is defined as the charge fluctuations per unit of time. One can analyze the noise by measuring the mean square deviation of the number of current-carrying charges. Determine the current noise S_I due to the discreteness of the current-carrying charges in terms of λ and τ.

(iii) (**1 point**) Calculate the noise-to-current ratio S_I/I_{B}, which was verified by R. de-Picciotto *et al.* and L. Saminadayar *et al.* in 1997. (One year later, Tsui and Stormer shared the Nobel Prize in Physics with R. B. Laughlin, who proposed an elegant ansatz for the ground state wave function at $\nu = 1/3$.)

Question 2

How are aurora ignited by the solar wind?

Figure 1

The following questions are designed to guide you to find the answer one step by one step.

Background information of the interaction between the solar wind and the Earth's magnetic field

It is well known that the Earth has a substantial magnetic field. The field lines defining the structure of the Earth's magnetic field is similar to that of a simple bar magnet, as shown in Figure 2. The Earth's magnetic field is disturbed by the solar wind, whichis a high-speed stream of hot plasma. (The plasma is the quasi-neutralionized gas.)The plasma blows outward from the Sun and varies in intensity with the amount of surface activity on the Sun. The solar wind compresses the Earth's magnetic field. On the other hand, the Earth's magnetic field shields the Earth from much of the solar wind. When the solar wind encounters the Earth's magnetic field, it is deflected like water around the bow of a ship, as illustrated in Figure 3.

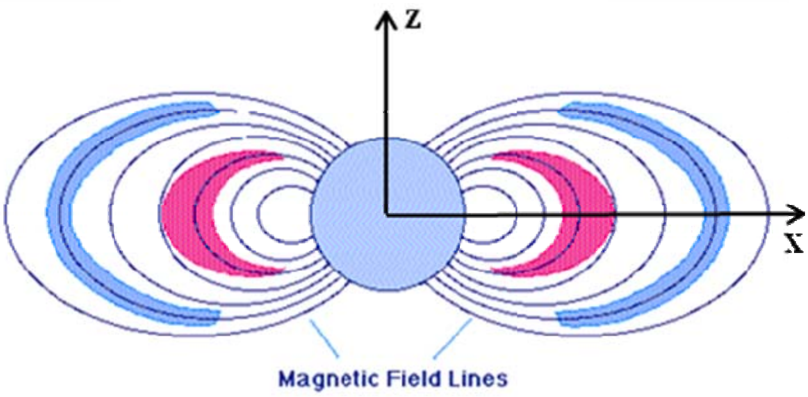


Figure 2

Question 2

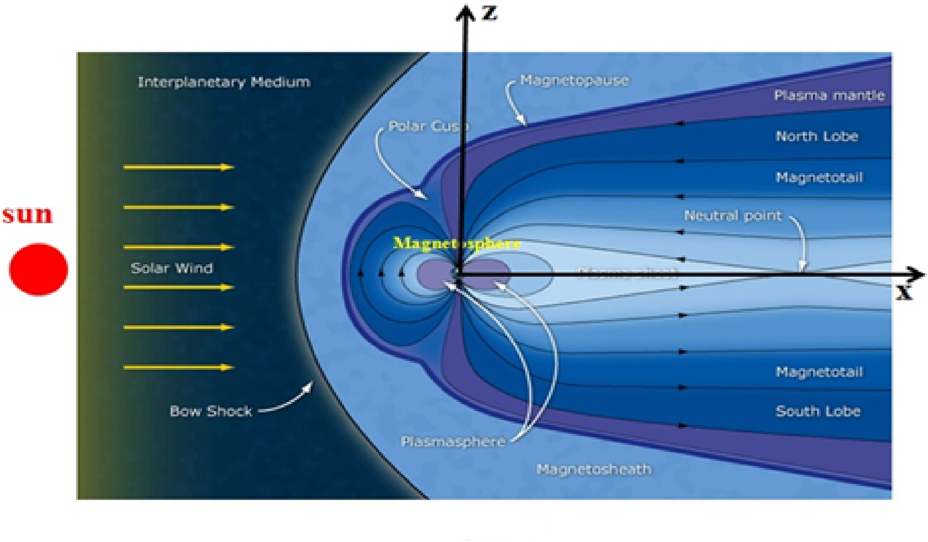
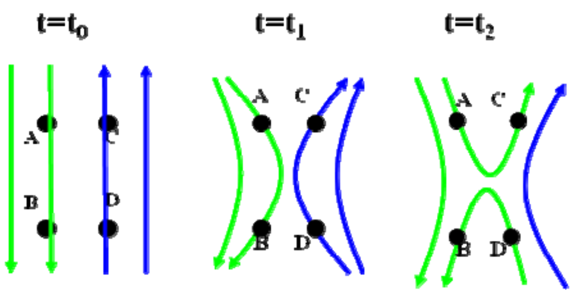


Figure 3

The curved surface at which the solar wind is first deflected is called the *bow shock*. The corresponding region behind the bow shock andfront of the Earth's magnetic field is called the*magnetosheath*. The region surroundedby the solar wind is called the *magnetosphere*.The Earth's magnetic field largely prevents the solar wind from enteringthe magnetosphere. The contact region between the solar wind and the Earth's magnetic field is named the *magnetopause*. The location of the magnetopause is mainly determined by the intensity and the magnetic field direction of the solar wind. When the magnetic field in the solar wind is antiparallel to the Earth's magnetic field, magnetic reconnection as shown in Figure 4 takes place at the dayside magnetopause, which allows some charged particles ofthe solar wind in the region "A" to move into the magnetotail "P" on the night side as illustrated in Figure 5. A powerful solar wind can push the dayside magnetopause tovery close to the Earth, which could cause a high-orbit satellite (such as a geosynchronous satellite) to be fully exposed to the solar wind. The energetic particles in the solar wind could damage high-tech electronic components in a satellite.Therefore, it isimportant to study the motion of charged particles in magnetic fields, which will give an answer of the aurora generation and could help us to understand the mechanism of the interaction between the solar wind and the Earth's magnetic field.



Question 2

Figure 4

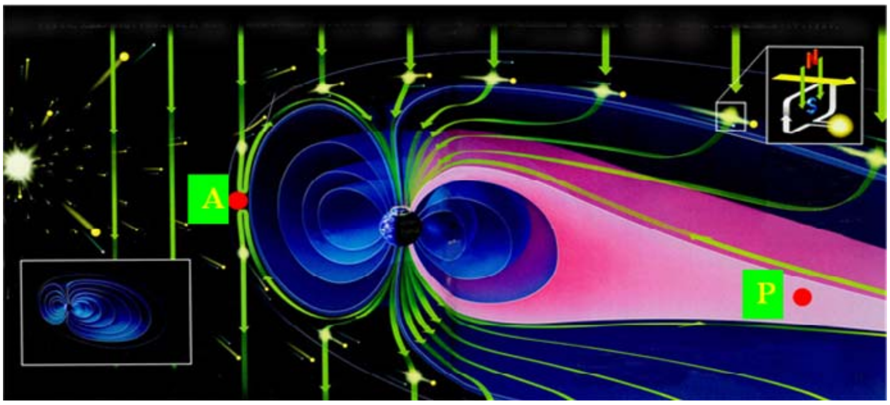


Figure 5

Numerical values of physical constants and the Earth's dipole magnetic field:

Speed of light in vacuum: $c = 2.998 \times 10^8 \text{m/s}$;

Permittivity in vacuum: $\varepsilon_0 = 8.9 \times 10^{-12} \text{C}^2 / (\text{N} \cdot \text{m}^2)$;

Permeability in vacuum: $\mu_0 = 4\pi \times 10^{-7} \text{N/A}^2$;

Charge of a proton: $e = 1.6 \times 10^{-19} \text{C}$;

Mass of an electron: $m = 9.1 \times 10^{-31} \text{kg}$;

Mass of a proton: $m_p = 1.67 \times 10^{-27} \text{kg}$;

Boltzmann's constant: $k = 1.38 \times 10^{-23} \text{J/K}$;

Gravitational acceleration: $g = 9.8 \text{m/s}^2$;

Planck's constant: $h = 6.626 \times 10^{-34} \text{J} \cdot \text{s}$

Earth's radius $R_E = 6.4 \times 10^6 \text{m}$.

The Earth's dipole magnetic field can be expressed as

$$\vec{B}_d = \frac{B_0 R_E^3}{r^5}[3xz\hat{x} - 3yz\hat{y} + (x^2 + y^2 - 2z^2)\hat{z}] \quad , (r \geq R_E) \tag{1}$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $B_0 = 3.1 \times 10^{-5} \text{T}$, and $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors in the x, y, z directions, respectively.

Questions:

(a) (3 Points)

(i) (1 Point) Before we study the motion of a charged particle in the Earth's dipole magnetic field,

Question 2

we first considerthe motion of an electron in a uniform magnetic field $\vec{B}$. When the initial electron velocity $\vec{v}$ is perpendicular to the uniform magnetic field as shown in Figure 6, please calculate the electron trajectory.The electron is initially located at (x,y,z)=(0,0,0).

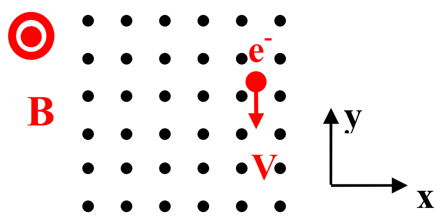


Figure 6

(ii) (1 Point) Please determine the electric current of the electron motion and calculate the magnetic moment $\vec{\mu} = I\vec{A}$, where $\vec{A}$ is the area of the electron circular orbit and the direction of $\vec{A}$ is determined by the right-hand rule of theelectric current.

(iii) (1 Point) If the initial electron velocity $\vec{v}$ is not perpendicular to the uniform magnetic field, i.e, the angle θ between $\vec{B}$ and $\vec{v}$ is $0^0 < \theta < 90^0$, please give the screw pitch (the distance along the z-axis between successive orbits) of the electron trajectory.

(b) (4 Points) In the uniform background magnetic field as shown in Figure 6, theplasma density isnonuniform in *x*. For simplicity, we assume that the temperature and the distribution of the ions andelectrons are the same. Thus, the plasma pressure can be expressed as

$$p(x) = kT[n_i(x) + n_e(x)] = 2kTn(x) = 2kT(n_0 + \alpha x),$$

Where *B*, *T*, *k*, n_0, and α are positive constants, $n_i(x)$ and $n_e(x)$ are the number densities of the ions and electrons.

(i) (2 Points) Please explain the generation mechanism of the electric current by a schematic drawing.

(ii) (2 Points) If both the ions and electrons have a Maxwellian distribution, the ion distribution is

$$f_i(x, v_\perp, v_\parallel) = n_i(x)\left(\frac{m_i}{2\pi kT}\right)^{3/2} e^{-m_i(v_\perp^2 + v_\parallel^2)/2kT},$$

please calculate the constant β in the magnetization $M = \beta n(x)\frac{kT}{B}$, where the magnetization

Question 2

$M$is the magnetic moment per unit volume. (Hint: We have $\int_0^\infty x\exp(-x)dx = 1$ and

$\int_{-\infty}^{\infty}\exp(-x^2)dx = \sqrt{\pi}$.)

(c) (1 Point)Now let's go back to the Earth's dipole magnetic field. Please apply the result from Question(b) to calculate the ratio of the diamagnetic field and the Earth's dipole magnetic field in Equation (1) at the position (x=10R$_E$，y=0, z=1 R$_E$). The plasma pressure is assumed to be $p(z) = p_0 e^{-(z/a)^2}$, where $p_0 = 3\times10^{-10}\text{pa}$ and $a = 2R_E$ .The magnetic field around this position is also assumed to be uniform. Be aware of the difference in the coordinate systemsin Questions (b) and (c). (Hint: The diamagnetic field is given by $B_{mx} = \mu_o M$.)

(d) (4 Points) From Figures 2, 3, and 5, it can be clearly seen that the Earth's magnetic fieldstrength along a magnetic field line is the largest at the poles and the smallest in the equatorial plane.Since the Earth's dipole magnetic field is axially symmetric and slowly varying along a magnetic field line, it can for simplicitybe treated as a magnetic-mirror field as shown in Figure 7.The magnetic field strength along a magnetic field line is the smallest (B_0) at the point "P$_2$" and the largest (B_m) at the points "P$_1$" and "P$_3$". An electron with an initial velocity $\vec{v}$ is located at the point "P$_2$" and drifts towards the point "P$_3$". The angle between the initial velocity $\vec{v}$ and the magnetic field at the point "P$_2$" is $0^0 < \theta < 90^0$. For the magnetic-mirror field $\vec{B} = B_r\hat{r} + B_z\hat{z}$ (with $B_r << B_z$), we can assume $\frac{dB}{dz} = \frac{dB}{ds}$, where $\frac{dB}{ds}$ is the spatial derivative of B along a magnetic field line. Since there is no evidence of the existence of magnetic monopoles, we have $\langle B_r \rangle = -\frac{1}{2}\frac{dB}{dz} r_c << B_z$, where $\langle B_r \rangle$ is the gyro-average of B_r and r_c is the electron gyroradius.

Question 2

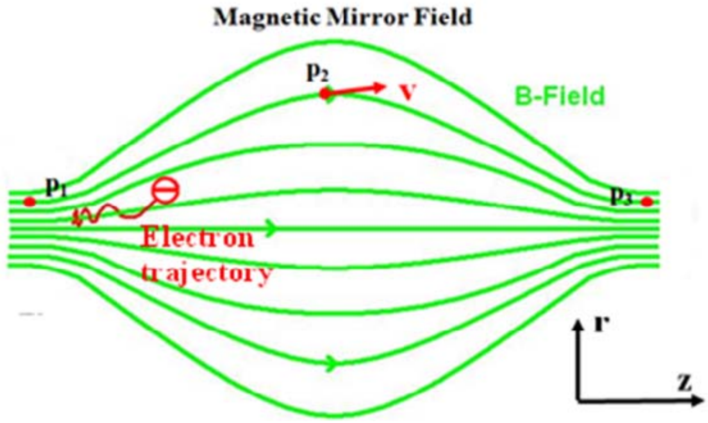
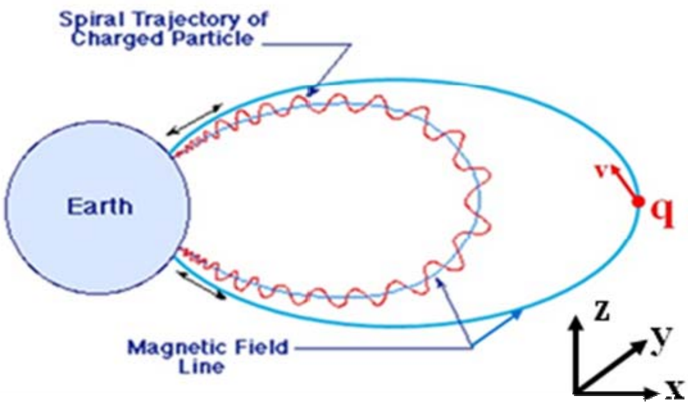


Figure 7

(i) (3 Points) Please givethe gyro-averaged magnetic-field force along the magnetic field lines on an electron and show that the magnetic moment is a motion constant, i.e., $\frac{d\mu}{dt}=0$, based on the law of the total kinetic energy conservation.

(ii) (1 Point) Based on the motion constant of magnetic moment, please determinewhat the condition should be satisfied for the angle θ between the initial electron velocity $\vec{v}$ and the magnetic field at the point "P_2"ifan electron will not escape from the magnetic mirror field.

(e) (1 Point) Earth's dipole magnetic field lines (blue lines) are shown in Figure 8. The spiral trajectory of a charged particle (red curve) is assumed to be confined in the y=0 plane since the gradient and the curvature of the magnetic field can be ignored. If a charged particle with the mass m, charge q, and velocity $\vec{v}$ is initially located at the equatorial position [x=6R_E, y=0, z=0] and the angle between the electron velocity $\vec{v}$ and the magnetic field is θ initially, please determine what the condition should be satisfied for θ if the charged particle arrives below 200km of its altitude at the latitude 60^0.



Question 2

Figure 8

(f)(5 Points) As shown in Figure 5, when magnetic reconnection takes place at the dayside magnetopause, reconnected magnetic field lines drift towards the nightside region because the solar wind flows tailward. Thus, some solar wind electrons in the region "A" also move towards the magnetotail in the region "P". After the electrons arrive in the region "P", some electrons can be accelerated to around 1keV. If energetic electrons drift down to the thermosphere (The altitude of the thermosphere is about 85km-800km.), energetic electrons can collide with the neutral atoms, which could cause the neutral atoms to jump into excited states. A photon is emitted when the higher excited state of aneutral atom returnsto its lower excited state or ground state. Splendid aurora (Figure 1) is generated in the aurora oval due to photons with different wavelengths. It is found that the auroras mainly resulted fromphotonsemittedby oxygen atoms.The energy levels in the first and second excited states relative to the ground stateare 1.96eV and 4.17eV, respectively. The lifetimes of the two excited states of an oxygen atomare 110s and 0.8s as shown in Figure 9.

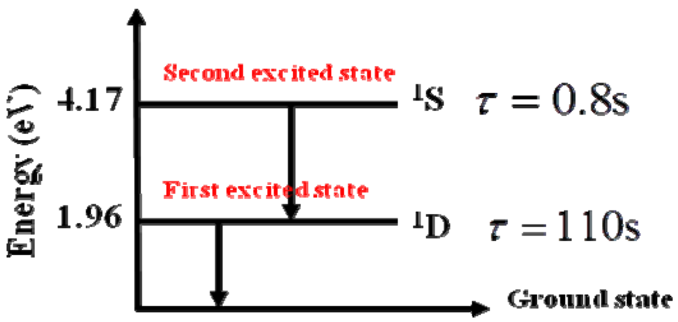


Figure 9

(i) (2 Points) Please give the atmospheric density as a function of the altitude and the ratio of the oxygen density at the altitudes H=160km and H=220km. For simplicity, we assume that the atmospheric temperature is independent of the altitude and the air is an ideal gas.

($\rho_0 g/P_0 = 0.13/\mathrm{km}$, where ρ_0 and P_0 are the atmospheric density and pressure at sea level.)

(ii) (3 Points)Please give the colors of auroras at the altitudes H=160km and H=220km. (Hint: The dependence of the collision frequency of atmosphericmolecules on the atmospheric density is $\nu = \nu_0 \rho/\rho_0$, where $\nu_0 \approx 10^9/s$ is the collision frequency of atmosphericmolecules at sea level.The excited oxygen atom will lose a part of its energy when it collides with other neutral molecules.)

(g) (2 Points) As mentioned above, a powerful solar wind can push the dayside magnetopause tovery close to the Earth, which could cause a high-orbit satelliteto be fully exposed to the solar wind. The energetic particles in the solar wind could damage high-tech electronic components in a

Question 2

satellite.For simplicity, the Earth's dipole magnetic field is assumed to remain unchanged when the solar wind compresses it and that the plasma density is ignorable in the magnetosphere. Please give the minimum solar wind speed to cause a damage of a geosynchronous satellite if the magnetic field strength and the plasma density of the solar wind are $B_s = 5\times10^{-9}\,\mathrm{T}$ and $\rho_s = 50\ \mathrm{proton/cm^3}$, respectively. (Hint: The force per unit area associated with the magnetic field is $f = B^2/2\mu_0$. We only consider the variation in *x* for all physical quantities, i.e., the physical quantities are independent of *y* and *z*.)

Question 3

Figure 1 shows a Fabry-Perot (F-P) etalon, in which air pressure is tunable. The F-P etalon consists of two glass plates with high-reflectivity inner surfaces. The two plates form a cavity in which light can be reflected back and forth. The outer surfaces of the plates are generally not parallel to the inner ones and do not affect the back-and-forth reflection. The air density in the etalon can be controlled. Light from a Sodium lamp is collimated by the lens L1 and then passes through the F-P etalon. The transmitivity of the etalon is given by $T = \frac{1}{1 + F\sin^2(\delta/2)}$, where $F = \frac{4R}{(1-R)^2}$, R is the reflectivity of the inner surfaces, $\delta = \frac{4\pi nt\cos\theta}{\lambda}$ is the phase shift of two neighboring rays, n is the refractive index of the gas, t is the spacing of inner surfaces, θ is the incident angle, and λ is the light wavelength.

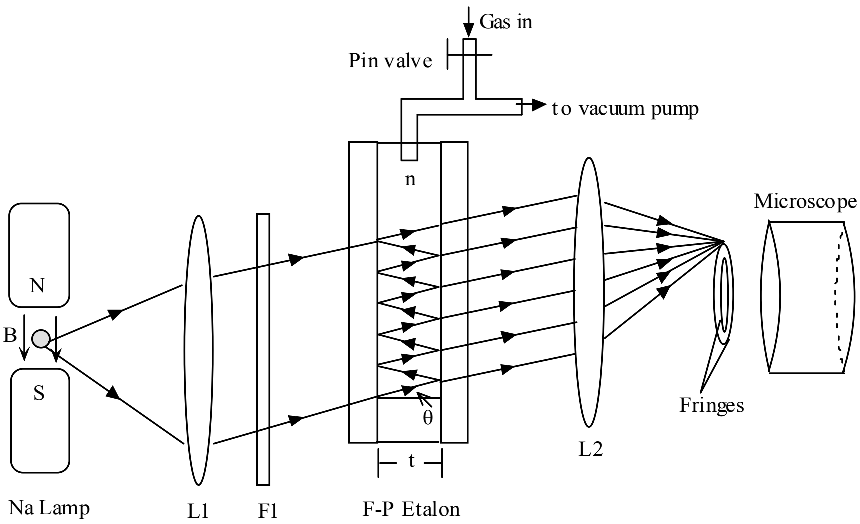


Figure 1

The Sodium lamp emits D1 ($\lambda = 589.6nm$) and D2 ($589nm$) spectral lines and is located in a tunable uniform magnetic field. For simplicity, an optical filter F1 is assumed to only allow the D1 line to pass through. The D1 line is then collimated to the F-P etalon by the lens L1. Circular interference fringes will be present on the focal plane of the lens L2 with a focal length f=30cm. Different fringes have the different incident angle θ. A microscope is used to observe the fringes. We take the reflectivity R= 90% and the inner-surface spacing t=1cm.

Some physical constants: $h = 6.626\times10^{-34} J\cdot s$, $e = 1.6\times10^{-19} C$, $m_e = 9.1\times10^{-31} kg$, $c = 3.0\times10^{8} ms^{-1}$.

Question 3

（a） （**3points**） The D1 line ($\lambda = 589.6nm$) is collimated to the F-P etalon. For the vacuum case (n=1.0), please calculate (i) interference orders m_i, (ii) incidence angle θ_i and (iii) diameter D_i for the first three（i=1，2，3）fringes from the center of the ring patterns on the focal plane.

（b）（**3 points**）As shown in Fig. 2, the width ε of the spectral line is defined as the full width of half maximum (FWHM) of light transmitivity T regarding the phase shift δ. The resolution of the F-P etalon is defined as follows: for two wavelengths λ and $\lambda+\Delta\lambda$, when the central phase difference $\Delta\delta$ of both spectral lines is larger than ε, they are thought to be resolvable; then the etalon resolution is $\lambda/\Delta\lambda$ when $\Delta\delta=\varepsilon$. For the vacuum case, the D1 line ($\lambda = 589.6nm$), and because of the incident angle $\theta \approx 0$, take $\cos\theta \approx 1.0$, please calculate：

（i）the width ε of the spectral line.

（ii）the resolution $\lambda/\Delta\lambda$ of the etalon.

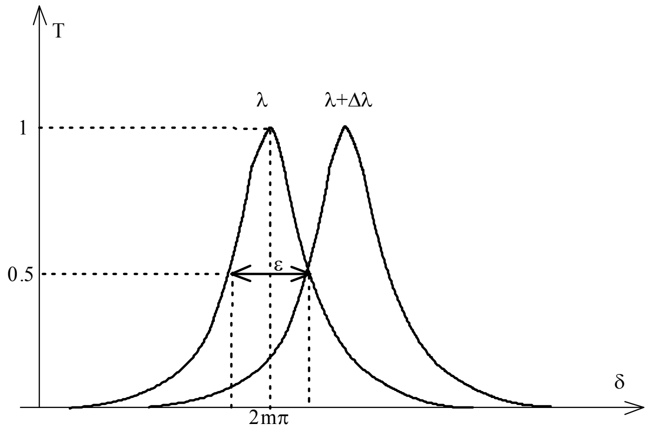


Figure 2

（c）（**1 point**）As shown in Fig. 1, the initial air pressure is zero. By slowly tuning the pin valve, air is gradually injected into the F-P etalon and finally the air pressure reaches the standard atmospheric pressure. On the same time, ten new fringes are observed to produce from the center of the ring patterns on the focal plane. Based on this phenomenon, calculate the refractive index of air n_{air} at the standard atmospheric pressure.

（d） **(2 points)** Energy levels splitting of Sodium atoms occurs when they are placed in a magnetic field. This is called as the Zeeman effect. The energy shift given by $\Delta E = m_j g_k \mu_B B$, where the quantum number m_j can be J，J-1，…，-J+1，-J，J is the total angular quantum number,

Question 3

g_k is the Landé factor, $\mu_B = \dfrac{he}{4\pi m_e}$ is Bohr magneton, h is the Plank constant, e is the electron charge, m_e is the electron mass, B is the magnetic field. As shown in Fig. 3, the D1 spectral line is emitted when Sodium atoms jump from the energy level $^2P_{1/2}$ down to $^2S_{1/2}$. We have $J = \dfrac{1}{2}$ for both $^2P_{1/2}$ and $^2S_{1/2}$. Therefore, in the magnetic field, each energy level will be split into two levels. We define the energy gap of two splitting levels as ΔE_1 for $^2P_{1/2}$ and ΔE_2 for $^2S_{1/2}$ respectively ($\Delta E_1 < \Delta E_2$). As a result, the D1 line is split into 4 spectral lines (a, b, c, and d), as showed in Fig. 3. Please write down the expression of the frequency (ν) of four lines a, b, c, and d.

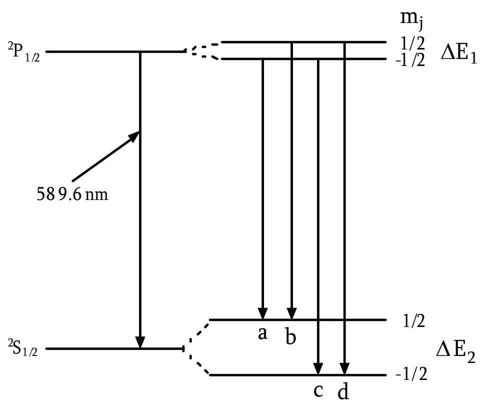


Figure 3

(e) **(3 points)** As shown in Fig. 4, when the magnetic field is turned on, each fringe of the D1 line will split into four sub-fringes (1, 2, 3, and 4). The diameter of the four sub-fringes near the center is measured as D_1, D_2, D_3, and D_4. Please give the expression of the splitting energy gap ΔE_1 of $^2P_{1/2}$ and ΔE_2 of $^2S_{1/2}$.

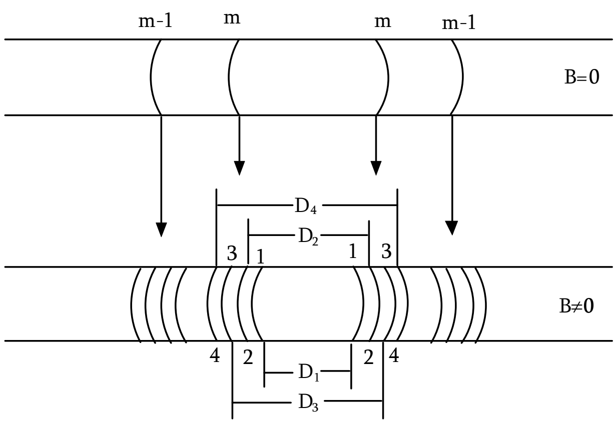


Figure 4

Question 3

（f）（**3 points**）For the magnetic field B=0.1T，the diameter of four sub-fringes is measured as：$D_1 = 3.88mm$， $D_2 = 4.05mm$， $D_3 = 4.35mm$， and $D_4 = 4.51mm$. Please calculate the Landé factor g_{k1} of $^2P_{1/2}$ and g_{k2} of $^2S_{1/2}$.

（g）**(2 points)** The magnetic field on the sun can be determined by measuring the Zeeman effect of the Sodium D1 line on some special regions of the sun. One observes that, in the four split lines, the wavelength difference between the shortest and longest wavelength is 0.012nm by a solar spectrograph. What is the magnetic field B in this region of the sun?

(h)（**3 points**）A Light- Emitting Diode (LED) source with a central wavelength $\lambda = 650nm$ and spectral width $\Delta\lambda = 20nm$ is normally incident ($\theta = 0$) into the F-P etalon shown in Fig. 1. For the vacuum case, find (i) the number of lines in transmitted spectrum and (ii) the frequency width $\Delta\nu$ of each line?