Review Session
Stable Matching

Yifei Huang
yifeih@usc.edu

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Crucial Concepts

When talking about matching, we usually talk about 2 sets **A** and **B** of the same size n

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- **Stable Matching Problem**: Given the preference list of A and B, find a perfect matching S that is not unstable.

G-S Algorithm

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Uniqueness

G-S algorithm returns a unique solution. But the problem instance might have multiple solutions.

Question 1

Find an instance of stable matching problem where there are multiple solutions and point out the solution that G-S algorithm will return.

Question 2

Find an instance of statble matching problem of size n, such that G-S algorithm terminates in $O(n)$ iteration.

Question 3

If every man has identical preference list, how many iteration does it take for G-S algorithm to terminate, give the precise answer in n.