Review Session
Stable Matching

Yifei Huang
yifeih@usc.edu

February 7, 2024

When talking about matching, we usually talk about 2 sets **A** and **B** of the same size n

- **Matching**: A matching S is a set of pairs (a, b) where $a \in A$ and $b \in B$, and no two pairs share the same a or b.

When talking about matching, we usually talk about 2 sets **A** and **B** of the same size n

- **Matching**: A matching S is a set of pairs (a, b) where $a \in A$ and $b \in B$, and no two pairs share the same a or b.
- **Perfect Matching**: A matching S is perfect if it is of size n. In other word, every $a \in A$ and $b \in B$ is matched with something.

When talking about matching, we usually talk about 2 sets **A** and **B** of the same size n

- **Matching**: A matching S is a set of pairs (a, b) where $a \in A$ and $b \in B$, and no two pairs share the same a or b.
- **Perfect Matching**: A matching S is perfect if it is of size n. In other word, every $a \in A$ and $b \in B$ is matched with something.
- **Unstable Matching**: A matching S is unstable if there exist 2 pairs (a_1, b_1) and (a_2, b_2) such that a_1 prefers b_2 than b_1 and b_2 prefers a_1 than a_2.

When talking about matching, we usually talk about 2 sets **A** and **B** of the same size n

- **Matching**: A matching S is a set of pairs (a, b) where $a \in A$ and $b \in B$, and no two pairs share the same a or b.
- **Perfect Matching**: A matching S is perfect if it is of size n. In other word, every $a \in A$ and $b \in B$ is matched with something.
- **Unstable Matching**: A matching S is unstable if there exist 2 pairs (a_1, b_1) and (a_2, b_2) such that a_1 prefers b_2 than b_1 and b_2 prefers a_1 than a_2.
- **Stable Matching Problem**: Given the preference list of A and B, find a perfect matching S that is not unstable.

Termination

G-S algorithm terminates in $O(n^2)$ iterations as each man can only propose at most n times.

G-S Algorithm

Termination

G-S algorithm terminates in $O(n^2)$ iterations as each man can only propose at most n times.

Uniqueness

G-S algorithm returns a unique solution. But the problem instance might have multiple solutions.

Question 1

Find an instance of stable matching problem where there are multiple solutions and point out the solution that G-S algorithm will return.

Answer 1

	1st	2nd
M 1	W1	W2
M 2	W2	W1
W 1	M2	M1
W 2	M1	M2

Table: Table caption.

1. (M1,W1), (M2,W2)

Answer 1

	1st	2nd
M 1	W1	W2
M 2	W2	W1
W 1	M2	M1
W 2	M1	M2

Table: Table caption.

1. (M1,W1), (M2,W2)
2. (M1,W2), (M2,W1)

Question 2

Find an instance of statble matching problem of size n, such that G-S algorithm terminates in $O(n)$ iteration.

Answer 2

Simple assign each man with different most preferred woman. E.g. m_i prefers w_i the most. In this case G-S algorithm will run exactly n iterations as each man will propose to different woman.

Question 3

If every man has identical preference list, how many iteration does it take for G-S algorithm to terminate, give the precise answer in n.

Answer 3

With out lose of generality, let's assume that every man's preference list is exactly $(w_1, w_2, ..., w_n)$. G-S algorithm returns a stable matching $S = \{(m'_1, w_1), (m'_2, w_2), ..., (m'_n, w_n)\}$. Since every man has the same preference list. m'_i must have proposed exactly i times. Then the total number of iteration is $\sum_{i=1}^{n} i = \frac{(n+1)n}{2}$.

Question 4

Is it true that for every $n > 2$, there exists an instance of stable matching problem that has only one solution?

Answer 4

Yes, simple make m_i's i-th preferred woman w_i and vice versa. The solution can only be $S = \{(m_i, w_i) | \forall i \in [1, n]\}$.Proof is just the extended version of HW1 Q4.