LogReturnApproximationProof

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For small enough percentage returns log returns provide a decent approximation of return with nice mathematical properties.

$$\begin{aligned} R_t &= \frac{P_t - t_{t-1}}{P_{t-1}} \\ &= \frac{P_t}{P_{t-1}} - 1 \end{aligned} \tag{1}$$

The Taylor expansion for log(1+x) is

$$log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + O(x^4) \tag{2}$$

Consider what happens when x is a small number:

$$log(1+x) \approx x \tag{3}$$

Substituting R_t forr x gives us $log(1_+R_t) \approx R_t$

$$\begin{aligned} log\left(1 + \frac{P_t}{P_{t-1}} - 1\right) &\approx R_t \\ log\left(\frac{P_t}{P_{t-1}}\right) = log(P_t) - log(P_{t-1}) &\approx R_t \end{aligned} \tag{4}$$