Problems

Expectations:

- **You present your independent work.**
- **Possibilities of plagiarism will be thoroughly checked for.**
- **All steps in the solution are clearly explained.**
- **If existing results are used (e.g. expansion in orthogonal functions), you may want to cite the literature as e.g. Jackson, Eq. (3.70).**
- **The solution is written or typed, with all equations written or typed by you (e.g. if you need a formula, write it down, do not paste a picture of it from Jackson's book).**

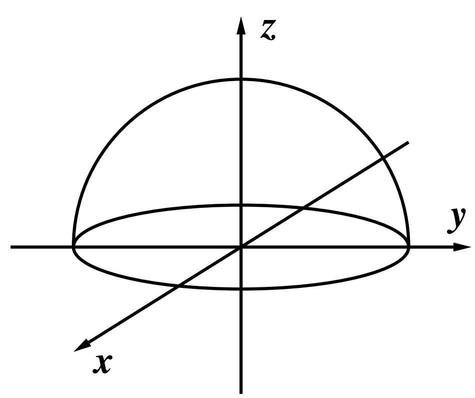
1. **A hemisphere:** Consider a cavity that has a shape of a hemisphere of radius R closed at the bottom, as shown in the figure. With our convention on the inclination angle it occupies the region of space with $\theta \in [0, \pi/2]$. The end goal is to calculate the electrostatic potential in the cavity.

   (a) (30 pts) Write down the Green's function for this problem. Explain how you arrive at it.

   (b) (40 pts) The spherical part of the cavity is maintained at the potential

   $$V(\theta, \phi) = V_0 \cos\theta,$$

   and the flat bottom part at $V = 0$. Evaluate the potential inside the cavity.



Do not try to evaluate the integrals on θ' in closed form, they may be too complicated. Rather, use properties of the spherical harmonics and the input in the problem to simplify as much as possible, so that you could make a statement e.g. like this: the result is presented as an expansion, the expansion coefficients A_{lm} are given in terms of the associated Legendre functions as $A_{lm} = \int_0^1 dx x P_l^m(x)$.

2. **A rotating sphere:** A sphere of radius R carries a uniform surface-charge distribution σ in the region of $\theta \in [\pi/2 - \alpha, \pi/2 + \alpha]$, α is a fixed parameter. The rest of the sphere has no charge. The sphere is rotated about the z axis with constant angular velocity ω.

   (a) (20 pts) Find the magnetic dipole moment $\vec{m}$.

   (b) (10 pts) Demonstrate that your solution is correct by considering two limits: $\alpha \to \pi/2$, $\alpha \to 0$. In the second limit assume that the total current is held fixed.