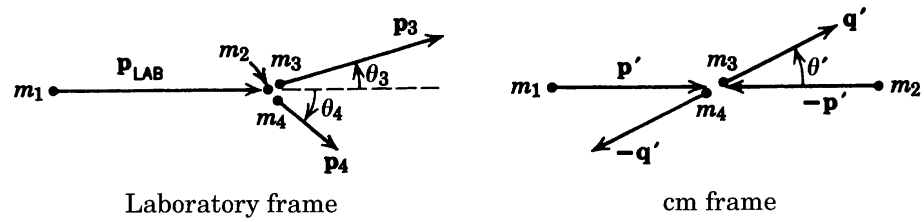
Physics 841 - Homework 3 **due Fri., Jan. 31, 2020**

Problems

1. **Collision with a particle at rest, Jackson, 11.23:** In a collision process a particle of mass m_2, at rest in the laboratory, is struck by a particle of mass m_1, momentum $\mathbf{p}_{LAB}$ and total energy E_{LAB}. In the collision the two initial particles are transformed into two others of mass m_3 and m_4. The configurations of the momentum vectors in the center of momentum (cm) frame (traditionally called the center-of-mass frame) and the laboratory frame are shown in the figure.



   (a) (10 pts) Use invariant scalar products to show that the total energy W in the cm frame has its square given by

$$W^2 = m_1^2 + m_2^2 + 2m_2 E_{LAB} \tag{1}$$

   and that the cms 3-momentum $\mathbf{p}'$ is

$$\mathbf{p}' = \frac{m_2 \mathbf{p}_{LAB}}{W} \tag{2}$$

   (b) (8 pts) Show that the Lorentz transformation parameters $\boldsymbol{\beta}_{cm}$ and γ_{cm} describing the velocity of the cm frame in the laboratory are

$$\boldsymbol{\beta}_{cm} = \frac{\mathbf{p}_{LAB}}{m_2 + E_{LAB}}, \quad \gamma_{cm} = \frac{m_2 + E_{LAB}}{W} \tag{3}$$

   (c) (8 pts) Show that the results of parts (a) and (b) reduce in the nonrelativistic limit to the familiar expressions,

$$W \simeq m_1 + m_2 + \left(\frac{m_2}{m_1 + m_2}\right) \frac{p_{LAB}^2}{2m_1} \tag{4}$$

$$\mathbf{p}' \simeq \left(\frac{m_2}{m_1 + m_2}\right) \mathbf{p}_{LAB}, \quad \boldsymbol{\beta}_{cm} \simeq \frac{\mathbf{p}_{LAB}}{m_1 + m_2} \tag{5}$$

2. **Converting photons to electron and positron:** Consider two photons with different energies that annihilate (in the vacuum) and produce an electron-positron pair. (I.e. a reaction with two photons – in, and electron and positron – out.)

   (a) (8 pts) For what ranges of initial photon energies and angles between their directions of propagation can this reaction take place? (In other words, give a relation, perhaps an inequality, that may contain photon energies, the angle, electron mass, speed of light, etc.)

(b) (6 pts) Consider now a head-on collision (the angle is π radians) and the photons of the same energy. Calculate the numerical value of the minimal photon energy required for the reaction to take place. Express the answer in SI units (Joules).

3. **Field tensor:** Consider the electromagnetic field tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ in the conventions of our course (SI units, $(+,-,-,-)$ metric tensor).

   (a) (15 pts) Starting from the definition in terms of the potential A^μ, *derive* the matrix $(F^{\mu\nu})$ in terms of the components of the electric and magnetic fields, E_x, E_y, E_z, B_x, B_y, B_z.

   (b) (15 pts) Consider a Lorentz boost with relativistic velocity β in the positive x direction. *Derive* the components of the electric and magnetic fields in the moving frame in terms of the corresponding quantities in the original frame by transforming $F^{\mu\nu}$.

   (c) (10 pts) *Derive* the transformation properties of the electric and magnetic fields under parity (space inversion) and time reversal.

4. **To $\vec{B}\vec{E}$ or not to $\vec{B}\vec{E}$:** In a reference frame K there are a constant electric $\vec{E}$ and a magnetic $\vec{B}$ fields such that $\vec{E}\perp\vec{B}$.

   (a) (10 pts) With what velocity a reference frame K' should be moving with respect to K so that in K' there is only electric or only magnetic field ? *Derive* the value of the corresponding field in the K' frame as function of the original fields.

   (b) (10 pts) Does the solution always exist ? Is it unique ?