Physics 841 - Homework 12 **due Fri., Apr. 17, 2020**

Problems

1. **Non-relativistic particle in homogeneous magnetic field:** Consider the motion of a non-relativistic point particle in a static homogeneous magnetic field, ignoring radiation. Assume $\vec{B}$ is in the z direction, $\vec{B}(\vec{r},t) = B_0\hat{z}$.

   (a) (20 pts) Starting from the Lorenz force, derive the trajectory $\vec{r}_0(t)$ of the particle and identify the non-relativistic cyclotron frequency. What is the change of kinetic energy of the particle with time ?

2. **Fields in a hollow cylinder:** Using cylindrical coordinates (ρ, φ, z), consider electric and magnetic fields

   $$\vec{E} = \hat{\rho}\frac{c_1}{\rho}, \qquad \vec{B} = \hat{\varphi}\frac{c_2}{\rho},$$

   with constant c_1 and c_2 inside the volume bounded by $a \leq \rho \leq b$, i.e. inside an infinitely long cylinder with a hole.

   (a) (10 pts) Determine the Poynting vector $\vec{S}$ inside the volume.

   (b) (10 pts) Determine the total flux of energy in the fields through a cross-sectional surface with $a \leq \rho \leq b$.

   (c) (10 pts) Determine the energy per unit length $d\mathcal{E}/dz$ and the momentum per unit length $d\vec{p}/dz$ in the fields (for $a \leq \rho \leq b$).

3. **Force due to a plane wave:** An incident monochromatic plane wave described by a vector potential $\vec{A} = \vec{A}_0 \cos(\omega t - \vec{k}\vec{r})$ is completely absorbed by a sphere of radius R.

   (a) (10 pts) Find the electric and magnetic fields. (Take into account that for a plane wave $\vec{k}\vec{A} = 0$.)

   (b) (20 pts) Determine the Maxwell's stress tensor.

   (c) (20 pts) Find the force $\vec{F}$ exerted by the wave on the sphere averaged over the period $T = 2\pi/\omega$ using the result of part (b).