Physics 841 - Homework 4 **due Fri., Feb. 7, 2020**

Problems

1. **Spherical cavity:** Consider a sphere of radius R that has a hollow spherical cavity of radius b inside it. The center of the big sphere is at the origin, the center of the cavity is at $\vec{a}$. The volume of the big sphere (excluding the cavity) is uniformly charged with a charge density ρ_0.

   (a) (25 pts) Derive the electric field (both magnitude and direction) at an arbitrary point inside the cavity. (Find a compact expression in terms of the given parameters.)

2. **Hydrogen atom, Jackson 1.5 (25 pts):** The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right) \tag{1}$$

   where q is the magnitude of electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

3. **Field of a thin disc:** An infinitely thin round disk of radius R has its symmetry axis on the z-axis. It is uniformly charged with total charge q.

   (a) (10 pts) Write an expression for the charge density of the disk $\rho(\vec{r})$ using appropriate coordinate variables.

   (b) (5 pts) Determine the cartesian surface density $\sigma(x, y)$ (from your expression for $\rho(\vec{r})$).

   (c) (10 pts) Calculate by direct integration the electric field $\vec{E}(\vec{r})$ at an arbitrary point on the z-axis (from your expression for $\rho(\vec{r}')$).

   (d) (5 pts) Find the limits of the field for $z \gg R$ and for $z \ll R$ and explain the results.

4. **Equipotential surface (20 pts):** Two opposite point charges q_1 and $-q_2$ are positioned distance d apart. (Here q_1 and q_2 are unequal positive numbers.) Show that the equipotential surfaces in this system include a sphere of finite radius. Find the location of the center of the sphere and its radius. What is the value of the potential on the surface of this sphere ? (We use such a normalization that the value of the potential at infinity is zero.)