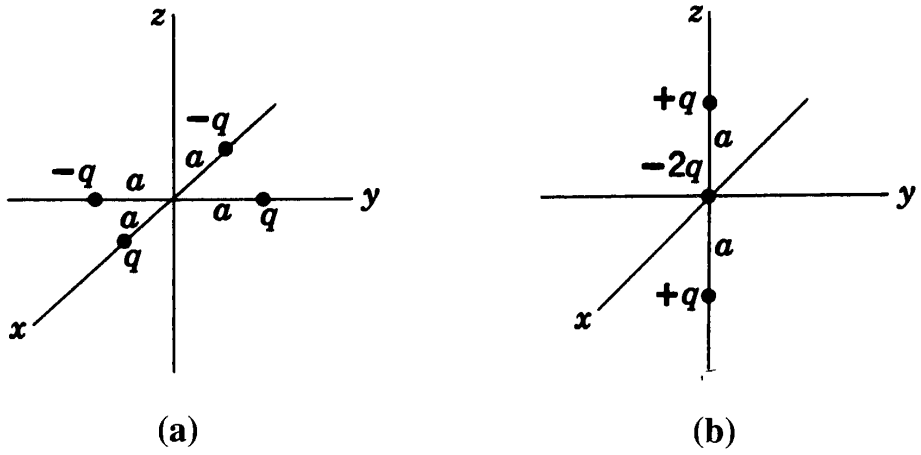
Physics 841 - Homework 7 **due Fri., Feb. 28, 2020**

Problems

1. **Spherical multipoles, Jackson, 4.1:** Calculate the multipole moments q_{lm} of the charge distributions shown as parts (a) (15 pts) and (b) (15 pts). Try to obtain results for the nonvanishing moments valid for all l, but in each case find the first *two* sets of nonvanishing moments at the very least.



   (c) (10 pts) For the charge distribution of the second set (b) write down the multipole expansion for the potential. Keeping only the lowest-order term in the expansion, plot the potential in the $x-y$ plane as a function of distance from the origin for distances greater than a.

   (d) (10 pts) Calculate directly from Coulomb's law the exact potential for (b) in the $x-y$ plane. Plot it as a function of distance and compare with the result found in part (c).

   Divide out the asymptotic form in parts (c) and (d) to see the behavior at large distances more clearly.

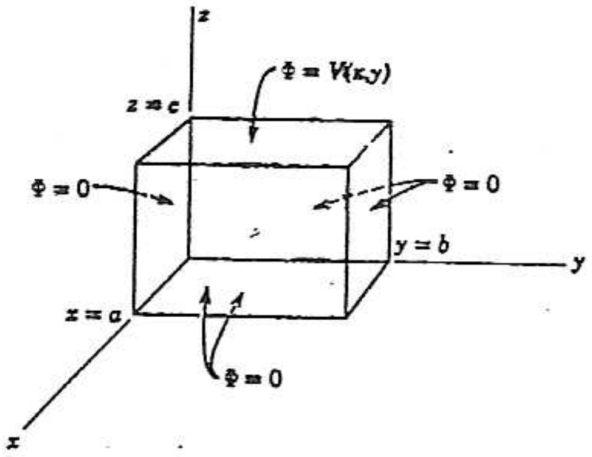
2. **The potential outside a charged disk, Zangwill 4.22:** The z-axis is the symmetry axis of a disk of radius R which lies in the $x-y$ plane and carries a uniform charge per unit area σ. Let Q be the total charge on the disk.

   (a) (15 pts) Evaluate the exterior multipole moments and show that

$$\phi(r,\theta) = \frac{Q}{4\pi\epsilon_0 r}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^l \frac{2}{l+2}P_l(0)P_l(\cos\theta) \qquad r > R. \tag{1}$$

   (b) (10 pts) Compute the potential at any point on the z-axis by elementary means and confirm that your answer agrees with the part (a) when $z > R$. Note: $P_l(1) = 1$.

3. **Potential in a box:** Consider a rectangular empty box with lengths (a, b, c) in (x, y, z) direction. All surfaces of the box have zero potential, except for the side at $z = c$, where the potential is $V(x, y) = d_0\, x\, y$ with a constant d_0.



(a) (20 pts) Solve the Laplace equation in Cartesian coordinates using a product ansatz and separation of variables. Derive a general solution for the potential with generic boundary constants.

(b) (5 pts) Determine the boundary constants by requiring the potential to take the prescribed values on the surfaces of the box.