Physics 841 - Homework 5 **due Fri., Feb. 14, 2020**

Problems

1. **Divergence theorem:** By using the divergence theorem evaluate the following integrals:

   (a) (5 pts)

   $$\oiint \vec{r}\,(\vec{a}\cdot\vec{n})\,dA, \tag{1}$$

   (b) (5 pts)

   $$\oiint (\vec{a}\cdot\vec{r})\,\vec{n}\,dA, \tag{2}$$

   where $\vec{a}$ is a constant vector and $\vec{n}$ is a vector normal to the surface element dA.

2. **Charged rod:** A uniformly charged rod with the total charge Q is placed along the z axis so that its ends are at $z=-a$ and $z=a$.

   (a) (15 pts) Find the potential $\phi(\vec{r})$ at an arbitrary point $\vec{r}=(x,y,z)$.

   (b) (15 pts) Find the electric field $\vec{E}(\vec{r})$.

   (c) (10 pts) Find the dipole and quadrupole moments of this system.

3. **Cartesian multipole moments of a charged ring:** A thin circular ring of radius R located in the xy-plane and centered at the z-axis has line charge density $+\lambda_0$ for $0\le\varphi<\pi$ and line charge density $-\lambda_0$ for $\pi\le\varphi<2\pi$, where φ is the azimuthal angle around the z-axis.

   (a) (15 pts) Calculate the components of the dipole moment of the ring.

   (b) (15 pts) Calculate the components of the quadrupole tensor of the ring. (Hint: exploit the symmetries of the tensor and the ring.)

   (c) (10 pts) Do your results depend on the choice of the origin of the coordinate frame?

   (d) (10 pts) Another point-like dipole $\vec{p}_2$ is located on the z-axis at a large distance $z\gg R$. Calculate the torque acting on this dipole.