

It's more convenient to start with 3. (b):

Originally we have the objective function

$$\|b - Ax\|_2^2 = (b - Ax)^* (b - Ax)$$

Which according to Theorem 11.1 is minimized if and only if

$$A^* Ax = A^* b,$$

We now add the L_2 normalization term to get the new objective function:

$$\begin{aligned} & \|b - Ax\|_2^2 + \alpha^2 \|x\|_2^2 \\ &= (b - Ax)^* (b - Ax) + \alpha^2 x^* x \\ &= (\tilde{b} - \tilde{A}x)^* (\tilde{b} - \tilde{A}x) \end{aligned}$$

$$\text{with } \tilde{A} = \begin{pmatrix} A \\ \alpha I \end{pmatrix}, \tilde{b} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

3. (a) Applying Theorem 11.1 to the new objective function immediately yields the new normal equations

$$\tilde{A}^* Ax = \tilde{A}^* \tilde{b} = A^* b$$

$$\Rightarrow (A^* A + \alpha^2 I)x = A^* b$$

□