Since the column of Q are orthonormal.
So we can allways construct an orthonormal
$m \times m$ matrix (Q_1, B), wit B $m \times (m-n)$.
(Like for the full QR-decomposition).

Then for any $x \in \mathbb{C}^m$: orthonormal, conserves norm

$$\|Q_1 Q^* x\|_2 \le \left\| (Q_1, B) \begin{pmatrix} Q^* x \\ 0 \end{pmatrix} \right\|_2 = \left\| \begin{pmatrix} Q^* x \\ 0 \end{pmatrix} \right\|_2 \le \left\| \begin{pmatrix} Q^* x \\ B^* x \end{pmatrix} \right\|_2$$

$$= \|(Q_1, B)^* x\|_2 = \|x\|_2$$

$$\Rightarrow \|Q_1 Q^*\|_2 \le 1$$

$$\Rightarrow \|A^+\|_2 \le \|A_1^{-1}\|_2 \|Q_1 Q^*\|_2 \le \|A_1^{-1}\|_2 \quad \square$$