4. Using single precision, evaluate the expression by hand only,

$$a = 1000\left(\frac{c}{\sqrt{b^2+c}-b} - 2b\right)$$

when $b = 1$ and $c = 0.004004$. Compare the computed value of a with the exact value $a = 2$. Show that a can be written

$$a = \frac{1000c}{\sqrt{b^2+c}+b}.$$

Now evaluate a again when $b = 1$ and $c = 0.004004$. Explain why this second expression is more accurate.

Since the floating point standard is not specified, I'll simply use

For all $x \in \mathbb{R}$, there exists ϵ with $|\epsilon| \le \epsilon_{\text{machine}}$ such that $\text{fl}(x) = x(1+\epsilon)$. **(13.5)**

and

Fundamental Axiom of Floating Point Arithmetic

For all $x, y \in \mathbf{F}$, there exists ϵ with $|\epsilon| \le \epsilon_{\text{machine}}$ such that

$$x \circledast y = (x * y)(1+\epsilon). \qquad (13.7)$$

With a typical $\epsilon_{\text{macine}} =: \epsilon_0 = 2^{-24}$ for single precision.

I'll do a worst case scenario analysis when $\epsilon = \epsilon_0$ in every calculation. It is easy to see then why the second expression is more accurate.