

## 1) MPI point-to-point vs collectives

a)

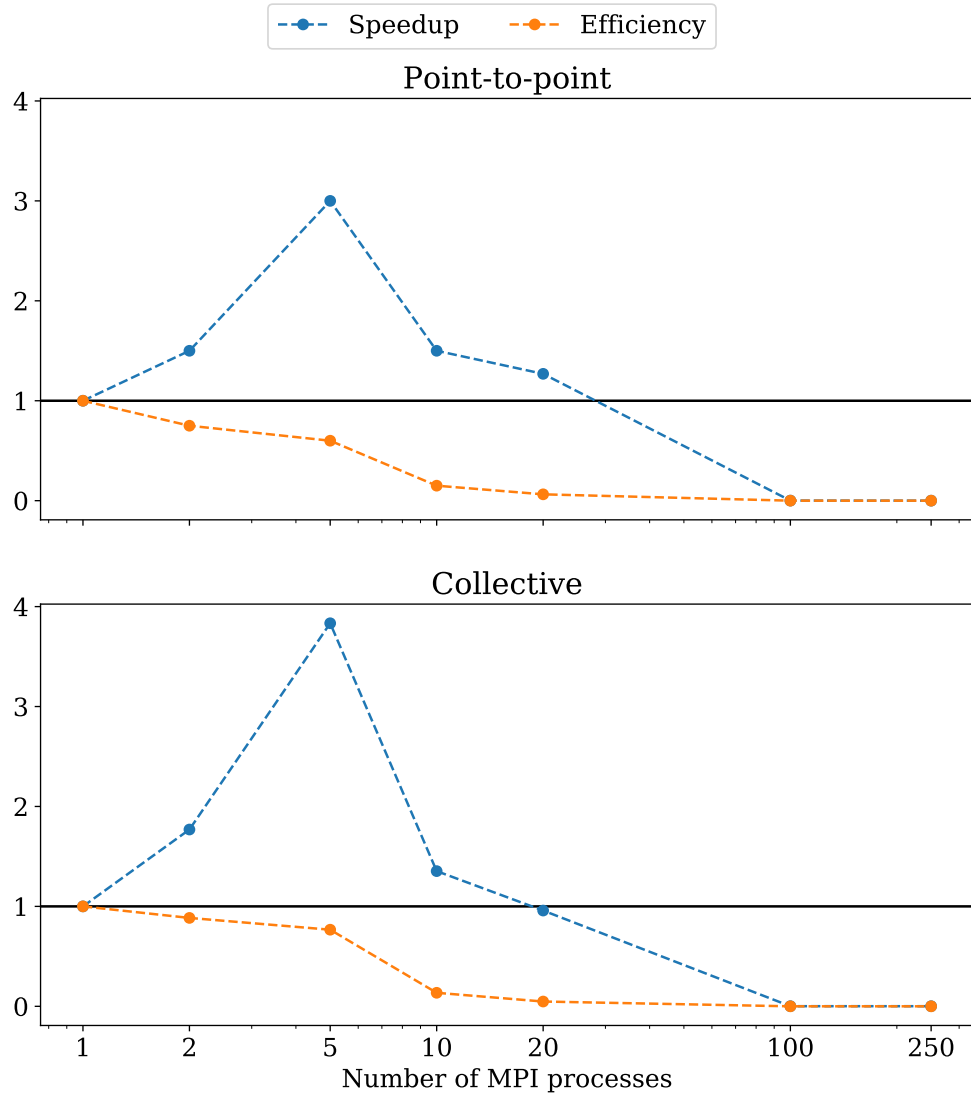
See code.

b)

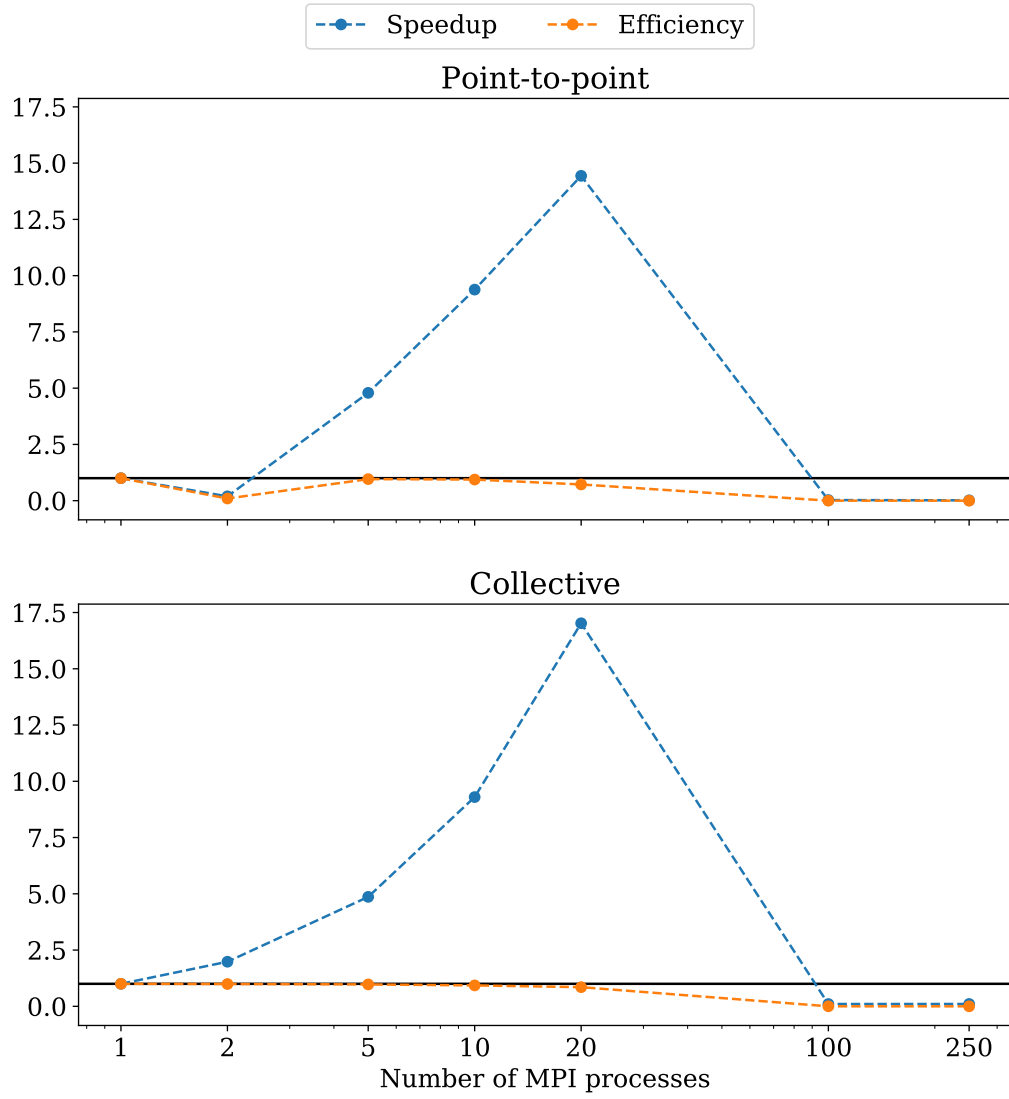
The implementation has been timed for various numbers of sampling points ( $n = 1000$ ,  $100000$  and  $10M$ ) and MPI processes ( $p = 1, 2, 5, 10, 20, 100, 250$ ). Using this data, the speed-up and efficiency for every data point was computed. The results can be seen in Figure 1, Figure 2 and Figure 3.

These plots have several features that can be easily understood: In all cases (independent of the communication method used) the implementation does not scale well with the number of used processes after a certain threshold. The overhead produced by the parallelization is not worth it any more after that threshold for  $p$  has been reached. Where that threshold is depends on the complexity of the problem. For a higher complexity (larger  $n$ ) the threshold is also larger. This threshold can be clearly seen by a steep drop in efficiency, which stays close to one before the threshold.

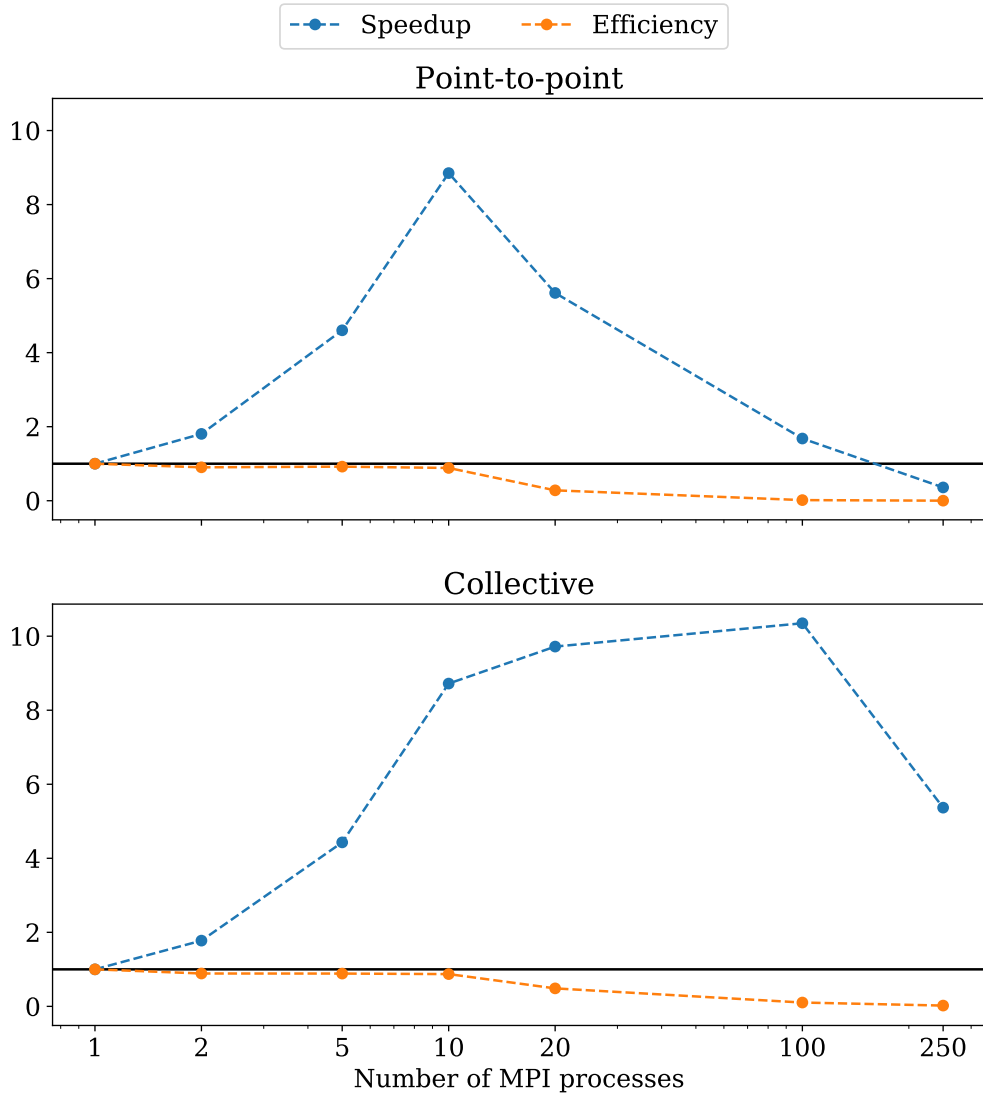
It is also clear that The collective implementation generally clearly outperforms the point-to-point implementation. However, that effect is a lot stronger for larger values of  $p$ , which makes sense as the amount of required communication is directly proportional to  $p$ . For small values of  $p$  there is almost no difference between the implementations.



**Figure 1:** Speedup and efficiency using point-to-point and collective communication for  $n = 1000$  sampling points.



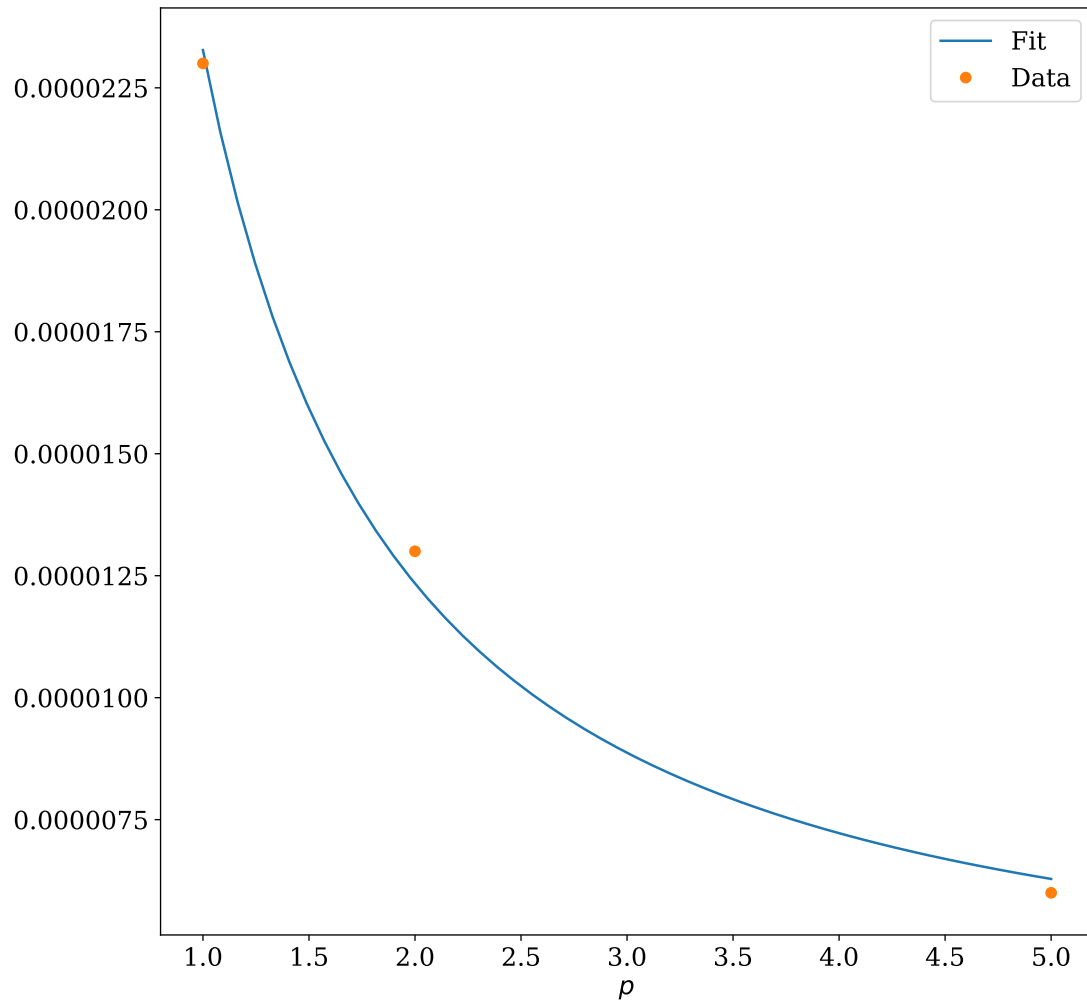
**Figure 2:** Speedup and efficiency using point-to-point and collective communication for  $n = 10^5$  sampling points.



**Figure 3:** Speedup and efficiency using point-to-point and collective communication for  $n = 10^7$  sampling points.

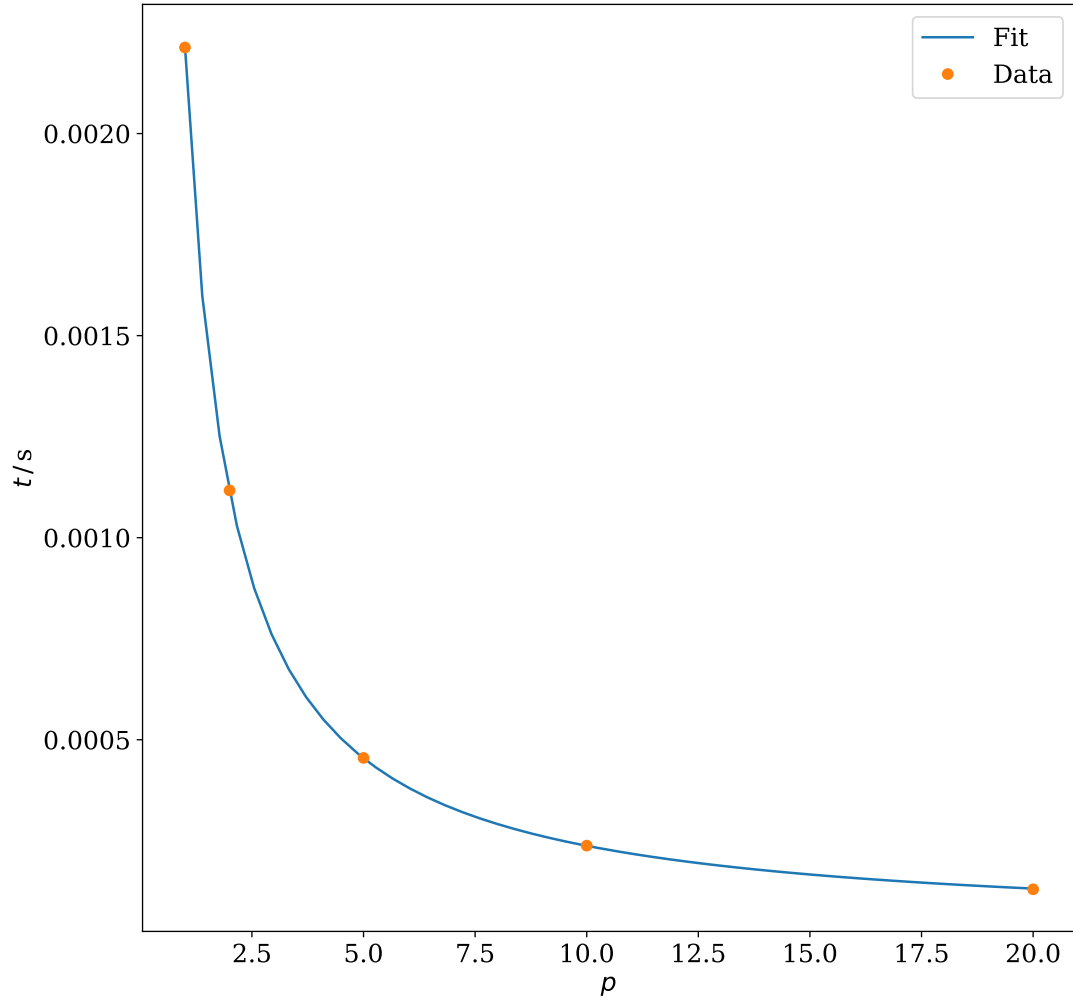
c)

The fitted parameters are listed in Table 1. The corresponding fit results are visualized by Figure 4, Figure 5 and Figure 6. The fitted model does not take overhead into account. For that reason it is necessary to exclude data points that can not be described by the model. So for all fits only data points have been used up until to the first value where the speed-up dropped (see Figures in last subsection). In that range the model fits the data with high accuracy, as can be seen in the figures and by looking at the small standard deviations of the fitted parameters in Table 1.

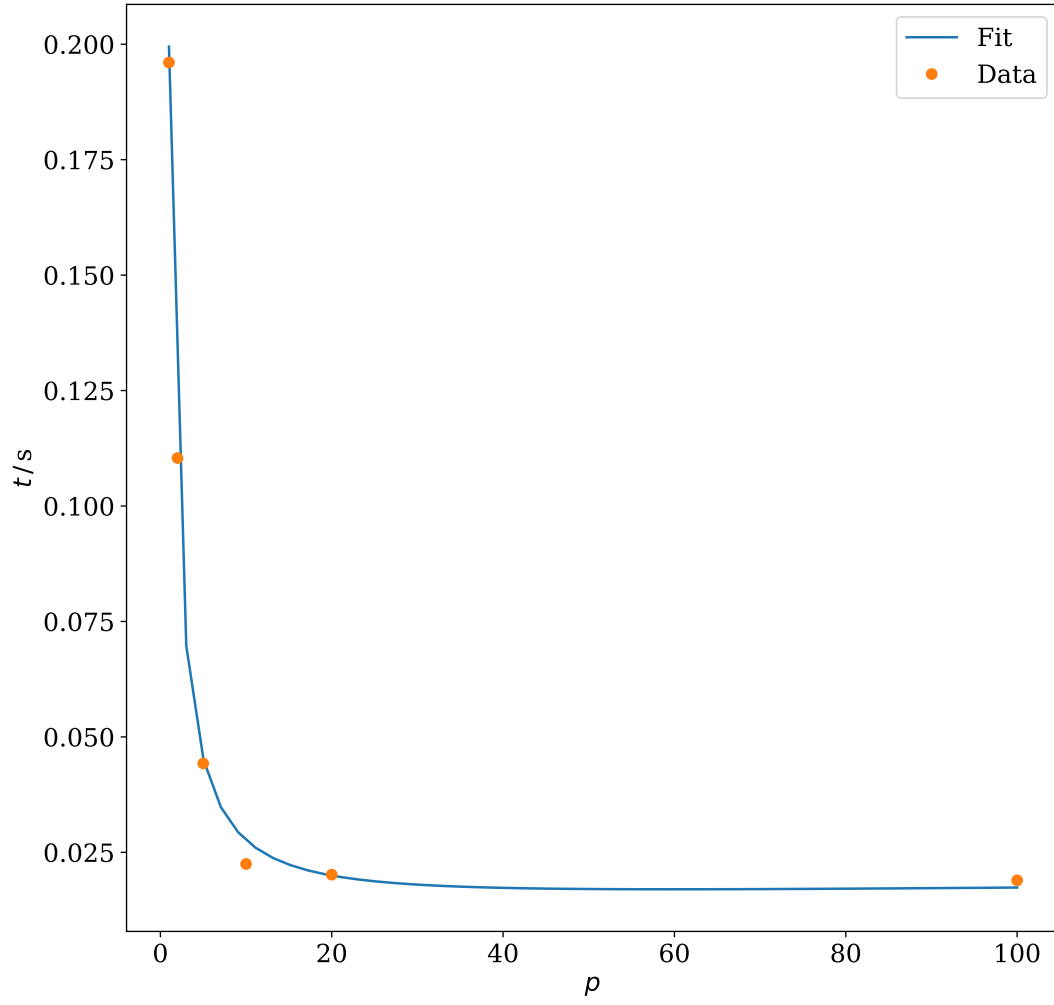


**Figure 4:** Least square fit for  $n = 1000$ .

**BONUS: MPI/OpenMP parallelization**



**Figure 5:** Least square fit for  $n = 10^5$ .



**Figure 6:** Least square fit for  $n = 10^7$ .

$n$	$a / s$	$b / s$
$10^3$	$2.327363481111708\text{e-}08 \pm 7.198806692920759\text{e-}10$	$7.020128452971986\text{e-}07 \pm 3.234141860026084\text{e-}07$
$10^5$	$2.2153350279079064\text{e-}08 \pm 2.7982074194404577\text{e-}11$	$4.843159937388218\text{e-}06 \pm 5.3147319516767\text{e-}07$
$10^7$	$1.9949962301415306\text{e-}08 \pm 4.67814210500844\text{e-}10$	$0.0023154202059335717 \pm 0.0005960271517657326$

**Table 1:** Fitted parameters for the run time using collective communication.

## 2) Code Breaking