A29

a) $-\ln \mathcal{L} = -\ln\left(\frac{\lambda^{13}}{13!}\cdot\frac{\lambda^{8}}{8!}\cdot\frac{\lambda^{9}}{9!}\, e^{-3\lambda}\right)$

$= -30\ln(\lambda) + 3\lambda + \underbrace{\ln(13!\,8!\,9!)}_{A}$

b) $(-\ln\mathcal{L})' = -\frac{30}{\lambda} + 3 \overset{!}{=} 0$

$\Leftrightarrow \lambda = 10$

$(-\ln\mathcal{L})''\Big|_{\lambda=10} = \frac{30}{\lambda^2}\Big|_{\lambda=10} = 0{,}3 > 0$

c) $-\ln\mathcal{L}(10) = -30\ln(10) + 30 + A \approx 6{,}88 =: -\ln\mathcal{L}_{max}$

$-\ln\mathcal{L}(\lambda_{1/2}) \overset{!}{=} 6{,}88 + 1/2 = -30\ln(\lambda_{1/2}) + 3\lambda_{1/2} + A$

$-\ln\mathcal{L}(\lambda_2) = 6{,}88 + 2$; $-\ln\mathcal{L}(\lambda_{9/2}) = 6{,}88 + 9/2$

Python: $\lambda_{1/2} = 8{,}28$ $\quad \lambda_2 = 6{,}78$ $\quad \lambda_{9/2} = 5{,}47$

$\lambda_{1/2} = 11{,}9$ $\quad \lambda_2 = 14{,}1$ $\quad \lambda_{9/2} = 16{,}5$

Dies sind die 1σ-, 2σ-, bzw. 3σ-Umgebungen.

d) $-\ln\mathcal{L} = -\ln\mathcal{L}_{max} + 0\cdot(\lambda-10) + \frac{1}{2}\cdot 0{,}3\,(\lambda-10)^2 + \sigma((\lambda-10)^3)$

$= -\ln\mathcal{L}_{max} + 0{,}15\,(\lambda-10)^2 + \sigma((\lambda-10)^3)$

$\lambda'_{1/2} = 8{,}17$ $\quad \lambda'_2 = 6{,}35$ $\quad \lambda'_{9/2} = 4{,}52$

$\lambda'_{1/2} = 11{,}8$ $\quad \lambda'_2 = 13{,}7$ $\quad \lambda'_{9/2} = 15{,}5$

Die Taylorentwicklung vereinfacht das analytische Rechnen um das Minimum.

A 30

a) $f(\varphi) = A \cdot a$

$$= \begin{pmatrix} \cos\varphi_0 & \sin\varphi_0 \\ \cos\varphi_1 & \sin\varphi_1 \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -1 & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -1 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

b) $a = (A^T A)^{-1} A^T y$

$$(A^T A)^{-1} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{pmatrix}$$

$$(A^T A)^{-1} A^T = \frac{1}{6} A^T$$

$$\Rightarrow a = \frac{1}{6} A^T y$$

$$= \begin{pmatrix} -0{,}0375 \\ 0{,}0774 \end{pmatrix}$$

$$V[a] = \chi_r^2 \, (A^T A)^{-1}$$

Berechnung von $\chi_r = \frac{\chi}{2}$ mit Python

$$\Rightarrow V[a] = 0{,}267 \begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix}$$

$\Rightarrow$ Fehler von a_i:

$\Delta a_i = 0{,}211$

d) $a_1 \cos(\varphi) + a_2 \sin(\varphi) = A_0 \cos(\varphi + \delta)$

$$= A_0 \cos(\varphi)\cos(\delta) - A_0 \sin(\varphi)\sin(\delta)$$

$$\Rightarrow \quad a_1 = A_0 \cos(\delta) \quad \mathrm{I} \quad \Leftrightarrow \quad A_0 = \frac{a_1}{\cos(\delta)}$$

$$a_2 = -A_0 \sin(\delta) \quad \mathrm{II} \quad \Leftrightarrow \quad A_0 = -\frac{a_2}{\sin(\delta)}$$

$$\mathrm{I} = \mathrm{II} \Rightarrow \quad \frac{a_1}{a_2} = -\frac{\cos(\delta)}{\sin(\delta)} = -\cot(\delta)$$

$$\Rightarrow \quad \delta = \operatorname{arccot}\left(-\frac{a_1}{a_2}\right) \approx 64{,}1^\circ$$

$$\Rightarrow \quad A_0 = \frac{a_1}{\cos\left(\operatorname{arccot}\left(-\frac{a_1}{a_2}\right)\right)} \approx -0{,}0860$$

$$\Delta\delta = \sqrt{\left(\frac{\partial\delta}{\partial a_1}\Delta a_1\right)^2 + \left(\frac{\partial\delta}{\partial a_2}\Delta a_2\right)^2}$$

$$= \sqrt{\left(-\frac{a_2}{a_1^2 + a_2^2}\Delta a_1\right)^2 + \left(\frac{a_1}{a_1^2 + a_2^2}\Delta a_2\right)^2} \approx 141^\circ$$

$$\Delta A_0 = \sqrt{\left(\frac{\partial A_0}{\partial a_1}\Delta a_1\right)^2 + \left(\frac{\partial A_0}{\partial\delta}\Delta\delta\right)^2}$$

$$= \sqrt{\left(\frac{1}{\cos(\delta)}\Delta a_1\right)^2 + \left(a_1 \tan(\delta)\sec(\delta)\Delta\delta\right)^2} \approx 0{,}651$$