A32

a) $-F(0,b) = N_{off}\ln(b) + N_{on}\ln(\alpha b) - (1+\alpha)b - \ln(N_{off}!) - \ln(N_{on}!)$

$$-\frac{dF(0,b)}{db} = \frac{N_{off}}{b} + \frac{N_{on}}{b} - 1 - \alpha \overset{!}{=} 0$$

$$\Rightarrow b_0 = \frac{N_{off}+N_{on}}{1+\alpha}$$

$$\sigma^2 = \left(\frac{d^2F(0,b)}{db^2}\right)^{-1}\Bigg|_{b=b_0} = \left(\frac{N_{off}+N_{on}}{b_0^2}\right)^{-1} = \frac{N_{off}+N_{on}}{(1+\alpha)^2}$$

b) $\lambda = \frac{L_0}{\hat{L}} = \exp\left(\ln\left(\frac{L_0}{\hat{L}}\right)\right) = \exp\left(\ln(L_0) - \ln(\hat{L})\right) = \exp\left(-F_0 - (-\hat{F})\right)$

$$= \exp\left[N_{off}\ln\left(\frac{b_0}{\hat{b}}\right) + N_{on}\ln\left(\frac{\alpha b_0}{\hat{s}+\alpha\hat{b}}\right) - (1+\alpha)(b_0-\hat{b}) + \hat{s}\right]$$

$$= \exp\Bigg[N_{off}\ln\left(\frac{1+\frac{N_{on}}{N_{off}}}{1+\alpha}\right) + N_{on}\ln\left(\alpha\frac{N_{off}+N_{on}}{1+\alpha}\cdot\frac{1}{N_{on}-\alpha N_{off}+\alpha N_{off}}\right)$$
$$- (1+\alpha)\left(\frac{N_{off}+N_{on}}{1+\alpha} - N_{off}\right) + N_{on} - \alpha N_{off}\Bigg]$$

$$= \exp\left[N_{off}\ln\left(\frac{1+\frac{N_{on}}{N_{off}}}{1+\alpha}\right) + N_{on}\ln\left(\alpha\cdot\frac{1+\frac{N_{off}}{N_{on}}}{1+\alpha}\right) - N_{off} - N_{on} + N_{off} + \alpha N_{off} + N_{on} - \alpha N_{off}\right]$$

$$= \exp\left[N_{off}\ln\left(\frac{1+\frac{N_{on}}{N_{off}}}{1+\alpha}\right) + N_{on}\ln\left(\alpha\cdot\frac{1+\frac{N_{off}}{N_{on}}}{1+\alpha}\right)\right]$$

c) u^2 ist $\chi^2(1)$ verteilt

$\Rightarrow u^2 \sim \chi^2 \ \wedge \ D \sim \chi^2$

$\Rightarrow \sqrt{D} \sim |u| \sim |\mathcal{N}(0,1)|$

$\Rightarrow \sqrt{D} \equiv$ Konfidenzschranke in Einheiten von σ

d) i) $\sqrt{D} \simeq 1{,}8363\,\sigma$ Signifikanz Level = p-Value = p

$$p = 1 - 2\cdot\int_0^{\sqrt{D}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx \simeq 0{,}0663 = 6{,}63\,\%$$

ii) $\sqrt{D} \approx 4{,}3774\,\sigma$

$p \approx 0{,}0000120 = 0{,}00120\%$

A33)

$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma_i^2}$$

a) $$\chi^2 = \frac{(31{,}6\,\mathrm{meV} - 31{,}3\,\mathrm{meV})^2}{(0{,}5\,\mathrm{meV})^2} + \frac{(32{,}2\,\mathrm{meV} - 31{,}3\,\mathrm{meV})^2}{(0{,}5\,\mathrm{meV})^2} + \dots$$

$$\approx 6{,}08$$

$\chi^2(p=0{,}05,\ \mathrm{DoF}=7) = 14{,}067 > 6{,}08$

$\Rightarrow H_0$ kann nicht abgelehnt werden

b) $\chi^2 \approx 21{,}9$

$\chi^2(p=0{,}05,\ \mathrm{DoF}=7) = 14{,}067 < 21{,}9$

$\Rightarrow H_0$ wird abgelehnt

A34 a)

$$\Gamma = \frac{\sup_{\theta\in\Theta_0} \mathcal{L}(\theta|X)}{\sup_{\theta\in\Theta} \mathcal{L}(\theta|X)}$$

$$= \frac{\sup_{\sigma^2} \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-1}{2\sigma^2}(X_i-\mu_0)^2\right]}{\sup_{\sigma^2,\mu} \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-1}{2\sigma^2}(X_i-\mu)^2\right]}$$

$\Gamma < k_\alpha \Rightarrow H_0$ verwerfen

$\Gamma \geq k_\alpha \Rightarrow H_0$ annehmen

b) $$\mathcal{L} = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N \exp\left[\frac{-1}{2\sigma^2}\sum_i (X_i-\mu)^2\right]$$

$$-\ln(\mathcal{L}) = F = N\ln(\sqrt{2\pi}) + N\underbrace{\ln(\sqrt{\sigma^2})}_{=\frac{1}{2}\ln(\sigma^2)} + \frac{1}{2\sigma^2}\sum_i (x_i-\mu)^2$$

$$\frac{\partial F}{\partial \sigma^2} = \frac{N}{2\sigma^2} - \frac{1}{2\sigma^4}\sum_i (x_i-\mu)^2 \overset{!}{=} 0$$

$$\Leftrightarrow \sigma^2 = \frac{1}{N}\sum_i (x_i-\mu)^2$$

$$\frac{\partial F}{\partial \mu} = \frac{-1}{\sigma^2}\sum_i (x_i-\mu) \overset{!}{=} 0$$

$$\Leftrightarrow \sum_i x_i = N\mu$$

$$\Leftrightarrow \mu = \frac{1}{N}\sum_i x_i = \bar{X}$$

c)

$$\Gamma = \frac{\left(\frac{1}{N}\sum_i (x_i-\mu_0)^2\right)^{-N/2}}{\left(\frac{1}{N}\sum_i (x_i-\bar{x})^2\right)^{-N/2}} \exp\left[-\frac{1}{2}\left(\left(\frac{1}{N}\sum_i (x_i-\mu_0)^2\right)^{-1}\sum_i (x_i-\mu_0)^2 - \left(\frac{1}{N}\sum_i (x_i-\bar{x})^2\right)^{-1}\sum_i (x_i-\bar{x})^2\right)\right]$$

$$= \left(\frac{\sum_i (x_i-\mu_0)^2}{\sum_i (x_i-\bar{x})^2}\right)^{-\frac{N}{2}} \exp\left[-\frac{1}{2}(N-N)\right]$$

$$= \left(\frac{\frac{1}{N-1}\sum_i (x_i-\mu_0)^2}{s^2}\right)^{-\frac{N}{2}}$$

$$\Rightarrow \Gamma^{-\frac{2}{N}} = \frac{1}{N-1}\sum_i (x_i-\mu_0)^2 \Big/ s^2$$

$$= \frac{1}{s^2(N-1)}\sum_i \left(x_i^2 - 2x_i\mu_0 + \mu_0^2\right)$$

$$= \frac{1}{s^2(N-1)}\left(\sum_i (x_i^2) \underbrace{- 2N\bar{x}\mu_0 + N\mu_0^2 + N\bar{x}^2}_{N(\mu_0-\bar{x})^2} - \underbrace{N\bar{x}^2}_{N\bar{x}\frac{1}{N}\sum_i x_i}\right)$$

$$\Gamma^{-\frac{2}{N}} = \frac{1}{s^2(N-1)}\Big(\sum_i (x_i^2) \underbrace{- 2N\bar{x}\mu_0 + N\mu_0^2 + N\bar{x}^2}_{N(\mu_0-\bar{x})^2} - \underbrace{N\bar{x}^2}_{N\bar{x}\frac{1}{N}\sum_i x_i}\Big)$$

$$= \frac{\sum_i (x_i^2 - \bar{x}x_i) + N(\mu_0-\bar{x})^2}{(N-1)\cdot s^2}$$

$$= \frac{\sum_i (x_i^2 - \bar{x}x_i - \bar{x}x_i + \bar{x}x_i) + N(\mu_0-\bar{x})^2}{(N-1)\, s^2}$$

$$= \frac{\sum_i (x_i^2 - 2\bar{x}x_i) + N\bar{x}^2 + N(\mu_0-\bar{x})^2}{(N-1)\, s^2}$$

$$= \frac{\sum_i (x_i^2 - 2\bar{x}x_i + \bar{x}^2) + N(\mu_0-\bar{x})^2}{(N-1)\, s^2}$$

$$= \frac{\sum_i (x_i-\bar{x})^2}{(N-1)\, s^2} + \frac{N(\mu_0-\bar{x})^2}{(N-1)\, s^2}$$

$$= 1 + \frac{T^2}{N-1}$$

$$\Leftrightarrow \sqrt{(N-1)\left(\Gamma^{-\frac{2}{N}}-1\right)} = T$$

$$\Gamma \overset{!}{\geq} k_\alpha \iff T \overset{!}{\leq} k_\alpha^*$$

$$\left(\Gamma \overset{!}{<} k_\alpha \iff T \overset{!}{>} k_\alpha^*\right)$$

k_α^* kann in Tabellen nachgeschlagen werden

d) $T = \dfrac{\sqrt{25}\,(205\,ml - 200\,ml)}{10\,ml} = 2{,}5$

$\alpha = 5\%$; $DoF = 24$; einseitiger Test

$\Rightarrow k_\alpha^* = 1{,}711 < T \Rightarrow H_0$ wird nicht abgelehnt

zweiseitiger Test:

$\Rightarrow k_\alpha^* = 2{,}064 < |T| \Rightarrow H_0$ wird nicht abgelehnt

A35 Bayes Theorem:

$$p(H_i | D, I) = \frac{p(H_i | I)\, p(D | H_i, I)}{p(D | I)}$$

$p(H_\pi | I) = 0{,}8$

$p(H_k | I) = 0{,}1$

$p(H_p | I) = 0{,}1$

a) $p(D | I) = \sum_i p(H_i | I)\, p(D | H_i, I)$

$= 0{,}8 \cdot 0{,}13 + 0{,}1 \cdot 1{,}5 + 0{,}1 \cdot 0{,}5 = 0{,}304$

$\Rightarrow p(H_\pi | D, I) = \frac{0{,}8 \cdot 0{,}13}{0{,}304} \approx 34{,}2\%$

$p(H_k | D, I) = \frac{0{,}1 \cdot 1{,}5}{0{,}304} \approx 49{,}3\%$

$p(H_p | D, I) = \frac{0{,}1 \cdot 0{,}5}{0{,}304} \approx 16{,}4\%$

b) $p(H_\pi | D, I) = 96{,}7\%$

$p(H_k | D, I) = 3{,}02\%$

$p(H_p | D, I) = 0{,}302\%$

c) $p(H_\pi | D, I) = 23{,}7\%$

$p(H_k | D, I) = 21{,}2\%$

$p(H_p | D, I) = 55{,}2\%$