Aufgabe 11a

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11.a) $\Phi = \Phi_0 \left(\frac{E}{1\,TeV}\right)^{-\gamma}, \quad \gamma = 2,7$

$$u = g(E) = \int_{1\,TeV}^{E} \Phi_0 \left(\frac{E'}{1\,TeV}\right)^{-\gamma} dE' \quad \left(\tilde{E} = \frac{E'}{1\,TeV}, \quad \frac{d\tilde{E}}{dE'} = \frac{1}{1\,TeV}\right)$$

$$g(E) = \int_{1}^{\frac{E}{1\,TeV}} \Phi_0 \cdot 1\,TeV \; \tilde{E}^{-\gamma} \, d\tilde{E}$$

$$= \frac{1}{-\gamma+1} \Phi_0 \cdot 1\,TeV \left(\left(\frac{E}{1\,TeV}\right)^{-\gamma+1} - 1\right) = u$$

$$\Rightarrow u + \frac{\Phi_0 \cdot 1\,TeV}{1-\gamma} = \frac{\Phi_0 \cdot 1\,TeV}{1-\gamma} \left(\frac{E}{1\,TeV}\right)^{-\gamma+1}$$

$$\Leftrightarrow \left(\frac{E}{1\,TeV}\right)^{-\gamma+1} = \frac{u(1-\gamma)}{\Phi_0 \cdot 1\,TeV} + 1$$

$$\Leftrightarrow \frac{E}{1\,TeV} = \left[1 + \frac{u(1-\gamma)}{\Phi_0 \cdot 1\,TeV}\right]^{\frac{1}{1-\gamma}}, \quad [\Phi_0] = TeV^3$$

$$u(1\,TeV) = \frac{\Phi_0 \cdot 1\,TeV}{1-\gamma} (1-1) = 0$$

$$u(\infty) = \frac{\Phi_0 \cdot 1\,TeV}{1-\gamma} (0-1) = \frac{\Phi_0 \cdot 1\,TeV}{\gamma - 1}$$

$$\Rightarrow u \in \left[0, \frac{\Phi_0 \cdot 1\,TeV}{\gamma - 1}\right]$$