21. a)

$$H(Y) = -\sum_{z \in Z} P(Y=z) \log_2(P(Y=z))$$

$Z = \{\text{False}, \text{True}\}$

$P(Y=\text{False}) = \frac{5}{14}, \quad P(Y=\text{True}) = \frac{9}{14}$

$$\Rightarrow H(Y) = -\frac{5}{14}\log_2\left(\frac{5}{14}\right) - \frac{9}{14}\log_2\left(\frac{9}{14}\right) \approx 0,9403$$

b) Entropie vorher: $H(Y)$

Entropie nachher: $H(Y|X)$

$$\Rightarrow H(Y|X) = -\sum_{m \in M} P(X=m) \sum_{z \in Z} P(Y=z|X=m) \cdot \log\left(P(Y=z|X=m)\right)$$

$M = \{\text{False}, \text{True}\}$

$P(X=\text{False}) = \frac{8}{14}, \quad P(X=\text{True}) = \frac{6}{14}$

$P(Y=\text{False}|X=\text{False}) = \frac{2}{14}$

$P(Y=\text{False}|X=\text{True}) = \frac{3}{14}$

$P(Y=\text{True}|X=\text{False}) = \frac{6}{14}$

$P(Y=\text{True}|X=\text{True}) = \frac{3}{14}$

$$\Rightarrow H(Y|X) = -\frac{8}{14}\left(\frac{2}{14}\log_2\left(\frac{2}{14}\right) + \frac{6}{14}\log_2\left(\frac{6}{14}\right)\right) - \frac{6}{14}\left(\frac{3}{14}\log_2\left(\frac{3}{14}\right) + \frac{3}{14}\log_2\left(\frac{3}{14}\right)\right)$$

$$\Rightarrow H(Y|X) \approx 0,9367$$

$$\Rightarrow H(Y) - H(Y|X) \approx 0,00356$$