PHY 831: Statistical Mechanics
Exam 1

October 2nd, 2020

Possibly useful results

$$\int_{-\infty}^{\infty} dx e^{-ax^2+bx} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}} \text{ (for } a > 0)$$
$$\Gamma(n) = (n-1)! = \int_0^{\infty} dx x^{n-1} e^{-x}$$
$$\ln N! \approx N \ln N - N \text{ (for } N \gg 1)$$

1. (a) (2 points) Prove the relationship

$$\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial P}{\partial T}\right)_{V,N}$$

(b) (4 points) Using only the properties $P = NT/V$ and $C_V = T\left(\frac{\partial S}{\partial T}\right)_{N,V} = \frac{3}{2}N$ for an ideal gas, find the adiabatic sound speed squared

$$c_s^2 = -\frac{V^2}{Nm}\left(\frac{\partial P}{\partial V}\right)_{S,N}$$

in terms of T, N, and V using partial derivative relations. Here, m is the mass of the particles in the gas. The result of part (a) should be useful.

2. Consider a classical, non-interacting gas of N indistinguishable particles that can only move in one-dimension and obey the single-particle dispersion relation $\epsilon = |\vec{p}|c$ (i.e. the Hamiltonian is given by $H = \sum_{i=1}^{N} |\vec{p}_i|c$) and are confined to a length L.

(a) (8 points) Find the canonical partition function when $N = 1$.

(b) (2 points) Find the canonical partition function for arbitrary N.

(c) (2 points) Calculate the entropy of the N particle system.

3. (8 points) Consider a systems with an equation of state given by

$$P = \frac{E}{V}\left(\frac{V}{V_0}\right)^{\lambda}\left(\frac{E}{E_0}\right)^{\lambda}$$

and a temperature given by

$$T = E\left(\frac{E}{E_0}\right)^{\lambda},$$

where E_0, V_0, and λ are constants. Given the entropy at E_0 and V_0 is $S(E_0, V_0) = 0$, find the entropy at arbitrary energy E and volume V.