

1. Show that the language  $L = \{w : w \in \{0, 1\}^* \text{ is a palindrome}\}$  is not regular.

Proof : Assume  $L$  is regular and  $p$  is the number from the pumping Lemma.

Consider  $S = 0^p 10^p$

Since  $S \in L$  and  $|S| \geq P$  then the conditions of the pumping lemma must hold for  $S = xyz$ .

Let  $y = 0^k$  for some  $k > 0$ ,

Then  $S = 0^{p-k}0^k10^p$  (relative to  $S = xyz$ ) for  $|xy| \leq P$

Consider the string  $S' = xy^2z = 0^{p-k}0^{2k}10^p$ .

Then  $p-k+2k = p+k \neq p$  so  $|xy| > p$

Therefore,  $S'$  breaks the pumping lemma, thus,  $L$  is not a regular language.

2. Show that the language  $L = \{0^n 1^m 0^n : m, n \geq 0\}$  is not regular.

Proof : Assume  $L$  is regular and  $p$  is the number from the pumping Lemma.

Consider  $S = 0^p 10^p$

Since  $S \in L$  and  $|S| \geq P$  then the conditions of the pumping lemma must hold for  $S = xyz$ .

Let  $y = 0^k$  for some  $k > 0$ ,

Then  $S = 0^{p-k}0^k10^p$  (relative to  $S = xyz$ ) for  $|xy| \leq P$

Consider the string  $S' = xy^2z = 0^{p-k}0^{2k}10^p$ .

Then  $p-k+2k = p+k \neq p$  so  $|xy| > p$

Therefore,  $S'$  breaks the pumping lemma, thus,  $L$  is not a regular language.

3. Show that the language  $L = \{www : w \in \{0, 1\}^*\}$  is not regular.

Proof : Assume  $L$  is regular and  $p$  is the number from the pumping Lemma.

Consider  $S = 0^p 1^p$

Since  $S \in L$  and  $|S| \geq P$  then the conditions of the pumping lemma must hold for  $S = xyz$ .

Let  $y = 0^k$  for some  $k > 0$ ,

Then  $S = 0^{p-k}0^k1^p$  for  $|xy| \leq P$

Consider the string  $S' = xy^2z = 0^{p-k}0^{2k}1^p$ .

Then  $p-k+2k = p+k \neq p$  so  $|xy| > p$

Consider the string  $S' = xy^3z = 0^{p-k}0^{3k}1^p$ .

Then  $p-k+3k = p+2k \neq p$  so  $|xy| > p$

Consider the string  $S' = xy^4z = 0^{p-k}0^{4k}1^p$ .

Then  $p-k+4k = p+3k \neq p$  so  $|xy| > p$

Therefore,  $S'$  for all provided  $i$ 's, breaks the pumping lemma, thus,  $L$  is not a regular language.

4. Design a context-free grammar for the language  $L = \{w : w \text{ contains more 0s than 1s}\}$ .

$S \rightarrow 0S1 \mid 1S0 \mid 01S \mid 10S \mid S0 \mid 0$