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1. Show that INDEPENDENT-SET is NP-C.

Suppose $V' \subseteq V$ and V' is in INDEPENDENT-SET.

M = on input<V', k>

1. Check if |V'| >= k, if not, reject.
2. For each vertex ∈ V'
3. Reject if there ∃ another vertex such that an edge exists between them.
4. Accept

Therefore, IS is in O (V' + E) and IS ∈ NP

Suppose v' is a VC, then, every edge contains an endpoint from V'

Suppose two vertices of V\V' share an edge. Then there is a contradiction.

Since V', every edge in V is connteced to a vertice from V'. Taking the set differnce of V from V' cannot share an edge. Therefore, V\V' is an IS

Suppose U is IS and U⊆V, then no pairs of vertices in U share an edge.

Suppose V' = V\U and there is an edge that contains no endpoints from V'.

Then, this is a contradiction.Eliminating the U from V must result in vertices from V' being connected.

Thus V' is VC.

There for $IS \leq_p VC$

2.

$Claim: 4SAT \in NP-C$

$Proof:$

1. $Show\ 4SAT \in NP.$

$\phi = (x_1 \vee x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4) \wedge (x_1 \vee \neg x_2 \vee x_3 \vee \neg x_4)$

$M = on\ input\ (\phi, True)$

1. Check $if\ \phi = True, if\ not\ reject$

$Therefore, M\ runs \in O(n)$ and $4SAT \in NP$

2. $Show\ 3SAT \leq_P 4SAT.$

$Suppose\ \phi_{3SAT}\ is\ an\ instance\ of\ 3SAT. Turn\ it\ into\ a\ \phi_{4SAT}\ instance\ by\ converting\ each\ clause\ as\ follows:$

$(x_1 \vee x_2 \vee x_3) \rightarrow (x_1 \vee x_2 \vee x_3 \vee z) \wedge (x_1 \vee x_2 \vee x_3 \vee \neg z)$

$Need$ to $show\ \phi_{3SAT}\ is\ satisfiable \Leftrightarrow \phi_{4SAT}\ is\ satisfiable:$

$Say\ x_1 = T, x_2 = F, x_3 = F, z = T,\ \phi_{3SAT} = (x_1 \vee x_2 \vee x_3),\ \phi_{4SAT} = (x_1 \vee x_2 \vee x_3 \vee z) \wedge (x_1 \vee x_2 \vee x_3 \vee \neg z)$

$Suppose\ \phi_{3SAT}\ is\ satisfiable.$

$\phi_{3SAT} = (T \vee F \vee F) = True$

$\phi_{4SAT} = (T \vee F \vee F \vee T) \wedge (T \vee F \vee F \vee F) = True$

$\phi_{3SAT} == \phi_{4SAT}$

$Then\ \phi_{4SAT}\ is\ satisfiable.$

$Suppose\ \phi_{4SAT}\ is\ satisfiable.$

$\phi_{4SAT} = (T \vee F \vee F \vee T) \wedge (T \vee F \vee F \vee F) = True$

$\phi_{3SAT} = (T \vee F \vee F) = True$

$\phi_{4SAT} == \phi_{3SAT}$

$Then\ \phi_{3SAT}\ is\ satisfiable.$

$Therefore\ \phi_{3SAT}\ is\ satisfiable \Leftrightarrow \phi_{4SAT}$