

Problem 1:

Prove by Counter Example

Let  $L1 = \{ \omega : \omega \text{ is odd} \}$  and  $L2 = \{ \omega : \omega \text{ is odd and contains } 0^n : n \text{ is prime} \}$

$L2 \subset L1$ , but  $L2$  is not regular by the pumping lemma

Suppose  $L2$  is regular and contains string  $S = 0^n$ ,  $y = 0^k$

$$S = 0^{(n-k)} 0^k$$

Let  $i = n+1$ , then  $S = xy^{(n+1)}z = 0^{(n-k)} 0^{((n+1)k)}$

So,  $n-k + (n+1)k = n-k + nk + k = n + nk = n(k+1)$ , which is divisible by  $n$  and  $k+1$  meaning  $n$  is not prime and  $S' \notin L2$

Therefore  $L2$  is not regular.

Problem 2:

Proof by Counter Example

If  $L1 = \{ 0^n 1^m : n \geq m \}$  and  $L2 = \{ 0^{+1} \}$ ,

then  $L2 \subset L1$  and  $L2$  is regular.

Problem 3:

For the string  $0^p 1^0 0^p$  the values for  $x = 0^p$ ,  $y = 1^0$  and  $z = 0^p$ .

With  $y = 1^0$ ,  $k = 0$ , we are not doing any pumping to the string. Because of this, we get

$p + 0 = p$ . Because the number of  $x$ 's 0s and  $y$ 's 1s are equal to the number of  $z$ 's 0's ( $p$ ), we cannot show that the language is not regular.