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1.
Proof: Suppose EQ_{CFG} is undecidable. Let TM H be its decider:

$H(\langle A,B\rangle)=\{accept, if\ A\ accepts\ L(A)=L(B).\ Decline, if\ A\ does\ not\ accept\ L(A)=L(B)\}$
Make a new TM D:
D = on input $\langle N\rangle$, *for TM N*
1. Run H on $\langle N,\langle N\rangle\rangle$.
2. If H accepts, reject. If H rejects, accept.
$D(\langle D\rangle)=\{accept, if\ D\ does\ not\ accept\ \langle D\rangle.\ reject, if\ D\ accepts\ \langle D\rangle\}$
D accepts $\langle D\rangle$, *so long as D does not accept* $\langle D\rangle$. → *TM D cannot exist* → *TM H cannot exist* → EQ_{CFG} *is undecidable*

2. Proof: Suppose $HALT-\varepsilon_{TM}=\{\langle M\rangle: M\ is\ a\ TM$ and *halts on blank input* $(i.e.\ \varepsilon)\}$ is undecidable.
Let TM H be its decider:

$H(\langle M\rangle)=\{$accept, if M accepts . Decline, if A does not halt on blank input$\}$
Make a new TM D:
D = on input $\langle N\rangle$, *for TM N*
1. Run H on $\langle N,\langle N\rangle\rangle$.
2. If H accepts, reject. If H rejects, accept.
$D(\langle D\rangle)=\{accept, if\ D\ does\ not\ accept\ \langle D\rangle.\ reject, if\ D\ accepts\ \langle D\rangle\}$
D accepts $\langle D\rangle$, *so long as D does not accept* $\langle D\rangle$. → *TM D cannot exist* → *TM H cannot exist* → $HALT-\varepsilon_{TM}$ *is undecid*

3. Proof: Suppose $EVEN_{TM}=\{\langle M\rangle: M\ is\ a\ TM$ and $L(M)$ *contains every string of even length* $\}$ is undecidable. Let TM H be its decider:

$H(\langle M\rangle)=\{$accept, if M accepts . Decline, if L(M) contains every string of even length$\}$
Make a new TM D:
D = on input $\langle N\rangle$, *for TM N*
1. Run H on $\langle N,\langle N\rangle\rangle$.
2. If H accepts, reject. If H rejects, accept.
$D(\langle D\rangle)=\{accept, if\ D\ does\ not\ accept\ \langle D\rangle.\ reject, if\ D\ accepts\ \langle D\rangle\}$
D accepts $\langle D\rangle$, *so long as D does not accept* $\langle D\rangle$. → *TM D cannot exist* → *TM H cannot exist* → $EVEN_{TM}$ *is undecidabl*