

Problem 1:

Prove by Counter Example

Let $L1 = \{ \omega : \omega \text{ is odd} \}$ and $L2 = \{ \omega : \omega \text{ is odd and contains } 0^n : n \text{ is prime} \}$

$L2 \subset L1$, but $L2$ is not regular by the pumping lemma

Suppose $L2$ is regular and contains string $S = 0^n$, $y = 0^k$

$$S = 0^{(n-k)} 0^k$$

Let $i = n+1$, then $S = xy^{(n+1)}z = 0^{(n-k)} 0^{((n+1)k)}$

So, $n-k + (n+1)k = n-k + nk + k = n + nk = n(k+1)$, which is divisible by n and $k+1$
meaning n is not prime and $S' \notin L2$

Therefore $L2$ is not regular.

Problem 2:

Proof by Counter Example

If $L1 = \{ 0^n 1^m : n \geq m \}$ and $L2 = \{ 0^{+1} \}$,

then $L2 \subset L1$ and $L2$ is regular.

Problem 3: