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LOGIC AND THEORY OF COMPUTATION EXAM
20.12.2018

Information:

The first part includes 10 logic and 10 theory of computation questions. For the logic questions (PL) denotes a question on propositional logic, and (FOL) denotes a question on first-order logic.

For each answer you can get up to 3 points. Some questions have several subquestions, pay attention to answer all of them. Do not forget to give examples for the logic questions.

The second part consists of a topic from logic and a proof from theory of computation for 12 points each.

There are 120 minutes for the exam.

Marks:

5: $57 \leq$ points ≤ 84,
4: $46 \leq$ points < 56,
3: $35 \leq$ points < 44,
2: $24 \leq$ points < 34,
1: $0 \leq$ points < 23.

PART I

1. (PL) Define what is a subformula in propositional logic. Give an example for a propositional formula having at least one of each of the four operators. List all the subformulas of your example.

2. (PL) Define what is meant by a satisfiable formula in propositional logic. Give an example for a satisfiable formula. Is there a decision procedure for satisfiability in propositional logic? If yes, name (or describe) one such procedure.

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3. (PL) What does it mean in propositional logic that a formula B is a logical consequence of a set of formulas U? Let $U = \{A_1, \ldots, A_n\}$ be a set of formulas and B be a formula. Give at least two equivalent statements with $U \models B$.

4. (PL) What are called the α and β formulas in the semantic tableux method? Demonstrate the semantic tableaux method on the example $A = (\neg(p \vee \neg q) \wedge (p \rightarrow \neg q))$. Is this formula A unsatisfiable? How do you decide unsatisfiability in general from the completed tableau?

5. (PL) What is meant by a literal? What is conjunctive normal form? Give a propositional formula in conjunctive normal form with at least two atoms and three terms! Give also the clausal form representation of your example.

6. (PL) What is a semantic tree? Draw a semantic tree for $\mathcal{S} = \{p\bar{q}, \bar{p}\bar{q}\}$. Show an open and a closed branch (if there is any).

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7. (FOL) What is called a free and what is called a bound variable is first order logic? What is a closed formula? Give a first order formula that has exactly one free and two bound variable.

8. (FOL) Define what is meant by logical equivalence of two closed formulas in first order logic. Give at least 2 pairs of formulas of first order logic containing at least one quantifier each that are logically equivalent.

9. (FOL) Define what is meant by prenex conjunctive normal form (PCNF). Give a PCNF containing all the following operators and quantifiers: at least one $\exists$, at least one $\forall$, at least one $\neg$, at least one $\vee$.

10. (FOL) What will be the Herbrand universe for the following set S of clauses of first order logic? $\mathcal{P} = \{p\}$, $\mathcal{F} = \{f\}$,$\mathcal{A} = \{b\}$, $\mathcal{V} = \{x, y, \ldots\}$, $S = \{\{\neg p(b, f(y, y))\}, \{p(x, y)\}\}$.

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11. Let f and g be $\mathbb{N} \to \mathbb{R}_0^+$ functions. Define $f = \Omega(g)$. Let $f(n) = \log_4 n$ and $g(n) = 2\log_2 n$. Does $f = \Omega(g)$ hold? And $g = \Omega(f)$?

12. Define k-tape, deterministic Turing machine ($k \geq 1$). Give the meaning of its components (including the transition function).

13. Define what does it mean that a nondeterministic Turing machine decides a language. Suppose that a nondeterministic Turing machine decide a language L in $n^2 + 2^n$ time. Does it follow, that $L \in$NP. And does it follow that $L \notin$NP? Why?

14. What does it mean, that a Turing machine computes a function $f : \Sigma^* \to \Delta^*$ (Σ, Δ are alphabets)? What is meant by a computable function? Define what is meant by reduction of a language to an other.

15. Define the language L_u (universal language). Is the complement of this language recognisable? Why?

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16. Define NP-completeness. What would be a consequence for time complexity classes if someone were find a polynomial time deterministic algorithm for an NP-complete problem?

17. Define the language kSAT ($k \geq 1$). State the theorem we learned on its time complexity. What is the conjecture on whether the language in the theorem is in P or not in P?

18. Define what is a vertex cover of an undirected graph. Define the formal language Vertex Cover. In what complexity class is it complete?

19. Define the space complexity classes SPACE($f(n)$), NSPACE($f(n)$), NL and coNL.

20. State Savitch's theorem. Is there a language that can be decided by a nondeterministic offline Turing machine in polynomial space but can not be decided by a deterministic one in polynomial space? If yes, give one such language, if there is no such language, why is it so?

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PART II

Logic question: (PL) Give the resolution procedure in propositional logic (including the related definitions as well). How can we use the resolution procedure for deciding logical consequence?

Theory of computation question: What is called an independent set in a simple, undirected graph? Define the formal language INDEPENDENT SET. State and proof the theorem on its time compexity.