EVOLUTIONARY ALGORITHMS

The evolutionary algorithm is a search system

- It handles several elements (individuals) of the problem space at each time and permanently modifies this population that becomes better and better until the solution (the goal, the best individual) appears.
- An individual is better than the other if it is somehow closer to the correct answer. The fitness function tries to measure this „distance” between an individual and the solution.
- Firstly an initial population is selected mostly at random. In every turn the population is updated with the mutated offspring of the individuals that are selected for reproduction. This change of the population is an irreversible step, so the evolution is controlled by an irrevocable strategy.

Evolutionary operators and termination condition

- *Selection*: better individuals are selected for reproduction
- *Recombination (crossover)*: pairs of the selected individuals (parents) are mated in order to create their offspring
- *Mutation*: offspring can be changed a bit
- *Replacement*: a new population is constructed from the elder one and the mutated offspring

- Termination condition:
  - either a goal individual appears in the population
  - or the overall fitness value of the population is not being changed

DATA := *initial value*
while ¬ *termination condition*(DATA) **loop**
SELECT R FROM *rules that can be applied*
DATA := R(DATA)
endloop

Evolutionary algorithm

Procedure EA

population := initial population

while *not termination condition* ***loop***

parents := selection(population)

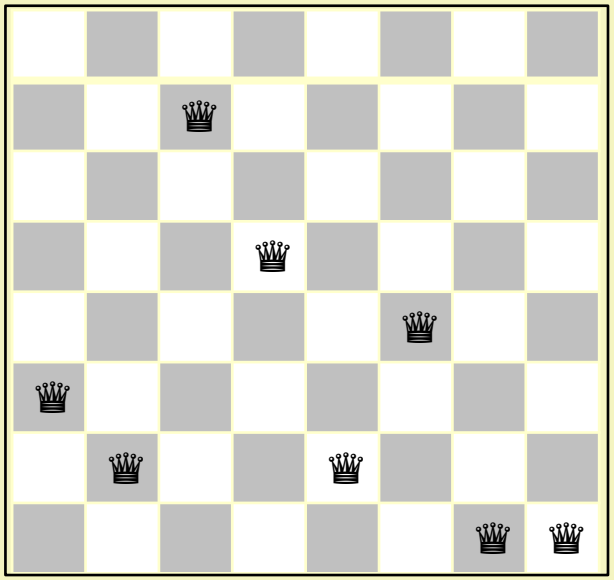
offspring:= recombination(parents)

offspring := mutation(offspring)

population := replacement(population , offspring)

endloop

n-queens problem 1

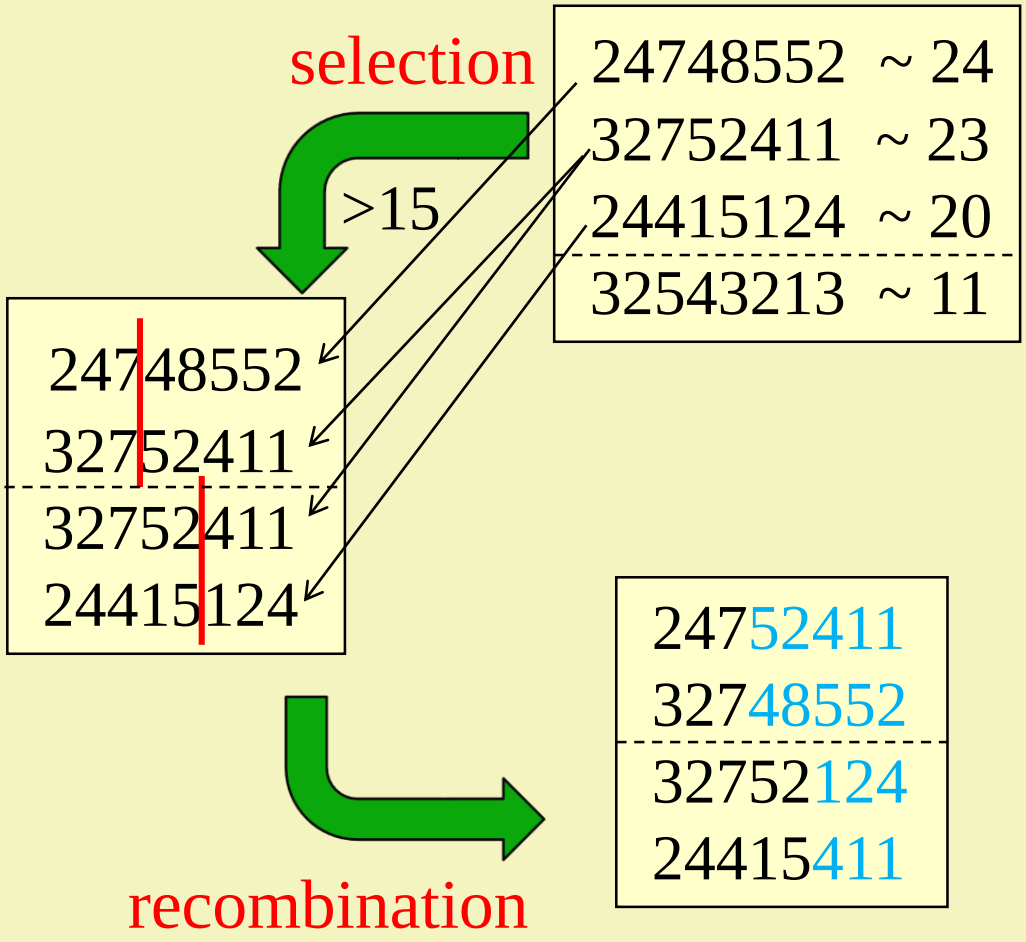


3	2	7	5	2	4	1	1

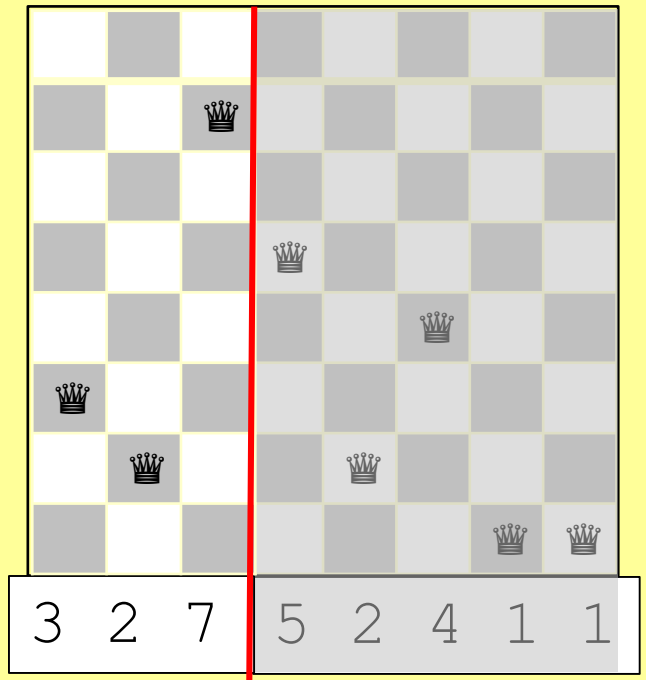
fitness value: 23

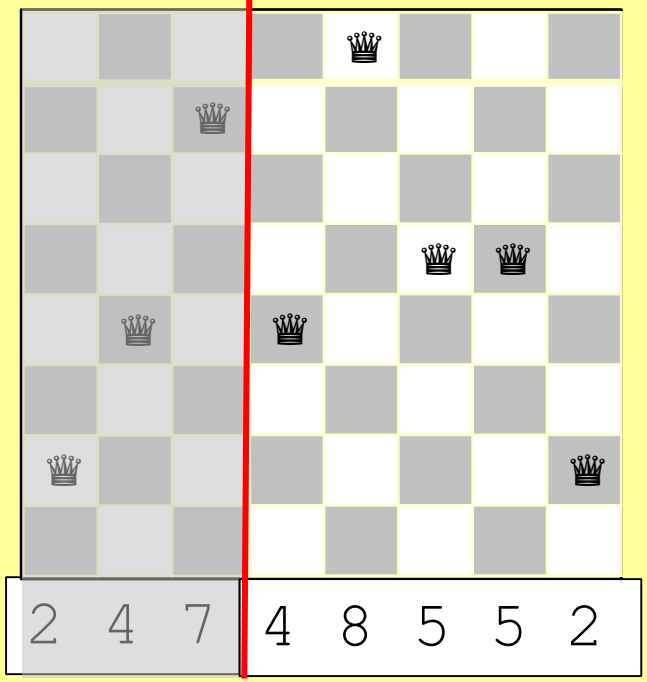
- Individual: a possible placement of the queens where each column contains exactly one queen
- Representation: the sequence of the row positions of the queens
- Fitness function: number of nonattacking pairs of queens

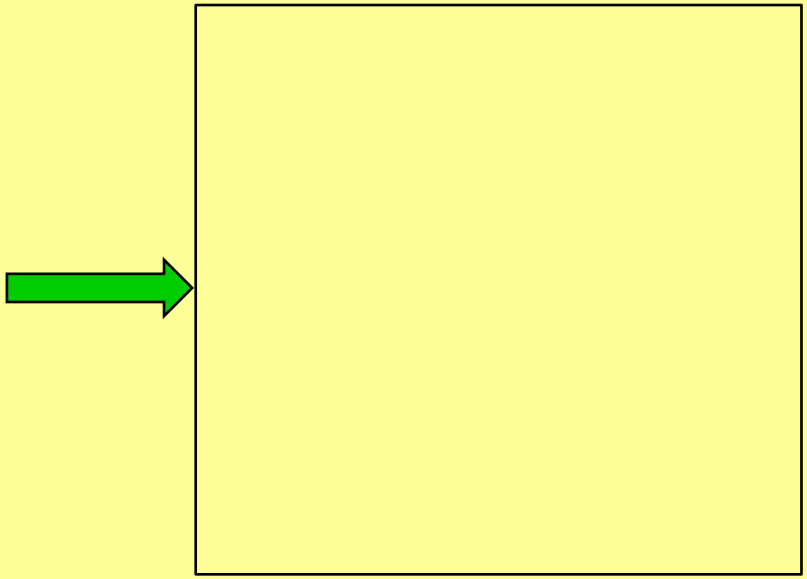
Evolutionary cycle



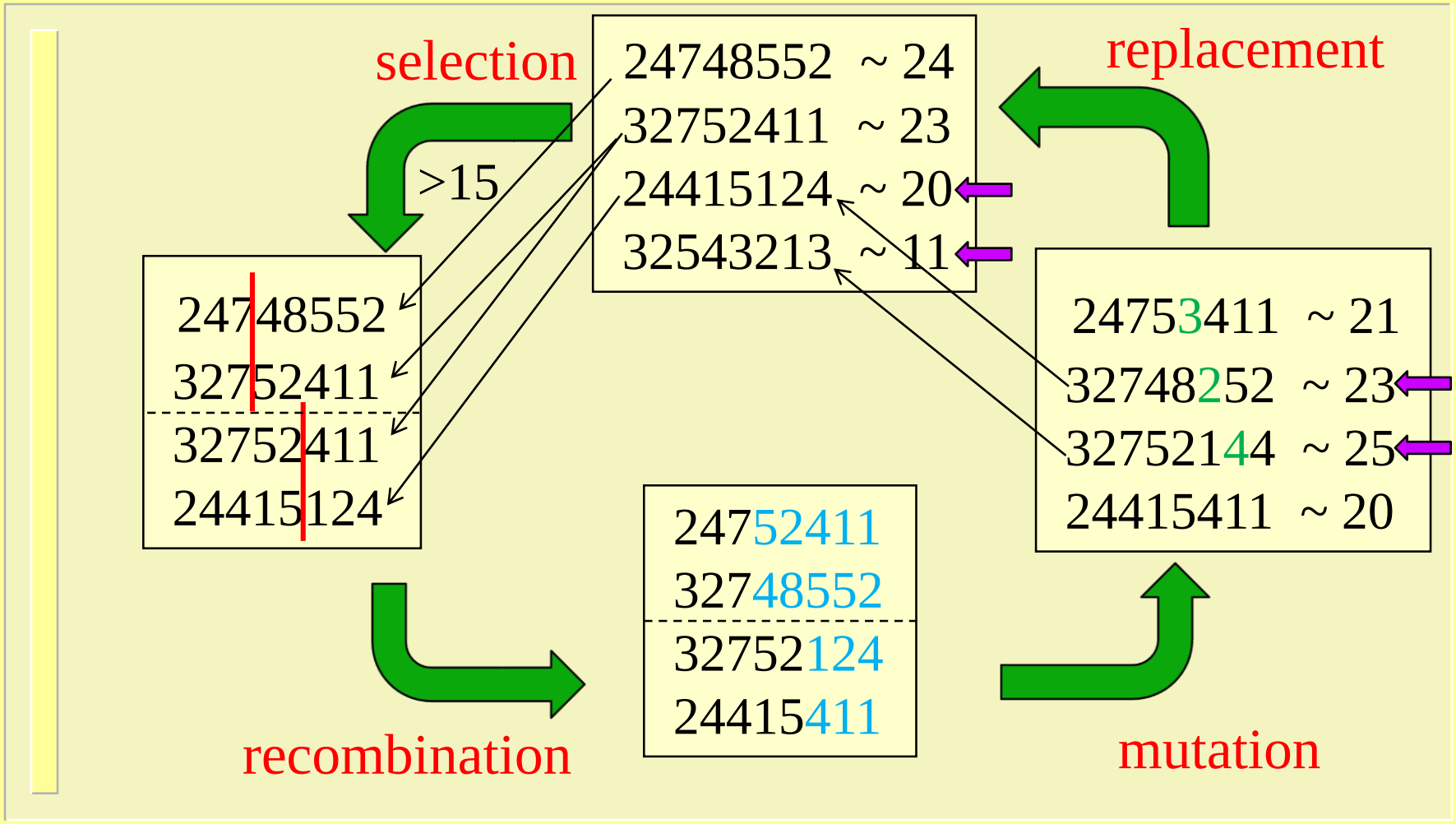
Crossover



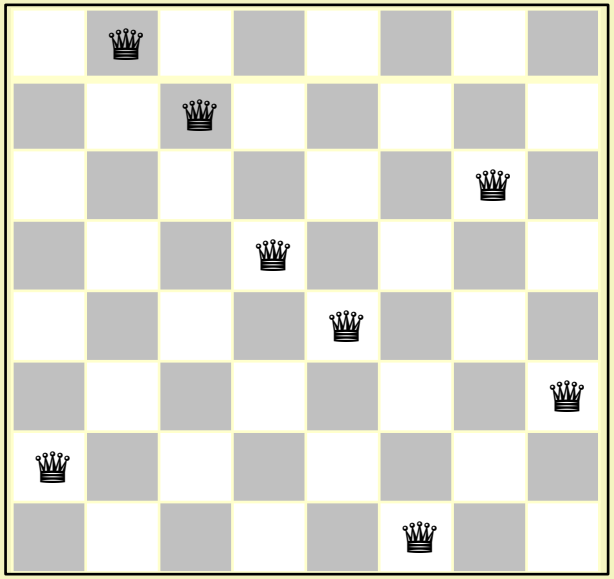




Evolutionary cycle



n-queens problem 2

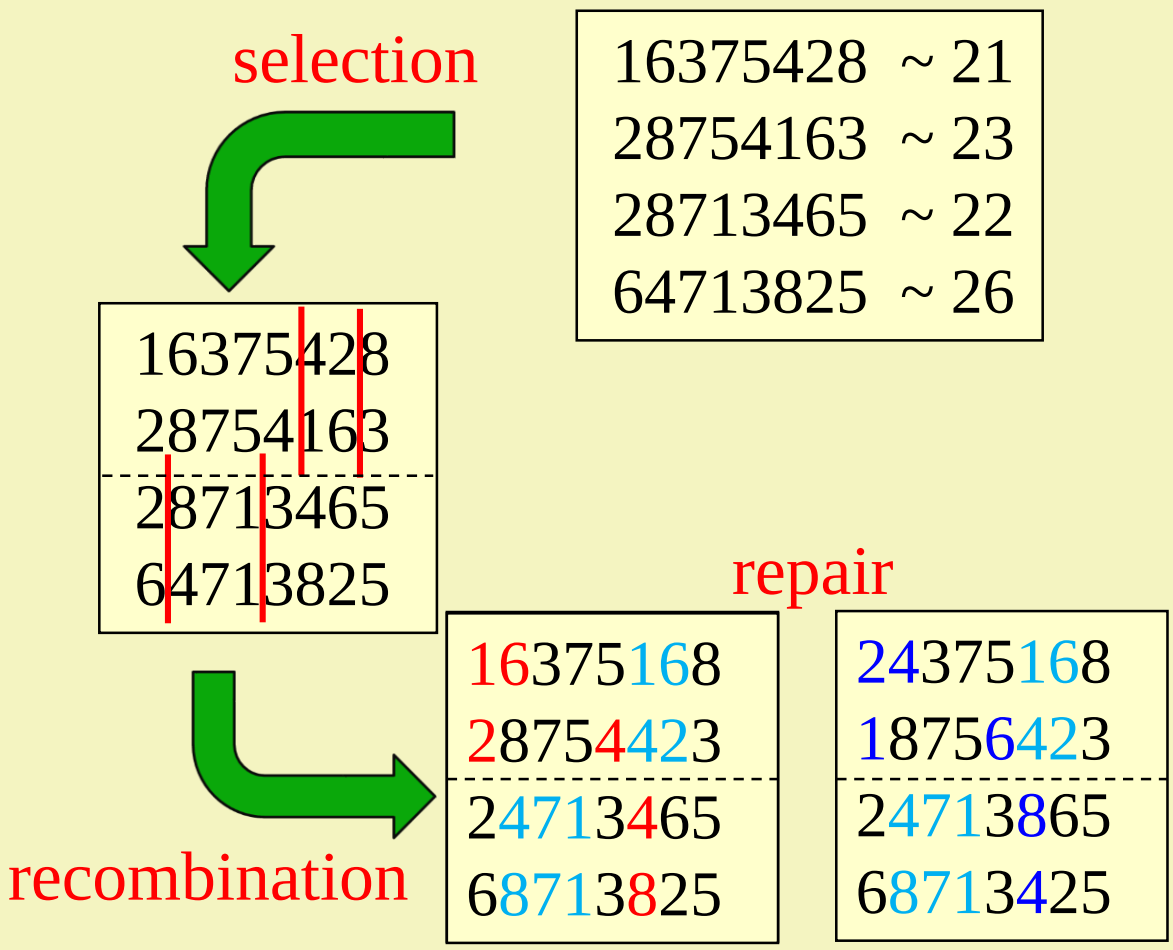


2	8	7	5	4	1	6	3

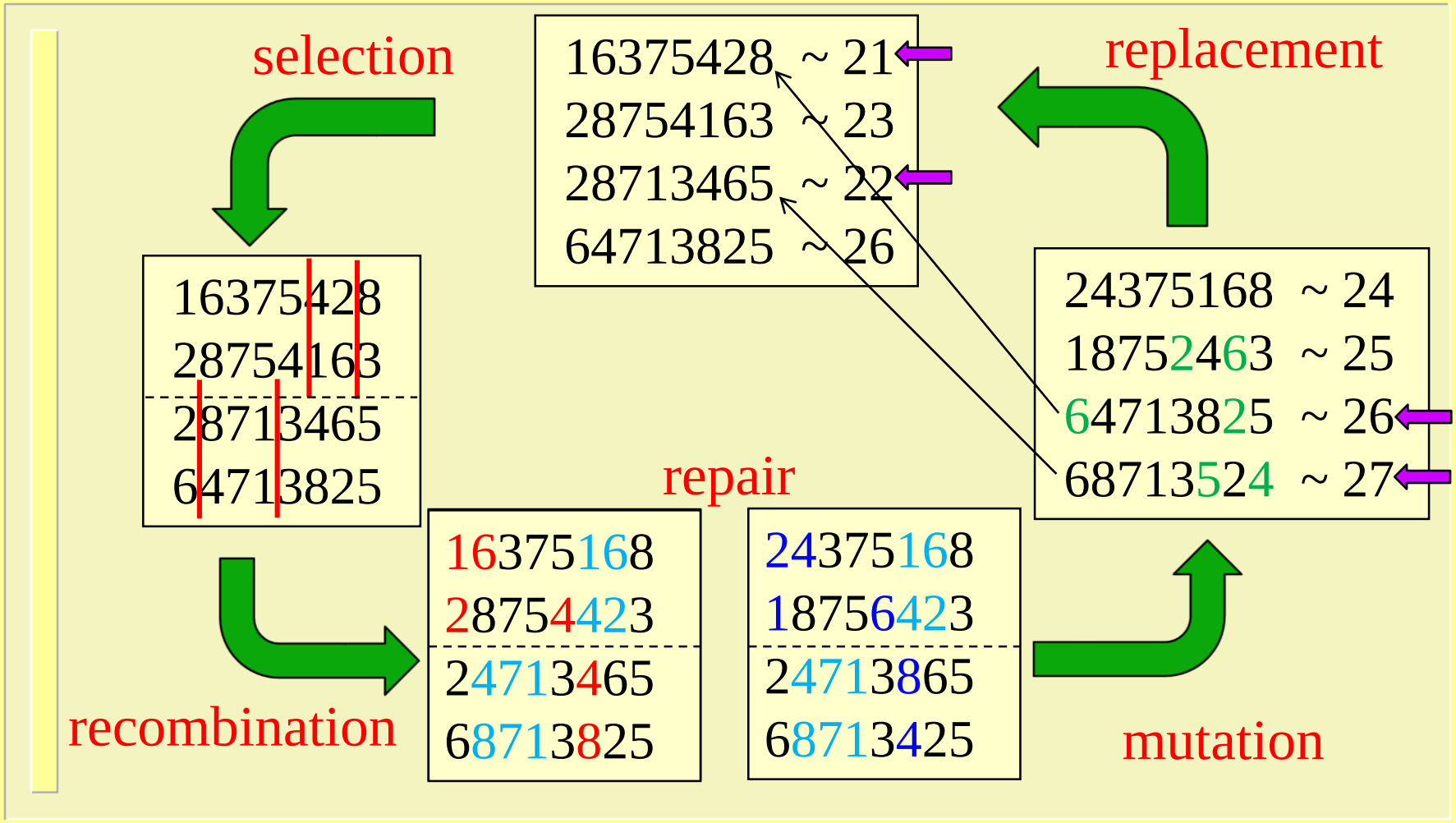
fitness value: 23

- Individual: a possible placement of the queens where each column and each row contains exactly one queen
- Representation: permutation of the row positions: permutation invariant
- Fitness function: number of nonattacking pairs of queens

Evolutionary cycle



Evolutionary cycle



Satisfiability problem (SAT)

There is given a Boolean statement in CNF with n Boolean variables. Find a truth assignment for the variables so that the formula be true.

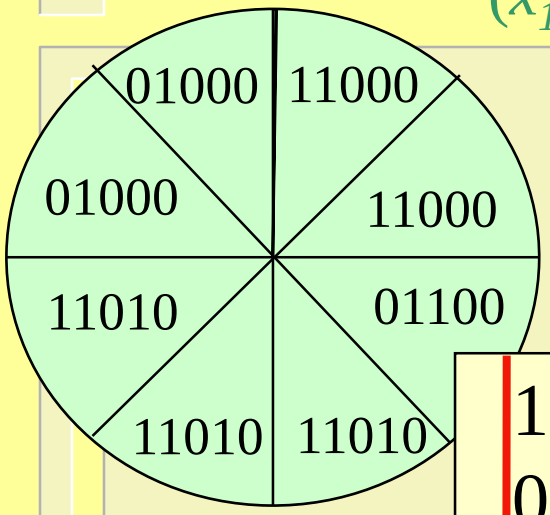
E.g.: $(x_1 \vee \neg x_2 \vee x_5) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1 \vee x_4) \wedge (\neg x_2 \vee x_5)$

one solution: $x_1 = false, x_2 = true, x_3 = false, x_4 = true, x_5 = true$

- Individual : a possible truth assignment
- Representation: a sequence of bits
- Fitness function: the number of the clauses of the formula that has got true value in the given truth assignment

Evolutionary cycle

$(x_1 \vee \neg x_2 \vee x_5) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1 \vee x_4) \wedge (\neg x_2 \vee x_5)$



roulette wheel

selection

01000 ~ 2
11010 ~ 3
01100 ~ 1
11000 ~ 2

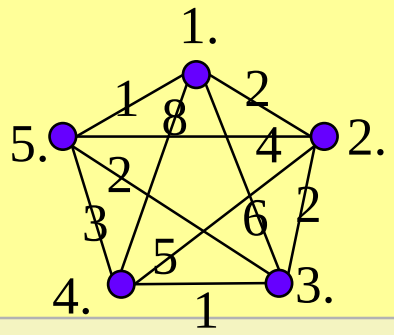
11010
01000
11010
11000

recombination

01010
11000
11010
11000

mutation

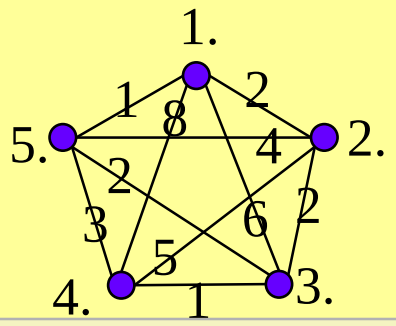
01011 ~ 4
11110 ~ 3
11010 ~ 3
11001 ~ 3



Traveling salesman problem

The traveling salesman must visit every city in his territory exactly once and then return to home city covering the shortest distance. (All distances between cities are known)

- Individual : one tour
- Representation: permutation of the cities (numbers) without the home city
- Fitness function: cost of the tour



Evolutionary cycle

selection

4352 ~ 17
3542 ~ 19
2543 ~ 16
5234 ~ 17

replacement

3452 ~ 16
5342 ~ 11
5234 ~ 16
2534 ~ 17

4352
3542
2543
5234

repair

4342
3552
5543
2234

3342
4552
5243
2534

3542
4352

recombination

mutation

Elements of the evolutionary algorithm

- Model: individuals and their representation
- Fitness function (heuristics)
- Evolutionary operators
  - selection, recombination, mutation, replacement
- Initial population, termination condition (goal)
- Settings of the strategy parameters
  - size of the population, probability of mutation, rate of the offspring, rate of the replacement

Representation

- An individual is represented by a code (chromosome) that is most commonly a sequence of signals.
- One signal or one section of signals with its position in the code (gene) can describe one property of the individual.
  - Thus the structure of the code can be dismembered property by property. If one signal is changed, then one property of the individual is changed a bit, too.
- Frequently coding methods:
  - Array: fix length sequence of real numbers or integers
  - Binary code: fix length sequence of bits
  - Permutation of the elements of a finite set

Representations and fitness functions of graph coloring problem

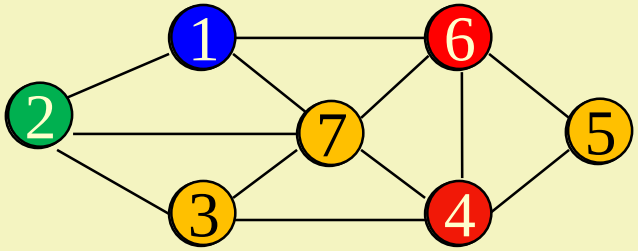
Color the vertices of a simple finite undirected graph with four colors so that no two adjacent vertices share the same color?

Direct encoding

1. 2. 3. 4. 5. 6. 7.

$x[i]$ is the color of the i^{th} vertex.

f is the number of the correct edges.



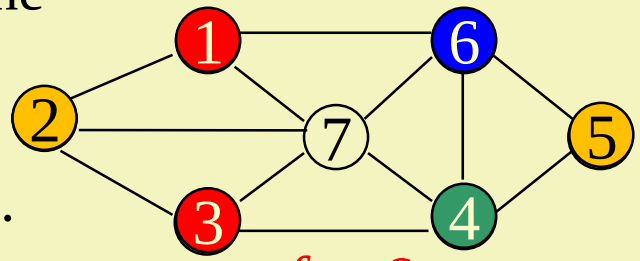
f = 10

Indirect encoding

2	5	1	3	4	6	7

In the i^{th} step of the coloring process the $x[i]^{th}$ vertex must be colored with the possible lightest color regarding the colors of neighbor vertices, if possible.

f is the number of the colored vertices.



f = 6

Representation and fitness functions of problem of rock-paper-scissors

Find a good strategy for the rock-paper-scissors world championship!

- A strategy should give our next step based on a few (say two) previous combats.
  - For example:

    Antecedent: *Me*: P R *Suggestion*: S
    Op: S P
  - This is only one slice of the strategy. We need a suggestion for each possible antecedent.

Representation

There are 3^{81} different strategies (individuals) and their codes are the elements of $\{0,1,2\}^{0..80}$

Signals

```
R ~ 0
P ~ 1
S ~ 2
```

Antecedents (MeOpMeOp)

```
RRRR ~ 0000 ~ 0
RRRP ~ 0001 ~ 1
RRRS ~ 0002 ~ 2
RRPR ~ 0010 ~ 3
…
SSSP ~ 2221 ~ 79
SSSS ~ 2222 ~ 80
```

Suggestions

```
P ~ 1
S ~ 2
R ~ 0
P ~ 1
…
S ~ 2
R ~ 0
```

Code of the strategy: `1201 … 20`

Fitness function

Strategy: 1201 ... 20

Specimen battle:

Player: 0002221222001000

Op: 0102011222010111

Signal

R ~ 0

P ~ 1

S ~ 2

Case	→	*Suggestion*	*Opponent*	*Value*	
0001	→	2	0	defeat	-1
0001	→	2	1	victory	+1
0100	→	1	1	draw	0
	...				
2221	→	2	1	victory	+1
2222	→	0	0	draw	0

Selection

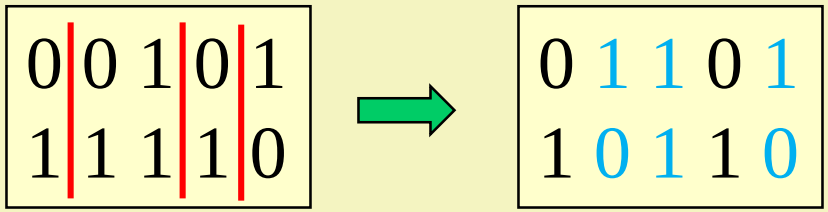
- The better individuals must be selected but the worse ones must be given a chance to choose. (stochastic method)
  - **Roulette wheel** : the probability of selection of an individual is proportional to its fitness function value
  - **Ranking**: the probability of selection of an individual is proportional to its ranking based on the fitness function values
  - **Tournament**: the selected individuals are the best individuals of randomly selected groups of the population
  - **Culling**: all individuals below a given threshold are discarded and then the individuals are selected randomly from the remaining individuals

Recombination

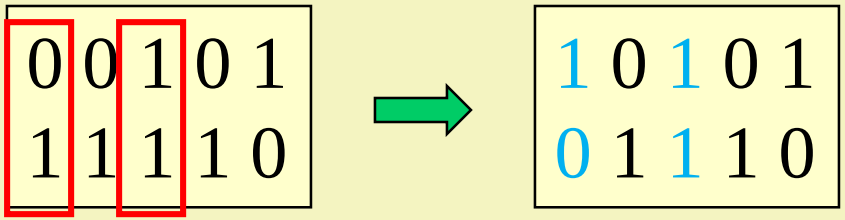
- Offspring must be created from the pairs of parents so that the offspring inherit the properties of their parents.
  - **Crossover**: signals of the parent codes are exchanged at the positions chosen randomly
  - **Recombination**: the corresponding signals of the parent codes are combined
- If the code has got some extra invariant condition (permutation), it must be preserved.

Crossover

- **Single or multiple pointed crossover**
  - Some crossover points are chosen at random and the signals are exchanged in every second section.

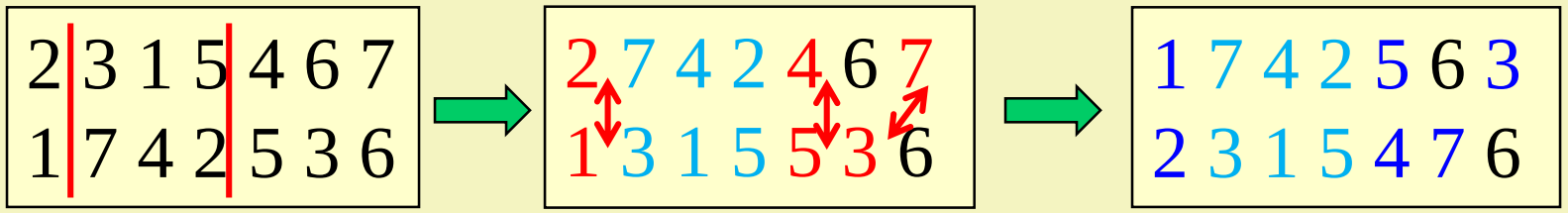


- **Uniform crossover**
  - Some positions are chosen at random and only the signals on these positions are exchanged.



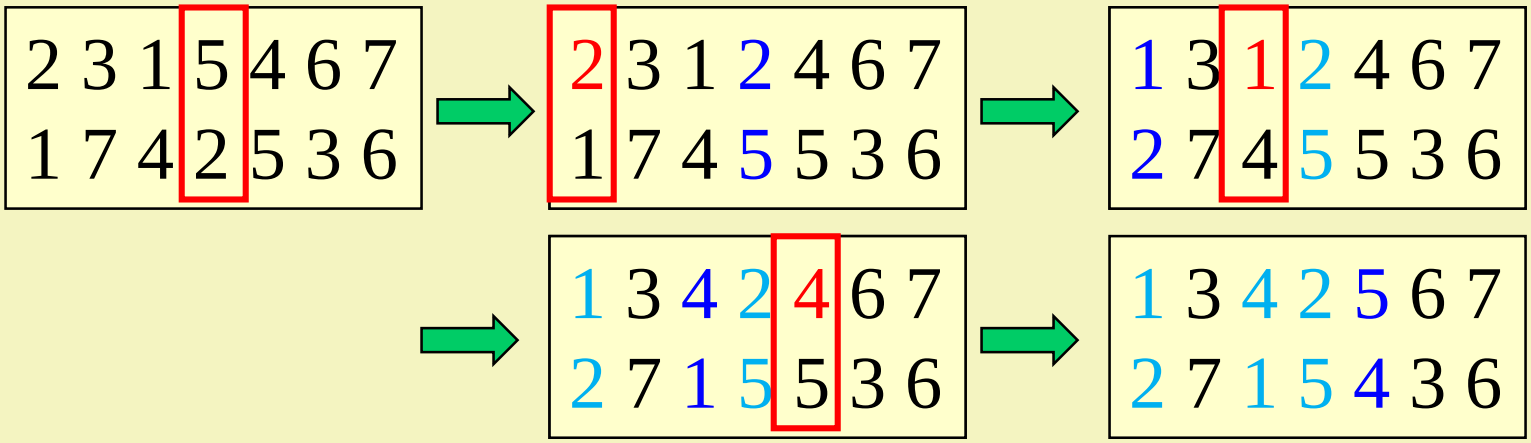
Crossover for permutations 1

- **Partial crossover**
  - Signals are exchanged in one section chosen at random and then the duplicated signals of the first and the second parent besides the crossover section are coupled and exchanged.

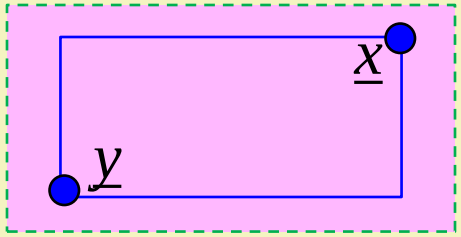


Crossover for permutations 2

- **Cycle crossover**
  1. Select $i \in [1..length]$ at random
  2. Change $x_i \leftrightarrow y_i$
  3. Find $j \in [1..length]$ $(j \neq i)$ where $x_j = x_i$,
  4. If it is not found then over else $i := j$
  5. goto 2.



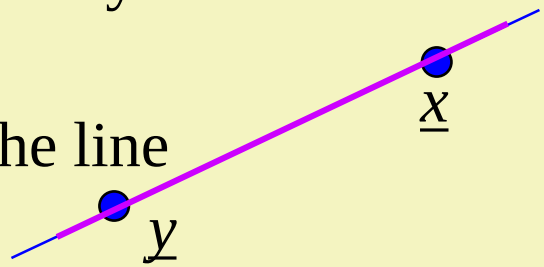
Recombinations of real vectors



- **General recombination**
  - The offspring ($\underline{u}$) is somewhere around or in the hyper cube spanned by the parents ($\underline{x}$, $\underline{y}$)
  - $\forall i \in 1 \ldots n : \; u_i = c_i \cdot x_i + (1 - c_i) \cdot y_i$

    where $c_i \in [-h, 1+h]$ selected randomly
- **Linear recombination**
  - The offspring ($\underline{u}$) is somewhere on the line matching the parents ($\underline{x}$, $\underline{y}$)
  - $\forall i \in 1 \ldots n : \; u_i = c \cdot x_i + (1 - c) \cdot y_i$

    where $c \in [-h, 1+h]$ selected randomly



Mutation

- Each position of the code is subject to random change with a small independent probability (p).
- When the code is an array including real numbers:
  - $\forall i=1...n : z_i = x_i \pm domain_i \cdot p$
- When the code is a binary code:
  - $\forall i=1...n : z_i = 1 - x_i$ if $random[0..1]<p$
- When the code is a permutation:
  - swapping signal-pairs
  - shifting the signals in a section cyclically
  - reversing or reordering the signals in a section

Replacement

- Creation of the new population from the elder one and the mutated offspring: the selected individuals of the population are substituted for the selected offspring.

two selections are needed

$$\text{offspring rate}(u) = \frac{\text{number of the offspring to be put in}}{\text{size of the population}}$$

$$\text{replacement rate}(v) = \frac{\text{number of the individuals to be removed}}{\text{size of the population}}$$

  - u=v : total exchange

additional selection is needed

  - u<v : some selected offspring must be used several times

additional selection is needed

  - u>v : some selected offspring must be eliminated