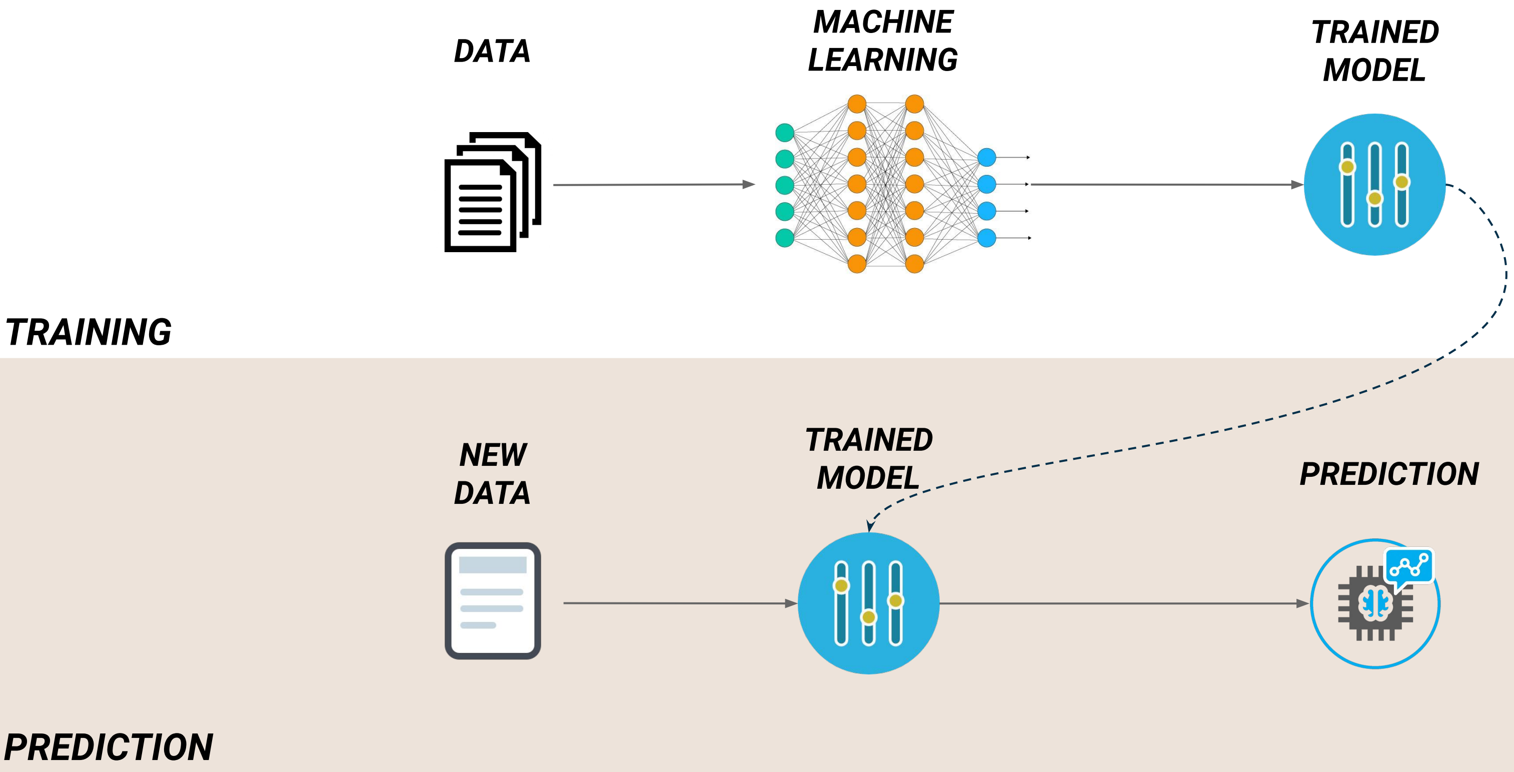
ECE ING4
MACHINE LEARNING

Jeremy Cohen

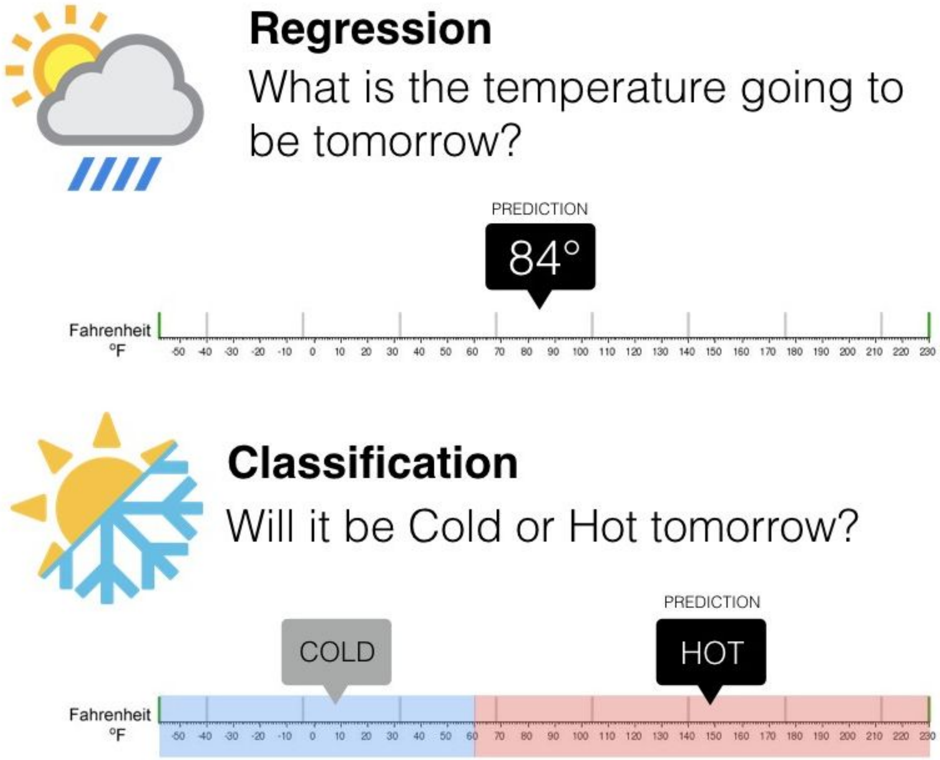
Logistic Regression

Week 2 Review

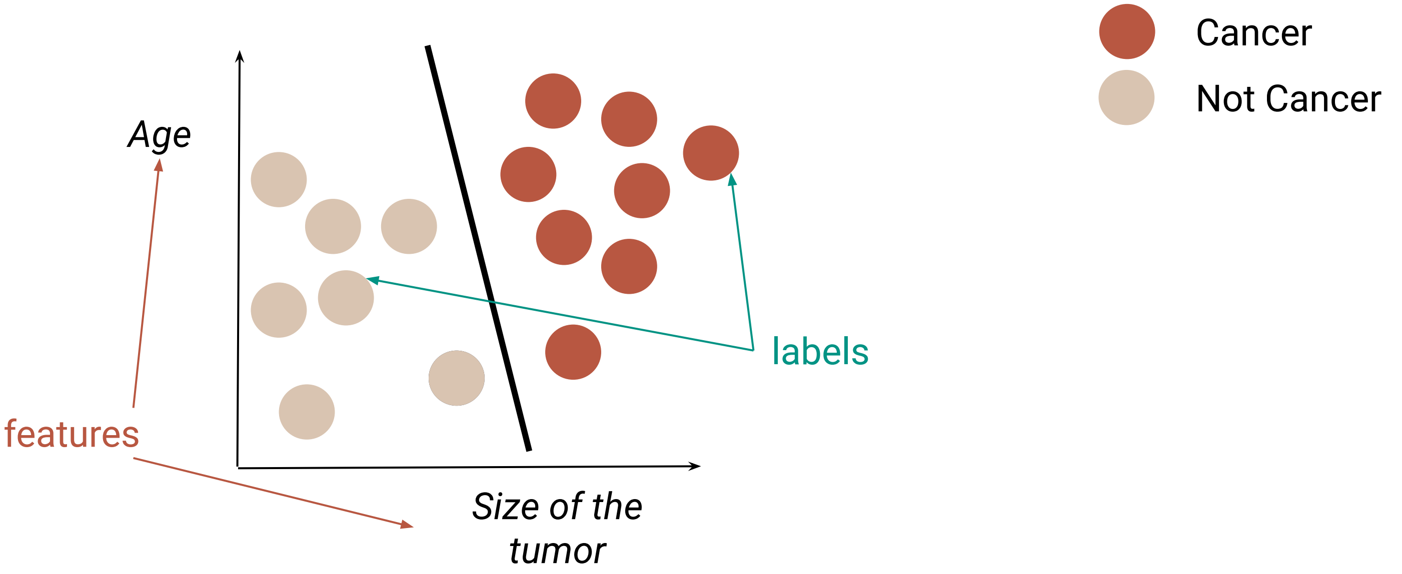
Machine Learning Process



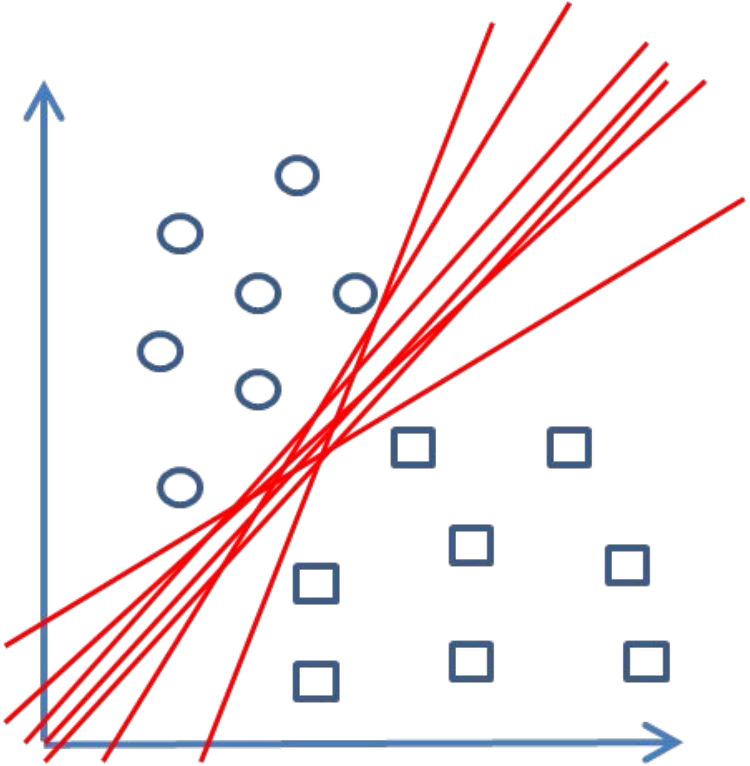
Classification vs Regression



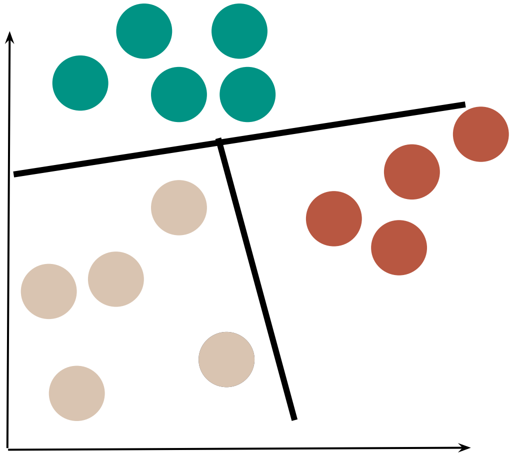
Classification



Classification



Multi-Class Classification



3 Datasets

Training Set

~70% of the dataset

Used to **train the model**

Validation Set

~20% of the dataset

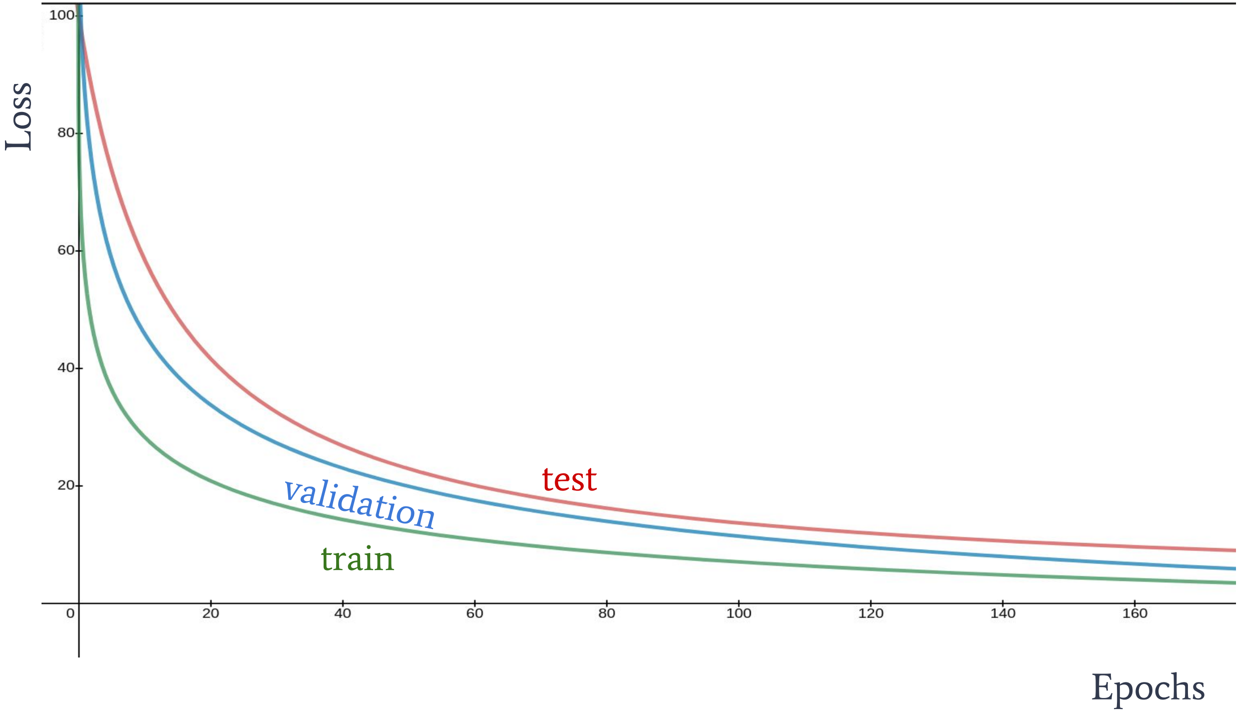
Used to **test** the model and **generalize better to new data**

Test Set

~10% of the dataset

Used **only once** when both accuracies are good and ready for real-world

3 Datasets


Loss
100
80
60
40
20
0
20
40
60
80
100
120
140
160
test
validation
train
Epochs

Linear Regression formula

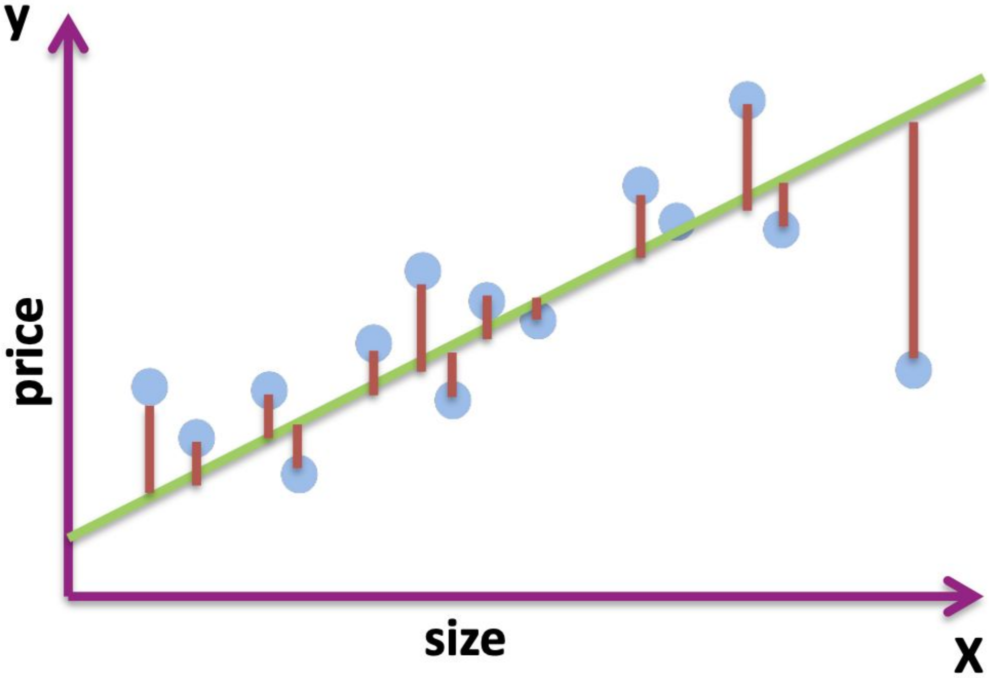
$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$ $(x_0^{(i)} = 1)$

$$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

$$h_\theta(x) = \sum_{i=1}^{n} \theta_i x_i = \theta^T x$$

Mean Squared Error

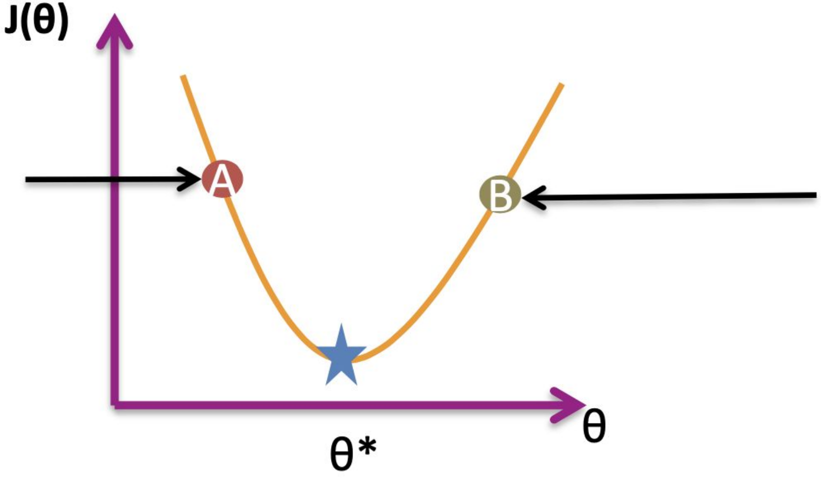


$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

Gradient Descent Recap

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

In this case, the derivative (gradient) $\partial J(\theta) / \partial\theta < 0$.
$\theta^A - \alpha * \partial J(\theta) / \partial\theta > \theta^A$
θ^A is moving to the right
θ is increasing



In this case, the derivative (gradient) $\partial J(\theta) / \partial\theta > 0$.
$\theta^B - \alpha * \partial J(\theta) / \partial\theta < \theta^B$
θ^B is moving to the left
θ is decreasing

Gradient Descent Recap

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \dots, n$)

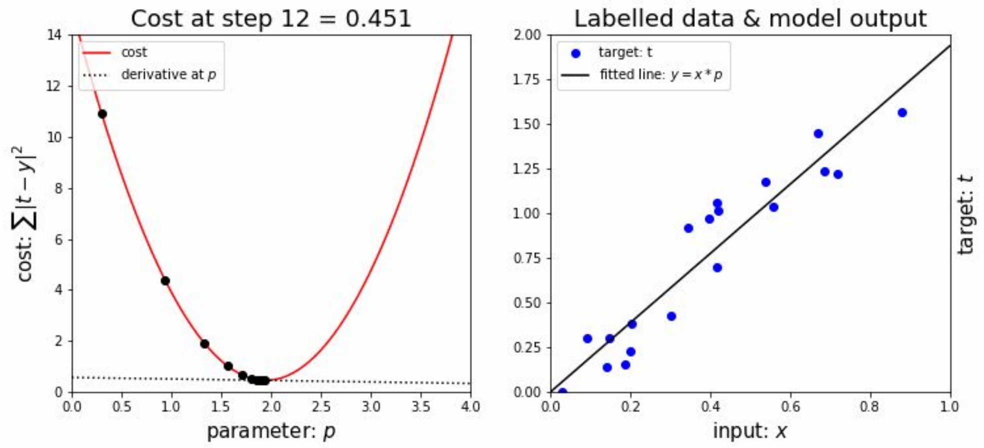
}

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

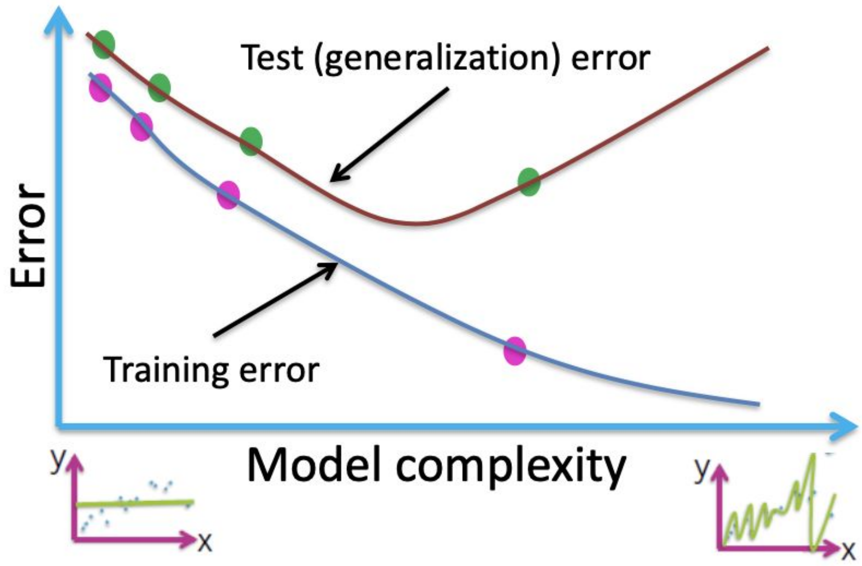
$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

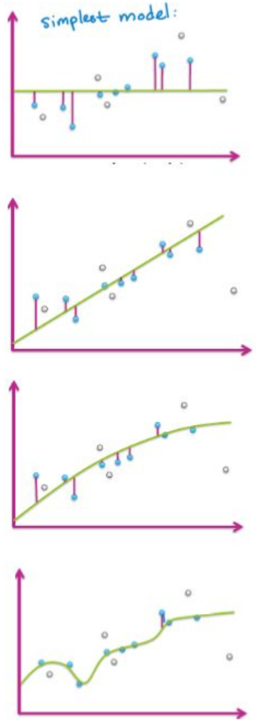
$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$\dots$



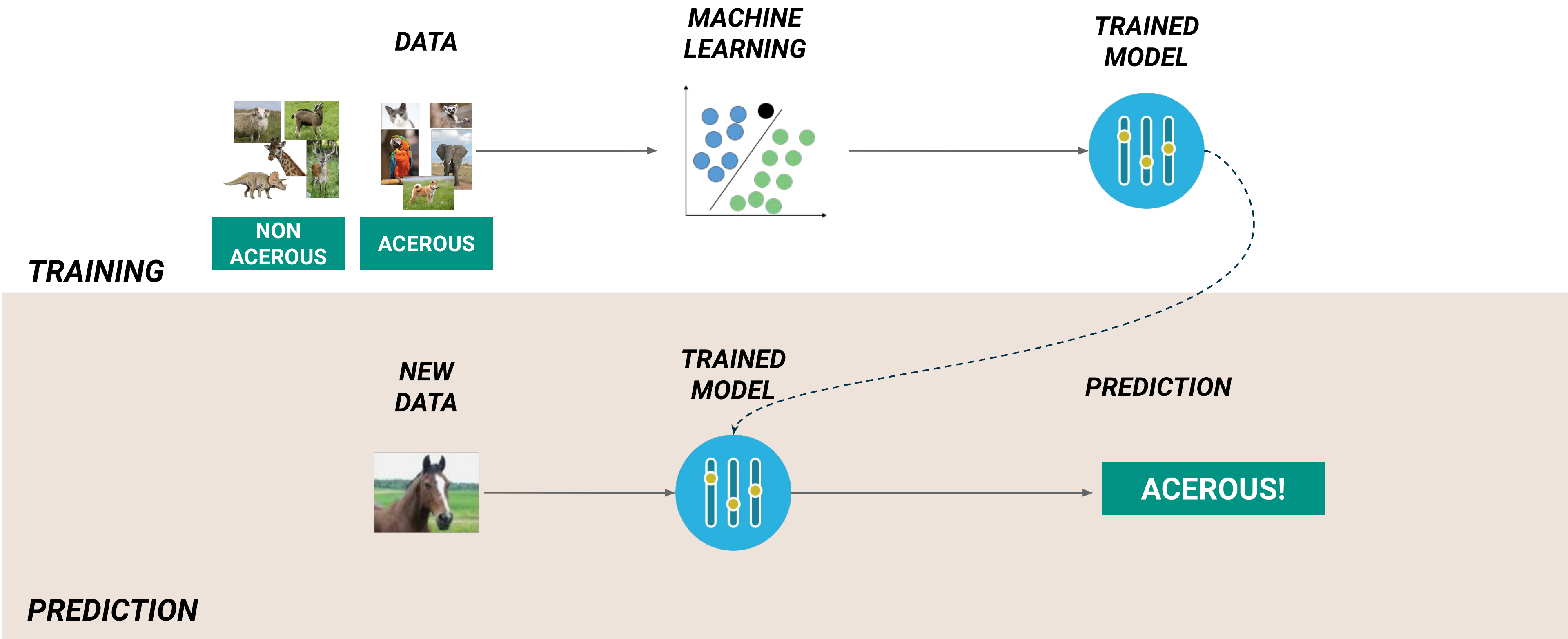
Performance





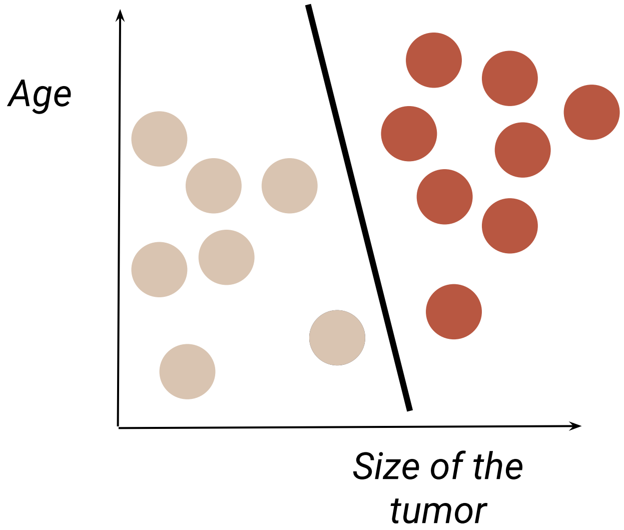
Classification

Classification Example



Binary vs Multi-Class Classification

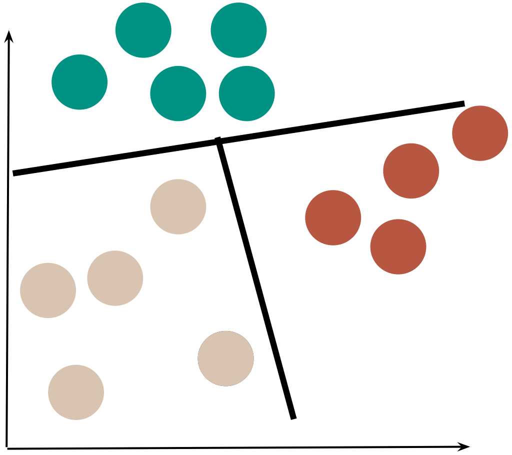
Output has 2 categories



Cancer

Not Cancer

Output has more than categories

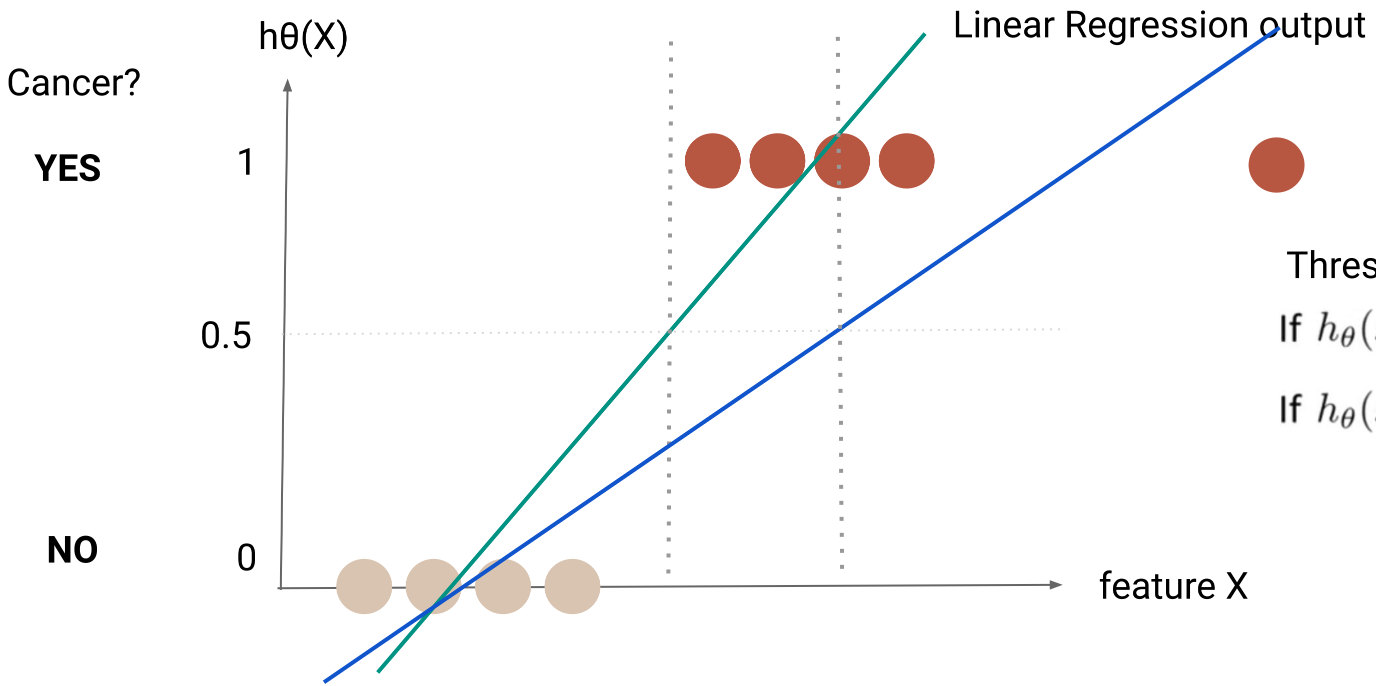


Benign

Malignant

Not Cancer

Can Linear Regression help?



$$h_\theta(x) = \sum_{i=1}^{n} \theta_i x_i = \theta^T x$$

Threshold the classifier at 0.5

If $h_\theta(x) \geq 0.5$, predict "y = 1"

If $h_\theta(x) < 0.5$, predict "y = 0"

Linear Regression vs Classification

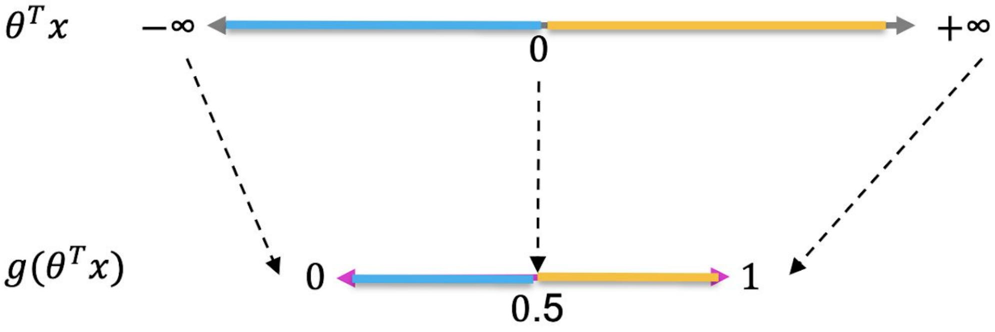
Linear Regression output : $- \infty < h_{\theta}(x) < + \infty$

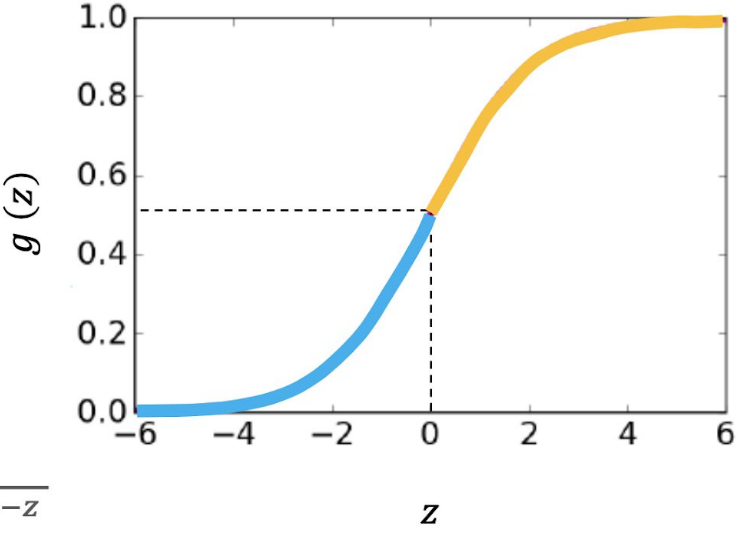
Classification output: $y = 0$ or 1

Logistic Regression output : $0 \leqq h_{\theta}(x) \leqq 1$

Linear Regression can’t help

We want $0 \leq h_\theta(x) \leq 1$





- Use the Sigmoid / Logistic Function : $g(z) = \frac{1}{1+e^{-z}}$

$$h_\theta(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

Logistic Regression

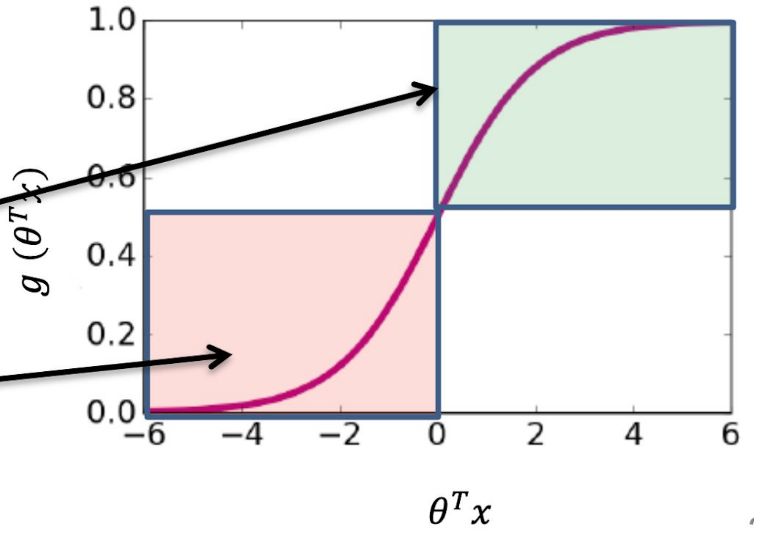
$h_\theta(x)$ = **estimated probability that y = 1** given the input x parameterized by θ

$h_\theta(x) = \mathrm{P}(\mathrm{y} = 1|\, \mathrm{x}; \theta) = 1 - P(\mathrm{y} = 0|\, \mathrm{x}; \theta)$

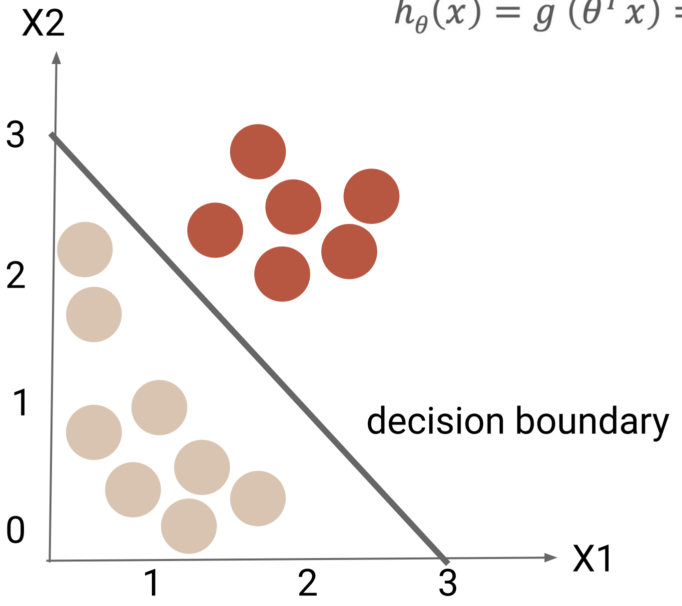
$\mathrm{P}(\mathrm{y} = 1|\, \mathrm{x}; \theta) + P(\mathrm{y} = 0|\, \mathrm{x}; \theta) = 1$

Hypothesis:

- ❑ $y = 1\ when\ g(\theta^T x) \geq 0{,}5 \rightarrow \theta^T x \geq 0$
- ❑ $y = 0\ when\ g(\theta^T x) < 0{,}5 \rightarrow \theta^T x < 0$



Logistic Regression



$$h_\theta(x) = g\left(\theta^T x\right) = \frac{1}{1 + e^{-\theta^T x}}$$

If $h_\theta(x) \geq 0.5$, predict "y = 1" $\quad \theta^T x \geq 0$

If $h_\theta(x) < 0.5$, predict "y = 0" $\quad \theta^T x < 0$

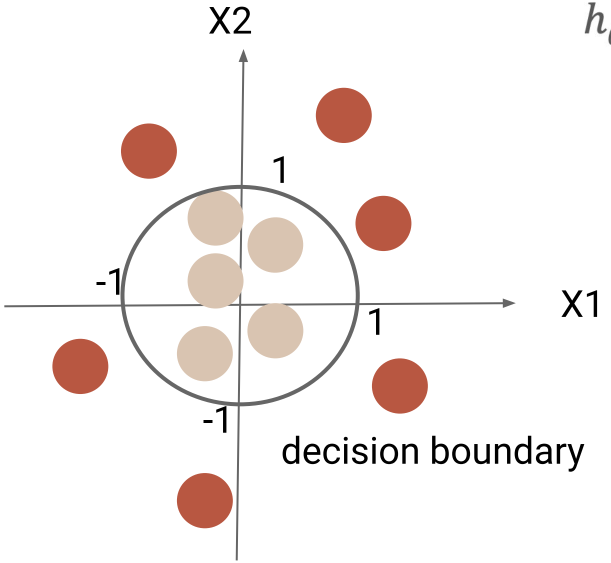
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

$$\theta = \begin{matrix} -3 \\ 1 \\ 1 \end{matrix}$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

x1 + x2 ≧ 3

Non-Linearities



$$h_\theta(x) = g\left(\theta^T x\right) = \frac{1}{1 + e^{-\theta^T x}}$$

Example: $h_\theta(x) = g\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 {x_1}^2 + \theta_4 {x_2}^2\right)$

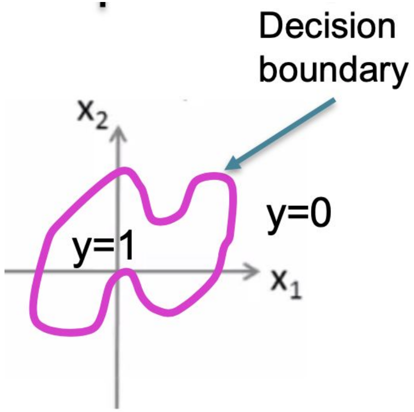
- $\theta = [-1, 0, 0, 1, 1]$
- Predict 1 if $-1 + {x_1}^2 + {x_2}^2 \geq 0 \Rightarrow {x_1}^2 + {x_2}^2 \geq 1$
- Predict 0 if $-1 + {x_1}^2 + {x_2}^2 < 0 \Rightarrow {x_1}^2 + {x_2}^2 < 1$

We can work with non-linearly separable data

Non-Linearities

As with polynomial regression, we can have more complex decision boundaries by adding higher polynomial terms

$$h_\theta(x) = g\left(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^{\,2} + \theta_4 x_2^{\,2} + \theta_5 x_1^{\,3} + \theta_6 x_2^{\,3}\right)$$



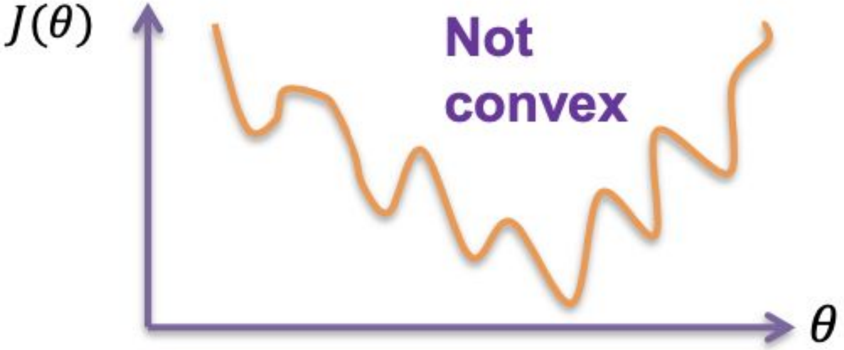
How to choose θ ?

Cost Function

Cost Function for Linear Regression

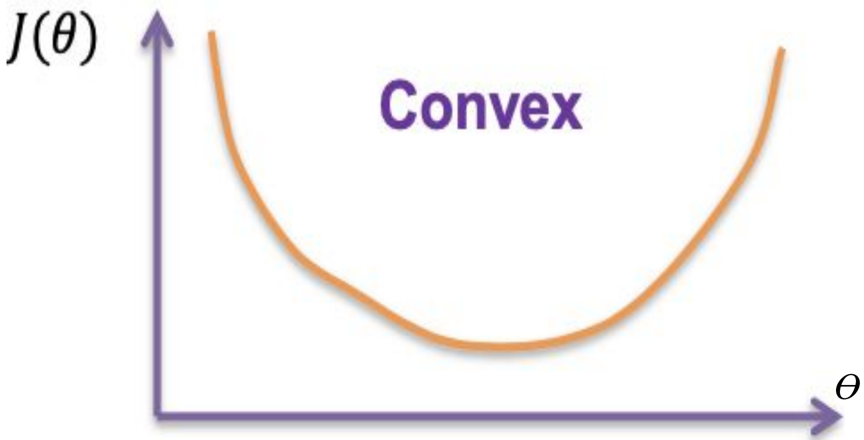
$$J(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$\boxed{\frac{1}{1 + e^{-\theta^T x}}}$$

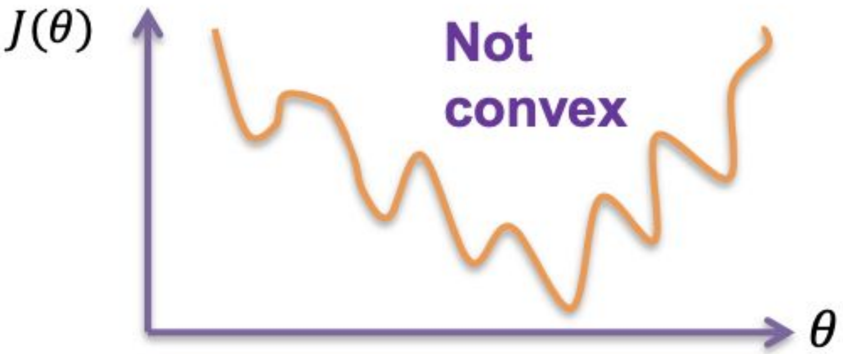


Cost Function for Linear Regression

Possible to run Gradient Descent



Impossible to run Gradient Descent



Cost Function for Logistic Regression

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Cost Function for Logistic Regression

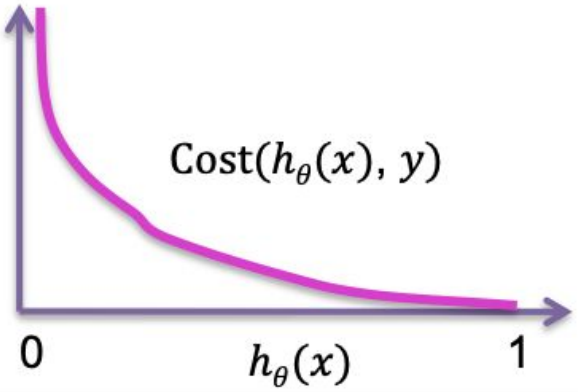
$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

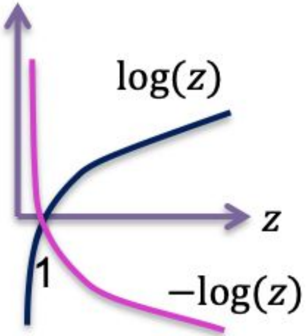
$$\mathrm{Cost} = 0 \text{ if } y = 1, h_\theta(x) = 1$$

But as $h_\theta(x) \rightarrow 0$

$$Cost \rightarrow \infty$$

y=1





Cost Function for Logistic Regression

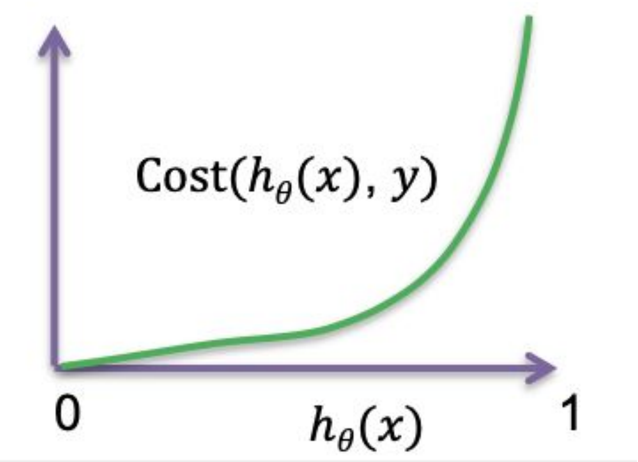
$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

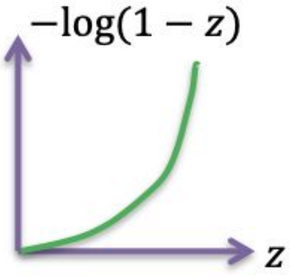
$\text{Cost} = 0$ if $y = 0, h_\theta(x) = 0$

But as $h_\theta(x) \to 1$

$Cost \to \infty$

y=0





Cost Function for Logistic Regression

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \text{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))]$$

Cost Function for Logistic Regression

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

(simultaneously update all θ_j)

$\}$

Cost Function for Logistic Regression

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

linear regression $h_\theta(x) = \theta^T x$

logistic regression

Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

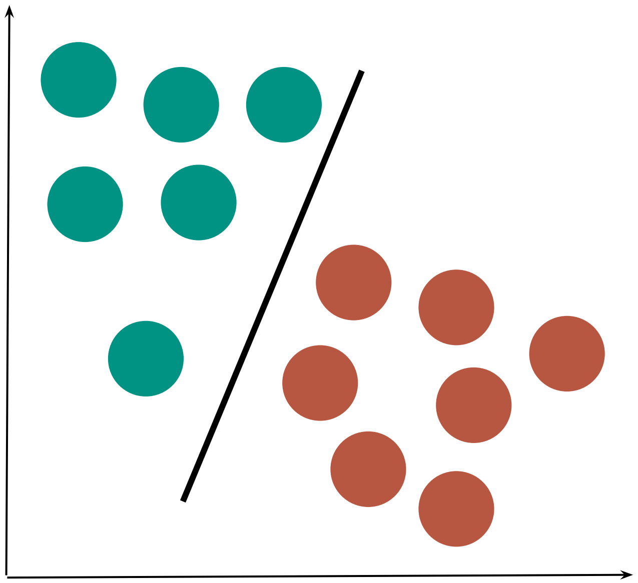
$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

(simultaneously update all θ_j)

}

Support Vector Machine

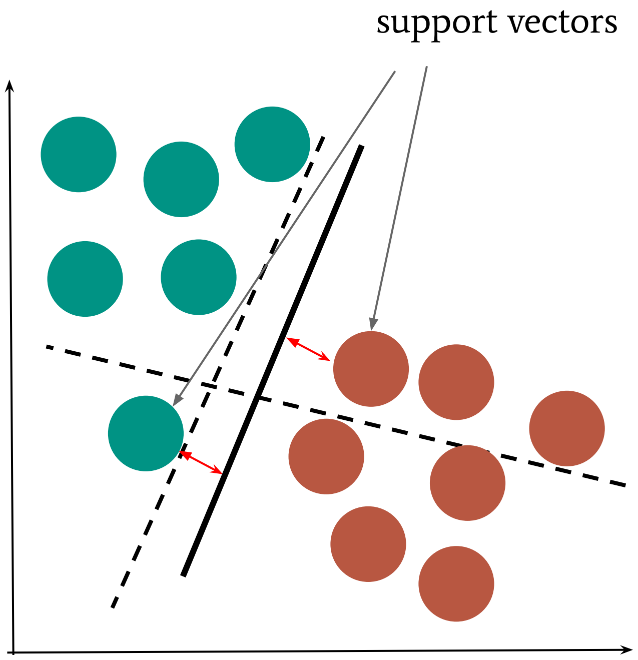
SVM



GOAL

Separate the dataset into classes with as much correctly classified points as possible

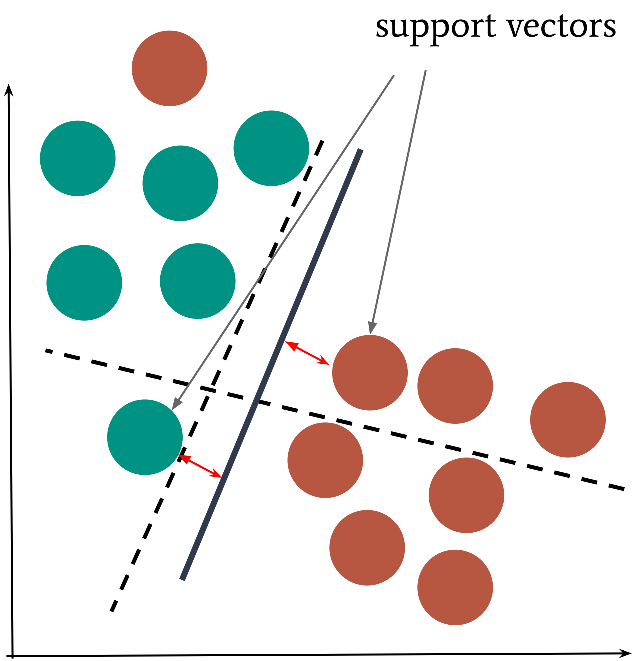
SVM



PROCESS

- Select the line that correctly classifies as many points as possible
- Select the line that maximizes distance with support vectors

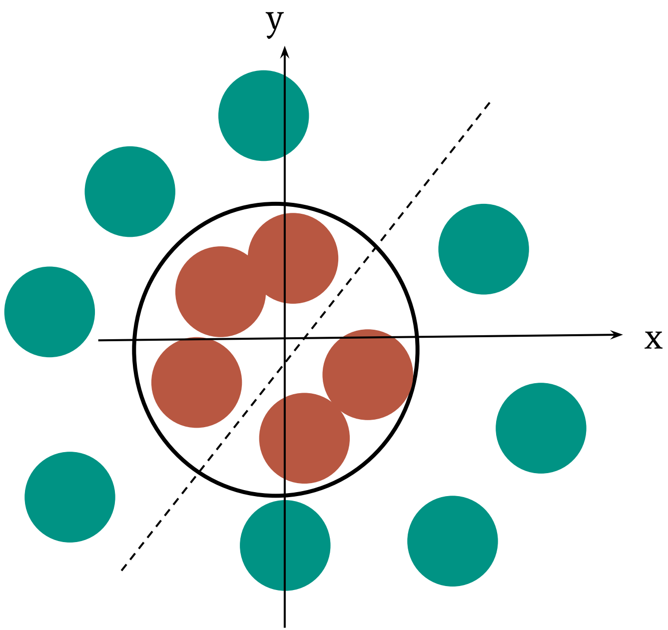
SVM



NOTABLE

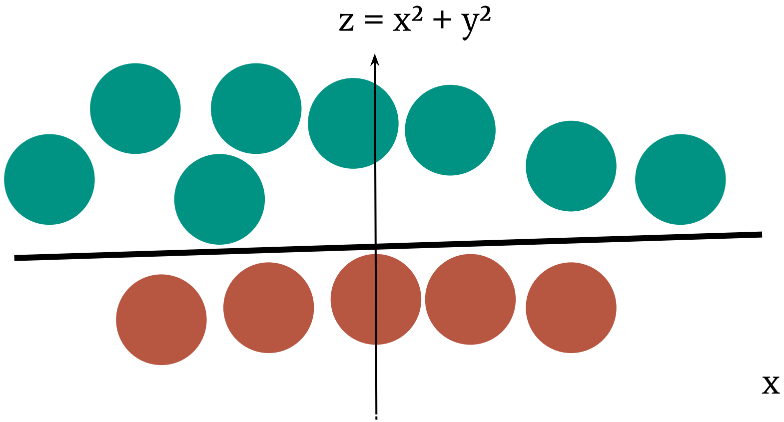
- Robust to outliers/exceptions; line will not be affected

SVM



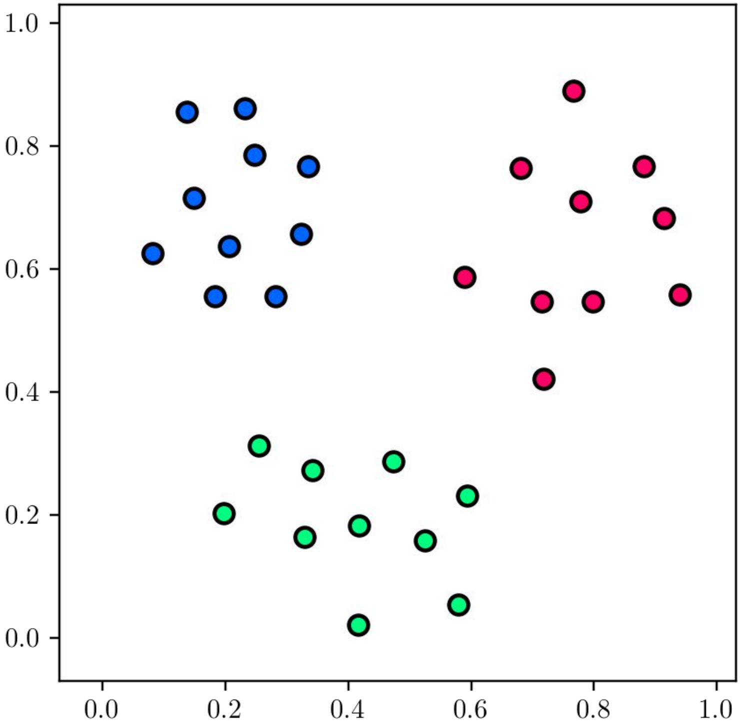
NOTABLE

- If the data is non linearly separable, we can make it



Multiclass Classification

MultiClass Classification: One vs All



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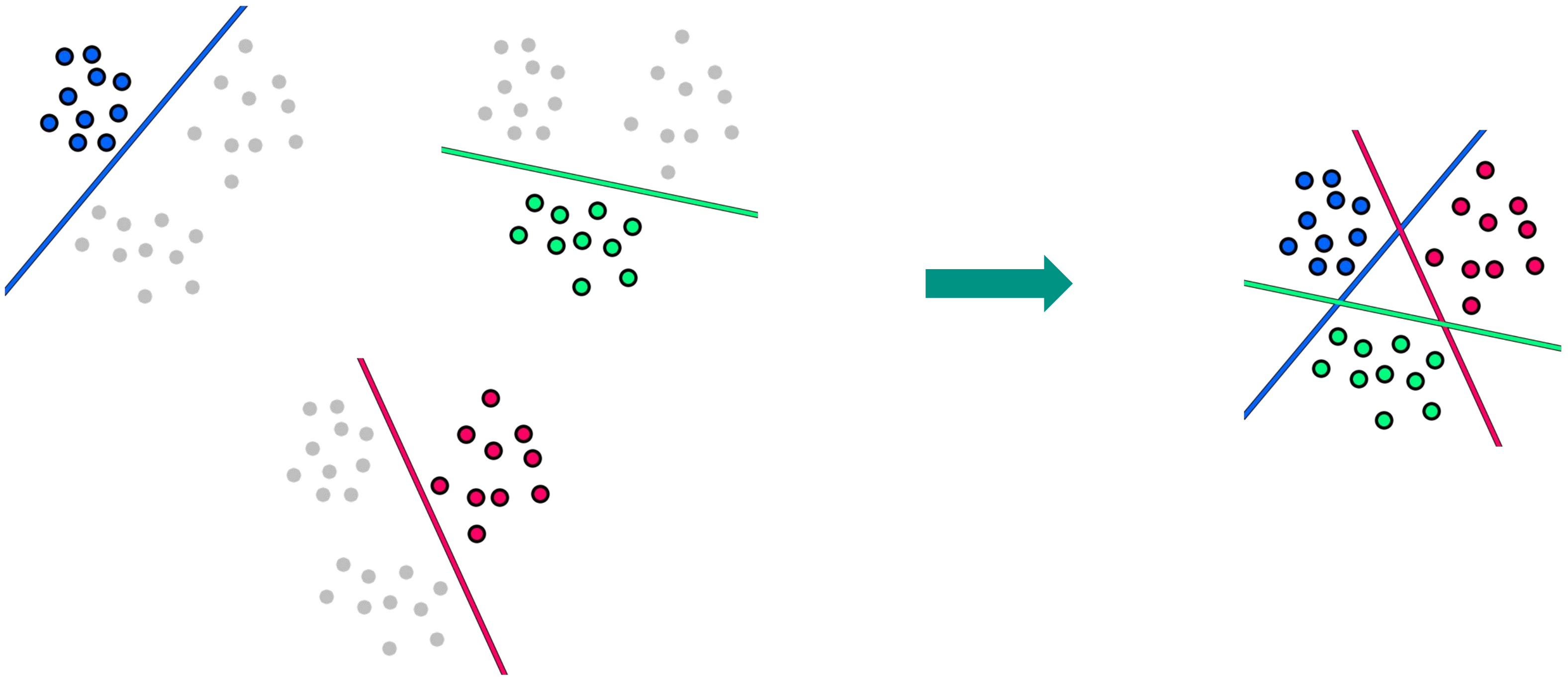
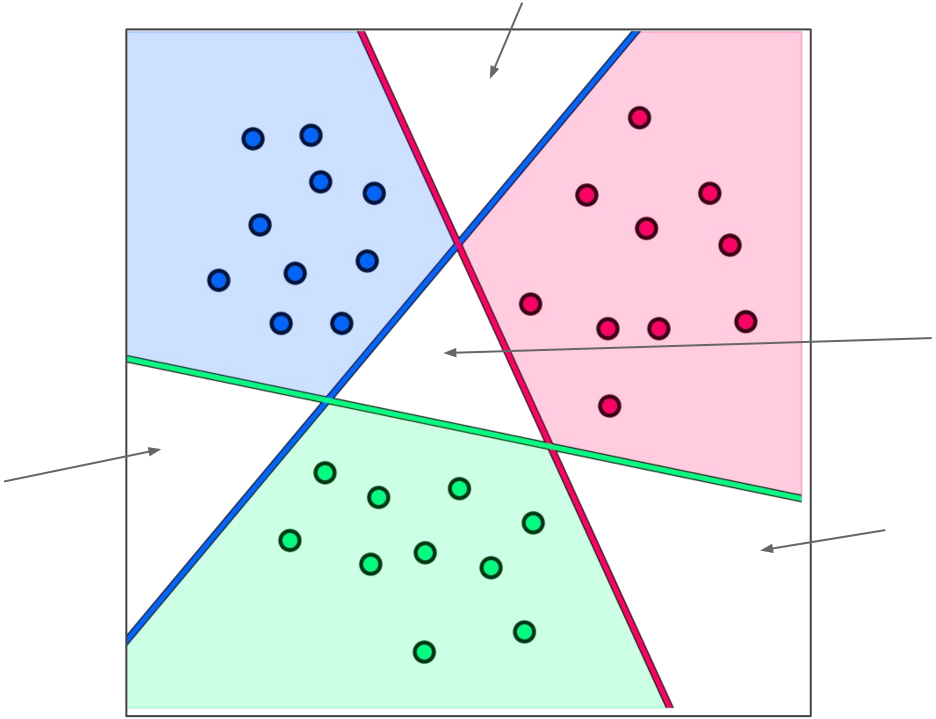
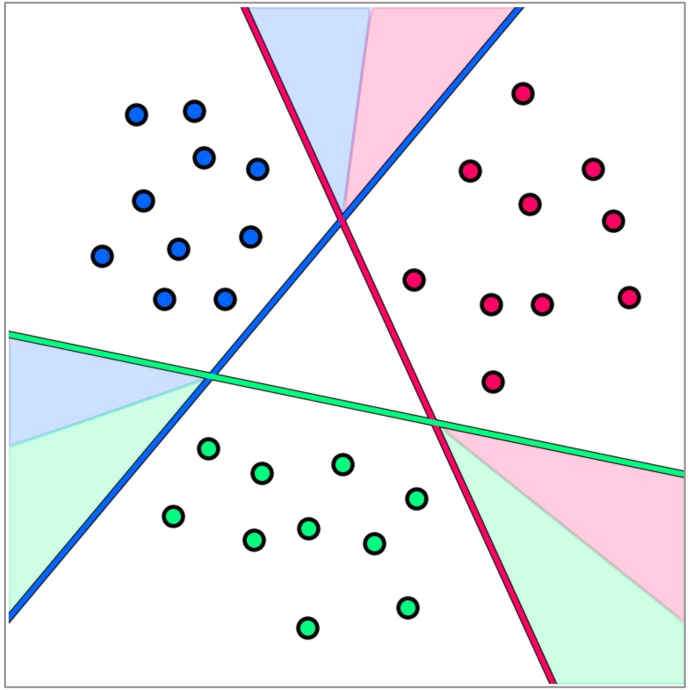


MultiClass Classification: One vs All

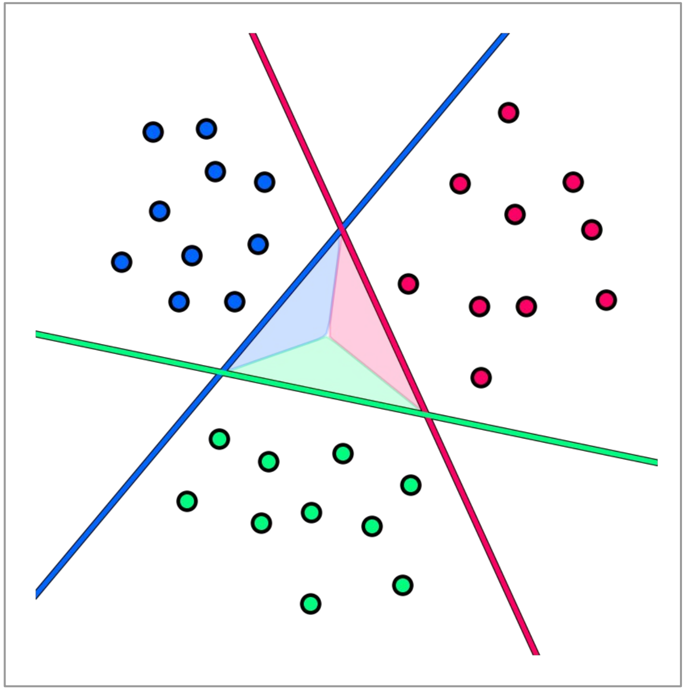


MultiClass Classification: One vs All



On a new input, to make a prediction, pick the class that maximizes: $max_i \ h_\theta{}^i(x)$

MultiClass Classification: One vs All



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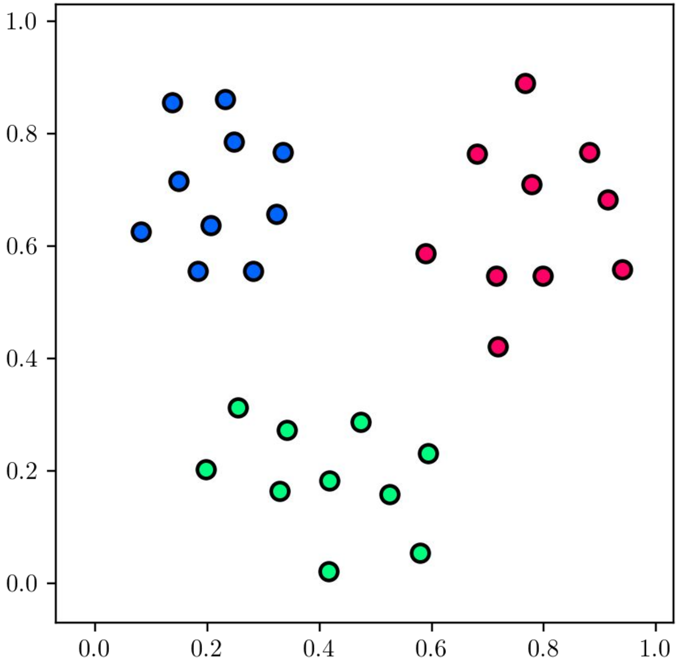
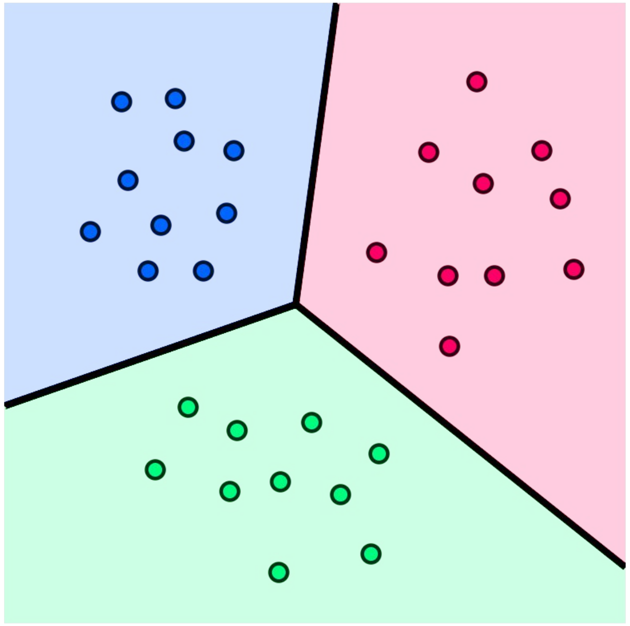
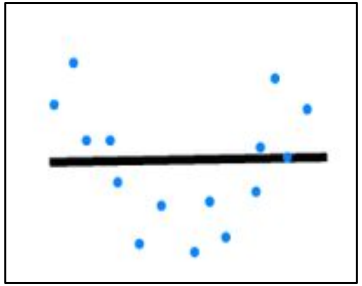
MultiClass Classification: One vs All





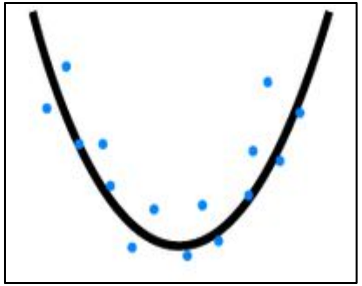
Overfitting

Linear Regression



UNDERFITTING

high bias

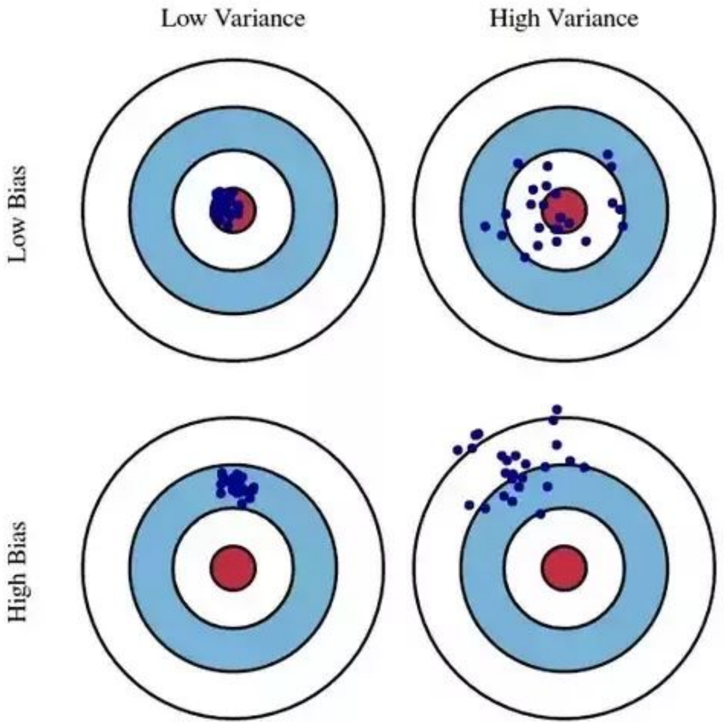


JUST RIGHT

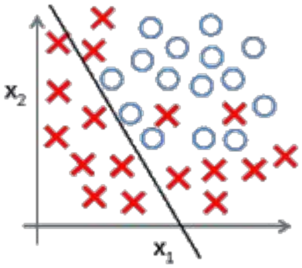
OVERFITTING

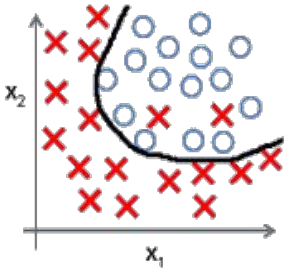
high variance

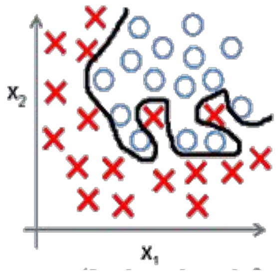
Linear Regression



Logistic Regression







UNDERFITTING

JUST RIGHT

OVERFITTING

high bias

high variance

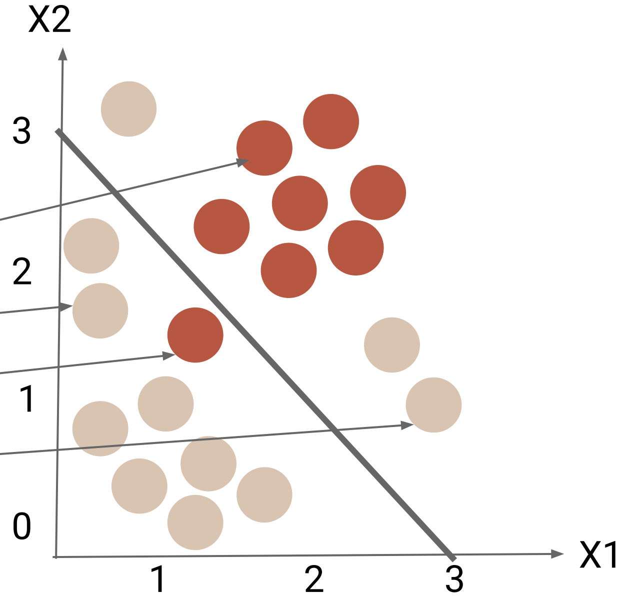
Performance

Accuracy

$$\text{Accuracy} = \frac{number\ of\ data\ points\ classified\ correctly}{all\ data\ points}$$

Confusion Matrix

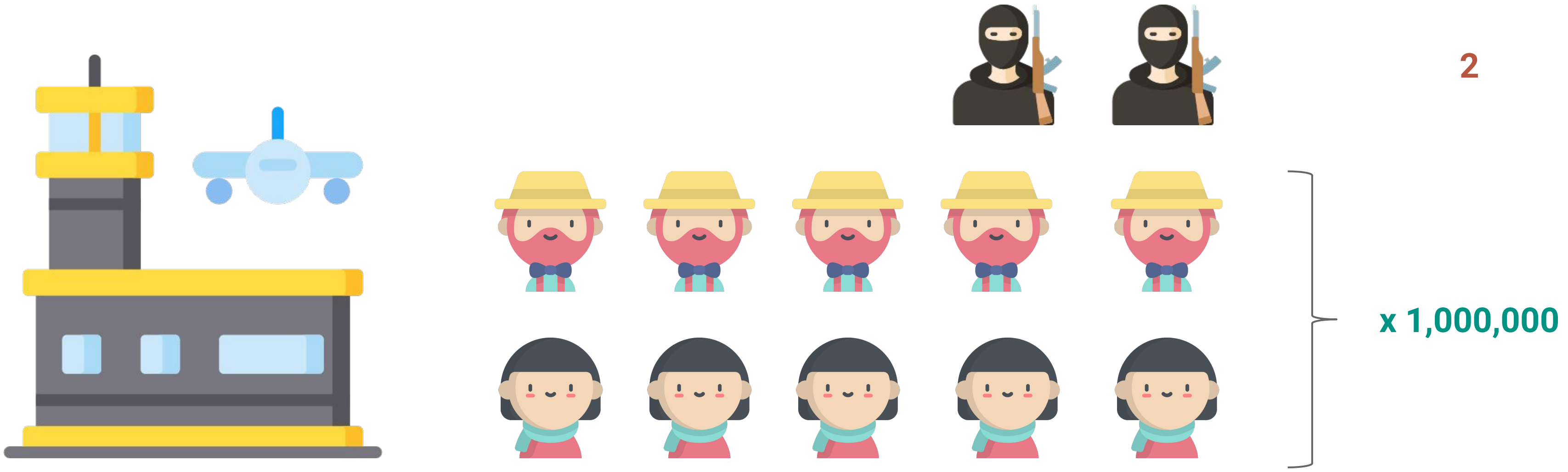
	Positive Examples	Negative Examples
Correct	TRUE POSITIVE	TRUE NEGATIVE
Wrong	FALSE NEGATIVE	FALSE POSITIVE



Imbalanced Classification Problem

If a classifier labels everyone as not terrorist:

Accuracy = 99.9998%



Confusion Matrix

TRUE POSITIVE

TRUE NEGATIVE

	Positive Examples	Negative Examples
Number of examples	50	50
Correct	**49**	**47**
Wrong	**1**	**3**

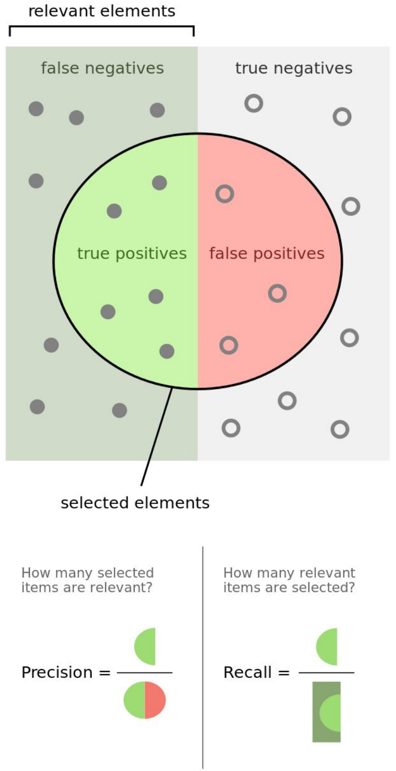
FALSE NEGATIVE

FALSE POSITIVE

Precision & Recall

$$\text{Precision} = \frac{\textit{True positive}}{\textit{True positive} + \textit{False positive}}$$

$$\text{Recall} = \frac{\textit{True positive}}{\textit{True positive} + \textit{False negative}}$$



Precision & Recall for a Balanced Dataset

	Positive	Negative
Num examples	53	47
Correct	**48**	**40**
Wrong	**5**	**7**

$$\textbf{Accuracy} = \frac{\text{True Positives (48) + True Negatives (40)}}{\text{All Examples (100)}} = 88\%$$

$$\textbf{Precision} = \frac{\text{True Positives (48)}}{\text{True Positives (48) + False Positive (7)}} = 87\%$$

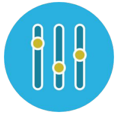
$$\textbf{Recall} = \frac{\text{True Positives (48)}}{\text{True Positives (48) + False Negative (5)}} = 90\%$$

Precision & Recall for an imbalanced Dataset

90

10

PREDICTED/ ACTUAL	Positive	Negative
Positive	0 (TP)	0 (FP)
Negative	10 (FN)	90 (TN)



100

$$\textbf{Accuracy} = \frac{\text{True Positives (0) + True Negatives (99)}}{\text{All Examples (100)}} = 90\%$$

$$\textbf{Precision} = \frac{\text{True Positives (0)}}{\text{True Positives (0) + False Positive (0)}}$$

$$\textbf{Recall} = \frac{\text{True Positives (0)}}{\text{True Positives (0) + False Negative (10)}} = 0\%$$

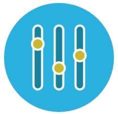
If we label everyone as non-terrorists, our recall and precision are 0 or not computable

Precision & Recall for an imbalanced Dataset

90

10

PREDICTED/ACTUAL	Positive	Negative
Positive	10 (TP)	90 (FP)
Negative	0 (FN)	0 (TN)



100

$$\textbf{Accuracy} = \frac{\text{True Positives (10) + True Negatives (0)}}{\text{All Examples (100)}} = 10\%$$

$$\textbf{Precision} = \frac{\text{True Positives (10)}}{\text{True Positives (10) + False Positive (90)}} = 10\%$$

$$\textbf{Recall} = \frac{\text{True Positives (10)}}{\text{True Positives (10) + False Negative (0)}} = 100\%$$

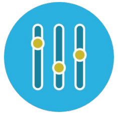
Recall is the ability to find relevant cases in a dataset!

Precision & Recall for an imbalanced Dataset

90

10

PREDICTED / ACTUAL	Positive	Negative
Positive	3 (TP)	0 (FP)
Negative	7 (FN)	90 (TN)



3

97

$$\textbf{Accuracy} = \frac{\text{True Positives (3) + True Negatives (90)}}{\text{All Examples (100)}} = 93\%$$

$$\textbf{Precision} = \frac{\text{True Positives (3)}}{\text{True Positives (3) + False Positive (0)}} = 100\%$$

Precision is the ability to find only the relevant data points

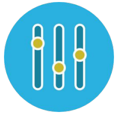
$$\textbf{Recall} = \frac{\text{True Positives (3)}}{\text{True Positives (3) + False Negative (7)}} = 30\%$$

Precision & Recall for an imbalanced Dataset

 90

 10

PREDICTED / ACTUAL	Positive	Negative
Positive	4 (TP)	3 (FP)
Negative	6 (FN)	87 (TN)



 13

 87

$$\text{Accuracy} = \frac{\text{True Positives (4) + True Negatives (87)}}{\text{All Examples (100)}} = 91\%$$

$$\text{Precision} = \frac{\text{True Positives (4)}}{\text{True Positives (4) + False Positive (3)}} = 57\%$$

$$\text{Recall} = \frac{\text{True Positives (4)}}{\text{True Positives (4) + False Negative (6)}} = 40\%$$

F1 Score

$$F_1 = 2 * \frac{precision * recall}{precision + recall}$$

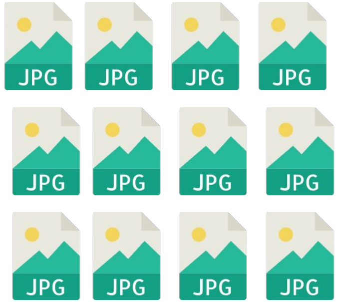
Accuracy is used when the True Positives and True negatives are **more important** while **F1-score** is used when the False Negatives and False Positives are crucial.

Accuracy can be used when the class distribution is similar while **F1-score** is a **better** metric when there are imbalanced classes as in the above case

K-Fold Cross-Validation

Problem

Training Set

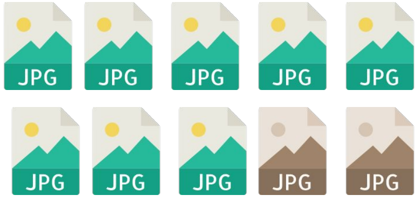


Validation Set

A part of the dataset is not used for training

K-Fold Cross Validation

Here, the test set is 20% of the dataset.
We loose ⅕ of the dataset.



Dataset

Iteration 1	Test	Train	Train	Train	Train
Iteration 2	Train	Test	Train	Train	Train
Iteration 3	Train	Train	Test	Train	Train
Iteration 4	Train	Train	Train	Test	Train
Iteration 5	Train	Train	Train	Train	Test

Run K-Experiments in which we:

- Choose a test set
- Train
- Test

After all the experiments, we average the results

Thank You

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