Curso de Tecnologia em Sistemas de Computação
Disciplina: Fundamentos de Algoritmos para Computação
Professoras: Sulamita Klein e Fernanda Couto
AD2-1 - Primeiro Semestre de 2022

Caderno de Respostas

Nome: Alexssandro Santana da Silva Júnior
Matrícula: 21213050314

Questão 1

O painel tem 32767 ou 2^{15} -1 maneiras distintas de estar aceso.

$$\binom{15}{0}+\binom{15}{1}+\binom{15}{2}+\binom{15}{3}+\binom{15}{4}+\binom{15}{5}+\binom{15}{6}+\binom{15}{7}+\binom{15}{8}+\binom{15}{9}+\binom{15}{10}+\binom{15}{11}+\binom{15}{12}+\binom{15}{13}+\binom{15}{14}+\binom{15}{15}$$

$$\binom{15}{1}=15 \qquad \binom{15}{2}=\frac{15!}{2!*13!}=\frac{15*14*\cancel{13!}}{2!*\cancel{13!}}=\frac{210}{2}=105 \qquad \binom{15}{3}=\frac{15!}{3!*12!}=\frac{15*14*13*\cancel{12!}}{3*2*\cancel{12!}}=\frac{2730}{6}=455$$

$$\binom{15}{0}=1$$

$$\binom{15}{4}=\frac{15!}{4!*11!}=\frac{15*14*13*12*\cancel{11!}}{4*3*2*\cancel{11!}}=\frac{32760}{24}=1365$$

$$\binom{15}{5}=\frac{15!}{5!*10!}=\frac{15*14*13*12*11*\cancel{10!}}{5*4*3*2*\cancel{10!}}=\frac{360360}{120}=3003$$

$$\binom{15}{6}=\frac{15!}{6!*9!}=\frac{15*14*13*12*11*10*\cancel{9!}}{6*5*4*3*2*\cancel{9!}}=\frac{3603600}{720}=5005$$

$$\binom{15}{7}=\frac{15!}{7!*8!}=\frac{15*14*13*12*11*10*9*\cancel{8!}}{7*6*5*4*3*2*\cancel{8!}}=\frac{32432400}{5040}=6435$$

$$\binom{15}{8}=\frac{15!}{8!*7!}=\frac{15*14*13*12*11*10*9*8*\cancel{7!}}{8*7*6*5*4*3*2*\cancel{7!}}=\frac{259459200}{40320}=6435$$

$$\binom{15}{9}=\frac{15!}{9!*6!}=\frac{15*14*13*12*11*10*9*8*7*\cancel{6!}}{9*8*7*6*5*4*3*2*\cancel{6!}}=\frac{1.816.214.400}{362.880}=5.005$$

$$\binom{15}{10}=\frac{15!}{10!*5!}=\frac{15*14*13*12*11*10*9*8*7*6*\cancel{5!}}{10*9*8*7*6*5*4*3*2*\cancel{5!}}=\frac{10.897.286.400}{3.628.800}=3003$$

$$\binom{15}{11}=\frac{15!}{11!*4!}=\frac{15*14*13*12*11*10*9*8*7*6*5*\cancel{4!}}{11*10*9*8*7*6*5*4*3*2*\cancel{4!}}=\frac{54.486.432.000}{39.916.800}=1365$$

$$\binom{15}{12}=\frac{15!}{12!*3!}=\frac{15*14*13*12*11*10*9*8*7*6*5*4*\cancel{3!}}{12*11*10*9*8*7*6*5*4*3*2*\cancel{3!}}=\frac{217.945.728.000}{479.001.600}=455$$

$$\binom{15}{13}=\frac{15!}{13!*2!}=\frac{15*14*13*12*11*10*9*8*7*6*5*4*3*\cancel{2!}}{13*12*11*10*9*8*7*6*5*4*3*2*\cancel{2!}}=\frac{653.837.184.000}{6.227.020.800}=105 \qquad \binom{15}{14}=15$$

Questão 2

$$\left(x^2 - \frac{x}{2y^3}\right)^n = \sum_{i=0}^{n} \left((x2)^{n-i}\left(-\frac{x}{2y^3}\right)^i\right) = \sum_{i=0}^{n} \left(\frac{x^{2n-2i}(-1)^i x^i}{2y^{3i}}\right)$$

$$\sum_{i=0}^{n} \frac{(-1)^i x^{2n-i}}{2y^{3i}}$$

Dessa forma temos: $2n - i \in (n, 2n) \notin 0$ por tanto, não é possível que o desenvolvimento deste binômio de Newton apresente termo independente.

Questão 3

a)

a1 = 100 + 2 = 102
a2 = 102 + 102 + 2^2 = 208
a3 = 208 + 208 + 2^3 = 424

an = $2a_n$ -1+2^n

b)

$a_4 = 2a_3 + 2^4$
$a_4 = 2(2a_2 + 2^3) + 2^4$
$a_4 = 2^2\, a_2 + 2 * 2^4$
$a_4 = 2^2(2a_1 + 2^2) + 2 * 2^4$
$a_4 = 2^2$ a1 + $3 * 2^4$

a5 = $2_{a4} + 2^5$
a5 = $2(2^3{}_{a1} + 3 * 2^4) + 2^5$
a5 = $2^4{}_{a1} + 4 * 2^5$
an = $2^{n-7} * 102 + (n-1)2^n$
an = $2^n * 51 + (n-1)2^n$

an = $(n+50)2^n$