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HW #4

1.

a. Each vertex is a state

- a = pints in 10 pint container $(0 \le a \le 10)$
- b = pints in 7 pint container $(0 \le b \le 7)$
- C = pints in 4 pint container $(0 \le c \le 4)$
- $a+b+c = 11$ (constraint)

- Initial state is $(0, 7, 4)$
- Goal state is where b or c $= 2$
- Directed edge from (a_1, b_1, c_1) to (a_2, b_2, c_2) if one pouring operation transforms the first state to the second. There are 6 pour types. The amount poured = min
- Does there exist a path from $(0, 7, 4)$ to a goal state?

b. Breadth first Search should be applied. It finds the shortest path in an unweighted graph. Because each pour has equal cost, and we want the minimum number of operations, Breadth first Search is perfect. It goes through all states level by level, and is guaranteed to find a solution if a solution exists.

2.

a. Vertices intersect the city. The directed edges are one way streets. But is the directed graph strongly connected? It is linear time because of Tarjan's algorithm. Uses $O(V+E)$ time. Strongly connected iff there exists one SCC containing all vertices

Algorithm:

1. Run DFS/BFS to check if all vertices reachable from starting $O(V+E)$
2. Reverse all edges $O(V+E)$
3. Run DFS/BFS again from the same starting vertex on the reversed graph $O(V+E)$
4. Strongly connected iff all vertices reachable in both steps, $O(V+E)$

b. vertex t = town hall. Is there a directed from t to all reachable vertices from t, and a directed path back from every vertex to t? Do all vertices reachable from t belong to the same strongly connected component as t?

To check in linear time:

1. Run DFS/BFS from t to find all reachable vertices from t. Set X. $O(V+E)$
2. Reverse all edges in entire graph $O(E)$
3. Run DFS/BFS from t in reversed graph to find all vertices which can reach t in the original graph. Set Y. $O(V+E)$
4. Check if $X = Y$. If yes, Property holds. $O(V)$

Total time is $O(V+E)$, which is linear.

- This works because vertex v satisfies the property iff there's a path from t to v, and a path from v to t. Mayor's claim is true iff $X = Y$