Lab 6 solutions (Calculus II)

#1 $\displaystyle\int \frac{x+3}{(x+1)(x^2+1)}\,dx$

partial fraction

$$\frac{x+3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$x+3 = A(x^2+1) + (Bx+C)(x+1)$$

$x=-1 \quad 2 = 2A \quad A=1$

$x=0 \quad 3 = A + C \quad C=2$

$x=1 \quad 4 = 2A + 2(B+C) \quad A+B+C=2$

$B=-1$

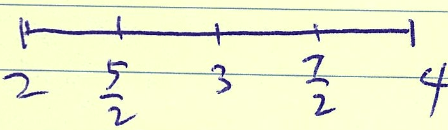
$$\int \frac{1}{x+1} + \frac{-x+2}{x^2+1}\,dx$$

$$= \int \frac{1}{x+1} - \frac{x}{x^2+1} + \frac{2}{x^2+1}\,dx$$

$$= \ln|x+1| - \frac{1}{2}\ln|x^2+1| + 2\tan^{-1}x + C$$

#2. $\int_2^4 x^2\,dx$ $\qquad \Delta x = \frac{4-2}{4} = \frac{1}{2}, \quad f(x) = x^2$

Trapezoidal Rule



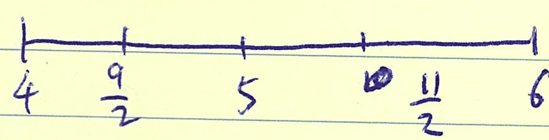
$$\frac{\Delta x}{2}\left(f(2) + 2f\left(\tfrac{5}{2}\right) + 2f(3) + 2f\left(\tfrac{7}{2}\right) + f(4)\right)$$

$$= \frac{1}{4}\left(4 + \frac{25}{2} + 18 + \frac{49}{2} + 16\right)$$

$$= \frac{75}{4} = 18.75$$

#3 $\int_4^6 \frac{1}{x}\,dx$

Simpson's Rule $\qquad \Delta x = \frac{6-4}{4} = \frac{1}{2} \qquad f(x) = \frac{1}{x}$



$$\frac{\Delta x}{3}\left(f(4) + 4f\left(\tfrac{9}{2}\right) + 2f(5) + 4f\left(\tfrac{11}{2}\right) + f(6)\right)$$

$$= \frac{1}{6}\left(\frac{1}{4} + \frac{8}{9} + \frac{2}{5} + \frac{8}{11} + \frac{1}{6}\right)$$

$$= \frac{4814}{11880} \approx 0.4055$$

#4 $\int_1^\infty \frac{x}{(x^2+1)^3}\,dx$

Substitution $\quad u = x^2+1 \quad \frac{du}{dx} = 2x \quad dx = \frac{du}{2x}$

$x = 1 \to \infty$

$u = x^2+1 = 2 \to \infty$

$$\int_2^\infty \frac{\cancel{x}}{u^3} \cdot \frac{du}{2\cancel{x}}$$

$$= \frac{1}{2}\int_2^\infty u^{-3}\,du = \frac{1}{2} \left.\frac{u^{-2}}{-2}\right|_2^\infty$$

$$= -\frac{1}{4}\cdot\left.\frac{1}{u^2}\right|_2^\infty = 0 - \left(-\frac{1}{4}\cdot\frac{1}{4}\right) = \frac{1}{16}$$

This improper integral is convergent to $\frac{1}{16}$.

#5 $\quad \int_2^5 \frac{1}{(x-2)^{3/2}}\,dx \qquad$ ($x=2$ is a vertical asymptote.)

$u = x-2 \quad \frac{du}{dx} = 1 \quad du = dx$

$x = 2 \to 5$

$u = x-2 = 0 \to 3$

$$\int_0^3 \frac{1}{u^{\frac{3}{2}}}\,du = \int_0^3 u^{-\frac{3}{2}}\,du$$

$$= \left.\frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}\right|_0^3 = -2\cdot\left.\frac{1}{\sqrt{u}}\right|_0^3$$

$$\left(-2\cdot\frac{1}{\sqrt{3}}\right) - \left(-2\cdot\frac{1}{\sqrt{0}}\right) = \infty$$

This improper integral is divergent!