Cal II Lab 5 Solution

#1. $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

Trig substitution: $x = \sin\theta \quad dx = \cos\theta\,d\theta$

$$\int \frac{\sin^3\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta = \int \frac{\sin^3\theta}{\sqrt{\cos^2\theta}}\cos\theta\,d\theta$$

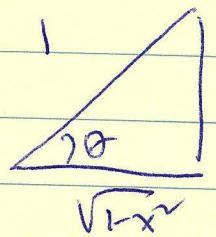
$$= \int \sin^3\theta\,d\theta \qquad u = \cos\theta \qquad \frac{du}{d\theta} = \sin\theta \qquad d\theta = \frac{du}{\sin\theta}$$

$$= \int \sin^3\theta \frac{du}{\sin\theta} = -\int \sin^2\theta\,du$$

$$= \int 1-\cos^2\theta\,du = -\int 1-u^2\,d\theta$$

$$= -\left(u - \frac{u^3}{3}\right) + C$$

$$= -\left(\cos\theta - \frac{\cos^3\theta}{3}\right) + C$$



$\sin\theta = x$

$\cos\theta = \sqrt{1-x^2}$

$$= -\left(\sqrt{1-x^2} - \frac{\left(\sqrt{1-x^2}\right)^3}{3}\right) + C = -\sqrt{1-x^2} + \frac{\left(\sqrt{1-x^2}\right)^3}{3} + C$$

#2 $\int x^2\sqrt{x^2+4}\,dx$

Try substitution $x = 2\tan\theta \qquad dx = 2\sec^2\theta\,d\theta$

$$= \int (2\tan\theta)^2\sqrt{(2\tan\theta)^2+4}\cdot 2\sec^2\theta\,d\theta$$

$$= \int 4\tan^2\theta\cdot 2\sec\theta\cdot 2\sec^2\theta\,d\theta$$

$$= 16\int \tan^2\theta\sec^3\theta\,d\theta$$

#3 $\int \frac{x^3 - 2x^2 + x + 5}{x - 2}\, dx$

Long division

$$x-2 \;\overline{)\; x^3 - 2x^2 + x + 5}$$

$x^2 + 1$ → Quotient

$x^3 - 2x^2$

$0 + x + 5$

$x - 2$

7 → Remainder

$$= \int x^2 + 1 + \frac{7}{x-2}\, dx$$

$$= \frac{x^3}{3} + x + 7\ln|x-2| + C$$

#4 $\int_1^3 \frac{1}{x^2+3x+2}\, dx$

Partial fraction $\frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$1 = A(x+2) + B(x+1)$

$x = -1, \quad A = 1, \qquad x = -2, \quad 1 = -B \quad B = -1$

$$\int_1^3 \frac{1}{x+1} + \frac{-1}{x+2}\, dx$$

$$\ln|x+1| - \ln|x+2| \Big|_1^3$$

$$(\ln 4 - \ln 5) - (\ln 2 - \ln 3)$$

$$\ln 4 - \ln 5 - \ln 2 + \ln 3$$

$$\ln \frac{6}{5}$$

#5 $\int \frac{2x+1}{x(x-1)^2}\,dx$

partial fraction

$$\frac{2x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)}$$

$$2x+1 = A(x-1)^2 + Bx + Cx(x-1)$$

$x=0, \quad A=1$

$x=1 \quad B=3$

$x=2, \quad 5 = A+2B+2C \quad 5 = 1+6+2C \quad C=-1$

$$\int \frac{1}{x} + \frac{3}{(x-1)^2} + \frac{-1}{x-1}\,dx$$

$$\ln|x| - \frac{3}{x-1} - \ln|x-1| + C$$