Confidence Intervals on Mean, Unknown Variance

-- what if you don't know population variance?

-- if your sample is "sufficently large," you may substitute sample variance (s^2) for pop. variance with little effect

-- what is "sufficiently large?"

$n \geq 40$ (book)

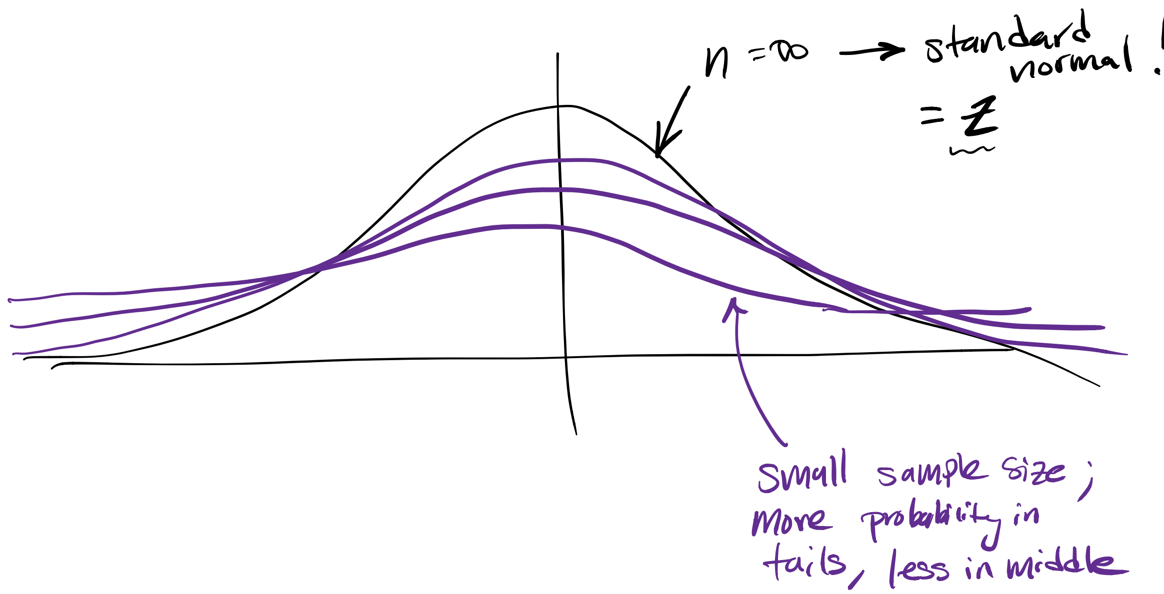
$n \geq 30$ (Kender) ✓

-- if $n < 30$, we need a new sampling distribution that compensates for this!

T-Distribution

Table V in textbook

- Gaussian-family distribution that places more probability in the "tails" for small sample sizes due to more uncertainty!



- T-table is very differently organized from z-table!
  - z-table gives cumulative probabilities given z
  - t-table gives t-values given specific values of α and the degrees of freedom

$\therefore$ d.o.f. $= n-1$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$\therefore$ we simple rewrite it into an interval on μ the same way we did our z formula

$\therefore$ a $(1-\alpha) \times 100\%$ confidence interval on mean with unknown pop. variance and $n-1$ degrees of freedom is:

$$\mu: \quad \bar{x} \pm \underbrace{t_{\alpha/2, n-1}}_{\text{two subscripts}} s/\sqrt{n}$$

final answer always an interval:

$$\bar{x} - t_{\alpha/2, n-1}\, s/\sqrt{n} < \mu < \bar{x} + t_{\alpha/2, n-1}\, s/\sqrt{n}$$

- if one-sided, use appropriate upper or lower bound with $t_{\alpha, n-1}$, not $t_{\alpha/2, n-1}$.
(just like Z distribution)

Ex: tensile adhesion test performed on $n = 22$ samples

test results: $\bar{x} = 13.71$

$s = 3.55$

(psi? kPa?)

- write a 95% C.I. on μ

- first: "95% C.I." means two-sided
[it will ask for a "bound" if one-sided]
- second: $\alpha = 0.05$
- third: $n = 22 \longrightarrow$ d.o.f. $= 21$
- need $t_{\alpha/2, n-1} = t_{.025, 21}$
$= 2.080$ (table)

$\therefore$ if this had been a z-problem (known σ^2), we would have used

$$Z_{\alpha/2} = Z_{.025} = 1.96$$

$\therefore$ $t = 2.080$ results in a little wider C.I. than $z = 1.96$ would have given us, due to $n < 30$

$$\mu: \quad 13.71 \pm 2.080 \cdot 3.55/\sqrt{22}$$

all of n!

$$12.14 < \mu < 15.27$$

(psi? kPa?)

.. cool trick you can do with T-table!

.. look at bottom row : $v = \infty$

→ implies population → z-distribution

.. this effectively gives us exact z-values, given α!

.. ex: $Z_{.025} = t_{.025,\infty} = 1.960$ ← hot dog!

$Z_{.05} = t_{.05,\infty} = 1.645$ ← looky there!

.. in light of this development, your exams will only include a t-table, not z-table!