25

ISE 2211 • Exam II • 31 May 2018 NAME SOLUTION

1) Fact: Joe Tritschler is a nerd. When he and his wife found out last fall that they were going to have twins, Joe immediately recognized that the probabilities of having two boys, two girls, or one of each would be easy to represent with a probability mass function. He now asks you to do the same.

First, write the sample space of possible outcomes. Use *b* for boy and *g* for girl.

$S \{bb \quad bg \quad gb \quad gg\}$ +1

Now, assume random variable *X* represents the number of girls in an outcome. List the values of *X* associated with each of the outcomes you listed above.

$bb \rightarrow X = 0$
$bg \rightarrow X = 1$
$gb \rightarrow X = 1$ +1
$gg \rightarrow X = 2$

Assuming the probability of a girl is exactly 50%, determine the probabilities associated with each outcome in the sample space. Hint: use the multiplication rule.

$P\{bb\} = (1-0.5)(1-0.5) = 0.25$
$P\{bg\} = (1-0.5)(0.5) = 0.25$ +2
$P\{gb\} = (0.5)(1-0.5) = 0.25$
$P\{gg\} = (0.5)(0.5) = 0.25$

Write the probability distribution; i.e., the probabilities associated with possible values of *X*. Based on this, should you expect two boys, two girls, or one-of-each?

$$\left.\begin{aligned} P(0) &= P\{bb\} = 0.25 \\ P(1) &= P\{bg\} + P\{gb\} = 0.25 + 0.25 = 0.5 \\ P(2) &= P\{gg\} = 0.25 \end{aligned}\right\} +2$$

one of each +1

Write the cumulative distribution of the number of girls.

$$F(0) = f(0) = 0.25$$
$$F(1) = f(0) + f(1) = 0.25 + 0.5 = 0.75$$
$$F(2) = f(0) + f(1) + f(2) = 1$$

+2

Determine the expected value and variance of the number of girls, with units for each answer.

Formulae:

$\mu = \sum x\, f(x)$

$\sigma^2 = \sum x^2 f(x) - \mu^2$

$$E(x) = \mu = 0 \cdot 0.25 + 1 \cdot 0.5 + 2 \cdot 0.25 = 1 \text{ girl}$$

+1

$$V(x) = \sigma^2 = 0^2 \cdot 0.25 + 1^2 \cdot 0.5 + 2^2 \cdot 0.25 - 1^2 = 0.5\ (\text{girl})^2$$

+1

2) Now, let's look at the problem a different way. Historical data suggests that there is a 50% probability that a newborn baby will be a girl. Let the binomially-distributed random variable X represent the number of girls. If two babies are born, determine the probability of at least one girl.

Formulae: $\rightarrow n=2$ $\qquad \rightarrow = f(1) + f(2)$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$\binom{2}{1} = \frac{2!}{1!(2-1)!} = 1$$

$$\therefore f(1) = 1 \cdot 0.5^1 (1-0.5)^{2-1} = 0.5 \quad +1$$

$$\binom{2}{2} = \frac{2!}{2!(2-2)!} = 1$$

$$\therefore f(2) = 1 \cdot 0.5^2 (1-0.5)^{2-2} = 0.25 \quad +1$$

$$\therefore P(X \geq 1) = 0.25 + 0.5 = 0.75 \quad +1$$

If two babies are born, what is the expected value, variance, and standard deviation of the number of girls?

$$\mu = E(X) = np = 2 \cdot 0.5 = 1 \text{ girl} \quad +1$$

$$\sigma^2 = V(X) = np(1-p) = 2 \cdot 0.5(1-0.5) = 0.5 \ (\text{girl})^2 \quad +1$$

3) Fact: Joe Tritschler is very particular when it comes to painting rooms. He gets a little aggravated when a piece of lint from a new roller boogers up his paint job. He has determined that, on average, there will be three lint boogers on a 100-square-foot wall. If the discrete random variable X represents the number of lint boogers in a paint job, determine the probability of three or fewer lint boogers in a room with 380 square feet of total wall area.

Formulae:

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\mu = \lambda$

$\sigma^2 = \lambda$

$\lambda = (3 \text{ boogers}/100 \text{ sq.ft.})\, 380 \text{ sq. ft.}$

$= 11.4 \text{ boogers}$ +1

exactly zero

$f(0) = e^{-11.4}\, 11.4^0/0! = 1.120 \times 10^{-5}$

$f(1) = e^{-11.4}\, 11.4^1/1! = 1.276 \times 10^{-4}$

$f(2) = e^{-11.4}\, 11.4^2/2! = 7.275 \times 10^{-4}$ +2

$f(3) = e^{-11.4}\, 11.4^3/3! = 0.002764$

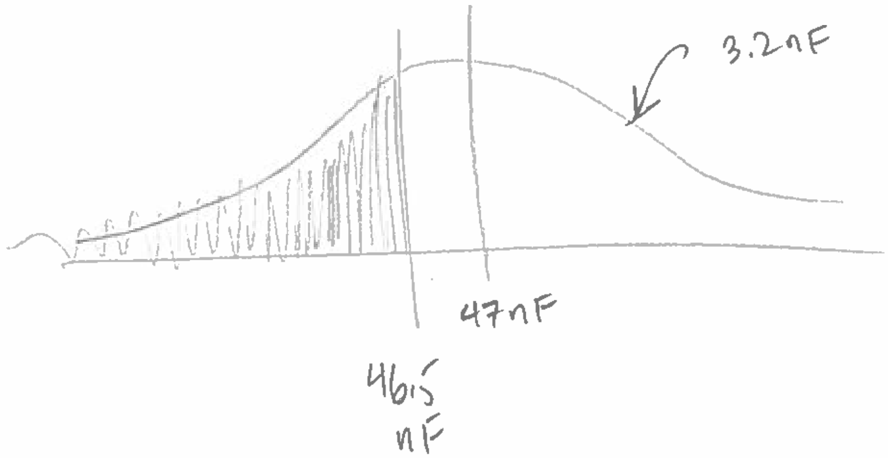
$P(X \leq 3) = 0.003631$ +1

Determine the expected value and variance of the number of lint boogers in 380 square feet of wall area.

(already did it)

$\sigma^2 = V(X) = 11.4 \text{ (boogers)}^2$ +1

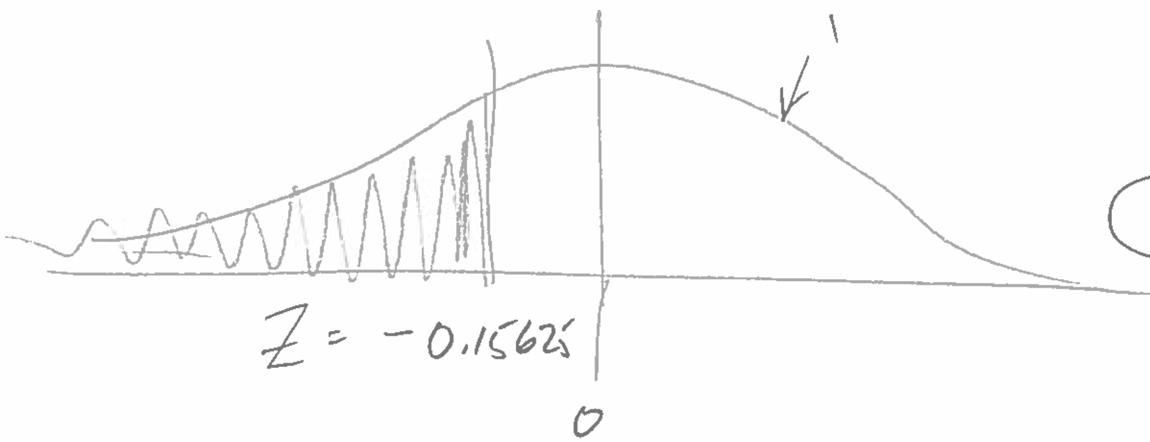
4) A manufacturer of capacitors states that the value of its product is normally distributed with a mean value of 47 nF and a standard deviation of 3.2 nF. Determine the probability that a capacitor will have a value less than 46.5 nF. Shade this probability on rough sketches of both the normal and standard normal distributions.



+1

$$Z = \frac{x-\mu}{\sigma} = \frac{46.5 - 47}{3.2} = -0.15625$$

+1



+1

close enough

$P(Z < -0.16) = 0.4364$ [table]

or 43.64%

+1