ISE 2211 • Exam VI • 27 November 2017 NAME SOLUTION

1) It is speculated that a relationship exists between exam scores (y) and the number of minutes since Joe Tritschler last ate anything when grading (x). Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model.

* Change R equation to prevent negative R values from occuring

Formulae:

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

currently → $R = \frac{S_{xy}}{\sqrt{S_{xx} SS_T}}$

	Exam Score (y)	Time Since Eating (x)
1	91	12
2	88	27
3	72	29
4	83	14
5	58	40
6	88	22
7	89	16
8	74	49
9	63	27
10	80	33

$\sum y_i x_i = 20365$ (+1)

$\sum x_i^2 = 8469$, $\sum y_i^2 = 62972$ (+1)

$\sum x_i = 269$, $\sum y_i = 786$ (+1)

$\bar{x}_i = 26.9$, $\bar{y}_i = 78.6$

$$S_{XY} = 20365 - \frac{269 \cdot 786}{10} = -778.4 \quad (+1)$$

$$S_{XX} = 8469 - \frac{269^2}{10} = 1232.9 \quad (+1)$$

$$\hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = \frac{-778.4}{1232.9} = -0.6314 \quad (+1)$$

$$\hat{\beta}_0 = 78.6 - (-0.6314)\,26.9 = 95.58 \quad (+1)$$

Write an equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \Rightarrow \hat{y} = 95.58 - 0.6314x \quad (+1)$$

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Write a 95% confidence interval on the mean exam score if Joe Tritschler hasn't eaten anything for half an hour when he grades it. $\Rightarrow$ $x_0 = 30$ min.

$$\hat{\mu}_{Y|30} = 95.58 - 0.6314 \cdot 30 = 76.64 \quad (+1)$$

$$SS_T = 62972 - 10 \cdot 78.6^2 = 1192 \quad (+1)$$

$$SS_E = 1192 - (-0.6314)(-778.4) = 700.5 \quad (+1)$$

$$\hat{\sigma}^2 = \frac{700.5}{8} = 87.56 \quad (+1)$$

$$t_{\alpha/2,\, n-2} = t_{.025,8} = 2.306 \quad (+1)$$

$$\therefore \mu_{Y|30}: 76.64 \pm 2.306 \sqrt{87.56\left[\frac{1}{10} + \frac{(30-26.9)^2}{1232.9}\right]}$$

$$69.56 < \mu_{Y_{30}} < 83.72 \quad (+1)$$

Now write a 95% prediction interval for the 11th exam score if Joe Tritschler hasn't eaten anything for half an hour when he grades it.

$$y_0: \quad 76.64 \pm 2.306 \sqrt{87.56\left[1 + \frac{1}{10} + \frac{(30-26.9)^2}{1232.9}\right]}$$

$$53.93 < Y_0 < 99.35 \quad (+1)$$

Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero. What does your conclusion imply about the relationship between exam scores and time since eating?

H_0: $\hat{\beta}_1 = 0$

H_1: $\hat{\beta}_1 \neq 0$

$$\beta_1 : \quad -0.6314 \pm 2.306 \sqrt{87.56 / 1232.9}$$

$$-1.246 < \beta_1 < -0.01686 \quad (+1)$$

-- barely reject Ho; C.I. does not contain zero (+1)

-- exam scores do have a significant linear relationship to time since eating (+1)

Determine the coefficient of regression and use it to test the following hypotheses on correlation coefficient, if *y* and *x* may both be considered random variables. What does your conclusion imply about the relationship between exam scores and time since eating?

H_0: $\rho = 0$

H_1: $\rho \neq 0$

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{700.5}{1192} = 0.4123$$

(+1)

$$t_0 = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}} = \frac{\sqrt{.4123} \cdot \sqrt{8}}{\sqrt{1-0.4123}} = 2.369$$

(+1)

$$t_{crit.} = t_{\alpha/2, n-2} = t_{.025, 8} = 2.306$$

$t_0 > t_{crit.}$ (barely)

∴ reject Ho (+1)

they are significantly correlated! (+1)

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