Axioms of Probability

premise or starting point of reasoning

1) $P(S) = 1$ or 100%

∴ some outcome of the sample space occurs on every trial of an experiment

2) $0 \leq P(E) \leq 1$ or 100%

∴ relative frequency must be between 0 and 1

3) If $E_1 \cap E_2 = \emptyset$, null set

then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

due to mutual exclusivity

useful concepts:

$$P(\emptyset) = 0$$

∴ probability of <u>nothing</u> happening is zero!

$$\ast \quad P(E') = 1 - P(E)$$

we'll use this one!

∴ if E_1 is contained in E_2,

$$P(E_1) \leq P(E_2)$$

Addition Rule

$$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}$$

area in which they intersect.
so you don't count twice!

∵ if events A & B are mutually exclusive:

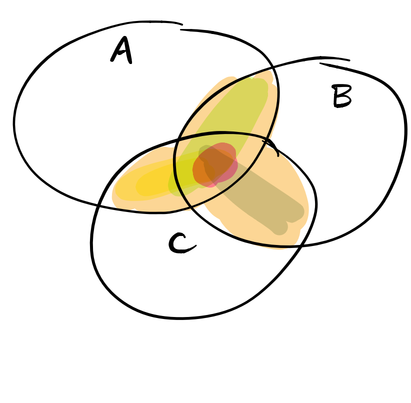
$$P(A \cap B) = \emptyset$$

$$\therefore \quad P(A \cup B) = P(A) + P(B)$$

three events:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(A \cap B)$$
$$- P(A \cap C)$$
$$- P(B \cap C)$$
$$+ P(A \cap B \cap C)$$

have to add this back; we subtracted it too many times!



- *if* we know probabilities associated with these events, we can determine total probability *much* more efficiently using these concepts than first determining outcomes associated w/ operation!

Conditional Probability

- outcome of an event depends on the outcome of some other event

notation:

$$P(B|A)$$ "the probability of B, given A"

$$P(B|A) = P(A \cap B) / P(A)$$

$$\boxed{\begin{aligned} &\text{Total Probability Rule:} \\ P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}}$$

<u>**exhaustive**</u> set or sets:

$$E_1 \cup E_2 \cup E_3 \cup \dots E_k = S$$

"events contain <u>all</u> outcomes"

∴ if $E_1, E_2, \dots E_k$ are exhaustive <u>and</u> mutually exclusive, then:

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + \dots + P(B \cap E_k)$$
$$= P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + \dots + P(B|E_k)P(E_k)$$

<u>independence</u> : two events are <u>independent</u> if any one of the following is true :

1.) $P(A|_B) = P(A)$

means B doesn't affect A

2.) $P(B|_A) = P(B)$

(same deal)

3.) $P(A \cap B) = P(A)\,P(B)$