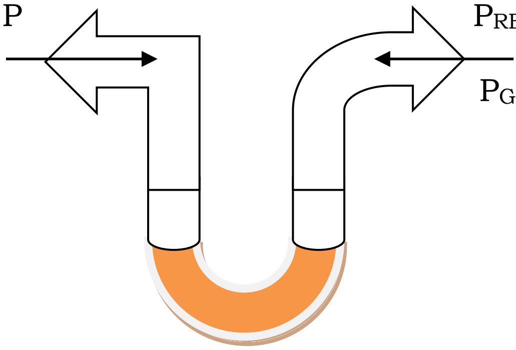
Name **KEY**

PHY 1110 – Summer 2020 - Study Guide #12 – Hydrostatics

1. A U-tube manometer filled with mercury is sketched below. An absolute pressure of 151300 N/m^2 is applied to the tube on the left.
a) The gauge pressure applied to the tube on the left is 50,000 N/m^2.



$P_{GAUGE} = P_{ABS} – P_A = 151{,}300 – 101{,}300 = 50{,}000\ N/m^2$

b) When the reference pressure in the tube on the right is standard atmospheric pressure, the difference in the elevations of the liquid levels on the two sides would be 0.375 m with the liquid level on the (1) right, or **(2)** left side being lower.

$\Delta P = P_{ABS} – P_{REF} = \rho gh$, $5 \times 10^4 = 13.6 \times 10^3 \times 9.8\ h$, $h = 0.375$ m

c) When the reference pressure in the tube on the right is 0 Pa because it was evacuated with a vacuum pump, the difference in the elevations of the liquid levels on the two sides would be 1.135 m with the liquid level on the (1) right, or **(2)** left side being lower.

$P_{ABS} – P_{REF} = \rho gh$, $1.513 \times 10^5 - 0 = 13.6 \times 10^3 \times 9.8\ h$, $h = 1.135$ m

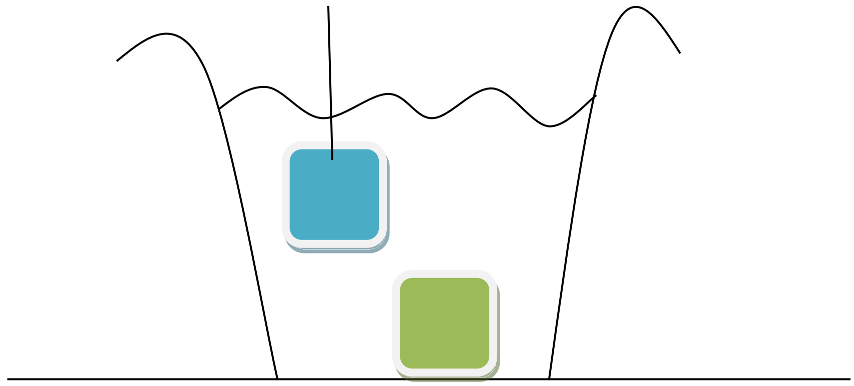
d) If the manometer had been filled with water rather than mercury in case (c), the difference in the elevations of the liquid levels on the two sides would be 15.439 m.

$P_{ABS} – P_{REF} = \rho gh$, $1.513 \times 10^5 - 0 = 10^3 \times 9.8\ h$, $h = 15.439$ m

NOTE: This is 13.6 times the height of the Hg column

2. (a) A piece of wood is placed in water and it immediately sinks. Its density must be **(1)** greater than, (2) equal to, or (3) less than the density of water.
(b) The piece of wood is then placed in another liquid and it floats. The density of this liquid is **(1)** greater than, (2) equal to, or (3) less than the density of water.

The sketch shows two cylinders in a tank of water.



3. The cylinder suspended by a thread is 5 cm in diameter, 5 cm high, and has a mass of 120 grams. Its upper face is parallel to the upper surface of the water and 10 cm below it.

a) The volume of the cylinder is 9.82×10^{-5} m^3 and the density is 1,222 kg/m^3.

$V = (\pi d^2/4)\ h = (\pi\ 0.05^2/4)\ 0.05 = 9.82 \times 10^{-5}\ m^3$

$\rho = m/V = 120 \times 10^{-3}\ kg/\ 9.82 \times 10^{-5}\ m^3 = 1{,}222\ kg/m^3$

b) The pressure acting on the cylinder is the sum of two contributions. One contribution is due to the weight of the water. The pressure exerted on the <u>upper face</u> of the cylinder due to the weight of the water is 980 N/m^2. The force is 1.924 N and is directed **DOWN**.

$P = \rho gh = 1{,}000\ kg/m^3 \times 9.8\ m/s^2 \times 0.1\ m = 980\ N/m^2$

$F = PA = 980\ N/m^2 \times 3.14 \times (0.05^2/4)\ m^2 = 1.924\ N$

c) The second contribution is due to the weight of the atmosphere pushing down on the upper surface of the water (see Pascal's principle). The additional pressure acting on the upper face of the cylinder due to the weight of the atmosphere is $1.013\text{x}10^5$ N/m^2 and the force is 198.902 N and is directed **DOWN**.

$F = PA = 1.013\text{x}10^5\ N/m^2 \times 3.14 \times (0.05^2/4)\ m^2 = 198.902\ N$

d) The actual pressure/force acting on the <u>upper face</u> of the cylinder is the sum of the two contributions. The actual pressure is 102,280 N/m^2 and the force is 200.826 N and is directed **DOWN**.

$980 + 101{,}300 = 102{,}280\ N/m^2$

$198.902 + 1.924 = 200.926\ N$

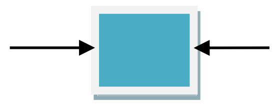
e) The actual pressure/force acting on the <u>lower face</u> of the cylinder may be calculated in a similar manner. The actual pressure is $1.0277\text{x}10^5$ N/m^2 (or Pa) and the force is 201.788 N and is directed **UP**.

$P = P_A + \rho gh = 1.013 \times 10^5\ N/m^2 + (1{,}000\ kg/m^3) \times 9.8\ m/s^2 \times 0.15\ m = 102{,}770\ Pa$

$F = PA = 1.0277 \times 10^5\ N/m^2 \times 3.14 \times (0.05^2/4)\ m^2 = 201.788\ N$

f) The average pressure acting on the sides of the cylinder is the pressure half way down the side of the cylinder and is 102,525 Pa. The net force acting on the sides of the cylinder is 0 N. Explain.

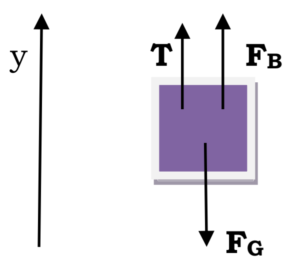
Forces acting on opposite sides at the same depth cancel out.



g) The net force acting on the cylinder by the surrounding water is the vector sum of the forces acting on each of the faces and is called the buoyant force. It is equal to 0.962 N and is directed **UP**.

$F_B = F_{UP} - F_{DOWN} = 201.788 - 200.826 = 0.962\ N$

h) Make a free body diagram of the suspended cylinder showing all three forces acting on the cylinder (buoyant force, weight, and tension) and find the tension in the thread.



$F_{NET} = T + F_B - F_G = ma_y = 0,\ T = F_G - F_B =$

$= 0.12\ kg \times 9.8\ m/s^2 - 0.962\ N = 0.214\ N$

i) The weight of a cylinder of water having the same dimensions as the submerged cylinder is 0.962 N. The magnitude of this force is (1) smaller than, **(2)** equal to, or (3) greater than, the buoyant force acting on the cylinder.

$F_G = mg = \rho_W Vg = 1{,}000\ kg/m^3 \times 9.82 \times 10^{-5}\ m^3 \times 9.8\ m/s^2 = 0.962\ N$

j) Imagine that another cylinder with the same dimensions but with a mass of 300 grams is suspended in the water tank at the same depth. In this case:

- the pressures exerted on the faces of the cylinder will be (1) larger, **(2)** the same, or (3) smaller;
- the buoyant force acting on the cylinder will be (1) larger, **(2)** the same, or (3) smaller;
- the tension in the thread will be **(1)** larger, (2) the same, or (3) smaller.

k) Imagine that another cylinder with the same dimensions but with a mass of only 85 grams is suspended at the same depth in the tank. In this case:

- the density of this cylinder is 866.2 kg/m^3 which is (1) greater than, (2) the same as, or **(3)** smaller than, the density of water,
- the buoyant force acting on the cylinder will be (1) larger, **(2)** the same, or (3) smaller;

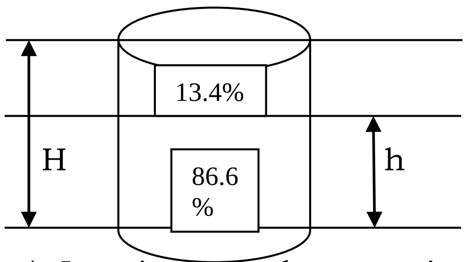
- the tension in the thread will be -0.129 (ZERO) N. Explain.

$\rho = m/V = 0.085\ kg/9.8125x10^{-5}\ m^3 = 866.2\ kg/m^3$

$T = F_G - F_B = 0.085\ kg\ x\ 9.8\ m/s^2 - 0.962\ N = -\ 0.129\ N$: ZERO

Cylinder floats in water $F_G < F_B$

l) The 85-gram cylinder will float in water when the weight of the displaced water is equal to the weight of the cylinder which occurs when 13.4 % of the volume of the cylinder is above the water surface.



F_G (displaced water) = F_G (cylinder)

$\rho_{WATER}\ (\pi d^2/4)h = \rho_{CYL}\ (\pi d^2/4)H$

$h = (\rho_{CYL}/\rho_{WATER})H = 0.866\ H$

m) Imagine that the water in the tank is replaced by a liquid of a density of 1600 kg/m^3 and the original cylinder is suspended at the same depth. Compared with immersion in water:

- the pressures exerted on the faces of the cylinder will be **(1)** larger, (2) the same, or (3) smaller;
- the buoyant force acting on the cylinder will be **(1)** larger, (2) the same, or (3) smaller;
- the tension in the thread will be (1) larger, (2) the same, or **(3)** smaller.