ISE 2211 • Exam II • 5 February 2021 16 pts NAME SOLUTION

1) My son Chuck will only eat a pretzel if it is completely intact and not broken in any way. He seems to get this trait from a certain person I happen to live with and from whom he happens to get half his DNA. Upon selecting a broken one, he throws it on the floor, but upon selecting an intact one, he puts it in his mouth. Determine the probability that out of three selections, at least two will make it into his mouth. Given: in a bag of 4800 pretzels, it is known that 312 are broken. Hint: you will need to determine whether this is a sampling-with-replacement or sampling-without-replacement problem. Another hint: it may help to determine the eight possible outcomes of three sampled pretzels, and these will resemble the eight outcomes of the three-camera flash example. [Also: we all know that there will be more broken pretzels in the bottom of the bag than in the top, but let's assume they are all randomly distributed in this example!]

Sampling without replacement!

b = broken, i = intact

sample space:

bbb
bbi
bib
bii +2
ibb
ibi
iib
iii

∴ need outcomes for which at least two are intact (= i)

∴ {bii ibi iib iii} +2

$$P\{bii\} = P\{ibi\} = P\{iib\}$$

all intact ones still in bag!

$$= \frac{312}{4800} \cdot \frac{4488}{4799} \cdot \frac{4487}{4798} = 0.05685$$ +2

$$P\{iii\} = \frac{4488}{4800} \cdot \frac{4487}{4799} \cdot \frac{4486}{4798} = 0.8174$$ +2

$$\therefore P(X \geq 2) = 0.05685 \cdot 3 + 0.8174 = 0.9879$$

or 98.79% +2

2) The number of broken pretzels on the floor of our 320-square-foot living room may be modelled as a discrete Poisson process. If it is known that, on any given day, there is an average of 36 broken pretzels on the floor, determine the probability that a 10x10 section of the floor will have zero broken pretzels. Bonus: use colorful descriptive language to express how likely this event is as a result.

+2

Formulae:

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\mu = \lambda$

$\sigma^2 = \lambda$

need probability of exactly zero broken pretzels per 100 square feet

+2

$\therefore$ need average value per 100 sq. ft. $\Downarrow$ = interval

$$\therefore \lambda = \frac{36 \text{ pretzels}}{320 \text{ sq. ft.}} \cdot \frac{100 \text{ sq. ft.}}{\text{interval}} = 11.25 \frac{\text{pretzels}}{\text{interval}}$$

+2

$$f(0) = \frac{e^{-11.25}\,\cancel{11.25^0}}{\underbrace{0!}_{=1}}$$

$= 0.0001301$

+2

or $0.001301\,\%$