Tests on σ^2

$H_0: \sigma^2 = \sigma_0^2$

some hypothesized value of pop. variance

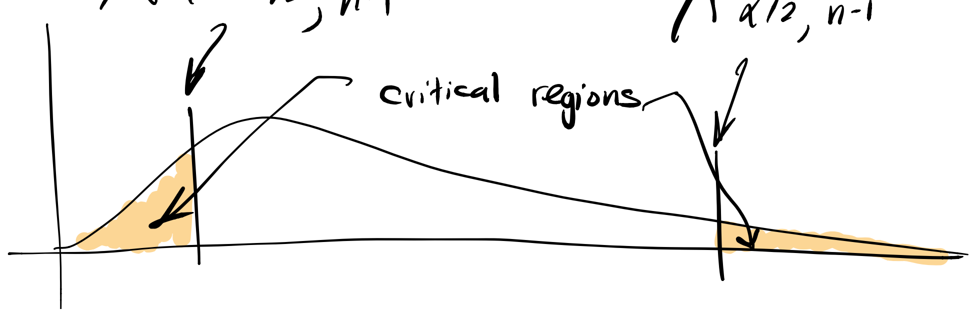
$H_1: \sigma^2 \neq \sigma_0^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

note: involves ratio $\frac{s^2}{\sigma_0^2}$, not difference $\bar{x} - \mu_0$!

∴ critical values for fixed-α approach:

$\chi^2_{1-\alpha/2,\, n-1}$ and $\chi^2_{\alpha/2,\, n-1}$



$\therefore$ reject H_0 if

$$\chi_0^2 > \chi^2_{\alpha/2,\, n-1}$$

OR $$\chi_0^2 < \chi^2_{1-\alpha/2,\, n-1}$$

- use $\chi^2_{\alpha, n-1}$ and $\chi^2_{1-\alpha, n-1}$ for one-sided H_1's.

- p-value approach: similar situation as tests on μ using t_0
  - best we can do is get range for p-value
  - may have to look at upper OR lower values!
  - either way, search for boundary values around χ_0^2 at your degrees of freedom

ex: fill volume of detergent bottles

if σ^2 exceeds 0.01 fl. oz.^{2}, too many bottles rejected

test $H_0: \sigma^2 = 0.01 \ (\text{fl. oz})^2$

$H_1: \sigma^2 > 0.01 \ (\text{fl. oz})^2$

using fixed-α approach @ $\alpha = 0.05$

test data:

$n = 20$

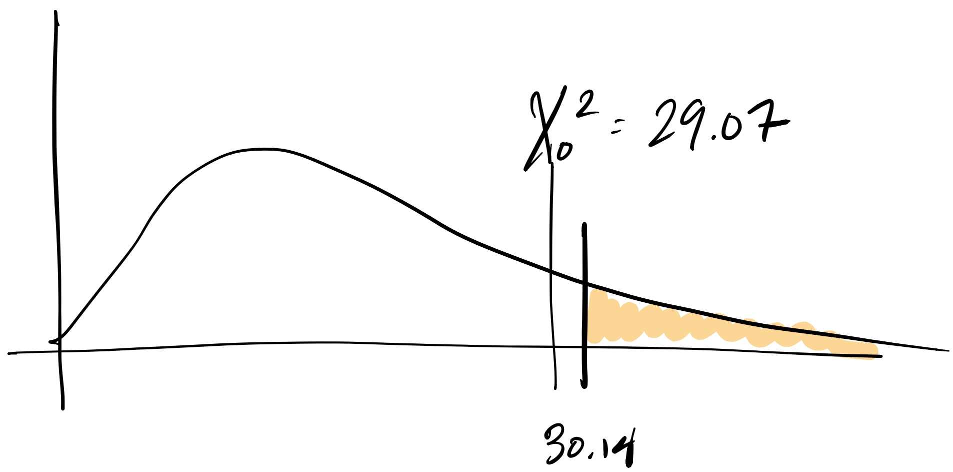
$s^2 = 0.0153$ fl. oz.^{2}

$$\chi_0^2 = \frac{(n-1)\, s^2}{\sigma^2} = \frac{19 \cdot 0.0153}{0.01}$$

$$\chi_0^2 = 29.07$$

Critical value for upper one-sided H_1 :

$$\chi^2_{\alpha, n-1} = \chi^2_{0.05, 19} = 30.14$$



$$\chi_0^2 \not> \chi^2_{\alpha, n-1}$$

$\therefore$ fail to reject H_0 (barely!)

Tests on Population Proportion (p)

$$H_0 : p = \underbrace{p_0}_{\text{hypothesized value}}$$

$$H_1 : p \neq p_0$$

$\therefore$ an approximate test statistic for $n \geq 30$:
(and proportions "not too close to zero or one")

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \quad \text{where } \hat{p} = \frac{x}{n}$$

remind you of binomial distribution? It should!

alt. version: $$Z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$$

(no need to compute $\hat{p}$)

∴ rejection criteria for fixed-α, p-value, etc.: <u>same</u> as z-tests on μ

ex: semiconductor manufacturer

customer: proportion of bad I.C.'s <u>can't</u> exceed 0.05 (i.e., 5%)

$n = 200$, $X = 4$ were found to be defective

test $H_0: p = 0.05$

$H_1: p < 0.05$

∴ why <u>lower</u> one-sided H_1?

∴ Probably chosen by manufacturer so that <u>rejecting</u> H_0 would suggest $p < 0.05$

∴ test @ $\alpha = 0.05$

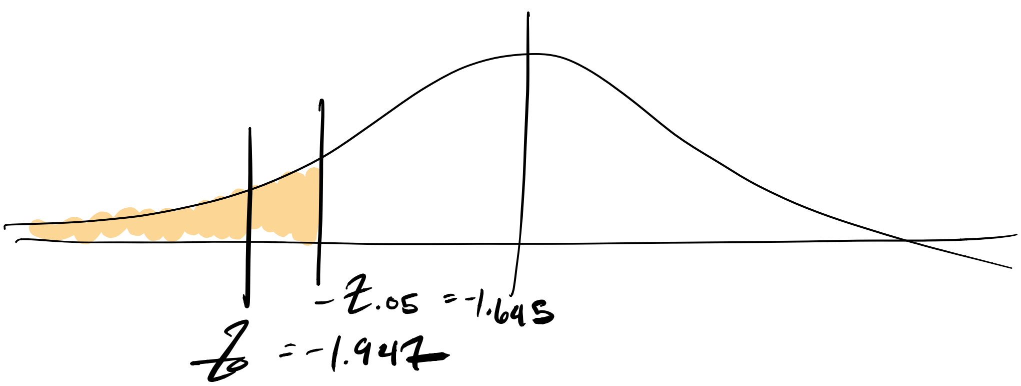
(fixed significance level approach)

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

$$\hat{p} = \frac{x}{n} = \frac{4}{200} = 0.02 \quad [2\%]$$

$$Z_0 = \frac{0.02 - 0.05}{\sqrt{0.05(.95)/200}} = \underline{-1.947}$$

∴ almost two standard deviations out!

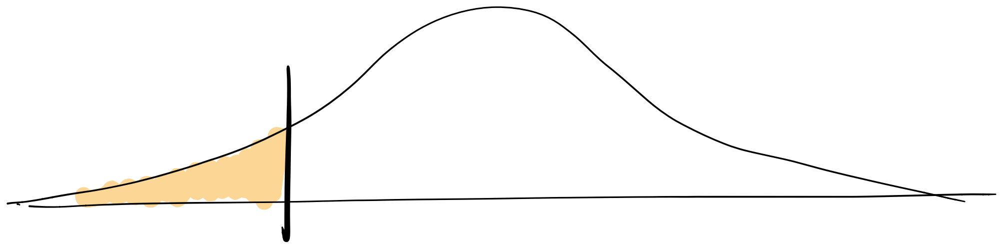


Critical value : $-Z_\alpha = -Z_{.05} = \underline{-1.645}$

$Z_0 < -Z_\alpha$ ∴ reject H_0

∴ data suggests $p < 0.05$

∴ if the p-value approach had been taken:



$z_0 = -1.947$

remember: fixed-α → shaded area is beyond critical value(s)

p-value → shaded area is beyond test statistic

$$\text{p-value} = P(Z < -1.947)$$

[note: because it's one-sided, this is the full p-value, not $\frac{\text{p-value}}{2}$!]

$$P(Z < -1.95) = 0.025588$$

p-value $< \alpha = 0.05$, reject H_0