## EE 2010 Circuit Analysis

**Module 21: Transient Response: Dynamic Circuits First-Order Problems**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

**Recall:**

Every first-order linear differential equation with constant coefficients has exactly the same solution:

$$\text{quantity}(t) = \text{quantity}(\infty) + [\text{quantity}(t_0) - \text{quantity}(\infty)]e^{-\frac{(t-t_0)}{\tau}}, \quad t \geq t_0$$

where $t_0$ it the time we are initially interested in the system, and $\tau$ is the *time constant* of the system.

We're ready to solve problems.

### Problem 7.5:

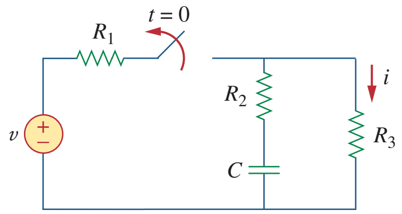


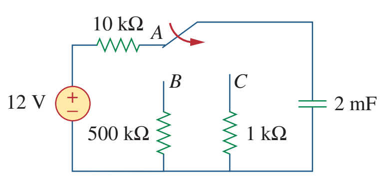

**Figure 7.85**
For Prob. 7.5.

We have $i(0) = \frac{v}{R_1+R_3}$, $i(\infty) = 0$, $\tau = (R_2 + R_3)C$, so that:

$$i(t) = \frac{v}{R_1 + R_3} e^{-\frac{t}{(R_2+R_3)C}}, \qquad t \geq 0$$

### Problem 7.7:

**7.7** Assuming that the switch in Fig. 7.87 has been in position $A$ for a long time and is moved to position $B$ at $t = 0$, Then at $t = 1$ second, the switch moves from $B$ to $C$. Find $v_C(t)$ for $t \geq 0$.




This is called a sequential transient, where the final condition of the present epoch is the initial condition of the next epoch. Each of these is simple, just a matter of keeping track of time. So... for each time epoch, consider that circuit from it's initial time to $t = \infty$.

*For* $0 \leq t \leq 1$*:* We have $v_C(0) = 12$, $v_C(\infty) = 0$, $\tau = RC = 500k\Omega \times 2mF = 1000$, so that:

$$v(t) = 12e^{-\frac{t}{1000}}, \qquad 0 \leq t \leq 1$$

*For* $1 \leq t$: We have $v_C(1) = 12e^{-\frac{1}{1000}} \approx 11.988$, $v_C(\infty) = 0$, $\tau = RC = 1k\Omega \times 2mF = 2$, so that:

$$v(t) = 11.988e^{-\frac{(t-1)}{2}}, \qquad 1 \leq t$$

... and we MUST have that $v(1)$ from the first part matches $v(1)$ from the second part. Check! Hence:

$$v(t) = \begin{cases} 12e^{-\frac{t}{1000}}, & 0 \leq t \leq 1 \\ 11.988e^{-\frac{(t-1)}{2}}, & 1 \leq t \end{cases}$$

**7.11** For the circuit in Fig. 7.91, find $i_o$ for $t > 0$.

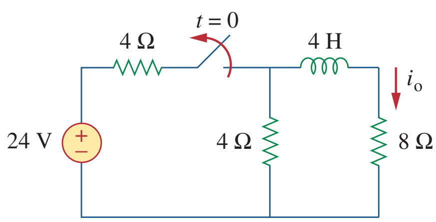



**Figure 7.91**
For Prob. 7.11.

We note, via a quick node equation that $v_o = 48/5$ so that $i_o(0) = 1.2$, $i_o(\infty) = 0$ $L = 4$ and $R_{eq} = 12$, so that:

$$i_o(t) = 1.2e^{-3t}, \qquad t \geq 0$$

**7.19** In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 6$ A.

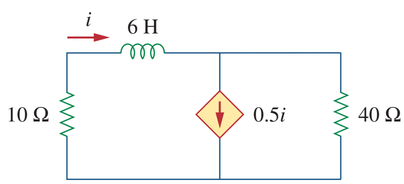



**Figure 7.99**
For Prob. 7.19.

We can't do 7.19. But if we knew the Thevenin equivalent of 7.19 as seen by the inductor, we could find $i(t)$ for the inductor driving the Thevenin equivalent. No independent sources, so $V_{th} = 0$. I would find $R_{th}$ by removing the inductor and applying a 1Amp source so that $i = 1$. The voltage induced is $v = 0.5 \times 40 + 1 \times 10 = 30V$, so that $R_{th} = 30$.

So now we have that $i(0) = 6$, $i_o(\infty) = 0$ $L = 6$ and $R_{th} = 30$, so that:

$$i(t) = 6e^{-5t}, \qquad t \geq 0$$

**7.23** Consider the circuit in Fig. 7.103. Given that $v_o(0) = 10$ V, find $v_o$ and $v_x$ for $t > 0$.

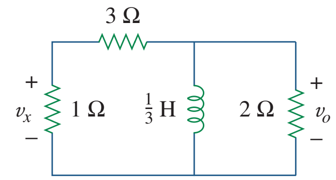



**Figure 7.103**
For Prob. 7.23.

Here, we could find the inductor current, and then do a current divider to find $v_s$ and $v_o$. But, since every voltage and every current in the circuit has a first-order solution, lets just go directly to:

We have that $v_o(0) = 10$, $v_o(\infty) = 0$, $L = 1/3$ and $R_{eq} = 2||4 = 4/3$, so that:

$$v_o(t) = 10e^{-4t}, \qquad t \geq 0$$

... and then by a voltage divider, $v_s(t) = v_o(t)/4$.

**7.39** Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.

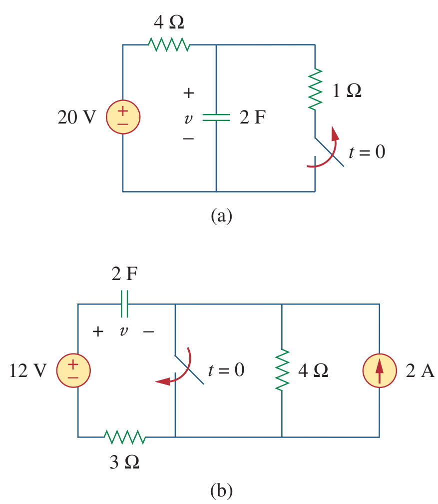



**Figure 7.106**
For Prob. 7.39.

We'll just ignore the $t < 0$ goofiness. We note, for part a) that $v(0) = 4$, $v(\infty) = 20$, $C = 2$ and $R = 4$, so that:

$$v(t) = 20 + (4 - 20)e^{-\frac{t}{8}}, \qquad t \geq 0$$

For part b) we note that that $v(0) = 12 - 8 = 4$, $v(\infty) = 12$, $C = 2$ and $R = 3$, so that:

$$v(t) = 12 + (4 - 12)e^{-\frac{t}{6}}, \qquad t \geq 0$$

**Problem 7.40:**

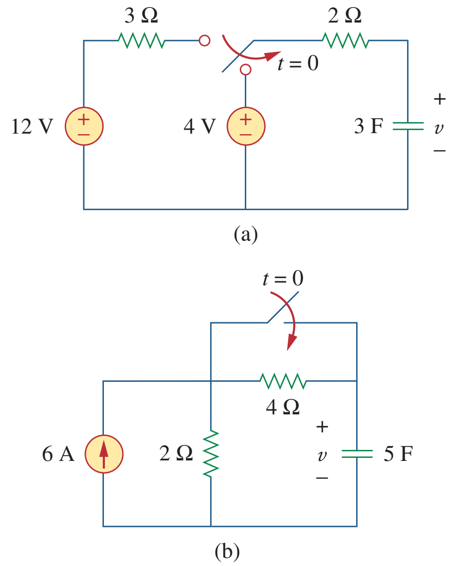



**Figure 7.107**
For Prob. 7.40.

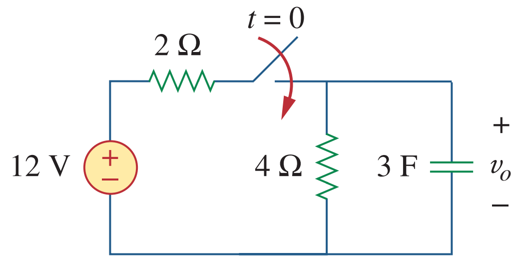
We note, for part a) that $v(0) = 12$, $v(\infty) = 4$, $C = 3$ and $R = 2$, so that:

$$v(t) = 4 + (12 - 4)e^{-\frac{t}{6}}, \qquad t \geq 0$$

For part b) we note that $v(0) = 12$, $v(\infty) = 12$, so that this is a really stupid problem since:

$$v(t) = 12$$

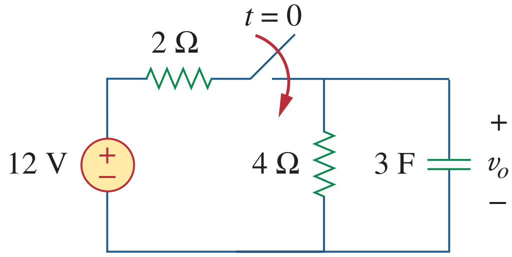
**Problem 7.42:**
Example: Find $v_0(t)$ for $t > 0$.




**The $t$-land approach:**
We observe that $v_o(0) = 0$, $v_o(\infty) = 8$, $C = 3$ and $R_{eq} = 4||2 = 4/3$, so that:

$$v(t) = 8 - 8e^{-\frac{t}{4}}, \qquad t \geq 0$$

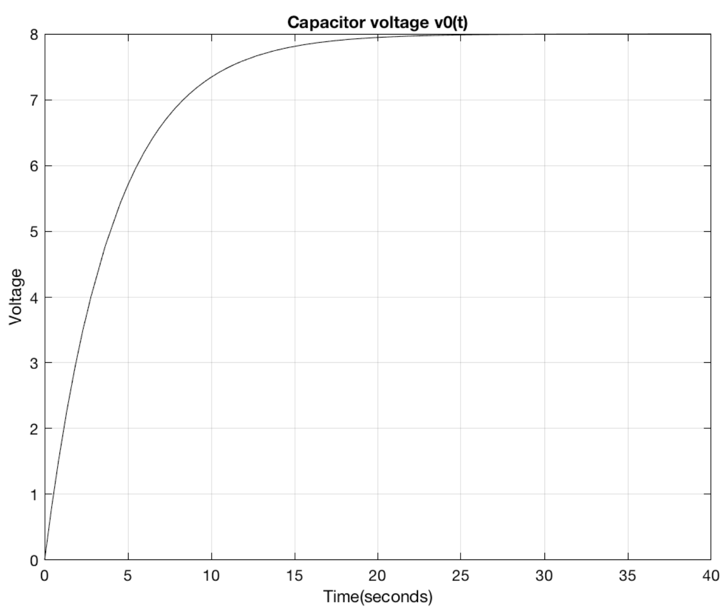
**Problem 7.42:**
Example: Find $v_0(t)$ for $t > 0$.




**The $s$-land approach:**

```
%% EE 2010 — Circuit Analysis
clear all
syms s t V0 v0
% Node equation
V0(s) = solve((V0—(12/s))/2 + V0/4 + 3*s*V0==0,V0)
% and the time—domain response
v0(t) = ilaplace(V0(s))
%
fplot(v0(t),[0,40]), grid
title('Capacitor voltage v0(t)'); xlabel('Time(seconds)'); ylabel('Voltage')
%%
```

and the result: `V0(s) = 8/(4*s^2 + s), v0(t) = 8 - 8*exp(-t/4)`



### Problem 7.56:

**7.56** For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

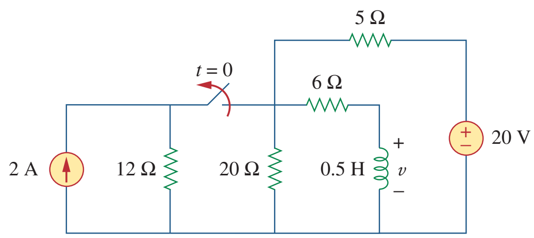



**Figure 7.122**
For Prob. 7.56.

This problem can be a bit confusing since one is inclined to jump to a first-order solution for $v(t)$ using the two steady-state values $v(0^-) = 0$ and $v(\infty) = 0$. But this is incorrect because while the inductor **current** cannot instantaneously change, the inductor **voltage** can indeed change instantaneously. That is, while $v(0^-) = 0$, $v(0^+) \neq 0$!

One could find the inductor **current** and then find the voltage as $v(t) = Ldi(t)/dt$, or one could find the inductor voltage directly. If we persue the inductor current, we have a node equation before the switch opens:

```
%% Problem 7.56 a
% Node equation
clear all
syms va
va = solve(-2 + va/12 + va/20 + va/6 + (va-20)/5 ==0,va)
i = va/6
```

from which we obtain $i(0) = 2A$. Again, after the switch opens, we have

```
%% Problem 7.56 b
% Node equation
clear all
syms va
va = solve(va/20 + va/6 + (va-20)/5 ==0,va)
i = va/6
```

which yields $i(\infty) = 8/5 = 1.6A$. We then have (with $R = 6 + 20||5 = 10$ so that $\tau = 0.05$):

$$i(t) = 1.6 + (2 - 1.6)e^{-20t}, \qquad 0 \leq t$$

so that

$$v(t) = 0.5\frac{di(t)}{dt} = -4e^{-20t}, \qquad 0 \leq t$$

## Problem 7.62:

**7.62** For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

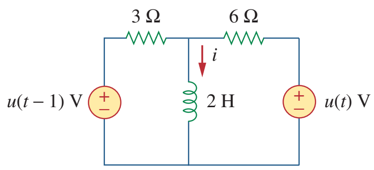



**Figure 7.127**
For Prob. 7.62.

Here is another sequential transient, where the final condition of the one epoch is the initial condition of the next epoch. Each of these is simple, just a matter of keeping track of time. So... for each time epoch, consider that circuit from it's initial time to $t = \infty$.

*For* $0 \leq t \leq 1$*:* We have $i(0) = 0$, $i(\infty) = 1/6$, $\tau = \frac{L}{R} = \frac{2}{6||3} = 1$, so that:

$$i(t) = 1/6 - 1/6e^{-t}, \qquad 0 \leq t \leq 1$$

*For* $1 \leq t$*:* We have $i(1) = 1/6 - 1/6e^{-1} = 0.1054$, $i(\infty) = 1/6 + 1/3 = 1/2$, $\tau = \frac{L}{R} = \frac{2}{6||3} = 1$, so that:

$$i(t) = 0.1054e^{-(t-1)}, \qquad 1 \leq t$$

... and we MUST have that $i(1)$ from the first part matches $i(1)$ from the second part. Check! Hence:

$$i(t) = \begin{cases} 1/6 - 1/6e^{-t}, & 0 \leq t \leq 1 \\ 0.1054e^{-(t-1)}, & 1 \leq t \end{cases}$$