ISE 2211 • Exam II • 6 June 2019 NAME SOLUTION

1) **Historical data suggests** that there is a 76% probability that Joe Tritschler's son will want to go on a bike ride to Corner Cone for frozen yogurt on any given summer evening. Over the course of 30 summer evenings, what is the probability that he will want to go to Corner Cone on at least 25 of them? Also determine the expected value, variance, and standard deviation of the number of evenings he will want to go to Corner Cone. Hint: let the Binomially-distributed random variable *X* represent the number of evenings Joe's son will want to go to Corner Cone.

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$P(X \geq 25) = P(25) + P(26) + P(27) + P(28) + P(29) + P(30)$$

+3 ← no points if only P(25) computed!

$P(25) = \binom{30}{25} .76^{25} (1-.76)^{30-25} = \underline{0.1189}$

$P(26) = \binom{30}{26} .76^{26} (1-.76)^{4} = \underline{0.07241}$

$P(27) = \binom{30}{27} .76^{27} (.24)^{3} = \underline{0.03397}$

$P(28) = \binom{30}{28} .76^{28} (.24)^{2} = \underline{0.01153}$

$P(29) = \binom{30}{29} .76^{29} (.24) = \underline{0.002517}$

$P(30) = \binom{30}{30} .76^{30} \cancel{(.24)^{0}} = \underline{0.0002657}$

+1 combinations calculated correctly

+1 p = 0.76, 1-p = 0.24

+1 no computation errors

(no double jeopardy)

$\therefore P(X \geq 25) = \sum = 0.2396$ or $\approx 24\%$ +1

$$\mu = E(X) = np = 30 \cdot .76 = 22.8$$ +1

evenings +1

$$\sigma^2 = V(X) = np(1-p) = 30 \cdot .76(.24)$$

$$= 5.472 \text{ evenings}^2$$ +1 +1

$$\sigma = +\sqrt{\sigma^2} = 2.339 \text{ evenings}$$ +1

2) On average, Joe's son will encounter seven barking dogs over the course of the 0.9-mile bike ride to Corner Cone. If the number of barking dogs may be modelled as a discrete Poisson process, determine the probability of encountering two or fewer barking dogs within the first 1,000 feet. (Note: 1 mile = 5280 feet.) Also determine the expected value and variance of the number of barking dogs in that first 1,000 feet of the ride.

Formulae:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$$\left(7 \text{ dogs}/.9 \cancel{\text{miles}}\right)\left(\frac{1 \cancel{\text{mile}}}{5280 \text{ ft.}}\right)\left(1000 \text{ ft.}\right)$$

$$\rightarrow \lambda = 1.473 \text{ dogs} \qquad +2 \qquad = E(x)$$

$$P(X \le 2) = P(2) + P(1) + P(0) \qquad +1$$

$$P(2) = \frac{e^{-1.473} \cdot 1.473^2}{2!} = 0.2487 \qquad +2$$

$$P(1) = 0.3377 \qquad +2$$

$$P(0) = 0.2292 \qquad +2$$

$$\therefore P(X \le 2) = 0.2487 + 0.3377 + 0.2292$$

$$= 0.8156 \text{ or } 81.56\% \qquad +1$$

$$\sigma^2 = V(x) = \lambda$$

$$= 1.473 \text{ dogs}^2 \qquad +1$$

3) What is a probability mass function? To which type of random variable does it correspond?

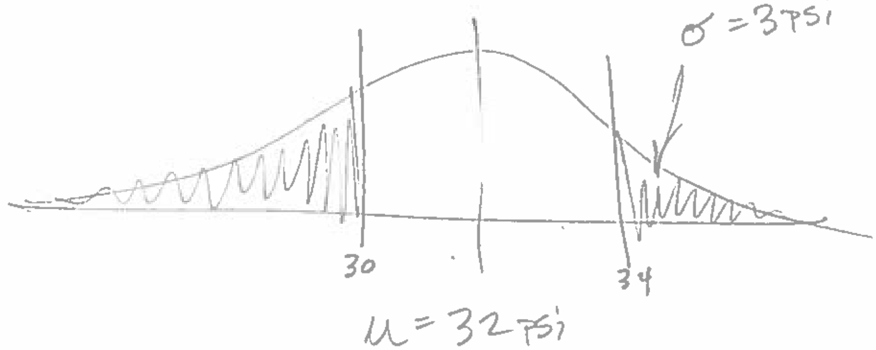
probability distribution in which

$\sum f(x) = 1$ (most important characteristic) +1

discrete +1

4) On any given summer evening, the mean tire pressure of one of the wheels on his son's bike trailer is 32 psi with a standard deviation of 3 psi, normally-distributed. What is the probability of a tire pressure *outside* the range of 30 to 34 psi? Shade this probability on rough sketches of both the normal and standard-normal distributions.

$$Z = \frac{X-\mu}{\sigma}$$

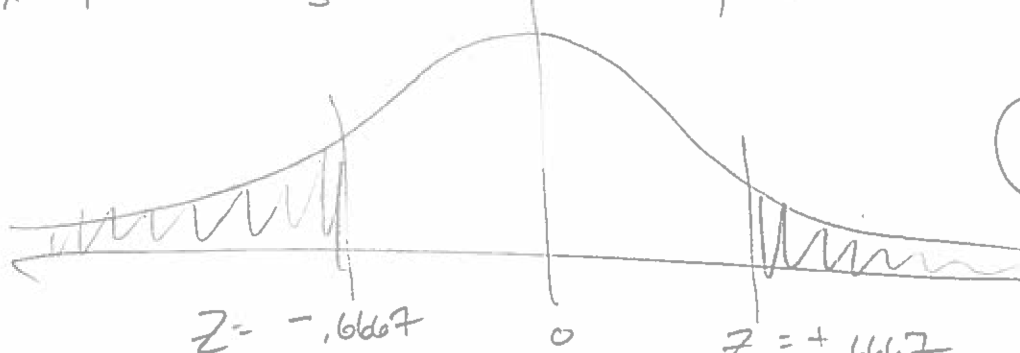


+2

$$Z\Big|_{X=30} = \frac{30-32}{3} = -.6667$$

$$Z\Big|_{X=34} = \frac{34-32}{3} = +.6667$$

+2



+2

close enough!

$P(Z < -.67) = 0.251429$ +1 (table)

symmetry: $P(Z > +.67) = 0.251429$ +1

$\therefore P(X<30,\ X>34) = 0.251429 \times 2 = 0.5029$ or 50% +1