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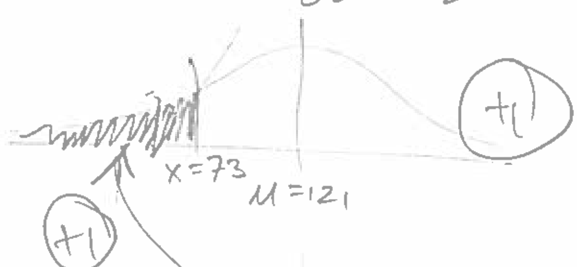
ISE 2211 • Exam II • 20 September 2019 NAME SOLUTION

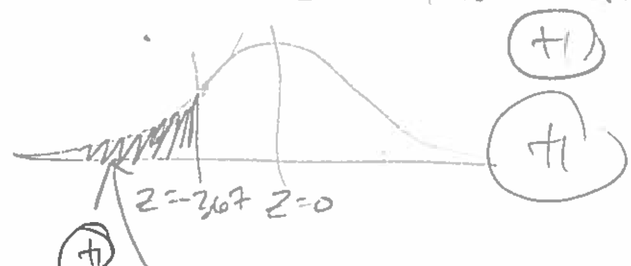
1) You just called Dr. Miller in his office to see how you did on the Differential Equations final exam. He said, "you got a 73..." Awesome! "...out of 150." Ouch. If the population mean final exam score is 121 points with a standard deviation of 18 points and final exam scores are normally-distributed, determine how many standard deviations away from the mean score you are by computing the *z*-value. Sketch both your score and the standardized score against the two distributions.

$$Z = \frac{X - \mu}{\sigma} \quad (+1)$$

$$= \frac{73 - 121}{18} = -2.67 \quad (+1) \qquad \text{(ouch indeed)}$$

$\therefore$ $2\frac{2}{3}$ standard deviations below mean (+1)





Determine the probability of getting a 73 or below on the exam. Sketch this probability against the normal and standard-normal distributions.

$$P(X < 73) = P(Z < -2.67) \quad (+1)$$

$$= 0.003793 \quad (+1) \qquad \text{(table)}$$

or 0.3793%

<9

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2) Fact: my children are deranged. My son Harold likes to climb onto the back of the couch and roll down the front, which he does successfully (i.e., without rolling onto the floor, hitting his head, knocking over his sister, etc.) 90 percent of the time. (For the record, we do not encourage this behavior; but we are outnumbered.) Determine the probability that over the course of an evening, in which he attempts this stunt 39 times in a row, that he ~~will only be~~ unsuccessful four or fewer times. Let the binomially-distributed random variable X represent the number of successful stunts. Also determine the expected value and variance of the number of successful couch-rolls.

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

$p = 0.90$ +1

$n = 39$ +1

Unsuccessful four or fewer times out of 39

$\equiv P(39) + P(38) + P(37) + P(36) + P(35)$ +1

unsuccessful zero times

one time

$P(39) = \binom{39}{39}^{=1} 0.9^{39} (1-p)^{39-39} = 0.01642$

$P(38) = \binom{39}{38}^{=39} 0.9^{38} (0.1)^1 = 0.07117$

$P(37) = \binom{39}{37}^{=741} 0.9^{37} (0.1)^2 = 0.1502$

$P(36) : \binom{39}{36}^{=9139} 0.9^{36} (0.1)^3 = 0.2059$

$P(35) : \binom{39}{35}^{=82251} 0.9^{35} (0.1)^4 = 0.2059$

+1 combinations

+1 Probabilities

$\%P = 0.6496$ +1 or ~65%

$\mu = E(x) = np$

$= 39 \cdot 0.9$

$= 35.1$ stunts

+1

$\sigma^2 = np(1-p)$

$= 35.1\ (0.1)$

$= 3.51$ stunts^2

+1

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3) The number of knotholes in a piece of lumber has a discrete Poisson distribution, with the mean value for a specific grade of 2x4 being three knotholes in an eight-foot-long board. What is the probability of getting six knotholes in the twelve-foot piece of garbage you just bought at Lowe's?

Formulae:

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\mu = \lambda$

$\sigma^2 = \lambda$

$\lambda = \frac{3 \text{ knotholes}}{8\text{ft.}} \times 12\text{ft.} = 4.5 \text{ knotholes}$ +1

$f(6) = \frac{e^{-4.5} \cdot 4.5^6}{6!} = 0.1281$ +1

or 12.81 %

If you would *really* like two or fewer knotholes in said twelve-foot 2x4, determine the probability of this actually happening. Suggest a course of action. get a life? give up?

$P(x \leq 2) = P(0) + P(1) + P(2)$ +1

$f(0) = \frac{e^{-4.5} \cdot 4.5^0}{0!} = 0.01111$

$f(1) = \frac{e^{-4.5} \cdot 4.5^1}{1!} = 0.04999$

$f(2) = \frac{e^{-4.5} \cdot 4.5^2}{2!} = 0.1125$ +1

$\therefore P(x \leq 2) = 0.1736$ +1