B3

ISE 2211 • Exam V • 10 July 2018 NAME SOLUTION

1) Two different brands of wood glue are being investigated in a shop that specializes in custom cabinetry. Ten test cabinets were prepared and both brands tried on biscuit joints in each cabinet. A tensile adhesion test was performed on each joint and the mean difference in tensile strengths between the two wood glues was determined to be $\bar{d}$ = 10.4 lbs with a variance of s_d = 17.8 lbs. What type of test is described in this problem? 6

paired t-test +1

Test the following hypotheses and state whether the results suggests one brand of wood glue has a different tensile strength than the other at the α = 0.05 fixed level of significance:

H_0: $\mu_D = 0$
H_1: $\mu_D \neq 0$

$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{10.4}{17.8/\sqrt{10}} = 1.848$$

+1

critical values : $\pm t_{\alpha/2,\, n-1} = \pm t_{.025,\, 9}$ +1

↑ pairs (i.e., cabinets)

$= 2.262$ (table)

+1

$t_0 \not> {}^{+}t_{.025,9}$; ∴ fail to reject H_0 +1

data suggests they do not differ +1

2) Two different grades of precision tape transport bearings were tested to determine if there is a significant difference in the variance of runout in microinches^2. Sixteen grade-7 bearings were tested and the mean runout was $\bar{x}_1 = 3.339$ µin with variance $s_1^2 = 0.5672$ µin^2. 25 grade-8 bearings were tested with $\bar{x}_2 = 2.846$ µin and $s_2^2 = 0.2248$ µin^2. Test the following hypotheses and state whether you would reject or fail to reject the null hypothesis that the two different grades of bearings have equal variances of runout at the $\alpha = 0.05$ fixed level of significance. <u>Sketch</u> the appropriate distribution, indicating the test statistic and critical values.

6

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

$$f_0 = s_1^2 / s_2^2 = 0.5672 / 0.2248 = \underline{2.523}$$

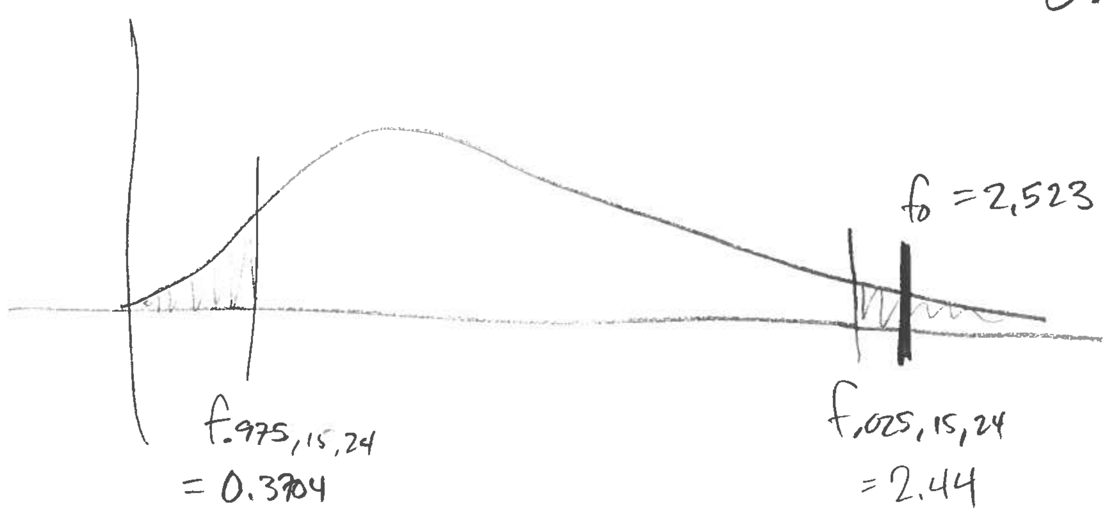
+1

$$f_{\alpha/2,\, n_1-1,\, n_2-1} = f_{.025,\, 15,\, 24} = 2.44$$ +1

$$f_{1-\alpha/2,\, n_1-1,\, n_2-1} = \frac{1}{f_{\alpha/2,\, n_2-1,\, n_1-1}} = \frac{1}{f_{.025,\, 24,\, 15}} = \frac{1}{2.70}$$

+1

$$= 0.3704$$ +1



+1

$f_0 > f_{.025,\,15,\,24}$ ∴ reject H_0 +1

data suggests variances of runout differ

Write a 95% C.I. on the ratio of population variances and verify that it draws the same conclusion as the fixed-α hypothesis test above.

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$$f_{\alpha/2,\, n_2-1,\, n_1-1} = f_{.025,\, 24,\, 15} = 2.70 \quad +1$$

$$f_{1-\alpha/2,\, n_2-1,\, n_1-1} = \frac{1}{f_{\alpha/2,\, n_1-1,\, n_2-1}} = \frac{1}{f_{.025,\, 15,\, 24}} = \frac{1}{2.44}$$

+1

$$= 0.4098 \quad +1$$

$$2.523 \cdot 0.4098 < \frac{\sigma_1^2}{\sigma_2^2} < 2.523 \cdot 2.70$$

$$1.034 < \frac{\sigma_1^2}{\sigma_2^2} < 6.8121 \quad +2$$

$[min^2]$

∴ the C.I. does not contain unity ; ∴ reject that $\sigma_1^2 = \sigma_2^2$ +1

Now test the mean runouts of the two grades of bearings using the p-value approach. Population variances are assumed to be unequal.

10

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 \neq 0$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3.339 - 2.846}{\sqrt{\frac{0.5672}{16} + \frac{0.2248}{25}}}$$

$$t_0 = 2.339 \quad (+1)$$

$$V = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} = \frac{0.04444^2}{\frac{0.03545^2}{15} + \frac{0.008992^2}{24}}$$

$$= 22.67 \quad (+1)$$

round down to 22 degrees of freedom (+1)

$t_{.025, 22} = 2.074$ (+1) (table) $\therefore 0.01 < \frac{p}{2} < 0.025$ (+1)

$t_{.01, 22} = 2.508$

$0.02 < p < 0.05$ (+1)

reject H_0 @ $\alpha = 0.05$ (+1)

Write a 95% confidence interval on the difference in means of the two grades of bearings. With respect to the hypotheses in the above problem, verify that it draws the same conclusion and state why.

$$\mu_1 - \mu_2 : \bar{x}_1 - \bar{x}_2 \pm \underbrace{t_{\alpha/2, v}}_{t_{.025,22}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$3.339 - 2.846 \pm 2.074 \sqrt{\frac{0.5672}{16} + \frac{0.2248}{25}}$$

$$0.05577 < \mu_1 - \mu_2 < 0.9302 \quad (+2)$$

[min.]

– C.I. does not contain zero; reject H_0 (+1)

3) Last week, a canning session at everyone's favorite brewery resulted in twelve out of 144 rejected cans due to under-filling. Chris Hutson blew his top and went on a completely unnecessary diatribe about corporate waste and the degeneration of societal values. Jon Vanderglas says it was <u>sabotage</u>!!! This week only 9 out of 144 cans were rejected and Jayson Hartings would like to demonstrate to Mr. Hutson that this is a significant improvement by testing the following hypotheses using the *p*-value approach: 5

H_0: $p_1 = p_2$

H_1: $p_1 > p_2$

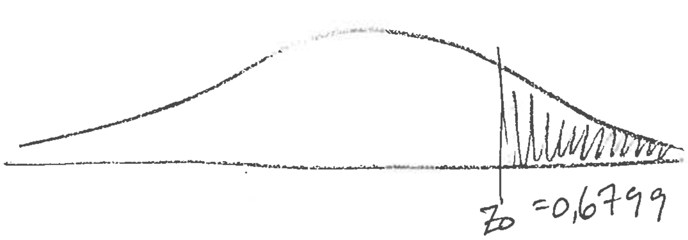
Hint: use the more recent canning session as population 2.

$$\left.\begin{aligned}\hat{p}_1 &= \frac{12}{144} = 0.08333 \\ \hat{p}_2 &= \frac{9}{144} = 0.0625\end{aligned}\right\} +1$$

$$\hat{p} = \frac{12+9}{144+144} = 0.07292 \quad +1$$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}} = \frac{0.08333 - 0.0625}{\sqrt{0.07292(1-0.07292)\left[\frac{1}{144}+\frac{1}{144}\right]}}$$

$$= \underline{0.6798} \quad +1$$



$$\text{P-value} = P(Z > 0.6798) = 0.248252 \quad +1$$

<u>reject</u> Ho for all $\alpha > p$ +1

Is this a significant improvement @ *α* = 0.05? Who should get a life – Chris, Jon, Jayson, or all three?

$P \ggg \alpha$; ∴ this is <u>not</u> a significant improvement!!!

(all three)