Paired T-Test

- special case of two-sample t-test when observations are collected in pairs from the beginning!

ex: two methods used to determine shear strength of steel plate girders;

<u>Lehigh</u> and <u>Karlsruhe</u>

- the problem: methods appear to give different results!
  - <u>not</u> good for decision-making; Which method do you trust?!

- we <u>could</u> test, say, 9 girders with Lehigh and 9 w/ Karlsruhe; compute sample means and standard deviations for each sample; do regular two-sample hypothesis test on difference in sample means

- how do we know if any differences we find are due to the methods, or the girders?

  i.e., the test specimens

- Paired t-test is an alternative way of doing this
  - test both methods on same test specimen
    - look for differences on each specimen, compute mean and std. dev. on differences

- **mean difference** rather than difference in means!

∴ this computation of mean difference is called $\bar{d}$ and std. dev. S_d

∴ we then test hypotheses that μ_d is some hypothesized value

$$H_0 : \mu_D = \Delta_0$$

$$H_1 : \mu_D \neq \Delta_0$$

$$\boxed{\text{test statistic: } T_0 = \frac{\bar{d} - \Delta_0}{S_d / \sqrt{n}}}$$

∴ rejection criteria for fixed-α:

# of pairs of observations

$$\text{critical values} = \pm t_{\alpha/2,\, n-1}$$

i.e., # of test specimens

· let's look @ data:

$$\bar{d} = 0.276889$$

$$S_d = 0.135027$$

$$n = 9 \quad \text{girders}$$

test: $H_0: \mu_d = 0$ $= \Delta_0$ @ $\alpha = 0.05$

$H_1: \mu_d \neq 0$

$$t_0 = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{0.2769}{0.1350/\sqrt{9}}$$

$$\boxed{t_0 = 6.08}$$ → big!!!!....

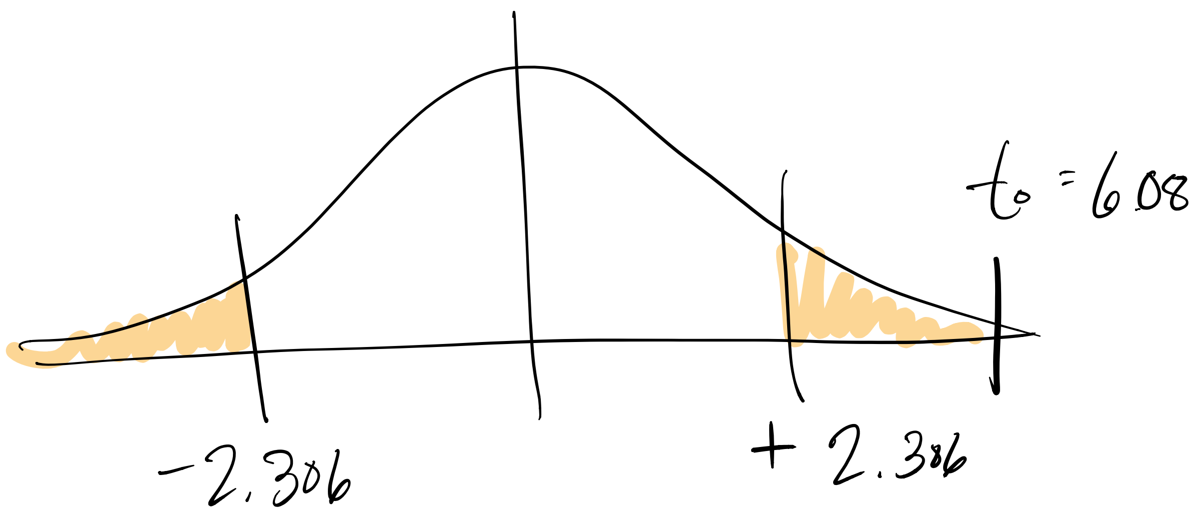
critical values: $\pm t_{\alpha/2,\, n-1} = \pm t_{.025,8}$

pairs; i.e., test specimens $= \underline{2.306}$

side note: if this had been a regular two-sample hypothesis test,

$$d.o.f. = n_1 + n_2 - 2 = 16$$

∴ smaller d.o.f. is minor disadvantage of paired t-test!



$$t_0 \ggg +t_{\alpha/2, n-1}$$

Strongly reject H_0

∴ data suggests Karlsrhue method gives bigger number; use Lehigh to be safe?

∴ on your exam: may need to compute differences to then get $\bar{d}$ and s_d.

C.I. on μ_d mean difference in paired t-test

$$\mu_d: \quad \bar{d} \pm t_{\alpha/2, n-1} \, s_d/\sqrt{n}$$

ex: time to parallel park two very different-sized vehicles

∴ we could get ten people to park one model, and ten to park the other model

∴ how do we know any measured difference in parking time is due to vehicle and not aptitude of driver?

- solution: have **same** person park each vehicle, compute differences !

$\longrightarrow$ paired t-test

- let's look at data: $n = 14$ subjects

$$\bar{d} = 1.21 \text{ s} \leftarrow \text{not very big!}$$

$$S_d = 12.68 \text{ s} \leftarrow \text{big std. dev.!}$$

confidence interval
$\rightarrow$ two-sided

- write 90% C.I. on μ_d

$$t_{\alpha/2,\, n-1} = t_{.10/2,\, 14-1}$$

$$t_{.05,\, 13} = 1.771$$

$$\bar{d} \pm t_{\alpha/2,\, n-1}\, S_d/\sqrt{n}$$

$$1.21 \pm 1.771 \cdot 12.68/\sqrt{14}$$

$$-4.79 < \mu_d < 7.21 \text{ (s)}$$

∴ note: C.I. does contain zero!

∴ therefore, we would fail to reject

$$H_0 : \mu_d = 0 \quad \Delta_0$$

∴ we cannot conclude that there is a difference in parking times due to vehicle size