Discrete Uniform Distribution

.. each of n values has <u>equal probability</u>

$$f(x_i) = \frac{1}{n}$$

ex: coin toss; $p(\text{heads}) = \frac{1}{2}$
heads/tails

fair six-sided die: $S\{1\ 2\ 3\ 4\ 5\ 6\}$

$P(4) = \frac{1}{6}$

ex: 1st digit of a part's serial number may be
$S\{0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\}$

.. assume <u>uniformly</u> distributed

.. what is the probability that the first digit is a 7?

$\frac{1}{n} = \frac{1}{10} = 0.1$
or 10%

mean or expected value of a discrete uniform random variable w/ consecutive integers:

$$\mu = E(x) = \frac{b+a}{2}$$ ← simple arithmetic mean

where $S\{a \leq X \leq b\}$

serial number problem: $S\{0 \ldots 9\}$ (a = 0, b = 9)

$$\mu = E(x) = \frac{9+0}{2} = 4.5$$

∴ ok that this is not a possible digit!

$$\sigma^2 = V(x) = \frac{(b-a+1)^2 - 1}{12}$$ ← if consecutive

$$\sigma^2 = \frac{(9-0+1)^2 - 1}{12} = 8.25 \ (\text{digit})^2$$

standard deviation: $\sigma = +\sqrt{\sigma^2} = 2.872$

∴ if range of values in sample space is not consecutive, use regular formulae!

Binomial Distribution

Bernouilli Trial : only two outcomes

heads/tails   pass/fail   good/bad

etc..

1.) trials are independent

2.) each trial only has two possible outcomes, defined as "success" or "failure"

3.) probability of "success", denoted as p, is constant.

... then the binomial random variable X has the probability distribution

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Combinations! (unordered)

... where p is the probability of "success" ex: water samples

- known: each sample has 10% chance of containing some pollutant
- samples are considered independent
- find probability that out of 18 samples, 2 will contain the pollutant

let X : # of "successfully" polluted samples

$n = 18$

$\therefore\ p = 0.1$

exactly two samples contain pollutant

$$P(2) = f(2) = \binom{n}{x} p^x (1-p)^{n-x}$$

probability of non-success!

$$= \binom{18}{2} 0.1^2 (1-0.1)^{18-2}$$

multiplication rule for two successes!

the rest of the samples!

$$\frac{n!}{x!(n-x)!}$$

$$\frac{18!}{2!(18-2)!} = \frac{18 \cdot 17 \cdot \cancel{16!}}{2!\ \cancel{16!}} = 153$$

$$= 153 \cdot .1^2 \cdot .9^{16}$$

$$= 0.2835 \text{ or } 28.35\%$$

∴ what about the probability of at least four samples polluted?

$$P(X \geq 4) = P(4) + P(5) + P(6) \ldots + P(18)$$

too much work !!!

$$P(X \geq 4) = 1 - \underbrace{P(X < 4)}_{\text{Compliment of } P(X \geq 4)}$$

$$P(X < 4) = P(0) + P(1) + P(2) + P(3)$$

way better

$$= \underbrace{0.150 + 0.3 + 0.2835 + 0.168}_{\text{see if you get these!}}$$

$$= 0.902$$

$$\therefore P(X \geq 4) = 1 - 0.902 = 0.098$$

or 9.8%

.. turns out the camera flash recharge rate example was a Bernouilli trial problem!

let X : # of $\underbrace{\text{passing}}_{}$ cameras

↪ = "success"

$\underbrace{P = 0.8}_{}$ ← beginning of problem: known 80% pass rate!

$n = 3$ $\underbrace{\text{independent}}_{}$ cameras

zero successes! failure

$$P(0) = \binom{3}{0} 0.8^{0} (1 - 0.8)^{3-0}$$

three failures!

exactly zero cameras passing

$$= 0.2^{3} = \underbrace{0.008}_{} \checkmark \text{ yep}$$

$$\frac{3!}{0!(3-0)!} = 1$$

$$P(1) = \binom{3}{1} 0.8^{1} (1 - 0.8)^{3-1}$$

$$\frac{3!}{1!(3-1)!} = \frac{3 \cdot \cancel{2!}}{\cancel{2!}} = 3$$

$$= 3 \cdot 0.8 \cdot 0.2^{2}$$

$$= 0.096$$

three outcomes for which one camera passed!

$P(2) = 0.384$
$P(3) = 0.512$ } check these!

∴ the sampling without replacement example with 850 manufactured parts is not a Bernouilli Trial
∴ not independent because parts aren't independently selected

mean and variane of binomial random variable:

$$\mu = E(x) = np$$

water sample example: $n = 18$, $p = 0.1$

$$\therefore E(x) = 18 \cdot 0.1 = 1.8 \text{ polluted samples}$$

$$\sigma^2 = V(x) = np(1-p)$$

$$\therefore V(x) = 18 \cdot 0.1 (1 - 0.1) = 1.62 \text{ (polluted samples)}^2$$

Poisson Distribution

poisson process: X : # of flaws

- average # of flaws in an interval : λ

$$\lambda = n p$$

probability of flaw in interval

then $f(x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$

exact # of flaws

key: interpreting original problem to get λ!

ex: CD manufacturing process

# of flaws on disc : Poisson process

- known : average # of flaws per square cm is 0.1
- disc under study is 100 cm^2
- find probability of 12 flaws on disc

exactly

∴ trick is to find λ !

∴ we want probability of 12 flaws on disc

∴ interval is disc !

∴ need average (known) # of flaws on disc, not square cm !

$$0.1 \frac{\text{flaws}}{\cancel{cm^2}} \cdot \frac{100 \cancel{cm^2}}{\text{disc}} = 10 \text{ flaws/disc} = \lambda$$

$$P(12) = \frac{e^{-10}\, 10^{12}}{12!} = 0.09478$$

or 9.478 %

∴ what about the probability of zero flaws?

$$P(0) = \frac{e^{-10}\, 10^{0}}{\underbrace{0!}_{=1}} = 0.0000454$$

not very likely!

$$P(X \leq 12) = P(0) + P(1) + P(2) + \ldots P(12)$$

up to 12

$$= 0.792$$

check!