ISE 2211 • Exam III • 14 June 2018 NAME SOLUTION 20

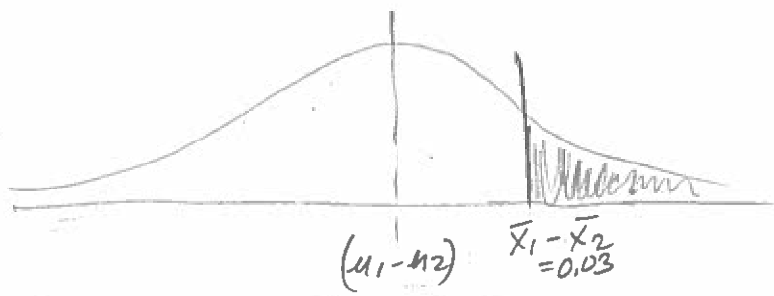
1) A transformer manufacturer wants to improve the line regulation of one of its core products and may accept a new design if it could increase the efficiency at full load by 3%. It is known that the mean efficiency of the old transformer design is 0.910 with standard deviation 0.030, and the new transformer has a mean efficiency of 0.925 and standard deviation 0.028. If 25 examples of each of the old and new transformers are tested, determine the probability that the new transformer has an efficiency at least 0.03 greater than the old transformer. Sketch these probabilities on both the normal and standard normal distributions.

call pop. 1 "new", pop. 2 "old"

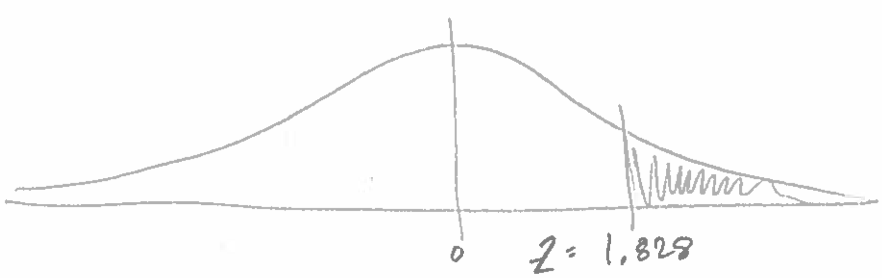
$$\therefore Z = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{0.03 - (0.925 - 0.910)}{\sqrt{\frac{.028^2}{25} + \frac{.030^2}{25}}}$$

$$Z = 1.828$$

+2



+1



+1

$$P(Z > 1.828) = 1 - P(Z < 1.828)$$ +1

$$= 1 - 0.966375 = 0.033625$$ +1

$P(Z < 1.83)$

2) 25 power transformers were tested for efficiency at full load current and the results were $\bar{x} = 0.9134$ and $s = 0.04117$. Write a 95% prediction interval on the efficiency of the 26th power transformer.

$$X_{26}: \quad \bar{x} \pm t_{\alpha/2,\, n-1}\, s\sqrt{1+\frac{1}{n}}$$

correct n +1

$$t_{\alpha/2,\, n-1} = t_{.025,\, 24} = 2.064$$

(Table) +1

$$0.9134 \pm 2.064 \cdot .04117 \sqrt{1+\frac{1}{25}}$$

$$0.8267 \leq X_{26} \leq 1.000$$

+2

3) At Yellow Springs Brewery, a few of their craft beers are sold in 12-oz. cans which are filled and packaged on-site. It is *essential* to the integrity of the operation that under-filled cans be taken home and consumed <u>immediately</u> by YSB employees!!! During a special midnight canning session, twelve out of 144 cans produced sadly fell below the rejection threshold and had to be removed from the premises. Write a 95% confidence interval on the population proportion of rejected cans.

$$\hat{p} = \frac{x}{n} = \frac{12}{144} = 0.08333 \quad +1$$

$$p: \quad \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$Z_{\alpha/2} = Z_{.025} = 1.960 \quad +1$$

$$0.08333 \pm 1.960\sqrt{\frac{.08333(1-0.08333)}{144}}$$

$$0.03819 < p < 0.1285 \quad +2$$

If it is very important to know the proportion of rejected cans within ±2%, determine the minimum sample size needed to accomplish this.

$$n = 0.25\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= 0.25\left(\frac{1.96}{0.02}\right)^2$$

+1

$$n = 2401 \text{ cans} \quad +1$$

4) Joe Tritschler used to have a 1970 Cadillac Coupe DeVille that burned an alarming amount of oil. The number of miles it took to burn a quart seemed to vary a lot; he checked this over the course of 12 quarts of oil and found the sample mean number of miles to be 472, with a sample standard deviation of 119. Write a 95% upper confidence bound on the standard deviation of the number miles it took to burn through a quart of oil.

$n = 12$

$\bar{x} = 472$

$s = 119$

Upper conf. bound:

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}}$$ +1

$$\chi^2_{1-0.05, 12-1} = \chi^2_{.95, 11} = 4.57$$ +1

$$\sigma^2 \leq \frac{11 \cdot 119^2}{4.57}$$

$$\sigma^2 \leq 34086$$ +1

$$\therefore \sigma \leq +\sqrt{34086}$$

$$\sigma \leq 185 \text{ miles}$$ +1