HW 4 1.8 # 2,4,9,11,14,15,16,17,19 Alex Yeah
1.9 # 1,3,6,7,8,9,15,19,20,22

1.8

2) $\begin{bmatrix}0.5\\0\\-2\end{bmatrix}, \begin{bmatrix}0.5a\\0.5b\\0.5c\end{bmatrix}$

4) $\left[\begin{array}{ccc|c}1&-3&2&6\\0&1&-4&-7\\3&-5&-9&-9\end{array}\right] \xrightarrow{R_3-3R_1} \left[\begin{array}{ccc|c}1&-3&2&6\\0&1&-4&-7\\0&4&-15&-27\end{array}\right] \xrightarrow{R_3-4R_2} \left[\begin{array}{ccc|c}1&-3&2&6\\0&1&-4&-7\\0&0&1&1\end{array}\right]$

$x_3 = 1$ $\quad x_2 - 4 = -7$ $\quad x_2 = -3$ $\quad x_1 - 3(-3) + 2(1) = 6$ $\quad x_1 + 9 + 2 = 6$ $\quad x_1 = -5$

$x = \begin{bmatrix}-5\\-3\\1\end{bmatrix}$ unique

9) $\left[\begin{array}{cccc|c}1&-4&7&-5&0\\0&1&-4&3&0\\2&-6&6&4&0\end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{cccc|c}1&-4&7&-5&0\\0&1&-4&3&0\\0&2&-8&6&0\end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{cccc|c}1&-4&7&-5&0\\0&1&-4&3&0\\0&0&0&0&0\end{array}\right]$ ↓

$\left[\begin{array}{cccc|c}1&-4&7&-5&0\\0&1&-4&3&0\\0&0&0&0&0\end{array}\right]$

Set x_4 free $\quad$ set x_3 free $\quad x_2 - 4x_3 + 3x_4 = 0$ $\quad x_2 = 4x_3 - 3x_4$

$x_1 - 4(4x_3 - 3x_4) + 7x_3 - 5x_4 = 0$

$x_1 - 16x_3 + 12x_4 + 7x_3 - 5x_4 = 0$

$x_1 - 9x_3 + 7x_4 = 0$

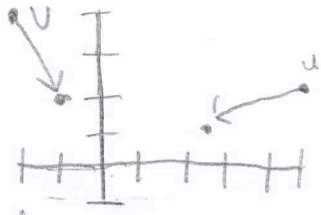
$x_1 = 9x_3 - 7x_4$

$x = \begin{pmatrix}9x_3 - 7x_4\\4x_3 - 3x_4\\x_3\\x_4\end{pmatrix} = x_3\begin{bmatrix}9\\4\\1\\0\end{bmatrix} + x_4\begin{bmatrix}-7\\-3\\0\\1\end{bmatrix}$

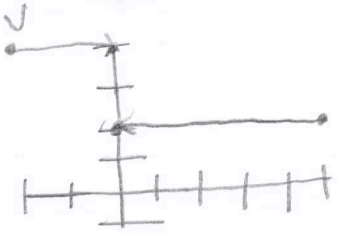
11) $\left[\begin{array}{cccc|c}1&-4&7&-5&-1\\0&1&-4&3&1\\2&-6&6&4&0\end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{cccc|c}1&-4&7&-5&-1\\0&1&-4&3&1\\0&2&-8&6&2\end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{cccc|c}1&-4&7&-5&-1\\0&1&-4&3&1\\0&0&0&0&0\end{array}\right] \rightarrow \left[\begin{array}{cccc|c}1&-4&7&-5&-1\\0&1&-4&3&1\end{array}\right]$

Yes

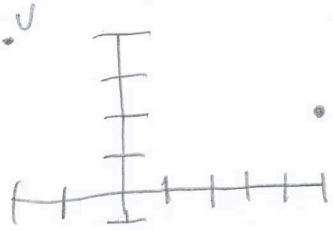
1.8 continued

14) 

scaled by $\frac{1}{2}$

15) 

projected to origin

16) 

no change

17) $T(u)=\begin{bmatrix}2\\1\end{bmatrix}$ $T(v)=\begin{bmatrix}-1\\3\end{bmatrix}$

$3T(u)=\begin{bmatrix}6\\3\end{bmatrix}$ $2T(v)=\begin{bmatrix}-2\\6\end{bmatrix}$

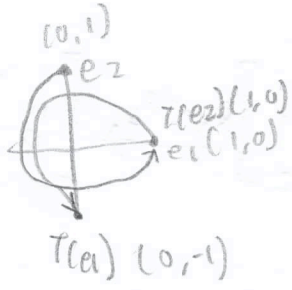
$3T(u)+2T(v)=\begin{bmatrix}4\\9\end{bmatrix}$

19) $\begin{bmatrix}2&-1\\5&6\end{bmatrix}\cdot\begin{bmatrix}5\\-3\end{bmatrix}=\begin{bmatrix}2(5)-1(-3)\\5(5)+6(-3)\end{bmatrix}=\begin{bmatrix}10+3\\25-18\end{bmatrix}=\begin{bmatrix}13\\7\end{bmatrix}$

$\begin{bmatrix}2&-1\\5&6\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}2x_1-x_2\\5x_1+6x_2\end{bmatrix}$

1.9

1) $\begin{bmatrix}3&-5\\1&2\\3&0\\1&0\end{bmatrix}$

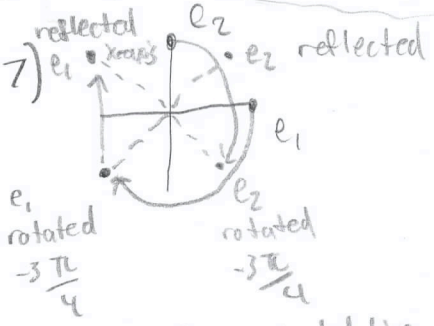
3) 

$\begin{matrix}e_1&e_2\end{matrix}$
$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
standard

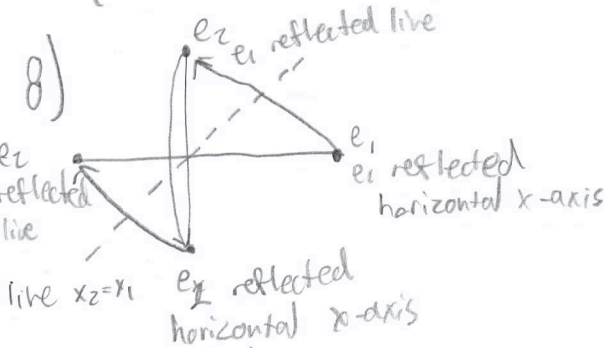
$\begin{matrix}T(e_1)&T(e_2)\end{matrix}$
$\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

1.9 continued

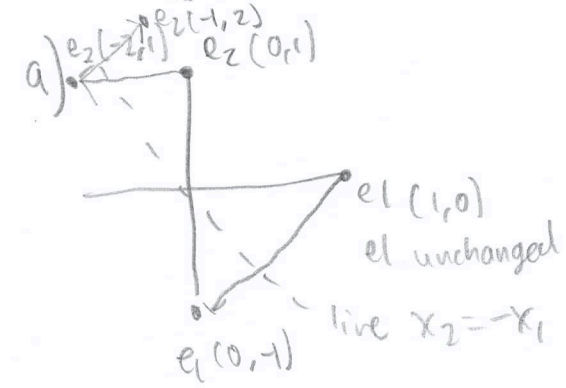
6) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $T(e_1) = e_1$, $T(e_2) = e_2 + 3e_1$ $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

7) 

$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

8) 

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9) 

$e_1: \begin{bmatrix} 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$e_2: \begin{bmatrix} 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} -2 \\ 1 \end{bmatrix} \to \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

15) $\begin{bmatrix} 3 & 0 & -2 \\ 4 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

19) $\begin{bmatrix} x_1 - 5x_2 + 4x_3 \\ x_2 - 6x_3 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

20) $\begin{bmatrix} 2 & 0 & 3 & -4 \end{bmatrix}$

~~22) $\begin{bmatrix} x_1 - 2x_2 \\ -x_1 + 3x_2 \\ 3x_1 - 2x_2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix}$~~

1.9 continued

22) $\left[\begin{array}{cc|c} 1 & -2 & -1 \\ -1 & 3 & 4 \\ 3 & -2 & 9 \end{array}\right] \xrightarrow[R_3+3R_2]{R_2+R_1} \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 7 & 21 \end{array}\right] \xrightarrow{R_3-7R_2} \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \end{array}\right]$

$x_2 = 3$

$x_1 - 2(3) = -1$

$x_1 - 6 = -1$

$x_1 = 5$

$x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$