Hypothesis Testing

- often we are tasked with choosing between two competing statements called hypotheses a population parameter, such as μ.

ex: an ejection seat should have a mean propellant burning rate of 50 cm/s

.. too low, and pilot might not be ejected

.. too high, and pilot might be injured

.. hypotheses are always set up with a null hypothesis:

$$H_0: \mu = 50 \text{ cm/s}$$

... always an equality about an unknown population parameter

... and an alternative hypothesis:

$$H_1: \mu \neq 50 \text{ cm/s}$$ (this H_1 happens to be two-sided)

.. how do we test these hypotheses so we know which one to support?

.. in this example, H_1 suggests that μ could be <u>higher</u> or <u>lower</u> than 50 cm/s

.. we <u>could</u> have proposed either of two <u>one-sided</u> alt. hypotheses:

$$H_1: \mu < 50 \text{ cm/s} \quad [\text{lower one-sided}]$$

OR

$$H_1: \mu > 50 \text{ cm/s} \quad [\text{upper one-sided}]$$

.. testing hypotheses utilizes information from a <u>random sample</u> of the population

.. only two possible outcomes:

1.) <u>reject the null hypothesis (H_0)</u>

OR

2.) <u>fail to reject H_0</u>

- we never say that we support a hypothesis, or say it's true

- we have three methods available for testing hypotheses :
  1.) confidence interval method
  2.) fixed significance level method
  3.) p-value method

- we'll start with the C.I. method

Tests on μ, σ Known

- test the following hypotheses at the $\alpha = 0.05$ level of significance :

$$H_0 : \mu = 50 \text{ cm/s}$$
$$H_1 : \mu \neq 50 \text{ cm/s}$$

test data: a sample of $n = 25$ ejection seats yielded a mean propellant burning rate of

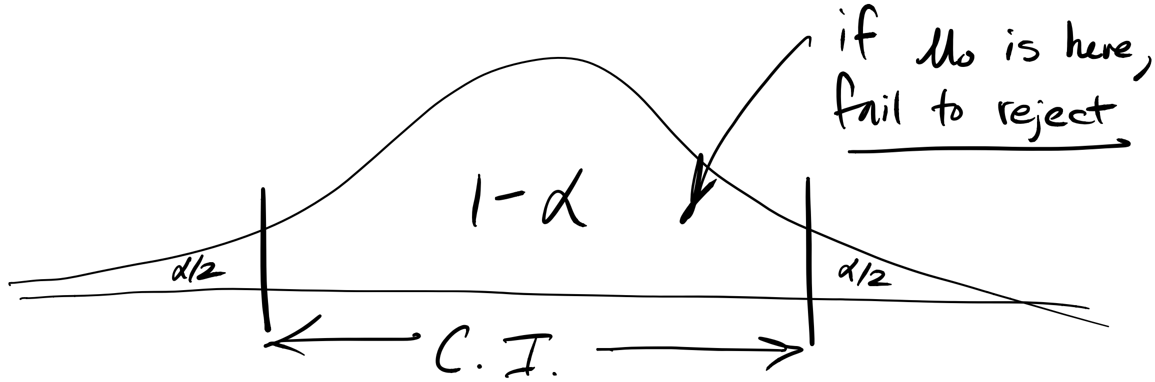
$$\bar{x} = 51.3 \text{ cm/s}$$

$\therefore$ known : $\sigma = 2$ cm/s

$\therefore$ 51.3 sounds pretty close to 50, right?

$\therefore$ C.I. method : if a hypothesized value of μ is inside a $(1-\alpha) \times 100\%$ confidence interval on μ, we fail to reject

$$H_0 : \mu = \mu_0$$



∴ if μ_0 is <u>outside</u> the C.I., <u>reject H_0</u>.

∴ first, write the C.I.:

$$\mu: \quad \bar{x} \pm Z_{\alpha/2}\,\sigma/\sqrt{n}$$

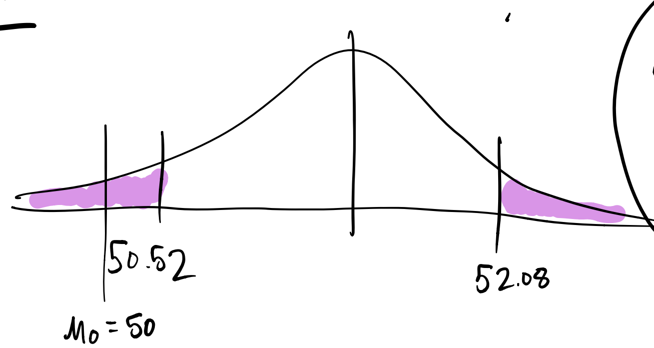
$$Z_{\alpha/2} = Z_{.05/2} = Z_{.025} = 1.960$$

(bottom row of t-table)

$$51.3 \pm 1.960 \cdot 2/\sqrt{25}$$

$$\boxed{50.52 < \mu < 52.08 \text{ (cm/s)}}$$

∴ the hypothesized value of $\mu_0 = 50$ cm/s <u>is not</u> inside the C.I.!



∴ <u>reject H_0</u>

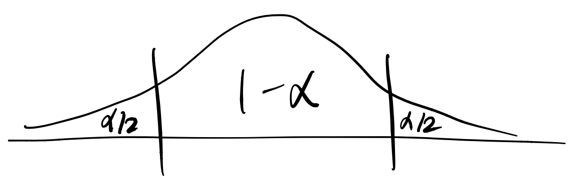
@ $\alpha = 0.05$

∴ So 51.3 cm/s is not close to 50 cm/s!

Fixed-significance level method

a/k/a fixed-α

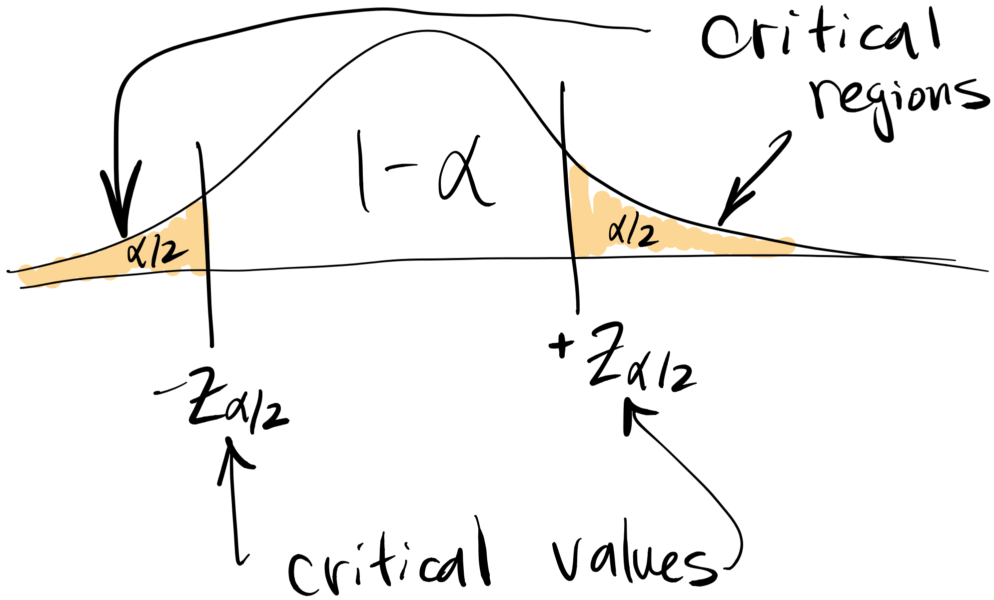
∴ What if we don't need a C.I. on μ and just want to know if μ_0 is "significant" with respect to some α?



∴ where is μ_0 in here?

∴ Compute test statistic, which normallizes our sample data into some z-value

∴ Compare this test statistic to z-values corresponding to α; called critical values.



test statistic :

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

reject H_0 if $Z_0 > +Z_{\alpha/2}$

or $Z_0 < -Z_{\alpha/2}$

i.e., reject if Z_0 is in either critical region.

μ_0

recall: $H_0 : \mu = 50 \text{ cm/s}$
$H_1 : \mu \neq 50 \text{ cm/s}$

test at $\alpha = 0.05$ using fixed-α approach

$\bar{x} = 51.3 \text{ cm/s}$ $n = 25$

known $\sigma = 2 \text{ cm/s}$

test statistic:

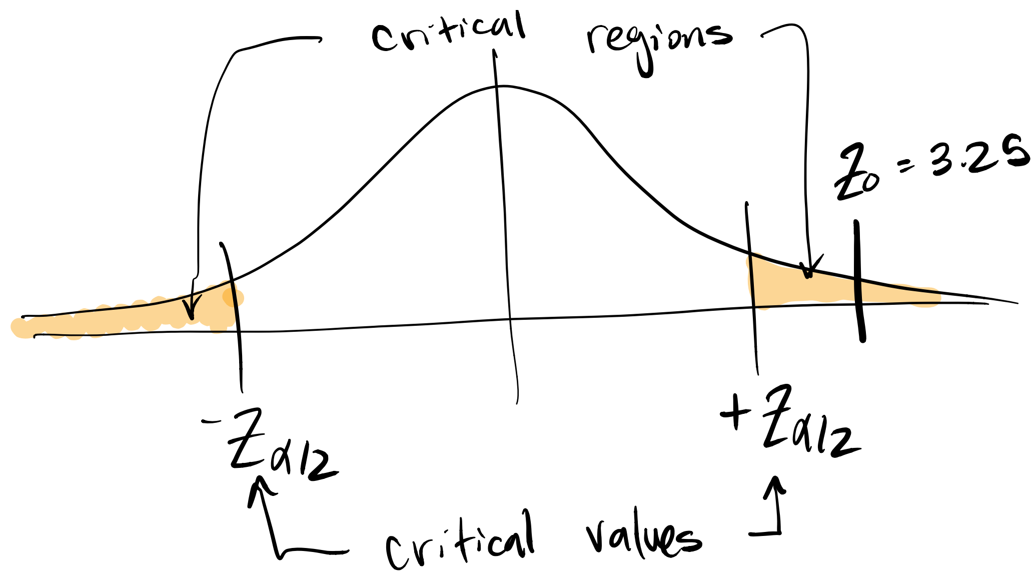
$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{51.3 - 50}{2/\sqrt{25}}$$

$$Z_0 = 3.25$$

red flag!!! > three standard deviations above mean !!!

Critical values: $\pm Z_{\alpha/2} = \pm Z_{.025} = \pm 1.960$

$Z_0 > +Z_{\alpha/2}$; $\therefore$ reject H_0

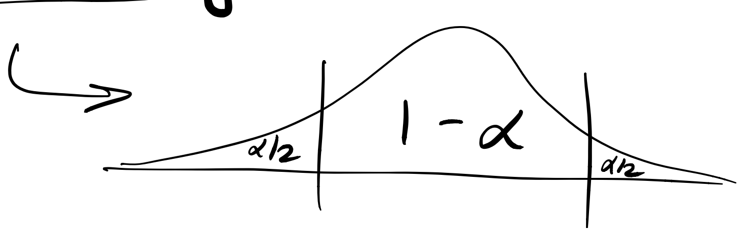


∴ Z_0 is in upper critical region!!! ∴ reject H_0

P-value approach

∴ What if we don't care to compute critical values and compare Z_0 to them?

∴ We can compute a probability associated with Z_0 and compare that to a reference probability.

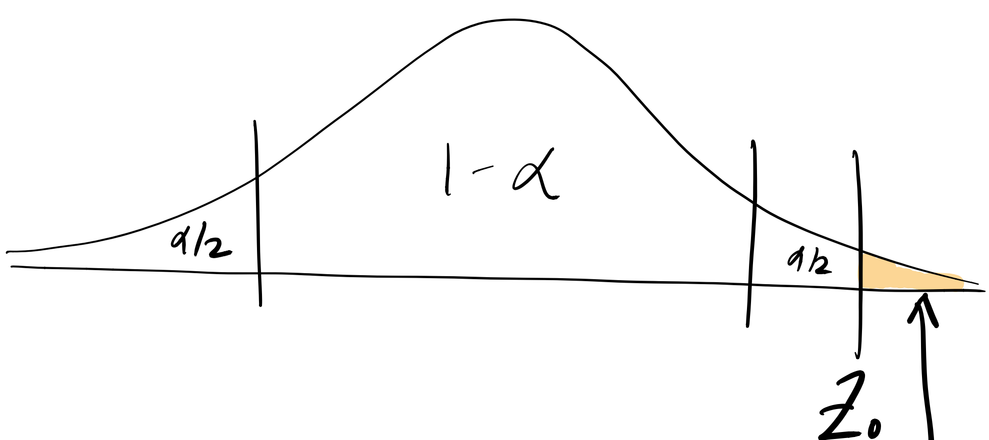


- Probability associated with our test statistic is called a <u>p-value</u>.

- we would reject H_0 for any p-value less than α.

  - small p-value $\rightarrow Z_0$ is "out there" $\therefore$ reject

- in reality, it's a little more complicated for a two-sided H_1



this would be $\dfrac{\text{p-value}}{2}$ and we would double it.

ex: test $H_0: \mu = 50$ cm/s

$H_1: \mu \neq 50$ cm/s

using p-value approach

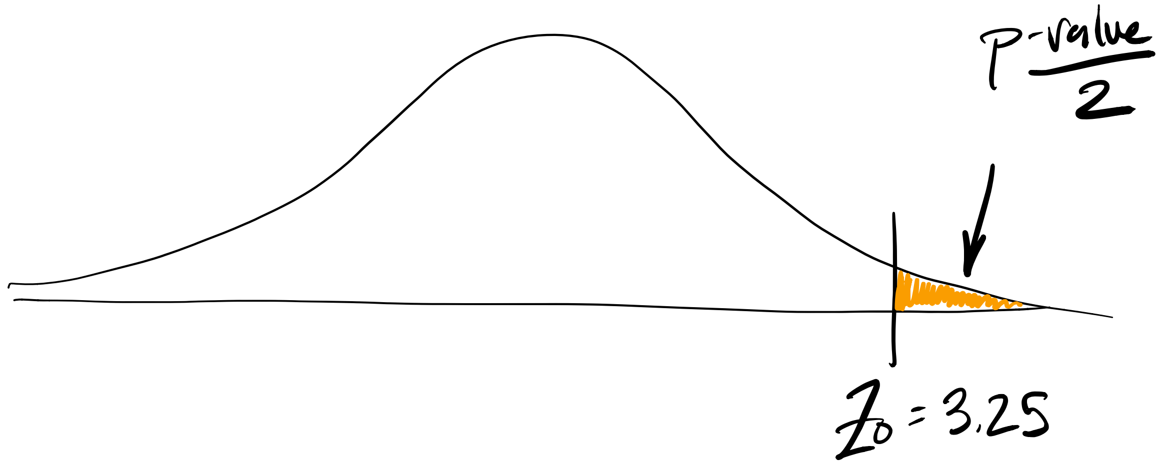
write final conclusion with respect to $\alpha = 0.05$

first: compute test statistic, Z_0;

same as fixed-α approach

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{51.3 - 50}{2/\sqrt{25}}$$

$$Z_0 = 3.25$$



$$P(Z > 3.25) = 1 - \underbrace{P(Z < 3.25)}_{\text{Z-table gives cumulative probabilities!}}$$

$$1 - 0.999423 = 0.000577$$

$$\frac{\text{p-value}}{2} = 0.000577$$

$\therefore$ p-value = 0.001154

$\therefore$ this is a very small probability!

p-value $\lll \alpha = 0.05$

reject H_0