EE 2010 Circuit Analysis

Module 06: Conservation Laws Yielding Systems of Equations: Node Equations Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

We now formally introduce techniques drawn from conservation laws that are the foundation of circuit analysis.

Definition: *Kirchhoff's current law (KCL):* The algebraic sum of currents entering a node (or a closed boundary) equals zero.

$$\sum_{n=1}^{N} i_n = 0$$

Yep! It has to go somewhere! This result is a consequence of **conservation of charge.**

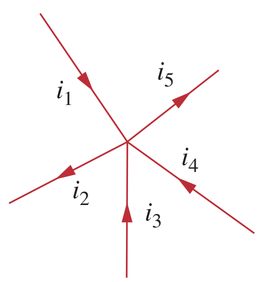


Figure 2.16
Currents at a node illustrating KCL.

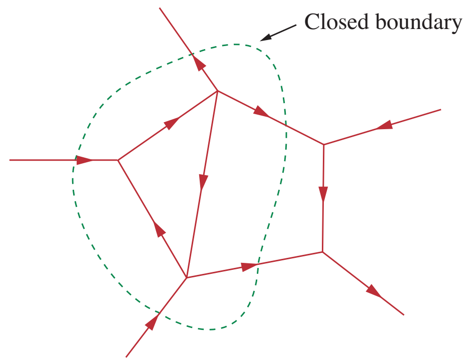
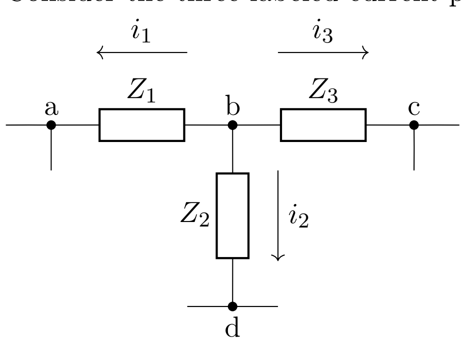


Figure 2.17
Applying KCL to a closed boundary.

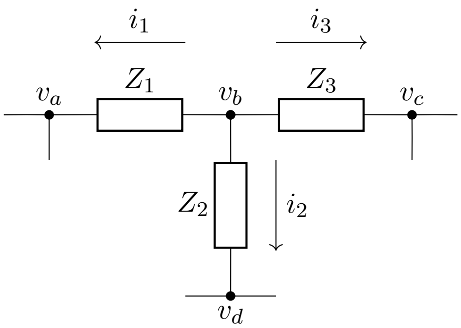
Consider the three labeled current paths in the subregion of the circuit below:



Clearly, KCL applies, so that we have $i_1 + i_2 + i_3 = 0$. Why is this alone not sufficient for a complete analysis of the circuit?

Since we (always) have impedance specifications for a modeled circuit, these **MUST** be incorporated if we are to arrive at a complete solution – but the impedance models are of use **ONLY** if current-voltage relationships are incorporated.

So ... with the impedance models specified, we *arbitrarily label* the node voltages in this subregion as:



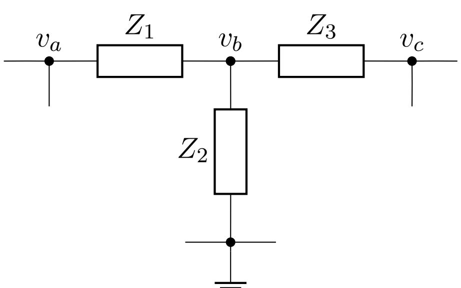
Recall that for *Passive Sign Convention*, current flows in the direction of a voltage drop. We will *assume* all currents are flowing away from node b and hence *assume* all other voltages $\{v_a, v_c, v_d\}$ are less than v_b because when this is not true, the algebra will take care of itself.

In this case, the KCL for this subregion becomes:

$$\begin{aligned} 0 &= i_1 \qquad + i_2 \qquad + i_3 \qquad (1) \\ 0 &= \frac{v_b - v_a}{Z_1} + \frac{v_b - v_d}{Z_2} + \frac{v_b - v_c}{Z_3} \end{aligned}$$

which are termed **"Node Equations"**.

Just one more simplification. Notice that all arbitrarily-labeled node voltages in this subregion are implicitly relative to some voltage in particular. We can reduce the number of voltage unknowns (without changing the voltage-difference unknowns) by arbitrarily-assigning one of the voltages as 0. We'll select $v_d = 0$ for this circuit:



so that the In this case, the node-equation describing this subregion becomes:

$$\frac{v_b - v_a}{Z_1} + \frac{v_b - 0}{Z_2} + \frac{v_b - v_c}{Z_3} = 0$$

where we have assumed (as I will always do) that all currents are leaving the node.

The technique we have just derived can be used to produce a *complete system of linear equations* describing any circuit. We can then apply algebraic tools to *answer any question* posed about any quantity anywhere in the circuit. More formally:

Definition: *The Node Technique (or Nodal Analysis):* A *methodical procedure* for composing a *complete set of linear equations*, based on Kirchoff's Current Laws (KCL) describing all relevant *voltages* of a circuit.

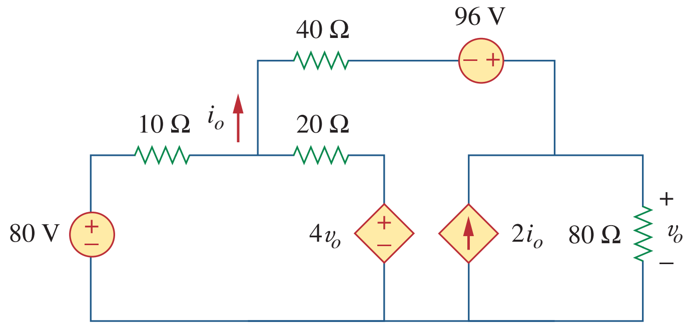


Figure 3.79
For Prob. 3.30.

In the circuit above, we can identify 6 *nodes* ($\geq$ 2-element connections) in this circuit. We will typically reduce the complexity by considering only the 3 ($\geq$ 3-element connections) = **essential nodes**. Further, by assigning a **reference node** ($v = 0$), this circuit can be described by just 2 essential unknown voltages. At that point, we will be able to completely describe this circuit in 4 (two node voltages, and two auxiliary equations for the dependent sources) linearly independent equations.

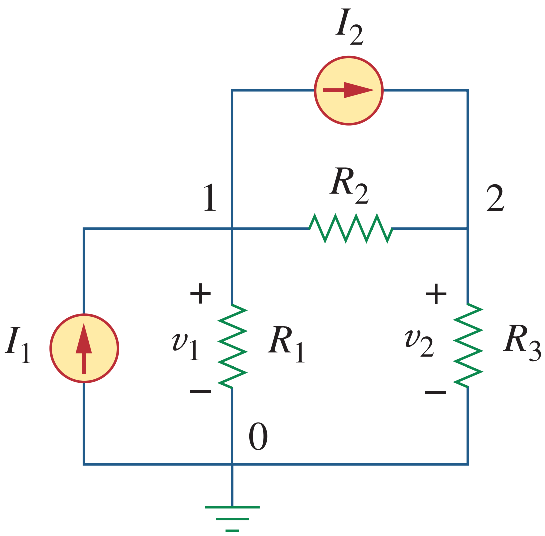
Node Voltage Procedure:

1. Identify the essential ($\geq$ 3-element connections) nodes
2. Select a node as the reference node = the node at *ground potential* = 0 Volts
3. Identify and label the voltages at nodes that are readily deduced
4. Assign voltage variables $v_a, v_b, \ldots$ to the remaining essential nodes where voltages are not readily deduced.
   (These voltages, or potential differences, are referenced with respect to the reference node.)
5. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
6. Enjoy the thrill of ending the consideration of each node with the powerful "$= 0$"
7. Add one additional equation for each dependent source specification if necessary

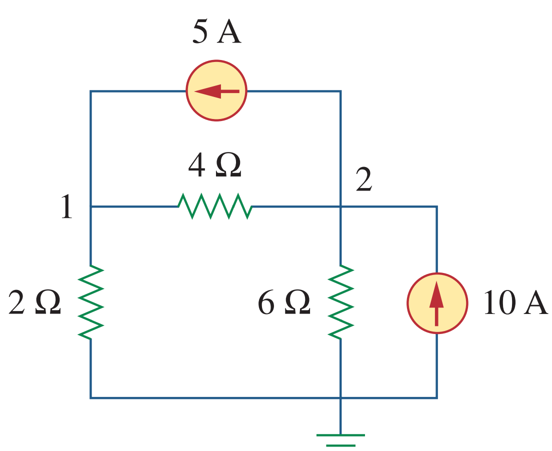
   Circuit analysis is now complete! But you may be asked to:
8. Invoke the power of algebra to solve for every assigned variable in the resulting system of equations.
9. Answer whatever questions are asked about the circuit using the solved values of knowledge of $v_a, v_b, \ldots$.

This technique solves EVERY lumped, linear circuit problem on the planet!

Let's do one.



Your turn:



(a)

Same technique, using $Z_R = R$, $Z_C = \frac{1}{sC}$, $Z_L = sL$.
Use symbolic I_s for the current source.

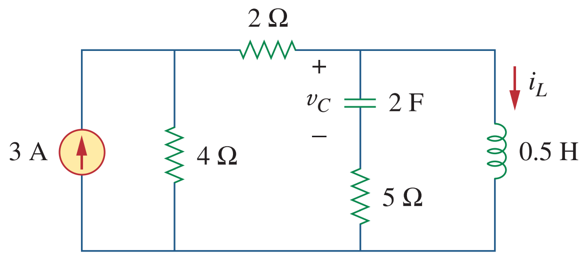


Figure 6.69
For Prob. 6.46.

Now with voltage sources. (Ignore the current labels.)

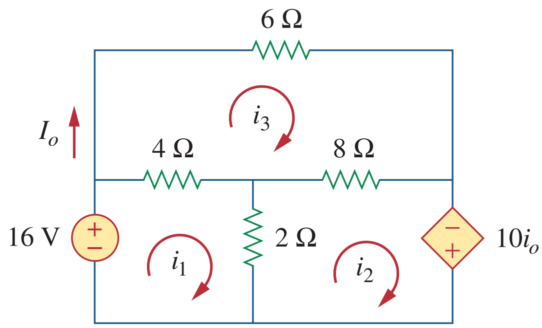


Figure 3.21
For Practice Prob. 3.6.

Your turn (ignore the current labels):

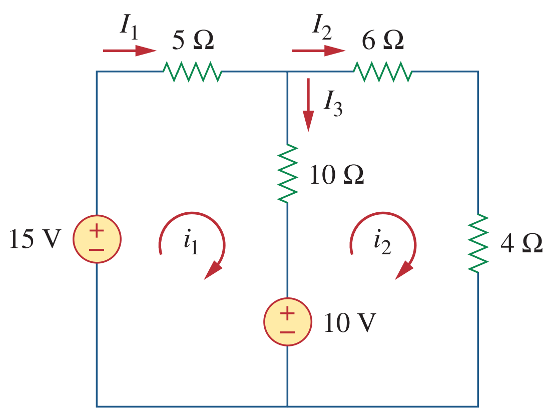


Figure 3.18
For Example 3.5.

A bit more complex. Same technique, more equations – call for a solver = Matlab.

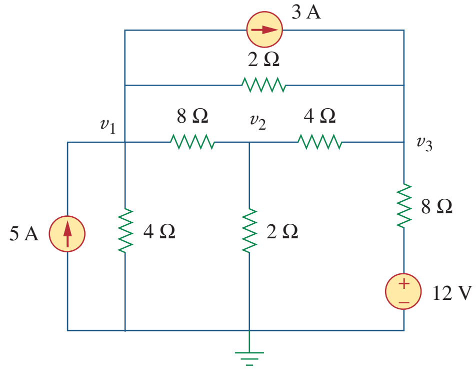


Figure 3.68
For Prob. 3.19.

```
%% Problem 3.19: Node equations
clear all
% Declare symbolic variables
syms v1 v2 v3
% Write node equations at v1, v2, v3 and solve
[v1,v2,v3]=solve(v1/4 - 5 + 3 + (v1-v3)/2 + (v1-v2)/8 == 0,...
  v2/2 + (v2-v1)/8 + (v2-v3)/4 == 0,...
  (v3-12)/8 + (v3-v2)/4 + (v3-v1)/2 - 3 == 0, v1,v2,v3)
double([v1,v2,v3])
```

Which yields:

```
v1 = 10
v2 = 74/15
v3 = 184/15
ans =  10.0000    4.9333   12.2667
```

Another dependent source problem.

3.23 Use nodal analysis to find V_o in the circuit of Fig. 3.72.

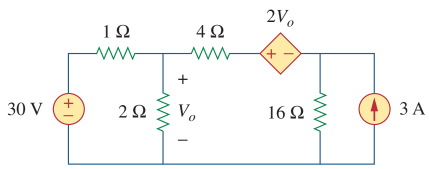


Figure 3.72
For Prob. 3.23.

```
%% Problem 3.23: Node equations
clear all
% Declare symbolic variables (for nodes a, b, c, d across top)
syms va vb Vo
% Write node equations at a, b, and an auxiliary equation for the
% dependent source (this one is simple)
% Solve for va, vb, Vo
[va, vb, Vo]=solve(va/2 + (va-30)/1 + (va - (vb + 2*Vo))/4 == 0,...
  -3 + vb/16 + ((vb + 2*Vo)-va)/4 == 0, Vo == va, va,vb,Vo)
Vo = double(Vo)
```

Which yields:

```
va = 648/29
vb = -240/29
Vo = 648/29 =  22.3448
```

Another .

3.24 Use nodal analysis and *MATLAB* to find V_o in the circuit of Fig. 3.73.

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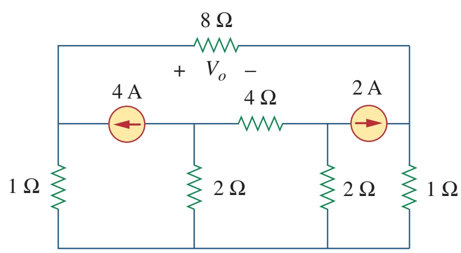

Figure 3.73
For Prob. 3.24.

```
%% Problem 3.24: Node equations
clear all
% Declare symbolic variables (for nodes a, b, c, d across top)
syms va vb vc vd
% Solve for va, vb, vc, vd
[va, vb, vc, vd] = solve(va/1 + (va-vd)/8 - 4 == 0,...
  vb/2 + 4 + (vb-vc)/4 == 0,...
  vc/2 + (vc-vb)/4 + 2 == 0,...
  vd/1 -2 + (vd-va)/8 == 0, va,vb,vc,vd)
% so in this case, Vo is
Vo = va - vd
```

Which yields:

```
va = 19/5
vb = -7
vc = -5
vd = 11/5
Vo = 8/5
```

Homework: Chapter 3 # 2, 4, 6, 8, 10, 12, 14, 22, 24, 26, 28, 30

3.7 In the circuit of Fig. 3.49, current i_1 is:

(a) 4 A (b) 3 A (c) 2 A (d) 1 A

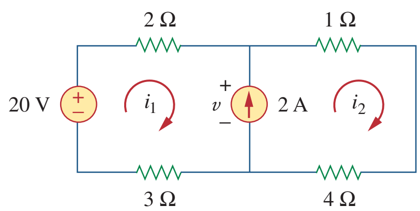

Figure 3.49
For Review Questions 3.7 and 3.8.

3.8 The voltage v across the current source in the circuit of Fig. 3.49 is:

(a) 20 V (b) 15 V (c) 10 V (d) 5 V

3.9 The *PSpice* part name for a current-controlled voltage source is:

(a) EX (b) FX (c) HX (d) GX

3.10 Which of the following statements are not true of the pseudocomponent IPROBE:

(a) It must be connected in series.
(b) It plots the branch current.
(c) It displays the current through the branch in which it is connected.
(d) It can be used to display voltage by connecting it in parallel.
(e) It is used only for dc analysis.
(f) It does not correspond to a particular circuit element.

Answers: 3.1a, 3.2c, 3.3a, 3.4c, 3.5c, 3.6a, 3.7d, 3.8b, 3.9c, 3.10b,d.

Problems

Sections 3.2 and 3.3 Nodal Analysis

3.1 Using Fig. 3.50, design a problem to help other students better understand nodal analysis.
e2d

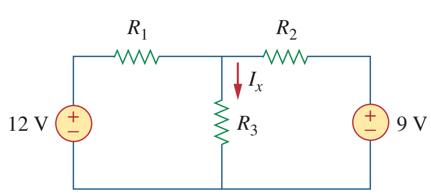

Figure 3.50
For Prob. 3.1 and Prob. 3.39.

3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2.

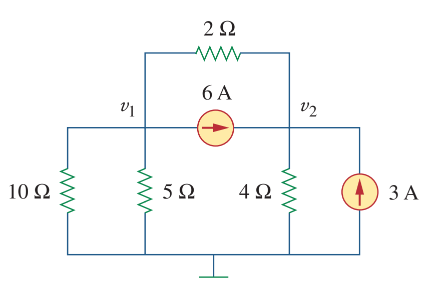

Figure 3.51
For Prob. 3.2.

3.3 Find the currents I_1 through I_4 and the voltage v_o in the circuit of Fig. 3.52.

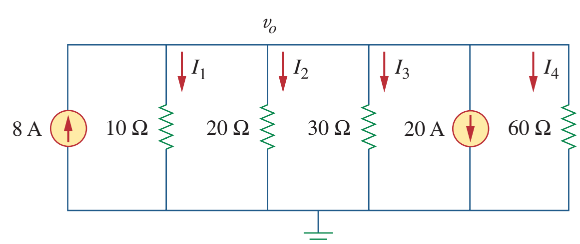

Figure 3.52
For Prob. 3.3.

3.4 Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4.

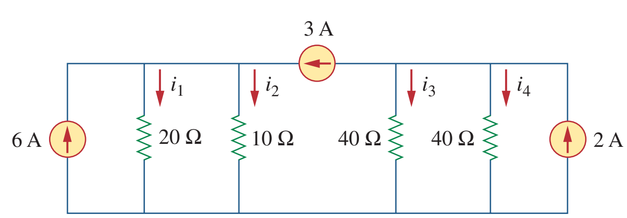

Figure 3.53
For Prob. 3.4.

3.5 Obtain v_o in the circuit of Fig. 3.54.

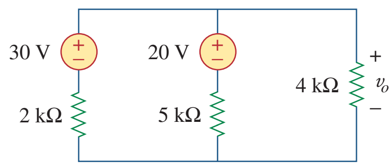

Figure 3.54
For Prob. 3.5.

3.6 Solve for V_1 in the circuit of Fig. 3.55 using nodal analysis.

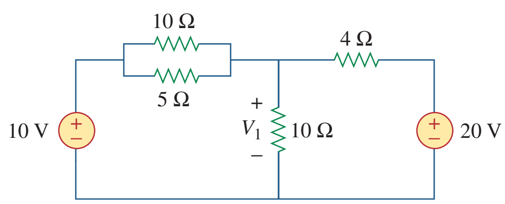

Figure 3.55
For Prob. 3.6.

3.7 Apply nodal analysis to solve for V_x in the circuit of Fig. 3.56.

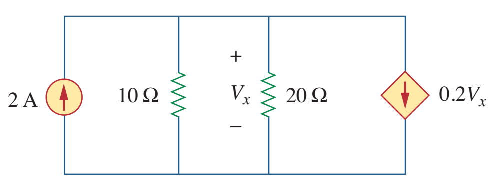

Figure 3.56
For Prob. 3.7.

3.8 Using nodal analysis, find v_o in the circuit of Fig. 3.57.

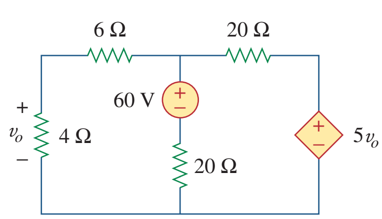

Figure 3.57
For Prob. 3.8 and Prob. 3.37.

3.9 Determine I_b in the circuit in Fig. 3.58 using nodal analysis.

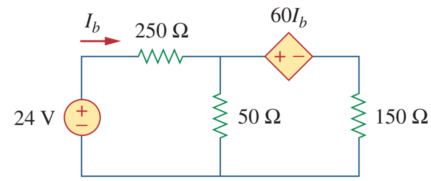

Figure 3.58
For Prob. 3.9.

3.10 Find I_o in the circuit of Fig. 3.59.

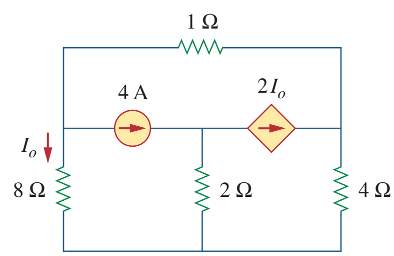

Figure 3.59
For Prob. 3.10.

3.11 Find V_o and the power dissipated in all the resistors in the circuit of Fig. 3.60.

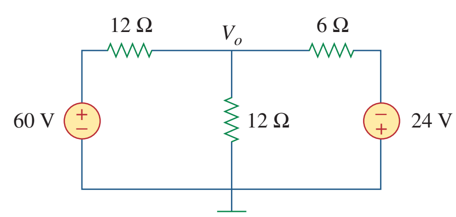

Figure 3.60
For Prob. 3.11.

3.12 Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

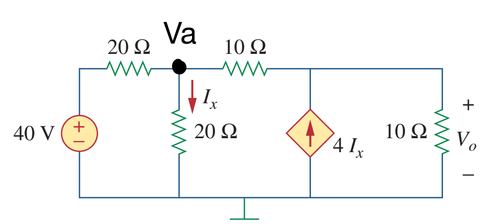

Figure 3.61 (Va-0)/20*20
For Prob. 3.12.

3.13 Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

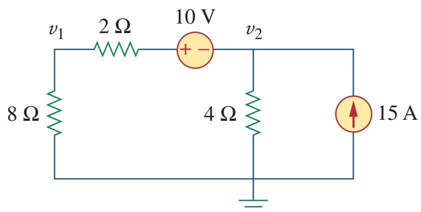

Figure 3.62
For Prob. 3.13.

3.14 Using nodal analysis, find v_o in the circuit of Fig. 3.63.

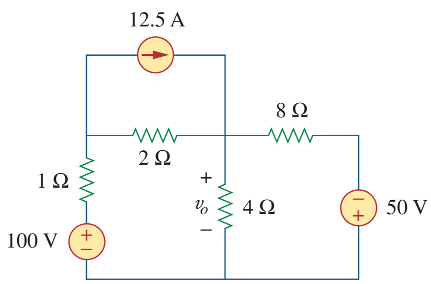

Figure 3.63
For Prob. 3.14.

3.15 Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

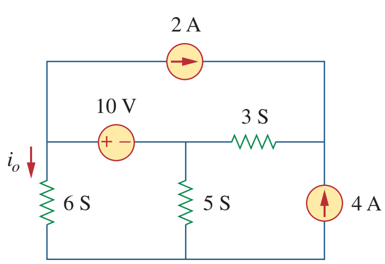

Figure 3.64
For Prob. 3.15.

3.16 Determine voltages v_1 through v_3 in the circuit of Fig. 3.65 using nodal analysis.

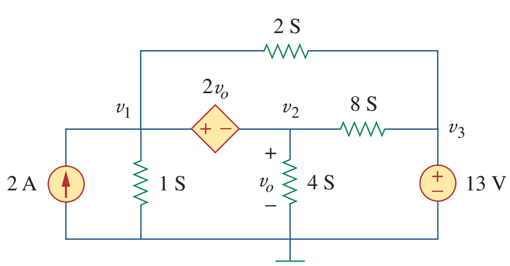

Figure 3.65
For Prob. 3.16.

3.17 Using nodal analysis, find current i_o in the circuit of Fig. 3.66.

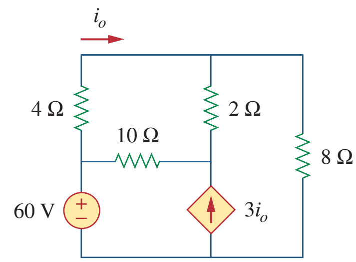

Figure 3.66
For Prob. 3.17.

3.18 Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

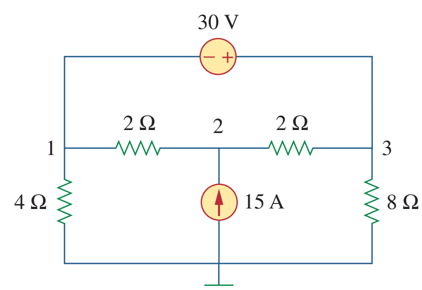

Figure 3.67
For Prob. 3.18.

3.19 Use nodal analysis to find v_1, v_2, and v_3 in the circuit of Fig. 3.68.

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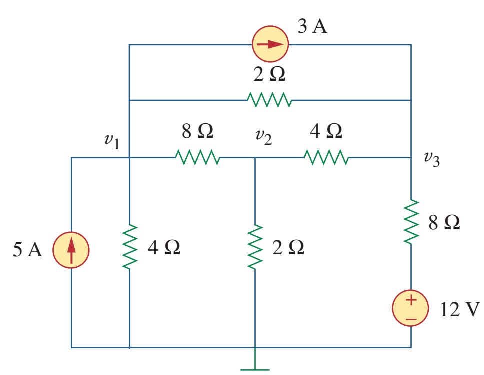

Figure 3.68
For Prob. 3.19.

3.20 For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

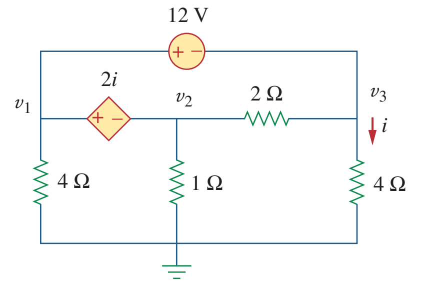

Figure 3.69
For Prob. 3.20.

3.21 For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

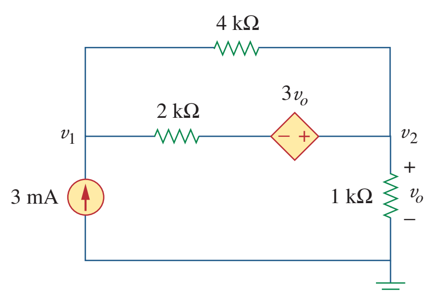

Figure 3.70
For Prob. 3.21.

3.22 Determine v_1 and v_2 in the circuit of Fig. 3.71.

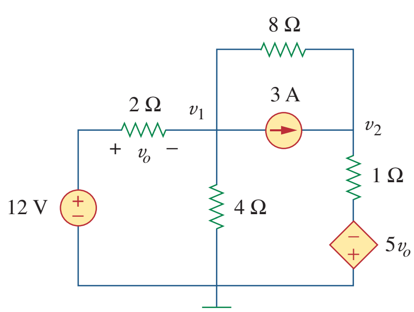

Figure 3.71
For Prob. 3.22.

3.23 Use nodal analysis to find V_o in the circuit of Fig. 3.72.

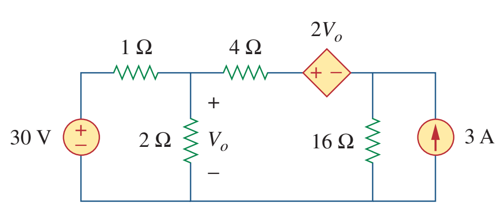

Figure 3.72
For Prob. 3.23.

3.24 Use nodal analysis and *MATLAB* to find V_o in the circuit of Fig. 3.73.

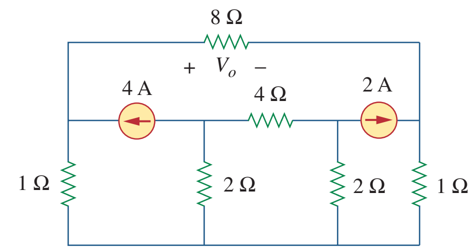

Figure 3.73
For Prob. 3.24.

3.25 Use nodal analysis along with *MATLAB* to determine the node voltages in Fig. 3.74.

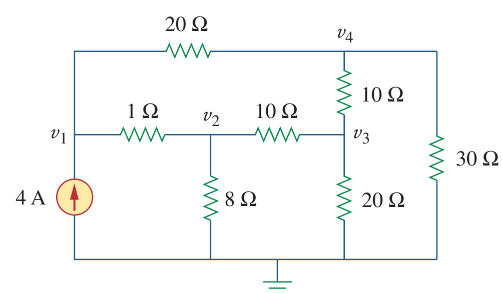

Figure 3.74
For Prob. 3.25.

3.26 Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.75.

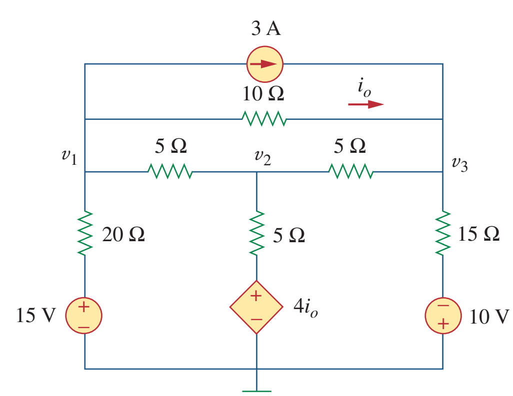

Figure 3.75
For Prob. 3.26.

***3.27** Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit of Fig. 3.76.

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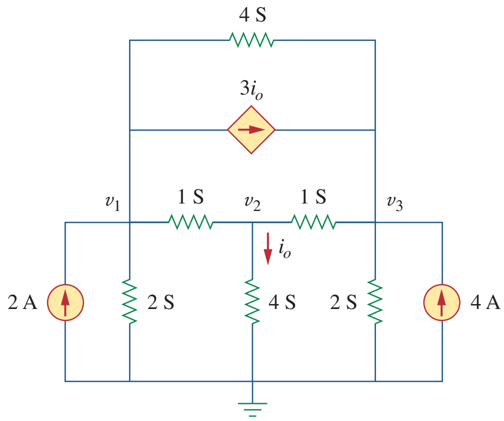

Figure 3.76
For Prob. 3.27.

***3.28** Use *MATLAB* to find the voltages at nodes *a*, *b*, *c*, and *d* in the circuit of Fig. 3.77.

ML

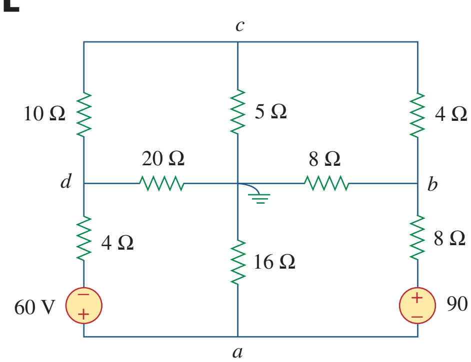

Figure 3.77
For Prob. 3.28.

3.29 Use *MATLAB* to solve for the node voltages in the circuit of Fig. 3.78.

ML

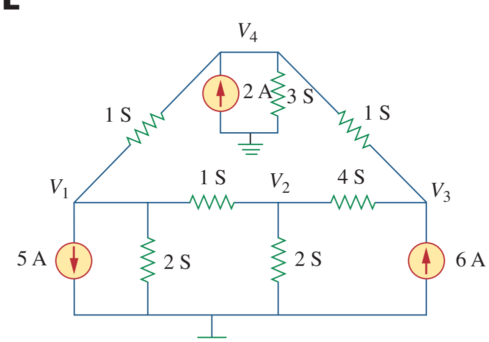

Figure 3.78
For Prob. 3.29.

* An asterisk indicates a challenging problem.

3.30 Using nodal analysis, find v_o and i_o in the circuit of Fig. 3.79.

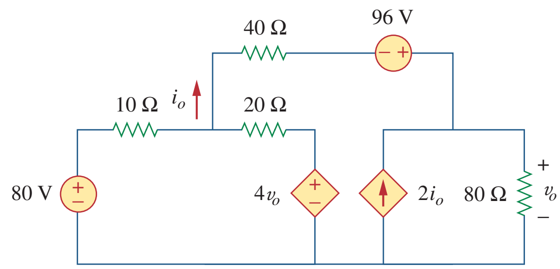

Figure 3.79
For Prob. 3.30.

3.31 Find the node voltages for the circuit in Fig. 3.80.

ML

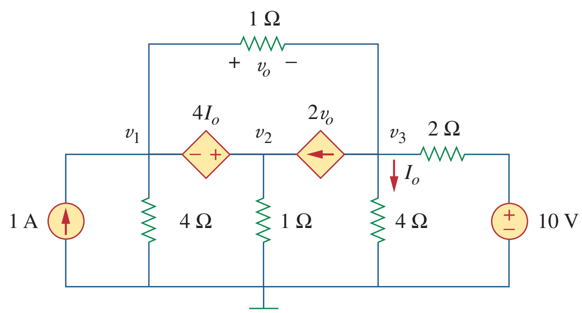

Figure 3.80
For Prob. 3.31.

3.32 Obtain the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.81.

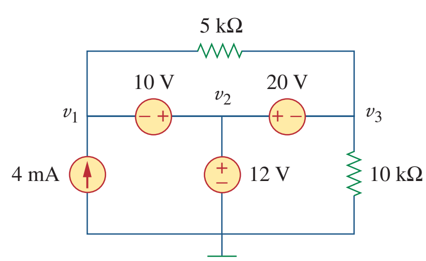

Figure 3.81
For Prob. 3.32.