C.I. on Slope

- a $(1-\alpha) \times 100\%$ C.I. on β_1 is:

$$\beta_1 : \quad \hat{\beta}_1 \pm t_{\alpha/2,\, n-2} \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$$

↑ denominator term of $\hat{\beta}_1$

ex: O_2 purity vs. hydrocarbon level example

- Write 95% C.I. on β_1

$\hat{\beta}_1 = 14.947$ ← from estimated regression line

$\left.\begin{array}{l} t_{.025,18} = 2.101 \\ \hat{\sigma}^2 = 1.18 \end{array}\right\}$ ← last time

$S_{xx} = 0.68088$

$$14.947 \pm 2.101 \sqrt{\frac{1.118}{0.68088}}$$

$$12.181 < \beta_1 < 17.713 \quad \left[\frac{\%\ O_2\ \text{purity}}{\%\ \text{hydrocarbon level}}\right]$$

- note: this C.I. does not include zero;

  $\therefore$ we would reject $H_0: \beta_1 = 0$ @ $\alpha = 0.05$

C.I. on Intercept

- a $(1-\alpha) \times 100\%$ C.I. on β_0 is:

$$\beta_0 : \quad \hat{\beta}_0 \pm t_{\alpha/2, n-2} \sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right]}$$

recall: $\hat{\beta}_0 = 74.28$ [% O_2 purity], $\bar{x} = 1.196$ [% hydrocarbon level]

$$74.28 \pm 2.101 \sqrt{1.18 \left[\frac{1}{20} + \frac{1.196^2}{0.68088} \right]} \quad \Leftarrow \text{95\% C.I.}$$

$$70.93 < \beta_0 < 77.63$$

[% O_2 purity]

C.I. on Mean Response

- mean response : $\mu_{Y|x_0}$
  - in other words: value computed by our estimated regression line at some value of x

$$\mu_{Y|x_0} : \hat{\mu}_{Y|x_0} \pm t_{\alpha/2,\, n-2} \sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]}$$

note: the further away from $\bar{x}$ our chosen x_0 is, the wider the C.I.

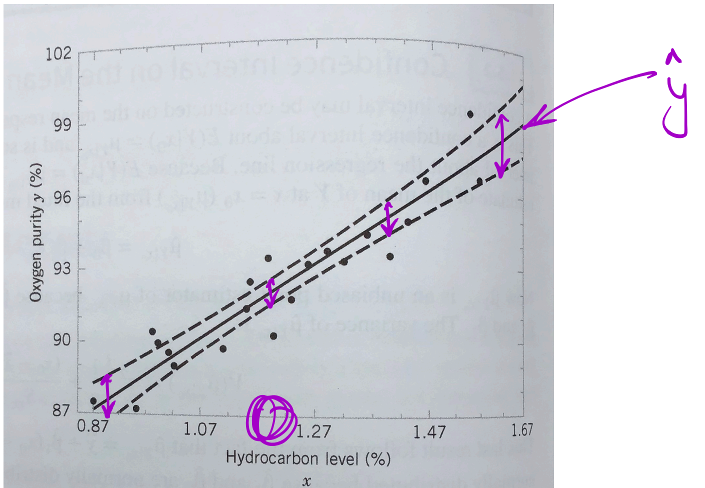

FIGURE 11.7

Scatter diagram of oxygen purity data from Example 11.1 with fitted regression line and 95% confidence limits on $\mu_{Y|x_0}$.

∴ Write 95% C.I. on mean response @ $x = 1.00$
% hydrocarbon level

∴ first need $\hat{\mu}_{Y|1.00}$!!

$$\hat{y} = \hat{\beta}_1 x + \hat{\beta}_0 = 14.95x + 74.28$$

$$\therefore \hat{\mu}_{Y|1.00} = 14.95 \cdot 1.00 + 74.28 = 89.23$$

(% O_2 purity)

$$89.23 \pm 2.101 \sqrt{1.18 \left[\frac{1}{20} + \frac{(1.00 - \overset{\bar{x}}{1.960})^2}{0.68088} \right.}$$

$$\boxed{88.49 < \mu_{Y|1.00\%} < 89.97}$$

(% O_2 purity)

∴ Very narrow range; estimated regression line does good job of computing $\hat{y}$ due to small amount of scatter in the data!

Prediction of New Observations

∴ rather than mean response @ some x_0,
What can we expect the next data point to return?

∴ a $(1-\alpha) \times 100\%$ prediction interval is:

$$Y_0 : \hat{y}_0 \pm t_{\alpha/2, n-2} \sqrt{\hat{\sigma}^2 \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]}$$

this term widens interval over C.I. on $\mu_{Y|x_0}$

ex: write a 95% P.I. on the 21st observation of O_2 purity @ $x_0 = 1.00\%$ hydrocarbon level

$$\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_0 = 89.23 \quad [\text{same as } \mu_{Y|1.00\%}]$$

$$89.23 \pm 2.101 \sqrt{1.18\left[1 + \frac{1}{20} + \frac{(1.00 - 1.196)^2}{0.6809}\right]}$$

$$86.83 < Y_0 < 91.63$$

(% O_2 purity)

$\therefore$ wider than 95% C.I. on $\mu_{Y|1.00\%}$