HW 7 4.1 # 1, 2, 3, 5, 6, 9, 11, 13, 15, 17 Alex Yeoh
4.2 # 1, 5, 7, 11, 15, 18, 22, 24

4.1

1a) Yes, both have positive x & y values, adding positives cannot make a negative

1b) $c = -1$ $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $cu = -1\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

2a) yes, both x & y must have the same sign to be in either quadrant

2b) $u = \begin{bmatrix} 100 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ -100 \end{bmatrix}$, $u + v = \begin{bmatrix} 99 \\ -99 \end{bmatrix}$

3) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot 100$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in H, $100\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is not

5) $[at^2] = a[t^2]$, t^2 is the span, yes

6) $[a + t^2] = a[1] + [t^2]$, span $[1], [t^2]$, yes

9) $\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix} = s\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ $H = \text{span}\left\{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}\right\}$, v is in $\mathbb{R}^3$ $\therefore$ H is a subspace of $\mathbb{R}^3$

11) $\begin{bmatrix} 5b + 2c \\ b \\ c \end{bmatrix} = b\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $H = \text{span}\left\{\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$, b & c are in $\mathbb{R}^3$

13a) No, w is linearly independent from v_1, v_2, and v_3. 3

13b) ∞

13c) Yes, $w = v_1 + v_2$

15) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ does not exist in span{w}

17) $a\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + c\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ $S = \left\{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}\right\}$

4.2

1) $\begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 - 15 + 12 \\ 6 - 6 + 0 \\ -8 + 12 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Yes, W is in Nul A

4.2 continued

5) $\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & -9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$ $x_5 = 0$ free x_4 $x_3 - 9x_4 = 0$ free x_2 $x_1 - 2x_2 + 4x_4 = 0$

$x_3 = 9x_4$ $x_1 = 2x_2 - 4x_4$

$$\vec{x} = \begin{bmatrix} 2x_2 - 4x_4 \\ x_2 \\ 9x_4 \\ x_4 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} \quad \text{Span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} \right\}$$

7) No, $0 + 0 + 0 \neq 2$

11) No, $5 + d \neq d \therefore \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ does not exist

15) $r\begin{bmatrix} 0 \\ 1 \\ 4 \\ 3 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix} + t\begin{bmatrix} 3 \\ -2 \\ 0 \\ -1 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -2 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix}$

18a) 3 18b) 4

22) $\left[\begin{array}{cccc|c} 1 & 3 & 5 & 0 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right]$ free x_4 free x_3 $x_2 + 4x_3 - 2x_4 = 0$ $x_1 + 3(-4x_3 + 2x_4) + 5x_3 = 0$

$x_2 = -4x_3 + 2x_4$ $x_1 - 12x_3 + 5x_3 + 6x_4 = 0$

$x_1 = 7x_3 - 6x_4$

$$\vec{x} = \begin{bmatrix} 7x_3 - 6x_4 \\ -4x_3 + 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -6 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix}$ in Nul A, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in Col A

24) Not in either