HW 2 1.3 # 5, 6, 9, 10, 11, 13, 19, 25 Alex Yeoh

5) $6x_1 + -3x_2 = 1$

$-x_1 + 4x_2 = -7$

$5x_1 = -5$

6) $-2x_1 + 8x_2 + x_3 = 0$

$3x_1 + 5x_2 - 6x_3 = 0$

9) $x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

10) $x_1 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}$

11) $\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{R_2+2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \end{array}\right]$

free $x_3$

$1x_2 + 4x_3 = 3$

$x_2 = 3 - 4x_3$

$x_1 + 5x_3 = 2$

$x_1 = 2 - 5x_3$

$\vec{x} = \begin{pmatrix} x_1 = 2 - 5x_3 \\ x_2 = 3 - 4x_3 \\ x_3 = x_3 \end{pmatrix}$

b is a linear combination of $a_1$, $a_2$, and $a_3$

13) $\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \xrightarrow{R_3+2R_1} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$

no solution b/c $0x_1 + 0x_2 + 0x_3 = 3$ ∴ b is not a linear combination of the vectors formed from the columns of the matrix A

19) $\frac{3}{2} v_1 = v_2$ ∴ Span$(v_1, v_2)$ is a set of points on a line through $v_1$, $v_2$, and the origin

25) $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \xrightarrow{R_3+2R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right]$

$-x_3 = 2$

$x_3 = -2$

$3x_2 - 2(-2) = 1$

$3x_2 + 4 = 1$

$3x_2 = -3$

$x_2 = -1$

$x_1 - 4(-2) = 4$

$x_1 + 8 = 4$

$x_1 = -4$

$\vec{x} = \begin{pmatrix} x_1 = -4 \\ x_2 = -1 \\ x_3 = -2 \end{pmatrix}$

a) b is not in $\{a_1, a_2, a_3\}$ b/c there is no constant c where $\left|\begin{matrix} ca_1 = b \\ ca_2 = b \\ ca_3 = b \end{matrix}\right|$,

3 vectors b/c there is no constant c where $\begin{matrix} ca_1 = a_2 \\ ca_2 = a_3 \\ ca_3 = a_1 \end{matrix}$

b) b is in W, see work above.

infinitely many vectors in W b/c the vectors don't have to be linearly independent

c) $1a_1 + 0a_2 + 0a_3 = a_1$