ISE 2211 • Exam VI • 23 July 2020 NAME SOLUTION

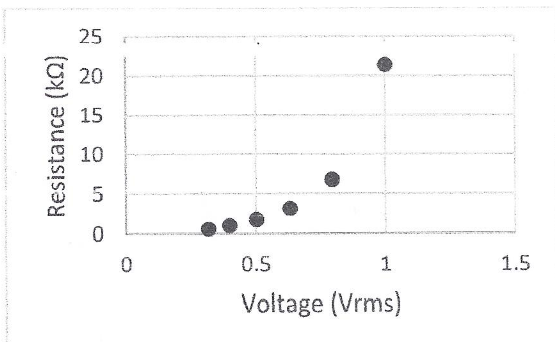
An unusual audio engineering experiment was performed in which a resistance was varied in order to achieve an exact output voltage and then measured, for six different pre-determined voltages. At first glance the resistance would appear to be the control variable and the output voltage the response, but since we are measuring the required resistance for pre-determined values of voltage, it's the other way around.

Test #	Resistance, kΩ (y)	Voltage, V_{rms} (x)
1	21.40	1.000
2	6.800	0.7943
3	3.190	0.6309
4	1.787	0.5011
5	1.059	0.3981
6	0.622	0.3162

Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model for resistance vs. output voltage, and use them to write the estimated regression line ($\hat{y}$). Include a unit.

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$



$S_{XY} = 30.33 - \frac{34.86 \cdot 3.641}{6} = 9.176$

+1 $\sum y_i x_i = 30.33$

$S_{XX} = 2.539 - \frac{3.641^2}{6} = 0.3295$

+1 $\sum y_i = 34.86$

$\sum x_i = 3.641$

$\hat{\beta}_1 = 27.85$ +2

+1 $\sum x_i^2 = 2.539$

$\sum y_i^2 = 519.1$

$\hat{\beta}_0 = 5.810 - 27.85 \cdot 0.6068$

$\hat{\beta}_0 = -11.09$ +1

+1 $\bar{y} = 5.810$

$\bar{x} = 0.6068$

$\hat{y} = 27.85x - 11.09$ +1 kΩ +1

Write a 95% prediction interval on the value of resistance necessary to achieve an output voltage of 0.2512 v_{rms}.

$$\hat{y}_0 = 27.85 \cdot 0.2512 - 11.09$$

$$\hat{y}_0 = -4.094 \ (k\Omega) \quad (!)$$

+1

$$t_{\alpha/2, n-2} = t_{.025, 4} = 2.776 \quad +1$$

$$SS_T = 519.1 - 6 \cdot 5.810^2 = 316.6$$

$$SS_E = 316.6 - 27.85 \cdot 9.176 = 61.05$$

$$\hat{\sigma}^2 = \frac{61.05}{6-2} = 15.26 \quad +1$$

$$Y_0 : \quad -4.094 \pm 2.776 \sqrt{15.26 \left[1 + \frac{1}{6} + \frac{(0.2512 - 6.6068)^2}{0.3295}\right]}$$

$$-17.60 < Y_0 < 9.409 \quad k\Omega$$

+2 +1

Test the following hypotheses on slope using the fixed-significance-level approach at $\alpha = 0.05$.

H_0: $\beta_1 = 0$

H_1: $\beta_1 \neq 0$

$$t_0 = \frac{27.85 - 0}{\sqrt{15.26 / 0.3295}} = 4.092 \; (!)$$

+1

critical values:

$$\pm t_{\alpha/2, n-2} = \pm t_{.025, 4} = 2.776$$

$$t_0 > +t_{.024, 4}$$

+1

$\therefore$ reject H_0

+1

Compute the coefficient of determination for this relationship.

$$R^2 = 1 - \frac{SS_E}{SS_T}$$

$$= 1 - \frac{61.05}{316.6}$$

$$R^2 = 0.8072$$

+1

Suppose we rejected the null hypothesis that slope is zero, and computed a relatively large value of R^2, and yet looking at the data, it is clearly non-linear. How could this have happened, and how does this warn us of the dangers of extrapolation beyond the range of original data? Is the P.I. we computed for an output voltage of 0.2512 V_{rms} likely to be useful?

a line may fit OK to the six original data points, but the negative $\hat{y}_0$ value of -4.094 kΩ

@ $X = 0.2512$ is a dead giveaway that the relationship falls apart very quickly.

+2