HW 5 2.1 # 2,4,9,10,13 Alex Yeoh
2.2 # 3,6,7,8,31,33,35
2.3 # 4,5,6,7,19,33

2.1

2) $\begin{bmatrix} -16 & -10 & 1 \\ -6 & -13 & -4 \end{bmatrix}$, undefined different size, $\begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$, undefined Sizes: $E: 2\times1$, $B: 2\times3$ $E \times B$ $\underbrace{2\times1 \cdot 2\times3}_{\text{different}}$

4) $5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $A - 5I_3 = \begin{bmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{bmatrix}$ $(5I_3)A: \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$

9) AB BA

$\begin{bmatrix} 8+15 & -10+5k \\ & \end{bmatrix} = \begin{bmatrix} 8+15 & 20-5 \\ & \end{bmatrix}$

$-10+5k = 20-5$
$5k = 25$
$k = 5$

10) AB AC

$\begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$ Yep, $AC = AB$ but $B \neq C$

13) $Q = \begin{bmatrix} & \end{bmatrix}_{m\times n} \cdot \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix}_{n\times1} = [Qr_1 \cdots Qr_p]$

2.2

3) $\left[\begin{array}{cc|cc} 8 & 5 & 1 & 0 \\ -7 & -5 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ -7 & -5 & 0 & 1 \end{array}\right] \xrightarrow{R_2+7R_1} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & -5 & 7 & 8 \end{array}\right] \xrightarrow{R_2/-5} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & \frac{-7}{5} & \frac{-8}{5} \end{array}\right]$ $\begin{bmatrix} 1 & 1 \\ \frac{-7}{5} & \frac{-8}{5} \end{bmatrix}$

6) $\begin{bmatrix} 1 & 1 \\ \frac{-7}{5} & \frac{-8}{5} \end{bmatrix}\begin{bmatrix} -9 \\ 11 \end{bmatrix} = \begin{bmatrix} -9+11 \\ \frac{63}{5} - \frac{88}{5} \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{-25}{5} \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ $x_1 = 2$, $x_2 = -5$

7a) A^{-1} $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 5 & 12 & 0 & 1 \end{array}\right] \xrightarrow{R_2-5R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -5 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 2 & -5 & 1 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 1 & \frac{-5}{2} & \frac{1}{2} \end{array}\right]$ $A^{-1} = \begin{bmatrix} 6 & -1 \\ \frac{-5}{2} & \frac{1}{2} \end{bmatrix}$

$Ax = b_1$
$b_1 = Ax$
$A^{-1}b_1 = x(AA^{-1})$
$A^{-1}b_1 = xI$

$\begin{bmatrix} 6 & -1 \\ \frac{-5}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$

$A^{-1}b_2 = xI$
$\begin{bmatrix} 6 & -1 \\ \frac{-5}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ -5 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \end{bmatrix}$

$A^{-1}b_3 = xI$
$\begin{bmatrix} 6 & -1 \\ \frac{-5}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$A^{-1}b_4 = xI$
$\begin{bmatrix} 6 & -1 \\ \frac{-5}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$

2.2 continued

7b)
$$\left[\begin{array}{cc|cccc} 1 & 2 & -1 & 1 & 2 & 3 \\ 5 & 12 & 3 & -5 & 6 & 5 \end{array}\right] \xrightarrow{R_2-5R_1} \left[\begin{array}{cc|cccc} 1 & 2 & -1 & 1 & 2 & 3 \\ 0 & 2 & 8 & -10 & -4 & -10 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|cccc} 1 & 0 & -9 & 11 & 6 & 13 \\ 0 & 2 & 8 & -10 & -4 & -10 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{cc|cccc} 1 & 0 & -9 & 11 & 6 & 13 \\ 0 & 1 & 4 & -5 & -2 & -5 \end{array}\right]$$

8) $AD = I$

$A^{-1}AD = IA^{-1}$ multiply both sides by A^{-1}

$ID = IA^{-1}$ $\quad A^{-1}A = I$

$D = A^{-1}$ $\quad$ Identity matrix

31)
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+3R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_3+3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\xrightarrow[R_2+R_3]{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \xrightarrow{R_3/2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{array}\right]$$

$$\begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

33)
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \qquad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_4-R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-R_1} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

35)
$$\left[\begin{array}{ccc|c} -2 & -7 & -9 & 0 \\ 2 & 5 & 6 & 0 \\ 1 & 3 & 4 & 1 \end{array}\right] \xrightarrow[R_2+R_1]{R_1+2R_1} \left[\begin{array}{ccc|c} 0 & -1 & -1 & 2 \\ 0 & -2 & -3 & 0 \\ 1 & 3 & 4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 4 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & -2 & -3 & 0 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & -1 & -4 \end{array}\right]$$

$$\xrightarrow{R_1+3R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & -1 & -4 \end{array}\right] \xrightarrow{R_2-R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & -1 & -4 \end{array}\right] \xrightarrow[R_3/-1]{R_2/-1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

HW 5 2.3 # 4, 5, 6, 7, 19, 33

4) not invertable, A^T is not invertable b/c A^T cannot have n pivot points due to an all 0 row

5) $\left[\begin{array}{ccc|ccc} 0 & 3 & -5 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ -4 & -9 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+3R_1]{R_3+4R_2} \begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ not invertable, cannot have n pivot points

6) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix} \xrightarrow{R_3+3R_1} \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & -9 & -12 \end{bmatrix}$ not invertable, columns are not linearly independent

7) $\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow[R_3-2R_1]{R_2+3R_1} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow{R_3/3} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow[R_4-2R_3]{R_2-8R_3} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

$\xrightarrow{R_2/4} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \xrightarrow[R_4+R_2]{R_1+3R_2} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1-R_4} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1/-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

can stop here b/c 4 pivot points

invertable

19) Invertable by invertible matrix theorm where all properties are true or none are which has columns being linearly independent as a property

33) $T(x) = \begin{bmatrix} -5 & 9 \\ 4 & -7 \end{bmatrix}$ invertible by invertible matrix theorm for ↑ reason

T^{-1} $\left[\begin{array}{cc|cc} -5 & 9 & 1 & 0 \\ 4 & -7 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|cc} -1 & 2 & 1 & 1 \\ 4 & -7 & 0 & 1 \end{array}\right] \xrightarrow{R_2+4R_1} \left[\begin{array}{cc|cc} -1 & 2 & 1 & 1 \\ 0 & 1 & 4 & 5 \end{array}\right] \xrightarrow{R_1\cdot -1} \left[\begin{array}{cc|cc} 1 & -2 & -1 & -1 \\ 0 & 1 & 4 & 5 \end{array}\right]$

$\xrightarrow{R_1+2R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & 9 \\ 0 & 1 & 4 & 5 \end{array}\right]$

$T^{-1} = \begin{bmatrix} 7 & 9 \\ 4 & 5 \end{bmatrix}$