EE 2010 Circuit Analysis

Module # 12: Fixed, Single-Frequency Inputs: Phasor Representations Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, WikipediA, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Learning Objective: In this module, we demonstrate how we can use *EXACTLY THE SAME* nodal analysis for circuits with dynamic elements to find the *STEADY STATE RESPONSE* for a specific *SINUSOIDAL INPUT AT A SINGLE FIXED FREQUENCY* by replacing sources and dynamic elements with their *PHASOR REPRESENTATION.*

While *phasors* are a throwback to the sliderule days decades ago when symbolic computation was not possible, phasors still have some utility in analyzing systems *with a single, fixed frequency.* However, phasors are inherently limited in their analytic generality in that *the phasor representation for every dynamic element must be recalculated for any change of the excitation frequency.*

From WikipediA: In physics and engineering, a *phasor* (a portmanteau of phase vector), is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. It is related to analytic representation, which decomposes a sinusoid into the product of a complex constant and a factor that encapsulates the frequency and time dependence.

$$A\cos(\omega t+\theta) = \Re\{Ae^{j(\omega t+\theta)}\} = \Re\{\underbrace{Ae^{j\theta}}_{\text{Phasor}} \cdot e^{j(\omega t)}\}$$

which is denoted compactly as

$$A\cos(\omega t+\theta) \longrightarrow Ae^{j\theta} \longrightarrow A\angle\theta$$

The complex constant, which encapsulates amplitude and phase dependence, is known as a *phasor* or complex amplitude. Simply put, *a phasor is a complex number that represents the amplitude and phase of a sinusoid at a fixed frequency.*

The utility of **phasors** is a direct consequence of **sinusoidal in** $\rightarrow$ **sinusoidal out** for linear systems since at a particular frequency, ω_0, if

$$v_{in}(t) = A\cos(\omega_0 t+\theta) \longrightarrow A\angle\theta$$

the output is found as:

$$v_{out}(t) = A|H(\omega_0)|\Re\left\{e^{j(\omega_0 t+\theta+\angle H(\omega_0))}\right\} \longrightarrow A\angle\theta \cdot |H(\omega_0)|\angle H(\omega_0), \qquad \text{which represents}$$
$$v_{out}(t) = A|H(\omega_0)|\cos(\omega_0 t+\theta+\angle H(\omega_0))$$

Glossing over some mathematical details, the phasor transform can also be seen as a particular case of the Fourier transform. However, the Fourier transform is mathematically more difficult to apply and the effort may be unjustified if only a steady state analysis at only a single frequency is required.

Phasors, with their relationship to the Fourier transform, are most useful for the analysis of the operation of dynamic systems on fixed-frequency sinusoidal excitations since the system operation is a confluence of differentials, integrals, constant multiplies, and additions (all linear operations) which are compactly modeled by phasor operations:

Phasor Operations (single frequency ω_0)

Time-domain	**Phasor-domain**
$\frac{dv}{dt}$	$j\omega V$
$\int v dt$	$\frac{V}{j\omega}$
av	aV
$a_1 v_1 + a_2 v_2$	$a_1 V_1 + a_2 V_2$

A common situation in some electrical networks is the existence of multiple sinusoids all with the same frequency, but different amplitudes and phases. The only difference in their analytic representations is the complex amplitude (phasor). A linear combination of such functions can be factored into the product of a linear combination of phasors (known as phasor arithmetic) and the time/frequency dependent factor that they all have in common.

In analysis of *three phase AC power systems*, usually a set of phasors is defined as the three complex cube roots of unity, graphically represented as unit magnitudes at angles of 0, 120 and 240 degrees. By treating polyphase AC circuit quantities as phasors, balanced circuits can be simplified and unbalanced circuits can be treated as an algebraic combination of symmetrical components. This approach greatly simplifies the work required in electrical calculations of voltage drop, power flow, and short-circuit currents. In the context of power systems analysis, the phase angle is often given in degrees, and the magnitude in rms value rather than the peak amplitude of the sinusoid.

Phasor Representation:

In like manner as the Laplace transform changes integro-differential equations into algebraic equations (as polynomials in s), phasor representations change integro-differential equations into algebraic equations (as polynomials in $j\omega$ or $j2\pi f$). We can obtain phasor representations through Laplace, or (for fixed, single-frequency systems), we can move directly to a phasor representation with excitations modeled as:

$$A\cos(2\pi f_0 t + \theta) \Rightarrow Ae^{j\theta} \Rightarrow A\angle\theta$$

In phasor notation, *sources are referenced to a cosine waveform* and s-domain impedance models reduce to a complex number.

Phasor Representations

Sources		
t-domain	**Computation @** ω_0, f_0	**Shorthand @** ω_0, f_0
$A\cos(2\pi f_0 t + \theta)$	$Ae^{j\theta}$	$A\angle\theta$
$-A\cos(2\pi f_0 t + \theta)$	$Ae^{j(\theta+\pi)}$	$A\angle(\theta+\pi)$
$B\sin(2\pi f_0 t + \phi)$	$Be^{j(\phi-\pi/2)}$	$B\angle(\phi-\pi/2)$
$-B\sin(2\pi f_0 t + \phi)$	$Be^{j(\phi-\pi/2+\pi)}$	$B\angle(\phi-\pi/2+\pi)$
Impedances		
s-domain	**Computation @** ω_0, f_0	**Shorthand @** ω_0, f_0
$Z_R = R$	$Z_R = Re^{j0}$	$Z_R = R\angle 0$
$Z_C = \frac{1}{s\cdot C}$	$Z_C = \frac{1}{j\omega_0 C} = \frac{-j}{\omega_0 C} = \frac{e^{-j\pi/2}}{\omega_0 C}$	$Z_C = \frac{1}{\omega_0 C}\angle(-\pi/2)$
$Z_C = \frac{1}{s\cdot C}$	$Z_C = \frac{1}{j2\pi f_0 C} = \frac{-j}{2\pi f_0 C} = \frac{e^{-j\pi/2}}{2\pi f_0 C}$	$Z_C = \frac{1}{2\pi f_0 C}\angle(-\pi/2)$
$Z_L = s\cdot L$	$Z_L = j\omega_0 L = \omega_0 L e^{j\pi/2}$	$Z_L = \omega_0 L\angle\pi/2$
$Z_L = s\cdot L$	$Z_L = j2\pi f_0 L = 2\pi f_0 L e^{j\pi/2}$	$Z_L = 2\pi f_0 L\angle\pi/2$

Table 1: Phasor Representations

Then circuit analysis techniques (all of them) proceed as usual by incorporating phasor representations.

Procedure for Phasor Notation Circuit Analysis:

1. Replace *all independent sinusoidal sources with their phasor representations in radians or with their symbolic V_{in} or I_{in} (see below).*
2. Replace *all impedances with their phasor representations for the specified frequency.*
3. Identify the essential ($\geq$ 3-element connections) nodes
4. Select a node as the reference node = the node at *ground potential* = 0 Volts
5. Identify and label the voltages at nodes that are readily deduced
6. Note the node-pairs linked by a *voltage source* and simplify accordingly
7. Assign voltage variables $v_a, v_b, \ldots$ to the remaining nodes with only one assignment for each linked node-pair
8. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
9. Enjoy the thrill of ending the consideration of each node with the powerful "= 0"
10. Add one additional equation for each dependent source specification if necessary
11. Depending on the type of analysis at hand, either:
    (a) Solve node equations directly using phasor representations of independent sources, **OR**
    (b) Use algebra or an algebra solver to find the transfer function: $H = \frac{V_{out}}{V_{in}}$, or whatever output-inputs function is dictated by the problem and,
    Convert the desired result back to a cosine representation as:

$$V_{out} = \Re\left\{|V_{in}| \cdot |H| \cdot e^{j((\angle V_{in} + \angle H)}\right\} = |V_{in}| \cdot |H| \cdot \cos(\omega_0 t + \angle V_{in} + \angle H)$$

Note that if the source frequency changes even a little bit, we would need to *begin the entire process all over again* including recalculation of the phasor representations for all dynamic elements.

So then: *Using phasor models is just like node equations with resistors at DC except with complex numbers in shorthand!*

Complex algebra operations such as multiplication and division are more convenient in *polar* form, while addition and subtraction are more convenient in rectangular form. That is, with $z = a + jb = Ae^{j\theta}$ and $w = c + jd = Ce^{j\phi}$

$$
\begin{aligned}
z &= a + jb = Ae^{j\theta} \\
w &= c + jd = Ce^{j\phi}
\end{aligned}
$$

where:

$$
\begin{aligned}
A &= |z| = \sqrt{a^2 + b^2} \\
\theta &= \angle z = \arctan\left(\frac{b}{a}\right)
\end{aligned}
$$

then:

$$
\begin{aligned}
zw &= Ae^{j\theta}Ce^{j\phi} = ACe^{j\theta}e^{j\phi} = ACe^{j(\theta+\phi)} \\
\frac{z}{w} &= \frac{Ae^{j\theta}}{Ce^{j\phi}} = \frac{A}{C}e^{j(\theta-\phi)} \\
z + w &= a + jb + c + jd = a + c + j(b + d) \\
z - w &= a + jb - c + jd = a - c + j(b - d) \\
z^* &= a - jb = Ae^{-j\theta} \\
zz^* &= |z|^2 = A^2 = (a - jb)(a + jb) = a^2 + b^2
\end{aligned}
$$

and, in relationships to trig functions,

$$
\begin{aligned}
\Re(z) &= a = A\cos(\theta) \\
\Im(z) &= b = A\sin(\theta) \\
e^{\pm j\theta} &= \cos(\theta) \pm j\sin(\theta) \\
\cos(\theta) &= \Re\left\{e^{j\theta}\right\} = \frac{e^{j\theta} + e^{-j\theta}}{2} \\
\sin(\theta) &= \Im\left\{e^{j\theta}\right\} = \frac{e^{j\theta} - e^{-j\theta}}{2j}
\end{aligned}
$$

For simple problems, we'll typically do the complex algebra ourselves with confirmation from Matlab. For more complex problems, we will do the circuit analysis part (node equations, voltage dividers, etc.) and use Matlab as a computational aid. †

Problem 9.9: A little complex number prqactice:

9.9 Evaluate the following complex numbers and leave your results in polar form:

(a) $5\underline{/30^\circ}\left(6 - j8 + \dfrac{3\underline{/60^\circ}}{2 + j}\right)$

(b) $\dfrac{(10\underline{/60^\circ})(35\underline{/-50^\circ})}{(2 + j6) - (5 + j)}$

```
%% Problem 9.9
clear all
% converting to radians
A = 5*exp(j*30*2*pi/360)*(6-j*8+(3*exp(j*60*2*pi/360))/(2+j))
[abs(A),angle(A)]
% in radians
B = (10*exp(j*60*2*pi/360)*35*exp(j*-50*2*pi/360))/((2+6*j)-(5+j*12))
[abs(B),angle(B)]
%
```

...Yielding...

```
A =   48.9808 -13.6410i
ans =    50.8448   -0.2716
B =  -31.0824 +41.9059i
ans =   52.1749    2.2090
```

Problem 9.16: Sources to phasors:

9.16 Transform the following sinusoids to phasors:

(a) $-20\cos(4t + 135°)$ (b) $8\sin(20t + 30°)$

(c) $20\cos(2t) + 15\sin(2t)$

```
%% Problem 9.16
clear all
C = 20 + 15*exp(-j*pi/2)
[abs(C),angle(C)]
%
```

...Yielding...

```
C =   20.0000 -15.0000i
ans =    25.0000   -0.6435
```

Problem 9.21: Trig with phasors:

9.21 Simplify the following:

(a) $f(t) = 5\cos(2t + 15°) - 4\sin(2t - 30°)$

(b) $g(t) = 8\sin t + 4\cos(t + 50°)$

(c) $h(t) = \int_0^t (10\cos 40t + 50\sin 40t)\,dt$

```
%% Problem 9.21
clear all
A = 5*exp(j*15*2*pi/360) + 4*exp(j*(-30*2*pi/360+pi/2))
[abs(A),angle(A)]
%
B = 8*exp(-j*pi/2) + 4*exp(j*50*2*pi/360)
[abs(B),angle(B)]
%
C = 10/(j*40) + 50*exp(-j*pi/2)/(j*40)
[abs(C),angle(C)]
%
```

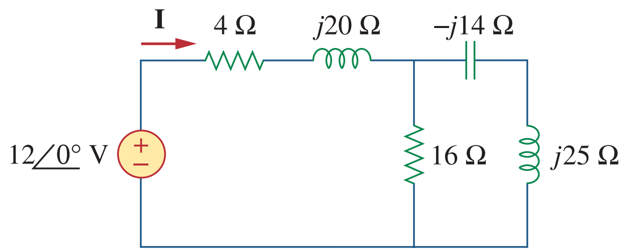
...Yielding...

```
A =    6.8296 + 4.7582i
ans =     8.3237    0.6085
B =    2.5712 - 4.9358i
ans =     5.5654   -1.0906
C =   -1.2500 - 0.2500i
ans =     1.2748   -2.9442
```

Notice for part C) there is no need to integrate!

Problem 9.39: This circuits has already been converted to phasor representation.

9.39 For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current **I**. Let $\omega = 10$ rad/s.




```
%% Problem 9.39
clear all
Z= (−j*14+j*25)*16/(−j*14+j*25+16) + j*20 + 4
[abs(Z),angle(Z)]
%
I = 12/Z
[abs(I),angle(I)]
%
```

...Yielding...

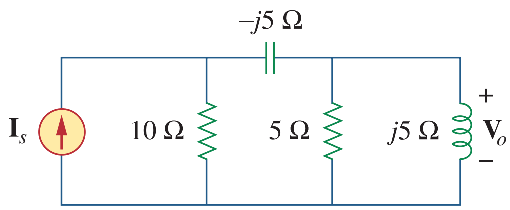
```
Z =    9.1353 +27.4695i
ans =    28.9487    1.2497
I =    0.1308 − 0.3933i
ans =     0.4145   −1.2497
```

So that $I(t) = 0.4145\cos(10t - 1.2497)$.

Problem 9.52: This circuits has already been converted to phasor representation.

Sort of an inverse transfer function problem. Interesting!

9.52 If $\mathbf{V}_o = 8\underline{/30^\circ}$ V in the circuit of Fig. 9.59, find $\mathbf{I}_s$.



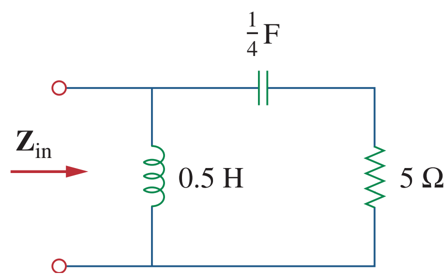

```
%% Problem 9.52
clear all
% Declare symbolic variables
syms Iin va Vout
% Nodal analysis directly in solve()
[va,Vout]=solve(-Iin + va/10 + (va-Vout)/(-j*5) == 0,...
  Vout/(j*5) + Vout/5 + (Vout-va)/(-j*5) == 0)
%
H = Vout/Iin
% but we are given that
Vout = 8*exp(j*30*2*pi/360);
% so then
Iin = double(Vout/H)
[abs(Iin),angle(Iin)]
%
```

...Yielding...

```
va = Iin*(20/13 + 30i/13)
Vout = Iin*(30/13 - 20i/13)
H = 30/13 - 20i/13
Iin = 1.2785 + 2.5856i
ans = 2.8844    1.1116
```

Problem 9.59: An unconverted circuit.

9.59 For the network in Fig. 9.66, find $\mathbf{Z}_{\text{in}}$. Let ω = 10 rad/s.




```
%%  Problem 9.59
clear all
% Impedance combination
Z = (0.5*j*10 * (4/(j*10) + 5))/(0.5*j*10 + (4/(j*10) + 5))
%
[abs(Z),angle(Z)]
%
```

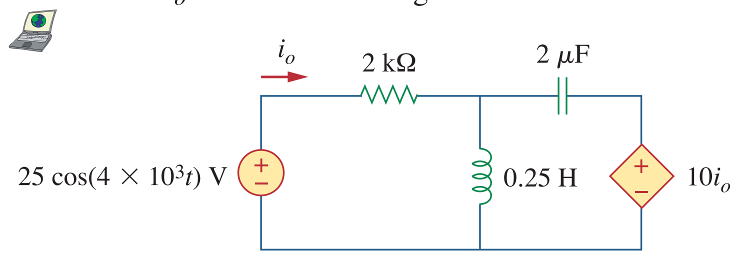
...Yielding...

```
Z =   2.7080 + 2.5087i
ans =    3.6914    0.7472
```

Problem 10.05 Direct-source approach:

We'll do this two ways as an illustration. First, the *Direct-source approach:*

10.5 Find i_o in the circuit of Fig. 10.54.




```
%% Problem 10.05 — Direct Source Approach
clear all
% Declare symbolic variables
syms va i0
% Nodal analysis directly in solve() with Vin = 20, and impedance model
% values computed at omega = 4x10^3 = 4e3
[va,i0]=solve((va-25)/2000 + va/(0.25*j*4e3) + (va-10*i0)*j*4e3*2e-6 == 0,...
  i0 == (25-va)/2000, va,i0)
% Answer :
formatSpec = 'i0(t) = %4.2f cos(4000*t %+4.2f )\n';
fprintf(formatSpec,double(abs(i0)),angle(i0))
%
```

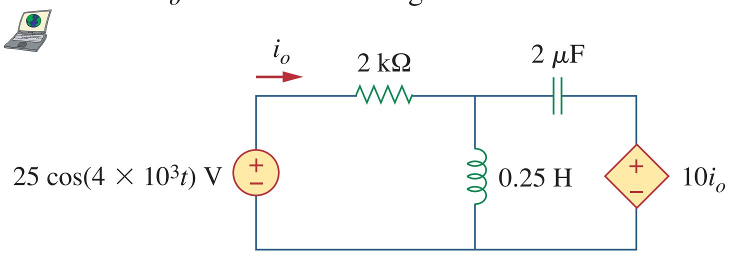
...Yielding...

```
va = 0.2668 - 1.7566i
i0 = 0.0124 + 8.7831e-04i
i0(t) = 0.01 cos(4000*t +0.07 )
```

Problem 10.05 Transfer-function approach:

Now, the *Transfer-function approach:*

10.5 Find i_o in the circuit of Fig. 10.54.




```
%% Problem 10.05 -- Transfer-function approach
clear all
% Declare symbolic variables
syms Vin va i0
% Nodal analysis directly in solve() with Vin = 20, and impedance model
% values computed at omega = 4x10^3 = 4e3
[va,i0]=solve((va-Vin)/2000 + va/(0.25*j*4e3) + (va-10*i0)*j*4e3*2e-6 == 0,...
  i0 == (Vin-va)/2000, va,i0)
%
H = double(i0/Vin)
% Then i0 = Vin * H
i0 = 25*H
% and
formatSpec = 'i0(t) = %4.2f cos(4000*t %+4.2f )\n';
fprintf(formatSpec,double(abs(i0)),angle(i0))
%
```

...Yielding...

```
va = Vin*(0.0107 - 0.0703i)
i0 = Vin*(4.9466e-04 + 3.5132e-05i)
H =  0.00049466 + 3.5132e-05i
i0 =  0.012366 +  0.0008783i
i0(t) = 0.01 cos(4000*t +0.07 )
```

Problem 10.07:

This is a two-source problem, so we'll use a direct-source approach.

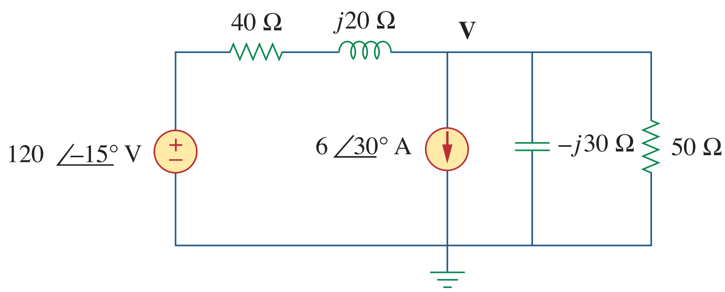


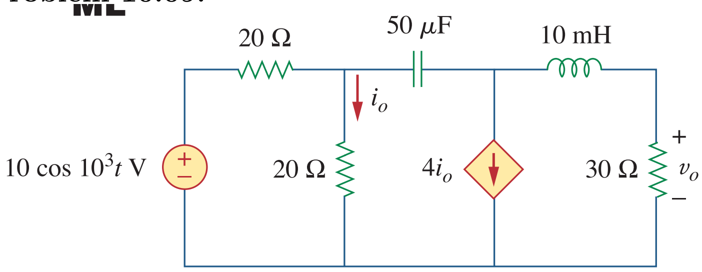
Figure 10.56

```
%% Problem 10.07
clear all
% Declare symbolic variables
syms va
% Nodal analysis directly in solve()
va=solve((va-120*exp(-j*15*2*pi/360))/(40+j*20)...
  + 6*exp(j*30*2*pi/360) + va/(-j*30) + va/50 == 0,va);
va=double(va)
% Answer :
[abs(va),angle(va)]
%
```

...Yielding...

```
va =  -1.1149e+02 - 5.4472e+01i
ans =  124.0840   -2.6871
```

Problem 10.09:




```
%%  Problem 10.09
clear all
syms Vin va vb i0
% Nodal analysis directly in solve()
[va,vb,i0]=solve((va-Vin)/20 + va/20 + (va-vb)*j*1000*50*10^(-6) == 0,...
  (vb-va)*j*1000*50*10^(-6) + 4*i0 + vb/(0.01*j*1000+30) == 0,...
  i0==va/20, va,vb,i0)
% and
v0 = vb*30/(0.01*j*1000+30)
%
H = v0/Vin
v0 = 10*H
formatSpec = 'v0(t) = %4.2f cos(1000*t %+4.2f )\n';
fprintf(formatSpec,double(abs(v0)),angle(v0))
%
```

...Yielding...

```
va = Vin*(0.1436 - 0.0644i)
vb = Vin*(0.0149 + 0.6485i)
i0 = Vin*(0.0072 - 0.0032i)
v0 = Vin*(0.2079 + 0.5792i)
H = 0.2079 + 0.5792i
v0 = 2.0792 + 5.7921i
v0(t) = 6.15 cos(1000*t +1.23 )
```

Problem 10.16: A two-source problem, use direct-source.

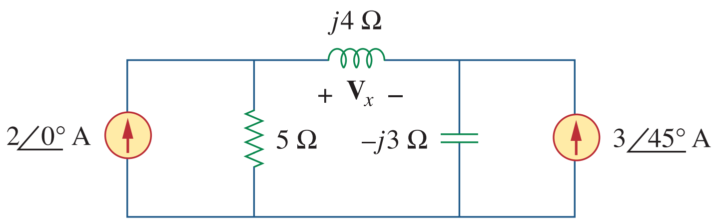


Figure 10.65

```
%% Problem 10.16
clear all
syms va vb
% Nodal analysis directly in solve()
[va,vb]=solve(va/5 - 2 + (va-vb)/(j*4) == 0,...
  vb/(-j*3) + (vb-va)/(j*4) - 3*exp(j*45*2*pi/360)== 0, va,vb)
% and
vx = va-vb
double([abs(vx),angle(vx)])
%
```

...Yielding...

```
va = 5.2800 - 5.4200i
vb = 9.6159 - 9.1960i
vx = - 4.3360 + 3.7760i
ans =  5.7497  2.4251
```

Homework: Chapter 9 # 8, 19, 35, 43, 47, 50
Chapter 10 # 6, 11, 14

→ Remember that only independent sources are replaced with symbolic labels.

→ Homework deliverables MUST be a pdf file generated using a solver.

→ The resulting .pdf file is to be uploaded to the Pilot Dropbox using the naming convention: First 4 letters of Lastname, First initial, year, title. For example, my .pdf file would be named: GarbF2020HW12.pdf

7. The techniques of voltage/current division, series/parallel combination of impedance/admittance, circuit reduction, and Y-Δ transformation all apply to ac circuit analysis.
8. AC circuits are applied in phase-shifters and bridges.

Review Questions

9.1 Which of the following is *not* a right way to express the sinusoid $A \cos \omega t$?

(a) $A \cos 2\pi ft$ (b) $A \cos(2\pi t/T)$
(c) $A \cos\omega(t - T)$ (d) $A \sin(\omega t - 90°)$

9.2 A function that repeats itself after fixed intervals is said to be:

(a) a phasor (b) harmonic
(c) periodic (d) reactive

9.3 Which of these frequencies has the shorter period?

(a) 1 krad/s (b) 1 kHz

9.4 If $v_1 = 30 \sin(\omega t + 10°)$ and $v_2 = 20 \sin(\omega t + 50°)$, which of these statements are true?

(a) v_1 leads v_2 (b) v_2 leads v_1
(c) v_2 lags v_1 (d) v_1 lags v_2
(e) v_1 and v_2 are in phase

9.5 The voltage across an inductor leads the current through it by 90°.

(a) True (b) False

9.6 The imaginary part of impedance is called:

(a) resistance (b) admittance
(c) susceptance (d) conductance
(e) reactance

9.7 The impedance of a capacitor increases with increasing frequency.

(a) True (b) False

9.8 At what frequency will the output voltage $v_o(t)$ in Fig. 9.39 be equal to the input voltage $v(t)$?

(a) 0 rad/s (b) 1 rad/s (c) 4 rad/s
(d) ∞ rad/s (e) none of the above

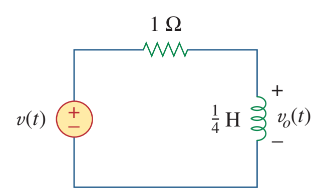


Figure 9.39
For Review Question 9.8.

9.9 A series *RC* circuit has $|V_R| = 12$ V and $|V_C| = 5$ V. The magnitude of the supply voltage is:

(a) −7 V (b) 7 V (c) 13 V (d) 17 V

9.10 A series *RCL* circuit has $R = 30\ \Omega$, $X_C = 50\ \Omega$, and $X_L = 90\ \Omega$. The impedance of the circuit is:

(a) $30 + j140\ \Omega$ (b) $30 + j40\ \Omega$
(c) $30 - j40\ \Omega$ (d) $-30 - j40\ \Omega$
(e) $-30 + j40\ \Omega$

Answers: 9.1d, 9.2c, 9.3b, 9.4b,d, 9.5a, 9.6e, 9.7b, 9.8d, 9.9c, 9.10b.

Problems

Section 9.2 Sinusoids

9.1 Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10°)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t = 10$ ms.

9.2 A current source in a linear circuit has

$$i_s = 15 \cos(25\pi t + 25°) \text{ A}$$

(a) What is the amplitude of the current?
(b) What is the angular frequency?
(c) Find the frequency of the current.
(d) Calculate i_s at $t = 2$ ms.

9.3 Express the following functions in cosine form:

(a) $10 \sin(\omega t + 30°)$ (b) $-9 \sin(8t)$
(c) $-20 \sin(\omega t + 45°)$

9.4 Design a problem to help other students better understand sinusoids.

9.5 Given $v_1 = 45 \sin(\omega t + 30°)$ V and $v_2 = 50 \cos(\omega t - 30°)$ V, determine the phase angle between the two sinusoids and which one lags the other.

9.6 For the following pairs of sinusoids, determine which one leads and by how much.

(a) $v(t) = 10 \cos(4t - 60°)$ and $i(t) = 4 \sin(4t + 50°)$

(b) $v_1(t) = 4 \cos(377t + 10°)$ and $v_2(t) = -20 \cos 377t$

(c) $x(t) = 13 \cos 2t + 5 \sin 2t$ and $y(t) = 15 \cos(2t - 11.8°)$

Section 9.3 Phasors

9.7 If $f(\phi) = \cos\phi + j \sin\phi$, show that $f(\phi) = e^{j\phi}$.

9.8 Calculate these complex numbers and express your results in rectangular form:

(a) $\dfrac{60\angle 45°}{7.5 - j10} + j2$

(b) $\dfrac{32\angle -20°}{(6 - j8)(4 + j2)} + \dfrac{20}{-10 + j24}$

(c) $20 + (16\angle -50°)(5 + j12)$

9.9 Evaluate the following complex numbers and leave your results in polar form:

(a) $5\angle 30°\left(6 - j8 + \dfrac{3\angle 60°}{2 + j}\right)$

(b) $\dfrac{(10\angle 60°)(35\angle -50°)}{(2 + j6) - (5 + j)}$

9.10 Design a problem to help other students better understand phasors.

9.11 Find the phasors corresponding to the following signals:

(a) $v(t) = 21 \cos(4t - 15°)$ V

(b) $i(t) = -8 \sin(10t + 70°)$ mA

(c) $v(t) = 120 \sin(10t - 50°)$ V

(d) $i(t) = -60 \cos(30t + 10°)$ mA

9.12 Let $\mathbf{X} = 4\angle 40°$ and $\mathbf{Y} = 20\angle -30°$. Evaluate the following quantities and express your results in polar form:

(a) $(\mathbf{X} + \mathbf{Y})\mathbf{X}^*$

(b) $(\mathbf{X} - \mathbf{Y})^*$

(c) $(\mathbf{X} + \mathbf{Y})/\mathbf{X}$

9.13 Evaluate the following complex numbers:

(a) $\dfrac{2 + j3}{1 - j6} + \dfrac{7 - j8}{-5 + j11}$

(b) $\dfrac{(5\angle 10°)(10\angle -40°)}{(4\angle -80°)(-6\angle 50°)}$

(c) $\begin{vmatrix} 2 + j3 & -j2 \\ -j2 & 8 - j5 \end{vmatrix}$

9.14 Simplify the following expressions:

(a) $\dfrac{(5 - j6) - (2 + j8)}{(-3 + j4)(5 - j) + (4 - j6)}$

(b) $\dfrac{(240\angle 75° + 160\angle -30°)(60 - j80)}{(67 + j84)(20\angle 32°)}$

(c) $\left(\dfrac{10 + j20}{3 + j4}\right)^2 \sqrt{(10 + j5)(16 - j20)}$

9.15 Evaluate these determinants:

(a) $\begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix}$

(b) $\begin{vmatrix} 20\angle -30° & -4\angle -10° \\ 16\angle 0° & 3\angle 45° \end{vmatrix}$

(c) $\begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix}$

9.16 Transform the following sinusoids to phasors:

(a) $-20 \cos(4t + 135°)$ (b) $8 \sin(20t + 30°)$

(c) $20 \cos(2t) + 15 \sin(2t)$

9.17 Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10 \cos(50t - \pi/3)$ V and $v_2 = 12 \cos(50t + 30°)$ V, find v.

9.18 Obtain the sinusoids corresponding to each of the following phasors:

(a) $\mathbf{V}_1 = 60\angle 15°$ V, $\omega = 1$

(b) $\mathbf{V}_2 = 6 + j8$ V, $\omega = 40$

(c) $\mathbf{I}_1 = 2.8e^{-j\pi/3}$ A, $\omega = 377$

(d) $\mathbf{I}_2 = -0.5 - j1.2$ A, $\omega = 10^3$

9.19 Using phasors, find:

(a) $3 \cos(20t + 10°) - 5 \cos(20t - 30°)$

(b) $40 \sin 50t + 30 \cos(50t - 45°)$

(c) $20 \sin 400t + 10 \cos(400t + 60°) - 5 \sin(400t - 20°)$

9.20 A linear network has a current input $7.5 \cos(10t + 30°)$ A and a voltage output $120 \cos(10t + 75°)$ V. Determine the associated impedance.

9.21 Simplify the following:

(a) $f(t) = 5\cos(2t + 15°) - 4\sin(2t - 30°)$

(b) $g(t) = 8\sin t + 4\cos(t + 50°)$

(c) $h(t) = \int_0^t (10\cos 40t + 50\sin 40t)\,dt$

9.22 An alternating voltage is given by $v(t) = 55\cos(5t + 45°)$ V. Use phasors to find

$$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)\,dt$$

Assume that the value of the integral is zero at $t = -\infty$.

9.23 Apply phasor analysis to evaluate the following:

(a) $v = [110\sin(20t + 30°) + 220\cos(20t - 90°)]$ V

(b) $i = [30\cos(5t + 60°) - 20\sin(5t + 60°)]$ A

9.24 Find $v(t)$ in the following integrodifferential equations using the phasor approach:

(a) $v(t) + \int v\,dt = 10\cos t$

(b) $\frac{dv}{dt} + 5v(t) + 4\int v\,dt = 20\sin(4t + 10°)$

9.25 Using phasors, determine $i(t)$ in the following equations:

(a) $2\frac{di}{dt} + 3i(t) = 4\cos(2t - 45°)$

(b) $10\int i\,dt + \frac{di}{dt} + 6i(t) = 5\cos(5t + 22°)$ A

9.26 The loop equation for a series *RLC* circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^{t} i\,dt = \cos 2t \text{ A}$$

Assuming that the value of the integral at $t = -\infty$ is zero, find $i(t)$ using the phasor method.

9.27 A parallel *RLC* circuit has the node equation

$$\frac{dv}{dt} + 50v + 100\int v\,dt = 110\cos(377t - 10°) \text{ V}$$

Determine $v(t)$ using the phasor method. You may assume that the value of the integral at $t = -\infty$ is zero.

Section 9.4 Phasor Relationships for Circuit Elements

9.28 Determine the current that flows through an 8-Ω resistor connected to a voltage source $v_s = 110\cos 377t$ V.

9.29 What is the instantaneous voltage across a 2-μF capacitor when the current through it is $i = 4\sin(10^6 t + 25°)$ A?

9.30 A voltage $v(t) = 100\cos(60t + 20°)$ V is applied to a parallel combination of a 40-kΩ resistor and a 50-μF capacitor. Find the steady-state currents through the resistor and the capacitor.

9.31 A series *RLC* circuit has $R = 80\ \Omega$, $L = 240$ mH, and $C = 5$ mF. If the input voltage is $v(t) = 10\cos 2t$, find the currrent flowing through the circuit.

9.32 Using Fig. 9.40, design a problem to help other students better understand phasor relationships for circuit elements.

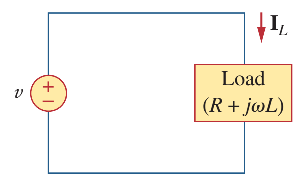


Figure 9.40
For Prob. 9.32.

9.33 A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

9.34 What value of ω will cause the forced response, v_o, in Fig. 9.41 to be zero?

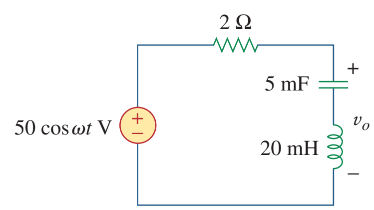


Figure 9.41
For Prob. 9.34.

Section 9.5 Impedance and Admittance

9.35 Find current i in the circuit of Fig. 9.42, when $v_s(t) = 50\cos 200t$ V.

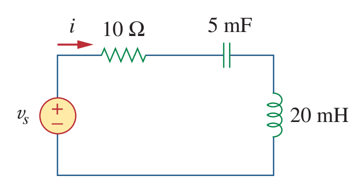


Figure 9.42
For Prob. 9.35.

9.36 Using Fig. 9.43, design a problem to help other students better understand impedance.

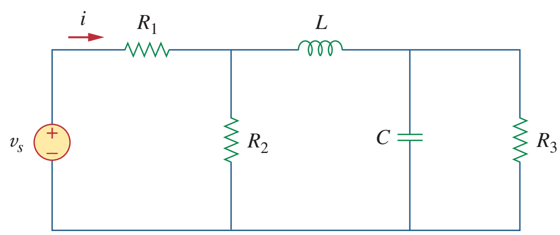


Figure 9.43
For Prob. 9.36.

9.37 Determine the admittance **Y** for the circuit in Fig. 9.44.

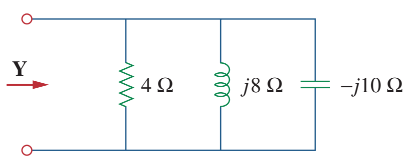


Figure 9.44
For Prob. 9.37.

9.38 Using Fig. 9.45, design a problem to help other students better understand admittance.

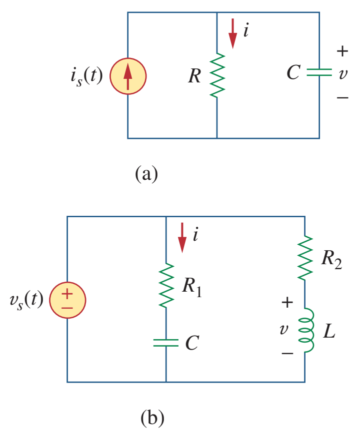


Figure 9.45
For Prob. 9.38.

9.39 For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current **I**. Let $\omega = 10$ rad/s.

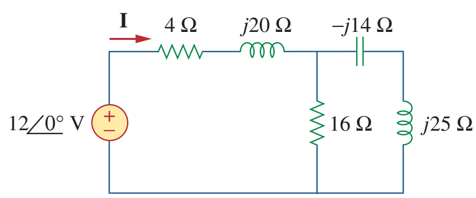


Figure 9.46
For Prob. 9.39.

9.40 In the circuit of Fig. 9.47, find i_o when:

(a) $\omega = 1$ rad/s (b) $\omega = 5$ rad/s
(c) $\omega = 10$ rad/s

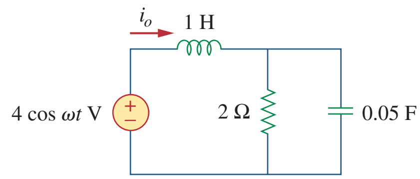


Figure 9.47
For Prob. 9.40.

9.41 Find $v(t)$ in the RLC circuit of Fig. 9.48.

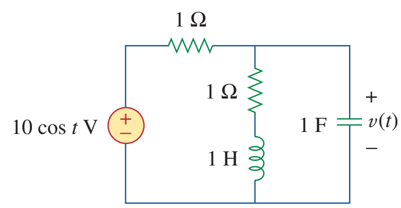


Figure 9.48
For Prob. 9.41.

9.42 Calculate $v_o(t)$ in the circuit of Fig. 9.49.

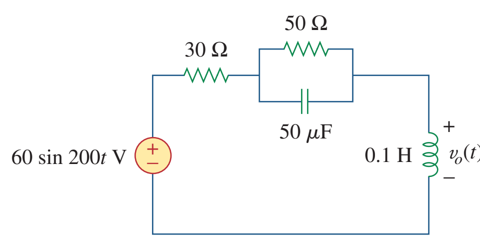


Figure 9.49
For Prob. 9.42.

9.43 Find current $\mathbf{I}_o$ in the circuit shown in Fig. 9.50.

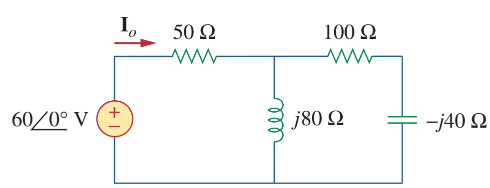


Figure 9.50
For Prob. 9.43.

9.44 Calculate $i(t)$ in the circuit of Fig. 9.51.

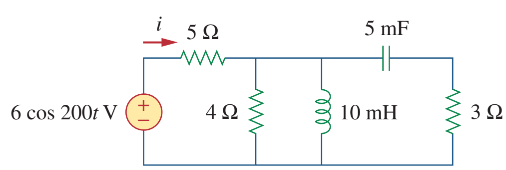


Figure 9.51
For prob. 9.44.

9.45 Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

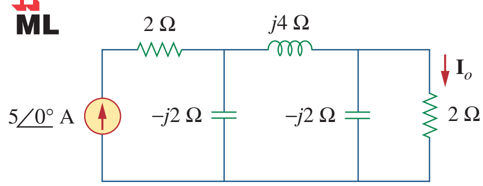


Figure 9.52
For Prob. 9.45.

9.46 If $i_s = 5\cos(10t + 40°)$ A in the circuit of Fig. 9.53, find i_o.

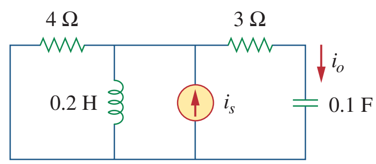


Figure 9.53
For Prob. 9.46.

9.47 In the circuit of Fig. 9.54, determine the value of $i_s(t)$.

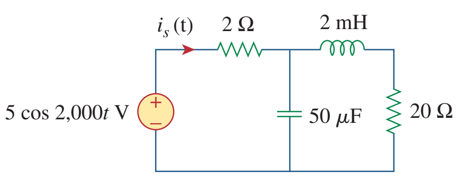


Figure 9.54
For Prob. 9.47.

9.48 Given that $v_s(t) = 20\sin(100t - 40°)$ in Fig. 9.55, determine $i_x(t)$.

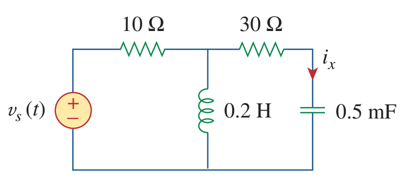


Figure 9.55
For Prob. 9.48.

9.49 Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is $0.5\sin 200t$ A.

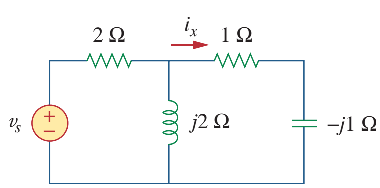


Figure 9.56
For Prob. 9.49.

9.50 Determine v_x in the circuit of Fig. 9.57. Let $i_s(t) = 5\cos(100t + 40°)$ A.

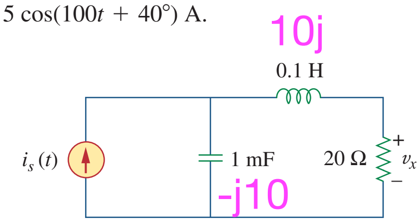


Figure 9.57
For Prob. 9.50.

9.51 If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is $10\cos 2t$ V, obtain i_s.

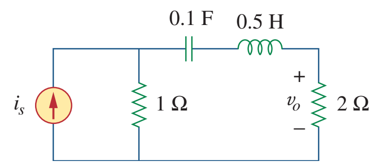


Figure 9.58
For Prob. 9.51.

9.52 If $\mathbf{V}_o = 8\angle 30°$ V in the circuit of Fig. 9.59, find $\mathbf{I}_s$.

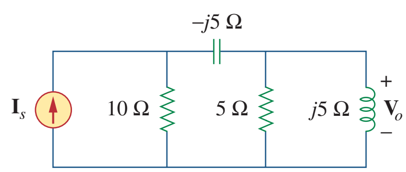


Figure 9.59
For Prob. 9.52.

9.53 Find $\mathbf{I}_o$ in the circuit of Fig. 9.60.

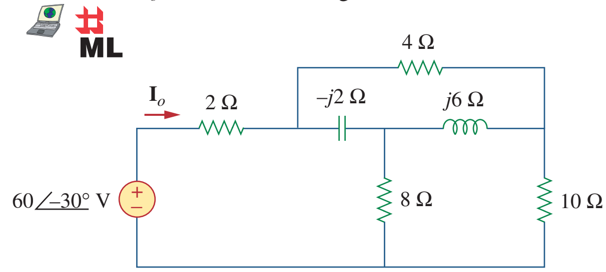


Figure 9.60
For Prob. 9.53.

9.54 In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 2\angle 0°$ A.

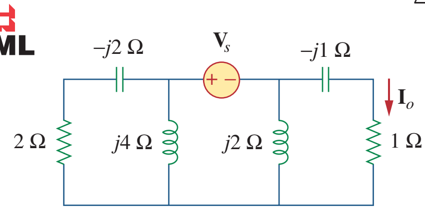


Figure 9.61
For Prob. 9.54.

***9.55** Find $\mathbf{Z}$ in the network of Fig. 9.62, given that $\mathbf{V}_o = 4\underline{/0^\circ}$ V.

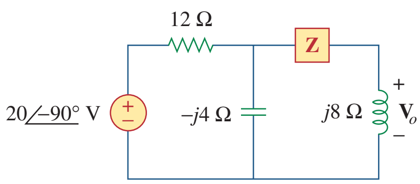


Figure 9.62
For Prob. 9.55.

Section 9.7 Impedance Combinations

9.56 At $\omega = 377$ rad/s, find the input impedance of the circuit shown in Fig. 9.63.

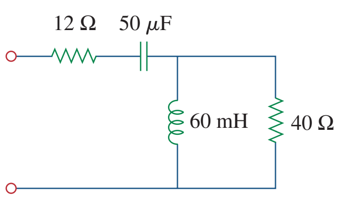


Figure 9.63
For Prob. 9.56.

9.57 At $\omega = 1$ rad/s, obtain the input admittance in the circuit of Fig. 9.64.

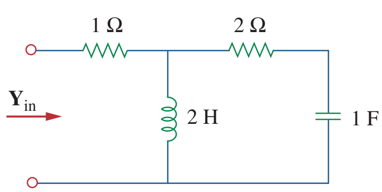


Figure 9.64
For Prob. 9.57.

9.58 Using Fig. 9.65, design a problem to help other students better understand impedance combinations.

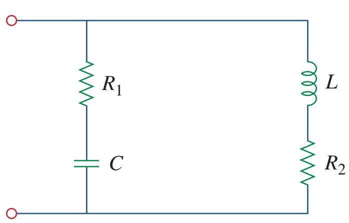


Figure 9.65
For Prob. 9.58.

9.59 For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 10$ rad/s.

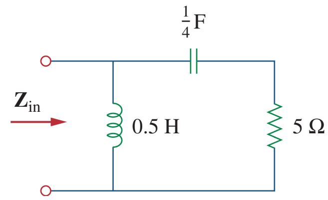


Figure 9.66
For Prob. 9.59.

9.60 Obtain $\mathbf{Z}_{in}$ for the circuit in Fig. 9.67.

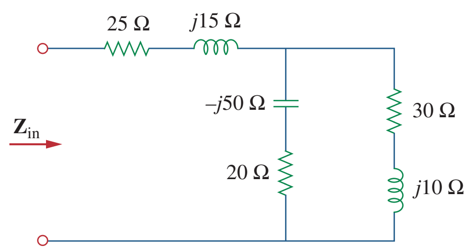


Figure 9.67
For Prob. 9.60.

9.61 Find $\mathbf{Z}_{eq}$ in the circuit of Fig. 9.68.

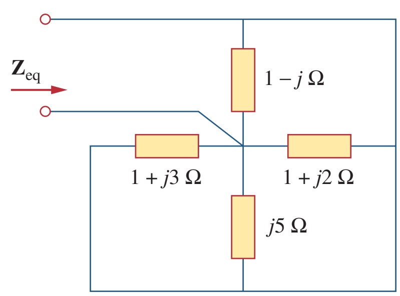


Figure 9.68
For Prob. 9.61.

9.62 For the circuit in Fig. 9.69, find the input impedance $\mathbf{Z}_{in}$ at 10 krad/s.

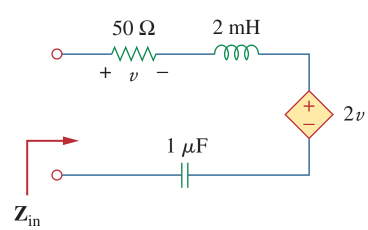


Figure 9.69
For Prob. 9.62.

* An asterisk indicates a challenging problem.

9.63 For the circuit in Fig. 9.70, find the value of $\mathbf{Z}_T$.

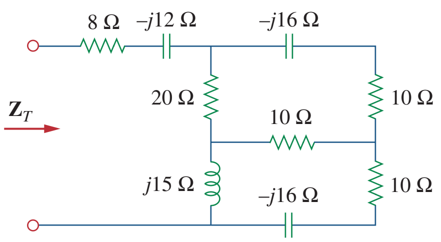


Figure 9.70
For Prob. 9.63.

9.64 Find $\mathbf{Z}_T$ and $\mathbf{I}$ in the circuit in Fig. 9.71.

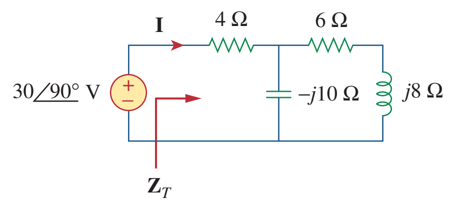


Figure 9.71
For Prob. 9.64.

9.65 Determine $\mathbf{Z}_T$ and $\mathbf{I}$ for the circuit in Fig. 9.72.

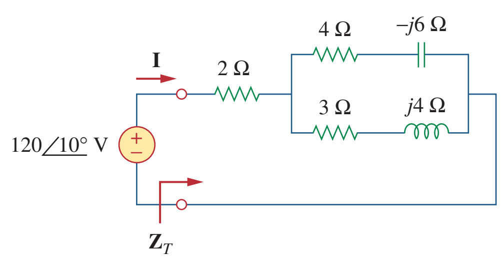


Figure 9.72
For Prob. 9.65.

9.66 For the circuit in Fig. 9.73, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.

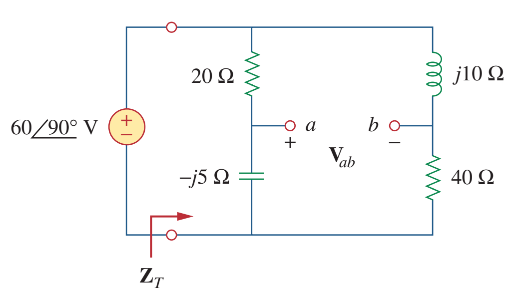


Figure 9.73
For Prob. 9.66.

9.67 At $\omega = 10^3$ rad/s, find the input admittance of each of the circuits in Fig. 9.74.

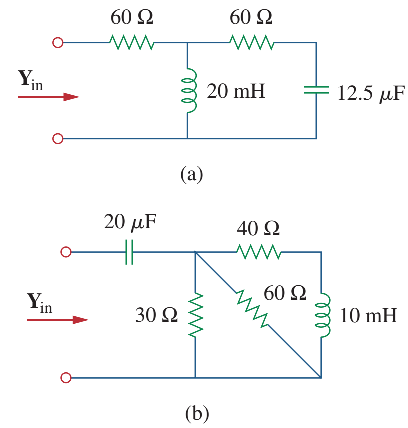


Figure 9.74
For Prob. 9.67.

9.68 Determine $\mathbf{Y}_{eq}$ for the circuit in Fig. 9.75.

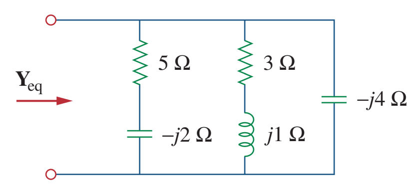


Figure 9.75
For Prob. 9.68.

9.69 Find the equivalent admittance $\mathbf{Y}_{eq}$ of the circuit in Fig. 9.76.

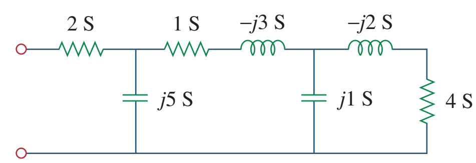


Figure 9.76
For Prob. 9.69.

9.70 Find the equivalent impedance of the circuit in Fig. 9.77.

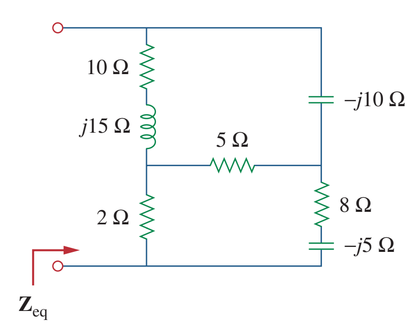


Figure 9.77
For Prob. 9.70.

9.71 Obtain the equivalent impedance of the circuit in Fig. 9.78.

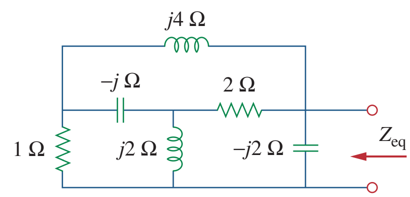


Figure 9.78
For Prob. 9.71.

9.72 Calculate the value of $\mathbf{Z}_{ab}$ in the network of Fig. 9.79.

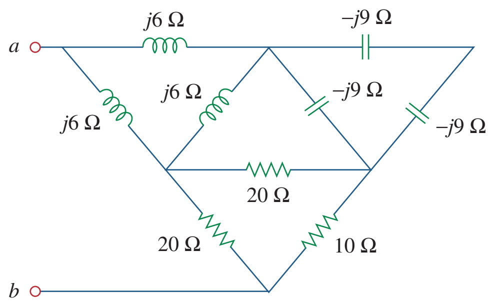


Figure 9.79
For Prob. 9.72.

9.73 Determine the equivalent impedance of the circuit in Fig. 9.80.

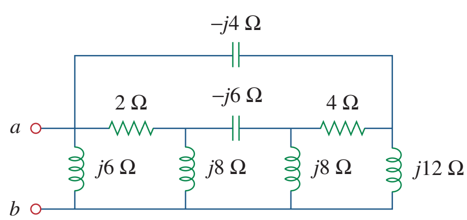


Figure 9.80
For Prob. 9.73.

Section 9.8 Applications

9.74 Design an *RL* circuit to provide a 90° leading phase shift.

9.75 Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

9.76 For the following pairs of signals, determine if v_1 leads or lags v_2 and by how much.

(a) $v_1 = 10\cos(5t - 20°)$, $v_2 = 8\sin 5t$

(b) $v_1 = 19\cos(2t + 90°)$, $v_2 = 6\sin 2t$

(c) $v_1 = -4\cos 10t$, $v_2 = 15\sin 10t$

9.77 Refer to the *RC* circuit in Fig. 9.81.

(a) Calculate the phase shift at 2 MHz.

(b) Find the frequency where the phase shift is 45°.

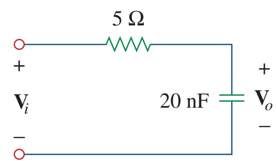


Figure 9.81
For Prob. 9.77.

9.78 A coil with impedance $8 + j6\ \Omega$ is connected in series with a capacitive reactance X. The series combination is connected in parallel with a resistor R. Given that the equivalent impedance of the resulting circuit is $5\angle 0°\ \Omega$, find the value of R and X.

9.79 (a) Calculate the phase shift of the circuit in Fig. 9.82.
(b) State whether the phase shift is leading or lagging (output with respect to input).
(c) Determine the magnitude of the output when the input is 120 V.

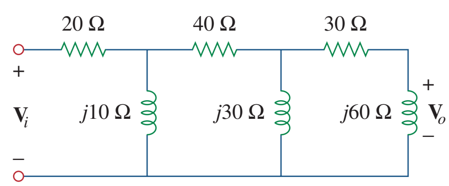


Figure 9.82
For Prob. 9.79.

9.80 Consider the phase-shifting circuit in Fig. 9.83. Let $\mathbf{V}_i = 120$ V operating at 60 Hz. Find:

(a) $\mathbf{V}_o$ when R is maximum

(b) $\mathbf{V}_o$ when R is minimum

(c) the value of R that will produce a phase shift of 45°

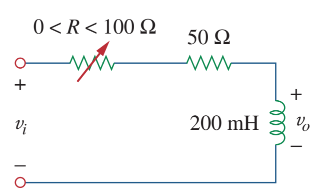


Figure 9.83
For Prob. 9.80.

9.81 The ac bridge in Fig. 9.37 is balanced when $R_1 = 400\ \Omega$, $R_2 = 600\ \Omega$, $R_3 = 1.2\ \text{k}\Omega$, and $C_2 = 0.3\ \mu\text{F}$. Find R_x and C_x. Assume R_2 and C_2 are in series.

9.82 A capacitance bridge balances when $R_1 = 100\ \Omega$, $R_2 = 2\ \text{k}\Omega$, and $C_s = 40\ \mu\text{F}$. What is C_x, the capacitance of the capacitor under test?

9.83 An inductive bridge balances when $R_1 = 1.2\ \text{k}\Omega$, $R_2 = 500\ \Omega$, and $L_s = 250$ mH. What is the value of L_x, the inductance of the inductor under test?

9.84 The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance C_s. Show that when the bridge is balanced,

$$L_x = R_2R_3C_s \qquad \text{and} \qquad R_x = \frac{R_2}{R_1}R_3$$

Find L_x and R_x for $R_1 = 40\ \text{k}\Omega$, $R_2 = 1.6\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$.

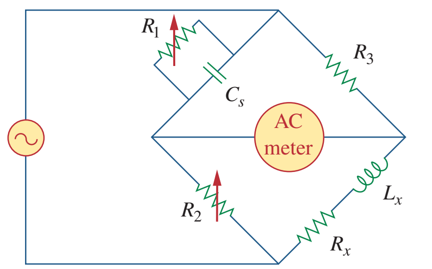


Figure 9.84
Maxwell bridge; For Prob. 9.84.

9.85 The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi\sqrt{R_2R_4C_2C_4}}$$

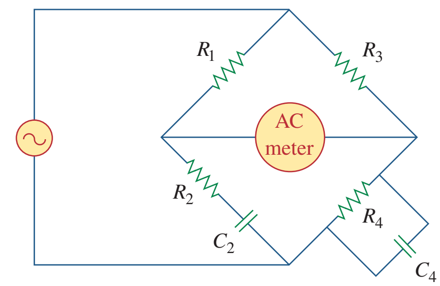


Figure 9.85
Wien bridge; For Prob. 9.85.

Comprehensive Problems

9.86 The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?

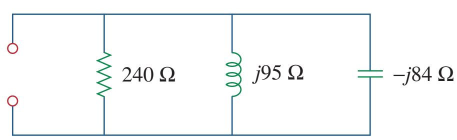


Figure 9.86
For Prob. 9.86.

9.87 The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

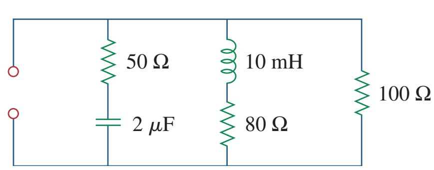


Figure 9.87
For Prob. 9.87.

9.88 A series audio circuit is shown in Fig. 9.88.

(a) What is the impedance of the circuit?

(b) If the frequency were halved, what would be the impedance of the circuit?

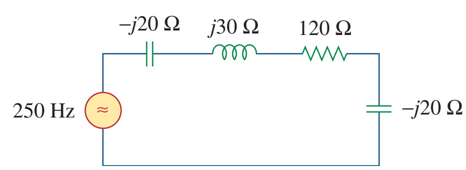


Figure 9.88
For Prob. 9.88.

9.89 An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor C across the series combination so that the net impedance is resistive at a frequency of 2 kHz.

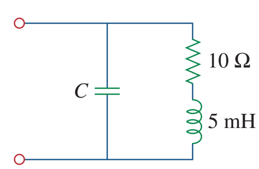


Figure 9.89
For Prob. 9.89.

9.90 An industrial coil is modeled as a series combination of an inductance L and resistance R, as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz when the circuit operates in the steady state:

$$|\mathbf{V}_s| = 145 \text{ V}, \qquad |\mathbf{V}_1| = 50 \text{ V}, \qquad |\mathbf{V}_o| = 110 \text{ V}$$

Use these measurements to determine the values of L and R.

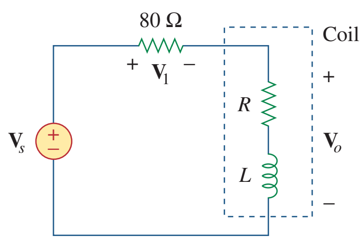


Figure 9.90
For Prob. 9.90.

9.91 Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of C?

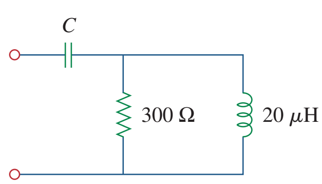


Figure 9.91
For Prob. 9.91.

9.92 A transmission line has a series impedance of $\mathbf{Z} = 100\underline{/75^\circ}$ Ω and a shunt admittance of $\mathbf{Y} = 450\underline{/48^\circ}$ μS. Find: (a) the characteristic impedance $\mathbf{Z}_o = \sqrt{\mathbf{Z}/\mathbf{Y}}$, (b) the propagation constant $\gamma = \sqrt{\mathbf{Z}\mathbf{Y}}$.

9.93 A power transmission system is modeled as shown in Fig. 9.92. Given the source voltage and circuit elements

$\mathbf{V}_s = 115\underline{/0^\circ}$ V, source impedance
$\mathbf{Z}_s = (1 + j0.5)$ Ω, line impedance
$\mathbf{Z}_\ell = (0.4 + j0.3)$ Ω, and load impedance
$\mathbf{Z}_L = (23.2 + j18.9)$ Ω, find the load current $\mathbf{I}_L$.

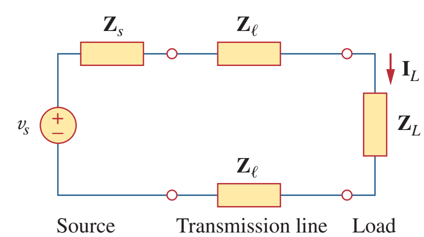


Figure 9.92
For Prob. 9.93.

Practice Problem 10.16

In the Wien-bridge oscillator circuit in Fig. 10.42, let $R_1 = R_2 = 2.5\ \text{k}\Omega$, $C_1 = C_2 = 1$ nF. Determine the frequency f_o of the oscillator.

Answer: 63.66 kHz.

10.10 Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_N$ $(=\mathbf{Z}_{\text{Th}})$.
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

Review Questions

10.1 The voltage $\mathbf{V}_o$ across the capacitor in Fig. 10.43 is:

(a) $5\angle 0^\circ$ V (b) $7.071\angle 45^\circ$ V
(c) $7.071\angle -45^\circ$ V (d) $5\angle -45^\circ$ V

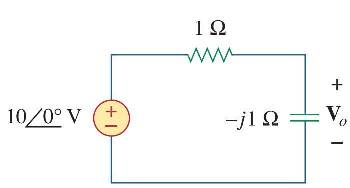


Figure 10.43
For Review Question 10.1.

10.2 The value of the current $\mathbf{I}_o$ in the circuit of Fig. 10.44 is:

(a) $4\angle 0^\circ$ A (b) $2.4\angle -90^\circ$ A
(c) $0.6\angle 0^\circ$ A (d) -1 A

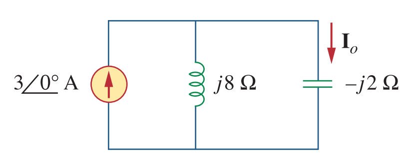


Figure 10.44
For Review Question 10.2.

10.3 Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit of Fig. 10.45 is:

(a) -24 V (b) -8 V
(c) 8 V (d) 24 V

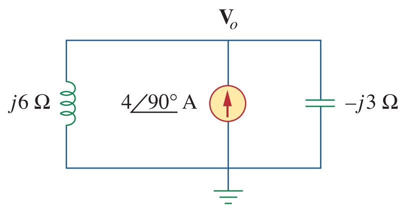


Figure 10.45
For Review Question 10.3.

10.4 In the circuit of Fig. 10.46, current $i(t)$ is:

(a) 10 cos t A (b) 10 sin t A (c) 5 cos t A
(d) 5 sin t A (e) 4.472 cos$(t - 63.43°)$ A

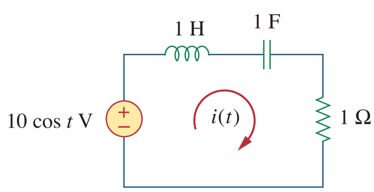


Figure 10.46
For Review Question 10.4.

10.5 Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current $i_x(t)$ can be obtained by:

(a) source transformation
(b) the superposition theorem
(c) *PSpice*

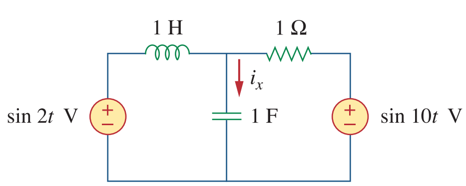


Figure 10.47
For Review Question 10.5.

10.6 For the circuit in Fig. 10.48, the Thevenin impedance at terminals a-b is:

(a) 1 Ω (b) $0.5 - j0.5\ \Omega$
(c) $0.5 + j0.5\ \Omega$ (d) $1 + j2\ \Omega$
(e) $1 - j2\ \Omega$

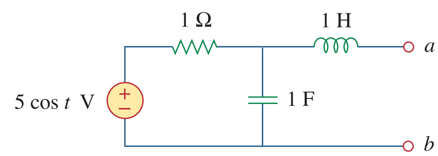


Figure 10.48
For Review Questions 10.6 and 10.7.

10.7 In the circuit of Fig. 10.48, the Thevenin voltage at terminals a-b is:

(a) $3.535\angle -45°$ V (b) $3.535\angle 45°$ V
(c) $7.071\angle -45°$ V (d) $7.071\angle 45°$ V

10.8 Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals a-b is:

(a) $-j4\ \Omega$ (b) $-j2\ \Omega$
(c) $j2\ \Omega$ (d) $j4\ \Omega$

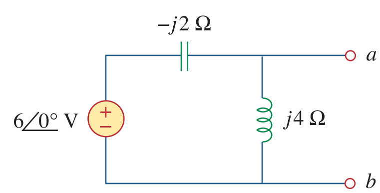


Figure 10.49
For Review Questions 10.8 and 10.9.

10.9 The Norton current at terminals a-b in the circuit of Fig. 10.49 is:

(a) $1\angle 0°$ A (b) $1.5\angle -90°$ A
(c) $1.5\angle 90°$ A (d) $3\angle 90°$ A

10.10 *PSpice* can handle a circuit with two independent sources of different frequencies.

(a) True (b) False

Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.

Problems

Section 10.2 Nodal Analysis

10.1 Determine i in the circuit of Fig. 10.50.

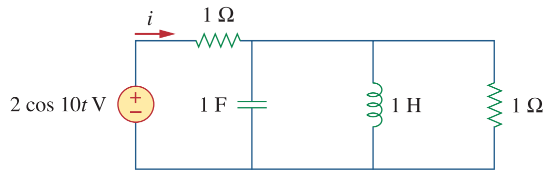


Figure 10.50
For Prob. 10.1.

10.2 Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

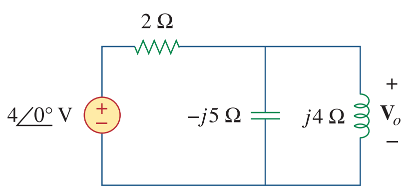


Figure 10.51
For Prob. 10.2.

10.3 Determine v_o in the circuit of Fig. 10.52.

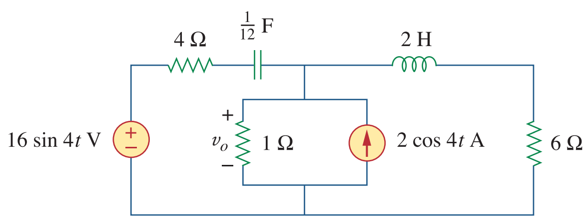


Figure 10.52
For Prob. 10.3.

10.4 Compute $v_o(t)$ in the circuit of Fig. 10.53.

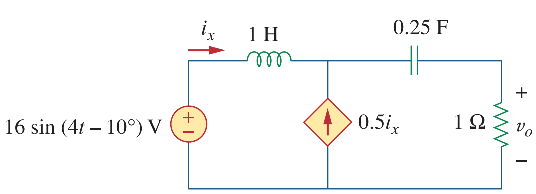


Figure 10.53
For Prob. 10.4.

10.5 Find i_o in the circuit of Fig. 10.54.

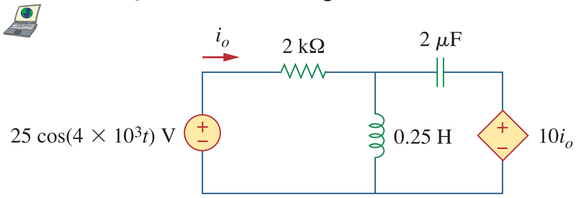


Figure 10.54
For Prob. 10.5.

10.6 Determine $\mathbf{V}_x$ in Fig. 10.55.

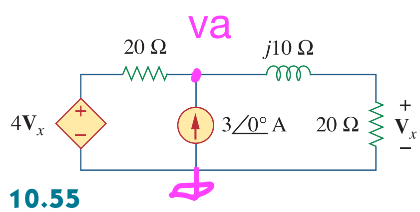


Figure 10.55
For Prob. 10.6.

10.7 Use nodal analysis to find $\mathbf{V}$ in the circuit of Fig. 10.56.

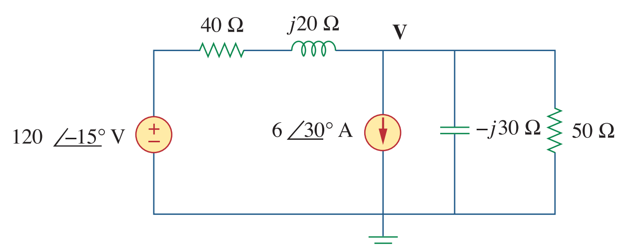


Figure 10.56
For Prob. 10.7.

10.8 Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6 \cos(200t + 15°)$ A.

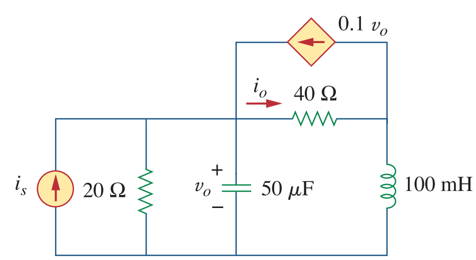


Figure 10.57
For Prob. 10.8.

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.

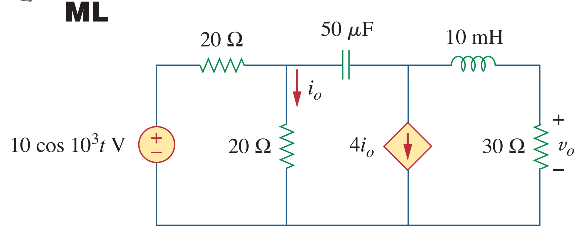


Figure 10.58
For Prob. 10.9.

10.10 Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

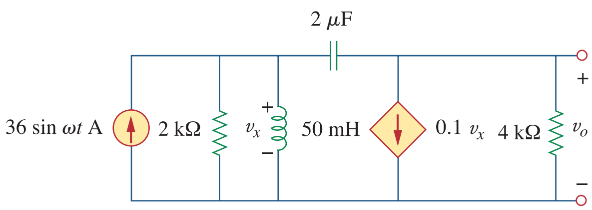


Figure 10.59
For Prob. 10.10.

10.11 Using nodal analysis, find $i_o(t)$ in the circuit in Fig. 10.60.

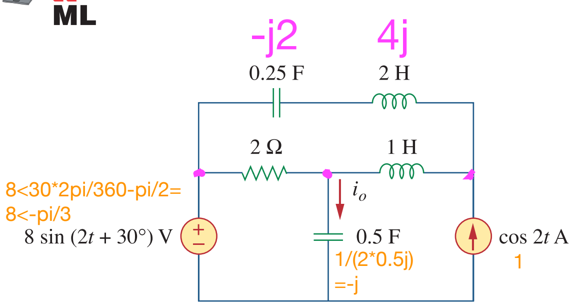


Figure 10.60
For Prob. 10.11.

10.12 Using Fig. 10.61, design a problem to help other students better understand nodal analysis.

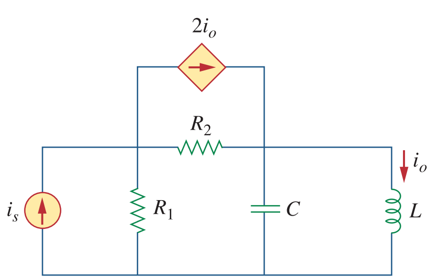


Figure 10.61
For Prob. 10.12.

10.13 Determine $\mathbf{V}_x$ in the circuit of Fig. 10.62 using any method of your choice.

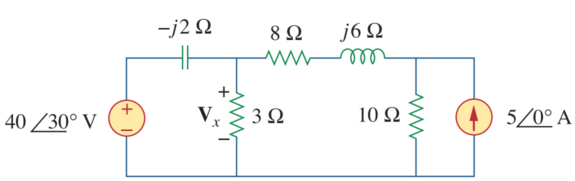


Figure 10.62
For Prob. 10.13.

10.14 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

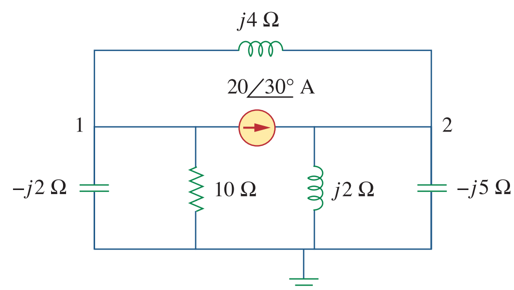


Figure 10.63
For Prob. 10.14.

10.15 Solve for the current **I** in the circuit of Fig. 10.64 using nodal analysis.

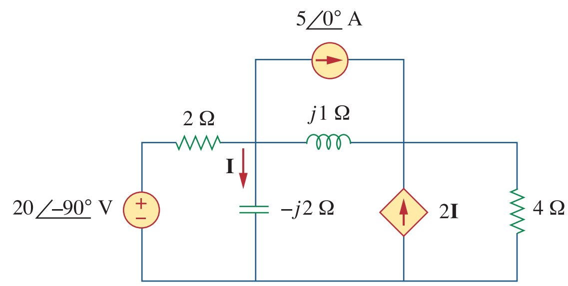


Figure 10.64
For Prob. 10.15.

10.16 Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

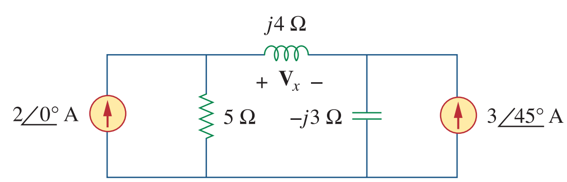


Figure 10.65
For Prob. 10.16.

10.17 By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

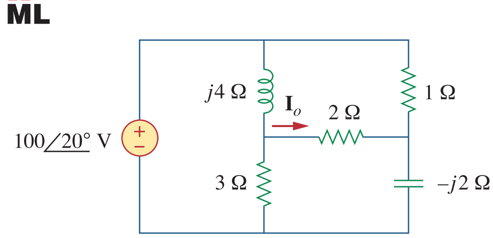


Figure 10.66
For Prob. 10.17.

10.18 Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

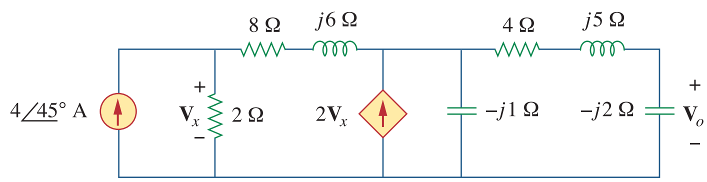


Figure 10.67
For Prob. 10.18.

10.19 Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

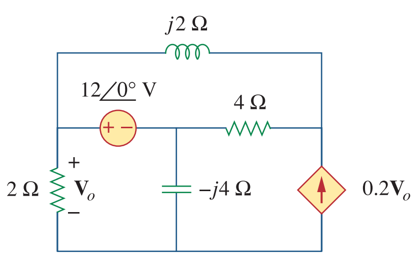


Figure 10.68
For Prob. 10.19.

10.20 Refer to Fig. 10.69. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A \sin(\omega t + \phi)$, derive the expressions for A and ϕ.

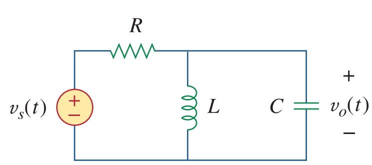


Figure 10.69
For Prob. 10.20.

10.21 For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0$, $\omega \rightarrow \infty$, and $\omega^2 = 1/LC$.

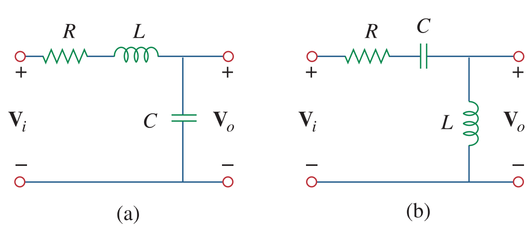


Figure 10.70
For Prob. 10.21.

10.22 For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

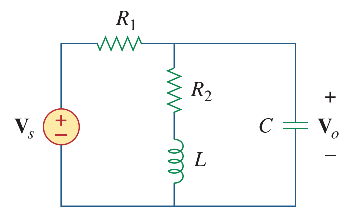


Figure 10.71
For Prob. 10.22.

10.23 Using nodal analysis obtain $\mathbf{V}$ in the circuit of Fig. 10.72.

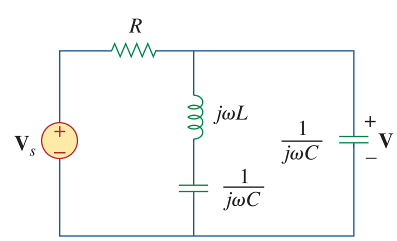


Figure 10.72
For Prob. 10.23.

Section 10.3 Mesh Analysis

10.24 Design a problem to help other students better understand mesh analysis.

10.25 Solve for i_o in Fig. 10.73 using mesh analysis.

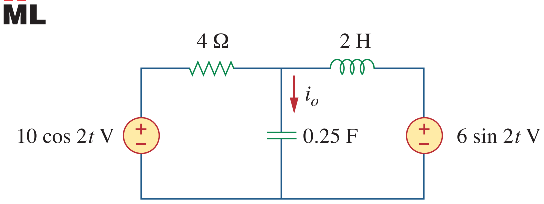


Figure 10.73
For Prob. 10.25.

10.26 Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

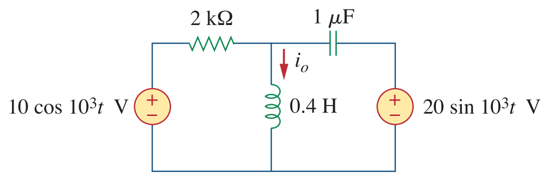


Figure 10.74
For Prob. 10.26.

10.27 Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

ML

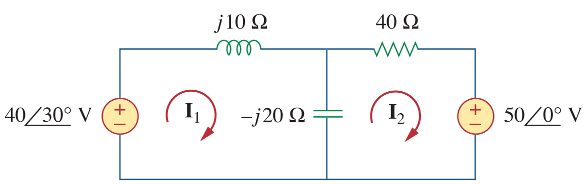


Figure 10.75
For Prob. 10.27.

10.28 In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10 \cos 4t$ **V** and $v_2 = 20 \cos(4t - 30°)$ V.

ML

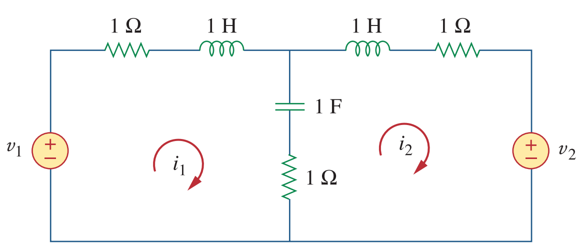


Figure 10.76
For Prob. 10.28.

10.29 Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

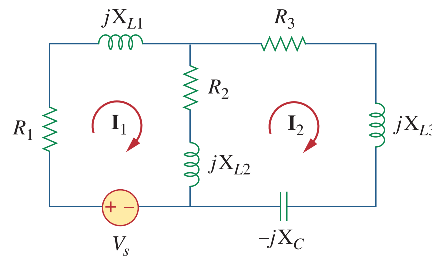


Figure 10.77
For Prob. 10.29.

10.30 Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120 \cos(100t + 90°)$ V, $v_{s2} = 80 \cos 100t$ V.

ML

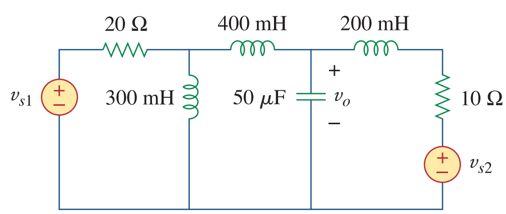


Figure 10.78
For Prob. 10.30.

10.31 Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

ML

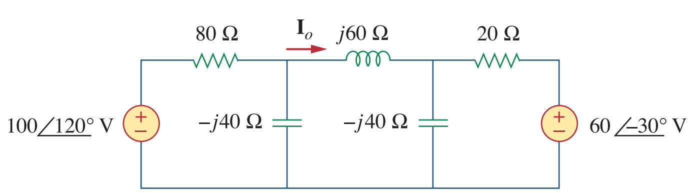


Figure 10.79
For Prob. 10.31.

10.32 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

ML

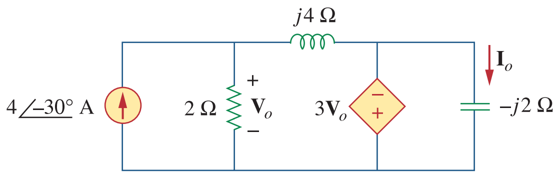


Figure 10.80
For Prob. 10.32.

10.33 Compute **I** in Prob. 10.15 using mesh analysis.

ML

10.34 Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

ML

10.35 Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

ML

10.36 Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

ML

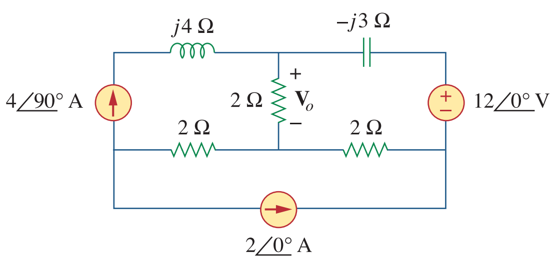


Figure 10.81
For Prob. 10.36.

10.37 Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

ML

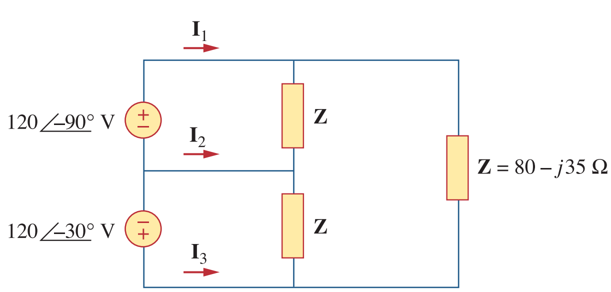


Figure 10.82
For Prob. 10.37.

10.38 Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

ML

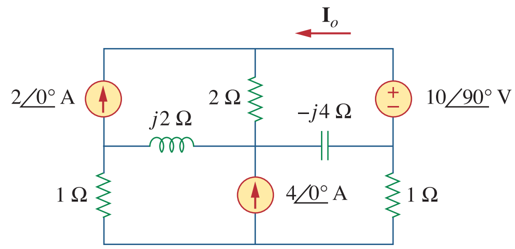


Figure 10.83
For Prob. 10.38.

10.39 Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

ML

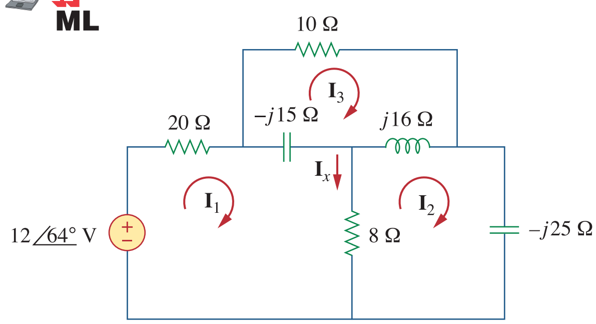


Figure 10.84
For Prob. 10.39.

Section 10.4 Superposition Theorem

10.40 Find i_o in the circuit shown in Fig. 10.85 using superposition.

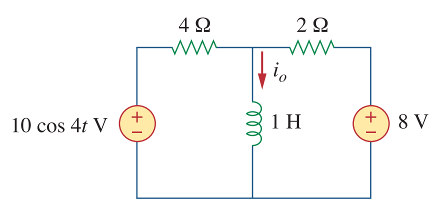


Figure 10.85
For Prob. 10.40.

10.41 Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 6 \cos 2t + 4 \sin 4t$ V.

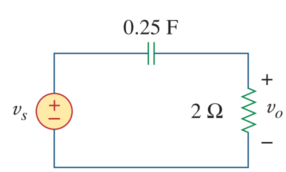


Figure 10.86
For Prob. 10.41.

10.42 Using Fig. 10.87, design a problem to help other students better understand the superposition theorem.

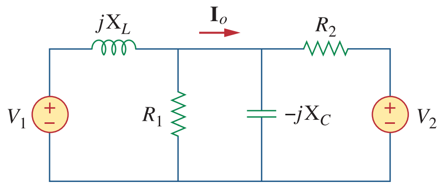


Figure 10.87
For Prob. 10.42.

10.43 Using the superposition principle, find i_x in the circuit of Fig. 10.88.

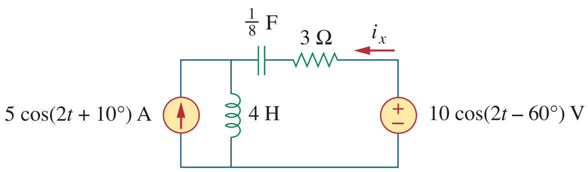


Figure 10.88
For Prob. 10.43.

10.44 Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 50 \sin 2t$ V and $i_s = 12 \cos(6t + 10°)$ A.

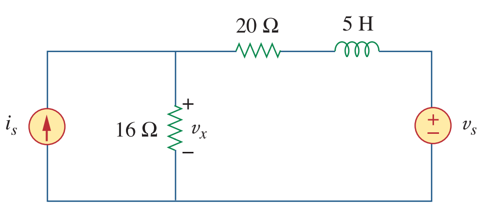


Figure 10.89
For Prob. 10.44.

10.45 Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

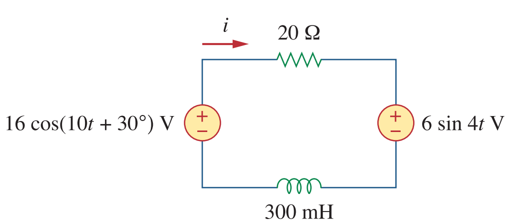


Figure 10.90
For Prob. 10.45.

10.46 Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

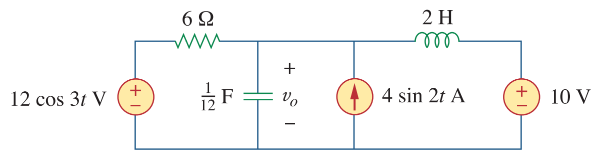


Figure 10.91
For Prob. 10.46.

10.47 Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

ML

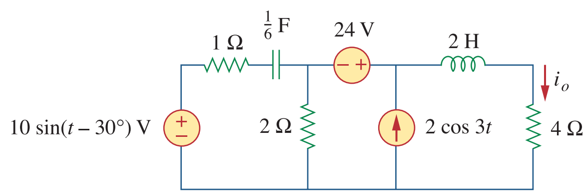


Figure 10.92
For Prob. 10.47.

10.48 Find i_o in the circuit of Fig. 10.93 using superposition.

ML

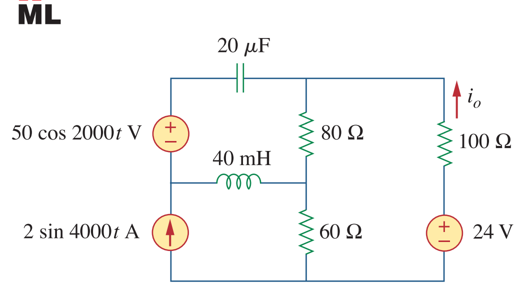


Figure 10.93
For Prob. 10.48.

Section 10.5 Source Transformation

10.49 Using source transformation, find i in the circuit of Fig. 10.94.

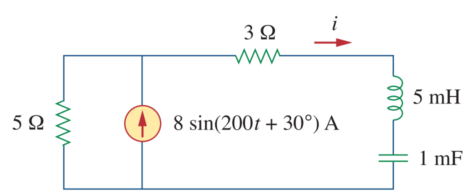


Figure 10.94
For Prob. 10.49.

10.50 Using Fig. 10.95, design a problem to help other students understand source transformation.

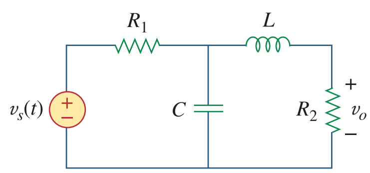


Figure 10.95
For Prob. 10.50.

10.51 Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

10.52 Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

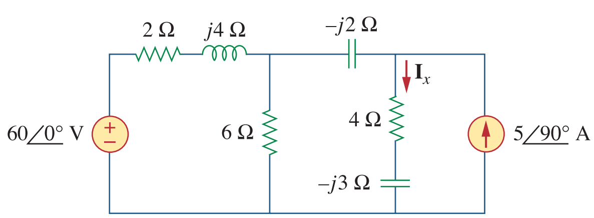


Figure 10.96
For Prob. 10.52.

10.53 Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

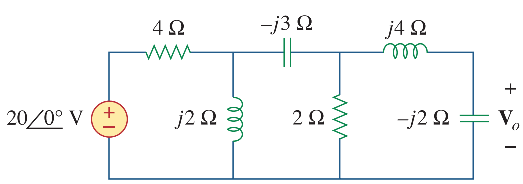
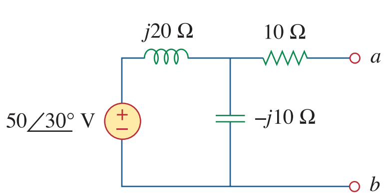


Figure 10.97
For Prob. 10.53.

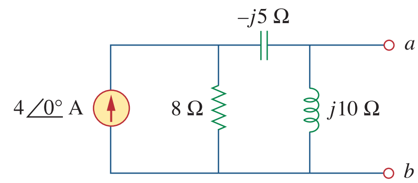
10.54 Rework Prob. 10.7 using source transformation.

Section 10.6 Thevenin and Norton Equivalent Circuits

10.55 Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



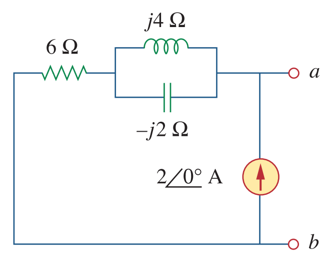
(a)



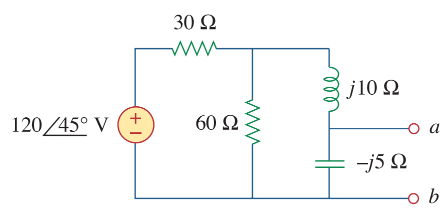
(b)

Figure 10.98
For Prob. 10.55.

10.56 For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.



(a)



(b)

Figure 10.99
For Prob. 10.56.

10.57 Using Fig. 10.100, design a problem to help other students better understand Thevenin and Norton equivalent circuits.

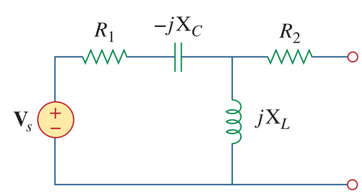


Figure 10.100
For Prob. 10.57.

10.58 For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

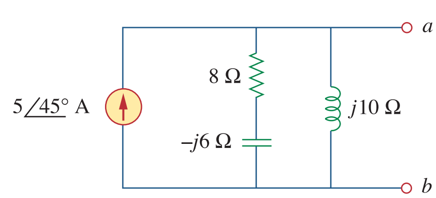


Figure 10.101
For Prob. 10.58.

10.59 Calculate the output impedance of the circuit shown in Fig. 10.102.

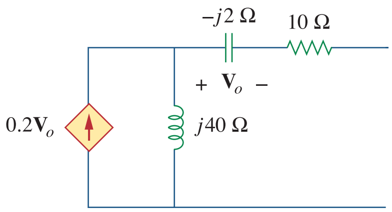


Figure 10.102
For Prob. 10.59.

10.60 Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b* (b) terminals *c-d*

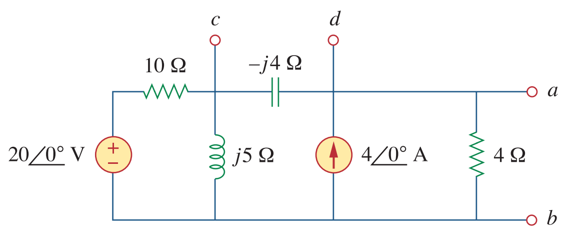


Figure 10.103
For Prob. 10.60.

10.61 Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.104.

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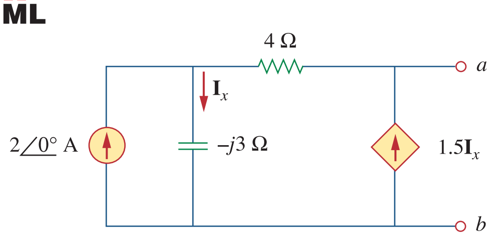

Figure 10.104
For Prob. 10.61.

10.62 Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

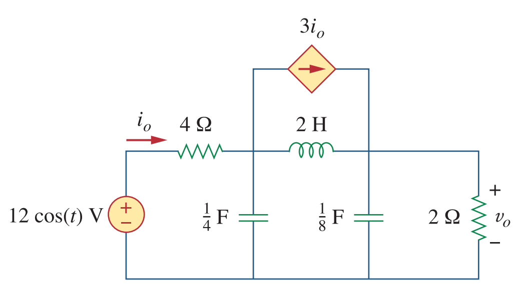

Figure 10.105
For Prob. 10.62.

10.63 Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

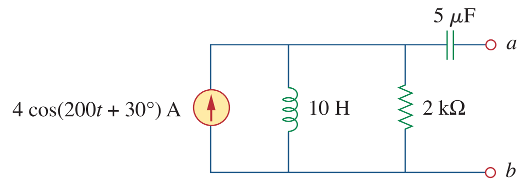

Figure 10.106
For Prob. 10.63.

10.64 For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

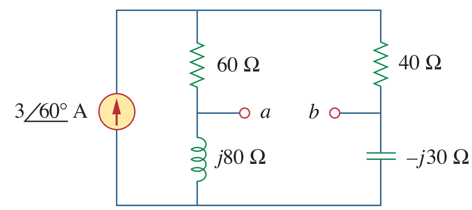

Figure 10.107
For Prob. 10.64.

10.65 Using Fig. 10.108, design a problem to help other students better understand Norton's theorem.

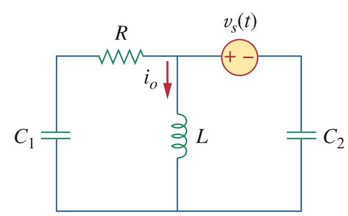

Figure 10.108
For Prob. 10.65.

10.66 At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

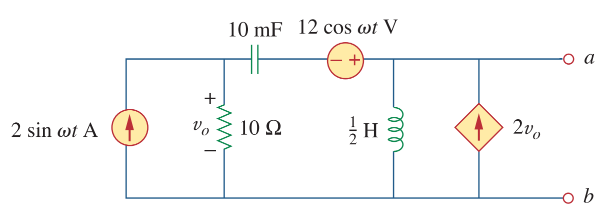

Figure 10.109
For Prob. 10.66.

10.67 Find the Thevenin and Norton equivalent circuits at terminals a-b in the circuit of Fig. 10.110.

ML

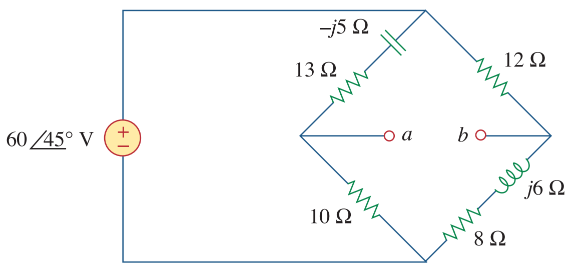

Figure 10.110
For Prob. 10.67.

10.68 Find the Thevenin equivalent at terminals a-b in the circuit of Fig. 10.111.

ML

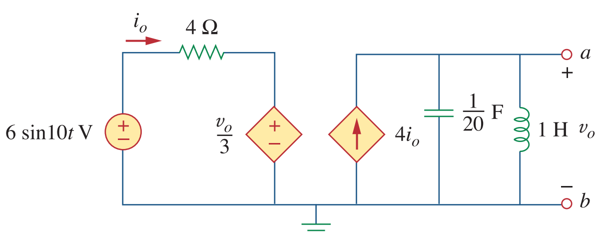

Figure 10.111
For Prob. 10.68.

Section 10.7 Op Amp AC Circuits

10.69 For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin \omega t$ and $\omega = 1/RC$.

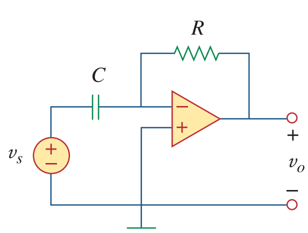

Figure 10.112
For Prob. 10.69.

10.70 Using Fig. 10.113, design a problem to help other students better understand op amps in AC circuits.

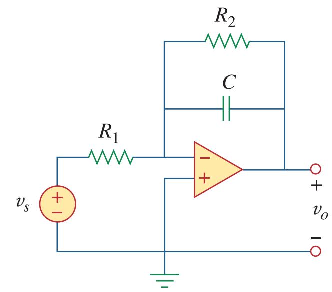

Figure 10.113
For Prob. 10.70.

10.71 Find v_o in the op amp circuit of Fig. 10.114.

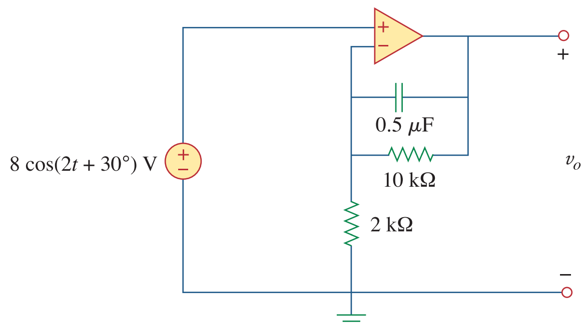

Figure 10.114
For Prob. 10.71.

10.72 Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 4\cos(10^4 t)$ V.

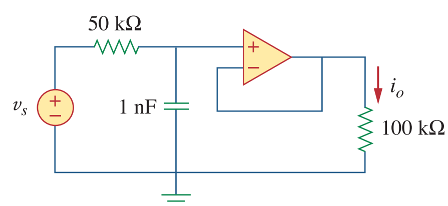

Figure 10.115
For Prob. 10.72.

10.73 If the input impedance is defined as $\mathbf{Z}_{\text{in}} = \mathbf{V}_s/\mathbf{I}_s$, find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $C_1 = 10$ nF, $C_2 = 20$ nF, and $\omega = 5000$ rad/s.

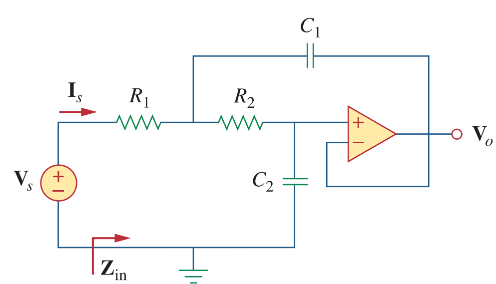

Figure 10.116
For Prob. 10.73.

10.74 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \rightarrow \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

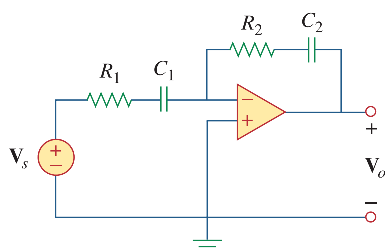

Figure 10.117
For Prob. 10.74.

10.75 In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 40$ kΩ, and $\omega = 2000$ rad/s.

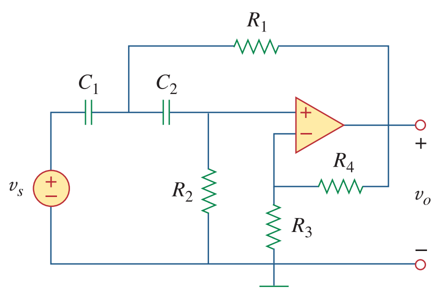

Figure 10.118
For Prob. 10.75.

10.76 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

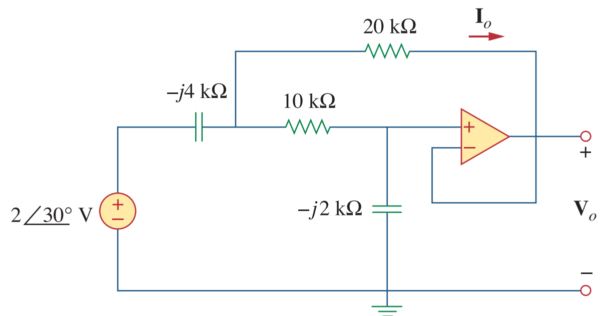

Figure 10.119
For Prob. 10.76.

10.77 Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

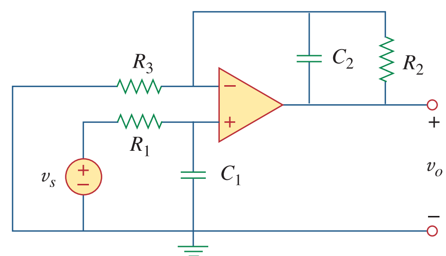

Figure 10.120
For Prob. 10.77.

10.78 Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

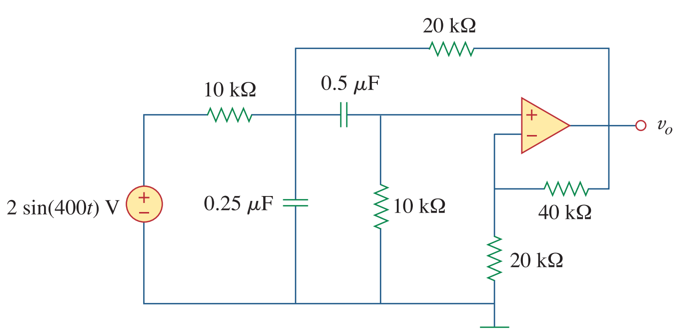

Figure 10.121
For Prob. 10.78.

10.79 For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

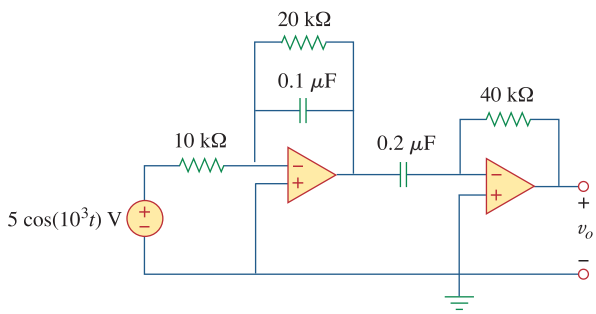

Figure 10.122
For Prob. 10.79.

10.80 Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

ML

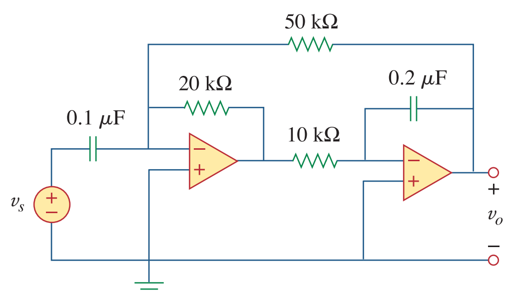

Figure 10.123
For Prob. 10.80.

Section 10.8 AC Analysis Using *PSpice*

10.81 Use *PSpice or MultiSim* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

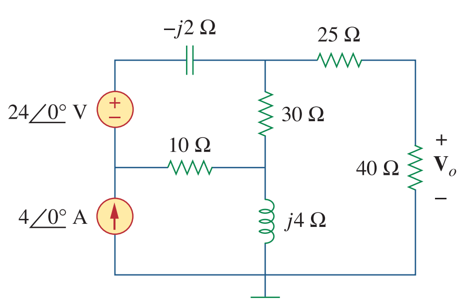

Figure 10.124
For Prob. 10.81.

10.82 Solve Prob. 10.19 using *PSpice or MultiSim.*

10.83 Use *PSpice or MultiSim* to find $v_o(t)$ in the circuit of Fig. 10.125. Let $i_s = 2\cos(10^3t)$ A.

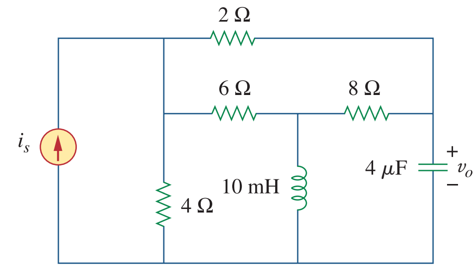

Figure 10.125
For Prob. 10.83.

10.84 Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.126 using *PSpice or MultiSim.*

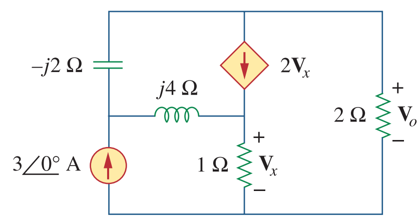

Figure 10.126
For Prob. 10.84.

10.85 Using Fig. 10.127, design a problem to help other students better understand performing AC analysis with *PSpice or MultiSim.*

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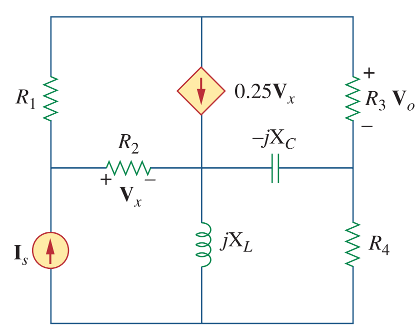

Figure 10.127
For Prob. 10.85.

10.86 Use *PSpice or MultiSim* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.128.

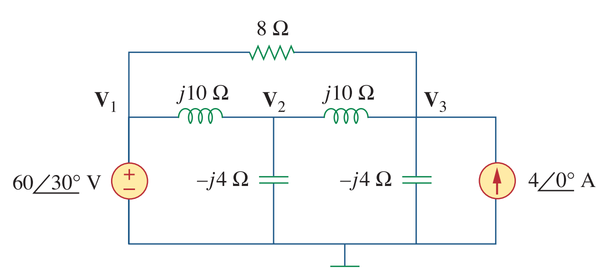

Figure 10.128
For Prob. 10.86.

10.87 Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.129 using *PSpice or MultiSim.*

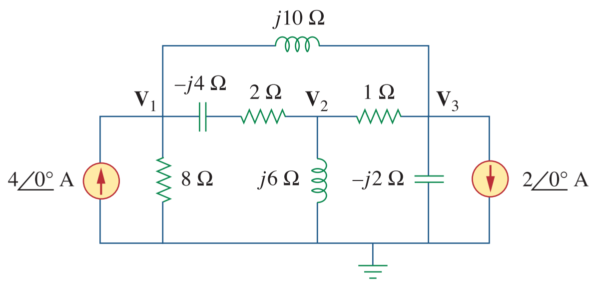

Figure 10.129
For Prob. 10.87.

10.88 Use *PSpice or MultiSim* to find v_o and i_o in the circuit of Fig. 10.130 below.

10.90 Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi\, RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

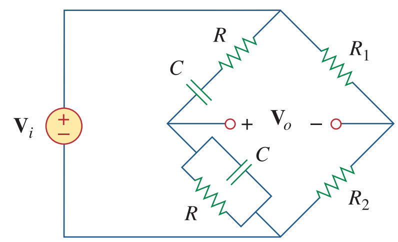

Figure 10.132
For Prob. 10.90.

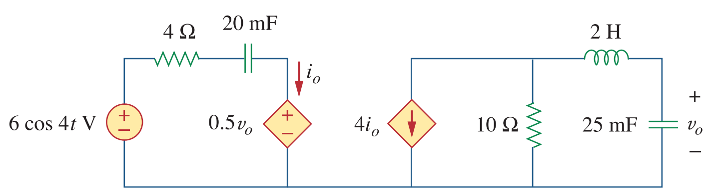

Figure 10.130
For Prob. 10.88.

Section 10.9 Applications

10.89 The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1R_3R_4}{R_2}C$$

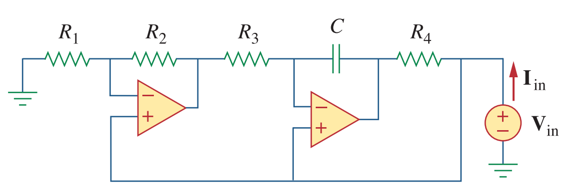

Figure 10.131
For Prob. 10.89.

10.91 Consider the oscillator in Fig. 10.133.

(a) Determine the oscillation frequency.

(b) Obtain the minimum value of R for which oscillation takes place.

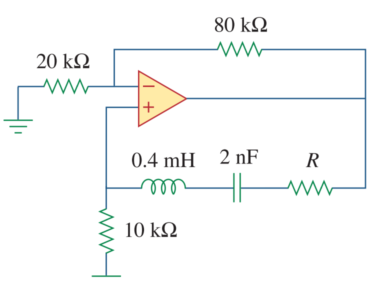

Figure 10.133
For Prob. 10.91.

10.92 The oscillator circuit in Fig. 10.134 uses an ideal op amp.

(a) Calculate the minimum value of R_o that will cause oscillation to occur.

(b) Find the frequency of oscillation.

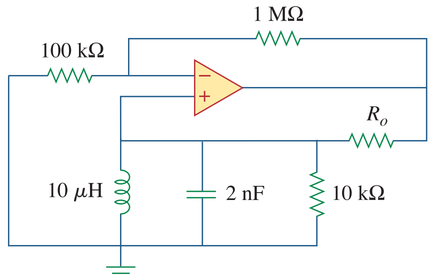

Figure 10.134
For Prob. 10.92.

10.93 Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i \gg X_{C_2}$.

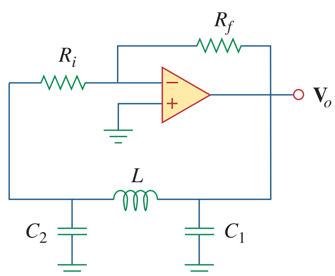

Figure 10.135
A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

10.94 Design a Colpitts oscillator that will operate at 50 kHz.

10.95 Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

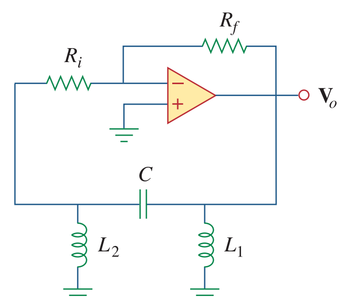

Figure 10.136
A Hartley oscillator; for Prob. 10.95.

10.96 Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

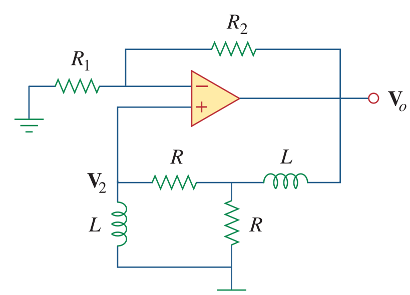

Figure 10.137
For Prob. 10.96.