C.I. on Treatment Mean

∴ if we reject the null hypothesis that treatment means are equal, we can write C.I.s on treatment means:

$(1-\alpha) \times 100\%$ C.I. on μ_i :

some level

μ_i : $\quad \bar{y}_{i\cdot} \pm t_{\alpha/2,\, a(n-1)} \sqrt{\dfrac{MS_E}{n}}$

ex: 20% hardwood concentration had

$\bar{y}_{4\cdot} = 21.167$ p.s.i.

4th level!

$MS_E = 6.51 \qquad t_{\alpha/2,\, a(n-1)} = t_{.025,\, 4 \cdot 5}$
$= 2.086$

$21.167 \pm 2.086 \cdot \sqrt{\dfrac{6.51}{6}}$

$19.00 < \mu_4 < 23.34$ (psi)

C.I. on Difference in Treatment Means

- this can tell us if a pair of treatments significantly differs!
  - if C.I. contains zero, then that pair does not significantly differ

$$\mu_i - \mu_j : \quad \bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm t_{\alpha/2,\, a(n-1)} \sqrt{\frac{2 MS_E}{n}}$$

not the same i and j!
just two different levels!

15% hardwood concentration
10% hardwood concentration

ex: $\mu_3 - \mu_2$

$\bar{y}_{3\cdot} = 17$ psi
$\bar{y}_{2\cdot} = 15.67$ psi

treatment means do differ

$$17 - 15.67 \pm 2.086 \sqrt{\frac{2 \cdot 6.51}{6}}$$

$$-1.743 < \mu_3 - \mu_2 < 4.403 \quad \text{(psi)}$$

- C.I. contains zero! What?!
  - So there is no significant difference in tensile strength between 15% and 10%!
  - So what pairs of concentrations are significant?
- testing every pair would be computationally expensive
- need better multiple comparison method

Fisher's Least Significant Difference

- Fisher recognized that most of the terms in a C.I. on difference in means don't change;

$$\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm \boxed{t_{\alpha/2,\, a(n-1)}\sqrt{\frac{2MSE}{n}}}$$

compute this once

- compare that to $\bar{y}_{i\cdot} - \bar{y}_{j\cdot}$

easy!

Fisher's $LSD = t_{\alpha/2, N-a}\sqrt{\frac{2MSE}{n}}$

∴ compare this to $|\bar{y}_{i\cdot} - \bar{y}_{j\cdot}|$ for every pair, then any difference $< LSD$ is equivalent to a <u>zero</u> in the C.I.

→ insignificant

$$LSD = \underbrace{t_{.025, 24-4}}_{2.086}\sqrt{\frac{2 \cdot 6.51}{6}}$$

$= 3.07$

inside pairs

5% vs. 10% $= |\bar{y}_{1\cdot} - \bar{y}_{2\cdot}| = |10 - 15.67| = 5.67 > LSD$

10% vs. 15% $= |\bar{y}_{2\cdot} - \bar{y}_{3\cdot}| = |15.67 - 17| = \boxed{1.33 < LSD}$

15% vs. 20% $= |\bar{y}_{3\cdot} - \bar{y}_{4\cdot}| = |17 - 21.17| = 4.17 > LSD$

5% vs. 15% $= |\bar{y}_{1\cdot} - \bar{y}_{3\cdot}| = |10 - 17| = 7 > LSD$

10% vs. 20% $= |\bar{y}_{2\cdot} - \bar{y}_{4\cdot}| = |15.67 - 21.17| = 5.5 > LSD$

5% vs. 20% $= |\bar{y}_{1\cdot} - \bar{y}_{4\cdot}| = |10 - 21.17| = 11.17 > LSD$

∴ all pairs of hardwood concentrations show significant difference in tensile strength <u>except</u> the 10% vs. 15% pair

∴ go back to box and whisker plot !

∴ it was evident that this pair exhibited less difference than other pairs