EE 2010 Circuit Analysis

Module 19: Transient Responses: Zero-State, s-domain Problems

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, Michael Richmond and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

We employ the s-domain impedance models,

Zero-State Impedance Models

t-domain	s-domain
$v(t) = i(t) \cdot R$	$V(s) = I(s) \cdot R$
$v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau$	$V(s) = \frac{I(s)}{sC}$
$v(t) = L \frac{di(t)}{dt}$	$V(s) = Ls \cdot I(s)$

Table 1: Models for R, C, and L

and s-domain models for common input stimuli,

Laplace transform pairs

Function	time-domain	s-domain
	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
Unit impulse	$\delta(t)$	1
Delayed impulse	$\delta(t-\tau)$	$e^{-\tau s}$
Unit step	$u(t)$	$\frac{1}{s}$
Delayed unit step	$u(t-\tau)$	$\frac{1}{s}e^{-\tau s}$
Rectangular pulse	$p_T(t) = u(t) - u(t-T)$	$\frac{1}{s}(1 - e^{-Ts})$
Unit ramp	$t \cdot u(t)$	$\frac{1}{s^2}$
nth power	$t^n \cdot u(t)$	$\frac{n!}{s^{n+1}}$
nth root	$\sqrt[n]{t} \cdot u(t)$	
Exponential decay	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$
Exponential approach	$(1 - e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$
Sine	$\sin(\omega t) \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t) \cdot u(t)$	$\frac{s}{s^2+\omega^2}$
Decaying sine	$e^{-\alpha t}\sin(\omega t) \cdot u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
Decaying cosine	$e^{-\alpha t}\cos(\omega t) \cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Table 2: Laplace transform table of common input functions

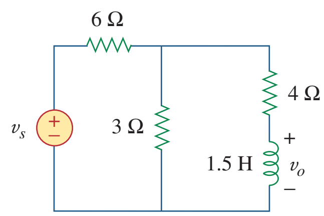
†

Procedure for s-domain Analysis of Zero-State Transient Responses:

1. Replace *all independent sources with symbolic representations such as* $V_{in}(s)$
2. Employ s-domain impedance models
3. Analyze the circuit using appropriate techniques to find the output: $V_{out}(s)$, or whatever output function is dictated by the problem
4. Find the transfer function: $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$, or whatever input-output function is dictated by the problem
5. Find the output $V_{out}(s) = H(s) \cdot V_{in}(s)$, where $V_{in}(s)$ is an s-domain input function
6. Find the output $v_{out}(t) = \text{ilaplace}(V_{out}(s))$ using the inverse Laplace transform solver
7. Plot $v_{out}(t)$ to observe the response of the system to that input

Problem 7.59::

7.59 Determine the step response $v_o(t)$ to $v_s = 18u(t)$ in the circuit of Fig. 7.124.



Comments:

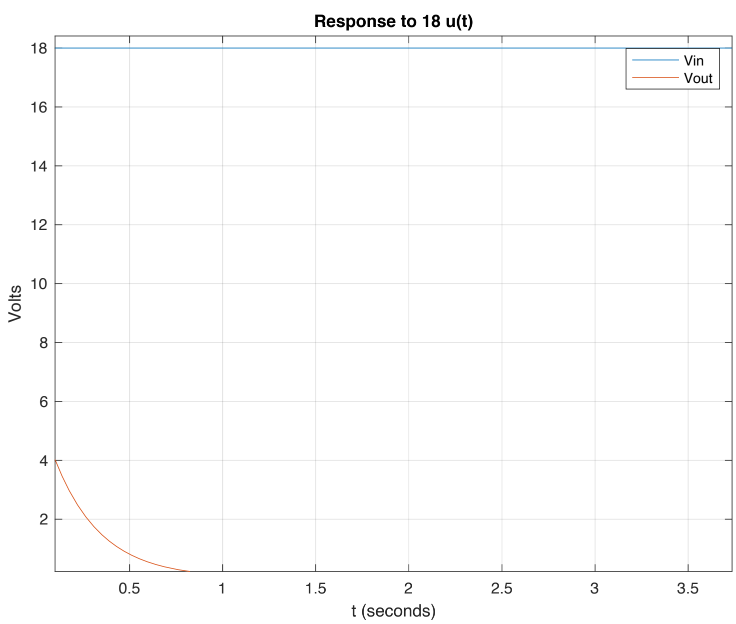
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab:

```
%% Problem 7.59
clear all
syms Vin Va Vout s t
[Va, Vout] = solve((Va-Vin)/6 + Va/3 + Va/(4+1.5*s) == 0,...
  Vout == Va*1.5*s/(1.5*s+4), Va, Vout)
%
H(s) = simplify(Vout/Vin)
% Define unitstep for plotting
unitstep = @(t) +(t>0);
clf
figure(59)
tmax = 4;
% The response to 18u(t)
vout(t) = ilaplace(H(s)*18/s)
fplot(@(t) 18*unitstep(t), [0,tmax])
hold on
fplot(vout(t), [0,tmax]), grid
axis([0, tmax, 0, 20])
legend('Vin','Vout')
title('Response to 18 u(t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

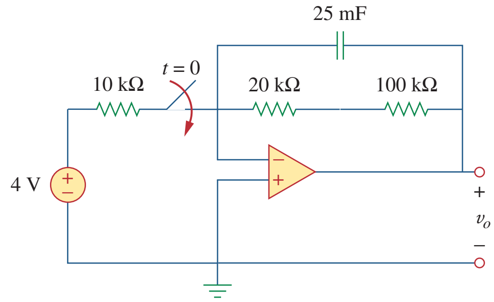
... which yields

```
H(s) = (0.3333*s)/(s + 4)
vout(t) = 6*exp(-4*t)
```



Which is an *exponential response.*

Problem 7.69::



Comments:

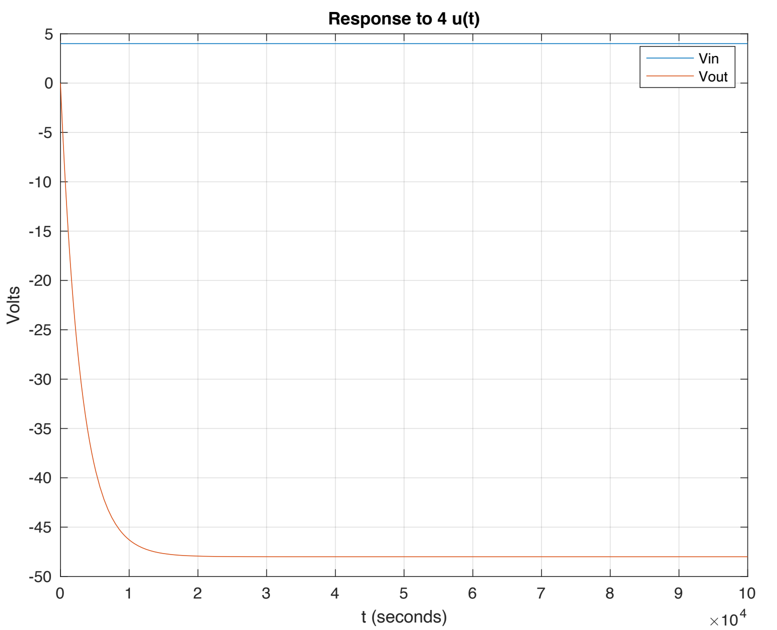
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab:

```
%% Problem 7.69
clear all
syms Vin Vout s t
Vout = solve((0-Vin)/10000 + (0-Vout)/120000 + (0-Vout)*s*25e-3 == 0, Vout)
%
H(s) = simplify(Vout/Vin)
clf
figure(69)
tmax = 100000;
% Plot the input and output
vin(t) = ilaplace(4/s);
vout(t) = ilaplace(H(s)*(4/s))
fplot(vin(t), [0,tmax])
hold on
fplot(vout(t), [0,tmax]), grid
axis([0, tmax, -50, 5])
legend('Vin','Vout')
title('Response to 4 u(t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

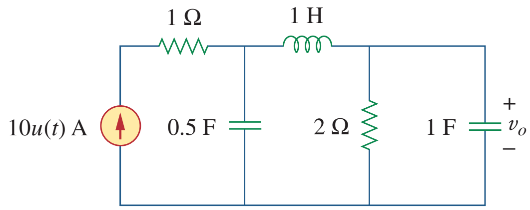
... which yields

```
H(s) = -12/(3000*s + 1)
vout(t) = 48*exp(-3.3333e-04*t) - 48
```



Which is an *exponential response* with a very long time constant.

Problem 16.61::



Comments:

Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

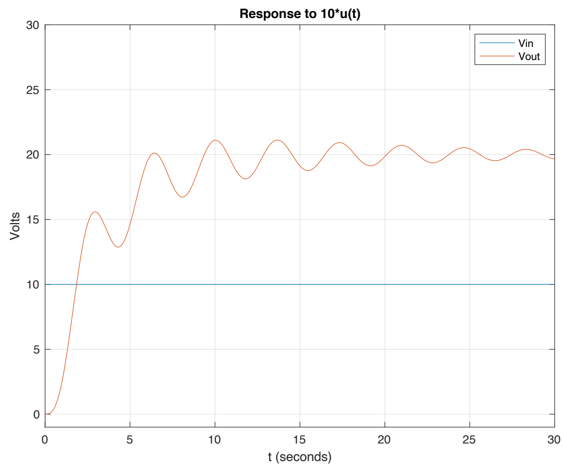
We'll verify the particulars in Matlab with two node equations and a voltage divider:

```
%% Problem 16.61
clear all
syms Iin Va Vout s t
[Va, Vout] = solve(-Iin + Va*s*0.5 + (Va-Vout)/s == 0,...
  (Vout-Va)/s + Vout/2 + Vout*s == 0, Va, Vout);
%
H(s) = simplify(Vout/Iin)
figure(66)
tmax = 30;
% Plot the input and output
vin(t) = ilaplace(10/s);
Vout(s) = H(s)*10/s;
% Doing an ilaplace direcrtly does not work here.
% Make it more tractable by doing a variable-precision arithmetic operation
vout(t) = vpa(ilaplace(Vout,s,t))
%
fplot(vin(t), [0,tmax])
hold on
fplot(vout(t), [0,tmax]), grid
axis([0, tmax, -1, 30])
legend('Vin','Vout')
title('Response to 10*u(t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

... which yields

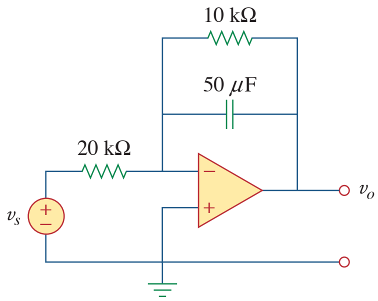
```
H(s) = 4/(2*s^3 + s^2 + 6*s + 2)
vout(t) = 20 - 0.4043*exp(-0.0803*t)*cos(1.7144*t) - 3.8995*exp(-0.0803*t)*sin
    (1.7144*t) - 19.5957*exp(-0.3395*t)
```

which is a 3-rd order system with a VERY interesting step response!



Problem 16.66::

16.66 For the op amp circuit in Fig. 16.89, find $v_0(t)$ for $t > 0$. Take $v_s = 3\,e^{-5t}u(t)$ V.



Comments:

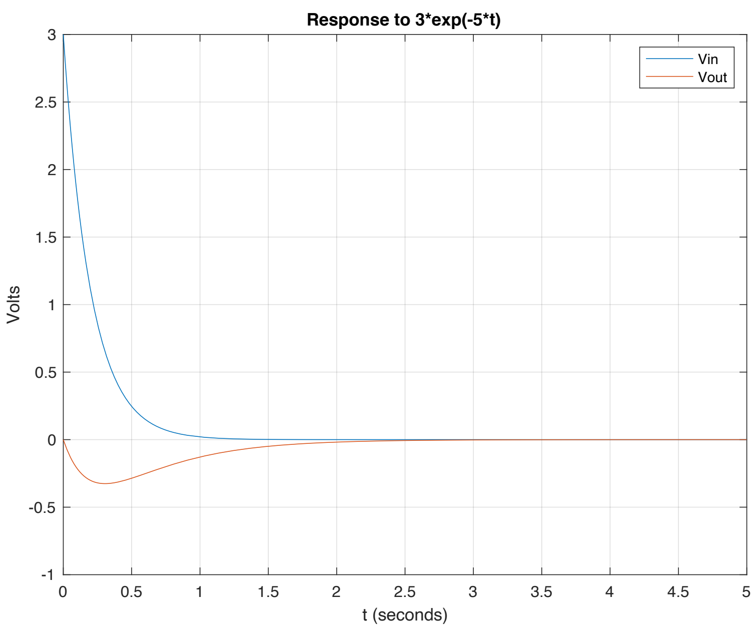
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab with one node equation:

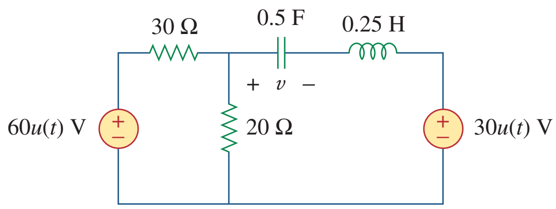
```
%% Problem 16.66
clear all
syms Vin Vout s t
Vout = solve((0-Vin)/20000 + (0-Vout)/10000 + (0-Vout)*s*50e-6 == 0, Vout)
%
H(s) = simplify(Vout/Vin)
clf
figure(66)
tmax = 5;
% Plot the input and output
vin(t) = 3*exp(-5*t);
vout(t) = ilaplace(H(s)*(3/(s+5)))
fplot(vin(t), [0,tmax])
hold on
fplot(vout(t), [0,tmax]), grid
axis([0, tmax, -1, 3])
legend('Vin','Vout')
title('Response to 3*exp(-5*t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

... which yields

```
H(s) = -1/(s + 2)
vout(t) = exp(-5*t) - exp(-2*t)
```



Problem 8.39::



Comments:

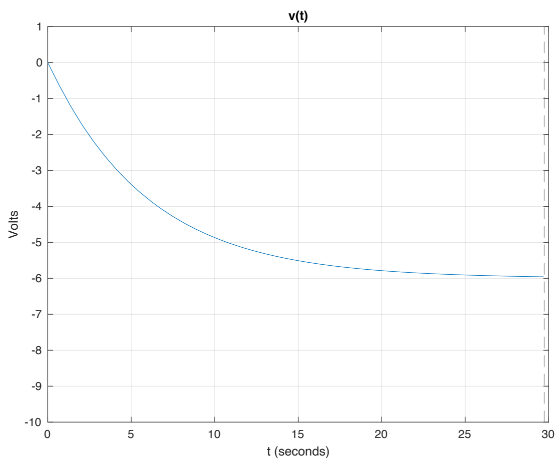
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab with two sources, one node equation:

```
%% Problem 8.39
clear all
syms Vin60 Vin30 Va s t
Va = solve(Va/20 + (Va-Vin60)/30 + (Va-Vin30)/(s*0.25+(1/(0.5*s)))== 0, Va)
V(s) = (Va-Vin30)*(1/(0.5*s))/(s*0.25+(1/(0.5*s)))
% We could separate this via superposition, but going right to the answer:
clf
figure(39)
tmax = 30;
% Find and plot the output
v(t) = ilaplace(subs(V(s),{Vin60,Vin30}, {60/s, 30/s}))
fplot(v(t), [0,tmax]), grid
axis([0, tmax, -10, 1])
title('v(t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

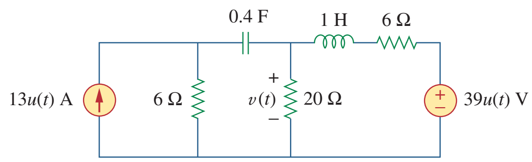
... which yields

```
Va = (2*Vin60*s^2 + 240*Vin30*s + 16*Vin60)/(5*s^2 + 240*s + 40)
V(s) = -(2*(Vin30 - (2*Vin60*s^2 + 240*Vin30*s + 16*Vin60)/(5*s^2 + 240*s + 40)))/(s
    *(0.2500*s + 2/s))
v(t) = 6*exp(-24*t)*(cosh(23.8328*t) + 1.0070*sinh(23.8328*t)) - 6
```



Problem 8.71::

8.71 Obtain $v(t)$ for $0 < t < 4$ s in the circuit of Fig. 8.116 using *PSpice* or *MultiSim*.



Comments:

Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab with two sources, two node equations:

```
%% Problem 8.71
clear all
syms Iin13 Vin39 Va Vb s t
[Va, Vb] = solve(Va/6 - Iin13 + (Va-Vb)*s*0.4 == 0,...
  Vb/20 + (Vb-Va)*s*0.4 + (Vb-Vin39)/(s+6) == 0, Va, Vb)
% We could separate this via superposition, but going right to the answer:
clf
figure(71)
tmax = 30;
% Find and plot the output
v(t) = ilaplace(subs(Vb,{Iin13,Vin39}, {13/s, 39/s}))
fplot(v(t), [0,tmax]), grid
axis([0, tmax, 20, 70])
title('v(t)'); xlabel('t (seconds)'); ylabel('Volts')
% ... and were done.
```

... which yields

```
Va = (30*(26*Iin13 + 49*Iin13*s + 8*Vin39*s + 8*Iin13*s^2))/(52*s^2 + 557*s + 130)
Vb = (20*(5*Vin39 + 72*Iin13*s + 12*Vin39*s + 12*Iin13*s^2))/(52*s^2 + 557*s + 130)
v(t) = 30*exp(-5.3558*t)*(cosh(5.1171*t) + 0.3777*sinh(5.1171*t)) + 30
%
```