ISE 2211 • Exam III • 20 June 2019 NAME SOLUTION

1)Explain how the Central Limit Theorem facilitates the construction of a confidence interval on mean.

→ states that as $n \uparrow$, $\bar{x}$ is normally distributed

$\therefore$ C.I. is range for μ representing some area under normal distribution w/ mean $\bar{x}$

+2

2) The B+ power supply voltage in a tube guitar amplifier may vary according to a number of factors, including AC line voltage, winding tolerances of the power transformer, bias currents of the tubes, and thermal drift. A sample of [redacted] 41 voltage measurements was taken and the results are $\bar{x}$ = 441.2 V and s = 7.23 V, with unknown population variance. Write a 95% confidence interval on the population mean B+ power supply voltage. Please include a unit with your answer.

$n > 30$; $\therefore$ use Z distribution +1

(No penalty for using $t_{\alpha/2,n-1} = t_{.025,40}$

95% C.I. → $\alpha = 0.05$

$\therefore Z_{\alpha/2} = Z_{.025} = 1.960$ +1

$\mu: \quad \bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}} \longrightarrow 441.2 \pm 1.960 \frac{7.23}{\sqrt{41}}$

+1 +1

$439.0 < \mu < 443.4$ V

Now write a 95% prediction interval on the [redacted] 42nd measured power supply voltage. Please include a unit.

$t_{\alpha/2,n-1} = t_{.025,40} = 2.021$ +1

$X_{42}: \bar{x} \pm t_{\alpha/2,n-1}\, s\sqrt{1+\frac{1}{n}}$

$441.2 \pm 2.021 \cdot 7.23\sqrt{1+\frac{1}{41}}$

+1

$426.4 < X_{42} < 456.0$ V +1

Write an upper 95% confidence bound on the standard deviation of B+ power supply voltage. Why would an upper confidence bound be appropriate for this parameter?

Because we would mostly be concerned with how much the voltage could vary; not how little. +1

upper 95% bound : need $\chi^2_{1-\alpha,\, n-1}$ ← not $\alpha/2$! +1

$$= \chi^2_{.95,\, 40} = 26.51 \quad +1$$

+1 eqn $\quad \sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{1-\alpha,\, n-1}} \rightarrow \sigma^2 \leq \dfrac{40 \cdot 7.23^2}{26.51}$

$$\sigma^2 \leq 78.87 \ V^2 \quad +1$$

$$\therefore \ \sigma \leq 8.881 \ V \quad +1$$

41

If six out of the [redacted] voltage measurements are out of specification according to the manufacture, write a 95% confidence interval on the proportion of out-of-spec voltages. Also determine the minimum sample size needed for the width of this C.I. to be ±1%.

$$\hat{p} = \frac{x}{n} = \frac{6}{41} = 0.1463 \quad +1$$

$$Z_{\alpha/2} = Z_{.025} = 1.960 \quad +1$$

$$\therefore \ p : \ \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad +1 \text{ eqn}$$

$$0.1463 \pm 1.960 \sqrt{\frac{.1463(1-.1463)}{41}}$$

$$0.0381 < p < 0.254 \quad +1$$

$$n = 0.25 \left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.25 \left(\frac{1.960}{.01}\right)^2$$

$$n = 9604 \quad +1$$

(ludicrous)