ISE 2211 • Exam V • 26 March 2018 NAME Solution

1) A few weeks ago, a sample of 25 [redacted] craft beers from Yellow Springs Brewery was tested for tannin content with a sample mean of $\bar{x}_1 = 82.68$ µg/mL and sample standard deviation $s_1 = 7.809$ µg/mL. In the interest of gauging consistency of the manufacturing process, a new sample of 16 [redacted] beers was prepared and the results were $\bar{x}_2 = 94.71$ µg/mL and $s_2 = 6.874$ µg/mL. Test the following hypotheses and state whether you would reject or fail to reject the null hypothesis that the two batches of craft beers have equal standard deviations of tannin content at the $\alpha = 0.05$ level of significance.

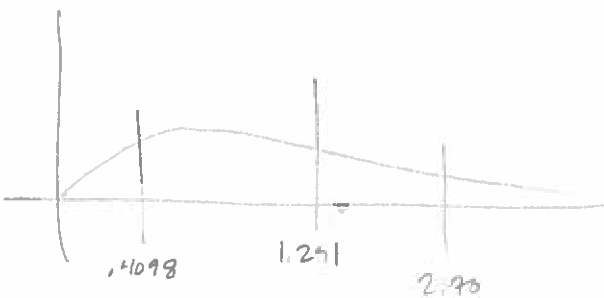
$H_0: \sigma_1 = \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$

$$f_0 = \frac{s_1^2}{s_2^2} = \frac{7.809^2}{6.874^2} = \underline{\underline{1.291}} \quad (+1)$$

$$f_{\alpha/2, n_1-1, n_2-1} = f_{.025, 24, 15} \ (+1) = 2.70 \quad (+1)$$

$$f_{1-\alpha/2, n_1-1, n_2-1} = \frac{1}{f_{\alpha/2, n_2-1, n_1-1}} = \frac{1}{f_{.025, 15, 24}} \ (+1)$$

$$= \frac{1}{2.44} = 0.4098 \quad (+1)$$



$$\left.\begin{array}{l} f_0 \not> f_{\alpha/2, n_1-1, n_2-1} \\ f \not< f_{1-\alpha/2, n_1-1, n_2-1} \end{array}\right\} (+1)$$

∴ fail to reject H_0 (+1)

Write a 95% C.I. on the ratio of population standard deviations and verify that it draws the same conclusion as the fixed-α hypothesis test above.

$$\frac{s_1^2}{s_2^2} f_{1-\alpha/2, n_2-1, n_1-1} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2, n_2-1, n_1-1}$$

$$\Rightarrow \frac{7.809^2}{6.874^2} \cdot \frac{1}{2.70} \ (+1) < \frac{\sigma_1^2}{\sigma_2^2} < \frac{7.809^2}{6.874^2} \cdot 2.44 \ (+1)$$

$$0.478 < \frac{\sigma_1^2}{\sigma_2^2} < 3.149 \Rightarrow 0.691 < \frac{\sigma_1}{\sigma_2} < 1.775$$

(+1) contains 1 ∴ fail (+1) to reject (+1)

Now test the same batches of craft beer to determine if the mean tannin content is equal batch-to-batch. Population variances are unknown but assumed to be equal. Use a two-sided alternative hypothesis @ $\alpha = 0.05$.

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

$$S_p = \sqrt{\frac{(n_1-1)\, s_1^2 + (n_2-1)\, s_2^2}{n + n_2 - 2}}$$

$$= \sqrt{\frac{24 \cdot 7.809^2 + 15 \cdot 6.874^2}{25 + 16 - 2}}$$

$$= 7.463 \quad +1$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - \overset{=0}{\Delta_0}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{82.68 - 94.71}{7.463\sqrt{\frac{1}{25} + \frac{1}{16}}}$$

$$= -5.035 \quad +1$$

critical value : $t_{\alpha/2,\, n_1+n_2-2} = t_{.025,39}$

closest : $t_{.025,40} = 2.021$

$t_0 >>> t_{.025,40}$ +1

REJECT Ho that means are equal +1

2) A <u>paired t-test</u> was performed in which two different grades of cellulose nitrate conformal coating were applied to each of five billets and the coating thickness measured after curing. The results are presented below:

Assembly #	Coating #1 (mm)	Coating #2 (mm)	differences
1	0.214	0.228	−0.014
2	0.318	0.300	+0.018
3	0.267	0.271	−0.004
4	0.371	0.429	−0.058
5	0.229	0.315	−0.086

$\}$ +1

Test the following hypotheses and state whether one type of coating produces a different net film thickness than the other at the $\alpha = 0.05$ level of significance:

H_0: $\mu_D = \Delta_0 = 0$

H_1: $\mu_D \neq 0$

Hint: $s^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$

$$\bar{d} = \frac{\sum d}{n} = \frac{-0.144}{5} = -0.0288$$

+1

$$S_d = 0.042275$$ +2

$$\therefore \; t_0 = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{-.0288}{.042275/\sqrt{5}}$$

$$t_0 = -1.523$$ +1

critical value: $t_{\alpha/2,\,n-1} = t_{.025,4}$ pairs!

+1

$= 2.776$

$t_0 \not> t_{\alpha/2,\,n-1}$ +1

$\therefore$ fail to reject H_0 +1 (no significant difference in coatings