ISE 2211 • Exam VII • 26 July 2018 NAME SOLUTION

It is believed that there is a relationship between atmospheric relative humidity and the hardness of a certain applied finish. Five guitar bodies were finished in each of four different relative humidity environments in random order and subjected to a Vickers hardness test. The results are presented below (+1) (+1)

RH (%)	Vickers Hardness (VH)					Totals	Averages
	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5		
20	31.1	31.7	32.0	30.9	33.8	159.5	31.9
40	30.1	30.4	29.9	28.0	31.6	150	30
60	28.2	30.1	29.4	27.3	26.8	141.8	28.36
80	26.2	28.2	25.7	23.9	26.6	130.6	26.12
						581.9	29.095

(+1)

Use Analysis of Variance (ANOVA) to test the null hypothesis that the treatment means are equal at the $\alpha = 0.01$ level of significance. Fill in the ANOVA table.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0
Treatments	90.3895	3	30.12983	16.397
Error	29.4	16	1.8375	-
Total	119.7895	19	-	-

$MS_{Treatments} = \frac{90.3895}{3}$

$= 30.12983$ (+1)

$MS_E = \frac{29.4}{16} = 1.8375$

$f_0 = \frac{MS\ Treatments}{MS_E}$

$= 16.397$ (+1)

$\sum y_{ij}^2 = 17050.17$

$\sum y_{i\cdot}^2 = 85103.85$ (+1)

$SS_T = \sum\sum y_{ij}^2 - \frac{y_{\cdot\cdot}^2}{N} = 17050.17 - \frac{581.9^2}{20}$

$= 119.7895$ (+1) $an-1 = 19$ d.o.f (+1)

$SS_{Treatments} = \sum \frac{y_{i\cdot}^2}{n} - \frac{y_{\cdot\cdot}^2}{N} = \frac{85103.85}{5} - \frac{581.9^2}{20}$

$= 90.3895$ (+1) $a-1 = 3$ d.o.f. (+1)

$SS_E = SS_T - SS_{Treatments} = 29.4$ (+1)

$a(n-1)$ = 16 d.o.f (+1)

$f_{critical} = f_{.01,3,16} = 5.29$ (+1)

$f_0 >>> f_{critical}$

reject H_0 @ $\alpha = 0.01$ (+1)

Write a 99% confidence interval on Vickers hardness at the 80% relative humidity level.

μ_4: $\bar{y}_{4\cdot} \pm t_{\alpha/2,\, a(n-1)} \sqrt{\frac{MS_E}{n}}$

$t_{.005,16} = 2.921$ (+1)

$26.12 \pm 2.921 \sqrt{\frac{1.8375}{5}}$

$24.35 < \mu_{80\%} < 27.89$ (+2)

Use Fisher's Least Significant Difference to determine which, if any, pairs of relative humidities show significant difference at $\alpha = 0.01$.

$$LSD = t_{\alpha/2,\, a(n-1)} \sqrt{\frac{2MS_E}{n}}$$

$$= 2.921 \sqrt{\frac{2 \cdot 1.8375}{5}}$$

$$= 2.504$$

+1

$\lvert \bar{y}_{20} - \bar{y}_{40} \rvert = \lvert 31.9 - 30 \rvert$	$= 1.9$	$\not>$ LSD	
$\lvert \bar{y}_{40} - \bar{y}_{60} \rvert = \lvert 30 - 28.36 \rvert$	$= 1.639$	$\not>$ LSD	
$\lvert \bar{y}_{60} - \bar{y}_{80} \rvert = \lvert 28.36 - 26.12 \rvert$	$= 2.24$	$\not>$ LSD	
$\lvert \bar{y}_{20} - \bar{y}_{60} \rvert = \lvert 31.9 - 28.36 \rvert$	$= 3.54$	$>$ LSD	
$\lvert \bar{y}_{40} - \bar{y}_{80} \rvert = \lvert 30 - 26.12 \rvert$	$= 3.88$	$>$ LSD	
$\lvert \bar{y}_{20} - \bar{y}_{80} \rvert = \lvert 31.9 - 26.12 \rvert$	$= 5.78$	$>$ LSD	

+1 +1

20% vs. 60%, 40% vs. 80%, and 20% vs. 80% RH show significant differences in Vickers hardness @ $\alpha = 0.01$

+1