ISE 2211 • Exam III • 21 February 2020 /21 NAME SOLUTION

1) In *Bioelectronics II* lab this week, students are attempting to correct the power factor of an inductive load by adding shunt capacitance to the circuit. The winding resistance of the inductor affects the computation of power factor but is not a parameter stated by the manufacturer; thus all population parameters on winding resistance are unknown. Joe Tritschler measured the winding resistance of ten 100-mH inductors with a Fluke 77-series multimeter and got the following results, all in ohms (Ω):

{231.8 231.1 229.9 230.5 230.3 229.8 230.7 232.1 230.1 230.8}

...and because Joe is a *really nice guy*, he calculated the following sample parameters for you:

$\bar{x}$ = 230.7 Ω and s = 0.7709 Ω.

Include a unit with each answer. Note that this data has not been altered in any way to make "nice" results!

Write a 95% confidence interval on the mean value of winding resistance.

unknown σ and $n < 30$ → t-distribution (+1)

$$t_{\alpha/2,n-1} = t_{.025,9} = 2.262 \quad (+2)$$

$\therefore$ C.I. on μ: $\bar{X} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

$$230.7 \pm 2.262\frac{0.7709}{\sqrt{10}}$$

$$230.15 < \mu < 231.25 \quad \Omega$$

(+2) (+U)

/6

2) Write a 95% prediction interval on the eleventh value of winding resistance.

$$X_{n+1}: \quad \bar{x} \pm t_{\alpha/2, n-1}\, s\sqrt{1+\frac{1}{n}}$$

$$230.7 \pm 2.262 \cdot 0.7709 \cdot \sqrt{1+\frac{1}{10}}$$

$$228.87 < X_{11} < 232.53 \quad \Omega$$

+2

+1

3) Write an upper 95% confidence bound on population standard deviation of winding resistance.

upper bound:

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}}$$

$$\chi^2_{1-\alpha, n-1} = \chi^2_{.95, 9} = 3.33$$

+2

$$\sigma^2 \leq \frac{9 \cdot .7709^2}{3.33}$$

$$\sigma^2 \leq 1.606$$

+1

/8

$$\sigma \leq 1.267 \quad \Omega$$

+1

+1

4) If any winding resistance over 231 Ω is considered out-of-spec, write a 95% confidence interval on the proportion of out-of-spec inductors. [Yes, the sample is too small for this to be valid; do it anyway.]

$x = 3$ inductors over 231 Ω

$\therefore \quad \hat{p} = \frac{x}{n} = \frac{3}{10} = 0.3$ (+1)

C.I. on p: $\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$Z_{\alpha/2} = Z_{.025} = 1.960$ (+1)

$0.3 \pm 1.960\sqrt{\frac{0.3(1-0.3)}{10}}$

$0.01597 < p < 0.5840$ (+2)

or $1.6\% < p < 58\%$

If the proportion of out-of-spec inductors must be known within ±10%, determine the sample size necessary to achieve this.

$n = 0.25\left(\frac{Z_{\alpha/2}}{E}\right)^2$

$= 0.25\left(\frac{1.960}{0.1}\right)^2$

$n = 96.04$ (+2)

use $n = 97$ (+1)

(although n=96 will probably be fine)

/7