ISE 2211 • Exam V • 26 March 2021 NAME SOLUTION

Joe Tritschler is still obsessed with low temperatures on February 17th. We l00ked at historical data for this date in Dayton, Ohio on our last two exams. Well, for most of those years, Joe was living in Mad River Township in Clark County and swore it was even colder. Refer to the data on low temperatures for both Dayton and Springfield:

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Low Temp Dayton	16	37	19.9	21.9	50.0	27.0	12.0	12.2	(no data)	29.1	30.2	28.2
Low Temp Springfield	-2.9	15.1	16.0	21.0	34.0	28.0	8.1	0.0	-6.0	28.0	23.0	28.9

Call the Dayton, Ohio temperatures *Population 1* and Springfield, Ohio temperatures *Population 2*, and test the following hypotheses on the difference in mean low temperature for February 17th at the fixed $\alpha = 0.05$ level of significance, stating whether you would reject or fail to reject H_0:

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

$\bar{X}_1 = 25.77$ °F +1

$S_1 = 11.27$ °F +1

$n_1 = 11$ +1

$\bar{X}_2 = 16.1$ °F +1

$S_2 = 13.53$ °F +1

$n_2 = 12$ +1

OK if

$\bar{X}_2 = 18.11$ °F

$S_2 = 12.16$ °F

$n_2 = 11$

$V = 20$

$t_0 = 1.934$

$t_{.05,20} = 1.725$

unknown and unequal variances; need v.

$\frac{S_1^2}{n_1} = 11.55$ $\frac{S_2^2}{n_2} = 15.26$

$$V = \frac{(11.55 + 15.26)^2}{\frac{11.55^2}{10} + \frac{15.26^2}{11}} = 20.83$$ +2

round down to $V = 20$ +1

$$t_0 = \frac{25.77 - 16.1 - 0}{\sqrt{11.55 + 15.26}} = 1.868$$ +1

$t_{\alpha,v} = t_{.05,20} = 1.725$ +1

$t_0 > t_{\alpha,v}$ +1 $\therefore$ reject H_0 +1

data suggests it is colder in Spgfld!

An astute observer will notice that this data is actually a perfect candidate for a *paired t-test*. Determine whether there is a significant difference in low temperatures between Dayton and Springfield using the *p*-value approach, and state your final conclusion with regards to the $\alpha = 0.05$ level of significance. Again, is Joe Tritschler right on, or way off? For your convenience, here is the data again (hint: use the empty row!).

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Low Temp Dayton	16	37	19.9	21.9	50.0	27.0	12.0	12.2	(no data)	29.1	30.2	28.2
Low Temp Springfield	-2.9	15.1	16.0	21.0	34.0	28.0	8.1	0.0	-6.0	28.0	23.0	28.9
	18.9	21.9	3.9	0.9	16	-1.0	3.9	12.2		1.1	7.2	-0.7

H_0: $\mu_D = \Delta_0 = 0$

H_1: $\mu_D > 0$

$\bar{d} = 7.664$ °F +1

$S_D = 8.255$ °F +1

$n = 11$ pairs +1

$$t_0 = \frac{7.664}{8.255/\sqrt{11}} = 3.079$$ +1

$n - 1 = 10$ degrees of freedom

+2

$t_{.01,\,10} = 2.764$

$t_{.005,\,10} = 3.169$

$\therefore$

$0.005 <$ p-value < 0.01 +1

$\ll \alpha = 0.05$ +1

$\therefore$ reject Ho +1 and Tritschler ain't just whistlin' Dixie! +1