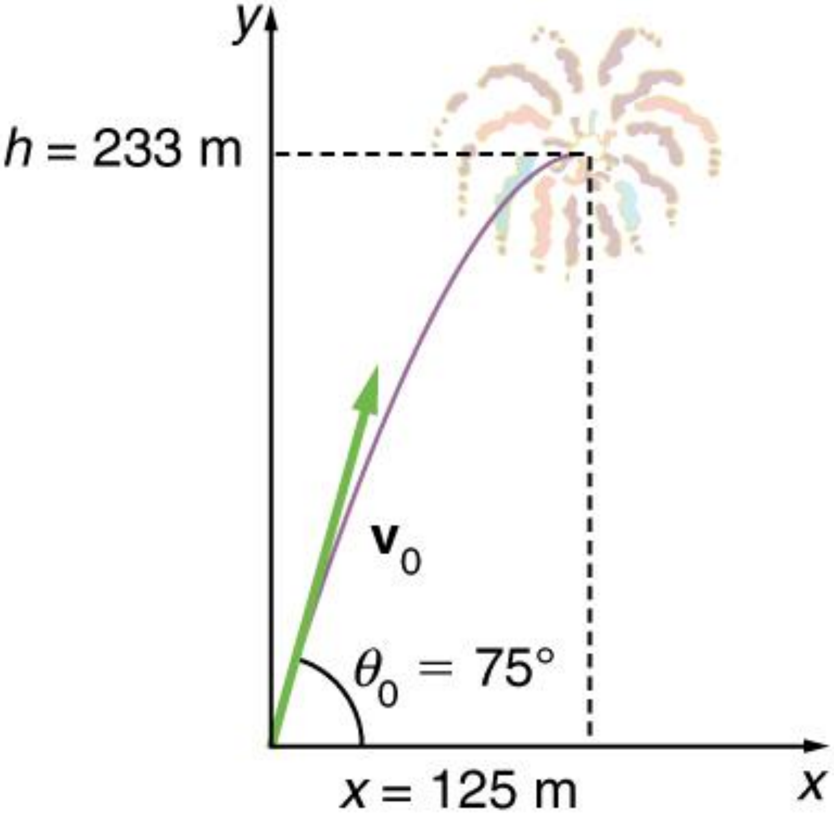
Circular Motion and Gravitation

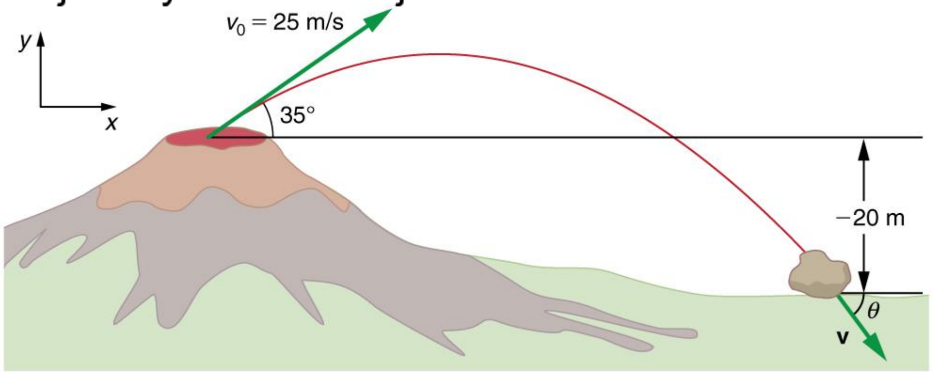
- Kinematics of Uniform Circular Motion
- Dynamics of Uniform Circular Motion
- Highway Curves, Banked and Unbanked
- Nonuniform Circular Motion
- Centrifugation
- Newton’s Law of Universal Gravitation

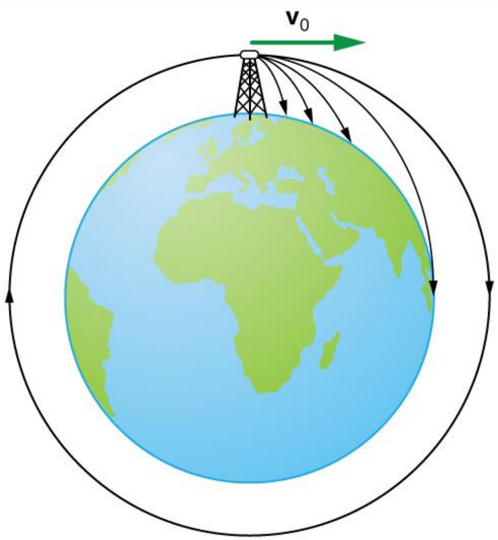
- **Gravity Near the Earth’s Surface; Geophysical Applications**
- **Kepler’s Laws**
- **Types of Forces in Nature**

The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.



The trajectory of a rock ejected from the Kilauea volcano.



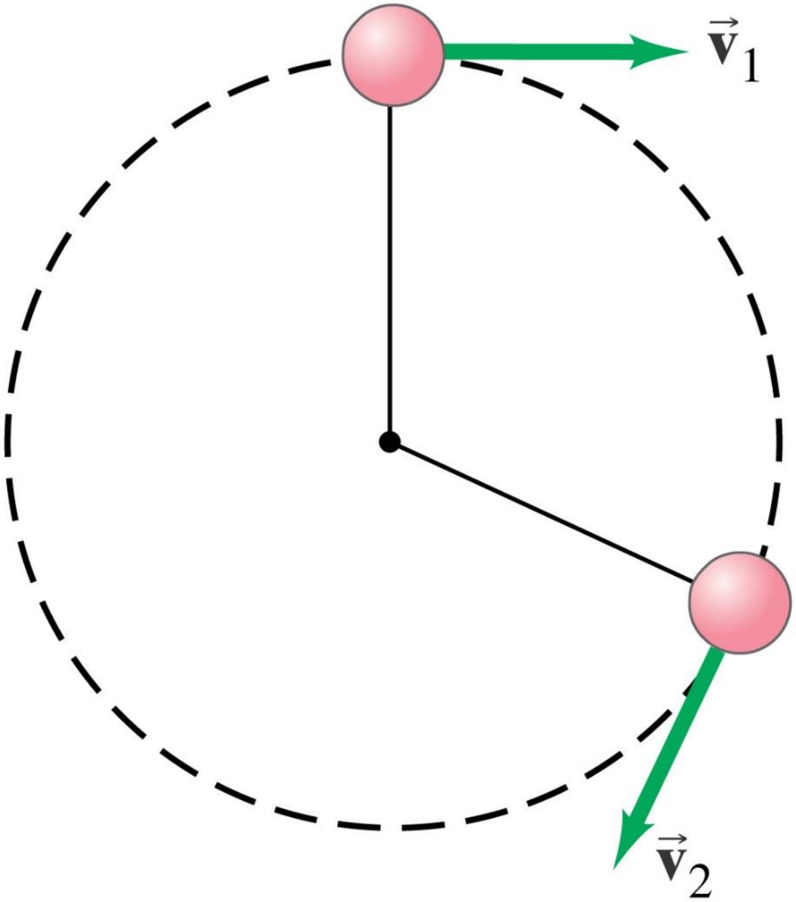


Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

Kinematics of Uniform Circular Motion

Uniform circular motion: motion in a circle of constant radius at constant speed.

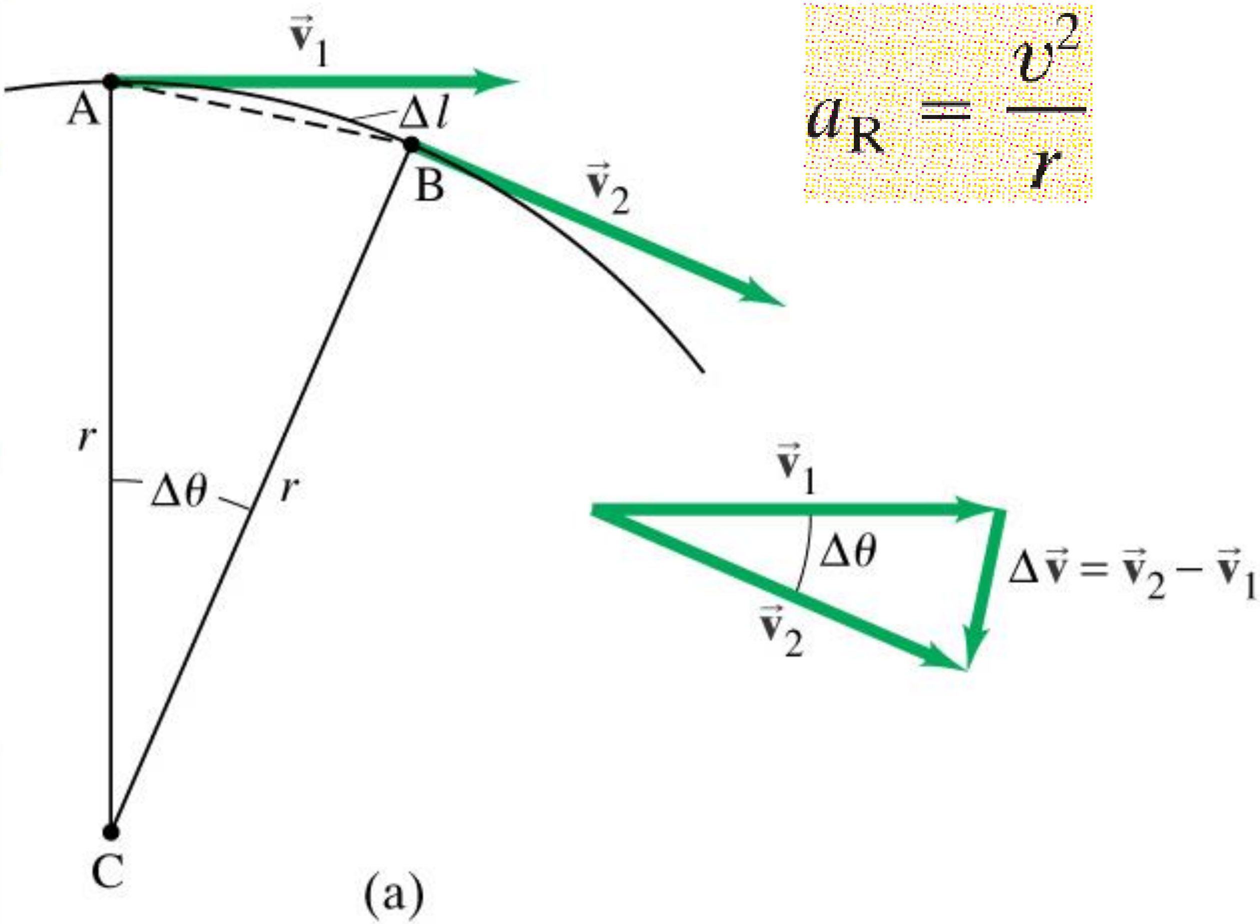
Instantaneous velocity is always tangent to circle.

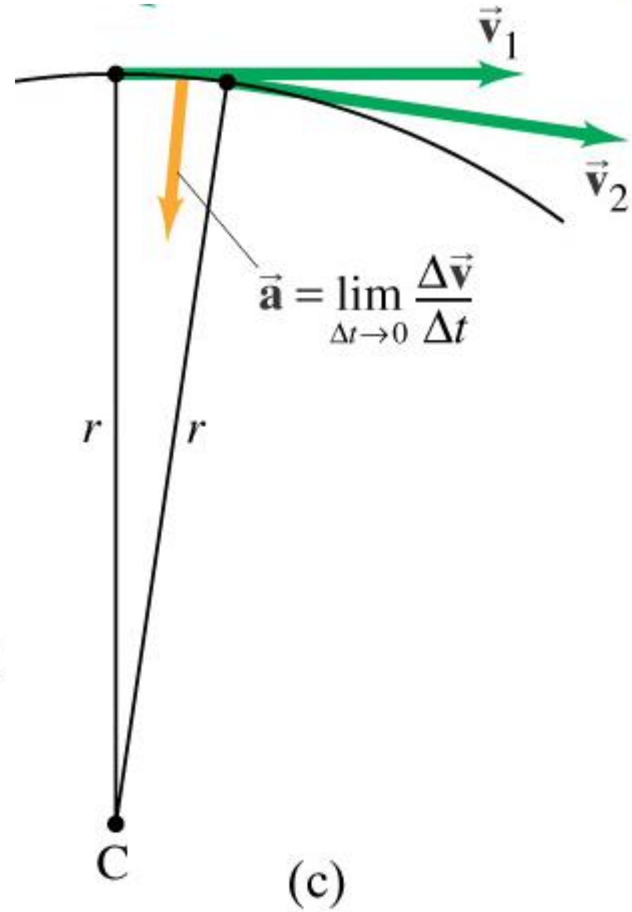


Kinematics of Uniform Circular Motion

Looking at the change in velocity in the limit that the time interval becomes infinitesimally small, we see that

$$a_R = \frac{v^2}{r}$$





Kinematics of Uniform Circular Motion

This acceleration is called the centripetal, or radial, acceleration, and it points towards the center of the circle.

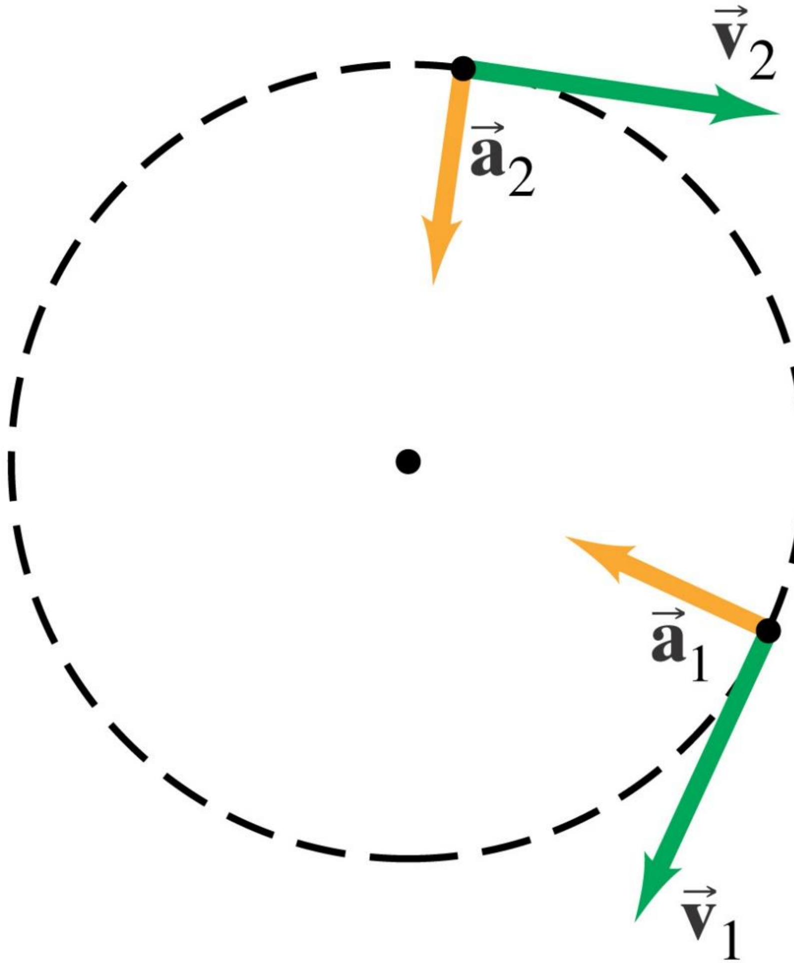
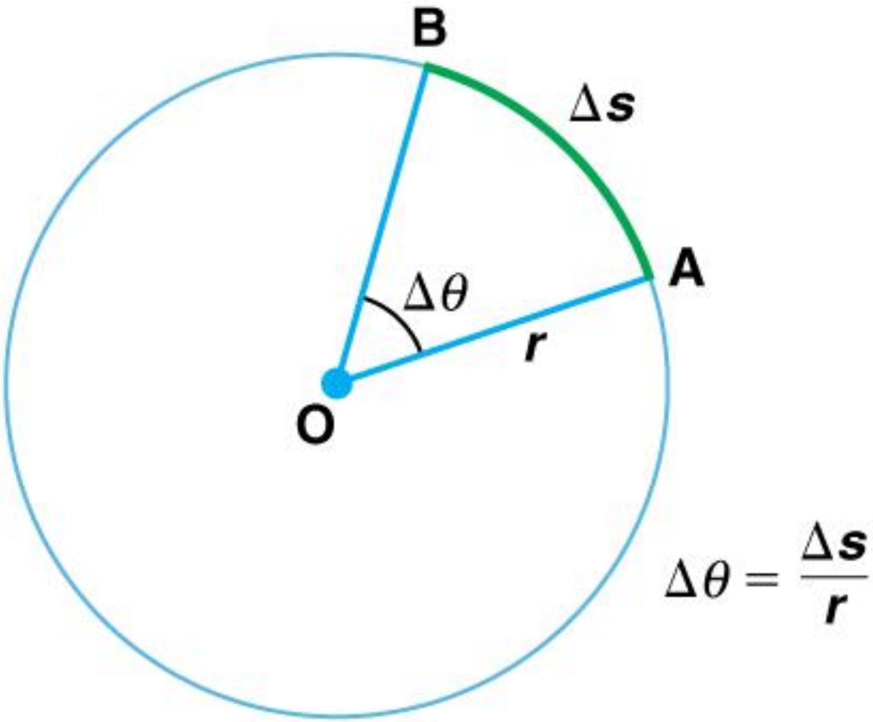


FIGURE 6.2

ANGLE

All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ in a time Δt .

FIGURE 6.3

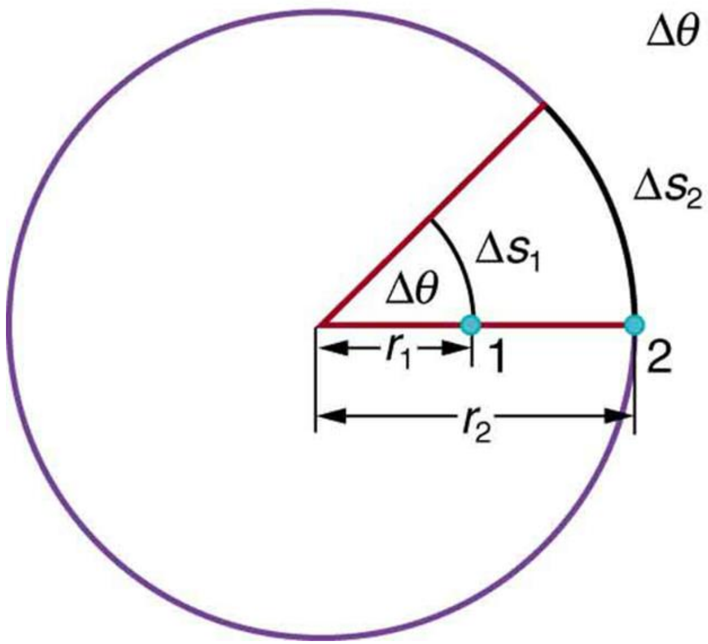


The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

FIGURE 6.4

$$\Delta\theta = \frac{\Delta s_1}{r_1}$$

$$\Delta\theta = \frac{\Delta s_2}{r_2}$$



Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs) because it is at a greater distance from the center of rotation (r) .

Table 6.1 Comparison of Angular Units

Degree Measures	Radian Measure
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

$$2\pi \text{ rad} = 360^\circ$$

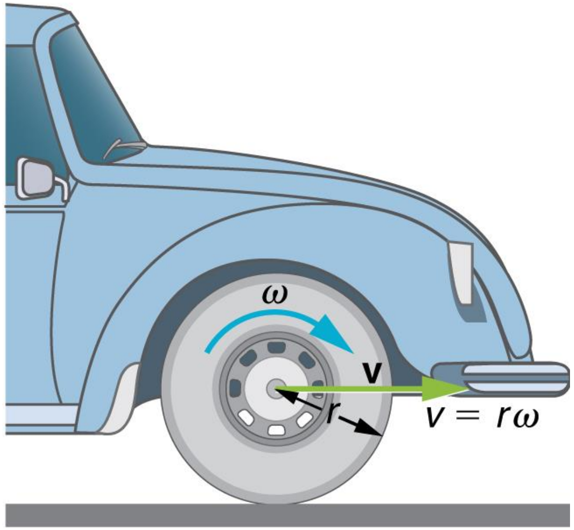
$$1 \text{ rad} = \frac{360^\circ}{2\pi} \approx 57.3^\circ.$$

Angular Velocity

How fast is an object rotating? We define **angular velocity** ω as the rate of change of an angle, $\omega = \Delta\vartheta/\Delta t$, where an angular rotation $\Delta\vartheta$ takes place in a time Δt . The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt , and so it has a linear velocity $v = \Delta s/\Delta t$. From $\Delta\vartheta = \Delta s/r$ we see that $\Delta s = r\Delta\vartheta$. Substituting this into the expression for v gives

$v = r\Delta\vartheta/\Delta t = r\omega$.



A car moving at a velocity *v* to the right has a tire rotating with an angular velocity *ω*. The speed of the tread of the tire relative to the axle is *v*, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where *r* is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

We can also call this linear speed v of a point on the rim the *tangential speed*. The relationship in $v = r\omega$ or $\omega = v/r$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. So the faster the car moves, the faster the tire spins—large v means a large ω, because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity ω will produce a greater linear speed v for the car.

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at 7.5×10^4 rev/min. Determine the ratio of this acceleration to that due to gravity.

Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω . Because r is given, we can use the second expression in the equation $a_c = v^2/r;\ a_c = r\omega^2$ to calculate the centripetal acceleration.

Solution

To convert 7.50×10^4 rev/min to radians per second, we use the facts that one revolution is 2πrad and one minute is 60.0s. Thus,

$\omega = 7.50\times10^4$ (rev/min)×(2π rad/1rev)×(1min/60.0s) = 7854 rad/s.

Now the centripetal acceleration is given by the second expression in $a_c = v^2/r;\ a_c = r\omega^2$ as $a_c = v^2/r;$

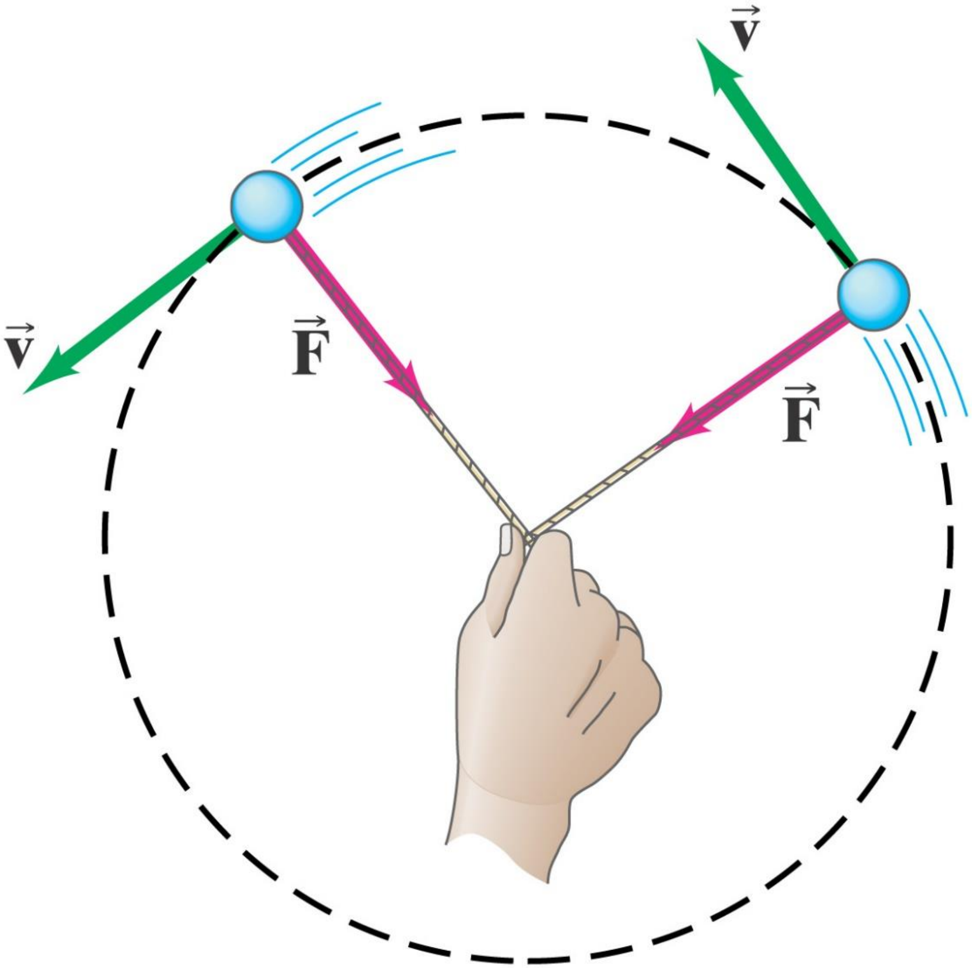
$$a_c = r\omega^2.$$

Converting 7.50 cm to meters and substituting known values gives

$a_c = (0.0750\ \text{m})(7854\ \text{rad/s})^2 = 4.63\times10^6\ \text{m/s}^2.$

Dynamics of Uniform Circular Motion

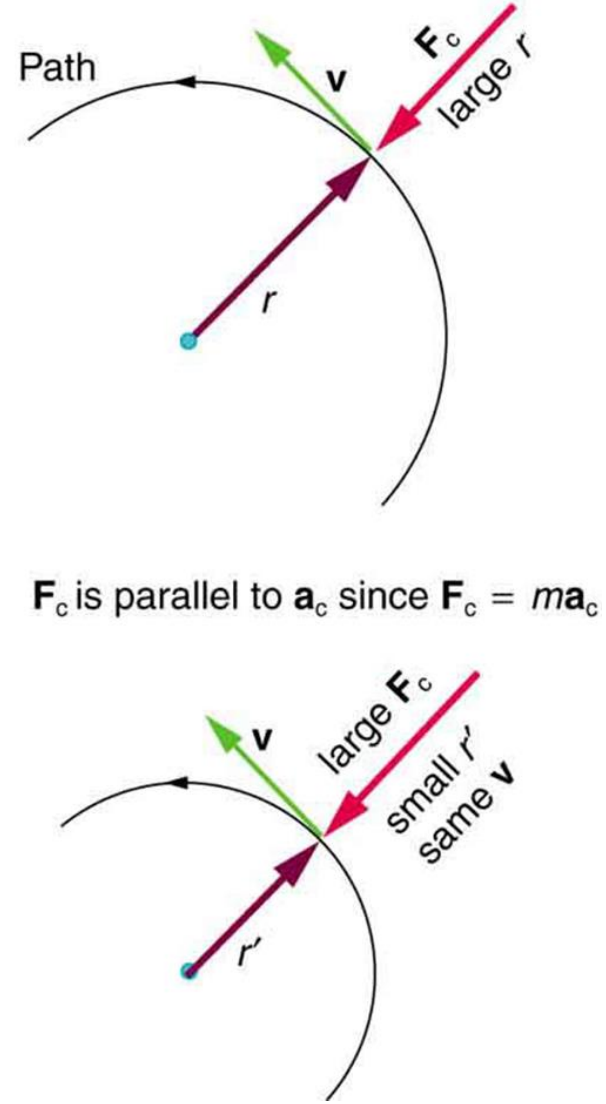
For an object to be in uniform circular motion, there must be a net force acting on it.



We already know the acceleration, so can immediately write the force:

$$\Sigma F_R = m a_R = m \frac{v^2}{r}$$

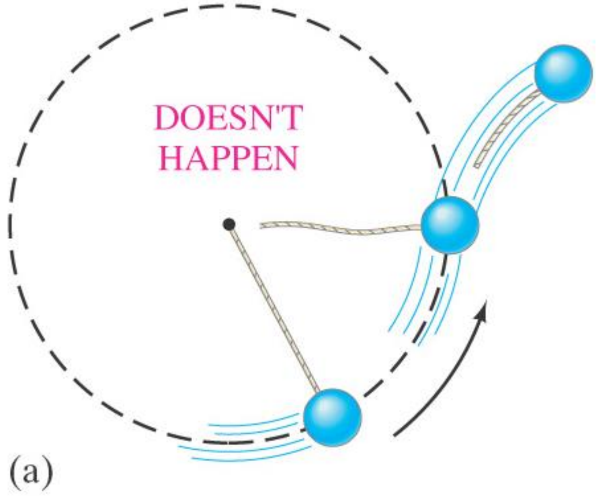
Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature *r* and the sharper the curve. The second curve has the same *v*, but a larger F_c produces a smaller *r*′.



Dynamics of Uniform Circular Motion

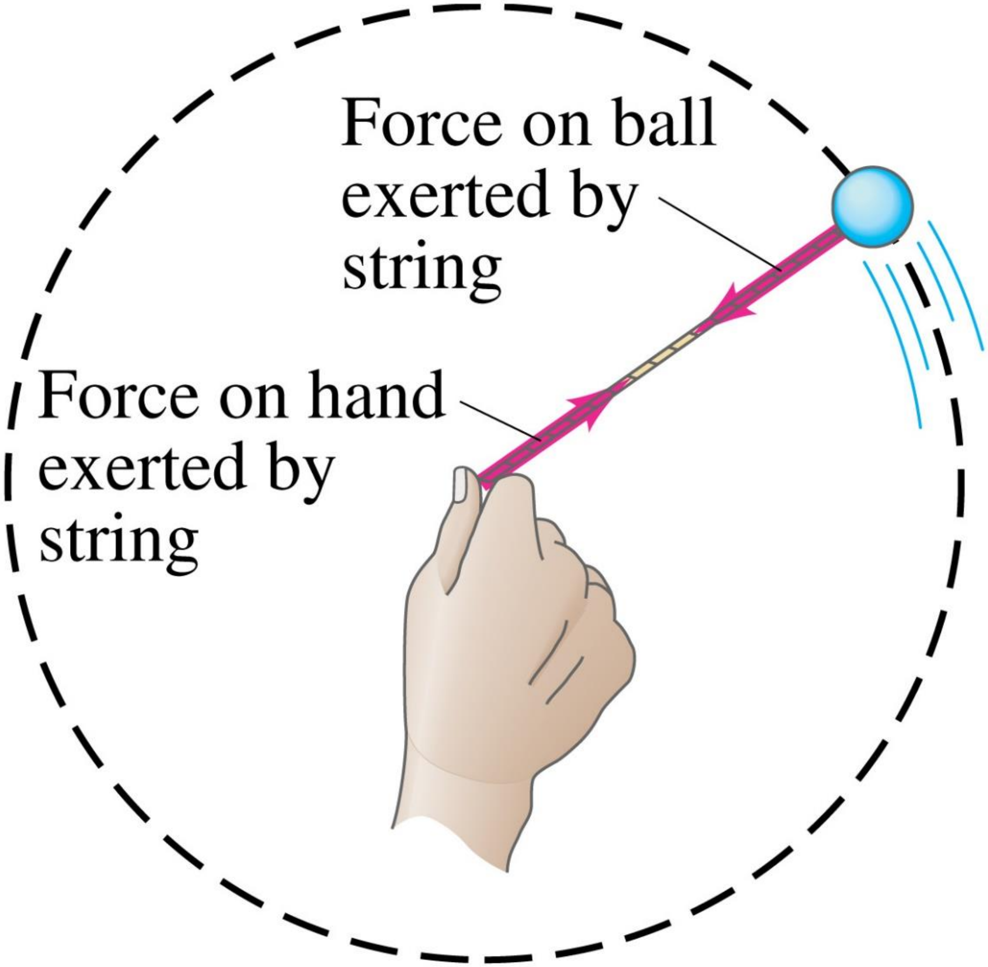
There is no centrifugal force pointing outward; what happens is that the natural tendency of the object to move in a straight line must be overcome.

If the centripetal force vanishes, the object flies off tangent to the circle.



Dynamics of Uniform Circular Motion

We can see that the force must be inward by thinking about a ball on a string:



Any force or combination of forces can cause a centripetal or radial acceleration:

the tension in the rope on a tether ball,

the force of Earth's gravity on the Moon,

friction between roller skates and a rink floor,

a banked roadway's force on a car,

and forces on the tube of a spinning centrifuge.

Highway Curves, Banked and Unbanked

When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by friction.

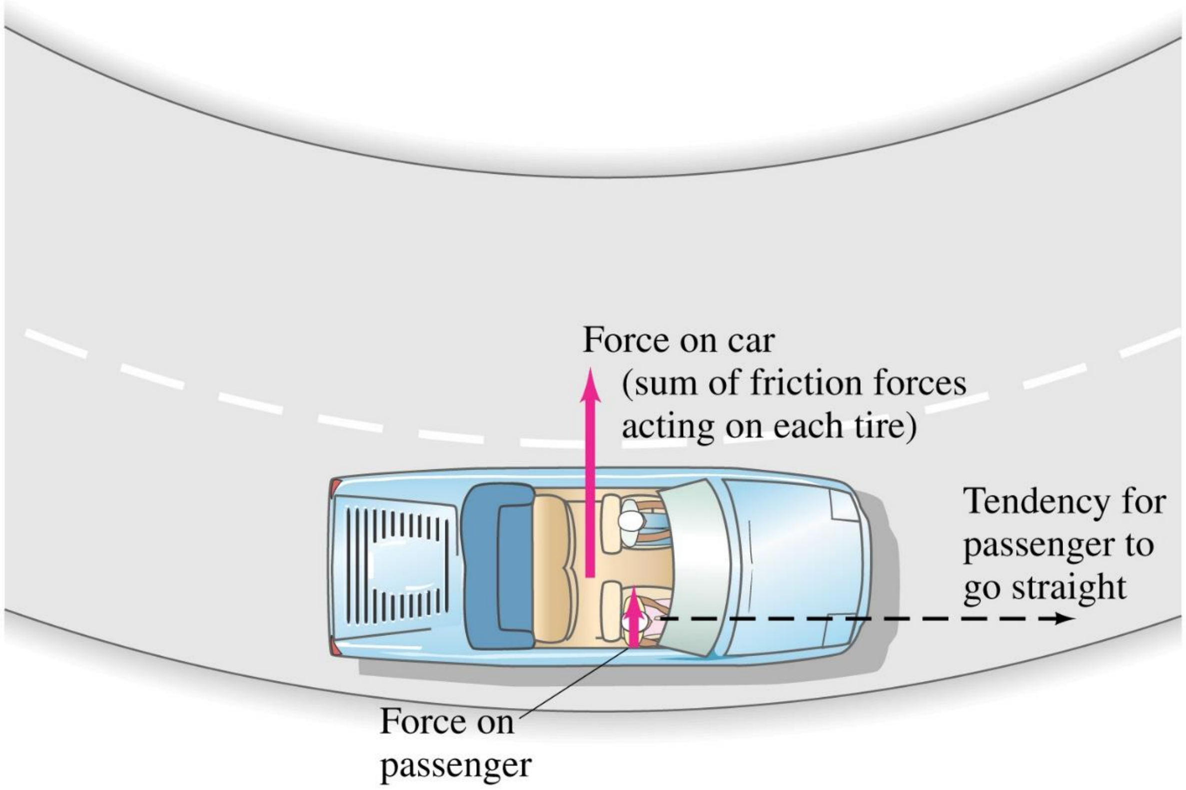
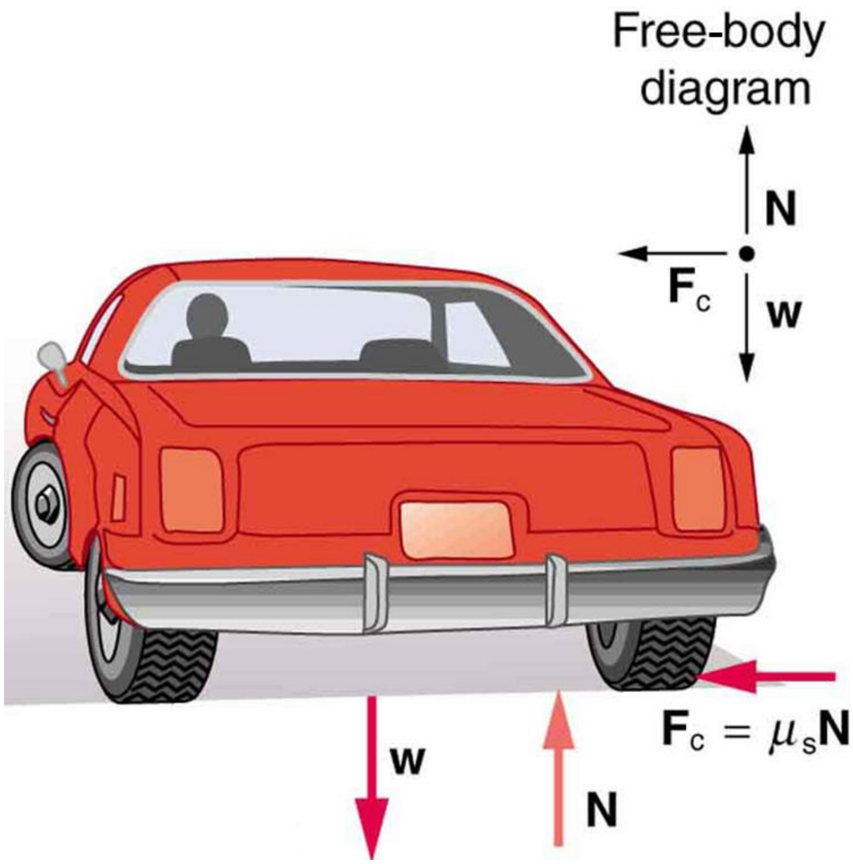


FIGURE 6.12



This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Highway Curves, Banked and Unbanked

If the frictional force is **insufficient**, the car will tend to move more nearly in a **straight line**, as the skid marks show.

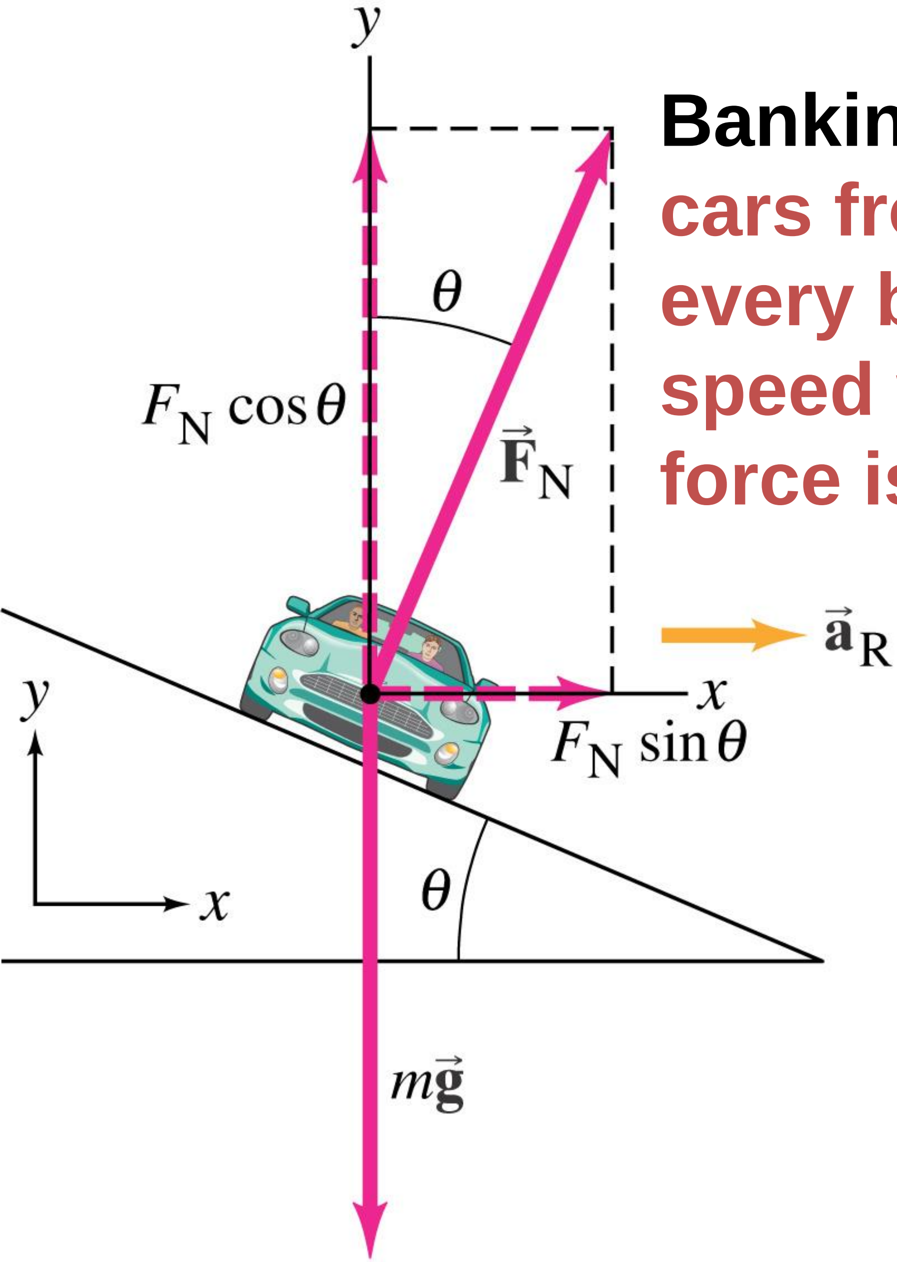
Highway Curves, Banked and Unbanked

As long as the tires do not slip, the friction is **static**. If the tires do start to slip, the friction is **kinetic**, which is bad in two ways:

1. The kinetic frictional force is **smaller** than the static.

2. The static frictional force can point towards the center of the circle, but the kinetic frictional force **opposes** the direction of motion, making it very difficult to regain control of the car and continue around the curve.

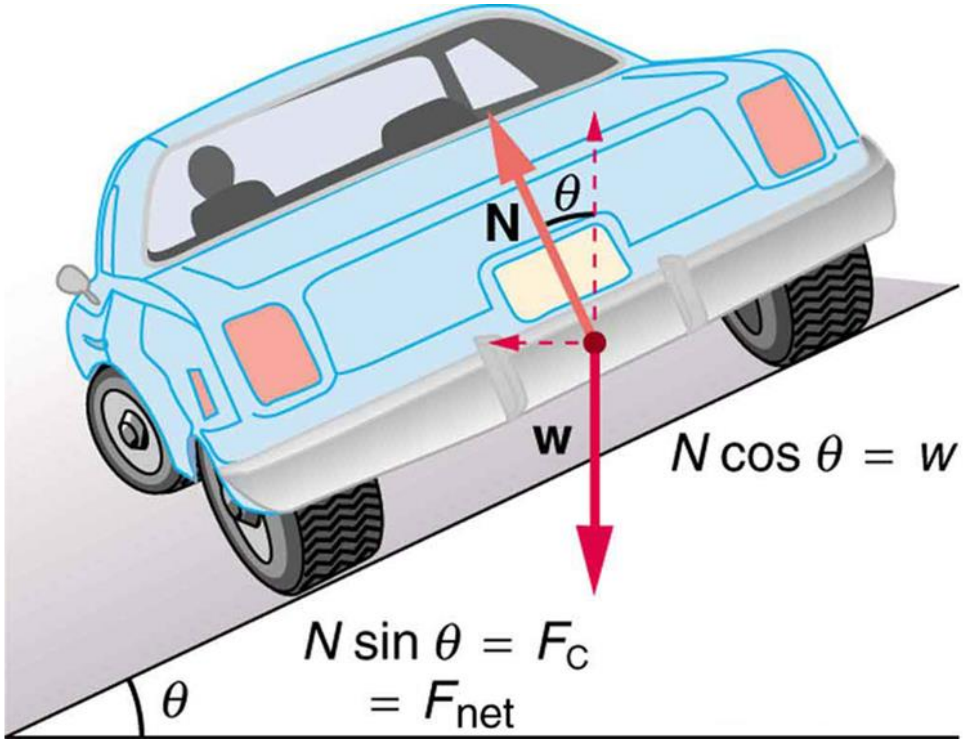
Highway Curves, Banked and Unbanked



Banking the curve can help keep cars from skidding. In fact, for every banked curve, there is one speed where the entire centripetal force is supplied by the horizontal component of the **normal** force, and no friction is required. This occurs when:

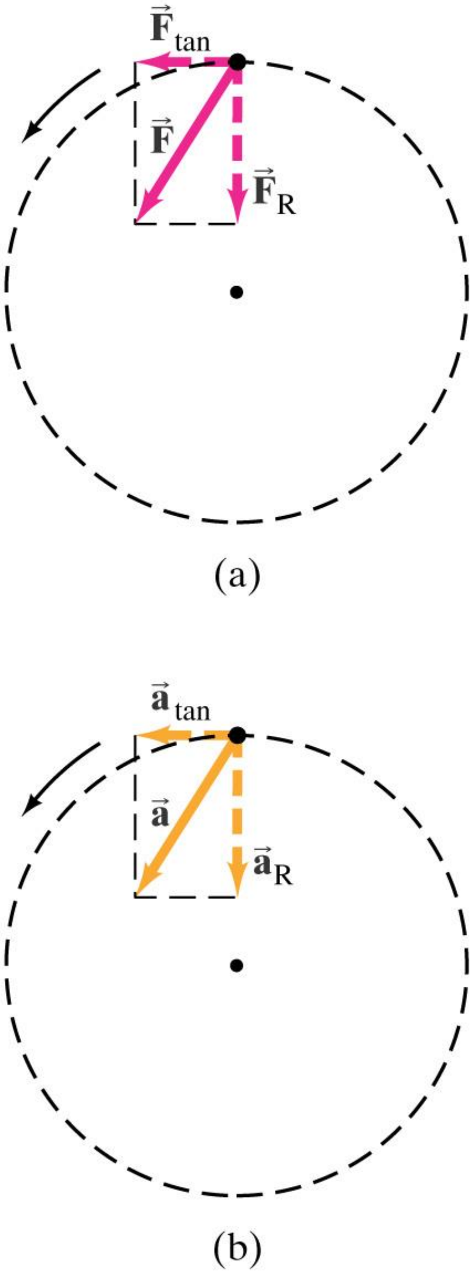
$$F_N \sin\theta = m\frac{v^2}{r}$$

FIGURE 6.13



The car on this banked curve is moving away and turning to the left.

Nonuniform Circular Motion



If an object is moving in a circular path but at varying **speeds**, it must have a **tangential** component to its acceleration as well as the **radial** one.

Nonuniform Circular Motion

This concept can be used for an object moving along any curved path, as a small segment of the path will be approximately circular.

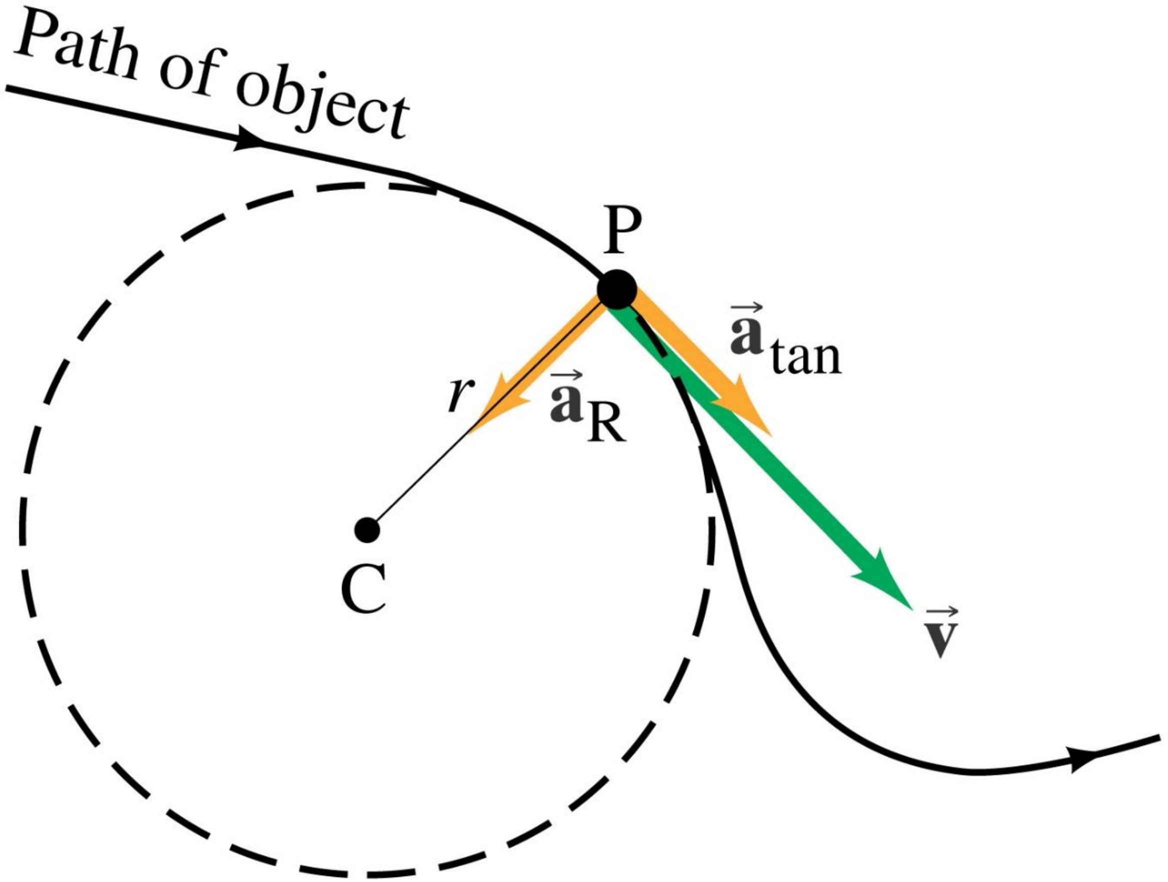
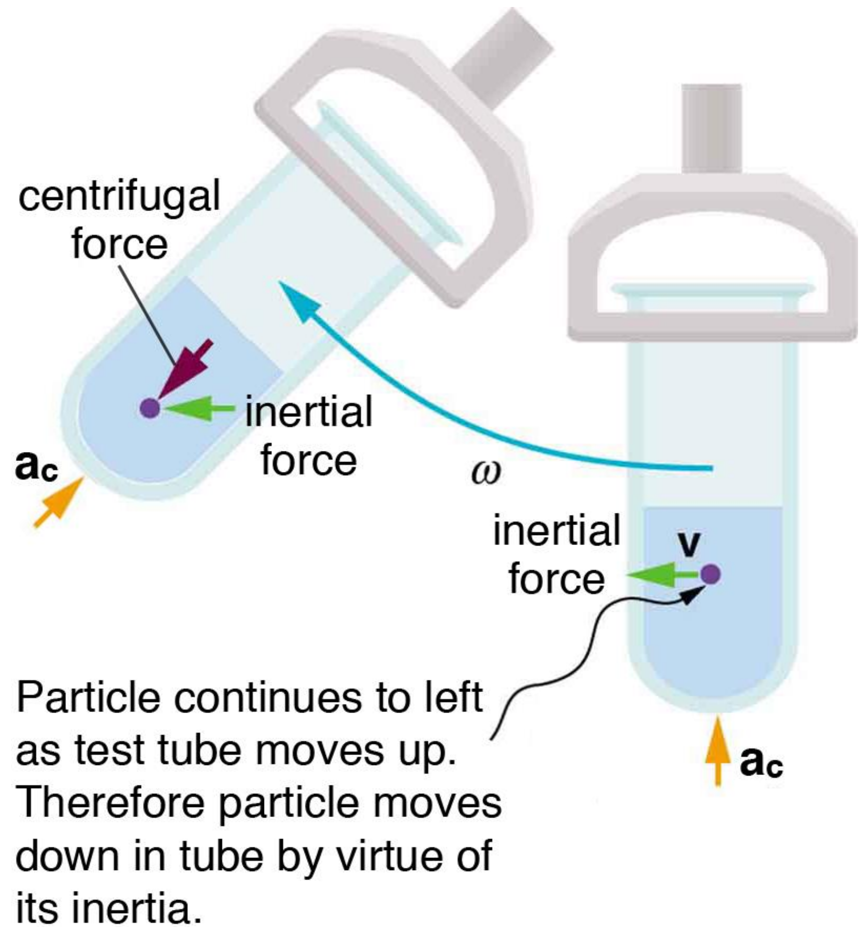
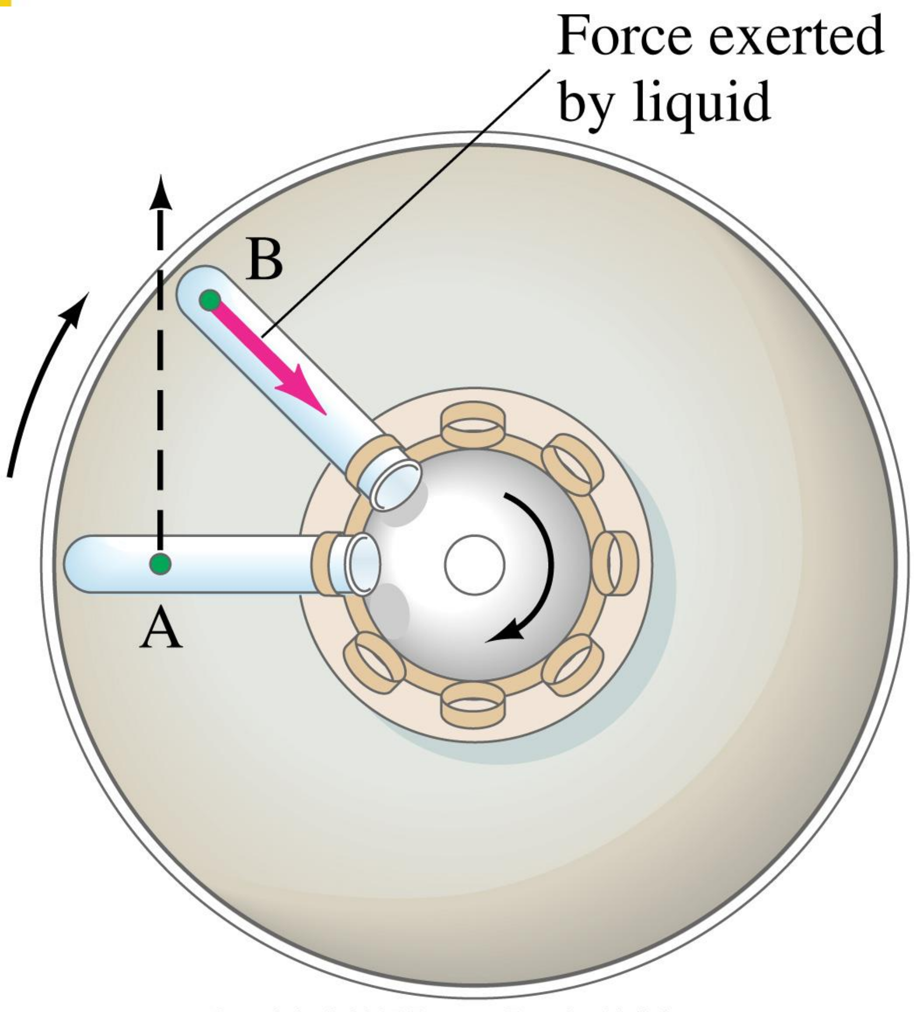


FIGURE 6.17

Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.



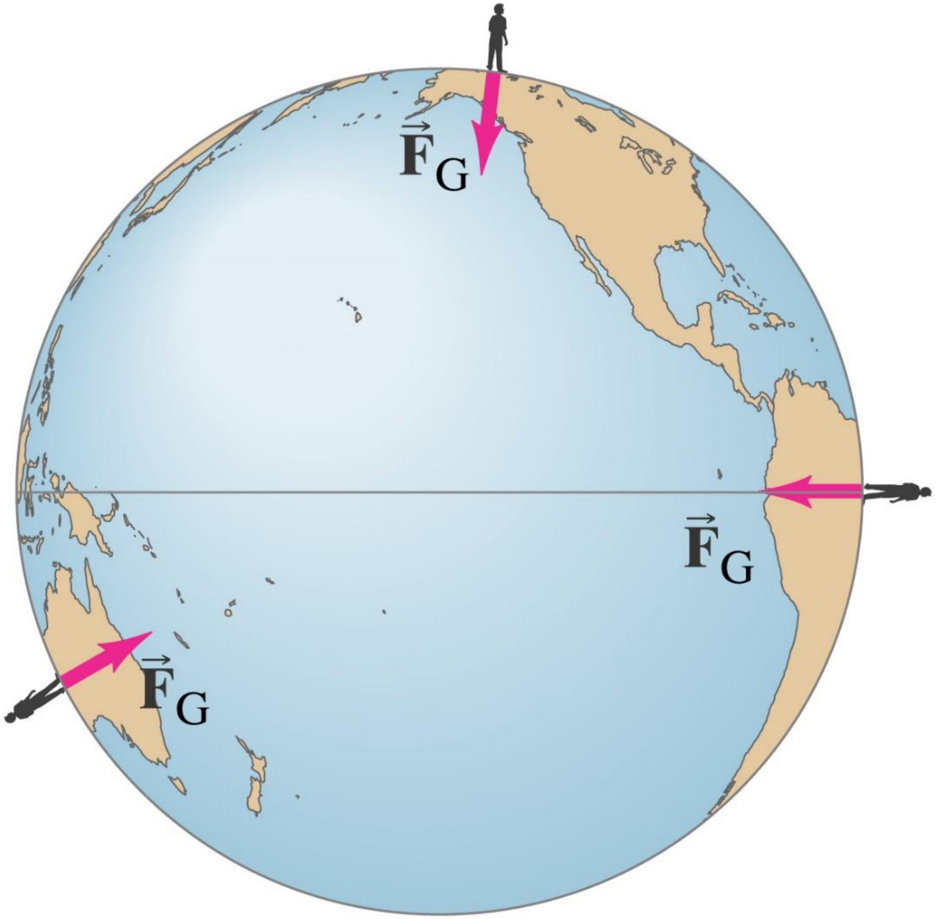
Centrifugation



A **centrifuge** works by spinning very fast. This means there must be a very large **centripetal** force. The object at A would go in a straight line but for this force; as it is, it winds up at B.

Newton's Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the **origin** of that force?

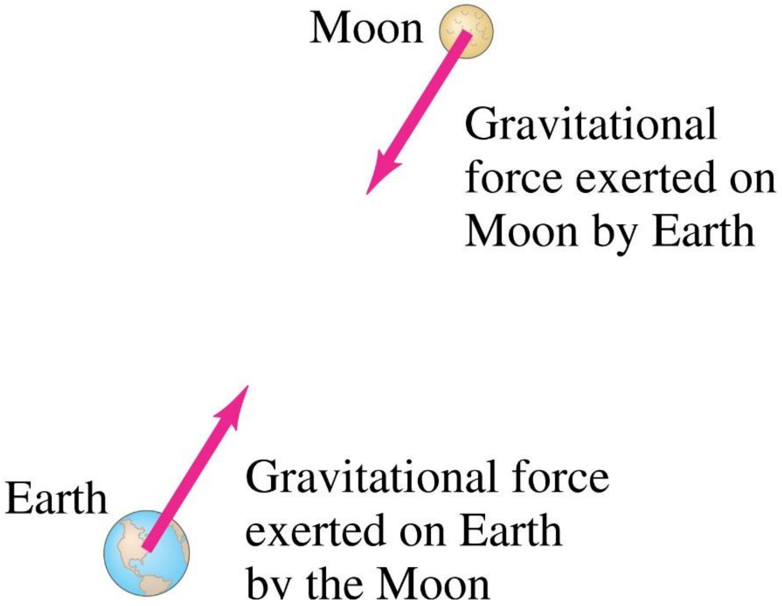


Newton's realization was that the force must come from the **Earth**.

He further realized that this force must be what keeps the **Moon** in its orbit.

Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a Third Law pair: the Earth exerts a downward force on you, and you exert an upward force on the Earth. When there is such a disparity in masses, the reaction force is undetectable, but for bodies more equal in mass it can be significant.



Newton’s Law of Universal Gravitation

Therefore, the gravitational force must be proportional to **both** masses.

By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the **inverse** of the **square** of the distance between the masses.

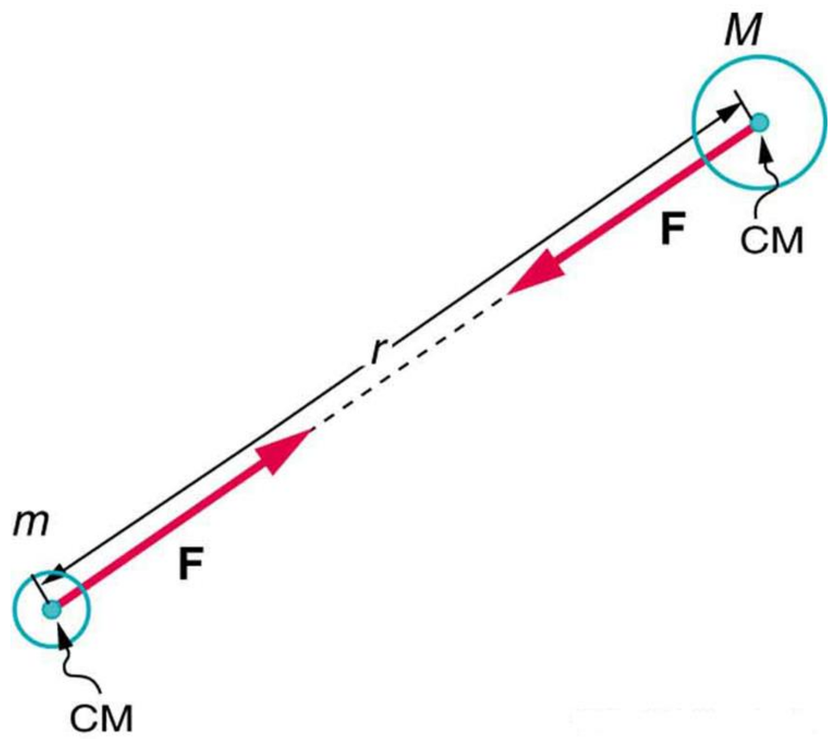
In its final form, the Law of Universal Gravitation reads:

$$F = G\frac{m_1 m_2}{r^2}$$

where

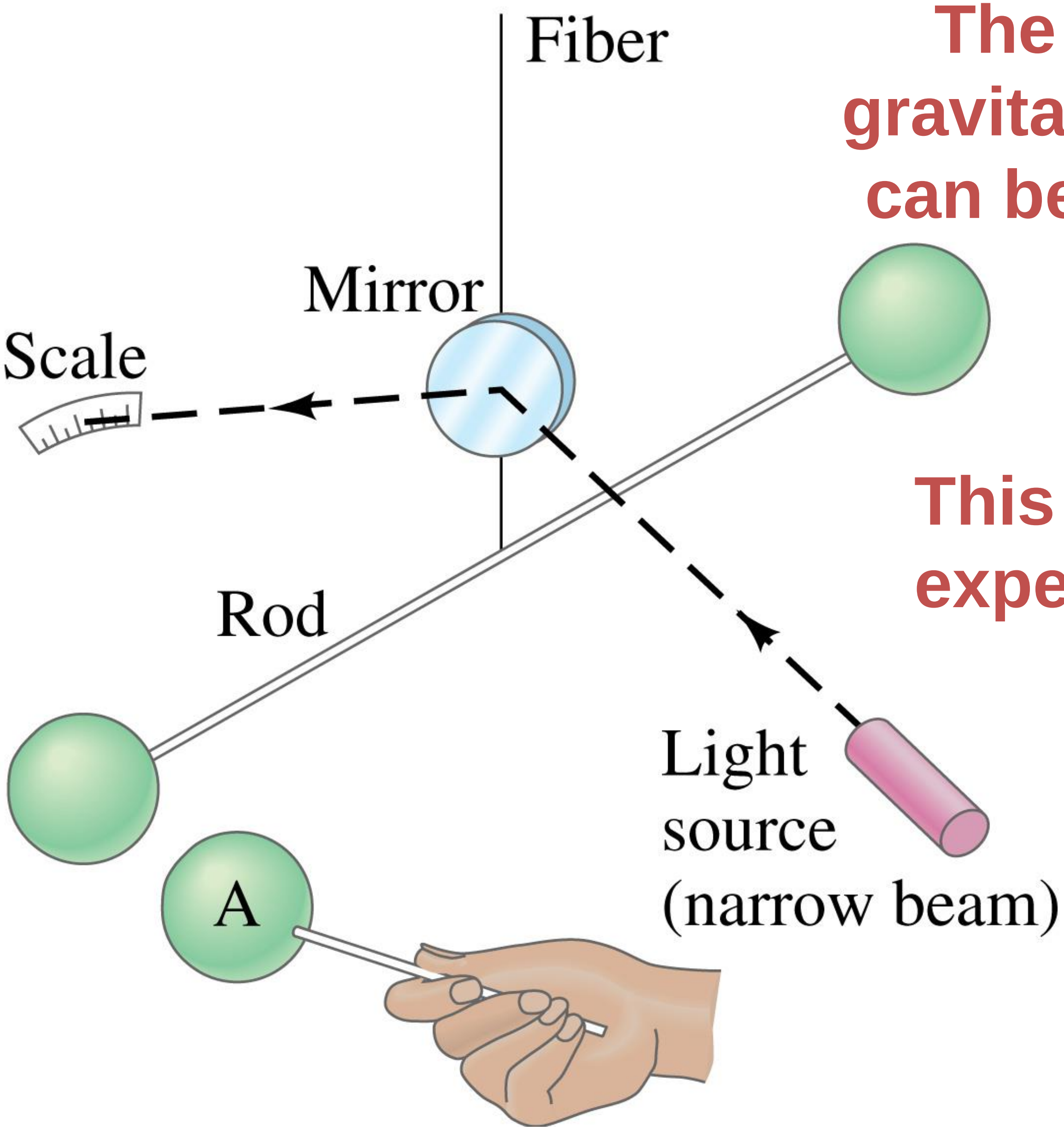
$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

FIGURE 6.21



Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton’s third law.

Newton's Law of Universal Gravitation



The **magnitude** of the gravitational constant *G* can be measured in the laboratory.

This is the **Cavendish** experiment.

Gravity Near the Earth's Surface; Geophysical Applications

Now we can relate the **gravitational constant** to the **local acceleration** of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{mm_{\mathrm{E}}}{r_{\mathrm{E}}^2}$$

Solving for *g* gives:

$$g = G\frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^2}$$

Now, knowing *g* and the radius of the Earth, the mass of the Earth can be calculated:

$$m_{\mathrm{E}} = \frac{gr_{\mathrm{E}}^2}{G} = \frac{(9.80\,\mathrm{m/s^2})(6.38 \times 10^6\,\mathrm{m})^2}{6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}} = 5.98 \times 10^{24}$$

Geophysical Applications

Acceleration Due to Gravity at Various Locations on Earth

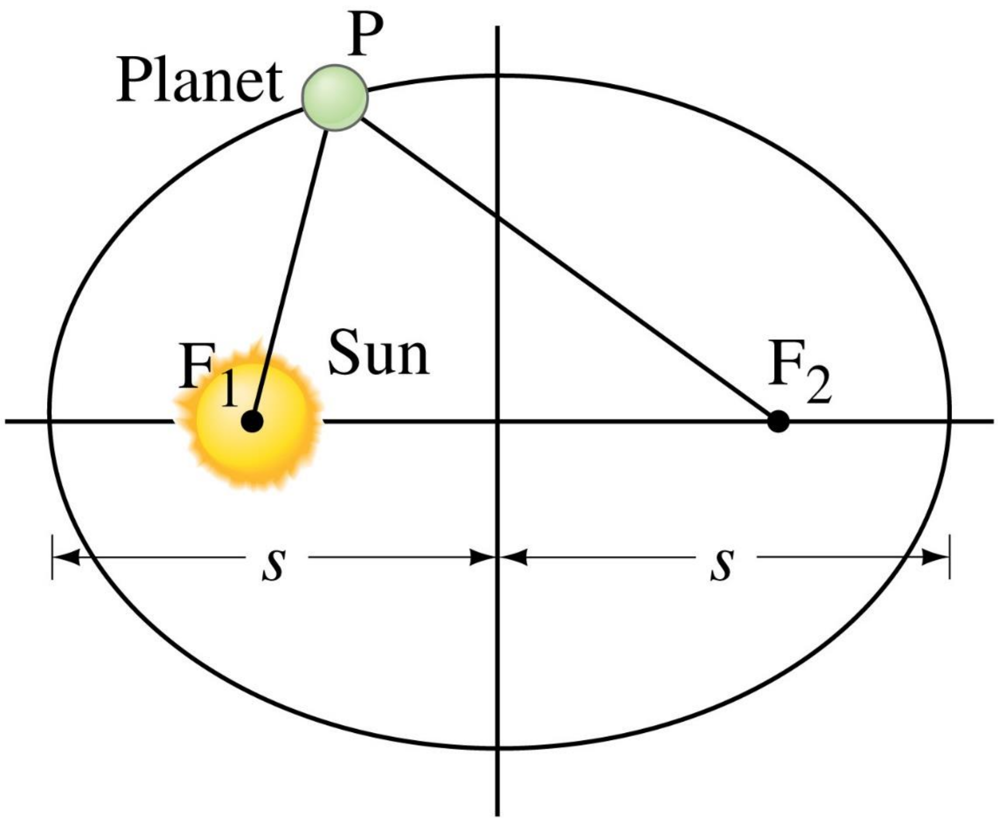
Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

Kepler's Laws

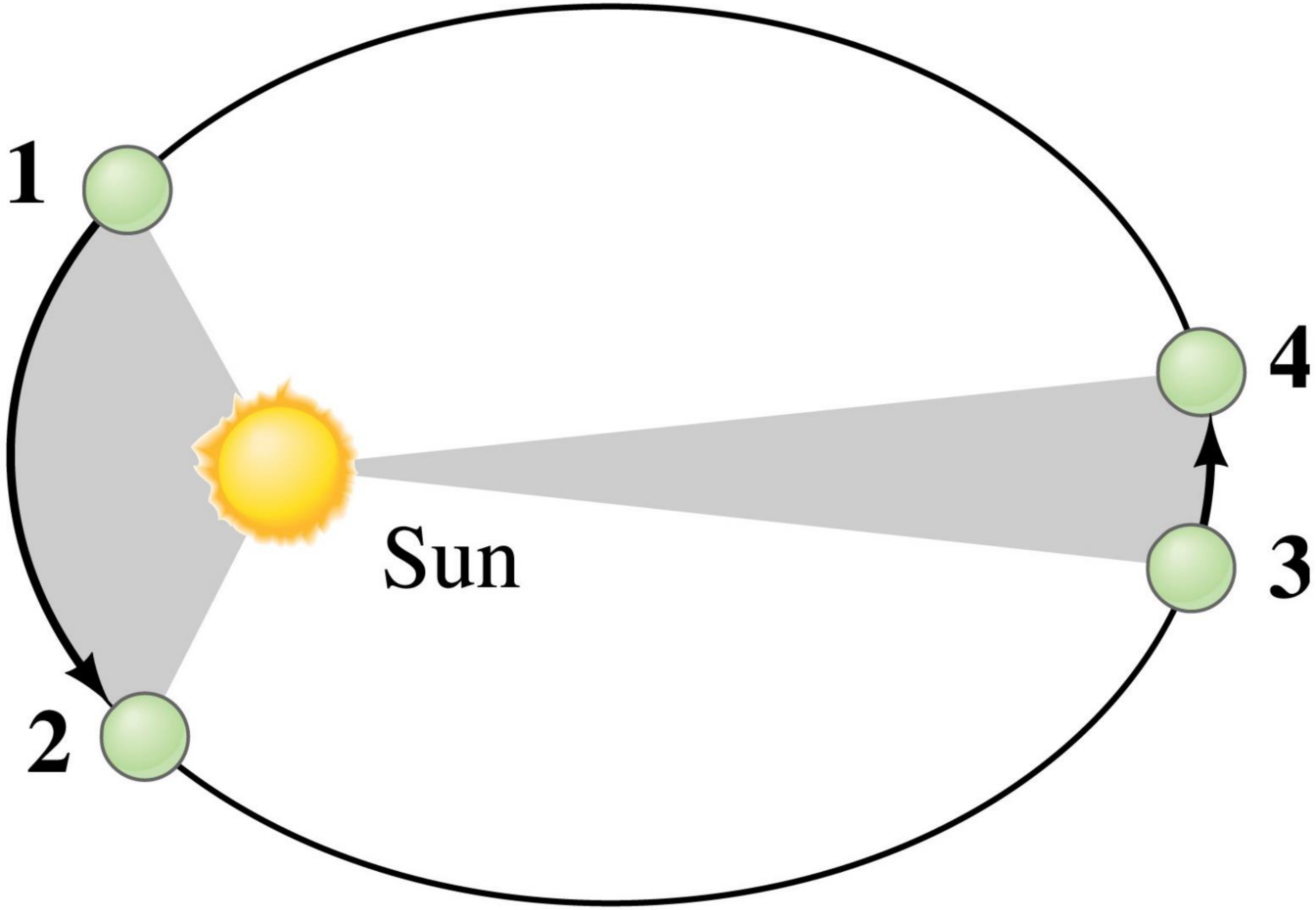
Kepler's laws describe planetary motion.

1. **The orbit of each planet is an ellipse, with the Sun at one focus.**



Kepler's Laws

2. An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.



Kepler's Laws

The ratio of the square of a planet's orbital period is proportional to the cube of its mean distance from the Sun.

Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth years)	s^3/T^2 (10^{24} km^3/y^2)
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. **Gravity**
2. **Electromagnetism**
3. **Weak nuclear force** (responsible for some types of radioactive decay)
4. **Strong nuclear force** (binds protons and neutrons together in the nucleus)

Types of Forces in Nature

So, what about **friction**, the **normal** force, **tension**, and so on?

Except for gravity, the forces we experience every day are due to **electromagnetic** forces acting at the **atomic** level.

Summary

- An object moving in a circle at constant speed is in uniform circular motion.

- It has a centripetal acceleration $$a_R = \frac{v^2}{r}$$

- There is a centripetal force given by

$$\Sigma F_R = ma_R = m\frac{v^2}{r}$$

- The centripetal force may be provided by friction, gravity, tension, the normal force, or others.