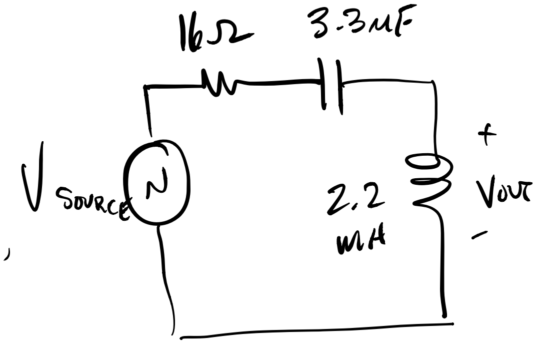
homework: RLC high-pass

$L = 2.2\ mH$

$R = 16\,\Omega$

$C = 3.3\,\mu F$

∴ sketch circuit, determine ω_0, f_0, and Q, describe freq. response (i.e., Butterworth etc..), describe transient response (i.e., overdamped, etc..), determine $H(\omega_0)$, $H(2\omega_0)$, and $H(0.5\,\omega_0)$ (mag. in dB and phase), sketch $|H(\omega)|$ and $\angle H(\omega)$.



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.2m \cdot 3.3\mu}}$$

$$= \underline{11.74\ krad/s}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

OR $\frac{\omega_0}{2\pi} \Rightarrow \underline{1.868\ kHz}$

$$Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{16}\sqrt{\frac{2.2m}{3.3\mu}} = \underline{1.614}$$

∴ quite underdamped in time domain!

∴ ringing and overshoot in transient response

$Q > 0.707 \Rightarrow$ Chebychev

$$|H(\omega_0)|\,(\text{dB}) = 20\log_{10} Q = 20\log_{10} 1.614$$

$$= \underbrace{+4.2 \text{ dB}}\ @\ \omega_0$$

"+4 Chebychev" or whatever

$$\underbrace{H(\omega) = \frac{(j\omega)^2}{(j\omega)^2 + j\omega\frac{\omega_0}{Q} + \omega_0^2}}$$

2nd-order HPF transfer function

$$H(2\omega_0) = \frac{(j\cdot 2\omega_0)^2}{(j\cdot 2\omega_0)^2 + j\cdot 2\omega_0\frac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{\cancel{j^2}^{-1}\cdot 4\,\cancel{\omega_0^2}}{j^2\cdot 4\,\cancel{\omega_0^2} + j\cdot\frac{2\,\cancel{\omega_0^2}}{Q} + \cancel{\omega_0^2}}$$

$$H(2\omega_0) = \frac{-4}{-4 + \frac{2j}{Q} + 1}$$

$$H(2\omega_0) = \frac{-4}{-3 + \frac{2j}{1.614}} \Rightarrow 1.232 \angle +22.44^\circ$$

$$\Downarrow$$

$$1.8 \text{ dB}$$

$$H(0.5\,\omega_0) = \frac{(j\,0.5\omega_0)^2}{(j \cdot 0.5\omega_0)^2 + j \cdot 0.5\omega_0 \frac{\omega_0}{Q} + \omega_0^2}$$

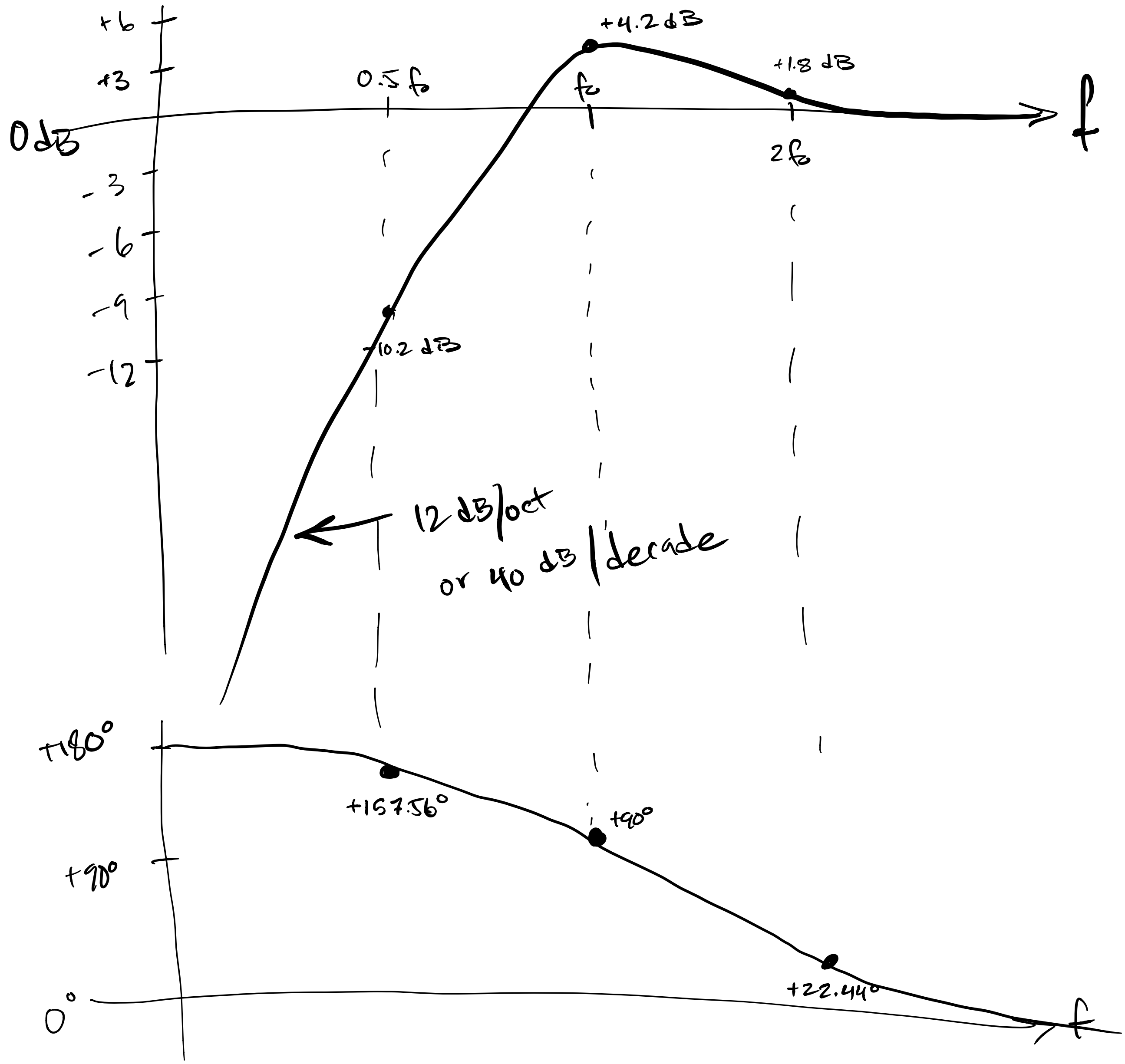
$$= \frac{-.25\,\cancel{\omega_0^2}}{-.25\,\cancel{\omega_0^2} + \frac{j\,0.5\,\cancel{\omega_0^2}}{Q} + \cancel{\omega_0^2}}$$

$$H(0.5\,\omega_0) = \frac{-.25}{0.75 + \frac{j\,0.5}{1.614}} \Rightarrow 0.309 \angle 157.56^\circ$$

$$\Downarrow$$

$$-10.2 \text{ dB}$$

$\therefore$ we now have three data points with which to sketch $|H(\omega)|$ and $\angle H(\omega)$!



.. check this in LTSPICE !!!

homework: parallel RLC bandpass

$L = 2.2\,H$ $C = 2.0\,nF$ $R = 47\,k\Omega$

solution:

$$f_0 = 2.4\ kHz$$

$$Q_{\parallel} = R\sqrt{\frac{C}{L}} = 1.486$$

∴ underdamped w/ ringing + overshoot

" + 3.44 Chebychev"

∴ we know $Q = \dfrac{f_0}{f_H - f_L}$

we also know $f_0 = \sqrt{f_H f_L}$

$$\therefore\ f_H - f_L = \frac{f_0}{Q} = \frac{2400}{1.486} = 1615\ Hz$$

$$\rightarrow f_H = 1615 + f_L$$

$$f_H f_L = f_0^2 = 2400^2 = 5760000$$

$$\therefore\ f_L = \frac{5760000}{f_H} = \frac{5760000}{1615 + f_L}$$

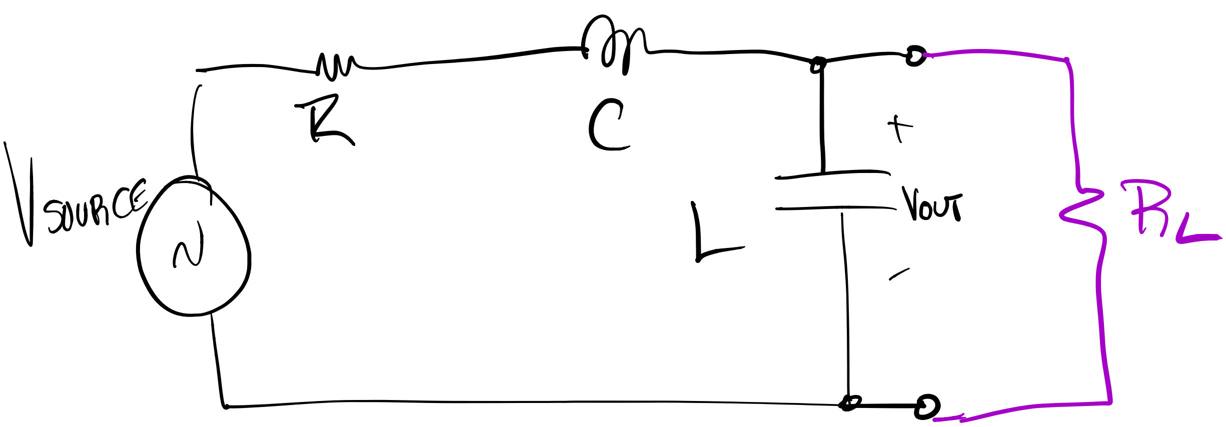
$f_L = 1725$ $\therefore\ f_H = 1725 + 1615 = 3340$

Active Filters

- let's look at a classic series-RLC Butterworth low-pass filter w/ $f_c = 10\,Hz$

$\Downarrow$

Q = 0.707, flattest magnitude response



pick $C = 1\mu F$; then

$$Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow L = \frac{1}{\omega_0^2 C}$$

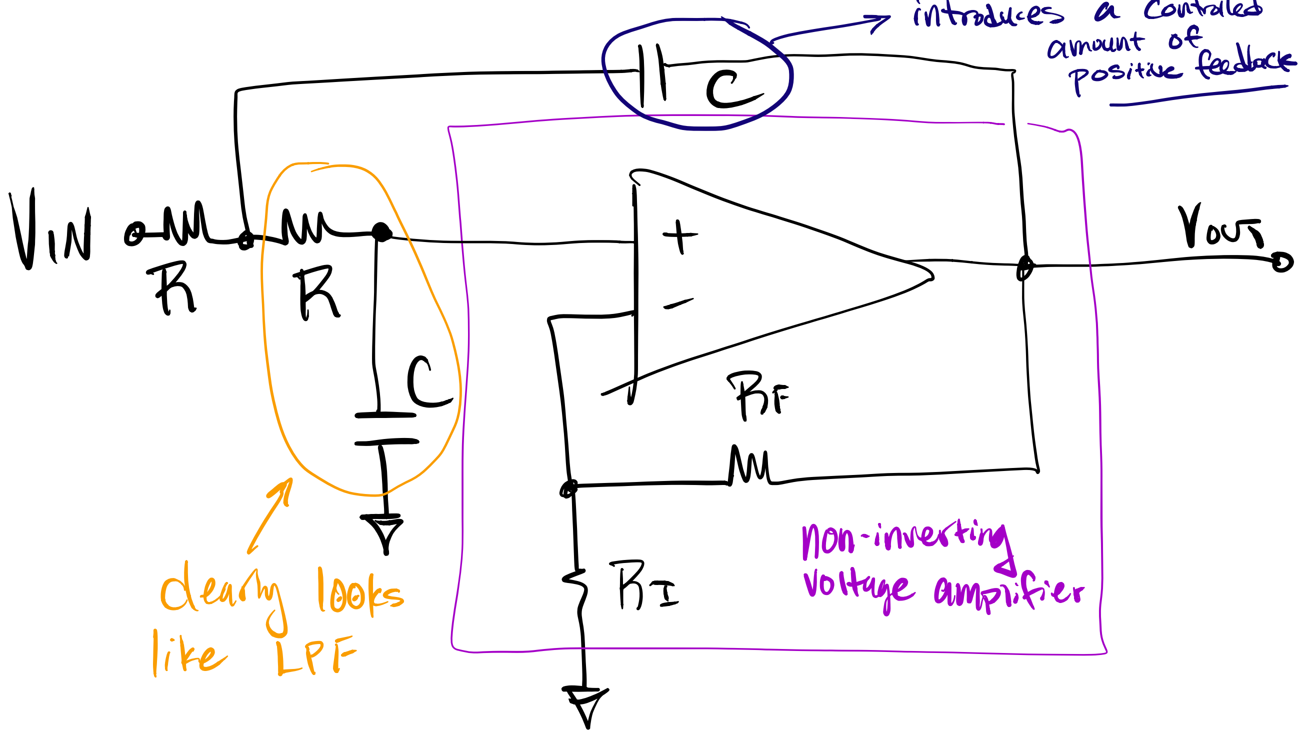
then $L = \dfrac{1}{(2\pi \cdot 10)^2 \cdot 1\times10^{-6}} = 253\ H$!!!...

.. this is an alarmingly large value !!!...

- big
- heavy
- expensive
  - probably lots of winding resistance (mess up filter response) and stray capacitance (will introduce an entirely separate RLC circuit itself!)
  - subsequent loading of the circuit makes it all worse!!! (scaling factor, Q needs recalculated, etc.)
- the solution: get rid of that inductor!!!
- in 1955, R.P. Sallen and E.L. Key developed a universal active filter algorithm in which many different filters may be developed by placing four impedances around a voltage amplifier in different configurations
  - original paper: pretty rough!

∴ many implementations; we're going to discuss the

Sallen-Key Equal-Component 2nd Order Low-Pass Filter



1.) pick C

2.) then $R = \frac{1}{2\pi f_0 C}$ OR $R = \frac{1}{\omega_0 C}$

3.) set R_F and R_I to the exact op-amp gain needed to acheive the desired Q

∴ for this circuit only:

$$A_v = 3 - \frac{1}{Q}$$

∴ design active filter using the Sallen-Key Equal-Component configuration to realize a 2nd-order Butterworth LPF w/ $f_c = 10\ \text{Hz}$

1.) pick $C = 1\ \mu F$

2.) then $R = \dfrac{1}{2\pi \cdot 1\times10^{-6} \cdot 10} = 15915\ \Omega$

∴ nearest E96 value:

15.8 kΩ 1%

3.) $A_v = 3 - \dfrac{1}{Q} = 3 - \dfrac{1}{0.707} = 1.586$

$$\therefore\ 1.586 = \underbrace{1 + \frac{R_F}{R_I}}_{\text{non-inverting!}}$$

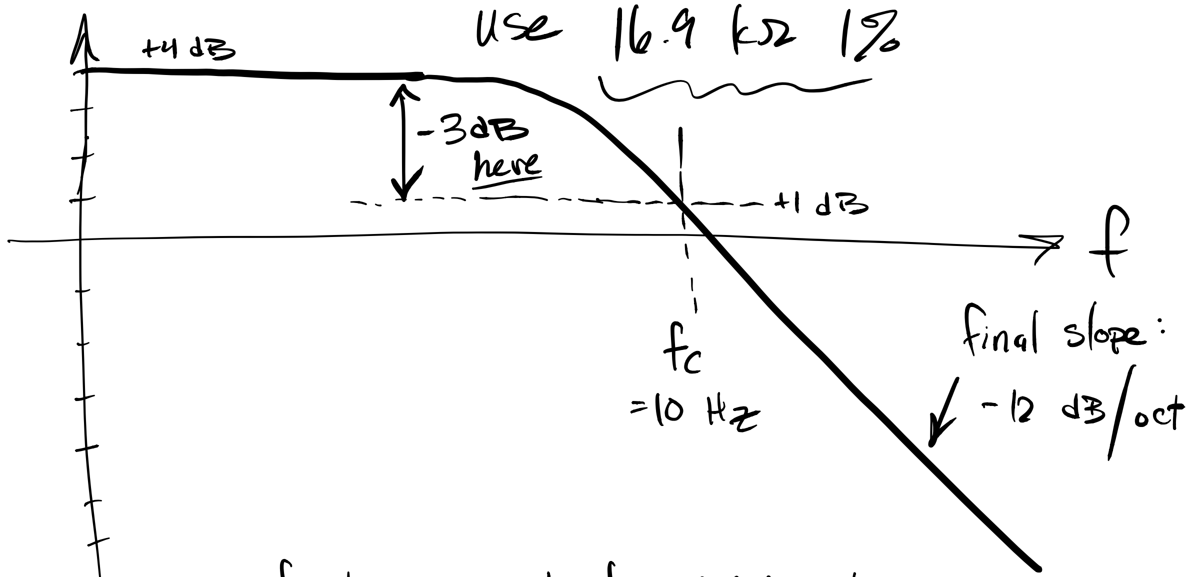
⇒ sets midband gain to $20 \log_{10} 1.586 \cong 4\ \text{dB}$

$$\frac{R_F}{R_I} = 0.586$$

pick $\underbrace{R_F = 10\,k\Omega \;\; 1\%}$

then $R_I = \dfrac{10k}{0.586} = 17.078\ k\Omega$

use $\underbrace{16.9\ k\Omega \;\; 1\%}$



∴ passband gain must be +4 dB to get the desired Q value!

∴ often, we specify filters with a -3dB corner frequency f_c that is different from f_0

∴ for Butterworth, $|H(\omega)|$ is -3dB @ f_0!

∴ for other Q-values, need frequency scaling factor (FSF):

Bessel	1.2736
Butterworth	1.000
3dB Chebychev	0.7194

where $FSF_{(\text{low-pass})} = \dfrac{f_0}{f_c}$

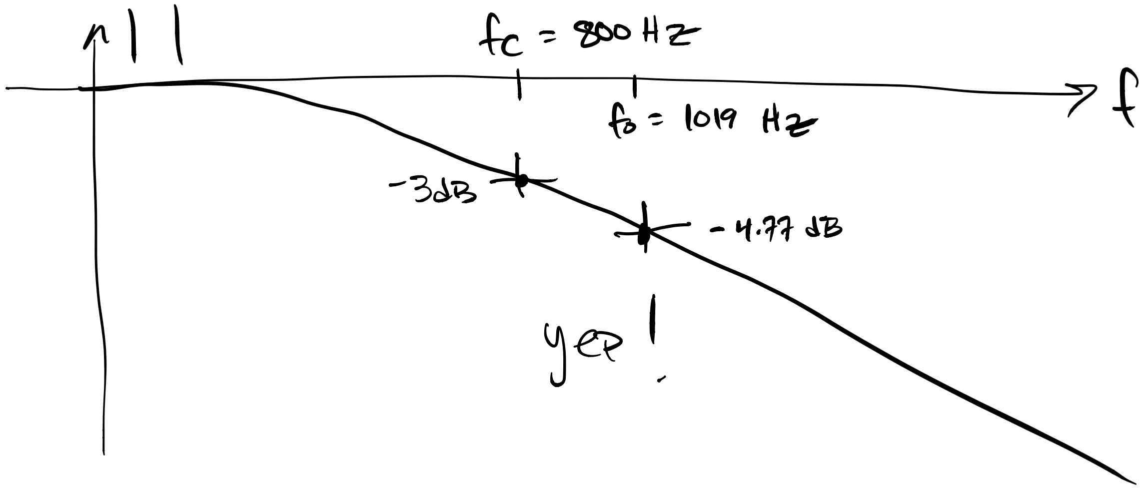
$FSF_{(\text{high-pass})} = \dfrac{f_c}{f_0}$

∴ ex: 2nd-order Bessel w/ $f_c = 800\ Hz$

$FSF_{(\text{low-pass})} = \dfrac{f_0}{f_c} \Rightarrow f_0 = 1.2736 \cdot 800$

$= 1019\ Hz$

∴ for Bessel, we know $|H(\omega_0)| = -4.77\ dB$


|H|
fc = 800 Hz
fo = 1019 Hz
f
-3dB
-4.77 dB
yep!