27 pts.

ISE 2211 • Exam V • 29 October 2021 NAME SOLUTION

Last week, we looked at HRV data from Exam I. Now, we group it into two weeks; one before the onset of symptoms, and one after.

| Week | Day | Date | HRV | Notes |
|---|---|---|---|---|
| 1 | W | 7/28 | 33 | |
| | R | 7/29 | 43 | |
| | F | 7/30 | 43 | |
| | S | 7/31 | 39 | |
| | U | 8/1 | 23 | |
| | M | 8/2 | 47 | |
| | T | 8/3 | 31 | |
| 2 | W | 8/4 | 37 | Presentation of COVID symptoms |
| | R | 8/5 | 40 | Confirmed positive COVID test |
| | F | 8/6 | 31 | |
| | S | 8/7 | 39 | |
| | U | 8/8 | 52 | Reported feeling "loads better" |
| | M | 8/9 | 48 | |
| | T | 8/10 | 49 | |

First, compute sample means and variances for the two different weeks.

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$\bar{X}_1 = 37$ ms $\qquad \bar{X}_2 = 42.29$ ms $\qquad$ +2

$$S_1^2 = \frac{10007 - \frac{259^2}{7}}{6} \qquad S_2^2 = \frac{12860 - \frac{296^2}{7}}{6}$$

$S_1^2 = 70.67\ (\text{ms})^2 \qquad S_2^2 = 57.24\ (\text{ms})^2$

+3

Test the following hypotheses on the difference in mean HRV using the *p*-value approach. State your final conclusion with respect to a significance level of $\alpha = 0.05$. Population variances are unknown and assumed unequal.

$H_0$: $\mu_1 - \mu_2 = 0$

$H_1$: $\mu_1 - \mu_2 \neq 0$

Based on available evidence, state whether you think HRV is a useful indicator of a COVID infection.

unknown and unequal $\rightarrow$ need $v$

$$\frac{S_1^2}{n_1} = \frac{70.67}{7} = 10.10 \qquad \frac{S_2^2}{n_2} = \frac{57.24}{7} = 8.177$$

+2

$$V = \frac{(10.10 + 8.177)^2}{\frac{10.10^2}{6} + \frac{8.177^2}{6}} = 11.87$$

+2

round down to $v = 11$

$$t_0 = \frac{37 - 42.29 - 0}{\sqrt{10.10 + 8.177}} = -1.237$$

+2

$$\left.\begin{aligned} t_{.25,11} &= 0.697 \\ t_{.10,11} &= 1.363 \end{aligned}\right\}$$

+1

$\therefore\ .10 < \frac{P}{2} < .25$

+1

$0.20 < P < .50$ +1

range is $\gg 0.05$; fail to reject $H_0$ +2

evidence does not suggest HRV is useful! +1

Write a 95% confidence interval on the equality of HRV standard deviations and use it to test the following hypotheses:

$H_0: \sigma_1 \ = \sigma_2$

$H_1: \sigma_1 \ \neq \sigma_2$

Based on available evidence, was the assumption of unequal population standard deviations justified in the first problem?

$$f_{\alpha/2,\, n_2-1,\, n_1-1} = f_{.025,6,6} = 5.82 \quad +2$$

$$f_{1-\alpha/2,\, n_2-1,\, n_1-1} = \frac{1}{f_{\alpha/2,\, n_1-1,\, n_2-1}} = \frac{1}{f_{.025,6,6}} = \frac{1}{5.82}$$

$$= 0.1718 \quad +2$$

$$\frac{70.67}{57.24} \cdot 0.1718 < \frac{\sigma_1^2}{\sigma_2^2} < \frac{70.67}{57.24} \cdot 5.82$$

$$0.2121 < \frac{\sigma_1^2}{\sigma_2^2} < 7.186 \quad +2$$

$$0.4606 < \frac{\sigma_1}{\sigma_2} < 2.681 \quad +1 \quad \left(\frac{ms^2}{ms^2}\right) \quad +2$$

C. I. contains unity; ∴ fail to reject Ho

No; Probably should have said equal variances! +1