ISE 2211 • Exam IV • 9 March 2018 NAME Solution

1) Joe Tritschler recently investigated the mean thickness of twenty 1x4s after he suspected they weren't planed correctly by the lumberyard. The thickness is supposed to be 3/4", but the sample mean was determined to be 0.69" with a sample standard deviation of 0.12". Test the following hypotheses on mean thickness using the p-value approach, assuming population variance is unknown. Sketch the corresponding distribution curve, indicating the test statistic value and region(s) corresponding to the *p*-value. State whether you would reject or fail to reject the null hypothesis @ $\alpha = 0.05$.

H_0: $\mu = 3/4''$
H_1: $\mu \neq 3/4''$

$n = 20$

$\bar{x} = 0.69$ $s = 0.12$

unknown variance, small sample $\longrightarrow$ t-test

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.69 - 0.75}{0.12/\sqrt{20}} = -2.2361$$

2 points (equation + value)

Reject if p-value is $< \alpha$ 1 point (rejection rule)

dof $= n-1 = 20-1 = 19$

Since $|-2.2361|$ is between $2.093 + 2.536$

p-value should be between $2(0.025) + 2(0.01)$

$\Downarrow$

$0.05 + 0.02$

$0.02 < \text{p-value} < 0.05$

1 point (p-value)

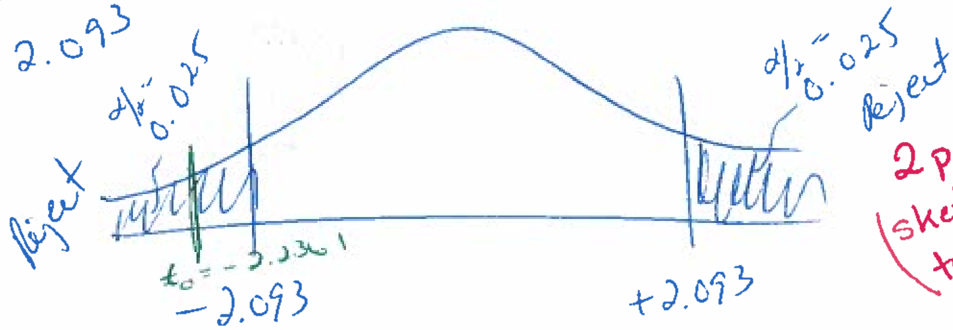
Since p-value is $\leq \alpha$

We reject the null hypothesis

$\rightarrow$ Conclude sufficient evidence to support claim that the mean is not 3/4"

1 point (reject H_0)

$t_{\alpha/2, 19} = t_{0.025, 19} = 2.093$



2 points (sketch + indicate test stat and region)

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2) In testing a random sample of 36 bags of Gardetto's®-brand snack mix pieces, Joe Tritschler measured a mean unit weight of 15.39 ounces with a variance of 1.437 ounces^2. The manufacturer claims these are 16-ounce bags. (Joe Tritschler is tired of these snack manufacturers constantly perpetrating insidious conspiracies against him and he wants answers!!!) Test the following hypotheses on the mean weight of Gardetto's®-brand snack mix using the fixed-significance-level approach at $\alpha = 0.05$, assuming population variance is unknown. Sketch the corresponding distribution curve, indicating the critical region(s) and your test statistic value. State whether you would reject or fail to reject the null hypothesis.

H_0: $\mu = 16.00$

H_1: $\mu < 16.00$

$n = 36 \rightarrow$ large sample

$\bar{x} = 15.39 \quad s^2 = 1.437 \rightarrow s = 1.199$

Unknown variance, large sample $\rightarrow$ Z-test

$$Z_0 = \frac{15.39 - 16}{1.199/\sqrt{36}} = -3.0525$$

2 points (equation+value)

Reject if $Z_0 < -Z_\alpha$

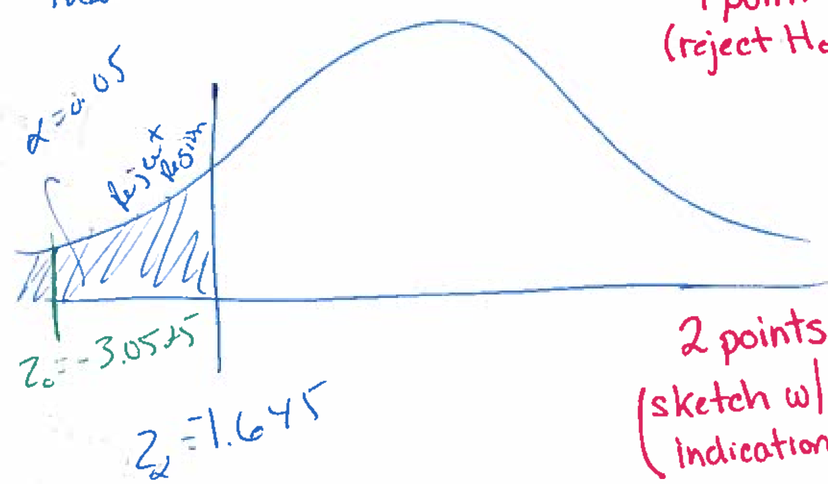
1 point (rejection rule)

$-Z_\alpha = -Z_{0.05} = -1.645$

Since $-3.0525 < -1.645$

Reject Ho
$\rightarrow$ conclude sufficient evidence to support claim that the mean is less than 16 oz

1 point (reject Ho)



2 points (sketch w/ indications)

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3) Here's some data nobody cares about.

$\bar{x}$ = 37.94
s = 4.928
n = 29
σ = ? (unknown)

Test the following hypotheses using the fixed-significance-level approach at α = 0.05:
H_0: σ = 5
H_1: σ < 5

Sketch the distribution, indicating the critical region(s) and your test statistic value. State whether you would reject or fail to reject the null hypothesis.

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(29-1)(4.928)^2}{5^2} = 27.199$$

2 points (equation and value)

one sided test

reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,\,n-1}$

1 point (rejection rule)

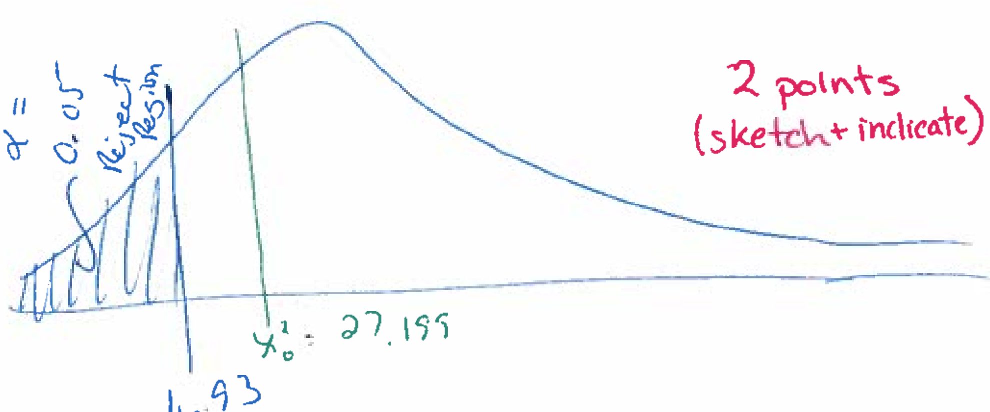
$$\chi^2_{1-\alpha,\,n-1} = \chi^2_{1-0.05,\,29-1} = \chi^2_{0.95,\,28} = 16.93$$

Since $27.199 \not< 16.93$

Fail to Reject H_0

1 point (Fail to reject H_0)

→ Conclude insufficient evidence to support claim that st dev. is less than 5



2 points (sketch + indicate)

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Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$
2.	$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2,n-1}$
			$H_1: \mu > \mu_0$	$t_0 > t_{\alpha,n-1}$
			$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha,n-1}$
3.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi_{\alpha/2,n-1}^2$ or $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi_{\alpha,n-1}^2$
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi_{1-\alpha,n-1}^2$
4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$
			$H_1: p > p_0$	$z_0 > z_\alpha$
			$H_1: p < p_0$	$z_0 < -z_\alpha$