MT2 Review, MTH 2530, Nov.19, 2022

Show all of your work for full credit.

Name (print): ______________________

1. Let $A = \begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix} = [\vec{a_1} \quad \vec{a_2} \quad \vec{a_3} \quad \vec{a_4} \quad \vec{a_5}]$.

   (a) (5 points) Find a basis for Row$A$, and determine its dimension.

   (b) (5 points) Find a basis for Col$A$, and determine its dimension.

   (c) (10 points) Find a basis for Nul$A$, and determine its dimension. Use back-substitution to select the free variables.

2. (a) (20 points) Diagonalize the matrix: $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ in the PDP^{-1} form. Clearly present P and D.

   (b) (5 points) If a vector $\vec{u}$ is in the Nul$A$, what is $A\vec{u}$?

   (c) (5 points) What is $E_{\lambda=0}$?

3. (a) (15 points) Use the Gram-Schmidt process to construct an orthogonal basis for Col$A$ with

$$A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix} = [\vec{a_1} \quad \vec{a_2} \quad \vec{a_3}].$$

   Follow the given order as in $\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$.

   (b) (10 points) Find the orthogonal projection $\hat{b}$ of a point $\vec{b} = (1, 0, 1, 0)$ onto Col$A$. Evaluate the distance from $\vec{b}$ to Col$A$.

4. (25 points) Solve the least-squares problem of $A\vec{x} = \vec{b}$, where

$$\vec{b} = (5, 1, 0)^T$$

and

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

Calculate the least squares error for this estimation.