Tests on Difference in Pop. Proportions

$H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

... where $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$, $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$

Z-distribution $\rightarrow$ $n \geq 30$

critical values : $\pm Z_{\alpha/2}$

p-value : same as any z-test; exact value is available ex: St. John's Wort → herbal extract

∴ is it effective in treating some condition?

200 human subjects;

$n_1 = 100$ received St. John's Wort

$x_1 = 27$ had improved symptoms after eight weeks

$n_2 = 100$ received placebo

$x_2 = 19$ had improved symptoms after 8 weeks

∴ looks like statistically-significant difference!

let's find out...

proportion improved with extract

test $H_0: p_1 = p_2$

proportion improved w/ placebo

$H_1: p_1 \neq p_2$

④ $\alpha = 0.05$

Sample proportions:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{27}{100} = 0.27 \quad [27\%]$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{19}{100} = 0.19 \quad [19\%]$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{27 + 19}{100 + 100} = 0.23 \quad [23\%]$$

pooled estimator of pop. proportion:

total proportion improved

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.27 - 0.19}{\sqrt{.23(1-.23)\left(\frac{1}{100} + \frac{1}{100}\right)}}$$

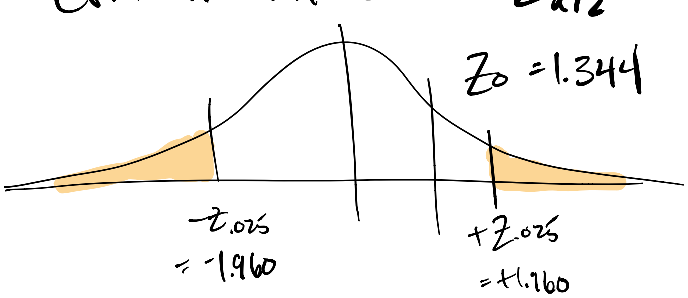
$$Z_0 = 1.344$$

hmmm......

only 1.3 std. dev. from mean!

critical values: $\pm Z_{\alpha/2} = \pm Z_{.025} = \pm 1.960$

(bottom row of t-table!)

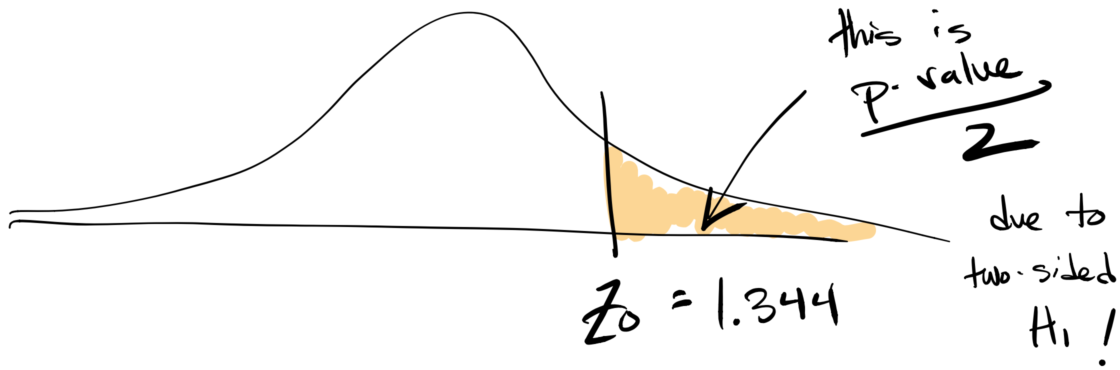


$Z_0 \not> +Z_{\alpha/2}$

fail to reject H_0

∴ insufficient evidence that extract vs. placebo is significant!

∴ if the p-value approach had been taken:



$$\frac{\text{p-value}}{2} = P(Z > 1.344)$$

$$= 1 - \underbrace{P(Z < 1.344)}_{\text{cumulative distribution table!}}$$

$\Rightarrow$ close enough to 1.34

$$\frac{\text{p-value}}{2} = 0.090123$$

$$\text{p-value} \approx 0.180$$

$$\boxed{\begin{array}{l}\text{p-value} \not< 0.05 \\ \text{fail to reject } H_0 \text{ @ } \alpha = 0.05\end{array}}$$

C.I. on Difference in Pop. Proportions

$$p_1 - p_2 : \quad \hat{p}_1 - \hat{p}_2 \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

ex: Crankshaft bearings:

$n_1 = 85$ bearings

$x_1 = 10$ bad ones

$\therefore \hat{p}_1 = \frac{10}{85} = 0.1176$ [or 11.76%]

new process: [surface finish?]

$n_2 = 85$

$x_2 = 8$ bad bearings

$\therefore \hat{p}_2 = \frac{8}{85} = 0.0941$ [9.41%]

∴ on surface, looks like an improvement!

"20% improvement"

∴ Write 95% C.I. on difference in proportion of bad bearings

$$p_1 - p_2 : .1176 - .0941 \pm 1.960 \sqrt{\frac{.1176(1-.1176)}{85} + \frac{.0941(1-.0941)}{85}}$$

$$\boxed{\underset{-6.89\%}{-0.0689} < p_1 - p_2 < \underset{11.55\%}{0.1155}}$$

∴ C.I. includes zero; ∴ we would <u>fail to reject</u> $H_0 : p_1 - p_2 = 0$

i.e., $H_0 : p_1 = p_2$

∴ <u>insufficient evidence</u> to claim that the new process reduces the proportion of bad bearings