LECTURE NO. 2

1.7 Integrals Resulting in Inverse Trig Functions

Wright State University

Two Integration Formulas Resulting Inverse Trig Functions

•

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

•

$$\int \frac{1}{x^2 + 1} dx = \tan^{-1} x + C$$

• We may use substitution and these formulas to calculate some other similar integrals.

$$\int \frac{1}{\sqrt{1-4x^2}} dx$$

$$\int \frac{1}{1-x^2} dx = \sin^4 x + C$$

$$\int \frac{1}{1-(2x)^2} dx$$

$$\int \frac{1}{1-(2x)^2} dx$$

$$\int \frac{1}{1-u^2} \frac{du}{2} = \frac{1}{2} \int \frac{1}{1-u^2} du = \frac{1}{2} \sin^4 u + C$$

$$= \int \frac{1}{1-u^2} \frac{du}{2} = \frac{1}{2} \int \frac{1}{1-u^2} du = \frac{1}{2} \int \sin^4 u + C$$

$$= \frac{1}{2} \int \sin^4 (2x) + C$$

FWAL ANSWER

$$\int_0^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}}$$

First let's do
$$\int \frac{dx}{4-qx^2} = \int \frac{1}{\sqrt{4-qx^2}} \cdot dx$$

$$\int \frac{1}{\sqrt{4-qx^2}} dx = \int \frac{1}{2} \int \frac{1}{\sqrt{1-(\frac{3x}{2})^2}} dx$$

$$\int \frac{1}{\sqrt{4-(1-\frac{qx^2}{4})}} dx = \int \frac{1}{2} \int \frac{1}{\sqrt{1-(\frac{3x}{2})^2}} dx$$
Substitution $u = \frac{3x}{2} = \frac{du}{dx} = \frac{3}{2} = \frac{du}{3} = \frac{2}{3} du$

$$\frac{1}{2} \int \frac{1}{1-u^2} \frac{2}{3} du = \frac{1}{3} \int \int \frac{1}{1-u^2} du = \frac{1}{3} \int \sin^4 u + C = \frac{1}{3} \int \sin^4 \frac{3x}{2} + C$$

$$\int \frac{1}{3} \frac{dx}{\sqrt{4-qx^2}} = \frac{1}{3} \int \sin^4 \frac{3x}{2} = \frac{1}{3} \int \sin^4 \frac{1}{2} + C$$

$$\int \frac{1}{3} \frac{dx}{\sqrt{4-qx^2}} = \frac{1}{3} \int \sin^4 \frac{3x}{2} = \frac{1}{3} \int \sin^4 \frac{1}{2} + C$$

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$$\int \frac{1}{3} \int \frac{dx}{\sqrt{4-qx^2}} = \frac{1}{3} \int \sin^4 \frac{3x}{2} = \frac{1}{3} \int \sin^4 \frac{1}{2} + C$$

$$\int \frac{1}{1+(2x+1)^2} dx$$

$$\int \frac{1}{1+x^2} dx = +an^{-1}x + C$$

Substitution
$$u=2x+1$$
 $\frac{dy}{dx}=2$ $dx=\frac{dy}{2}$

$$\left(\frac{1}{1+11^{2}} \frac{du}{2} = \frac{1}{2} \left(\frac{1}{1+41^{2}} du = \frac{1}{2} + an^{-1}u + C \right)$$

$$=\left(\frac{1}{2} + an^{-1}(2x+1) + C\right)$$

TWAL ANSWER.

$$\int \frac{dx}{25+16x^2}$$

$$\int \frac{dx}{25 + 16x^{2}} = \int \frac{1}{25 + 16x^{2}} dx = \int \frac{1}{1 + (\frac{4}{5})^{2}} dx = \int \frac{1}{1 + (\frac{4}{5})^{2}} dx$$

$$= \int \frac{1}{25(1 + \frac{16x^{2}}{25})} dx = \frac{1}{25} \int \frac{1}{1 + (\frac{4}{5})^{2}} dx$$

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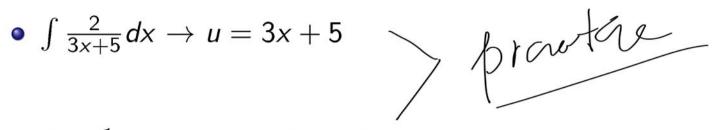
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Substitution Techniques for Integral of Simple Rational Functions



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$$\int \frac{1}{(2x+1)^3} dx \to u = 2x + 1$$

• What about $\int \frac{4x+3}{x^2+9} dx$?

x79 irreducible quadratu factor!

$$\int \frac{4x+3}{x^2+9} dx$$

$$\int \frac{4x}{x^{2}+9} + \frac{3}{x^{2}+9} dx = \int \frac{4x}{x^{2}+9} dx + \int \frac{3}{x^{2}+9} dx = 2\ln|x^{2}+9| + \tan(\frac{x}{3}) + C$$
TWAL ANSWER

$$\int \frac{4x}{x^{2}+9} dx \quad u = x^{2}+9 \quad \frac{dy}{dx^{2}=2x} dx = \frac{du}{2x}$$

$$\int \frac{4x}{u} \frac{du}{2x} = \int \frac{2}{u} du = 2 \int \frac{1}{u} dy$$

$$= 2 \ln|u| + C = 2 \ln|x^{2}+9| + C$$