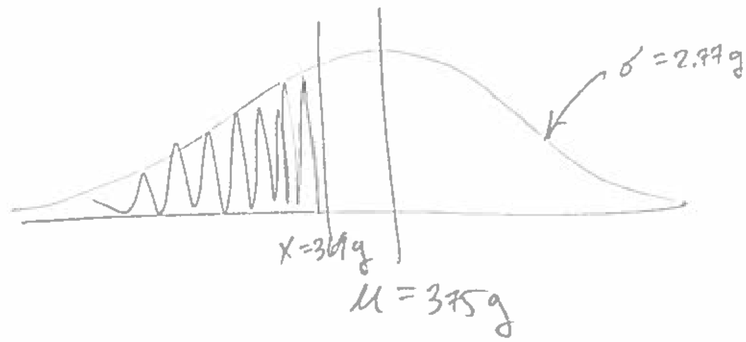
ISE 2211 • Exam II • 8 February 2019 NAME SOLUTION

1) Yellow Springs Brewery offers several of their award-winning craft beverages in cans. Fill volume is monitored very closely by measuring the finished can's mass. It is known that that mass is normally-distributed with a mean value of 375 g and a standard deviation of 2.77 g. Determine the probability that a can will have a mass of 369 g or fewer, thus necessitating the employee doing the measurement to drink it on the spot. Shade this probability on rough sketches of both the normal and standard normal distributions.

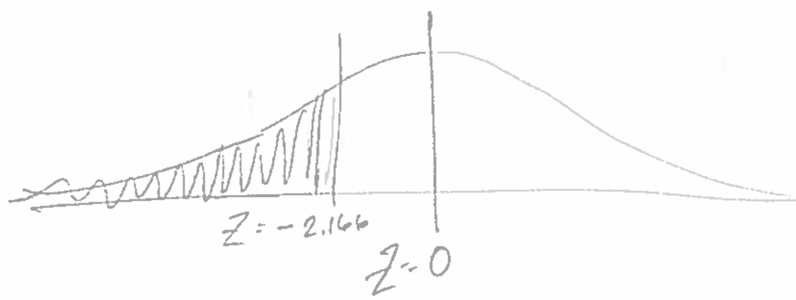
$$Z = \frac{X-\mu}{\sigma}$$




+1

$$Z = \frac{369 - 375}{2.77} = -2.166$$

+1




+1

from table:

0.015386 also acceptable
↳ $P(Z < -2.16)$

$$P(Z < -2.17) = 0.015003$$

$$= P(m < 369\ g)$$

+1

2) In a plastic tub filled with 24 canned beverages from Yellow Springs Brewery, twelve are the kind you like. If you sample three without replacement, determine the probability that all three are the kind you like. Hint: use the multiplication rule.

$$P = \frac{12}{24} \cdot \frac{11}{23} \cdot \frac{10}{22} = 0.1087 \quad (+1)$$

or 10.87%

3) Now, consider the same tub of 24 canned beverages, of which twelve are the kind you like. If you sample three with replacement, determine the probability that all three are the kind you like. Let the binomially-distributed random variable *X* represent the number of successful selections of the kind of beverage you like. Hint: you'll need to determine the probability of a "success" from the information given above.

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$p = \frac{12}{24} = 0.5 \quad (+1)$$

$$\left.\begin{array}{l} n = 3 \\ x = 3 \end{array}\right\} \quad (+1)$$

$$f(3) = \cancel{\binom{3}{3}}^{\,1} \, 0.5^3 \, \cancel{(1-0.5)^{3-3}}^{\,1}$$

$$\frac{3!}{3!\,(0!)} \to 1$$

$$= 0.125 \quad (+1)$$

or 12.5%

Determine the probability of selecting at least one beverage of the kind you like.

$$= f(1) + f(2) + f(3)$$

$$f(1) = \binom{3}{1} 0.5^1 (1-0.5)^{3-1} = 3 \cdot 0.125 = 0.375$$

+1

$$\binom{3}{1} = \frac{3!}{1!(3-1)!} = 3$$

$$f(2) = \binom{3}{2} 0.5^2 (1-0.5)^{3-2} = 3 \cdot 0.125 = 0.375$$

+1

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = 3$$

$$\therefore f(x \geq 1) = 0.375 + 0.375 + 0.125$$

+1

$$= 0.875 \quad \text{or} \quad 87.5\%$$

Determine the expected value and variance of the number of successful selections. Include a unit with each answer.

$$E(x) = np = 3 \cdot 0.5 = 1.5$$ beverages you like

+1 +1

$$V(x) = np(1-p) = 1.5(0.5) = 0.75$$ +1

$$(\text{beverages you like})^2$$

+1