Module 13: **Linearity and its Corollaries** **Notes**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, WIKIPEDIA, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Learning Objective: In this module, we demonstrate how we can use *EXACTLY THE SAME* nodal analysis for circuits with dynamic elements to find the *STEADY STATE RESPONSE* for a specific *SINUSOIDAL INPUT AT A SINGLE FIXED FREQUENCY* by replacing sources and dynamic elements with their *PHASOR REPRESENTATION.*

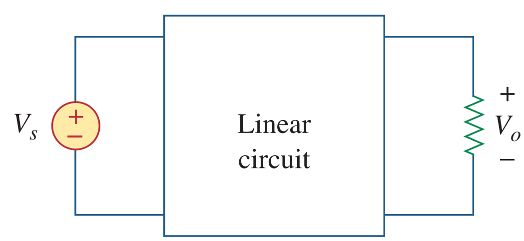
While *phasors* are a throwback to the sliderule days decades ago when symbolic computation was not possible, phasors still have some utility in analyzing systems *with a single, fixed frequency.* However, phasors are inherently limited in their analytic generality in that *the phasor representation for every dynamic element must be recalculated for any change of the excitation frequency.*

Definition: *Linearity:* A system $\mathcal{S}$ is *Linear* if for constants a and b:

1. $\mathcal{S}(ax) = a\mathcal{S}(x)$
2. $\mathcal{S}(ax_1 + bx_2) = a\mathcal{S}(x_1) + b\mathcal{S}(x_2)$

We have been implicitly employing both properties of linearity at the component level and in our analysis approach for the entire course. We can exploit linearity at the system level in a number of ways. Here's an example of the first property:

Experiment	V_s	V_o
1	12 V	4 V
2		16 V
3	1 V	
4		−2 V



How about power?

An immediate consequence of the second property of linearity is *superposition* - which we have observed before in the analysis of systems with symbolic inputs. In particular, for a system with a single input, we have

$$\text{Output} = \text{Input} \cdot \mathcal{H}$$

where the transfer function is given by

$$\mathcal{H} = \frac{\text{Output}}{\text{Input}}$$

We have also seen that when there are multiple inputs, the output solution takes the form:

$$\text{Output} = \text{Input}_1 \cdot \mathcal{H}_1 + \text{Input}_2 \cdot \mathcal{H}_2$$

and so on. It is a natural consequence of linearity to state:

Definition: *The superposition principle:* The result of a sum of stimuli is the sum of the outputs due to the individual stimuli. More specifically, the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

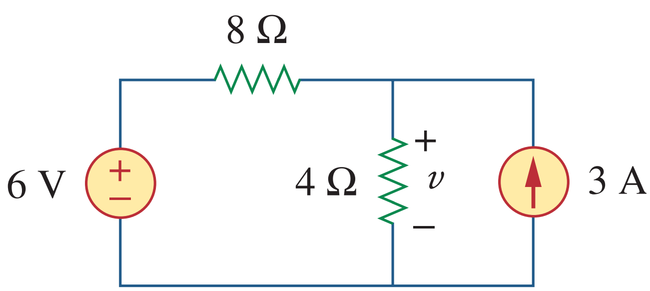
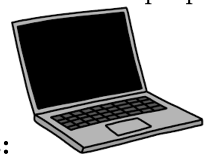


Figure 4.6
For Example 4.3.

$$V_{\text{total}} = V_{\text{due to 6V}} + V_{\text{due to 3A}}$$

The most general and powerful approach to quantify and understand superposition is:

Superposition Attribution using Generalized Sources: 

1. Replace *all independent sources with symbolic representations*
2. Apply nodal analysis using these generalized sources.
3. Simplify (if necessary) to observe:

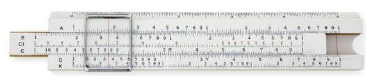
   $$\text{Output} = \text{Input}_1 \cdot \mathcal{H}_1 + \text{Input}_2 \cdot \mathcal{H}_2$$

   Circuit analysis is now complete! But you may be asked to answer whatever questions are asked about the circuit using the above characterizations.

For example, to find the output due only to Input_1, we trivially have:

$$\text{Output} = \text{Input}_1 \cdot \mathcal{H}_1 + 0 \cdot \mathcal{H}_2 = \text{Input}_1 \cdot \mathcal{H}_1$$

Note: *The symbolic analysis using generalized sources renders the following procedure obsolete.* Nonetheless, we will dabble through a few examples to illustrate.



Old-fashion Superposition Procedure:

1. *Zero* all independent sources except for the one under consideration.
   (a) **Voltage sources** are replaced with **shorts**
   (b) **Current sources** are replaced with **open circuits.**
2. Do *not* modify any of the dependent sources.
3. Find the output (voltage or current) due to that active source.
4. Repeat for each independent source - Yes, go back to 1. and start all over!
5. Find the total contribution by adding all contributions from individual independent sources.

Generalized Sources: 

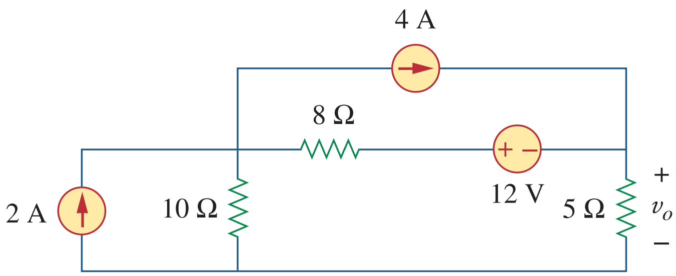


Figure 4.81
For Prob. 4.13.

```
%% Problem 4.13
clear all
syms Iin2 Iin4 Vin va vo
[va,vo] = solve(va/10 -Iin2 + Iin4 + (va-(vo+Vin))/8 == 0,...
  vo/5 + ((vo+Vin)-va)/8 - Iin4 == 0, va, vo)
% Which exhibits the superposition result!
% Individual contributions include:
vo2A = eval(subs(vo, [Iin2, Iin4, Vin], [2, 0, 0]))
vo4A = eval(subs(vo, [Iin2, Iin4, Vin], [0, 4, 0]))
vo12V = eval(subs(vo, [Iin2, Iin4, Vin], [0, 0, 12]))
% And altogether
voTot = eval(subs(vo, [Iin2, Iin4, Vin], [2, 4, 12]))
% Compare to
voTot2 = vo2A + vo4A + vo12V
%
```

which yields...

```
vo = (50*Iin2)/23 + (40*Iin4)/23 - (5*Vin)/23
vo2A =     4.3478
vo4A =     6.9565
vo12V = -2.6087
voTot =    8.6957
voTot2 =  8.6957
```

which is self-explanatory.

Old-fashioned way (to demonstrate):

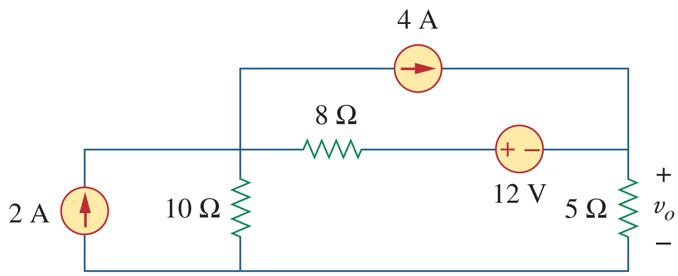


Figure 4.81
For Prob. 4.13.

Generalized Sources: 

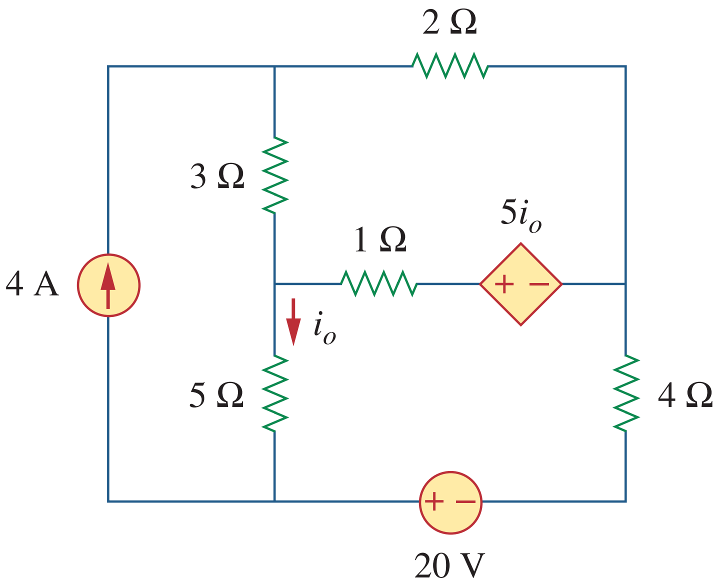


Figure 4.9
For Example 4.4.

```
%% Example 4.4
clear all
syms Iin Vin io va vb vc
[va,vb,vc,io] = solve(va/5 + (va-(vb+5*io))/1 + (va-vc)/3 == 0,...
  (vb+Vin)/4 + ((vb+5*io)-va)/1 + (vb-vc)/2 == 0,...
  -Iin + (vc-va)/3 + (vc-vb)/2 == 0, io==va/5, va, vb, vc, io)
%
io
% Which exhibits the superposition result!
% Individual contributions include:
io4A = eval(subs(io, [Iin, Vin], [4, 0]))
io20V = eval(subs(io, [Iin, Vin], [0, 20]))
% And altogether
ioTot = eval(subs(io, [Iin, Vin], [4, 20]))
% Compare to
ioTot2 = io4A + io20V
%
```

which yields...

```
io4A =    3.0588
io20V =-3.5294
ioTot =  -0.4706
ioTot2 =-0.4706
```

Old-fashioned way:

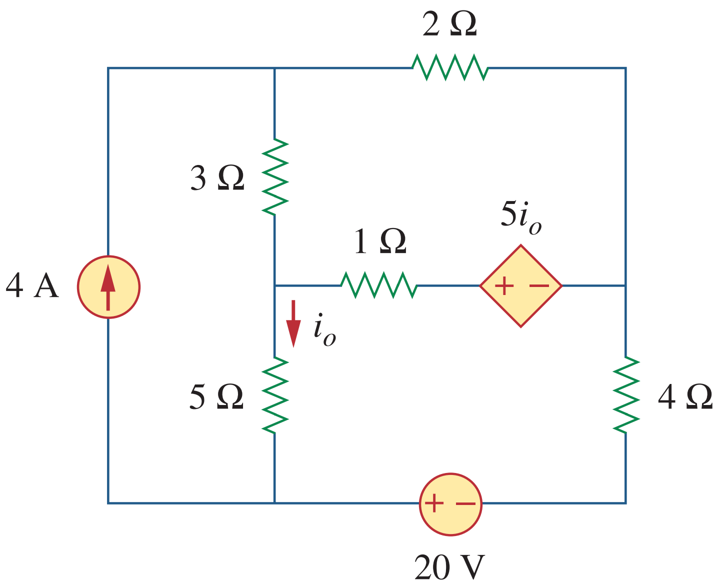


Figure 4.9
For Example 4.4.

Generalized Sources: 

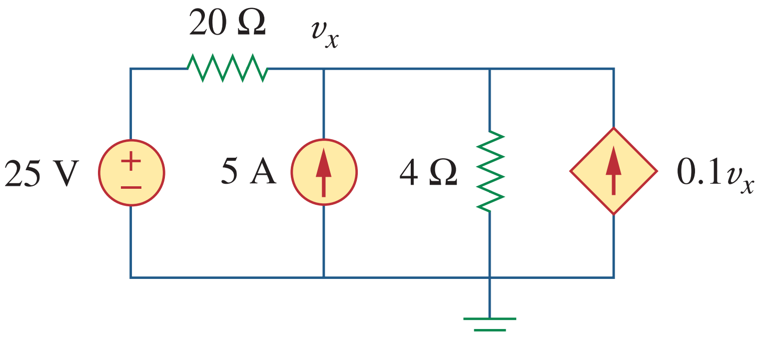


Figure 4.11
For Practice Prob. 4.4.

```
%% Practice Problem 4.4
clear all
syms Iin Vin vx
[vx] = solve(-0.1*vx + vx/4 - Iin + (vx-Vin)/20 == 0, vx)
% Which exhibits the superposition result!
% Individual contributions include:
vx5A = eval(subs(vx, [Iin, Vin], [5, 0]))
vx25V = eval(subs(vx, [Iin, Vin], [0, 25]))
% And altogether
vxTot = eval(subs(vx, [Iin, Vin], [5, 25]))
% Compare to
vxTot2 = vx5A + vx25V
%
```

which yields...

```
vx = 5*Iin + 0.2500*Vin
vx5A =     25
vx25V =          6.25
vxTot =         31.25
vxTot2 =         31.25
```

Old-fashioned way:

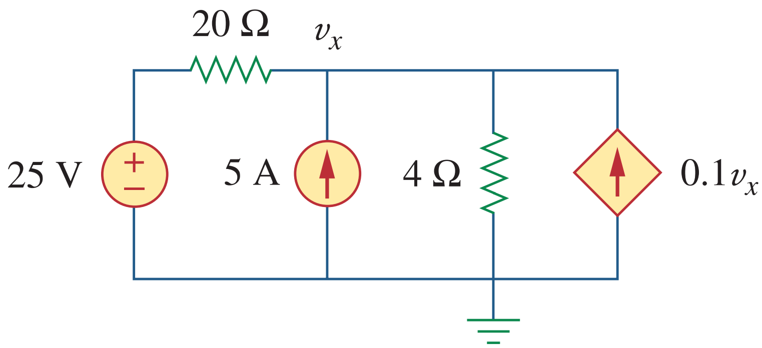


Figure 4.11
For Practice Prob. 4.4.

Homework 13: Chapter 4 # 8, 11, 18 **Note:** Do these both the Generalized Source way AND the Old-fashioned way.

→ Remember that only independent sources are replaced with symbolic labels.

→ Homework deliverables MUST be a pdf file generated using a solver.

→ The resulting .pdf file is to be uploaded to the Pilot Dropbox using the naming convention: First 4 letters of Lastname, First initial, year, title. For example, my .pdf file would be named: GarbF2020HW13.pdf

Problems

Section 4.2 Linearity Property

4.1 Calculate the current i_o in the circuit of Fig. 4.69. What value of input voltage is necessary to make i_o equal to 5 amps?

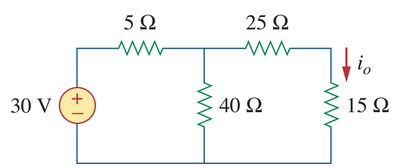


Figure 4.69
For Prob. 4.1.

4.2 Using Fig. 4.70, design a problem to help other students better understand linearity.

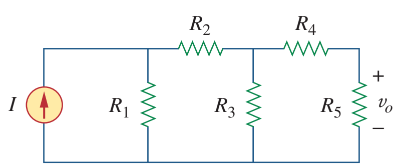


Figure 4.70
For Prob. 4.2.

4.3 (a) In the circuit of Fig. 4.71, calculate v_o and i_o when $v_s = 1$ V.

(b) Find v_o and i_o when $v_s = 10$ V.

(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

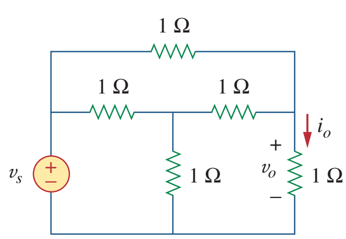


Figure 4.71
For Prob. 4.3.

4.4 Use linearity to determine i_o in the circuit of Fig. 4.72.

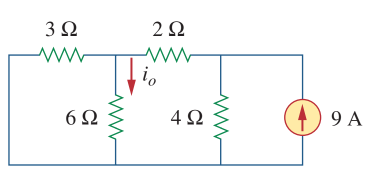


Figure 4.72
For Prob. 4.4.

4.5 For the circuit in Fig. 4.73, assume $v_o = 1$ V, and use linearity to find the actual value of v_o.

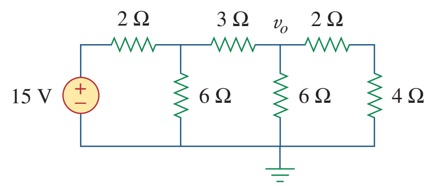


Figure 4.73
For Prob. 4.5.

4.6 For the linear circuit shown in Fig. 4.74, use linearity to complete the following table.

Experiment	V_s	V_o
1	12 V	4 V
2	48	16 V
3	1 V	1/3
4	-6	−2 V

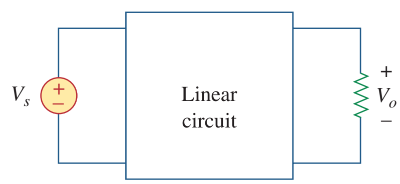


Figure 4.74
For Prob. 4.6.

4.7 Use linearity and the assumption that $V_o = 1$ V to find the actual value of V_o in Fig. 4.75.

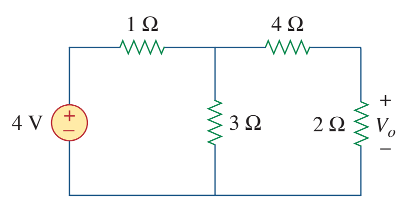


Figure 4.75
For Prob. 4.7.

Section 4.3 Superposition

4.8 Using superposition, find V_o in the circuit of Fig. 4.76. Check with *PSpice or MultiSim.*

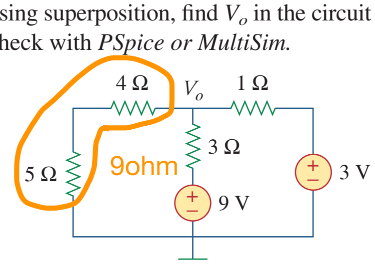


Figure 4.76
For Prob. 4.8.

4.9 Given that $I = 4$ amps when $V_s = 40$ volts and $I_s = 4$ amps and $I = 1$ amp when $V_s = 20$ volts and $I_s = 0$, use superposition and linearity to determine the value of I when $V_s = 60$ volts and $I_s = -2$ amps.

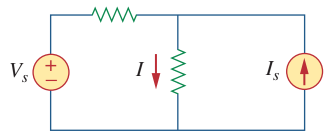


Figure 4.77
For Prob. 4.9.

4.10 Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter k is a gain you can specify to make the problem easier to solve but must not be zero.

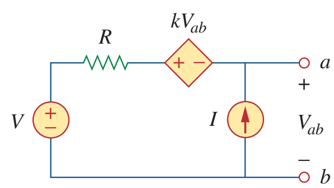


Figure 4.78
For Prob. 4.10.

4.11 Use the superposition principle to find i_o and v_o in the circuit of Fig. 4.79.

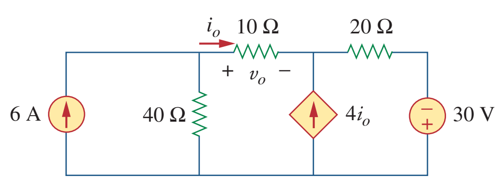


Figure 4.79
For Prob. 4.11.

4.12 Determine v_o in the circuit of Fig. 4.80 using the superposition principle.

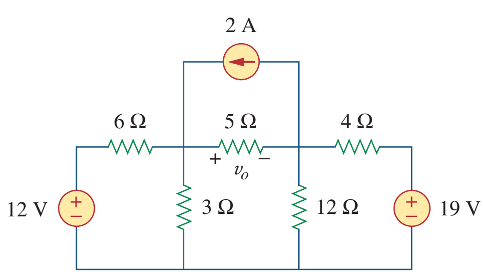


Figure 4.80
For Prob. 4.12.

4.13 Use superposition to find v_o in the circuit of Fig. 4.81.

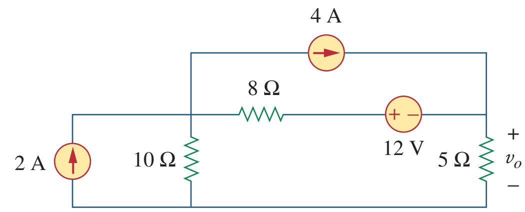


Figure 4.81
For Prob. 4.13.

4.14 Apply the superposition principle to find v_o in the circuit of Fig. 4.82.

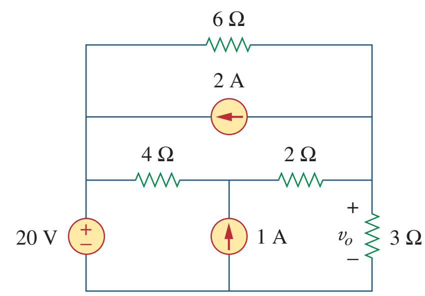


Figure 4.82
For Prob. 4.14.

4.15 For the circuit in Fig. 4.83, use superposition to find i. Calculate the power delivered to the 3-Ω resistor.

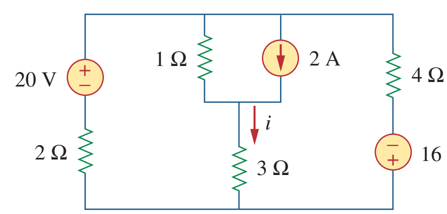


Figure 4.83
For Probs. 4.15 and 4.56.

4.16 Given the circuit in Fig. 4.84, use superposition to obtain i_o.

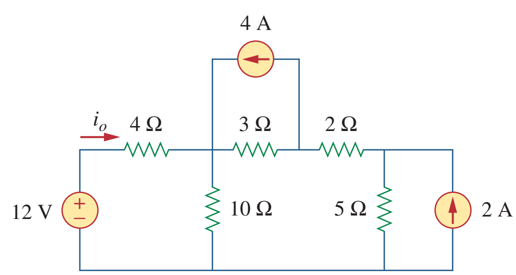


Figure 4.84
For Prob. 4.16.

4.17 Use superposition to obtain v_x in the circuit of Fig. 4.85. Check your result using *PSpice or MultiSim*.

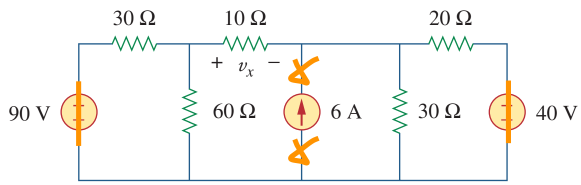


Figure 4.85
For Prob. 4.17.

4.18 Use superposition to find V_o in the circuit of Fig. 4.86.

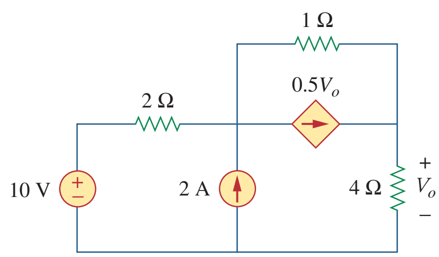


Figure 4.86
For Prob. 4.18.

4.19 Use superposition to solve for v_x in the circuit of Fig. 4.87.

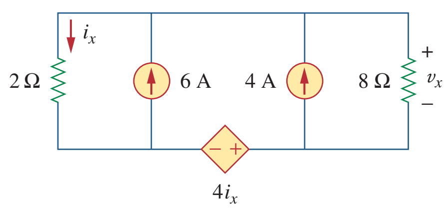


Figure 4.87
For Prob. 4.19.

Section 4.4 Source Transformation

4.20 Use source transformation to reduce the circuit in Fig. 4.88 to a single voltage source in series with a single resistor.

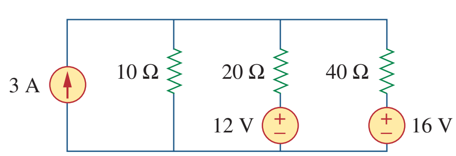


Figure 4.88
For Prob. 4.20.

4.21 Using Fig. 4.89, design a problem to help other students better understand source transformation.

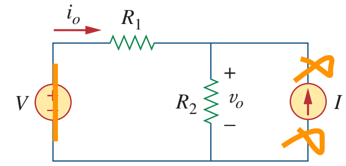


Figure 4.89
For Prob. 4.21.

when I=0
v0=v*r2/(r1+r2)
i0=v/(r1+r2)

when V=0
v0=-r1r2/(r1+r0)
? figure this part out

4.22 For the circuit in Fig. 4.90, use source transformation to find i.

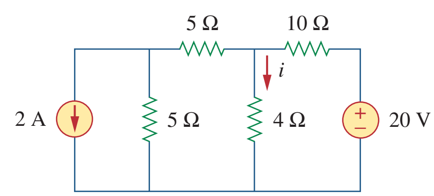


Figure 4.90
For Prob. 4.22.

4.23 Referring to Fig. 4.91, use source transformation to determine the current and power absorbed by the 8-Ω resistor.

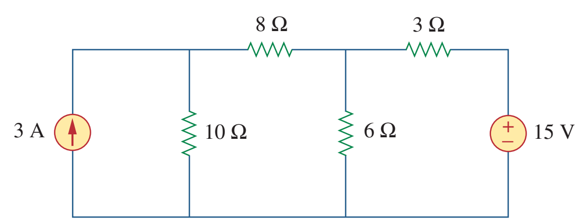


Figure 4.91
For Prob. 4.23.

4.24 Use source transformation to find the voltage V_x in the circuit of Fig. 4.92.

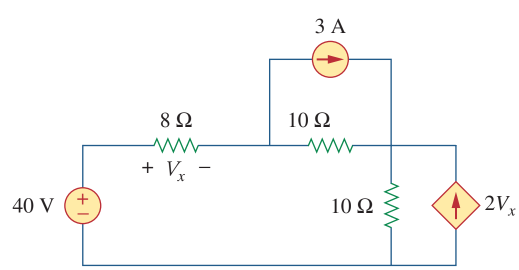


Figure 4.92
For Prob. 4.24.