EE 2010 Circuit Analysis

Module 07: System of Independent Equations: Node Equations; Linked Nodes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Definition: *Voltage-Linked-Nodes (or Super Nodes):* Occasionally, we encounter a circuit where two nodes are linked by a voltage source (with no other element in series with that source). This presents an opportunity to solve the circuit using one fewer node equation for every voltage-source-linked pair of nodes.

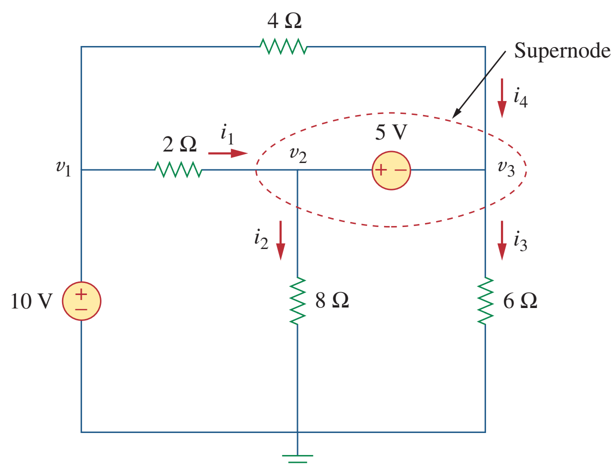

Figure 3.7
A circuit with a supernode.

We could, of course follow the standard
Node Voltage Procedure:

1. Identify the essential ($\geq$ 3-element connections) nodes
2. Select a node as the reference node = the node at *ground potential* = 0 Volts
3. Identify and label the voltages at nodes that are readily deduced
4. Assign voltage variables $v_a, v_b, \ldots$ to the remaining essential nodes where voltages are not readily deduced.
   (These voltages, or potential differences, are referenced with respect to the reference node.)
5. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
6. Enjoy the thrill of ending the consideration of each node with the powerful "= 0"
7. Add one additional equation for each dependent source specification if necessary

   Circuit analysis is now complete! But you may be asked to:

8. Invoke the power of algebra to solve for every assigned variable in the resulting system of equations.

9. Answer whatever questions are asked about the circuit using the solved values of knowledge of $v_a, v_b, \ldots$.

Noting that there is, in general, a current flowing through every voltage source.

Let's do one the hard way.

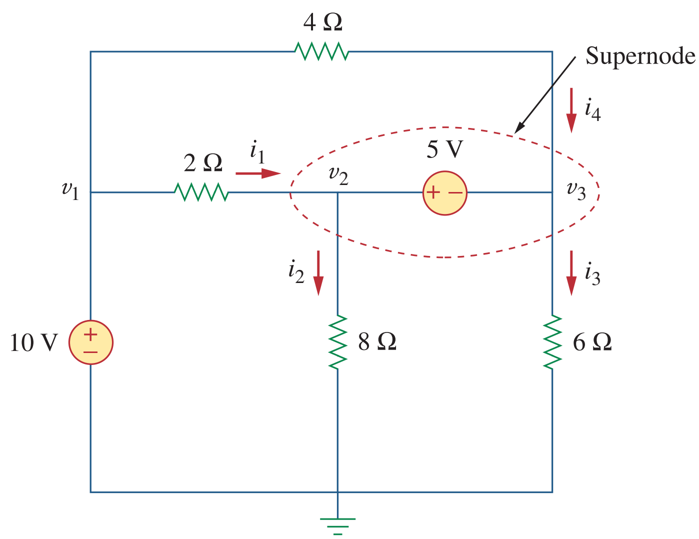

Figure 3.7
A circuit with a supernode.

There is a better way. Noting that Step 4. says: "Assign voltage variables $v_a, v_b, \ldots$ to the remaining $M \leq N$ nodes where voltages are not readily deduced." Implying there *may be* other nodes where the voltages *are readily deduced*. Let's look again:

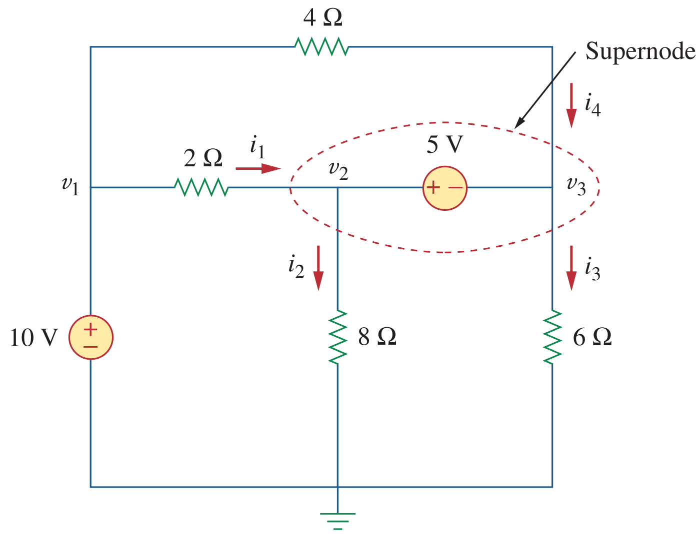

Figure 3.7
A circuit with a supernode.

Another one:

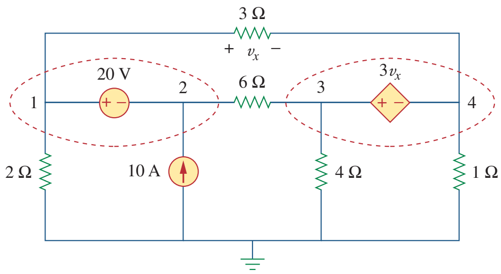

Figure 3.12
For Example 3.4.

Node Voltage Procedure with Linked Nodes (or Super Nodes):

1. Identify the essential ($\geq$ 3-element connections) nodes
2. Select a node as the reference node = the node at *ground potential* = 0 Volts
3. Identify and label the voltages at nodes that are readily deduced
4. Note the node-pairs linked by a *voltage source*
5. Assign voltage variables $v_a, v_b, \ldots$ to the remaining nodes with only one assignment for each linked node-pair, the other node in that pair assigned voltages such as (see above) "$v_1 - 20$" or "$v_4 + 3v_x$".
6. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
7. Enjoy the thrill of ending the consideration of each node with the powerful "$= 0$"
8. Add one additional equation for each dependent source specification if necessary

   Circuit analysis is now complete! But you may be asked to:

9. Invoke the power of algebra to solve for every assigned variable in the resulting system of equations.
10. Answer whatever questions are asked about the circuit using the solved values of knowledge of $v_a, v_b, \ldots$.

Homework, Ignore the authors directions. Do these as Node Equation problems.
Chapter 3 # 18, 20, 32, 41, 44, 55, 72

3.7 In the circuit of Fig. 3.49, current i_1 is:

(a) 4 A (b) 3 A (c) 2 A (d) 1 A

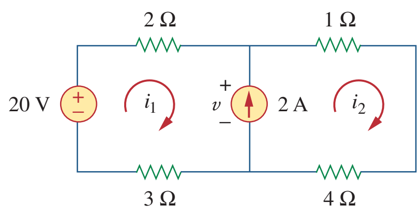

Figure 3.49
For Review Questions 3.7 and 3.8.

3.8 The voltage v across the current source in the circuit of Fig. 3.49 is:

(a) 20 V (b) 15 V (c) 10 V (d) 5 V

3.9 The *PSpice* part name for a current-controlled voltage source is:

(a) EX (b) FX (c) HX (d) GX

3.10 Which of the following statements are not true of the pseudocomponent IPROBE:

(a) It must be connected in series.
(b) It plots the branch current.
(c) It displays the current through the branch in which it is connected.
(d) It can be used to display voltage by connecting it in parallel.
(e) It is used only for dc analysis.
(f) It does not correspond to a particular circuit element.

Answers: 3.1a, 3.2c, 3.3a, 3.4c, 3.5c, 3.6a, 3.7d, 3.8b, 3.9c, 3.10b,d.

Problems

Sections 3.2 and 3.3 Nodal Analysis

3.1 Using Fig. 3.50, design a problem to help other students better understand nodal analysis.

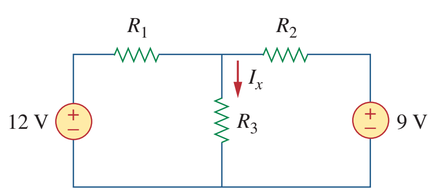

Figure 3.50
For Prob. 3.1 and Prob. 3.39.

3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2.

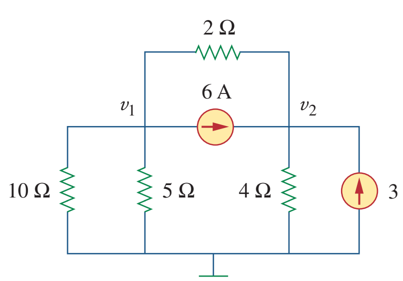

Figure 3.51
For Prob. 3.2.

3.3 Find the currents I_1 through I_4 and the voltage v_o in the circuit of Fig. 3.52.

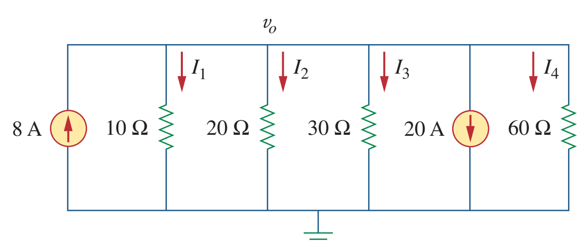

Figure 3.52
For Prob. 3.3.

3.4 Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4.

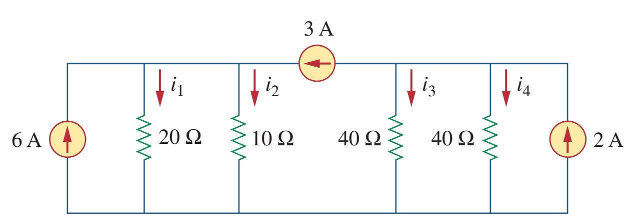

Figure 3.53
For Prob. 3.4.

3.5 Obtain v_o in the circuit of Fig. 3.54.

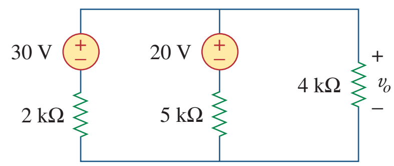

Figure 3.54
For Prob. 3.5.

3.6 Solve for V_1 in the circuit of Fig. 3.55 using nodal analysis.

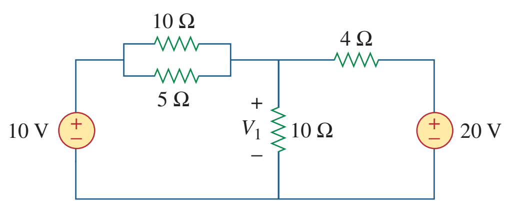

Figure 3.55
For Prob. 3.6.

3.7 Apply nodal analysis to solve for V_x in the circuit of Fig. 3.56.

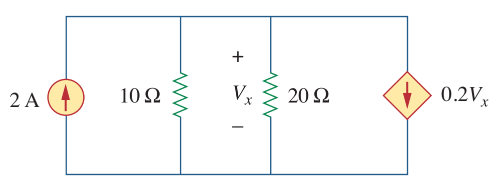

Figure 3.56
For Prob. 3.7.

3.8 Using nodal analysis, find v_o in the circuit of Fig. 3.57.

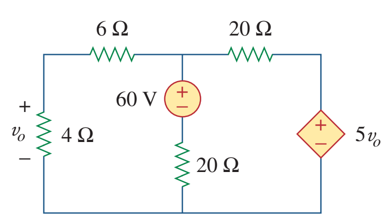

Figure 3.57
For Prob. 3.8 and Prob. 3.37.

3.9 Determine I_b in the circuit in Fig. 3.58 using nodal analysis.

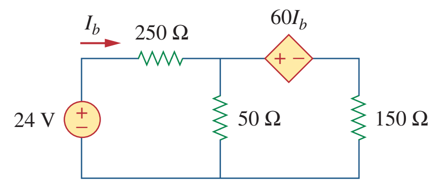

Figure 3.58
For Prob. 3.9.

3.10 Find I_o in the circuit of Fig. 3.59.

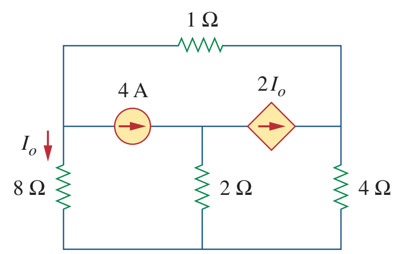

Figure 3.59
For Prob. 3.10.

3.11 Find V_o and the power dissipated in all the resistors in the circuit of Fig. 3.60.

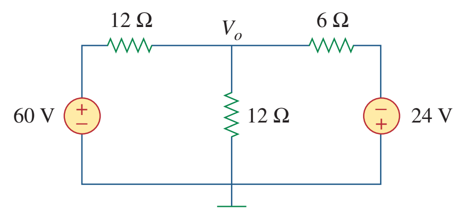

Figure 3.60
For Prob. 3.11.

3.12 Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

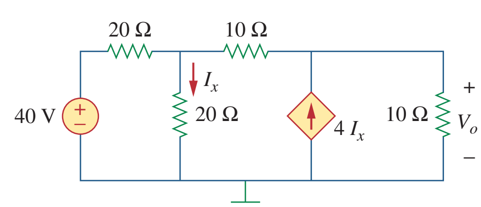

Figure 3.61
For Prob. 3.12.

3.13 Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

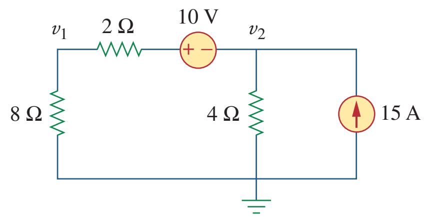

Figure 3.62
For Prob. 3.13.

3.14 Using nodal analysis, find v_o in the circuit of Fig. 3.63.

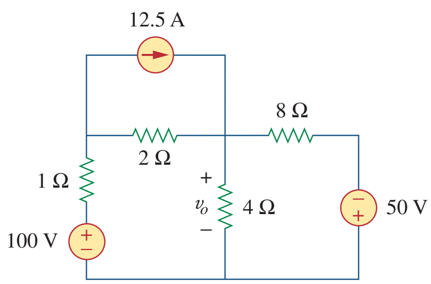

Figure 3.63
For Prob. 3.14.

3.15 Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

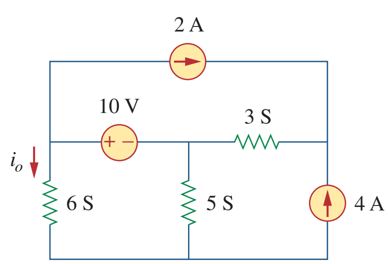

Figure 3.64
For Prob. 3.15.

3.16 Determine voltages v_1 through v_3 in the circuit of Fig. 3.65 using nodal analysis.

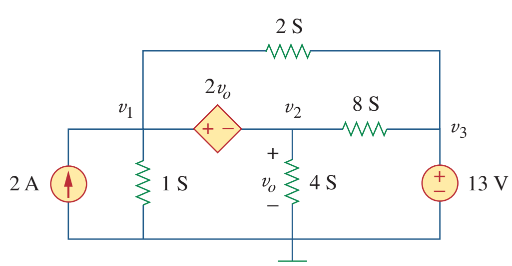

Figure 3.65
For Prob. 3.16.

3.17 Using nodal analysis, find current i_o in the circuit of Fig. 3.66.

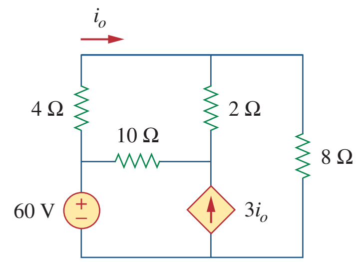

Figure 3.66
For Prob. 3.17.

3.18 Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

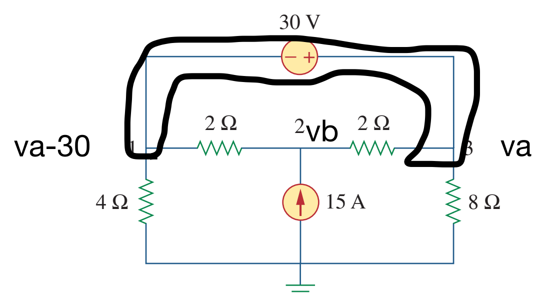

Figure 3.67
For Prob. 3.18.

va: (va-30)/4+((va-30)

3.19 Use nodal analysis to find v_1, v_2, and v_3 in the circuit of Fig. 3.68.

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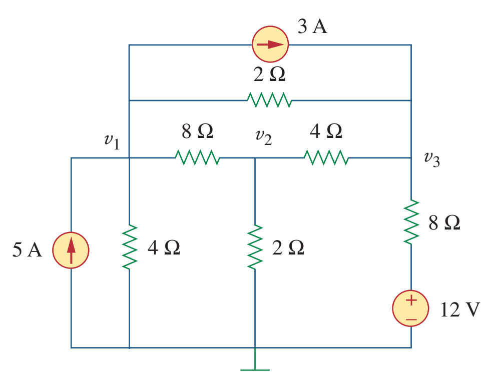

Figure 3.68
For Prob. 3.19.

3.20 For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

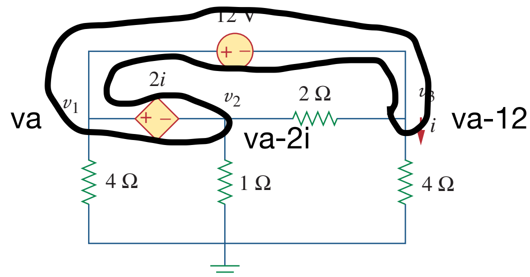

Figure 3.69
For Prob. 3.20.

va: va/4+(va-2i)/1+(va-12)/4=0

i: (va-12)/4-i=0

3.21 For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

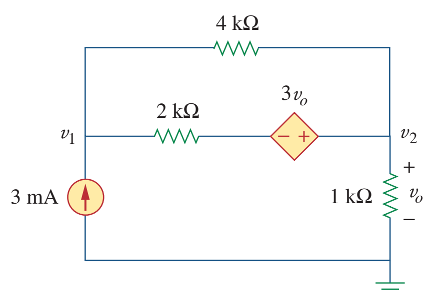

Figure 3.70
For Prob. 3.21.

3.22 Determine v_1 and v_2 in the circuit of Fig. 3.71.

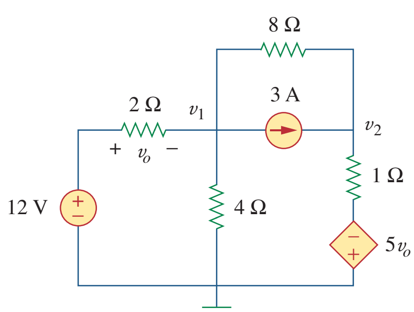

Figure 3.71
For Prob. 3.22.

3.23 Use nodal analysis to find V_o in the circuit of Fig. 3.72.

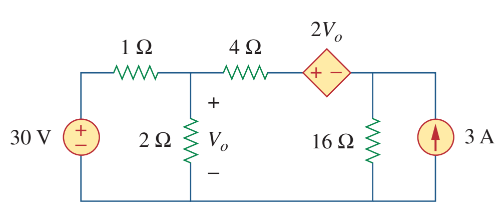

Figure 3.72
For Prob. 3.23.

3.24 Use nodal analysis and *MATLAB* to find V_o in the circuit of Fig. 3.73.

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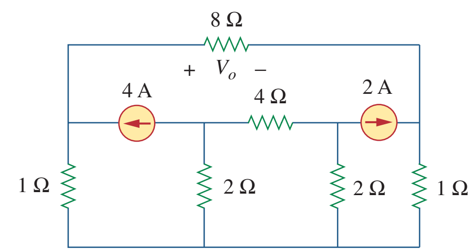

Figure 3.73
For Prob. 3.24.

3.25 Use nodal analysis along with *MATLAB* to determine the node voltages in Fig. 3.74.

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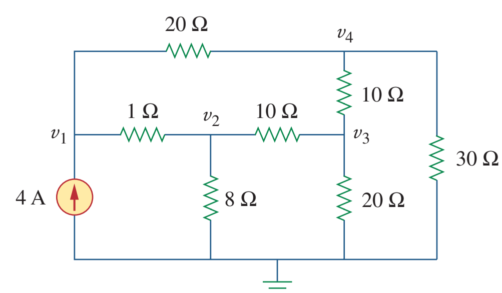

Figure 3.74
For Prob. 3.25.

3.26 Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.75.

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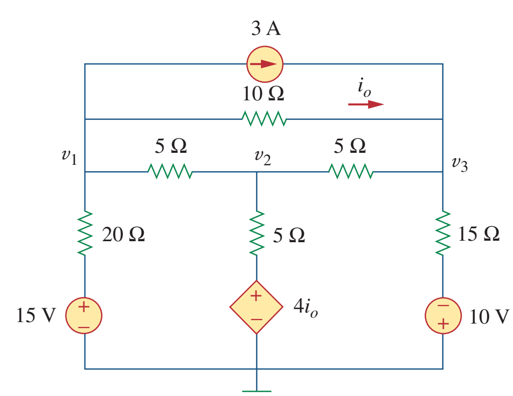

Figure 3.75
For Prob. 3.26.

***3.27** Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit of Fig. 3.76.

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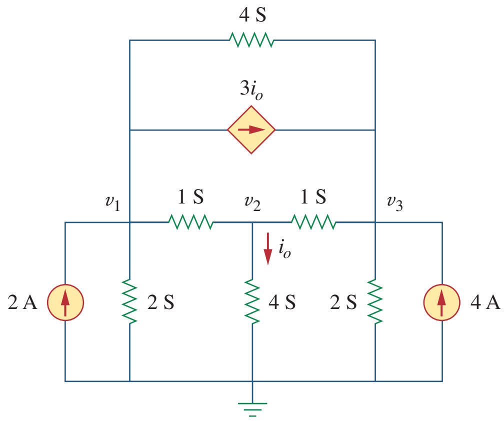

Figure 3.76
For Prob. 3.27.

***3.28** Use *MATLAB* to find the voltages at nodes *a*, *b*, *c*, and *d* in the circuit of Fig. 3.77.

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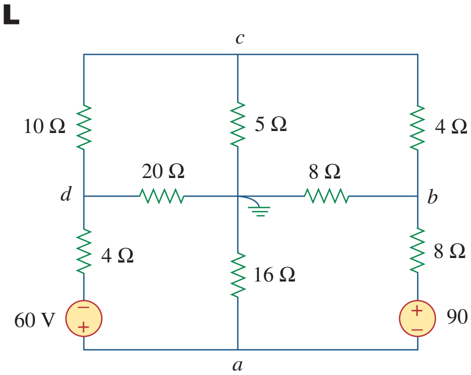

Figure 3.77
For Prob. 3.28.

3.29 Use *MATLAB* to solve for the node voltages in the circuit of Fig. 3.78.

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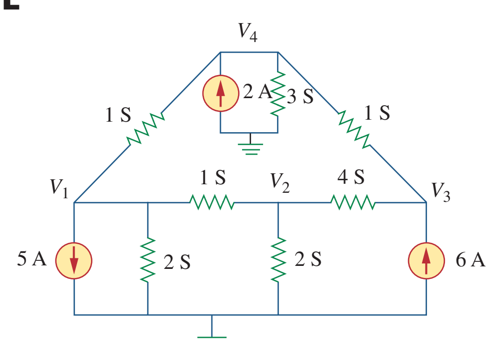

Figure 3.78
For Prob. 3.29.

3.30 Using nodal analysis, find v_o and i_o in the circuit of Fig. 3.79.

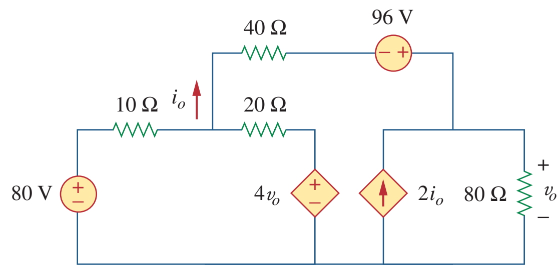

Figure 3.79
For Prob. 3.30.

3.31 Find the node voltages for the circuit in Fig. 3.80.

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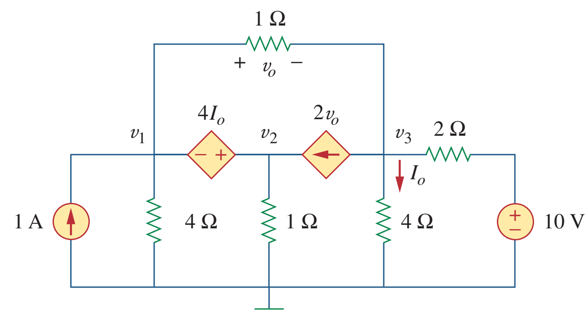


Figure 3.80
For Prob. 3.31.

3.32 Obtain the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.81.

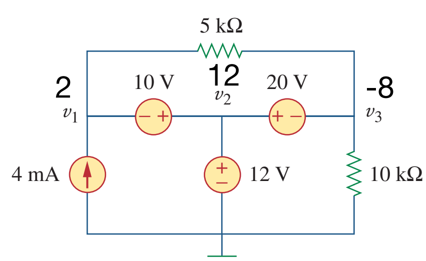


Figure 3.81
For Prob. 3.32.

* An asterisk indicates a challenging problem.

Sections 3.4 and 3.5 Mesh Analysis

3.33 Which of the circuits in Fig. 3.82 is planar? For the planar circuit, redraw the circuits with no crossing branches.

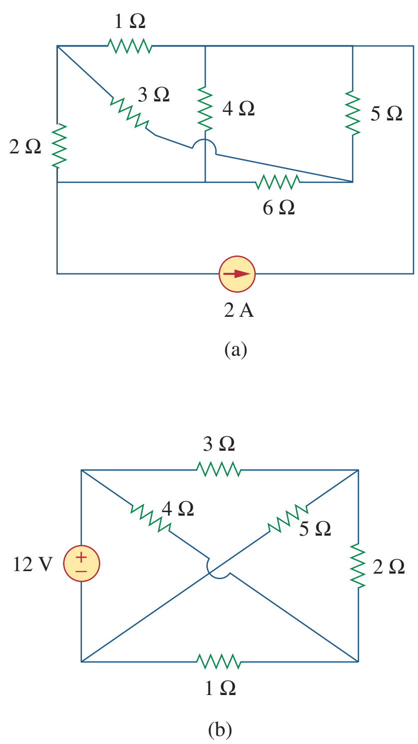
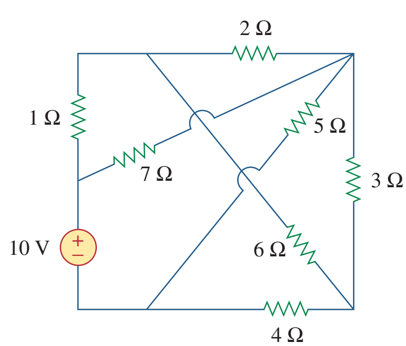
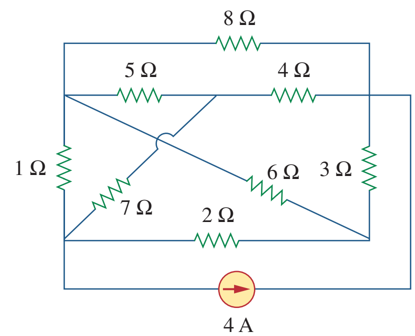


Figure 3.82
For Prob. 3.33.

3.34 Determine which of the circuits in Fig. 3.83 is planar and redraw it with no crossing branches.



(a)



(b)

Figure 3.83
For Prob. 3.34.

3.35 Rework Prob. 3.5 using mesh analysis.

3.36 Use mesh analysis to obtain i_1, i_2, and i_3 in the circuit in Fig. 3.84.

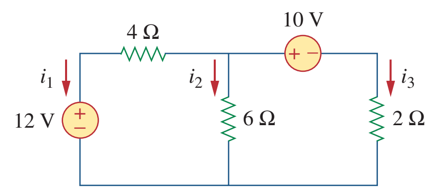


Figure 3.84
For Prob. 3.36.

3.37 Solve Prob. 3.8 using mesh analysis.

3.38 Apply mesh analysis to the circuit in Fig. 3.85 and obtain I_o.

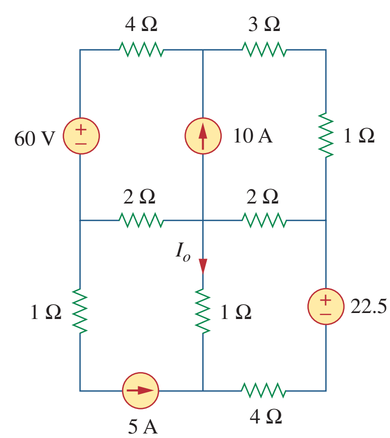


Figure 3.85
For Prob. 3.38.

3.39 Using Fig. 3.50 from Prob. 3.1, design a problem to help other students better understand mesh analysis.

3.40 For the bridge network in Fig. 3.86, find i_o using mesh analysis.

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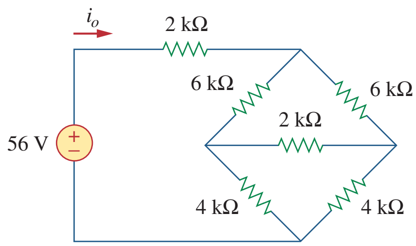

Figure 3.86
For Prob. 3.40.

3.41 Apply mesh analysis to find i in Fig. 3.87.

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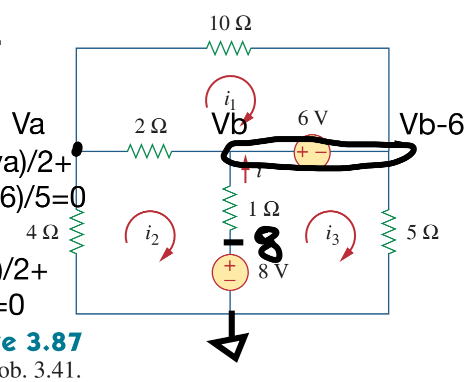

vb: (vb-8)/1+(vb-va)/2+ ((vb-6)-va)/10+(vb-6)/5=0

va: va/4+(va-vb)/2+ (va-(vb-6))/10=0

Figure 3.87
For Prob. 3.41.

3.42 Using Fig. 3.88, design a problem to help students better understand mesh analysis using matrices.

e2d

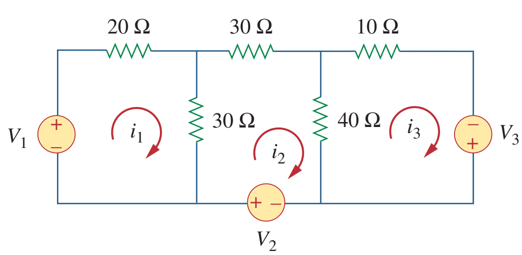

Figure 3.88
For Prob. 3.42.

3.43 Use mesh analysis to find v_{ab} and i_o in the circuit of Fig. 3.89.

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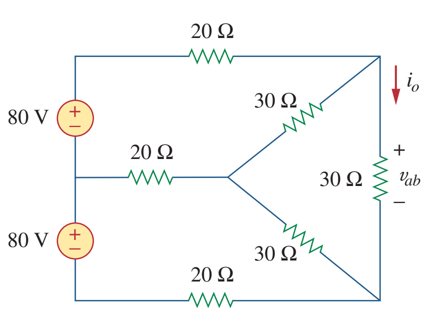

Figure 3.89
For Prob. 3.43.

3.44 Use mesh analysis to obtain i_o in the circuit of Fig. 3.90.

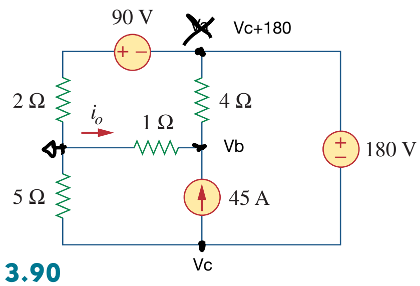

Figure 3.90
For Prob. 3.44.

Vb: -45+(vb-(vc+180))/4+vb/1=0

Vc: 45+vc/5+((vc+180)-(-90))/2+((vc+180)-vb)/4=0

3.45 Find current i in the circuit of Fig. 3.91.

ML

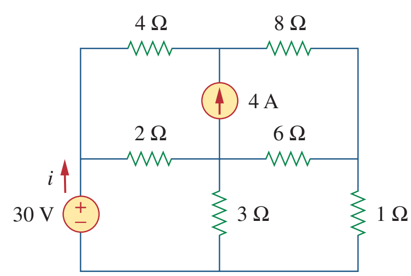

Figure 3.91
For Prob. 3.45.

3.46 Calculate the mesh currents i_1 and i_2 in Fig. 3.92.

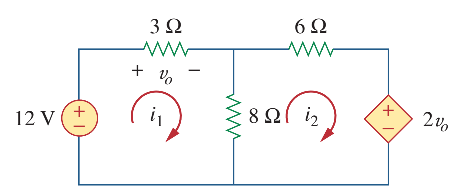

Figure 3.92
For Prob. 3.46.

3.47 Rework Prob. 3.19 using mesh analysis.

ML

3.48 Determine the current through the 10-kΩ resistor in the circuit of Fig. 3.93 using mesh analysis.

ML

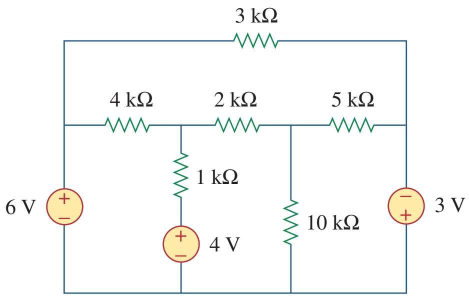

Figure 3.93
For Prob. 3.48.

3.49 Find v_o and i_o in the circuit of Fig. 3.94.

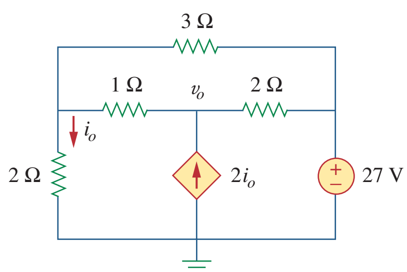

Figure 3.94
For Prob. 3.49.

3.50 Use mesh analysis to find the current i_o in the circuit of Fig. 3.95.

ML

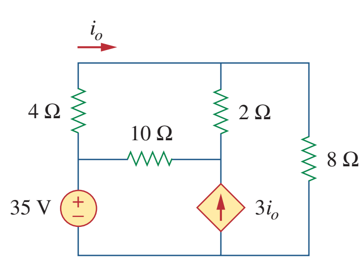

Figure 3.95
For Prob. 3.50.

3.51 Apply mesh analysis to find v_o in the circuit of Fig. 3.96.

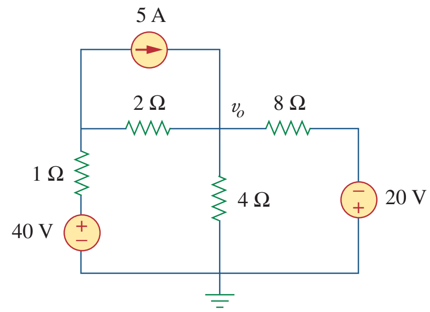

Figure 3.96
For Prob. 3.51.

3.52 Use mesh analysis to find i_1, i_2, and i_3 in the circuit of Fig. 3.97.

ML

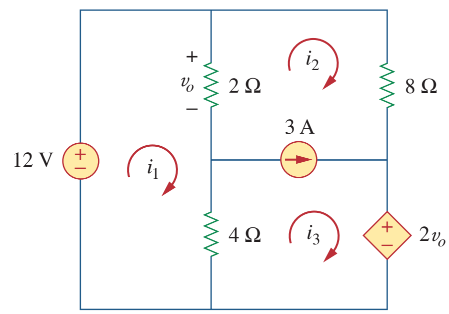

Figure 3.97
For Prob. 3.52.

3.53 Find the mesh currents in the circuit of Fig. 3.98 using *MATLAB*.

ML

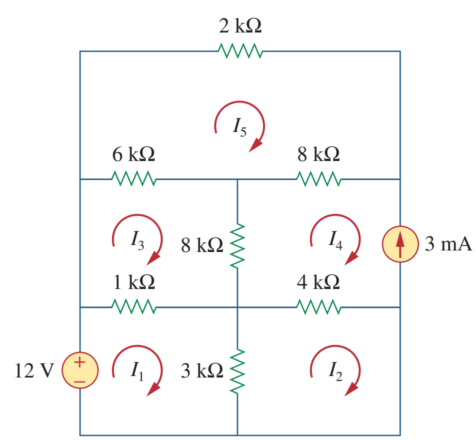

Figure 3.98
For Prob. 3.53.

3.54 Find the mesh currents i_1, i_2, and i_3 in the circuit in Fig. 3.99.

ML

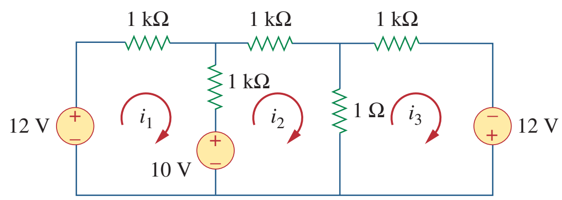

Figure 3.99
For Prob. 3.54.

***3.55** In the circuit of Fig. 3.100, solve for I_1, I_2, and I_3.

ML

vb: -4+(vb+10)/6+1+(vb-vd)/2=0

vd: 4+(vb+8)/12+vd/4+(vd-vb)/2=0

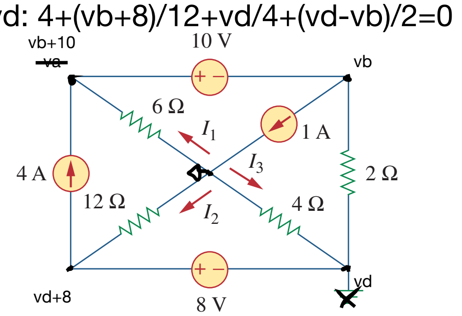

Figure 3.100
For Prob. 3.55.

vb: (vb

3.56 Determine v_1 and v_2 in the circuit of Fig. 3.101.

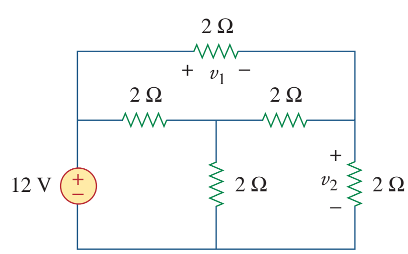

Figure 3.101
For Prob. 3.56.

3.57 In the circuit of Fig. 3.102, find the values of R, V_1, and V_2 given that $i_o = 15$ mA.

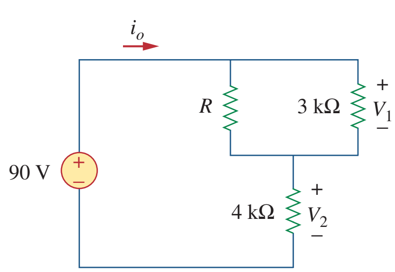

Figure 3.102
For Prob. 3.57.

3.58 Find i_1, i_2, and i_3 in the circuit of Fig. 3.103.

ML

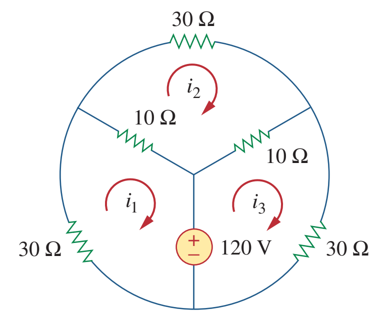

Figure 3.103
For Prob. 3.58.

3.59 Rework Prob. 3.30 using mesh analysis.

ML

3.60 Calculate the power dissipated in each resistor in the circuit of Fig. 3.104.

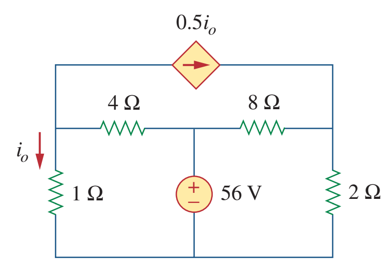

Figure 3.104
For Prob. 3.60.

3.61 Calculate the current gain i_o/i_s in the circuit of Fig. 3.105.

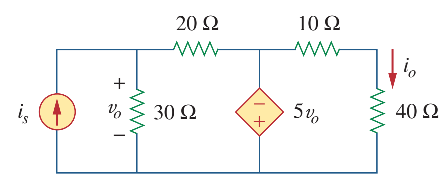

Figure 3.105
For Prob. 3.61.

3.62 Find the mesh currents i_1, i_2, and i_3 in the network of Fig. 3.106.

ML

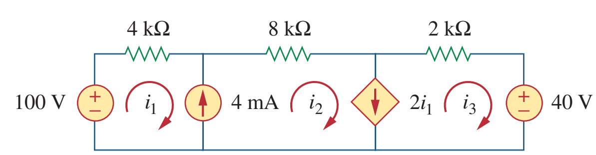

Figure 3.106
For Prob. 3.62.

3.63 Find v_x and i_x in the circuit shown in Fig. 3.107.

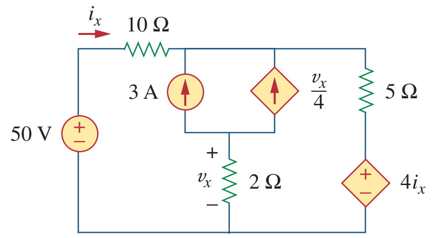

Figure 3.107
For Prob. 3.63.

3.64 Find v_o and i_o in the circuit of Fig. 3.108.

ML

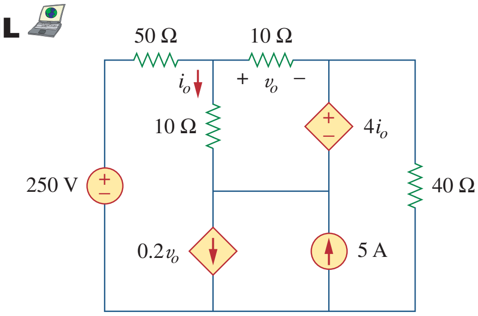

Figure 3.108
For Prob. 3.64.

3.65 Use *MATLAB* to solve for the mesh currents in the circuit of Fig. 3.109.

ML

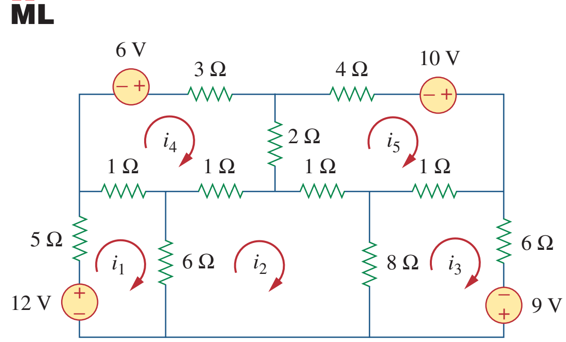

Figure 3.109
For Prob. 3.65.

3.66 Write a set of mesh equations for the circuit in Fig. 3.110. Use *MATLAB* to determine the mesh currents.

ML

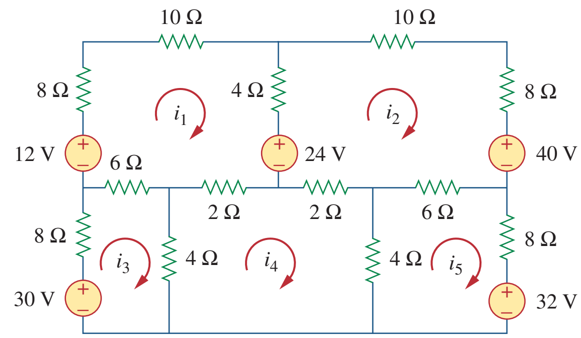

Figure 3.110
For Prob. 3.66.

Section 3.6 Nodal and Mesh Analyses by Inspection

3.67 Obtain the node-voltage equations for the circuit in Fig. 3.111 by inspection. Then solve for V_o.

ML

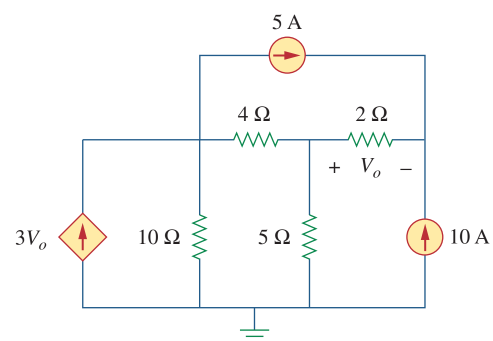

Figure 3.111
For Prob. 3.67.

3.68 Using Fig. 3.112, design a problem, to solve for V_o, to help other students better understand nodal analysis. Try your best to come up with values to make the calculations easier.

e2d

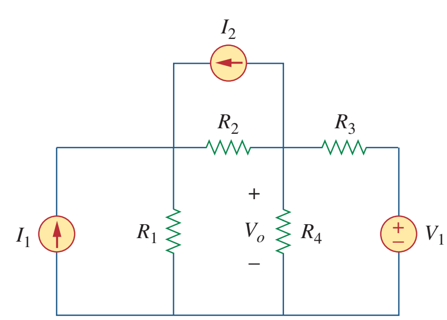

Figure 3.112
For Prob. 3.68.

3.69 For the circuit shown in Fig. 3.113, write the node-voltage equations by inspection.

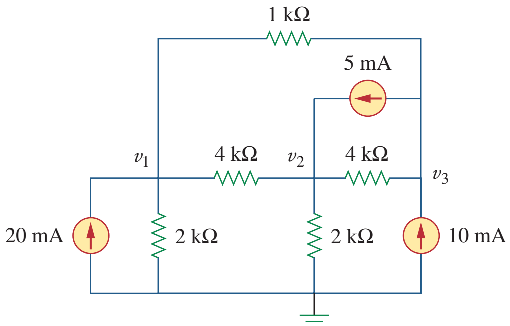

Figure 3.113
For Prob. 3.69.

3.70 Write the node-voltage equations by inspection and then determine values of V_1 and V_2 in the circuit of Fig. 3.114.

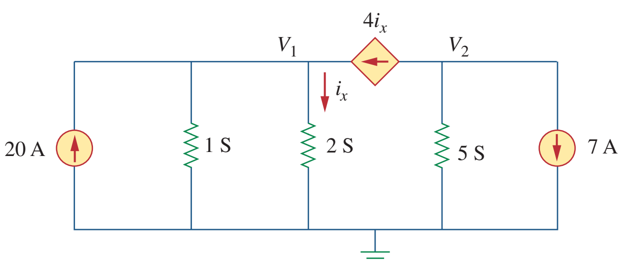

Figure 3.114
For Prob. 3.70.

3.71 Write the mesh-current equations for the circuit in Fig. 3.115. Next, determine the values of i_1, i_2, and i_3.

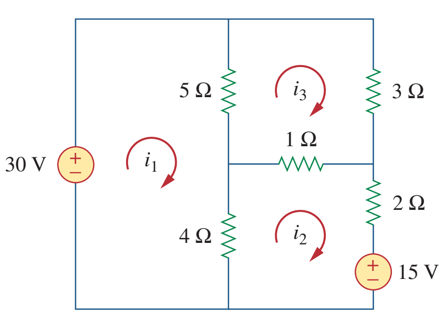

Figure 3.115
For Prob. 3.71.

3.72 By inspection, write the mesh-current equations for the circuit in Fig. 3.116.

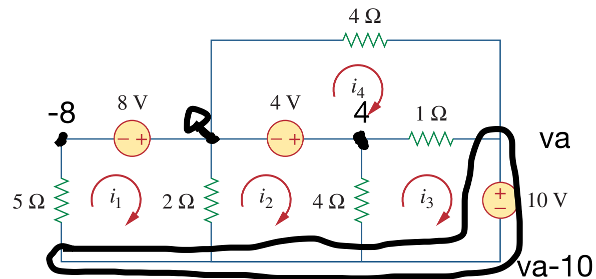

Figure 3.116
For Prob. 3.72.

va:va/4+(va-4)/1+((va-10)-4)/4+((va-10)+8)/5+(va-10)/2=0

3.73 Write the mesh-current equations for the circuit in Fig. 3.117.

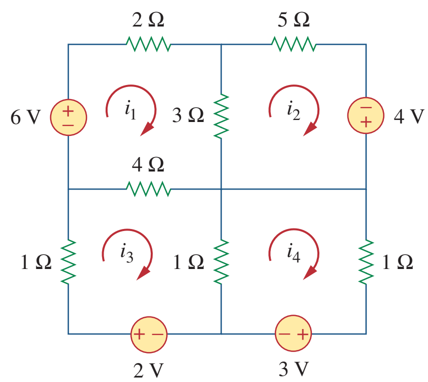

Figure 3.117
For Prob. 3.73.

3.74 By inspection, obtain the mesh-current equations for the circuit in Fig. 3.118.

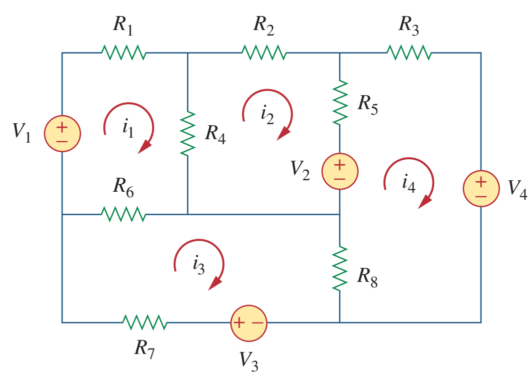

Figure 3.118
For Prob. 3.74.

Section 3.8 Circuit Analysis with *PSpice* or *MultiSim*

3.75 Use *PSpice* or *MultiSim* to solve Prob. 3.58.

3.76 Use *PSpice* or *MultiSim* to solve Prob. 3.27.

3.77 Solve for V_1 and V_2 in the circuit of Fig. 3.119 using *PSpice* or *MultiSim*.

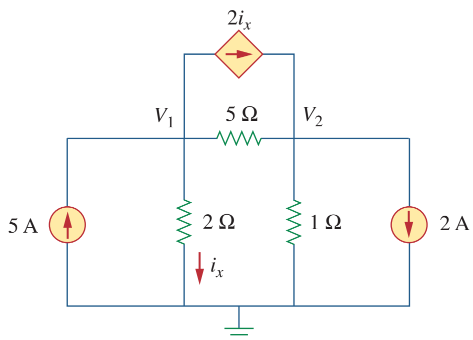

Figure 3.119
For Prob. 3.77.

3.78 Solve Prob. 3.20 using *PSpice* or *MultiSim*.

3.79 Rework Prob. 3.28 using *PSpice* or *MultiSim*.

3.80 Find the nodal voltages v_1 through v_4 in the circuit of Fig. 3.120 using *PSpice* or *MultiSim*.

ML

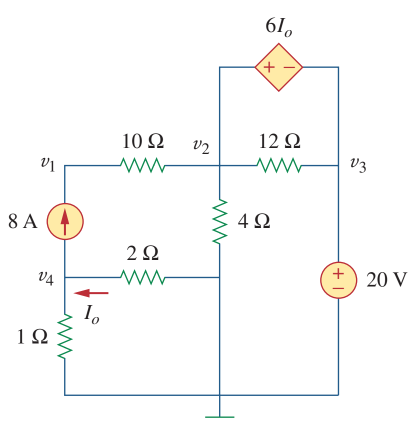

Figure 3.120
For Prob. 3.80.

3.81 Use *PSpice* or *MultiSim* to solve the problem in Example 3.4.

3.82 If the Schematics Netlist for a network is as follows, draw the network.

```
R_R1   1  2  2K
R_R2   2  0  4K
R_R3   3  0  8K
R_R4   3  4  6K
R_R5   1  3  3K
V_VS   4  0  DC     100
I_IS   0  1  DC     4
F_F1   1  3  VF_F1  2
VF_F1  5  0  0V
E_E1   3  2  1      3    3
```

3.83 The following program is the Schematics Netlist of a particular circuit. Draw the circuit and determine the voltage at node 2.

```
R_R1  1  2  20
R_R2  2  0  50
R_R3  2  3  70
R_R4  3  0  30
V_VS  1  0  20V
I_IS  2  0  DC  2A
```

Section 3.9 Applications

3.84 Calculate v_o and I_o in the circuit of Fig. 3.121.

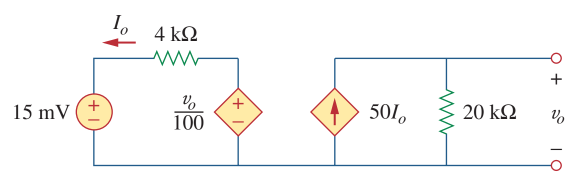

Figure 3.121
For Prob. 3.84.

3.85 An audio amplifier with a resistance of 9 Ω supplies power to a speaker. What should be the resistance of the speaker for maximum power to be delivered?

3.86 For the simplified transistor circuit of Fig. 3.122, calculate the voltage v_o.

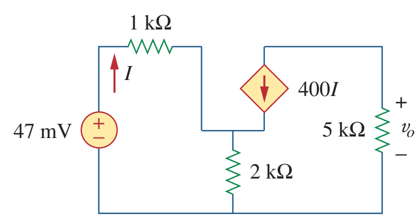

Figure 3.122
For Prob. 3.86.

3.87 For the circuit in Fig. 3.123, find the gain v_o/v_s.

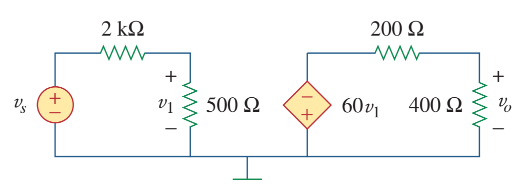

Figure 3.123
For Prob. 3.87.

***3.88** Determine the gain v_o/v_s of the transistor amplifier circuit in Fig. 3.124.

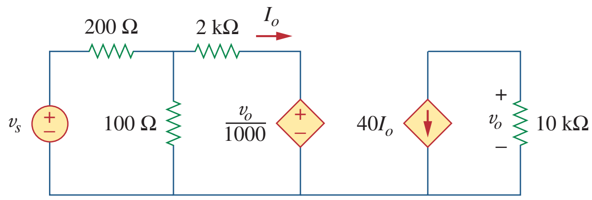

Figure 3.124
For Prob. 3.88.

3.89 For the transistor circuit shown in Fig. 3.125, find I_B and V_{CE}. Let $\beta = 100$, and $V_{BE} = 0.7$ V.

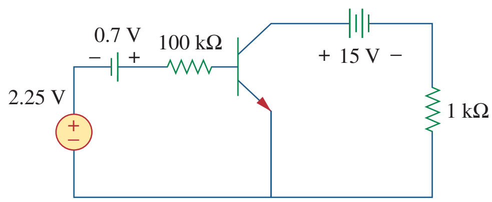

Figure 3.125
For Prob. 3.89.

3.90 Calculate v_s for the transistor in Fig. 3.126 given that $v_o = 4$ V, $\beta = 150$, $V_{BE} = 0.7$ V.

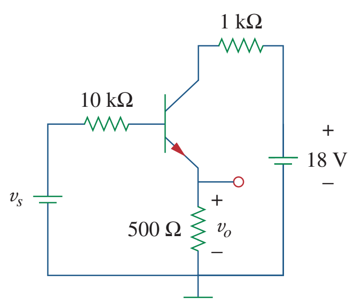

Figure 3.126
For Prob. 3.90.

3.91 For the transistor circuit of Fig. 3.127, find I_B, V_{CE}, and v_o. Take $\beta = 200$, $V_{BE} = 0.7$ V.

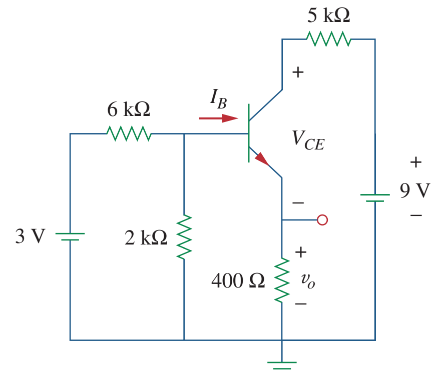

Figure 3.127
For Prob. 3.91.

3.92 Using Fig. 3.128, design a problem to help other students better understand transistors. Make sure you use reasonable numbers!

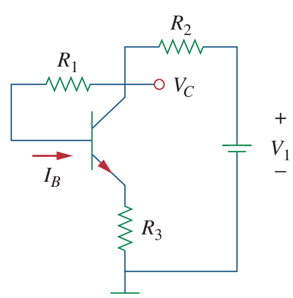

Figure 3.128
For Prob. 3.92.

Comprehensive Problem

***3.93** Rework Example 3.11 with hand calculation.