ISE 2211 • Exam III • 19 February 2021 NAME SOLUTION

The following is real data on daily low temperature in Dayton, Ohio in °F for February 17th, according to https://www.usclimatedata.com/climate/dayton/ohio/united-states/usoh0245 :

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Low Temp	16	37	19.9	21.9	50.0	27.0	12.0	12.2	(no data)	29.1	30.2	28.2

First, determine the sample mean and sample variance of daily low temperature. Include a unit with each answer.

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$\bar{x} = 25.77$ °F (+2)

$n = 11$

$$s^2 = \frac{8577 - \frac{283.5^2}{11}}{10} = 127.0\ (°F)^2$$ (+3)

Write a 95% confidence interval on mean low temperature.

-- Population variance is not given and assumed unknown;

$n < 30 \longrightarrow$ use t-distribution

$\mu : \bar{x} \pm t_{\alpha/2, n-1}\ s/\sqrt{n}$ (+2)

$s = +\sqrt{127.0} = 11.27$

$t_{.025, 10} = 2.228$ (+2) (table)

$\mu : 25.77 \pm 2.228 \cdot 11.27/\sqrt{11}$

$18.20 < \mu < 33.34$ (°F) (+1)

Write a 95% prediction interval on mean low temperature. (+2)

$X_{n+1} : \bar{x} \pm t_{\alpha/2, n-1}\, s\sqrt{1 + \frac{1}{n}}$ (+2)

$25.77 \pm 2.228 \cdot 11.27 \sqrt{1 + \frac{1}{11}}$

$-0.4561 < \mu < 52.00$ (+2) (°F) (+1)

wide range -- lot of variance!

Write an upper 95% confidence bound on the standard deviation of low temperature.

$$\sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha,\,n-1}}$$ (+2)

$$\chi^2_{.95,\,10} = 3.94$$ (+2) (table)

$$\sigma^2 < \frac{10 \cdot 127.0}{3.94}$$

$$\sigma^2 < 322.3$$

$$\sigma < 17.95 \text{ °F}$$ (+1) (+1)

Bonus: This year, it was –1°F the morning of February 17th. In the "comments" field when submitting your exam, write a subjective, qualitative assessment of this temperature using whatever colorful language you feel is necessary to express your feelings adequately.

(+2)