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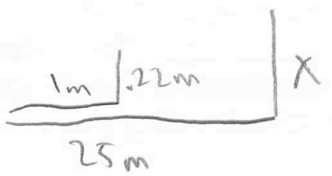
PHY 1110 - Summer 2020 - Study guide #3 - Trigonometry/Vectors

1. When you make one complete rotation you are said to have turned through an angle of ___360___ °. A rotation through ¼ of a complete circle is a turn through an angle of ___90___ °. An angle of 90° is called a ___right___ angle.

2. A triangle is a closed polygon having ___3___ sides. There are ___3___ angles enclosed within a triangle and the sum of these enclosed angles is ___180___ °. A right triangle must have one enclosed angle which is ___90___ °.

3. Plane trigonometry is based upon the properties of similar right triangles. Two right triangles are said to be similar if their corresponding internal angles are ___equal___ even though their corresponding sides are ___different___. Since they are right triangles, one internal angle in each triangle must be ___90___ °.

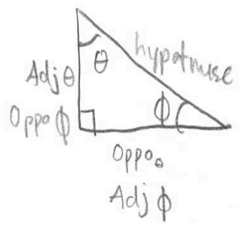
4. Two girl scouts wish to use the method described in the "Handbook for Girls" to find the height of a flag pole at their school. Amy lies on the ground 25 m from the pole. Helen holds a meter stick perpendicular to the ground at a point on a line between Amy's eye and the base of the flag pole. The distance from Amy's eye to the base of the meter stick is 1 m. Amy looks up with her eye nearer to the ground and observes that the top of the pole appears next to the 22 cm mark on the meter stick. The flag pole must be ___5.5 m___ tall. (5.5 m)



$$\frac{1}{25} = \frac{.22}{X}$$

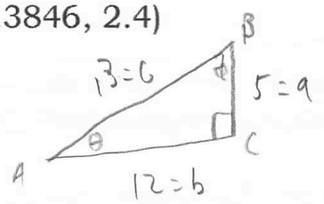
$$X = 5.5$$

5. Since trigonometry is based upon the properties of right triangles, the sides of right triangles are given names. Consider the sketch below. If you are concerned with the angle θ, the side opposite the angle θ is called the ___opposite___, and the side that forms part of the angle θ and is not the longest side of the triangle is called the ___adjacent___. The side opposite the 90° (or right) angle, which is always the longest side of the triangle, is called the ___hypotnuse___. If you are interested in the angle ϕ, the sides called the ___opposite___ and the ___adjacent___ exchange names but the side called the ___hypotnuse___ is still the same.



6. Although plane trigonometry is based upon the properties of similar right triangles, calculations are usually done by making use of the values of the trigonometric functions. Using the names of the sides, the sin θ = __opposite__ / __hypothenuse__, the cos θ = __adjacent__ / __hypothenuse__, and the tan θ = __opposite__ / __adjacent__.

7. In the triangle shown below, sin θ = __0.385__, cos θ = __0.923__, tan θ = __0.417__, sin ϕ = __0.923__, cos ϕ = __0.385__, tan ϕ = __2.40__. (0.3846, 0.9231, 0.4167, 0.9231, 0.3846, 2.4)



$\theta = \sin^{-1}\left(\frac{5}{13}\right) = 22.6°$

$\phi = \sin^{-1}\left(\frac{12}{13}\right) = 67.4°$

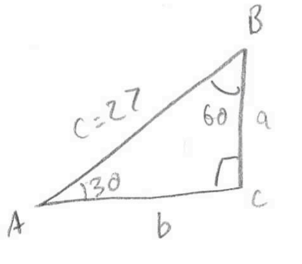
sin θ = 0.385　　sin ϕ = 0.923
cos θ = 0.923　　cos ϕ = 0.385
tan θ = 0.417　　tan ϕ = 2.40

8. You will normally use your scientific calculator to find the values of the trigonometric functions. Use your calculator to find the numerical values of the following functions: sin 20° = __0.342__, cos 35° = __0.819__, tan 77° = __4.33__. Now use the table inside of the back cover of your textbook to check the values that you found with your calculator. (0.3420, 0.8192, 4.3315)

9. Sometimes you will find an angle by "back calculating" from the value of a trigonometric function. If the sin 60° = 0.866, then the $\sin^{-1}$ 0.866 = 60°. (Pronounce $\sin^{-1}$ 0.866 as "arc sine of 0.866".) Find the following using your calculator: $\sin^{-1}$ 0.982 = __79.1°__, $\cos^{-1}$ 0.996 = __5.13°__, $\tan^{-1}$ 0.488 = __26.0°__. The values of these functions listed in the table inside the back cover of your textbook are (a) in good agreement with, (b) close to, or (c) in disagreement with the values found using your calculator. (79.11°, 5.126°, 26.01°)

10. Sometimes you will find the length of one or two of the sides of a right triangle when you know an angle. Consider the triangle below.

a) Calculate the length of a using the sin 30° (a = __13.5°__).
b) Calculate the length of a using the cos 60° (a = __13.5°__).
c) Calculate the length of b using a trigonometric function (b = __23.38__).
d) According to the Pythagorean Theorem, $a^2 + b^2 = c^2$ for all right triangles. Use your calculated values of a and b to find the length of c. (c = __27.0__)
e) This value of c is (1) exactly equal to, (2) close to, or (3) not equal to the value given in the sketch. (13.50°, 13.50°, 23.38, 27.00)



$\sin(30) = \frac{a}{27}$　　$\cos(60) = \frac{a}{27}$

a = 13.5　　a = 13.5

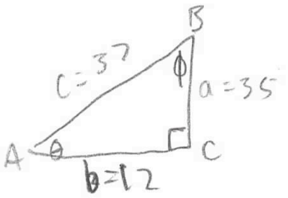
$13.5^2 + b^2 = 27^2$

$b = \sqrt{27^2 - 13.5^2} = 23.4$

$c = \sqrt{13.5^2 + 23.4^2} = 27.01$

(rounding)

11. In the right triangle below, the lengths of all three sides age given. Using the lengths given, sin ϕ = 12/37 and $\sin^{-1}\phi$ = 18.9 °. Using the lengths given, tan ϕ = 12/35 and $\tan^{-1}\phi$ = 18.9 °. These two values of ϕ (1)agree closely, (2) are fairly close, or (3) disagree with one another. Using the lengths given, sin θ = 35/37 which means that θ = 71.1 °. θ + ϕ = 90 ° which is (1) consistent, or (2)inconsistent with the value of 90° which is expected.
(0.3243, 18.92°, 0.3429, 18.92°, 0.9459, 71.08°, 89.995°)



$\sin^{-1}\left(\frac{12}{37}\right) = 18.9°$

$\tan^{-1}\left(\frac{12}{35}\right) = 18.9°$

$\sin^{-1}(35/37) = 71.1°$

$18.9 + 71.1 = 90°$

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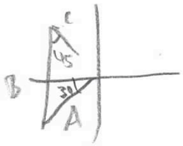
12. Consider three displacement vectors:
A = 22 m 30° South of West **B** = 27 m North **C** = 9 m Southeast

a) Find the vector sum, **R** = **A** + **B** + **C**, using graphical methods. (R = 20.1 m and θ = 37.2° N of W. Clearly specify what direction the angle is measured with respect to.) (approximately 19 m and 40° N of W)



A: $\sin(30) = \frac{Y}{22}$ $\cos(30) = \frac{X}{30}$

$Y = 11$ $X = 19.1$

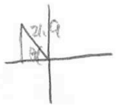
C: $\sin(45) = \frac{X}{9}$ $\cos(45) = \frac{Y}{9}$

$X = 6.36$ $Y = 6.36$

$X = -19.1 + 6.36 = -12.7$

$Y = -11 + 27 - 6.36 = 9.64$

$R = \sqrt{(12.7)^2 + 9.64^2} = 20.1$



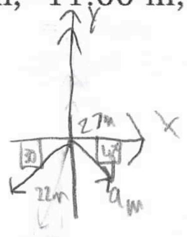
$\theta = \tan^{-1}\left(\frac{9.64}{12.7}\right) = 37.2°$

b) Sketch a Cartesian coordinate system with North in the +y direction and East in the +x direction. Sketch the vectors **A**, **B**, and **C** on this coordinate system with their tails at the origin. Then calculate the x and y components of each of these vectors.

A_x = -19.05 B_x = 0 C_x = 6.36

A_y = -11.00 B_y = 27.00 C_y = -6.36

(-19.05 m, -11.00 m; 0 m, +27.00 m; +6.36 m, -6.36 m)



$A_x = -A\cos 30° = -19.05$ m

$A_y = -A\sin(30) = -11$ m

$B_x = B\cos 90 = 0$ m

$B_y = B\sin 90 = 27$ m

$C_x = C\cos 45 = 6.36$ m

$C_y = -C\sin 45 = -6.36$ m

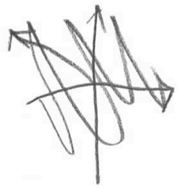
c) Find the vector **R**, where **R** = **A** + **B** + **C**, using the method of components. (R_x = -12.69 m, R_y = 9.64 m, R = 15.94 m and θ = 37.22° N of W.) This is in (a)poor, (b) good, or (c) excellent agreement with the values found in part (a). (15.94 m, 37.22° N of W)

$R_x = 6.36 - 19.05 = -12.69\,m$

$R_y = 27 - 11 - 6.36 = 9.64\,m$

$R = \sqrt{(-12.69)^2 + 9.64^2} = 15.94$

$\theta = \tan^{-1}\left(\frac{9.64}{12.69}\right) = 37.22^\circ$



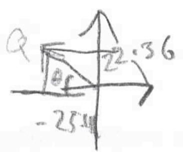
d) Find the vector **Q**, where **Q** = **A** + **B** - **C**, using the method of components. (Q = 33.85 m and θ = 41.35° N of W. Make a sketch of your results. (33.85 m, 41.35° N of W)

$Q_x = -19.05 - 25.41 = -25.41\,m$

$Q_y = 27 + 6.36 - 11 = 22.36\,m$

$Q = \sqrt{(-25.41)^2 + 22.36^2} = 33.85\,m$

$\theta = \tan^{-1}\left(\frac{22.36}{25.41}\right) = 41.35^\circ$



e) Find the vector **P**, where **P** = 5 **A** - 2 **B**, using the method of components. (P_x = -95.25 m, P_y = -109 m, P = 144.75 m and θ = 48.85° S of W. Make a sketch of your results. (144.75 m, 48.85° S of W)

$P_x = 5(-19.05) - 2(0) = -95.25\,m$

$P_y = 5(-11) - 2(27) = -109\,m$

$P = \sqrt{(-95.25)^2 + (-109)^2} = 144.75$

$\theta = \tan^{-1}\left(\frac{109}{95.25}\right) = 48.85^\circ$ S of W