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SOLUTION

Wait times in the drive-thru at Skyline Chili seem to differ considerably between their locations at 1223 E. Dayton-Yellow Springs Road in Fairborn and 2805 Centre Drive in Beavercreek, with the Fairborn location (#1) always seeming much worse than Beavercreek (#2). After an infuriating experience waiting close to twenty minutes for a regular five-way at the Fairborn location one Tuesday at 7pm, Joe Tritschler decided to conduct a rigorous controlled engineering experiment to determine if the two locations differ significantly in terms of drive-thru wait-times. The same regular five-way was ordered every time, randomized during the 5:00 – 8:00pm dinner cycle on different days, with the following test results (all times in minutes):

$$\begin{aligned} n_1 &= 8 & n_2 &= 5 \\ \bar{x}_1 &= 8.48 & \bar{x}_2 &= 5.67 \\ s_1 &= 4.80 & s_2 &= 2.12 \end{aligned}$$

Test the following hypotheses on the difference in mean wait times in the drive-thru at Skyline using the fixed-significance level approach at $\alpha = 0.05$: Population variances are unknown and assumed unequal.

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 \neq 0$$

State your final conclusion in terms of the original locations and support your process with a sketch.

Unknown and unequal variances \rightarrow need v .

$$\frac{s_1^2}{n_1} = \frac{4.80^2}{8} = 2.88$$

$$\frac{s_2^2}{n_2} = \frac{2.12^2}{5} = 0.89888$$

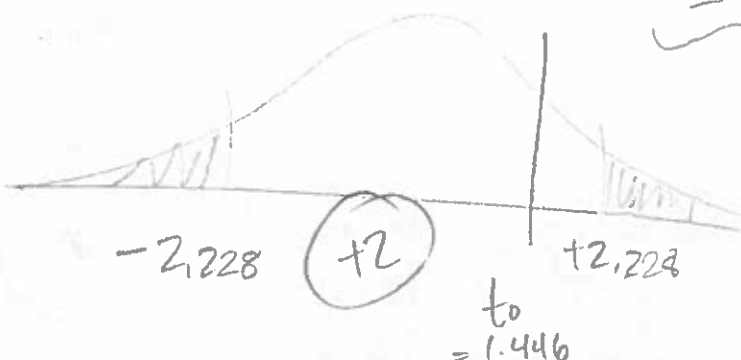
$$v = \frac{(2.88 + 0.89888)^2}{\frac{2.88^2}{7} + \frac{0.89888^2}{4}} = 10.29 \quad (+1)$$

$$= 10.29 \quad (+1)$$

round down to $v = 10$ (+1)

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{8.48 - 5.67}{\sqrt{2.88 + 0.89888}} = 1.446 \quad (+1)$$

Critical values: $\pm t_{\alpha/2, v} = \pm t_{0.025, 10} = \pm 2.228 \quad (+1)$



$$t_0 < +t_{0.025, 10} \quad (+1)$$

fail to reject H_0 (+1)

the two locations do not significantly differ! (+1)

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Write a 95% confidence interval on the ratio of population standard deviations of drive-thru wait times and use it to test the null hypothesis that they are equal. State your final conclusion in terms of the original locations and support your process with a sketch. Include a unit with your C.I.

$$f_{\alpha/2, n_2-1, n_1-1} = f_{.025, 4, 7} = \underline{5.52} \quad (+1)$$

$$f_{1-\alpha/2, n_2-1, n_1-1} = \frac{1}{f_{\alpha/2, n_1-1, n_2-1}} = \frac{1}{f_{.025, 7, 4}} = \frac{1}{9.07} = 0.1103 \quad (+1)$$

$$\frac{s_1^2}{s_2^2} = \frac{4.80^2}{2.12^2} = \underline{5.126} \quad (+1)$$

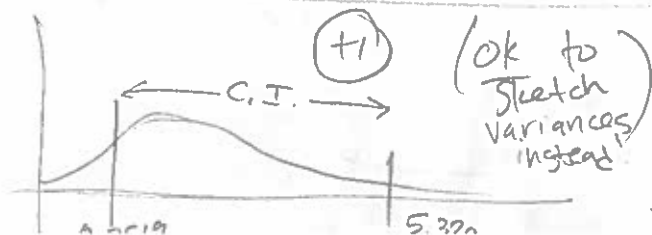
C.I.:

$$5.126 \cdot 0.1103 < \frac{\sigma_1^2}{\sigma_2^2} < 5.126 \cdot 5.52$$

$$0.5654 < \frac{\sigma_1^2}{\sigma_2^2} < 28.30 \quad (+2)$$

$$0.7519 < \frac{\sigma_1}{\sigma_2} < 5.320 \quad (+1)$$

min.
min.
(+1)



-- C.I. contains 1;
oo fail to reject H_0 (+2)
-- locations do not significantly differ. (+1)

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Realizing that a different approach might help mitigate confounding variables, Joe Tritschler decided to conduct an experiment in which he and his wife visited the two locations at exactly the same time to reduce the influence of time and day of the week on drive-thru wait times.* This resulted in the following data:

Visit #	Location #1	Location #2	differences
1	5.67	3.81	1.86
2	3.42	4.68	-1.26
3	2.25	6.62	-4.37
4	16.4	2.49	13.91
5	8.23	3.15	5.08

What type of test is this?

paired t-test

(+1)

Test the following hypotheses and state whether there is a significant difference in drive-thru wait times between the two Skyline Chili locations using the p-value approach. State your final conclusion with regards to the $\alpha = 0.05$ level of significance.

$$H_0: \mu_D = \Delta_0 = 0$$

$$H_1: \mu_D \neq 0$$

$$\text{Hint: } s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

$$\bar{d} = 3.044$$

(+1)

$$s_d = 7.020$$

(+2)

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}} = \frac{3.044}{7.020 / \sqrt{5}} = 0.9696$$

(+1)

$$t_{.25, 4} = 0.741$$

$$t_{.10, 4} = 1.533$$

(+1)

$$0.10 < \frac{P}{2} < 0.25$$

(+1)

$$0.20 < P < 0.50$$

(+1)

$$P < 0.05 \quad \therefore \text{fail to reject } H_0$$

*Joe's wife would never be a party to anything this moronic. In fact, it was her sitting in the drive-thru for 20 minutes, getting food for her husband. Sorry, babe. And thanks.

(+1)

(+1)