BODE ANALYSIS

FREQUENCY RESPONSE: $G(j\omega)$

BODE PLOTS: $|G(j\omega)|$ vs. ω ; $\angle G(j\omega)$ vs. ω

$|G(j\omega)|$: usually logarithmic ($dB = 20 \log_{10} |G(j\omega)|$)

ω: logarithmic

$$G(j\omega) = \frac{K(j\omega + z_1)(j\omega + z_2)\cdots(j\omega + z_m)}{(j\omega + p_1)(j\omega + p_2)\cdots(j\omega + p_n)}$$

$$= \frac{\left[K \prod_{i=1}^{m} z_i \Big/ \prod_{i=1}^{n} p_i\right](1 + j\omega/z_1)(1 + j\omega/z_2)\cdots(1 + j\omega/z_m)}{(1 + j\omega/p_1)(1 + j\omega/p_2)\cdots(1 + j\omega/p_n)}$$

Bode gain

EX. $G(j\omega) = \dfrac{K(j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)} = \dfrac{\frac{z_1}{p_1 p_2} K (1 + j\omega/z_1)}{(1 + j\omega/p_1)(1 + j\omega/p_2)}$

break (corner) frequencies

$$\log\left(\frac{ab}{cd}\right) = \log ab - \log cd = \log a + \log b - \log c - \log d$$

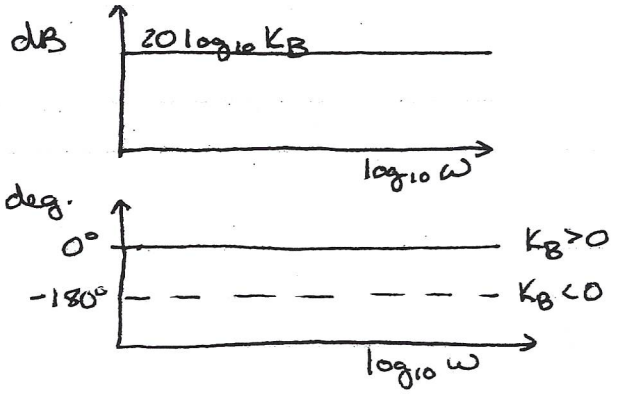
$$20 \log |G(j\omega)| = 20 \log \left|\left(K \frac{z_1}{p_1 p_2}\right)\right| + 20 \log \left|1 + \frac{j\omega}{z_1}\right| - 20 \log \left|1 + \frac{j\omega}{p_1}\right| - 20 \log \left|1 + \frac{j\omega}{p_2}\right|$$

CONSTANT GAIN

K_B

$|K_B| \angle 0°$ if $K_B \geq 0$

$|K_B| \angle -180°$ if $K_B \leq 0$



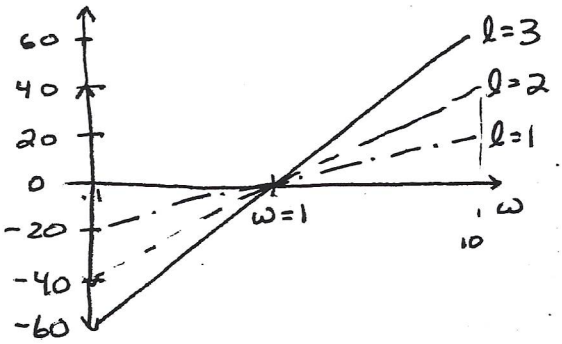
NB: if $|K_B| > 1$ dB +

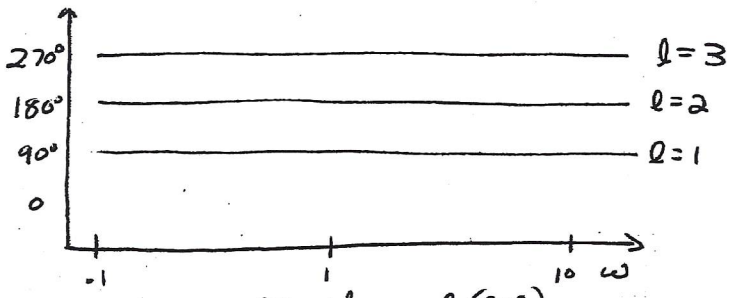
if $|K_B| < 1$ dB −

POLES AND ZEROS @ THE ORIGIN

zeros @ origin: $(j\omega)^\ell \Rightarrow dB = 20\log|(j\omega)^\ell| = 20\ell\log\omega$ ($\to 20\log|\omega^\ell|$)

BODE MAGNITUDE PLOT: straight line that intersects the ω-axis @ $\omega=1$ w/ slope of 20ℓ dB/decade

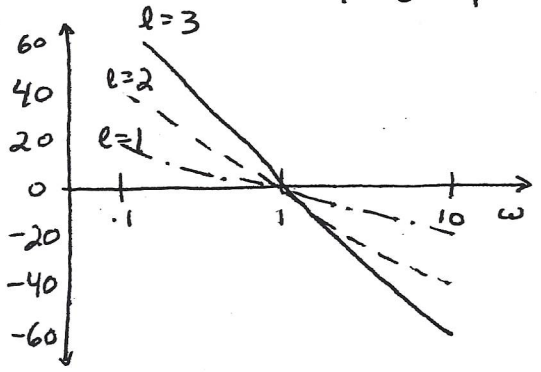


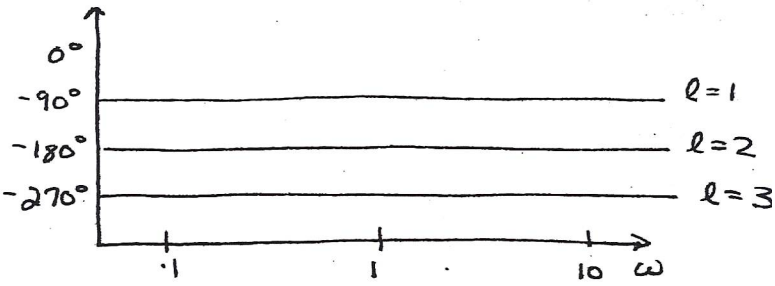


$\angle(j\omega)^\ell = \ell(90°)$

pole @ origin $\frac{1}{(j\omega)^\ell} \Rightarrow dB = -20\ell\log\omega$

MAG. PLOT: Straight line that intersects the ω-axis @ $\omega=1$ w/ slope of -20ℓ dB/decade





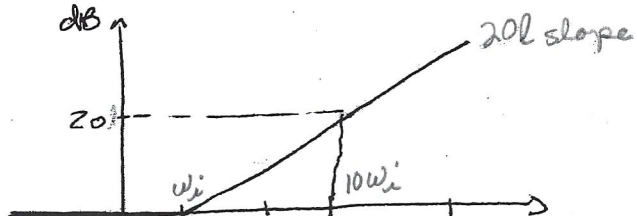
NONZERO REAL POLES AND ZEROS

ZERO @ ω_i: $20\log\left|1+\frac{j\omega}{\omega_i}\right| = 20\log\sqrt{1+\left(\frac{\omega}{\omega_i}\right)^2}$

$$\approx \begin{cases} 0 & \omega \le \omega_i \\ 20\log\omega - 20\log\omega_i & \omega > \omega_i \end{cases}$$

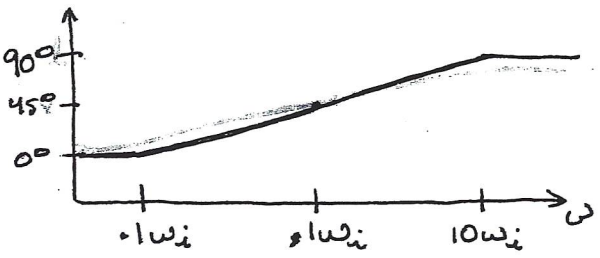
$\to \approx 20\log_{10}\left|\frac{j\omega}{\omega_i}\right|$

$\Rightarrow 20\log_{10}\left(\frac{\omega}{\omega_i}\right)$ as $\frac{\omega}{\omega_i} \to \infty$



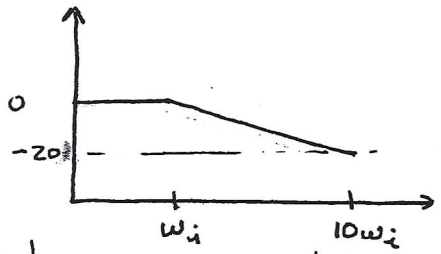
ℓ zeros @ ω_i: $20 \log \left| \left(1 + \frac{j\omega}{\omega_i}\right)^\ell \right| = 20 \log \left[1 + \left(\frac{\omega}{\omega_i}\right)^2\right]^{\ell/2}$

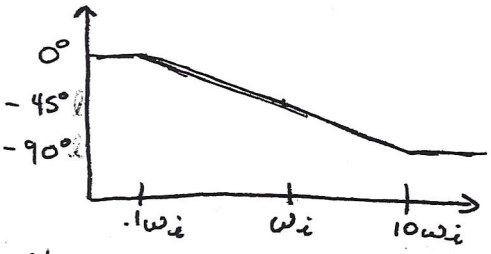
$$\cong \begin{cases} 0 & \omega \le \omega_i \\ 20\ell \log\left(\frac{\omega}{\omega_i}\right) & \omega > \omega_i \end{cases}$$



$\angle \left(1 + \frac{j\omega}{\omega_i}\right) = \tan^{-1} \frac{\omega}{\omega_i}$

ℓ poles @ ω_i





Mag: $20 \log_{10} \left| \frac{1}{\left(1 + j\frac{\omega}{p_1}\right)^\ell} \right| = -20 \log_{10} \left| \left(1 + j\frac{\omega}{p_1}\right)^\ell \right|$

COMPLEX POLES AND ZEROS

complex poles: $\frac{1}{\left(1 + j\,2\zeta\,\omega/\omega_n - (\omega/\omega_n)^2\right)}$ → factored out ω_n^2 $\quad 0 \le \zeta \le 1$

shown on readable

complex zeros: reflections about 0 dB & 0° lines for c.p.

NONMINIMUM PHASE ZERO

$(j\omega + z_1) \sim (j\omega - z_1) \sim \left(\frac{j\omega}{z_1} - 1\right)$

magnitude unchanged: $\left| \frac{j\omega}{z_1} + 1 \right| = \left| \frac{j\omega}{z_1} - 1 \right| = \sqrt{1^2 + \left(\frac{\omega}{z_1}\right)^2}$

phase changes: $\angle \frac{j\omega}{z_1} - 1 = \tan^{-1}\left(-\frac{\omega}{z_1}\right)$ phase is like pol

DMT

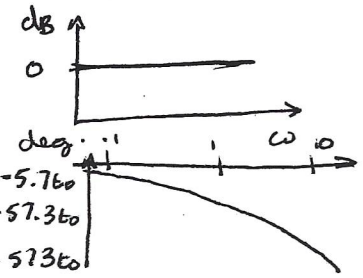
IDEAL TIME DELAY

$G(s) = e^{-t_0 s} \Rightarrow G(j\omega) = 1 \angle -\omega t_0$

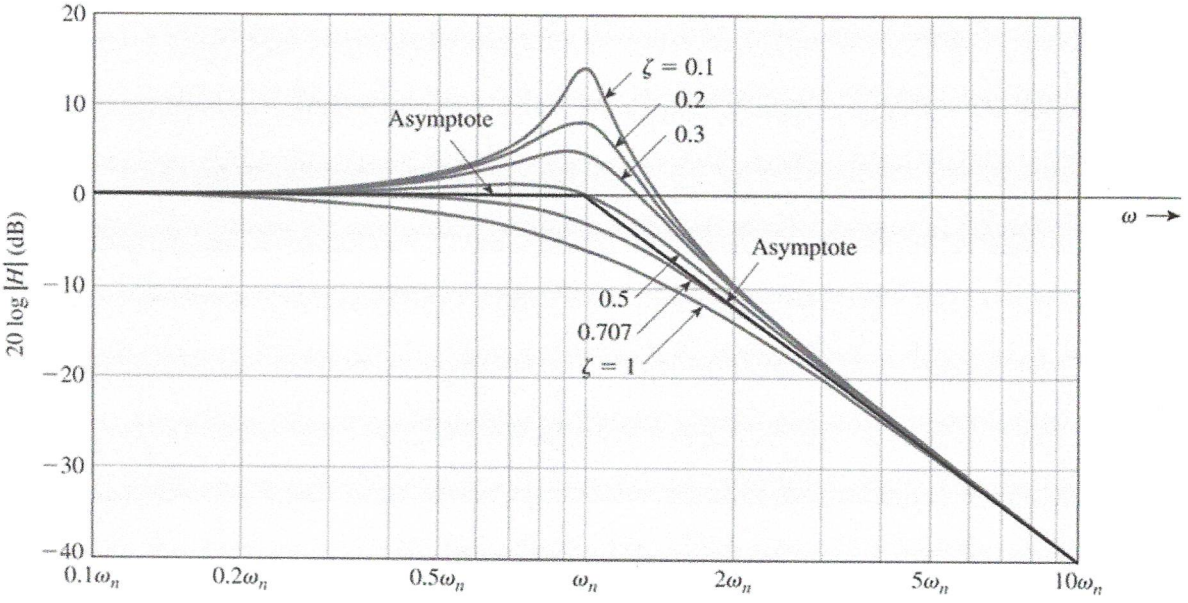
magnitude is unity → $dB = 0$

phase is linear w.r.t. ω

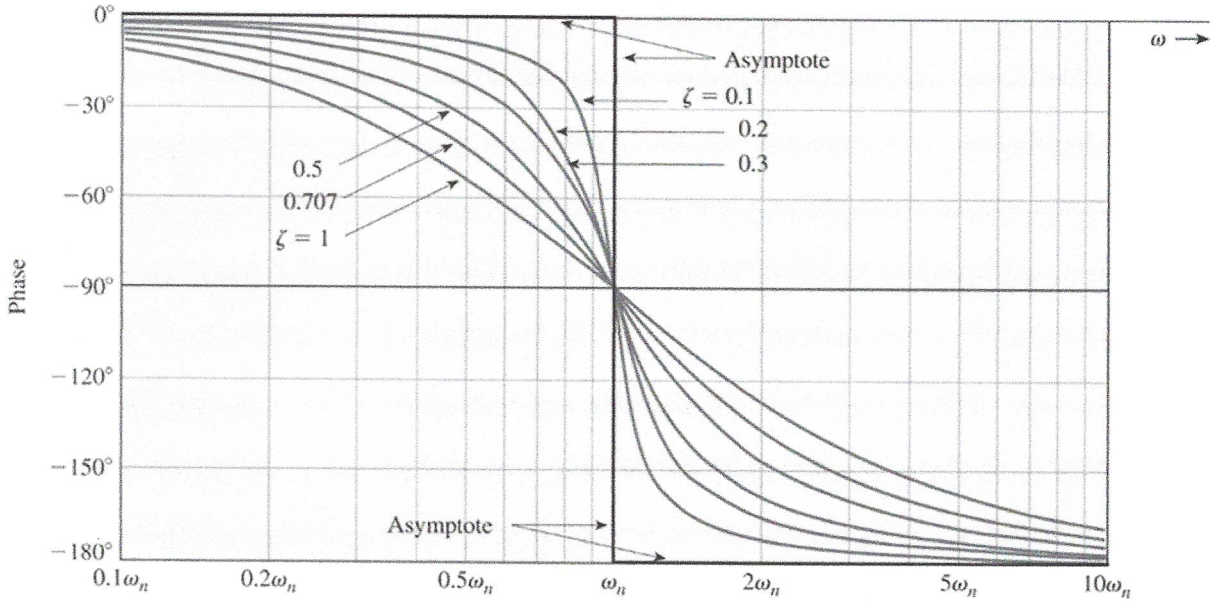
phase lag ↑ as $\omega \to \infty$



Phase = $-57.3\omega t_0$


20
10
0
−10
−20
−30
−40
20 log |H| (dB)
ω →
ζ = 0.1
0.2
0.3
Asymptote
Asymptote
0.5
0.707
ζ = 1
0.1ωn
0.2ωn
0.5ωn
ωn
2ωn
5ωn
10ωn

(a)

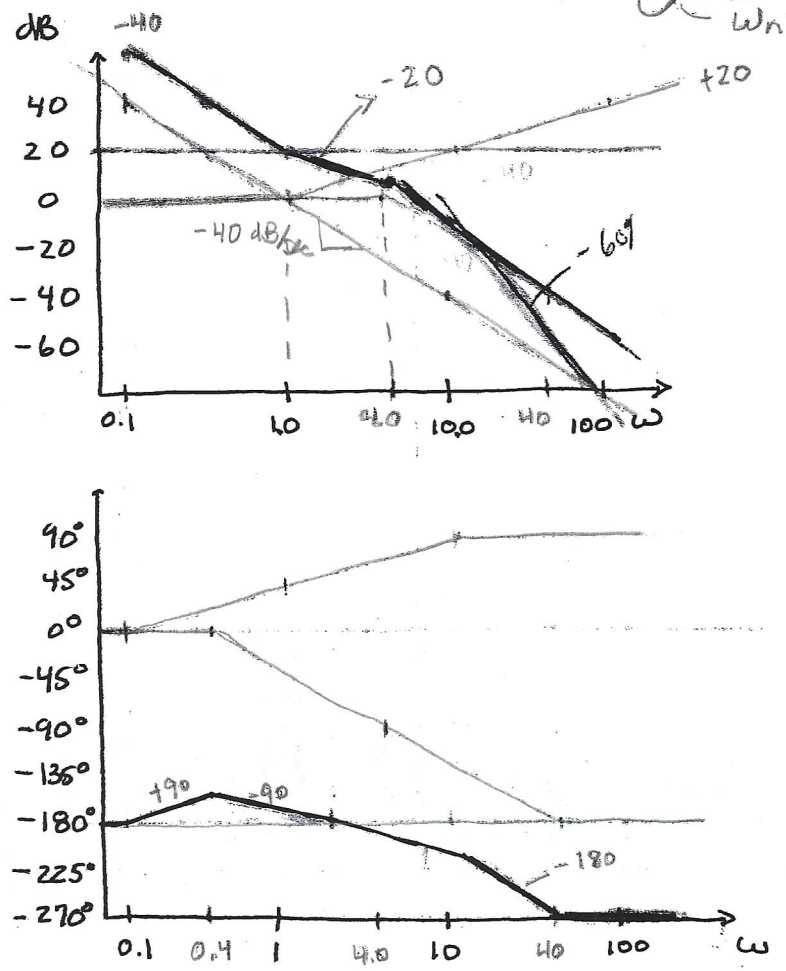

0°
−30°
−60°
−90°
−120°
−150°
−180°
Phase
ω →
Asymptote
ζ = 0.1
0.2
0.3
0.5
0.707
ζ = 1
Asymptote
0.1ωn
0.2ωn
0.5ωn
ωn
2ωn
5ωn
10ωn

(b)

EX. $GH(j\omega) = \dfrac{10(1+j\omega)}{(j\omega)^2\left[1+\frac{j\omega}{4}-\left(\frac{\omega}{4}\right)^2\right]}$

$\omega_n = 4$ $\quad \xi = 0.5$

$20\log_{10}(10) = 20$



- straight line approximatio
- for more accurate
  - plot drawn in exa
  - plot on computer

RELATIVE STABILITY

gain margin: # dB that $|GH(j\omega)|$ is below 0dB @ pha crossover freq. ($\omega = 180°$)

phase margin: # deg. that $\arg GH(j\omega)$ is above $-180°$ @ gain crossover freq. ($|GH(j\omega)| = 1$)

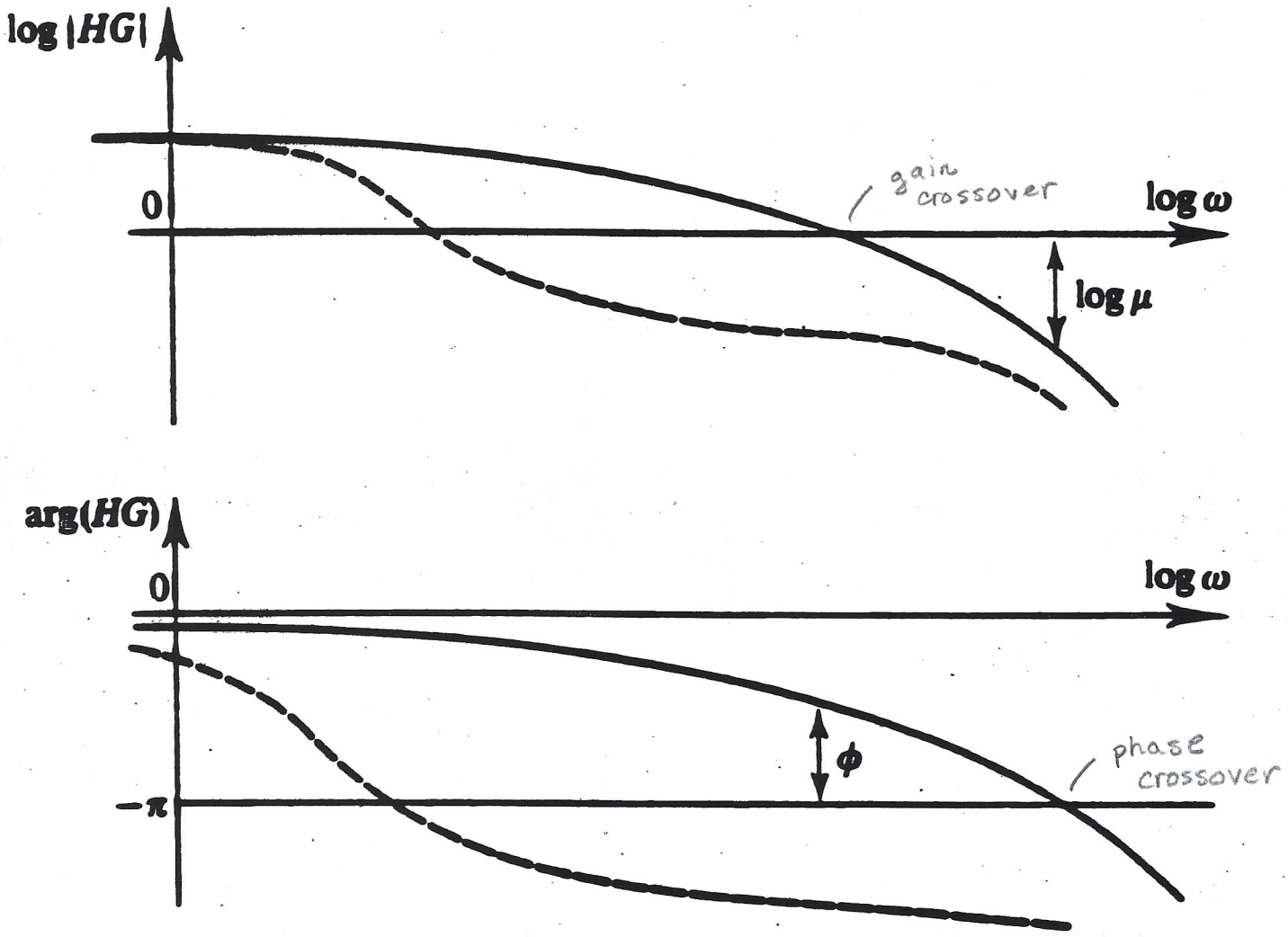


Figure 1.3 Bode plot form of Figure 1.2.