EE 3310L/5310L · Electronic Devices and Circuits Laboratory

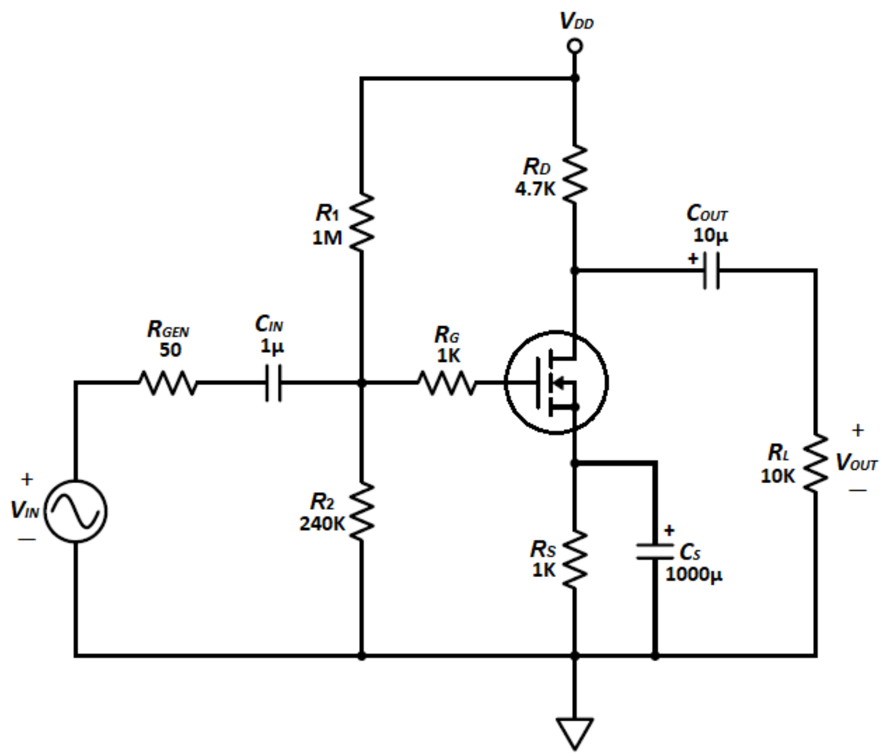
Lab 8: Nodal Analysis of Common-Source Small-Signal Model

Purpose

The purpose of this lab is to plot the frequency response of a common-source MOSFET voltage amplifier using *s*-domain nodal analysis of the small-signal model. Results will be compared to those obtained from multifrequency analysis, laboratory measurement, and simulation.

Background

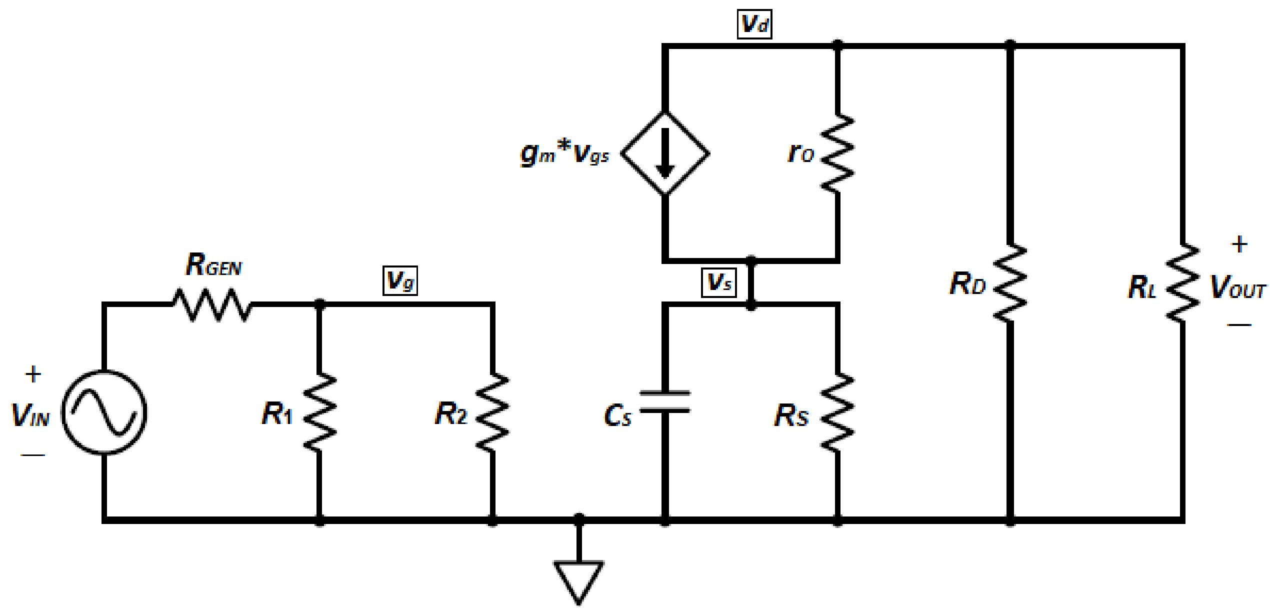
We constructed, tested, simulated, and analyzed the following MOSFET voltage amplifier in Lab 7:



Conventional analysis of small-signal amplifiers is performed in multiple frequency ranges in order to employ simplified models and demonstrate the mechanisms by which individual circuit elements contribute to the overall response of the amplifier. For example, mid-frequency analysis is usually performed first, in which the impedances of external capacitors are assumed to be sufficiently low to be replaced by short circuits, while the impedances of internal parasitic capacitances are assumed to be sufficiently high to be replaced by open circuits, greatly simplifying the small-signal model and facilitating easy computation of passband gain and terminal impedances. Then, for high- and low-frequency models, different assumptions are made regarding treatment of these impedances; frequency response is computed for each model; and a composite total frequency response is assembled for the complete amplifier. This approach is a powerful way to understand how different circuit components each contribute to the total amplifier response, and makes the design process for determining suitable component values for desired system response straightforward.

However, there are limitations to the conventional approach. One potential problem is the accumulation of errors in the total response due to several iterations of assumptions in different frequency ranges. For example, it is assumed that external coupling capacitors never interact with parasitic capacitances, but that might not be the case when, say, an input capacitor is used as part of a filter in which the shunt element includes Miller Effect input capacitance. A solution to these limitations is to perform nodal analysis of the entire amplifier with *s*-domain impedance models to develop a complete transfer function. This is quite easy today with computational solvers such as MATLAB and was not possible 100 years ago when the fundamentals of small-signal analysis were developed.

Consider the following **partial** small-signal model for the common-source MOSFET voltage amplifier. Not all external capacitors are represented, and *none* of the internal ones are, but it will serve as a suitable basis for developing node equations and solving them.



Essential node voltages corresponding to the FET terminals of gate, source, and drain are defined as v_g, v_s, and v_d, respectively. In this rudimentary example, we can make the simplification of $v_d = V_{OUT}$. Thus, we can write a set of node voltages describing the circuit. [Note that, by definition, $v_{gs} = v_g - v_s$.]

$$\frac{v_g - V_{IN}}{R_{GEN}} + \frac{v_g - 0}{R_1} + \frac{v_g - 0}{R_2} = 0$$

$$\frac{v_s - 0}{1/_{s * C_S}} + \frac{v_s - 0}{R_S} - g_m * (v_g - v_s) + \frac{v_s - V_{OUT}}{r_o} = 0$$

$$g_m * (v_g - v_s) + \frac{V_{OUT} - v_s}{r_o} + \frac{V_{OUT} - 0}{R_D} + \frac{V_{OUT} - 0}{R_L} = 0$$

We can now use a computational solver to determine an *s*-domain transfer function $H(s) = V_{OUT} / V_{IN}$. The following is sample MATLAB code that will get you started.

```
clear all

% Declare all symbolic variables, including the Laplace variable s and frequency f

syms VOUT VIN vg vs vd RGEN R1 R2 CS RS gm ro RD RL s f

% Enter node voltage equations, solve for outputs VOUT, vg, vs, and vd (note we don't actually use vd yet)

[VOUT vg vs vd] = solve ((vg-VIN)/RGEN + vg/R1 + vg/R2 ==0, ...

vs*s*CS + vs/RS - gm*(vg-vs) + (vs-VOUT)/ro ==0, ...

gm*(vg-vs) + (VOUT-vs)/ro + VOUT/RD + VOUT/RL ==0, VOUT, vg, vs, vd)

% Compute the s-domain transfer function H(s)

H(s) = simplify (VOUT/VIN)

% Plug in external component values and dynamic parameters gm and ro

RGEN = 50;

R1 = 1000000;

R2 = 240000;

CS = 0.001;

RS = 1000;

gm = 0.017;

ro = 110000;

RD = 4700;

RL = 10000;

% Determine transfer function with component values

H(s)=subs(H(s))

% Symbolically define the gain at 1 kHz as 'm,' meaning 'midfrequency'

m=[abs(H(j*2*pi*1000)),angle(H(j*2*pi*1000))]
```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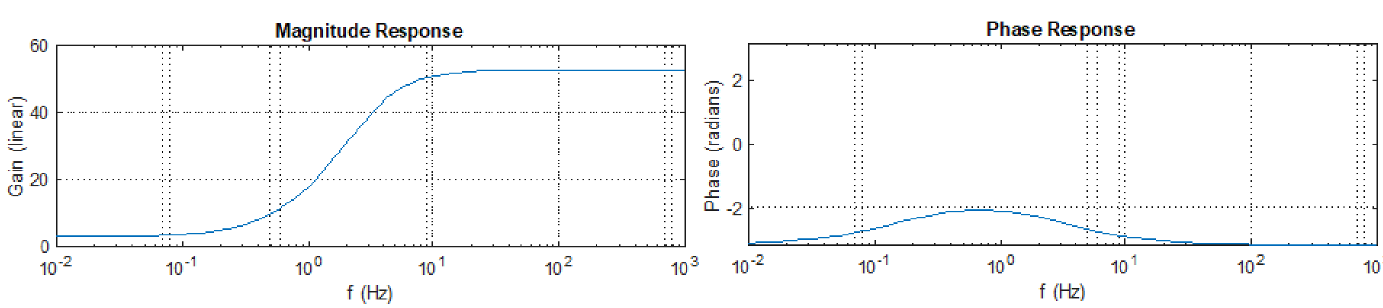
```
% determine the 1 kHz magnitude and phase

double (abs(m))
```

Note that the computed magnitude of 52.8047 is consistent with the approximately 34 dB of gain found experimentally in Lab 7, and the phase of 3.1390 radians indicates the polarity inversion inherent in the common-source amplifier configuration, with very little additional phase shift at mid frequencies.

```
% plot the frequency response from 0.01 Hz to 1 kHz on a semilogarithmic frequency scale

fmin = 0.01;

fmax = 1000;

figure(8)

subplot(2,1,1)

fplot(abs(H(j*2*pi*f)),[fmin,fmax]), grid

set(gca, 'XScale','log')

axis([fmin,fmax, 0, 60])

title('Magnitude Response'); xlabel('f (Hz)'); ylabel('Gain (linear)')

subplot(2,1,2)

fplot(angle(H(j*2*pi*f)),[fmin,fmax]), grid

set(gca, 'XScale','log')

axis([fmin,fmax, -pi,pi])

title('Phase Response'); xlabel('f (Hz)'); ylabel('Phase (radians)')

snapnow;
```



The resulting frequency response perfectly demonstrates the shelved high-pass characteristic due to C_S predicted by the low-frequency model, with one pole located near

$$f_p \cong \frac{1}{2\pi C_S * \frac{1}{g_m}} = 2.7\ Hz$$

and one zero at

$$f_z = \frac{1}{2\pi C_S R_S} = 0.16\ Hz.$$

The beauty of the complete-model approach is that we can discern the *exact* poles and zeroes of the system from the transfer function; look at the numerator of $H(s)$ and note that the zero-frequency *is* exactly as expected, while the exact pole location is more complicated than predicted by the simplified low-frequency analysis.

Of course we can - and should! – display our frequency response in more "engineer friendly" terms such as a dB scale for magnitude and degrees for phase. But, most importantly, we can now include *all* impedances in a complete model and get a total frequency response over any desired passband.

Prelab

Hand-draw a *complete* small-signal model for the common-source MOSFET amplifier, including all external and parasitic capacitances. Carefully write a full set of node equations using *s*-domain impedance models.

Procedure

Use MATLAB or the solver of your choice to determine the transfer function $H(s)$. Plot the magnitude response in dB and phase response in degrees vs. frequency in Hz from 0.1 Hz to 10 MHz using a logarithmic frequency axis. From the plot, determine the midband gain in dB and the high and low −3-dB corner frequencies of the total system.

Lab Report

Compare the gain and frequency response of the circuit using *s*-domain nodal analysis to the values obtained using a) hand computation, b) simulation, and c) laboratory execution. Include all models, node equations, and heavily-commented MATLAB code in your lab report.