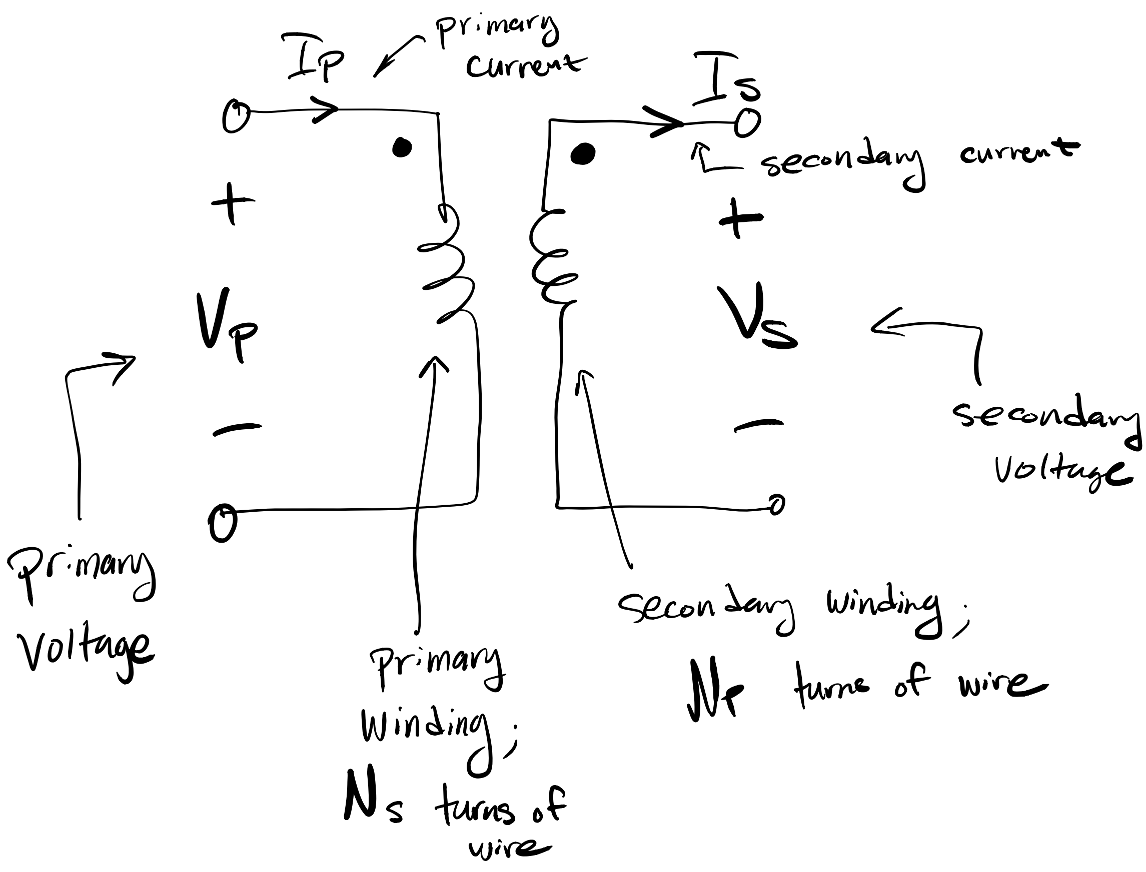
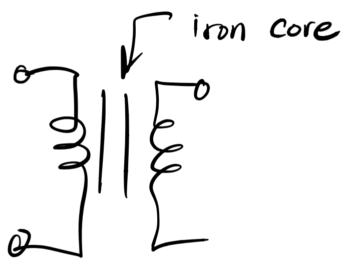
Ideal Transformers

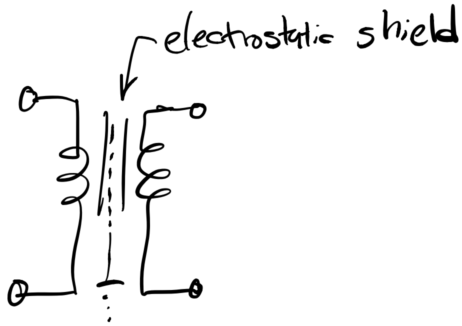
principle: two magnetically-coupled inductors may transform AC (only!) voltages or currents by a ratio proportional to the ratio of primary-to-secondary turns

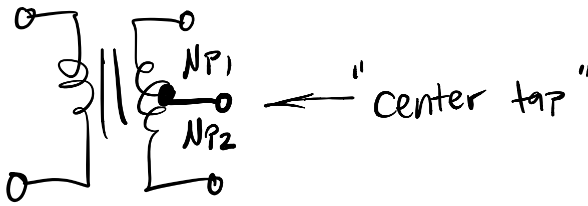
Michael Faraday: August 1831



symbolic variations :







∴ for an ideal (lossless) transformer :

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$\therefore \quad V_S = V_P\left(\frac{N_S}{N_P}\right)$$

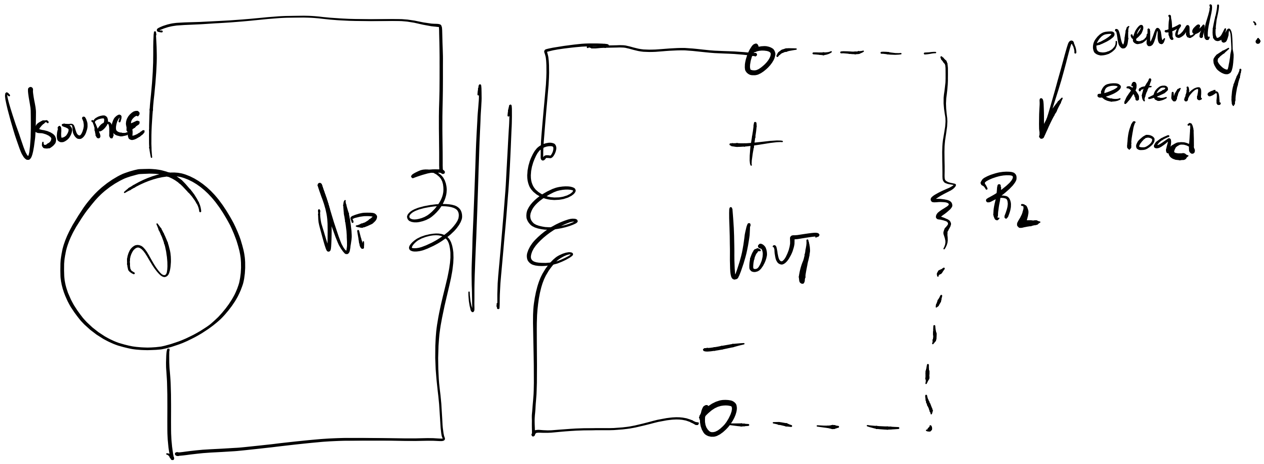
turns ratio

ex: $N_P = 500$ turns

$N_S = 1{,}000$ turns

$V_{SOURCE} = 120\,V_{RMS}$ @ 60 Hz

wall socket!



$$V_P = V_{SOURCE};$$

$$\therefore \quad V_{OUT} = V_S = V_P\left(\frac{N_S}{N_P}\right)$$

$$= V_{SOURCE}\left(\frac{N_S}{N_P}\right)$$

$$= 120\left(\frac{1000}{500}\right) = 240\ V_{RMS}$$

∴ when $N_S > N_P$, and therefore $V_S > V_P$, we call this a <u>step-up transformer</u>.

ex: $N_P = 100$ turns, $N_S = 10$ turns, $V_{SOURCE} = 120 V_{RMS}$

$$V_{OUT} = V_S = V_P\left(\frac{N_S}{N_P}\right) = V_{SOURCE}\left(\frac{N_S}{N_P}\right)$$

$$= 120\left(\frac{10}{100}\right) = 12\ V_{RMS}$$

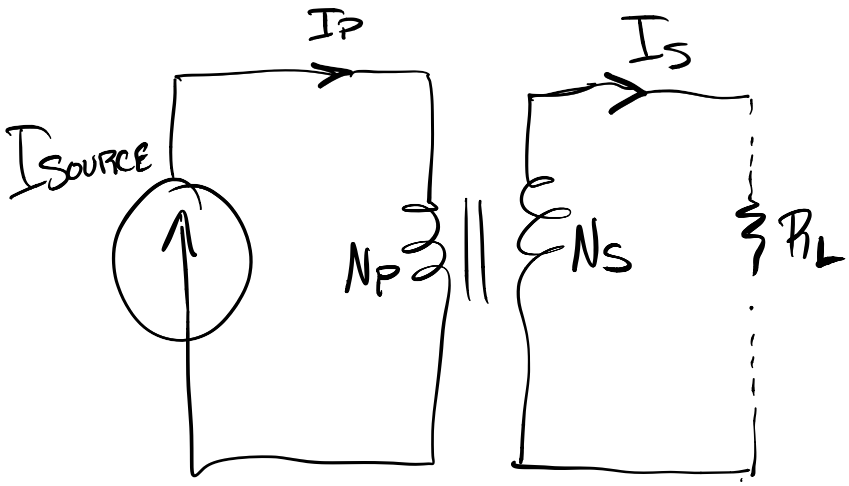
∴ when $N_P > N_S$, we call this a step down transformer.

∴ the current ratio is the inverse of the turns (and thus voltage) ratio

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

∴ $I_S = I_P\left(\frac{N_P}{N_S}\right)$

∴ you can use a transformer w/ a current source,



$I_{SOURCE} = 280\,mA_{RMS}$

$N_P = 300$ turns

$N_S = 1200$ turns

$$I_S = I_P\left(\frac{N_P}{N_S}\right)$$

$$= 280\left(\frac{300}{1200}\right)$$

$$= 70\ mA_{RMS}$$

$N_S > N_P$, so it would be a voltage <u>stepup</u> transformer; but acts as a <u>current stepdown transformer</u> due to inverse relationship

Power

- ideal transformer is 100% efficient

in reality: iron losses

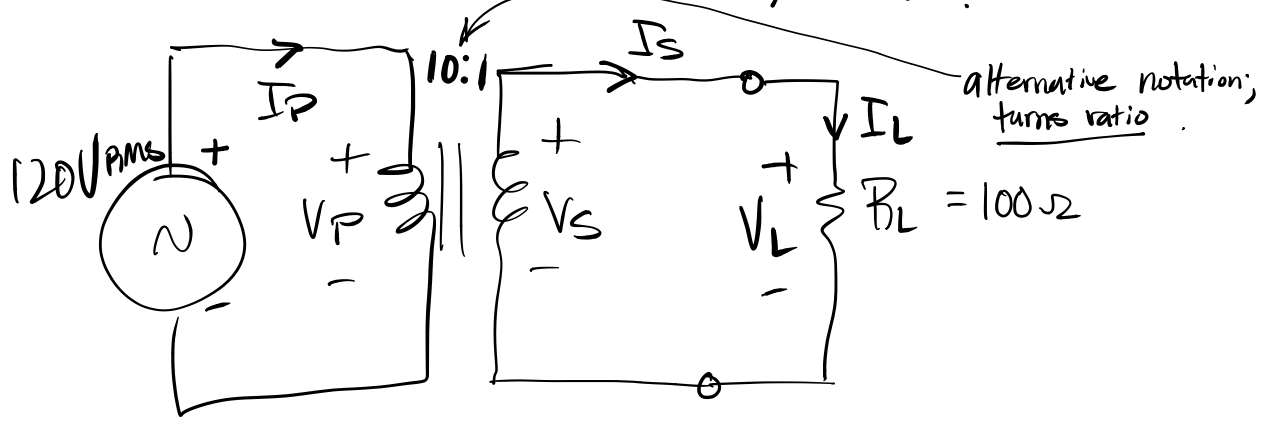
- imperfect magnetic coupling due to core properties
- Eddy currents
- leakage inductance

Copper losses

- wire used to wind transformer has non-zero resistance
- "I^2R losses"

- we assume ideal transformers in this course
- transformer manufacturers can estimate losses and somewhat compensate for them, if used as rated

.. let's hook a stepdown transformer to an external load resistance, R_L.



.. load voltage $V_L = V_S$

$$V_S = V_P\left(\frac{N_S}{N_P}\right) = 120\left(\frac{1}{10}\right) = 12\,V_{RMS}$$

10-to-1 stepdown ✓

.. load current $I_L = I_S = \frac{V_S}{R_L} = \frac{12}{100}$

$$I_L = 0.12\,A_{RMS}$$

.. load power:

dissipated in R_L

$$P_L = V_L I_L = V_S I_S = 12 \cdot 0.12 = 1.44\,W$$

$\because$ We know $I_S = I_P\left(\frac{N_P}{N_S}\right)$

$\therefore$ primary current is $I_P = I_S\left(\frac{N_S}{N_P}\right)$

$$= 0.12\left(\frac{1}{10}\right) = \underline{0.012\ A_{RMS}}$$

$\therefore$ power delivered by source:

$$P_{SOURCE} = \overset{\text{rise!}}{-}V_P \cdot I_P = -120 \cdot 0.012$$

$$= \underline{-1.44\ W} \quad \text{or } 1.44\ W \text{ generated}$$

$\because$ transformer is (losslessly) trading voltage for current w/ perfect efficiency!

Reflected Impedance

- in the previous example: load connected to transformer secondary was $R_L = 100\,\Omega$

- we calculated a primary current of $0.012\ A_{RMS}$ with a primary voltage of $120\ V_{RMS}$

- what equivalent impedance does the source "think" it's driving?

$$\frac{V_P}{I_P} = \frac{120}{0.012} = 10{,}000\,\Omega = 100\,R_L$$

- the impedance relationship in a transformer is

$$\frac{Z_P}{Z_S} = \left(\frac{N_P}{N_S}\right)^2$$

$$= \left(\frac{V_P}{V_S}\right)^2$$

∴ that $10\,k\Omega$ is called the reflected impedance.

Check: $$Z_P = Z_S \left(\frac{N_P}{N_S}\right)^2$$

$$= 100 \left(\frac{10}{1}\right)^2 = 10\,k\Omega \checkmark$$

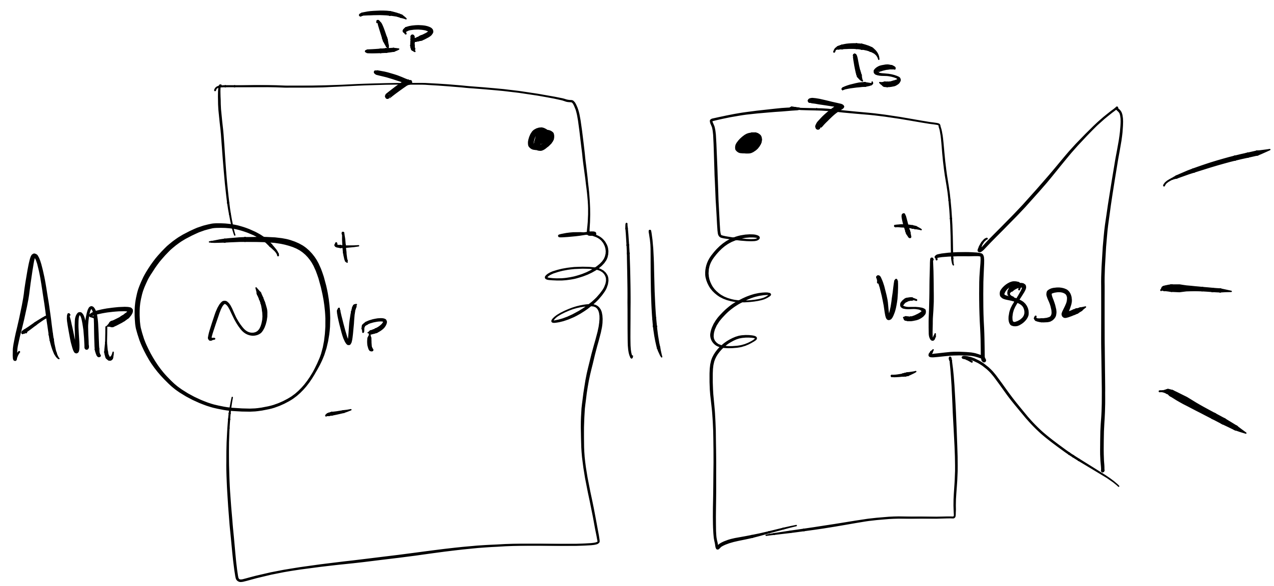
ex: a tube audio amplifier works best into a $3.2\,k\Omega$ load.

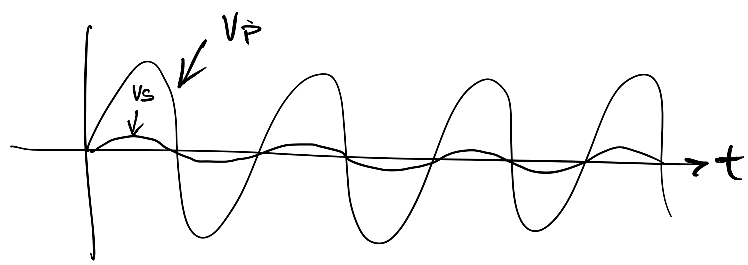
∴ problem: you have an $8\,\Omega$ speaker.

∴ solution: use a transformer to match the impedances

∴ what turns ratio is needed?

∴ what are the primary and secondary voltages and currents assuming $100W$ of audio power?





∴ no polarity inversion if two "dots" are on <u>positive</u> terminals of V_s and V_p

$$\frac{Z_p}{Z_s} = \left(\frac{N_p}{N_s}\right)^2$$

∴ turns ratio : $\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{3200}{8}}$

$= 20:1$

∴ 100W into 8Ω :

$$P_s = \frac{V_s^2}{R_s} \longrightarrow V_s^2 = P_s R_s$$

$$V_s = \sqrt{P_s R_s} = \sqrt{100 \cdot 8}$$

$$V_s = 28.28 \ V_{RMS}$$

$$\therefore \ I_s = \frac{V_s}{R_s} = \frac{28.28}{8} = \underset{\sim\sim\sim\sim}{3.535 \ A_{RMS}}$$

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \longrightarrow V_P = V_S\left(\frac{N_P}{N_S}\right)$$

$$= 28.28\left(\frac{20}{1}\right)$$

$$\underline{V_P = 565.6 \ V_{RMS}}$$

yikes!

to get Vpeak from RMS

$$565.6 \sqrt{2} \cdot 2 = \underline{1600 \ V_{pk\text{-}pk}} \ !!!$$

yes, real figure!

$$I_P = \frac{V_P}{R_P} = \frac{565.6}{3200} = 0.1768\ A_{RMS}$$

reflected load Z

or $176.8\ mA_{RMS}$

∴ we're taking a tube amplifier, which is comfortable @ kV and mA, and interfacing with a speaker that wants V and A

→ the magic of the transformer!