ISE 2211 • Exam III • 6 October 2017 NAME SOLUTION

1) The mean explosive yield of a W87 thermonuclear warhead (as used in Peacekeeper guided missiles) is 300 kt (kilotons TNT), based on a sample size of n = 24 warheads, with a sample ~~variance~~ std. dev. of 18.8 kt-TNT. The population variance is unknown. Write a 95% two-sided confidence interval on mean explosive yield, and include a unit with your answer.

$\alpha = 0.05$

$n < 30$, σ unknown $\rightarrow$ use t-distribution

$$t_{\alpha/2, n-1} = t_{.025, 23} = 2.069 \quad (+1) \; (+1) \; \text{(table)}$$

$$\bar{X} - t_{\alpha/2, n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{X} + t_{\alpha/2, n-1}\frac{s}{\sqrt{n}} \quad \Leftarrow (+1)$$

$$300 - 2.069\frac{18.8}{\sqrt{24}} \le \mu \le 300 + 2.069\frac{18.8}{\sqrt{24}}$$

$$292.1 \le \mu \le 307.9 \quad (+1)$$

(kt-TNT) (+1) [or just kt]

Now write a 95% prediction interval on the mean explosive yield of the 25th W87 warhead.

$$\bar{x} - t_{\alpha/2, n-1}\, s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2, n-1}\, s\sqrt{1+\frac{1}{n}} \quad (+1)$$

$$300 - 2.069 \cdot 18.8\sqrt{1+\tfrac{1}{24}} \le X_{25} \le 300 + 2.069 \cdot 18.8\sqrt{1+\tfrac{1}{24}}$$

$$261.9 \le X_{25} \le 338.1 \quad (+1) \; \text{(kt-TNT)}$$

2) At Yellow Springs Brewery, a few of their craft beers are sold in 12-oz. cans which are filled and packaged on-site. It is *essential* to the integrity of the operation that under-filled cans be taken home and consumed <u>immediately</u> by YSB employees!!! During a special midnight canning session, the following masses were recorded by an employee (all of which sadly fell below the rejection threshold and had to be removed from the premises):

{367 366 362 367 365} (g)

Compute the sample variance using $s^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}$ and use it to write a 90% upper confidence bound on the population variance of the operation. Also determine an upper bound on population standard deviation. Include <u>units</u> with all answers.

$$s^2 = \frac{667603 - \frac{1827^2}{5}}{4} = 4.3 \quad (+1)$$

$$\chi^2_{1-\alpha, n-1} = \underbrace{\chi^2_{.90,4}}_{(+1)} = \underline{\underline{1.06}} \quad (+1) \quad \text{(table)}$$

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}} \Leftarrow \underline{\text{upper}} \text{ bound } (+1)$$

$$\sigma^2 \leq \frac{4 \cdot 4.30}{1.06} =$$

$$\underline{\underline{\sigma^2 \leq 16.23}} \quad (+1) \quad g^2 \; (+1)$$

$$\sigma \leq \sqrt{16.23}$$

$$\sigma \leq 4.028 \text{ g} \quad (+1)$$

3) Everybody knows that the rye chips are the best thing in Gardetto's®-brand snack mix. (If you disagree, you get an *F* in this class.) Last weekend, Joe Tritschler felt he was being short-changed on rye chips so he got mad, bought 27 bags, and counted every piece. Out of a total of 328 snack pieces, 72 were rye chips. Write a 95% confidence interval on the population proportion of rye chips in Gardetto's®-brand snack mix.

$$\hat{p} = \frac{x}{n} = \frac{72}{328} = \underline{0.2195} \quad +1$$

$$Z_{\alpha/2} = 1.96 \quad +1$$

$$0.21995 - 1.96\sqrt{\frac{.2195(1-.2195)}{328}} \leq P \leq 0.2195 + 1.96\sqrt{\frac{.2195(1-.2195)}{328}}$$

$$0.1747 \leq P \leq 0.2643$$

+2

If it is very important to know the proportion of rye chips in Gardetto's®-brand snack mix within ±0.5%, determine the minimum sample size needed to accomplish this.

$\therefore E = 0.005$ +1

$$n = 0.25\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= 0.25\left(\frac{1.96}{0.005}\right)^2$$

$$n = 38{,}416 \quad +1$$