

SOLUTION

In a recent investigation into the quality control of 3.9-k $\Omega$  carbon film resistors for a stepped attenuator design, Joe Tritschler took samples from each of two manufacturers (Stackpole Electronics, Inc. and KOA Speer Electronics, Inc.) and measured their resistances. Population variances are unknown and assumed unequal. The results were as follows:  $\bar{x}_1 = 3.894$  k $\Omega$ ,  $s_1 = 0.04582$  k $\Omega$ ,  $\bar{x}_2 = 3.915$  k $\Omega$  and  $s_2 = 0.02248$  k $\Omega$ . If the Stackpole resistors represent sample #1 with eight units measured and the KOA Speer resistors represent sample #2 with ten units measured, test the following hypotheses regarding the mean values of resistance between the two brands using the fixed-significance-level approach at  $\alpha = 0.05$ . Sketch the appropriate distribution, indicating the test statistic and critical values.

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

$$n_1 = 8 \quad n_2 = 10$$

need  $\checkmark$

$$\frac{s_1^2}{n_1} = \frac{0.04582^2}{8} = 2.624 \times 10^{-4}$$

$$\frac{s_2^2}{n_2} = \frac{0.02248^2}{10} = 5.054 \times 10^{-5}$$

$$V = \frac{(2.624 \times 10^{-4} + 5.054 \times 10^{-5})^2}{\frac{(2.624 \times 10^{-4})^2}{7} + \frac{(5.054 \times 10^{-5})^2}{9}}$$

$$V = 9.677$$

round down,

Use  $V = 9$  d.o.f.

(+2)

(+1)

$$t_0 = \frac{3.894 - 3.915 - 0}{\sqrt{\frac{0.04582^2}{8} + \frac{0.02248^2}{10}}} = -1.187$$

(+1)

Critical values:  $\pm t_{\alpha/2, V} = \pm t_{0.025, 9}$

(+1)

$t_0 < -2.262$  (table)

$= \pm 2.262$

(+1)

fail to reject  $H_0$

(+1)



(+2)

Write a 95% C.I. on the ratio of population standard deviations and use it to test the following hypotheses:

$$H_0: \sigma_1 = \sigma_2$$

$$H_1: \sigma_1 \neq \sigma_2$$

Include a unit with the C.I.

$$f = \frac{S_1^2}{S_2^2} = \frac{.04582^2}{.02248^2}$$

$$f = 4.154 \quad (+)$$

$$f_{d/2, n_2-1, n_1-1} = f_{.025, 9, 7} = 4.82 \text{ (table)}$$

$$f_{.025, 9, 7} = 4.82 \quad (+)$$

$$f_{1-d/2, n_2-1, n_1-1} = \frac{1}{f_{d/2, n_1-1, n_2-1}} = \frac{1}{f_{.025, 7, 9}} = \frac{1}{4.20} = 0.238 \text{ (table)}$$

$$f_{.025, 7, 9} = 4.20 \quad (+)$$

$$0.238 \quad (+)$$

$$4.154 \cdot 0.238 < \frac{\sigma_1^2}{\sigma_2^2} < 4.154 \cdot 4.82$$

$$0.9887 < \frac{\sigma_1^2}{\sigma_2^2} < 20.02 \quad (+)$$

$$0.9943 < \frac{\sigma_1}{\sigma_2} < 4.475 \quad (+)$$

$$\left( \frac{k_{\alpha/2}}{k_{\alpha/2}} \right) \quad (+)$$

C.I. contains 1 (+) fail to reject  $H_0$  (+)

Two of the Stackpole resistors and one of the KOA Speer resistors failed to meet spec. Test the following hypotheses on the population proportion of out-of-spec resistors using the  $p$ -value approach and state your conclusion with respect to  $\alpha = 0.05$ .

$$H_0: p_1 = p_2$$

$$H_1: p_1 \neq p_2$$

$$\begin{aligned} \hat{p}_1 &= \frac{2}{8} = 0.25 \\ \hat{p}_2 &= \frac{1}{10} = 0.1 \end{aligned} \quad \left. \vphantom{\begin{aligned} \hat{p}_1 &= \frac{2}{8} \\ \hat{p}_2 &= \frac{1}{10} \end{aligned}} \right\} (+)$$

$$\hat{p} = \frac{1+2}{8+10} = 0.1667 \quad (+)$$

$$Z = \frac{0.25 - 0.1}{\sqrt{.1667(1-.1667) \left(\frac{1}{8} + \frac{1}{10}\right)}} = 0.8485 \quad (+)$$

$$P(Z < -0.85) = 0.197662 \quad (+)$$

$$\begin{aligned} \therefore P\text{-value} &= 2 - 0.197662 \\ &= 0.395 \quad (+) \end{aligned}$$

$$P \not< 0.05 \quad (+)$$

$\therefore$  (strongly) fail to reject  $H_0$  (+)

Based on the results of these three hypothesis tests, briefly state what you think about the equivalency of the two brands of carbon film resistors with regards to quality control.

means, variances, and proportion of out-of-spec  
can't be proven unequal. They are statistically  
equivalent. (+)