

## EE 2010 Circuit Analysis

### Module 09: Node Equations with Generalized Sources & Impedances

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These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

We now turn to circuits with *general impedances* and follow *exactly the same generalized-source procedure* we have used with resistors and add the impedance models  $Z_R(s) = R$ ,  $Z_C(s) = \frac{1}{s \cdot C}$ , and  $Z_L(s) = s \cdot L$ .

#### Node Voltage Procedure for Generalized Sources with Impedances:

1. Replace *all independent sources with symbolic representations*
2. Identify the essential ( $\geq 3$ -element connections) nodes
3. Select a node as the reference node = the node at *ground potential* = 0 Volts
4. Identify and label the voltages at nodes that are readily deduced
5. Note the node-pairs linked by a *voltage source* and simplify accordingly
6. Assign voltage variables  $v_a, v_b, \dots$  to the remaining nodes with only one assignment for each linked node-pair, the other node in that pair assigned voltages such as " $v_1 - 20$ " or " $v_4 + 3v_x$ ".
7. Employ  $s$ -domain impedance models:  $Z_R(s) = R$ ,  $Z_C(s) = \frac{1}{s \cdot C}$ , and  $Z_L(s) = s \cdot L$ .
8. Apply  $I_{\text{out}} = V_{\text{difference}}/Z$  for each branch leaving the node
9. Enjoy the thrill of ending the consideration of each node with the powerful " $= 0$ "
10. Add one additional equation for each dependent source specification if necessary

*Circuit analysis is now complete! But you may be asked to:*

11. Find the transfer function:  $H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$ , or whatever input-output function is dictated by the problem.
12. Use circuit analysis techniques to find the output:  $V_{\text{out}}(s)$ , or whatever output function is dictated by the problem.

**Pattern of Results:** In all that follows, we will consistently find a result that can be written in the form

$$\text{Output}(s) = H(s) \cdot \text{Input}(s)$$

where  $H(s)$  is the *transfer function* particular to the system. The transfer function  $H(s)$  will be characterized by ratios of polynomials in  $s$ .

**Superposition:** In cases where there are multiple independent excitations, we will consistently find results that can be written in the form

$$\text{Output}(s) = H_1(s) \cdot \text{Input}_1(s) + H_2(s) \cdot \text{Input}_2(s) + \dots$$

where  $H_i(s)$  are the *transfer functions* particular to each excitation of the system. The transfer functions  $H(s)$  will be characterized by ratios of polynomials in  $s$ .

We employ the steady-state  $s$ -domain impedance models:

### Steady-State Impedance Models

$s$ -domain	Steady State @ $\omega$ R/s	Steady State @ $f$ Hz
$Z_R = R$	$Z_R = R$	$Z_R = R$
$Z_C = \frac{1}{s \cdot C}$	$Z_C = \frac{1}{j\omega \cdot C}$	$Z_C = \frac{1}{j2\pi f \cdot C}$
$Z_L = s \cdot L$	$Z_L = j\omega \cdot L$	$Z_L = j2\pi f \cdot L$

Table 1: Steady-State Impedance Models for  $R$ ,  $C$ , and  $L$

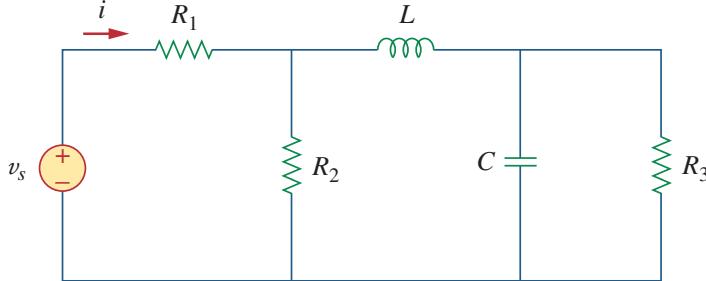
**Recall that:** This implies *Capacitors are open circuits at DC* and *Inductors are short circuits at DC*

Later, we'll more completely address some of the homework problems by including particular details of the excitation signals. For now, we'll focus ONLY on writing a complete set of equations describing the circuit at hand.



**Note:** The book sometimes labels voltage and current sources as  $v_s$  or  $i_s$ . To avoid confusion with s-domain impedance models, I will replace these labels with  $V_{in}$  or  $I_{in}$ , etc. I will *generally* use **UPPERCASE LETTERS** to denote sources, and *lowercase letters* to denote circuit variables.

**Problem 9.36:** Find  $I_{in}$ . We could use circuit reduction here. I will use Node Equations with 2 nodes,  $v_a$  and  $v_b$ .



### %% Problem 9.36

```
clear all
% Declare symbolic variables, including the Laplace variable ``s''
syms Vin va vb R1 R2 L C R3 s
% Nodal analysis directly in solve()
[va vb]=solve(va/R2 + (va-Vin)/R1 + (va-vb)/(L*s) == 0, ...
vb/R3 + vb*C*s + (vb-va)/(L*s) ==0,va,vb)
Iin=(Vin-va)/R1
simplify(Iin)
% Transfer function
H(s) = simplify(Iin/Vin)
```

Which yields:

```
va = (R2*Vin*(C*L*R3*s^2 + L*s + R3))/(R1*R2 + R1*R3 + R2*R3 + L*R1*s + L*R2*s + C*
R1*R2*R3*s + C*L*R1*R3*s^2 + C*L*R2*R3*s^2)
vb = (R2*R3*Vin)/(R1*R2 + R1*R3 + R2*R3 + L*R1*s + L*R2*s + C*R1*R2*R3*s + C*L*R1*R3
*s^2 + C*L*R2*R3*s^2)
Iin = (Vin - (R2*Vin*(C*L*R3*s^2 + L*s + R3))/(R1*R2 + R1*R3 + R2*R3 + L*R1*s + L*R2
*s + C*R1*R2*R3*s + C*L*R1*R3*s^2 + C*L*R2*R3*s^2))/R1
ans = (Vin*(R2 + R3 + L*s + C*L*R3*s^2 + C*R2*R3*s))/(R1*R2 + R1*R3 + R2*R3 + L*R1*s
+ L*R2*s + C*R1*R2*R3*s + C*L*R1*R3*s^2 + C*L*R2*R3*s^2)
H(s) = (R2 + R3 + L*s + C*L*R3*s^2 + C*R2*R3*s)/(R1*R2 + R1*R3 + R2*R3 + L*R1*s + L*
R2*s + C*R1*R2*R3*s + C*L*R1*R3*s^2 + C*L*R2*R3*s^2)
```

We can evaluate the transfer function at DC steady state ( $f = 0$ ), or any other frequency.

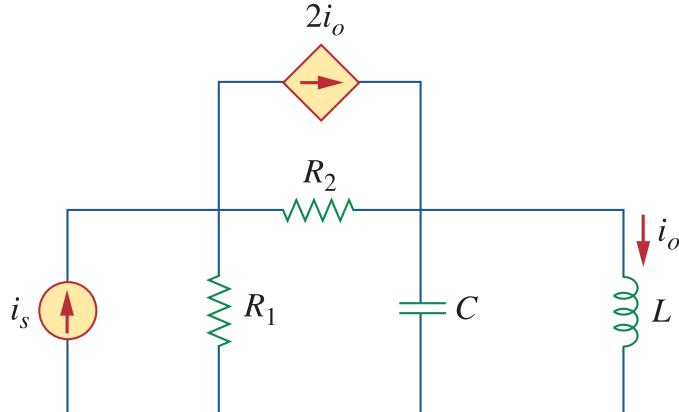
### %% Evaluate at f = 0 (DC steady state)

```
H(j*0)
```

which yields

```
ans = (R2 + R3)/(R1*R2 + R1*R3 + R2*R3)
```

Problem 10.12:



**%% Problem 10.12**

```

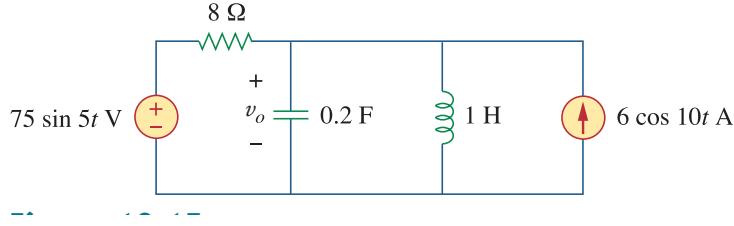
clear all
% Declare symbolic variables
syms Iin va vb i0 R1 R2 L C s
% Nodal analysis directly in solve()
[vb vb i0]=solve(va/R1 - Iin + 2*i0 + (va-vb)/R2 == 0, ...
vb/(L*s) + vb*C*s + (vb-va)/R2 - 2*i0 == 0, ...
i0 == vb/(L*s),va,vb, i0)
% Transfer function
H(s) = i0/Iin
%
```

Which yields:

```

va = (Iin*R1*(C*L*R2*s^2 + L*s - R2))/(R1 - R2 + L*s + C*L*R1*s^2 + C*L*R2*s^2)
vb = (Iin*L*R1*s)/(R1 - R2 + L*s + C*L*R1*s^2 + C*L*R2*s^2)
i0 = (Iin*R1)/(R1 - R2 + L*s + C*L*R1*s^2 + C*L*R2*s^2)
H(s) = R1/(R1 - R2 + L*s + C*L*R1*s^2 + C*L*R2*s^2)
>>
```

**Practice Problem 10.6:** (Two sources)



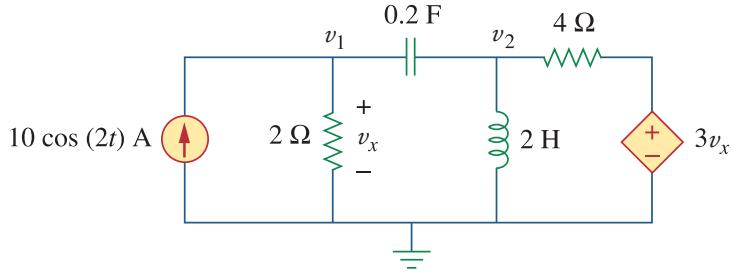
```
%% Practice Problem 10.6
clear all
% Declare symbolic variables
syms Iin Vin vo s
% Nodal analysis directly in solve()
[vo]=solve(-Iin + Vin/s + vo*0.2*s + (vo-Vin)/8 == 0, vo)
%
```

Which yields:

$$vo = \frac{(40*Iin*s - 40*Vin + 5*Vin*s)}{(8*s^2 + 5*s)}$$

We'll see later how to admit sinusoidal inputs (through superposition) to find the output for sinusoidal steady-state excitations.

### Practice Problem 10.1:



```
%% Practice Problem 10.1
clear all
% Declare symbolic variables
syms Iin vx v2 s
% Nodal analysis directly in solve()
[vx, v2]=solve(vx/2 - Iin + (vx-v2)*0.2*s == 0, ...
    (v2-3*vx)/4 + v2/(2*s) + (v2-vx)*0.2*s == 0,vx,v2)
% Find the transfer function
Hvx(s)=vx/Iin
```

Which yields:

$$\begin{aligned} vx &= (2*Iin*(4*s^2 + 5*s + 10))/(9*s + 10) \\ v2 &= (2*Iin*s*(4*s + 15))/(9*s + 10) \\ Hv(x)(s) &= (2*(4*s^2 + 5*s + 10))/(9*s + 10) \end{aligned}$$

**Homework: Ignore the authors directions.**

**Do these as Node Equation problems using  $s$ -domain impedance models:**

$$Z_R(s) = R, Z_C(s) = \frac{1}{sC}, \text{ and } Z_L(s) = s \cdot L.$$

**Replace all *independent sources* with symbolic sources.**

Chapter 9 # 40, 44, 46, 48

Chapter 10 # 1, 4, 5, 8, 9, 10

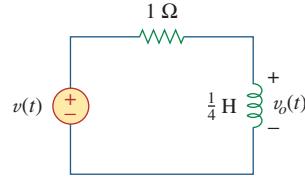
→ Homework deliverables MUST be a pdf file generated using a solver.

→ Examples of construction for Matlab, Python, and L<sup>A</sup>T<sub>E</sub>X are in the Pilot folder.

7. The techniques of voltage/current division, series/parallel combination of impedance/admittance, circuit reduction, and  $Y$ - $\Delta$  transformation all apply to ac circuit analysis.
8. AC circuits are applied in phase-shifters and bridges.

## Review Questions

- 9.1** Which of the following is *not* a right way to express the sinusoid  $A \cos \omega t$ ?
- $A \cos 2\pi ft$
  - $A \cos(2\pi t/T)$
  - $A \cos \omega(t - T)$
  - $A \sin(\omega t - 90^\circ)$
- 9.2** A function that repeats itself after fixed intervals is said to be:
- a phasor
  - harmonic
  - periodic
  - reactive
- 9.3** Which of these frequencies has the shorter period?
- 1 krad/s
  - 1 kHz
- 9.4** If  $v_1 = 30 \sin(\omega t + 10^\circ)$  and  $v_2 = 20 \sin(\omega t + 50^\circ)$ , which of these statements are true?
- $v_1$  leads  $v_2$
  - $v_2$  leads  $v_1$
  - $v_2$  lags  $v_1$
  - $v_1$  lags  $v_2$
  - $v_1$  and  $v_2$  are in phase
- 9.5** The voltage across an inductor leads the current through it by  $90^\circ$ .
- True
  - False
- 9.6** The imaginary part of impedance is called:
- resistance
  - admittance
  - susceptance
  - conductance
  - reactance
- 9.7** The impedance of a capacitor increases with increasing frequency.
- True
  - False
- 9.8** At what frequency will the output voltage  $v_o(t)$  in Fig. 9.39 be equal to the input voltage  $v(t)$ ?
- 0 rad/s
  - 1 rad/s
  - 4 rad/s
  - $\infty$  rad/s
  - none of the above



**Figure 9.39**

For Review Question 9.8.

- 9.9** A series  $RC$  circuit has  $|V_R| = 12$  V and  $|V_C| = 5$  V. The magnitude of the supply voltage is:
- 7 V
  - 7 V
  - 13 V
  - 17 V
- 9.10** A series  $RCL$  circuit has  $R = 30 \Omega$ ,  $X_C = 50 \Omega$ , and  $X_L = 90 \Omega$ . The impedance of the circuit is:
- $30 + j140 \Omega$
  - $30 + j40 \Omega$
  - $30 - j40 \Omega$
  - $-30 - j40 \Omega$
  - $-30 + j40 \Omega$

*Answers: 9.1d, 9.2c, 9.3b, 9.4b,d, 9.5a, 9.6e, 9.7b, 9.8d, 9.9c, 9.10b.*

## Problems

### Section 9.2 Sinusoids

- 9.1** Given the sinusoidal voltage  $v(t) = 50 \cos(30t + 10^\circ)$  V, find: (a) the amplitude  $V_m$ , (b) the period  $T$ , (c) the frequency  $f$ , and (d)  $v(t)$  at  $t = 10$  ms.
- 9.2** A current source in a linear circuit has

$$i_s = 15 \cos(25\pi t + 25^\circ) \text{ A}$$

- What is the amplitude of the current?
  - What is the angular frequency?
  - Find the frequency of the current.
  - Calculate  $i_s$  at  $t = 2$  ms.
- 9.3** Express the following functions in cosine form:
- $10 \sin(\omega t + 30^\circ)$
  - $-9 \sin(8t)$
  - $-20 \sin(\omega t + 45^\circ)$

- 9.4** Design a problem to help other students better understand sinusoids.

- 9.5** Given  $v_1 = 45 \sin(\omega t + 30^\circ)$  V and  $v_2 = 50 \cos(\omega t - 30^\circ)$  V, determine the phase angle between the two sinusoids and which one lags the other.
- 9.6** For the following pairs of sinusoids, determine which one leads and by how much.

(a)  $v(t) = 10 \cos(4t - 60^\circ)$  and  
 $i(t) = 4 \sin(4t + 50^\circ)$

(b)  $v_1(t) = 4 \cos(377t + 10^\circ)$  and  
 $v_2(t) = -20 \cos 377t$

(c)  $x(t) = 13 \cos 2t + 5 \sin 2t$  and  
 $y(t) = 15 \cos(2t - 11.8^\circ)$

### Section 9.3 Phasors

- 9.7** If  $f(\phi) = \cos\phi + j \sin\phi$ , show that  $f(\phi) = e^{j\phi}$ .
- 9.8** Calculate these complex numbers and express your results in rectangular form:

(a)  $\frac{60/45^\circ}{7.5 - j10} + j2$

(b)  $\frac{32/-20^\circ}{(6 - j8)(4 + j2)} + \frac{20}{-10 + j24}$

(c)  $20 + (16/-50^\circ)(5 + j12)$

- 9.9** Evaluate the following complex numbers and leave your results in polar form:

(a)  $5/30^\circ \left( 6 - j8 + \frac{3/60^\circ}{2 + j} \right)$

(b)  $\frac{(10/60^\circ)(35/-50^\circ)}{(2 + j6) - (5 + j)}$

- 9.10** Design a problem to help other students better understand phasors.

- 9.11** Find the phasors corresponding to the following signals:

(a)  $v(t) = 21 \cos(4t - 15^\circ)$  V

(b)  $i(t) = -8 \sin(10t + 70^\circ)$  mA

(c)  $v(t) = 120 \sin(10t - 50^\circ)$  V

(d)  $i(t) = -60 \cos(30t + 10^\circ)$  mA

- 9.12** Let  $\mathbf{X} = 4/40^\circ$  and  $\mathbf{Y} = 20/-30^\circ$ . Evaluate the following quantities and express your results in polar form:

(a)  $(\mathbf{X} + \mathbf{Y})\mathbf{X}^*$

(b)  $(\mathbf{X} - \mathbf{Y})^*$

(c)  $(\mathbf{X} + \mathbf{Y})/\mathbf{X}$

- 9.13** Evaluate the following complex numbers:

(a)  $\frac{2 + j3}{1 - j6} + \frac{7 - j8}{-5 + j11}$

(b)  $\frac{(5/10^\circ)(10/-40^\circ)}{(4/-80^\circ)(-6/50^\circ)}$

(c)  $\begin{vmatrix} 2 + j3 & -j2 \\ -j2 & 8 - j5 \end{vmatrix}$

- 9.14** Simplify the following expressions:

(a)  $\frac{(5 - j6) - (2 + j8)}{(-3 + j4)(5 - j) + (4 - j6)}$

(b)  $\frac{(240/75^\circ + 160/-30^\circ)(60 - j80)}{(67 + j84)(20/32^\circ)}$

(c)  $\left( \frac{10 + j20}{3 + j4} \right)^2 \sqrt{(10 + j5)(16 - j20)}$

- 9.15** Evaluate these determinants:

(a)  $\begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix}$

(b)  $\begin{vmatrix} 20/-30^\circ & -4/-10^\circ \\ 16/0^\circ & 3/45^\circ \end{vmatrix}$

(c)  $\begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix}$

- 9.16** Transform the following sinusoids to phasors:

(a)  $-20 \cos(4t + 135^\circ)$       (b)  $8 \sin(20t + 30^\circ)$

(c)  $20 \cos(2t) + 15 \sin(2t)$

- 9.17** Two voltages  $v_1$  and  $v_2$  appear in series so that their sum is  $v = v_1 + v_2$ . If  $v_1 = 10 \cos(50t - \pi/3)$  V and  $v_2 = 12 \cos(50t + 30^\circ)$  V, find  $v$ .

- 9.18** Obtain the sinusoids corresponding to each of the following phasors:

(a)  $\mathbf{V}_1 = 60/15^\circ$  V,  $\omega = 1$

(b)  $\mathbf{V}_2 = 6 + j8$  V,  $\omega = 40$

(c)  $\mathbf{I}_1 = 2.8e^{-j\pi/3}$  A,  $\omega = 377$

(d)  $\mathbf{I}_2 = -0.5 - j1.2$  A,  $\omega = 10^3$

- 9.19** Using phasors, find:

(a)  $3 \cos(20t + 10^\circ) - 5 \cos(20t - 30^\circ)$

(b)  $40 \sin 50t + 30 \cos(50t - 45^\circ)$

(c)  $20 \sin 400t + 10 \cos(400t + 60^\circ) - 5 \sin(400t - 20^\circ)$

- 9.20** A linear network has a current input  $7.5 \cos(10t + 30^\circ)$  A and a voltage output  $120 \cos(10t + 75^\circ)$  V. Determine the associated impedance.

**9.21** Simplify the following:

(a)  $f(t) = 5 \cos(2t + 15^\circ) - 4 \sin(2t - 30^\circ)$

(b)  $g(t) = 8 \sin t + 4 \cos(t + 50^\circ)$

(c)  $h(t) = \int_0^t (10 \cos 40t + 50 \sin 40t) dt$

**9.22** An alternating voltage is given by  $v(t) = 55 \cos(5t + 45^\circ)$  V. Use phasors to find

$$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^t v(t) dt$$

Assume that the value of the integral is zero at  $t = -\infty$ .

**9.23** Apply phasor analysis to evaluate the following:

(a)  $v = [110 \sin(20t + 30^\circ) + 220 \cos(20t - 90^\circ)]$  V

(b)  $i = [30 \cos(5t + 60^\circ) - 20 \sin(5t + 60^\circ)]$  A

**9.24** Find  $v(t)$  in the following integrodifferential equations using the phasor approach:

(a)  $v(t) + \int v dt = 10 \cos t$

(b)  $\frac{dv}{dt} + 5v(t) + 4 \int v dt = 20 \sin(4t + 10^\circ)$

**9.25** Using phasors, determine  $i(t)$  in the following equations:

(a)  $2\frac{di}{dt} + 3i(t) = 4 \cos(2t - 45^\circ)$

(b)  $10 \int i dt + \frac{di}{dt} + 6i(t) = 5 \cos(5t + 22^\circ)$  A

**9.26** The loop equation for a series RLC circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^t i dt = \cos 2t$$

Assuming that the value of the integral at  $t = -\infty$  is zero, find  $i(t)$  using the phasor method.

**9.27** A parallel RLC circuit has the node equation

$$\frac{dv}{dt} + 50v + 100 \int v dt = 110 \cos(377t - 10^\circ)$$

Determine  $v(t)$  using the phasor method. You may assume that the value of the integral at  $t = -\infty$  is zero.

## Section 9.4 Phasor Relationships for Circuit Elements

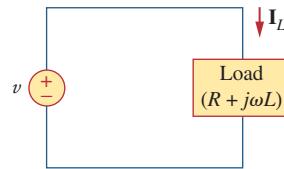
**9.28** Determine the current that flows through an 8- $\Omega$  resistor connected to a voltage source  $v_s = 110 \cos 377t$  V.

**9.29** What is the instantaneous voltage across a 2- $\mu$ F capacitor when the current through it is  $i = 4 \sin(10^6 t + 25^\circ)$  A?

**9.30** A voltage  $v(t) = 100 \cos(60t + 20^\circ)$  V is applied to a parallel combination of a 40-k $\Omega$  resistor and a 50- $\mu$ F capacitor. Find the steady-state currents through the resistor and the capacitor.

**9.31** A series RLC circuit has  $R = 80 \Omega$ ,  $L = 240$  mH, and  $C = 5$  mF. If the input voltage is  $v(t) = 10 \cos 2t$ , find the current flowing through the circuit.

**e&d 9.32** Using Fig. 9.40, design a problem to help other students better understand phasor relationships for circuit elements.

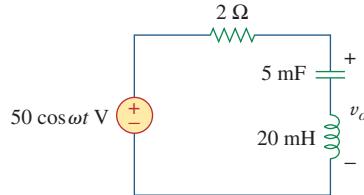


**Figure 9.40**

For Prob. 9.32.

**9.33** A series RL circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

**9.34** What value of  $\omega$  will cause the forced response,  $v_o$ , in Fig. 9.41 to be zero?

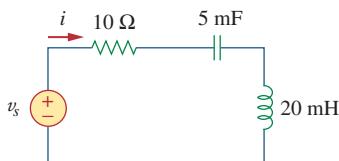


**Figure 9.41**

For Prob. 9.34.

## Section 9.5 Impedance and Admittance

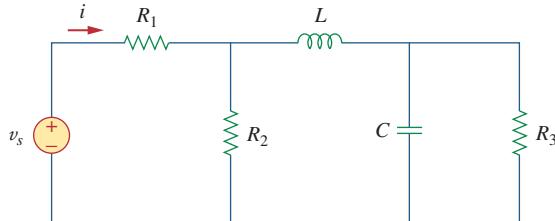
**9.35** Find current  $i$  in the circuit of Fig. 9.42, when  $v_s(t) = 50 \cos 200t$  V.



**Figure 9.42**

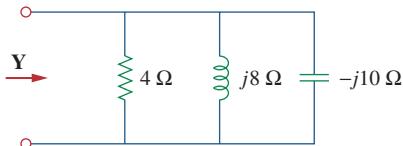
For Prob. 9.35.

- 9.36** Using Fig. 9.43, design a problem to help other students better understand impedance.

**Figure 9.43**

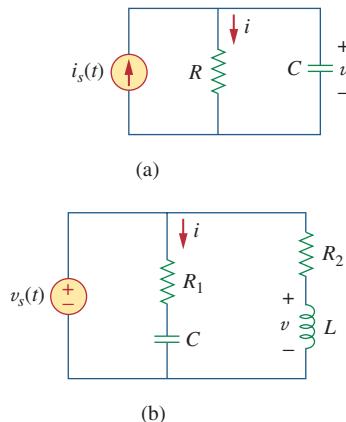
For Prob. 9.36.

- 9.37** Determine the admittance  $\mathbf{Y}$  for the circuit in Fig. 9.44.

**Figure 9.44**

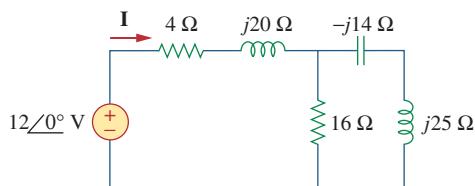
For Prob. 9.37.

- 9.38** Using Fig. 9.45, design a problem to help other students better understand admittance.

**Figure 9.45**

For Prob. 9.38.

- 9.39** For the circuit shown in Fig. 9.46, find  $Z_{eq}$  and use that to find current  $\mathbf{I}$ . Let  $\omega = 10 \text{ rad/s}$ .

**Figure 9.46**

For Prob. 9.39.

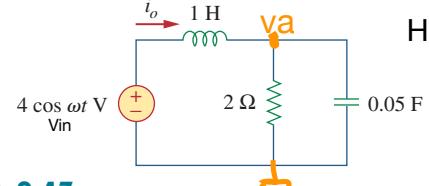
- 9.40** In the circuit of Fig. 9.47, find  $i_o$  when:

- (a)  $\omega = 1 \text{ rad/s}$     (b)  $\omega = 5 \text{ rad/s}$   
 (c)  $\omega = 10 \text{ rad/s}$

$$va = (va - Vin)/s + va/2 + va * 0.05 * s = 0$$

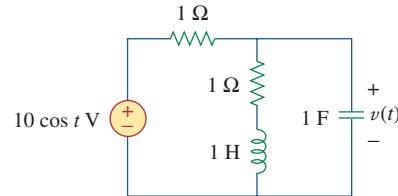
$$io = (Vin - va)/s - io = 0$$

$$Hio(s) = io/Vin$$

**Figure 9.47**

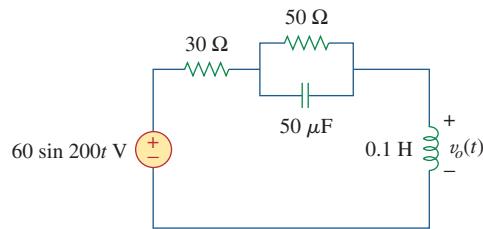
For Prob. 9.40.

- 9.41** Find  $v(t)$  in the RLC circuit of Fig. 9.48.

**Figure 9.48**

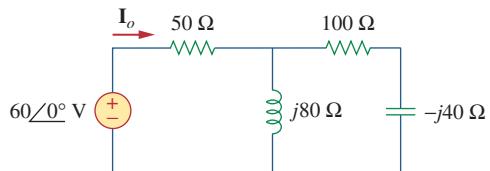
For Prob. 9.41.

- 9.42** Calculate  $v_o(t)$  in the circuit of Fig. 9.49.

**Figure 9.49**

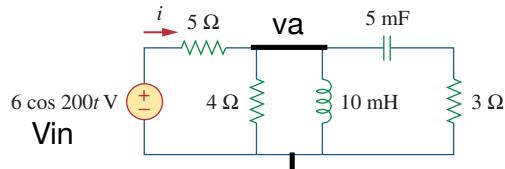
For Prob. 9.42.

- 9.43** Find current  $I_o$  in the circuit shown in Fig. 9.50.

**Figure 9.50**

For Prob. 9.43.

- 9.44** Calculate  $i(t)$  in the circuit of Fig. 9.51.

**Figure 9.51**

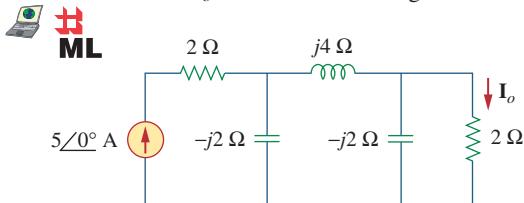
For Prob. 9.44.

$$va: (va - Vin)/5 + va/4 + va/(10e-3 * s) + va/(1/(5e-3s) + 3)$$

$$i: (Vin - va)/5 - i = 0$$

$$Hi(s) = i/Vin$$

**9.45** Find current  $I_o$  in the network of Fig. 9.52.



**Figure 9.52**

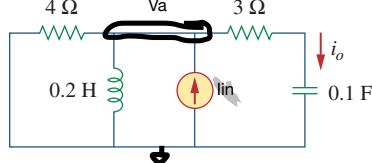
For Prob. 9.45

**9.46** If  $i_s = 5 \cos(10t + 40^\circ)$  A in the circuit of Fig. 9.53, find  $i_o$ .

$$\text{va: } \text{va}/4 + \text{va}/(0.2s) - \text{lin} + \text{va}/(3 + (1/0.1s)) = 0$$

$$\text{io: } \text{va}/(3 + (1/0.1s)) - \text{io} = 0$$

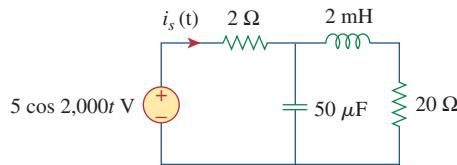
$$\text{Hio}(s) = \text{io}/\text{vin}$$



**Figure 9.53**

For Prob. 9.46.

**9.47** In the circuit of Fig. 9.54, determine the value of  $i_s(t)$ .



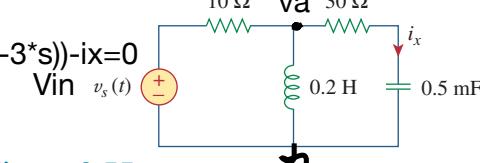
**Figure 9.54**

For Prob. 9.47

**9.48** Given that  $v_s(t) = 20 \sin(100t - 40^\circ)$  in Fig. 9.55, determine  $i_x(t)$ .

$$\text{va: } (\text{va} - \text{vin})/10 + \text{va}/(0.2s) + \text{va}/(30 + 1/(0.5e^{-3}s)) = 0$$

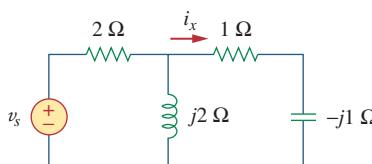
$$\text{ix: } \text{va}/(30 + 1/(0.5e^{-3}s)) - \text{ix} = 0$$



**Figure 9.55**

For Prob. 9.48.

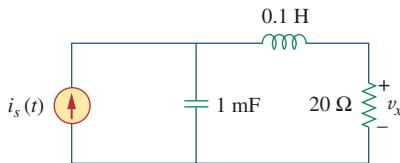
**9.49** Find  $v_s(t)$  in the circuit of Fig. 9.56 if the current  $i_x$  through the 1-Ω resistor is  $0.5 \sin 200t$  A.



**Figure 9.56**

For Prob. 9.49.

**9.50** Determine  $v_x$  in the circuit of Fig. 9.57. Let  $i_s(t) = 5 \cos(100t + 40^\circ)$  A.

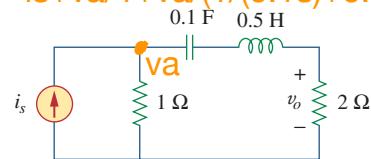


**Figure 9.57**

For Prob. 9.50.

**9.51** If the voltage  $v_o$  across the 2-Ω resistor in the circuit of Fig. 9.58 is  $10 \cos 2t$  V, obtain  $i_s$ .

$$\text{va: } -is + \text{va}/1 + \text{va}/(1/(0.1s) + 0.5s + 2) = 0$$



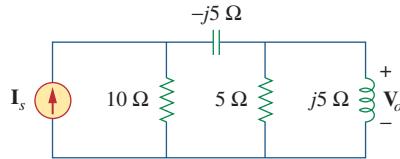
**Figure 9.58**

For Prob. 9.51.

$$\text{vo: } \text{va}/(1/(0.1s) + 0.5s + 2)^*2 - \text{vo} = 0$$

$$\text{Hvo}(s) = \text{vo}/is$$

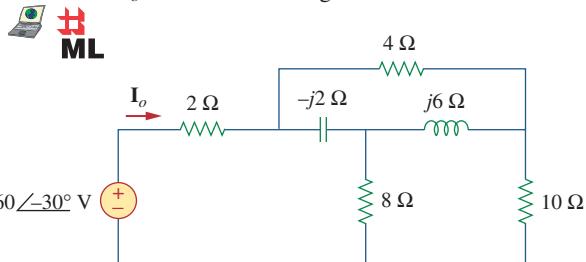
**9.52** If  $\text{V}_o = 8 \angle 30^\circ$  V in the circuit of Fig. 9.59, find  $\text{I}_s$ .



**Figure 9.59**

For Prob. 9.52.

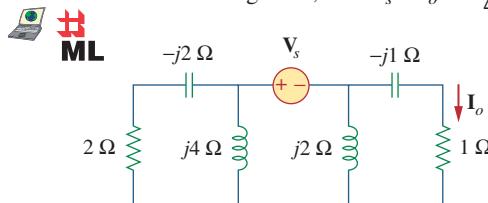
**9.53** Find  $\text{I}_o$  in the circuit of Fig. 9.60.



**Figure 9.60**

For Prob. 9.53.

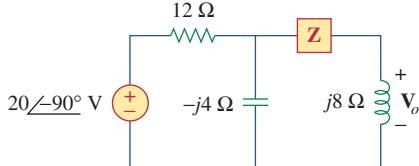
**9.54** In the circuit of Fig. 9.61, find  $\text{V}_s$  if  $\text{I}_o = 2 \angle 0^\circ$  A.



**Figure 9.61**

For Prob. 9.54.

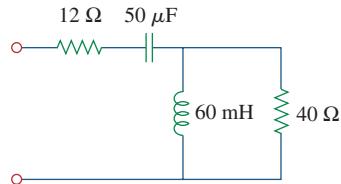
- \*9.55 Find  $Z$  in the network of Fig. 9.62, given that  
**ML**  
 $\text{V}_o = 4\angle 0^\circ \text{ V}$ .

**Figure 9.62**

For Prob. 9.55.

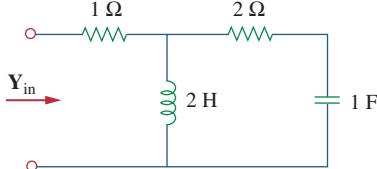
### Section 9.7 Impedance Combinations

- 9.56 At  $\omega = 377 \text{ rad/s}$ , find the input impedance of the circuit shown in Fig. 9.63.

**Figure 9.63**

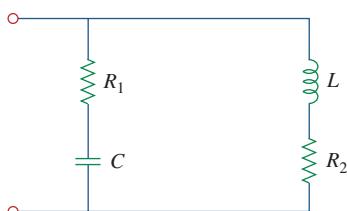
For Prob. 9.56.

- 9.57 At  $\omega = 1 \text{ rad/s}$ , obtain the input admittance in the circuit of Fig. 9.64.

**Figure 9.64**

For Prob. 9.57.

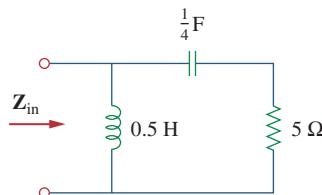
- 9.58 Using Fig. 9.65, design a problem to help other  
**eTd** students better understand impedance combinations.

**Figure 9.65**

For Prob. 9.58.

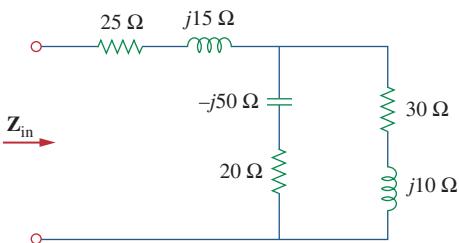
\* An asterisk indicates a challenging problem.

- 9.59 For the network in Fig. 9.66, find  $Z_{in}$ . Let  $\omega = 10 \text{ rad/s}$ .

**Figure 9.66**

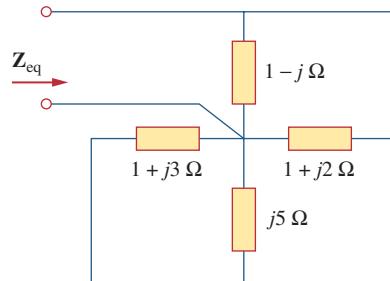
For Prob. 9.59.

- 9.60 Obtain  $Z_{in}$  for the circuit in Fig. 9.67.

**Figure 9.67**

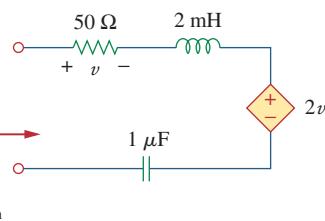
For Prob. 9.60.

- 9.61 Find  $Z_{eq}$  in the circuit of Fig. 9.68.

**Figure 9.68**

For Prob. 9.61.

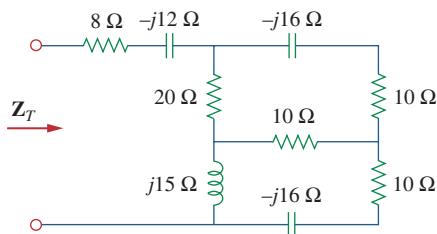
- 9.62 For the circuit in Fig. 9.69, find the input impedance  $Z_{in}$  at 10 krad/s.

**Figure 9.69**

For Prob. 9.62.

**9.63** For the circuit in Fig. 9.70, find the value of  $Z_T$ .

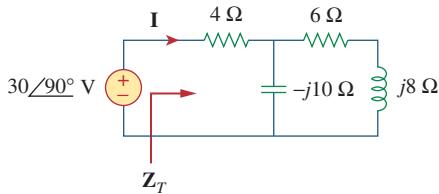
**ML**



**Figure 9.70**

For Prob. 9.63.

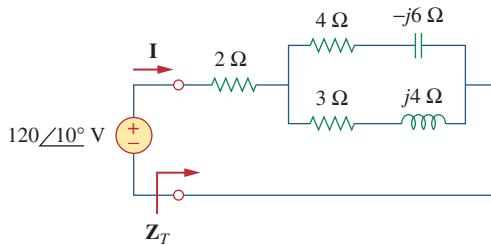
**9.64** Find  $Z_T$  and  $\mathbf{I}$  in the circuit in Fig. 9.71.



**Figure 9.71**

For Prob. 9.64.

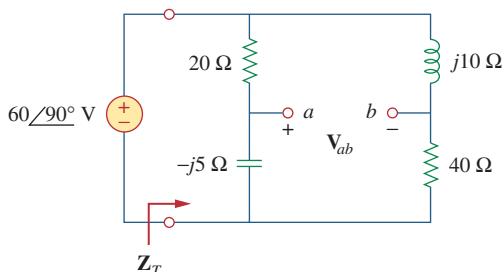
**9.65** Determine  $Z_T$  and  $\mathbf{I}$  for the circuit in Fig. 9.72.



**Figure 9.72**

For Prob. 9.65.

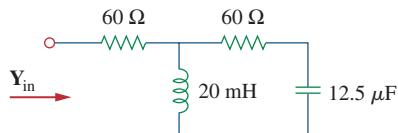
**9.66** For the circuit in Fig. 9.73, calculate  $Z_T$  and  $\mathbf{V}_{ab}$ .



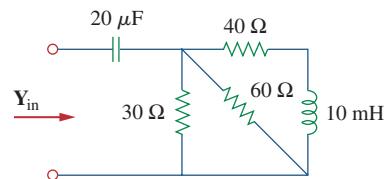
**Figure 9.73**

For Prob. 9.66.

**9.67** At  $\omega = 10^3$  rad/s, find the input admittance of each of the circuits in Fig. 9.74.



(a)

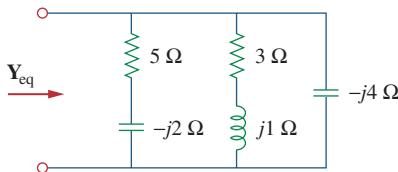


(b)

**Figure 9.74**

For Prob. 9.67.

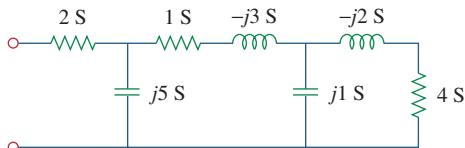
**9.68** Determine  $\mathbf{Y}_{eq}$  for the circuit in Fig. 9.75.



**Figure 9.75**

For Prob. 9.68.

**9.69** Find the equivalent admittance  $\mathbf{Y}_{eq}$  of the circuit in Fig. 9.76.

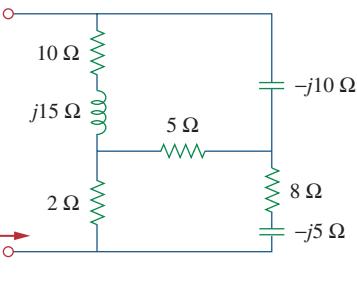


**Figure 9.76**

For Prob. 9.69.

**9.70** Find the equivalent impedance of the circuit in Fig. 9.77.

**ML**

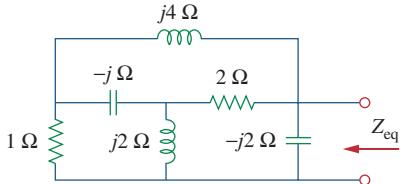


**Figure 9.77**

For Prob. 9.70.

- 9.71** Obtain the equivalent impedance of the circuit in Fig. 9.78.

**ML**

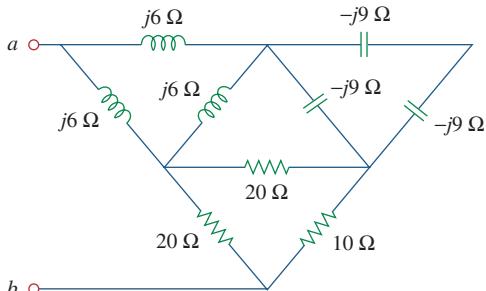


**Figure 9.78**

For Prob. 9.71.

- 9.72** Calculate the value of  $Z_{ab}$  in the network of Fig. 9.79.

**ML**

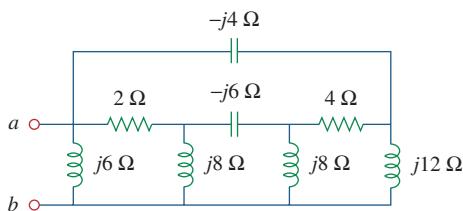


**Figure 9.79**

For Prob. 9.72.

- 9.73** Determine the equivalent impedance of the circuit in Fig. 9.80.

**ML**



**Figure 9.80**

For Prob. 9.73.

## Section 9.8 Applications

- 9.74** Design an  $RL$  circuit to provide a  $90^\circ$  leading phase shift.

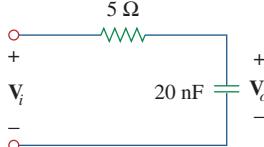
- 9.75** Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

- 9.76** For the following pairs of signals, determine if  $v_1$  leads or lags  $v_2$  and by how much.

- $v_1 = 10 \cos(5t - 20^\circ)$ ,  $v_2 = 8 \sin 5t$
- $v_1 = 19 \cos(2t + 90^\circ)$ ,  $v_2 = 6 \sin 2t$
- $v_1 = -4 \cos 10t$ ,  $v_2 = 15 \sin 10t$

- 9.77** Refer to the  $RC$  circuit in Fig. 9.81.

- Calculate the phase shift at 2 MHz.
- Find the frequency where the phase shift is  $45^\circ$ .

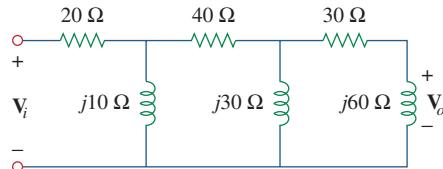


**Figure 9.81**

For Prob. 9.77.

- 9.78** A coil with impedance  $8 + j6 \Omega$  is connected in series with a capacitive reactance  $X$ . The series combination is connected in parallel with a resistor  $R$ . Given that the equivalent impedance of the resulting circuit is  $5 \angle 0^\circ \Omega$ , find the value of  $R$  and  $X$ .

- 9.79** (a) Calculate the phase shift of the circuit in Fig. 9.82.  
 (b) State whether the phase shift is leading or lagging (output with respect to input).  
 (c) Determine the magnitude of the output when the input is 120 V.

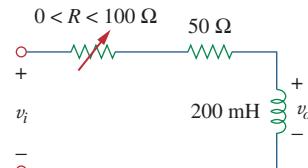


**Figure 9.82**

For Prob. 9.79.

- 9.80** Consider the phase-shifting circuit in Fig. 9.83. Let  $V_i = 120 \text{ V}$  operating at 60 Hz. Find:

- $V_o$  when  $R$  is maximum
- $V_o$  when  $R$  is minimum
- the value of  $R$  that will produce a phase shift of  $45^\circ$



**Figure 9.83**

For Prob. 9.80.

- 9.81** The ac bridge in Fig. 9.37 is balanced when  $R_1 = 400 \Omega$ ,  $R_2 = 600 \Omega$ ,  $R_3 = 1.2 \text{ k}\Omega$ , and  $C_2 = 0.3 \mu\text{F}$ . Find  $R_x$  and  $C_x$ . Assume  $R_2$  and  $C_2$  are in series.

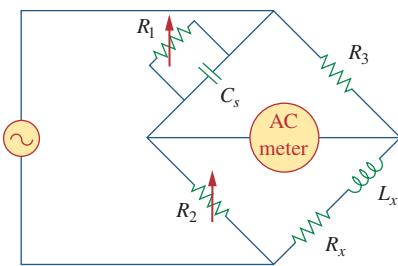
- 9.82** A capacitance bridge balances when  $R_1 = 100 \Omega$ ,  $R_2 = 2 \text{ k}\Omega$ , and  $C_s = 40 \mu\text{F}$ . What is  $C_x$ , the capacitance of the capacitor under test?

- 9.83** An inductive bridge balances when  $R_1 = 1.2 \text{ k}\Omega$ ,  $R_2 = 500 \Omega$ , and  $L_s = 250 \text{ mH}$ . What is the value of  $L_x$ , the inductance of the inductor under test?

- 9.84** The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance  $C_s$ . Show that when the bridge is balanced,

$$L_x = R_2 R_3 C_s \quad \text{and} \quad R_x = \frac{R_2}{R_1} R_3$$

Find  $L_x$  and  $R_x$  for  $R_1 = 40 \text{ k}\Omega$ ,  $R_2 = 1.6 \text{ k}\Omega$ ,  $R_3 = 4 \text{ k}\Omega$ , and  $C_s = 0.45 \mu\text{F}$ .

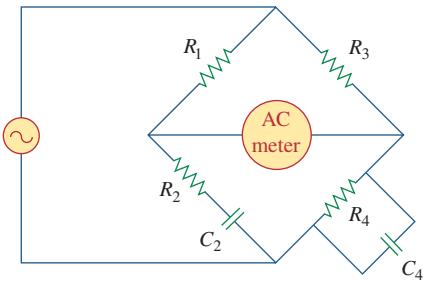


**Figure 9.84**

Maxwell bridge; For Prob. 9.84.

- 9.85** The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi \sqrt{R_2 R_4 C_2 C_4}}$$

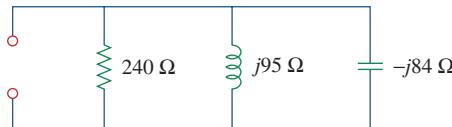


**Figure 9.85**

Wien bridge; For Prob. 9.85.

## Comprehensive Problems

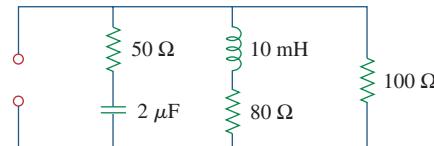
- 9.86** The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?



**Figure 9.86**

For Prob. 9.86.

- 9.87** The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

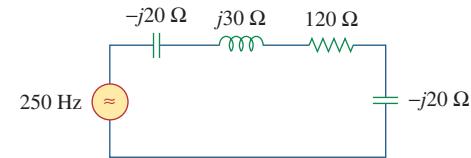


**Figure 9.87**

For Prob. 9.87.

- 9.88** A series audio circuit is shown in Fig. 9.88.

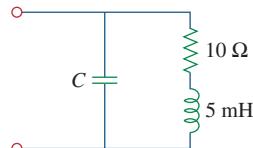
- What is the impedance of the circuit?
- If the frequency were halved, what would be the impedance of the circuit?



**Figure 9.88**

For Prob. 9.88.

- 9.89** An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor  $C$  across the series combination so that the net impedance is resistive at a frequency of 2 kHz.



**Figure 9.89**

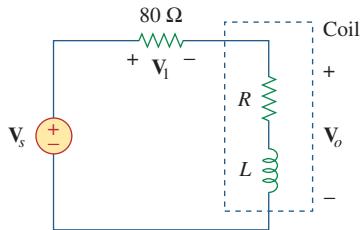
For Prob. 9.89.

- 9.90** An industrial coil is modeled as a series combination of an inductance  $L$  and resistance  $R$ , as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following

measurements are taken at 60 Hz when the circuit operates in the steady state:

$$|V_s| = 145 \text{ V}, \quad |V_1| = 50 \text{ V}, \quad |V_o| = 110 \text{ V}$$

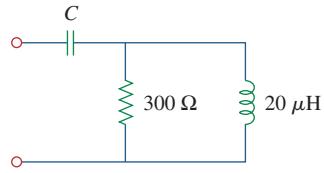
Use these measurements to determine the values of  $L$  and  $R$ .



**Figure 9.90**

For Prob. 9.90.

- 9.91** Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of  $C$ ?



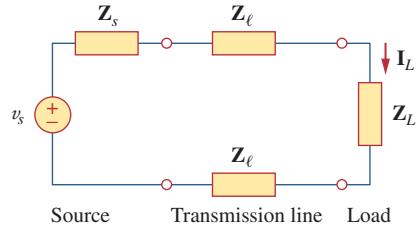
**Figure 9.91**

For Prob. 9.91.

- 9.92** A transmission line has a series impedance of  $Z = 100\angle 75^\circ \Omega$  and a shunt admittance of  $\mathbf{Y} = 450\angle 48^\circ \mu\text{S}$ . Find: (a) the characteristic impedance  $Z_o = \sqrt{Z/Y}$ , (b) the propagation constant  $\gamma = \sqrt{ZY}$ .

- 9.93** A power transmission system is modeled as shown in Fig. 9.92. Given the source voltage and circuit elements

$$\begin{aligned} V_s &= 115\angle 0^\circ \text{ V}, && \text{source impedance} \\ Z_s &= (1 + j0.5) \Omega, && \text{line impedance} \\ Z_t &= (0.4 + j0.3) \Omega, && \text{and load impedance} \\ Z_L &= (23.2 + j18.9) \Omega, && \text{find the load current } \mathbf{I}_L. \end{aligned}$$



**Figure 9.92**

For Prob. 9.93.

In the Wien-bridge oscillator circuit in Fig. 10.42, let  $R_1 = R_2 = 2.5 \text{ k}\Omega$ ,  $C_1 = C_2 = 1 \text{ nF}$ . Determine the frequency  $f_o$  of the oscillator.

### Practice Problem 10.16

**Answer:** 63.66 kHz.

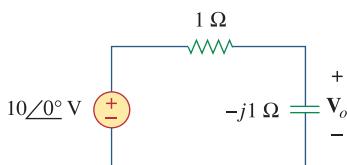
## 10.10 Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source  $\mathbf{V}_{\text{Th}}$  in series with the Thevenin impedance  $\mathbf{Z}_{\text{Th}}$ .
5. The Norton equivalent of an ac circuit consists of a current source  $\mathbf{I}_N$  in parallel with the Norton impedance  $\mathbf{Z}_N (= \mathbf{Z}_{\text{Th}})$ .
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

## Review Questions

**10.1** The voltage  $\mathbf{V}_o$  across the capacitor in Fig. 10.43 is:

- (a)  $5\angle 0^\circ \text{ V}$       (b)  $7.071\angle 45^\circ \text{ V}$   
 (c)  $7.071\angle -45^\circ \text{ V}$       (d)  $5\angle -45^\circ \text{ V}$

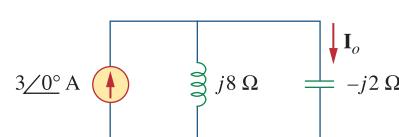


**Figure 10.43**

For Review Question 10.1.

**10.2** The value of the current  $\mathbf{I}_o$  in the circuit of Fig. 10.44 is:

- (a)  $4\angle 0^\circ \text{ A}$       (b)  $2.4\angle -90^\circ \text{ A}$   
 (c)  $0.6\angle 0^\circ \text{ A}$       (d)  $-1 \text{ A}$

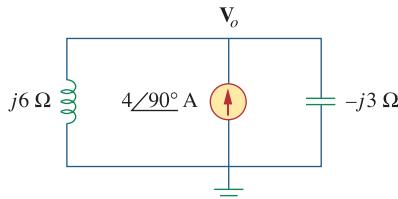


**Figure 10.44**

For Review Question 10.2.

- 10.3** Using nodal analysis, the value of  $V_o$  in the circuit of Fig. 10.45 is:

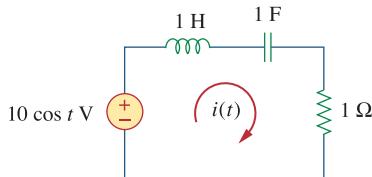
- (a) -24 V      (b) -8 V  
 (c) 8 V      (d) 24 V

**Figure 10.45**

For Review Question 10.3.

- 10.4** In the circuit of Fig. 10.46, current  $i(t)$  is:

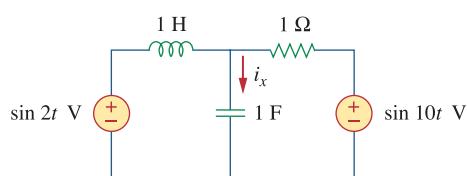
- (a)  $10 \cos t$  A      (b)  $10 \sin t$  A      (c)  $5 \cos t$  A  
 (d)  $5 \sin t$  A      (e)  $4.472 \cos(t - 63.43^\circ)$  A

**Figure 10.46**

For Review Question 10.4.

- 10.5** Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current  $i_x(t)$  can be obtained by:

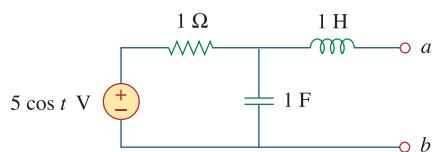
- (a) source transformation  
 (b) the superposition theorem  
 (c) PSpice

**Figure 10.47**

For Review Question 10.5.

- 10.6** For the circuit in Fig. 10.48, the Thevenin impedance at terminals  $a-b$  is:

- (a)  $1 \Omega$       (b)  $0.5 - j0.5 \Omega$   
 (c)  $0.5 + j0.5 \Omega$       (d)  $1 + j2 \Omega$   
 (e)  $1 - j2 \Omega$

**Figure 10.48**

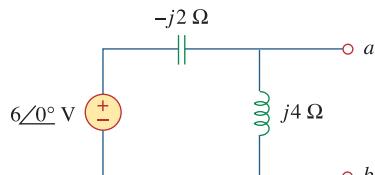
For Review Questions 10.6 and 10.7.

- 10.7** In the circuit of Fig. 10.48, the Thevenin voltage at terminals  $a-b$  is:

- (a)  $3.535 \angle -45^\circ$  V      (b)  $3.535 \angle 45^\circ$  V  
 (c)  $7.071 \angle -45^\circ$  V      (d)  $7.071 \angle 45^\circ$  V

- 10.8** Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals  $a-b$  is:

- (a)  $-j4 \Omega$       (b)  $-j2 \Omega$   
 (c)  $j2 \Omega$       (d)  $j4 \Omega$

**Figure 10.49**

For Review Questions 10.8 and 10.9.

- 10.9** The Norton current at terminals  $a-b$  in the circuit of Fig. 10.49 is:

- (a)  $1 \angle 0^\circ$  A      (b)  $1.5 \angle -90^\circ$  A  
 (c)  $1.5 \angle 90^\circ$  A      (d)  $3 \angle 90^\circ$  A

- 10.10** PSpice can handle a circuit with two independent sources of different frequencies.

- (a) True      (b) False

*Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.*

## Problems

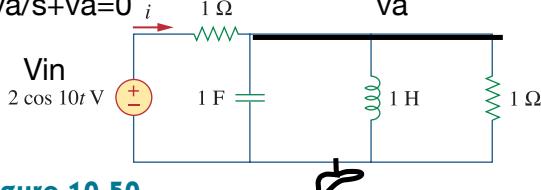
**Section 10.2 Nodal Analysis**

**10.1** Determine  $i$  in the circuit of Fig. 10.50.

$$va:(va-Vin)+va*s+va/s+va=0$$

$$i: Vin-va-i=0$$

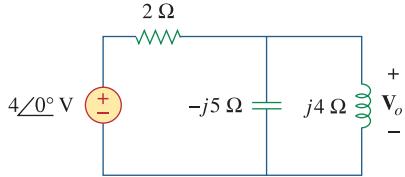
$$Hi(s):i/Vin$$



**Figure 10.50**

For Prob. 10.1.

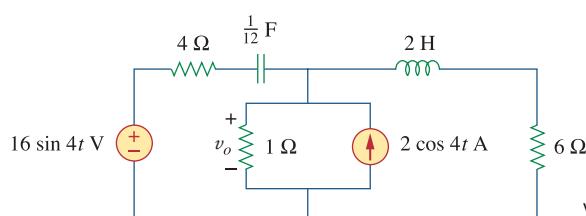
**10.2** Using Fig. 10.51, design a problem to help other students better understand nodal analysis.



**Figure 10.51**

For Prob. 10.2.

**10.3** Determine  $v_o$  in the circuit of Fig. 10.52.



**Figure 10.52**

For Prob. 10.3.

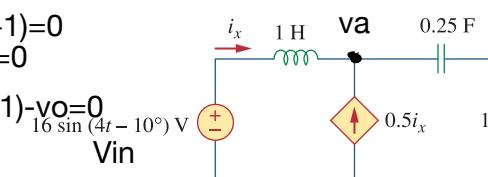
**10.4** Compute  $v_o(t)$  in the circuit of Fig. 10.53.

$$+va/(1/(0.25s)+1)=0$$

$$ix: (Vin-va)/s-ix=0$$

$$vo: va/(1/(0.25s)+1)-vo=0$$

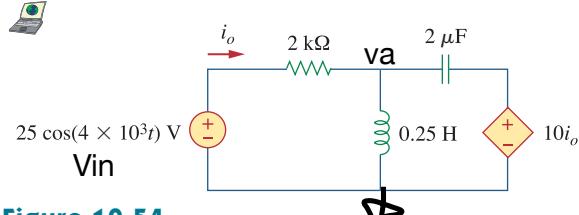
$$Hvo(s)=vo/Vin$$



**Figure 10.53**

For Prob. 10.4.

**10.5** Find  $i_o$  in the circuit of Fig. 10.54.



**Figure 10.54**

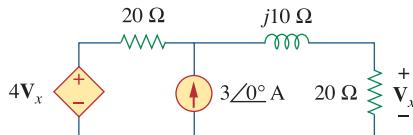
For Prob. 10.5.

$$va:(va-Vin)/2000+va/(0.25*s)+(va-10*i_o)2e-6*s$$

$$io:(Vin-va)/2000-io=0$$

$$Hio(s)=io/Vin$$

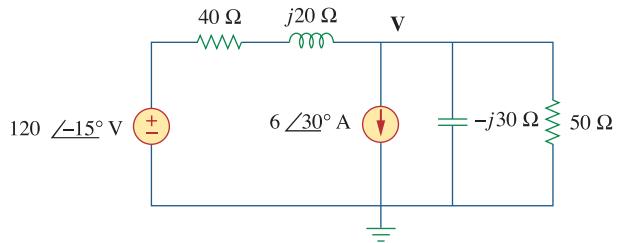
**10.6** Determine  $V_x$  in Fig. 10.55.



**Figure 10.55**

For Prob. 10.6.

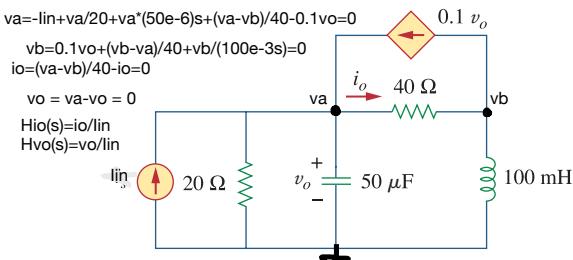
**10.7** Use nodal analysis to find  $V$  in the circuit of Fig. 10.56.



**Figure 10.56**

For Prob. 10.7.

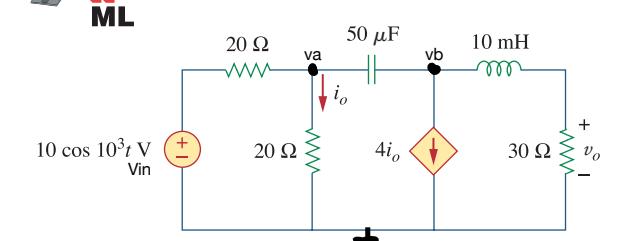
**10.8** Use nodal analysis to find current  $i_o$  in the circuit of Fig. 10.57. Let  $i_s = 6 \cos(200t + 15^\circ)$  A.



**Figure 10.57**

For Prob. 10.8.

**10.9** Use nodal analysis to find  $v_o$  in the circuit of Fig. 10.58.



**Figure 10.58**

For Prob. 10.9.

$$va:(va-Vin)/20+va/20+(va-vb)*(50e-6)s=0$$

$$vb:(vb-vb)/(50e-6)s+4io+vb/(10e-3*s+30)=0$$

$$io: va/20-io=0$$

$$vo:vb/(10e-3*s+30)-vo=0$$

$$Hvo(s)=vo/Vin$$

$$va - lin + va/(50e-3*s) + (va - vo)2e-6*s = 0$$

$$vo: 0.1vx + vo/4000 + (vo - va)2e-6*s = 0$$

$$vx: va/(50e-3*s) - vx = 0$$

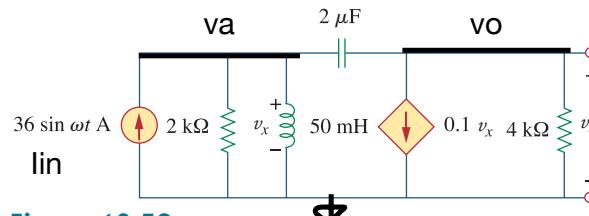
$$Hvo(s) = vo/lin$$

444

Chapter 10

Sinusoidal Steady-State Analysis

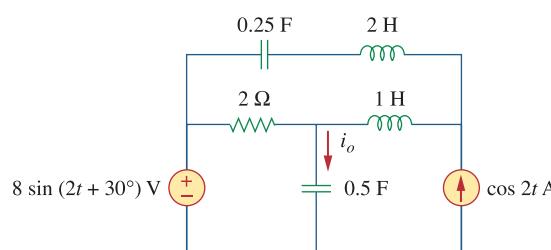
- ML** 10.10 Use nodal analysis to find  $v_o$  in the circuit of Fig. 10.59. Let  $\omega = 2$  krad/s.



**Figure 10.59**

For Prob. 10.10.

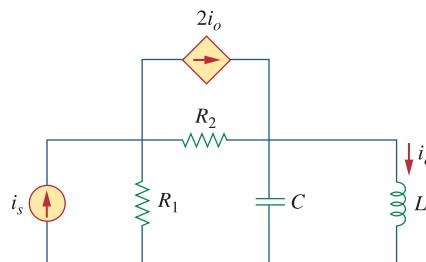
- ML** 10.11 Using nodal analysis, find  $i_o(t)$  in the circuit in Fig. 10.60.



**Figure 10.60**

For Prob. 10.11.

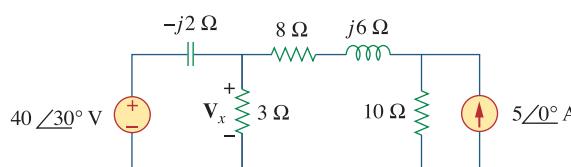
- 10.12 Using Fig. 10.61, design a problem to help other students better understand nodal analysis.



**Figure 10.61**

For Prob. 10.12.

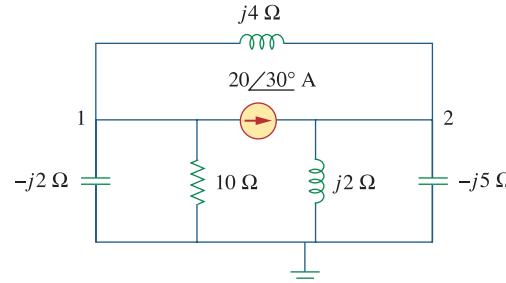
- ML** 10.13 Determine  $V_x$  in the circuit of Fig. 10.62 using any method of your choice.



**Figure 10.62**

For Prob. 10.13.

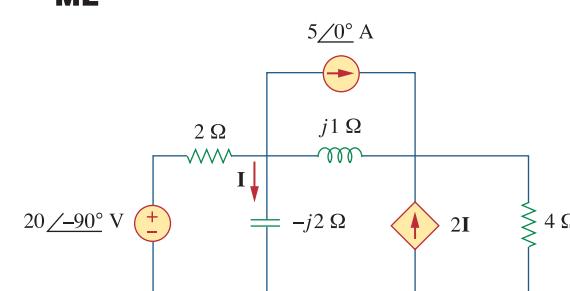
- ML** 10.14 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.



**Figure 10.63**

For Prob. 10.14.

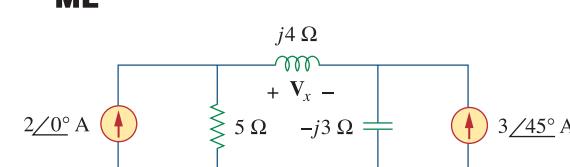
- ML** 10.15 Solve for the current  $I$  in the circuit of Fig. 10.64 using nodal analysis.



**Figure 10.64**

For Prob. 10.15.

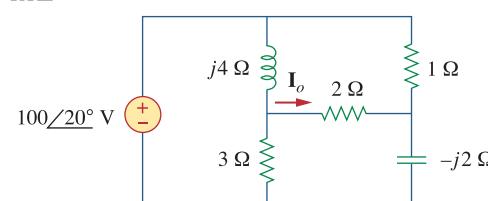
- ML** 10.16 Use nodal analysis to find  $V_x$  in the circuit shown in Fig. 10.65.



**Figure 10.65**

For Prob. 10.16.

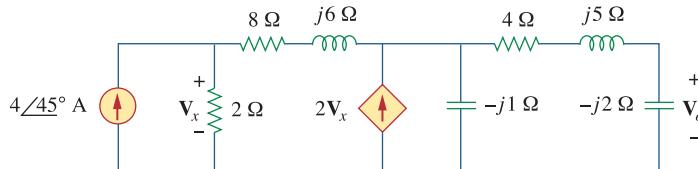
- ML** 10.17 By nodal analysis, obtain current  $I_o$  in the circuit of Fig. 10.66.



**Figure 10.66**

For Prob. 10.17.

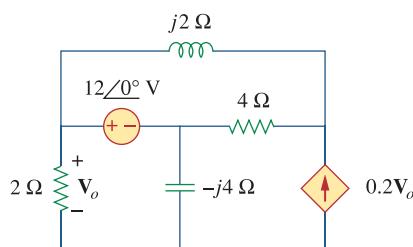
**10.18** Use nodal analysis to obtain  $\mathbf{V}_o$  in the circuit of Fig. 10.67 below.



**Figure 10.67**

For Prob. 10.18.

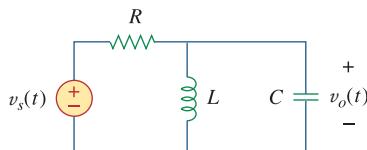
**10.19** Obtain  $\mathbf{V}_o$  in Fig. 10.68 using nodal analysis.



**Figure 10.68**

For Prob. 10.19.

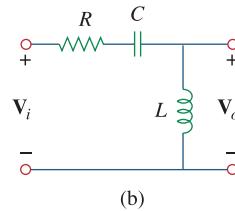
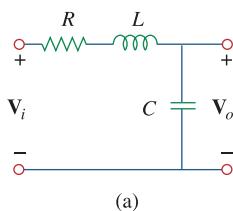
**10.20** Refer to Fig. 10.69. If  $v_s(t) = V_m \sin \omega t$  and  $v_o(t) = A \sin(\omega t + \phi)$ , derive the expressions for  $A$  and  $\phi$ .



**Figure 10.69**

For Prob. 10.20.

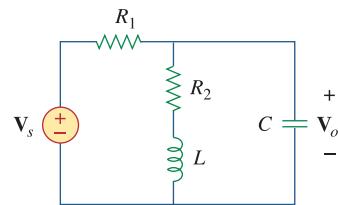
**10.21** For each of the circuits in Fig. 10.70, find  $\mathbf{V}_o/\mathbf{V}_i$  for  $\omega = 0$ ,  $\omega \rightarrow \infty$ , and  $\omega^2 = 1/LC$ .



**Figure 10.70**

For Prob. 10.21.

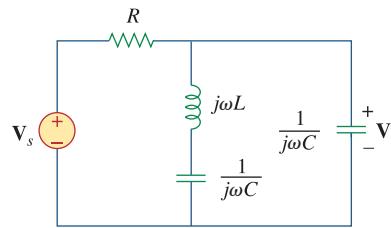
**10.22** For the circuit in Fig. 10.71, determine  $\mathbf{V}_o/\mathbf{V}_s$ .



**Figure 10.71**

For Prob. 10.22.

**10.23** Using nodal analysis obtain  $\mathbf{V}$  in the circuit of Fig. 10.72.



**Figure 10.72**

For Prob. 10.23.

### Section 10.3 Mesh Analysis

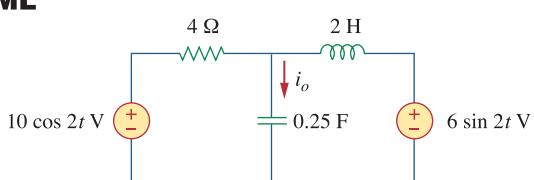
**10.24** Design a problem to help other students better understand mesh analysis.



**10.25** Solve for  $i_o$  in Fig. 10.73 using mesh analysis.



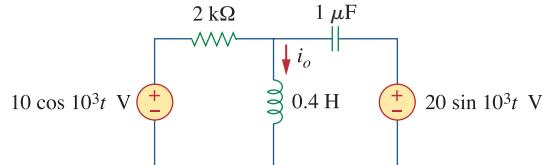
**ML**



**Figure 10.73**

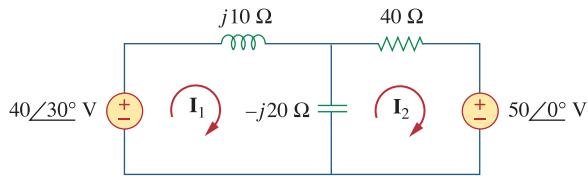
For Prob. 10.25.

- 10.26** Use mesh analysis to find current  $i_o$  in the circuit of Fig. 10.74.

**Figure 10.74**

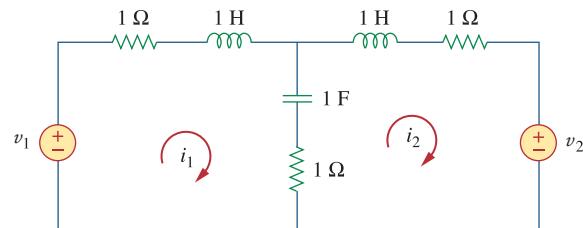
For Prob. 10.26.

- 10.27** Using mesh analysis, find  $\mathbf{I}_1$  and  $\mathbf{I}_2$  in the circuit of Fig. 10.75.

**ML****Figure 10.75**

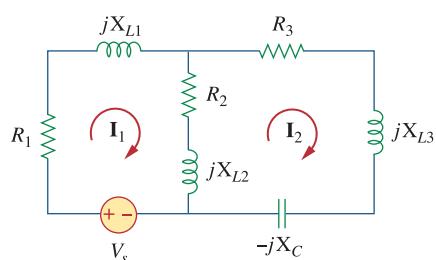
For Prob. 10.27.

- 10.28** In the circuit of Fig. 10.76, determine the mesh currents  $i_1$  and  $i_2$ . Let  $v_1 = 10 \cos 4t$  V and  $v_2 = 20 \cos(4t - 30^\circ)$  V.

**Figure 10.76**

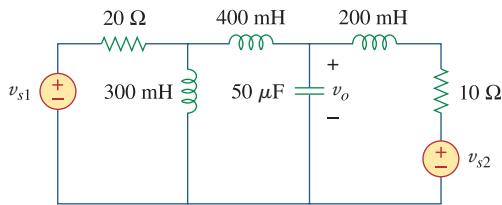
For Prob. 10.28.

- 10.29** Using Fig. 10.77, design a problem to help other **end** students better understand mesh analysis.

**Figure 10.77**

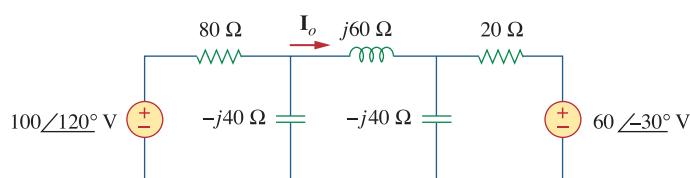
For Prob. 10.29.

- 10.30** Use mesh analysis to find  $v_o$  in the circuit of Fig. 10.78. Let  $v_{s1} = 120 \cos(100t + 90^\circ)$  V,  $v_{s2} = 80 \cos 100t$  V.

**Figure 10.78**

For Prob. 10.30.

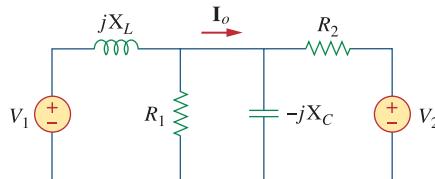
- 10.31** Use mesh analysis to determine current  $\mathbf{I}_o$  in the **end** circuit of Fig. 10.79 below.

**ML****Figure 10.79**

For Prob. 10.31.



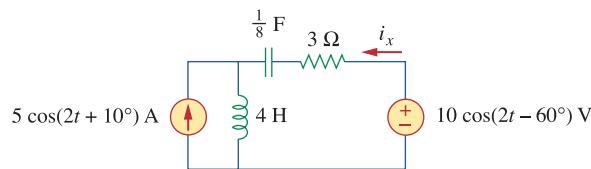
- 10.42** Using Fig. 10.87, design a problem to help other students better understand the superposition theorem.



**Figure 10.87**

For Prob. 10.42.

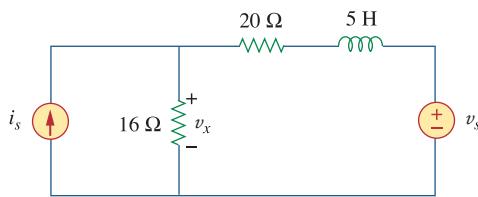
- 10.43** Using the superposition principle, find  $i_x$  in the circuit of Fig. 10.88.



**Figure 10.88**

For Prob. 10.43.

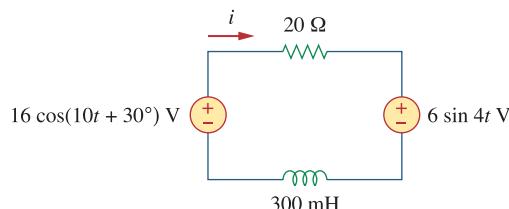
- 10.44** Use the superposition principle to obtain  $v_x$  in the circuit of Fig. 10.89. Let  $v_s = 50 \sin 2t$  V and  $i_s = 12 \cos(6t + 10^\circ)$  A.



**Figure 10.89**

For Prob. 10.44.

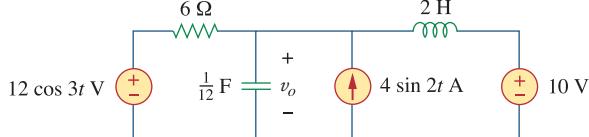
- 10.45** Use superposition to find  $i(t)$  in the circuit of Fig. 10.90.



**Figure 10.90**

For Prob. 10.45.

- 10.46** Solve for  $v_o(t)$  in the circuit of Fig. 10.91 using the superposition principle.

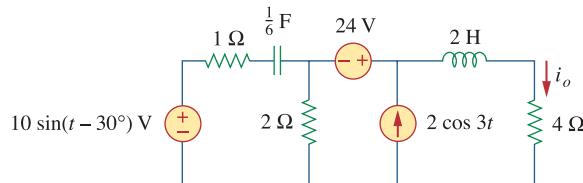


**Figure 10.91**

For Prob. 10.46.

- 10.47** Determine  $i_o$  in the circuit of Fig. 10.92, using the superposition principle.

**ML**

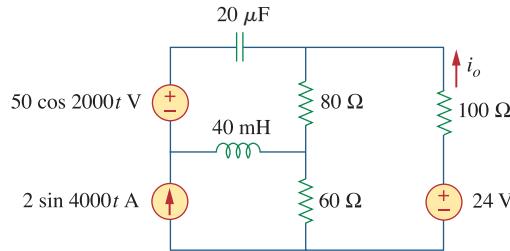


**Figure 10.92**

For Prob. 10.47.

- 10.48** Find  $i_o$  in the circuit of Fig. 10.93 using superposition.

**ML**

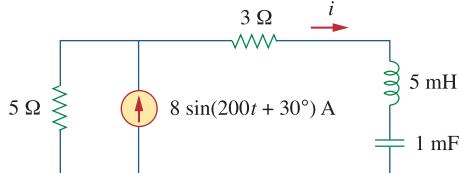


**Figure 10.93**

For Prob. 10.48.

## Section 10.5 Source Transformation

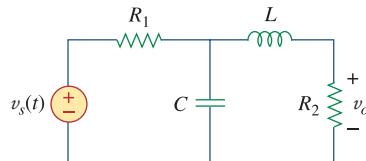
- 10.49** Using source transformation, find  $i$  in the circuit of Fig. 10.94.



**Figure 10.94**

For Prob. 10.49.

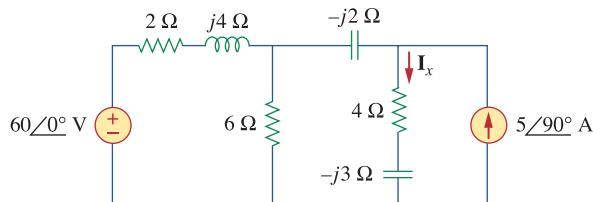
- 10.50** Using Fig. 10.95, design a problem to help other **end** students understand source transformation.

**Figure 10.95**

For Prob. 10.50.

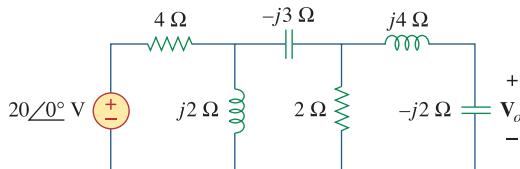
- 10.51** Use source transformation to find  $\mathbf{I}_o$  in the circuit of Prob. 10.42.

- 10.52** Use the method of source transformation to find  $\mathbf{I}_x$  in the circuit of Fig. 10.96.

**Figure 10.96**

For Prob. 10.52.

- 10.53** Use the concept of source transformation to find  $\mathbf{V}_o$  in the circuit of Fig. 10.97.

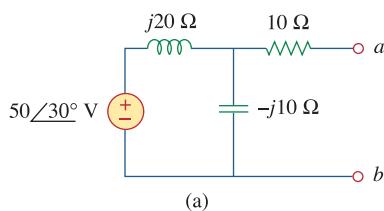
**Figure 10.97**

For Prob. 10.53.

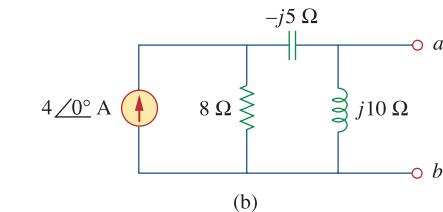
- 10.54** Rework Prob. 10.7 using source transformation.

### Section 10.6 Thevenin and Norton Equivalent Circuits

- 10.55** Find the Thevenin and Norton equivalent circuits at terminals  $a-b$  for each of the circuits in Fig. 10.98.



(a)

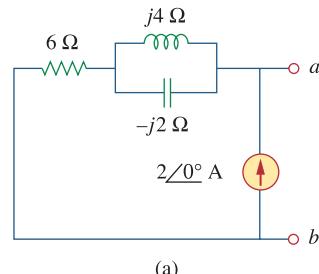


(b)

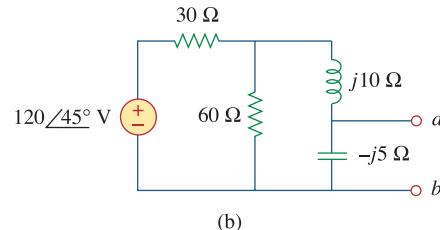
**Figure 10.98**

For Prob. 10.55.

- 10.56** For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals  $a-b$ .



(a)

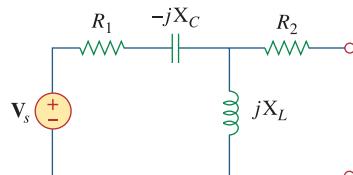


(b)

**Figure 10.99**

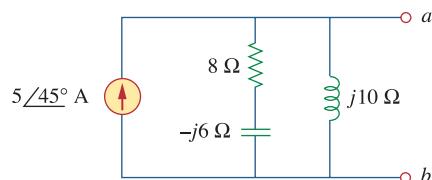
For Prob. 10.56.

- 10.57** Using Fig. 10.100, design a problem to help other **end** students better understand Thevenin and Norton equivalent circuits.

**Figure 10.100**

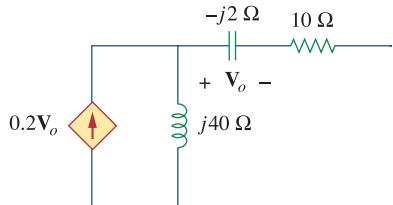
For Prob. 10.57.

- 10.58** For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals  $a-b$ .

**Figure 10.101**

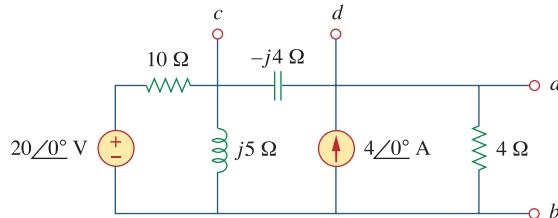
For Prob. 10.58.

- 10.59** Calculate the output impedance of the circuit shown in Fig. 10.102.

**Figure 10.102**

For Prob. 10.59.

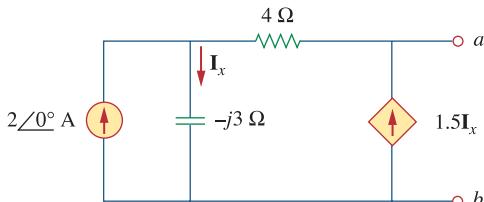
- 10.60** Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b*    (b) terminals *c-d***Figure 10.103**

For Prob. 10.60.

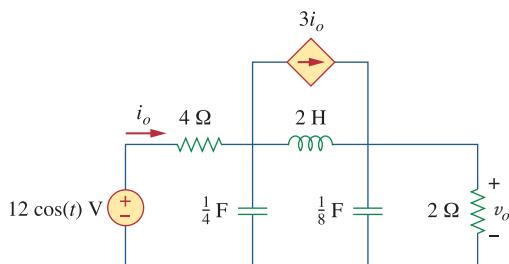
- 10.61** Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.104.

ML

**Figure 10.104**

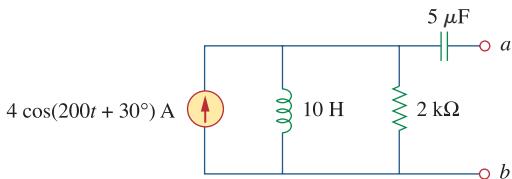
For Prob. 10.61.

- 10.62** Using Thevenin's theorem, find  $v_o$  in the circuit of Fig. 10.105.

**Figure 10.105**

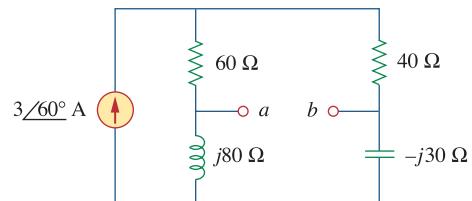
For Prob. 10.62.

- 10.63** Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

**Figure 10.106**

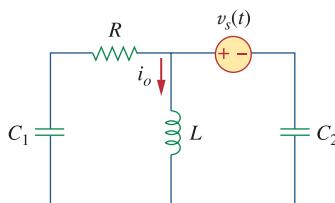
For Prob. 10.63.

- 10.64** For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

**Figure 10.107**

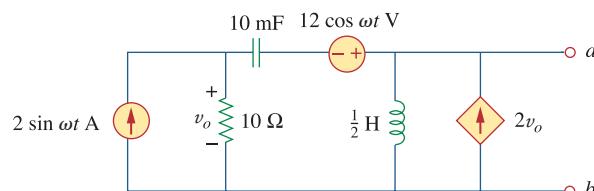
For Prob. 10.64.

- 10.65** Using Fig. 10.108, design a problem to help other students better understand Norton's theorem.

**Figure 10.108**

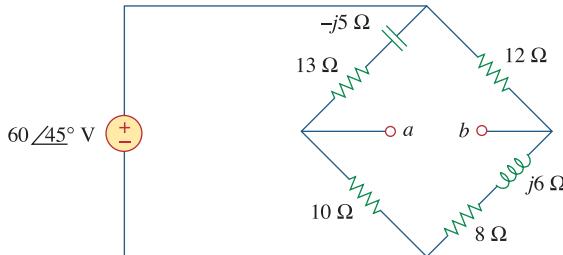
For Prob. 10.65.

- 10.66** At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take  $\omega = 10 \text{ rad/s}$ .

**Figure 10.109**

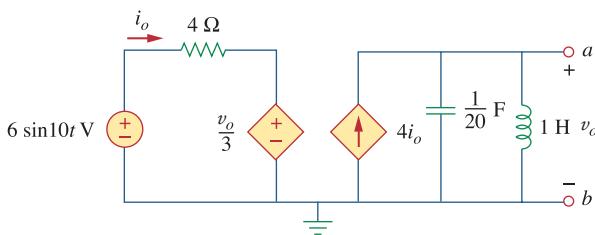
For Prob. 10.66.

- 10.67** Find the Thevenin and Norton equivalent circuits at terminals *a-b* in the circuit of Fig. 10.110.
-   **ML**

**Figure 10.110**

For Prob. 10.67.

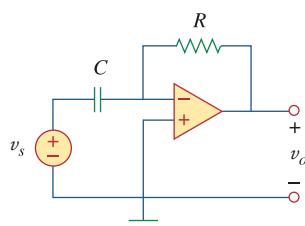
- 10.68** Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.
-   **ML**

**Figure 10.111**

For Prob. 10.68.

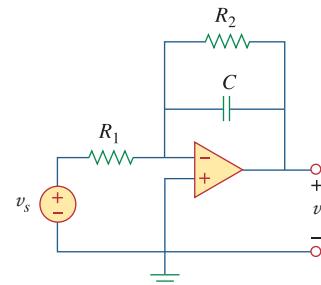
### Section 10.7 Op Amp AC Circuits

- 10.69** For the differentiator shown in Fig. 10.112, obtain  $V_o/V_s$ . Find  $v_o(t)$  when  $v_s(t) = V_m \sin \omega t$  and  $\omega = 1/RC$ .

**Figure 10.112**

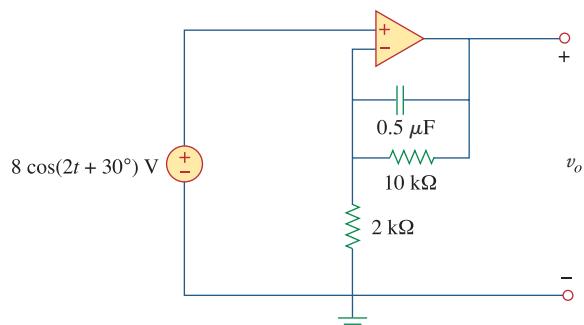
For Prob. 10.69.

- 10.70** Using Fig. 10.113, design a problem to help other  students better understand op amps in AC circuits.

**Figure 10.113**

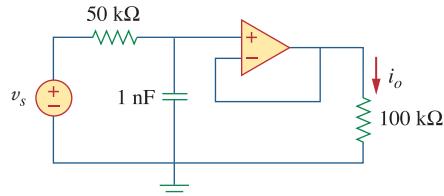
For Prob. 10.70.

- 10.71** Find  $v_o$  in the op amp circuit of Fig. 10.114.

**Figure 10.114**

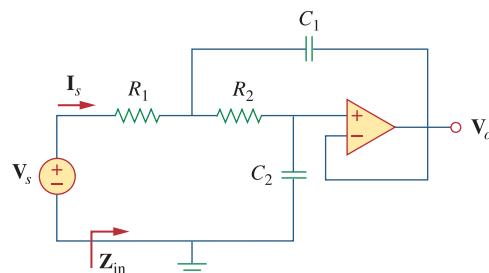
For Prob. 10.71.

- 10.72** Compute  $i_o(t)$  in the op amp circuit in Fig. 10.115 if  $v_s = 4 \cos(10^4 t)$  V.

**Figure 10.115**

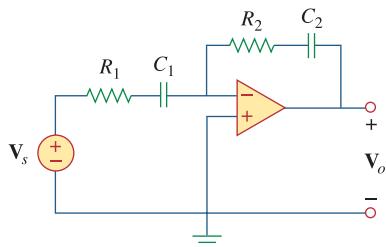
For Prob. 10.72.

- 10.73** If the input impedance is defined as  $Z_{in} = V_s/I_s$ , find the input impedance of the op amp circuit in Fig. 10.116 when  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 20 \text{ k}\Omega$ ,  $C_1 = 10 \text{ nF}$ ,  $C_2 = 20 \text{ nF}$ , and  $\omega = 5000 \text{ rad/s}$ .

**Figure 10.116**

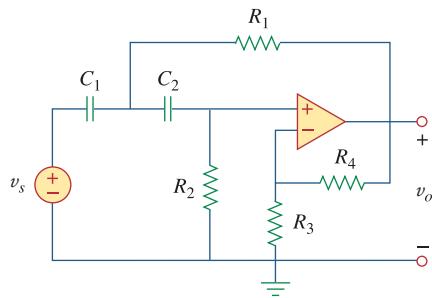
For Prob. 10.73.

- 10.74** Evaluate the voltage gain  $A_v = V_o/V_s$  in the op amp circuit of Fig. 10.117. Find  $A_v$  at  $\omega = 0$ ,  $\omega \rightarrow \infty$ ,  $\omega = 1/R_1C_1$ , and  $\omega = 1/R_2C_2$ .

**Figure 10.117**

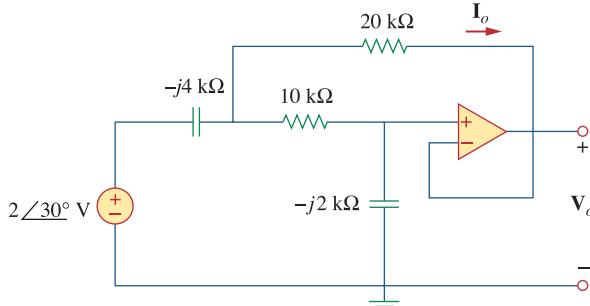
For Prob. 10.74.

- 10.75** In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if  $C_1 = C_2 = 1 \text{ nF}$ ,  $R_1 = R_2 = 100 \text{ k}\Omega$ ,  $R_3 = 20 \text{ k}\Omega$ ,  $R_4 = 40 \text{ k}\Omega$ , and  $\omega = 2000 \text{ rad/s}$ .

**Figure 10.118**

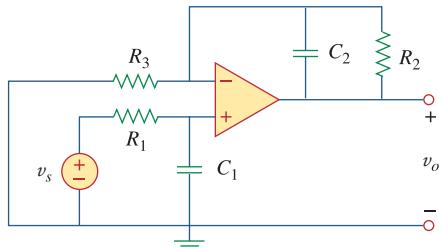
For Prob. 10.75.

- 10.76** Determine  $V_o$  and  $I_o$  in the op amp circuit of Fig. 10.119.

**ML****Figure 10.119**

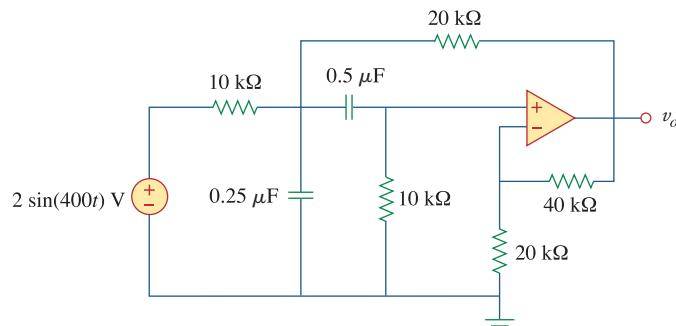
For Prob. 10.76.

- 10.77** Compute the closed-loop gain  $V_o/V_s$  for the op amp circuit of Fig. 10.120.

**ML****Figure 10.120**

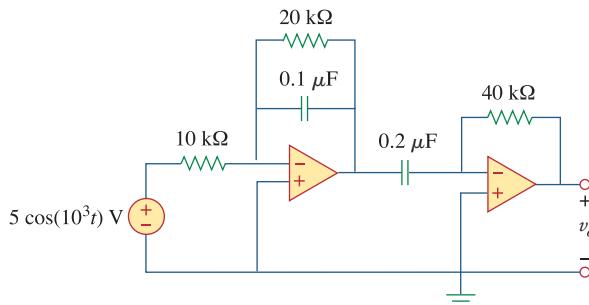
For Prob. 10.77.

- 10.78** Determine  $v_o(t)$  in the op amp circuit in Fig. 10.121 below.

**ML****Figure 10.121**

For Prob. 10.78.

- 10.79** For the op amp circuit in Fig. 10.122, obtain  $v_o(t)$ .

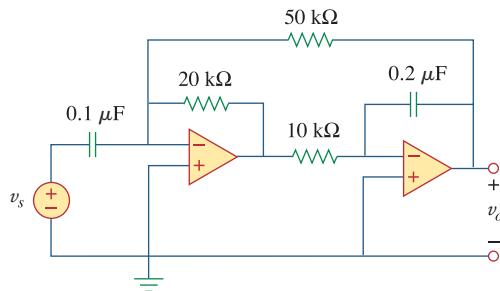


**Figure 10.122**

For Prob. 10.79.

- 10.80** Obtain  $v_o(t)$  for the op amp circuit in Fig. 10.123 if  $v_s = 4 \cos(1000t - 60^\circ)$  V.

**ML**



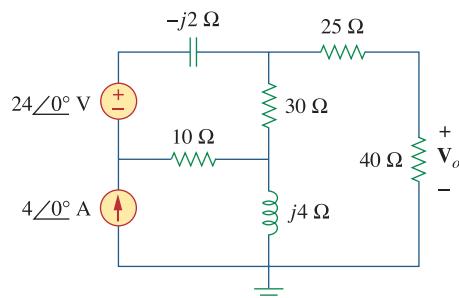
**Figure 10.123**

For Prob. 10.80.

### Section 10.8 AC Analysis Using PSpice



- 10.81** Use PSpice or MultiSim to determine  $\mathbf{V}_o$  in the circuit of Fig. 10.124. Assume  $\omega = 1$  rad/s.

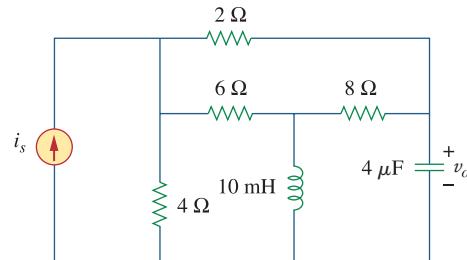


**Figure 10.124**

For Prob. 10.81.

- 10.82** Solve Prob. 10.19 using PSpice or MultiSim.

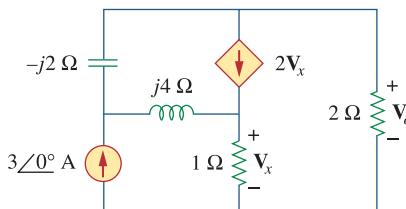
- 10.83** Use PSpice or MultiSim to find  $v_o(t)$  in the circuit of Fig. 10.125. Let  $i_s = 2 \cos(10^3 t)$  A.



**Figure 10.125**

For Prob. 10.83.

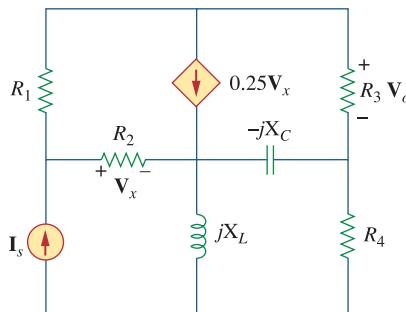
- 10.84** Obtain  $\mathbf{V}_o$  in the circuit of Fig. 10.126 using PSpice or MultiSim.



**Figure 10.126**

For Prob. 10.84.

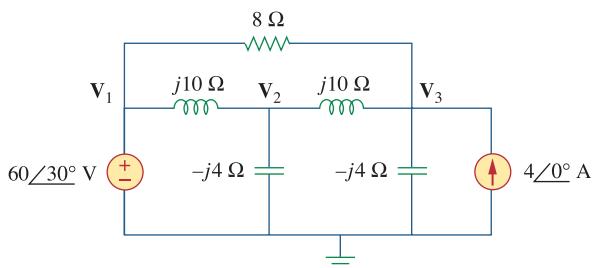
- 10.85** Using Fig. 10.127, design a problem to help other **e2d** students better understand performing AC analysis with PSpice or MultiSim.



**Figure 10.127**

For Prob. 10.85.

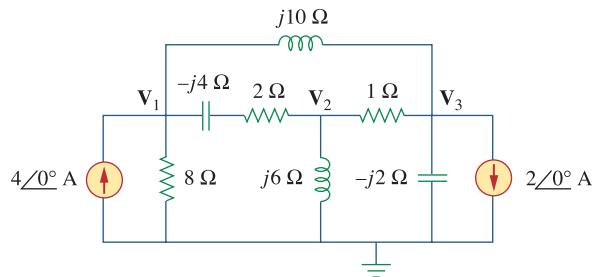
- 10.86** Use PSpice or MultiSim to find  $\mathbf{V}_1$ ,  $\mathbf{V}_2$ , and  $\mathbf{V}_3$  in the network of Fig. 10.128.



**Figure 10.128**

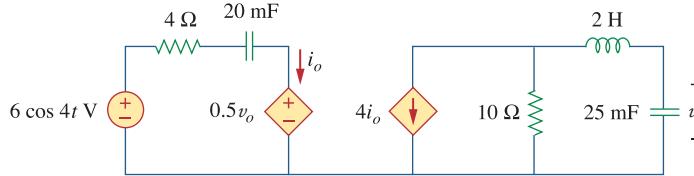
For Prob. 10.86.

- 10.87** Determine  $\mathbf{V}_1$ ,  $\mathbf{V}_2$ , and  $\mathbf{V}_3$  in the circuit of Fig. 10.129 using *PSpice or MultiSim*.

**Figure 10.129**

For Prob. 10.87.

- 10.88** Use *PSpice or MultiSim* to find  $v_o$  and  $i_o$  in the circuit of Fig. 10.130 below.

**Figure 10.130**

For Prob. 10.88.

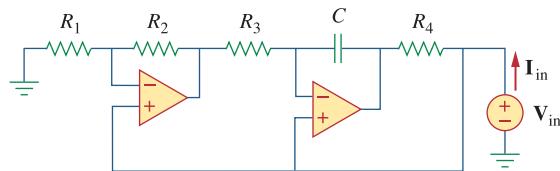
## Section 10.9 Applications

- 10.89** The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$Z_{in} = \frac{V_{in}}{I_{in}} = j\omega L_{eq}$$

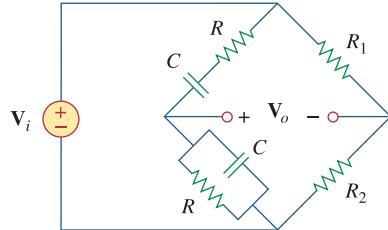
where

$$L_{eq} = \frac{R_1 R_3 R_4}{R_2} C$$

**Figure 10.131**

For Prob. 10.89.

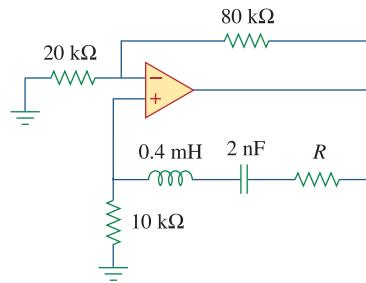
- 10.90** Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is  $f = \frac{1}{2\pi} RC$ , and that the necessary gain is  $A_v = V_o/V_i = 3$  at that frequency.

**Figure 10.132**

For Prob. 10.90.

- 10.91** Consider the oscillator in Fig. 10.133.

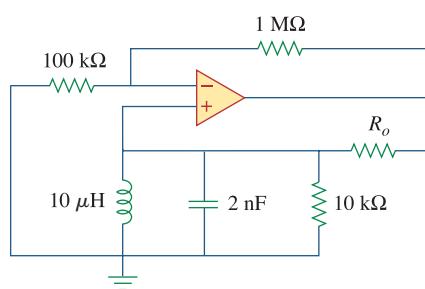
- Determine the oscillation frequency.
- Obtain the minimum value of  $R$  for which oscillation takes place.

**Figure 10.133**

For Prob. 10.91.

- 10.92** The oscillator circuit in Fig. 10.134 uses an ideal op amp.

- (a) Calculate the minimum value of  $R_o$  that will cause oscillation to occur.  
 (b) Find the frequency of oscillation.



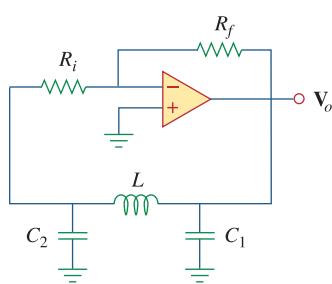
**Figure 10.134**

For Prob. 10.92.

- 10.93** Figure 10.135 shows a *Colpitts oscillator*. Show that **eod** the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where  $C_T = C_1 C_2 / (C_1 + C_2)$ . Assume  $R_i \gg X_{C_2}$ .



**Figure 10.135**

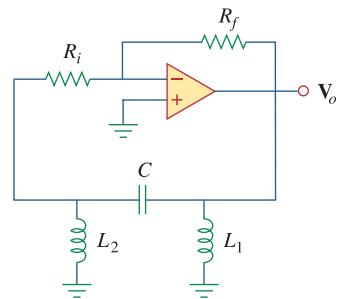
A Colpitts oscillator; for Prob. 10.93.

(Hint: Set the imaginary part of the impedance in the feedback circuit equal to zero.)

- 10.94** Design a Colpitts oscillator that will operate at 50 kHz. **eod**

- 10.95** Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$



**Figure 10.136**

A Hartley oscillator; for Prob. 10.95.

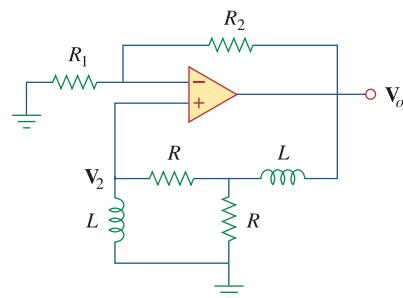
- 10.96** Refer to the oscillator in Fig. 10.137.

- (a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

- (b) Determine the oscillation frequency  $f_o$ .

- (c) Obtain the relationship between  $R_1$  and  $R_2$  in order for oscillation to occur.



**Figure 10.137**

For Prob. 10.96.