HW9 5.1 #3,6,15,16,19,20 Alex Yeoh

5.2 #3,6,9,11,15,16

5.3 #3,5,8,11,12,19

5.1

3) $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -3+4 \\ -3+32 \end{bmatrix} = \begin{bmatrix} 1 \\ 29 \end{bmatrix}$ No

6) $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-12+7 \\ 3-6+7 \\ 5-12+5 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, yes, -2

15) $\left[\begin{array}{ccc|c} 4-3 & 2 & 3 & 0 \\ -1 & 1-3 & -3 & 0 \\ 2 & 4 & 9-3 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & -2 & -3 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

free x_3, x_2 $\quad x_1+2x_2+3x_3=0$, $x_1=-2x_2-3x_3$ $\quad \vec{x} = \begin{bmatrix} -2x_2-3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ basis: $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

16) $\left[\begin{array}{cccc|c} 3-4 & 0 & 2 & 0 & 0 \\ 1 & 3-4 & 1 & 0 & 0 \\ 0 & 1 & 1-4 & 0 & 0 \\ 0 & 0 & 0 & 4-4 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} -1 & 0 & 2 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{cccc|c} -1 & 0 & 2 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_3+R_2} \left[\begin{array}{cccc|c} -1 & 0 & 2 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_2\cdot -1]{R_1\cdot -1} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$

free x_4, x_3 $\quad x_2-3x_3=0$, $x_2=3x_3$ $\quad x_1-2x_3=0$, $x_1=2x_3$ $\quad \vec{x} = \begin{bmatrix} 2x_3 \\ 3x_3 \\ x_3 \\ x_4 \end{bmatrix} = x_3\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ basis: $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

19) 0, 0 is an eigenvalue for noninvertable matrix

20) 0, 0 is an eigenvalue for noninvertable matrix $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\}$ All numbers in the matrix are the same, two variables must be freed when $\lambda=0$

5.2

3) $\begin{vmatrix} 3-\lambda & -2 \\ 1 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-\lambda)-(-2)(1) = -3-2\lambda+\lambda^2+2 = \lambda^2-2\lambda-1$ $\quad \frac{2\pm\sqrt{2^2-4(-1)(1)}}{2} = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm\sqrt{4\cdot 2}}{2} = \frac{2\pm 2\sqrt{2}}{2} = 1\pm\sqrt{2}$

6) $\begin{vmatrix} 3-\lambda & -4 \\ 4 & 8-\lambda \end{vmatrix} = (3-\lambda)(8-\lambda)-(-4)(4) = 24-12\lambda+\lambda^2+16 = \lambda^2-12\lambda+40$ $\quad \frac{12\pm\sqrt{12^2-4(40)(1)}}{2} = \frac{12\pm\sqrt{-16}}{2}$, no real eigenvalues

9) $\begin{vmatrix} 1-\lambda & 0 & -1 \\ 2 & 3-\lambda & -1 \\ 0 & 6 & -\lambda \end{vmatrix} = (1-\lambda)(3-\lambda)(-\lambda)+0+(-1)(2)(6)-((1-\lambda)(-1)(6)+0+0) = (3-4\lambda+\lambda^2)(-\lambda)-12-(-6+6\lambda) = -3\lambda+4\lambda^2-\lambda^3-12+6-6\lambda$

$= -\lambda^3+4\lambda^2-9\lambda-6$

11) $\begin{vmatrix} 4-\lambda & 0 & 0 \\ 5 & 3-\lambda & 2 \\ -2 & 0 & 2-\lambda \end{vmatrix} = (-1)^{1+1}(4-\lambda)\begin{vmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = (4-\lambda)\left((3-\lambda)(2-\lambda)-2(0)\right) = (4-\lambda)(6-5\lambda+\lambda^2) = 24-20\lambda-6\lambda+5\lambda^2+4\lambda^2-\lambda^3$

$= -\lambda^3+9\lambda^2-26\lambda+24$

5.2 continued

15) $\begin{vmatrix} 4-\lambda & -7 & 0 & 2 \\ 0 & 3-\lambda & -4 & 6 \\ 0 & 0 & 3-\lambda & -8 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 3-\lambda & -4 & 6 \\ 0 & 3-\lambda & -8 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda)\begin{vmatrix} 3-\lambda & -8 \\ 0 & 1-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda)(3-\lambda)(1-\lambda)$ 4,3,3,1

16) for the same reason above, $(5-\lambda)(-4-\lambda)(1-\lambda)(1-\lambda)$ 5,-4,1,1

5.3

3) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} a^k & 0 \\ 0 & b^k \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} a^k & 0 \\ 3a^k & b^k \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} a^k & 0 \\ 3a^k-3b^k & b^k \end{bmatrix} = \begin{bmatrix} a^k & 0 \\ 3(a^k-b^k) & b^k \end{bmatrix}$

5) As seen in D, eigenvalues are 5,1,1

@ $\lambda=5$

$\left[\begin{array}{ccc|c} 2-5 & 2 & 1 & 0 \\ 1 & 3-5 & 1 & 0 \\ 1 & 2 & 2-5 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right] \xrightarrow[R_3-R_2]{R_1+3R_3} \left[\begin{array}{ccc|c} 0 & 8 & -8 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right] \xrightarrow[R_3/4]{R_1-2R_3} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2+2R_3} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

free x_3 $\quad x_2-x_3=0 \quad x_1-x_3=0 \quad \vec{x} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ basis: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = x_3 \quad x_1 = x_3$

@ $\lambda=1$

$\left[\begin{array}{ccc|c} 2-1 & 2 & 1 & 0 \\ 1 & 3-1 & 1 & 0 \\ 1 & 2 & 2-1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \end{array}\right]$

free x_3, x_2 $\quad x_1+2x_2+x_3=0 \quad \vec{x} = \begin{bmatrix} -2x_2-x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ basis: $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$x_1 = -2x_2 - x_3$

8) $\begin{vmatrix} 5-\lambda & 1 \\ 0 & 5-\lambda \end{vmatrix} = (5-\lambda)^2$ eiganvalues of 5,5

$\left[\begin{array}{cc|c} 5-5 & 1 & 0 \\ 0 & 5-5 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x_2=0$ free x_1 $\quad \vec{x} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ not diagonalizable

11) eigenvalues = 1,2,3

@ $\lambda=1$

$\left[\begin{array}{ccc|c} -2 & 4 & -2 & 0 \\ -3 & 3 & 0 & 0 \\ -3 & 1 & 2 & 0 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} -2 & 4 & -2 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & -7 & 6 & 0 \end{array}\right] \xrightarrow[R_2+3R_3]{R_1+2R_3} \left[\begin{array}{ccc|c} 0 & -10 & 10 & 0 \\ 0 & -18 & 18 & 0 \\ 1 & -7 & 6 & 0 \end{array}\right] \xrightarrow{R_1/-10} \left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & -18 & 18 & 0 \\ 1 & -7 & 6 & 0 \end{array}\right] \xrightarrow{R_2+18R_1} \left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -7 & 6 & 0 \end{array}\right] \xrightarrow{R_3 \Leftrightarrow R_1} \left[\begin{array}{ccc|c} 1 & -7 & 6 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{R_1+7R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

free x_3 $\quad x_2-x_3=0 \quad x_1-x_3=0 \quad \vec{x} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix}$ basis: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = x_3 \quad x_1 = x_3$

@ $\lambda=2$

$\left[\begin{array}{ccc|c} -3 & 4 & -2 & 0 \\ -3 & 2 & 0 & 0 \\ -3 & 1 & 1 & 0 \end{array}\right] \xrightarrow[R_2-R_3]{R_1-R_3} \left[\begin{array}{ccc|c} 0 & 3 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ -3 & 1 & 1 & 0 \end{array}\right] \xrightarrow[R_3-R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -3 & 0 & 2 & 0 \end{array}\right] \xrightarrow{R_3 \Leftrightarrow R_2} \left[\begin{array}{ccc|c} -3 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

free x_3 $\quad x_2-x_3=0 \quad -3x_1-2x_3=0 \quad \vec{x} = \begin{bmatrix} \frac{2}{3}x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ basis: $\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$

$x_2 = x_3 \quad x_1 = \frac{2}{3}x_3$

@ $\lambda=3$

$\left[\begin{array}{ccc|c} -4 & 4 & -2 & 0 \\ -3 & 1 & 0 & 0 \\ -3 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|c} -4 & 4 & -2 & 0 \\ -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1/-1} \left[\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ -3 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -3 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_2+3R_1} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -8 & 6 & 0 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 4 & -3 & 0 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 4 & -3 & 0 \end{array}\right]$

free x_3 $\quad 4x_2-3x_3=0 \quad x_1+\frac{3}{4}x_3-x_3=0 \quad \vec{x} = \begin{bmatrix} \frac{1}{4}x_3 \\ \frac{3}{4}x_3 \\ x_3 \end{bmatrix}$ basis: $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$x_2 = \frac{3}{4}x_3 \quad x_1 = \frac{1}{4}x_3$

$\boxed{D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}}$ ← Ans for 11

5.3 continued

12) eigenvalues = 2,8

@ $\lambda=2$

$$\left[\begin{array}{ccc|c}2&2&2&0\\2&2&2&0\\2&2&2&0\end{array}\right]\xrightarrow[R_2-R_1]{R_3-R_1}\left[\begin{array}{ccc|c}2&2&2&0\\0&0&0&0\\0&0&0&0\end{array}\right]\xrightarrow{R_1/2}\left[\begin{array}{ccc|c}1&1&1&0\\0&0&0&0\\0&0&0&0\end{array}\right]$$

free x_3, x_2 $\quad x_1+x_2+x_3=0$, $\quad x_1=-x_2-x_3$

$$\vec{x}=\begin{bmatrix}-x_2-x_3\\x_2\\x_3\end{bmatrix}=x_2\begin{bmatrix}-1\\1\\0\end{bmatrix}+\begin{bmatrix}-1\\0\\1\end{bmatrix}x_3 \quad \text{basis: }\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

@ $\lambda=8$

$$\left[\begin{array}{ccc|c}-4&2&2&0\\2&-4&2&0\\2&2&-4&0\end{array}\right]\xrightarrow[R_2-R_3]{R_1+2R_3}\left[\begin{array}{ccc|c}0&6&-6&0\\0&-6&6&0\\2&2&-4&0\end{array}\right]\xrightarrow{R_1+R_2}\left[\begin{array}{ccc|c}0&0&0&0\\0&-6&6&0\\2&2&-4&0\end{array}\right]\xrightarrow{R_3\Leftrightarrow R_2}\left[\begin{array}{ccc|c}2&2&-4&0\\0&-6&6&0\end{array}\right]\xrightarrow[R_2/-6]{R_1/2}\left[\begin{array}{ccc|c}1&1&-2&0\\0&1&-1&0\end{array}\right]$$

free x_3 $\quad x_2-x_3=0$, $\quad x_2=x_3$ $\quad x_1+x_3-2x_3=0$, $\quad x_1=x_3$

$$\vec{x}=\begin{bmatrix}x_3\\x_3\\x_3\end{bmatrix} \quad \text{basis}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$D=\begin{bmatrix}2&0&0\\0&2&0\\0&0&8\end{bmatrix} \quad P=\begin{bmatrix}-1&-1&1\\1&0&1\\0&1&1\end{bmatrix}$$

19) eigenvalues = 5,3,2,2

@ $\lambda=5$

$$\left[\begin{array}{cccc|c}0&-3&0&9&0\\0&-2&1&-2&0\\0&0&-3&0&0\\0&0&0&-3&0\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{cccc|c}0&-3&0&9&0\\0&1&1&-11&0\\0&0&-3&0&0\\0&0&0&-3&0\end{array}\right]\xrightarrow[R_1+R_3,\ R_1-8R_4]{R_1+3R_2}\left[\begin{array}{cccc|c}0&0&0&0&0\\0&1&1&-11&0\\0&0&-3&0&0\\0&0&0&-3&0\end{array}\right]\xrightarrow[R_4/-3]{R_3/-3}\left[\begin{array}{cccc|c}0&1&1&-11&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right]\xrightarrow[R_1+11R_3]{R_1-R_2}\left[\begin{array}{cccc|c}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right]$$

$x_4=0 \quad x_3=0 \quad x_2=0$ free x_1

$$\vec{x}=\begin{bmatrix}x_1\\0\\0\\0\end{bmatrix} \quad \text{basis}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

@ $\lambda=3$

$$\left[\begin{array}{cccc|c}2&-3&0&9&0\\0&0&1&-2&0\\0&0&-1&0&0\\0&0&0&-1&0\end{array}\right]\xrightarrow[R_2-2R_4]{R_2+R_3}\left[\begin{array}{cccc|c}2&-3&0&9&0\\0&0&0&0&0\\0&0&-1&0&0\\0&0&0&-1&0\end{array}\right]\xrightarrow[R_4/-1]{R_3/-1}\left[\begin{array}{cccc|c}2&-3&0&9&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{array}\right]$$

$x_4=0 \quad x_3=0$ free x_2 $\quad 2x_1-3x_2+0=0$, $\quad x_1=\frac{3}{2}x_2$

$$\vec{x}=\begin{bmatrix}\frac{3}{2}x_2\\x_2\\0\\0\end{bmatrix}=x_2\begin{bmatrix}3\\2\\0\\0\end{bmatrix} \quad \text{basis}=\begin{bmatrix}3\\2\\0\\0\end{bmatrix}$$

@ $\lambda=2$

$$\left[\begin{array}{cccc|c}3&-3&0&9&0\\0&1&1&-2&0\\0&0&0&0&0\\0&0&0&0&0\end{array}\right]\xrightarrow{R_1/3}\left[\begin{array}{cccc|c}1&-1&0&3&0\\0&1&1&-2&0\end{array}\right]$$

free x_4, x_3 $\quad x_2+x_3-2x_4=0$, $\quad x_2=-x_3+2x_4$

$x_1-(-x_3+2x_4)+3x_4=0$

$x_1+x_3-2x_4+3x_4=0$

$x_1=-x_3-x_4$

$$\vec{x}=\begin{bmatrix}-x_3-x_4\\-x_3+2x_4\\x_3\\x_4\end{bmatrix}=x_3\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix}+x_4\begin{bmatrix}-1\\2\\0\\1\end{bmatrix} \quad \text{basis}=\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\2\\0\\1\end{bmatrix}$$

$$D=\begin{bmatrix}5&0&0&0\\0&3&0&0\\0&0&2&0\\0&0&0&2\end{bmatrix} \quad P=\begin{bmatrix}1&3&-1&-1\\0&2&-1&2\\0&0&1&0\\0&0&0&1\end{bmatrix}$$