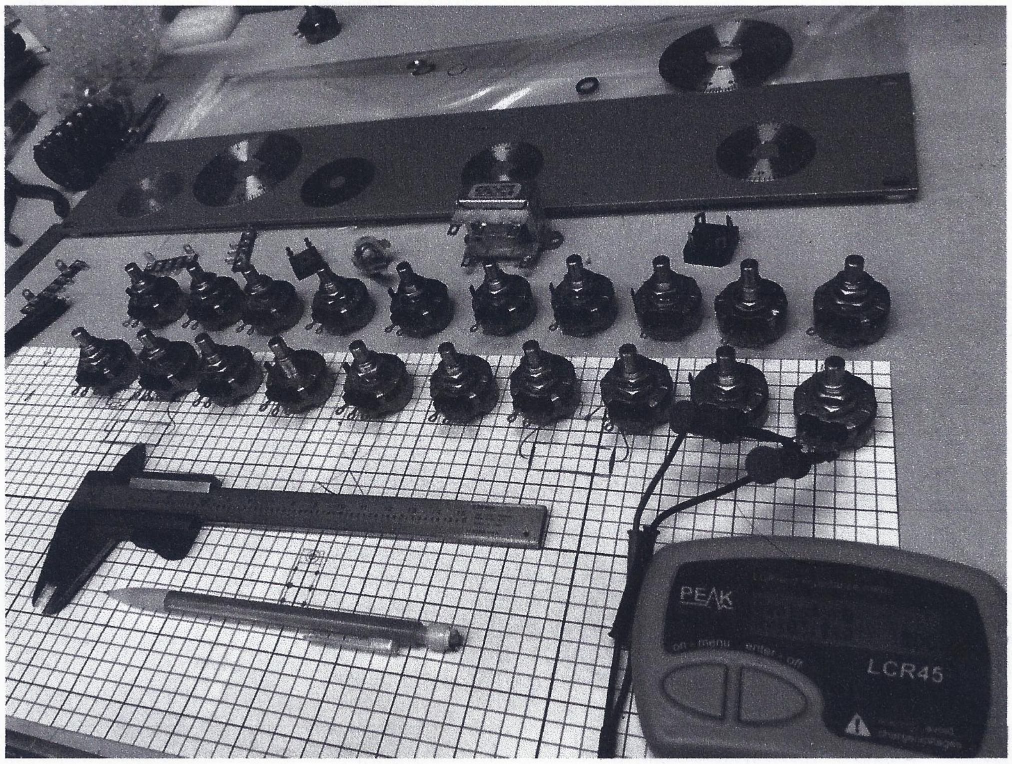
ISE 2211 • Exam III • 17 June 2021 29 pts. NAME SOLUTION

Joe Tritschler is working on a new equalizer design for his all-analog recording studio and is thinking about using the resistance of the 100-kΩ level control as part of the filter network. The issue here is that he's not sure how accurate that 100-kΩ value is and thus how likely it is to affect the filter response. So he bought a bunch of CM-series Clarostat 100k pots from Mr. Roger Hughes at Midwest Surplus Electronics in Fairborn, Ohio, his favorite electronics parts shop for over 25 years, and measured twenty of them with a Peak LCR45 impedance meter (all values in kilohms):

106.3 ✓
102.4
104.7
106.0 ✓
104.5
102.2
102.4
103.8
101.1
103.3
106.6 ✓
103.9
104.6
103.6
103.4
102.3
105.5 ✓
106.5 ✓
107.0 ✓
103.8



First, determine the sample mean and variance for these potentiometers. Include a unit with each answer. Feel free to use any electronic resources you wish to compute the sample parameters (calculator, MS Excel, etc.) but be advised that if your answer is WRONG I only give partial credit for work shown.

$n = 20$

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$\sum x_i = 2083.9$ $\qquad$ $\sum x_i^2 = 217186.6$

$$\bar{x} = \frac{2083.9}{20} = 104.195 \text{ k}\Omega \quad +2$$

$$s^2 = \frac{217186.6 - \frac{2083.9^2}{20}}{19} = 2.876 \text{ k}\Omega^2$$

+3

Write a 95% confidence interval on mean potentiometer resistance. Be sure to include a unit. Assume population variance is unknown. Sketch the appropriate probability distribution and indicate the location of the C.I.

$n<30$, unknown $\sigma^2 \Rightarrow$ T-distribution

$\alpha = 0.05$ +1

$t_{\alpha/2,\, n-1} = t_{.05/2,\, 19} = 2.093$ +2 (table)

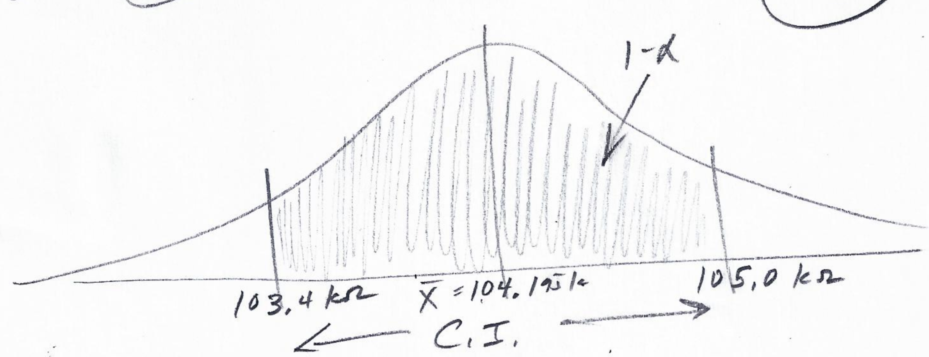
yes, a little bigger than $z_{\alpha/2} = 1.960$!

$\mu: \quad \bar{x} \pm t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}$

$104.195 \pm 2.093 \frac{\sqrt{2.876}}{\sqrt{20}}$

$103.4 < \mu < 105.0$ ($k\Omega$) +1

+2



+3

Write a 95% upper confidence bound on the standard deviation of potentiometer resistance; again, include a unit. Sketch the appropriate probability distribution and indicate the location of the bound.

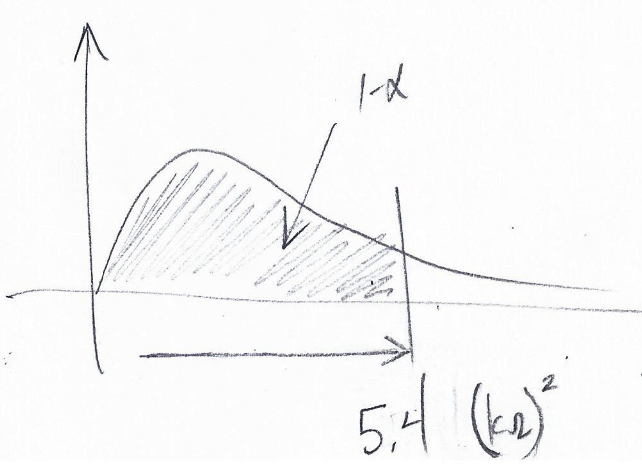
→ need $\sigma^2 < \dfrac{(n-1)s^2}{\chi^2_{1-\alpha,\, n-1}}$ (+1)

$\chi^2_{.95,\,19} = 10.12$ (+2) (table)

$$\sigma^2 < \frac{19 \cdot 2.876}{10.12}$$

$\sigma^2 < 5.3996$ (+1)

$\sigma < 2.324\ k\Omega$ (+2)



(+3)

← need upper bound on variance here because distribution is χ^2

Joe has determined that he will cull any pots whose resistance is more than 5% over the claimed value of 100 kΩ. Write a 95% confidence interval on the population proportion of culled pots.

means $> 105\ k\Omega$

+1

$X = 6$

$\therefore \hat{p} = \frac{6}{20} = 0.30$ +2

$Z_{\alpha/2} = Z_{.025} = 1.960$ +1

(bottom of t-table)

$p: \quad \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.30 \pm 1.960 \cdot \sqrt{\frac{0.30(1-.30)}{20}}$

$0.09916 < p < 0.5008$ +2

or 10 to 50%, not really useful info!