Lab 7 Solution

#1

Sequence $a_n = \cos\frac{1}{n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\frac{1}{n} = \cos 0 = 1$$

The sequece is convergent to 1

#2 Sequence $b_n = \frac{n^2}{n+1}$

$$\lim_{n\to\infty} \frac{n^2}{n+1} = \lim_{n\to\infty} \frac{\frac{n^2}{n}}{\frac{n}{n}+\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{1+\underset{0}{\underbrace{\frac{1}{n}}}} = \infty$$

The sequence b_n is divergent.

#3 Sequence $c_n = \frac{n^2}{2^n}$

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} \frac{n^2}{2^n} = \lim_{x\to\infty} \frac{x^2}{2^x} \overset{\text{LHR}}{=} \lim_{x\to\infty} \frac{2x}{2^x \ln 2} \overset{\text{LHR again}}{=}$$

$$= \lim_{x\to\infty} \frac{2}{2^x (\ln 2)^2} = 0$$

The sequence is convergent to 0.

#4 $3.2\overline{18} = 3.2181818\cdots$

$$= 3.2 + \underbrace{0.018 + 0.00018 + 0.0000018 + \cdots}_{\text{a geometric series with } r = \frac{1}{100}}$$

$$= \frac{16}{5} + \frac{\frac{18}{1000}}{1-\frac{1}{100}} = \frac{16}{5} + \frac{18}{1000-10}$$

$$= \frac{16}{5} + \frac{18}{990} = \frac{177}{55}$$

#5 Geometric Series

$$\sum_{n=1}^{\infty} \frac{3}{(-4)^n} = -\frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \cdots$$

So Common Ratio $r = -\frac{1}{4}$

Therefore, the Geometric Series is convergent.

and it is convergent to $\frac{\text{First term}}{1-r} = \frac{-\frac{3}{4}}{1-(-\frac{1}{4})} = -\frac{3}{5}$

#6 Geometric Series

$$\sum_{n=0}^{\infty} \frac{3^n}{2^n} = 1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \cdots$$

So common Ratio $r = \frac{3}{2}$

Therefore, this Geometric series is divergent.