ISE 2211 • Exam VII • 20 April 2018 NAME SOLUTION

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It is believed that there is a relationship between pH and tannin content in μg/mL of a specific craft beer made by Yellow Springs Brewery. 21 beers were tested for tannin content at three pH levels and then *heroically* and *altruistically* consumed by the on-site chemists -- in the name of science!! The results are presented below:

pH	Tannin Level (μg/mL)							totals	averages
	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7		
5.15	82.2	79.2	79.9	80.3	81.4	79.5	80.9	563.4	80.49
5.45	88.2	89.4	87.3	84.8	86.5	88.4	84.6	609.2	87.03
5.75	92.2	93.6	97.7	94.3	83.6	85.0	91.6	638	91.14
								1810.6 +1	86.22 +1

Use Analysis of Variance (ANOVA) to test the null hypothesis that the treatment means are equal at the $\alpha = 0.05$ level of significance. Fill in the ANOVA table.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0
Treatments	404.4	2	202.2	20.0
Error	182	18	10.11	-
Total	586.4	20	-	-

+1

$y_{ij}^2 = 156694.6$ +1

$y_{i\cdot}^2 = 1095588$ +1

$SS_T = 156694.6 - \frac{1810.6^2}{3\cdot 7} = 586.4$ +1 [d.o.f. $= 3\times 7 - 1 = 20$ +1]

$SS_{Treatments} = \frac{1095588}{7} - \frac{1810.6^2}{3\cdot 7} = 404.4$ +1 [d.o.f. $= 3-1 = 2$ +1]

$SS_E = 586.4 - 404.4 = 182$ +1

[d.o.f. $= 3(7-1) = 18$ +1]

$MS_{Treatments} = \frac{404.4}{2} = 202.2$ +1

$MS_E = \frac{182}{18} = 10.11$ +1

$f_0 = \frac{202.2}{10.11} = 20.0$ +1

$f_0 \ggg f_{critical}$ +1

$f_{critical} = f_{.05,2,18} = 3.55$ +1

reject H_0 +1

Write a 95% confidence interval on tannin content at the 5.15 pH level. Include a unit with your answer.

$$\mu_{5.15} : 80.49 \pm t_{.025,18}\sqrt{\frac{10.11}{7}} \quad +1$$

$$= 2.101 \quad +1$$

$$77.97 < \mu_{5.15} < 83.01 \quad +1$$

$$[mg/ml] \quad +1$$

7 1

Use Fisher's Least Significant Difference to determine which, if any, pairs of treatment means show significant difference at $\alpha = 0.05$.

$$LSD = \underbrace{t_{.025,18}}_{=2.101} \sqrt{\frac{2 \cdot 10.11}{7}}$$

$$= 3.57$$ +1

5.15 vs. 5.45 : $|\bar{y}_{1\cdot} - \bar{y}_{2\cdot}| = |80.49 - 87.03| = 6.54 > LSD$ +1

5.45 vs. 5.75 : $|\bar{y}_{2\cdot} - \bar{y}_{3\cdot}| = |87.03 - 91.14| = 4.11 > LSD$ +1

5.15 vs. 5.75 : $|\bar{y}_{1\cdot} - \bar{y}_{3}| = |80.49 - 91.14| = 10.65 > LSD$ +1

all pairs show significant diffference in tannin content +1

Formulae:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{N} \quad \textit{with } an-1 \textit{ degrees of freedom}$$

$$SS_{Treatments} = \sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{N} \quad \textit{with } a-1 \textit{ degrees of freedom}$$

$$SS_E = SS_T - SS_{Treatments} \quad \textit{with } a(n-1) \textit{ degrees of freedom}$$

$$MS_{Treatments} = \frac{SS_{Treatments}}{a-1}$$

$$MS_E = \frac{SS_E}{a(n-1)}$$

$$f_0 = \frac{MS_{Treatments}}{MS_E}$$

$$f_{critical} = f_{\alpha,a-1,a(n-1)}$$

$$CI \text{ on } \mu_i: \quad \bar{y}_{i.} \pm t_{\alpha/2,a(n-1)}\sqrt{\frac{MS_E}{n}}$$

$$CI \text{ on } \mu_i - \mu_j: \quad \bar{y}_{i.} - \bar{y}_{j.} \pm t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}}$$

$$LSD = t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}}$$