EE 2010 Circuit Analysis

Module 08: Node Equations with Generalized Symbolic Sources Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Learning Objective: In this module, we demonstrate how we can use *EXACTLY THE SAME* nodal analysis for circuits with dynamic elements with *SYMBOLIC SOURCES*, and then use the resulting *TRANSFER FUNCTION* to find the *STEADY STATE RESPONSE* for a specific *SINUSOIDAL INPUT*.

As we approach circuit analysis, almost always using *node equations*, replacing the specified *independent sources* with symbolic *generalized sources* (such a V_s and I_s, etc.) and performing *symbolic analyses* will provide several benefits. Chief among them are:

- → Allow the consideration of a variety of excitation waveforms for a particular circuit.
  For example If the circuit is given as a DC excitation, we may easily consider the response of that circuit to *sinusoidal inputs* or *transient waveforms* without requiring additional analyses.
- → Provide common analytical means for the consideration of both *steady-state* and *transient* responses for a particular circuit.
- → Symbolic analyses will unveil some of the more salient consequences of linearity such as *superposition*, *load lines*, and *equivalent reductions*.
  In particular, *superposition* will immediately become apparent in its form and function.
- → Create a *modus operandi* of viewing a circuit as a system operating on a stimulus to produce an output.
  This insight will provide valuable context for the considerations of other systems – *mechanical*, *biological*, *process*, etc.

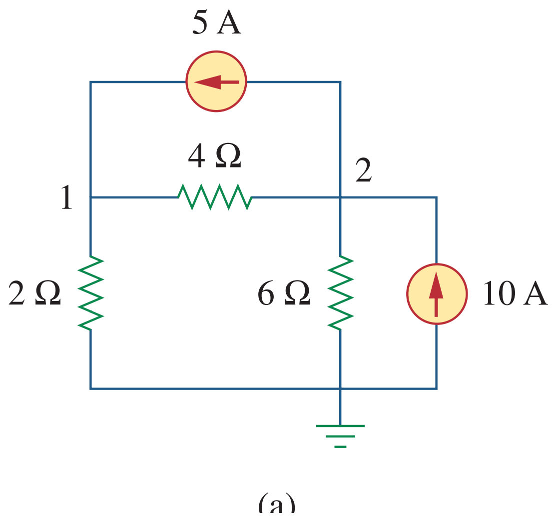
In our analyses of *linear* circuits, *we have thus far implicitly considered steady-state responses* in the special case of $f = 0$, that is, circuits at DC. A bit later, we shall consider identical and similar circuits at steady-state with inputs at $f > 0$, that is, with sinusoidal inputs.

From WIKIPEDIA: "In systems theory, a system or a process is in a *steady state* if the variables (called state variables) which define the behavior of the *system or the process are unchanging in time*."

We'll see much more on the concept of "steady-state," in contrast to the concept of "transient" in modules that follow.

Generalized Sources **with** ***Symbolic Analyses***

Let's illustrate with a simple circuit we've seen before. Find the voltages v_1 and v_2 for the circuit below, with the 5A and 10A current sources replaced by generalized, symbolic current sources I_5 and I_{10}



Here, there are 3 essential nodes. Identifying the bottom node as the reference node, we have:

```
%% Example 1: Node equations, generalized sources
clear all
% Declare symbolic variables
syms I5 I10 v1 v2
% Write node equations at v1, v2 and solve
[v1,v2]=solve(v1/2 - I5 + (v1-v2)/4 == 0,...
  v2/6 + (v2-v1)/4 + I5 - I10 == 0, v1,v2)
```

Which yields:

```
v1 = (2*I5)/3 + I10
v2 = 3*I10 - 2*I5
```

The insight gained here is amazing! Consider the implications:

1. We immediately see each response is the *linear combination of the individual responses due to each stimulus.*
   For example, the response $v2$ is immediately seen as a constant (=3) times the input $I10$ plus another constant (=-2) times the input $I5$.
   This *additive composition or responses* is referred to as **Superposition.**

2. We can easily calculate the response for each individual stimulus. For example v_1 due to I_5 is found by setting $I_{10} = 0$ which, of course, yields $v_1 = \frac{2}{3}I_5$.

3. We establish the *fundamental observation* that responses are generally of the form:

$$v_1 = H_1 * I_5 + H_2 * I_{10}$$
$$v_2 = H_3 * I_5 + H_4 * I_{10}$$

where the H_i are **Transfer Functions** (which happen to be numerical constants for this case) model the operation on the inputs relating inputs (I_5, I_{10}) to outputs (v_1, v_2).

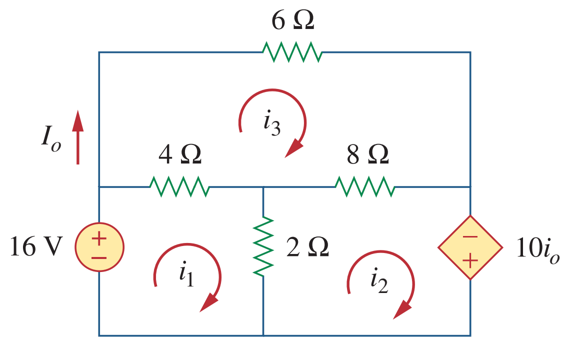
Finally, we can easily compute the results for the specific sources by substituting:

```
subs([v1,v2], [I5, I10], [5, 10])
```

which, in turn, yields

```
ans = [ 40/3, 20]
```

Let's look at another example.
Problem: Find the voltage v_a for the circuit below with generalized source V_{16}. Notice **only independent sources are "generalized"**. This because dependent sources are used to represent active devices that are part of the circuit and not part of the stimulus.



Again, applying node analysis, we have:

```
%% Problem 3.6: Node equations, generalized source
clear all
% Declare symbolic variables
syms V16 va i0
% Write node equation at va and solve
[va, i0] = solve(va/2 + (va−V16)/4 + (va+10∗i0)/8 == 0,...
  i0 == (V16+10∗i0)/6, va,i0)
```

Which yields:

```
va = (9∗V16)/14
i0 = −V16/4
```

Again, we see the form: $Output = Input \cdot Transfer\ Function$ Additionally, we can compute the results for the specified source voltage by substituting:

```
subs([va,i0], V16, 16)
```

which, in turn, yields

```
ans = [ 72/7, −4]
```

Key Observation: *For any linear system,* every quantity of interest can be expressed in the form:

$$\text{Output} = \mathcal{T}\{\text{Input}\}$$

where in the examples above, the operation is a multiplication with the transfer function $\mathcal{T}\{\text{Input}\} = \mathcal{H} * \text{Input}$ so that

$$\text{Output} = \mathcal{H} * \text{Input}$$

Our observations affirm the understanding of the role of **Transfer Functions** in **ALL** systems modeled as linear.

Henceforth, we will include analyses using *generalized sources* to:

1. Discover the transfer function operation producing the output(s) from inputs(s), and
2. Allow the consideration of a variety of excitation waveforms for a particular circuit

Node Voltage Procedure for Generalized Sources:

1. Replace *all independent sources with symbolic representations*
2. Identify the essential ($\geq$ 3-element connections) nodes
3. Select a node as the reference node = the node at *ground potential* = 0 Volts
4. Identify and label the voltages at nodes that are readily deduced
5. Note the node-pairs linked by a *voltage source* and simplify accordingly
6. Assign voltage variables $v_a, v_b, \ldots$ to the remaining nodes with only one assignment for each linked node-pair, the other node in that pair assigned voltages such as "$v_1 - 20$" or "$v_4 + 3v_x$".
7. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
8. Enjoy the thrill of ending the consideration of each node with the powerful "$= 0$"
9. Add one additional equation for each dependent source specification if necessary

   Circuit analysis is now complete! *But you may be asked to:*
10. Invoke the power of algebra to solve for every assigned variable in the resulting system of equations.
11. Answer whatever questions are asked about the circuit using the solved values of knowledge of $v_a, v_b, \ldots$.

One for practice:

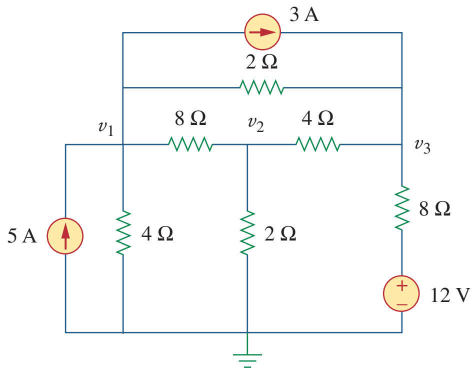


Figure 3.68
For Prob. 3.19.

```
%% Problem 3.19: Node equations, generalized sources
clear all
% Declare symbolic variables
syms I5 I3 V12 v1 v2 v3
% Write node equations at v1, v2, v3 and solve
[v1,v2,v3]=solve(v1/4 - I5 + I3 + (v1-v3)/2 + (v1-v2)/8 == 0,...
  v2/2 + (v2-v1)/8 + (v2-v3)/4 == 0,...
  (v3-V12)/8 + (v3-v2)/4 + (v3-v1)/2 - I3 == 0, v1,v2,v3)
%
subs([v1,v2,v3], [I5,I3,V12], [5,3,12])
```

Which yields:

```
v1 = 2*I5 - (2*I3)/3 + V12/6
v2 = (2*I3)/15 + (2*I5)/3 + V12/10
v3 = (4*I3)/5 + (4*I5)/3 + (4*V12)/15
ans = [ 10, 74/15, 184/15]
```

Homework with generalized sources: Chapter 3 # 2, 6, 8, 12, 22, 30

→ Remember that *only independent sources* are replaced with symbolic labels.

→ Homework deliverables MUST be a pdf file generated using a solver.

→ The resulting .pdf file is to be uploaded to the Pilot Dropbox using the naming convention: First 4 letters of Lastname, First initial, year, title. For example, my .pdf file would be named: GarbF2020HW08.pdf

→ Examples of construction for Matlab, Python, and LaTeX are in the Pilot folder.

3.7 In the circuit of Fig. 3.49, current i_1 is:

(a) 4 A (b) 3 A (c) 2 A (d) 1 A

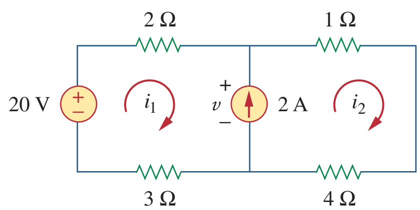


Figure 3.49
For Review Questions 3.7 and 3.8.

3.8 The voltage v across the current source in the circuit of Fig. 3.49 is:

(a) 20 V (b) 15 V (c) 10 V (d) 5 V

3.9 The *PSpice* part name for a current-controlled voltage source is:

(a) EX (b) FX (c) HX (d) GX

3.10 Which of the following statements are not true of the pseudocomponent IPROBE:

(a) It must be connected in series.
(b) It plots the branch current.
(c) It displays the current through the branch in which it is connected.
(d) It can be used to display voltage by connecting it in parallel.
(e) It is used only for dc analysis.
(f) It does not correspond to a particular circuit element.

Answers: 3.1a, 3.2c, 3.3a, 3.4c, 3.5c, 3.6a, 3.7d, 3.8b, 3.9c, 3.10b,d.

Problems

Sections 3.2 and 3.3 Nodal Analysis

3.1 Using Fig. 3.50, design a problem to help other students better understand nodal analysis.

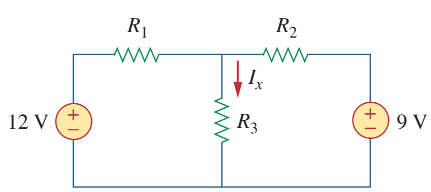


Figure 3.50
For Prob. 3.1 and Prob. 3.39.

3.2 For the circuit in Fig. 3.51, obtain v_1 and v_2.

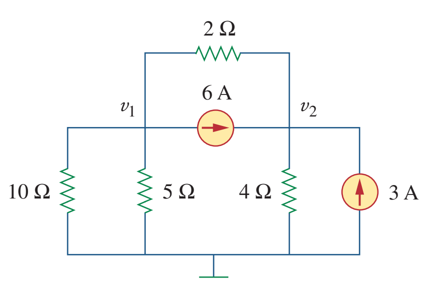


Figure 3.51
For Prob. 3.2.

v1:v1/10+v1/5+i6+(v1-v2)/2=0

v2: (v2-v1)/2–i6+v2/4-i3=0

3.3 Find the currents I_1 through I_4 and the voltage v_o in the circuit of Fig. 3.52.

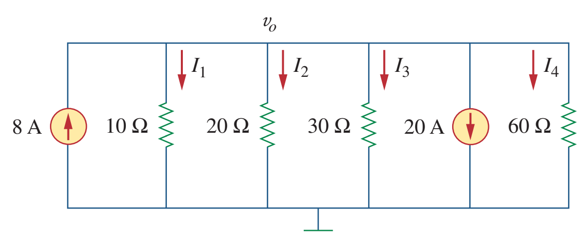


Figure 3.52
For Prob. 3.3.

3.4 Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4.

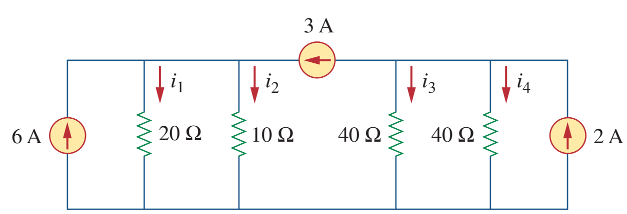


Figure 3.53
For Prob. 3.4.

3.5 Obtain v_o in the circuit of Fig. 3.54.

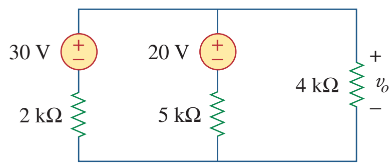


Figure 3.54
For Prob. 3.5.

3.6 Solve for V_1 in the circuit of Fig. 3.55 using nodal analysis.

va: (va-v10)/(10/3)+va/10+(va-v20)/4
v1: va/10*10-v1=0

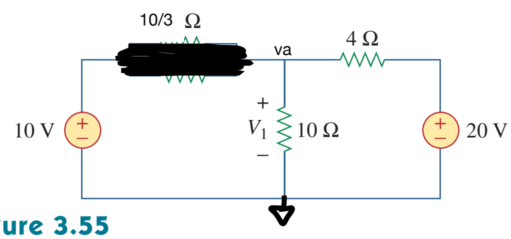


Figure 3.55
For Prob. 3.6.

3.7 Apply nodal analysis to solve for V_x in the circuit of Fig. 3.56.

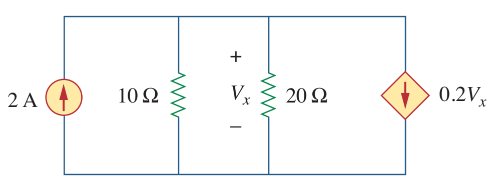


Figure 3.56
For Prob. 3.7.

3.8 Using nodal analysis, find v_o in the circuit of Fig. 3.57.

va: va/10+(va-v60)/20+(va-5vo)/20=0

vo = va/10*4-vo=0

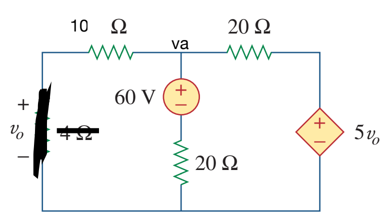


Figure 3.57
For Prob. 3.8 and Prob. 3.37.

3.9 Determine I_b in the circuit in Fig. 3.58 using nodal analysis.

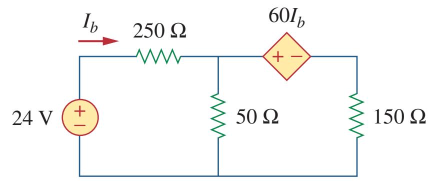


Figure 3.58
For Prob. 3.9.

3.10 Find I_o in the circuit of Fig. 3.59.

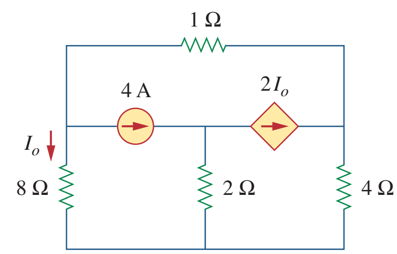


Figure 3.59
For Prob. 3.10.

3.11 Find V_o and the power dissipated in all the resistors in the circuit of Fig. 3.60.

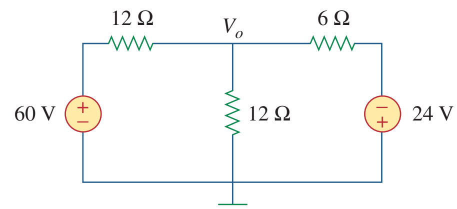


Figure 3.60
For Prob. 3.11.

3.12 Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

va: (va-v40)/20+va/20+(va-vb)/10=0
vb: (vb-va)/10+-4ix+vb/10=0
ix: va/20-ix=0
vo: vb/10*10-vo=0

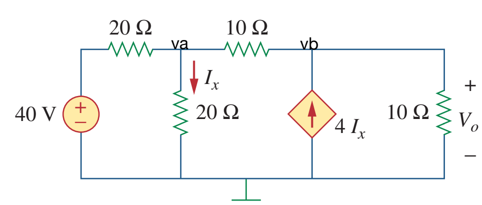


Figure 3.61
For Prob. 3.12.

3.13 Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

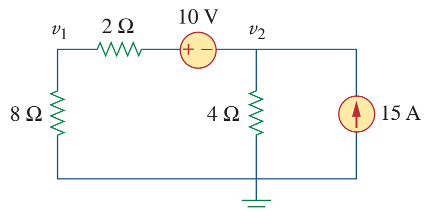


Figure 3.62
For Prob. 3.13.

3.14 Using nodal analysis, find v_o in the circuit of Fig. 3.63.

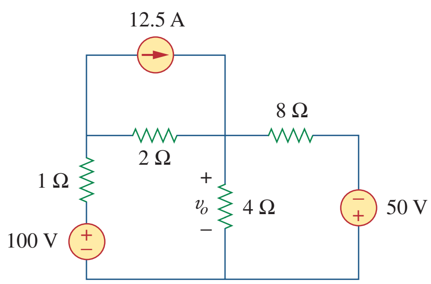


Figure 3.63
For Prob. 3.14.

3.15 Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

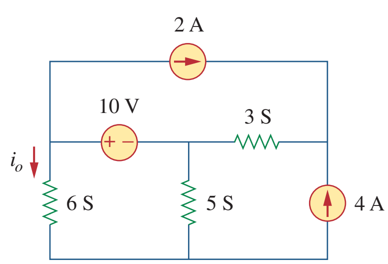


Figure 3.64
For Prob. 3.15.

3.16 Determine voltages v_1 through v_3 in the circuit of Fig. 3.65 using nodal analysis.

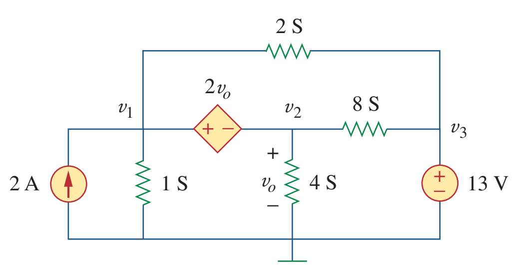


Figure 3.65
For Prob. 3.16.

3.17 Using nodal analysis, find current i_o in the circuit of Fig. 3.66.

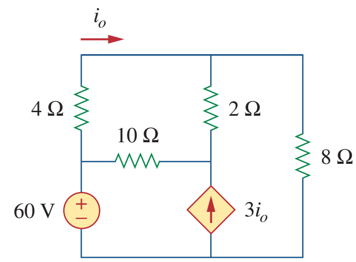


Figure 3.66
For Prob. 3.17.

3.18 Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

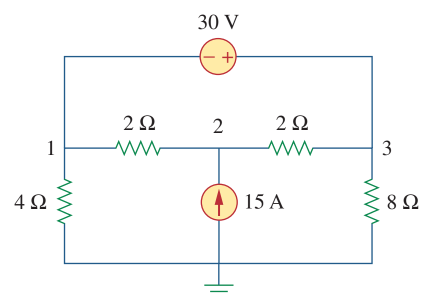


Figure 3.67
For Prob. 3.18.

3.19 Use nodal analysis to find v_1, v_2, and v_3 in the circuit of Fig. 3.68.

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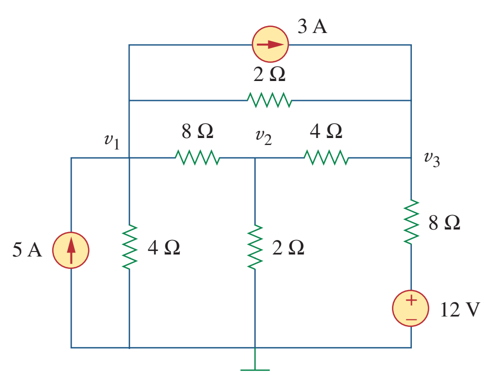

Figure 3.68
For Prob. 3.19.

3.20 For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

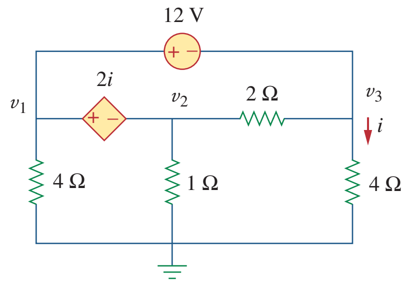

Figure 3.69
For Prob. 3.20.

3.21 For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

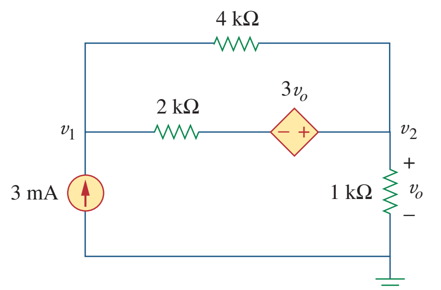

Figure 3.70
For Prob. 3.21.

3.22 Determine v_1 and v_2 in the circuit of Fig. 3.71.

v1: (v1-v12)/2+v1/4+i3+(v1-v2)/8=0
v2: -i3+(v2-v1)/8+(v2+5v0)/1=0
v0: (v1-v12)/2*2-v0=0

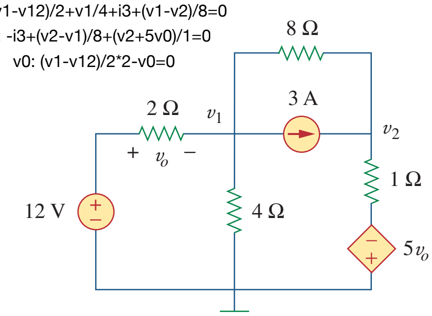

Figure 3.71
For Prob. 3.22.

3.23 Use nodal analysis to find V_o in the circuit of Fig. 3.72.

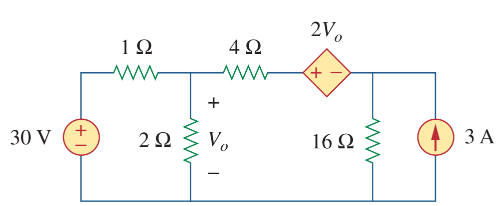

Figure 3.72
For Prob. 3.23.

3.24 Use nodal analysis and *MATLAB* to find V_o in the circuit of Fig. 3.73.
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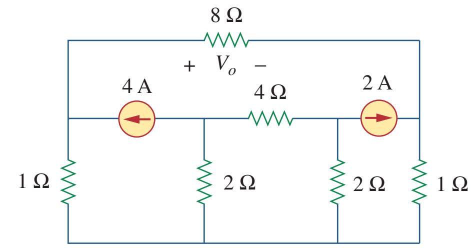

Figure 3.73
For Prob. 3.24.

3.25 Use nodal analysis along with *MATLAB* to determine the node voltages in Fig. 3.74.
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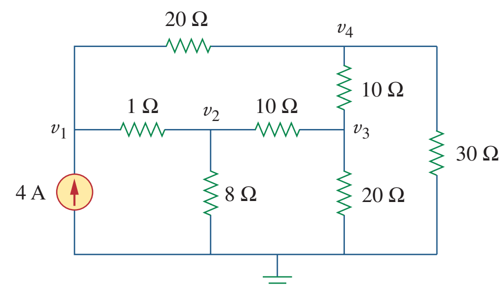

Figure 3.74
For Prob. 3.25.

3.26 Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.75.
ML

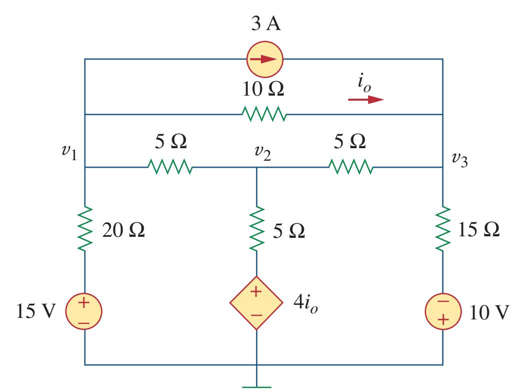

Figure 3.75
For Prob. 3.26.

***3.27** Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit of Fig. 3.76.

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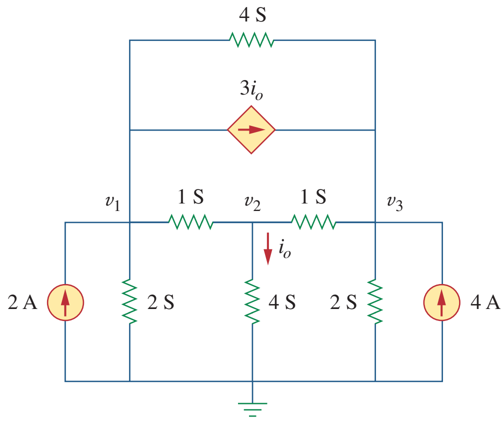

Figure 3.76
For Prob. 3.27.

***3.28** Use *MATLAB* to find the voltages at nodes *a*, *b*, *c*, and *d* in the circuit of Fig. 3.77.

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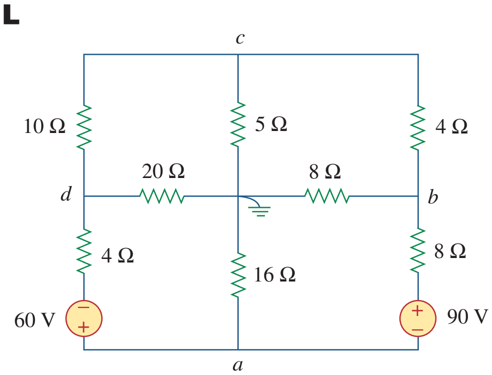

Figure 3.77
For Prob. 3.28.

3.29 Use *MATLAB* to solve for the node voltages in the circuit of Fig. 3.78.

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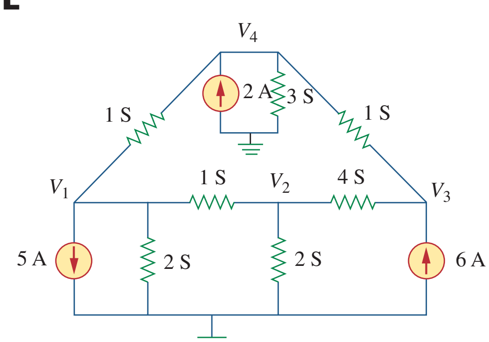

Figure 3.78
For Prob. 3.29.

3.30 Using nodal analysis, find v_o and i_o in the circuit of Fig. 3.79.

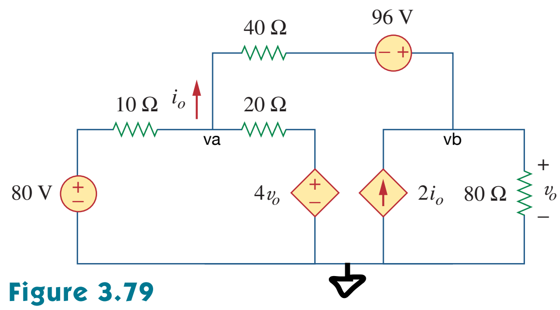

Figure 3.79
For Prob. 3.30.

va: (va-v80)/10+(va-4vo)/20+(va-(vb-v96))/40=0

vb: -2io+vb/80+((vb-v96)-va)/40=0

io: (va-(vb-v96))/40-io=0

vo:vb/80*80-vo=0

3.31 Find the node voltages for the circuit in Fig. 3.80.

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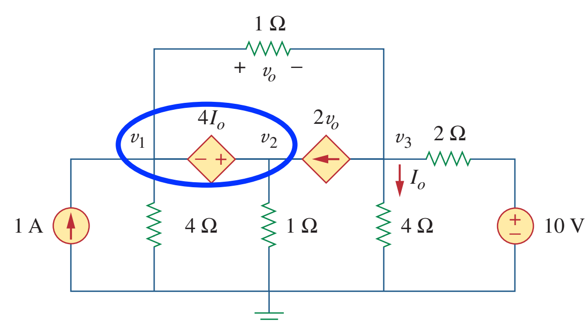

Figure 3.80
For Prob. 3.31.

v2=v1+4io

v1:-Iin+v1/4+v2/1-2vo+(v1-v3)/1=0

v3:2vo+(v3-v1)/1+v3/4+(v3-10)/2=0

3.32 Obtain the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.81.

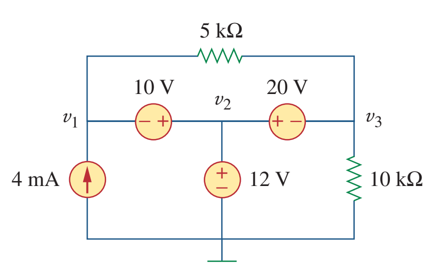

Figure 3.81
For Prob. 3.32.

* An asterisk indicates a challenging problem.