EE 2010 Circuit Analysis

Module 15: **Linearity and its Corollaries: Power Transfer** **Notes**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, WIKIPEDIA, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Learning Objective: In this module, we calculate the power delivered to an external load from a linear system with at least one source. We explore the competing notions of *Power Transfer Efficiency* and *Maximum Power Transfer* using a *Thevenin Equivalent* representation.

Recall: A second corollary of linearity is the aggregation utility of *Thevenin* and *Norton* representations. These representations are interchangeable via *Source transformation* (which follows immediately from above).

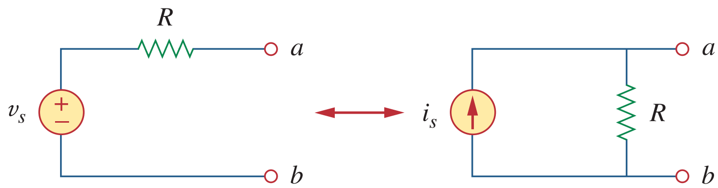
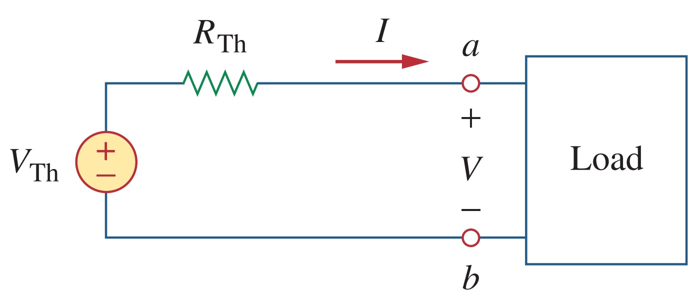


Figure 4.15
Transformation of independent sources.

with

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

1 Fixed Load Design



Design Requirements for Fixed Load: For a fixed load of R_L,

1. The system shall supply a terminal voltage of not less than 12VDC.
2. The system shall supply a terminal voltage of not more than 12.4VDC.
3. The system shall maximize power transmission efficiency defined as

$$\eta = \frac{R_L}{R_L + R_{Th}} = \frac{1}{1 + R_{Th}/R_L}$$

4. ...

Solution:

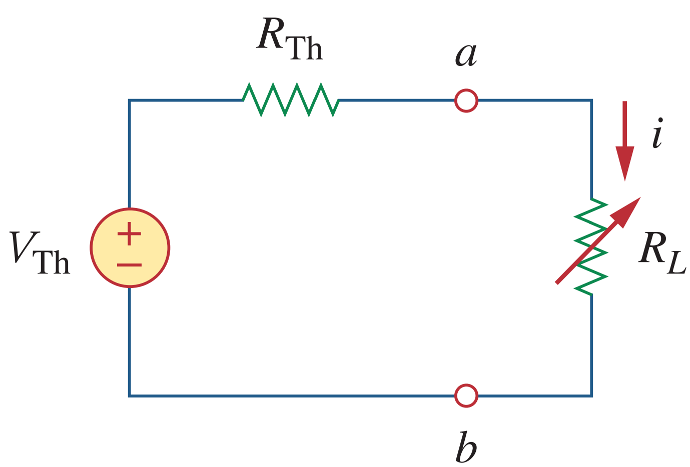
1. Design R_{Th} **as small as practical, and much smaller than** R_L
2. Design V_{Th} such that $12 = \frac{V_{Th} R_L}{R_L + R_{Th}}$

In this case, we see that "maximum power" corresponds to maximum efficiency which is achieved by minimizing R_{Th}. This is exactly the goal of every power company on the planet, and all good power supply designers.

2 Fixed Source Maximum Power Design

Note: I have never seen a practical application of this consideration in any engineering situation other than at RF frequencies where the goal is to eliminate reflected energy in waveguides. At these frequencies, systems are referred to as "distributed" as opposed to the "lumped" systems we consider here. I believe the reason this material appears in so many circuits books is that it is a "make believe" application of Thevenin - since most circuits books are so lacking in applications in general.

Notwithstanding, for completeness...



Design Requirements for Fixed Source: For fixed V_{Th} and R_{Th},

1. Maximize the power supplied to the load.

Solution:

1. Don't worry about burning up the power supply.
2. Don't worry about being grossly inefficient (as we shall see).
3. And don't worry about getting fired for placing the company in a position of negligence.

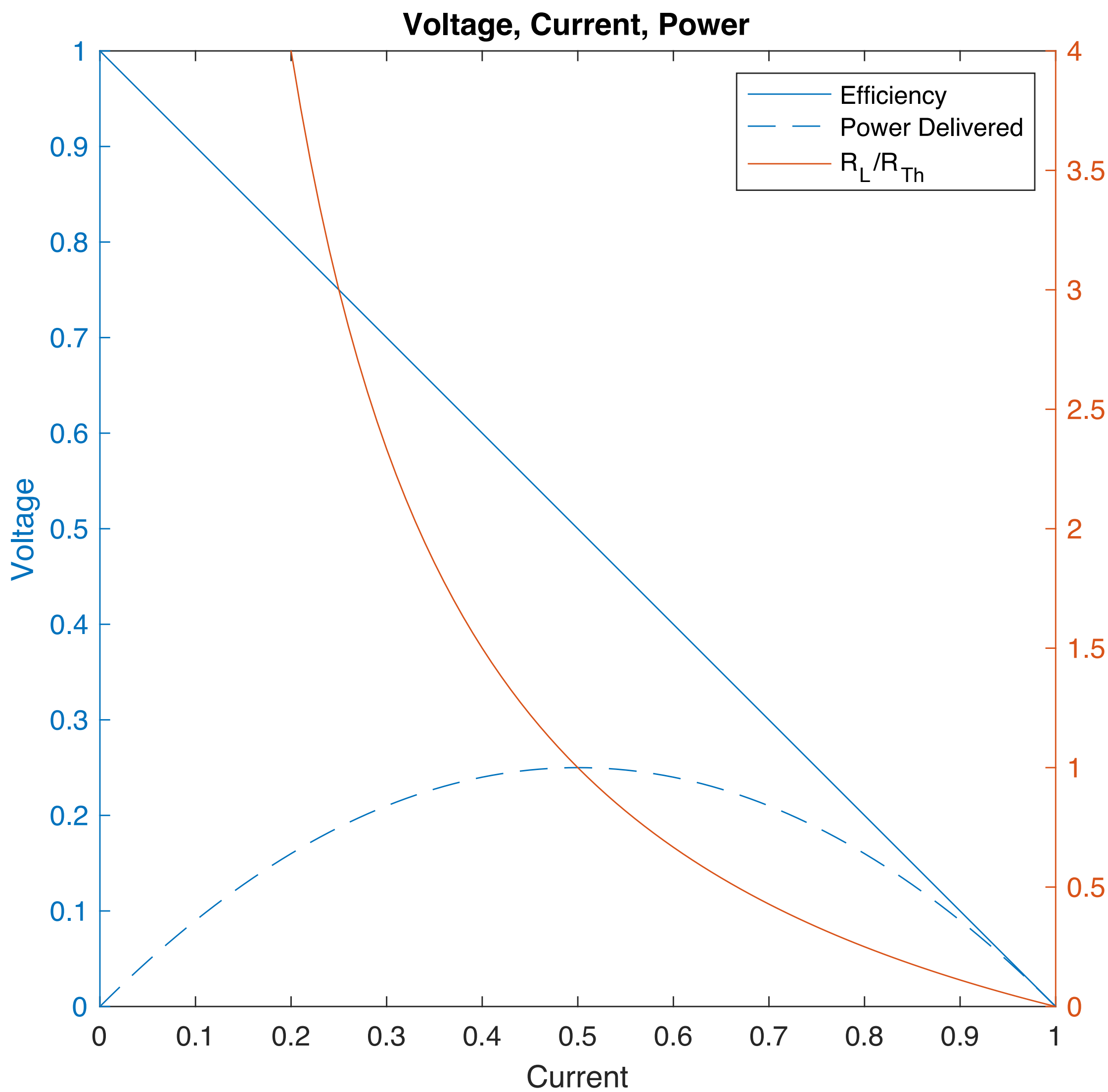
4. Set $R_L = R_{Th}$

Recall, for fixed V_{Th} and R_{Th}, the fraction of power dissipated by the load, R_L to the power developed by the source is easily found as

$$\eta = \frac{R_L}{R_L + R_{Th}} = \frac{1}{1 + R_{Th}/R_L}$$

where η is the *efficiency*.

The figure below shows the efficiency along with the power delivered and the ratio $\frac{R_L}{R_{Th}}$.



Consider three particular cases:

→ If $\frac{R_L}{R_{Th}} = 1$ then the Power Delivered is maximized but the Efficiency $\eta = 0.5$,

→ If $\frac{R_L}{R_{Th}} = \rightarrow \infty$ then the Efficiency $\eta = 1$,

→ If $\frac{R_L}{R_{Th}} = 0$ then the Efficiency $\eta = 0$.

Notice that:

The efficiency is only 50% when maximum power transfer is achieved, but approaches 100% as the load resistance approaches infinity, though the total power level tends towards zero.

Efficiency also approaches 100% if the source resistance approaches zero, and 0% if the load resistance approaches zero. In the latter case, all the power is consumed inside the source (unless the source also has no resistance), so the power dissipated in a *short circuit* is zero.

Question: What value of load impedance maximizes the power transferred to the external load?

Answer: $R_L = R_{Th}$ which is referred to as a **"matched load."**

Procedure for Determining Maximum Deliverable Power:

1. For fixed load, *minimize* R_{Th}
2. For fixed V_{Th} and R_{Th}, $R_L = R_{Th}$, the so-called "matched load."
3. In this latter case, $P_{Max} = \frac{V_{Th}^2}{4 \cdot R_{Th}}$, so that half the available power is dissipated internally and half the available power is delivered to the load. This is terribly *inneficient*!

Homework: None.