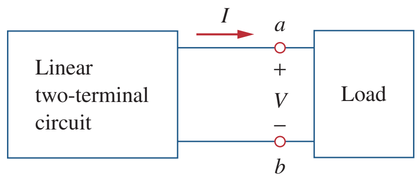
Module 14a: **Linearity and its Corollaries: Equivalents** **Notes**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, WIKIPEDIA, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

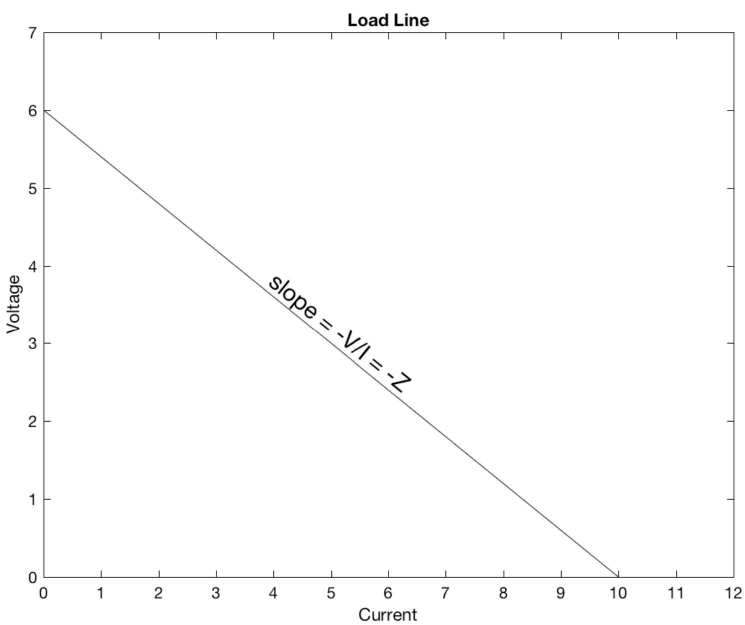
Learning Objective: In this module, we show that a linear circuit at its output terminals *can be modeled by a single voltage source and a single internal resistance.* The resulting simplification is referred to as the *Theven Equivalent.* This simplification is useful in certain analysis processes, and can be readily found using the same node techniques for a specific excitation.

A second corollary of linearity manifests as *equivalent system representations.* Consider the system below:



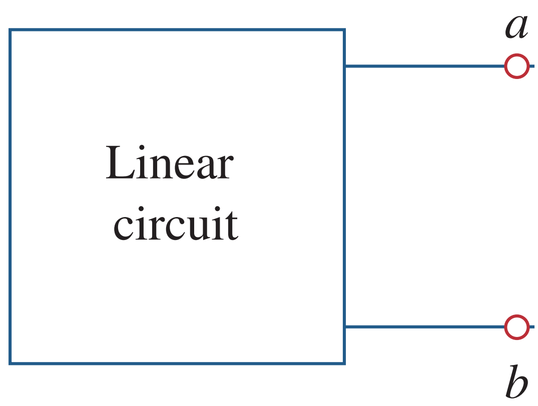
(a)

What happens to the voltage V and current I if the load resistance is varied?



Every linear circuit has such a load line! And ... *Every such load line can be uniquely determined by two distinct points on the line!*. But be very careful in loading circuits outside their load-range specifications.

Question: Are there simplified circuit forms that exhibit this same load line? Then any such simplified circuit would be *electrically equivalent* and indistinguishable to the observer!



Two Simple Equivalants Note: doesn't have to be a resistor, can be any impedence

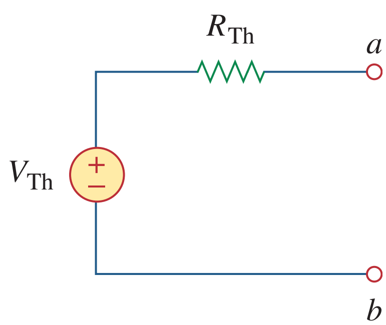


Figure 1: *Thevenin equivalent*

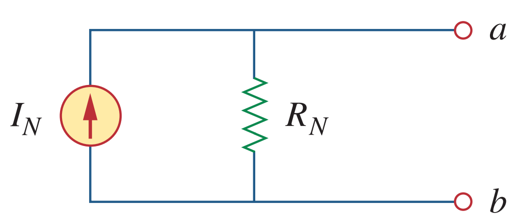


Figure 2: *Norton equivalent*

... and we also have the notion of **source transformation** - so that the simple equivalents are completely **interchangeable**.

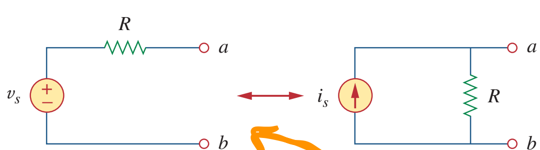


Figure 4.15
Transformation of independent sources.

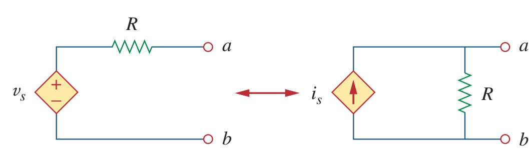


Figure 4.16
Transformation of dependent sources.

converting between
1) don't change impedance
2) if going voltage -> current, I=V/R
3) if going current -> voltage, V=IR

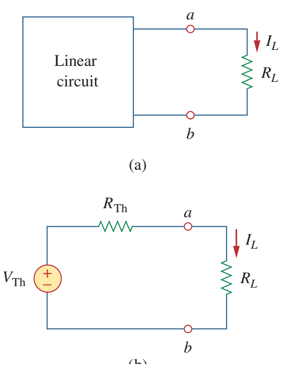
Best-Practice Procedure for Determining a Simplified Model: Closed-Box Systems

For modeling **physical closed-box systems,** we need to find two points to determine the load line empirically, and thereby determine a simplified equivalent circuit.

Test Load Procedure:

1. With the circuit *in-situ* and energized,
2. Measure the open-circuit voltage, obtain $V_{Th} = V_{\text{OPEN}}$.
3. Apply a reasonable load R_{LOAD} within range of the system specs **(NEVER A SHORT!)**
4. Measure the voltage, V_{LOAD}.
5. We can then find:

$$
\begin{aligned}
R_{Th} &= -\frac{V_{\text{OPEN}} - V_{\text{LOAD}}}{I_{\text{OPEN}} - I_{\text{LOAD}}} \\
&= \frac{V_{\text{OPEN}} - V_{\text{LOAD}}}{I_{\text{LOAD}} - 0} \\
&= \frac{V_{\text{OPEN}} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}
\end{aligned}
$$



Note: Accuracy increases if additional R_{LOAD} measurements are incorporated into a *linear regression estimate* of the load line.

Best-Practice Procedure for Determining a Simplified Model: Open-Box Systems

Test Load Procedure: 

1. Replace *all independent sources with symbolic representations such as* V_{in} *or* I_{in}
2. Add a "test-load" resistor R_L across the specified terminals, $a - b$.
3. Use circuit analysis techniques to determine V_{a-b} in terms of V_{in} and R_L.
4. The result will be of the form: $V_{a-b} = \text{Input}_1 \cdot H_1 + \text{Input}_2 \cdot H_2$ with R_L in the expressions for the transfer functions.
5. Compute V_{a-b} as $R_L \to \infty$ to obtain $V_{Th} = V_{a-b}|_{R_L \to \infty} = \text{Input}_1 \cdot H_1^* + \text{Input}_2 \cdot H_2^*$.
6. If asked, find V_{Th} for specific values of V_{in} or I_{in}.
7. To find R_{Th}, select a second reasonable value for R_{LOAD}
8. Compute the voltage, $V_{\text{LOAD}} = V_{a-b}|_{R_L = R_{\text{LOAD}}}$.
9. Then find:

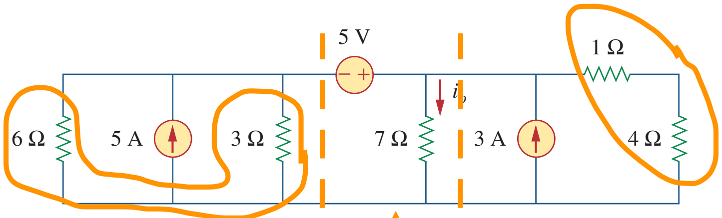
$$R_{Th} = \frac{V_{Th} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}$$

Note: R_{Th} is not dependent on the values of *independent* sources!

Source transformation is rarely used to solve real circuit problems. But to practice going back and forth, some **source transformation** problems.

Find i_o.

parallel, 2ohm



series, 5ohm

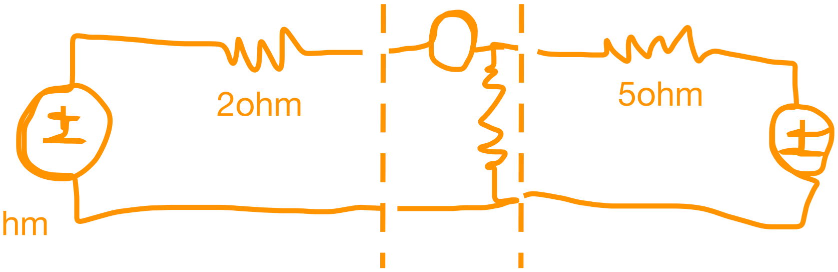
Step 1

Figure 4.19
For Practice Prob. 4.6.

treat this separate, transform this into voltage

same as

Step 2

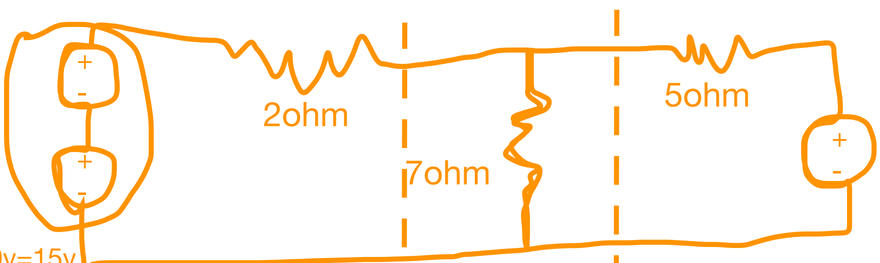


Step 3

V=15V

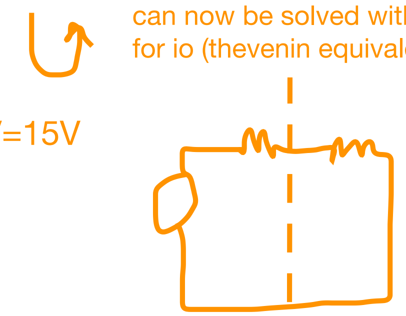
V=IR
=5A*2ohm
=10V
note: current points up = + on top

Step 4, swap the 2ohm resistor with the middle voltage source, they're still in series so no change



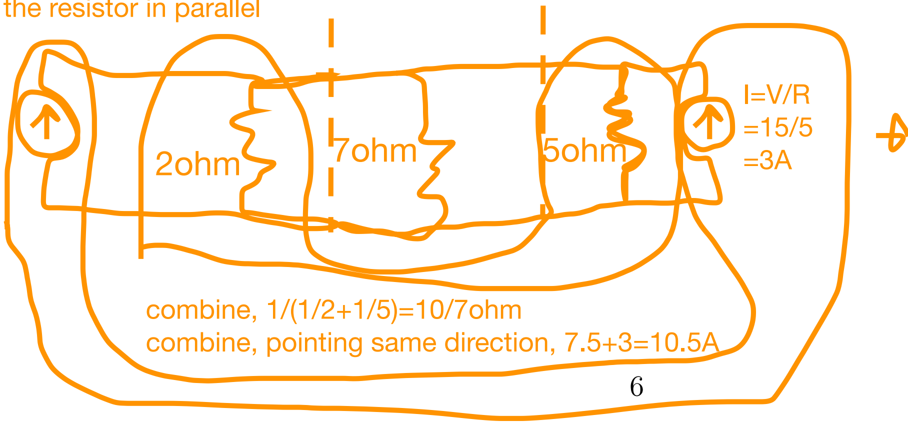
5v+10v=15v

can now be solved with I=V/R to solve for io (thevenin equivalent circuit)



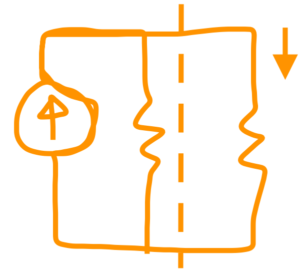
Step 5: nothing left to do, change it back to current bc it puts the resistor in parallel

I=V/R
=15/2
=7.5A



I=V/R
=15/5
=3A

io



combine, 1/(1/2+1/5)=10/7ohm
combine, pointing same direction, 7.5+3=10.5A

can now be solved with current division to solve for io (norton equivalent circuit)

Step 6: change a bit and solve

One more practice with source transformation: Find v_x.

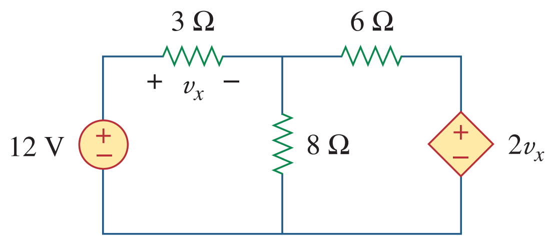
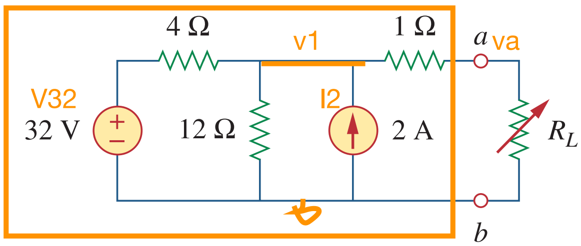


Figure 4.99
For Prob. 4.31.

And now some **Thevenin equivalents**.



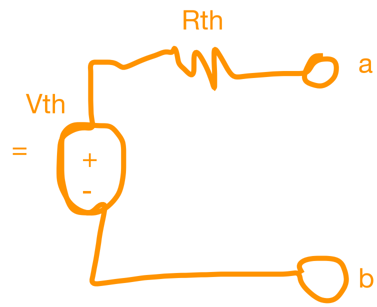


Figure 4.27
For Example 4.8.

v1: (v1-V32)/4+v1/12-I2+(v1-va)=0
va: (va-v1)+va/RL=0

Test Load Procedure:

1. Replace *all independent sources with symbolic representations such as* V_{in} *or* I_{in}
2. Compute V_{a-b} as $R_L \to \infty$ to obtain $V_{Th} = V_{a-b}|_{R_L\to\infty} = \text{Input}_1 \cdot H_1^* + \text{Input}_2 \cdot H_2^*$.
3. To find R_{Th}, select a second reasonable value for R_{LOAD}
4. Compute the voltage, $V_{\text{LOAD}} = V_{a-b}|_{R_L=R_{\text{LOAD}}}$.
5. Then find:

$$R_{Th} = \frac{V_{Th} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}$$

```
%% Example 4.8 Test Load
clear all
% Declare symbolic variables
syms Vin Iin vx RL
% Write node equation at vx
[vx]=solve((vx-Vin)/4 - Iin + vx/12 + vx/(1+RL) == 0, vx)
VTh = limit(vx*RL/(1+RL), RL, Inf)
subs(VTh,[Vin,Iin],[32,2])
% Find a second load-point, Pick RL = 10
V10 = subs(vx*RL/(1+RL),RL,10);
RTh = simplify((VTh - V10)/(V10/10))
%
```

Pick RL = 10 is just a random point that won't break the circuit

which yields...

```
vx = (Iin + 0.2500*Vin)/(1/(RL + 1) + 0.3333)
VTh = 3*Iin + 0.7500*Vin
ans = 30
RTh = 4
```

so that $V_{Th} = 30V$ and $R_{Th} = 4\Omega$.

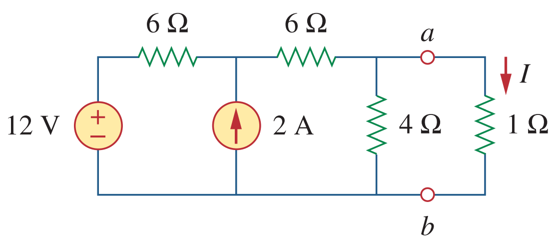


Figure 4.30
For Practice Prob. 4.8.

Test Load Procedure:

1. Replace the 1Ω resistor with a load resistor R_L. (Notice that R_L is in parallel with the 4Ω resistor).
2. Replace *all independent sources with symbolic representations such as* V_{in} *or* I_{in}
3. Compute V_{a-b} as $R_L \to \infty$ to obtain $V_{Th} = V_{a-b}|_{R_L \to \infty} = \text{Input}_1 \cdot H_1^* + \text{Input}_2 \cdot H_2^*$.
4. To find R_{Th}, select a second reasonable value for R_{LOAD}
5. Compute the voltage, $V_{\text{LOAD}} = V_{a-b}|_{R_L = R_{\text{LOAD}}}$.
6. Then find:

$$R_{Th} = \frac{V_{Th} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}$$

```
%% Practice Problem 4.8 Test Load
clear all
% Declare symbolic variables
syms Vin Iin vx RL
% Write node equation at vx
[vx]=solve((vx-Vin)/6 - Iin + vx/(6 + (4*RL)/(4+RL)) == 0, vx)
VTh = limit(vx*((4*RL)/(4+RL))/(6 + (4*RL)/(4+RL)), RL, Inf)
subs(VTh,[Vin,Iin],[12,2])
% Pick RL = 6
V6 = subs(vx*((4*RL)/(4+RL))/(6 + (4*RL)/(4+RL)),RL,6);
RTh = simplify((VTh - V6)/(V6/6))
%
```

which yields...

```
vx = (Iin + 0.1667*Vin)/(1/((4*RL)/(RL + 4) + 6) + 0.1667)
VTh = 1.5000*Iin + 0.2500*Vin
ans = 6
RTh = 3
```

so that $V_{Th} = 6V$ and $R_{Th} = 3\Omega$.

Now with dependent sources.

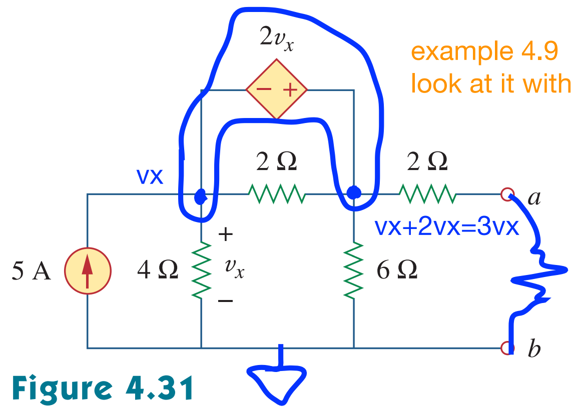


Figure 4.31
For Example 4.9.

example 4.9
look at it with an open circuit, then look at it with a load resistor there

RL (this is load resistor)

node equation vx:
vx/4-5+(vx-3vx)/2+(3vx-vx)/2+(3vx)/6+(3vx)/(2+RL)=0

Test Load Procedure:

1. Add a load resistor R_L to terminals $a - b$.
2. Replace *all independent sources with symbolic representations such as* V_{in} *or* I_{in}
3. Compute V_{a-b} as $R_L \to \infty$ to obtain $V_{Th} = V_{a-b}|_{R_L \to \infty} = \text{Input}_1 \cdot H_1^* + \text{Input}_2 \cdot H_2^*$.
4. To find R_{Th}, select a second reasonable value for R_{LOAD}
5. Compute the voltage, $V_{\text{LOAD}} = V_{a-b}|_{R_L = R_{\text{LOAD}}}$.
6. Then find:

$$R_{Th} = \frac{V_{Th} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}$$

```
%% Example 4.9 Test Load
clear all
% Declare symbolic variables
syms Iin vx RL
% Write node equation at vx
[vx]=solve(vx/4 − Iin + (vx + 2*vx)/6 + (vx+2*vx)/(2+RL) == 0, vx)
VTh = limit(3*vx*(RL/(2+RL)), RL, Inf)
subs(VTh,Iin,5)
% Pick RL = 6
V6 = subs(3*vx*(RL/(2+RL)),RL,6);
RTh = (VTh − V6)/(V6/6)
%
```

equation from using voltage divider
from node 3vx to get voltage at RL

which yields...

```
vx = Iin/(3/(RL + 2) + 0.7500)
VTh = 4*Iin
ans = 20
RTh = 6
```

so that $V_{Th} = 20V$ and $R_{Th} = 6\Omega$.

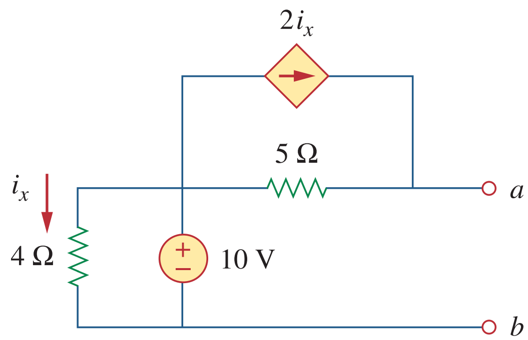


Figure 4.43
For Example 4.12.

Test Load Procedure:

1. Add a load resistor R_L to terminals $a - b$.
2. Replace *all independent sources with symbolic representations such as* V_{in} *or* I_{in}
3. Compute V_{a-b} as $R_L \to \infty$ to obtain $V_{Th} = V_{a-b}|_{R_L \to \infty} = \text{Input}_1 \cdot H_1^* + \text{Input}_2 \cdot H_2^*$.
4. To find R_{Th}, select a second reasonable value for R_{LOAD}
5. Compute the voltage, $V_{\text{LOAD}} = V_{a-b}|_{R_L = R_{\text{LOAD}}}$.
6. Then find:

$$R_{Th} = \frac{V_{Th} - V_{\text{LOAD}}}{V_{\text{LOAD}}/R_{\text{LOAD}}}$$

```
%% Example 4.12 Test Load
clear all
% Declare symbolic variables
syms Vin va RL ix
% Write node equation at vx
[va, ix] = solve(va/RL + (va - Vin)/5 - 2*ix == 0, ix == Vin/4, va, ix)
VTh = limit(va, RL, Inf)
subs(VTh,Vin,10)
% Pick RL = 10
V10 = subs(va,RL,10);
RTh = (VTh - V10)/(V10/10)
%
```

which yields...

```
va = (3.5000*RL*Vin)/(RL + 5)
ix = 0.2500*Vin
VTh = 3.5000*Vin
ans = 35
RTh = 5
```

so that $V_{Th} = 35V$ and $R_{Th} = 5\Omega$.

Homework: Chapter 4: Source transformation: # 20, 24 (Just do enough to be confident.)
Thevenin Test Load: # 33, 37, 39, 47, 50, 52, 53, 71

Note: If someone asks for Norton, I always do Thevenin and Source transform to Norton.

→ Homework deliverables MUST be a pdf file generated using a solver.

→ The resulting .pdf file is to be uploaded to the Pilot Dropbox using the naming convention: First 4 letters of Lastname, First initial, year, title. For example, my .pdf file would be named: GarbF2021HW14.pdf

6. For a given Thevenin equivalent circuit, maximum power transfer occurs when $R_L = R_{Th}$; that is, when the load resistance is equal to the Thevenin resistance.
7. The maximum power transfer theorem states that the maximum power is delivered by a source to the load R_L when R_L is equal to R_{Th}, the Thevenin resistance at the terminals of the load.
8. *PSpice* can be used to verify the circuit theorems covered in this chapter.
9. Source modeling and resistance measurement using the Wheatstone bridge provide applications for Thevenin's theorem.

Review Questions

4.1 The current through a branch in a linear network is 2 A when the input source voltage is 10 V. If the voltage is reduced to 1 V and the polarity is reversed, the current through the branch is:

(a) −2 A (b) −0.2 A (c) 0.2 A
(d) 2 A (e) 20 A

4.2 For superposition, it is not required that only one independent source be considered at a time; any number of independent sources may be considered simultaneously.

(a) True (b) False

4.3 The superposition principle applies to power calculation.

(a) True (b) False

4.4 Refer to Fig. 4.67. The Thevenin resistance at terminals *a* and *b* is:

(a) 25 Ω (b) 20 Ω
(c) 5 Ω (d) 4 Ω

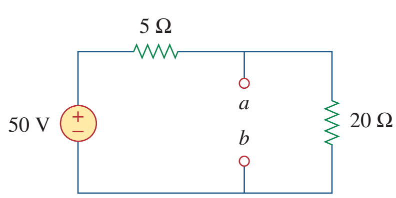


Figure 4.67
For Review Questions 4.4 to 4.6.

4.5 The Thevenin voltage across terminals *a* and *b* of the circuit in Fig. 4.67 is:

(a) 50 V (b) 40 V
(c) 20 V (d) 10 V

4.6 The Norton current at terminals *a* and *b* of the circuit in Fig. 4.67 is:

(a) 10 A (b) 2.5 A
(c) 2 A (d) 0 A

4.7 The Norton resistance R_N is exactly equal to the Thevenin resistance R_{Th}.

(a) True (b) False

4.8 Which pair of circuits in Fig. 4.68 are equivalent?

(a) a and b (b) b and d
(c) a and c (d) c and d

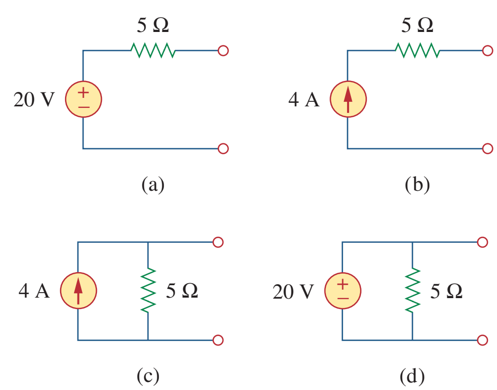


Figure 4.68
For Review Question 4.8.

4.9 A load is connected to a network. At the terminals to which the load is connected, $R_{Th} = 10\ \Omega$ and $V_{Th} = 40$ V. The maximum possible power supplied to the load is:

(a) 160 W (b) 80 W
(c) 40 W (d) 1 W

4.10 The source is supplying the maximum power to the load when the load resistance equals the source resistance.

(a) True (b) False

Answers: 4.1b, 4.2a, 4.3b, 4.4d, 4.5b, 4.6a, 4.7a, 4.8c, 4.9c, 4.10a.

Problems

Section 4.2 Linearity Property

4.1 Calculate the current i_o in the circuit of Fig. 4.69. What value of input voltage is necessary to make i_o equal to 5 amps?

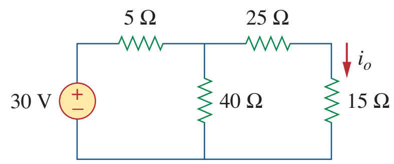


Figure 4.69
For Prob. 4.1.

4.2 Using Fig. 4.70, design a problem to help other students better understand linearity.

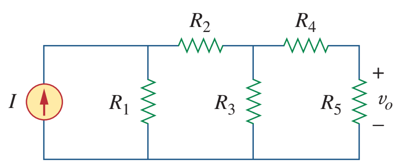


Figure 4.70
For Prob. 4.2.

4.3 (a) In the circuit of Fig. 4.71, calculate v_o and i_o when $v_s = 1$ V.

(b) Find v_o and i_o when $v_s = 10$ V.

(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

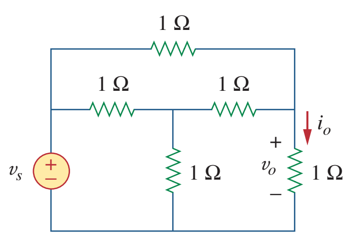


Figure 4.71
For Prob. 4.3.

4.4 Use linearity to determine i_o in the circuit of Fig. 4.72.

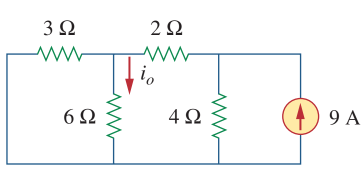


Figure 4.72
For Prob. 4.4.

4.5 For the circuit in Fig. 4.73, assume $v_o = 1$ V, and use linearity to find the actual value of v_o.

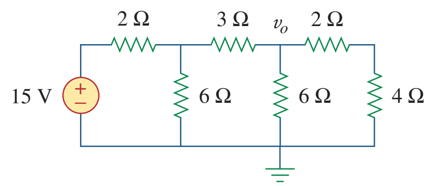


Figure 4.73
For Prob. 4.5.

4.6 For the linear circuit shown in Fig. 4.74, use linearity to complete the following table.

Experiment	V_s	V_o
1	12 V	4 V
2		16 V
3	1 V	
4		−2 V

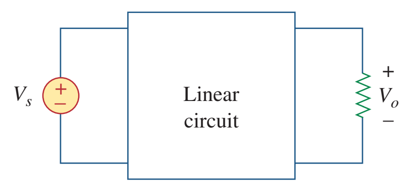


Figure 4.74
For Prob. 4.6.

4.7 Use linearity and the assumption that $V_o = 1$ V to find the actual value of V_o in Fig. 4.75.

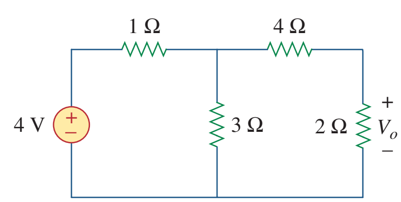


Figure 4.75
For Prob. 4.7.

Section 4.3 Superposition

4.8 Using superposition, find V_o in the circuit of Fig. 4.76. Check with *PSpice or MultiSim.*

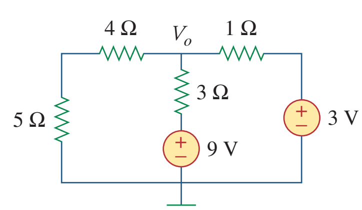


Figure 4.76
For Prob. 4.8.

4.9 Given that $I = 4$ amps when $V_s = 40$ volts and $I_s = 4$ amps and $I = 1$ amp when $V_s = 20$ volts and $I_s = 0$, use superposition and linearity to determine the value of I when $V_s = 60$ volts and $I_s = -2$ amps.

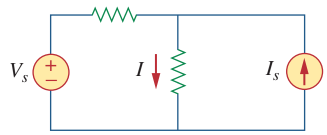


Figure 4.77
For Prob. 4.9.

4.10 Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter k is a gain you can specify to make the problem easier to solve but must not be zero.

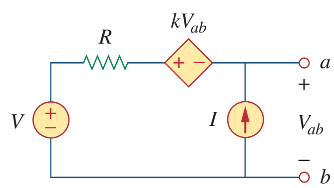


Figure 4.78
For Prob. 4.10.

4.11 Use the superposition principle to find i_o and v_o in the circuit of Fig. 4.79.

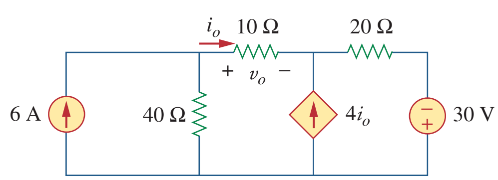


Figure 4.79
For Prob. 4.11.

4.12 Determine v_o in the circuit of Fig. 4.80 using the superposition principle.

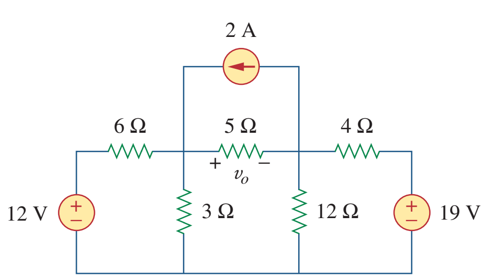


Figure 4.80
For Prob. 4.12.

4.13 Use superposition to find v_o in the circuit of Fig. 4.81.

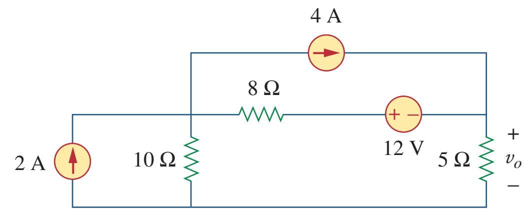


Figure 4.81
For Prob. 4.13.

4.14 Apply the superposition principle to find v_o in the circuit of Fig. 4.82.

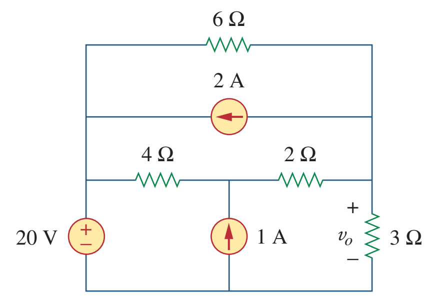


Figure 4.82
For Prob. 4.14.

4.15 For the circuit in Fig. 4.83, use superposition to find i. Calculate the power delivered to the 3-Ω resistor.

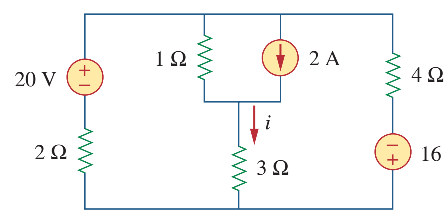


Figure 4.83
For Probs. 4.15 and 4.56.

4.16 Given the circuit in Fig. 4.84, use superposition to obtain i_o.

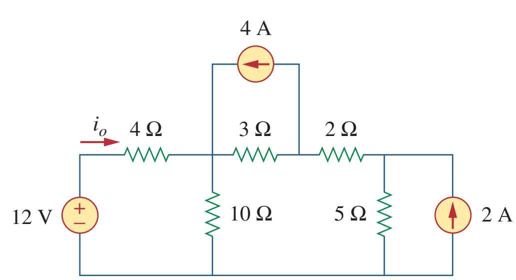


Figure 4.84
For Prob. 4.16.

4.17 Use superposition to obtain v_x in the circuit of Fig. 4.85. Check your result using *PSpice or MultiSim*.

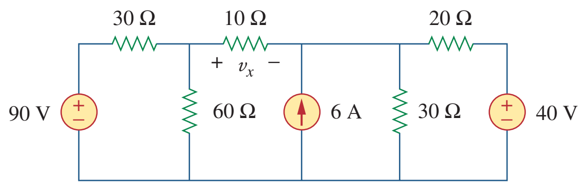


Figure 4.85
For Prob. 4.17.

4.18 Use superposition to find V_o in the circuit of Fig. 4.86.

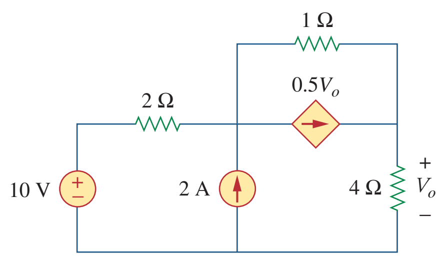


Figure 4.86
For Prob. 4.18.

4.19 Use superposition to solve for v_x in the circuit of Fig. 4.87.

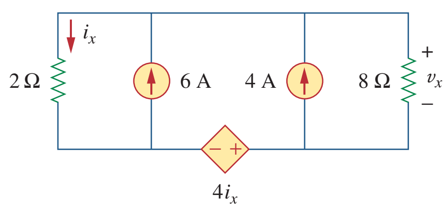


Figure 4.87
For Prob. 4.19.

Section 4.4 Source Transformation

4.20 Use source transformation to reduce the circuit in Fig. 4.88 to a single voltage source in series with a single resistor.

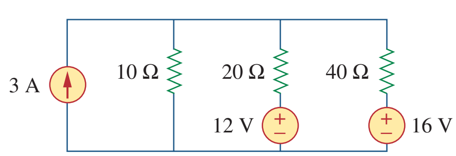


Figure 4.88
For Prob. 4.20.

4.21 Using Fig. 4.89, design a problem to help other students better understand source transformation.

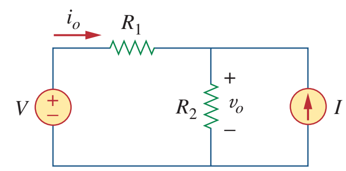


Figure 4.89
For Prob. 4.21.

4.22 For the circuit in Fig. 4.90, use source transformation to find i.

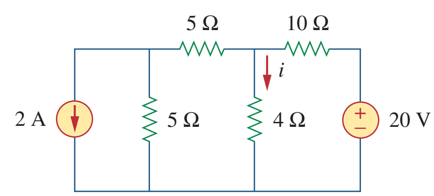


Figure 4.90
For Prob. 4.22.

4.23 Referring to Fig. 4.91, use source transformation to determine the current and power absorbed by the 8-Ω resistor.

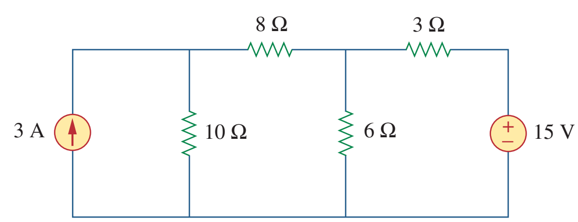


Figure 4.91
For Prob. 4.23.

4.24 Use source transformation to find the voltage V_x in the circuit of Fig. 4.92.

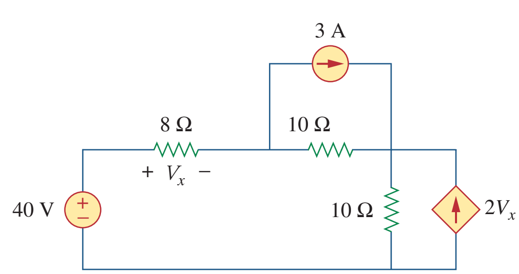


Figure 4.92
For Prob. 4.24.

4.25 Obtain v_o in the circuit of Fig. 4.93 using source transformation. Check your result using *PSpice or MultiSim.*

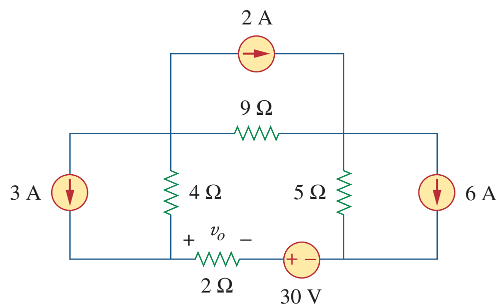


Figure 4.93
For Prob. 4.25.

4.26 Use source transformation to find i_o in the circuit of Fig. 4.94.

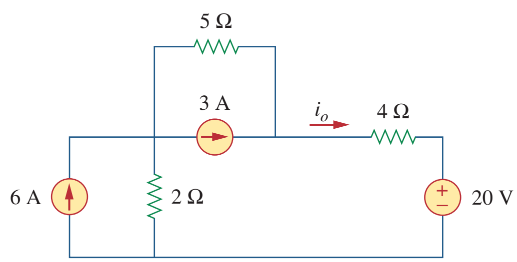


Figure 4.94
For Prob. 4.26.

4.27 Apply source transformation to find v_x in the circuit of Fig. 4.95.

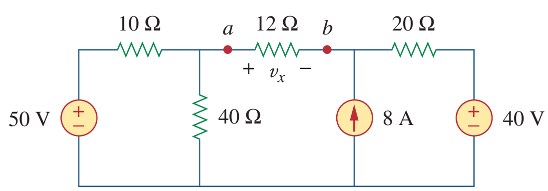


Figure 4.95
For Probs. 4.27 and 4.40.

4.28 Use source transformation to find I_o in Fig. 4.96.

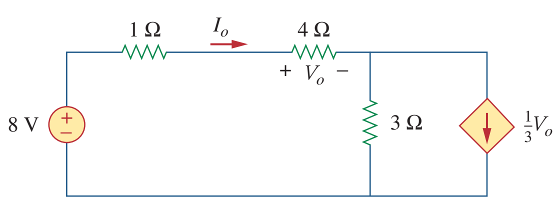


Figure 4.96
For Prob. 4.28.

4.29 Use source transformation to find v_o in the circuit of Fig. 4.97.

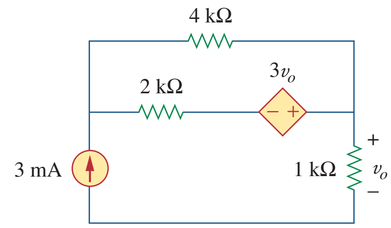


Figure 4.97
For Prob. 4.29.

4.30 Use source transformation on the circuit shown in Fig 4.98 to find i_x.

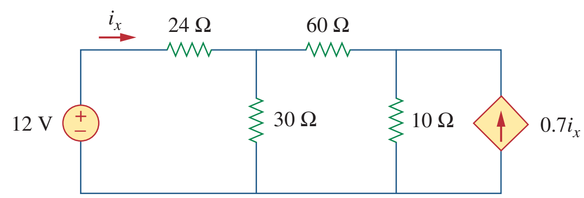


Figure 4.98
For Prob. 4.30.

4.31 Determine v_x in the circuit of Fig. 4.99 using source transformation.

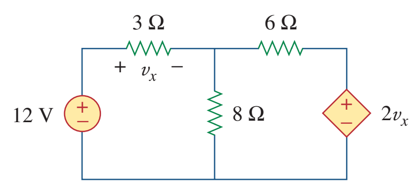


Figure 4.99
For Prob. 4.31.

4.32 Use source transformation to find i_x in the circuit of Fig. 4.100.

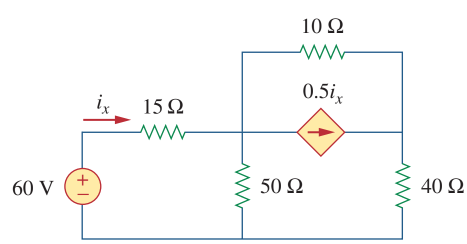


Figure 4.100
For Prob. 4.32.

Sections 4.5 and 4.6 Thevenin's and Norton's Theorems

4.33 Determine the Thevenin equivalent circuit, shown in Fig. 4.101, as seen by the 5-ohm resistor.
Then calculate the current flowing through the 5-ohm resistor.

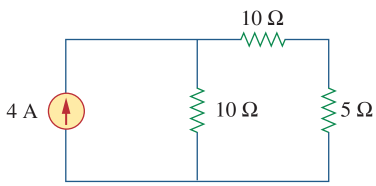


Figure 4.101
For Prob. 4.33.

4.34 Using Fig. 4.102, design a problem that will help other students better understand Thevenin equivalent circuits.

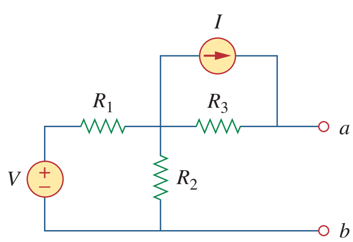


Figure 4.102
For Probs. 4.34 and 4.49.

4.35 Use Thevenin's theorem to find v_o in Prob. 4.12.

4.36 Solve for the current i in the circuit of Fig. 4.103 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent seen by the 12-Ω resistor.)

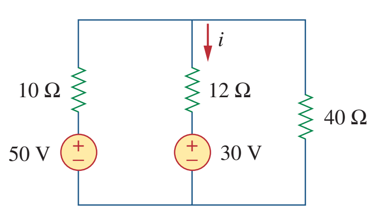


Figure 4.103
For Prob. 4.36.

4.37 Find the Norton equivalent with respect to terminals a-b in the circuit shown in Fig. 4.104.

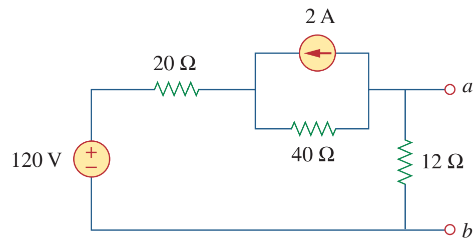


Figure 4.104
For Prob. 4.37.

4.38 Apply Thevenin's theorem to find V_o in the circuit of Fig. 4.105.

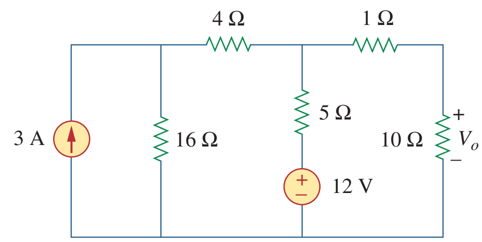


Figure 4.105
For Prob. 4.38.

4.39 Obtain the Thevenin equivalent at terminals a-b of the circuit shown in Fig. 4.106.

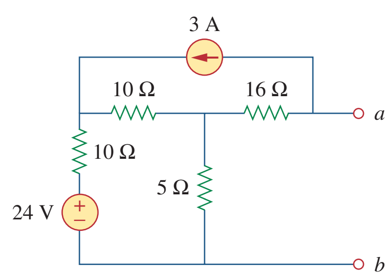


Figure 4.106
For Prob. 4.39.

4.40 Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 4.107.

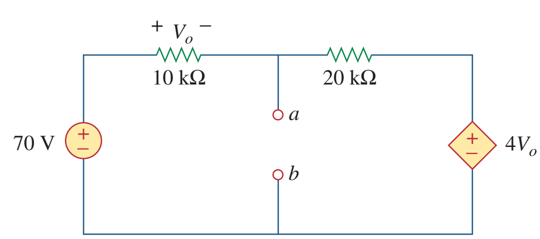


Figure 4.107
For Prob. 4.40.

4.41 Find the Thevenin and Norton equivalents at terminals *a-b* of the circuit shown in Fig. 4.108.

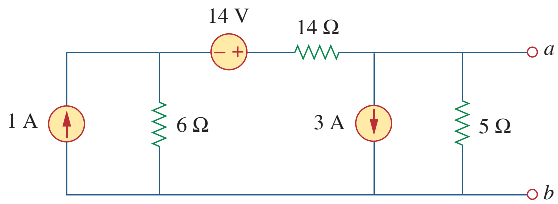


Figure 4.108
For Prob. 4.41.

***4.42** For the circuit in Fig. 4.109, find the Thevenin equivalent between terminals *a* and *b*.

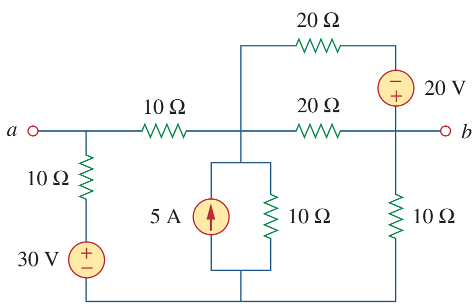


Figure 4.109
For Prob. 4.42.

4.43 Find the Thevenin equivalent looking into terminals *a-b* of the circuit in Fig. 4.110 and solve for i_x.

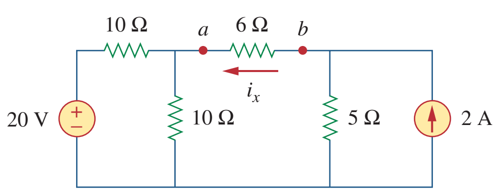


Figure 4.110
For Prob. 4.43.

4.44 For the circuit in Fig. 4.111, obtain the Thevenin equivalent as seen from terminals:

(a) *a-b* (b) *b-c*

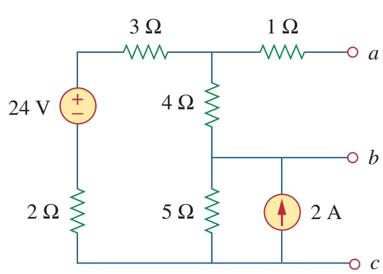


Figure 4.111
For Prob. 4.44.

* An asterisk indicates a challenging problem.

4.45 Find the Thevenin equivalent of the circuit in Fig. 4.112 as seen by looking into terminals *a* and *b*.

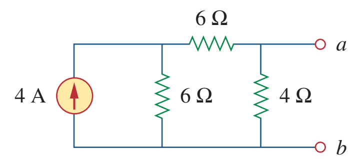


Figure 4.112
For Prob. 4.45.

4.46 Using Fig. 4.113, design a problem to help other students better understand Norton equivalent circuits.

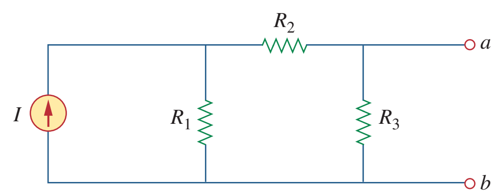


Figure 4.113
For Prob. 4.46.

4.47 Obtain the Thevenin and Norton equivalent circuits of the circuit in Fig. 4.114 with respect to terminals *a* and *b*.

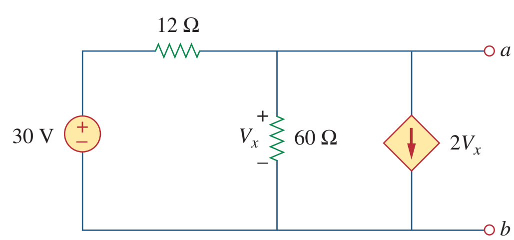


Figure 4.114
For Prob. 4.47.

4.48 Determine the Norton equivalent at terminals *a-b* for the circuit in Fig. 4.115.

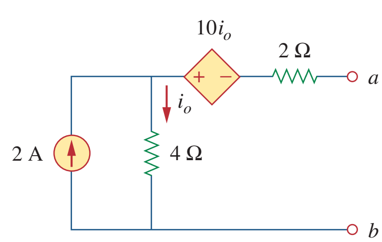


Figure 4.115
For Prob. 4.48.

4.49 Find the Norton equivalent looking into terminals *a-b* of the circuit in Fig. 4.102. Let $V = 40$ V, $I = 3$ A, $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$, and $R_3 = 20\ \Omega$.

4.50 Obtain the Norton equivalent of the circuit in Fig. 4.116 to the left of terminals *a-b*. Use the result to find current *i*.

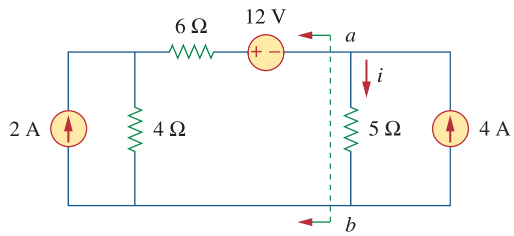


Figure 4.116
For Prob. 4.50.

4.51 Given the circuit in Fig. 4.117, obtain the Norton equivalent as viewed from terminals:

(a) *a-b* (b) *c-d*

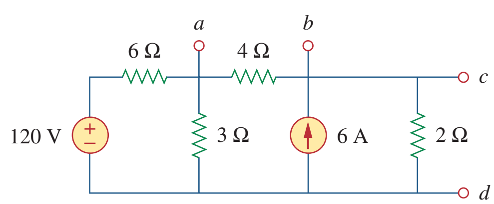


Figure 4.117
For Prob. 4.51.

4.52 For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals *a-b*.

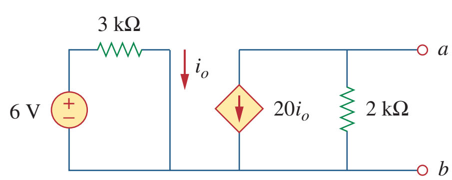


Figure 4.118
For Prob. 4.52.

4.53 Find the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.119.

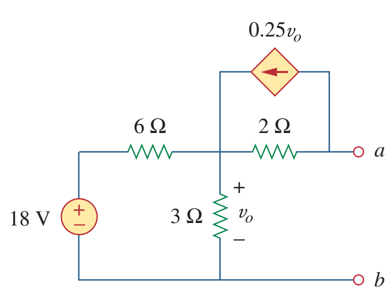


Figure 4.119
For Prob. 4.53.

4.54 Find the Thevenin equivalent between terminals *a-b* of the circuit in Fig. 4.120.

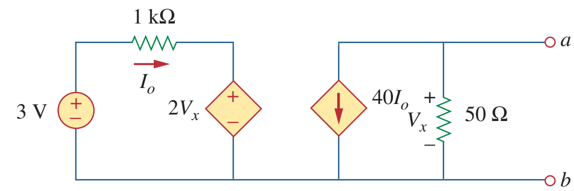


Figure 4.120
For Prob. 4.54.

***4.55** Obtain the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.121.

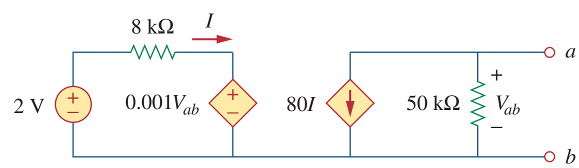


Figure 4.121
For Prob. 4.55.

4.56 Use Norton's theorem to find V_o in the circuit of Fig. 4.122.

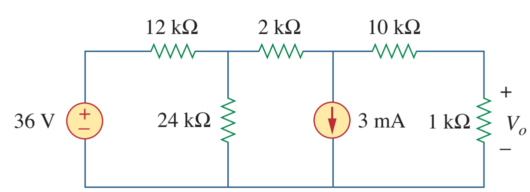


Figure 4.122
For Prob. 4.56.

4.57 Obtain the Thevenin and Norton equivalent circuits at terminals *a-b* for the circuit in Fig. 4.123.

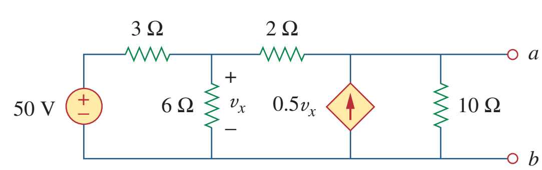


Figure 4.123
For Probs. 4.57 and 4.79.

4.58 The network in Fig. 4.124 models a bipolar transistor common-emitter amplifier connected to a load. Find the Thevenin resistance seen by the load.

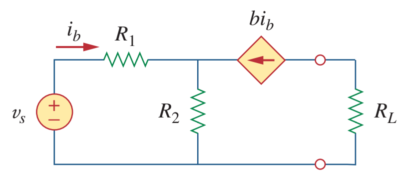


Figure 4.124
For Prob. 4.58.

4.59 Determine the Thevenin and Norton equivalents at terminals *a-b* of the circuit in Fig. 4.125.

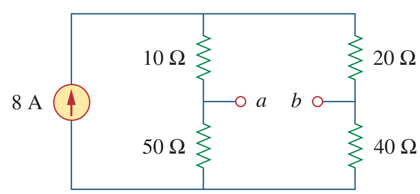


Figure 4.125
For Probs. 4.59 and 4.80.

***4.60** For the circuit in Fig. 4.126, find the Thevenin and Norton equivalent circuits at terminals *a-b*.

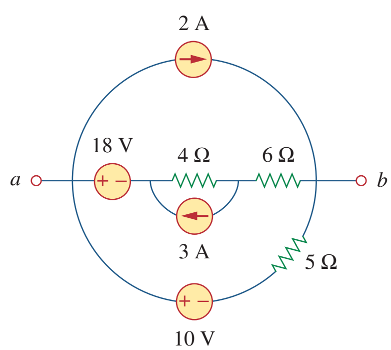


Figure 4.126
For Probs. 4.60 and 4.81.

***4.61** Obtain the Thevenin and Norton equivalent circuits at terminals *a-b* of the circuit in Fig. 4.127.

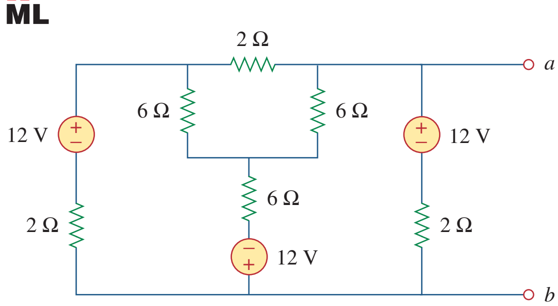


Figure 4.127
For Prob. 4.61.

***4.62** Find the Thevenin equivalent of the circuit in Fig. 4.128.

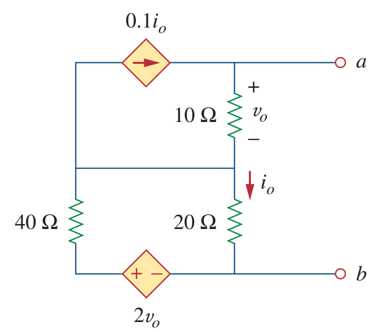


Figure 4.128
For Prob. 4.62.

4.63 Find the Norton equivalent for the circuit in Fig. 4.129.

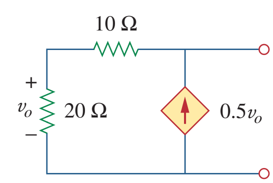


Figure 4.129
For Prob. 4.63.

4.64 Obtain the Thevenin equivalent seen at terminals *a-b* of the circuit in Fig. 4.130.

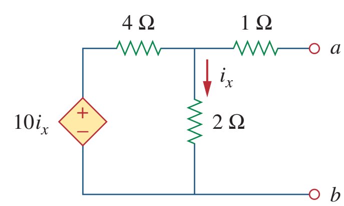


Figure 4.130
For Prob. 4.64.

4.65 For the circuit shown in Fig. 4.131, determine the relationship between V_o and I_o.

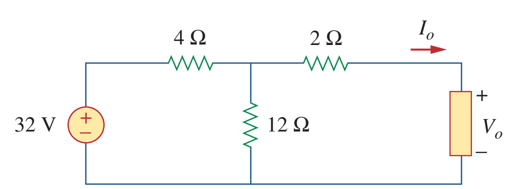


Figure 4.131
For Prob. 4.65.

Section 4.8 Maximum Power Transfer

4.66 Find the maximum power that can be delivered to the resistor R in the circuit of Fig. 4.132.

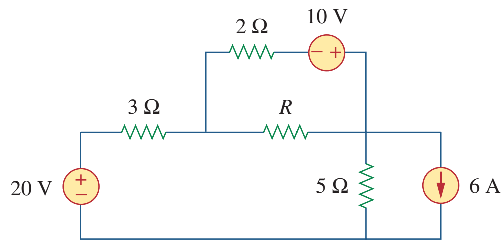


Figure 4.132
For Prob. 4.66.

4.67 The variable resistor R in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit.
(a) Calculate the value of R for maximum power.
(b) Determine the maximum power absorbed by R.

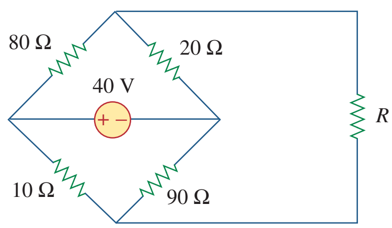


Figure 4.133
For Prob. 4.67.

***4.68** Compute the value of R that results in maximum power transfer to the 10-Ω resistor in Fig. 4.134. Find the maximum power.

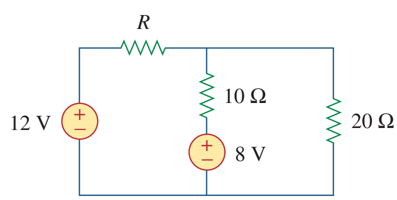


Figure 4.134
For Prob. 4.68.

4.69 Find the maximum power transferred to resistor R in the circuit of Fig. 4.135.

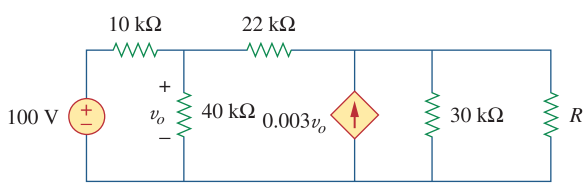


Figure 4.135
For Prob. 4.69.

4.70 Determine the maximum power delivered to the variable resistor R shown in the circuit of Fig. 4.136.

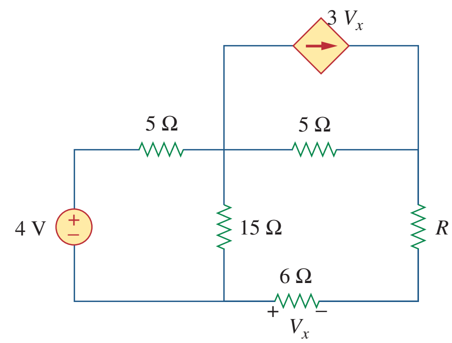


Figure 4.136
For Prob. 4.70.

4.71 For the circuit in Fig. 4.137, what resistor connected across terminals a-b will absorb maximum power from the circuit? What is that power?

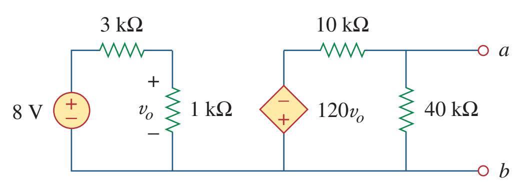


Figure 4.137
For Prob. 4.71.

4.72 (a) For the circuit in Fig. 4.138, obtain the Thevenin equivalent at terminals a-b.
(b) Calculate the current in $R_L = 8\ \Omega$.
(c) Find R_L for maximum power deliverable to R_L.
(d) Determine that maximum power.

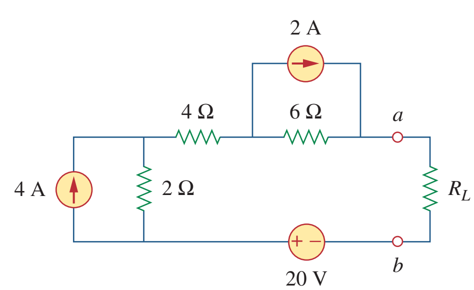


Figure 4.138
For Prob. 4.72.

4.73 Determine the maximum power that can be delivered to the variable resistor R in the circuit of Fig. 4.139.

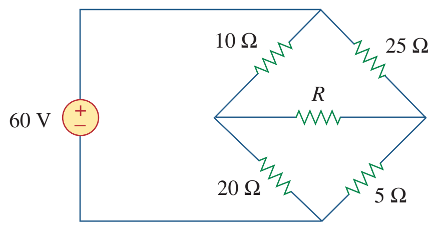


Figure 4.139
For Prob. 4.73.

4.74 For the bridge circuit shown in Fig. 4.140, find the load R_L for maximum power transfer and the maximum power absorbed by the load.

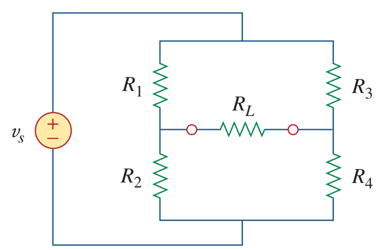


Figure 4.140
For Prob. 4.74.

***4.75** For the circuit in Fig. 4.141, determine the value of R such that the maximum power delivered to the load is 3 mW.

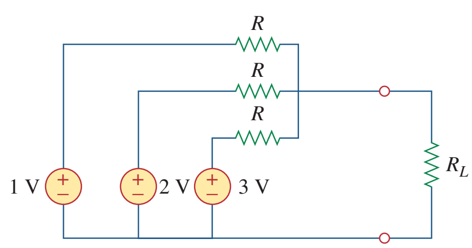


Figure 4.141
For Prob. 4.75.

Section 4.9 Verifying Circuit Theorems with *PSpice*

4.76 Solve Prob. 4.34 using *PSpice or MultiSim.* Let $V = 40$ V, $I = 3$ A, $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$, and $R_3 = 20\ \Omega$.

4.77 Use *PSpice or MultiSim* to solve Prob. 4.44.

4.78 Use *PSpice or MultiSim* to solve Prob. 4.52.

4.79 Obtain the Thevenin equivalent of the circuit in Fig. 4.123 using *PSpice or MultiSim.*

4.80 Use *PSpice or MultiSim* to find the Thevenin equivalent circuit at terminals *a-b* of the circuit in Fig. 4.125.

4.81 For the circuit in Fig. 4.126, use *PSpice or MultiSim* to find the Thevenin equivalent at terminals *a-b*.

Section 4.10 Applications

4.82 A battery has a short-circuit current of 20 A and an open-circuit voltage of 12 V. If the battery is connected to an electric bulb of resistance 2 Ω, calculate the power dissipated by the bulb.

4.83 The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	12 V	0 V
Terminal Current	0 A	1.5 A

Find the Thevenin equivalent of the network.

4.84 When connected to a 4-Ω resistor, a battery has a terminal voltage of 10.8 V but produces 12 V on an open circuit. Determine the Thevenin equivalent circuit for the battery.

4.85 The Thevenin equivalent at terminals *a-b* of the linear network shown in Fig. 4.142 is to be determined by measurement. When a 10-kΩ resistor is connected to terminals *a-b*, the voltage V_{ab} is measured as 6 V. When a 30-kΩ resistor is connected to the terminals, V_{ab} is measured as 12 V. Determine: (a) the Thevenin equivalent at terminals *a-b*, (b) V_{ab} when a 20-kΩ resistor is connected to terminals *a-b*.

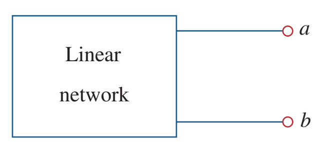


Figure 4.142
For Prob. 4.85.

4.86 A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.143. The results are shown in the table on the next page.

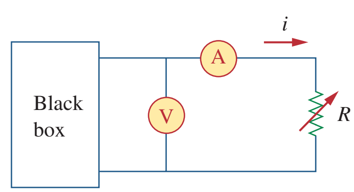


Figure 4.143
For Prob. 4.86.

(a) Find i when $R = 4\ \Omega$.

(b) Determine the maximum power from the box.

$R(\Omega)$	V(V)	i(A)
2	3	1.5
8	8	1.0
14	10.5	0.75

4.87 A transducer is modeled with a current source I_s and a parallel resistance R_s. The current at the terminals of the source is measured to be 9.975 mA when an ammeter with an internal resistance of 20 Ω is used.

(a) If adding a 2-kΩ resistor across the source terminals causes the ammeter reading to fall to 9.876 mA, calculate I_s and R_s.

(b) What will the ammeter reading be if the resistance between the source terminals is changed to 4 kΩ?

4.88 Consider the circuit in Fig. 4.144. An ammeter with internal resistance R_i is inserted between A and B to measure I_o. Determine the reading of the ammeter if: (a) $R_i = 500\ \Omega$, (b) $R_i = 0\ \Omega$. (*Hint*: Find the Thevenin equivalent circuit at terminals a-b.)

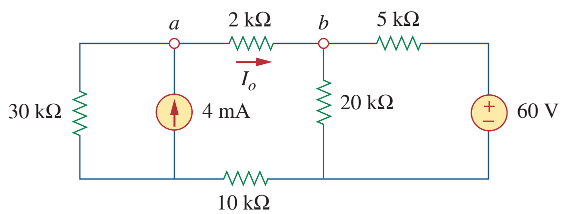


Figure 4.144
For Prob. 4.88.

4.89 Consider the circuit in Fig. 4.145. (a) Replace the resistor R_L by a zero resistance ammeter and determine the ammeter reading. (b) To verify the reciprocity theorem, interchange the ammeter and the 12-V source and determine the ammeter reading again.

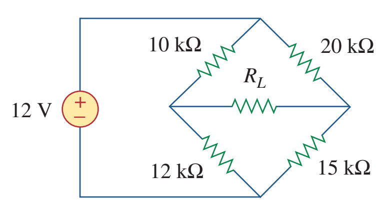


Figure 4.145
For Prob. 4.89.

4.90 The Wheatstone bridge circuit shown in Fig. 4.146 is used to measure the resistance of a strain gauge. The adjustable resistor has a linear taper with a maximum value of 100 Ω. If the resistance of the strain gauge is found to be 42.6 Ω, what fraction of the full slider travel is the slider when the bridge is balanced?

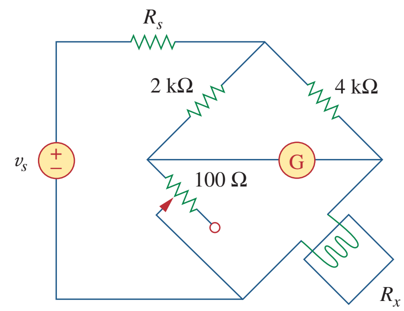


Figure 4.146
For Prob. 4.90.

4.91 (a) In the Wheatstone bridge circuit of Fig. 4.147, select the values of R_1 and R_3 such that the bridge can measure R_x in the range of 0–10 Ω.

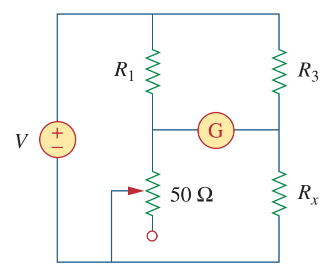


Figure 4.147
For Prob. 4.91.

(b) Repeat for the range of 0–100 Ω.

***4.92** Consider the bridge circuit of Fig. 4.148. Is the bridge balanced? If the 10-kΩ resistor is replaced by an 18-kΩ resistor, what resistor connected between terminals a-b absorbs the maximum power? What is this power?

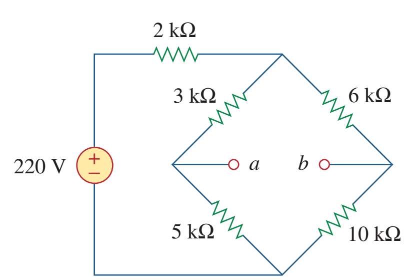


Figure 4.148
For Prob. 4.92.

Comprehensive Problems

4.93 The circuit in Fig. 4.149 models a common-emitter transistor amplifier. Find i_x using source transformation.

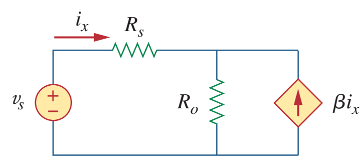


Figure 4.149
For Prob. 4.93.

4.94 An attenuator is an interface circuit that reduces the voltage level without changing the output resistance.

(a) By specifying R_s and R_p of the interface circuit in Fig. 4.150, design an attenuator that will meet the following requirements:

$$\frac{V_o}{V_g} = 0.125, \qquad R_{eq} = R_{Th} = R_g = 100\ \Omega$$

(b) Using the interface designed in part (a), calculate the current through a load of $R_L = 50\ \Omega$ when $V_g = 12$ V.

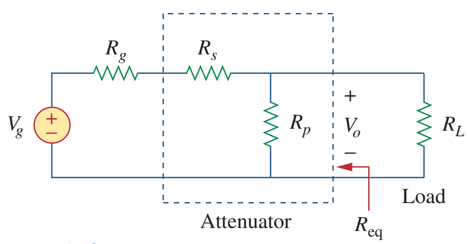


Figure 4.150
For Prob. 4.94.

***4.95** A dc voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:

(a) 0–10 V scale: 4 V (b) 0–50 V scale: 5 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

***4.96** A resistance array is connected to a load resistor R and a 9-V battery as shown in Fig. 4.151.

(a) Find the value of R such that $V_o = 1.8$ V.

(b) Calculate the value of R that will draw the maximum current. What is the maximum current?

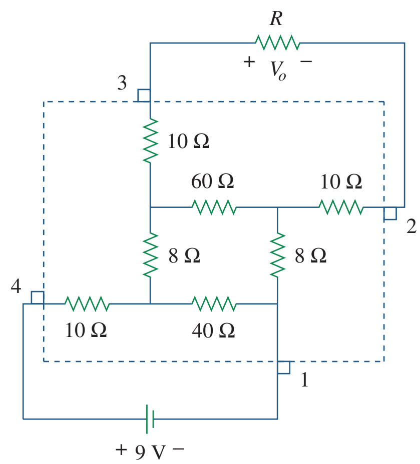


Figure 4.151
For Prob. 4.96.

4.97 A common-emitter amplifier circuit is shown in Fig. 4.152. Obtain the Thevenin equivalent to the left of points B and E.

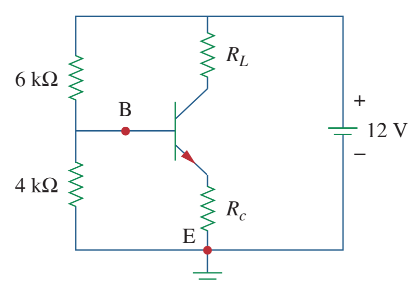


Figure 4.152
For Prob. 4.97.

***4.98** For Practice Prob. 4.18, determine the current through the 40-Ω resistor and the power dissipated by the resistor.