**EE 2010/2011 Circuit Analysis**

**Module 01: Electrical quantities and their relationships Notes**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

An example circuit:

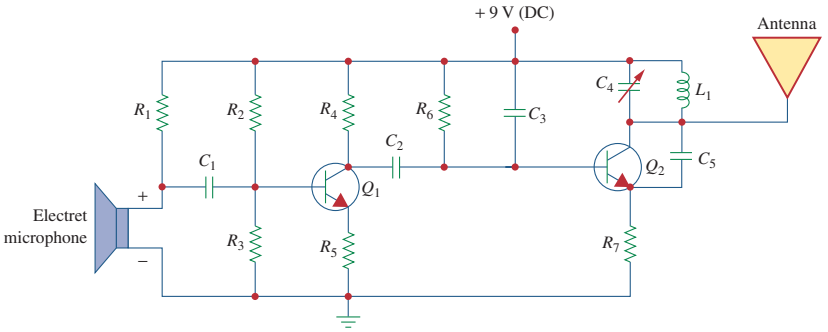



**Figure 1.2**
Electric circuit of a radio transmitter.

While most circuits of interest are more complex by orders of magnitude, this exemplar contains both active elements (transistors & sources), passive-instantaneous elements (resistors), passive-dynamic elements (capacitors and inductors), and both DC ($f = 0$) and AC ($f \neq 0$) signals. We will consider each of these in the sequence of topics to follow.

Engineers are primarily interested in first-order and second-order quantities. For the consideration and analysis of electrical systems, the first-order quantities of interest are *current* and *voltage*.

**Definition:** *Current in Amperes (A)* Electrical *flow*. (Water model)

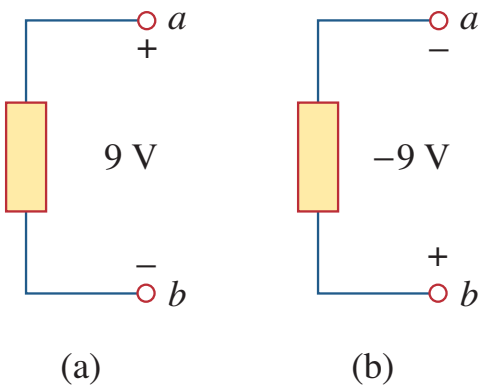
$$i(t) = \frac{dq(t)}{dt}$$

where $q(t)$ is charge (electrical volume) in Coulombs (C), and $t$ is time in seconds (s). Note that current is *bipolar*.

**Definition:** *Voltage in Volts (V)* Electrical *pressure difference.* (Water model)

$$v(t)_{ab} = \frac{dw(t)}{dq(t)}$$

where $a$ and $b$ are points on a circuit, $w(t)$ is energy in Joules (J), and $q(t)$ is charge (electrical volume) in Coulombs (C). Note that voltage is also *bipolar*.




(a) (b)

**Figure 1.7**
Two equivalent representations of the same voltage $v_{ab}$: (a) Point $a$ is 9 V above point $b$; (b) point $b$ is $-9$ V above point $a$.

For the consideration and analysis of most mechanical and electrical systems, the second-order quantities of interest are *power* and *energy*.

**Definition:** *Power in Watts (W)* – The rate of flow of energy.

$$p(t) = \frac{dw(t)}{dt}$$

where $w(t)$ is energy in Joules (J), and $t$ is time in seconds (s). Note that power is *bipolar* in the sense that we can have power delivered or power absorbed.

The electric company charges you for energy,

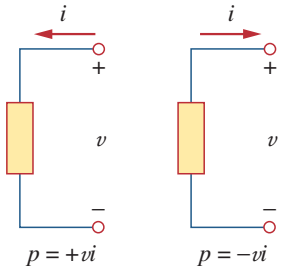
$$w = \int_{\text{first of month}}^{\text{end of month}} p(t)dt$$

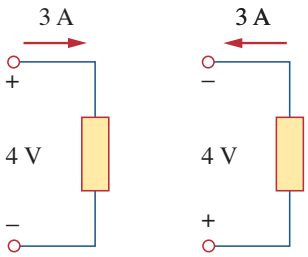
in units of kilo-Watt-hours where 1 KWH = 3,600,000 J (= $1000 \times 60 \times 60$ watt-seconds). The average home uses 910KWH/month. What is the average power?

Notice that in terms of voltage and current, we have

$$p(t) = \frac{dw(t)}{dt} = \frac{dw(t)}{dq(t)} \cdot \frac{dq(t)}{dt} = v(t) \cdot i(t)$$

So that power, in Watts, for an electrical system can be expressed in Volt-Amps. Notice that we also have a polarity ambiguity. So, we need a convention.




Hence, this way of viewing things is referred to as the *passive sign convention*.

**Homework 01:** Chapter 1 Review # 1.1 - 1.10, Problems # 8, 17, 19

## Review Questions

**1.1** One millivolt is one millionth of a volt.

(a) True (b) False

**1.2** The prefix *micro* stands for:

(a) $10^6$ (b) $10^3$ (c) $10^{-3}$ (d) $10^{-6}$

**1.3** The voltage 2,000,000 V can be expressed in powers of 10 as:

(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV

**1.4** A charge of 2 C flowing past a given point each second is a current of 2 A.

(a) True (b) False

**1.5** The unit of current is:

(a) coulomb (b) ampere
(c) volt (d) joule

**1.6** Voltage is measured in:

(a) watts (b) amperes
(c) volts (d) joules per second

**1.7** A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.

(a) True (b) False

**1.8** The voltage across a 1.1-kW toaster that produces a current of 10 A is:

(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V

**1.9** Which of these is not an electrical quantity?

(a) charge (b) time (c) voltage
(d) current (e) power

**1.10** The dependent source in Fig. 1.22 is:

(a) voltage-controlled current source
(b) voltage-controlled voltage source
(c) current-controlled voltage source
(d) current-controlled current source

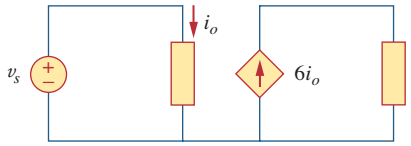



**Figure 1.22**
For Review Question 1.10.

*Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5b, 1.6c, 1.7a, 1.8c, 1.9b, 1.10d.*

## Problems

### Section 1.3 Charge and Current

**1.1** How many coulombs are represented by these amounts of electrons?

(a) $6.482 \times 10^{17}$ (b) $1.24 \times 10^{18}$
(c) $2.46 \times 10^{19}$ (d) $1.628 \times 10^{20}$

**1.2** Determine the current flowing through an element if the charge flow is given by

(a) $q(t) = (3t + 8)$ mC
(b) $q(t) = (8t^2 + 4t - 2)$ C
(c) $q(t) = (3e^{-t} - 5e^{-2t})$ nC
(d) $q(t) = 10 \sin 120\pi t$ pC
(e) $q(t) = 20e^{-4t} \cos 50t\,\mu$C

**1.3** Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3$ A, $q(0) = 1$ C
(b) $i(t) = (2t + 5)$ mA, $q(0) = 0$
(c) $i(t) = 20 \cos(10t + \pi/6)\,\mu$A, $q(0) = 2\,\mu$C
(d) $i(t) = 10e^{-30t} \sin 40t$ A, $q(0) = 0$

**1.4** A current of 7.4 A flows through a conductor. Calculate how much charge passes through any cross-section of the conductor in 20 s.

**1.5** Determine the total charge transferred over the time interval of $0 \le t \le 10$ s when $i(t) = \frac{1}{2}t$ A.

**1.6** The charge entering a certain element is shown in Fig. 1.23. Find the current at:

(a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

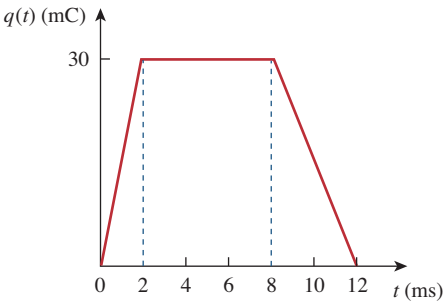



**Figure 1.23**
For Prob. 1.6.

**1.7** The charge flowing in a wire is plotted in Fig. 1.24. Sketch the corresponding current.

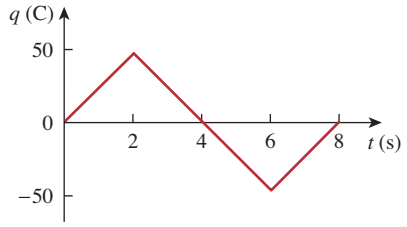



**Figure 1.24**
For Prob. 1.7.

**1.8** The current flowing past a point in a device is shown in Fig. 1.25. Calculate the total charge through the point.

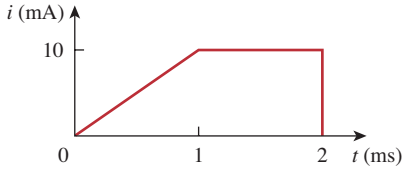



**Figure 1.25**
For Prob. 1.8.

**1.9** The current through an element is shown in Fig. 1.26. Determine the total charge that passed through the element at:

(a) $t = 1$ s (b) $t = 3$ s (c) $t = 5$ s

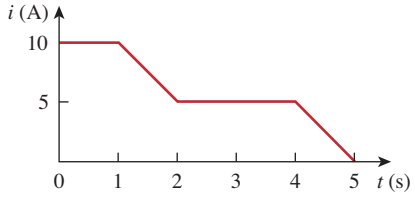



**Figure 1.26**
For Prob. 1.9.

## Sections 1.4 and 1.5 Voltage, Power, and Energy

**1.10** A lightning bolt with 10 kA strikes an object for 15 $\mu$s. How much charge is deposited on the object?

**1.11** A rechargeable flashlight battery is capable of delivering 90 mA for about 12 h. How much charge can it release at that rate? If its terminal voltage is 1.5 V, how much energy can the battery deliver?

**1.12** If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t\text{A}, & 0 \leqq t < 6\text{ s} \\ 18\text{A}, & 6 \leqq t < 10\text{ s} \\ -12\text{A}, & 10 \leqq t < 15\text{ s} \\ 0, & t \geqq 15\text{ s} \end{cases}$$

Plot the charge stored in the element over $0 < t < 20$ s.

**1.13** The charge entering the positive terminal of an element is

$$q = 5 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 3 \cos 4\pi t \text{ V}$$

(a) Find the power delivered to the element at $t = 0.3$ s.

(b) Calculate the energy delivered to the element between 0 and 0.6 s.

**1.14** The voltage $v$ across a device and the current $i$ through it are

$$v(t) = 10 \cos 2t \text{ V}, \qquad i(t) = 20(1 - e^{-0.5t}) \text{ mA}$$

Calculate:

(a) the total charge in the device at $t = 1$ s

(b) the power consumed by the device at $t = 1$ s.

**1.15** The current entering the positive terminal of a device is $i(t) = 6e^{-2t}$ mA and the voltage across the device is $v(t) = 10di/dt$ V.

(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

## Section 1.6 Circuit Elements

**1.16** Figure 1.27 shows the current through and the voltage across an element.

(a) Sketch the power delivered to the element for $t > 0$ .

(b) Fnd the total energy absorbed by the element for the period of $0 < t < 4$s.

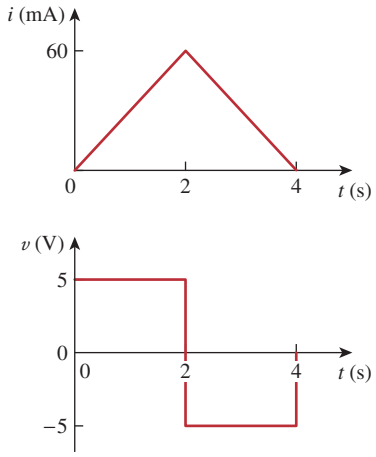



**Figure 1.27**
For Prob. 1.16.

**1.17** Figure 1.28 shows a circuit with five elements. If $p_1 = -205$ W, $p_2 = 60$ W, $p_4 = 45$ W, $p_5 = 30$ W, calculate the power $p_3$ received or delivered by element 3.

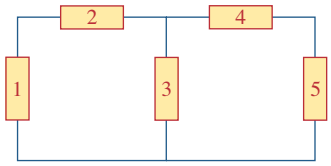



**Figure 1.28**
For Prob. 1.17.

**1.18** Find the power absorbed by each of the elements in Fig. 1.29.

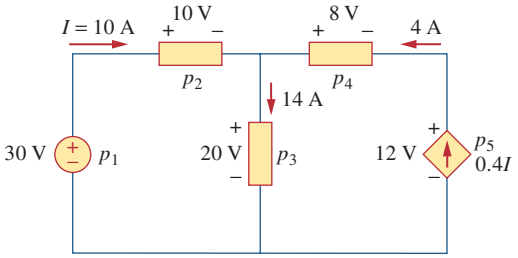



**Figure 1.29**
For Prob. 1.18.

**1.19** Find $I$ and the power absorbed by each element in the network of Fig. 1.30.

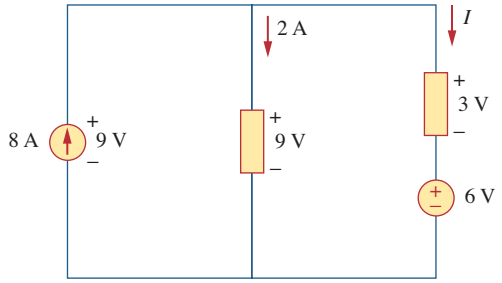



**Figure 1.30**
For Prob. 1.19.

**1.20** Find $V_o$ and the power absorbed by each element in the circuit of Fig. 1.31.

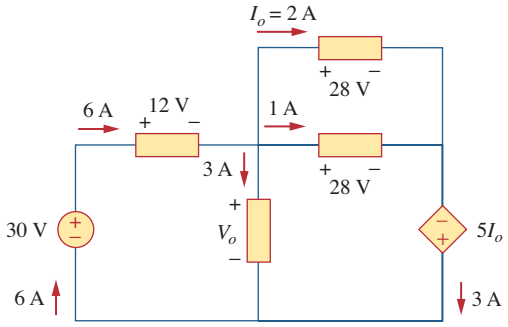



**Figure 1.31**
For Prob. 1.20.

## Section 1.7 Applications

**1.21** A 60-W incandescent bulb operates at 120 V. How many electrons and coulombs flow through the bulb in one day?

**1.22** A lightning bolt strikes an airplane with 40 kA for 1.7 ms. How many coulombs of charge are deposited on the plane?

**1.23** A 1.8-kW electric heater takes 15 min to boil a quantity of water. If this is done once a day and power costs 10 cents/kWh, what is the cost of its operation for 30 days?

**1.24** A utility company charges 8.2 cents/kWh. If a consumer operates a 60-W light bulb continuously for one day, how much is the consumer charged?

**1.25** A 1.5-kW toaster takes roughly 3.5 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 8.2 cents/kWh.

**1.26** A flashlight battery has a rating of 0.8 ampere-hours (Ah) and a lifetime of 10 hours.

(a) How much current can it deliver?

(b) How much power can it give if its terminal voltage is 6 V?

(c) How much energy is stored in the battery in Wh?

**1.27** A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$ V, where $t$ is in hours,

(a) how much charge is transported as a result of the charging?

(b) how much energy is expended?

(c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

**1.28** A 60-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:

(a) the current through the lamp.

(b) the cost of operating the light for one non-leap year if electricity costs 9.5 cents per kWh.

**1.29** An electric stove with four burners and an oven is used in preparing a meal as follows.

Burner 1: 20 minutes Burner 2: 40 minutes

Burner 3: 15 minutes Burner 4: 45 minutes

Oven: 30 minutes

If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

**1.30** Reliant Energy (the electric company in Houston, Texas) charges customers as follows:

Monthly charge $6

First 250 kWh @ $0.02/kWh

All additional kWh @ $0.07/kWh

If a customer uses 2,436 kWh in one month, how much will Reliant Energy charge?

**1.31** In a household, a 120-W personal computer (PC) is run for 4 h/day, while a 60-W bulb runs for 8 h/day. If the utility company charges $0.12/kWh, calculate how much the household pays per year on the PC and the bulb.

## Comprehensive Problems

**1.32** A telephone wire has a current of 20 $\mu$A flowing through it. How long does it take for a charge of 15 C to pass through the wire?

**1.33** A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

**1.34** Figure 1.32 shows the power consumption of a certain household in 1 day. Calculate:

(a) the total energy consumed in kWh,

(b) the average power per hour over the total 24 hour period.

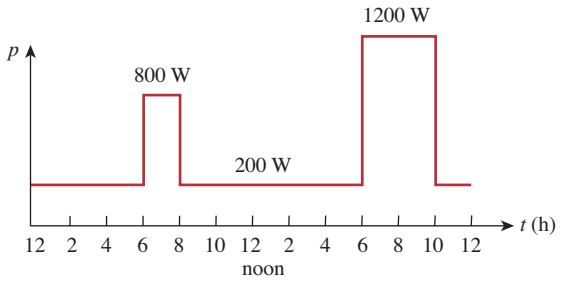



**Figure 1.32**
For Prob. 1.34.

**1.35** The graph in Fig. 1.33 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

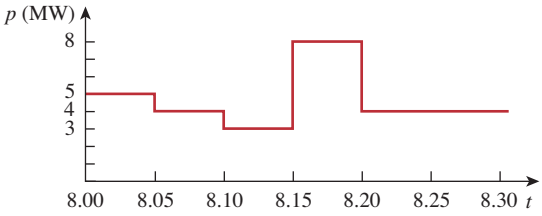



**Figure 1.33**
For Prob. 1.35.

**1.36** A battery may be rated in ampere-hours (Ah). A lead-acid battery is rated at 160 Ah.

(a) What is the maximum current it can supply for 40 h?

(b) How many days will it last if it is discharged at 1 mA?

**1.37** A 12-V battery requires a total charge of 40 ampere-hours during recharging. How many joules are supplied to the battery?

**1.38** How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

**1.39** A 600-W TV receiver is turned on for 4 h with nobody watching it. If electricity costs 10 cents/kWh, how much money is wasted?