ISE 2211 • Exam III • 2 October 2020 NAME SOLUTION

This week's stats exam problem comes from Tara Wilson, bioelectronics GTA extraordinaire. Excuse the Harry Potter references if you're not familiar.

61 Hogwarts students have taken their OWL exams, where the possible scores range from Outstanding (0) to Troll (5). The mean score was determined to be 2.082 with a standard deviation of 1.520.

Write a 90% lower confidence bound on mean exam score, since, in general, we are much more concerned with how *badly* we did on an exam, rather than how well. Assume unknown population standard deviation. Sketch the appropriate probability distribution and indicate the location of this lower bound.

$90\% \longrightarrow \alpha = 0.10$ +1

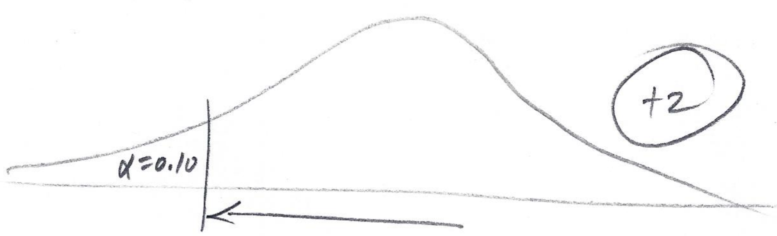
$n = 61 \geq 30$

$\therefore$ use Z-distribution +1

$Z_{\alpha} = Z_{0.10} = 1.282$ +1 +1

$\uparrow$ not $\alpha/2$

(bottom row of t-table)



+2

$\bar{x} - Z_{\alpha}\, s/\sqrt{n} < \mu$ +1

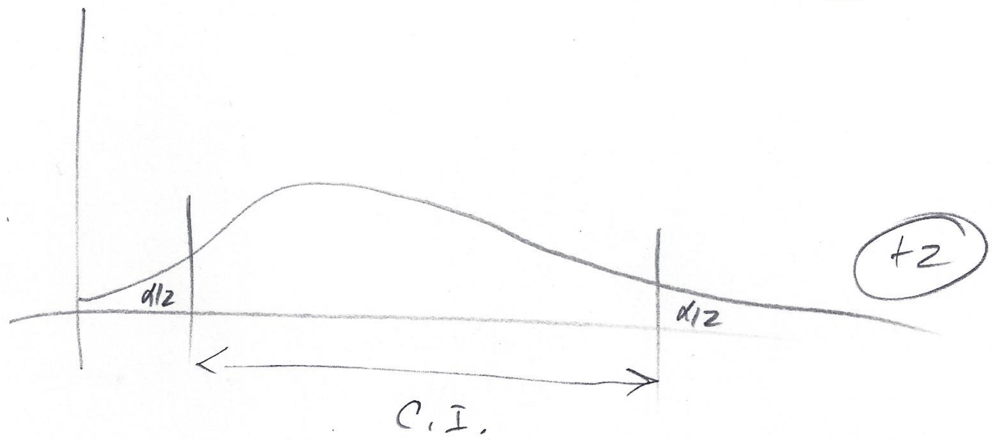
$2.082 - 1.282 \cdot 1.520/\sqrt{61} < \mu$

$1.833 < \mu$ +1 (score)

+8

Write a 95% confidence interval on the standard deviation of exam scores. Sketch the appropriate probability distribution and indicate the location of the interval.

$95\% \rightarrow \alpha = 0.05$ +1



$$\chi^2_{\alpha/2,\,n-1} = \chi^2_{.025,\,60} = 83.30$$

$$\chi^2_{1-\alpha/2,\,n-1} = \chi^2_{.975,\,60} = 40.48$$

+2 (table)

$$\frac{60 \cdot 1.520^2}{83.30} < \sigma^2 < \frac{60 \cdot 1.520^2}{40.48}$$

$$1.664 < \sigma^2 < 3.425$$ +2

$$1.290 < \sigma < 1.851$$ +1 (score)

+8

Six students got a score of Troll. Write a 95% confidence interval on the population proportion of students receiving a Troll.

$$\hat{p} = \frac{x}{n} = \frac{6}{61} = 0.09836 \quad +1$$

$$Z_{0.025} = 1.960 \quad +1$$

$$p: \quad \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.09836 \pm 1.960\sqrt{\frac{0.09836(1-0.09836)}{61}}$$

$$0.02363 < p < 0.1731 \quad +2$$

or

$$2.363\% < p < 17.31\%$$

+4