CEG 3320 - Digital System Design

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Review from previous modules: #1s greater than 2 device



Module III: Sequential Analysis and Design

State Devices
Parameterized Boolean Algebra
Sequential Function Analysis
State Diagrams
Sequential Timing Analysis
Design with State Diagrams
Sequential Synthesis

Introduction to Sequential Devices

State Memory
Bi-stable elements
Latches
Flip-flops
Parameterized Boolean Algebra
Timing Characteristics

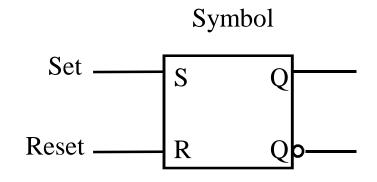
Logic Devices

- Logic devices divide into two major types:
- Combinational Logic
 - Current output depends on current input only
 - Examples: gates, decoders, multiplexors (MUXs), ALUs
 - Familiarity with combinational logic is a course prerequisite
- Sequential Logic
 - Current output depends on past inputs as well as current input
 - Thus has a memory (usually called the state)
 - Examples: latches, flip-flops, state machines, counters, shift registers

How can we 'store' a bit over time?



S-R Latch



Hold

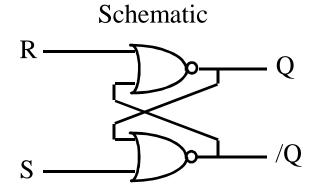
Reset

Set

ILLEGAL

Function Table				
S	R	Q	/Q	
0	0	Last Q	Last /Q	
0	1	0	1	
1	0	1	0	
1	1	0	0	

Erro otion Tolalo



Characteristic Equation:

$$Q(t+1) = S + R'Q(t)$$

Consider:

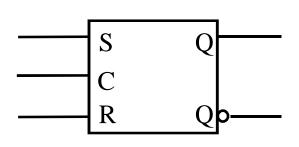
Timing Diagram

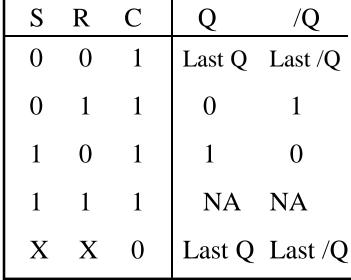
Propagation delay

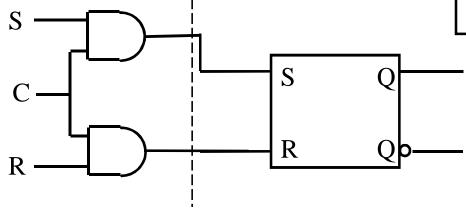
Minimum pulse width

Oscillation

S-R Latch with Enable

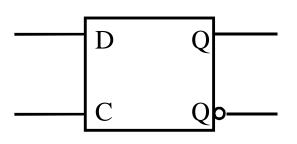




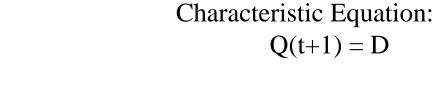


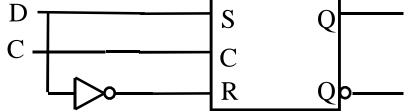
Only sensitive to S and R
when enabled (C=1)
Same oscillation problem
How does C effect the
minimum pulse width?

D Latch



С	D	Q	/Q
1	0	0	1
1	1	1	0
0	X	Last (Q Last /Q





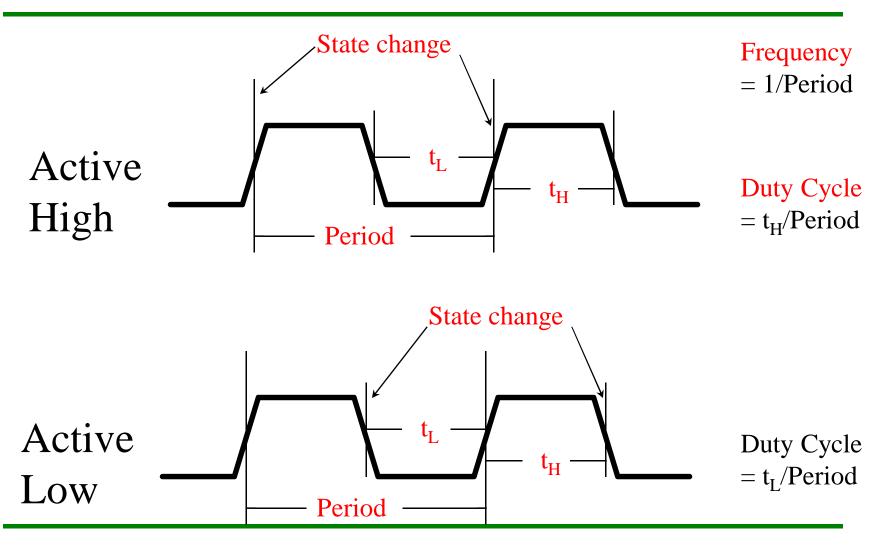
Store a data bit, not set/reset
The "Transparent" latch
No illegal operation problem

Types of Sequential Logic

- An Asynchronous Sequential Circuit uses ordinary gates and feedback loops to implement "memory" in a logic circuit.
 - Meeting minimum pulse width requirements may be 'tricky'
- A Synchronous Sequential Circuit uses flip-flops (internally, an asynchronous sequential device) to form useful sequential logic functions or applications.
 - The state variables and outputs of a synchronous system change with respect to a controlling <u>clock</u> signal
 - Meeting minimum pulse width requirements is simplified by restating all timing constraints in terms of the clock signal



Clock Characteristics



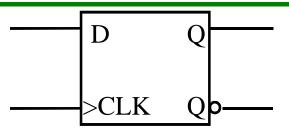


Sequential Logic Definitions

- Clock the master timing element behind the state changes of most sequential circuits.
 - a clock signal is <u>active high</u> if the state changes occur at the rising edge (for edge triggered devices) or in the logic 1 state (for pulse-triggered devices)
 - <u>active low</u> if state changes occur at the falling edge or in the logic 0 state.
- Clock Period time between successive transitions in the same direction
- Clock Frequency reciprocal of the clock period
- Clock Tick the first edge or pulse in a clock period, or the period itself
- Duty Cycle the percentage of time that a clock is at its assertion level



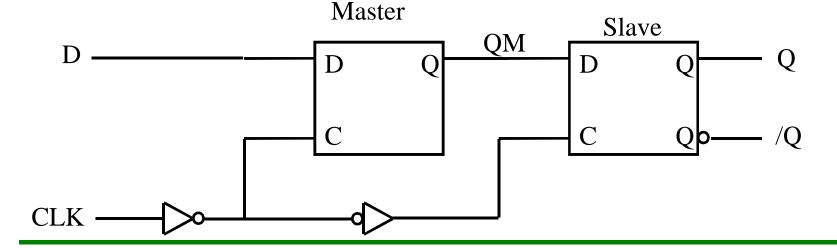
Positive-Edge-Triggered D Flip-Flop



Device samples inputs and changes state only on a clock edge.

Simplifies the meeting of setup/hold times. Master FF - transparent on CLK' (entire pulse) Slave FF - transparent on CLK (master fixed)

D	CLK	Q	/Q
0		0	1
1		1	0
X	0	Last	Q Last /Q
X	1	Last	Q Last /Q



What Are Flip-flops?

- Common asynchronous (feedback) sequential circuits
- Latch
 - Single-bit storage (memory)
 - Changes state at any time due to input change
 - Must guarantee a minimum pulse width to avoid metastability
 - Fast and cheap (small # of transistors)
 - Often used in high speed microprocessor design
- Flip-flop
 - Also single-bit storage
 - Changes state ONLY when a clock edge or pulse is applied
 - Uses setup and hold times before and after the clock pulse to avoid metastability
 - Clocking simplifies the design process



Characteristic Equations

- Describe the next state of a flip-flop as function of current state and inputs: Q(t+1) = f(Q(t), inputs)
 - t+1 represents the next clock tick
 - t represents the current clock tick
 - t-1 represents the previous clock tick
 - and so on...
- Standard practice is to use Q* to represent Q(t+1)
- Derived from basic function table for a given flip-flop type
- Very useful in state machine analysis and design
- EXERCISE: What is the equation of a D-type FlipFlop?



Characteristic Equations

D flip-flop

	D	Q(t)	Q(t+1)
_	0	0	0
	0	1	0
	1	0	1
	1	1	1

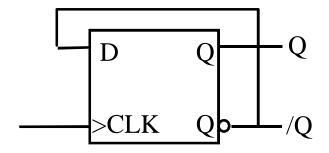
Characteristic Equation:

$$Q(t+1) = D$$

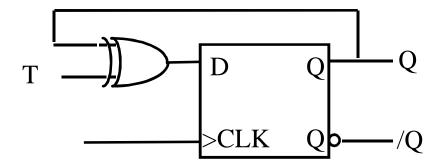
$$Q^* = D$$

T (toggle) Flip-Flop

- A T flip-flop changes state on every clock tick (if enabled)
- Possible circuit designs
 - T without enable

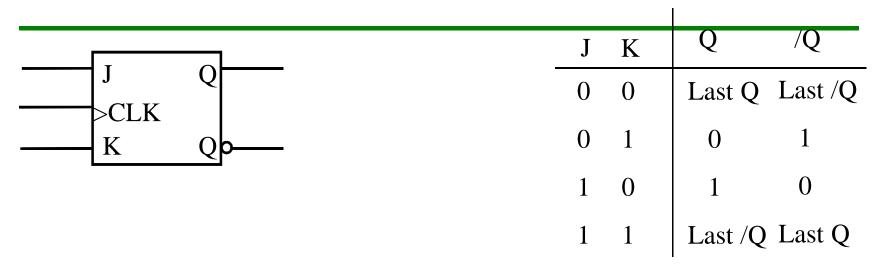


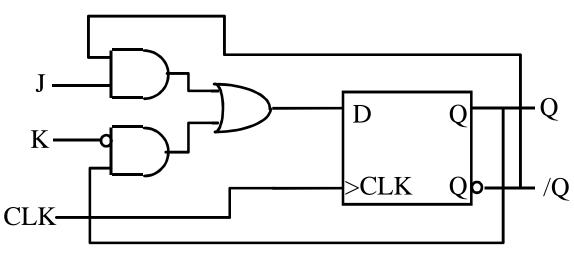
T with enable



Equation?

Edge-Triggered J-K Flip-Flop





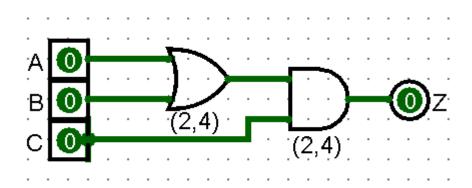
Equation?

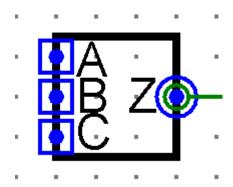


Functional Analysis of Clocked Synchronous State Machines

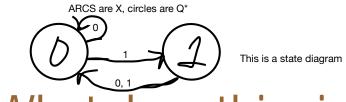
Analysis procedure
Moore Vs. Mealy
Methods of representation

Review: What does this circuit do?

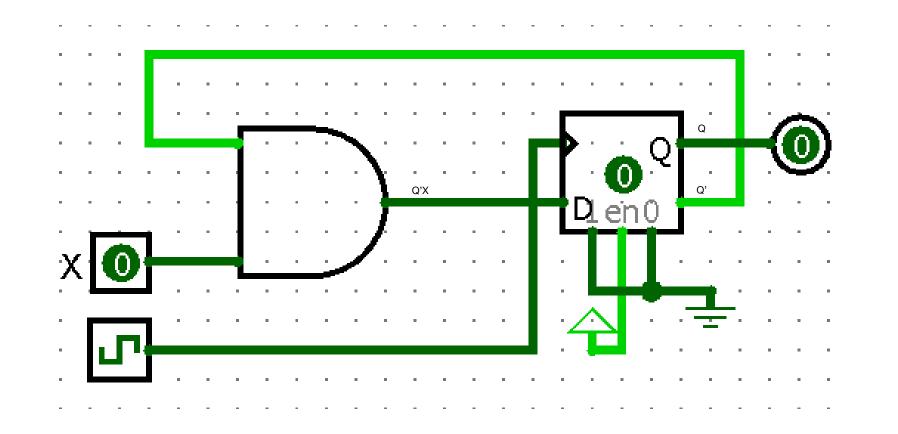




 $Q^* = Q'X$

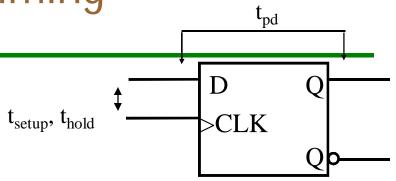


What does this circuit do?





Flip-flop Timing



Combinational

t_{pd,min} - minimum propagation delay, input to output

t_{pd,max} - maximum propagation delay, input to output

Latch

t_{pd,min} - minimum propagation delay, input to output

t_{pd.max} - maximum propagation delay, input to output

t_w - minimum pulse width, input to input

Flip-flop

t_{pd, min} - minimum propagation delay, CLK to output

t_{pd, max} - maximum propagation delay, CLK to output

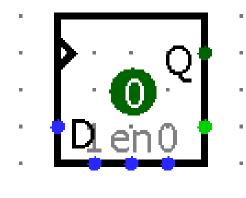
t_{setup} - required time of stable input before CLK, input before CLK

- required time of stable input after CLK, input after CLK



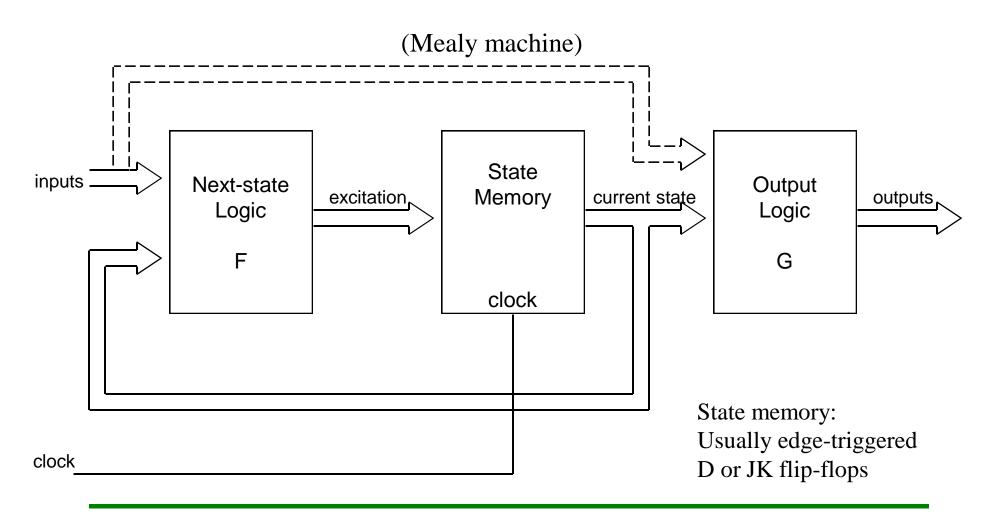
Synchronous & Asynchronous Inputs

- Synchronous inputs
 - are aware of and respect the timing restrictions of the clocked device
 - Behavior which follows from a Synchronous Input is itself Synchronous and deterministic
- Asynchronous inputs
 - Are not aware of the clock
 - Have effects that occur immediately,
 - State depends on order of events
- Example: Preset and Reset direct inputs



- Good design practice dictates:
 - NEVER use asynchronous inputs for logic functions, only for system initialization to a known state

Clocked Synchronous State-machine Model



Clocked Synchronous State Machine Analysis

Analysis

- How does a given circuit work? What does it do?
- How do input sequences map to output sequences?
- Clocked synchronous state-machine (CSSM)
 - Clocked: storage elements (flip-flops) use a clock input
 - Synchronous: all flip-flops use the same clock signal
- State-machine types
 - Mealy Machine (most general type):
 - Next state = F (current state, inputs)
 - Output = G (current state, inputs)
 - Moore Machine:
 - Next state = F (current state, inputs)
 - Output = G (current state)

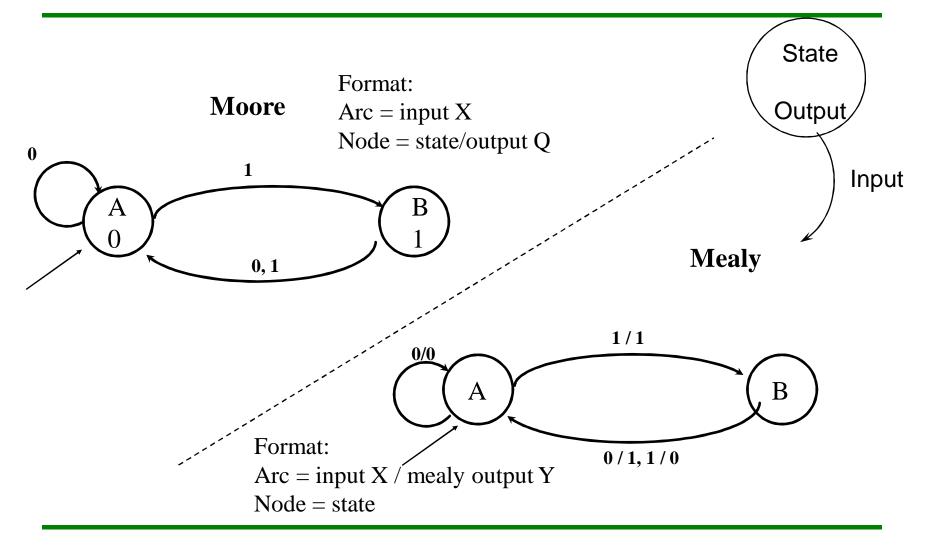


Basic Analysis of State Machines

- Determine next-state and output functions F and G
- Use F and G to construct a state/output table
- Draw a graphical representation of the state/output table
 - State Diagram
 - Common for small designs
 - Similar to a finite automata
 - Timing Diagram
 - Common for <u>all</u> designs

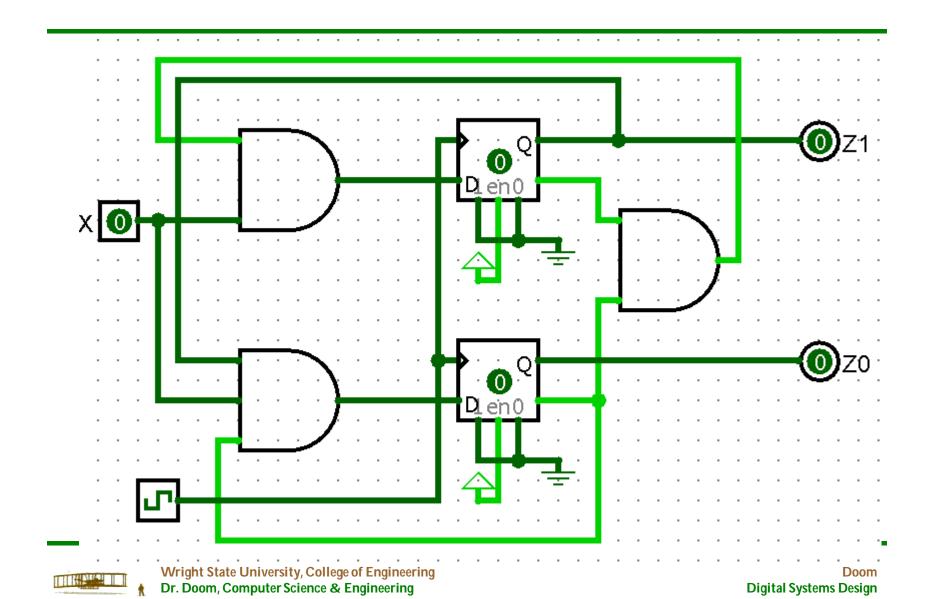
State Diagram

Basic Format:





What does this device do?





Timing Analysis of Clocked Synchronous State Machines

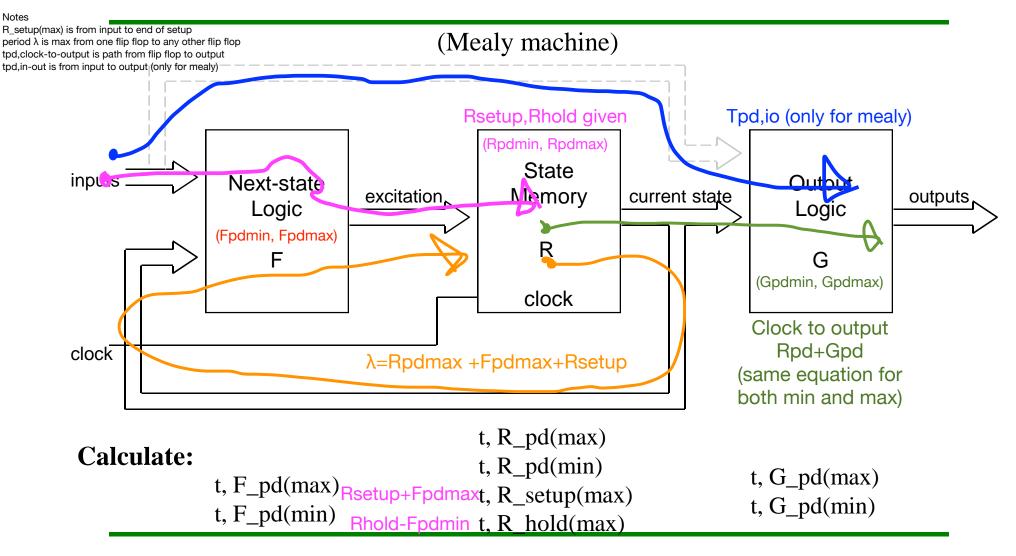
Analysis procedure
Moore Vs. Mealy
Methods of representation

Timing Analysis

- All digital devices have associated propagation delays (min,max)
- Sequential devices have setup and hold times that must be satisfied to avoid metastable behavior
- Providing a synchronous clock simplifies timing analysis
 - All devices produce effects within a well-defined range
- At what speeds will a device function?
 - One transition per clock
 - What is the maximum clock rate?
- Sequential devices require the following timing documentation:
 - Maximum propagation delay (clock to output)
 - Minimum propagation delay (clock to output)
 - Setup time (input before clock)
 - Hold time (input after clock)
 - Maximum Clock Frequency (minimum clock period)

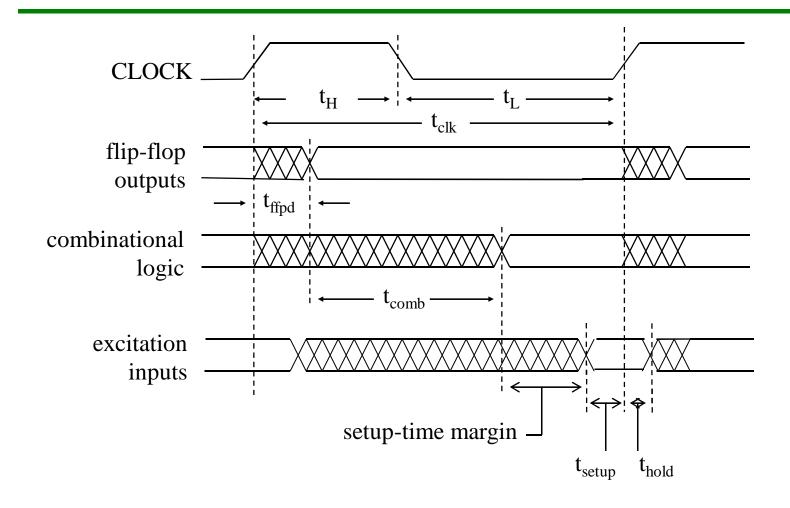


Clocked Synchronous State-machine Structure





Timing Diagram





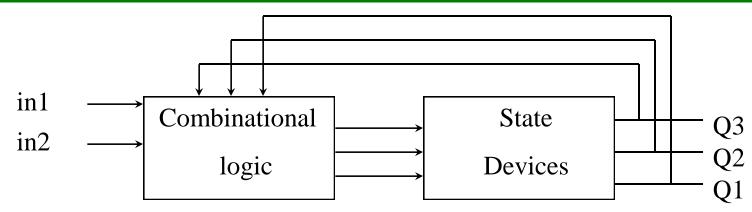
Calculating Sequential Device Timing Specs.

- Calculate the delay from clock edge to worst-case primary output:
 - t_pd,clock-to-output (min) = t,R_pd(min) + t,G_pd(min)
 - t_pd,clock-to-output (max) = t,R_pd(max) + t,G_pd(max)
- Calculate the delay from input to worst-case (Mealy) primary output:
 - t_pd,input-to-output (min) = t,G_pd(min)
 - t_pd,input-to-output (min) = t,G_pd(min)
- Calculate the worst-case setup time for any input:
 - t_setup = t,F_pd(max) + t,R_setup(max)
- Calculate the worst-case hold time for any input:
 - t_hold = t,R_hold(max) t,F_pd(min)
- Calculate the maximum clock rate by finding the minimum period:
 - min. period = t,R_pd(max) + t,F_pd(max) + t,R_setup(max)
- Make certain that the device works!
 - t,R_pd(min) + t,F_pd(min) > t,R_hold(max)





Synchronous System Example

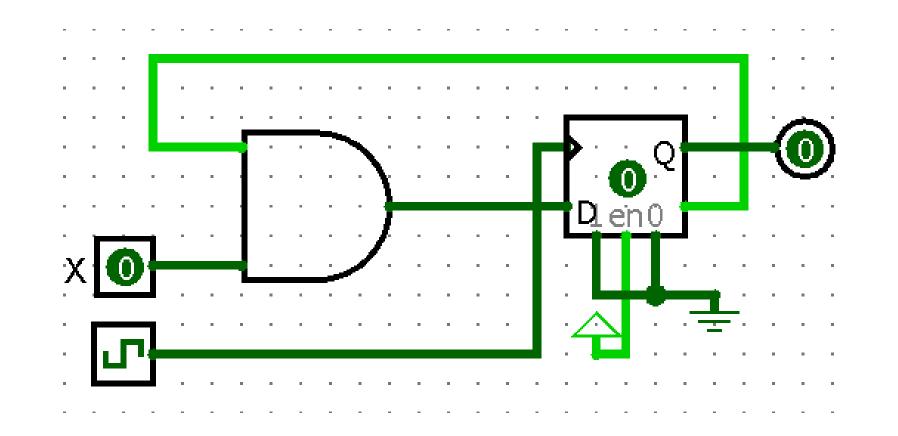


$$t_{pd,comb} = 2 \text{ ns (min) to } 20 \text{ ns (max)}$$
 $t_{pd,ff} = 3 \text{ ns (min) to } 15 \text{ ns (max)}$ $t_{setup} = 5 \text{ ns; } t_{hold} = 4 \text{ ns}$

time,prop.delay (clock->output)
time,prop.delay(input->output)
Setup/Hold Time:
Max Frequency?

$$t_{pd(co),min} = \frac{1}{100}; t_{pd(co),max} = \frac{1}{100}; t_{pd(io),max} = \frac{1}{100};$$

What does this circuit do?



get

tsetup: 5+6 = 11 (setup + fpdmax from 3input and)

thold: 1-2 = 0 (hold - fpdmin from 2input and)

pd,clk->out (min): 3+0 = 3 (rpdmin + gpdmin, there is no g because nothing block path to output)

pd,clk->out (max): 6+0 = 3 (rpdmax+gpdmax, there is no g because nothing block path to output)

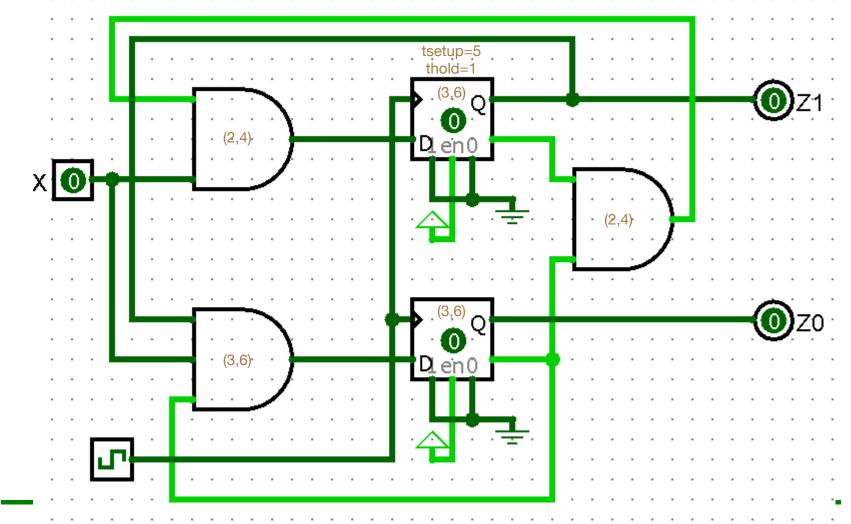
pd,in>out (min): pd,in->out (max): -

What does this device do?

period: 6+8+5 = 19 (Rpdmax + fpdmax through 2 2input and + tsetup)

Frequency = 1/period (can be in the form where period is exactly as calculated)

example: 1/(19ns)



Introduction to the design of Clocked Synchronous State Machines

Basic Design procedure
State diagrams
Synthesis process

State Machine Design Procedure

Most difficult and creative

- 1. Build state/output table (or state diagram) from word description
- 2. Minimize number of states
- 3. Choose state variables and assign bit combinations to named states
- 4. Build transition/output table from state/output table/diagram
- 5. Choose flip-flop type (D, J-K, etc.)
- 6. Build excitation table for flip-flop inputs from transition table
- 7. Derive excitation equations from excitation table
- 8. Derive output equations from transition/output table
- 9. Draw logic diagram with excitation logic, output logic, and state memory elements

Synthesis is generally followed by simulation and verification

Well-defined procedure - can be automated

Step 1: State Table Design from Word Description

- Much like writing a computer program:
 - Start with a vague description
 - Make decisions, sometimes using common sense, and sometimes arbitrarily
 - Handle all special cases (even those not covered in the word description)
 - Design may not work, so debug and iterate



Design



Synthesis



Unused States: Minimum Cost/Risk

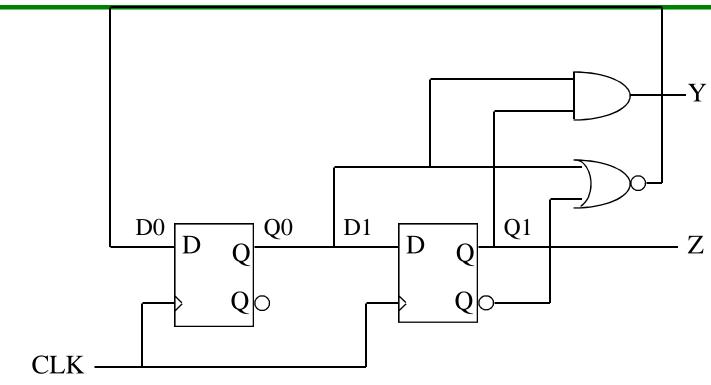
- If extra unused state codes exist (number of available states 2ⁿ > s), then two choices are common:
- Minimal Risk = most reliable but most expensive
 - Assume it is possible to enter unused state by noise, weird inputs, etc.
 - So include all unused states as present states, but all corresponding next states go to "initial" or "idle"
- Minimal Cost = somewhat risky but least expensive
 - Assume unused states NEVER entered accidentally.
 - So the next state and outputs of all unused states = "don't care" to reduce next state and output logic
- Achieving both minimal risk and minimal cost is sometimes possible!



Examples of analysis, design, and implementation

These examples are 'extra' problems for those who missed class or just want more problems to review

Example 1 - Circuit w/o Primary Inputs



Excitation: $D0 = (Q0 + Q1')' = Q0' \cdot Q1$

D1 = Q0

Output: $Y = Q0 \cdot Q1$

Z = Q1

Thus, Moore machine



Example 1 - Equations



$$D0 = Q0' \cdot Q1$$

$$D1 = Q0$$

Characteristic

$$Q0(t+1) = D0$$

$$Q1(t+1) = D1$$

Transition

$$Q0(t+1) = D0 = Q0' \cdot Q1$$

$$Q1(t+1) = D1 = Q0$$

Output

$$Y = Q0 \cdot Q1$$

$$Z = Q1$$

Example 1 - Tables

No inputs!

State Table:

Q1 Q0		ΥZ
0 0	00	0 0
0 1	10	0 1
1 0	01	0 0
1 1	01	1 1

Transition

$$Q1(t+1) = D1 = Q1' \cdot Q0$$

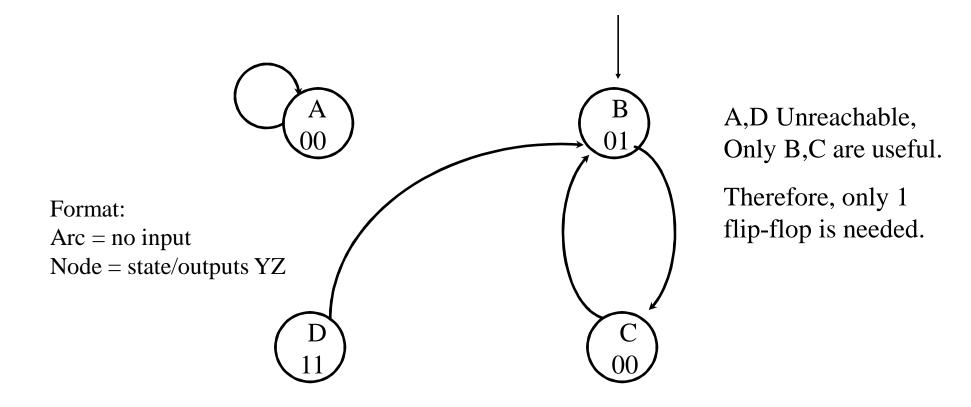
 $Q0(t+1) = D0 = Q1$

Q1(t+1)Q0(t+1)

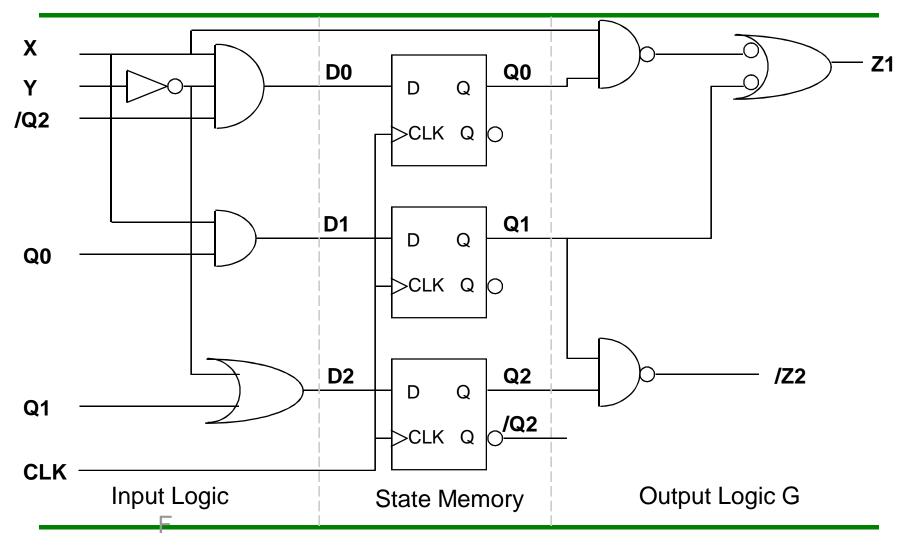
State Table w/named states:

S		YZ
A	A	0 0
В	C	0 1
C	В	0 0
D	В	1 1

Example 1 - State Diagram



Example 2 - State Machine with D Flip-flops





Example 2 - Equations

Excitation

$$D0 = X Y'Q2$$

$$D1 = X Q0$$

$$D2 = Y' + Q1$$

Characteristic

$$Q0(t+1) = D0$$

$$Q1(t+1) = D1$$

$$Q2(t+1) = D2$$

Transition

$$Q0(t+1) = D0 = X Y' Q2'$$

$$Q1(t+1) = D1 = X Q0$$

$$Q2(t+1) = D2 = Y' + Q1$$

Output

$$Z1 = X Q0 + Q1'$$

$$/Z2 = (Q1 Q2)'$$



Example 2 - Two-Dimensional State table

state					X	Υ	
name	Q2	Q1	Q0	00	01	11	10
A=	0	0	0	100, 11	000, 11	000, 11	101, 11
B=	0	0	1	100, 11	000, 11	010, 11	111, 11
C=	0	1	0	100, 01	100, 01	100, 01	101, 01
D=	0	1	1	100, 01	100, 01	110, 11	111, 11
E=	1	0	0	100, 11	000, 11	000, 11	100, 11
F=	1	0	1	100, 11	000, 11	010, 11	110, 11
G=	1	1	0	100, 00	100, 00	100, 00	100, 00
H=	1	1	1	100, 00	100, 00	110, 10	110,10

Q2(t+1) Q1(t+1) Q0(t+1), Z1 /Z2 (Next State, Outputs)

Transition Equations	X Y Q2 Q1 Q0	
Q0(t+1) = D0 = X Y' Q2'	1 0 0	Output Equations
Q1(t+1) = D1 = X Q0	1 1	Z1 = X Q0 + Q1'
Q2(t+1) = D2 = Y' + Q1	- 0 or	/Z2= (Q1 Q2)'

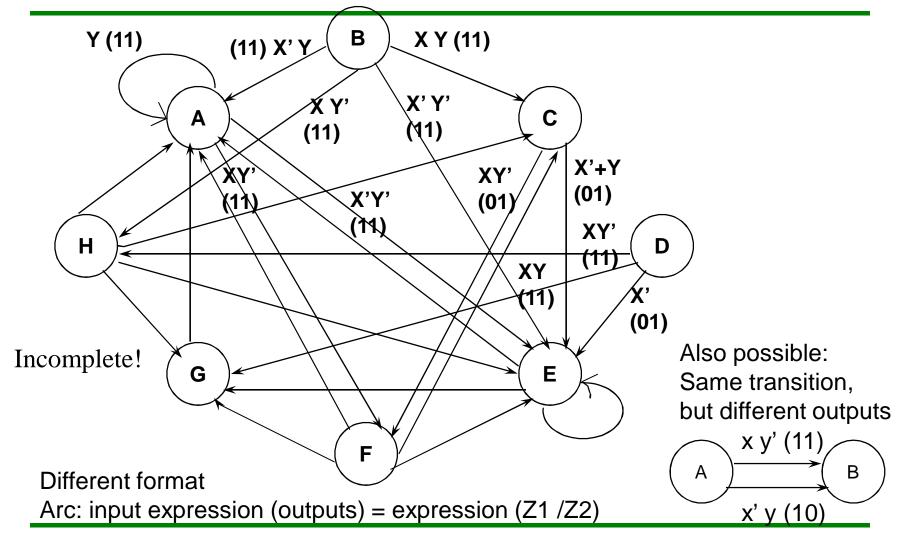


Example 2 - Named State / Output table

	XY					
S	00	01	11	10		
Α	E, 11	A, 11	A, 11	F, 11		
В	E, 11	A, 11	C, 11	H, 11		
С	E, 01	E, 01	E, 01	F, 01		
D	E, 01	E, 01	G, 11	H, 11		
E	E, 11	A, 11	A, 11	E, 11		
F	E, 11	A, 11	C, 11	G, 11		
G	E, 00	E, 00	E, 00	E, 00		
H	E, 00	E, 00	G, 10	G,10		
	S(t+1), Z1 /Z2					



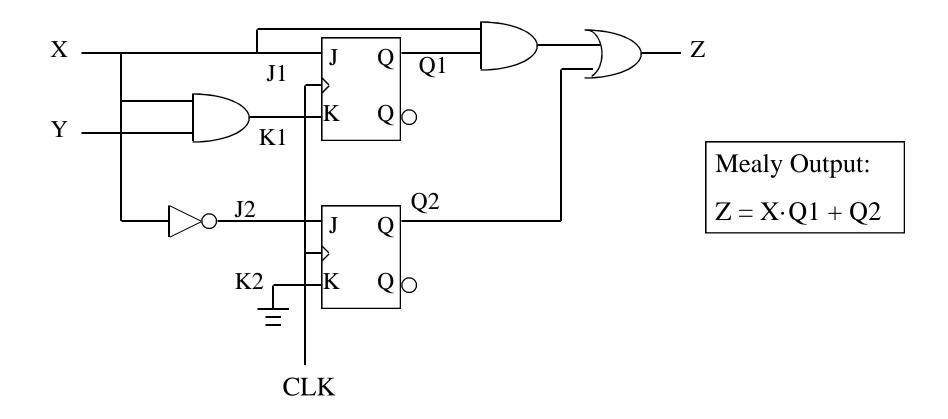
Example 2 - State Diagram



Analysis of J-K Flip-Flop State Machines

- There are two excitation equations per flip-flop (J, K)
- The characteristic equation : Q(t+1) = J·Q(t)' + K'·Q(t)
- Use the same analysis procedure shown previously

Example 3 - State Machine with J-K Flip-flops





Example 3 - Equations

Excitation

$$J1 = X$$

$$K1 = X \cdot Y$$

$$J2 = X'$$

$$K2 = 0$$

Characteristic

$$Q(t+1)=J\cdot Q'+K'\cdot Q$$

$$Q1(t+1) = J1 \cdot Q1' + K1' \cdot Q1$$

$$Q2(t+1) = J2 \cdot Q2' + K2' \cdot Q2$$

Transition

$$Q1(t+1) = X \cdot Q1' + (X \cdot Y)' \cdot Q1 = X \cdot Q1' + X' \cdot Q1 + Y' \cdot Q1$$

$$Q2(t+1) = X' \cdot Q2' + 0' \cdot Q2 = X' \cdot Q2' + Q2$$

Mealy Output

$$Z = X \cdot Q1 + Q2$$

Example 3 - State Table

		XY				
S	Q1 Q2	00	01	11	10	
A	0 0	01,0	01,0	10,0	10,0	
В	0 1	01,1	01,1	11,1	11,1	
C	1 0	11,0	11,0	00,1	10,1	
D	1 1	11,1	11,1	01,1	11,1	
	Q1(t+1) Q2(t+1), Z					

Transition

$$Q1(t+1) = X \cdot Q1' + (X \cdot Y)' \cdot Q1 = X \cdot Q1' + X' \cdot Q1 + Y' \cdot Q1$$

$$Q2(t+1) = X' \cdot Q2' + 0' \cdot Q2 = X' \cdot Q2' + Q2$$

Mealy Output

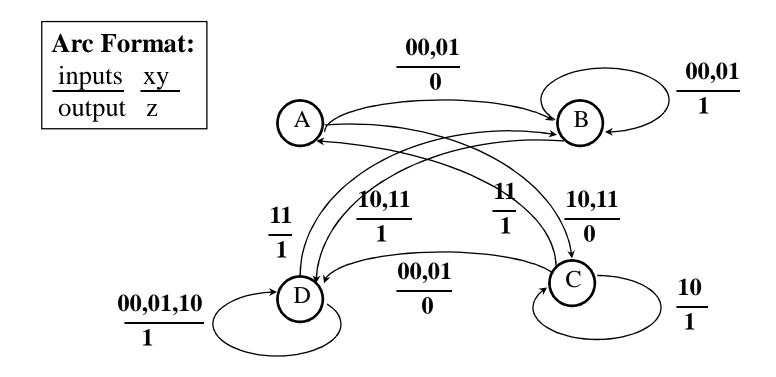
$$Z = X \cdot Q1 + Q2$$



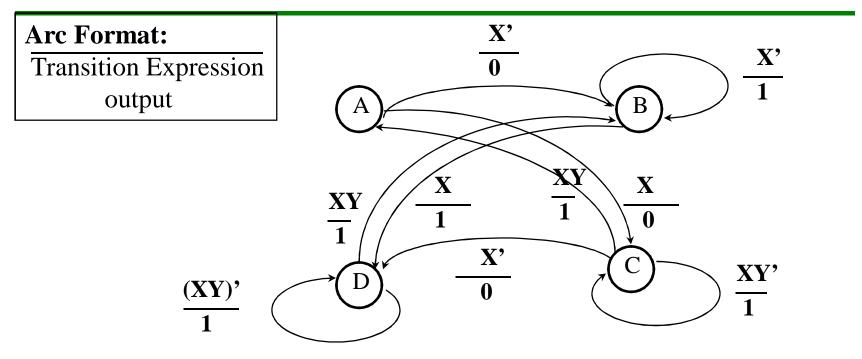
Example 3 - Named State/Output Table

	XY					
S	00	01	11	10		
A	В,0	В,0	C,0	C,0		
В	B,1	B,1	D,1	D,1		
C	D,0	D,0	A,1	C,1		
D	D,1	D,1	B,1	D,1		
	S(t+1), Z					

Example 3 - State Diagram



Example 3 - State Diagram



For each state/input combination there must be exactly one next-state (and output).

Mutual Exclusion: No more than one transition arc from any state can be satisfied by

any input assignment

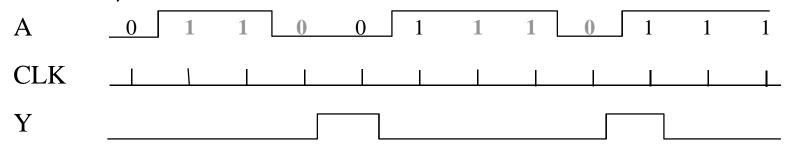
All Inclusion: At least one transition arc must exist from any state for any input

assignment



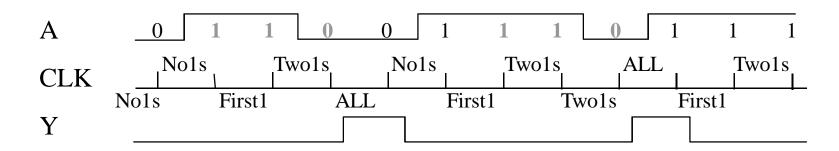
Design Example 1: 110 Detector

- Word description (input sequence detector)
 - Design a state machine with input A and output Y.
 - Y should be 1 whenever the sequence 1 1 0 has been detected on A on the last 3 consecutive clock ticks.
 - Otherwise, Y = 0
 - Note: this is a Moore machine, that is the output, Y, depends only on inputs at previous clocks, not on the current input.
- Interpretation of word description (only rising clock edges, or ticks, are shown)



Design Example 1: Choosing States

- Possible states (What do you need to remember?)
 - Initial : power up, no clocks yetY = 0
 - No1s : first 1 not found Y = 0
 - First 1 : first 1 found Y = 0
 - Two1s : at least 2 consecutive 1s found Y = 0
 - ALL : found 1 1 0 Y = 1
- Are all the states needed?
 - Notice: Initial is equivalent to NO1s
 - We can drop the state Initial and replace it with state No1s

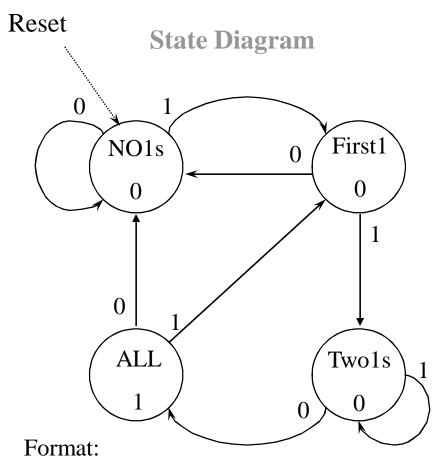




Design Example 1: State Table and Diagram

State Table

A						
S	0	1	Y			
NO1s	NO1s	First1	0			
First1	NO1s	Two1s	0			
Two1s	ALL	Two1s	0			
ALL	NO1s	First1	1			
S(t+1)						



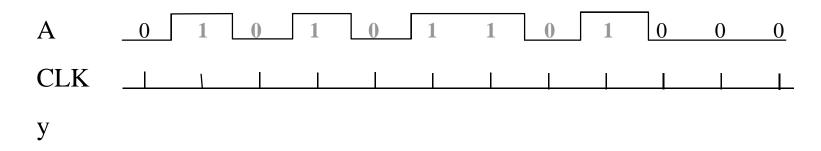
Arc: input A

Node: state/output Y



Design Example 2: 110/101 Detector

- Word description (input sequence detector)
 - Design a state machine with input A and output Y.
 - Y = 1 when either sequence 1 1 0 or 1 0 1 has been detected on A on the last 3 consecutive clock ticks.
 - Otherwise Y = 0
 - Note: Correct sequences <u>may overlap</u> and still be accepted
- Interpretation of word description (only rising clock ticks are shown)





Design Example 2: Choosing States

Possible states (What do you want to remember?)

Idle : Initial, no starting 1 yetY = 0

- Got1 : A = 1 on last tick Y = 0

Got10 : Sequence A = 10 on last two ticks Y = 0

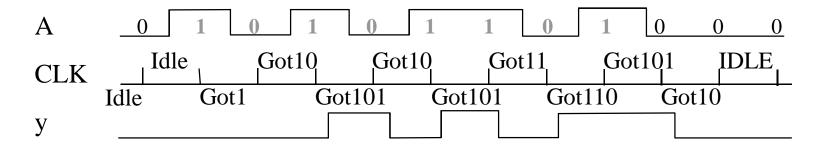
Got101 : Sequence A = 101 on last three ticksY =

1

- Got11 : Sequence A = 11 on last two ticks Y = 0

Got110 : Sequence A = 110 on last three ticks

1

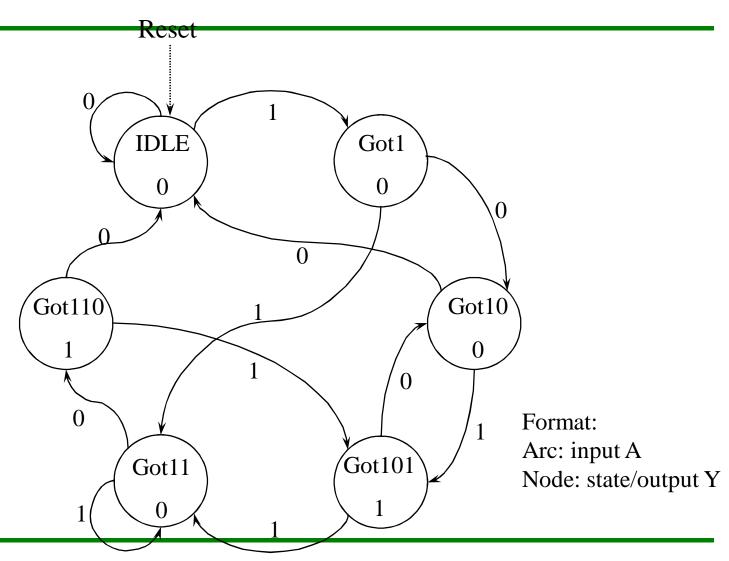


Design Example 2: State Table

A						
S	0	1	Y			
IDLE	IDLE	Got1	0			
Got1	Got10	Got11	0			
Got10	IDLE	Got101	0			
Got101	Got10	Got11	1			
Got11	Got110	Got11	0			
Got110	IDLE	Got101	1			
S(t+1)						



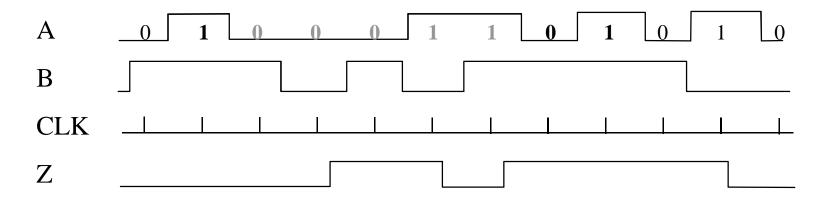
Design Example 2: State Diagram





Design Example 3: Interpretation

- Word description (input sequence detector)
 - Design a state machine with inputs A and B, and output Z.
 - -7 = 1 if either:
 - A had the same value for both previous clock ticks or
 - B has been 1 ever since the first condition occurred
 - Else Z = 0
- Interpretation of word description (only rising clock ticks are shown)



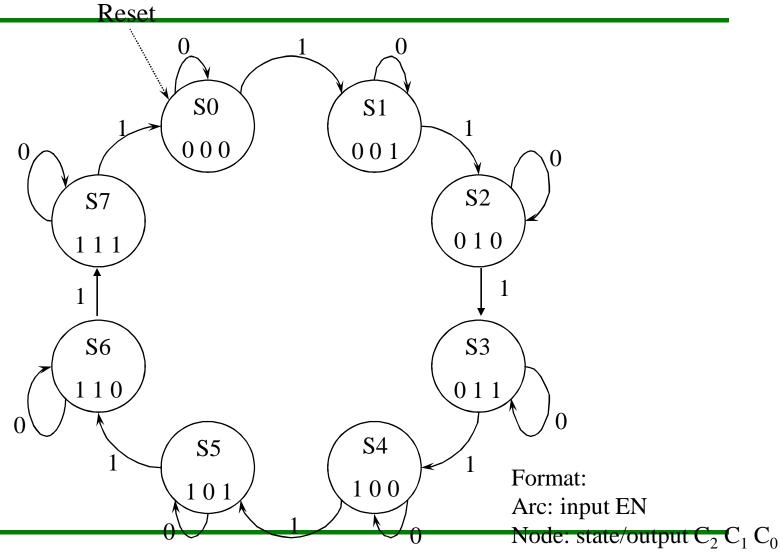
Design Example 4: State Table

- Word description (output sequence generator)
 - 3-bit counter with enable (0, 1, 2, 3, 4, 5, 6, 7, 0, 1 ...)

	Ŀ		OUI
S	0	1	$C_2 C_1 C_0$
S0	S0	S 1	0 0 0
S 1	S1	S 2	0 0 1
S 2	S2	S 3	0 1 0
S 3	S3	S 4	0 1 1
S 4	S4	S5	1 0 0
S 5	S5	S 6	1 0 1
S 6	S6	S 7	1 1 0
S7	S7	S0	1 1 1
		0	



Design Example 4: State Diagram





Design Example 5: State Table

- Word description (output sequence generator)
 - Design state machine with input GO and BCD output code B₃B₂B₁B₀
 - Anytime GO = 1 at a clock tick and machine is IDLE, output the BCD sequence 1, 2, 5, 9 during the next 4 clock ticks (regardless of GO)
 - Return to IDLE with BCD = 0 until GO = 1 is next detected

	. (Ο̈́C				
S	0	1	B_3	B_2	\mathbf{B}_1	B_0
IDLE	IDLE	S 1	0	0	0	0
S 1	S2	S 2	0	0	0	1
S2	S5	S5	0	0	1	0
S5	S 9	S 9	0	1	0	1
S 9	IDLE	IDLE	1	0	0	1