ISE 2211 • Exam VI • 9 April 2018 NAME Solution

Nine high-frequency inductors were wound with different turns of wire and the inductance of each measured to determine the relationship, if any, between the number of turns and inductance in millihenrys. Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model of inductance vs. turns.

Formulae:

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

# of Turns of Wire (x)	Inductance in mH (y)
400	27
500	34
600	39
700	46
800	55
900	62
1000	69
1100	75
1200	80

$\sum x_i = 7200 \qquad \sum y_i = 487$ (+1)

$\bar{x} = 800 \qquad \bar{y} = 54.\bar{1}$ (+1)

$\sum x_i^2 = 6360000 \qquad \sum y_i^2 = 29177$ (+1)

$\sum x_i y_i = 430700$ (+1)

$S_{XY} = 430700 - \frac{487 \cdot 7200}{9} = 41100$ (+1)

$S_{XX} = 6360000 - \frac{7200^2}{9} = 600000$ (+1)

$\hat{\beta}_1 = \frac{41100}{600000} = 0.0685$ (+1)

$\hat{\beta}_0 = 54.\bar{1} - 0.0685 \cdot 800 = -0.6\bar{8}$ (+1)

Write an equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$\hat{y} = 0.0685x - 0.6\bar{8}$ (+1)

Write a 95% confidence interval on the mean inductance @ 850 turns of wire.

$$\hat{\mu}_{Y|850} = 0.0685(850) - 0.6\overline{8} = 57.536\overline{1} \quad +1$$

$$t_{\alpha/2, n-2} = t_{.025,7} = 2.365 \quad +1$$

$$SS_T = \sum y_i^2 - n\bar{y}^2 = 29177 - 9 \cdot 54.\overline{1}^2 = 2824.\overline{8} \quad +1$$

$$SS_E = SS_T - \hat{\beta}_1 S_{XY} = 2824.\overline{8} - 0.0685 \cdot 41100$$
$$= 9.53\overline{8} \quad +1$$

$$\hat{\sigma}^2 = \frac{SS_E}{n-2} = \frac{9.53\overline{8}}{7} = 1.3627 \quad +1$$

$$\mu_{Y|850}: \quad 57.536\overline{1} \pm 2.365\sqrt{1.3627\left[\frac{1}{9} + \frac{(850-800)^2}{600000}\right]}$$

$$56.60 < \mu_{Y|850} < 58.47 \qquad [mH]$$

+1

Now write a 95% prediction interval on the inductance of the 10th high-frequency inductor, wound with 850 turns.

$$\hat{y}_0 = 57.536\overline{1} \quad +1$$

$$y_0: \quad 57.536\overline{1} \pm 2.365\sqrt{1.3627\left[1 + \frac{1}{9} + \frac{50^2}{600000}\right]}$$

$$54.62 < y_0 < 60.45 \qquad [mH]$$

+1

Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero. What does your conclusion imply about the relationship between turns and inductance?

H_0: $\hat{\beta}_1 = 0$

H_1: $\hat{\beta}_1 \neq 0$

$$\beta_1: \quad \hat{\beta}_1 \pm t_{\alpha/2, n-2}\sqrt{\hat{\sigma}^2 / S_{xx}}$$

$$0.0685 \pm 2.365\sqrt{1.3627/600000}$$

$$0.06493 < \beta_1 < 0.07206$$ (+1)

does not contain zero (+1) reject H_0 (+1)

the relationship is significant (+1)

Now suppose that both turns and inductance are random variables. Write a 95% C.I. on their correlation coefficient. (Don't worry about the low sample size; compute it anyway.) What does the confidence interval imply about the possible correlation between turns of wire and inductance?

$$\rho: \quad \tanh\left(\tanh^{-1} R \pm \frac{z_{\alpha/2}}{\sqrt{n-3}}\right)$$

$$z_{\alpha/2} = z_{.025} = 1.96$$ (+1)

$$R^2 = 1 - \frac{SS_E}{SS_T} = 1 - \frac{9.538}{7824.5} = 0.9966$$ (+1)

$$\tanh^{-1}\sqrt{.9966} = 3.5377$$ (+1)

$$\tanh\left(3.5377 + \frac{1.96}{\sqrt{6}}\right) = 0.99966$$ (+1)

$$\tanh\left(3.5377 - \frac{1.96}{\sqrt{6}}\right) = 0.99166$$ (+1)

$$0.99166 < \rho < 0.99966$$

they are significantly correlated (+1)