EE 2010 Circuit Analysis

Module 20: Transient Responses: Time-domain Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, Michael Richmond and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

From WIKIPEDIA: "In electrical engineering and mechanical engineering, a *transient response* or natural response is the *response* of a system *to a change* from an equilibrium or a steady state. The transient response is not necessarily tied to "on/off" events but to any event that affects the equilibrium of the system. The *impulse response* and *step response* are transient responses to a specific input."

and recall ...

Dynamic elements are passive elements with the capability of energy storage. They are referred to as *dynamic*, as opposed to *instantaneous*, because their present state depends on the present input as well as previous inputs.

Therefore,...

The **transient response** of dynamic systems must (in general) include the effects of both the **input** stimulus and any **stored energy** in any *dynamic element*. We will consider these two contributions separately, with the effects of the **input stimulus** termed the **zero-state response** (since there is assumed to be no stored energy) and the response due to the **stored energy** in any of the *dynamic elements* termed the **zero-input response** (since this response does not result from an input stimulus). The **total response** is the sum of the responses to these two stimuli.

In this section, we consider time-domain approaches that include responses to both an input stimulus *(The Zero-State Response)* and due to stored energy *(The Zero-Input Response)*. The latter being manifested in the *initial conditions* for the specification of the *particular solution* of the system differential equation.

We will find the integro-differential equation representation to be quite tedious, and intractable for higher-order systems (>2). To simplify, we then introduce **canonical forms** of solutions applicable to **all first-order systems** and others applicable to **some 2nd-order systems**.

For time-domain analysis we employ the time-domain voltage-current models,

Time-Domain Models

Impedance	**Admittance**
$v(t) = i(t) \cdot R$	$i(t) = \frac{v(t)}{R}$
$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$i(t) = C\frac{dv(t)}{dt}$
$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$

Table 1: Models for R, C, and L

†

1 Integro-Differential Procedure

Integro-Differential Procedure for t-domain Analysis of Transient Responses:

1. Analyze the circuit using appropriate techniques to find the initial conditions for every dynamic element (i.e, every L and C)
2. Analyze the circuit using appropriate techniques to find the output: $v_{out}(t)$, or whatever output function is dictated by the problem, in terms of integro-differential equations describing the circuit
3. Rewrite the descriptive equation into a differential-equation-only form (no integrals)
4. Find the undetermined constants by matching the initial conditions from the circuit with the same quantities in the equation describing the circuit

Later, we'll employ:

2 Canonical-Form Procedure

Canonical-Form Procedure for t-domain Analysis of Transient Responses:

1. Analyze the circuit using appropriate techniques to find the initial conditions for every dynamic element (i.e, every L and C)
2. If the circuit happens to match a 1st-order or 2nd-order canonical form,
3. Evaluate the parameters of the canonical-form solution to match the component values and initial conditions of the circuit.

2.1 First-Order System Response:

Every first-order linear dynamic system has exactly the same transient response solution:

$$\text{quantity}(t) = \text{quantity}(\infty) + [\text{quantity}(t_0) - \text{quantity}(\infty)]e^{-\frac{(t-t_0)}{\tau}}, \quad t \geq t_0$$

where t_0 it the time we are initially interested in the system, and τ is the *time constant* of the system.

2.2 Second-Order System Response:

Since, for a second-order system, the *characteristic equation* (= denominator of the transfer function) is a second-order polynomial in s and this second-order polynomial has exactly two roots, s_1 and

s_2, the characteristics of the system response to a transient stimulus follows directly from the roots s_1 and s_2 of the *characteristic equation* of the system. We will find these roots to have the form of

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

The particular form of solution is determined by the relationship of *damping rate* α to the *natural frequency* ω_0 as

$$\begin{aligned} \text{overdamped, if } \alpha > \omega_0, &\quad \text{the roots are real and distinct} \\ \text{critically-damped, if } \alpha = \omega_0, &\quad \text{the roots are real and repeated} \\ \text{underdamped, if } \alpha < \omega_0, &\quad \text{the roots are complex} \end{aligned}$$

and in the latter case $\omega_d = \sqrt{-(\omega_0^2 - \alpha^2)}$. Here, ω_0 is often called the *undamped natural frequency*, and ω_d is called the *damped natural frequency.*

The form of the roots of the characteristic equation (= the denominator of the transfer function) parametrically y determine the form of the solution for second-order linear differential equations with constant coefficients as:

$$\begin{aligned} \text{quantity}(t) &= \text{quantity}(\infty) + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}, \quad \text{roots} = -\alpha_1, -\alpha_2 \quad \text{overdamped} \\ \text{quantity}(t) &= \text{quantity}(\infty) + A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}, \quad \text{roots} = -\alpha, -\alpha \quad \text{critically-damped} \\ \text{quantity}(t) &= \text{quantity}(\infty) + e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)), \quad \text{roots} = -\alpha \pm j\omega_d \quad \text{underdamped} \end{aligned}$$

where A_i are derived from the (two) initial conditions (one for each energy-storage element) in the form of quantity($t = 0$) and quantity'($t = 0$).

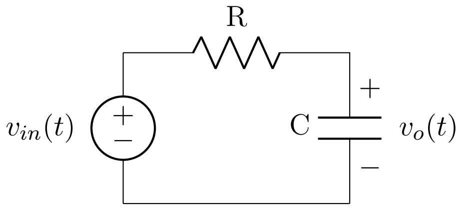
2.3 Third-Order (and higher) System Response:

No time-domain parametric forms available. Use s-domain analysis.

First-Order Dynamic Circuit Examples

We begin by considering a change from an equilibrium or a steady state of a circuit containing **exactly one first-order** dynamic element. There are only two of these: an RC and an RL circuit.

Example 1t: Find $v_o(t)$ where $v_{in}(t) = Au(t)$ is an A-volt step function:



Remember, we proceed as if the Laplace Transform does not exist. Instead, we'll use the capacitor model:

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v(t_0)$$

where $v(t_0)$ is the "initial" voltage across the capacitor at time t_0. Employing KVL around the loop for the common current $i(t)$, we have, beginning at $t = 0^+$ – just after the step turns on:

$$-A + i(t)R + \frac{1}{C}\int_0^t i(\tau)d\tau = 0$$

Taking the derivative $\frac{d}{dt}$, we have

$$\begin{aligned} i'(t)R + \frac{1}{C}i(t) &= 0 \\ i'(t) &= -\frac{1}{RC}i(t) \end{aligned}$$

which is a first-order linear differential equation with constant coefficients. The solution has the form

$$i(t) = Ke^{-\frac{t}{RC}}, \quad t \geq 0$$

where K is determined by the initial condition. If the initial voltage across the capacitor is $v_C(0)$ the initial condition $i((0) = \frac{(A-v_C(0))}{R}$.

So, how about the voltage? We have $v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v_C(0)$ which yields

$$\begin{aligned} v(t) &= \frac{(A - v_C(0)}{RC}\int_0^t e^{-\frac{t}{RC}}dt + v_C(0) \\ &= \frac{(A - v_C(0))}{RC}(-RC(e^{-\frac{t}{RC}} - 1)) + v_C(0) \\ &= (A - v_C(0))(1 - e^{-\frac{t}{RC}}) + v_C(0), \quad t \geq 0 \\ &= A + (v_C(0) - A)e^{-\frac{t}{RC}}, \quad t \geq 0 \end{aligned}$$

(which we could have deduced from

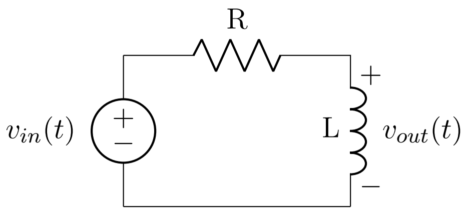
$$\text{quantity}(t) = \text{quantity}(\infty) + [\text{quantity}(t_0) - \text{quantity}(\infty)]e^{-\frac{(t-t_0)}{\tau}}, \quad t \geq t_0$$

noting that $\tau = RC$ is the *time constant* of the RC system).

We will notice a pattern of *exponential responses* for impulse and step inputs.

Does this result make sense? Check $v_C(0)$ and $v_C(\infty)$.

Example 2: Find $v_o(t)$ where $v_{in}(t) = Au(t)$:



We'll use the inductor model:

$$v(t) = L\frac{di(t)}{dt}$$

Employing KVL around the loop for the common current $i(t)$, we have for $t = 0^+$ – just after the unit step turns on:

$$-A + i(t)R + L\frac{di(t)}{dt} = 0$$

or

$$i'(t) = -\frac{R}{L}i(t) + \frac{A}{L}$$

The solution is

$$i(t) = \frac{A}{R} - \frac{A}{R}e^{-\frac{tR}{L}}, \quad t \geq 0$$

So, how about the voltage? We have $v(t) = L\frac{di(t)}{dt}$ which yields

$$v(t) = L\frac{A}{R}\frac{R}{L}e^{-\frac{tR}{L}} = Ae^{-\frac{tR}{L}}, \quad t \geq 0$$

(which, again, we could have also deduced from

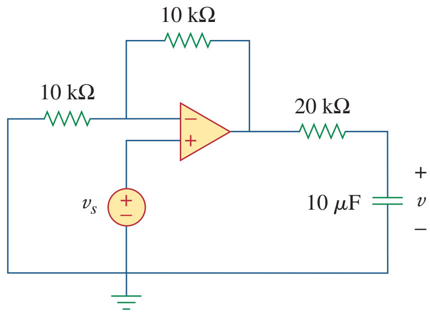
$$\text{quantity}(t) = \text{quantity}(\infty) + [\text{quantity}(t_0) - \text{quantity}(\infty)]e^{-\frac{(t-t_0)}{\tau}}, \quad t \geq t_0$$

noting that $\tau = \frac{L}{R}$ is the *time constant* of the RL system.) Which again is an *exponential response.*

Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

Problem 7.71 with $v_s = 3u(t)$::

7.71 For the op amp circuit in Fig. 7.136, suppose $v_0 =$
and $v_s = 3$ V. Find $v(t)$ for $t > 0$.



Labeling the Op-Amp output as v_{out} and writing a node equation at the inverting input, we have:

$$\frac{v_{in}}{10000} + \frac{(v_{in} - v_{out})}{10000} = 0$$

so that

$$V_{out} = 2V_{in} = 6u(t)$$

At $t = 0^+$ (just after the unit step turns on), we write a KVL on the right side of the circuit as:

$$-6 + 20000 * i(t) + \frac{1}{10^{-5}} \int_0^t i(\tau) d\tau = 0$$

so upon taking the derivative, we have,

$$20000 * i'(t) + \frac{1}{10^{-5}} i(t) = 0 = i'(t) + \frac{1}{10^{-5} * 20000} i(t) = i'(t) + 5i(t) = 0$$

which has solution

$$i(t) = c \cdot e^{-\frac{1}{10^{-5} * 20000} t} = c \cdot e^{-5t}$$

where the constant c satisfies the initial condition for the current. Since the capacitor voltage is zero, we must have $i(0) = 6/20000$, so

$$i(t) = \frac{6}{20000} e^{-5t}$$

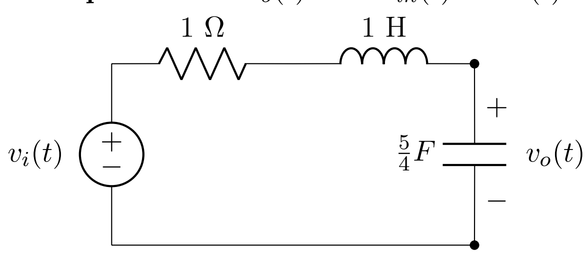
Getting back to the voltage, we have

$$\begin{aligned} v(t) &= \frac{1}{C} \int_0^t i(\tau) d\tau = \frac{1}{10^{-5}} \int_0^t \frac{6}{20000} e^{-5\tau} d\tau \\ &= \frac{6}{20000 * 10^{-5}} \int_0^t e^{-5\tau} d\tau = \frac{6}{0.2} \cdot \frac{1}{-5} * (e^{-5t} - 1) \\ &= 6 * (1 - e^{-5t}) \end{aligned}$$

Which is an *exponential response.*

Does this make sense? Check $v_o(0)$ and $v_o(\infty)$.

Example 4: Find $v_o(t)$ with $v_{in}(t) = Au(t)$:



This is a 2nd-order system for which we will use both the capacitor and inductor models. We'll also use symbolic values for clarity and plug in the component values later. Employing KVL around the loop for the common current $i(t)$, we have for $t = 0^+$ – just after the unit step turns on:

$$-A + i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau = 0$$

Taking a derivative and dividing by L, we have

$$i''(t) + \frac{R}{L}i'(t) + \frac{1}{LC}i(t) = 0$$

And that's as far as we can go. The next step is finding the roots of the characteristic equation:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

(which is the denominator of the transfer function H(s), but we're not supposed to know that).

This is a quadratic equation with roots:

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{(RL)^2 - 4\frac{1}{LC}}}{2}$$

For our component values, we have

$$\begin{aligned} s_{1,2} &= \frac{-1 \pm \sqrt{(1)^2 - 4\frac{4}{5}}}{2} \\ &= \frac{-1 \pm \sqrt{1 - \frac{16}{5}}}{2} \\ &= -0.5 \pm \sqrt{-\frac{11}{20}} \\ &= -0.5 \pm j * 0.7416 \end{aligned}$$

The complex pair roots suggests the solution of the form:

$$\text{quantity}(t) = \text{quantity}(\infty) + e^{-\alpha t}(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)), \quad \text{roots} = -\alpha \pm j\omega_d$$

with $\alpha = 0.5$ and $\omega_d = 0.7416$ and where A_i are derived from the (two) initial conditions (one for each energy-storage element) in the form of quantity($t = 0$) and quantity'($t = 0$).

Since we are interested in the voltage across the capacitor, we have the solution form

$$v_o(t) = v_o(\infty) + e^{-0.5t}(A_1 \cos(0.7416t) + A_2 \sin(0.7416t))$$

where A_i are derived from the (two) initial conditions in the form of $v_o(t = 0)$ and $v_o'(t = 0)$.

So, back to the circuit we go.

We easily see that $v_o(\infty) = A$ and that $v_o(t = 0) = 0$ (i.c. for Energy in C) (why?) To find $v_o'(t = 0)$, we need to look to some quantity and circuit element that helps determine $v_o'(t)$.

We know that for the capacitor, $i(t) = C\frac{dv(t)}{dt}$ so if we can find $i(t = 0)$ we can find $v_o'(t = 0)$. So what is $i(t = 0)$? Well, we know that in this case, the inductor current $i(t)$ was zero just BEFORE the step turned on. Then the inductor current $i(t)$ MUST also be zero just AFTER the step turned on.

So, since $i(t = 0) = 0$, we also have $i(t = 0) = C\frac{dv(t=0)}{dt} = 0$, so that $v_o'(t = 0) = 0$. OK, coming down the home stretch.

Initial Conditions Determine the Particular Solution

$v_o(t = 0)$:
From the circuit, we have:

$$v_o(t = 0) = 0$$

From the form of the solution, we have:

$$\begin{aligned} v_o(t) &= A + e^{-0.5t}(A_1 \cos(0.7416t) + A_2 \sin(0.7416t))\Big|_{t=0} \\ &= A + A_1 \end{aligned}$$

$v_o'(t = 0)$:
From the circuit, we have:

$$v_o'(t = 0) = 0$$

From the form of the solution, we have:

$$\begin{aligned} v_o'(t) &= -0.5e^{-0.5t}(A_1 \cos(0.7416t)) + e^{-0.5t}(A_2 * 0.7416 \cos(0.7416t))\Big|_{t=0} \\ &= -0.5A_1 + 0.7416A_2 \end{aligned}$$

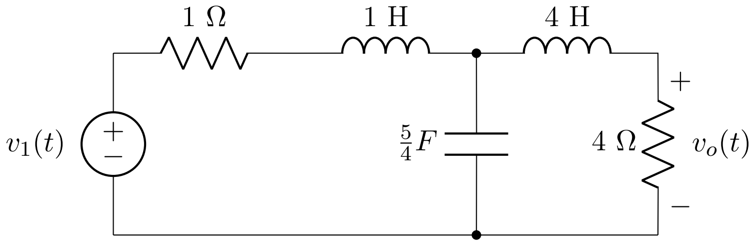
So we must have that $A_1 = -A$ and $A_2 = -0.6742A$. We then finally have:

$$v_o(t) = A - Ae^{-0.5t}(\cos(0.7416t) + 0.6742 \sin(0.7416t))$$

Which agrees with the (one-line) result from the previous module. Done!

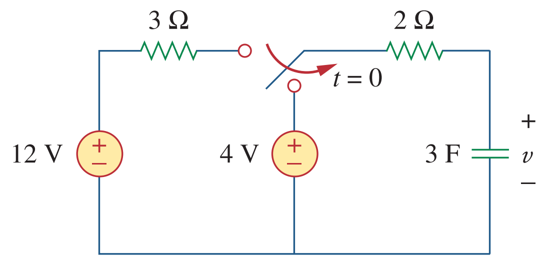
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

Example 5: A third-order system.



Time-domain? No way! **NOBODY** does a problem like this in time domain!

Problem 7.40a with $v_{in} = 4u(t)$ and $v(0) = 12$V::



(a)

For this circuit, the first switch position is used to compute the initial condition ($v_C(0) = 12$V), while the second switch position determines the voltage input and the circuit for which to find the transient response.

With the switch in the second position, we have a $Vin = 4$V source acting on a circuit with a $v_C(0) = 12$V charged capacitor. We'll use the voltage-source model for a charged capacitor.

Using

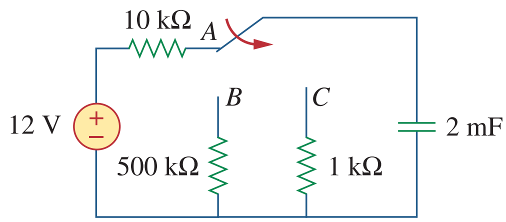
$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-\frac{(t-t_0)}{\tau}}, \quad t \geq t_0$$

We can find this directly with $t_0 = 0$, $v(0) = 12$, $v(\infty) = 4$, and time constant $\tau = RC = 6$ so that

$$v(t) = 4 + (12 - 4)e^{-t/6}, \quad t \geq 0$$

Problem 7.7:

7.7 Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at $t = 0$, Then at $t = 1$ second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.



For this circuit, the first switch position is used to compute the initial condition ($v_C(0) = 12$V), while the second and third switch positions govern the transient response, where the final condition of the circuit at switch B is the initial condition of the circuit at switch C.

At $t = 0$, switch is in position B and we $Vin = 0$ and a $v_C(0) = 12$V charged capacitor. We can find this directly with $t_0 = 0$, $v(0) = 12$, $v(\infty) = 0$, and time constant $\tau = RC = 1000$ so that

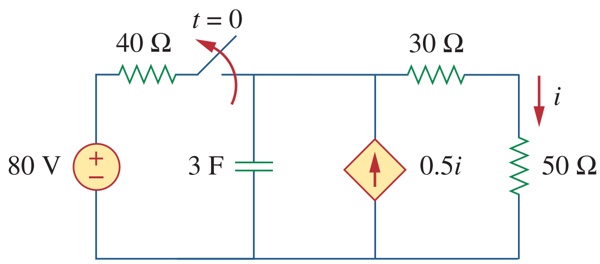
$$v(t) = 12e^{-t/1000}, \quad 0 \leq t \leq 1$$

At $t = 1$, the switch moves to C and $v_C(1) = 12 * exp(-1/1000) = 11.9880$V becomes the new initial condition for the next time interval. Again, we can find this directly with $t_0 = 1$, $v(1) = 11.9880$, $v(\infty) = 0$, and time constant $\tau = RC = 2$ so that

$$v(t) = 11.9880e^{-(t-1)/2}, \quad 1 \leq t$$

Problem 7.43:

7.43 Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.



The first switch position is used to compute the initial condition ($v_C(0)$), which we can find with a node equation:

```
%% Problem 7.43 (t<0)
clear all
syms va i50
% Node equation
[va, i50]=solve((va-80)/40 - 0.5*i50 + va/(30+50) == 0,...
  i50 == va/(30+50), va, i50)
vC0 = double(va)
```

from which we get $v_C(0) = 64$V.

At $t = 0$, switch opens and we have $v_C(0) = 64$V. So we have $t_0 = 0$, $v(0) = 64$, and $v(\infty) = 0$. To find the time constant $\tau = RC$ we must compute the equivalent resistance as seen by the capacitor. And yes, this is a Thevenin problem.

Homework: Chapter 7 # 2, 4, 11 , 39 42a, 45(with $v_0(0) = 0$), 46

Chapter 8 # 5, 45(with $i_0(0) = 0$)

7.5 In the circuit of Fig. 7.79, the capacitor voltage just before $t = 0$ is:

(a) 10 V (b) 7 V (c) 6 V
(d) 4 V (e) 0 V

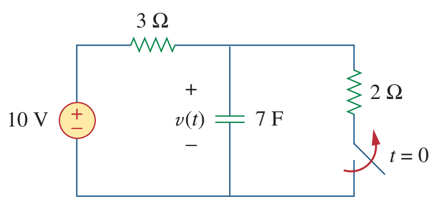


Figure 7.79
For Review Questions 7.5 and 7.6.

7.6 In the circuit in Fig. 7.79, $v(\infty)$ is:

(a) 10 V (b) 7 V (c) 6 V
(d) 4 V (e) 0 V

7.7 For the circuit in Fig. 7.80, the inductor current just before $t = 0$ is:

(a) 8 A (b) 6 A (c) 4 A
(d) 2 A (e) 0 A

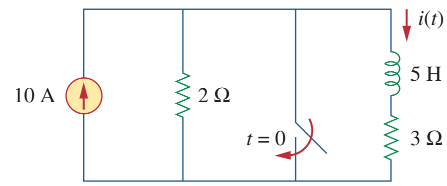


Figure 7.80
For Review Questions 7.7 and 7.8.

7.8 In the circuit of Fig. 7.80, $i(\infty)$ is:

(a) 10 A (b) 6 A (c) 4 A
(d) 2 A (e) 0 A

7.9 If v_s changes from 2 V to 4 V at $t = 0$, we may express v_s as:

(a) $\delta(t)$ V (b) $2u(t)$ V
(c) $2u(-t) + 4u(t)$ V (d) $2 + 2u(t)$ V
(e) $4u(t) - 2$ V

7.10 The pulse in Fig. 7.116(a) can be expressed in terms of singularity functions as:

(a) $2u(t) + 2u(t - 1)$ V (b) $2u(t) - 2u(t - 1)$ V
(c) $2u(t) - 4u(t - 1)$ V (d) $2u(t) + 4u(t - 1)$ V

Answers: 7.1d, 7.2b, 7.3c, 7.4b, 7.5d, 7.6a, 7.7c, 7.8e, 7.9c,d, 7.10b.

Problems

Section 7.2 The Source-Free *RC* Circuit

7.1 In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t}\text{ V}, \quad t > 0$$
$$i(t) = 8e^{-200t}\text{ mA}, \quad t > 0$$

(a) Find the values of R and C.

(b) Calculate the time constant τ.

(c) Determine the time required for the voltage to decay half its initial value at $t = 0$.

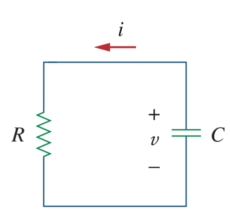


Figure 7.81
For Prob. 7.1.

7.2 Find the time constant for the *RC* circuit in Fig. 7.82.

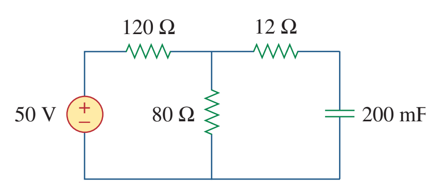


Figure 7.82
For Prob. 7.2.

7.3 Determine the time constant for the circuit in Fig. 7.83.

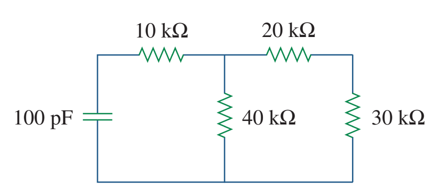


Figure 7.83
For Prob. 7.3.

7.4 The switch in Fig. 7.84 has been in position A for a long time. Assume the switch moves instantaneously from A to B at $t = 0$. Find v for $t > 0$.

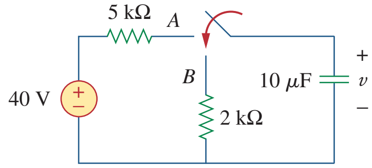


Figure 7.84
For Prob. 7.4.

7.5 Using Fig. 7.85, design a problem to help other students better understand source-free RC circuits.

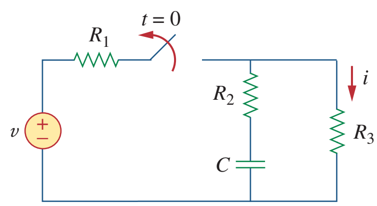


Figure 7.85
For Prob. 7.5.

7.6 The switch in Fig. 7.86 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

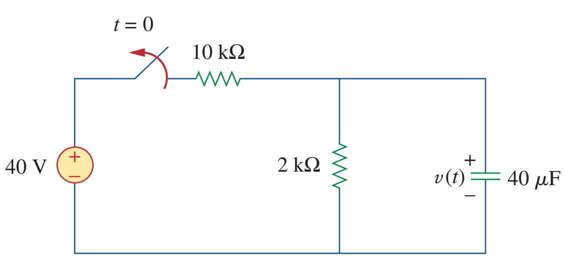


Figure 7.86
For Prob. 7.6.

7.7 Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at $t = 0$, Then at $t = 1$ second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.

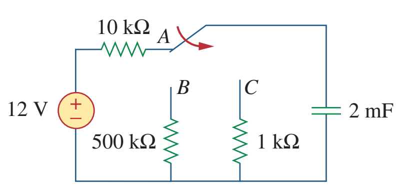


Figure 7.87
For Prob. 7.7.

7.8 For the circuit in Fig. 7.88, if

$$v = 10e^{-4t} \text{ V} \quad \text{and} \quad i = 0.2\,e^{-4t} \text{ A}, \quad t > 0$$

(a) Find R and C.

(b) Determine the time constant.

(c) Calculate the initial energy in the capacitor.

(d) Obtain the time it takes to dissipate 50 percent of the initial energy.

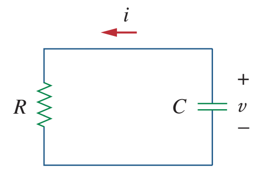


Figure 7.88
For Prob. 7.8.

7.9 The switch in Fig. 7.89 opens at $t = 0$. Find v_o for $t > 0$.

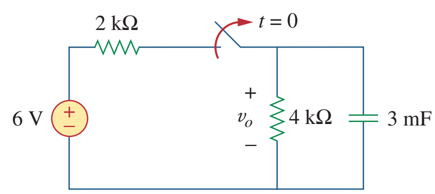


Figure 7.89
For Prob. 7.9.

7.10 For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

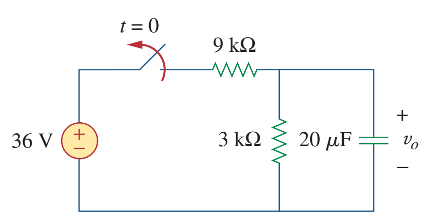


Figure 7.90
For Prob. 7.10.

Section 7.3 The Source-Free *RL* Circuit

7.11 For the circuit in Fig. 7.91, find i_o for $t > 0$.

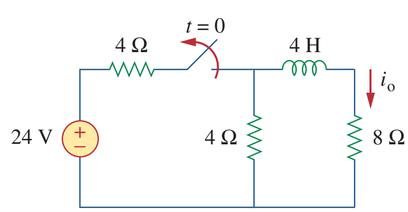


Figure 7.91
For Prob. 7.11.

7.12 Using Fig. 7.92, design a problem to help other students better understand source-free *RL* circuits.

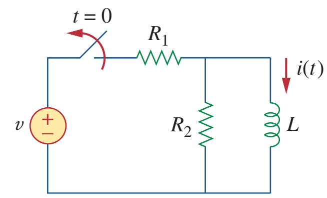


Figure 7.92
For Prob. 7.12.

7.13 In the circuit of Fig. 7.93,

$$v(t) = 80e^{-10^3 t}\ \text{V}, \quad t > 0$$
$$i(t) = 5e^{-10^3 t}\ \text{mA}, \quad t > 0$$

(a) Find R, L, and τ.

(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

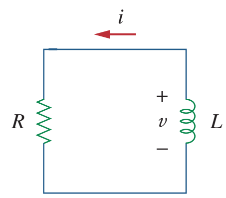


Figure 7.93
For Prob. 7.13.

7.14 Calculate the time constant of the circuit in Fig. 7.94.

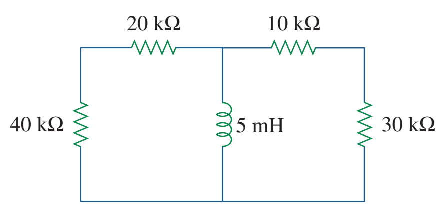


Figure 7.94
For Prob. 7.14.

7.15 Find the time constant for each of the circuits in Fig. 7.95.

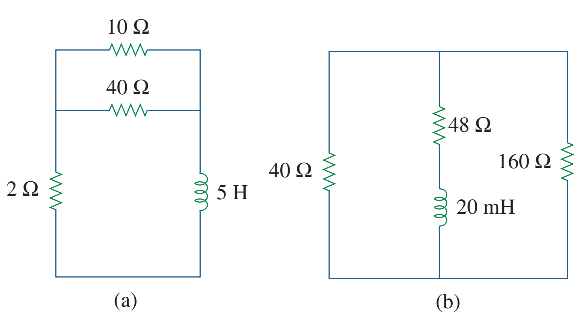


Figure 7.95
For Prob. 7.15.

7.16 Determine the time constant for each of the circuits in Fig. 7.96.

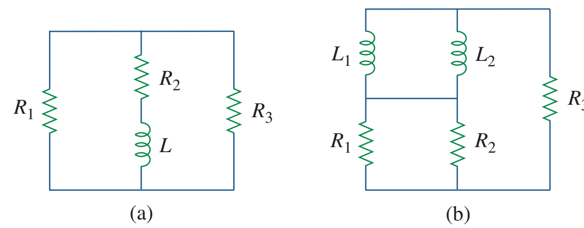


Figure 7.96
For Prob. 7.16.

7.17 Consider the circuit of Fig. 7.97. Find $v_o(t)$ if $i(0) = 6$ A and $v(t) = 0$.

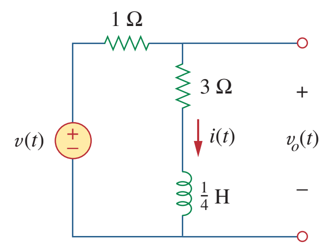


Figure 7.97
For Prob. 7.17.

7.18 For the circuit in Fig. 7.98, determine $v_o(t)$ when $i(0) = 5$ A and $v(t) = 0$.

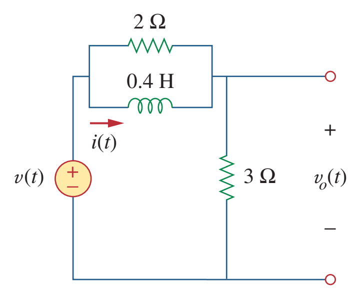


Figure 7.98
For Prob. 7.18.

7.19 In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 6$ A.

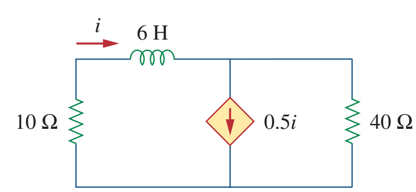


Figure 7.99
For Prob. 7.19.

7.20 For the circuit in Fig. 7.100,

$$v = 90e^{-50t} \text{ V}$$

and

$$i = 30e^{-50t} \text{ A}, \qquad t > 0$$

(a) Find L and R.

(b) Determine the time constant.

(c) Calculate the initial energy in the inductor.

(d) What fraction of the initial energy is dissipated in 10 ms?

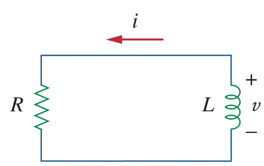


Figure 7.100
For Prob. 7.20.

7.21 In the circuit of Fig. 7.101, find the value of R for which the steady-state energy stored in the inductor will be 1 J.

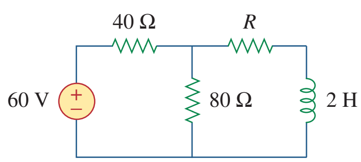


Figure 7.101
For Prob. 7.21.

7.22 Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 7.102 if $i(0) = 10$ A.

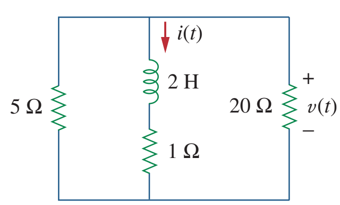


Figure 7.102
For Prob. 7.22.

7.23 Consider the circuit in Fig. 7.103. Given that $v_o(0) = 10$ V, find v_o and v_x for $t > 0$.

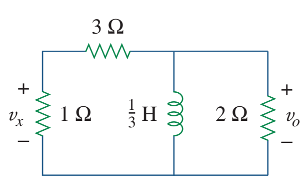


Figure 7.103
For Prob. 7.23.

Section 7.4 Singularity Functions

7.24 Express the following signals in terms of singularity functions.

(a) $v(t) = \begin{cases} 0, & t < 0 \\ -5, & t > 0 \end{cases}$

(b) $i(t) = \begin{cases} 0, & t < 1 \\ -10, & 1 < t < 3 \\ 10, & 3 < t < 5 \\ 0, & t > 5 \end{cases}$

(c) $x(t) = \begin{cases} t - 1, & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 4 - t, & 3 < t < 4 \\ 0, & \text{Otherwise} \end{cases}$

(d) $y(t) = \begin{cases} 2, & t < 0 \\ -5, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$

7.25 Design a problem to help other students better understand singularity functions.

7.26 Express the signals in Fig. 7.104 in terms of singularity functions.

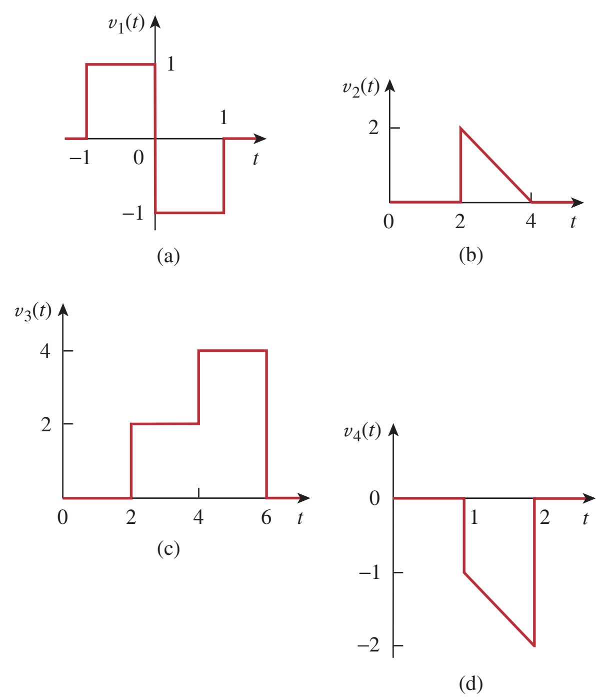


Figure 7.104
For Prob. 7.26.

7.27 Express $v(t)$ in Fig. 7.105 in terms of step functions.

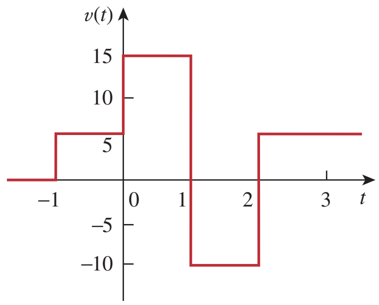


Figure 7.105
For Prob. 7.27.

7.28 Sketch the waveform represented by

$$i(t) = r(t) - r(t-1) - u(t-2) - r(t-2) + r(t-3) + u(t-4)$$

7.29 Sketch the following functions:

(a) $x(t) = 10e^{-t}u(t-1)$,
(b) $y(t) = 10e^{-(t-1)}u(t)$,
(c) $z(t) = \cos 4t\delta(t-1)$

7.30 Evaluate the following integrals involving the impulse functions:

(a) $\displaystyle\int_{-\infty}^{\infty} 4t^2\delta(t-1)\,dt$

(b) $\displaystyle\int_{-\infty}^{\infty} 4t^2 \cos 2\pi t\delta(t-0.5)\,dt$

7.31 Evaluate the following integrals:

(a) $\displaystyle\int_{-\infty}^{\infty} e^{-4t^2}\delta(t-2)\,dt$

(b) $\displaystyle\int_{-\infty}^{\infty} [5\delta(t) + e^{-t}\delta(t) + \cos 2\pi t\delta(t)]\,dt$

7.32 Evaluate the following integrals:

(a) $\displaystyle\int_{1}^{t} u(\lambda)\,d\lambda$

(b) $\displaystyle\int_{0}^{4} r(t-1)\,dt$

(c) $\displaystyle\int_{1}^{5} (t-6)^2\delta(t-2)\,dt$

7.33 The voltage across a 10-mH inductor is $15\delta(t-2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.

7.34 Evaluate the following derivatives:

(a) $\dfrac{d}{dt}[u(t-1)u(t+1)]$

(b) $\dfrac{d}{dt}[r(t-6)u(t-2)]$

(c) $\dfrac{d}{dt}[\sin 4tu(t-3)]$

7.35 Find the solution to the following differential equations:

(a) $\dfrac{dv}{dt} + 2v = 0, \qquad v(0) = -1$ V

(b) $2\dfrac{di}{dt} - 3i = 0, \qquad i(0) = 2$

7.36 Solve for v in the following differential equations, subject to the stated initial condition.

(a) $dv/dt + v = u(t), \qquad v(0) = 0$
(b) $2\,dv/dt - v = 3u(t), \qquad v(0) = -6$

7.37 A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

(a) What is the time constant of the circuit?
(b) What is $v(\infty)$, the final value of v?
(c) If $v(0) = 2$, find $v(t)$ for $t \geq 0$.

7.38 A circuit is described by

$$\frac{di}{dt} + 3i = 2u(t)$$

Find $i(t)$ for $t > 0$ given that $i(0) = 0$.

Section 7.5 Step Response of an *RC* Circuit

7.39 Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.

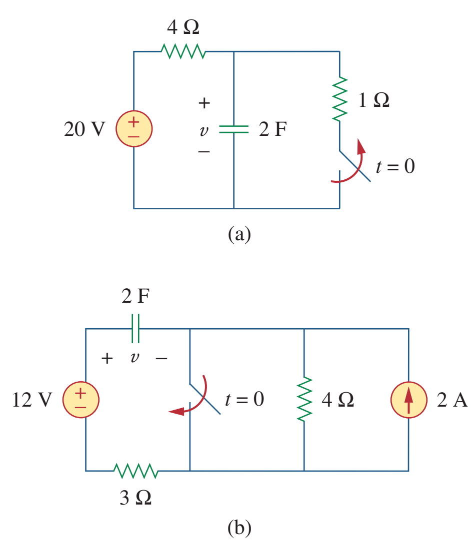


Figure 7.106
For Prob. 7.39.

7.40 Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.

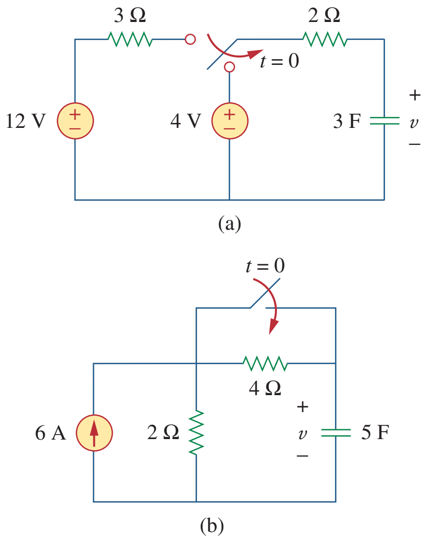


Figure 7.107
For Prob. 7.40.

7.41 Using Fig. 7.108, design a problem to help other students better understand the step response of an *RC* circuit.

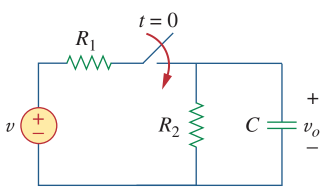


Figure 7.108
For Prob. 7.41.

7.42 (a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

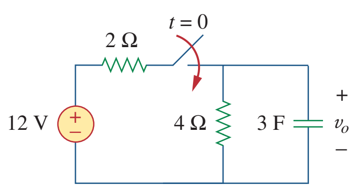


Figure 7.109
For Prob. 7.42.

7.43 Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.

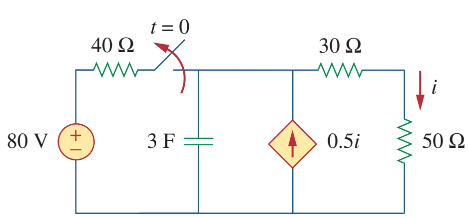


Figure 7.110
For Prob. 7.43.

7.44 The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$, it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

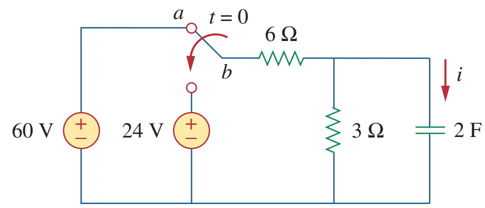


Figure 7.111
For Prob. 7.44.

7.45 Find v_o in the circuit of Fig. 7.112 when $v_s = 30u(t)$ V. Assume that $v_o(0) = 5$ V.

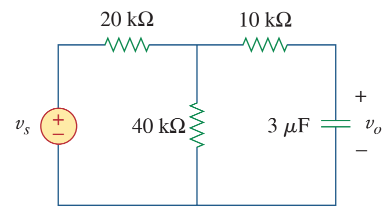


Figure 7.112
For Prob. 7.45.

7.46 For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$. Find $v(t)$.

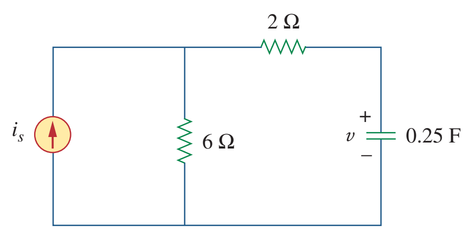


Figure 7.113
For Prob. 7.46.

7.47 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

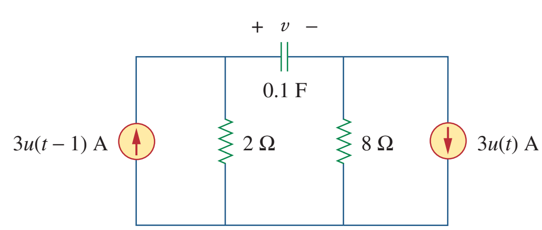


Figure 7.114
For Prob. 7.47.

7.48 Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

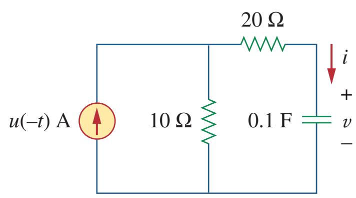


Figure 7.115
For Prob. 7.48.

7.49 If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.

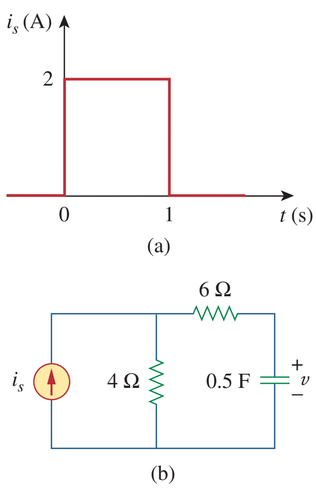


Figure 7.116
For Prob. 7.49 and Review Question 7.10.

***7.50** In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1$ kΩ, $R_3 = 2$ kΩ, and $C = 0.25$ mF.

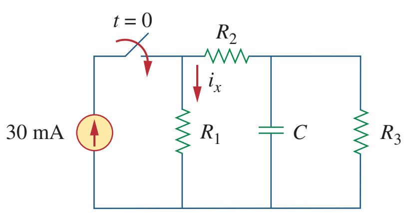


Figure 7.117
For Prob. 7.50.

Section 7.6 Step Response of an *RL* Circuit

7.51 Rather than applying the short-cut technique used in Section 7.6, use KVL to obtain Eq. (7.60).

7.52 Using Fig. 7.118, design a problem to help other students better understand the step response of an *RL* circuit.

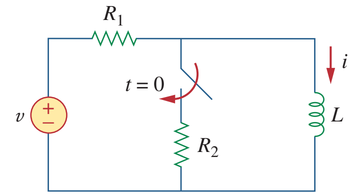


Figure 7.118
For Prob. 7.52.

7.53 Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.

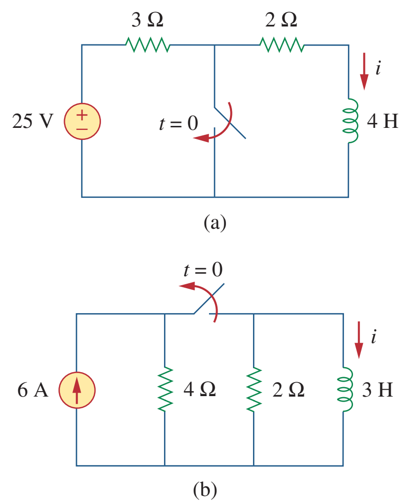


Figure 7.119
For Prob. 7.53.

7.54 Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.

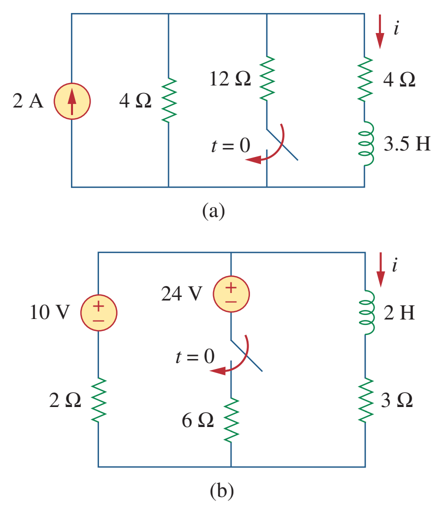


Figure 7.120
For Prob. 7.54.

* An asterisk indicates a challenging problem.

7.55 Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

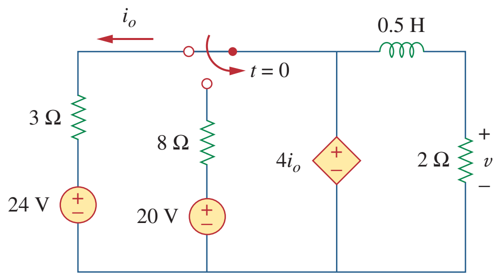


Figure 7.121
For Prob. 7.55.

7.56 For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

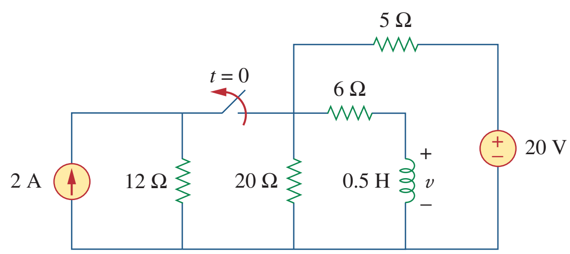


Figure 7.122
For Prob. 7.56.

***7.57** Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

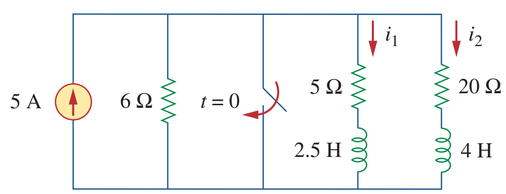


Figure 7.123
For Prob. 7.57.

7.58 Rework Prob. 7.17 if $i(0) = 10$ A and $v(t) = 20u(t)$ V.

7.59 Determine the step response $v_o(t)$ to $v_s = 18u(t)$ in the circuit of Fig. 7.124.

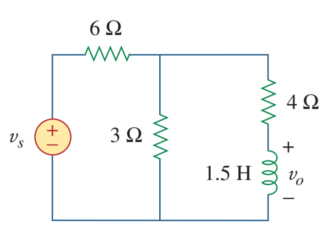


Figure 7.124
For Prob. 7.59.

7.60 Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.125 if the initial current in the inductor is zero.

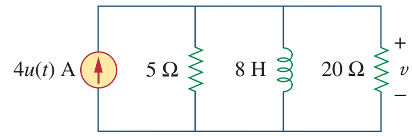


Figure 7.125
For Prob. 7.60.

7.61 In the circuit in Fig. 7.126, i_s changes from 5 A to 10 A at $t = 0$; that is, $i_s = 5u(-t) + 10u(t)$. Find v and i.

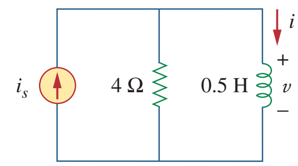


Figure 7.126
For Prob. 7.61.

7.62 For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

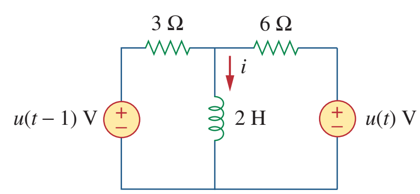


Figure 7.127
For Prob. 7.62.

7.63 Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

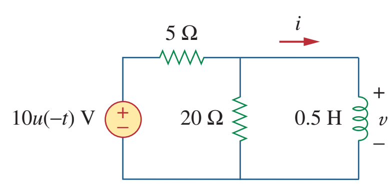


Figure 7.128
For Prob. 7.63.

7.64 Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from $t = 0$ *sec* to $t = \infty$ *sec*. The value of $v_{in}(t)$ is equal to $[40 - 40u(t)]$ volts.

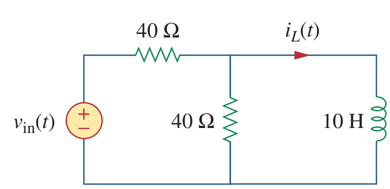
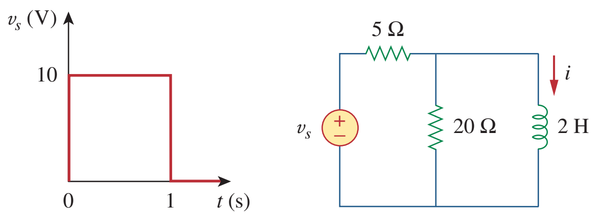


Figure 7.129
For Prob. 7.64.

7.65 If the input pulse in Fig. 7.130(a) is applied to the circuit in Fig. 7.130(b), determine the response $i(t)$.



(a) (b)

Figure 7.130
For Prob. 7.65.

Section 7.7 First-order Op Amp Circuits

7.66 Using Fig. 7.131, design a problem to help other students better understand first-order op amp circuits.

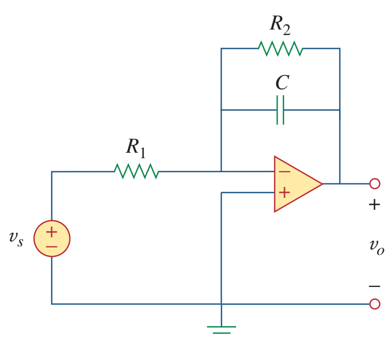


Figure 7.131
For Prob. 7.66.

7.67 If $v(0) = 5$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit in Fig. 7.132. Let $R = 10$ kΩ and $C = 1\ \mu$F.

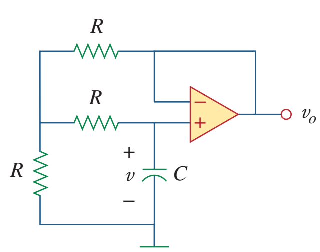


Figure 7.132
For Prob. 7.67.

7.68 Obtain v_o for $t > 0$ in the circuit of Fig. 7.133.

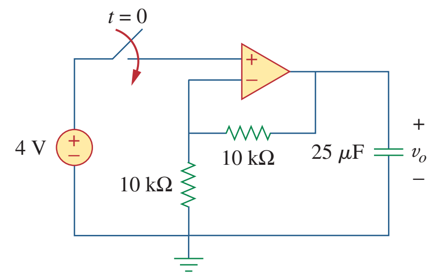


Figure 7.133
For Prob. 7.68.

7.69 For the op amp circuit in Fig. 7.134, find $v_o(t)$ for $t > 0$.

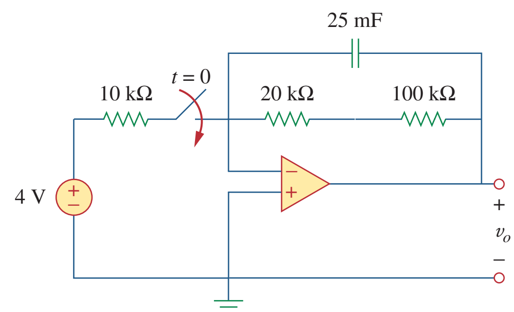


Figure 7.134
For Prob. 7.69.

7.70 Determine v_o for $t > 0$ when $v_s = 20$ mV in the op amp circuit of Fig. 7.135.

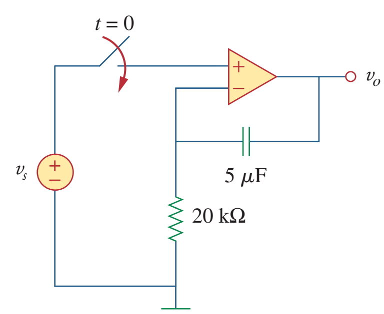


Figure 7.135
For Prob. 7.70.

7.71 For the op amp circuit in Fig. 7.136, suppose $v_0 = 0$ and $v_s = 3$ V. Find $v(t)$ for $t > 0$.

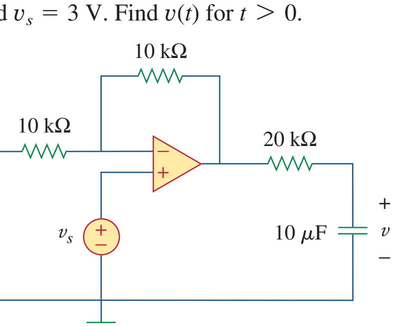


Figure 7.136
For Prob. 7.71.

7.72 Find i_o in the op amp circuit in Fig. 7.137. Assume that $v(0) = -2$ V, $R = 10$ kΩ, and $C = 10$ μF.

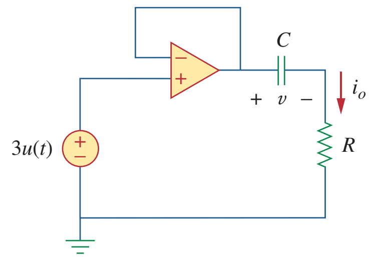


Figure 7.137
For Prob. 7.72.

7.73 For the op amp circuit of Fig. 7.138, let $R_1 = 10$ kΩ, $R_f = 20$ kΩ, $C = 20$ μF, and $v(0) = 1V$. Find v_0.

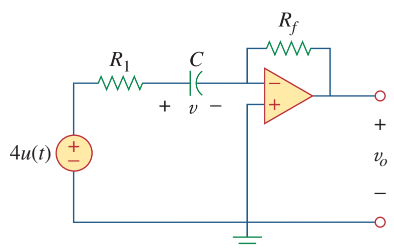


Figure 7.138
For Prob. 7.73.

7.74 Determine $v_o(t)$ for $t > 0$ in the circuit of Fig. 7.139. Let $i_s = 10u(t)$ μA and assume that the capacitor is initially uncharged.

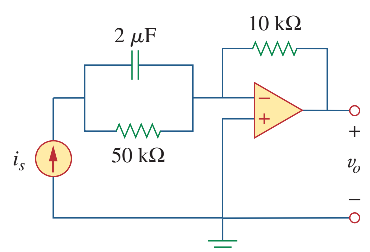


Figure 7.139
For Prob. 7.74.

7.75 In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.

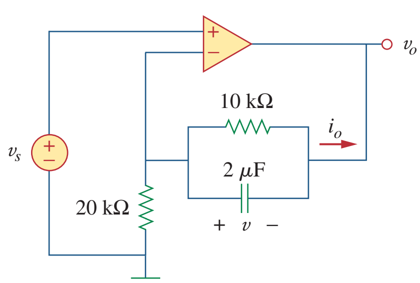


Figure 7.140
For Prob. 7.75.

Section 7.8 Transient Analysis with *PSpice*

7.76 Repeat Prob. 7.49 using *PSpice or MultiSim*.

7.77 The switch in Fig. 7.141 opens at $t = 0$. Use *PSpice or MultiSim* to determine $v(t)$ for $t > 0$.

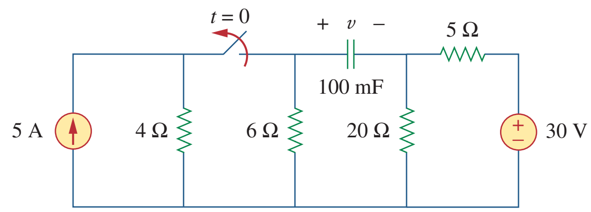


Figure 7.141
For Prob. 7.77.

7.78 The switch in Fig. 7.142 moves from position a to b at $t = 0$. Use *PSpice or MultiSim* to find $i(t)$ for $t > 0$.

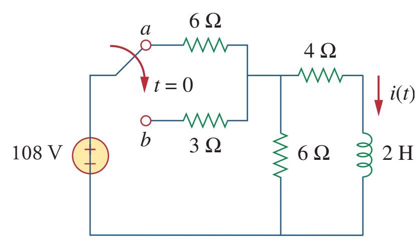


Figure 7.142
For Prob. 7.78.

7.79 In the circuit of Fig. 7.143, the switch has been in position a for a long time but moves instantaneously to position b at $t = 0$. Determine $i_o(t)$.

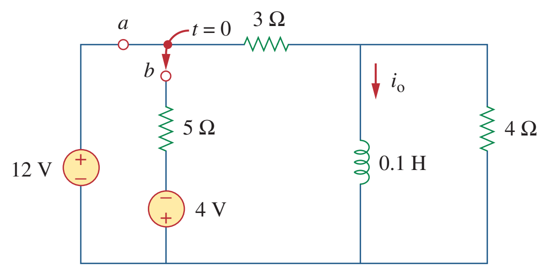


Figure 7.143
For Prob. 7.79.

7.80 In the circuit of Fig. 7.144, assume that the switch has been in position a for a long time, find:

(a) $i_1(0)$, i_2 (0), and $v_o(0)$

(b) $i_L(t)$

(c) $i_1(\infty)$, $i_2(\infty)$, and $v_o(\infty)$.

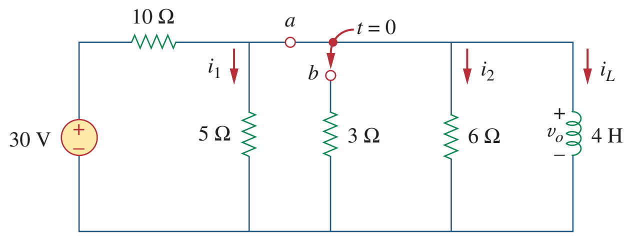


Figure 7.144
For Prob. 7.80.

7.81 Repeat Prob. 7.65 using *PSpice or MultiSim*.

Section 7.9 Applications

7.82 In designing a signal-switching circuit, it was found that a 100-μF capacitor was needed for a time constant of 3 ms. What value resistor is necessary for the circuit?

7.83 An *RC* circuit consists of a series connection of a 120-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

7.84 The resistance of a 160-mH coil is 8 Ω. Find the time required for the current to build up to 60 percent of its final value when voltage is applied to the coil.

7.85 A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is 120 Ω when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?

(b) What is the time interval between light flashes?

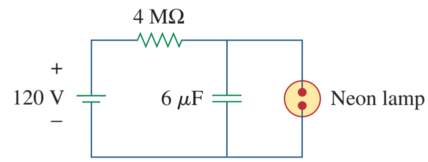


Figure 7.145
For Prob. 7.85.

7.86 Figure 7.146 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

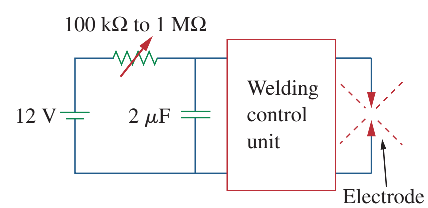


Figure 7.146
For Prob. 7.86.

7.87 A 120-V dc generator energizes a motor whose coil has an inductance of 50 H and a resistance of 100 Ω. A field discharge resistor of 400 Ω is connected in parallel with the motor to avoid damage to the motor, as shown in Fig. 7.147. The system is at steady state. Find the current through the discharge resistor 100 ms after the breaker is tripped.

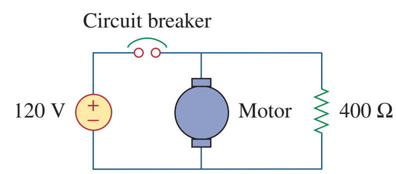


Figure 7.147
For Prob. 7.87.

Comprehensive Problems

7.88 The circuit in Fig. 7.148(a) can be designed as an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant $\tau = RC$ of the circuit and the width T of the input pulse in Fig. 7.148(b). The circuit is a differentiator if $\tau \ll T$, say $\tau < 0.1T$, or an integrator if $\tau \gg T$, say $\tau > 10T$.

(a) What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?

(b) If the output is to be an integrated form of the input, what is the maximum value the pulse width can assume?

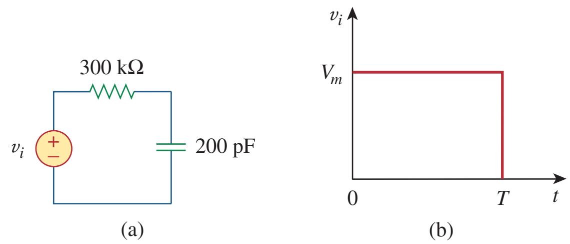


Figure 7.148
For Prob. 7.88.

7.89 An RL circuit may be used as a differentiator if the output is taken across the inductor and $\tau \ll T$ (say $\tau < 0.1T$), where T is the width of the input pulse. If R is fixed at 200 kΩ, determine the maximum value of L required to differentiate a pulse with $T = 10\ \mu$s.

7.90 An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s, while the probe has an internal resistance R_p. If R_p is fixed at 6 MΩ, find R_s and C_s for the circuit to have a time constant of 15 μs.

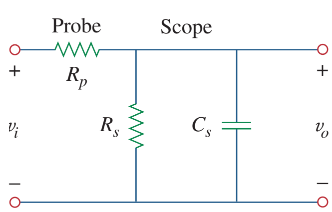


Figure 7.149
For Prob. 7.90.

7.91 The circuit in Fig. 7.150 is used by a biology student to study "frog kick." She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.

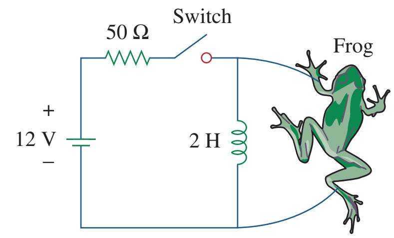


Figure 7.150
For Prob. 7.91.

7.92 To move a spot of a cathode-ray tube across the screen requires a linear increase in the voltage across the deflection plates, as shown in Fig. 7.151. Given that the capacitance of the plates is 4 nF, sketch the current flowing through the plates.

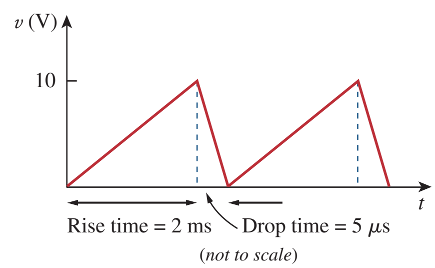


Figure 7.151
For Prob. 7.92.

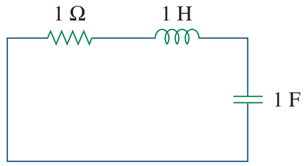


Figure 8.59
For Review Question 8.7.

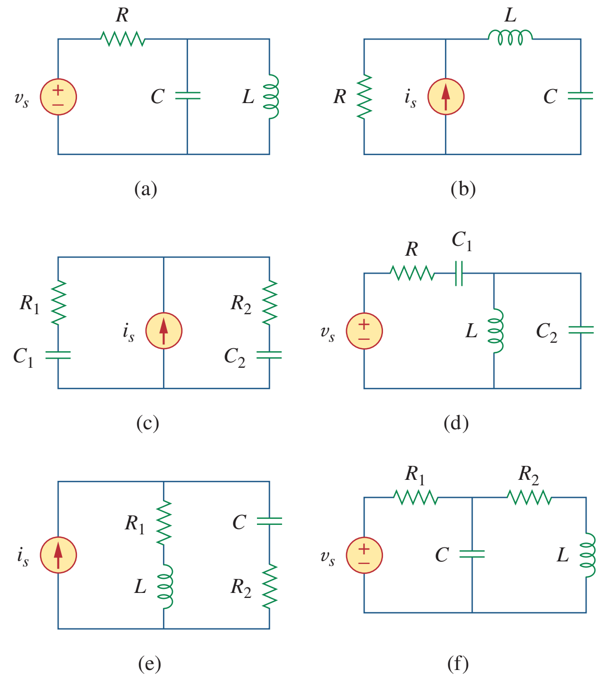


Figure 8.61
For Review Question 8.9.

8.8 Consider the parallel *RLC* circuit in Fig. 8.60. What type of response will it produce?

(a) overdamped
(b) underdamped
(c) critically damped
(d) none of the above

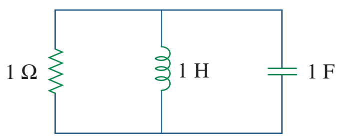


Figure 8.60
For Review Question 8.8.

8.9 Match the circuits in Fig. 8.61 with the following items:

(i) first-order circuit
(ii) second-order series circuit
(iii) second-order parallel circuit
(iv) none of the above

8.10 In an electric circuit, the dual of resistance is:

(a) conductance (b) inductance
(c) capacitance (d) open circuit
(e) short circuit

Answers: 8.1a, 8.2c, 8.3b, 8.4d, 8.5d, 8.6c, 8.7b, 8.8b, 8.9 (i)-c, (ii)-b, e, (iii)-a, (iv)-d, f, 8.10a.

Problems

Section 8.2 Finding Initial and Final Values

8.1 For the circuit in Fig. 8.62, find:

(a) $i(0^+)$ and $v(0^+)$,
(b) $di(0^+)/dt$ and $dv(0^+)/dt$,
(c) $i(\infty)$ and $v(\infty)$.

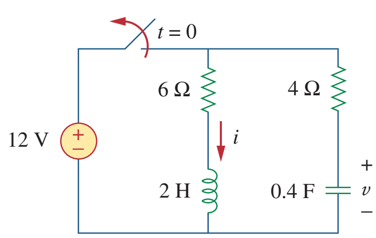


Figure 8.62
For Prob. 8.1.

8.2 e⊘d Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

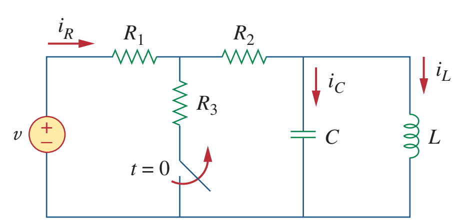


Figure 8.63
For Prob. 8.2.

8.3 Refer to the circuit shown in Fig. 8.64. Calculate:

(a) $i_L(0^+)$, $v_C(0^+)$, and $v_R(0^+)$,

(b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, and $dv_R(0^+)/dt$,

(c) $i_L(\infty)$, $v_C(\infty)$, and $v_R(\infty)$.

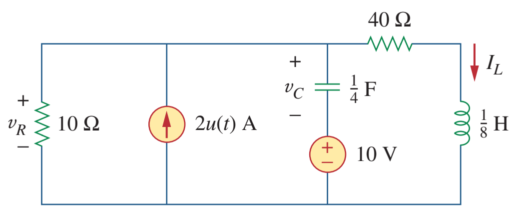


Figure 8.64
For Prob. 8.3.

8.4 In the circuit of Fig. 8.65, find:

(a) $v(0^+)$ and $i(0^+)$,

(b) $dv(0^+)/dt$ and $di(0^+)/dt$,

(c) $v(\infty)$ and $i(\infty)$.

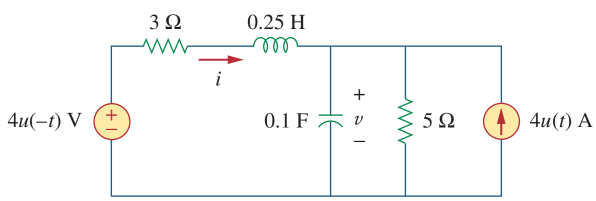


Figure 8.65
For Prob. 8.4.

8.5 Refer to the circuit in Fig. 8.66. Determine:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

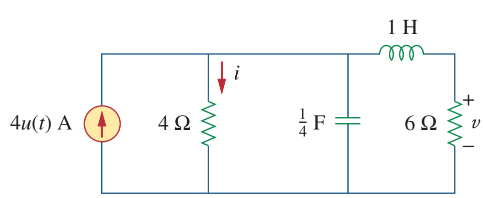


Figure 8.66
For Prob. 8.5.

8.6 In the circuit of Fig. 8.67, find:

(a) $v_R(0^+)$ and $v_L(0^+)$,

(b) $dv_R(0^+)/dt$ and $dv_L(0^+)/dt$,

(c) $v_R(\infty)$ and $v_L(\infty)$.

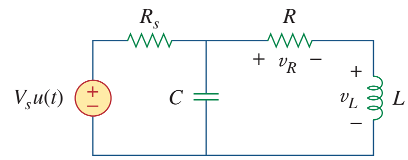


Figure 8.67
For Prob. 8.6.

Section 8.3 Source-Free Series *RLC* Circuit

8.7 A series *RLC* circuit has $R = 20$ kΩ, $L = 0.2$ mH, and $C = 5\ \mu$F. What type of damping is exhibited by the circuit?

8.8 Design a problem to help other students better understand source-free *RLC* circuits.

8.9 The current in an *RLC* circuit is described by

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0$$

If $i(0) = 10$ A and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.

8.10 The differential equation that describes the voltage in an *RLC* network is

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 10$ V/s, obtain $v(t)$.

8.11 The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 0$$

for which the initial conditions are $v(0) = 10$ V and $dv(0)/dt = 0$. Solve for $v(t)$.

8.12 If $R = 50\ \Omega$, $L = 1.5$ H, what value of C will make an *RLC* series circuit:

(a) overdamped,

(b) critically damped,

(c) underdamped?

8.13 For the circuit in Fig. 8.68, calculate the value of R needed to have a critically damped response.

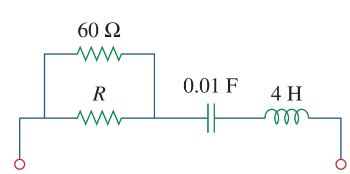


Figure 8.68
For Prob. 8.13.

8.14 The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.

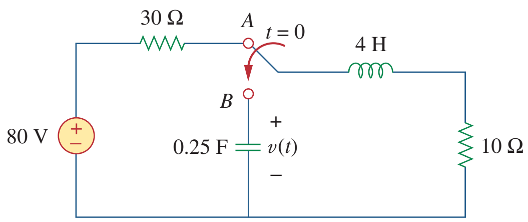


Figure 8.69
For Prob. 8.14.

8.15 The responses of a series RLC circuit are

$$v_C(t) = 30 - 10e^{-20t} + 30e^{-10t} \text{ V}$$
$$i_L(t) = 40e^{-20t} - 60e^{-10t} \text{ mA}$$

where v_C and i_L are the capacitor voltage and inductor current, respectively. Determine the values of R, L, and C.

8.16 Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.70.

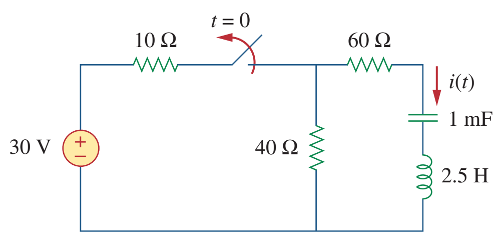


Figure 8.70
For Prob. 8.16.

8.17 In the circuit of Fig. 8.71, the switch instantaneously moves from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.

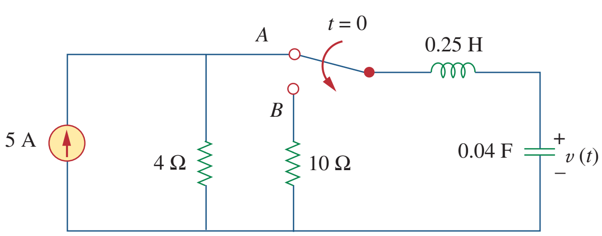


Figure 8.71
For Prob. 8.17.

8.18 Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 8.72. Assume steady-state conditions exist at $t = 0^-$.

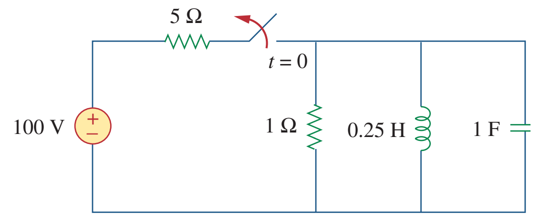


Figure 8.72
For Prob. 8.18.

8.19 Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 8.73.

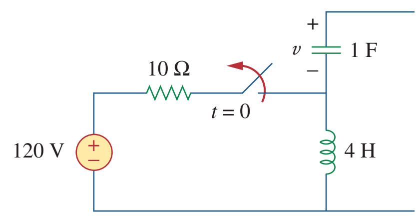


Figure 8.73
For Prob. 8.19.

8.20 The switch in the circuit of Fig. 8.74 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

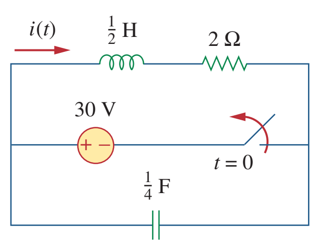


Figure 8.74
For Prob. 8.20.

***8.21** Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.

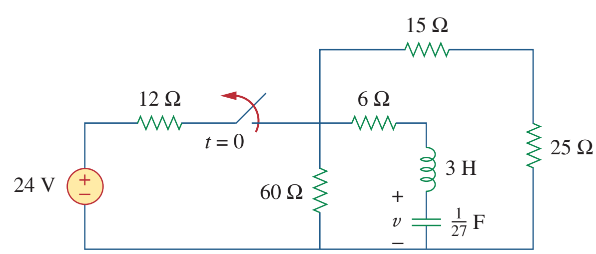


Figure 8.75
For Prob. 8.21.

* An asterisk indicates a challenging problem.

Section 8.4 Source-Free Parallel *RLC* Circuit

8.22 Assuming $R = 2\ \text{k}\Omega$, design a parallel *RLC* circuit that has the characteristic equation

$$s^2 + 100s + 10^6 = 0.$$

8.23 For the network in Fig. 8.76, what value of C is needed to make the response underdamped with unity damping factor ($\alpha = 1$)?

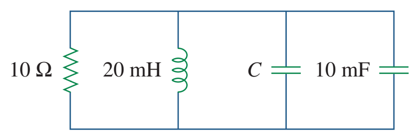


Figure 8.76
For Prob. 8.23.

8.24 The switch in Fig. 8.77 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Determine $i(t)$ for $t > 0$.

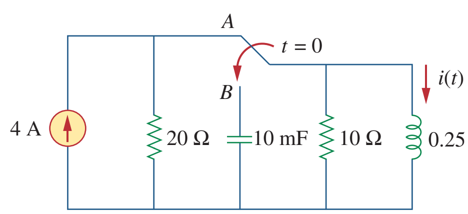


Figure 8.77
For Prob. 8.24.

8.25 Using Fig. 8.78, design a problem to help other students better understand source-free *RLC* circuits.

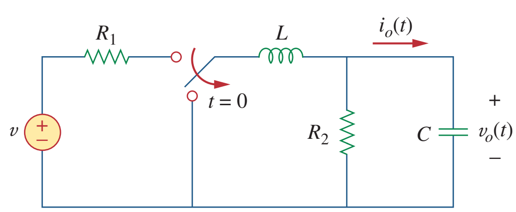


Figure 8.78
For Prob. 8.25.

Section 8.5 Step Response of a Series *RLC* Circuit

8.26 The step response of an *RLC* circuit is given by

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 2$ and $di(0)/dt = 4$, solve for $i(t)$.

8.27 A branch voltage in an *RLC* circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

8.28 A series *RLC* circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5$ H, $R = 4\ \Omega$, and $C = 0.2$ F. Let $i(0) = 1$, $di(0)/dt = 0$.

8.29 Solve the following differential equations subject to the specified initial conditions

(a) $d^2v/dt^2 + 4v = 12$, $v(0) = 0$, $dv(0)/dt = 2$

(b) $d^2i/dt^2 + 5\,di/dt + 4i = 8$, $i(0) = -1$, $di(0)/dt = 0$

(c) $d^2v/dt^2 + 2\,dv/dt + v = 3$, $v(0) = 5$, $dv(0)/dt = 1$

(d) $d^2i/dt^2 + 2\,di/dt + 5i = 10$, $i(0) = 4$, $di(0)/dt = -2$

8.30 The step responses of a series *RLC* circuit are

$$v_C = 40 - 10e^{-2000t} - 10e^{-4000t}\ \text{V}, \qquad t > 0$$
$$i_L(t) = 3e^{-2000t} + 6e^{-4000t}\ \text{mA}, \qquad t > 0$$

(a) Find C. (b) Determine what type of damping is exhibited by the circuit.

8.31 Consider the circuit in Fig. 8.79. Find $v_L(0^+)$ and $v_C(0^+)$.

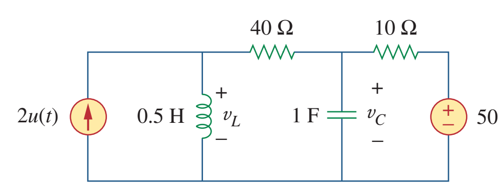


Figure 8.79
For Prob. 8.31.

8.32 For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

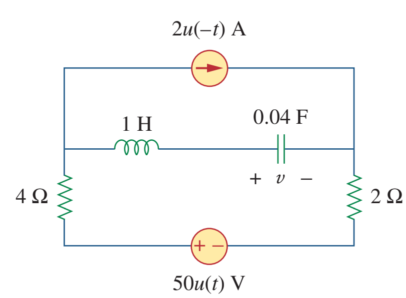


Figure 8.80
For Prob. 8.32.

8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

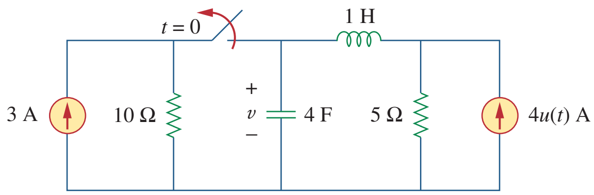


Figure 8.81
For Prob. 8.33.

8.34 Calculate $i(t)$ for $t > 0$ in the circuit of Fig. 8.82.

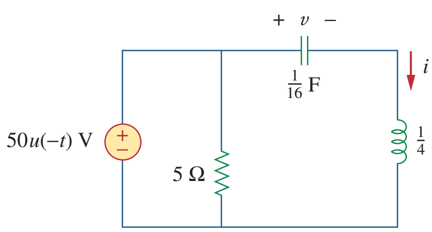


Figure 8.82
For Prob. 8.34.

8.35 Using Fig. 8.83, design a problem to help other students better understand the step response of series *RLC* circuits.

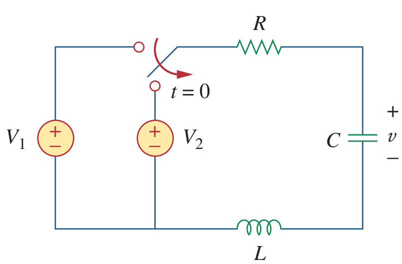


Figure 8.83
For Prob. 8.35.

8.36 Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 8.84.

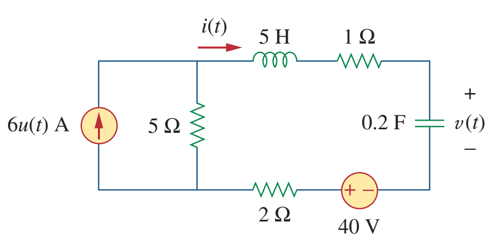


Figure 8.84
For Prob. 8.36.

***8.37** For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

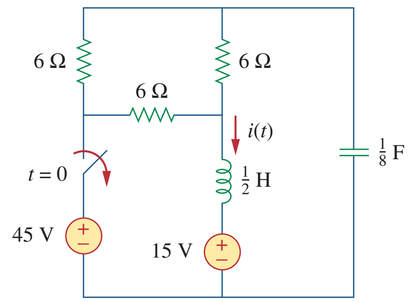


Figure 8.85
For Prob. 8.37.

8.38 Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$.

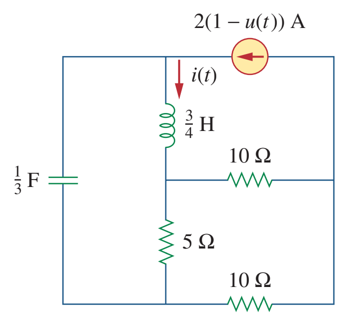


Figure 8.86
For Prob. 8.38.

8.39 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 8.87.

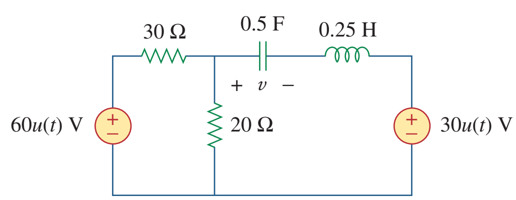


Figure 8.87
For Prob. 8.39.

8.40 The switch in the circuit of Fig. 8.88 is moved from position *a* to *b* at $t = 0$. Determine $i(t)$ for $t > 0$.

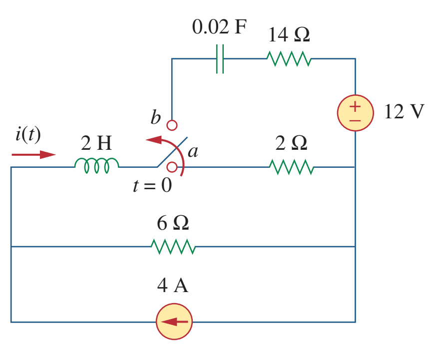


Figure 8.88
For Prob. 8.40.

***8.41** For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

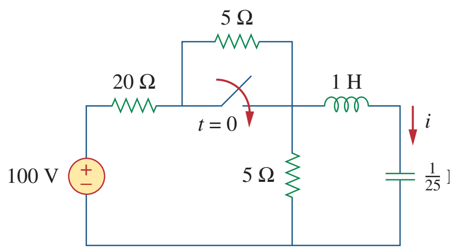


Figure 8.89
For Prob. 8.41.

***8.42** Given the network in Fig. 8.90, find $v(t)$ for $t > 0$.

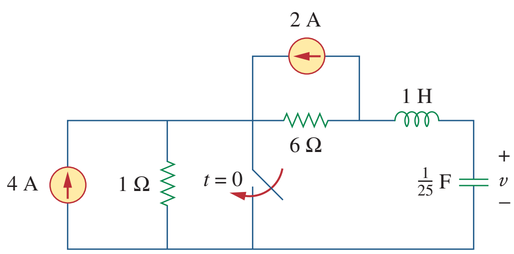


Figure 8.90
For Prob. 8.42.

8.43 The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

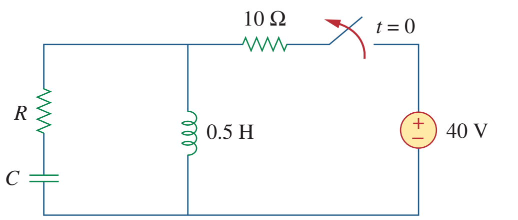


Figure 8.91
For Prob. 8.43.

8.44 A series *RLC* circuit has the following parameters: $R = 1$ kΩ, $L = 1$ H, and $C = 10$ nF. What type of damping does this circuit exhibit?

Section 8.6 Step Response of a Parallel *RLC* Circuit

8.45 In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0$ V and $i(0) = 1$ A.

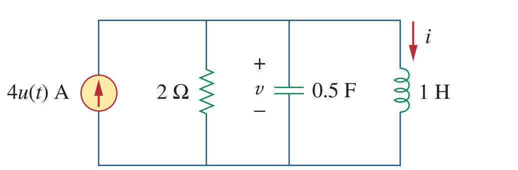


Figure 8.92
For Prob. 8.45.

8.46 Using Fig. 8.93, design a problem to help other students better understand the step response of a parallel *RLC* circuit.

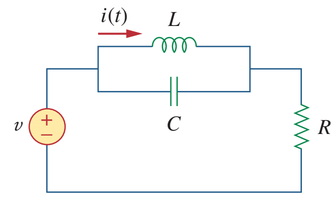


Figure 8.93
For Prob. 8.46.

8.47 Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

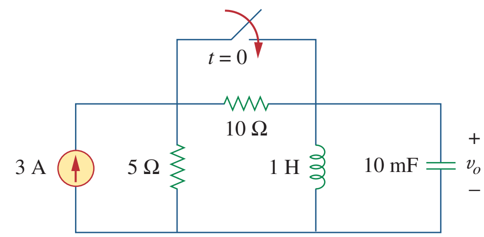


Figure 8.94
For Prob. 8.47.

8.48 Given the circuit in Fig. 8.95, find $i(t)$ and $v(t)$ for $t > 0$.

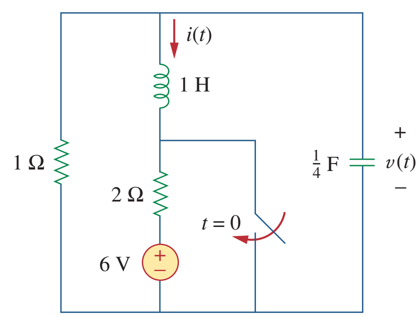


Figure 8.95
For Prob. 8.48.

8.49 Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

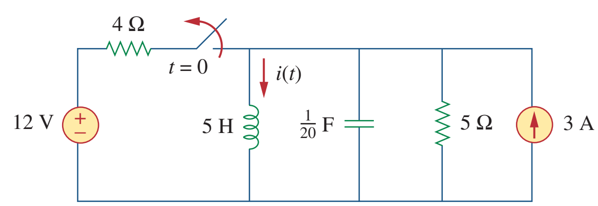


Figure 8.96
For Prob. 8.49.

8.50 For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

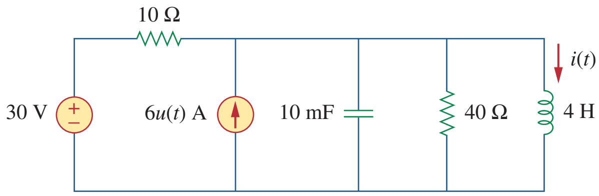


Figure 8.97
For Prob. 8.50.

8.51 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.98.

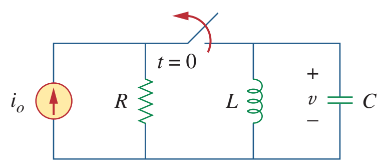


Figure 8.98
For Prob. 8.51.

8.52 The step response of a parallel *RLC* circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2 \sin 400t) \text{ V}, \qquad t \geq 0$$

when the inductor is 50 mH. Find R and C.

Section 8.7 General Second-Order Circuits

8.53 After being open for a day, the switch in the circuit of Fig. 8.99 is closed at $t = 0$. Find the differential equation describing $i(t)$, $t > 0$.

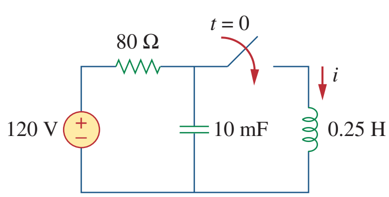


Figure 8.99
For Prob. 8.53.

8.54 Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

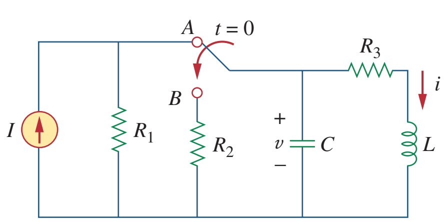


Figure 8.100
For Prob. 8.54.

8.55 For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.

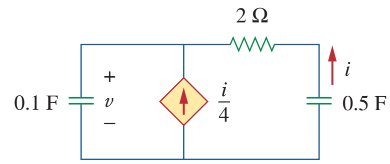


Figure 8.101
For Prob. 8.55.

8.56 In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

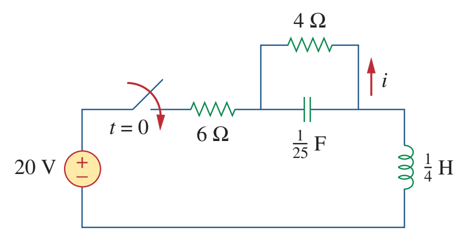


Figure 8.102
For Prob. 8.56.

8.57 If the switch in Fig. 8.103 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine:

(a) the characteristic equation of the circuit,

(b) i_x and v_R for $t > 0$.

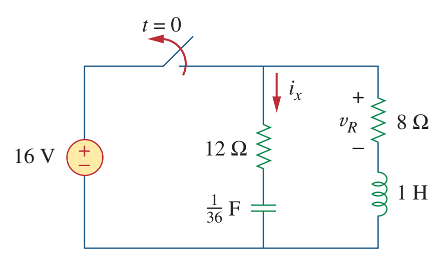


Figure 8.103
For Prob. 8.57.

8.58 In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$,

(b) $v(t)$ for $t \geq 0$.

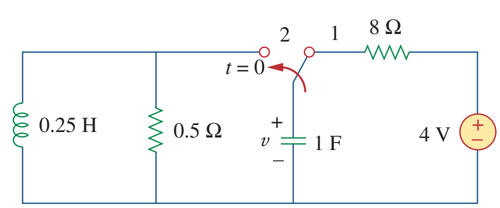


Figure 8.104
For Prob. 8.58.

8.59 The switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at $t = 0$. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

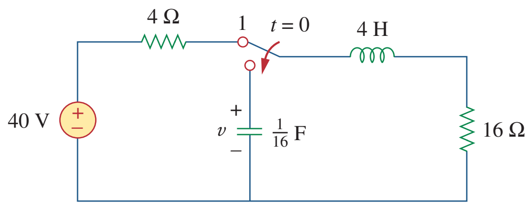


Figure 8.105
For Prob. 8.59.

8.60 Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

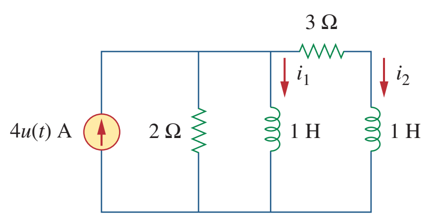


Figure 8.106
For Prob. 8.60.

8.61 For the circuit in Prob. 8.5, find i and v for $t > 0$.

8.62 Find the response $v_R(t)$ for $t > 0$ in the circuit of Fig. 8.107. Let $R = 3\ \Omega$, $L = 2$ H, and $C = 1/18$ F.

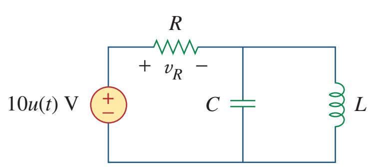


Figure 8.107
For Prob. 8.62.

Section 8.8 Second-Order Op Amp Circuits

8.63 For the op amp circuit in Fig. 8.108, find the differential equation for $i(t)$.

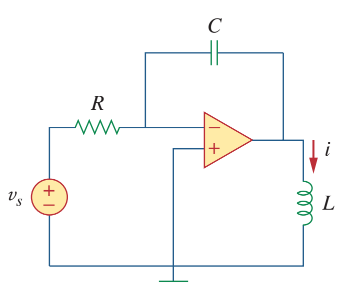


Figure 8.108
For Prob. 8.63.

8.64 Using Fig. 8.109, design a problem to help other students better understand second-order op amp circuits.

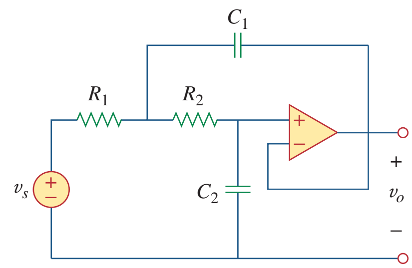


Figure 8.109
For Prob. 8.64.

8.65 Determine the differential equation for the op amp circuit in Fig. 8.110. If $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V, find v_o for $t > 0$. Let $R = 100$ kΩ and $C = 1\ \mu$F.

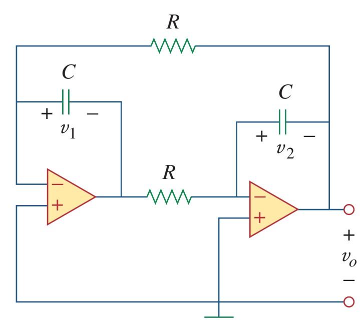


Figure 8.110
For Prob. 8.65.

8.66 Obtain the differential equations for $v_o(t)$ in the op amp circuit of Fig. 8.111.

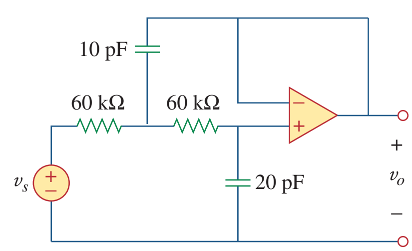


Figure 8.111
For Prob. 8.66.

***8.67** In the op amp circuit of Fig. 8.112, determine $v_o(t)$ for $t > 0$. Let $v_{in} = u(t)$ V, $R_1 = R_2 = 10$ kΩ, $C_1 = C_2 = 100$ μF.

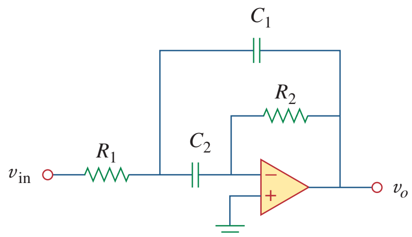


Figure 8.112
For Prob. 8.67.

Section 8.9 *PSpice* Analysis of *RLC* Circuit

8.68 For the step function $v_s = u(t)$, use *PSpice* or *MultiSim* to find the response $v(t)$ for $0 < t < 6$ s in the circuit of Fig. 8.113.

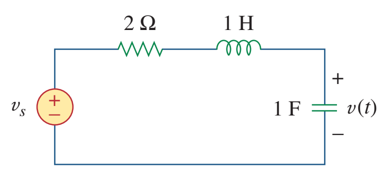


Figure 8.113
For Prob. 8.68.

8.69 Given the source-free circuit in Fig. 8.114, use *PSpice* or *MultiSim* to get $i(t)$ for $0 < t < 20$ s. Take $v(0) = 30$ V and $i(0) = 2$ A.

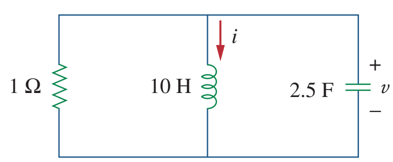


Figure 8.114
For Prob. 8.69.

8.70 For the circuit in Fig. 8.115, use *PSpice* or *MultiSim* to obtain $v(t)$ for $0 < t < 4$ s. Assume that the capacitor voltage and inductor current at $t = 0$ are both zero.

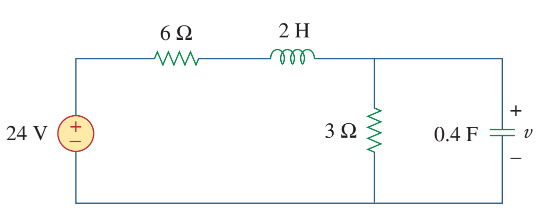


Figure 8.115
For Prob. 8.70.

8.71 Obtain $v(t)$ for $0 < t < 4$ s in the circuit of Fig. 8.116 using *PSpice* or *MultiSim*.

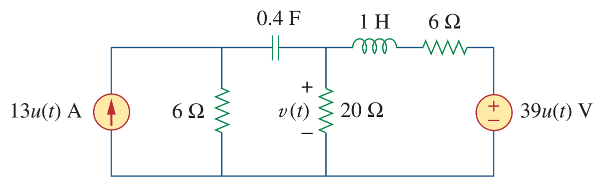


Figure 8.116
For Prob. 8.71.

8.72 The switch in Fig. 8.117 has been in position 1 for a long time. At $t = 0$, it is switched to position 2. Use *PSpice* or *MultiSim* to find $i(t)$ for $0 < t < 0.2$ s.

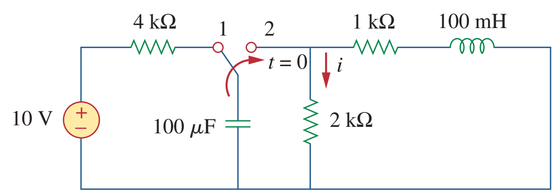


Figure 8.117
For Prob. 8.72.

8.73 Design a problem, to be solved using *PSpice* or *MultiSim*, to help other students better understand source-free *RLC* circuits.

Section 8.10 Duality

8.74 Draw the dual of the circuit shown in Fig. 8.118.

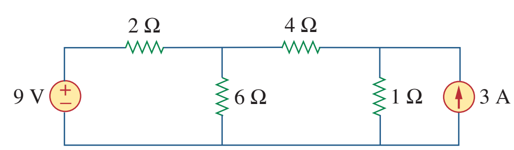


Figure 8.118
For Prob. 8.74.

8.75 Obtain the dual of the circuit in Fig. 8.119.

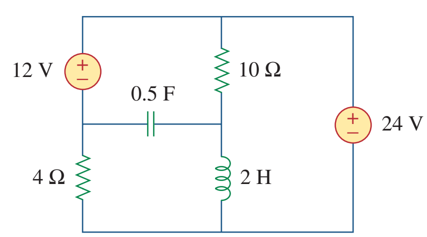


Figure 8.119
For Prob. 8.75.

8.76 Find the dual of the circuit in Fig. 8.120.

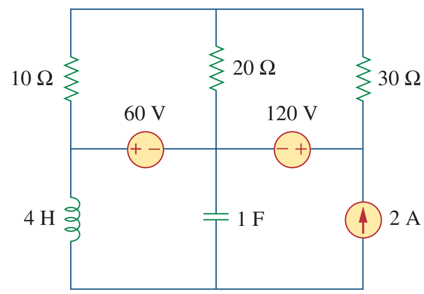


Figure 8.120
For Prob. 8.76.

8.77 Draw the dual of the circuit in Fig. 8.121.

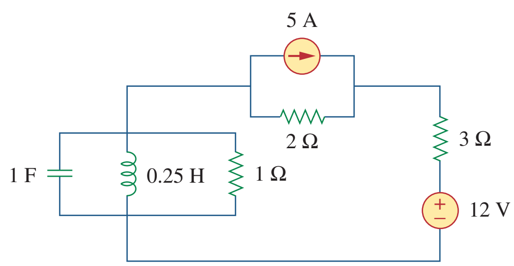


Figure 8.121
For Prob. 8.77.

Section 8.11 Applications

8.78 An automobile airbag igniter is modeled by the circuit in Fig. 8.122. Determine the time it takes the voltage across the igniter to reach its first peak after switching from A to B. Let $R = 3\ \Omega$, $C = 1/30$ F, and $L = 60$ mH.

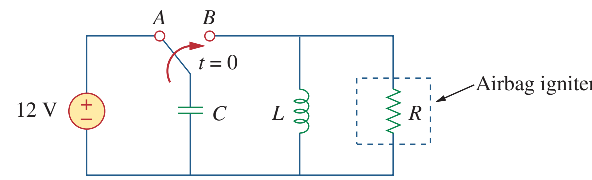


Figure 8.122
For Prob. 8.78.

8.79 A load is modeled as a 250-mH inductor in parallel with a 12-Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.

Comprehensive Problems

8.80 A mechanical system is modeled by a series RLC circuit. It is desired to produce an overdamped response with time constants 0.1 ms and 0.5 ms. If a series 50-kΩ resistor is used, find the values of L and C.

8.81 An oscillogram can be adequately modeled by a second-order system in the form of a parallel RLC circuit. It is desired to give an underdamped voltage across a 200-Ω resistor. If the damping frequency is 4 kHz and the time constant of the envelope is 0.25 s, find the necessary values of L and C.

8.82 The circuit in Fig. 8.123 is the electrical analog of body functions used in medical schools to study convulsions. The analog is as follows:

C_1 = Volume of fluid in a drug
C_2 = Volume of blood stream in a specified region
R_1 = Resistance in the passage of the drug from the input to the blood stream
R_2 = Resistance of the excretion mechanism, such as kidney, etc.
v_0 = Initial concentration of the drug dosage
$v(t)$ = Percentage of the drug in the blood stream

Find $v(t)$ for $t > 0$ given that $C_1 = 0.5\ \mu$F, $C_2 = 5\ \mu$F, $R_1 = 5$ MΩ, $R_2 = 2.5$ MΩ, and $v_0 = 60u(t)$ V.

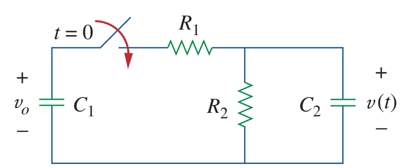


Figure 8.123
For Prob. 8.82.

8.83 Figure 8.124 shows a typical tunnel-diode oscillator circuit. The diode is modeled as a nonlinear resistor with $i_D = f(v_D)$, i.e., the diode current is a nonlinear function of the voltage across the diode. Derive the differential equation for the circuit in terms of v and i_D.

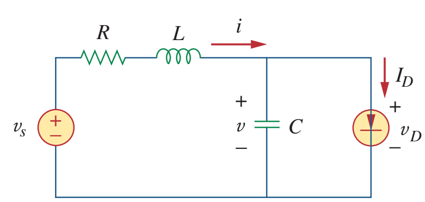


Figure 8.124
For Prob. 8.83.