ISE 2211 • Exam II • 18 September 2020 NAME SOLUTION

1. *The Anglers* is an exclusive recreational club of world-class fishing experts based in Pitchin, Ohio with a vast and loyal following on YouTube. Historical data has shown that they average four catches per hour on a fishing trip. If the continuous random variable *X* represents the time between catches and a may be modelled as a continuous Poisson process, determine the probability of *zero* catches in a three-hour fishing expedition. Huge hint: zero catches in three hours is the same thing as saying that the time between catches *exceeds* three hours.

Formulae:

$f(x) = \lambda e^{-\lambda x}$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mu = \frac{1}{\lambda}$

$\sigma^2 = \frac{1}{\lambda^2}$

$\because$ λ in form events per interval

$\therefore$ $\lambda = 4$ if interval = hour (+2)

$\because$ time between catches $>$ 3 hours

$\longrightarrow P(X > 3)$ (+1)

$= 1 - P(X < 3)$ (+1)

$= 1 - F(3)$ (+1)

$= 1 - \left[1 - e^{-4 \cdot 3}\right] = 6.144 \times 10^{-6}$ (+1)

or 0.0006144 %

+6

2. Joe Tritschler's deranged three-year-old son Chuck rarely naps anymore. He's apparently afraid of missing out on something important. Current data suggests that the probability of napping on any given day is 14.2%. Let the binomially-distributed random variable X represent the number of naps.
What is the probability that Chuck will nap every single day this week?

$\frac{7!}{7!(7-7)!} = 1$

$n = 7$ (days in week)
$x = 7$ (every day)
$p = 0.142$

$\therefore f(7) = \binom{7}{7} 0.142^7 (1-0.142)^{7-7} = 1.164 \times 10^{-6}$ or 0.0001164% +2

How about the probability that he will nap *zero* times this week?

$f(0) = \binom{7}{0} 0.142^0 (1-0.142)^{7-0}$

$= 0.3423$ or 34.23% +2

$\frac{7!}{0!(7-0)!} = 1$

How about at least twice?

$P(x \geq 2) = 1 - P(x < 2) = 1 - [P(0) + P(1)]$ +2

$f(1) = \binom{7}{1} 0.142^1 (1-0.142)^{7-1} = 0.3966$ or 39.66% +2

$\frac{7!}{1!(7-1)!} = 7$

$\therefore P(x \geq 2) = 1 - [0.3423 + 0.3966]$
$= 0.2611$ or 26.11% +1

Compute the expected value of the number of naps Chuck will take this week.

$\mu = np$
$= 7 \cdot 0.142$
$= 0.994$ naps +1

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

+10

3. Joe Tritschler was hanging 5/8" drywall one day and, whilst supporting a 12' sheet with one hand, reached into his pocket and TO HIS HORROR pulled out a 1-1/4" screw instead of the 1-5/8" he expected. Shouting obscenities, he careened it across the room and reached into his pocket again...and pulled out ANOTHER 1-1/4" screw. WHAT!? Shouting even worse language, he threw that one away and reached for a third screw...yep, 1-1/4". Calmly setting the sheet of drywall down, he knew he must have put the wrong screws in his pocket. Emptying his pocket, he was befuddled to find thirty-seven 1-5/8" screws. Not a single 1-1/4" to be found. What was the probability of this actual event happening? Hint: does this problem constitute *sampling with replacement*, or *sampling without replacement*? → throwing across room ≠ replacement

37 1-5/8" screws

3 1-1/4" screws

$n = 40$ +1

Probability of three fails in a row without replacement:

three 1-1/4" screws, two left, 1 left

$$\frac{3}{40} \cdot \frac{2}{39} \cdot \frac{1}{38}$$

total, total left

+3

$= 1.012 \times 10^{-4}$

or 0.01012 %

+1

+5