Module 04: **Impedance Combination Equivalents & Forms** **Notes**

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Definition: *Resistors in Series:*

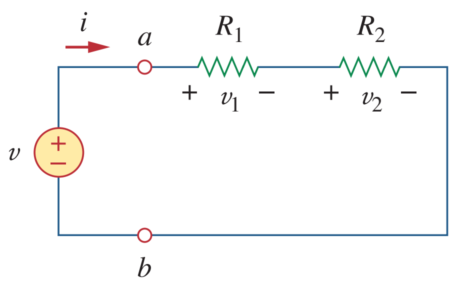


Figure 2.29
A single-loop circuit with two resistors in series.

Notice that KVL implies: $v = v_1 + v_2$ or $v = i(R_1 + R_2)$ so that $i = \frac{v}{(R_1+R_2)}$. From this result, we can find v_1 as a *voltage division*

$$v_1 = iR_1 = \frac{vR_1}{(R_1 + R_2)}$$

Also notice that the *electrically-equivalent* element is

$$R_{eq} = R_1 + R_2$$

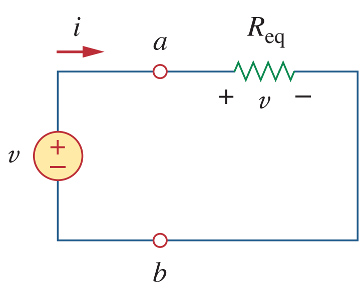


Figure 2.30
Equivalent circuit of the Fig. 2.29 circuit.

Definition: *Resistors in Parallel:*

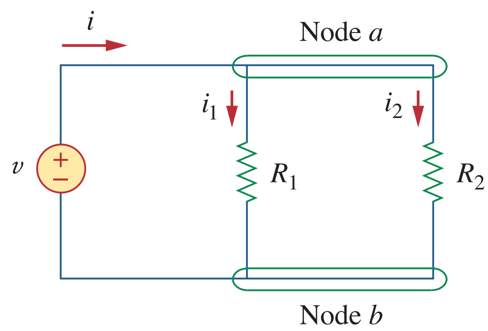


Figure 2.31
Two resistors in parallel.

Ohm's law and KCL implies: $i_1 = \frac{v}{R_1}$, $i_2 = \frac{v}{R_2}$, and $i = i_1 + i_2$. The total current can be rewritten as

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\frac{R_1 + R_2}{R_1 R_2} = \frac{v}{R_{eq}}$$

where the *electrically-equivalent* element is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

or more generally:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Now, since $v = iR_{eq} = i\frac{R_1 R_2}{R_1 + R_2}$ and $i_1 = \frac{v}{R_1}$, we can find i_1 as a *current division*,

$$i_1 = i\frac{R_2}{R_1 + R_2}$$

Impedance Element Combinations

We will soon include the consideration of *dynamic* elements, i.e., capacitors and inductors. These are modeled as more general *impedances*. In particular, for resistors with value R, capacitors with value C, and inductors with value L, these are given, respectively, as

$$Z_R = R, \qquad Z_C = \frac{1}{C \cdot s}, \qquad Z_L = L \cdot s$$

where s is the *Laplace Transform* variable. Nonetheless, the impedance combination rules reflect those for resistors as:

Impedances in Series:

$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots$$

Impedances in Parallel:

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots$$

Examples:

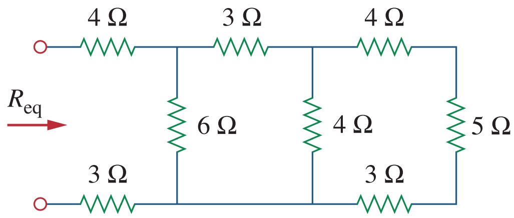


Figure 2.36
For Practice Prob. 2.9.

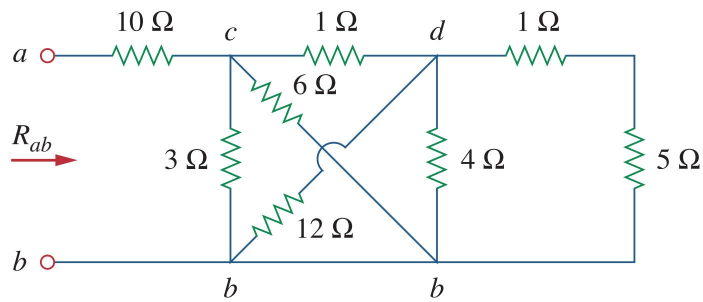


Figure 2.37
For Example 2.10.

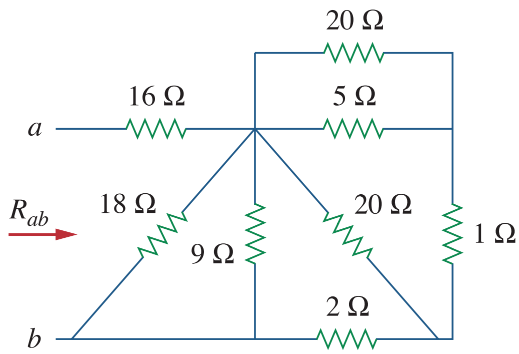
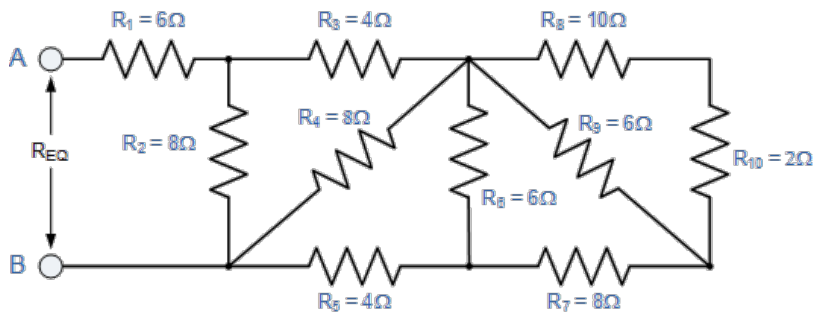


Figure 2.39
For Practice Prob. 2.10.

Numerical computation packages can be useful in quickly solving otherwise tedious tasks. For example:




```
>> SeriesParallelExample1
Req =
    10
>>
```

where the script, `SeriesParallelExample1.m` is given by

```
%% Example in notes
% For the circuit shown, we can move from right to left and simply dictate
% the resistor connections with + representing series and the function
% parallelos() (which is Latin for parallel) representing parallel.
% Note: I could not use the function name "parallel" since Matlab already
% has a function of this name, but not for circuit analysis.

Req = 6+parallelos(8,4+parallelos(8,4+parallelos(6,8+parallelos(6,10+2))))

% Here's the function definition:
function requivalent = parallelos(varargin)
   nargs = length(varargin);
   admittance = 0;
   for k = 1:nargs % add the admittances (the 1/R's)
      admittance = admittance + 1/varargin{k};
   end
   requivalent = 1/admittance; % do the inverse
end

% results in: Req = 10
```

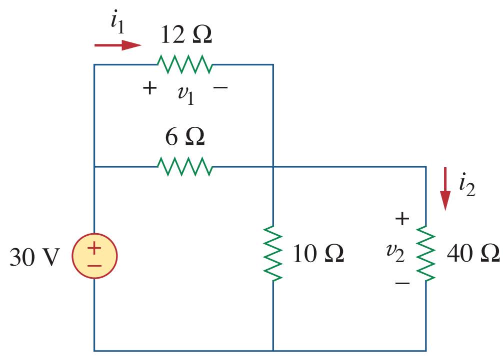


Figure 2.43
For Practice Prob. 2.12.

Homework 04: Chapter 2 # 23, 26, 27, 30, 31, 34, 35, 37, 38, 39, 42, 43

2.23 In the circuit shown in Fig. 2.87, determine v_x and the power absorbed by the 12-Ω resistor.

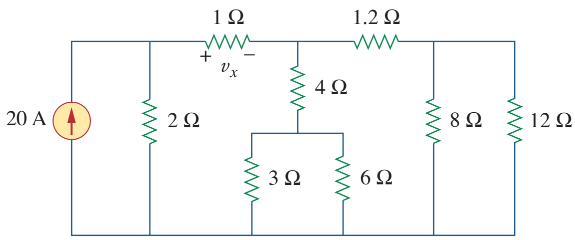


Figure 2.87
For Prob. 2.23.

2.24 For the circuit in Fig. 2.88, find V_o/V_s in terms of α, R_1, R_2, R_3, and R_4. If $R_1 = R_2 = R_3 = R_4$, what value of α will produce $|V_o/V_s| = 10$?

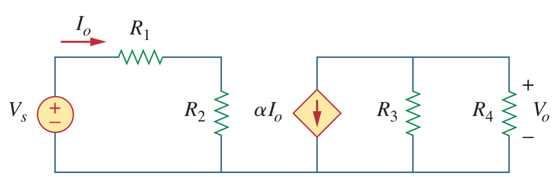


Figure 2.88
For Prob. 2.24.

2.25 For the network in Fig. 2.89, find the current, voltage, and power associated with the 20-kΩ resistor.

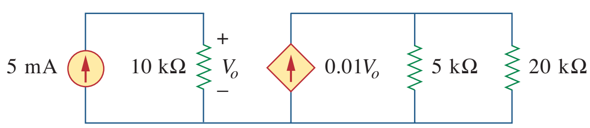


Figure 2.89
For Prob. 2.25.

Sections 2.5 and 2.6 Series and Parallel Resistors

2.26 For the circuit in Fig. 2.90, $i_o = 3$ A. Calculate i_x and the total power absorbed by the entire circuit.

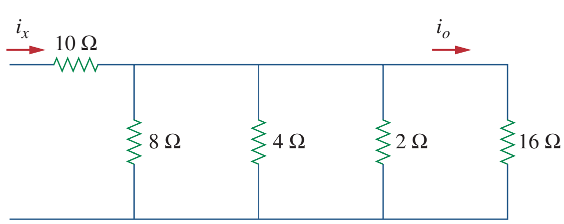


Figure 2.90
For Prob. 2.26.

2.27 Calculate I_o in the circuit of Fig. 2.91.

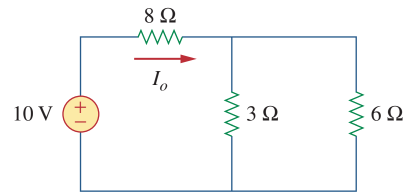


Figure 2.91
For Prob. 2.27.

2.28 Design a problem, using Fig. 2.92, to help other students better understand series and parallel circuits.

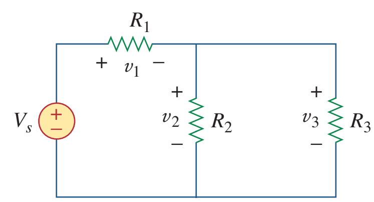


Figure 2.92
For Prob. 2.28.

2.29 All resistors in Fig. 2.93 are 5 Ω each. Find R_{eq}.

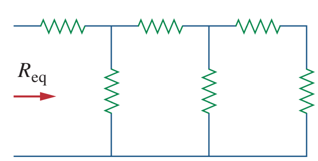


Figure 2.93
For Prob. 2.29.

2.30 Find R_{eq} for the circuit in Fig. 2.94.

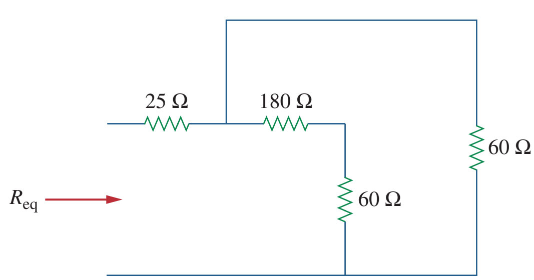


Figure 2.94
For Prob. 2.30.

2.31 For the circuit in Fig. 2.95, determine i_1 to i_5.

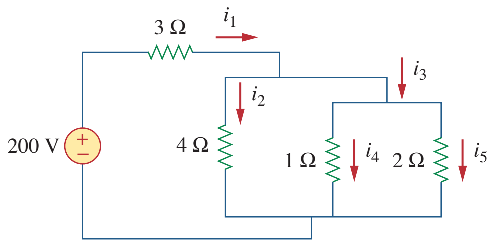


Figure 2.95
For Prob. 2.31.

2.32 Find i_1 through i_4 in the circuit in Fig. 2.96.

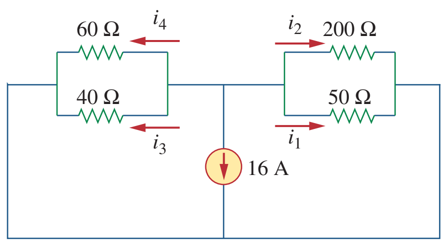


Figure 2.96
For Prob. 2.32.

2.33 Obtain v and i in the circuit of Fig. 2.97.

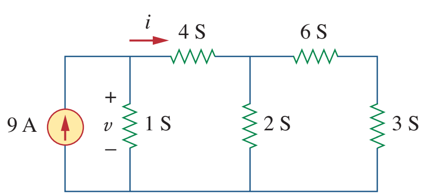


Figure 2.97
For Prob. 2.33.

2.34 Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall absorbed power by the resistor network.

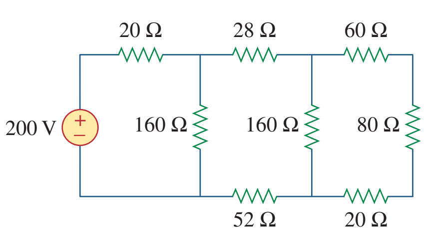


Figure 2.98
For Prob. 2.34.

2.35 Calculate V_o and I_o in the circuit of Fig. 2.99.

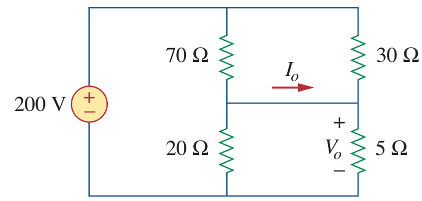


Figure 2.99
For Prob. 2.35.

2.36 Find i and V_o in the circuit of Fig. 2.100.

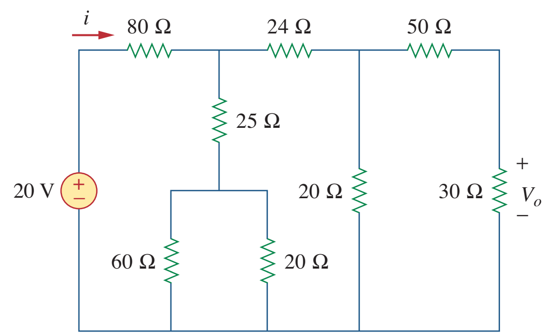


Figure 2.100
For Prob. 2.36.

2.37 Find R for the circuit in Fig. 2.101.

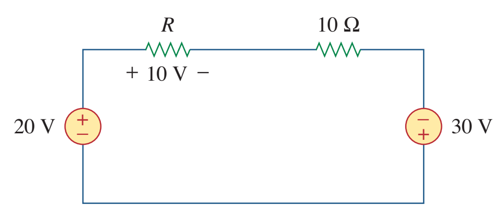


Figure 2.101
For Prob. 2.37.

2.38 Find R_{eq} and i_o in the circuit of Fig. 2.102.

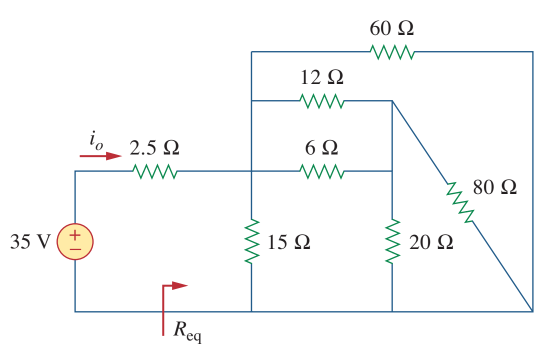


Figure 2.102
For Prob. 2.38.

2.39 Evaluate R_{eq} for each of the circuits shown in Fig. 2.103.

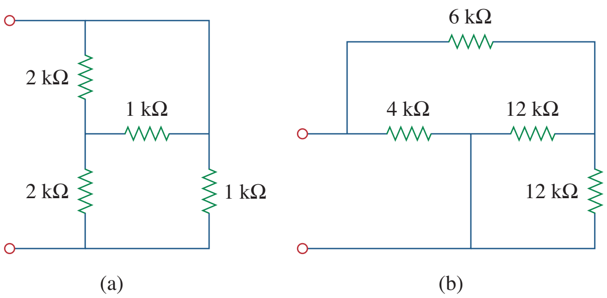


Figure 2.103
For Prob. 2.39.

2.40 For the ladder network in Fig. 2.104, find I and R_{eq}.

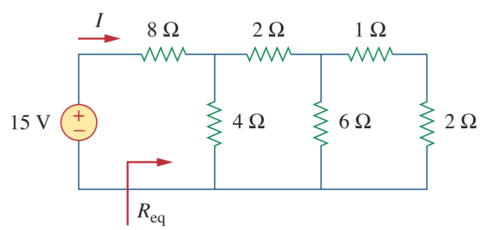


Figure 2.104
For Prob. 2.40.

2.41 If $R_{eq} = 50\ \Omega$ in the circuit of Fig. 2.105, find R.

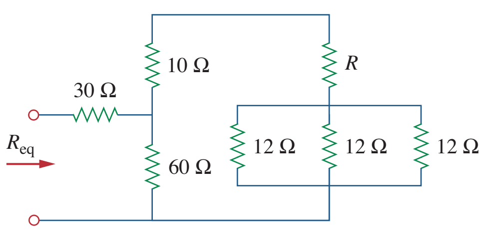
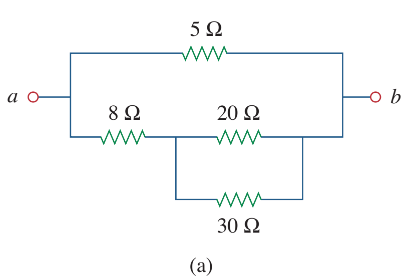


Figure 2.105
For Prob. 2.41.

2.42 Reduce each of the circuits in Fig. 2.106 to a single resistor at terminals *a-b*.



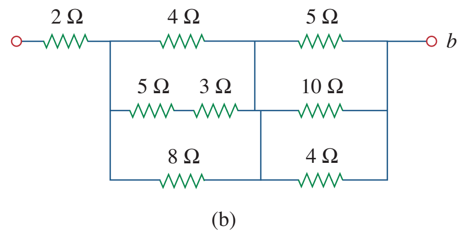


Figure 2.106
For Prob. 2.42.

2.43 Calculate the equivalent resistance R_{ab} at terminals *a-b* for each of the circuits in Fig. 2.107.

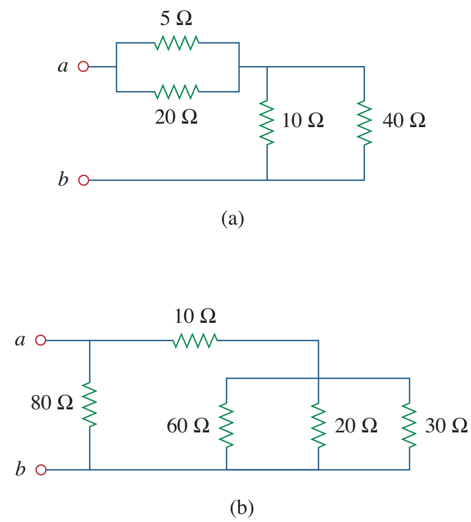


Figure 2.107
For Prob. 2.43.

2.44 For the circuits in Fig. 2.108, obtain the equivalent resistance at terminals *a-b*.

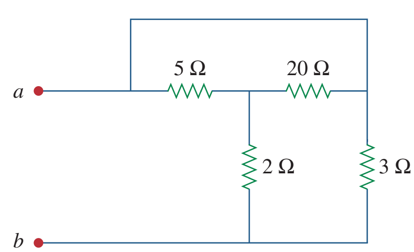


Figure 2.108
For Prob. 2.44.

2.45 Find the equivalent resistance at terminals *a-b* of each circuit in Fig. 2.109.

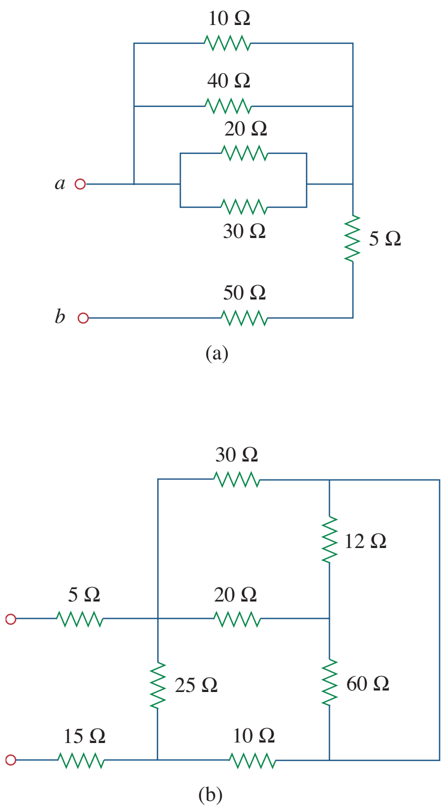


Figure 2.109
For Prob. 2.45.

2.46 Find I in the circuit of Fig. 2.110.

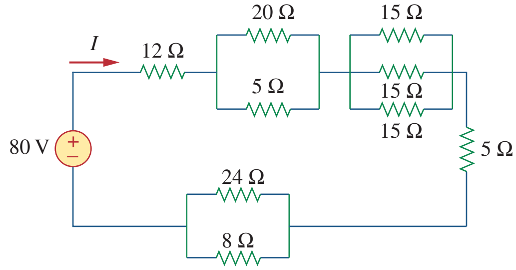


Figure 2.110
For Prob. 2.46.

2.47 Find the equivalent resistance R_{ab} in the circuit of Fig. 2.111.

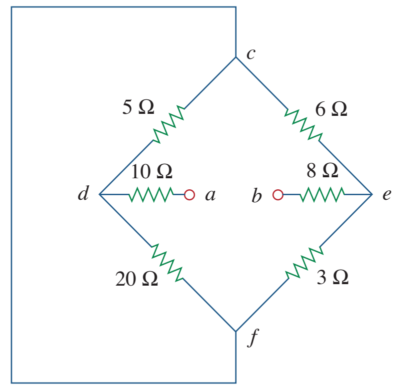


Figure 2.111
For Prob. 2.47.

Section 2.7 Wye-Delta Transformations

2.48 Convert the circuits in Fig. 2.112 from Y to Δ.

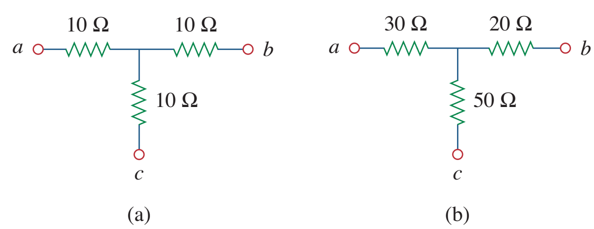


Figure 2.112
For Prob. 2.48.

2.49 Transform the circuits in Fig. 2.113 from Δ to Y.

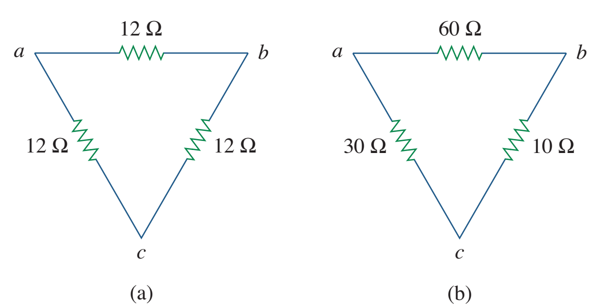


Figure 2.113
For Prob. 2.49.

2.50 Design a problem to help other students better understand wye-delta transformations using Fig. 2.114.

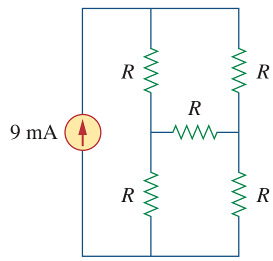


Figure 2.114
For Prob. 2.50.

2.51 Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits in Fig. 2.115.

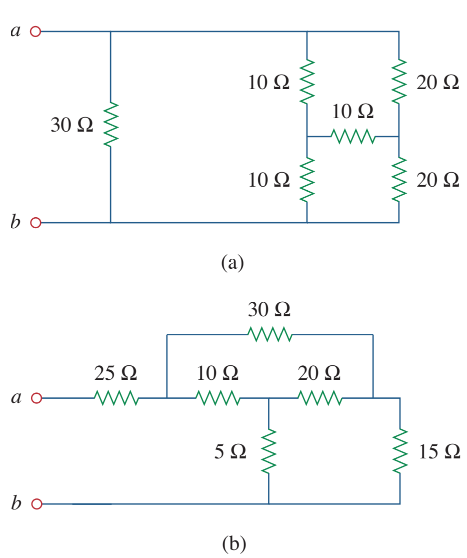


Figure 2.115
For Prob. 2.51.

***2.52** For the circuit shown in Fig. 2.116, find the equivalent resistance. All resistors are 3 Ω.

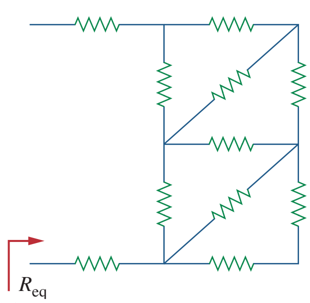


Figure 2.116
For Prob. 2.52.

***2.53** Obtain the equivalent resistance R_{ab} in each of the circuits of Fig. 2.117. In (b), all resistors have a value of 30 Ω.

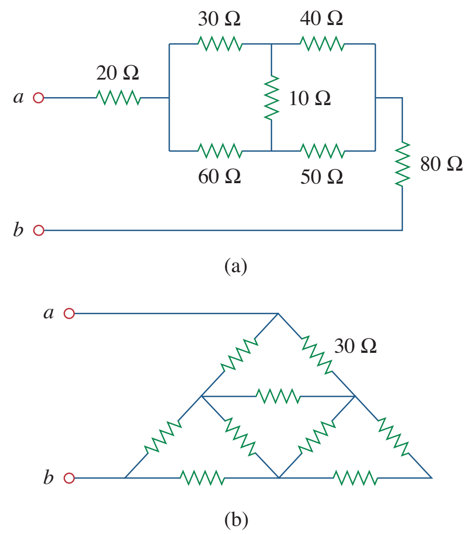


Figure 2.117
For Prob. 2.53.

2.54 Consider the circuit in Fig. 2.118. Find the equivalent resistance at terminals: (a) *a*-*b*, (b) *c*-*d*.

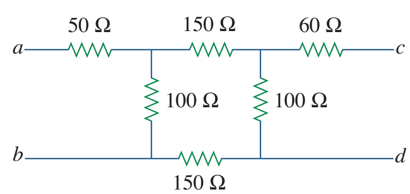


Figure 2.118
For Prob. 2.54.

2.55 Calculate I_o in the circuit of Fig. 2.119.

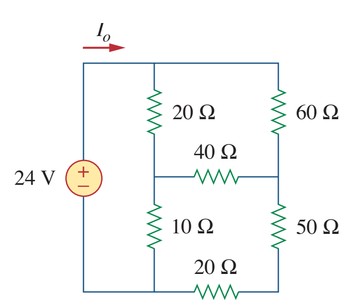


Figure 2.119
For Prob. 2.55.

* An asterisk indicates a challenging problem.

2.56 Determine V in the circuit of Fig. 2.120.

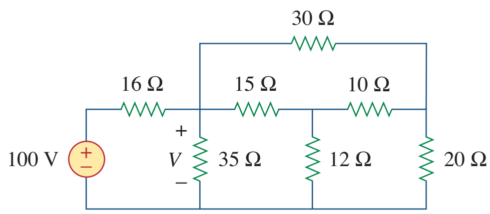


Figure 2.120
For Prob. 2.56.

***2.57** Find R_{eq} and I in the circuit of Fig. 2.121.

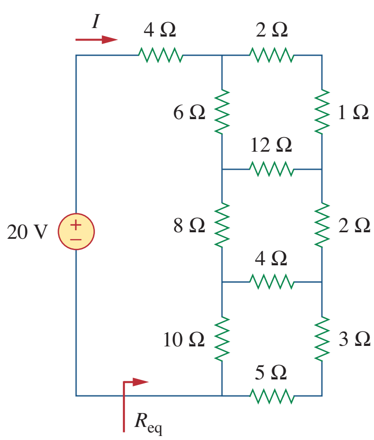


Figure 2.121
For Prob. 2.57.

Section 2.8 Applications

2.58 The 60 W light bulb in Fig. 2.122 is rated at 120 volts. Calculate V_s to make the light bulb operate at the rated conditions.

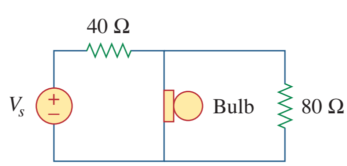


Figure 2.122
For Prob. 2.58.

2.59 Three light bulbs are connected in series to a 120-V source as shown in Fig. 2.123. Find the current I through the bulbs. Each bulb is rated at 120 volts. How much power is each bulb absorbing? Do they generate much light?

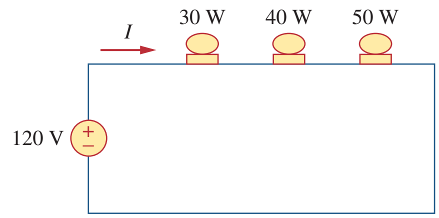


Figure 2.123
For Prob. 2.59.

2.60 If the three bulbs of Prob. 2.59 are connected in parallel to the 120-V source, calculate the current through each bulb.

2.61 As a design engineer, you are asked to design a lighting system consisting of a 70-W power supply and two light bulbs as shown in Fig. 2.124. You must select the two bulbs from the following three available bulbs.

$R_1 = 80\ \Omega$, cost $= \$0.60$ (standard size)
$R_2 = 90\ \Omega$, cost $= \$0.90$ (standard size)
$R_3 = 100\ \Omega$, cost $= \$0.75$ (nonstandard size)

The system should be designed for minimum cost such that lies within the range $I = 1.2\text{ A} \pm 5$ percent.

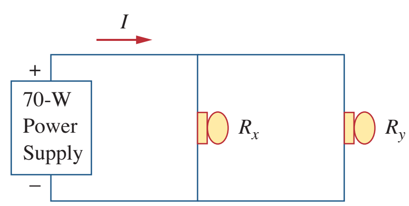


Figure 2.124
For Prob. 2.61.

2.62 A three-wire system supplies two loads A and B as shown in Fig. 2.125. Load A consists of a motor drawing a current of 8 A, while load B is a PC drawing 2 A. Assuming 10 h/day of use for 365 days and 6 cents/kWh, calculate the annual energy cost of the system.

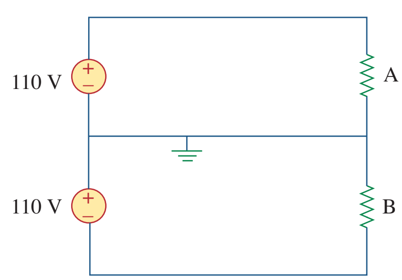


Figure 2.125
For Prob. 2.62.

2.63 If an ammeter with an internal resistance of 100 Ω and a current capacity of 2 mA is to measure 5 A, determine the value of the resistance needed.

Calculate the power dissipated in the shunt resistor.

2.64 The potentiometer (adjustable resistor) R_x in Fig. 2.126 is to be designed to adjust current i_x from 1 A to 10 A. Calculate the values of R and R_x to achieve this.

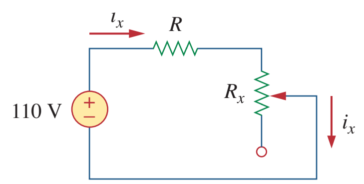


Figure 2.126
For Prob. 2.64.

2.65 A d'Arsonval meter with an internal resistance of 1 kΩ requires 10 mA to produce full-scale deflection. Calculate the value of a series resistance needed to measure 50 V of full scale.

2.66 A 20-kΩ/V voltmeter reads 10 V full scale.

(a) What series resistance is required to make the meter read 50 V full scale?

(b) What power will the series resistor dissipate when the meter reads full scale?

2.67 (a) Obtain the voltage V_o in the circuit of Fig. 2.127(a).

(b) Determine the voltage V_o' measured when a voltmeter with 6-kΩ internal resistance is connected as shown in Fig. 2.127(b).

(c) The finite resistance of the meter introduces an error into the measurement. Calculate the percent error as

$$\left| \frac{V_o - V_o'}{V_o} \right| \times 100\%$$

(d) Find the percent error if the internal resistance were 36 kΩ.

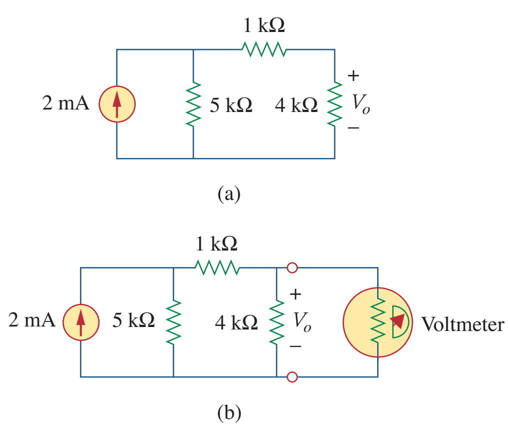


Figure 2.127
For Prob. 2.67.

2.68 (a) Find the current I in the circuit of Fig. 2.128(a).

(b) An ammeter with an internal resistance of 1 Ω is inserted in the network to measure I' as shown in Fig. 2.128(b). What is I'?

(c) Calculate the percent error introduced by the meter as

$$\left| \frac{I - I'}{I} \right| \times 100\%$$

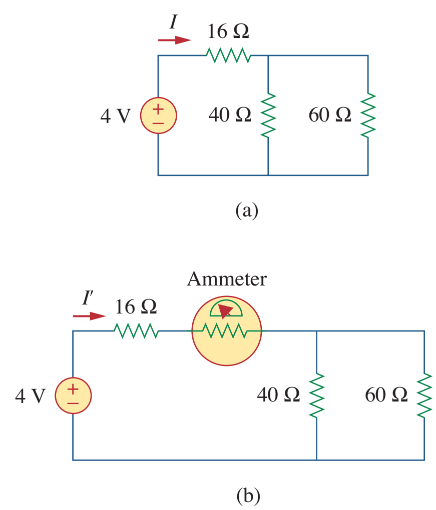


Figure 2.128
For Prob. 2.68.

2.69 A voltmeter is used to measure V_o in the circuit in Fig. 2.129. The voltmeter model consists of an ideal voltmeter in parallel with a 100-kΩ resistor. Let $V_s = 40$ V, $R_s = 10$ kΩ, and $R_1 = 20$ kΩ. Calculate V_o with and without the voltmeter when

(a) $R_2 = 1$ kΩ (b) $R_2 = 10$ kΩ

(c) $R_2 = 100$ kΩ

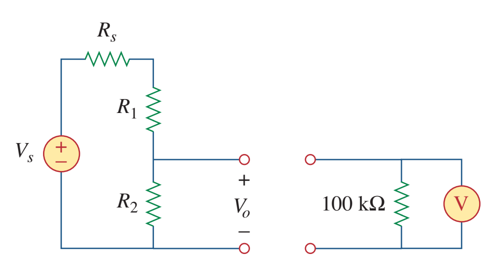


Figure 2.129
For Prob. 2.69.

2.70 (a) Consider the Wheatstone bridge shown in Fig. 2.130. Calculate v_a, v_b, and v_{ab}.
(b) Rework part (a) if the ground is placed at a instead of o.

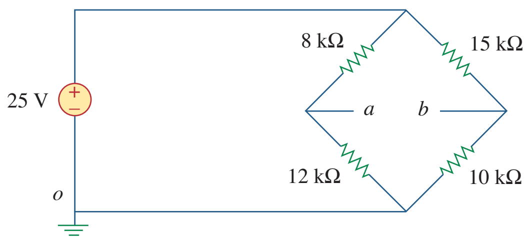


Figure 2.130
For Prob. 2.70.

2.71 Figure 2.131 represents a model of a solar photovoltaic panel. Given that $V_s = 30$ V, $R_1 = 20\ \Omega$, and $i_L = 1$ A, find R_L.

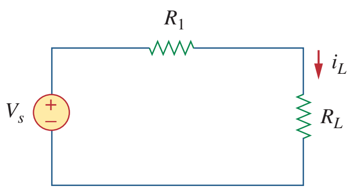


Figure 2.131
For Prob. 2.71.

2.72 Find V_o in the two-way power divider circuit in Fig. 2.132.

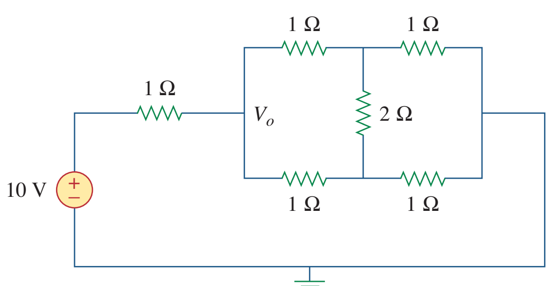


Figure 2.132
For Prob. 2.72.

2.73 An ammeter model consists of an ideal ammeter in series with a 20-Ω resistor. It is connected with a current source and an unknown resistor R_x as shown in Fig. 2.133. The ammeter reading is noted. When a potentiometer R is added and adjusted until the ammeter reading drops to one half its previous reading, then $R = 65\ \Omega$. What is the value of R_x?

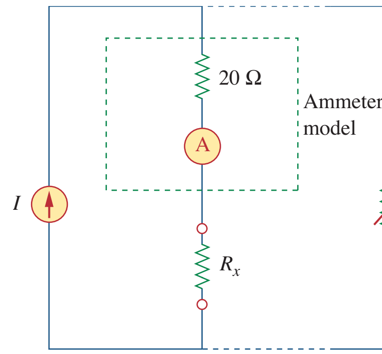


Figure 2.133
For Prob. 2.73.

2.74 The circuit in Fig. 2.134 is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 20 mΩ. Determine the series dropping resistances R_1, R_2, and R_3.

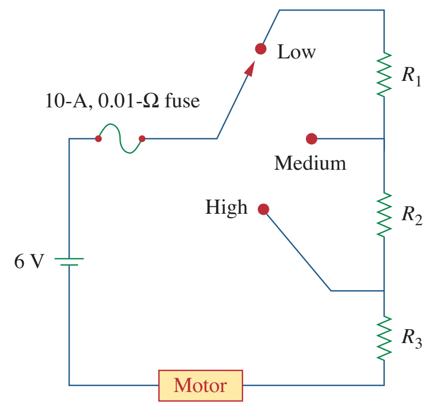


Figure 2.134
For Prob. 2.74.

2.75 Find R_{ab} in the four-way power divider circuit in Fig. 2.135. Assume each element is 1 Ω.

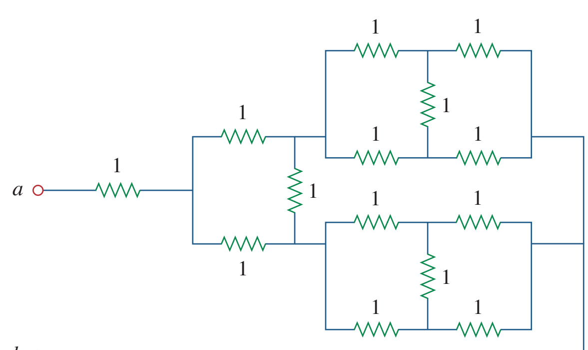


Figure 2.135
For Prob. 2.75.

Comprehensive Problems

2.76 Repeat Prob. 2.75 for the eight-way divider shown in Fig. 2.136.

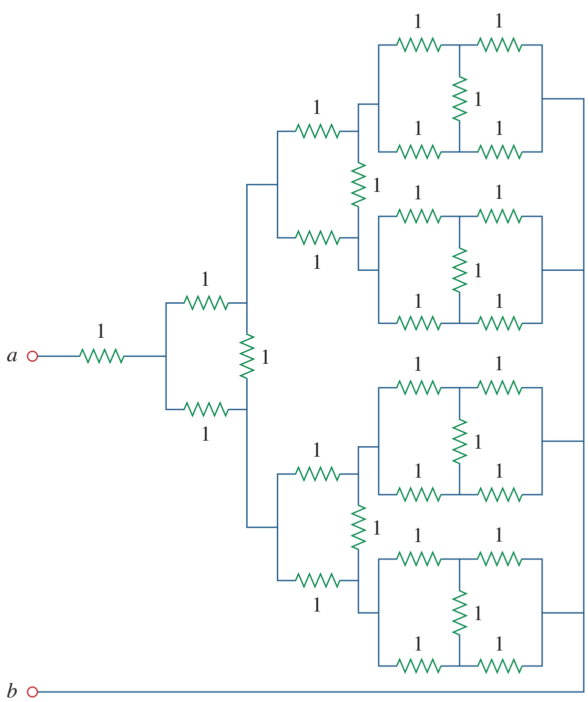


Figure 2.136
For Prob. 2.76.

2.77 Suppose your circuit laboratory has the following standard commercially available resistors in large quantities:

1.8 Ω   20 Ω   300 Ω   24 kΩ   56 kΩ

Using series and parallel combinations and a minimum number of available resistors, how would you obtain the following resistances for an electronic circuit design?

(a) 5 Ω   (b) 311.8 Ω

(c) 40 kΩ   (d) 52.32 kΩ

2.78 In the circuit in Fig. 2.137, the wiper divides the potentiometer resistance between αR and $(1 - \alpha)R$, $0 \leq \alpha \leq 1$. Find v_o/v_s.

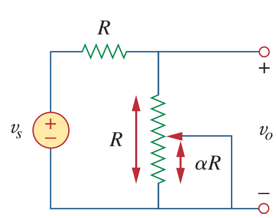


Figure 2.137
For Prob. 2.78.

2.79 An electric pencil sharpener rated 240 mW, 6 V is connected to a 9-V battery as shown in Fig. 2.138. Calculate the value of the series-dropping resistor R_x needed to power the sharpener.

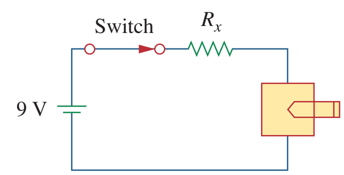


Figure 2.138
For Prob. 2.79.

2.80 A loudspeaker is connected to an amplifier as shown in Fig. 2.139. If a 10-Ω loudspeaker draws the maximum power of 12 W from the amplifier, determine the maximum power a 4-Ω loudspeaker will draw.

Figure 2.139
For Prob. 2.80.

2.81 In a certain application, the circuit in Fig. 2.140 must be designed to meet these two criteria:

(a) $V_o/V_s = 0.05$ (b) $R_{eq} = 40\ \text{k}\Omega$

If the load resistor 5 kΩ is fixed, find R_1 and R_2 to meet the criteria.

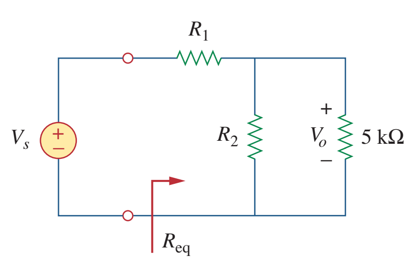


Figure 2.140
For Prob. 2.81.

2.82 The pin diagram of a resistance array is shown in Fig. 2.141. Find the equivalent resistance between the following:

(a) 1 and 2

(b) 1 and 3

(c) 1 and 4

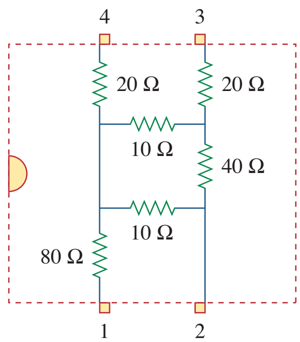


Figure 2.141
For Prob. 2.82.

2.83 Two delicate devices are rated as shown in Fig. 2.142. Find the values of the resistors R_1 and R_2 needed to power the devices using a 24-V battery.

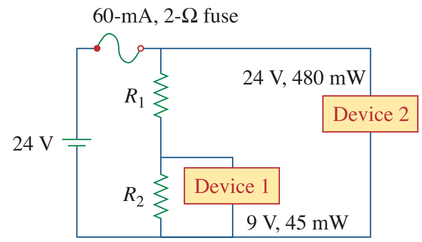


Figure 2.142
For Prob. 2.83.