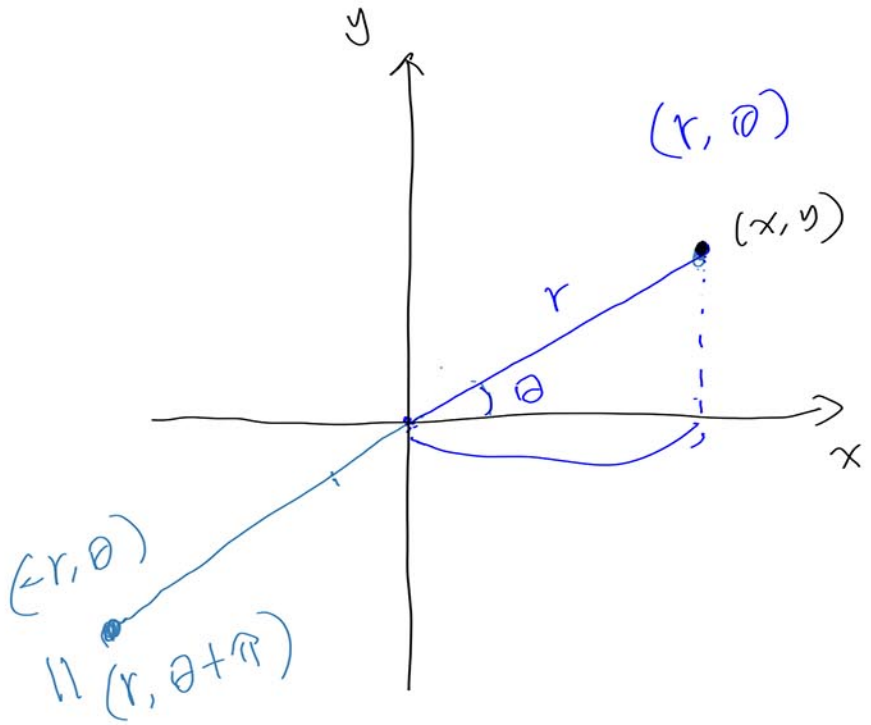
LECTURE NO. 25

7.3 Polar Coordinates

Wright State University

Defining Polar Coodinates



$x = r\cos\theta \qquad y = r\sin\theta$

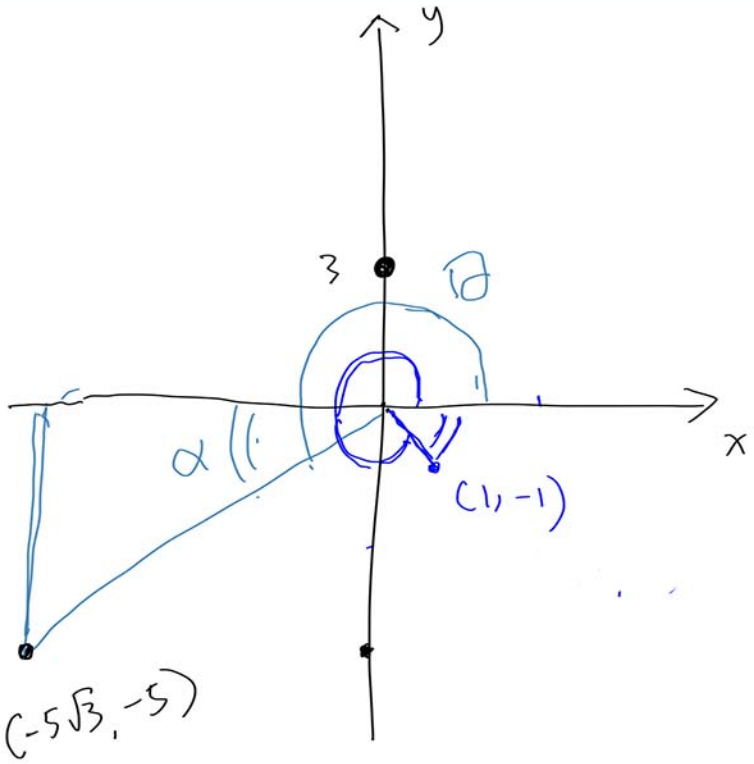
$r = \sqrt{x^2 + y^2}$

$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$(-r, \theta)$ is the symmetric image of (r, θ) around the orlgin.

$(-r, \theta)$ is the same point as $(r, \theta + \pi)$

Convert Rectangular Coodinates $(0,3)$, $(1,-1)$, $(-5\sqrt{3},-5)$ into Polar Coordinates



$(0,3)$: $r=3$, $\theta=\frac{\pi}{2}$ $\quad (3, \frac{\pi}{2})$

$(1,-1)$: $r=\sqrt{1^2+(-1)^2}=\sqrt{2}$ $\quad (\sqrt{2}, -\frac{\pi}{4})$

$\theta=-\frac{\pi}{4}$ $\quad (\sqrt{2}, \frac{7\pi}{4})$

$\pi+\frac{\pi}{2}+\frac{\pi}{4}$

$(-5\sqrt{3},-5)$: $r=\sqrt{(-5\sqrt{3})^2+(-5)^2}=\sqrt{75+25}=\sqrt{100}=10$

$\tan\alpha=\frac{-5}{-5\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3} \Rightarrow \alpha=\frac{\pi}{6}$

$\theta=\pi+\alpha=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$ $\quad (10, \frac{7\pi}{6})$

Convert Polar Coordinates $(3, \frac{\pi}{3})$, $(6, -\frac{5\pi}{6})$ into Rectangular Coordinates.

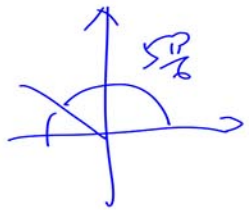
$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$(3, \frac{\pi}{3})$

$x = 3\cos\frac{\pi}{3} = 3\cdot\frac{1}{2} = \frac{3}{2}$

$y = 3\sin\frac{\pi}{3} = 3\cdot\frac{\sqrt{3}}{2} = \frac{3}{2}\sqrt{3}$

$\left(\frac{3}{2}, \frac{3}{2}\sqrt{3}\right)$



$(6, -\frac{5\pi}{6})$

$x = 6\cos(-\frac{5\pi}{6}) = 6\cos\frac{5\pi}{6}$

$x = -6\cos\frac{\pi}{6} = -6\cdot\frac{\sqrt{3}}{2} = -3\sqrt{3}$

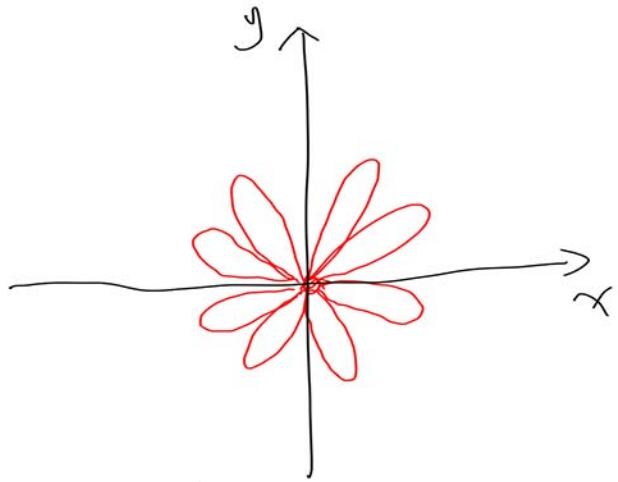
$y = 6\cdot\sin(-\frac{5\pi}{6}) = -6\sin\frac{5\pi}{6}$

$= -6\sin\frac{\pi}{6} = -6\cdot\frac{1}{2} = -3$

$(-3\sqrt{3}, -3)$

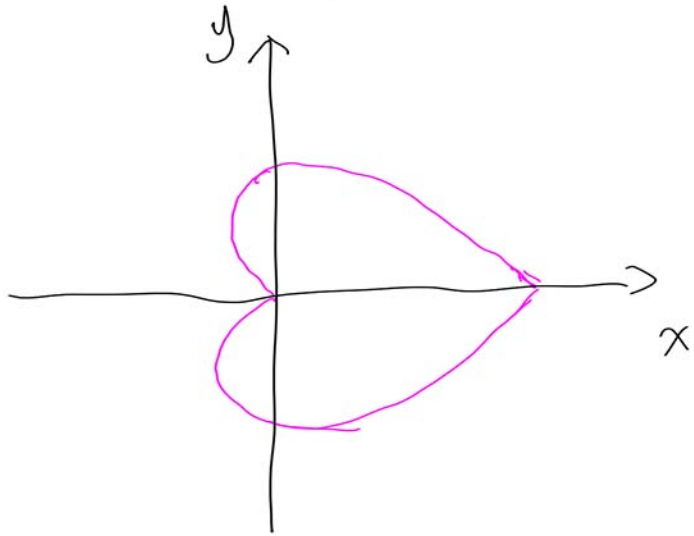
Use Calculator to Graph Polar Functions

- $r = 3\sin(4\theta)$.



flower

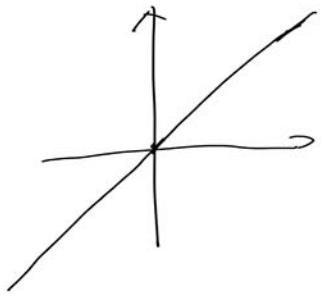
- $r = 3 + 3\cos\theta$



Heart

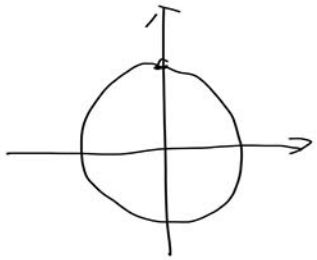
Transform Polar Equations into Rectangular Equations

- $\theta = \frac{\pi}{3}$.



$\tan\theta = \sqrt{3} = \frac{y}{x}$ $\quad y = \sqrt{3}\,x$

- $r = 3$



$r = 3 = \sqrt{x^2+y^2} \Rightarrow x^2+y^2 = 9$

$r^2 = x^2+y^2$ $\quad r\cos\theta = x$ $\quad r\sin\theta = y$

- $r = 2\cos\theta - 4\sin\theta$ $\xrightarrow{\text{multiply both sides by } r}$ $r^2 = r(2\cos\theta - 4\sin\theta)$

$r^2 = 2r\cos\theta - 4r\sin\theta$

$x^2+y^2 = 2x - 4y$ $\leftarrow$ a circle