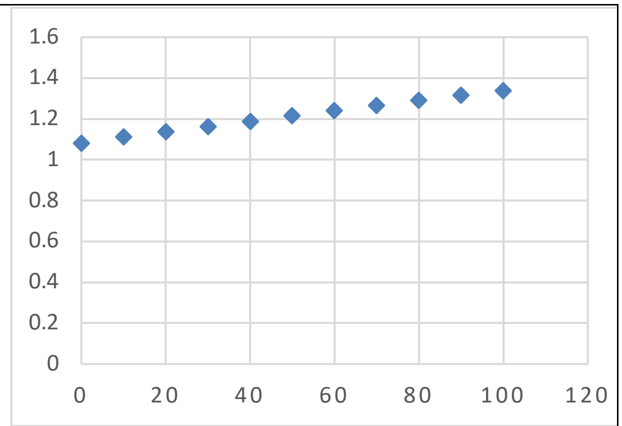
ISE 2211 • Exam VI • 15 November 2021 NAME ____

A bonded cantilever-type strain-gage sensor was connected to an instrumentation preamplifier and output voltages (y) measured for a series of applied masses in grams (x). The data and scatter plot are presented below, courtesy of Andrew Bleakley (BME 4550L, Fall 2021).

Mass (g)	Voltage (V)
0	1.082
10	1.112
20	1.138
30	1.164
40	1.189
50	1.215
60	1.24
70	1.265
80	1.291
90	1.317
100	1.339



Determine least-squares estimated linear regression coefficients for this data. Write the resulting regression equation (with unit!) and draw the line on the scatter plot.

An unknown mass was applied to the end of the cantilever and an output voltage of **1.177 V** recorded. Use your regression line to estimate the value of mass.

Write a 95% prediction interval on a 12^{th} measurement @ 120 g.

Determine the coefficient of determination for this relationship. Would you say there is a strong linear relationship between applied mass and output voltage for this strain gage system?

Formulae:

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$SS_T = \sum y_i^2 - n\bar{y}^2$$

$$SS_E = SS_T - \hat{\beta}_1 S_{XY}$$

$$\hat{\sigma}^2 = \frac{SS_E}{n-2}$$

$$R^2 = 1 - \frac{SS_E}{SS_T} = \hat{\beta}_1{}^2 \frac{S_{XX}}{SS_T} = \frac{\hat{\beta}_1 S_{XY}}{SS_T}$$

Parameter	**Test Statistic**	**Critical Value**	**C.I.**
β_1	$t_0 = \dfrac{\hat{\beta}_1 - \hat{\beta}_{1,0}}{\sqrt{\hat{\sigma}^2 / S_{XX}}}$	$\pm t_{\alpha/2, n-2}$	$\hat{\beta}_1 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2/S_{XX}}$
β_0	$t_0 = \dfrac{\hat{\beta}_0 - \hat{\beta}_{0,0}}{\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{XX}}\right]}}$	$\pm t_{\alpha/2, n-2}$	$\hat{\beta}_0 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{XX}}\right]}$
$\mu_{Y\|x_0}$			$\hat{\mu}_{Y\|x_0} \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{XX}}\right]}$
Y_0		P.I.:	$\hat{y}_0 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{XX}}\right]}$
ρ	$t_0 = \dfrac{R\sqrt{n-2}}{\sqrt{1-R^2}}$	$\pm t_{\alpha/2, n-2}$	$tanh\left(tanh^{-1}R \pm \frac{z_{\alpha/2}}{\sqrt{n-3}}\right)$ $(n \geq 30)$
ρ_0	$z_0 = (tanh^{-1}R - tanh^{-1}\rho_0)\sqrt{n-3}$	$\pm z_{\alpha/2}$	

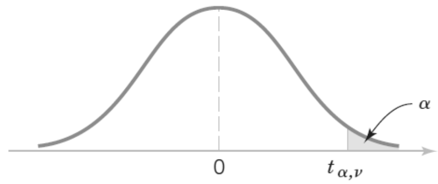


TABLE • V Percentage Points $t_{\alpha,\nu}$ of the t Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291