LECTURE NO. 2

1.7 Integrals Resulting in Inverse Trig Functions

Wright State University

Two Integration Formulas Resulting Inverse Trig Functions

- $$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$$

- $$\int \frac{1}{x^2+1}dx = \tan^{-1} x + C$$

- We may use substitution and these formulas to calculate some other similar integrals.

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$$

Substitution $u = 2x$ $\quad \frac{du}{dx} = 2$ $\quad dx = \frac{du}{2}$

$$\int \frac{1}{\sqrt{1-(2x)^2}}\, dx$$

$$= \int \frac{1}{\sqrt{1-u^2}} \frac{du}{2} = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}}\, du = \frac{1}{2} \sin^{-1} u + C$$

$$= \frac{1}{2} \sin^{-1}(2x) + C$$

FINAL ANSWER.

First let's do $\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{1}{\sqrt{4-9x^2}} \cdot dx$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1}x + C$$

$$\int \frac{1}{\sqrt{4\left(1-\frac{9x^2}{4}\right)}}\, dx = \int \frac{1}{2\sqrt{1-\frac{9x^2}{4}}}\, dx = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{3x}{2}\right)^2}}\, dx$$

Substitution $u = \frac{3x}{2}$ $\quad \frac{du}{dx} = \frac{3}{2}$ $\quad dx = \frac{du}{\frac{3}{2}} = \frac{2}{3}\, du$

antiderivative

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}} \frac{2}{3}\, du = \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\, du = \frac{1}{3}\sin^{-1}u + C = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$$

$$\int_0^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\Big|_0^{\frac{1}{3}} = \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\right) - \frac{1}{3}\sin^{-1}(0) = \frac{1}{3}\cdot\frac{\pi}{6} = \frac{\pi}{18}$$

$\int \frac{1}{1+(2x+1)^2} dx$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

Substitution $u = 2x+1 \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2}$

$$\int \frac{1}{1+u^2} \frac{du}{2} = \frac{1}{2} \int \frac{1}{1+u^2} du = \frac{1}{2} \tan^{-1} u + C$$

$$= \frac{1}{2} \tan^{-1}(2x+1) + C$$

FINAL ANSWER.

$$\int \frac{dx}{25+16x^2} = \int \frac{1}{25+16x^2} \cdot dx$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$= \int \frac{1}{25\left(1+\frac{16x^2}{25}\right)}\, dx = \frac{1}{25} \int \frac{1}{1+\left(\frac{4x}{5}\right)^2}\, dx$$

Substitution $u = \frac{4x}{5}$ $\quad \frac{du}{dx} = \frac{4}{5}$ $\quad dx = \frac{du}{\frac{4}{5}} = \frac{5}{4}\, du$

$$= \frac{1}{25} \int \frac{1}{1+u^2} \frac{5}{4}\, du = \frac{1}{20} \tan^{-1} u + C$$

$$= \boxed{\frac{1}{20} \tan^{-1}\left(\frac{4x}{5}\right) + C}$$

FINAL ANSWER.

Substitution Techniques for Integral of Simple Rational Functions

- $\int \frac{2}{3x+5} dx \to u = 3x + 5$
- $\int \frac{1}{(2x+1)^3} dx \to u = 2x + 1$

practice

x^2+9 irreducible quadratic factor!

- What about $\int \frac{4x+3}{x^2+9} dx$?
- Need to break up the NUMERATOR!

$$\int \frac{4x}{x^2+9} + \frac{3}{x^2+9} dx$$

$\int \frac{4x+3}{x^2+9}dx$

$$\int \frac{4x}{x^2+9} + \frac{3}{x^2+9}\,dx = \int \frac{4x}{x^2+9}\,dx + \int \frac{3}{x^2+9}\,dx = 2\ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + C$$

FINAL ANSWER

$$\int \frac{4x}{x^2+9}\,dx \quad u = x^2+9 \quad \frac{du}{dx} = 2x \quad dx = \frac{du}{2x}$$

$$\int \frac{4x}{u}\,\frac{du}{2x} = \int \frac{2}{u}\,du = 2\int \frac{1}{u}\,du$$

$$= 2\ln|u| + C = 2\ln|x^2+9| + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\int \frac{3}{x^2+9}\,dx = \int \frac{3}{9\left(\frac{x^2}{9}+1\right)}\,dx = \frac{1}{3}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$$

$$u = \frac{x}{3} \quad \frac{du}{dx} = \frac{1}{3} \quad dx = 3\,du$$

$$\frac{1}{3}\int \frac{1}{u^2+1}\,3\,du = \tan^{-1}u + C$$

$$= \tan^{-1}\left(\frac{x}{3}\right) + C$$