Continuous Random Variables

∴ <u>interval</u> of real numbers

"uncountably infinite"

∴ think <u>measurement</u>, rather than counting

∴ instead of probability mass function (discrete), <u>probability density function</u>:

1.) $f(x) \geq 0$

2.) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ ← rather than $\sum$

3.) $P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$

(area under curve)

fun facts!

∴ histograms approximate probability density functions

∴ the probability of <u>any one precise point</u> is zero

∴ we can be lazy with our inequalities!

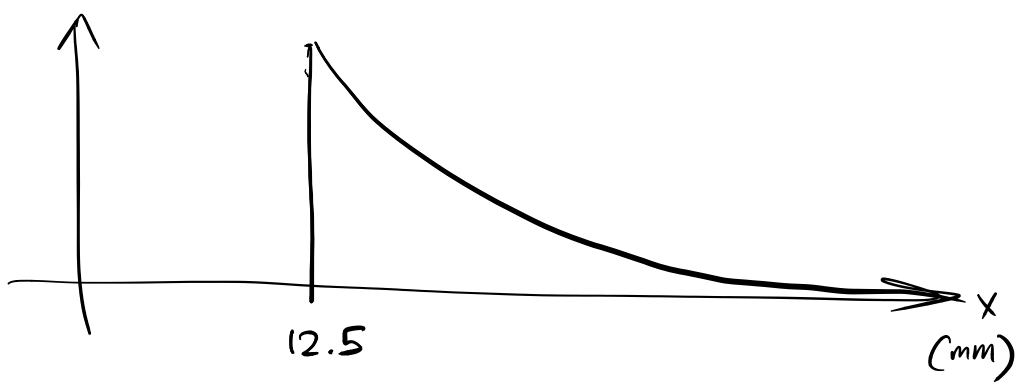
$$P(x_1 \leq X \leq x_2) = P(x_1 < X < x_2)$$

ex: hole drilled in sheetmetal

.. target diameter is 12.5 mm (~ ½")

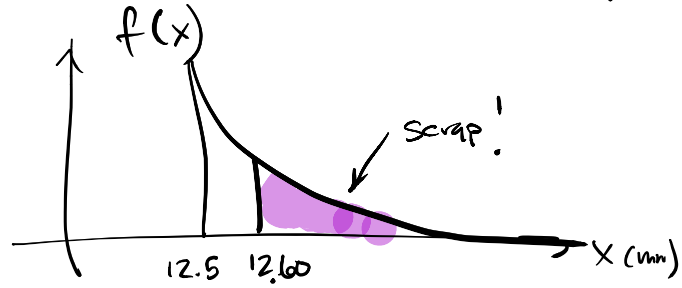
.. historical data suggests distribution of X (hole diameter) is modelled by the following probability density function:

$$f(x) = 20e^{-20(x-12.5)} \qquad x \geq 12.5$$



.. any diameter larger than 12.60 mm is scrapped; what percentage of workpieces is scrapped?

.. draw it!



$$P(x > 12.60) = \int_{12.60}^{\infty} f(x)\,dx = \int_{12.60}^{\infty} 20\,e^{-20(x-12.5)}\,dx$$

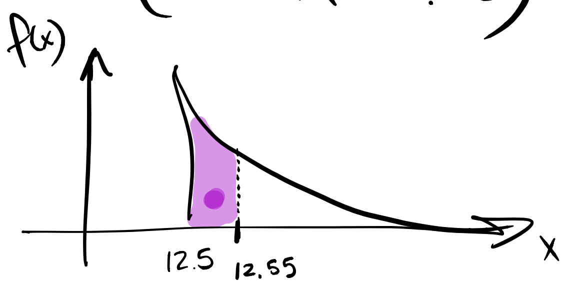
$$= \frac{1}{-\cancel{20}} \cdot \cancel{20}\, e^{-20(x-12.5)} \Big|_{12.6}^{\infty}$$

$$= \underbrace{-e^{-20(\infty)}}_{0} - {-e^{-20(12.6-12.5)}}$$

$$= 0.135 \quad \text{or} \quad 13.5\%$$

.. what proportion are between 12.5 and 12.55?

$$P(12.5 < X < 12.55) = \int_{12.5}^{12.55} 20e^{-20(x-12.5)}$$



$$= -e^{-20(12.55-12.5)} - \overbrace{-e^{-20(12.5-12.5)}}^{1}$$

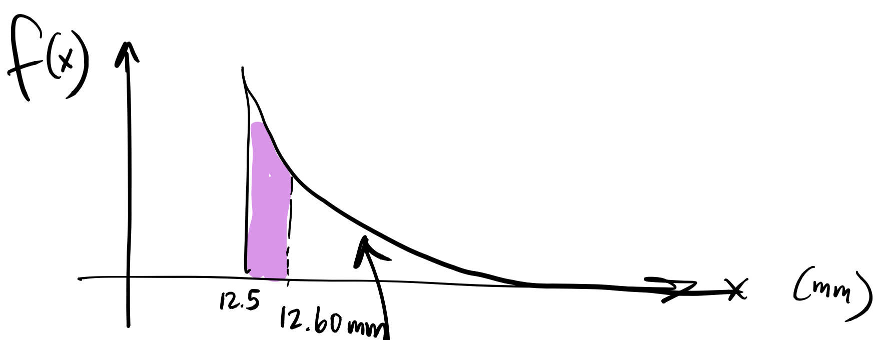
$$= -0.368 - {-1} = 0.632$$

or 63.2%

Cumulative Distribution Functions

$$F(x) = \underbrace{P(X \le x)}_{\text{up to some value of } x} = \underbrace{\int_{-\infty}^{x} f(x)\,dx}_{\text{area under curve up to that value!}}$$

.. what is the probability that a part has a diameter less than 12.60 mm?



$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{12.5}^{12.60} 20e^{-20(x-12.5)}\,dx$$

→ original problem constrains lower bound to 12.5 mm

$$-e^{-20(12.6-12.5)} - -e^{-20(12.5-12.5)} = \underbrace{0.865}_{\text{or } 86.5\%}$$

.. we already knew this area to be 0.135 from before; it was the scrapped proportion

.. all adds to 1 or 100%

Mean & Variance

$$\mu = \underbrace{E(x)}_{\text{expected value}} = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

∴ <u>weighted area</u> under f(x)!

$$\sigma^2 = \underbrace{V(x)}_{\text{variance}} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

Computational formula!

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

ex: hole drilling problem

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_{12.5}^{\infty} \overset{u}{x}\, \overbrace{20e^{-20(x-12.5)} dx}^{dv}$$

$-\infty$ → really 12.5 w/ this problem

integration by parts!

$$\int u\, dv = uv - \int v\, du$$

then

$du = 1$

$v = -e^{-20(x-12.5)}$

$$E(x) = x \cdot -e^{-20(x-12.5)} - \int -e^{-20(x-12.5)}\,dx \cdot 1$$

$$E(x) = \left[-x e^{-20(x-12.5)} - \frac{1}{20} e^{-20(x-12.5)} \right] \Bigg|_{12.5}^{\infty}$$

$$= -\infty e^{-20(\infty-12.5)} - \frac{1}{20} e^{-20(\infty-12.5)}$$

$$0 \quad 0 \quad 0, \qquad 1 \quad 1$$

$$- -12.5 e^{-20(12.5-12.5)} - -\frac{1}{20} e^{-20(12.5-12.5)}$$

$$E(x) = 12.5 + \frac{1}{20} = 12.55 \text{ (mm)}$$

$V(x) \longrightarrow$ requires two integrations by parts!

(no thank you!)

$$= 0.0025 \text{ (mm}^2\text{)}$$

note that the "scrap limit" of 12.6 mm is only one σ above the mean!

pretty tight variance!

$$\sigma = {}^{+}\sqrt{\sigma^2} = 0.05 \text{ (mm)}$$