ISE 2211 • Exam III • 22 February 2019 14 pts. NAME SOLUTION

1) Yellow Springs Brewery offers several of their award-winning craft beverages in cans. Fill volume is monitored very closely by measuring the finished can's mass. It is known that that can mass has a population standard deviation of 5.77 grams. After a midnight canning session of 24 beverages, the mean can mass was determined to be $\bar{x} = 370$ g with a sample standard deviation of 4.8 g. Write a 90% lower confidence bound on mean can mass. Also determine the minimum number of beverages that would need to be canned for the lower bound to be 1.0 g from the mean.

known σ → use Z-distribution formula (+1)

lower bound → $\bar{x} - Z_{\alpha} \frac{\sigma}{\sqrt{n}}$ (+1)

↑ not $\alpha/2$

90% bound → $\alpha = 0.10$

∴ $Z_{0.10} = 1.282$

(bottom row of t table) (+1)

$$370 - 1.282 \frac{5.77}{\sqrt{24}} \leq \mu$$

$$= 368.5 \text{ g} \leq \mu \quad (+1)$$

$$n = \left(\frac{Z_{\alpha}\sigma}{E}\right)^2 = \left(\frac{1.282 \cdot 5.77}{1}\right)^2 = 54.72$$

(+1)

Use $n = 55$ (+1)

2) Summarize the Central Limit Theorem in your own words. Why is it so important?

As $n \uparrow$, the distribution of $\bar{x} \longrightarrow$ normal. (+1)

means we have statistical tools to draw conclusions about μ regardless of the distribution of X. (+1)

3) The amount of time that a customer must wait in line at the Department of Motor Vehicles in Fairborn, Ohio seems to vary way too much, and for no damn good reason. Over the course of an afternoon at the Fairborn DMV, the wait times of 26 subjects were recorded with the following results: sample mean = 8.543 minutes, sample standard deviation = 9.816 minutes.* Write an upper 95% confidence bound on the standard deviation of wait time and include a unit with your answer. Additionally, indicate this bound on a rough sketch of the appropriate distribution.

upper bound $\longrightarrow$ $\sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{1-\alpha,\, n-1}}$ (+1)

↑ not $\alpha/2$

$\chi^2_{1-\alpha,\, n-1} = \chi^2_{.95,\, 25} = 14.61$ (table) (+1)

(+1)

$$\sigma^2 \leq \frac{25 \cdot 9.816^2}{14.61}$$

$$\sigma^2 \leq 164.9 \quad (+1)$$

$$\sigma \leq \sqrt{164.9}$$

$$\sigma \leq 12.84 \quad (+1) \qquad \text{minutes} \quad (+1)$$

*Joe Tritschler made this whole problem up. The people at the Fairborn DMV are actually very nice and the wait time is never too bad.