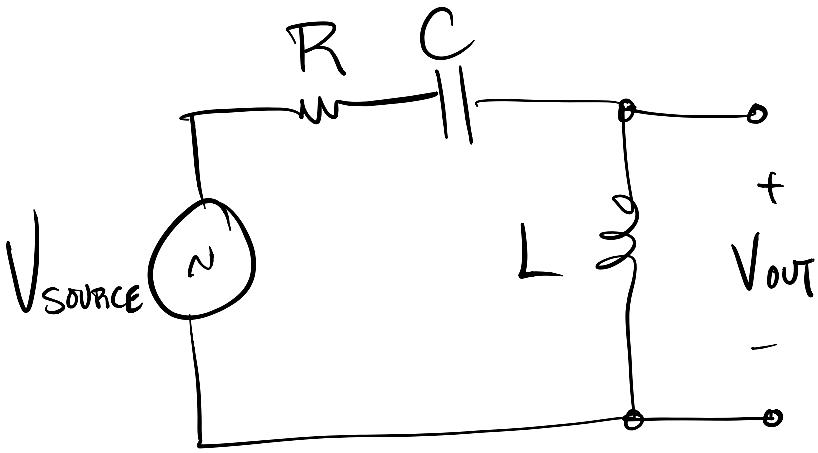
Series-RLC High-Pass Filter



$$H(\omega) = \frac{j\omega L}{j\omega L + R + \frac{1}{j\omega C}}$$

× top and bottom by $j\omega/L$

$$H(\omega) = \frac{(j\omega)^2}{(j\omega)^2 + j\omega R/L + \frac{1}{LC}}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$, $Q_{series} = \frac{1}{R}\sqrt{L/C}$

$$\boxed{H(\omega) = \frac{(j\omega)^2}{(j\omega)^2 + j\omega \frac{\omega_0}{Q} + \omega_0^2}}$$

as $\omega \to \infty$,

$$H(\omega) \to \frac{(j\,BIG)^2}{\omega_0^2 + j\omega \frac{BIG}{Q} + (j\,BIG)^2}$$

$$\approx \frac{(j\,BIG)^2}{(j\,BIG)^2} \to 1 \angle 0^\circ$$

∴ high frequencies are indeed passed!

as $\omega \to 0$,

$$H(\omega) \to \frac{(j\,SMALL)^2}{\omega_0^2 + j\,SMALL \frac{\omega_0}{Q} + (j\,SMALL)^2}$$

$$\to \frac{(j\,SMALL)^2}{\omega_0^2} \to \overset{-1}{\overset{\Uparrow}{j^2}}\, SMALL^2$$

$$\to 0 \angle +180^\circ$$

∴ low frequencies stopped, phase → 180°

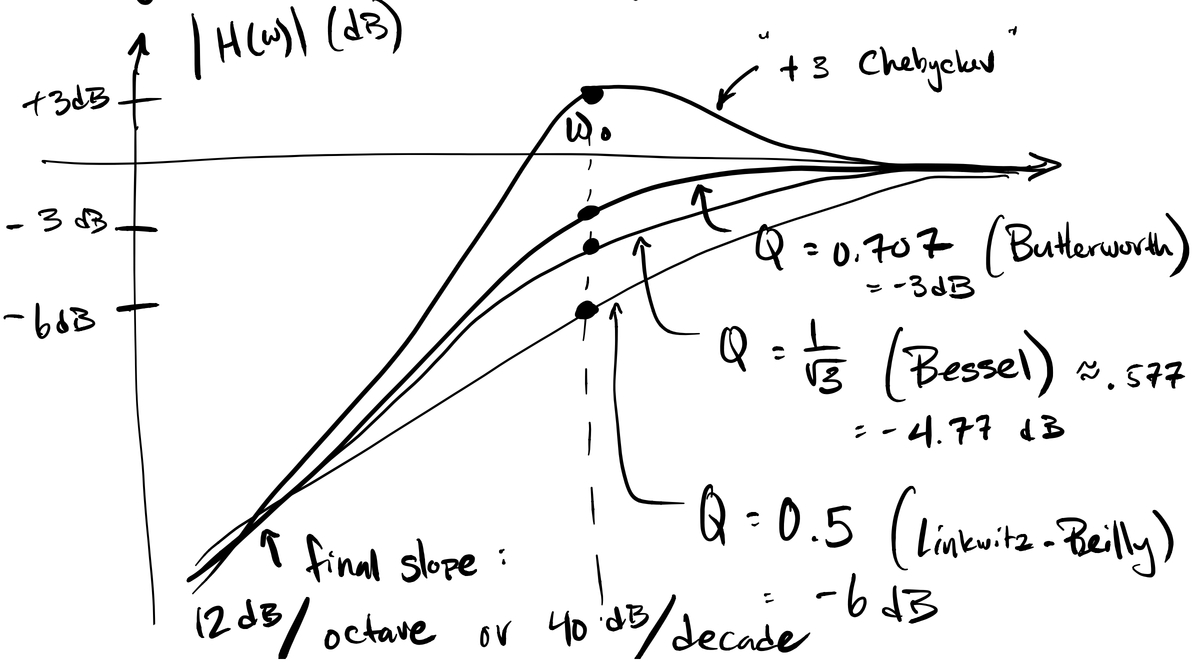
b ω_0 ← natural frequency

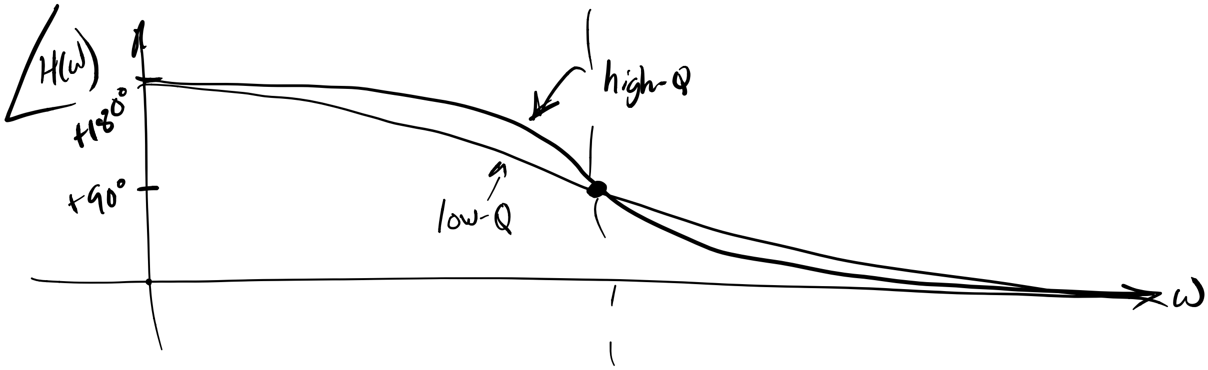
$$H(\omega_0) = \frac{(j\omega_0)^2}{\cancel{\omega_0^2} + j\omega_0 \frac{\omega_0}{Q} \cancel{+ (j\omega_0)^2}}$$

$$= \frac{j^2 \omega_0^2}{j \frac{\omega_0^2}{Q}} = jQ$$

$$\longrightarrow Q \angle +90^\circ$$

∴ again, magnitude at ω_0 is Q!





homework: RLC high-pass

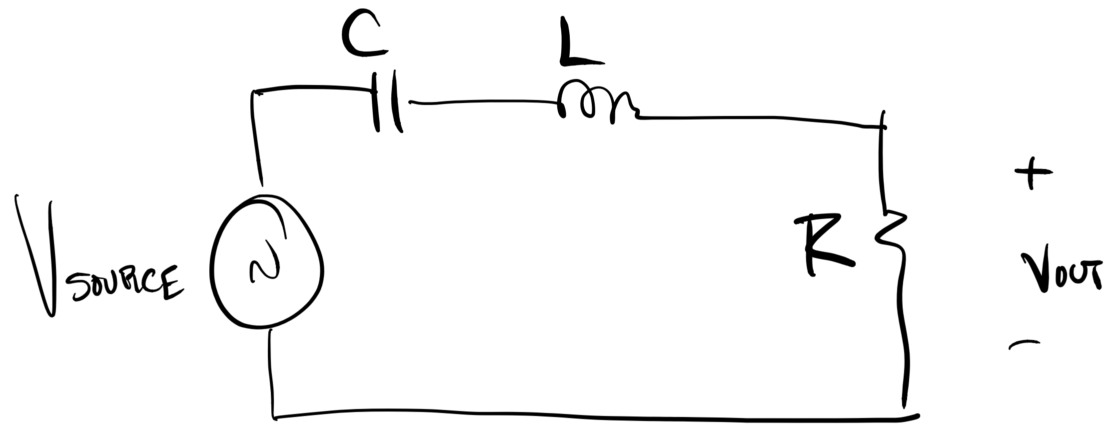
$L = 2.2\ mH$

$R = 16\ \Omega$

$C = 3.3\ \mu F$

- sketch circuit, determine ω_0, f_0, and Q,
describe freq. response (i.e., Butterworth etc..),
describe transient response (i.e., overdamped, etc..),
determine $H(\omega_0)$, $H(2\omega_0)$, and $H(0.5\,\omega_0)$
(mag. in dB <u>and</u> phase),
sketch $|H(\omega)|$ and $\angle H(\omega)$.

Series RLC Bandpass Filter



∴ only passes frequencies in some range

$$H(\omega) = \frac{Z_R}{Z_R + Z_C + Z_L}$$

$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

$$H(\omega) = \frac{j\omega R}{(j\omega)^2 L + \frac{1}{C} + j\omega R}$$

$$H(\omega) = \frac{j\omega\, R/L}{(j\omega)^2 + \frac{1}{LC} + j\omega\, R/L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

then $$H(\omega) = \frac{j\omega \frac{\omega_0}{Q}}{(j\omega)^2 + j\omega \frac{\omega_0}{Q} + \omega_0^2}$$

Same denominator !!!

.. ω_0 is the center frequency of the filter

.. Q has a special definition;

$$Q = \frac{f_0}{f_H - f_L}$$

... where $f_0 = \omega_0 / 2\pi$

f_H : upper -3dB point

f_L : lower -3dB point

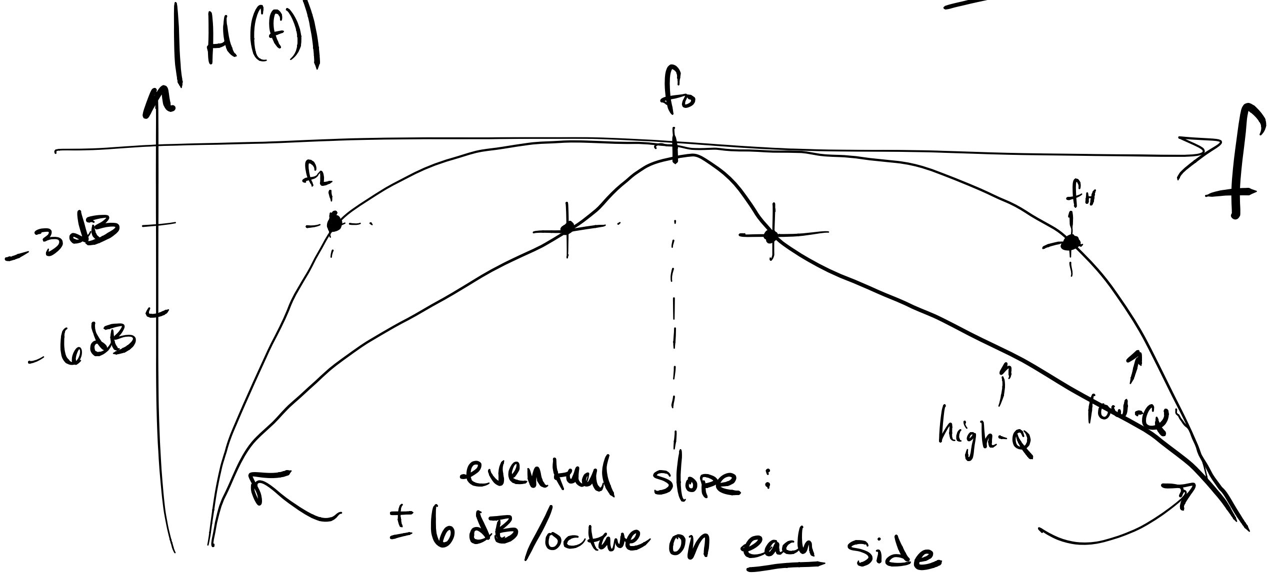
$$H(\omega_0) = \frac{j\omega_0 \frac{\omega_0}{Q}}{\cancel{(j\omega_0)^2} + j\omega_0 \frac{\omega_0}{Q} + \cancel{\omega_0^2}} = \frac{j\,\omega_0^2/Q}{j\,\omega_0^2/Q}$$

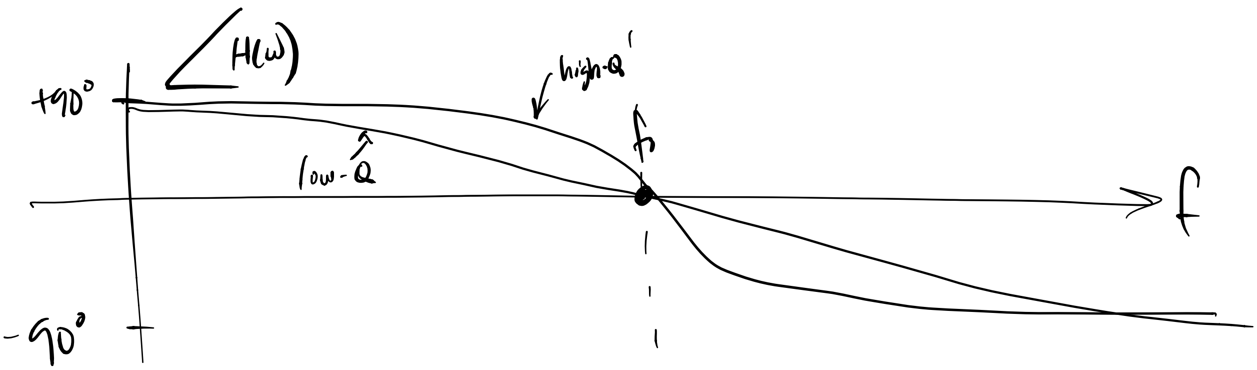
$$\longrightarrow \underline{1\angle 0^\circ} \quad \text{or} \quad \underline{0\,\text{dB}}$$

∴ perfect, lossless bandpass filter; naturally, some R in L, etc.

high-Q filter → small difference between −3 dB points → sharp (highly resonant @ ω_0)

low-Q filter → large difference between −3 dB points → broad





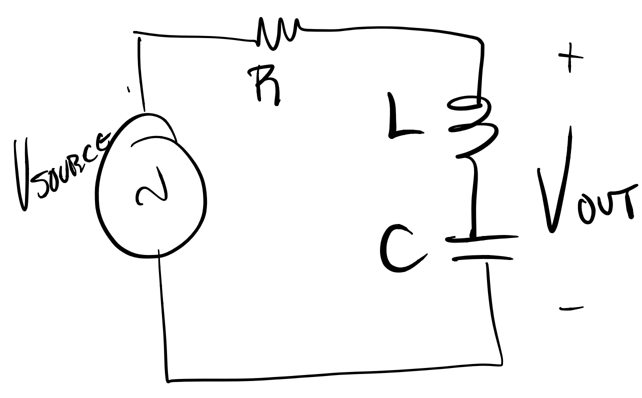
∴ if f_H and f_L are <u>known</u>, then

$$\underbrace{f_0 = \sqrt{f_H f_L}}$$

useful in design!

(geometric mean)

Series RLC Bandreject Filter



$$H(\omega) = \frac{Z_L + Z_C}{Z_L + Z_C + R}$$

$$H(\omega) = \frac{\frac{1}{j\omega C} + j\omega L}{j\omega L + \frac{1}{j\omega C} + R}$$

$$= \frac{\frac{1}{C} + (j\omega)^2 L}{(j\omega)^2 L + \frac{1}{C} + j\omega R}$$

$$= \frac{\frac{1}{LC} + (j\omega)^2}{(j\omega)^2 + \frac{1}{LC} + j\omega \frac{R}{L}}$$

$$\boxed{H(\omega) = \frac{\omega_o^2 + (j\omega)^2}{\underline{(j\omega)^2 + j\omega \frac{\omega_o}{Q} + \omega_o^2}}}$$

$\omega_o = \frac{1}{\sqrt{LC}}$

$Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}}$

⇓

again!

ω_o : center frequency

Q : sharpness of rejection

∴ high-Q bandreject is also called a <u>notch filter</u>.

again: $$Q = \frac{f_0}{f_H - f_L}$$

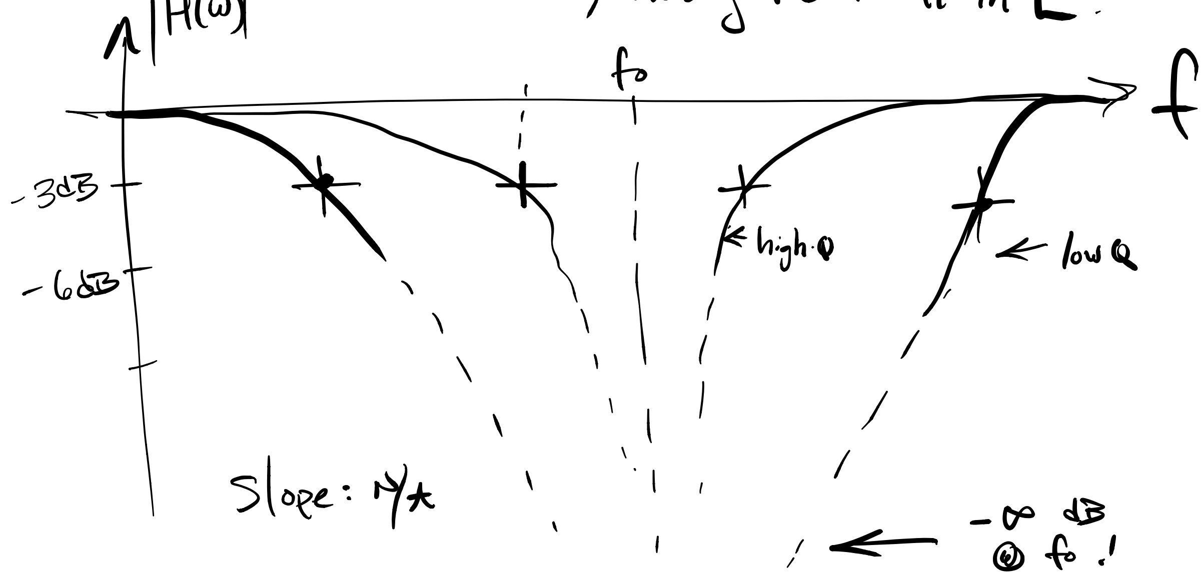
$$f_0 = \sqrt{f_H \, f_L}$$

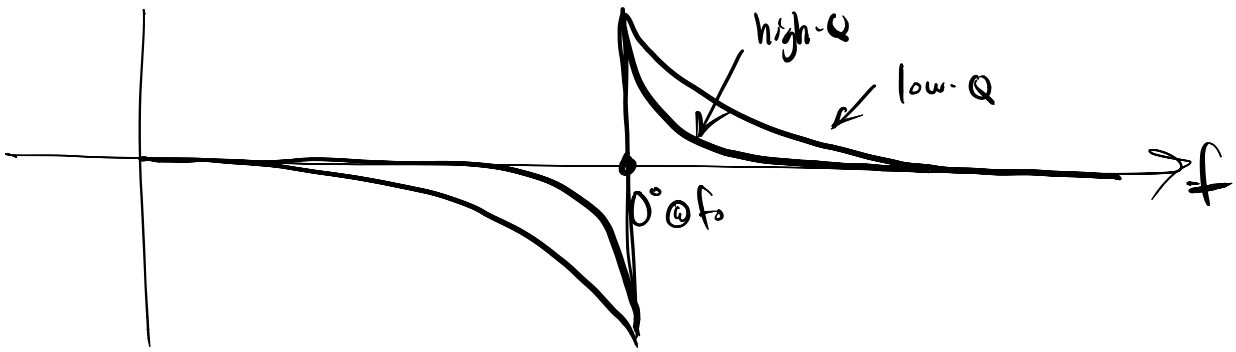
$$H(\omega_0) = \frac{\cancel{\omega_0^2 + (j\omega_0)^2}}{\cancel{(j\omega_0)^2} + j\frac{\omega_0^2}{Q} + \cancel{\omega_0^2}}$$

$$\longrightarrow 0 \angle 0^\circ$$

∴ Zero output @ ω_0 for perfect bandreject filter!

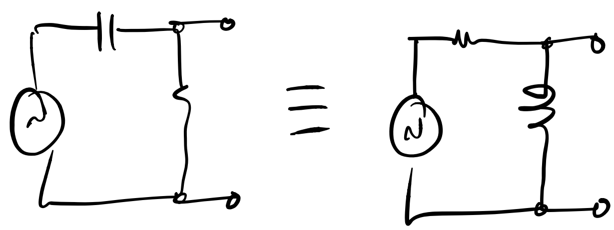
$|H(\omega)|$ ∵ never happens, mostly due to R in L.



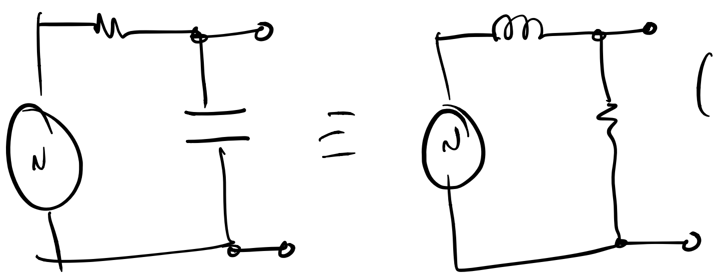


.. what about parallel RLC filters?

.. just like



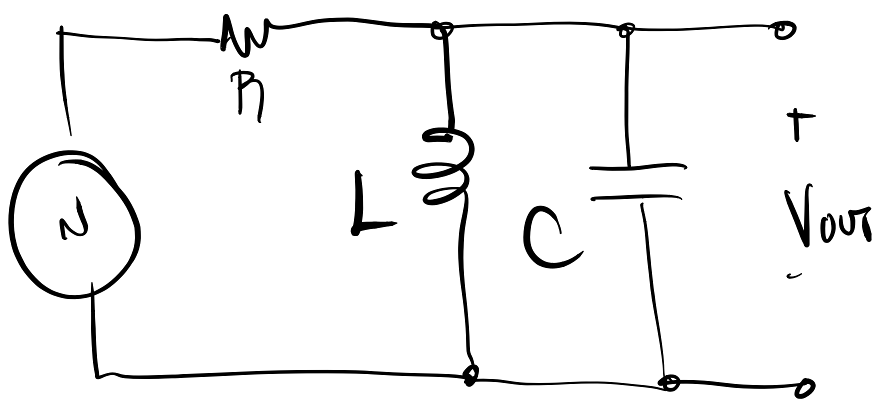
(1st-order HPF),



(1st-order LPF)

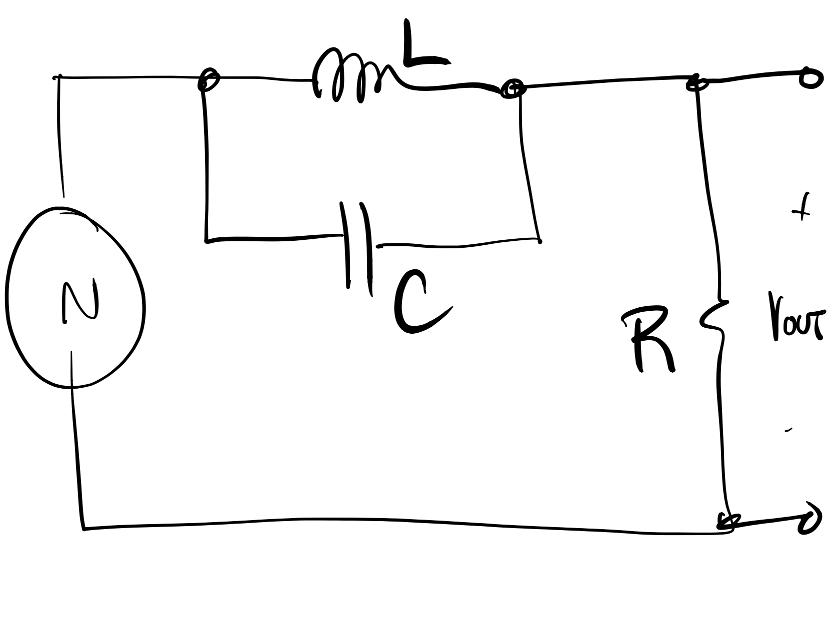
... we can build bandpass and bandreject filters w/ parallel LC combination, same transfer function as series version!

bandpass:



∴ same ω_0, but $Q_{parallel} = R\sqrt{\frac{C}{L}}$

bandreject:



homework: sketch parallel RLC bandpass filter

$L = 2.2H$, $C = 2.0\ nF$, $R = 47\ k\Omega$

∴ determine ω_o, f_o, Q, characterize frequency and transient response, sketch mag. and phase

∴ determine f_H and f_L

∴ Simulate both homework problems in

LTSPICE !