2310 LAB 3 Solution.

#1. $\text{Arc length} = \int_1^3 \sqrt{1+[f'(x)]^2}\,dx$

$f'(x) = x^2 - \frac{1}{4x^2}$

$\text{Arc length} = \int_1^3 \sqrt{1+(x^2-\frac{1}{4x^2})^2}\,dx$

$= \int_1^3 \sqrt{1+x^4-\frac{1}{2}+\frac{1}{16x^4}}\,dx$

$= \int_1^3 \sqrt{x^4+\frac{1}{2}+\frac{1}{16x^4}}\,dx$

$= \int_1^3 \sqrt{(x^2+\frac{1}{4x^2})^2}\,dx$

$= \int_1^3 x^2+\frac{1}{4x^2}\,dx$

anti derivative

$= \frac{x^3}{3} - \frac{1}{4x}\Big|_1^3$

$= (9-\frac{1}{12}) - (\frac{1}{3}-\frac{1}{4})$

$= 9 - \frac{1}{12} - \frac{1}{12} = \frac{53}{6}$

#2 $\frac{dy}{dx} = 3x^2$

$\text{Surface Area} = \int_0^1 2\pi y\sqrt{1+(\frac{dy}{dx})^2}\,dx$

$= \int_0^1 2\pi x^3 \sqrt{1+(3x^2)^2}\,dx$

$= \int_0^1 2\pi x^3 \sqrt{1+9x^4}\,dx$

<u>Substitution</u>

$u = 1+9x^4 \quad \frac{du}{dx} = 36x^3$

$dx = \frac{du}{36x^3}$

$x: 0 \to 1$

$u = 1+9x^4: 1 \to 10$

$= \int_1^{10} 2\pi\, x^3 \sqrt{u}\, \frac{du}{36x^3}$

$= \frac{\pi}{18}\int_1^{10} u^{\frac{1}{2}}\,du$

$= \frac{\pi}{18}\,\frac{2}{3}\,u^{\frac{3}{2}}\Big|_1^{10}$

$= \frac{\pi}{27}\cdot 10^{\frac{3}{2}} - \frac{\pi}{27}$

#3 $f(x) = 2x^2$

$A = \int_0^1 2x^2\,dx = \frac{2}{3}x^3\Big|_0^1 = \frac{2}{3}$

$\bar{x} = \frac{1}{A}\int_0^1 x\,f(x)\,dx \qquad (A=\frac{2}{3})$

$= \frac{3}{2}\int_0^1 x\cdot 2x^2\,dx$

$= \frac{3}{2}\int_0^1 2x^3\,dx$

$= \frac{3}{2}\cdot\frac{x^4}{2}\Big|_0^1 = \frac{3}{4}$

$\bar{y} = \frac{1}{A}\int_0^1 \frac{1}{2}[f(x)]^2\,dx$

$= \frac{3}{2}\cdot\frac{1}{2}\int_0^1 (2x^2)^2\,dx$

$= \frac{3}{4}\int_0^1 4x^4\,dx$

$= 3\cdot\frac{x^5}{5}\Big|_0^1 = \frac{3}{5}$

#4 $\int x^2 \sin(2x)\, dx$

IBP $u = x^2$ $\quad dv = \sin(2x)\, dx$

$du = 2x\, dx$ $\quad \cancel{v = \frac{\cos x}{2}}$

$v = -\frac{\cos(2x)}{2}$

$= -\frac{\cos(2x)}{2} \cdot x^2 - \int -\frac{\cos(2x)}{2}\, 2x\, dx$

$= -\frac{1}{2}x^2 \cos 2x + \underbrace{\int x \cos(2x)\, dx}$

IBP again

$u = x$ $\quad dv = \cos(2x)\, dx$

$du = dx$ $\quad v = \frac{\sin(2x)}{2}$

$= -\frac{1}{2}x^2\cos(2x) + x \cdot \frac{\sin(2x)}{2}$

$- \int \frac{\sin(2x)}{2}\, dx$

$= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x)$

$- \left(\frac{-\cos(2x)}{4}\right) + C$

$= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin 2x$

$+ \frac{1}{4}\cos(2x) + C$

5.

$\int x^4 \ln x\, dx$

$u = \ln x$ $\quad dv = x^4\, dx$

$du = \frac{1}{x}\, dx$ $\quad v = \frac{x^5}{5}$

$= \ln x \cdot \frac{x^5}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x}\, dx$

$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4\, dx$

$= \frac{1}{5}x^5 \ln x - \frac{1}{5} \cdot \frac{x^5}{5} + C$

or

$\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C$