19 pts.

ISE 2211 • Exam II • GROUNDHOG DAY 2018! NAME SOLUTION

1) Fact: Joe Tritschler goes through a lot of guitar picks. On average, he ruins a dozen of them in a 45-minute concert. (Not really.) If the continuous random variable X equals the average number of minutes between ruined picks with an exponential distribution, determine the probability of getting through the song In-A-Gadda-Da-Vida (17 minutes long) using the same pick. Hint: this implies that the time to the next ruined guitar pick is more than 17 minutes.

Formulae:

$f(x) = \lambda e^{-\lambda x}$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mu = \frac{1}{\lambda}$

$\sigma^2 = \frac{1}{\lambda^2}$

$$\mu = \frac{45 \text{ min.}}{12 \text{ picks}} = 3.75 \text{ min./pick} \quad +1$$

$$\therefore \mu = \frac{1}{\lambda} \rightarrow \lambda = \frac{1}{3.75} = 0.2667 \quad +1$$

need $P(x > 17)$ +1

$= 1 - P(x < 17)$ +1

$= 1 - [1 - e^{-\lambda x}]$ +1

$= 1 - [1 - e^{-.2667 \cdot 17}]$

$= 0.01074$ +1

or 1.0715%

+6

Now consider the discrete random variable X representing the number of ruined picks in a performance. Determine the probability of using the same pick during the 17-minute song. Hint: determine the average number of guitar picks ruined in 17 minutes, and then determine the probability that it happens zero times. You should get the same probability.

Formulae:

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\mu = \lambda$

$\sigma^2 = \lambda$

$$\mu = \frac{12 \text{ picks}}{45 \text{ min.}} \times 17 = 4.533 \text{ picks}/17 \text{ min.} \quad +1$$

$$\therefore \lambda = 4.533 \quad +1$$

Want $f(0)$ +1

$$\therefore \quad f(0) = \frac{e^{-4.533} \cdot 4.533^0}{0!}$$

$$= \underline{\underline{0.01074}} \quad +1$$

(same)

2) Fact: Punxsutawney Phil (the famous groundhog) is usually WRONG in his prognostications regarding the arrival of spring. According to Stormfax, the probability of successfully predicting the arrival of spring is only 39%. Determine the probability that Punxsutawney Phil will correctly predict the arrival of spring every single year for the next five years. Also determine the expected value and variance of the number of successful predictions in the next five years. Include units with all answers.

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$p = 0.39$ +1

$X = 5$ +1 (number of sucesses)

$n = 5$ +1 (number of trials)

$$\binom{5}{5} = \frac{5!}{5!\,(5-5)!} = 1 \quad +1$$

$$\therefore \quad f(5) = 1 \cdot .39^5 \cancel{(1-.39)^{5-5}}$$

$$= 0.009022 \quad +1$$

or $0.9022\ \%$

$E(x) = \mu = np$ +1

$= 5 \cdot 0.39 = 1.95$ +1 sucessful predictions

$V(x) = \sigma^2 = np(1-p) = 1.1895$ +1 (successful predictions)^2 +1

+9