ISE 2211 • Exam VI • 25 July 2019 NAME SOLUTION

A series of measurements has been taken in order to determine what, if any, relationship exists between the thickness in μm of a conformal coating applied to the body of a resistor (x) and the measured stray capacitance in pF (y).

Test #	Coating thickness, μm (x)	Stray capacitance, pF (y)
1	100	0.054
2	125	0.068
3	150	0.078
4	175	0.092
5	200	0.110
6	225	0.124
7	250	0.138

Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model for stray capacitance vs. coating thickness.

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$\left.\begin{aligned}\sum_1^7 x_i &= 1225 \\ \sum_1^7 y_i &= 0.664\end{aligned}\right\}$ +1

$\sum_1^7 x_i y_i = 126.1$ +1

$\left.\begin{aligned}\sum_1^7 x_i^2 &= 231875 \\ \sum_1^7 y_i^2 &= 0.068608\end{aligned}\right\}$ +1

$\left.\begin{aligned}\bar{x} &= \frac{1225}{7} = 175 \\ \bar{y} &= \frac{0.664}{7} = 0.094857\end{aligned}\right\}$ +1

$S_{XY} = 126.1 - \frac{0.664 \cdot 1225}{7} = 9.9$ +1

$S_{XX} = 231875 - \frac{1225^2}{7} = 17500$ +1

$\hat{\beta}_1 = \frac{9.9}{17500} = 5.657 \times 10^{-4}$ +1

$\hat{\beta}_0 = 0.094857 - 5.657 \times 10^{-4} \cdot 175 = -4.14 \times 10^{-3}$ +1

Write the equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$\hat{y} = -4.14 \times 10^{-3} + 5.657 \times 10^{-4}\, x$ +1

Write a 95% confidence interval on the mean stray capacitance at $x = 180$ μm.

$$\hat{\mu}_{Y|180} = -4.14 \times 10^{-3} + 5.657 \times 10^{-4} \cdot 180$$

$$= \underline{0.097686} \ (\text{pF})$$

+1

$$SS_T = 0.068608 - 7 \cdot 0.094857^2$$

$$= \underline{0.005623}$$ +1

$$SS_E = 0.005623 - 5.657 \times 10^{-4} \cdot 9.9$$

$$= \underline{2.262 \times 10^{-5}}$$ +1

$$\hat{\sigma}^2 = \frac{SS_E}{n-2} = \underline{4.523 \times 10^{-6}}$$ +1

$$t_{\alpha/2, n-2} = t_{.025, 5} = 2.571$$ +1

$$\mu_{Y|180}: \quad 0.097686 \pm 2.571 \sqrt{4.523 \times 10^{-6} \left[\frac{1}{7} + \frac{(180-175)^2}{17500}\right]}$$

$$\underline{0.09561 < \mu_{Y|180} < 0.09976} \quad (\text{pF})$$

+2

Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero. Include a unit with the C.I.

H_0: $\hat{\beta}_1 = 0$

H_1: $\hat{\beta}_1 \neq 0$

β_1: $\quad \hat{\beta}_1 \pm t_{\alpha/2, n-2} \sqrt{\hat{\sigma}^2 / Sxx}$

$5.657 \times 10^{-4} \pm 2.571 \sqrt{4.523 \times 10^{-6} / 17500}$

$5.244 \times 10^{-4} < \beta_1 < 6.070 \times 10^{-4} \left(\frac{pF}{\mu m}\right)$

+2 +1

C.I does not contain zero; reject H_0

+2

List two theoretical scenarios that would fail to reject H_0. What does *your* conclusion, above, imply about the relationship between coating thickness and stray capacitance?

1.) no significant relationship

2.) non-linear relationship

+2

there is a significant linear relationship between coating thickness and stray capacitance

+1

Write a 95% confidence interval on the correlation coefficient ρ, if y and x may both be considered random variables. (Ignore the fact that $n \ngtr 30$.)

$$R^2 = 1 - \frac{SSE}{SST}$$

$$= 1 - \frac{2.262 \times 10^{-5}}{0.005623}$$

$$= 0.9960 \qquad +1 \qquad \therefore R = 0.9979$$

$$\tanh^{-1}(.9979) = 3.453 \qquad +1$$

$$Z_{\alpha/2} = Z_{.025} = 1.960 \qquad +1$$

$$\rho : \quad \tanh\left(3.453 \pm \frac{1.960}{\sqrt{4}}\right)$$

$$.9859 < \rho < .9997$$

+2