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ISE 2211 • Exam II • 7 February 2020 NAME SOLUTION

1) You have a pile of 100 4.75-kΩ precision resistors, of which nine are out-of-spec. The printed circuit board you are assembling uses three of them. Write a probability distribution for the number of out-of-spec resistors on a board, assuming sampling without replacement. Hint: {ppp ppf pfp pff fpp fpf ffp fff}.

$P\{ppp\} = \frac{91}{100} \cdot \frac{90}{99} \cdot \frac{89}{98} = 0.7513$

$P\{ppf\} = \frac{91}{100} \cdot \frac{90}{99} \cdot \frac{9}{98} = 0.07597$

$P\{pfp\} = \frac{91}{100} \cdot \frac{9}{99} \cdot \frac{90}{98} = 0.07597$

$P\{pff\} = \frac{91}{100} \cdot \frac{9}{99} \cdot \frac{8}{98} = 0.006753$

$P\{fpp\} = \frac{9}{100} \cdot \frac{91}{99} \cdot \frac{90}{98} = 0.07597$

$P\{fpf\} = \frac{9}{100} \cdot \frac{91}{99} \cdot \frac{8}{98} = 0.006753$

$P\{ffp\} = \frac{9}{100} \cdot \frac{8}{99} \cdot \frac{91}{98} = 0.006753$ /12

$P\{fff\} = \frac{9}{100} \cdot \frac{8}{99} \cdot \frac{7}{98} = 0.0005195$

+4 basic concept of sampling w/out replacement

$P(0) = P\{ppp\} = 0.7513$ (+2)

$P(1) = P\{ppf\ pfp\ fpp\} = 0.07597 \times 3 = 0.2279$ (+2)

$P(2) = P\{pff\ fpf\ ffp\} = 0.006753 \times 3 = 0.02026$ (+2)

$P(3) = P\{fff\} = 0.0005195$ (+2)

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Write a cumulative distribution for the number of out-of-spec resistors on a circuit board.

$F(0) = P(0) = 0.7513$ +1

$F(1) = P(0) + P(1) = 0.7513 + 0.2279$
$= 0.9792$ +1

$F(2) = P(0) + P(1) + P(2) = 0.7513 + 0.2279 + 0.02026$
$= 0.99946$ +1

$F(3) = P(0) + P(1) + P(2) + P(3) = 1$ +1

What is the probability of at least one out-of-spec resistor on a circuit board?

$P(x \geq 1) = P(1) + P(2) + P(3)$
$= 0.2487$ +2

9H. : $P(x \geq 1) = 1 - P(x < 1) = 1 - P(0) = 1 - 0.7513$
$= 0.2487$

Compute the expected value and variance of the number of out-of-spec resistors on a circuit board.

$E(x) = \sum x f(x) = 0 \cdot 0.7513 + 1 \cdot 0.2279$
$+ 2 \cdot 0.02026 + 3 \cdot 0.0005195$
$= 0.27$ resistors +2

$V(x) = \sum x^2 f(x) - \mu^2 = 0^2 \cdot 0.7513 + 1^2 \cdot 0.2279$
$+ 2^2 \cdot 0.02026 + 3^2 \cdot 0.0005195 - (0.2$

$V(x) = 0.240715$ resistors^2 +2

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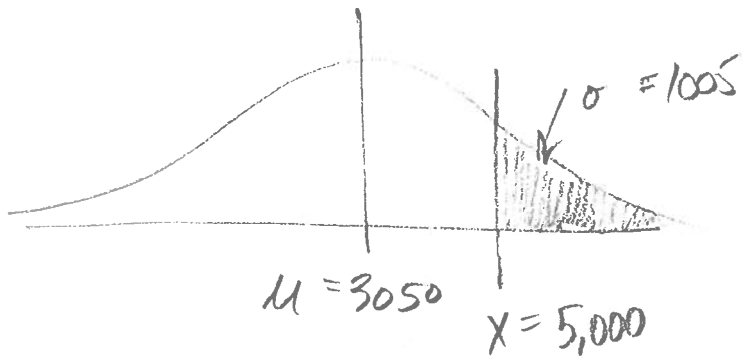
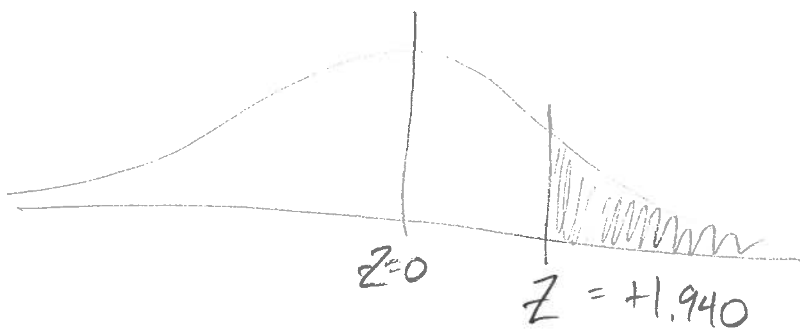
2) The lifespan of a Mullard EL34 vacuum tube is normally-distributed with a known mean of 3,050 hours and a standard deviation 1,005 hours. Determine the probability that an EL34 will last longer than 5,000 hours. Sketch this probability on both the normal and standard-normal distributions.



+2

$$Z = \frac{X-\mu}{\sigma} = \frac{5000-3050}{1005} = +1.940$$ +1



+2

$$P(x > 3050) = P(Z > 1.940)$$

$$= 1 - P(Z < 1.940)$$ +2

table +1

$$= 1 - .973810$$

$$= 0.02619$$

(not very likely)