Cumulative Probabilities

∴ alternative way of describing probabilities associated with a random variable

some value

$$P(X \leq x)$$

∴ "up to and including" some value of X, rather than the probability of an exact value.

$$= \sum_{x_i \leq x} f(x_i)$$

Sampling with replacement: item is selected; placed back into the batch (randomly!); next selection has same probability distribution as previous selection ∴ independent

<u>Sampling without replacement</u> : item is selected;

<u>not</u> placed back in ; probabilities associated with next selection <u>depend</u> on outcome of previous selection

·· <u>not</u> independent !!!

ex: 850 manufactured parts

·· known: <u>50</u> contain flaws

·· assembly requires two parts, selected at random without replacement

·· let X : <u>non</u>-conforming parts

·· write a cumulative distribution for the # of non-conforming parts in an assembly

first: write sample space of outcomes!

$$S\{gg \quad gb \quad bg \quad bb\}$$

·· now let's determine probabilities associated with these outcomes ·· sampling w/out replacement!

$$P\{gg\} = \frac{\overset{\text{good parts}}{800}}{\underset{\text{total parts}}{850}} \cdot \frac{\overset{\text{good parts left after 1st selection}}{799}}{\underset{\text{parts left after first selection!}}{849}}$$

$= 0.886$ or 88.6%

$$P\{gb\} = \frac{800}{850} \cdot \frac{50}{849} = 0.05543$$

all 50 bad ones still in batch

parts left

or 5.5%

$$P\{bg\} = \frac{50}{850} \cdot \frac{800}{849} = 0.05543$$

all 800 good ones still there

bad ones left

$$P\{bb\} = \frac{50}{850} \cdot \frac{49}{849} = 0.003$$

or 0.3%

.. now we can put together a probability distribution

non-conforming parts

↳ possible values of X

$S\{0\ 1\ 2\}$

$$P(0) = P\{gg\} = 0.886$$

$$P(1) = P\{gb\ bg\} = 0.05543 + 0.05543 \doteq 0.111$$

exactly

$$P(2) = P\{bb\} = 0.003$$

now we compute <u>cumulative</u> probabilities

$$P(X \le x) \quad \text{(up to and including)}$$

$$= F(x)$$

capital F notation

$$F(0) = P(X \le 0) = P(0) = \underline{0.886}$$

up to one bad one

$$F(1) = P(X \le 1) = P(0) + P(1) = 0.886 + 0.111 = \underline{0.997}$$

$$F(2) = P(X \le 2) = P(0) + P(1) + P(2) = \underline{1}$$

or <u>100%</u>

Mean and Variance of Discrete Random Variable

- we already know mean and variance as sample parameters $\bar{x}$ and s^2
- we can think of population mean μ and variance σ^2 in a new way

define mean or expected value of discrete random variable X:

$$\mu = E(X) = \sum x\, f(x)$$

- weighted sum of possible values of X
  - by respective probabilities

define variance of X:

$$\sigma^2 = V(X) = E(X-\mu)^2 = \sum (x-\mu)^2 f(x)$$

- weighted sum of squares of errors

-- quantifies dispersion or scatter in data

computational formula :

not in summation !!!

$$V(X) = \sum x^2 f(x) - \mu^2$$

ex : camera problem

recall probability distribution :

$P(0) = 0.008$

$P(1) = 0.096$

$P(2) = 0.384$

$P(3) = 0.512$

-- the expected value of the # of passing cameras $= X$

$$\mu = E(X) = \sum x f(x)$$

$$= 0 \cdot 0.008 + 1 \cdot 0.096 + 2 \cdot 0.384 + 3 \cdot 0.512$$

$E(X) = 2.4$ passing cameras

$\therefore$ it is perfectly okay that 2.4 passing cameras is not a possible outcome in the discrete sample space!

$$\sigma^2 = V(x) = \sum x^2 f(x) - \mu^2$$

$$= 0^2 \cdot 0.008 + 1^2 \cdot 0.096 + 2^2 \cdot 0.384 + 3^2 \cdot 0.512 - 2.4^2$$

$$\underline{V(x) = 0.48 \text{ (passing cameras)}^2}$$

$$\sigma = +\sqrt{\sigma^2} = \sqrt{0.48} = \underline{0.6928 \text{ passing cameras}}$$