HW 10 6.1 #2,6,11,14,17,18,27,28 Alex Yeoh

6.2 # 9,10,12,13,15,19,20

6.3 # 1,3,4,7,9,12,13,15,19

6.1

2) $w \cdot w = (3\cdot 3)+(-1\cdot -1)+(-5\cdot -5) = 9+1+25 = 35$

$x \cdot w = (6\cdot 3) + (-2\cdot -1) + (3\cdot -5) = 18+2-15 = 5$

$\frac{x\cdot w}{w\cdot w} = \frac{5}{35}$

6) $x \cdot x = (6\cdot 6)+(-2\cdot -2)+(3\cdot 3) = 36+4+9 = 49$

$\left(\frac{x\cdot w}{x\cdot x}\right)x = \frac{5}{49}\begin{pmatrix}6\\-2\\3\end{pmatrix} = \begin{bmatrix}\frac{30}{49}\\ \frac{-10}{49}\\ \frac{15}{49}\end{bmatrix}$

11) $\sqrt{\left(\frac{7}{4}\right)^2+\left(\frac{1}{2}\right)^2+1^2} = \sqrt{69/16} = \frac{\sqrt{69}}{4}$

$\frac{\begin{bmatrix}7/4\\1/2\\1\end{bmatrix}}{\sqrt{69}/4} = \begin{bmatrix}\frac{4}{\sqrt{69}}\cdot\frac{7}{4}\\ \frac{4}{\sqrt{69}}\cdot\frac{1}{2}\\ \frac{4}{\sqrt{69}}\cdot 1\end{bmatrix} = \begin{bmatrix}7/\sqrt{69}\\ 2/\sqrt{69}\\ 4/\sqrt{69}\end{bmatrix}$

14) $u - z = \begin{bmatrix}0 & +4\\ -5 & +1\\ 2 & -8\end{bmatrix} = \begin{bmatrix}4\\-4\\-6\end{bmatrix}$

$\|u-z\| = \sqrt{4^2+(-4)^2+(-6)^2} = \sqrt{68}$

17) $(3\cdot -4)+(2\cdot 1)+(-5\cdot -2)+(0\cdot 6) = -12+2+10+0 = 0$ orthogonal

18) $(-3\cdot 1)+(7\cdot -8)+(4\cdot 15)+(0\cdot -7) = -3-56+60+0 = 1$ not orthogonal

27) $y \cdot u = 0$

$y \cdot v = 0$

$y(u+v) = y\cdot u + y\cdot v = 0+0 = 0 \therefore$ orthogonal

28) $y \cdot u = 0$

$y \cdot v = 0$

$y \cdot w = y(c_1 u + c_2 v) = c_1 y\cdot u + c_2 y \cdot v = c_1\cdot 0 + c_2 \cdot 0 = 0 \therefore$ orthogonal

6.2

9) $(1\cdot -1)+(0\cdot 4)+(1\cdot 1) = -1+0+1 = 0$

$(1\cdot 2)+(0\cdot 1)+(1\cdot -2) = 2+0-2 = 0$

$(-1\cdot 2)+(4\cdot 1)+(1\cdot -2) = -2+4-2 = 0$

$\left[\begin{array}{ccc|c}1&-1&2&8\\0&4&1&-4\\1&1&-2&-3\end{array}\right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|c}1&-1&2&8\\0&4&1&-4\\0&2&-4&-11\end{array}\right] \xrightarrow{R_2-2R_3} \left[\begin{array}{ccc|c}1&-1&2&8\\0&0&9&18\\0&2&-4&-11\end{array}\right] \xrightarrow{R_2\Leftrightarrow R_3} \left[\begin{array}{ccc|c}1&-1&2&8\\0&2&-4&-11\\0&0&9&18\end{array}\right] \xrightarrow[R_3/9]{R_2/2} \left[\begin{array}{ccc|c}1&-1&2&8\\0&1&-2&-11/2\\0&0&1&2\end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c}1&0&0&5/2\\0&1&-2&-11/2\\0&0&1&2\end{array}\right] \xrightarrow{R_2+2R_1}$

$\left[\begin{array}{ccc|c}1&0&0&5/2\\0&1&0&-3/2\\0&0&1&2\end{array}\right]$

$\frac{5}{2}u_1 - \frac{3}{2}u_2 + 2u_3 = x$

6.2 continued

10) $(3\cdot 2)+(-3\cdot 2)+(0\cdot -1) = 6-6+0 = 0$

$(3\cdot 1)+(3\cdot -1)+(0\cdot 4) = 3-3+0 = 0$

$(2\cdot 1)+(2\cdot 1)+(-1\cdot 4) = 2+2-4 = 0$

$\left[\begin{array}{ccc|c}3&2&1&5\\-3&2&1&-3\\0&-1&4&1\end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|c}3&2&1&5\\0&4&2&2\\0&-1&4&1\end{array}\right] \xrightarrow{R_2+4R_3} \left[\begin{array}{ccc|c}3&2&1&5\\0&0&18&6\\0&-1&4&1\end{array}\right] \xrightarrow{R_2\Leftrightarrow R_3} \left[\begin{array}{ccc|c}3&2&1&5\\0&-1&4&1\\0&0&18&6\end{array}\right] \xrightarrow{R_1+2R_2} \left[\begin{array}{ccc|c}3&0&9&7\\0&-1&4&1\\0&0&18&6\end{array}\right] \xrightarrow{R_1/3,\ R_2/-1,\ R_3/18} \left[\begin{array}{ccc|c}1&0&3&7/3\\0&1&-4&-1\\0&0&1&-1/3\end{array}\right]$

$\xrightarrow{R_1-3R_3,\ R_2+4R_3} \left[\begin{array}{ccc|c}1&0&0&4/3\\0&1&0&1/3\\0&0&1&1/3\end{array}\right]$

$\frac{4}{3}u_1 + \frac{1}{3}u_2 + \frac{1}{3}u_3 = x$

12) $\frac{[1\ -1]\begin{bmatrix}-1\\3\end{bmatrix}}{[-1\ 3]\begin{bmatrix}-1\\3\end{bmatrix}}\begin{bmatrix}-1\\3\end{bmatrix} = \frac{(1\cdot -1)+(-1\cdot 3)}{(-1\cdot -1)+(3\cdot 3)}\begin{bmatrix}-1\\3\end{bmatrix} = \frac{-1-3}{1+9}\begin{bmatrix}-1\\3\end{bmatrix}$

$= \frac{-4}{10}\begin{bmatrix}-1\\3\end{bmatrix} = \frac{-2}{5}\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}2/5\\-6/5\end{bmatrix}$

13) let v be a vector orthogonal to u

$u \cdot v = 0$

$u_i v_i + u_j v_2 = 4\cdot v_1 - 7v_2 = 0$

$4v_1 = 7v_2$ let v_2 be 4

$v_1 = \frac{7}{4}v_2$ $\quad v_1 = 7$

$\left[\begin{array}{cc|c}4&7&2\\-7&4&3\end{array}\right] \xrightarrow{R_1/4} \left[\begin{array}{cc|c}1&7/4&1/2\\-7&4&3\end{array}\right] \xrightarrow{R_2+7R_1} \left[\begin{array}{cc|c}1&7/4&1/2\\0&65/4&13/2\end{array}\right] \xrightarrow{R_2\cdot\frac{4}{65}} \left[\begin{array}{cc|c}1&7/4&1/2\\0&1&2/5\end{array}\right] \xrightarrow{R_1-\frac{7}{4}R_2} \left[\begin{array}{cc|c}1&0&-1/5\\0&1&2/5\end{array}\right]$

$\frac{-1}{5}\begin{bmatrix}4\\-7\end{bmatrix} + \frac{2}{5}\begin{bmatrix}7\\4\end{bmatrix}$

$= \begin{bmatrix}-4/5\\7/5\end{bmatrix} + \begin{bmatrix}14/5\\8/5\end{bmatrix} = y$

6.2 continued

15) $\frac{yu}{uu}u = \frac{(3\cdot8)+(1\cdot6)}{(8\cdot8)+(6\cdot6)}\begin{bmatrix}8\\6\end{bmatrix} = \frac{24+6}{64+36}\begin{bmatrix}8\\6\end{bmatrix} = \frac{30}{100}\begin{bmatrix}8\\6\end{bmatrix} = \frac{3}{10}\begin{bmatrix}8\\6\end{bmatrix} = \begin{bmatrix}\frac{24}{10}\\\frac{18}{10}\end{bmatrix} = \begin{bmatrix}\frac{12}{5}\\\frac{9}{5}\end{bmatrix} = \hat{y}$

$y-\hat{y} = \begin{bmatrix}3\\1\end{bmatrix} - \begin{bmatrix}12/5\\9/5\end{bmatrix} = \begin{bmatrix}3/5\\-4/5\end{bmatrix}$

$\|y-\hat{y}\| = \sqrt{(3/5)^2+(-4/5)^2} = \sqrt{1} = 1$

19) $(-0.6\cdot0.8)+(0.8\cdot0.6) = (0.8\cdot0.6)-(0.8\cdot0.6) = 0$

$\sqrt{(-0.6\cdot-0.6)+(0.8\cdot0.8)} = \sqrt{0.36+0.64} = \sqrt{1} = 1$ they are orthonormal

$\sqrt{(0.8\cdot0.8)+(0.6\cdot0.6)} = \sqrt{0.64+0.36} = \sqrt{1} = 1$

20) $(-2/3\cdot1/3)+(1/3\cdot2/3)+(2/3\cdot0) = \frac{-2}{9}+\frac{2}{9}+0 = 0$

$\sqrt{(-2/3\cdot-2/3)+(1/3\cdot1/3)+(2/3\cdot2/3)} = \sqrt{\frac{4}{9}+\frac{1}{9}+\frac{4}{9}} = 1$ only orthogonal

$\sqrt{(1/3\cdot1/3)+(2/3\cdot2/3)+(0\cdot0)} = \sqrt{1/9+4/9+0} = \sqrt{5/9} = \sqrt{5}/3$

$\frac{\begin{bmatrix}1/3\\2/3\\0\end{bmatrix}}{\sqrt{5}/3} = \frac{3}{\sqrt{5}}\begin{bmatrix}1/3\\2/3\\0\end{bmatrix} = \begin{bmatrix}1/\sqrt{5}\\2/\sqrt{5}\\0\end{bmatrix}$ Orthonormal set: $\begin{bmatrix}-2/3\\1/3\\2/3\end{bmatrix}, \begin{bmatrix}1/\sqrt{5}\\2/\sqrt{5}\\0\end{bmatrix}$

6.3

1) $\hat{x} = \frac{(0\cdot10)+(1\cdot-8)+(-4\cdot2)+(-1\cdot0)}{(0\cdot0)+(1\cdot1)+(-4\cdot-4)+(-1\cdot-1)}\begin{bmatrix}0\\1\\-4\\-1\end{bmatrix} + \frac{(3\cdot10)+(5\cdot-8)+(1\cdot2)+(0\cdot1)}{(3\cdot3)+(5\cdot5)+(1\cdot1)+(1\cdot1)}\begin{bmatrix}3\\5\\1\\1\end{bmatrix} + \frac{(1\cdot10)+(0\cdot-8)+(1\cdot2)+(-4\cdot0)}{(1\cdot1)+(0\cdot0)+(1\cdot1)+(-4\cdot-4)}\begin{bmatrix}1\\0\\1\\-4\end{bmatrix} = \frac{0-8-8+0}{18}\begin{bmatrix}0\\1\\-4\\-1\end{bmatrix} + \frac{30-40+2+0}{36}\begin{bmatrix}3\\5\\1\\1\end{bmatrix} + \frac{10+0+2+0}{18}\begin{bmatrix}1\\0\\1\\-4\end{bmatrix}$

$= \frac{-16}{18}\begin{bmatrix}0\\1\\-4\\-1\end{bmatrix} + \frac{-8}{36}\begin{bmatrix}3\\5\\1\\1\end{bmatrix} + \frac{12}{18}\begin{bmatrix}1\\0\\1\\-4\end{bmatrix} = \frac{6}{9}\begin{bmatrix}1\\0\\1\\-4\end{bmatrix} - \frac{8}{9}\begin{bmatrix}0\\1\\-4\\-1\end{bmatrix} - \frac{2}{9}\begin{bmatrix}3\\5\\1\\1\end{bmatrix} = \begin{bmatrix}6/9\\0\\6/9\\-24/9\end{bmatrix} - \begin{bmatrix}0\\8/9\\-32/9\\-8/9\end{bmatrix} - \begin{bmatrix}6/9\\10/9\\2/9\\2/9\end{bmatrix} = \begin{bmatrix}0\\-18/9\\36/9\\-18/9\end{bmatrix} = \begin{bmatrix}0\\-2\\4\\-2\end{bmatrix}$

$z = x-\hat{x} = \begin{bmatrix}10\\-8\\2\\0\end{bmatrix} - \begin{bmatrix}0\\-2\\4\\-2\end{bmatrix} = \begin{bmatrix}10\\-6\\-2\\2\end{bmatrix}$ $x = z+\hat{x} = \begin{bmatrix}10\\-6\\-2\\2\end{bmatrix} + \begin{bmatrix}0\\-2\\4\\-2\end{bmatrix}$

3) $(1\cdot-1)+(1\cdot1)+(0\cdot0) = -1+1+0 = 0$ orthogonal

$\frac{y\cdot u_1}{u_1\cdot u_1}u_1 + \frac{y\cdot u_2}{u_2\cdot u_2}u_2 = \frac{(-1\cdot1)+(4\cdot1)+(3\cdot0)}{(1\cdot1)+(1\cdot1)+(0\cdot0)}u_1 + \frac{(-1\cdot-1)+(4\cdot1)+(3\cdot0)}{(-1\cdot-1)+(1\cdot1)+(0\cdot0)}u_2 = \frac{-1+4+0}{2}u_1 + \frac{1+4+0}{2}u_2 = \frac{3}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} + \frac{5}{2}\begin{bmatrix}-1\\1\\0\end{bmatrix} = \begin{bmatrix}\frac{3}{2}\\\frac{3}{2}\\0\end{bmatrix} + \begin{bmatrix}-\frac{5}{2}\\\frac{5}{2}\\0\end{bmatrix} = \begin{bmatrix}-\frac{2}{2}\\\frac{8}{2}\\0\end{bmatrix}$

$\hat{y} = \begin{bmatrix}-1\\4\\0\end{bmatrix}$

4) $(3\cdot-4)+(4\cdot3)+(0\cdot0) = -12+12+0 = 0$ orthogonal

$\frac{yu_1}{u_1u_1}u_1 + \frac{yu_2}{u_2u_2}u_2 = \frac{(6\cdot3)+(3\cdot4)+(-2\cdot0)}{(3\cdot3)+(4\cdot4)+(0\cdot0)}u_1 + \frac{(6\cdot-4)+(3\cdot3)+(-2\cdot0)}{(-4\cdot-4)+(3\cdot3)+(0\cdot0)}u_2 = \frac{18+12+0}{25}u_1 + \frac{-24+9+0}{25}u_2 = \frac{30u_1}{25} - \frac{15u_2}{25} = \frac{6}{5}\begin{bmatrix}3\\4\\0\end{bmatrix} - \frac{3}{5}\begin{bmatrix}-4\\3\\0\end{bmatrix} = \begin{bmatrix}18/5\\24/5\\0\end{bmatrix} - \begin{bmatrix}-12/5\\9/5\\0\end{bmatrix}$

$= \begin{bmatrix}30/5\\15/5\\0\end{bmatrix} = \begin{bmatrix}6\\3\\0\end{bmatrix}$

7) $(1\cdot5)+(3\cdot1)+(-2\cdot4) = 5+3-8 = 0$

$Yu_1 = (1\cdot1)+(3\cdot3)+(5\cdot-2) = 1+9-10 = 0$

$Yu_2 = (1\cdot5)+(3\cdot1)+(5\cdot4) = 5+3+20 = 28$

$u_1u_1 = (1\cdot1)+(3\cdot3)+(-2\cdot-2) = 1+9+4 = 14$

$u_2u_2 = (5\cdot5)+(1\cdot1)+(4\cdot4) = 25+1+16 = 42$

$\frac{Yu_1}{u_1u_1}u_1 + \frac{Yu_2}{u_2u_2}u_2 = \frac{0}{14}u_1 + \frac{28}{42}\begin{bmatrix}5\\1\\4\end{bmatrix} = \frac{2}{3}\begin{bmatrix}5\\1\\4\end{bmatrix} = \begin{bmatrix}10/3\\2/3\\8/3\end{bmatrix} = \hat{y}$

$z = y-\hat{y} = \begin{bmatrix}1\\3\\5\end{bmatrix} - \begin{bmatrix}10/3\\2/3\\8/3\end{bmatrix} = \begin{bmatrix}-7/3\\7/3\\7/3\end{bmatrix}$ $y = z+\hat{y} = \begin{bmatrix}-7/3\\7/3\\7/3\end{bmatrix} + \begin{bmatrix}10/3\\2/3\\8/3\end{bmatrix}$

6.3 continued

9) $u_1u_2 = (1\cdot-1)+(1\cdot3)+(0\cdot1)+(1\cdot-2) = -1+3+0+-2=0$

$u_1u_3 = (1\cdot-1)+(1\cdot0)+(0\cdot1)+(1\cdot1) = -1+0+0+1=0$

$u_2u_3 = (-1\cdot-1)+(3\cdot0)+(1\cdot1)+(-2\cdot1) = 1+0+1-2=0$

$u_1u_1 = (1\cdot1)+(1\cdot1)+(0\cdot0)+(1\cdot1) = 1+1+0+1 = 3$

$u_2u_2 = (-1\cdot-1)+(3\cdot3)+(1\cdot1)+(-2\cdot-2) = 1+9+1+4 = 15$

$u_3u_3 = (-1\cdot-1)+(0\cdot0)+(1\cdot1)+(1\cdot1) = 1+0+1+1 = 3$

$yu_1 = (4\cdot1)+(3\cdot1)+(3\cdot0)+(-1\cdot1) = 4+3+0-1 = 6$

$yu_2 = (4\cdot-1)+(3\cdot3)+(3\cdot1)+(-1\cdot-2) = -4+9+3+2 = 10$

$yu_3 = (4\cdot-1)+(3\cdot0)+(3\cdot1)+(-1\cdot1) = -4+0+3+-1 = -2$

$$\hat{y} = \frac{yu_1}{u_1u_1}u_1 + \frac{yu_2}{u_2u_2}u_2 + \frac{yu_3}{u_3u_3} = \frac{6}{3}\begin{bmatrix}1\\1\\0\\1\end{bmatrix} + \frac{10}{15}\begin{bmatrix}-1\\3\\1\\-2\end{bmatrix} - \frac{2}{3}\begin{bmatrix}-1\\0\\1\\1\end{bmatrix} = \begin{bmatrix}2\\2\\0\\2\end{bmatrix} + \begin{bmatrix}-2/3\\2\\2/3\\-4/3\end{bmatrix} - \begin{bmatrix}-2/3\\0\\2/3\\2/3\end{bmatrix} = \begin{bmatrix}2\\4\\0\\0\end{bmatrix}$$

$$z = y - \hat{y} = \begin{bmatrix}4\\3\\3\\-1\end{bmatrix} - \begin{bmatrix}2\\4\\0\\0\end{bmatrix} = \begin{bmatrix}2\\-1\\3\\-1\end{bmatrix} \quad y = z + \hat{y} = \begin{bmatrix}2\\-1\\3\\-1\end{bmatrix} + \begin{bmatrix}2\\4\\0\\0\end{bmatrix}$$

12) $v_1v_2 = (1\cdot-4)+(-2\cdot1)+(-1\cdot0)+(2\cdot3) = -4-2+0+6=0$

$yv_1 = (3\cdot1)+(-1\cdot-2)+(1\cdot-1)+(13\cdot2) = 3+2-1+26 = 30$ $\quad v_1v_1 = (1\cdot1)+(-2\cdot-2)+(-1\cdot-1)+(2\cdot2) = 1+4+1+4 = 10$

$yv_2 = (3\cdot-4)+(-1\cdot1)+(1\cdot0)+(13\cdot3) = -12-1+0+39 = 26$ $\quad v_2v_2 = (-4\cdot-4)+(1\cdot1)+(0\cdot0)+(3\cdot3) = 16+1+0+9 = 26$

$$\hat{y} = \frac{yv_1}{v_1v_1}v_1 + \frac{yv_2}{v_2v_2}v_2 = \frac{30}{10}v_1 + \frac{26}{26}v_2 = 3\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix} + \begin{bmatrix}-4\\1\\0\\3\end{bmatrix} = \begin{bmatrix}3\\-6\\-3\\6\end{bmatrix} + \begin{bmatrix}-4\\1\\0\\3\end{bmatrix} = \begin{bmatrix}-1\\-5\\-3\\9\end{bmatrix}$$

13) $v_1v_2 = (2\cdot1)+(-1\cdot1)+(-3\cdot0)+(1\cdot-1) = 2-1+0-1 = 0$

$zv_1 = (3\cdot2)+(-7\cdot-1)+(2\cdot-3)+(3\cdot1) = 6+7-6+3 = 10$ $\quad v_1v_1 = (2\cdot2)+(-1\cdot-1)+(-3\cdot-3)+(1\cdot1) = 4+1+9+1 = 15$

$zv_2 = (3\cdot1)+(-7\cdot1)+(2\cdot0)+(3\cdot-1) = 3-7+0-3 = -7$ $\quad v_2v_2 = (1\cdot1)+(1\cdot1)+(0\cdot0)+(-1\cdot-1) = 1+1+0+1 = 3$

$$\hat{z} = \frac{zv_1}{v_1v_1}v_1 + \frac{zv_2}{v_2v_2}v_2 = \frac{10}{15}\begin{bmatrix}2\\-1\\-3\\1\end{bmatrix} - \frac{7}{3}\begin{bmatrix}1\\1\\0\\-1\end{bmatrix} = \begin{bmatrix}4/3\\-2/3\\-2\\2/3\end{bmatrix} - \begin{bmatrix}7/3\\7/3\\0\\-7/3\end{bmatrix} = \begin{bmatrix}-1\\-3\\-2\\3\end{bmatrix}$$

15) $u_1u_2 = (-3\cdot-3)+(-5\cdot2)+(1\cdot1) = 9-10+1=0$

$yu_1 = (5\cdot-3)+(-9\cdot-5)+(5\cdot1) = -15+45+5 = 35$ $\quad u_1u_1 = (-3\cdot-3)+(-5\cdot-5)+(1\cdot1) = 9+25+1 = 35$

$yu_2 = (5\cdot-3)+(-9\cdot2)+(5\cdot1) = -15-18+5 = -28$ $\quad u_2u_2 = (-3\cdot-3)+(2\cdot2)+(1\cdot1) = 9+4+1 = 14$

$$\hat{y} = \frac{yu_1}{u_1u_1}u_1 + \frac{yu_2}{u_2u_2}u_2 = \frac{35}{35}\begin{bmatrix}-3\\-5\\1\end{bmatrix} - \frac{28}{14}\begin{bmatrix}-3\\2\\1\end{bmatrix} = \begin{bmatrix}-3\\-5\\1\end{bmatrix} - \begin{bmatrix}-6\\4\\2\end{bmatrix} = \begin{bmatrix}3\\-9\\-1\end{bmatrix}$$

$$y - \hat{y} = \begin{bmatrix}5\\-9\\5\end{bmatrix} - \begin{bmatrix}3\\-9\\-1\end{bmatrix} = \begin{bmatrix}2\\0\\6\end{bmatrix} \quad \|y-\hat{y}\| = \sqrt{2^2+0^2+6^2} = \sqrt{4+36} = \sqrt{40}$$

19) $u_3u_1 = -2 \quad u_1u_1 = 1+1+4 = 6$

$u_3u_2 = 2 \quad u_2u_2 = 25+1+4 = 30$

$$\hat{u}_3 = \frac{u_3u_1}{u_1u_1}u_1 + \frac{u_3u_2}{u_2u_2}u_2 = \frac{-2}{6}u_1 + \frac{2}{30}u_2 = \frac{1}{15}\begin{bmatrix}5\\-1\\2\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\1\\-2\end{bmatrix} = \begin{bmatrix}1/3\\-1/15\\2/15\end{bmatrix} - \begin{bmatrix}1/3\\1/3\\-2/3\end{bmatrix} = \begin{bmatrix}0\\-6/15\\12/15\end{bmatrix}$$

$$u_3 - \hat{u}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix} - \begin{bmatrix}0\\-6/15\\12/15\end{bmatrix} = \begin{bmatrix}0\\6/15\\3/15\end{bmatrix}$$