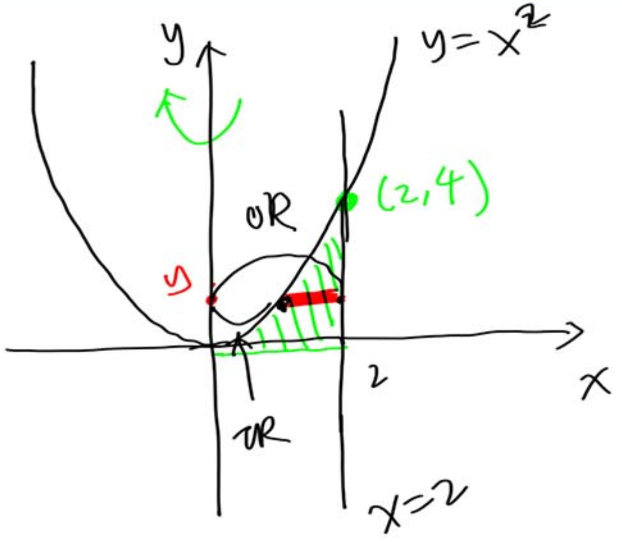
LECTURE NO. 5

2.3 Volume of Revolution: Cylindrical Shells

Wright State University

Let R be the region bound by $y = x^2$, $x = 2$, and x-axis. Find the volume of the solid formed by revolving R around y-axis.



$$\text{Volume} = \int_0^4 \text{cross sectional Area } dy$$

$$y = x^2 \longrightarrow x = \sqrt{y}$$ Washer Method

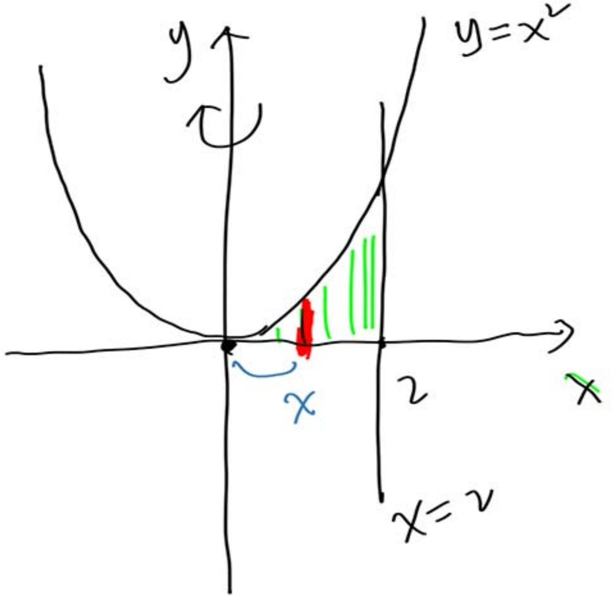
$$C.S.A = \pi (OR)^2 - \pi (IR)^2$$

$$IR = \sqrt{y} \qquad OR = 2$$

$$V = \int_0^4 \pi 2^2 - \pi(\sqrt{y})^2 \, dy = \pi \int_0^4 4 - y \, dy$$

$$V = \pi\left(4y - \frac{y^2}{2}\right)\Big|_0^4 = 8\pi$$ FINAL ANSWER

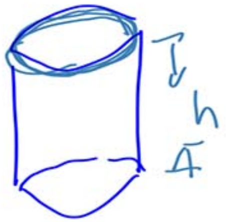
Can we integrate with respect to x?



Volume $= \int_0^2$ cross sectional Area dx

Shell Method

side of a cylinder = shell



height = length of cutting segment $= x^2$

radius = distance between the cutting segment and the axis of rotation $= x$

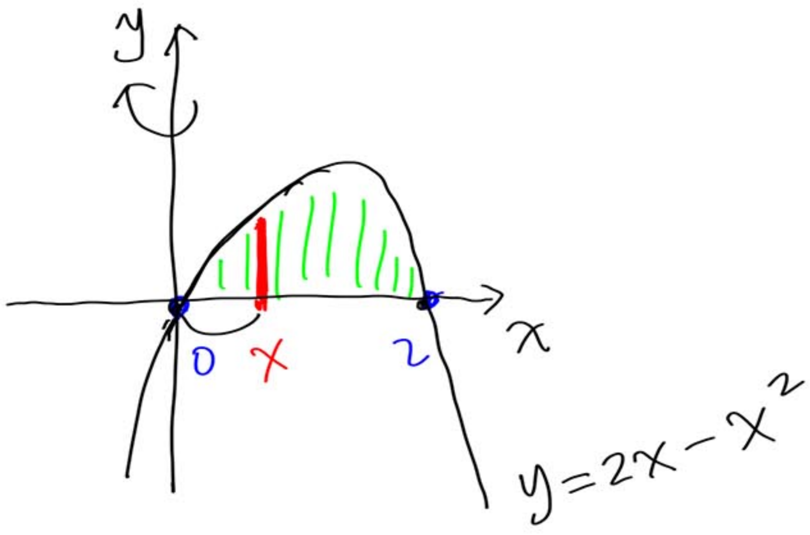
Area of shell $= 2\pi R \cdot h$

$$C.S.A = 2\pi x \cdot x^2 = 2\pi x^3$$

$$V = \int_0^2 2\pi x^3 \, dx = 2\pi \cdot \frac{x^4}{4} \Big|_0^2 = 8\pi$$

same answer.

Let R be the region bounded by $y = 2x - x^2$ and x-axis. Find the volume of solid formed by rotating R around y-axis.



Since it is not easy to solve for x from $y=2x-x^2$, we will choose x to integrate

$V = \int_0^2 C.S.A \, dx$ Shell Method

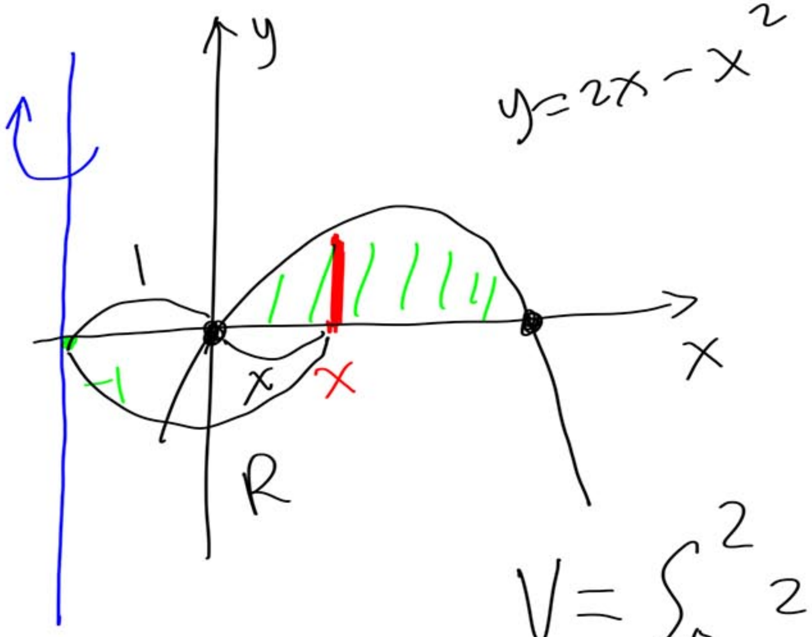
height $= 2x - x^2$ Radius $= x$

$$V = \int_0^2 2\pi x (2x - x^2) dx = \int_0^2 2\pi (2x^2 - x^3) dx$$

$$V = 2\pi \left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^2 = 2\pi\left(\frac{16}{3} - 4\right) = \frac{8}{3}\pi$$

FINAL ANSWER

What if R is rotated around the line $x = -1$?



$$V = \int_0^2 C.S.A \; dx$$

Shell Method

height $= 2x - x^2$ Radius $= x + 1$

$$C.S.A = 2\pi(x+1)(2x-x^2)$$

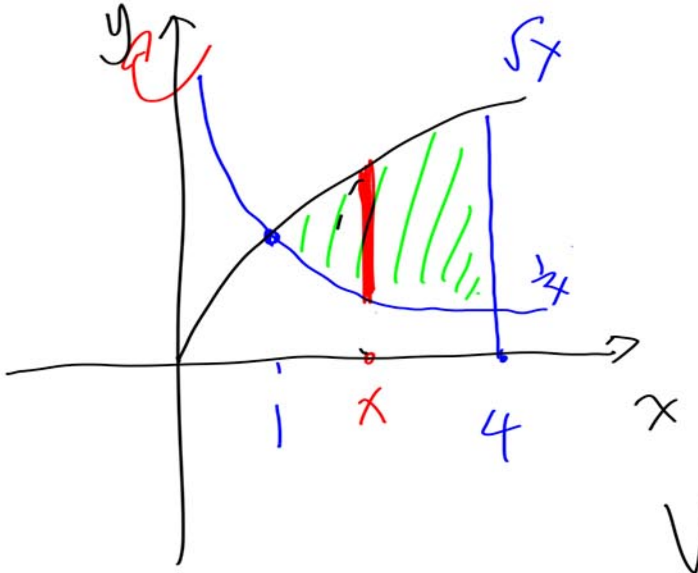
$$V = \int_0^2 2\pi(x+1)(2x-x^2)\,dx = 2\pi\int_0^2 2x^2 - x^3 + 2x - x^2\,dx$$

$$V = 2\pi\int_0^2 x^2 - x^3 + 2x\,dx = 2\pi\left(\frac{x^3}{3} - \frac{x^4}{4} + x^2\right)\Big|_0^2$$

$$V = 2\pi\left(\frac{8}{3} - \frac{16}{4} + 4\right) = \frac{16}{3}\pi$$

FINAL ANSWER

Let S be the region bounded by $y = \sqrt{x}$ and $y = \frac{1}{x}$ over $[1, 4]$. Find the volume of the solid generated by rotating S around y-axis.



$\text{Volume} = \int_1^4 C.S.A \, dx$ Shell Method

$\text{height} = \sqrt{x} - \frac{1}{x}$ $\quad \text{Radius} = x$

$C.S.A = 2\pi x \left(\sqrt{x} - \frac{1}{x}\right)$

$V = \int_1^4 2\pi x \left(\sqrt{x} - \frac{1}{x}\right) dx = 2\pi \int_1^4 x^{\frac{3}{2}} - 1 \, dx$

$V = 2\pi\left(\frac{2}{5} x^{\frac{5}{2}} - x\right)\Big|_1^4 = 2\pi\left(\frac{64}{5} - 4\right) - 2\pi\left(\frac{2}{5} - 1\right)$

$V = 2\pi\left(\frac{44}{5} + \frac{3}{5}\right) = \frac{94}{5}\pi$ FINAL ANSWER

What Method Should I Use?

- If the solid is generated by rotating around a horizontal line (ex. x-axis, $y = -1$, $y = 2$ etc):
  - If we integrate with respect to x, use Washer/Disk Method;
  - If we integrate with respect to y, use Shell Method.
- If the solid is generated by rotating around a vertical line (ex y-axis, $x = -2$, $x = 3$, etc):
  - If we integrate with respect to x, use Shell Method;
  - If we integrate with respect to y, use Washer/Disk Method.