HW 6 3.1 #5, 11, 13, 15, 37
3.2 #7, 9, 11, 13, 15, 17, 19, 21, 25, 40
3.3 #21, 27, 29

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3.1

5) $\begin{vmatrix} 2 & 1 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{vmatrix} = (-1)^{1+2} 3 \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} + (-1)^{3+2} 1 \begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = -3(4\cdot5-3\cdot6) + -1(2\cdot3 - -3\cdot4) = -3(20-18)-(6+12)$
$= -6-18 = -24$

11) $\begin{vmatrix} 3 & 5 & -6 & 4 \\ 0 & -2 & 3 & -3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 3 \end{vmatrix} = (-1)^{1+1} 3 \begin{vmatrix} -2 & 3 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{vmatrix} = 3\left((-1)^{1+1}(-2)\begin{vmatrix} 1 & 5 \\ 0 & 3 \end{vmatrix}\right) = 3(-2(3-0)) = -18$

13) $\begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix} = (-1)^{3+2} 3 \begin{vmatrix} 4 & -7 & 3 & -5 \\ 0 & 2 & 0 & 0 \\ 5 & 5 & 2 & -3 \\ 0 & 9 & -1 & 2 \end{vmatrix} = -3\left((-1)^{2+2} 2 \begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix}\right) = -6\left((-1)^{1+1} 4 \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} + (-1)^{2+1} 5 \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix}\right)$
$= -6(4(4-3)+-5(6-5)) = -6(4-5) = 6$

15) $\begin{vmatrix} 1 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -2 \end{vmatrix} = -6+0+40-0-10-0 = 40-16 = 24$

37) $\begin{vmatrix} 15 & 5 \\ 20 & 10 \end{vmatrix} = (15\cdot10-20\cdot5) = 150-100 = 50$ ≠ no

$\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = (3\cdot2-4\cdot1) = 2$ $2\cdot5 = 10$

3.2

7) $\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 4 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 4 \\ 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{vmatrix}$ b/c $|A| = 0$

9) $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 0 & 5 & 3 \\ 3 & -3 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 7 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 7 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 0 & -4 \end{vmatrix} = 1\cdot1\cdot\begin{vmatrix} 7 & 7 \\ 0 & -4 \end{vmatrix} = 7\cdot-4-0\cdot7 = -28$

11) $\begin{vmatrix} 3 & 4 & -3 & -1 \\ 3 & 0 & 1 & 3 \\ -6 & 0 & -4 & 3 \\ -6 & 8 & -4 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 4 & -4 & 2 \\ 3 & 0 & 1 & -3 \\ 0 & 0 & -2 & -3 \\ 0 & 8 & -6 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 4 & -4 & 2 \\ 3 & 0 & 1 & -3 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 2 & 1 \end{vmatrix} = -3\begin{vmatrix} 4 & -4 & 2 \\ 0 & -2 & -3 \\ 0 & 2 & 1 \end{vmatrix} = -12\begin{vmatrix} -2 & -3 \\ 2 & 1 \end{vmatrix} = -12(-2\cdot1 - -3\cdot2) = -12(-2+6) = -48$

13) $\begin{vmatrix} 2 & 5 & 4 & 1 \\ 4 & 7 & 6 & 2 \\ 6 & -2 & -4 & 0 \\ -6 & 7 & 7 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 4 & 1 \\ 0 & -3 & -2 & 0 \\ 6 & -2 & -4 & 0 \\ -6 & 7 & 7 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 4 & 1 \\ 0 & -3 & -2 & 0 \\ 6 & -2 & -4 & 0 \\ 0 & 5 & 3 & 0 \end{vmatrix} = -1\begin{vmatrix} 0 & -3 & -2 \\ 6 & -2 & -4 \\ 0 & 5 & 3 \end{vmatrix} = 6\begin{vmatrix} -3 & -2 \\ 5 & 3 \end{vmatrix} = 6(-3\cdot3 - -2\cdot5) = 6(-9+10) = 6$

3.2 continued

15) since $KR_i, k \neq 0$ $|A| = k|A|$; $\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix} = 3 \cdot 7 = 21$

17) Since $KR_i + R_j$ $|A| = |A|$; $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix} = 7$

19) since $KR_i + R_j$ $|A| = |A|$; $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix}$

since $KR_i, k \neq 0$ $|A| = k|A|$; $\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} = 2 \cdot 7 = 14$

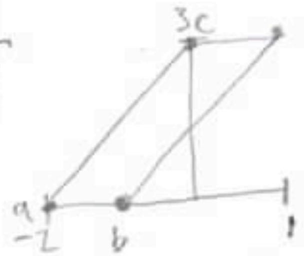
21) $\begin{vmatrix} 2 & 6 & 6 \\ 1 & 3 & 2 \\ 3 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 6 \\ 1 & 3 & 2 \\ 2 & 6 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 0 \\ 1 & 3 & 2 \\ 0 & 0 & 0 \end{vmatrix}$ ← all 0 row $\therefore |A| = 0 \therefore$ not invertable

25) $\begin{vmatrix} 7 & -8 & 7 \\ -4 & 5 & 0 \\ -6 & 7 & 5 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 7 \\ -4 & 1 & 0 \\ -11 & 1 & 5 \end{vmatrix} = -\begin{vmatrix} 0 & -1 & 7 \\ -4 & 0 & 7 \\ -11 & 0 & 12 \end{vmatrix} = (-1)^{1+2}(-1)\begin{vmatrix} -4 & 7 \\ -11 & 12 \end{vmatrix} \neq 0 \therefore$ invertable $\therefore$ linearly independent

40) a) $-3 \cdot -1 = 3$ b) $-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 = -1$ c) $16 \cdot -3 = -48$ d) $-3 \cdot -1 \cdot -3 = -9$ e) $\frac{1}{-1} \cdot -3 \cdot -1 = -3$

3.3

21)



$\vec{ab} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $\quad \left| \det \begin{vmatrix} -1 & -2 \\ 0 & -3 \end{vmatrix} \right| = 3$

$\vec{ac} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

27) $\left| \det \begin{vmatrix} -2 & -2 \\ 3 & 5 \end{vmatrix} \right| = |-2 \cdot 5 - -2 \cdot 3| = |-10+6| = |-4| = 4$

$\left| \det \begin{vmatrix} 6 & -3 \\ -3 & 2 \end{vmatrix} \right| = |6 \cdot 2 - -3 \cdot -3| = |12-9| = 3$

$4 \cdot 3 = 12$

29) area of a triangle $= \frac{1}{2} bh$, $b \cdot h =$ area of a rectangle $= |\det|ad||$

$\therefore \frac{1}{2}|\det|ad|| =$ area of a triangle $\therefore \frac{1}{2}|\det|v_1 v_2|| =$ area of this triangle