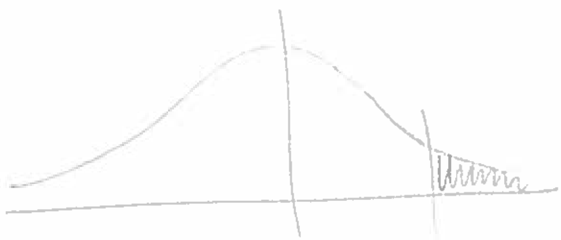
ISE 2211 • Exam II • 22 September 2017 NAME SOLUTION

1) The average seasonal snowfall in Enon, Ohio is normally distributed with a mean of 18.7 inches and a variance of 11.7 in^2. In the 1995-1996 season, there was a recorded total of 30.1 inches. Determine the probability of a seasonal snowfall of 30.1 inches or greater. Illustrate your answer by roughly sketching this probability against the standard normal distribution. Show all work.

$$Z = \frac{X-\mu}{\sigma}$$

$$Z = \frac{30.1 - 18.7}{\sqrt{11.7}} = 3.333$$ (+1)



(+1)

$Z = 3.333$

$$P(Z < 3.33) = 0.999566 \quad [\text{table}]$$ (+1)

$$\therefore P(Z > 3.33) = 1 - P(Z < 3.33) = 0.000434$$ (+1)

Determine the probability of receiving at least 8 inches of snowfall in a season.

$$Z = \frac{8 - 18.7}{\sqrt{11.7}} = -3.128$$ (+1)

$Z = -3.128$

$$P(Z < -3.13) = 0.000874$$ (+1)

$$\therefore P(Z > -3.13) = 1 - P(Z < -3.13) = 0.999126$$

or 99.9% (+1)

2) It is a fact that, on average, you receive 37 text messages per hour from your spouse. If the discrete random variable X equals the number of texts in an interval with a Poisson distribution, determine the probability of receiving exactly ~~###~~ seven texts in the next five minutes.

Formulae:

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\mu = \lambda$

$\sigma^2 = \lambda$

pick interval = 5 minutes (+1)

$\therefore$ want $f(7)$

$\frac{37 \text{ texts}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min.}} = .6167 \text{ text/min.}$

$\therefore \lambda = 0.6167 \cdot 5 = 3.083$ texts/interval (+1)

$f(7) = \frac{e^{-3.083} \, 3.083^7}{7!}$ (+1)

$f(7) = 0.02407$ or 2.4%

(+1)

If you haven't received a text from your spouse in six minutes, how does this change the probability of receiving one in the next five?

it doesn't; lack of memory property

(+1)

3) Historical data suggests that the probability of falling off your bicycle on the way home from Yellow Springs Brewery is 24.7%. Let the binomially-distributed random variable *X* represent the number of times you successfully fall off your bike. If you visit the Brewery 9 times this academic year, determine the probability of falling off your bike less than three times.

$$= f(0) + f(1) + f(2) \quad (+1)$$

$$f(0) = \binom{9}{0} .247^0 (.753)^9 = 1 \cdot 1 \cdot .753^9 = 0.077 \quad (+1)$$

$$\frac{9!}{0!\,(9-0)!} = 1$$

$$f(1) = \binom{9}{1} .247^1 (.753)^8 = (9)(.247)(.753)^8 \quad (+1)$$

$$= .2298$$

$$\frac{9!}{1!\,(8)!}$$

$$f(2) = \binom{9}{2} .247^2 (.753)^7 = (36)(.247)^2(.753)^7 \quad (+1)$$

$$= .3015$$

$$\frac{9!}{2!\,(7!)}$$

$$\therefore P(x < 3) = .6091$$

What is the expected value, variance, and standard deviation of the number of times you will fall off your bicycle on the way home from Yellow Springs Brewery?

$$\mu = np = 9 \cdot .247 = 2.223 \quad (+1)$$

$$\sigma^2 = 9 \cdot .247 (1 - .247) = 1.674 \quad (+1)$$

$$\sigma = 1.294 \quad (+1)$$

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$