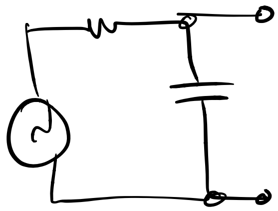
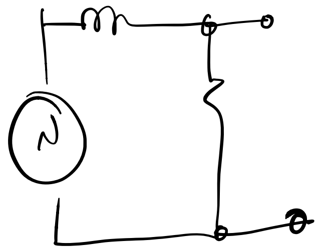
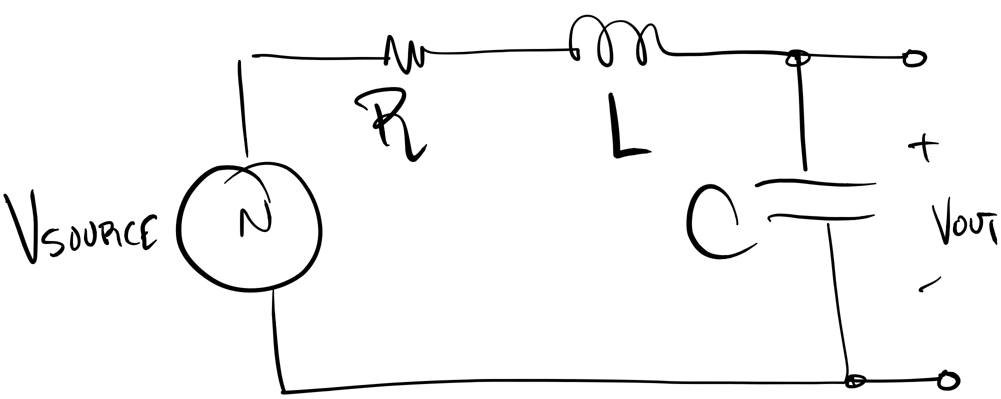
Series RLC Low-Pass Filter



1st-order RC low-pass



1st-order RL low-pass



2nd-order LPF !

complex voltage divider:

$$V_{OUT} = V_{SOURCE}\left[\frac{Z_C}{Z_R + Z_L + Z_C}\right]$$

$$\frac{V_{OUT}}{V_{SOURCE}} = \frac{Z_C}{Z_R + Z_L + Z_C} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$\times$ top and bottom by $j\omega C$:

$$\frac{V_{OUT}}{V_{SOURCE}} = \frac{1}{1 + \underbrace{(j\omega)^2}_{\hookrightarrow\ -\omega^2} LC + j\omega RC} \Rightarrow \text{signals class !!! ...}$$

$\div$ top and bottom by LC :

$$\frac{V_{OUT}}{V_{SOURCE}} = \frac{\frac{1}{LC}}{\frac{1}{LC} + (j\omega)^2 + j\omega \frac{R}{L}}$$

recall : $\underbrace{Q_{series} = \frac{1}{R}\sqrt{\frac{L}{C}}}$

all RLC $\rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}}$

correct final form for any 2nd-order low-pass filter !

$$\therefore \boxed{H(\omega) = \frac{\omega_0^2}{\omega_0^2 + j\omega \frac{\omega_0}{Q} + (j\omega)^2}}$$

- at <u>very low</u> frequencies ($\omega \to 0$; approaching DC!)

$$H(\omega) \to \frac{\omega_0^2}{\omega_0^2 + j\,\text{SMALL} + (j\,\text{SMALL})^2} \approx \frac{\omega_0^2}{\omega_0^2}$$

$$\to 1\angle 0^\circ$$

$\therefore$ low frequencies are <u>passed</u>

- at <u>very high</u> frequencies: ($\omega \to \infty$)

$$H(\omega) \to \frac{\omega_0^2}{\omega_0^2 + j\,\text{BIG} + (j\,\text{BIG})^2} = \frac{1}{\underbrace{j^2}_{-1}\text{BIG}^2}$$

$\Uparrow$ this term <u>dominates</u> denominator!

$$\to 0\angle -180^\circ$$

- let's back up: first-order LPF approached -90° in stopband

Okie dokie!

.. what happens @ $\omega = \omega_0$?

$\uparrow$ natural frequency

.. it's a little different from what happens @ ω_c ↙ corner frequency!
in a first-order filter!

$$H(j\omega_0) = \frac{\omega_0^2}{\omega_0^2 + j\omega_0 \frac{\omega_0}{Q} + (j\omega_0)^2}$$

$$= \frac{\omega_0^2}{\cancel{\omega_0^2} + \omega_0^2 \frac{j}{Q} + \cancel{\underbrace{j^2}_{=-1} \omega_0^2}} = \frac{\cancel{\omega_0^2}}{\cancel{\omega_0^2} \frac{j}{Q}}$$

$$H(j\omega_0) = \frac{Q}{j} = -jQ$$

$$\Rightarrow Q \angle -90^\circ$$

.. at ω_0, phase is -90°; a first-order LPF had -45° phase @ ω_c, makes sense

∴ much more importantly: the magnitude of the frequency response @ ω_0 isn't -3 dB like a 1st-order, but $= Q \rightarrow$ related to damping!

$$|H(\omega_0)|_{dB} = 20 \log_{10} Q$$

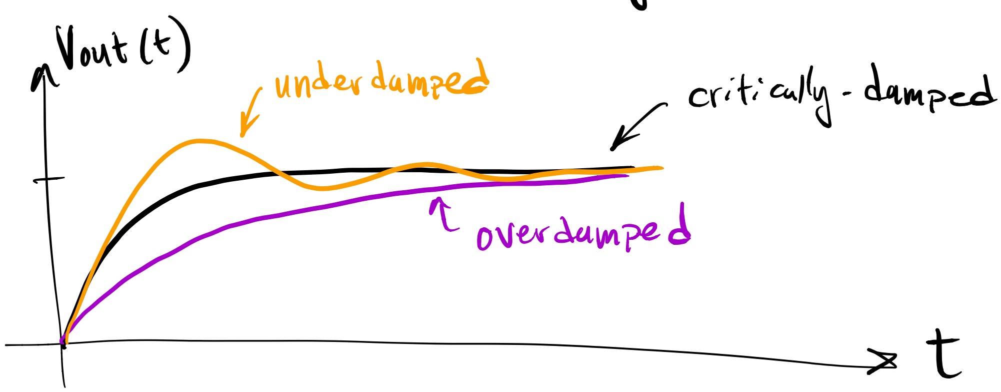
recall: in time-domain, we characterized transient response as follows:

overdamped if $Q < 0.5$

critically-damped if $Q = 0.5$

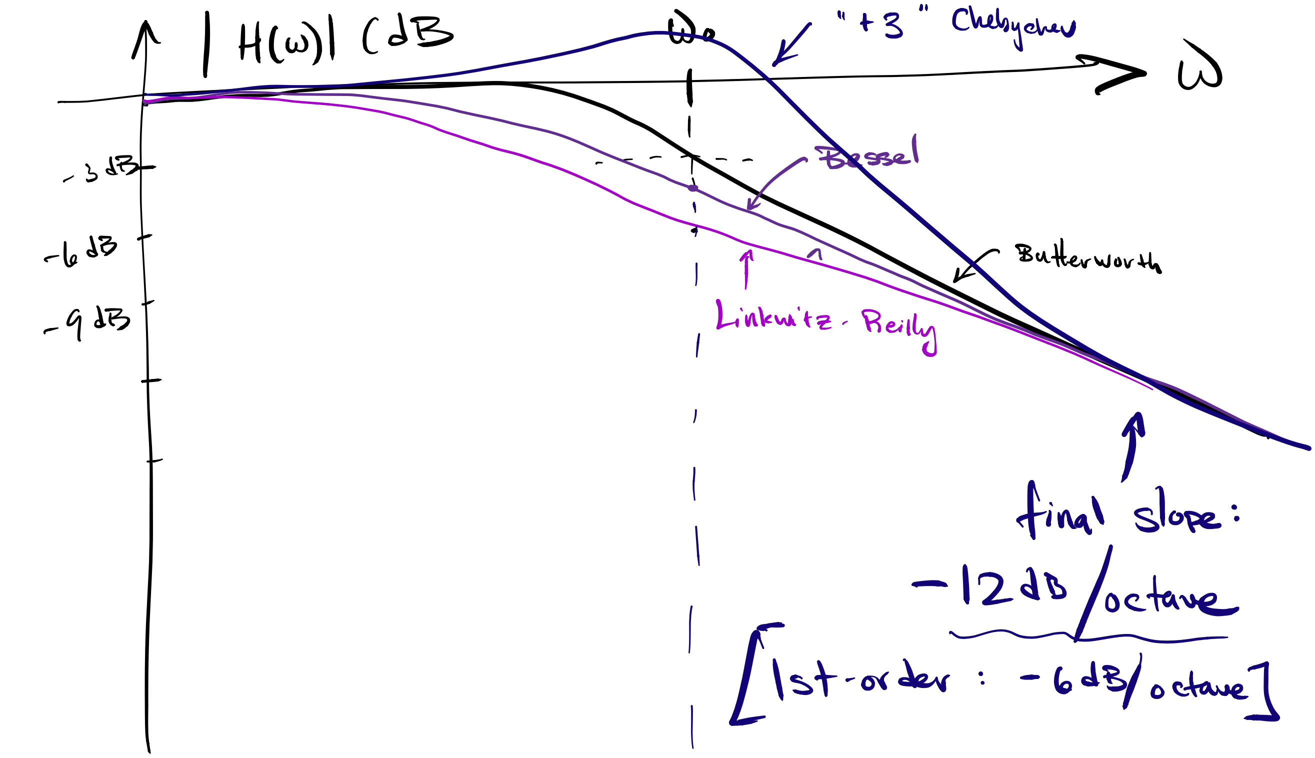
underdamped if $Q > 0.5$

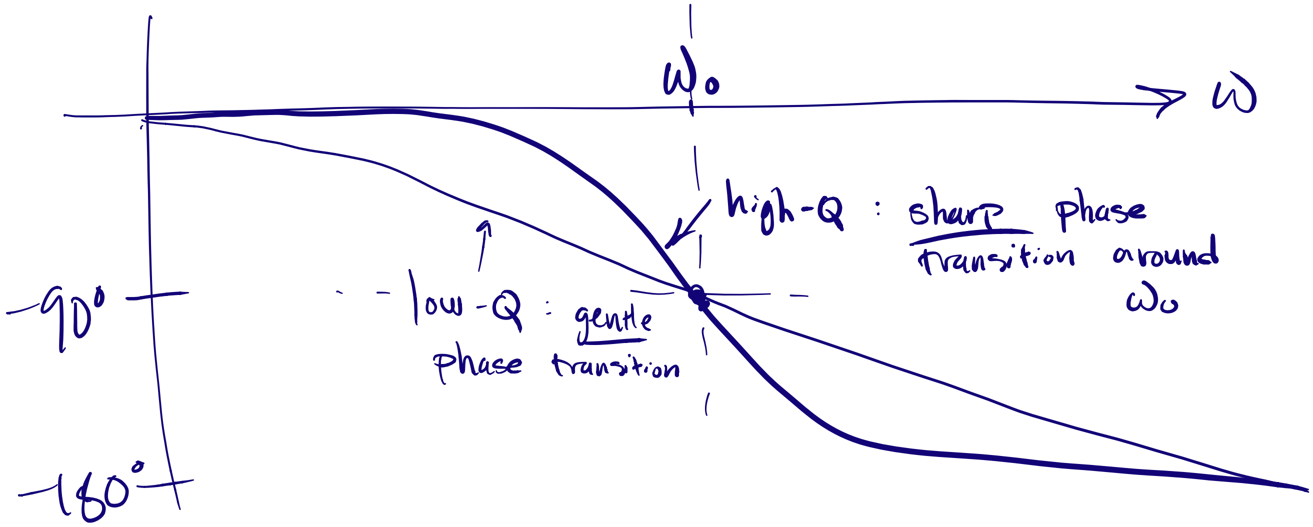
↳ overshoot, ringing, etc..



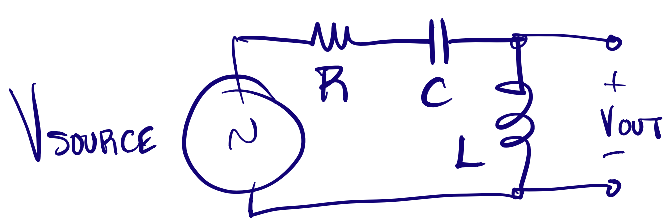
.. for frequency-domain analysis, we look at it entirely differently:

Q	filter type	freq./time response
0.5	Linkwitz-Reilly	$20\log_{10}(0.5)$ $= -6\text{ dB} @ \omega_0$ .. flattest time response (critically damped) .. no ringing or overshoot
$\frac{1}{\sqrt{3}}$ $=0.577$	Bessel	$-4.77\text{ dB} @ \omega_0$ .. flattest phase response
$\frac{1}{\sqrt{2}}$ $=0.707$	Butterworth *	$-3\text{ dB} @ \omega_0$ .. flattest amplitude response .. some overshoot, no ringing
>0.707	Chebychev	peak $@ \omega_0$ for $Q>1$ .. sharpest transition between pass and stop bands! .. ringing and overshoot





homework: write transfer function for the following circuit:



.. try to get it into final form in terms of Q