Lab 9 Solution

#1 $\sum_{n=0}^{\infty} \frac{n^3}{2^n}$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(n+1)^3}{2^{n+1}}}{\frac{n^3}{2^n}}\right|$$

$$=\lim_{n\to\infty} \frac{(n+1)^3}{\cancel{2^{n+1}}_{2}} \cdot \frac{\cancel{2^n}}{n^3}$$

$= \frac{1}{2} < 1 \Rightarrow$ Absolutely Convergent!

#2 $\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n+1}$

For abs convergent, we take absolute value

$$\sum_{n=1}^{\infty}\left|(-1)^n \frac{1}{2n+1}\right| = \sum_{n=1}^{\infty} \frac{1}{2n+1}$$ compare with $\sum_{n=1}^{\infty} \frac{1}{2n}$

$= \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$ divergent

$\lim_{n\to\infty} \frac{\frac{1}{2n+1}}{\frac{1}{2n}} = 1$. By comparison Test, $\sum_{n=1}^{\infty}\frac{1}{2n+1}$ is divergent

Therefore the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+1}$ is not absolutely convergent.

However $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+1}$ is an alternating series,

$\frac{1}{2n+1}$ goes to 0, decreasing, So by AST $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+1}$ is Convergent.

#3 $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

Alternating Series Test

$\frac{1}{3n+1}$: $\begin{cases} \lim_{n\to\infty} \frac{1}{3n+1} = 0 \\ \frac{1}{3n+1} \text{ is decreasing} \end{cases}$

Therefore $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$ is convergent by AST.

#4 $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n+1}$

Note that $\lim_{n\to\infty} \frac{2n}{3n+1} = \lim_{n\to\infty} \frac{2}{3+\frac{1}{n}} = \frac{2}{3} \neq 0$

So the series $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n+1}$ is divergent

by Test for Divergence.