EE 2010 Circuit Analysis

Module # 11: Steady-State Response: Sinusoidal Inputs Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, WIKIPEDIA, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

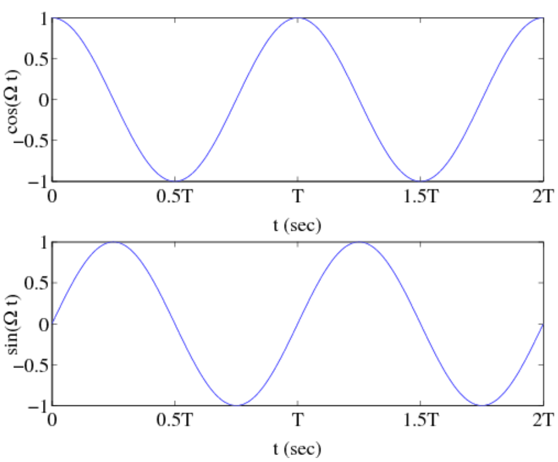
Learning Objective: In this module, we demonstrate how we can use *EXACTLY THE SAME* nodal analysis for circuits with dynamic elements with *SYMBOLIC SOURCES*, and then use the resulting *TRANSFER FUNCTION* to find the *STEADY STATE RESPONSE* for a specific *SINUSOIDAL INPUT*.

NO ENHANCEMENTS OR GENERALIZATIONS OF THE ANALYSIS PROCESS IS REQUIRED!

From WIKIPEDIA: "In systems theory, a system or a process is in a *steady state* if the variables (called state variables) which define the behavior of the *system or the process are unchanging in time*."

In our earlier work, we have already considered the steady-state response of circuits – either in the special case of $f = 0$ for circuits at DC, or in the unspecified case using s-domain models. We now consider the steady-state response of circuits with inputs specifically at $f > 0$, that is, with sinusoidal inputs. For time-varying waveform inputs, we note the subtle distinction that while the input signal is changing (periodically) with time and the output signal is changing (also periodically) with time, the system is unchanging in time and hence is in steady-state.

Definition: A *sinusoid* is a continuous function (of t) having the form of a cosine $A\cos(2\pi ft + \phi)$ or a sine $A\sin(2\pi ft + \theta)$, where A is the amplitude, f is the *frequency* in cycles/s or Hz, $\omega = 2\pi f$ is the *angular frequency* in radians/s, and ϕ and θ are the phases in radians. The *period* of the sinusoid is $T = \frac{1}{f} = \frac{2\pi}{2\pi f} = \frac{2\pi}{\omega}$.



The phase of a continuous-time sinusoid manifests as a kind of "time delay." Notice that $\sin(2\pi ft)$ can be written as $\cos(2\pi ft - \pi/2)$ – a time *delay* and that $\cos(2\pi ft)$ can be written as $\sin(2\pi ft + \pi/2)$ – a time *advance*. In what follows, we will typically express sinusoidal waveforms in their $\cos(\cdot)$ form.

Sinusoidal signals are of particular interest since any periodic signal, i.e., signals for which $s(t) = s(t+T)$, can be represented as a sum of sinusoids. The well-known *Fouries series* is one such representation. The smallest T for which $s(t) = s(t+T)$ is referred to as the *period* of the waveform.

Sinusoidal Steady-State Circuit Analysis:

Thus far in the course, we have focused on the *steady-state response* = *zero-state response* of circuit systems. In doing so, we have used the steady-state (also zero-state) impedance models:

Steady-State Impedance Models

s-domain	Steady State @ ω R/s	Steady State @ f Hz
$Z_R = R$	$Z_R = R$	$Z_R = R$
$Z_C = \frac{1}{s \cdot C}$	$Z_C = \frac{1}{j\omega \cdot C}$	$Z_C = \frac{1}{j2\pi f \cdot C}$
$Z_L = s \cdot L$	$Z_L = j\omega \cdot L$	$Z_L = j2\pi f \cdot L$

Table 1: Steady-State Impedance Models for R, C, and L

The response of any time-invariant linear system (modeled by $h(t) \stackrel{\mathcal{L}}{\longleftrightarrow} H(s)$) to a stimulus signal $f(t)$ is the *convolution*

$$(v * h)(t) = \int_0^t v(\tau)h(t-\tau)\,d\tau \stackrel{\mathcal{L}}{\longleftrightarrow} V(s) \cdot H(s)$$

Pretty messy in time-domain, but just a *functional multiply* in s-domain. Moreover, we have thus far constrained our attention to *sinusoidal steady-state* (including DC, $f = 0$) - so we don't have to use the Laplace representation of input signals to find $F(s)$ (as in the table above). Instead, we employ the additional simplification where if the input is sinusoidal at a particular frequency, ω_0,

$$v_{in}(t) = A\cos(\omega_0 t + \theta)$$

the output is found as:

$$v_{out}(t) = A|H(\omega_0)|\Re\left\{e^{j(\omega_0 t + \theta + \angle H(\omega_0))}\right\}$$
$$v_{out}(t) = A|H(\omega_0)|\cos(\omega_0 t + \theta + \angle H(\omega_0))$$

that is, the *amplitude* of the output is the *product* of the amplitude of the input signal and the magnitude of the transfer function, $A|H(\omega_0)|$ while the *phase* of the output is the *sum* of the phases of the input signal and the transfer function, $\theta + \angle H(\omega_0)$.

If the input is a DC voltage or current, i.e.,

$$v_{in}(t) = A$$

the output is found as:

$$v_{out}(t) = A|H(0)|\Re\left\{e^{(j\angle H(0))}\right\}$$
$$v_{out}(t) = A|H(0)|\cos(\angle H(0))$$

where for DC inputs, the "phase" $\angle H(0)$ will be 0 or π, that is, the systems can modify the output to be positive or negative.

One might ask, "Since we know the general response form:

$$V_{out}(s) = H(s)V_{in}(s)$$

or, in steady-state,

$$V_{out}(j\omega) = H(j\omega)V_{in}(j\omega)$$

why not just multiply $H(s)$ by the appropriate $V_{in}(s)$ and take the inverse Laplace transform where $V_{in}(s)$ is the Laplace transform of the input sinusoid? "

Good question! The answer is twofold:

First, following the above process would find the total response for $t \geq 0$. Separating the steady-state requires yet another step. *Additionally*, the appropriate $V_{in}(s)$ may be messy especially if it is a zero-phase sinusoid.

We will find the approach of moving directly to the steady-state time response to be **more straightforward and more flexible** in terms of gaining insight to the frequency-domain response of the circuit.

Summary:

We can consider the response of a linear system to a general periodic signal $s(t)$ by considering the response of the linear system to *sinusoidal inputs*, because:

1. All periodic signals can be modeled as a combination of sinusoidal inputs. Why?

2. The circuit elements considered in this course are modeled by *linear* voltage-current relationships. True?

3. The composite influence on a given input signal of any circuit so modeled is a *linear operation.*

4. For any linear operation or system, we have **sinusoidal in $\rightarrow$ sinusoidal out!** and at exactly the same frequency.

$$A\cos(2\pi f_0 t + \theta) \longrightarrow \boxed{\mathcal{H}} \text{———} (|H(f_0)| \cdot A)\cos(2\pi f_0 t + \theta + \angle H(f_0))$$

5. That is, the effect of *any linear operation on a sinusoid* is a change of *amplitude* $A \rightarrow (B \cdot A)$ and/or a change of *phase* $\theta \rightarrow \theta + \phi$, where $B = |H(f_0)|$ and $\phi = \angle H(f_0)$ are the magnitude and phase of the transfer function at f_0.

6. Then to analyze the operation of a linear system (circuit) on sinusoidal signals, we need only determine, as a function of frequency f, the affect of the system on the *amplitude* (i.e, $|H(f)|$) and the *phase* (i.e., $\angle H(f)$).

Hence, we need only find $|H(f)|$ and $\angle H(f)$; the **magnitude and phase of the Transfer Function** of the system!

A Convenient Representation:

You will *NEVER EVER* build or analyze any system in which a complex number exists anywhere in the system. However, the task of accounting for changes in amplitudes and phases of sinusoidal signals using only trigonometric relations and identities is VERY cumbersome.

It is expedient to employ a useful *short-hand* representation of the affect of system operations on the amplitudes and phases of signals applied to the system. In fact, *we have already done so* in our consideration of dynamic elements (Capacitors and Inductors).

Capacitors: Recall, the capacitor is a *dynamic* passive element (it can store energy, but not generate energy), thus, in steady-state (all transients have long gone), the dynamic impedance model is an integrator

$$v(t) = \frac{1}{C}\int i(t)dt$$

Here, if $i(t) = A\cos(2\pi ft + \theta)$, then $v(t) = \frac{A}{C2\pi f}\sin(2\pi ft + \theta) = \frac{A}{C2\pi f}\cos(2\pi ft + \theta - \pi/2)$, so that capacitors or capacitor networks alter both *amplitude* and *phase* of a sinusoidal input, but have no impact on *frequency.*

Thus, for a sinusoidal excitation, $i(t) = A\cos(2\pi ft+\theta)$, the voltage across the capacitor (impedance model) is

$$v(t) = \frac{A}{C2\pi f}\cos(2\pi ft + \theta - \pi/2)$$

which parallels

$$v(t) = \Re\left\{\frac{1}{j2\pi fC}Ae^{(j2\pi ft+\theta)}\right\} = \Re\left\{\frac{1}{2\pi fC}e^{(-j\pi/2)}Ae^{(j2\pi ft+\theta)}\right\}$$

at $s \to j2\pi f$ where $\Re\{\}$ denotes the *real part* of the complex argument. We note that $\frac{1}{j}$ accounts for the phase shift, and $\frac{1}{C2\pi f}$ accounts for the frequency-dependent amplitude modification for a capacitor.

Inductors: Likewise, recall the inductor is a *dynamic* passive element (it can store energy, but not generate energy), thus, in steady-state (all transients have long gone)

$$v(t) = L\frac{di(t)}{dt}$$

In this case, if $i(t) = A\cos(2\pi ft + \theta)$, then $v(t) = A \cdot L2\pi f[-\sin(2\pi ft + \theta)] = A \cdot L2\pi f\cos(2\pi ft + \theta + \pi/2)$, so that inductors or inductor networks also alter both *amplitude* and *phase*, but have no impact on *frequency.*

We can represent the weighted derivative effect of an inductor by the algebraic derivative-operator (Laplace transform) Ls. For a sinusoidal excitation in steady-state, this reduces to the substitution $s \to j2\pi f$. Thus, for a sinusoidal excitation, $i(t) = A\cos(2\pi ft + \theta)$, the voltage across the inductor (impedance model) is

$$v(t) = A2\pi fL\cos(2\pi ft + \theta + \pi/2)$$

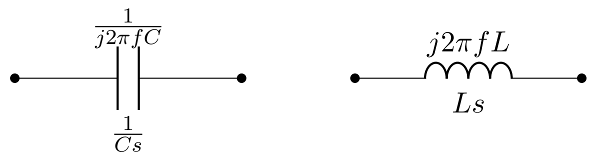
which parallels

$$v(t) = \Re\left\{j2\pi fLAe^{(j2\pi ft+\theta)}\right\} = \Re\left\{2\pi fLe^{(j\pi/2)}Ae^{(j2\pi ft+\theta)}\right\}$$

$s \rightarrow j2\pi f$, with the j accounting for the phase shift, and $L2\pi f$ accounting for the frequency-dependent amplitude modification of an inductor.

An Algebra of a Convenient Representation:

It follows that for capacitors and inductors with sinusoidal inputs at steady state, we have the convenient representations of the impedance models:



where, for circuit analysis, we will almost always use the shorthand parameter s and, after any algebraic manipulation, substitute $s = j2\pi f$ for computation of the affect of a system on inputs.

Intuitively, we observe that an *integral-operation* (the impedance model for a capacitor) is algebraically represented by a $\frac{1}{s}$ or $\frac{1}{j2\pi f}$ and, by virtue of the $\frac{1}{j} = -j = e^{(-j\pi/2)}$, retards the phase by $\pi/2$ while the *derivative-operation* (the impedance model for an inductor) is algebraically represented by a s or $j2\pi f$ and, by virtue of the $j = e^{(+j\pi/2)}$, advances the phase by $\pi/2$.

We will consider circuits containing a variety of elements and configurations. Nonetheless, circuits that **retard** the phase at a particular frequency may be considered to be **mostly capacitive** at that frequency and circuits that **advance** the phase at a particular frequency may be considered to be **mostly inductive** at that frequency. (In the examples below, we'll see what appear to be counter-intuitive results.) †

Procedure to Find Sinusoidal Steady-State Responses:

1. Replace *all independent sources with symbolic representations such as* $V_{in}(s)$
2. Identify the essential ($\geq$ 3-element connections) nodes
3. Select a node as the reference node = the node at *ground potential* = 0 Volts
4. Identify and label the voltages at nodes that are readily deduced
5. Note the node-pairs linked by a *voltage source* and simplify accordingly
6. Assign voltage variables $v_a, v_b, \ldots$ to the remaining nodes with only one assignment for each linked node-pair, the other node in that pair assigned voltages such as "$v_1 - 20$" or "$v_4 + 3v_x$".
7. Employ s-domain impedance models: $Z_R(s) = R$, $Z_C(s) = \frac{1}{s \cdot C}$, and $Z_L(s) = s \cdot L$.
8. Apply $I_{\text{out}} = V_{\text{difference}}/Z$ for each branch leaving the node
9. Enjoy the thrill of ending the consideration of each node with the powerful "$= 0$"
10. Add one additional equation for each dependent source specification if necessary
11. Use algebra or an algebra solver to find the output: $V_{out}(s)$, or whatever output function is dictated by the problem
12. Find the transfer function: $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$, or whatever input-output function is dictated by the problem.
13. We immediately have the steady-state transfer function $H(f)$ by letting $s = j2\pi f$ or $H(\omega)$ by letting $s = j\omega$
14. We can now compute $V_{out}(\cdot) = H(\cdot)V_{in}(\cdot)$ where $H(\cdot)$ at any particular f or ω is a complex number accounting for both the change in amplitude (through $|H(\cdot)|$) and change of phase (through $\angle H(\cdot)$)
15. If the output to a specific input, say $v_{in}(t) = A\cos(\omega_0 t + \theta)$ is desired, find $v_{out}(t) = A|H(\omega_0)|\cos(\omega_0 t + \theta + \angle H(\omega_0))$
16. If the *Amplitude* and *Phase* of the system *Transfer Function* is desired, construct a plot of $|H(\omega)|$ and $\angle H(\omega))$ versus the desired frequency range.

As before, we use the s-domain impedance models:

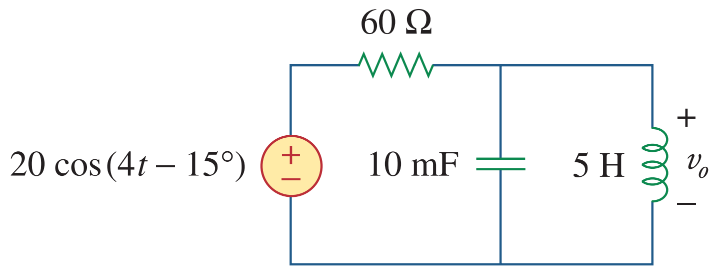
Steady-State Impedance Models

s-domain	**Steady State @ ω R/s**	**Steady State @ f Hz**
$Z_R = R$	$Z_R = R$	$Z_R = R$
$Z_C = \frac{1}{s \cdot C}$	$Z_C = \frac{1}{j\omega \cdot C}$	$Z_C = \frac{1}{j2\pi f \cdot C}$
$Z_L = s \cdot L$	$Z_L = j\omega \cdot L$	$Z_L = j2\pi f \cdot L$

Table 2: Steady-State Impedance Models for R, C, and L

We illustrate the simple procedure and the all-Laplace procedure with an example:

Example 9.11:




```
%% Example 9.11
clear all
% Declare symbolic variables
syms Vin v0 s
% Nodal analysIin directly in solve()
[v0]=solve((v0-Vin)/60 + v0*0.01*s + v0/(5*s)== 0)
% Transfer function
H(s) = v0/Vin
```

...Yielding...

```
v0 = (5*Vin*s)/(3*s^2 + 5*s + 60)
H(s) = (5*s)/(3*s^2 + 5*s + 60)
```

Having this result, we can move to a solution to this problem in two ways. In what follows, we illustrate these two approaches.

1. Use "sinusoidal-in : sinusoidal-out" steady-state procedure

In this problem, $v_{in}(t) = 20\cos(4t - 15^o)$. Hence, $v_{out}(t) = 20|H(s = j4)|\cos(4t - 15^o + \angle H(s = j4))$ which we find as:

```
%% 1. Proceed using sinusoidal-in : sinusoidal-out
% Transfer function @ excitation frequency
H(j*4) % We will use the magnitude and phase of this complex number:
[abs(H(j*4)),angle(H(j*4))]
% In the answer below:
% The source phase shift is in degrees so convert to radians
formatSpec = 'v0(t) = %4.2f cos(4*t %+4.2f )\n';
fprintf(formatSpec,20*abs(H(j*4)), -15*(2*pi)/360 + angle(H(j*4)))
%
```

...*Yielding...*

```
ans = 25/34 + 15i/34
ans =  0.8575    0.5404
v0(t) = 17.15 cos(4*t +0.28 )
```

In particular, $|H(s = j4)| = 0.8575$ and $\angle H(s = j4)) = 0.5404$ so that

$$v_{out}(t) = 20 * 0.8575\cos(4t - 0.2618 + 0.5404) = 17.15\cos(4t + 0.2786)$$

the latter angle being in radians, not degrees.

2. Use s-domain representations for the input and the transfer function

For this problem, the source, $v_{in}(t) = 20\cos(4t - 15^o)$. To find the s-domain representation, $V_{in}(s)$, we must use trig components of the sine and cosine transform pairs

$$\begin{aligned}\sin(\omega t)\cdot u(t) &\overset{\mathcal{L}}{\longleftrightarrow} \frac{\omega}{s^2+\omega^2}\\ \cos(\omega t)\cdot u(t) &\overset{\mathcal{L}}{\longleftrightarrow} \frac{s}{s^2+\omega^2}\end{aligned}$$

along with the second trig identity

$$\begin{aligned}\sin(\alpha\pm\beta) &= \sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)\\ \cos(\alpha\pm\beta) &= \cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)\end{aligned}$$

to arrive at

$$\begin{aligned}V_{in}(s) &= 20\cos(15^o)\frac{s}{s^2+4^2} + 20\sin(15^o)\frac{4}{s^2+4^2}\\ &= 20\cos(0.2618)\frac{s}{s^2+4^2} + 20\sin(0.2618)\frac{4}{s^2+4^2}\\ &= 20\cos(0.2618)\frac{s}{s^2+16} + 20\sin(0.2618)\frac{4}{s^2+16}\end{aligned}$$

Then, with

$$H(s) = (5*s)/(3*s^2+5*s+60)$$

we have

$$V_{out}(s) = H(s)V_{in}(s) = (5*s)/(3*s^2+5*s+60)\cdot\left(20\cos(0.2618)\frac{s}{s^2+16} + 20\sin(0.2618)\frac{4}{s^2+16}\right)$$

and finally

$$v_{out}(t) = \mathcal{L}^{-1}V_{out}(s)$$

which is the response from $t = 0$ onward.

```
%% 2. Calculate response in s-domain
syms Vout vout t
% Output in s
Vout(s) = H(s) * (20*cos(0.2618)*(s/(s^2 + 16)) + 20*sin(0.2618)*(4/(s^2 + 16)))
vout(t) = ilaplace(Vout(s))
%
```

...Yielding...

```
Vout(s) =
(5*s*((19.3185*s)/(s^2 + 16) + 20.7056/(s^2 + 16)))/(3*s^2 + 5*s + 60)
vout(t) = 16.4885*cos(4*t) - 4.7167*sin(4*t) - 16.4885*exp(-0.8333*t)*(cos(4.3938*t)
    - 0.5152*sin(4.3938*t))
>>
```

To arrive at the steady-state response, we need to let all the transients die out and see what's left. That would leave

```
vout(t) = 16.4885*cos(4*t) - 4.7167*sin(4*t)
```

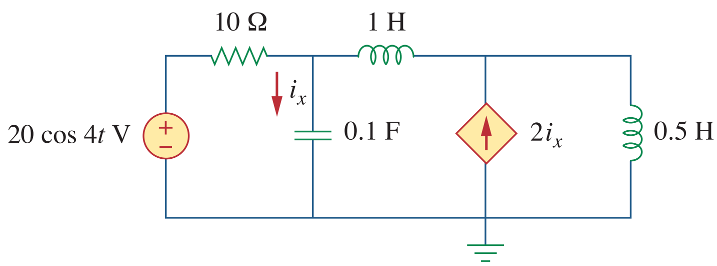
which is also written as

$$\begin{aligned} vout(t) &= 16.4885\cos(4t) - 4.7167\sin(4t) \\ &= 17.1499\cos(4t + 0.2786) \end{aligned}$$

which is the same result yielded by the time-domain approach.

I think it much better to use the "sinusoidal-in : sinusoidal-out" approach. What say ye?

Example 10.1:



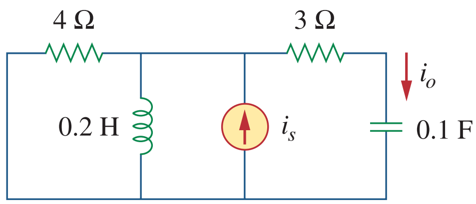

```
%% Example 10.1
clear all
% Declare symbolic variables
syms Vin va vb ix s
% Nodal analysIin directly in solve()
[va,vb,ix]=solve((va-Vin)/10 + va*0.1*s + (va-vb)/s == 0,...
  (vb-va)/s - 2*ix + vb/(0.5*s) == 0, ix == va*0.1*s, va,vb,ix)
% Let's say we are interested in vb
% Transfer function
H(s) = vb/Vin
% Transfer function @ excitation frequency
H(j*4) % We will use the magnitude and phase of this complex number:
[double(abs(H(j*4))),double(angle(H(j*4)))]
% In the answer below (in radians):
formatSpec = 'v0(t) = %4.2f cos(4*t %+4.2f )\n';
fprintf(formatSpec,20*double(abs(H(j*4))), angle(H(j*4)))
%
```

...Yielding...

```
va = (3*Vin*s)/(s^2 + 3*s + 20)
vb = (Vin*s*(s^2 + 5))/(5*(s^2 + 3*s + 20))
ix = (3*Vin*s^2)/(10*(s^2 + 3*s + 20))
H(s) = (s*(s^2 + 5))/(5*(s^2 + 3*s + 20))
ans = - 33/50 - 11i/50
ans =     0.6957   -2.8198
v0(t) = 13.91 cos(4*t -2.82 )
```

Problem 9.46:

9.46 If $i_s = 5\cos(10t + 40°)$ A in the circuit of Fig. 9.53, find i_o.



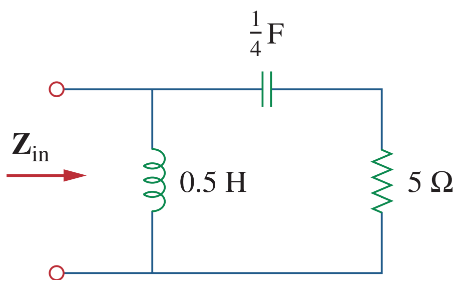

```
%% Problem 9.46
clear all
% Declare symbolic variables
syms Iin va s
% Nodal analysIin directly in solve()
[va]=solve(-Iin + va/(0.2*s) + va/4 + va/(3+10/s) == 0, va)
% and
i0 = va/(3+10/s)
% Transfer function
H(s) = i0/Iin
% Transfer function @ excitation frequency
omega = 10;
H(j*omega) % We will use the magnitude and phase of this complex number:
[double(abs(H(j*omega))),double(angle(H(j*omega)))]
% In the answer below (in degrees):
formatSpec = 'v0(t) = %4.2f cos(%4.2f*t %+4.2f )\n';
fprintf(formatSpec,5*double(abs(H(j*omega))),omega,40 + angle(H(j*omega))*360/(2*pi)
    )
%
```

...Yielding...

```
va = Iin/(5/s + 1/(10/s + 3) + 1/4)
i0 = Iin/((10/s + 3)*(5/s + 1/(10/s + 3) + 1/4))
H(s) = 1/((10/s + 3)*(5/s + 1/(10/s + 3) + 1/4))
ans = 10/37 + 14i/37
ans =     0.4650    0.9505
v0(t) = 2.32 cos(10.00*t +94.46 )
```

Problem 9.59:

9.59 For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 10$ rad/s.



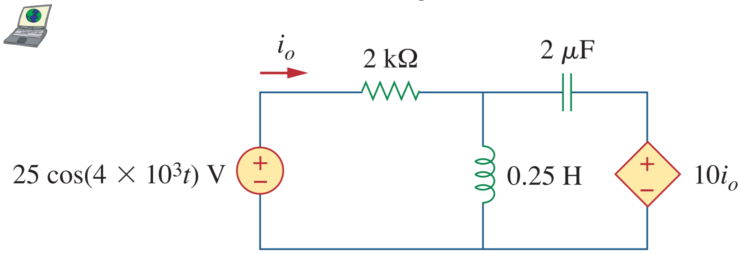

```
%% Problem 9.59
clear all
% Declare symbolic variables
syms Z s
% Impedance combination
Z(s) = (0.5*s * (4/s + 5))/(0.5*s + (4/s + 5))
% @ excitation frequency
omega = 10;
Z(j*omega) % We will use the magnitude and phase of this complex number:
[double(abs(Z(j*omega))),double(angle(Z(j*omega)))]
% In the answer below:
formatSpec = 'Z = %4.2f exp(j* %+4.2f )\n';
fprintf(formatSpec,double(abs(Z(j*omega))),angle(Z(j*omega)))
%
```

...Yielding...

```
Z(s) = (s*(4/s + 5))/(2*(s/2 + 4/s + 5))
ans = 3125/1154 + 2895i/1154
ans =     3.6914    0.7472
Z = 3.69 exp(j* +0.75 )
```

Problem 10.05:

10.5 Find i_o in the circuit of Fig. 10.54.



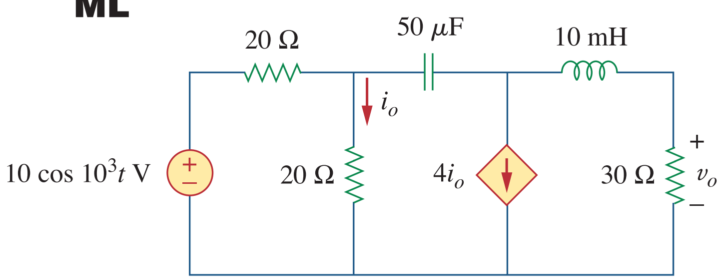

```
%% Problem 10.05
clear all
% Declare symbolic variables
syms Vin va i0 s
% Nodal analysIin directly in solve()
[va,i0]=solve((va-Vin)/2000 + va/(0.25*s) + (va-10*i0)*s*2*10^(-6) == 0,...
  i0 == (Vin-va)/2000, va,i0)
% Transfer function
H(s) = simplify(i0/Vin)
% Transfer function @ excitation frequency
omega = 4000;
H(j*omega) % We will use the magnitude and phase of this complex number:
[double(abs(H(j*omega))),double(angle(H(j*omega)))]
% In the answer below (in degrees):
formatSpec = 'v0(t) = %4.2f cos(%4.2f*t %+4.2f )\n';
fprintf(formatSpec,25*double(abs(H(j*omega))),omega,angle(H(j*omega)))
%
```

...Yielding...

```
va = (Vin*(s^2 + 50000*s))/(201*s^2 + 50000*s + 400000000)
i0 = (Vin*(s^2 + 2000000))/(10*(201*s^2 + 50000*s + 400000000))
H(s) = (s^2 + 2000000)/(10*(201*s^2 + 50000*s + 400000000))
ans = 308/622645 + 35i/996232
ans =     0.0005    0.0709
v0(t) = 0.01 cos(4000.00*t +0.07 )
```

Problem 10.09:

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.




```
%%  Problem 10.09
clear all
% Declare symbolic variables
syms Vin va vb i0 s
% Nodal analysIin directly in solve()
[va,vb,i0]=solve((va-Vin)/20 + va/20 + (va-vb)*s*50*10^(-6) == 0,...
  (vb-va)*s*50*10^(-6) + 4*i0 + vb/(0.01*s+30) == 0,...
  i0==va/20, va,vb,i0)
% and
v0 = vb*30/(0.01*s+30)
% Transfer function
H(s) = simplify(v0/Vin)
% Transfer function @ excitation frequency
omega = 1000;
H(j*omega) % We will use the magnitude and phase of this complex number:
[double(abs(H(j*omega))),double(angle(H(j*omega)))]
% In the answer below (in degrees):
formatSpec = 'v0(t) = %4.2f cos(%4.2f*t %+4.2f )\n';
fprintf(formatSpec,10*double(abs(H(j*omega))),omega,angle(H(j*omega)))
%
```

...Yielding...

```
va = (Vin*(s^2 + 3000*s + 2000000))/(2*(3*s^2 + 10000*s + 2000000))
vb = (Vin*(s + 3000)*(s - 4000))/(2*(3*s^2 + 10000*s + 2000000))
i0 = (Vin*(s^2 + 3000*s + 2000000))/(40*(3*s^2 + 10000*s + 2000000))
v0 = (15*Vin*(s + 3000)*(s - 4000))/((s/100 + 30)*(3*s^2 + 10000*s + 2000000))
H(s) = (1500*s - 6000000)/(3*s^2 + 10000*s + 2000000)
ans = 21/101 + 117i/202
ans =     0.6154    1.2261
v0(t) = 6.15 cos(1000.00*t +1.23 )
```

Homework: Chapter 9 # 40, 42, 44, 47, 48, 50, 56
Chapter 10 # 4, 8, 10

→ Remember that only independent sources are replaced with symbolic labels.

→ Homework deliverables MUST be a pdf file generated using a solver.

→ The resulting .pdf file is to be uploaded to the Pilot Dropbox using the naming convention: First 4 letters of Lastname, First initial, year, title. For example, my .pdf file would be named: GarbF2020HW11.pdf

7. The techniques of voltage/current division, series/parallel combination of impedance/admittance, circuit reduction, and Y-Δ transformation all apply to ac circuit analysis.
8. AC circuits are applied in phase-shifters and bridges.

Review Questions

9.1 Which of the following is *not* a right way to express the sinusoid $A \cos \omega t$?

(a) $A \cos 2\pi ft$ (b) $A \cos(2\pi t/T)$
(c) $A \cos\omega(t - T)$ (d) $A \sin(\omega t - 90°)$

9.2 A function that repeats itself after fixed intervals is said to be:

(a) a phasor (b) harmonic
(c) periodic (d) reactive

9.3 Which of these frequencies has the shorter period?

(a) 1 krad/s (b) 1 kHz

9.4 If $v_1 = 30 \sin(\omega t + 10°)$ and $v_2 = 20 \sin(\omega t + 50°)$, which of these statements are true?

(a) v_1 leads v_2 (b) v_2 leads v_1
(c) v_2 lags v_1 (d) v_1 lags v_2
(e) v_1 and v_2 are in phase

9.5 The voltage across an inductor leads the current through it by 90°.

(a) True (b) False

9.6 The imaginary part of impedance is called:

(a) resistance (b) admittance
(c) susceptance (d) conductance
(e) reactance

9.7 The impedance of a capacitor increases with increasing frequency.

(a) True (b) False

9.8 At what frequency will the output voltage $v_o(t)$ in Fig. 9.39 be equal to the input voltage $v(t)$?

(a) 0 rad/s (b) 1 rad/s (c) 4 rad/s
(d) ∞ rad/s (e) none of the above

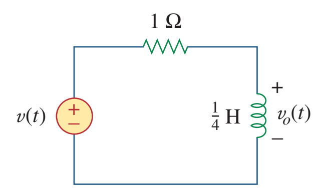


Figure 9.39
For Review Question 9.8.

9.9 A series RC circuit has $|V_R| = 12$ V and $|V_C| = 5$ V. The magnitude of the supply voltage is:

(a) −7 V (b) 7 V (c) 13 V (d) 17 V

9.10 A series RCL circuit has $R = 30\ \Omega$, $X_C = 50\ \Omega$, and $X_L = 90\ \Omega$. The impedance of the circuit is:

(a) $30 + j140\ \Omega$ (b) $30 + j40\ \Omega$
(c) $30 - j40\ \Omega$ (d) $-30 - j40\ \Omega$
(e) $-30 + j40\ \Omega$

Answers: 9.1d, 9.2c, 9.3b, 9.4b,d, 9.5a, 9.6e, 9.7b, 9.8d, 9.9c, 9.10b.

Problems

Section 9.2 Sinusoids

9.1 Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10°)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t = 10$ ms.

9.2 A current source in a linear circuit has

$$i_s = 15 \cos(25\pi t + 25°) \text{ A}$$

(a) What is the amplitude of the current?
(b) What is the angular frequency?
(c) Find the frequency of the current.
(d) Calculate i_s at $t = 2$ ms.

9.3 Express the following functions in cosine form:

(a) $10 \sin(\omega t + 30°)$ (b) $-9 \sin(8t)$
(c) $-20 \sin(\omega t + 45°)$

9.4 Design a problem to help other students better understand sinusoids.

9.5 Given $v_1 = 45\sin(\omega t + 30^\circ)$ V and $v_2 = 50\cos(\omega t - 30^\circ)$ V, determine the phase angle between the two sinusoids and which one lags the other.

9.6 For the following pairs of sinusoids, determine which one leads and by how much.

(a) $v(t) = 10\cos(4t - 60^\circ)$ and $i(t) = 4\sin(4t + 50^\circ)$

(b) $v_1(t) = 4\cos(377t + 10^\circ)$ and $v_2(t) = -20\cos 377t$

(c) $x(t) = 13\cos 2t + 5\sin 2t$ and $y(t) = 15\cos(2t - 11.8^\circ)$

Section 9.3 Phasors

9.7 If $f(\phi) = \cos\phi + j\sin\phi$, show that $f(\phi) = e^{j\phi}$.

9.8 Calculate these complex numbers and express your results in rectangular form:

(a) $\dfrac{60\angle 45^\circ}{7.5 - j10} + j2$

(b) $\dfrac{32\angle -20^\circ}{(6 - j8)(4 + j2)} + \dfrac{20}{-10 + j24}$

(c) $20 + (16\angle -50^\circ)(5 + j12)$

9.9 Evaluate the following complex numbers and leave your results in polar form:

(a) $5\angle 30^\circ\left(6 - j8 + \dfrac{3\angle 60^\circ}{2 + j}\right)$

(b) $\dfrac{(10\angle 60^\circ)(35\angle -50^\circ)}{(2 + j6) - (5 + j)}$

9.10 Design a problem to help other students better understand phasors.

9.11 Find the phasors corresponding to the following signals:

(a) $v(t) = 21\cos(4t - 15^\circ)$ V

(b) $i(t) = -8\sin(10t + 70^\circ)$ mA

(c) $v(t) = 120\sin(10t - 50^\circ)$ V

(d) $i(t) = -60\cos(30t + 10^\circ)$ mA

9.12 Let $\mathbf{X} = 4\angle 40^\circ$ and $\mathbf{Y} = 20\angle -30^\circ$. Evaluate the following quantities and express your results in polar form:

(a) $(\mathbf{X} + \mathbf{Y})\mathbf{X}^*$

(b) $(\mathbf{X} - \mathbf{Y})^*$

(c) $(\mathbf{X} + \mathbf{Y})/\mathbf{X}$

9.13 Evaluate the following complex numbers:

(a) $\dfrac{2 + j3}{1 - j6} + \dfrac{7 - j8}{-5 + j11}$

(b) $\dfrac{(5\angle 10^\circ)(10\angle -40^\circ)}{(4\angle -80^\circ)(-6\angle 50^\circ)}$

(c) $\begin{vmatrix} 2 + j3 & -j2 \\ -j2 & 8 - j5 \end{vmatrix}$

9.14 Simplify the following expressions:

(a) $\dfrac{(5 - j6) - (2 + j8)}{(-3 + j4)(5 - j) + (4 - j6)}$

(b) $\dfrac{(240\angle 75^\circ + 160\angle -30^\circ)(60 - j80)}{(67 + j84)(20\angle 32^\circ)}$

(c) $\left(\dfrac{10 + j20}{3 + j4}\right)^2\sqrt{(10 + j5)(16 - j20)}$

9.15 Evaluate these determinants:

(a) $\begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix}$

(b) $\begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix}$

(c) $\begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix}$

9.16 Transform the following sinusoids to phasors:

(a) $-20\cos(4t + 135^\circ)$ (b) $8\sin(20t + 30^\circ)$

(c) $20\cos(2t) + 15\sin(2t)$

9.17 Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$ V and $v_2 = 12\cos(50t + 30^\circ)$ V, find v.

9.18 Obtain the sinusoids corresponding to each of the following phasors:

(a) $\mathbf{V}_1 = 60\angle 15^\circ$ V, $\omega = 1$

(b) $\mathbf{V}_2 = 6 + j8$ V, $\omega = 40$

(c) $\mathbf{I}_1 = 2.8e^{-j\pi/3}$ A, $\omega = 377$

(d) $\mathbf{I}_2 = -0.5 - j1.2$ A, $\omega = 10^3$

9.19 Using phasors, find:

(a) $3\cos(20t + 10^\circ) - 5\cos(20t - 30^\circ)$

(b) $40\sin 50t + 30\cos(50t - 45^\circ)$

(c) $20\sin 400t + 10\cos(400t + 60^\circ) - 5\sin(400t - 20^\circ)$

9.20 A linear network has a current input $7.5\cos(10t + 30^\circ)$ A and a voltage output $120\cos(10t + 75^\circ)$ V. Determine the associated impedance.

9.21 Simplify the following:

(a) $f(t) = 5\cos(2t + 15°) - 4\sin(2t - 30°)$

(b) $g(t) = 8\sin t + 4\cos(t + 50°)$

(c) $h(t) = \int_0^t (10\cos 40t + 50\sin 40t)\,dt$

9.22 An alternating voltage is given by $v(t) = 55\cos(5t + 45°)$ V. Use phasors to find

$$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)\,dt$$

Assume that the value of the integral is zero at $t = -\infty$.

9.23 Apply phasor analysis to evaluate the following:

(a) $v = [110\sin(20t + 30°) + 220\cos(20t - 90°)]$ V

(b) $i = [30\cos(5t + 60°) - 20\sin(5t + 60°)]$ A

9.24 Find $v(t)$ in the following integrodifferential equations using the phasor approach:

(a) $v(t) + \int v\,dt = 10\cos t$

(b) $\frac{dv}{dt} + 5v(t) + 4\int v\,dt = 20\sin(4t + 10°)$

9.25 Using phasors, determine $i(t)$ in the following equations:

(a) $2\frac{di}{dt} + 3i(t) = 4\cos(2t - 45°)$

(b) $10\int i\,dt + \frac{di}{dt} + 6i(t) = 5\cos(5t + 22°)$ A

9.26 The loop equation for a series *RLC* circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^{t} i\,dt = \cos 2t \text{ A}$$

Assuming that the value of the integral at $t = -\infty$ is zero, find $i(t)$ using the phasor method.

9.27 A parallel *RLC* circuit has the node equation

$$\frac{dv}{dt} + 50v + 100\int v\,dt = 110\cos(377t - 10°) \text{ V}$$

Determine $v(t)$ using the phasor method. You may assume that the value of the integral at $t = -\infty$ is zero.

Section 9.4 Phasor Relationships for Circuit Elements

9.28 Determine the current that flows through an 8-Ω resistor connected to a voltage source $v_s = 110\cos 377t$ V.

9.29 What is the instantaneous voltage across a 2-μF capacitor when the current through it is $i = 4\sin(10^6 t + 25°)$ A?

9.30 A voltage $v(t) = 100\cos(60t + 20°)$ V is applied to a parallel combination of a 40-kΩ resistor and a 50-μF capacitor. Find the steady-state currents through the resistor and the capacitor.

9.31 A series *RLC* circuit has $R = 80\ \Omega$, $L = 240$ mH, and $C = 5$ mF. If the input voltage is $v(t) = 10\cos 2t$, find the currrent flowing through the circuit.

9.32 Using Fig. 9.40, design a problem to help other students better understand phasor relationships for circuit elements.

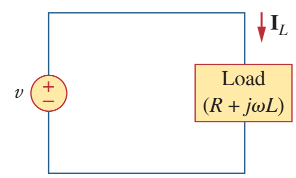


Figure 9.40
For Prob. 9.32.

9.33 A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

9.34 What value of ω will cause the forced response, v_o, in Fig. 9.41 to be zero?

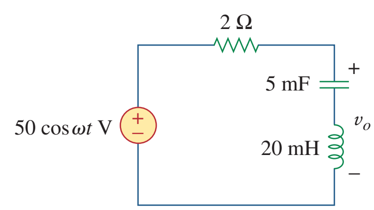


Figure 9.41
For Prob. 9.34.

Section 9.5 Impedance and Admittance

9.35 Find current i in the circuit of Fig. 9.42, when $v_s(t) = 50\cos 200t$ V.

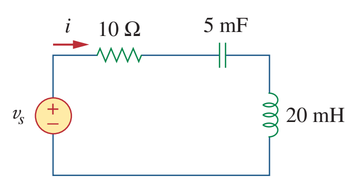


Figure 9.42
For Prob. 9.35.

9.36 Using Fig. 9.43, design a problem to help other students better understand impedance.

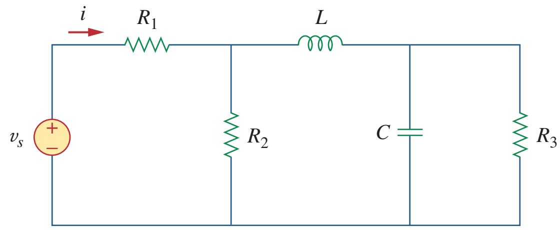


Figure 9.43
For Prob. 9.36.

9.37 Determine the admittance **Y** for the circuit in Fig. 9.44.

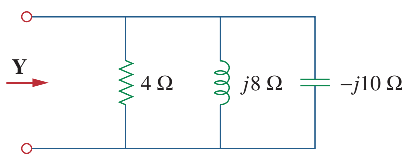
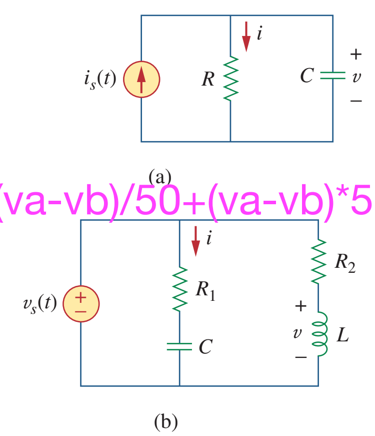


Figure 9.44
For Prob. 9.37.

9.38 Using Fig. 9.45, design a problem to help other students better understand admittance.



(va-Vin)/30+(va-vb)/50+(va-vb)*50e-6*s, (vb-va)/50+(vb-va)*50e-6*s+vb/(0.1*s)

Figure 9.45
For Prob. 9.38.

9.39 For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current **I**. Let $\omega = 10$ rad/s.

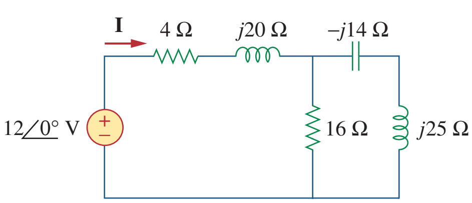


Figure 9.46
For Prob. 9.39.

9.40 In the circuit of Fig. 9.47, find i_o when:

(a) $\omega = 1$ rad/s (b) $\omega = 5$ rad/s
(c) $\omega = 10$ rad/s

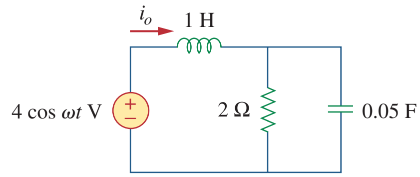


Figure 9.47
For Prob. 9.40.

9.41 Find $v(t)$ in the *RLC* circuit of Fig. 9.48.

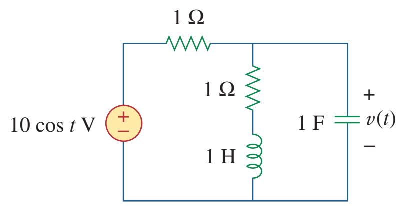


Figure 9.48
For Prob. 9.41.

9.42 Calculate $v_o(t)$ in the circuit of Fig. 9.49.

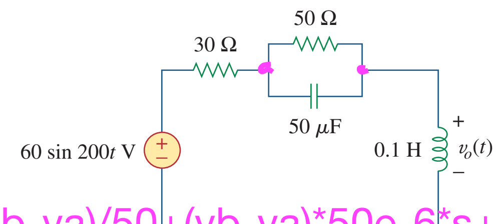


Figure 9.49
For Prob. 9.42.

9.43 Find current $\mathbf{I}_o$ in the circuit shown in Fig. 9.50.

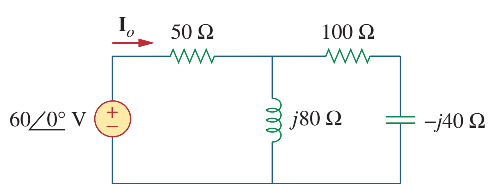


Figure 9.50
For Prob. 9.43.

9.44 Calculate $i(t)$ in the circuit of Fig. 9.51.

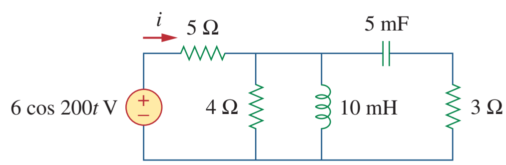


Figure 9.51
For prob. 9.44.

9.45 Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

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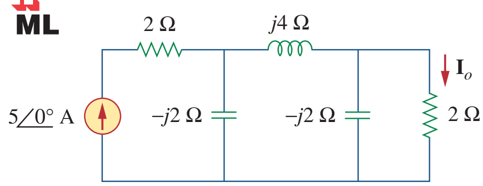

Figure 9.52
For Prob. 9.45.

9.46 If $i_s = 5\cos(10t + 40°)$ A in the circuit of Fig. 9.53, find i_o.

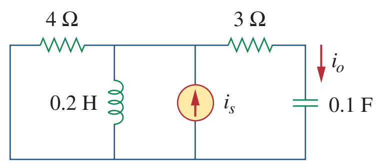

Figure 9.53
For Prob. 9.46.

9.47 In the circuit of Fig. 9.54, determine the value of $i_s(t)$.

va: va/-j10+(va-5)/2+va/(j4+20)

1/(J50e-6*2000)=1/(J0.1)=-J10

J2e-3*2000=J4

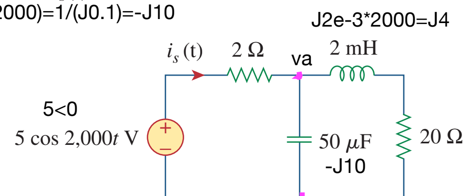

Figure 9.54
For Prob. 9.47.

9.48 Given that $v_s(t) = 20\sin(100t - 40°)$ in Fig. 9.55, determine $i_x(t)$.

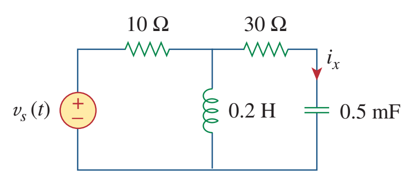

Figure 9.55
For Prob. 9.48.

9.49 Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is $0.5\sin 200t$ A.

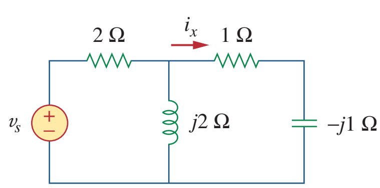

Figure 9.56
For Prob. 9.49.

9.50 Determine v_x in the circuit of Fig. 9.57. Let $i_s(t) = 5\cos(100t + 40°)$ A.

0.1J*100=20J

5<40*2pi/360

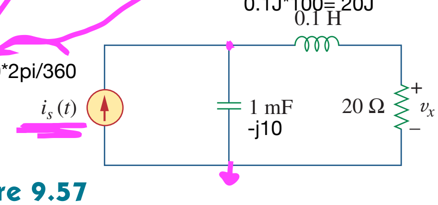

Figure 9.57
For Prob. 9.50.

1mF=1/(100*1e-3*j)=1/(0.1j)=-j10

9.51 If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is $10\cos 2t$ V, obtain i_s.

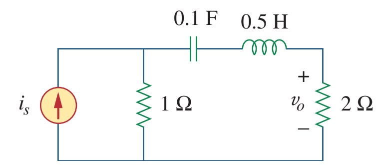

Figure 9.58
For Prob. 9.51.

9.52 If $\mathbf{V}_o = 8\angle 30°$ V in the circuit of Fig. 9.59, find $\mathbf{I}_s$.

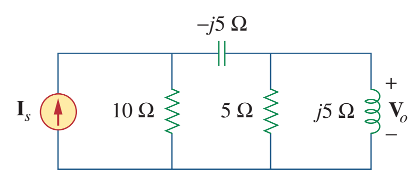

Figure 9.59
For Prob. 9.52.

9.53 Find $\mathbf{I}_o$ in the circuit of Fig. 9.60.

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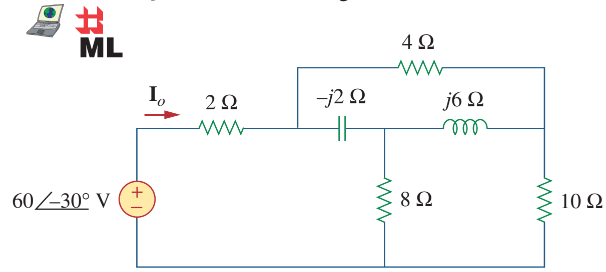

Figure 9.60
For Prob. 9.53.

9.54 In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 2\angle 0°$ A.

ML

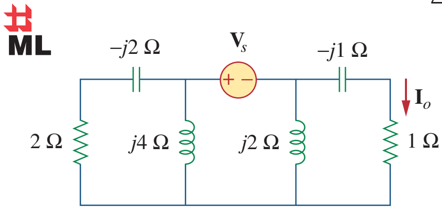

Figure 9.61
For Prob. 9.54.

***9.55** Find **Z** in the network of Fig. 9.62, given that $\mathbf{V}_o = 4\underline{/0^\circ}$ V.

ML

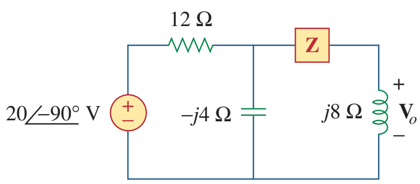

Figure 9.62
For Prob. 9.55.

Section 9.7 Impedance Combinations

9.56 At $\omega = 377$ rad/s, find the input impedance of the circuit shown in Fig. 9.63.

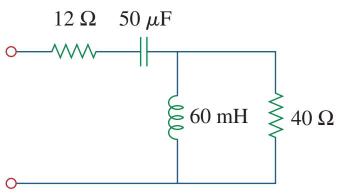

Figure 9.63
For Prob. 9.56.

9.57 At $\omega = 1$ rad/s, obtain the input admittance in the circuit of Fig. 9.64.

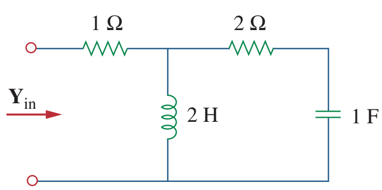

Figure 9.64
For Prob. 9.57.

9.58 Using Fig. 9.65, design a problem to help other students better understand impedance combinations.

e2d

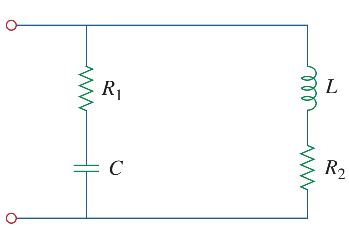

Figure 9.65
For Prob. 9.58.

9.59 For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 10$ rad/s.

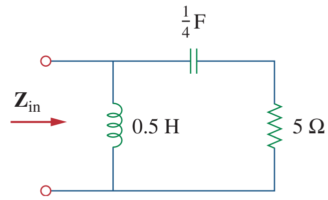

Figure 9.66
For Prob. 9.59.

9.60 Obtain $\mathbf{Z}_{in}$ for the circuit in Fig. 9.67.

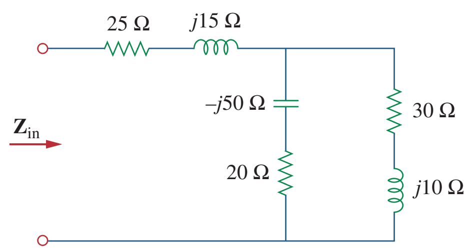

Figure 9.67
For Prob. 9.60.

9.61 Find $\mathbf{Z}_{eq}$ in the circuit of Fig. 9.68.

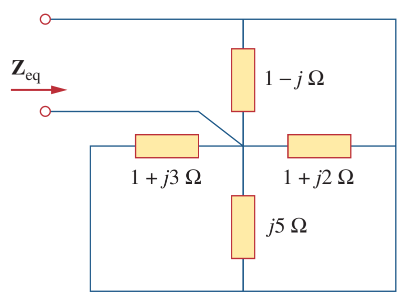

Figure 9.68
For Prob. 9.61.

9.62 For the circuit in Fig. 9.69, find the input impedance $\mathbf{Z}_{in}$ at 10 krad/s.

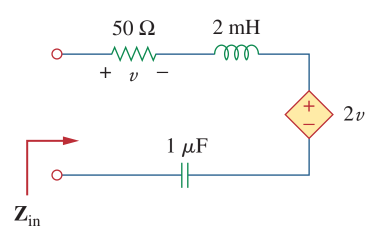

Figure 9.69
For Prob. 9.62.

* An asterisk indicates a challenging problem.

9.63 For the circuit in Fig. 9.70, find the value of $\mathbf{Z}_T$.

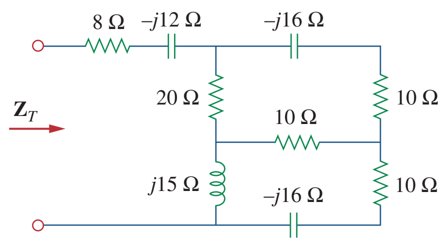

Figure 9.70
For Prob. 9.63.

9.64 Find $\mathbf{Z}_T$ and $\mathbf{I}$ in the circuit in Fig. 9.71.

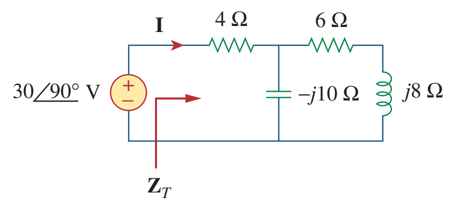

Figure 9.71
For Prob. 9.64.

9.65 Determine $\mathbf{Z}_T$ and $\mathbf{I}$ for the circuit in Fig. 9.72.

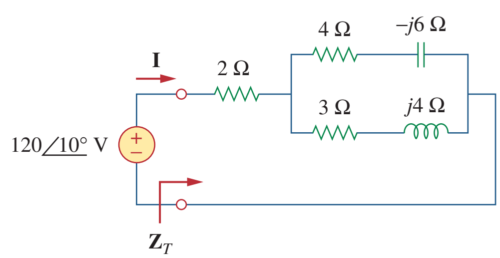

Figure 9.72
For Prob. 9.65.

9.66 For the circuit in Fig. 9.73, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.

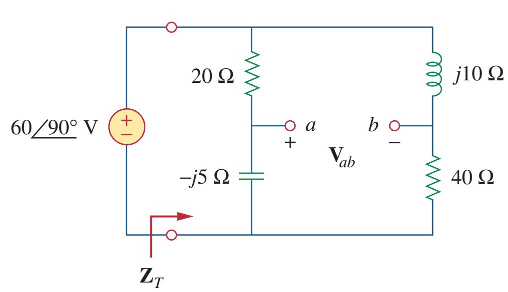

Figure 9.73
For Prob. 9.66.

9.67 At $\omega = 10^3$ rad/s, find the input admittance of each of the circuits in Fig. 9.74.

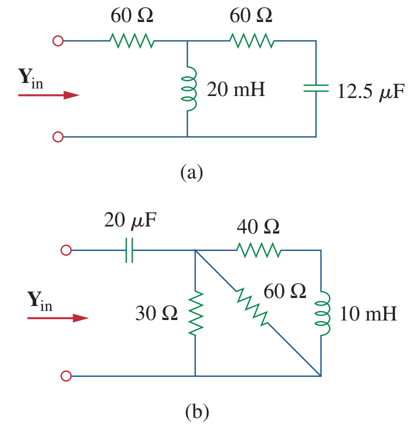

Figure 9.74
For Prob. 9.67.

9.68 Determine $\mathbf{Y}_{eq}$ for the circuit in Fig. 9.75.

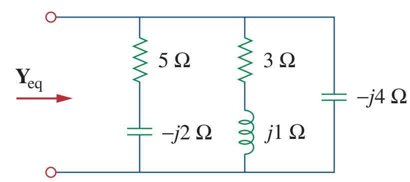

Figure 9.75
For Prob. 9.68.

9.69 Find the equivalent admittance $\mathbf{Y}_{eq}$ of the circuit in Fig. 9.76.

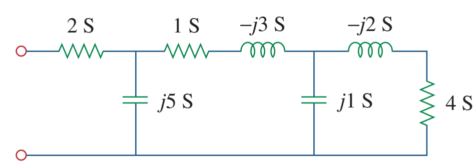

Figure 9.76
For Prob. 9.69.

9.70 Find the equivalent impedance of the circuit in Fig. 9.77.

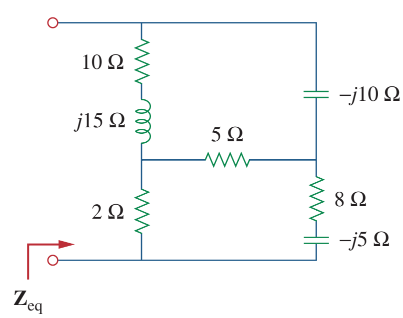

Figure 9.77
For Prob. 9.70.

9.71 Obtain the equivalent impedance of the circuit in Fig. 9.78.

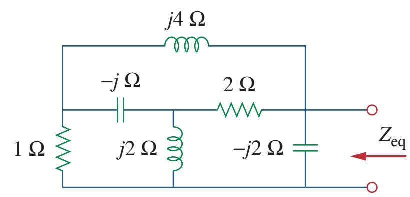

Figure 9.78
For Prob. 9.71.

9.72 Calculate the value of $\mathbf{Z}_{ab}$ in the network of Fig. 9.79.

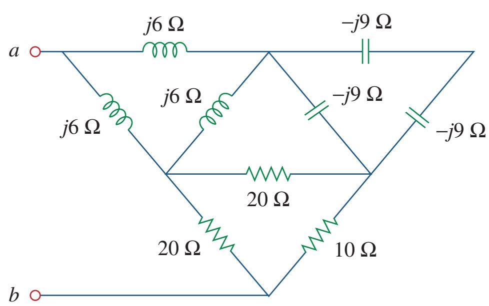

Figure 9.79
For Prob. 9.72.

9.73 Determine the equivalent impedance of the circuit in Fig. 9.80.

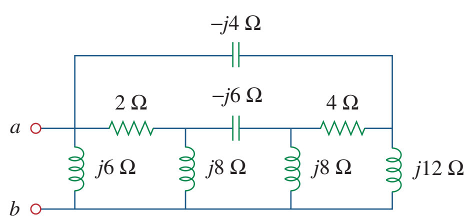

Figure 9.80
For Prob. 9.73.

Section 9.8 Applications

9.74 Design an *RL* circuit to provide a 90° leading phase shift.

9.75 Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

9.76 For the following pairs of signals, determine if v_1 leads or lags v_2 and by how much.

(a) $v_1 = 10\cos(5t - 20°)$, $v_2 = 8\sin 5t$

(b) $v_1 = 19\cos(2t + 90°)$, $v_2 = 6\sin 2t$

(c) $v_1 = -4\cos 10t$, $v_2 = 15\sin 10t$

9.77 Refer to the *RC* circuit in Fig. 9.81.

(a) Calculate the phase shift at 2 MHz.

(b) Find the frequency where the phase shift is 45°.

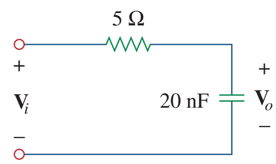

Figure 9.81
For Prob. 9.77.

9.78 A coil with impedance $8 + j6\ \Omega$ is connected in series with a capacitive reactance X. The series combination is connected in parallel with a resistor R. Given that the equivalent impedance of the resulting circuit is $5\angle 0°\ \Omega$, find the value of R and X.

9.79 (a) Calculate the phase shift of the circuit in Fig. 9.82.

(b) State whether the phase shift is leading or lagging (output with respect to input).

(c) Determine the magnitude of the output when the input is 120 V.

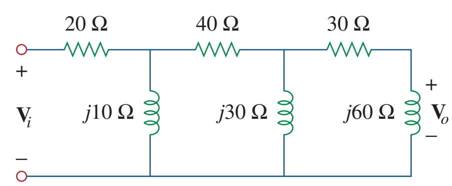

Figure 9.82
For Prob. 9.79.

9.80 Consider the phase-shifting circuit in Fig. 9.83. Let $\mathbf{V}_i = 120$ V operating at 60 Hz. Find:

(a) $\mathbf{V}_o$ when R is maximum

(b) $\mathbf{V}_o$ when R is minimum

(c) the value of R that will produce a phase shift of 45°

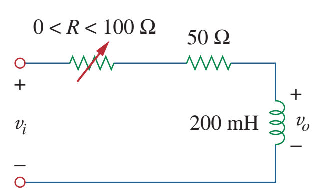

Figure 9.83
For Prob. 9.80.

9.81 The ac bridge in Fig. 9.37 is balanced when $R_1 = 400\ \Omega$, $R_2 = 600\ \Omega$, $R_3 = 1.2\ \text{k}\Omega$, and $C_2 = 0.3\ \mu\text{F}$. Find R_x and C_x. Assume R_2 and C_2 are in series.

9.82 A capacitance bridge balances when $R_1 = 100\ \Omega$, $R_2 = 2\ \text{k}\Omega$, and $C_s = 40\ \mu\text{F}$. What is C_x, the capacitance of the capacitor under test?

9.83 An inductive bridge balances when $R_1 = 1.2\ \text{k}\Omega$, $R_2 = 500\ \Omega$, and $L_s = 250$ mH. What is the value of L_x, the inductance of the inductor under test?

9.84 The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance C_s. Show that when the bridge is balanced,

$$L_x = R_2 R_3 C_s \qquad \text{and} \qquad R_x = \frac{R_2}{R_1} R_3$$

Find L_x and R_x for $R_1 = 40\text{ k}\Omega$, $R_2 = 1.6\text{ k}\Omega$, $R_3 = 4\text{ k}\Omega$, and $C_s = 0.45\ \mu\text{F}$.

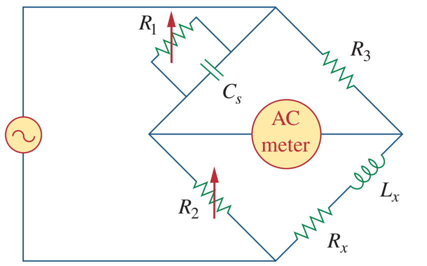

Figure 9.84
Maxwell bridge; For Prob. 9.84.

9.85 The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$

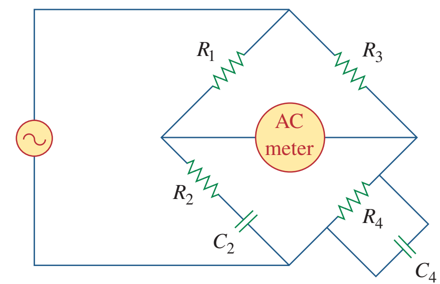

Figure 9.85
Wien bridge; For Prob. 9.85.

Comprehensive Problems

9.86 The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?

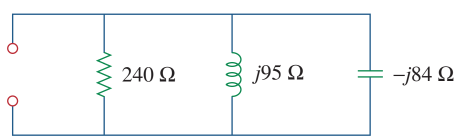

Figure 9.86
For Prob. 9.86.

9.87 The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

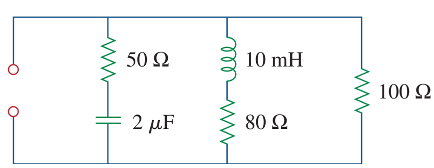

Figure 9.87
For Prob. 9.87.

9.88 A series audio circuit is shown in Fig. 9.88.

(a) What is the impedance of the circuit?

(b) If the frequency were halved, what would be the impedance of the circuit?

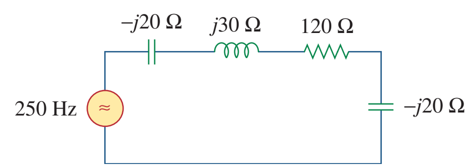

Figure 9.88
For Prob. 9.88.

9.89 An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor C across the series combination so that the net impedance is resistive at a frequency of 2 kHz.

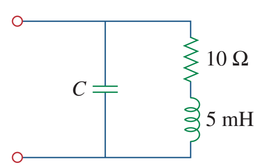

Figure 9.89
For Prob. 9.89.

9.90 An industrial coil is modeled as a series combination of an inductance L and resistance R, as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz when the circuit operates in the steady state:

$$|\mathbf{V}_s| = 145 \text{ V}, \qquad |\mathbf{V}_1| = 50 \text{ V}, \qquad |\mathbf{V}_o| = 110 \text{ V}$$

Use these measurements to determine the values of L and R.

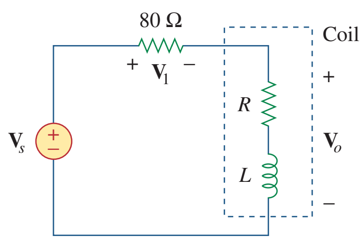

Figure 9.90
For Prob. 9.90.

9.91 Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of C?

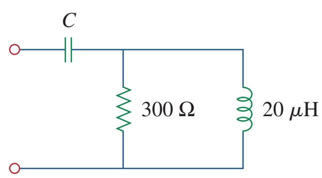

Figure 9.91
For Prob. 9.91.

9.92 A transmission line has a series impedance of $\mathbf{Z} = 100\angle 75^\circ \ \Omega$ and a shunt admittance of $\mathbf{Y} = 450\angle 48^\circ \ \mu\text{S}$. Find: (a) the characteristic impedance $\mathbf{Z}_o = \sqrt{\mathbf{Z}/\mathbf{Y}}$, (b) the propagation constant $\gamma = \sqrt{\mathbf{ZY}}$.

9.93 A power transmission system is modeled as shown in Fig. 9.92. Given the source voltage and circuit elements

$\mathbf{V}_s = 115\angle 0^\circ$ V, source impedance
$\mathbf{Z}_s = (1 + j0.5)\ \Omega$, line impedance
$\mathbf{Z}_\ell = (0.4 + j0.3)\ \Omega$, and load impedance
$\mathbf{Z}_L = (23.2 + j18.9)\ \Omega$, find the load current $\mathbf{I}_L$.

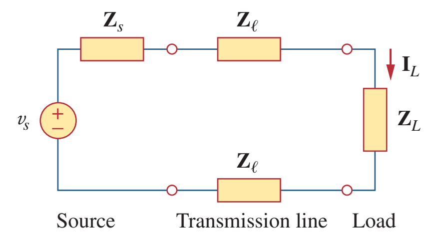

Figure 9.92
For Prob. 9.93.

Practice Problem 10.16

In the Wien-bridge oscillator circuit in Fig. 10.42, let $R_1 = R_2 = 2.5\ \text{k}\Omega$, $C_1 = C_2 = 1$ nF. Determine the frequency f_o of the oscillator.

Answer: 63.66 kHz.

10.10 Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_N\ (=\mathbf{Z}_{\text{Th}})$.
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

Review Questions

10.1 The voltage $\mathbf{V}_o$ across the capacitor in Fig. 10.43 is:

(a) $5\angle 0^\circ$ V (b) $7.071\angle 45^\circ$ V
(c) $7.071\angle -45^\circ$ V (d) $5\angle -45^\circ$ V

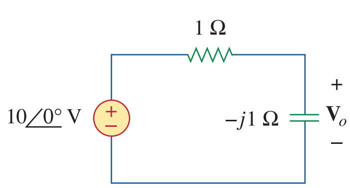

Figure 10.43
For Review Question 10.1.

10.2 The value of the current $\mathbf{I}_o$ in the circuit of Fig. 10.44 is:

(a) $4\angle 0^\circ$ A (b) $2.4\angle -90^\circ$ A
(c) $0.6\angle 0^\circ$ A (d) -1 A

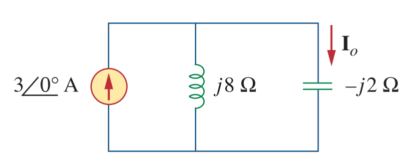

Figure 10.44
For Review Question 10.2.

10.3 Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit of Fig. 10.45 is:

(a) -24 V (b) -8 V
(c) 8 V (d) 24 V

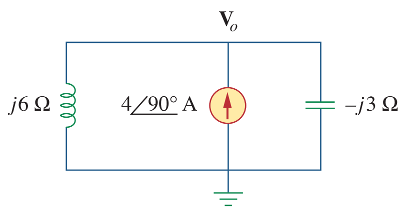

Figure 10.45
For Review Question 10.3.

10.4 In the circuit of Fig. 10.46, current $i(t)$ is:

(a) $10 \cos t$ A (b) $10 \sin t$ A (c) $5 \cos t$ A
(d) $5 \sin t$ A (e) $4.472 \cos(t - 63.43^\circ)$ A

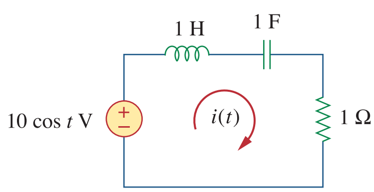

Figure 10.46
For Review Question 10.4.

10.5 Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current $i_x(t)$ can be obtained by:

(a) source transformation
(b) the superposition theorem
(c) *PSpice*

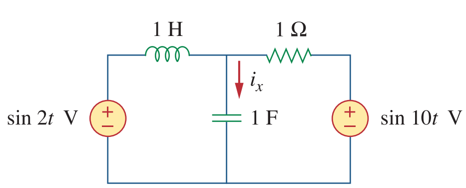

Figure 10.47
For Review Question 10.5.

10.6 For the circuit in Fig. 10.48, the Thevenin impedance at terminals *a-b* is:

(a) $1\ \Omega$ (b) $0.5 - j0.5\ \Omega$
(c) $0.5 + j0.5\ \Omega$ (d) $1 + j2\ \Omega$
(e) $1 - j2\ \Omega$

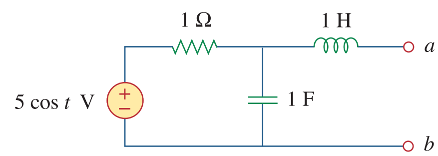

Figure 10.48
For Review Questions 10.6 and 10.7.

10.7 In the circuit of Fig. 10.48, the Thevenin voltage at terminals *a-b* is:

(a) $3.535\angle{-45^\circ}$ V (b) $3.535\angle 45^\circ$ V
(c) $7.071\angle{-45^\circ}$ V (d) $7.071\angle 45^\circ$ V

10.8 Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals *a-b* is:

(a) $-j4\ \Omega$ (b) $-j2\ \Omega$
(c) $j2\ \Omega$ (d) $j4\ \Omega$

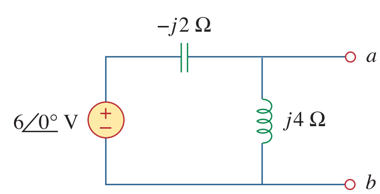

Figure 10.49
For Review Questions 10.8 and 10.9.

10.9 The Norton current at terminals *a-b* in the circuit of Fig. 10.49 is:

(a) $1\angle 0^\circ$ A (b) $1.5\angle{-90^\circ}$ A
(c) $1.5\angle 90^\circ$ A (d) $3\angle 90^\circ$ A

10.10 *PSpice* can handle a circuit with two independent sources of different frequencies.

(a) True (b) False

Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.

Problems

Section 10.2 Nodal Analysis

10.1 Determine i in the circuit of Fig. 10.50.

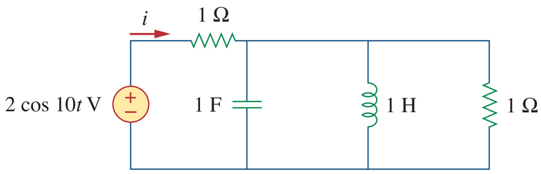

Figure 10.50
For Prob. 10.1.

10.2 Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

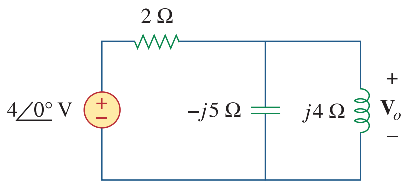

Figure 10.51
For Prob. 10.2.

10.3 Determine v_o in the circuit of Fig. 10.52.

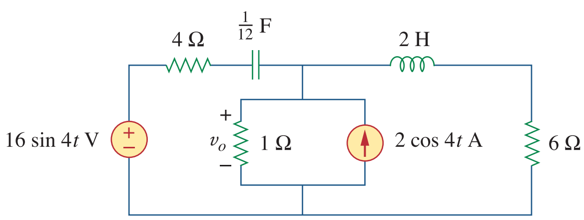

Figure 10.52
For Prob. 10.3.

10.4 Compute $v_o(t)$ in the circuit of Fig. 10.53.

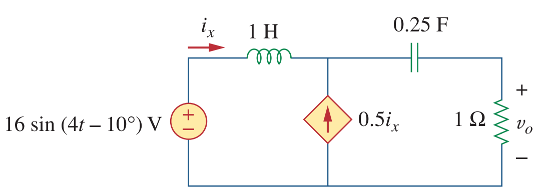

Figure 10.53
For Prob. 10.4.

10.5 Find i_o in the circuit of Fig. 10.54.

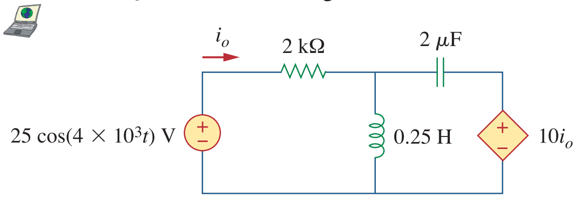

Figure 10.54
For Prob. 10.5.

10.6 Determine $\mathbf{V}_x$ in Fig. 10.55.

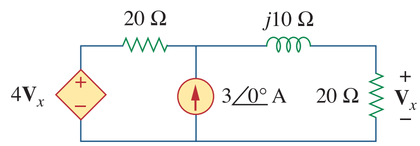

Figure 10.55
For Prob. 10.6.

10.7 Use nodal analysis to find $\mathbf{V}$ in the circuit of Fig. 10.56.

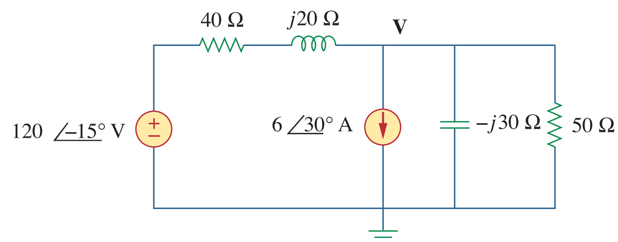

Figure 10.56
For Prob. 10.7.

10.8 Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6\cos(200t + 15°)$ A.

ML

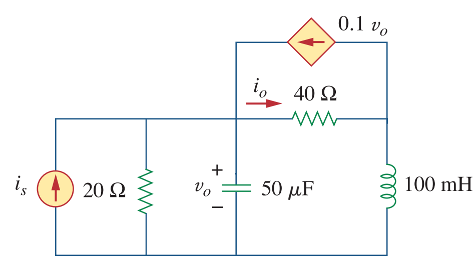

Figure 10.57
For Prob. 10.8.

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.

ML

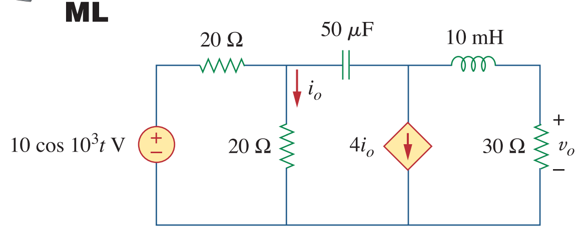

Figure 10.58
For Prob. 10.9.

10.10 Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

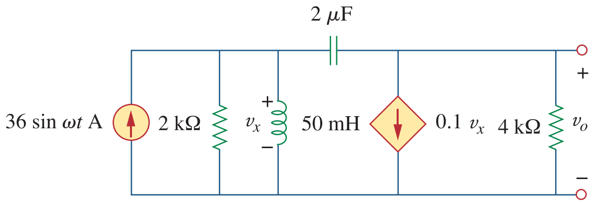

Figure 10.59
For Prob. 10.10.

10.11 Using nodal analysis, find $i_o(t)$ in the circuit in Fig. 10.60.

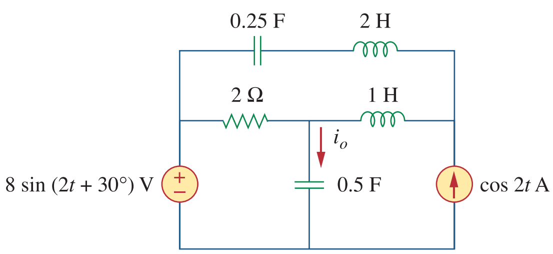

Figure 10.60
For Prob. 10.11.

10.12 Using Fig. 10.61, design a problem to help other students better understand nodal analysis.

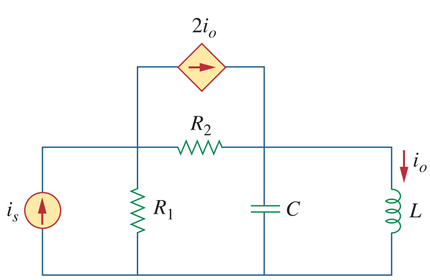

Figure 10.61
For Prob. 10.12.

10.13 Determine $\mathbf{V}_x$ in the circuit of Fig. 10.62 using any method of your choice.

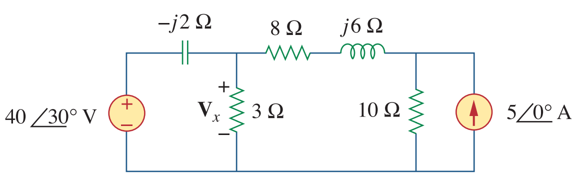

Figure 10.62
For Prob. 10.13.

10.14 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

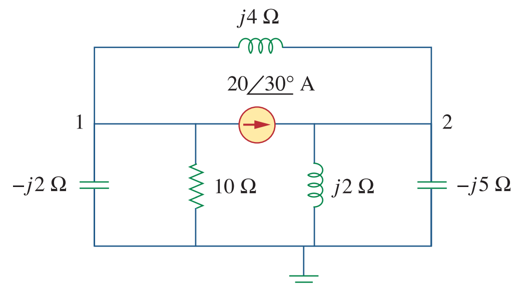

Figure 10.63
For Prob. 10.14.

10.15 Solve for the current $\mathbf{I}$ in the circuit of Fig. 10.64 using nodal analysis.

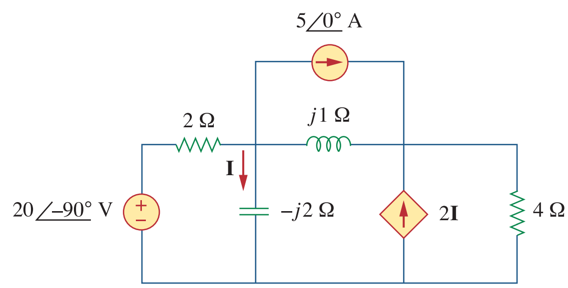

Figure 10.64
For Prob. 10.15.

10.16 Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

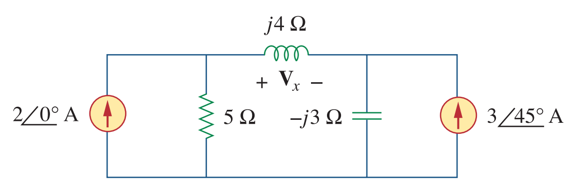

Figure 10.65
For Prob. 10.16.

10.17 By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

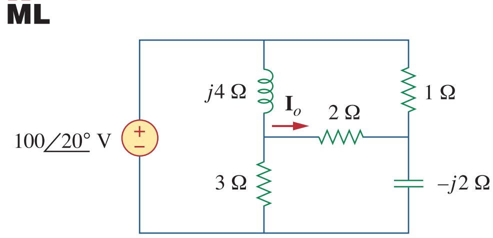

Figure 10.66
For Prob. 10.17.

10.18 Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

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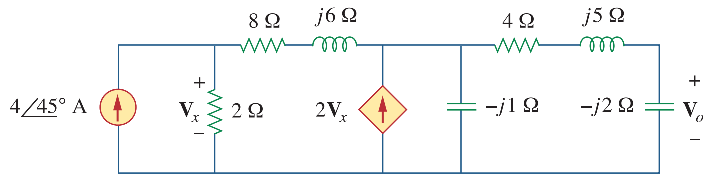

Figure 10.67
For Prob. 10.18.

10.19 Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

ML

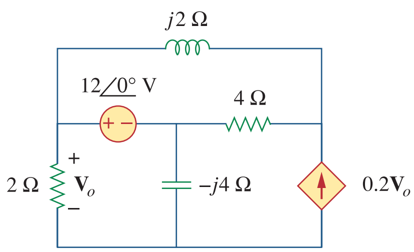

Figure 10.68
For Prob. 10.19.

10.20 Refer to Fig. 10.69. If $v_s(t) = V_m \sin\omega t$ and $v_o(t) = A\sin(\omega t + \phi)$, derive the expressions for A and ϕ.

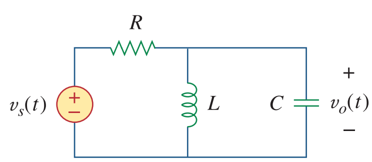

Figure 10.69
For Prob. 10.20.

10.21 For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0$, $\omega \to \infty$, and $\omega^2 = 1/LC$.

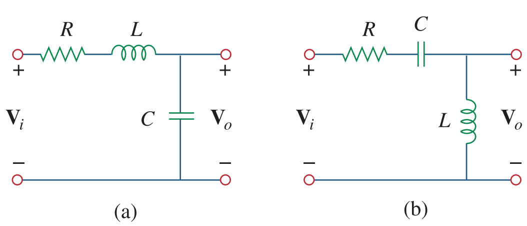

Figure 10.70
For Prob. 10.21.

10.22 For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

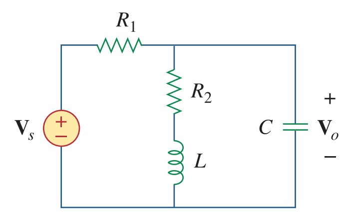

Figure 10.71
For Prob. 10.22.

10.23 Using nodal analysis obtain $\mathbf{V}$ in the circuit of Fig. 10.72.

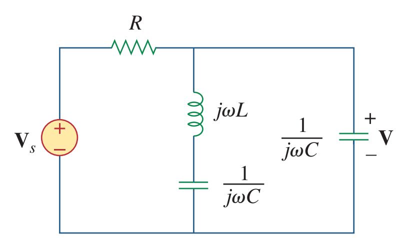

Figure 10.72
For Prob. 10.23.

Section 10.3 Mesh Analysis

10.24 Design a problem to help other students better understand mesh analysis.

10.25 Solve for i_o in Fig. 10.73 using mesh analysis.

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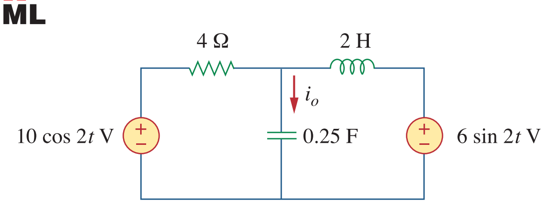


Figure 10.73
For Prob. 10.25.

10.26 Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

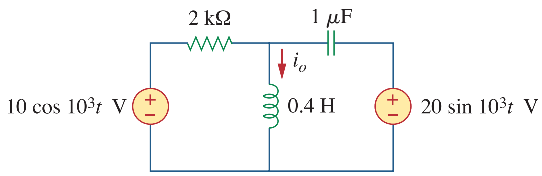


Figure 10.74
For Prob. 10.26.

10.27 Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

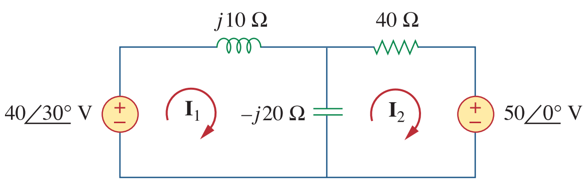


Figure 10.75
For Prob. 10.27.

10.28 In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10 \cos 4t$ **V** and $v_2 = 20 \cos(4t - 30°)$ V.

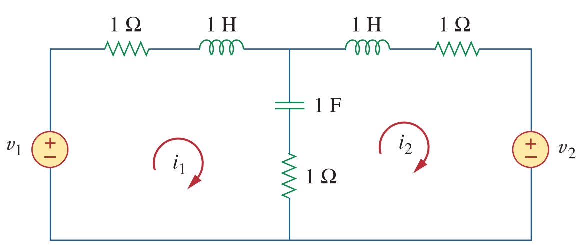


Figure 10.76
For Prob. 10.28.

10.29 Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

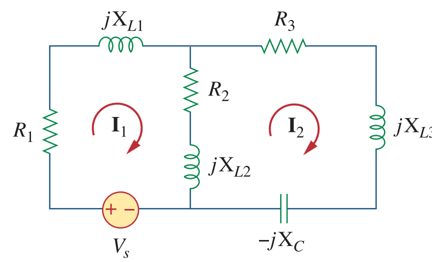


Figure 10.77
For Prob. 10.29.

10.30 Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120 \cos(100t + 90°)$ V, $v_{s2} = 80 \cos 100t$ V.

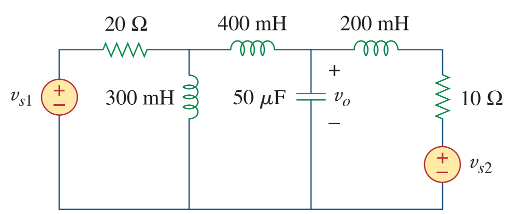


Figure 10.78
For Prob. 10.30.

10.31 Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

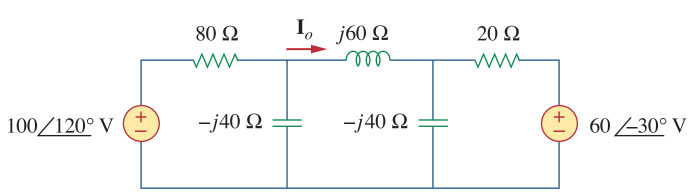


Figure 10.79
For Prob. 10.31.

10.32 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

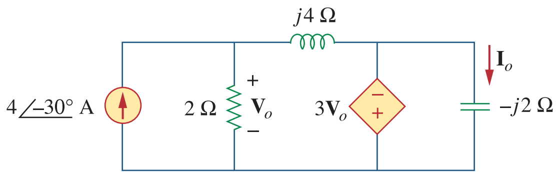


Figure 10.80
For Prob. 10.32.

10.33 Compute **I** in Prob. 10.15 using mesh analysis.

10.34 Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

10.35 Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

10.36 Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

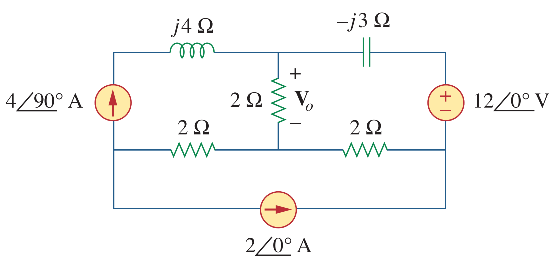


Figure 10.81
For Prob. 10.36.

10.37 Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

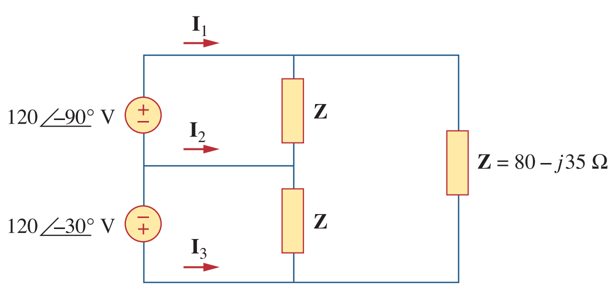


Figure 10.82
For Prob. 10.37.

10.38 Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

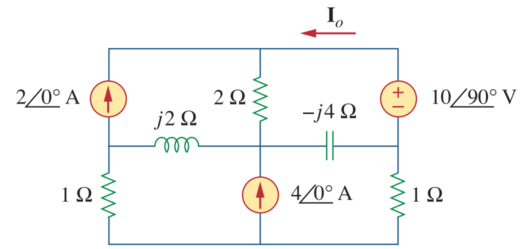


Figure 10.83
For Prob. 10.38.

10.39 Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

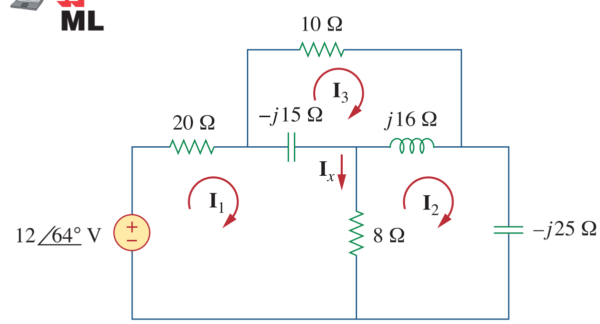


Figure 10.84
For Prob. 10.39.

Section 10.4 Superposition Theorem

10.40 Find i_o in the circuit shown in Fig. 10.85 using superposition.

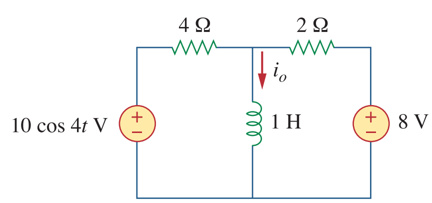


Figure 10.85
For Prob. 10.40.

10.41 Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 6 \cos 2t + 4 \sin 4t$ V.

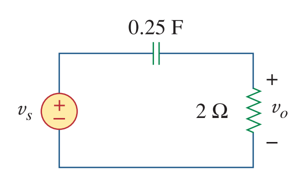


Figure 10.86
For Prob. 10.41.

10.42 Using Fig. 10.87, design a problem to help other students better understand the superposition theorem.

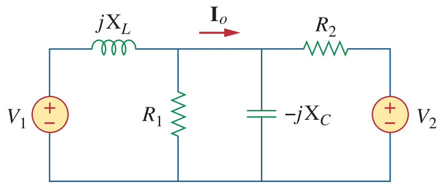


Figure 10.87
For Prob. 10.42.

10.43 Using the superposition principle, find i_x in the circuit of Fig. 10.88.

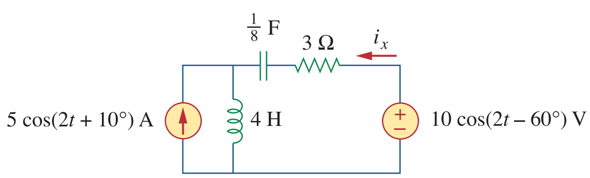


Figure 10.88
For Prob. 10.43.

10.44 Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 50 \sin 2t$ V and $i_s = 12 \cos(6t + 10°)$ A.

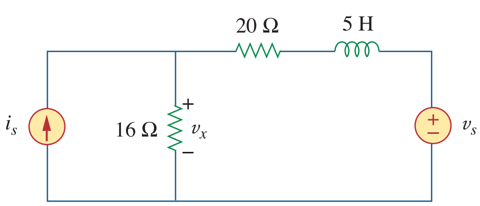


Figure 10.89
For Prob. 10.44.

10.45 Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

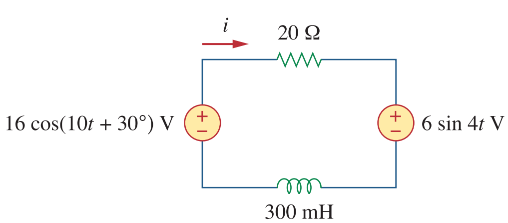


Figure 10.90
For Prob. 10.45.

10.46 Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

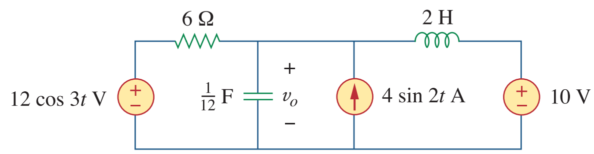


Figure 10.91
For Prob. 10.46.

10.47 Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

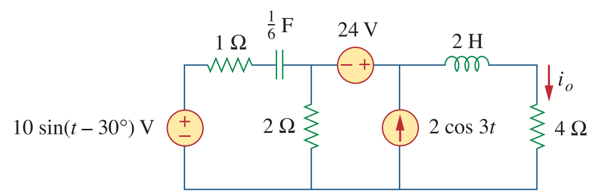


Figure 10.92
For Prob. 10.47.

10.48 Find i_o in the circuit of Fig. 10.93 using superposition.

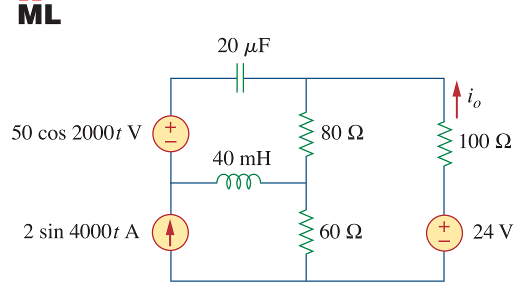


Figure 10.93
For Prob. 10.48.

Section 10.5 Source Transformation

10.49 Using source transformation, find i in the circuit of Fig. 10.94.

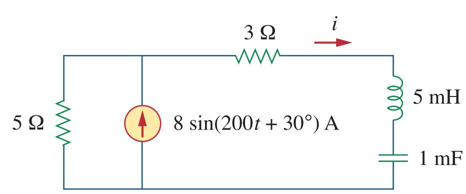


Figure 10.94
For Prob. 10.49.

10.50 Using Fig. 10.95, design a problem to help other students understand source transformation.

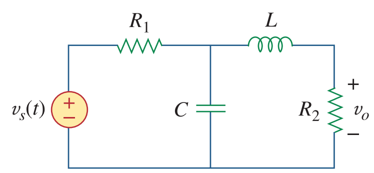


Figure 10.95
For Prob. 10.50.

10.51 Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

10.52 Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

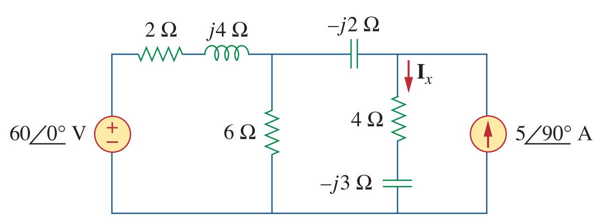


Figure 10.96
For Prob. 10.52.

10.53 Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

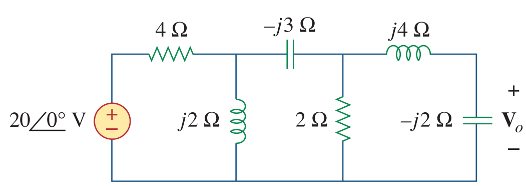
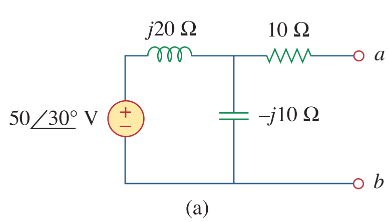


Figure 10.97
For Prob. 10.53.

10.54 Rework Prob. 10.7 using source transformation.

Section 10.6 Thevenin and Norton Equivalent Circuits

10.55 Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



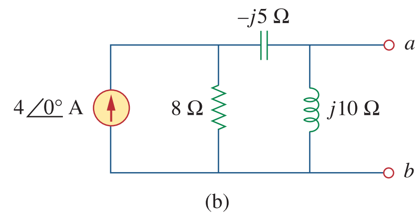


Figure 10.98
For Prob. 10.55.

10.56 For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.

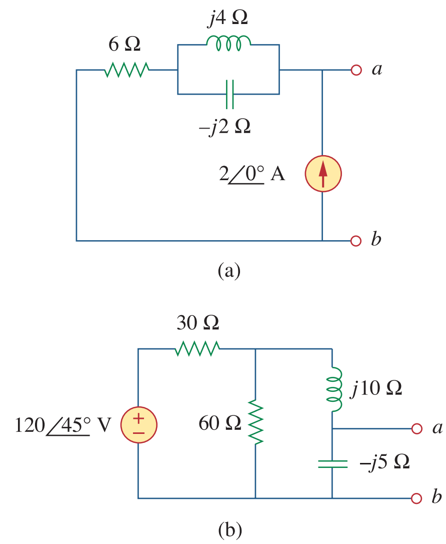


Figure 10.99
For Prob. 10.56.

10.57 Using Fig. 10.100, design a problem to help other students better understand Thevenin and Norton equivalent circuits.

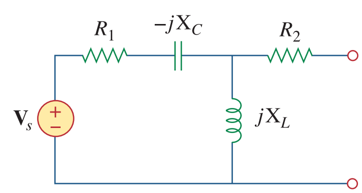


Figure 10.100
For Prob. 10.57.

10.58 For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

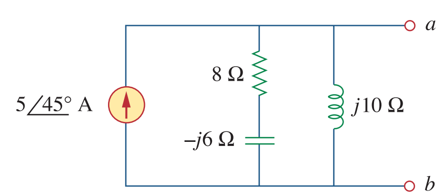


Figure 10.101
For Prob. 10.58.

10.59 Calculate the output impedance of the circuit shown in Fig. 10.102.

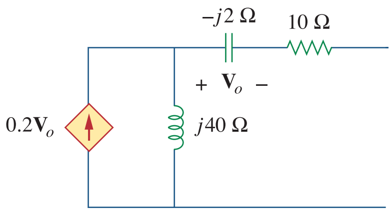


Figure 10.102
For Prob. 10.59.

10.60 Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b* (b) terminals *c-d*

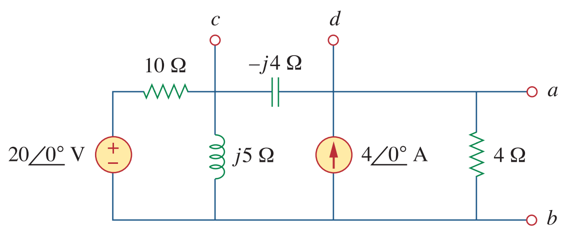


Figure 10.103
For Prob. 10.60.

10.61 Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.104.

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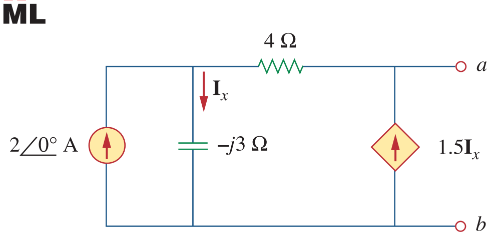

Figure 10.104
For Prob. 10.61.

10.62 Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

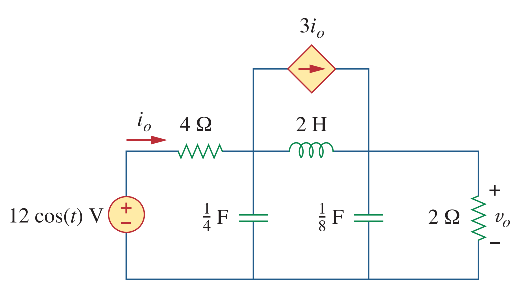

Figure 10.105
For Prob. 10.62.

10.63 Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

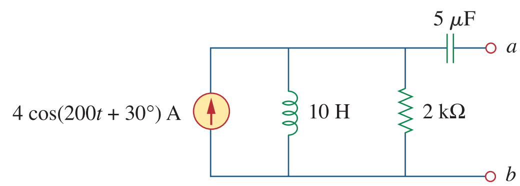

Figure 10.106
For Prob. 10.63.

10.64 For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

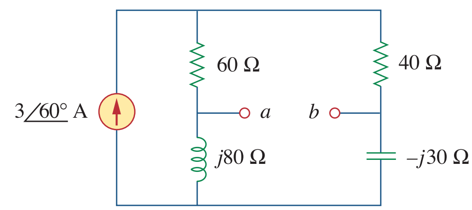

Figure 10.107
For Prob. 10.64.

10.65 Using Fig. 10.108, design a problem to help other students better understand Norton's theorem.

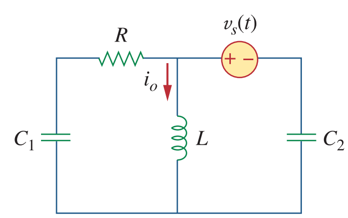

Figure 10.108
For Prob. 10.65.

10.66 At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

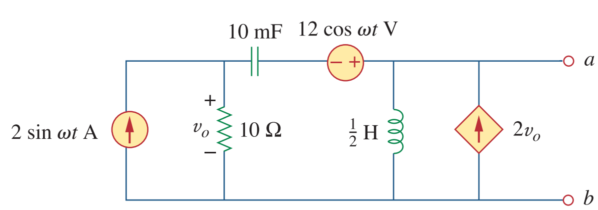

Figure 10.109
For Prob. 10.66.

10.67 Find the Thevenin and Norton equivalent circuits at terminals *a-b* in the circuit of Fig. 10.110.
ML

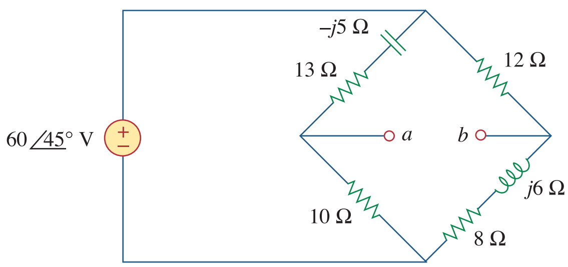

Figure 10.110
For Prob. 10.67.

10.68 Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.
ML

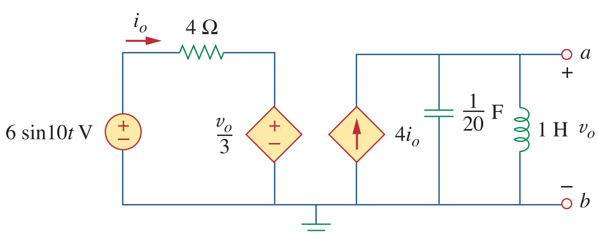

Figure 10.111
For Prob. 10.68.

Section 10.7 Op Amp AC Circuits

10.69 For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin \omega t$ and $\omega = 1/RC$.

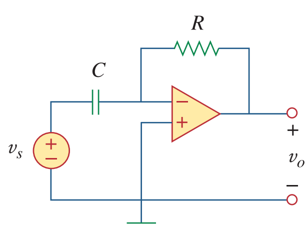

Figure 10.112
For Prob. 10.69.

10.70 Using Fig. 10.113, design a problem to help other students better understand op amps in AC circuits.
e⊘d

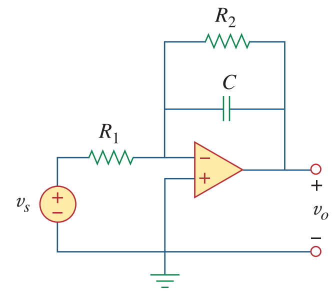

Figure 10.113
For Prob. 10.70.

10.71 Find v_o in the op amp circuit of Fig. 10.114.

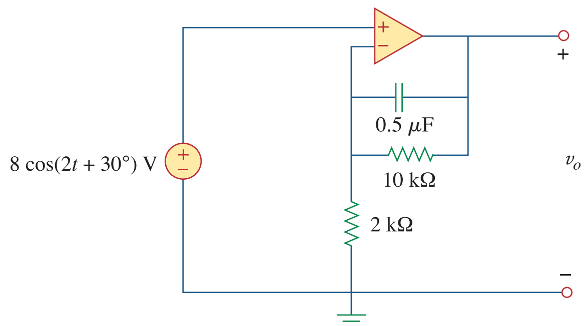

Figure 10.114
For Prob. 10.71.

10.72 Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 4 \cos(10^4 t)$ V.

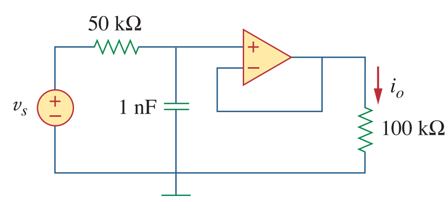

Figure 10.115
For Prob. 10.72.

10.73 If the input impedance is defined as $\mathbf{Z}_{in} = \mathbf{V}_s/\mathbf{I}_s$, find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $C_1 = 10$ nF, $C_2 = 20$ nF, and $\omega = 5000$ rad/s.

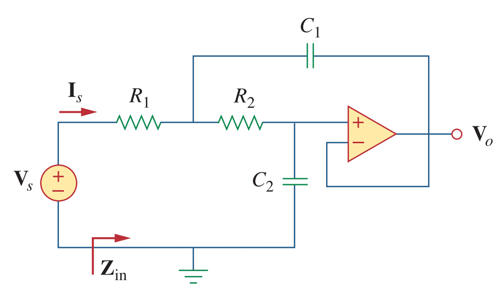

Figure 10.116
For Prob. 10.73.

10.74 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \to \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

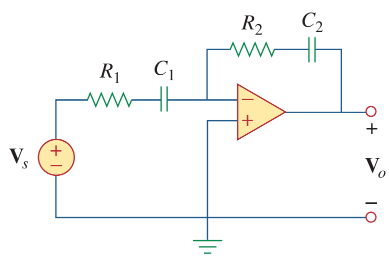

Figure 10.117
For Prob. 10.74.

10.75 In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 40$ kΩ, and $\omega = 2000$ rad/s.

ML

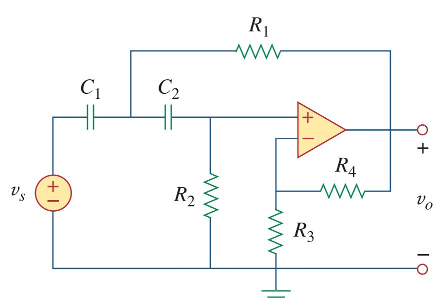

Figure 10.118
For Prob. 10.75.

10.76 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

ML

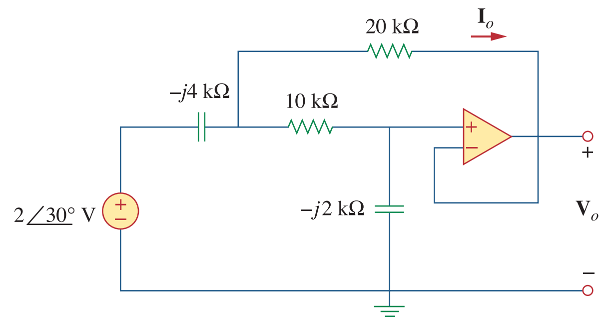

Figure 10.119
For Prob. 10.76.

10.77 Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

ML

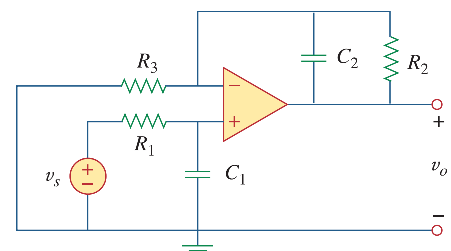

Figure 10.120
For Prob. 10.77.

10.78 Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

ML

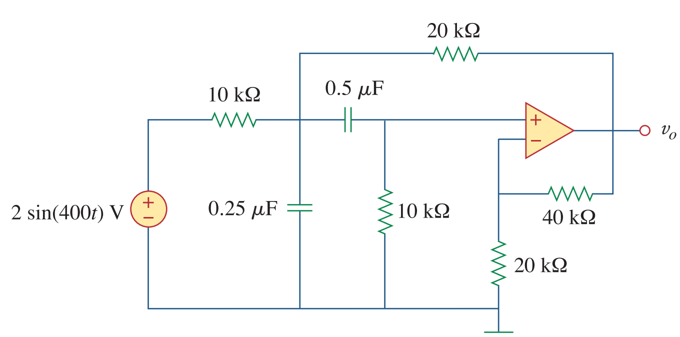

Figure 10.121
For Prob. 10.78.

10.79 For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

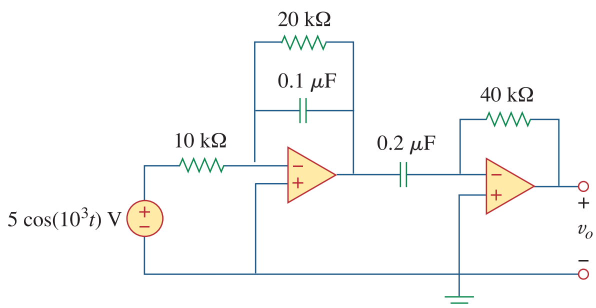

Figure 10.122
For Prob. 10.79.

10.80 Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

ML

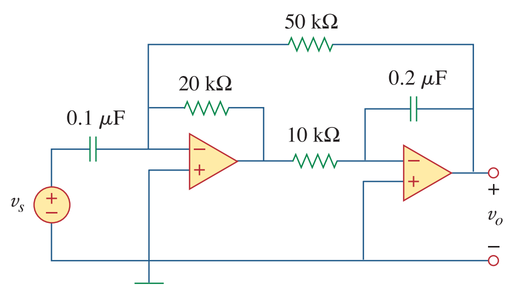

Figure 10.123
For Prob. 10.80.

Section 10.8 AC Analysis Using *PSpice*

10.81 Use *PSpice or MultiSim* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

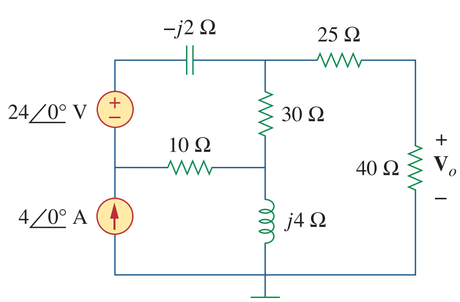

Figure 10.124
For Prob. 10.81.

10.82 Solve Prob. 10.19 using *PSpice or MultiSim*.

10.83 Use *PSpice or MultiSim* to find $v_o(t)$ in the circuit of Fig. 10.125. Let $i_s = 2\cos(10^3 t)$ A.

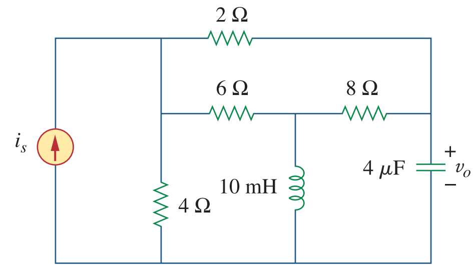

Figure 10.125
For Prob. 10.83.

10.84 Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.126 using *PSpice or MultiSim.*

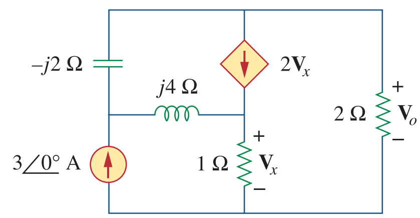

Figure 10.126
For Prob. 10.84.

10.85 Using Fig. 10.127, design a problem to help other students better understand performing AC analysis with *PSpice or MultiSim.*

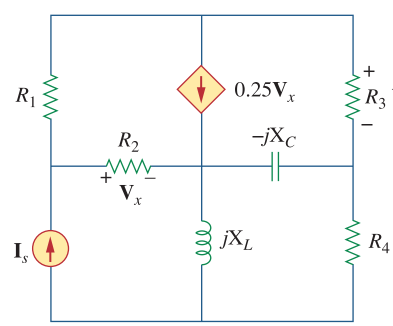

Figure 10.127
For Prob. 10.85.

10.86 Use *PSpice or MultiSim* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.128.

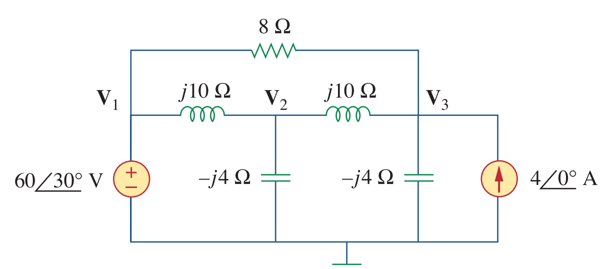

Figure 10.128
For Prob. 10.86.

10.87 Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.129 using *PSpice or MultiSim.*

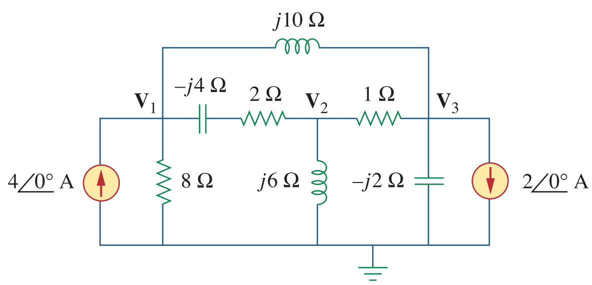

Figure 10.129
For Prob. 10.87.

10.88 Use *PSpice or MultiSim* to find v_o and i_o in the circuit of Fig. 10.130 below.

10.90 Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

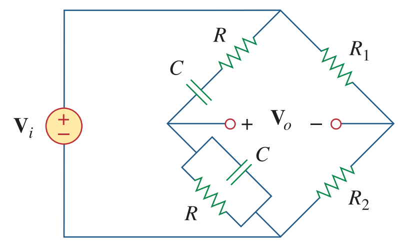

Figure 10.132
For Prob. 10.90.

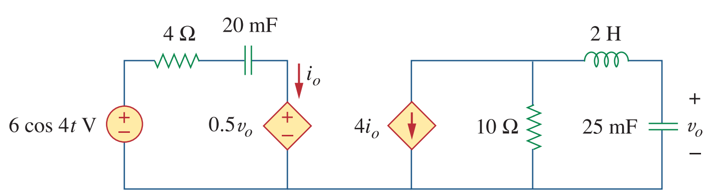

Figure 10.130
For Prob. 10.88.

Section 10.9 Applications

10.89 The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1 R_3 R_4}{R_2} C$$

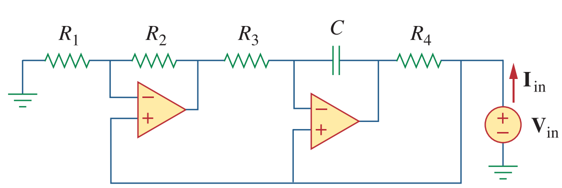

Figure 10.131
For Prob. 10.89.

10.91 Consider the oscillator in Fig. 10.133.

(a) Determine the oscillation frequency.

(b) Obtain the minimum value of R for which oscillation takes place.

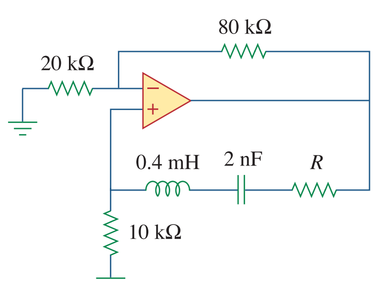

Figure 10.133
For Prob. 10.91.

10.92 The oscillator circuit in Fig. 10.134 uses an ideal op amp.

(a) Calculate the minimum value of R_o that will cause oscillation to occur.

(b) Find the frequency of oscillation.

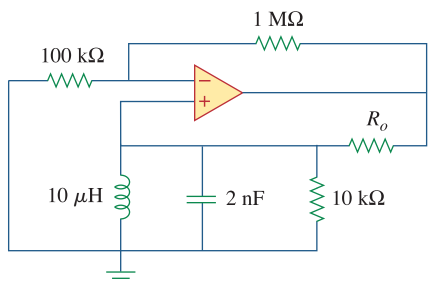

Figure 10.134
For Prob. 10.92.

10.93 Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i \gg X_{C_2}$.

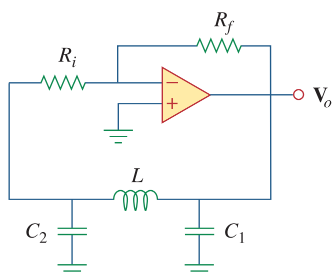

Figure 10.135
A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

10.94 Design a Colpitts oscillator that will operate at 50 kHz.

10.95 Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

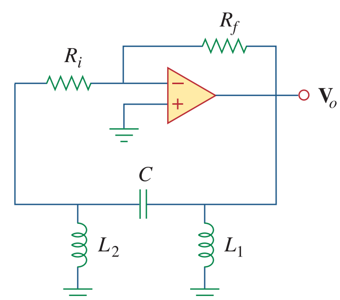

Figure 10.136
A Hartley oscillator; for Prob. 10.95.

10.96 Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

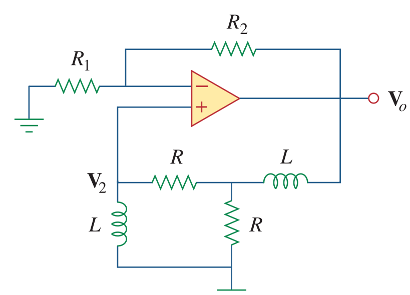

Figure 10.137
For Prob. 10.96.