Tests on Equality of Variances

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

recall: $H_0 : \mu_1 - \mu_2 = \Delta_0$

∴ if $\Delta_0 = 0$, then alt:

$H_0 : \mu_1 = \mu_2$

∴ for variance, alt. notation:

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

∴ all about ratio, not difference!

∴ need new probability distribution:

F - Distribution

∴ in gamma family, like chi-squared

∴ key feature of table:

<u>numerator</u> degrees of freedom (columns)

<u>denominator</u> degrees of freedom (rows)

∴ where does α fit in?

∴ seperate table for each α!

test statistic:

$$F_0 = \frac{S_1^2}{S_2^2}$$

∴ ratio of <u>sample</u> variances!

critical values for fixed-α tests:

upper: $f_{\alpha/2,\, n_1-1,\, n_2-1}$

num. d.o.f.

den. d.o.f.

lower: $f_{1-\alpha/2,\, n_1-1,\, n_2-1}$

- like chi-squared, <u>separate</u> upper and lower critical values
- we should have separate tables for different $1-\alpha/2$ values (.975, .95, etc..)
- they don't give 'em to us!

trick:

$$f_{1-\alpha/2,\, n_1-1,\, n_2-1} = \frac{1}{f_{\alpha/2,\, \underbrace{\overset{\text{num}}{n_2-1},\, \overset{\text{den}}{n_1-1}}_{\text{reversed!}}}}$$

this we have!

ex: two mixtures of gases used in semiconductor wafer etching process

- study: is one superior in reducing variability of oxide thickness?

test results:

$n_1 = 16$ $\qquad$ $n_2 = 16$

$S_1 = 1.96\,Å$ $\qquad$ $S_2 = 2.13\,Å$

.. test $H_0: \dfrac{\sigma_1^2}{\sigma_2^2} = 1$ ← unity

$H_1: \dfrac{\sigma_1^2}{\sigma_2^2} \neq 1$

(iii) $\alpha = 0.05$ [fixed-α]

$$f_0 = \frac{s_1^2}{s_2^2} = \frac{1.96^2}{2.13^2} = 0.8467$$

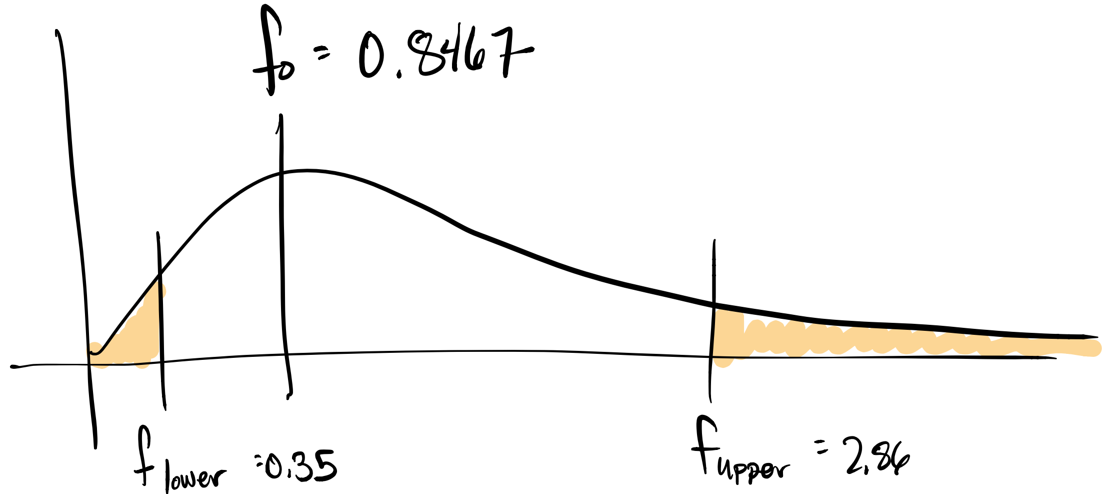
Critical values: $f_{\alpha/2,\, n_1-1,\, n_2-1} = f_{.025,15,15}$

$= 2.86$ (upper)

$$f_{1-\alpha/2,\, n_1-1,\, n_2-1} = \frac{1}{f_{\alpha/2,\, n_2-1,\, n_1-1}}$$

$$= \frac{1}{f_{.025,15,15}} = \frac{1}{2.86}$$

$= 0.35$ (lower)



$$f_0 \not\geq f_{\alpha/2,\, n_1-1,\, n_2-1}$$

$$f_0 \not\leq f_{1-\alpha/2,\, n_1-1,\, n_2-1}$$

$\boxed{\therefore \text{ fail to reject } H_0}$

$\therefore$ data does not suggest that the variance of oxide thickness differs between gas mixtures

C.I. on Ratio of Two Variances

$$\frac{s_1^2}{s_2^2} f_{1-\alpha/2,\,\underbrace{n_2-1,\,n_1-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2,\,\underbrace{n_2-1,\,n_1-1}_{\text{reversed}}}$$

with respect to upper critical value of hypothesis test!

$\therefore$ lower bound is computed by

$$f_{1-\alpha/2,\,n_2-1,\,n_1-1} = \frac{1}{f_{\alpha/2,\,\underbrace{n_1-1,\,n_2-1}_{\text{switched back !!!!}}}}$$

ex: manufacturer of jet-turbine impellers

.. needs to select grinding process for titanium alloy surface that offers <u>least variability</u> in surface roughness

process #1:

$n_1 = 11$

$s_1 = 5.1$ μin.

process #2:

$n_2 = 16$

$s_2 = 4.7$ μin.

.. write 90% C.I. on ratio of pop. standard deviations. $\alpha = 0.10$

$$f_{\alpha/2,\, n_2-1,\, n_1-1} = f_{.05,\, 15,\, 10} = \underline{2.85}$$

15 → num.   10 → den.

$$f_{1-\alpha/2,\, n_2-1,\, n_1-1} = \frac{1}{f_{\alpha/2,\, n_1-1,\, n_2-1}} = \frac{1}{f_{.05,\, 10,\, 15}} = \frac{1}{2.54}$$

$$= \underline{0.3937}$$

$$\frac{5.1^2}{4.7^2} \cdot 0.3937 < \frac{\sigma_1^2}{\sigma_2^2} < \frac{5.1^2}{4.7^2} \cdot 2.85$$

unit !!!

$$0.4636 < \frac{\sigma_1^2}{\sigma_2^2} < 3.356 \left[\frac{\mu in^2}{\mu in^2}\right]$$

$$0.6809 < \frac{\sigma_1}{\sigma_2} < 1.832 \left[\frac{\mu in}{\mu in}\right]$$

$\because$ this C.I. <u>does</u> include <u>unity</u> (1)

$\therefore$ We would <u>fail to reject</u>

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$\text{OR} \quad H_0 : \sigma_1^2 = \sigma_2^2$$

$\because$ data does <u>not</u> suggest that the grinding processes affect surface roughness of impellers