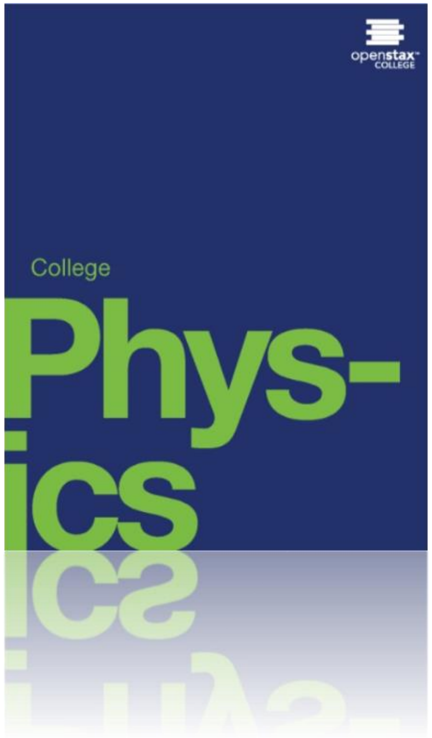
COLLEGE PHYSICS

Chapter 3 TWO-DIMENSIONAL KINEMATICS

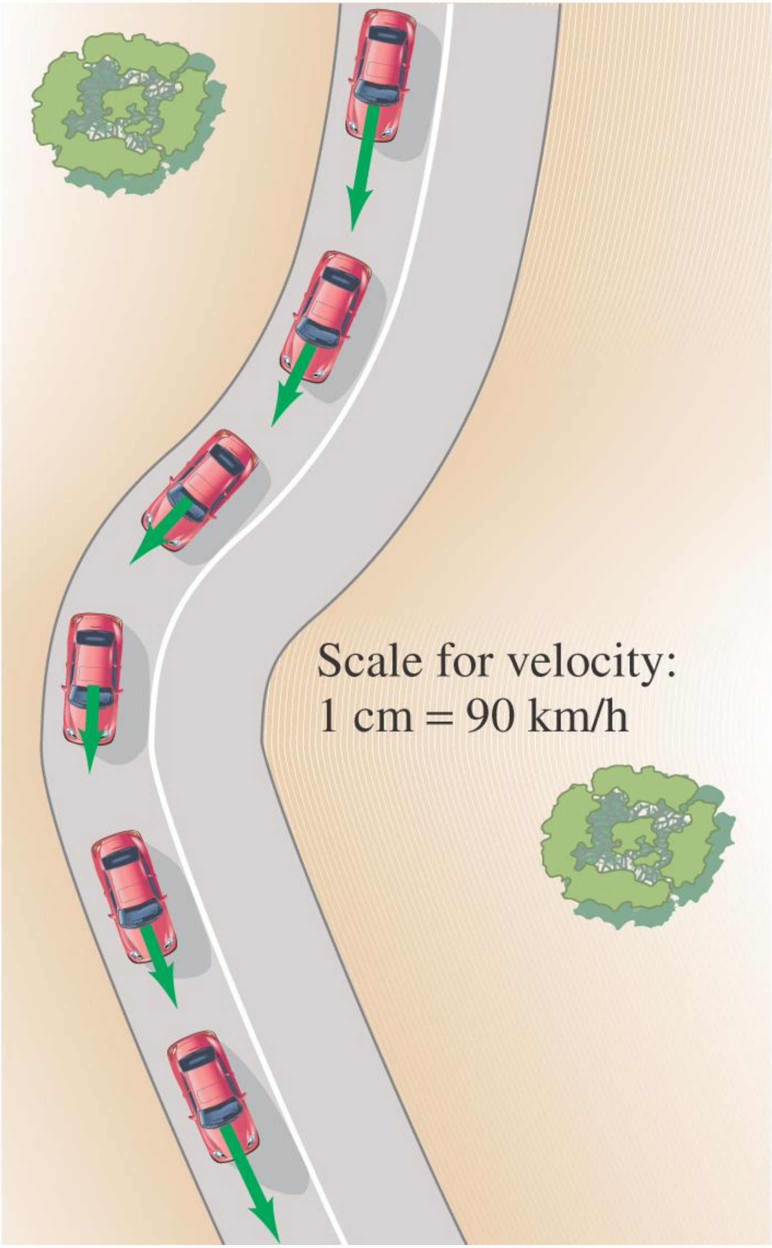
PowerPoint Image Slideshow



Chapter 3

- Vectors and Scalars
- Addition of Vectors – Graphical Methods
- Subtraction of Vectors, and Multiplication of a Vector by a Scalar
- Adding Vectors by Components

Vectors and Scalars



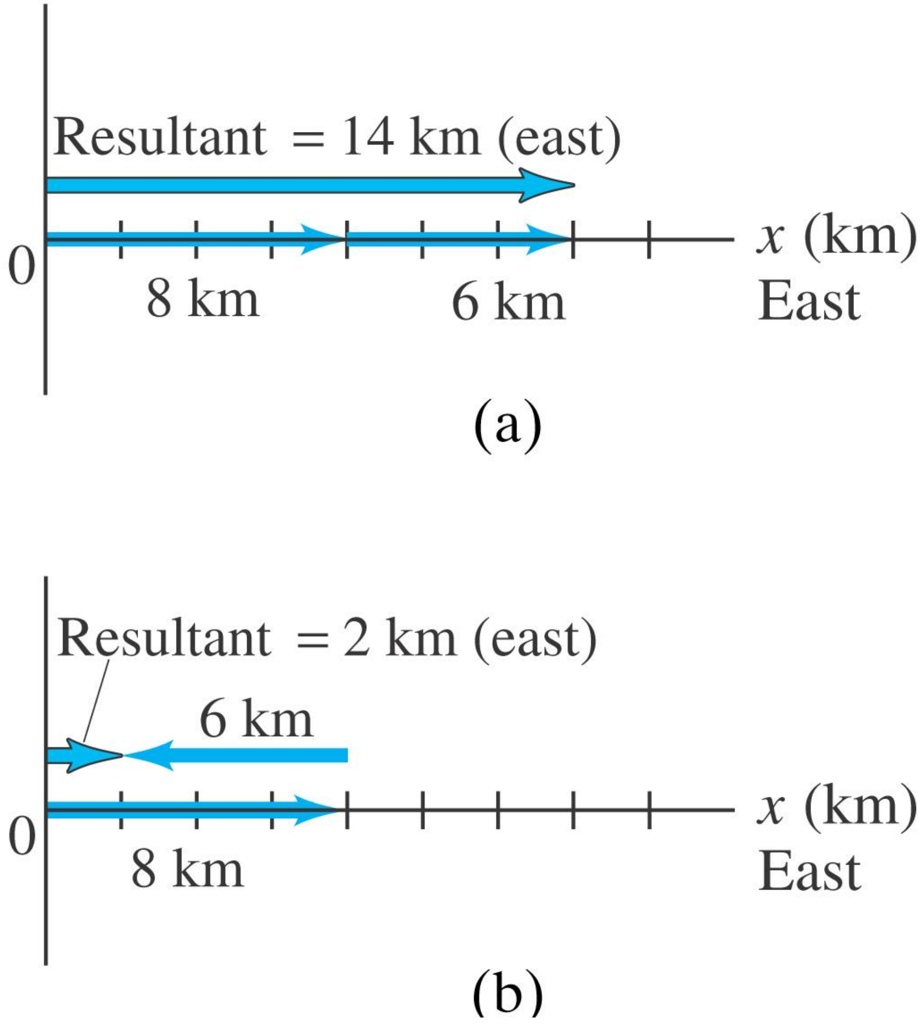
A vector has magnitude as well as direction.

Some vector quantities: displacement, velocity, force, momentum

A scalar has only a magnitude.

Some scalar quantities: mass, time, temperature

Addition of Vectors – Graphical Methods



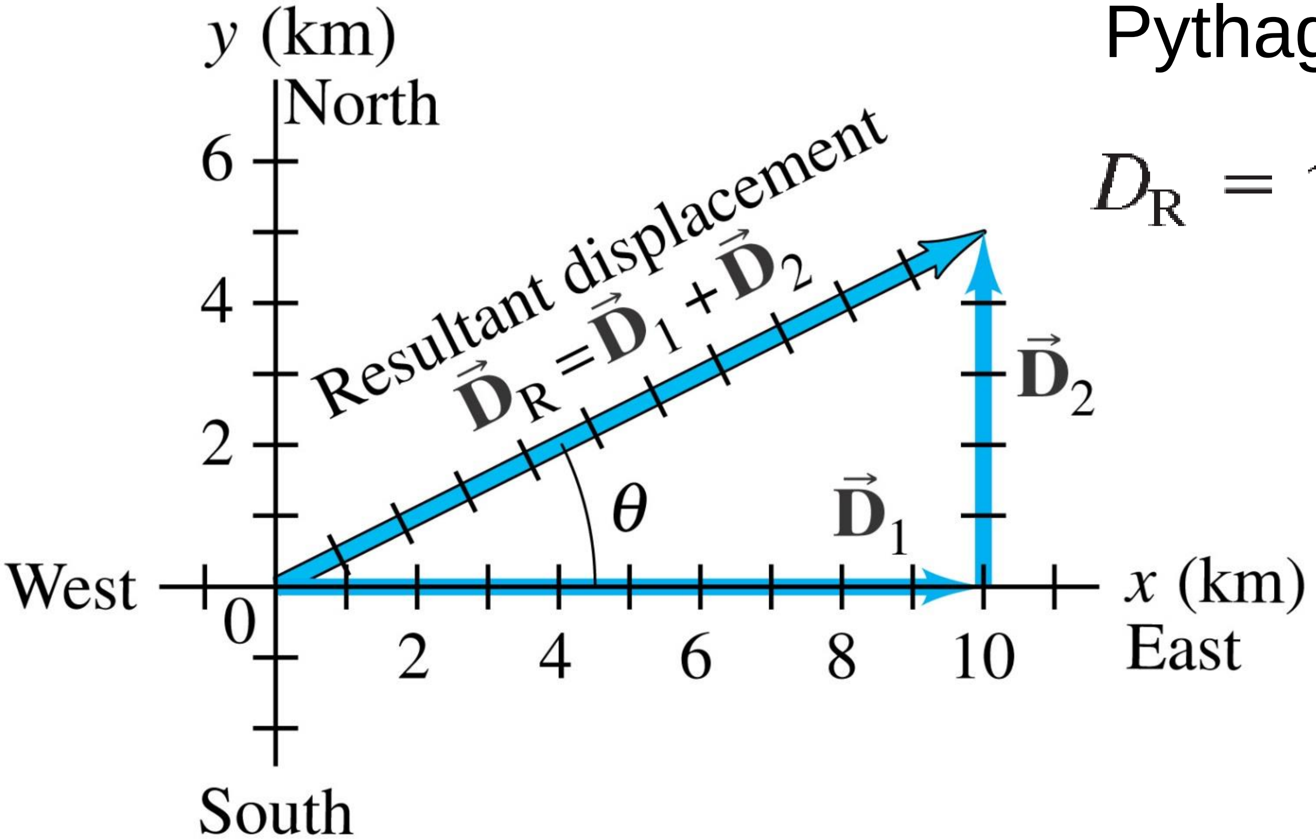
For vectors in one dimension, simple addition and subtraction are all that is needed.

You do need to be careful about the signs, as the figure indicates.

Addition of Vectors – Graphical Methods

If the motion is in two dimensions, the situation is somewhat more complicated.

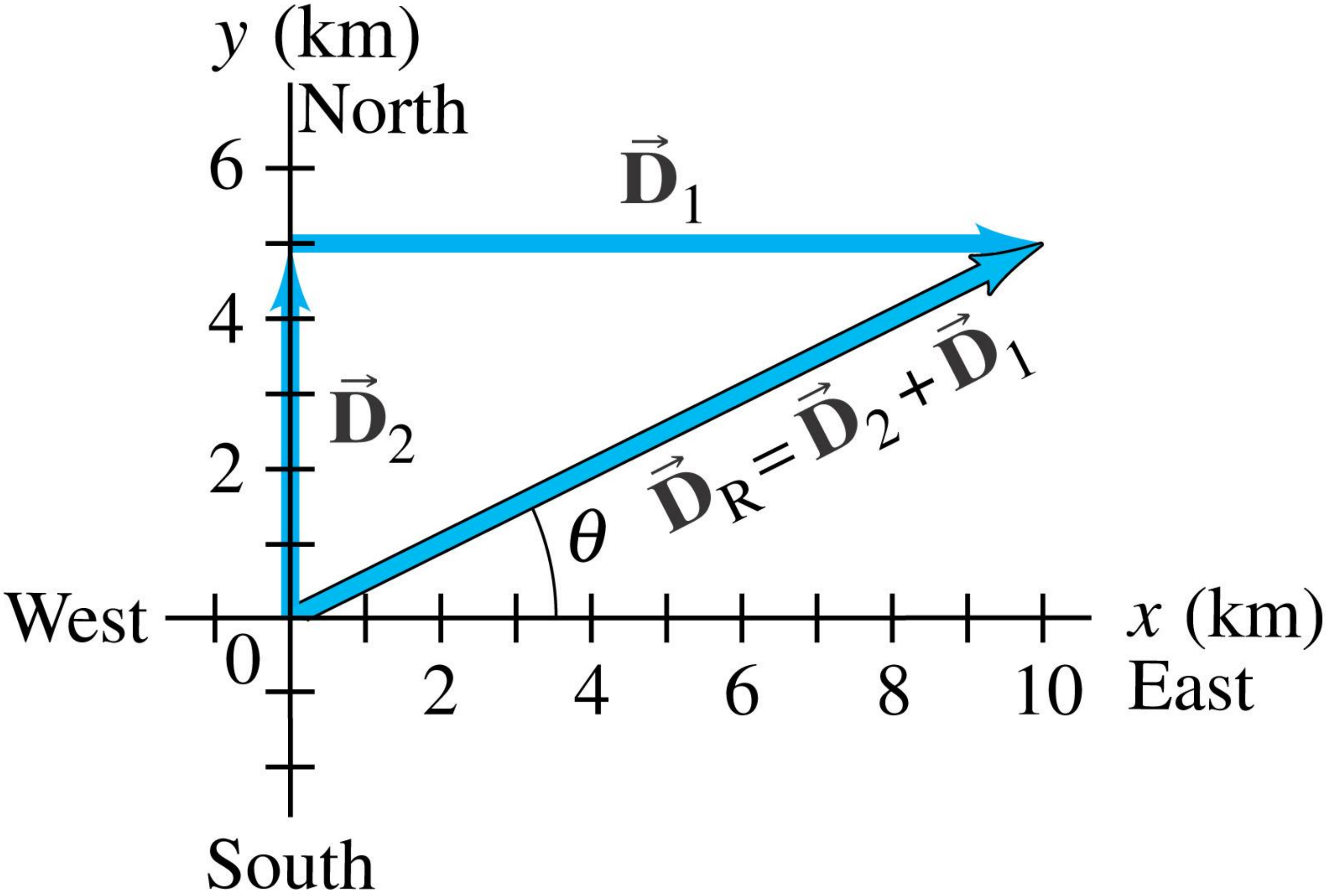
Here, the actual travel paths are at right angles to one another; we can find the displacement by using the Pythagorean Theorem.



$$D_R = \sqrt{D_1^2 + D_2^2}$$

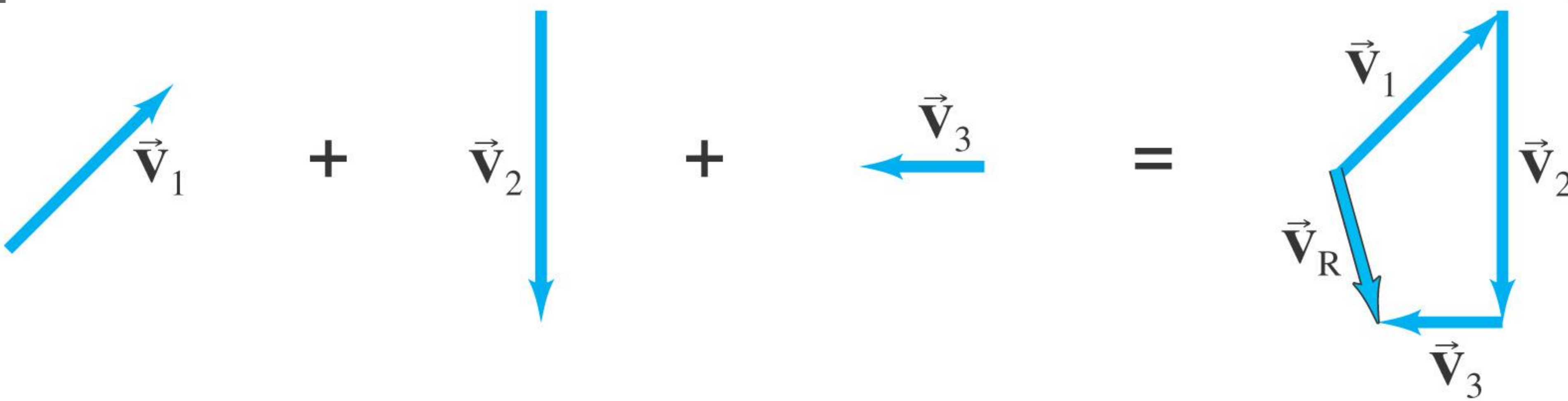
Addition of Vectors – Graphical Methods

Adding the vectors in the opposite order gives the same result: $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1$



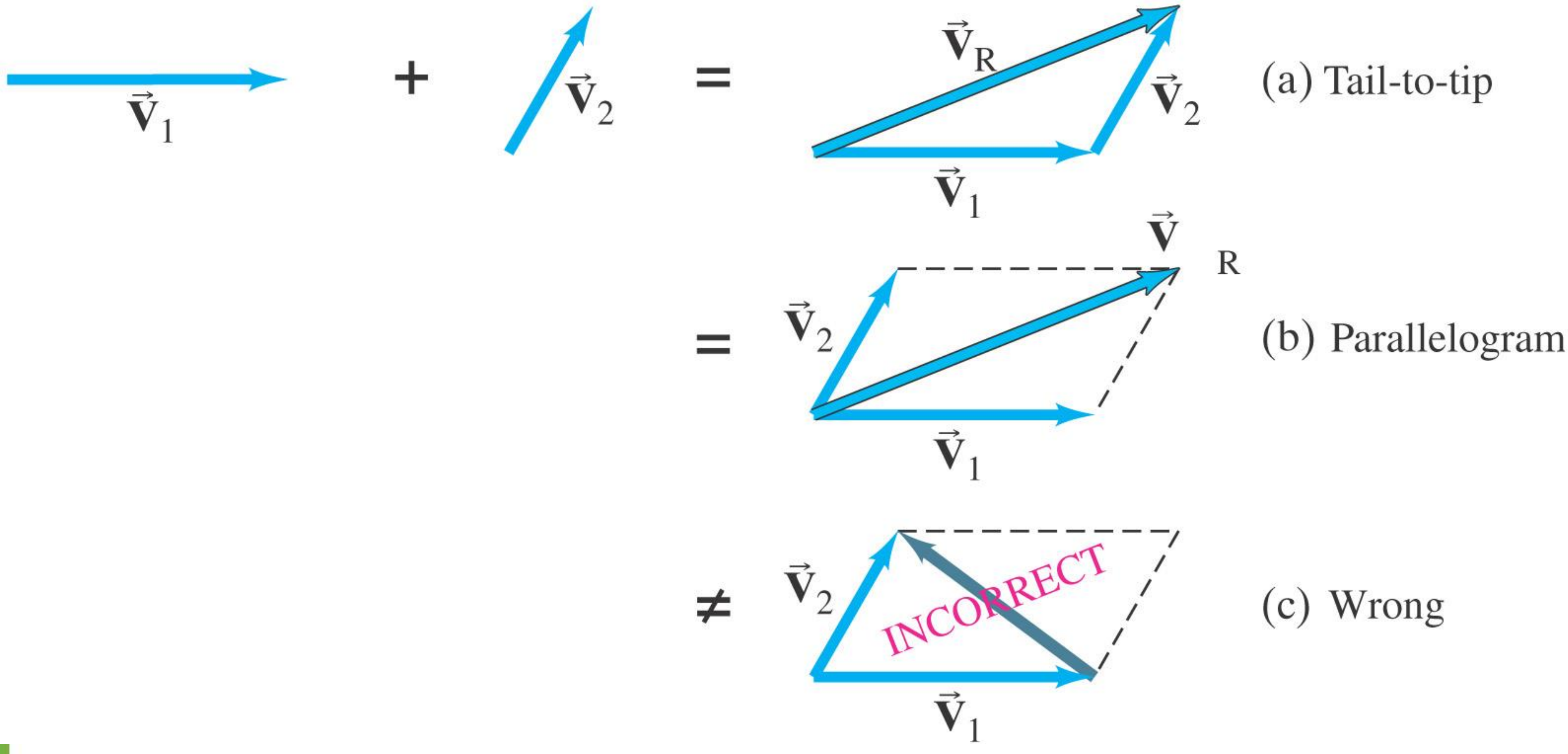
Addition of Vectors – Graphical Methods

Even if the vectors are not at right angles, they can be added graphically by using the "tail-to-tip" method.

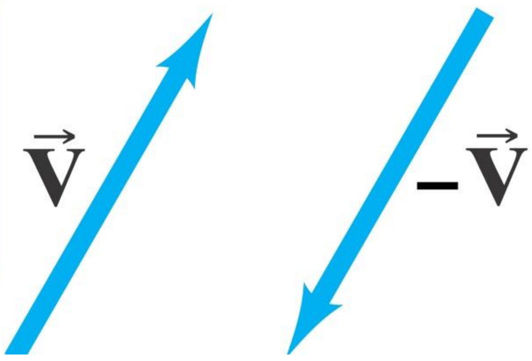


Addition of Vectors – Graphical Methods

The parallelogram method may also be used; here again the vectors must be "tail-to-tip."



Subtraction of Vectors, and Multiplication of a Vector by a Scalar

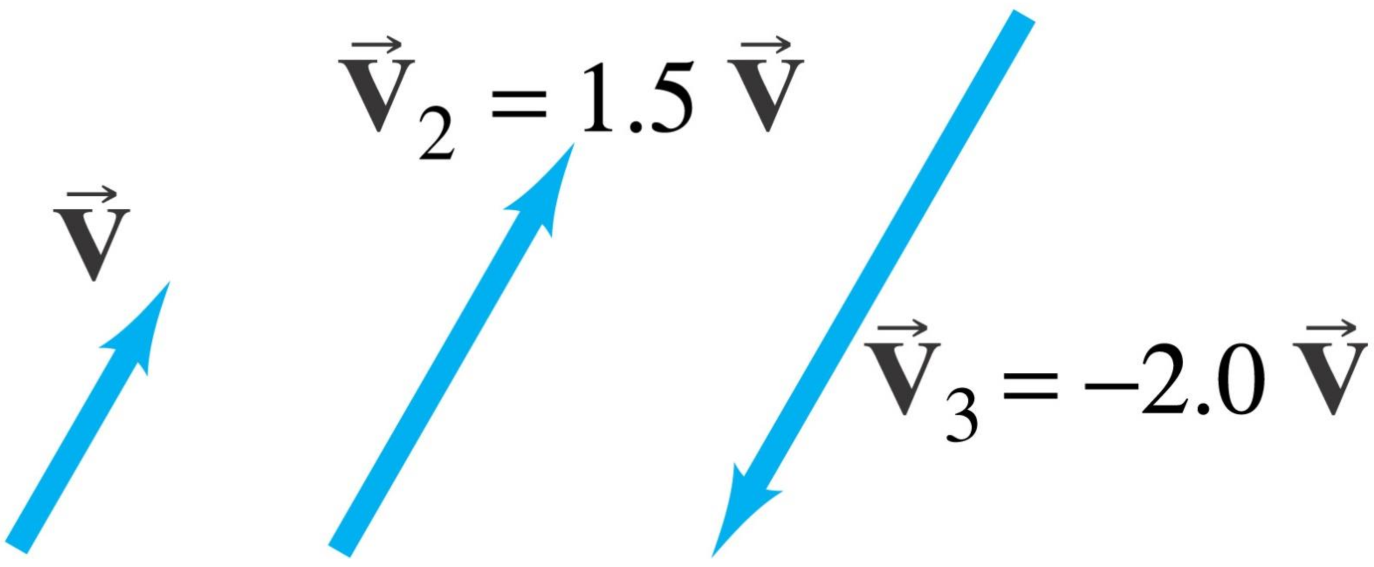


In order to subtract vectors, we define the negative of a vector, which has the same magnitude but points in the opposite direction.

Then we add the negative vector:

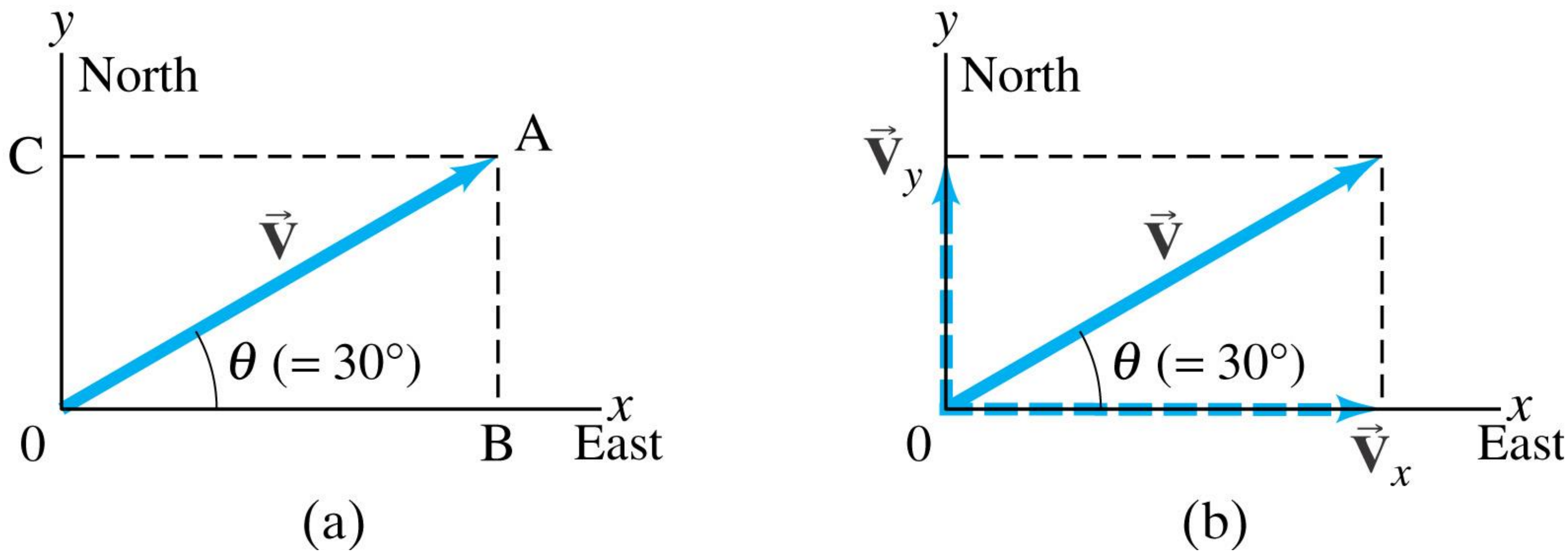
Subtraction of Vectors, and Multiplication of a Vector by a Scalar

A vector V can be multiplied by a **scalar** *c*; the result is a vector *c*V that has the same **direction** but a **magnitude** *cV*. If *c* is **negative**, the resultant vector points in the opposite direction.

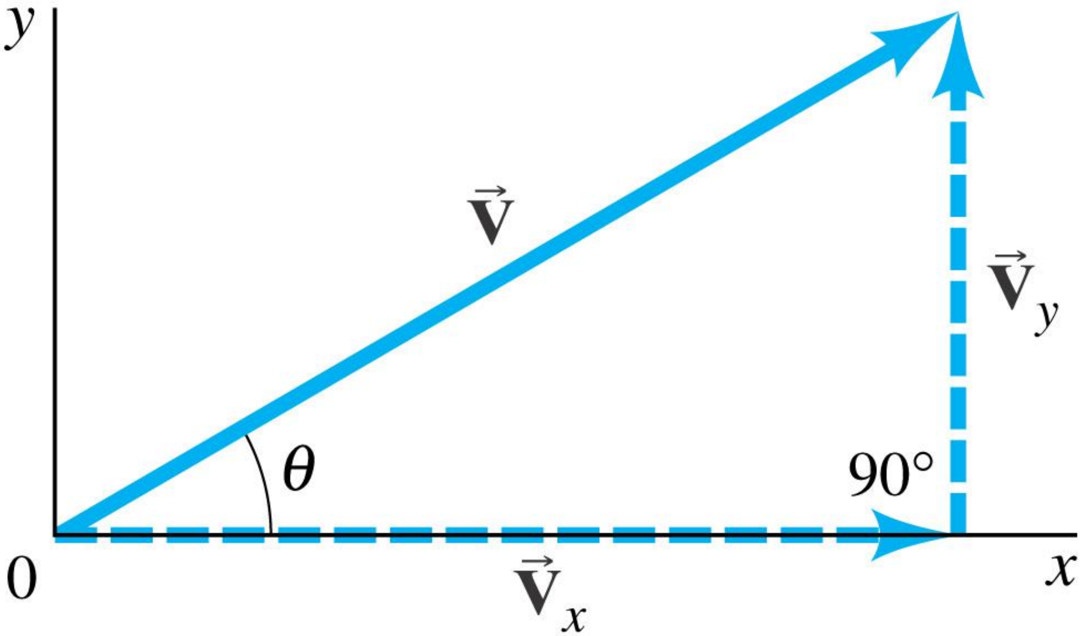


Adding Vectors by Components

Any vector can be expressed as the sum of two other vectors, which are called its components. Usually the other vectors are chosen so that they are perpendicular to each other.



Adding Vectors by Components



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

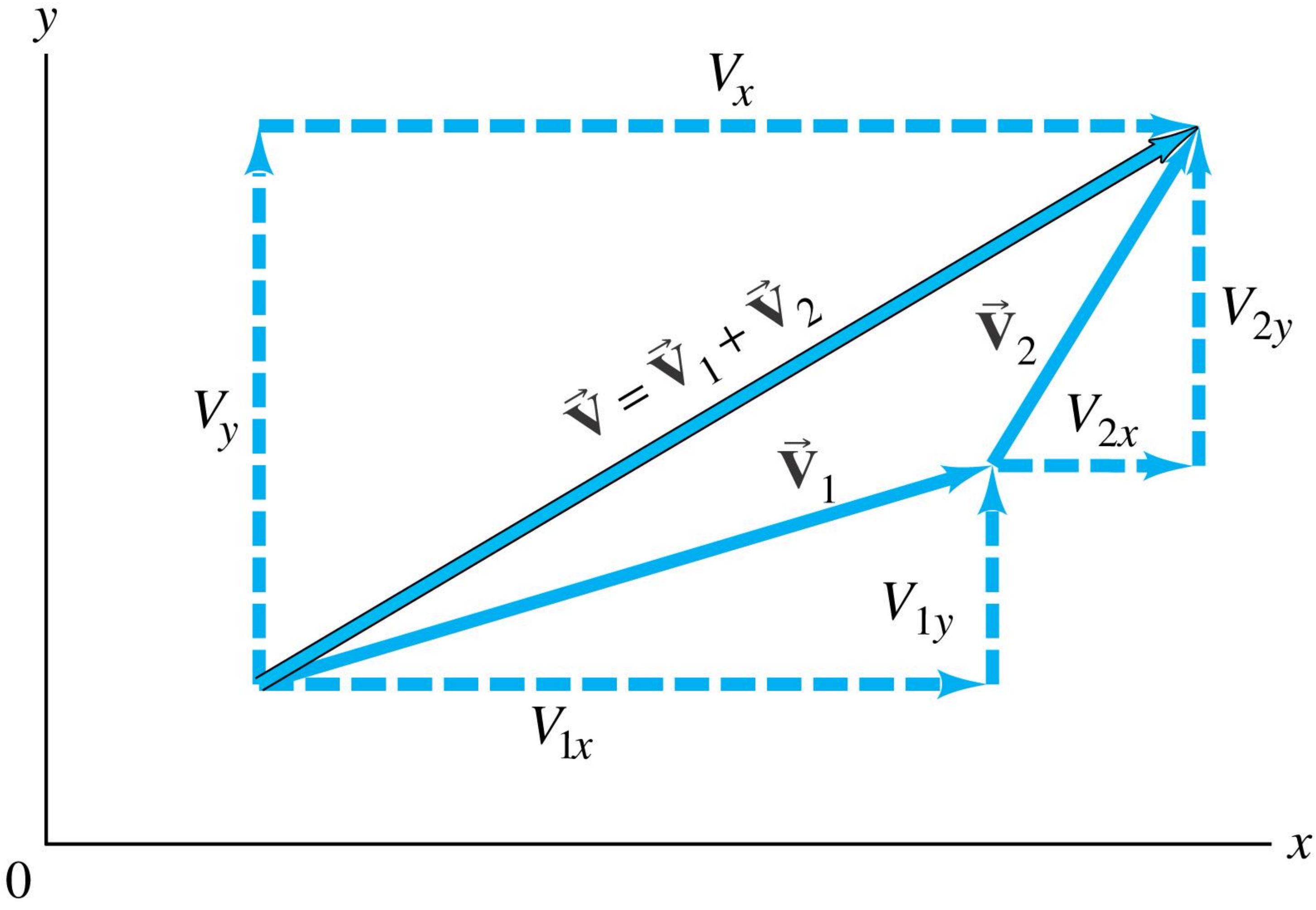
$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

If the components are perpendicular, they can be found using trigonometric functions.

Adding Vectors by Components

The components are effectively one-dimensional, so they can be added arithmetically:

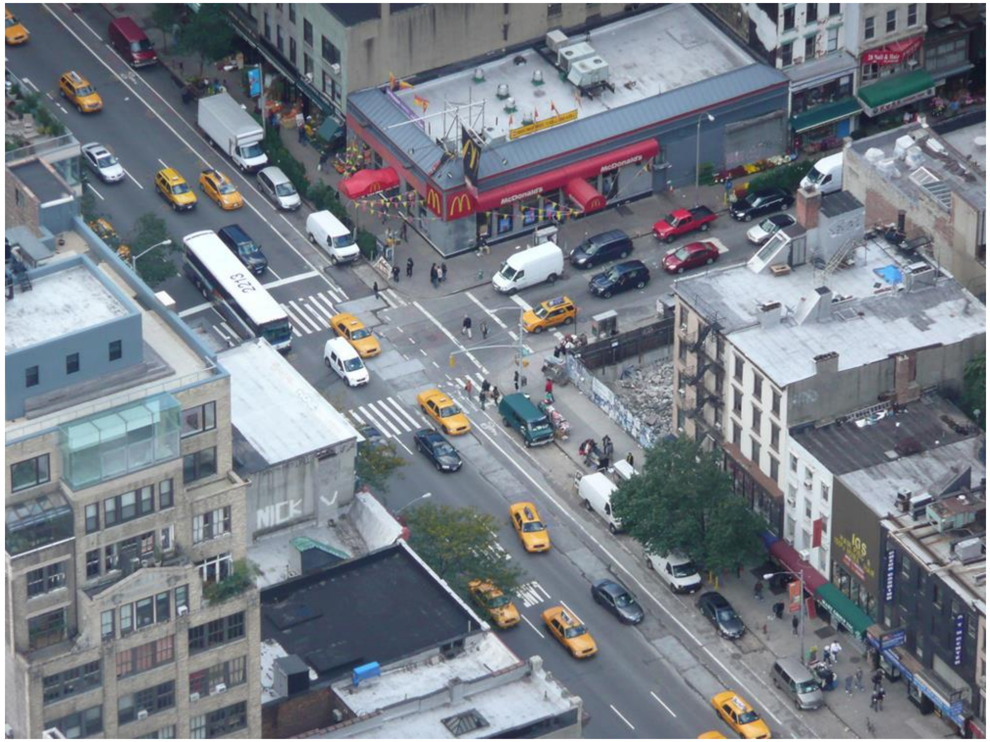


Adding Vectors by Components

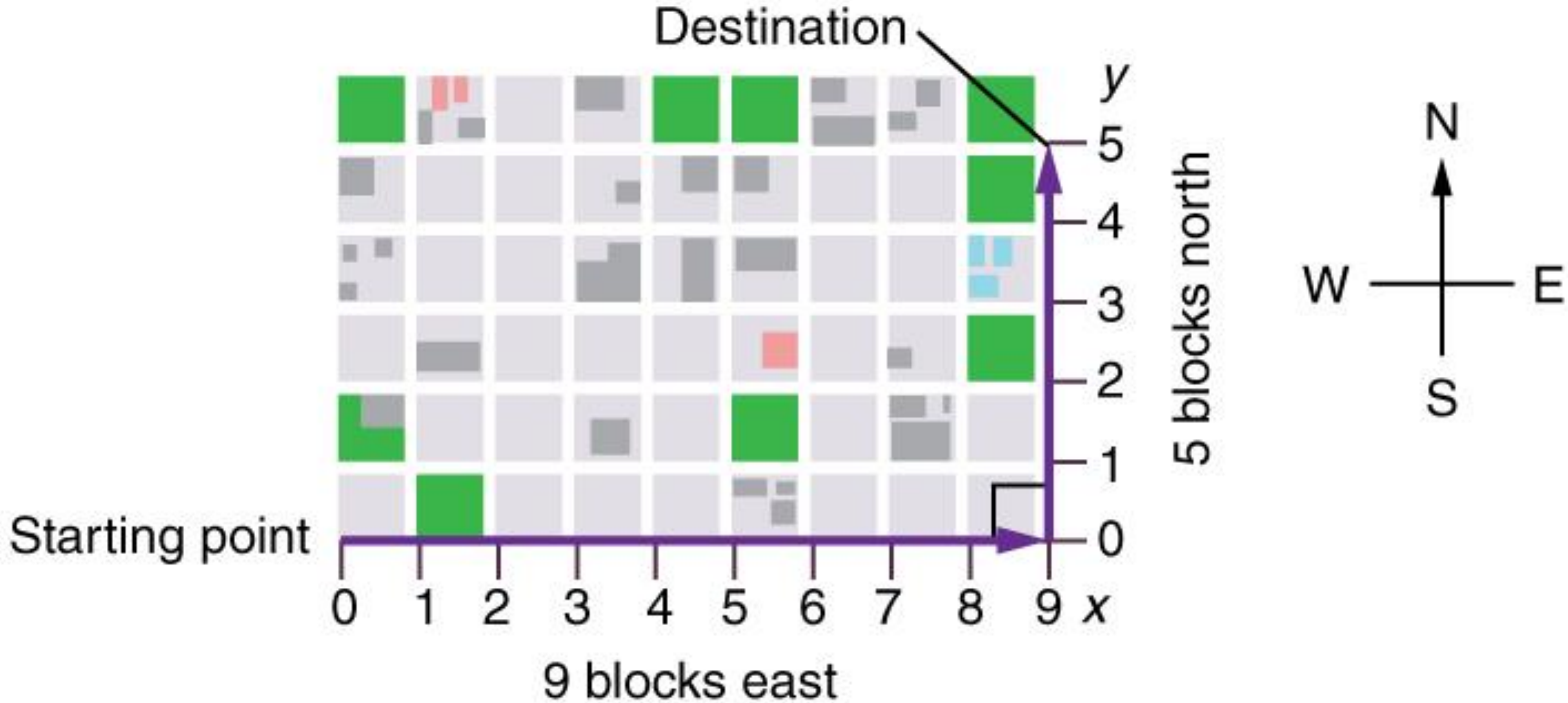
Adding vectors:

1. Draw a diagram; add the vectors graphically.
2. Choose *x* and *y* axes.
3. Resolve each vector into *x* and *y* components.
4. Calculate each component using sines and cosines.
5. Add the components in each direction.
6. To find the length and direction of the vector, use:

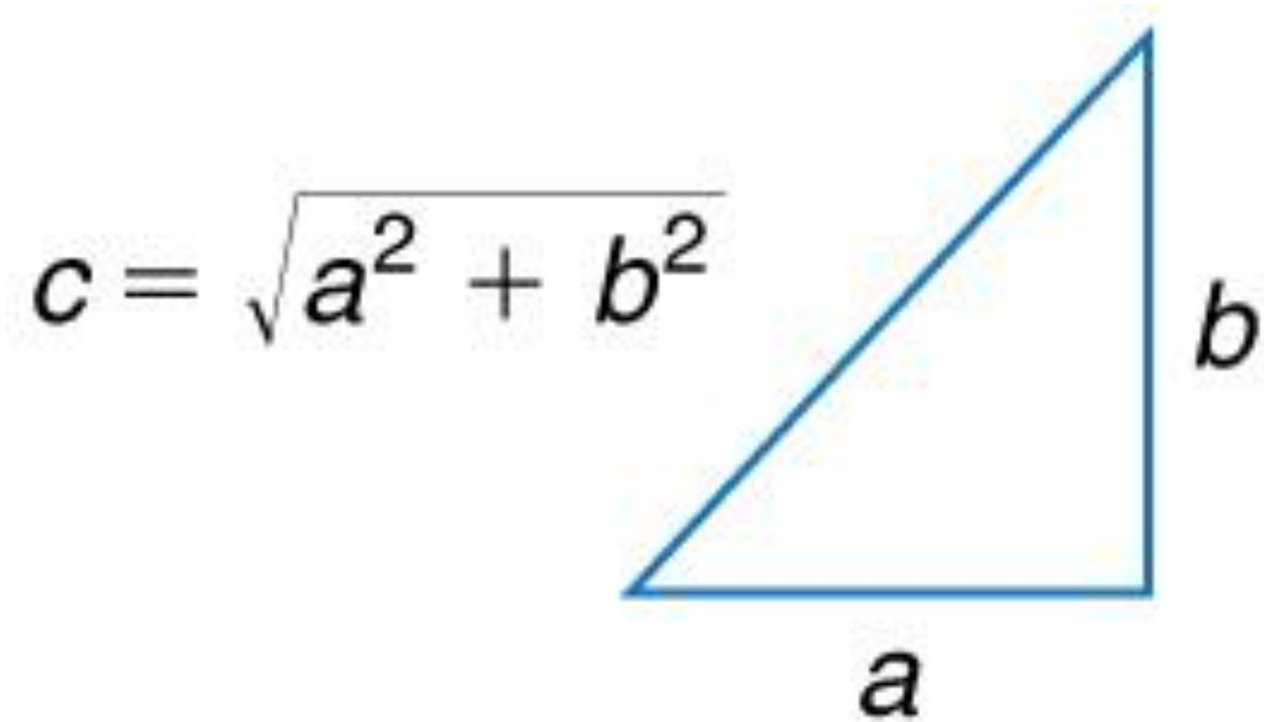
$$V = \sqrt{V_x^2 + V_y^2} \qquad \tan\theta = \frac{V_y}{V_x}$$



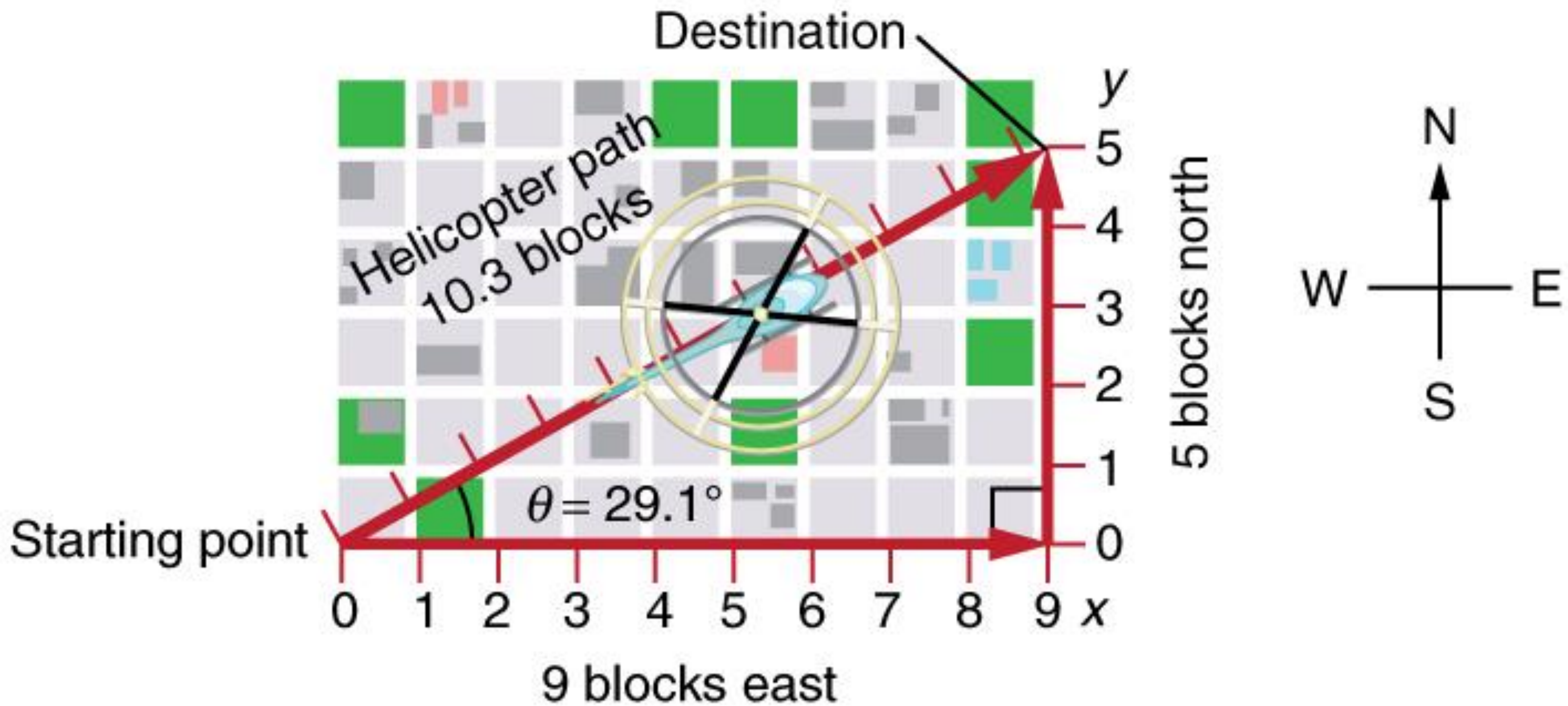
Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths.



A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

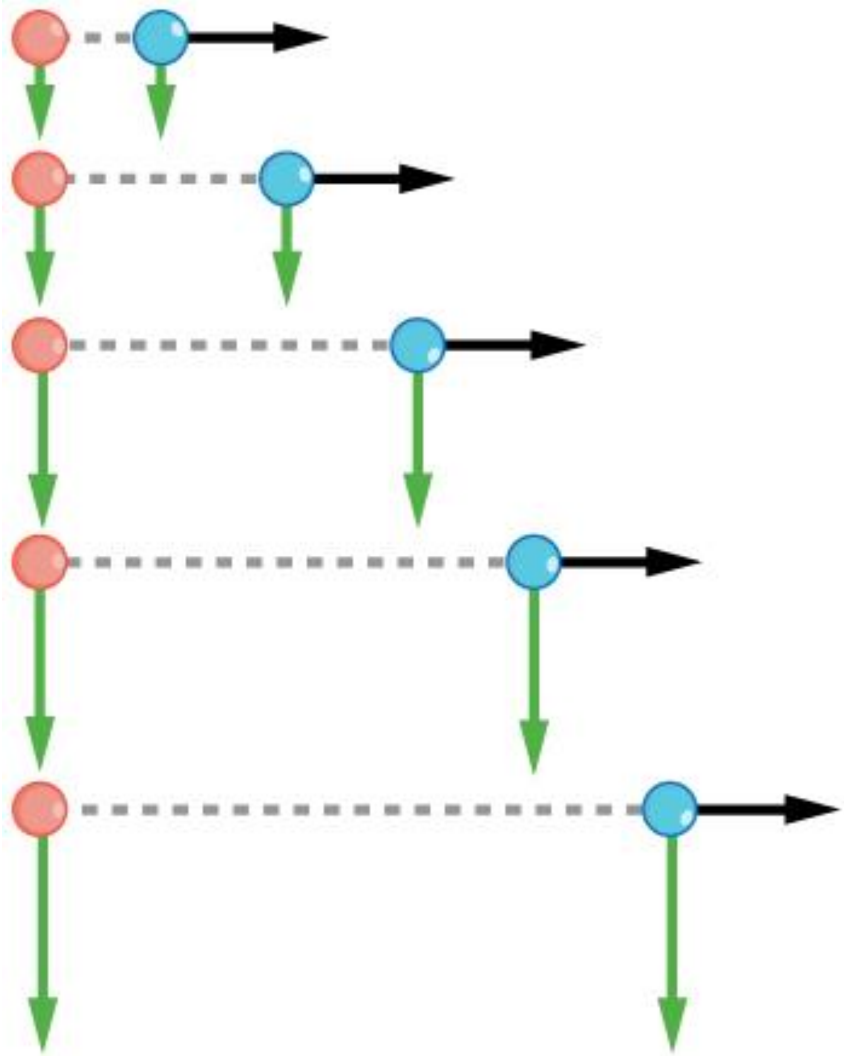


The Pythagorean theorem relates the length of the legs of a right triangle, labeled *a* and *b* , with the hypotenuse, labeled *c* . The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for $c = \sqrt{a^2 + b^2}$.



The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

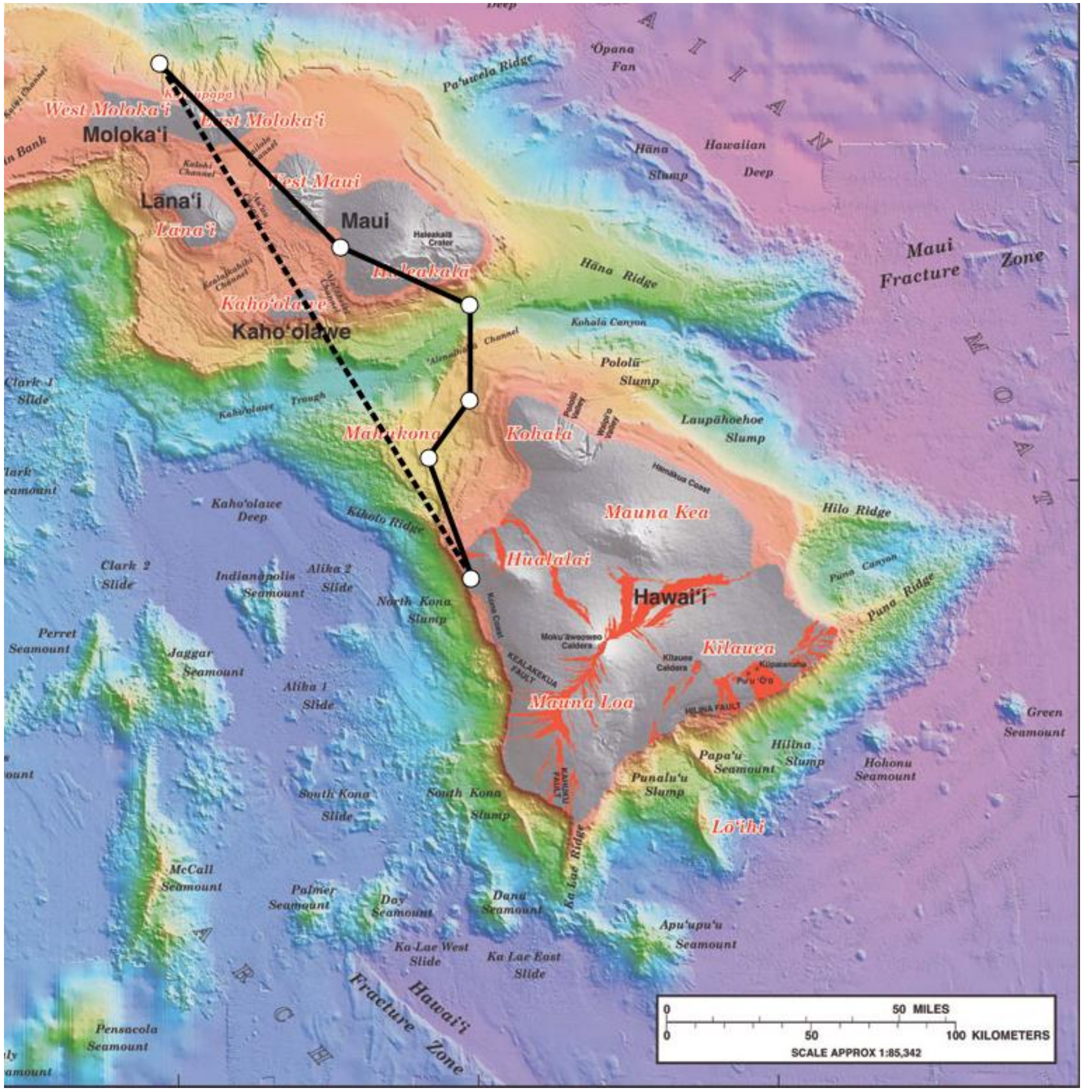
FIGURE 3.6



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval.

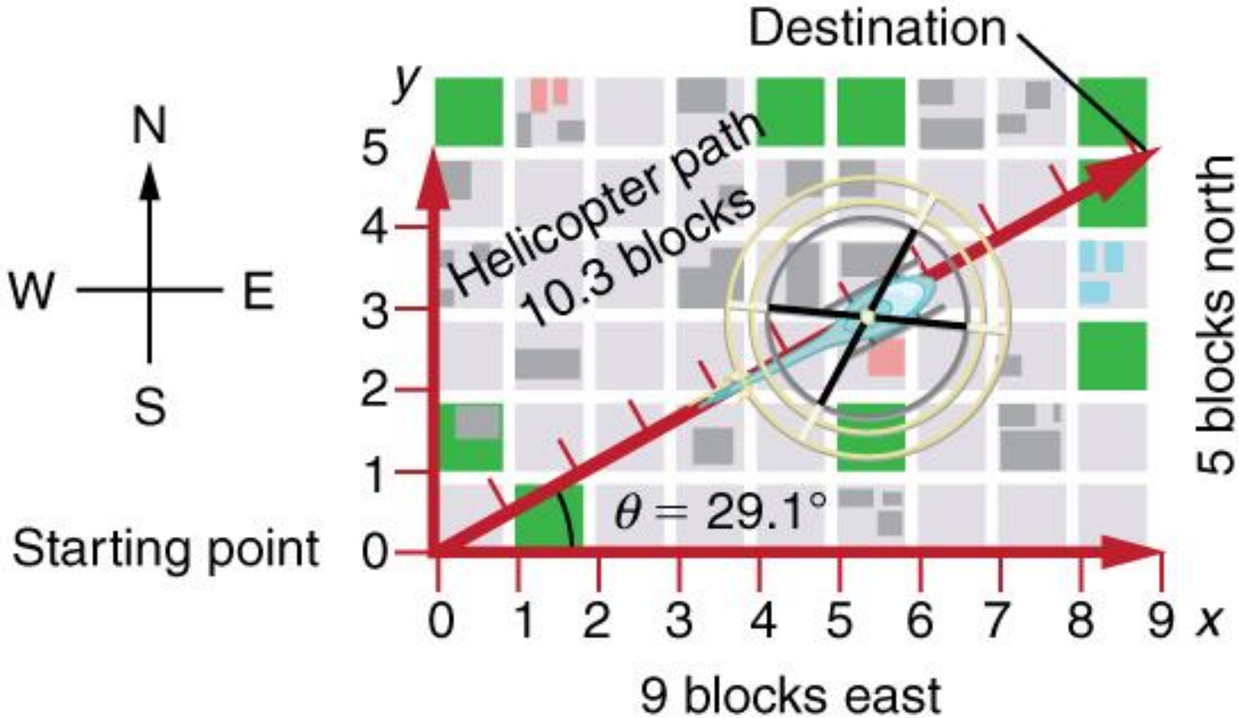
Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity.

Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.



Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey.

FIGURE 3.9

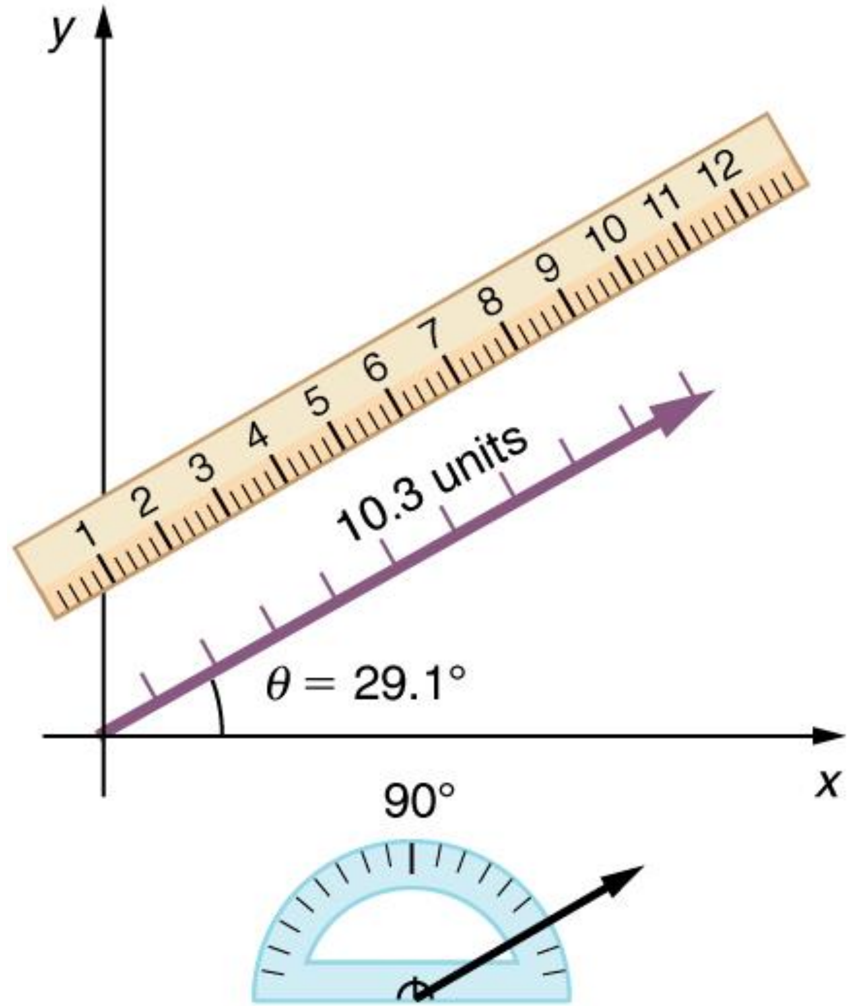


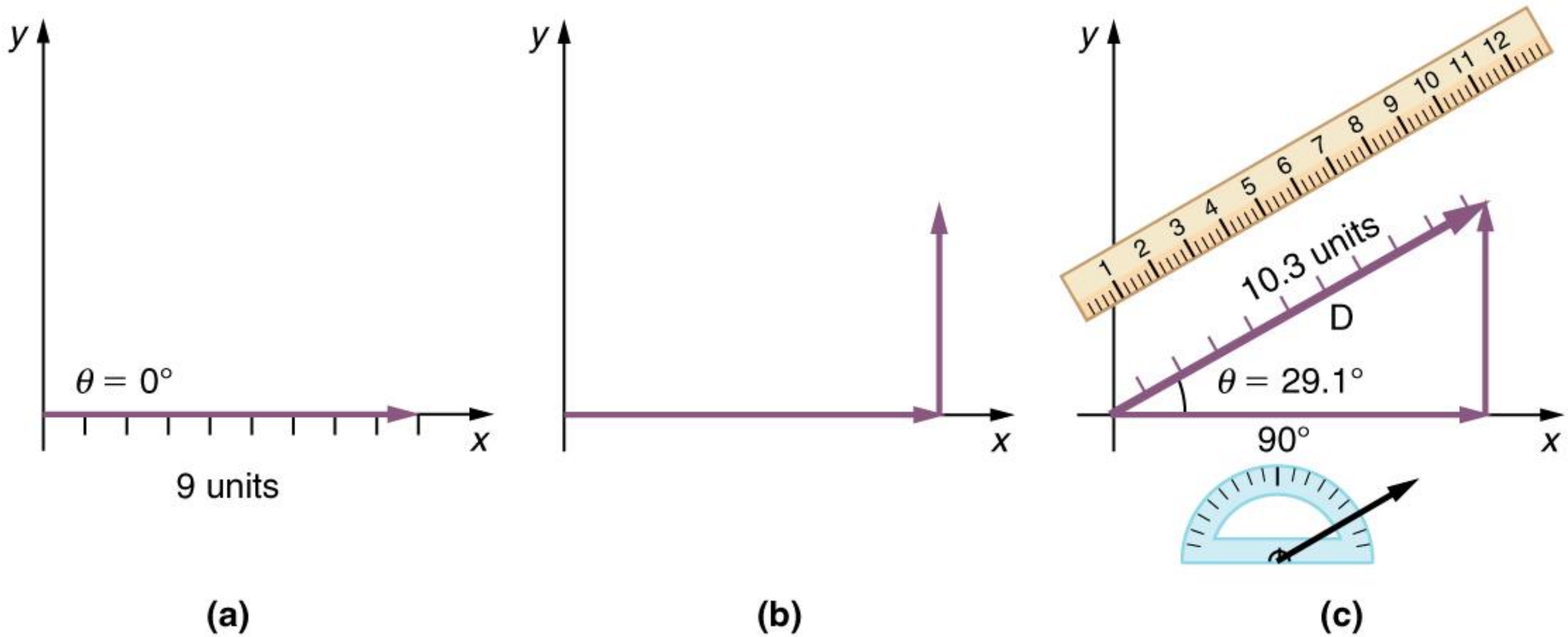
A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1º north of east.

FIGURE 3.10

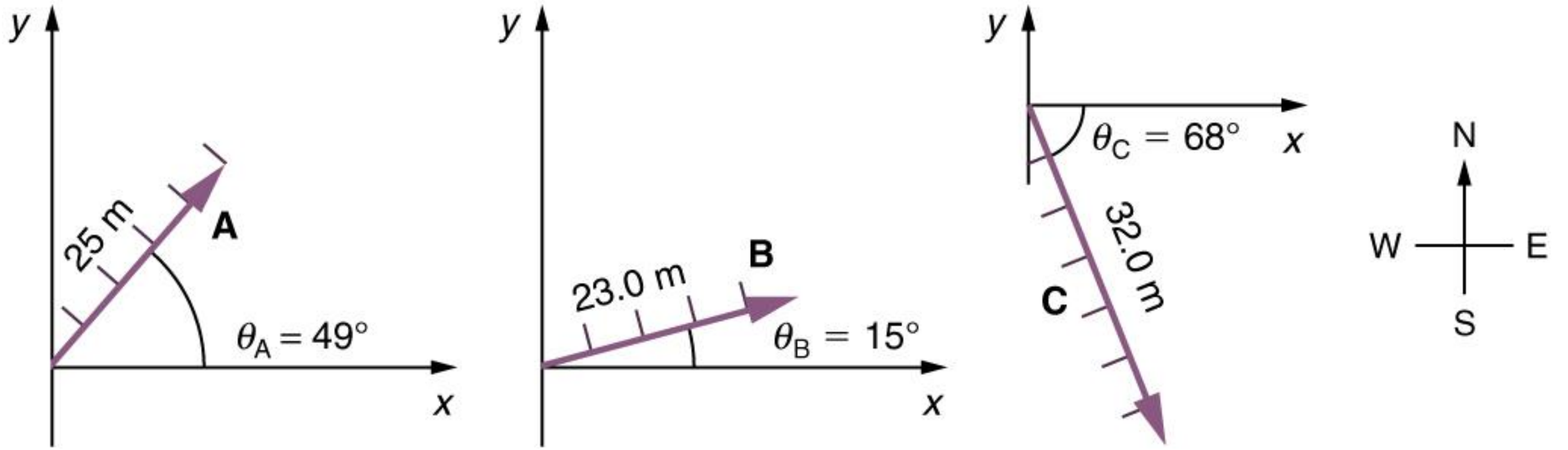
To describe the resultant vector for the person walking in a city considered in **Figure 3.9** graphically, draw an arrow to represent the total displacement vector **D**
.

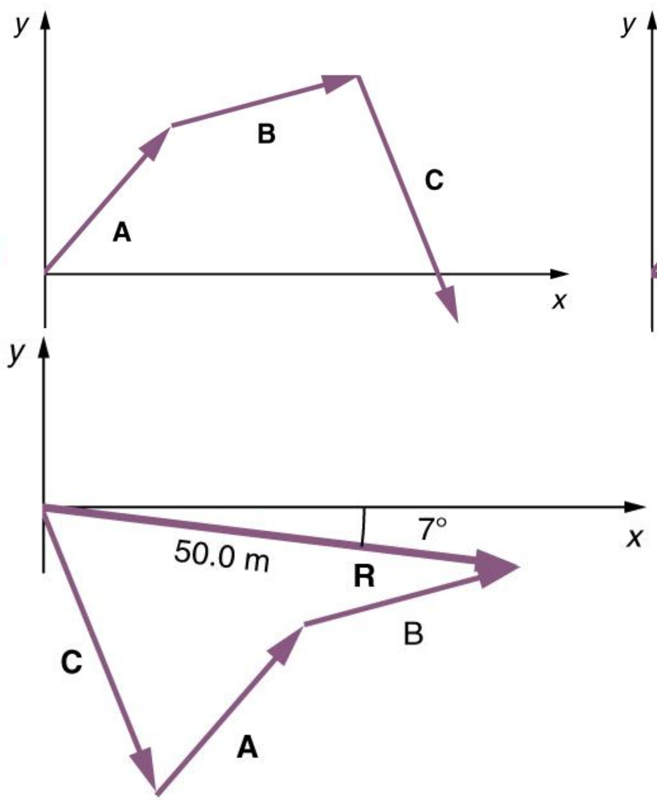
Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1º north of east.

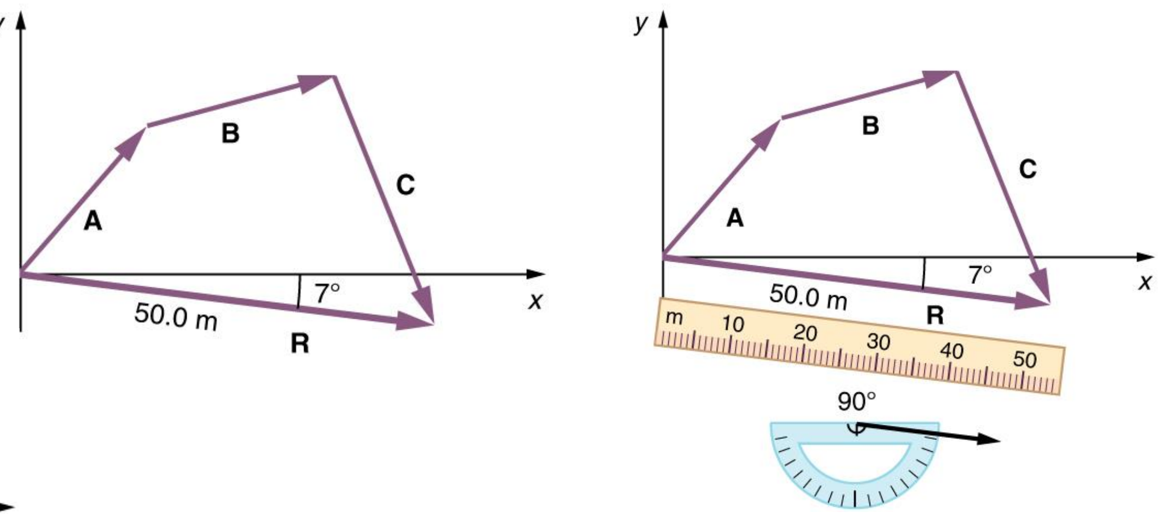




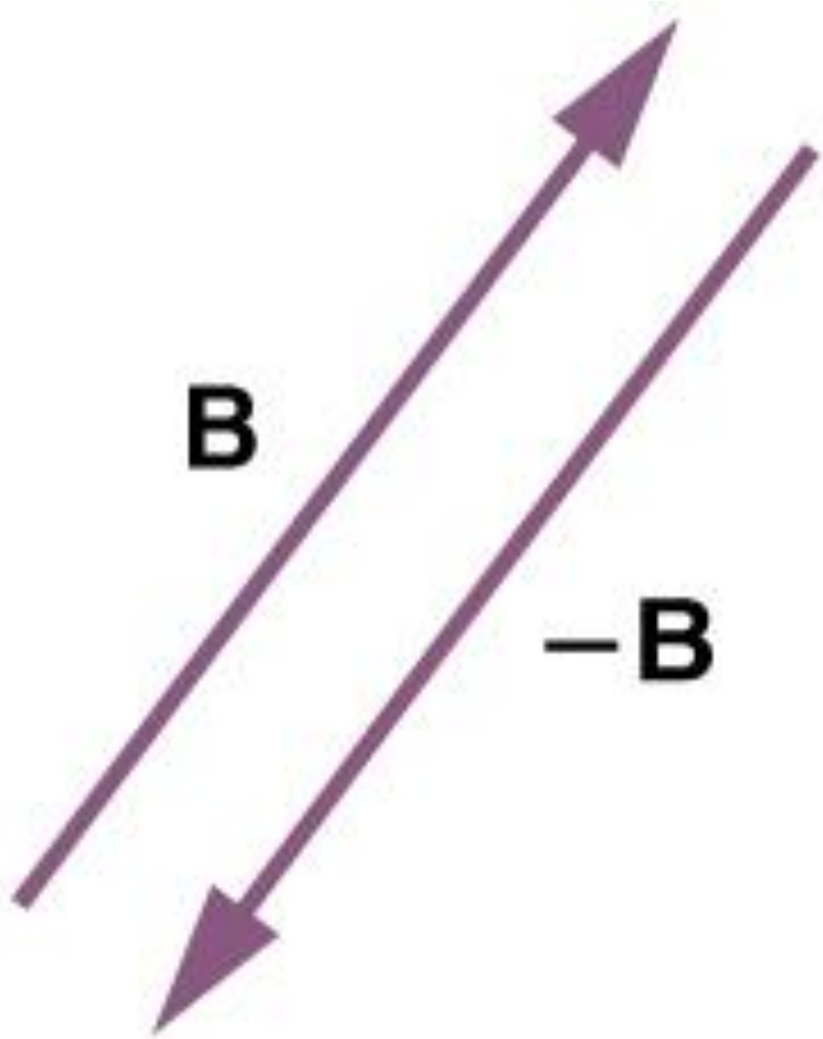
Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in **Figure 3.9**. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector D** . The length of the arrow **D** is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1º .


y
x
25 m
A
$\theta_A = 49°$
y
x
23.0 m
B
$\theta_B = 15°$
y
x
$\theta_C = 68°$
32.0 m
C
N
W
E
S

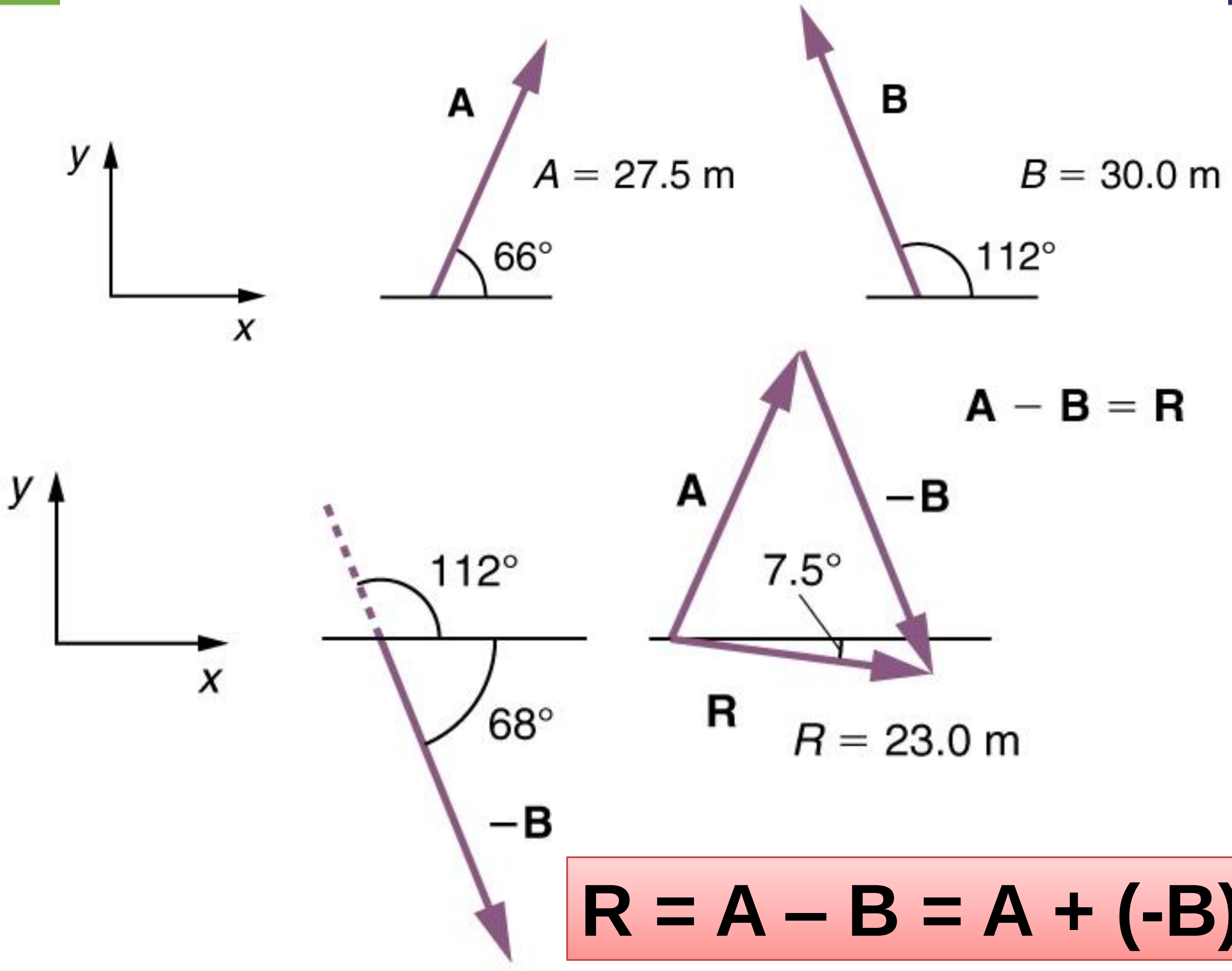

y
x
A
B
C
y
x
7°
50.0 m
R
C
B
A


y
x
A
B
C
7°
50.0 m
R
y
x
A
B
C
7°
50.0 m
R
m
10
20
30
40
50
90°

R = (A + B) + C = C + (A + B)



The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So **B** is the negative of **–B** ; it has the same length but opposite direction.



$$\mathbf{R} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

FIGURE 3.26

The vector **A** , with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

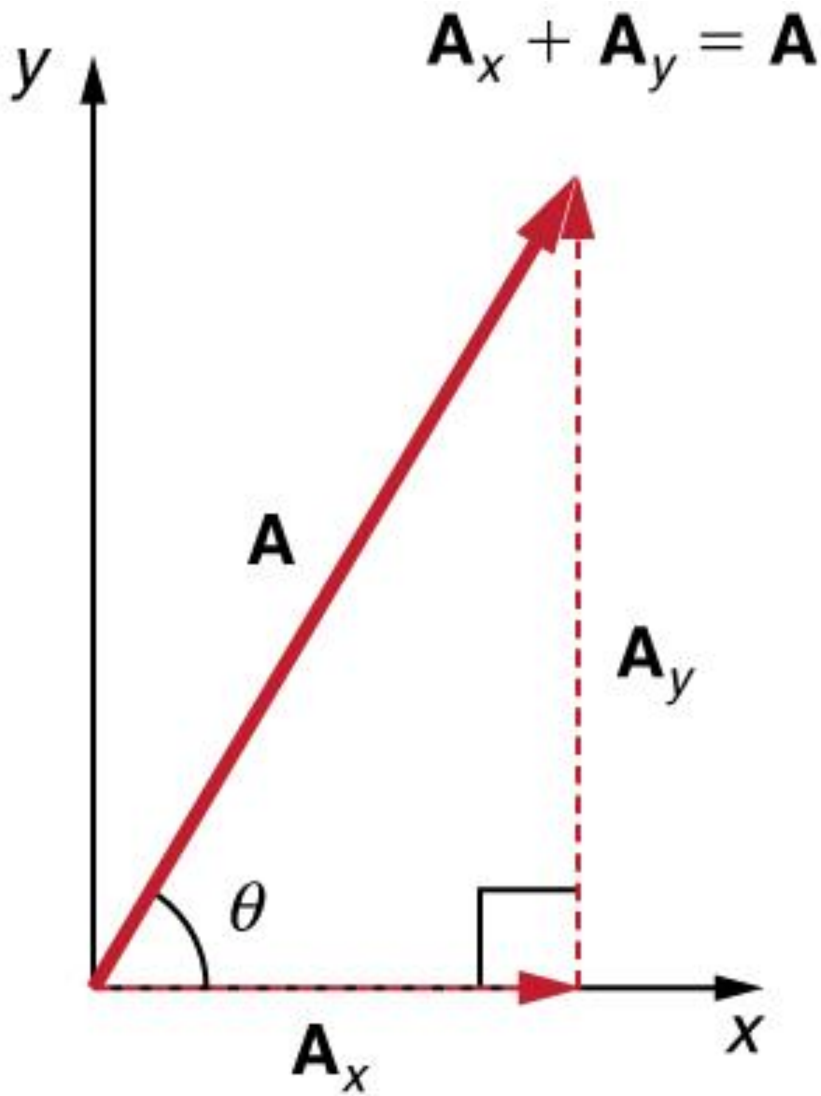
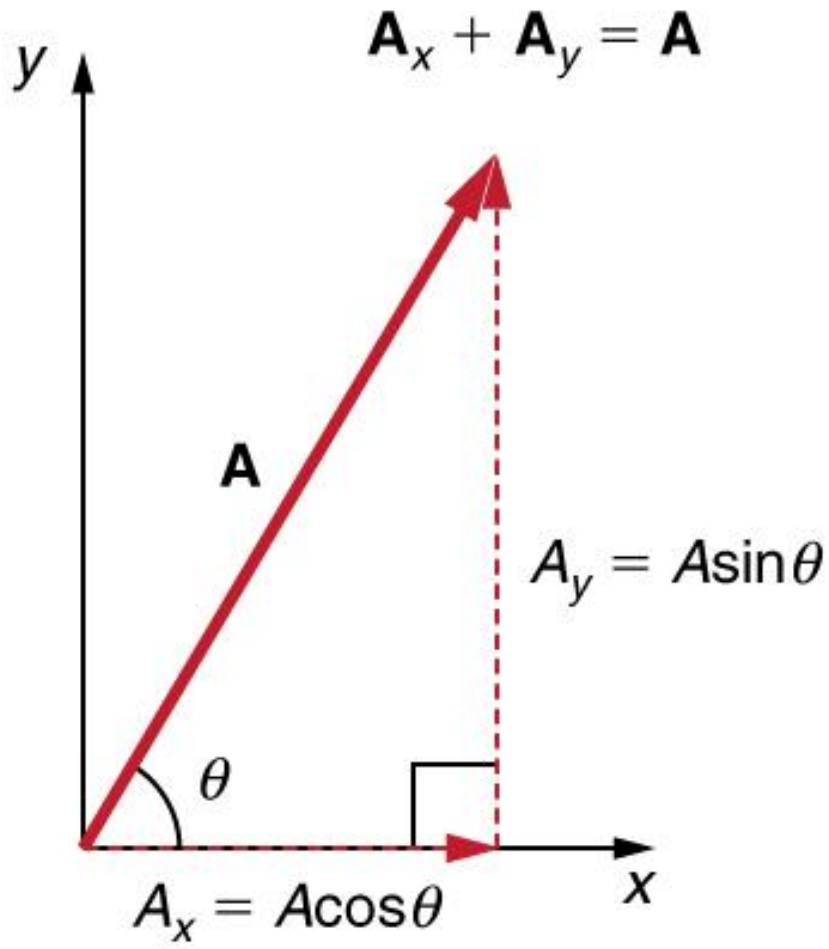
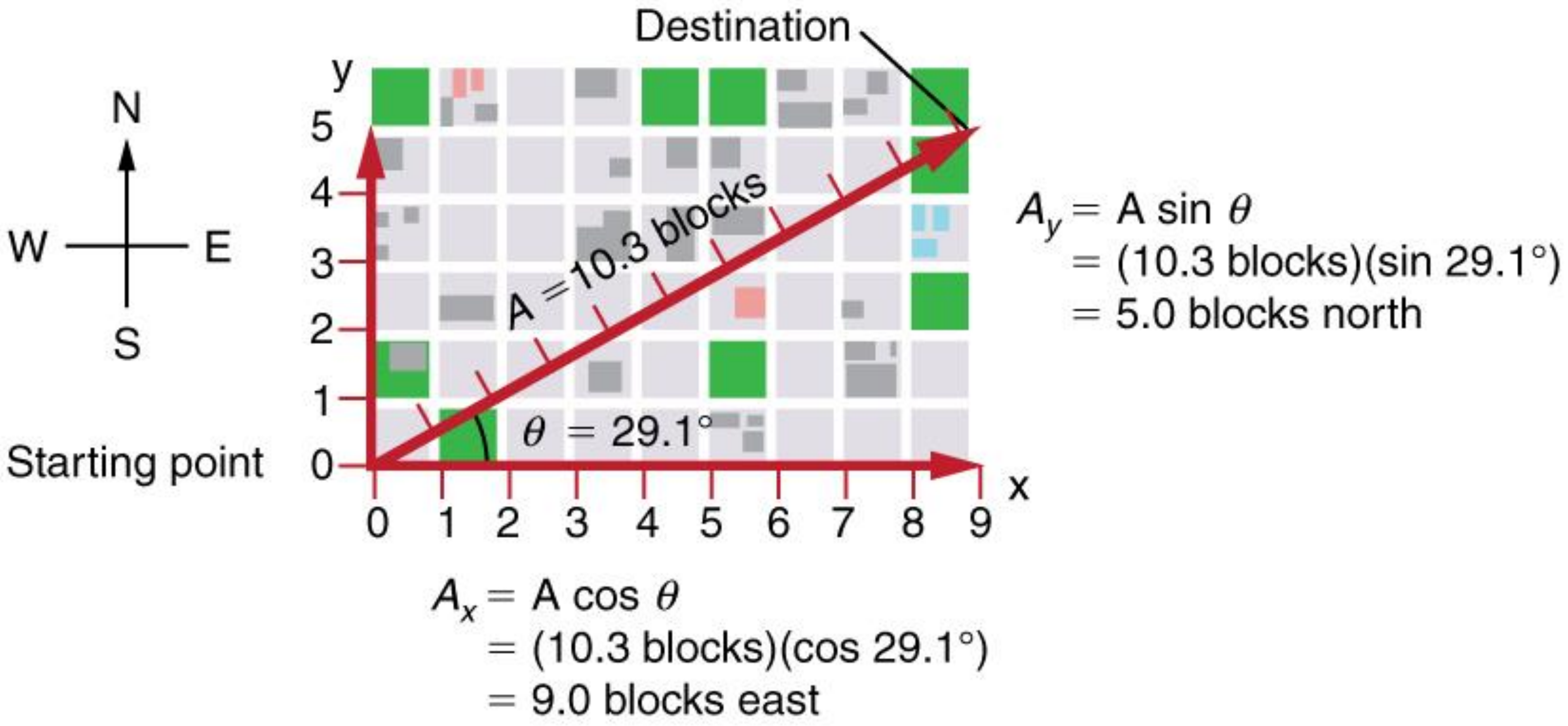


FIGURE 3.27



The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector **A** and the angle θ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin \theta$.

FIGURE 3.28



We can use the relationships $A_x = A \cos\theta$ and $A_y = A \sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

FIGURE 3.29

The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components *Ax* and *Ay* have been determined.

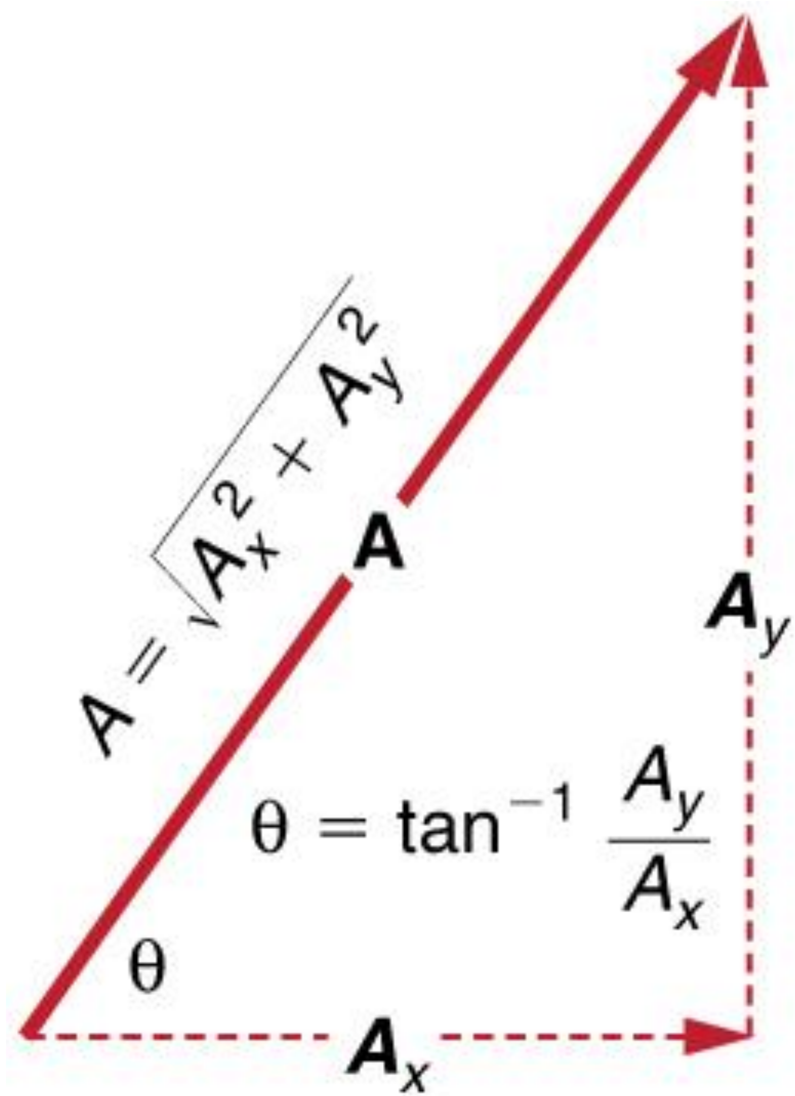
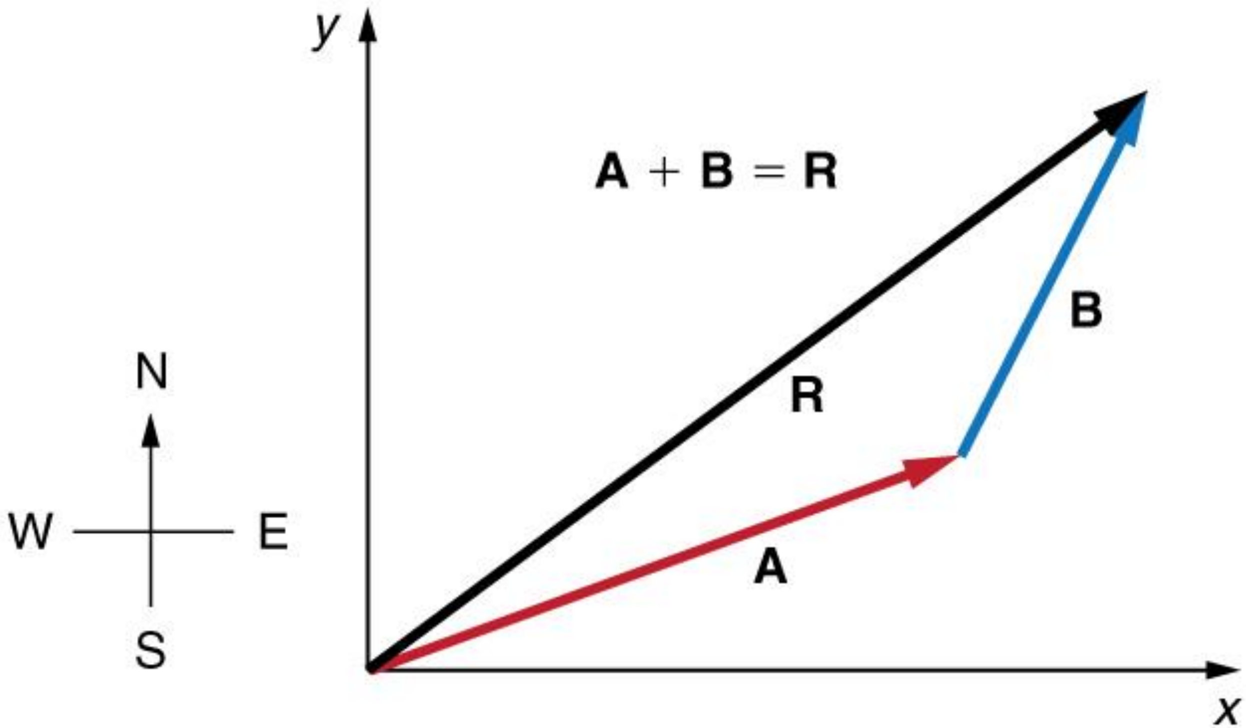
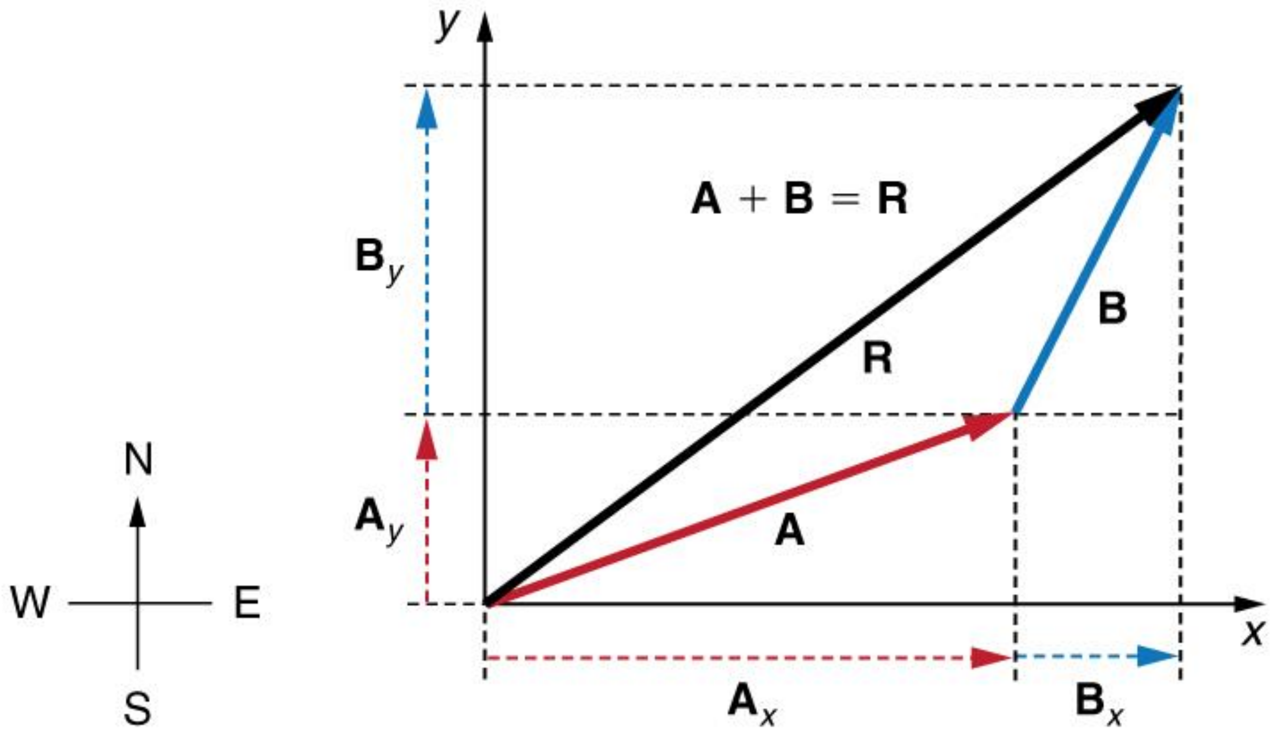


FIGURE 3.30



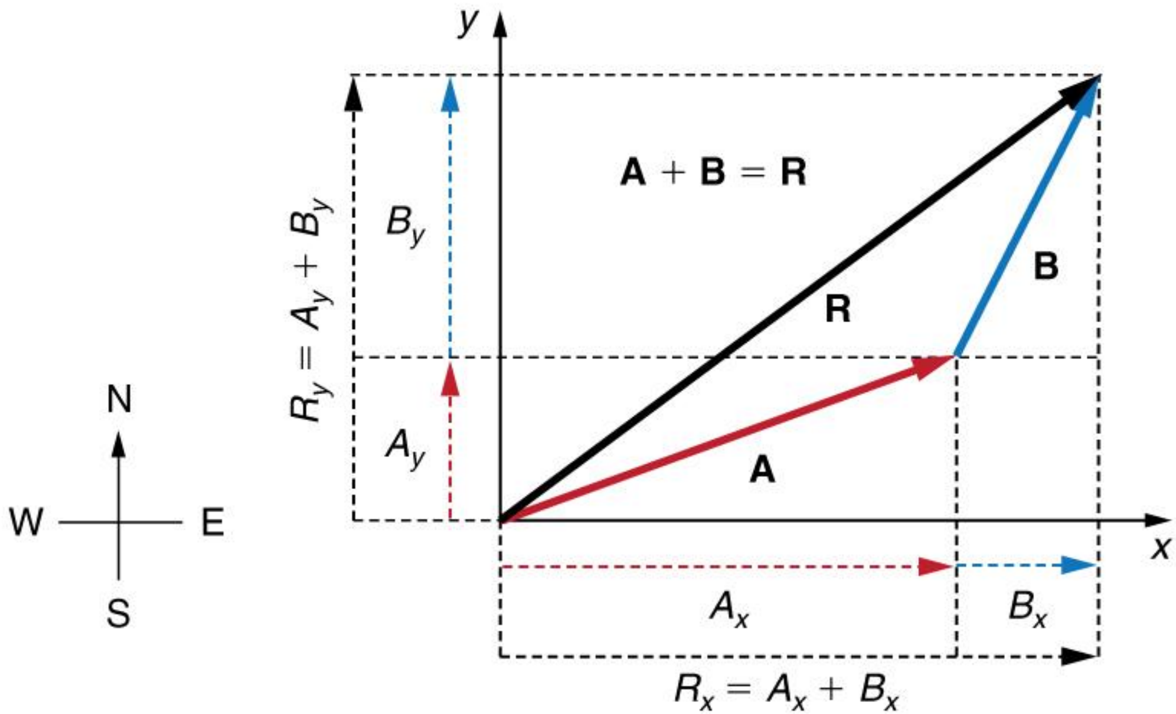
Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R** .

FIGURE 3.31



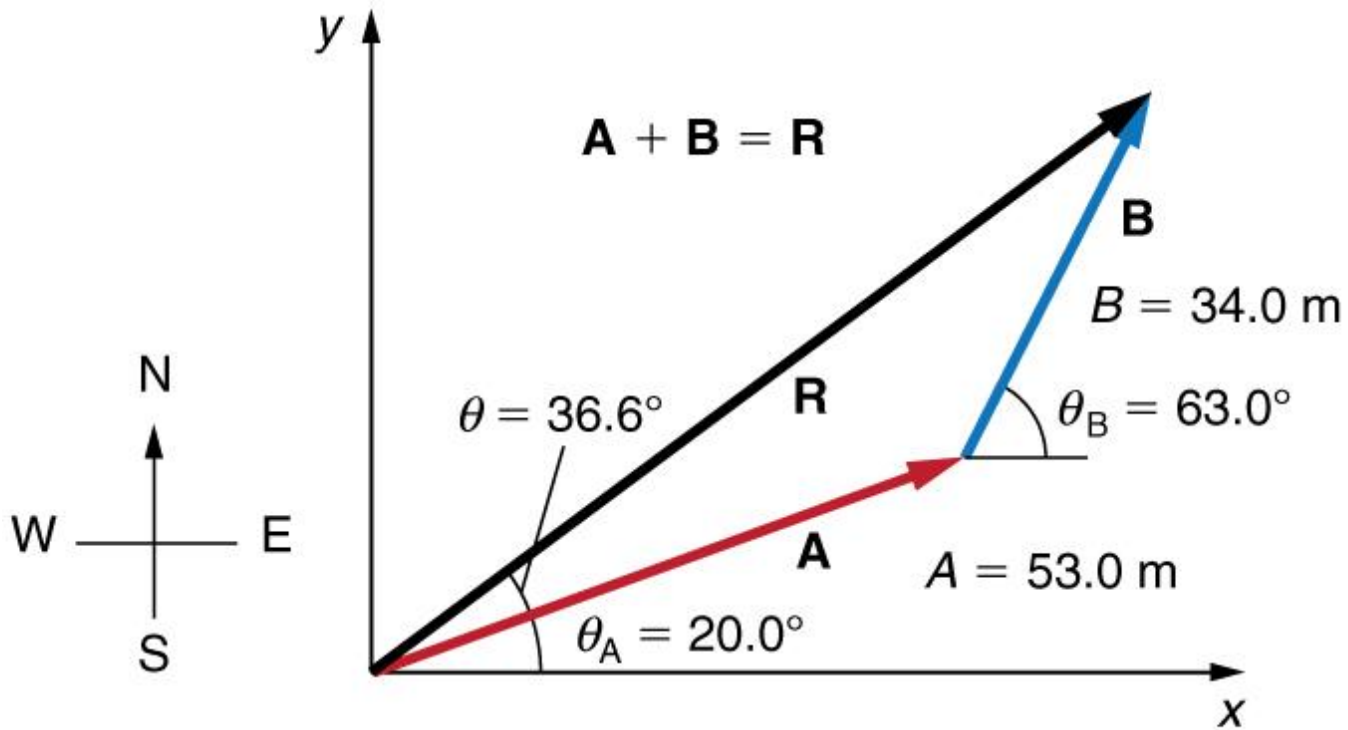
To add vectors **A** and **B** , first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

FIGURE 3.32



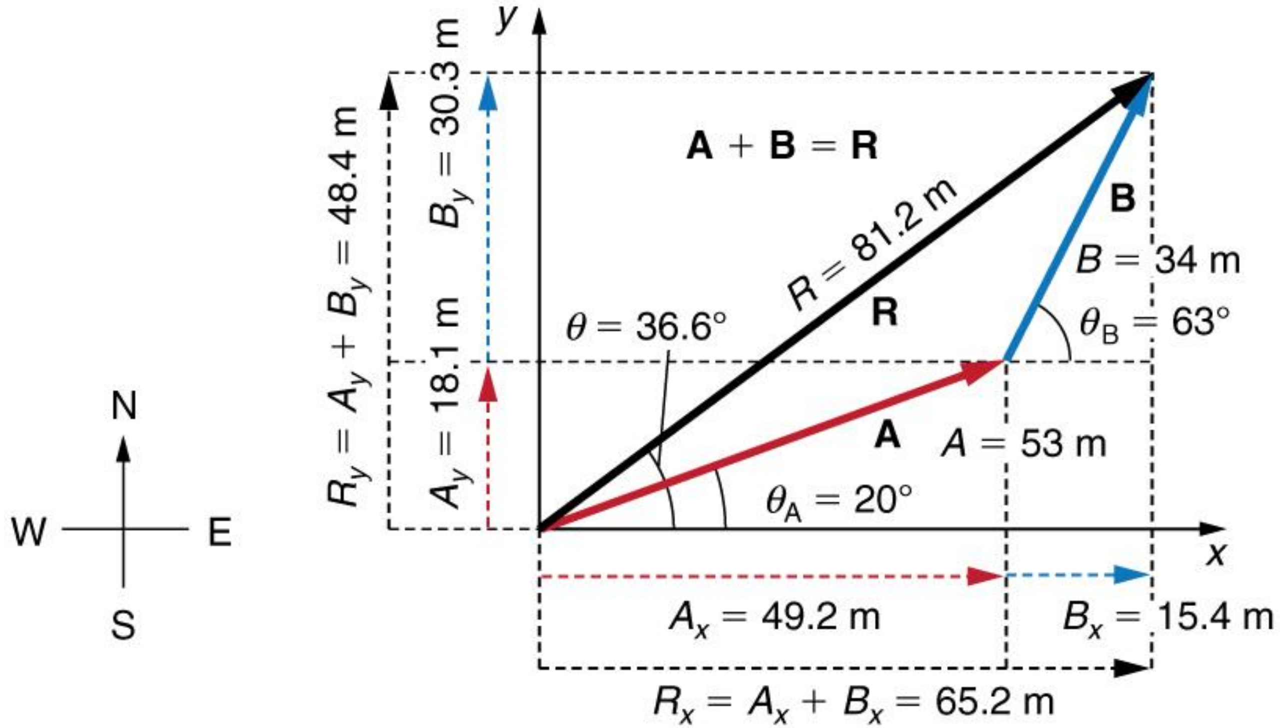
The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

FIGURE 3.33



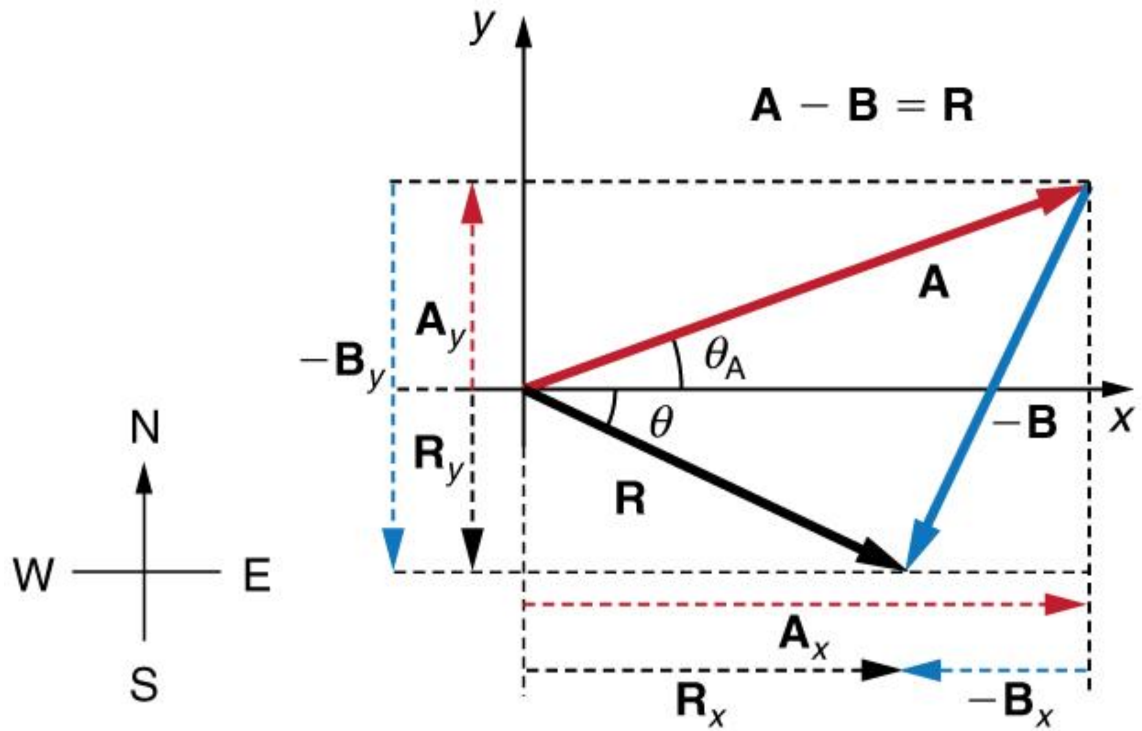
Vector **A** has magnitude 53.0 m and direction 20.0 º north of the *x*-axis. Vector **B** has magnitude 34.0 m and direction 63.0º north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of **R** .

FIGURE 3.34



Using analytical methods, we see that the magnitude of **R** is 81.2 m and its direction is 36.6º north of east.

FIGURE 3.35



The subtraction of the two vectors shown in **Figure 3.30**. The components of **–B** are the negatives of the components of **B** . The method of subtraction is the same as that for addition.