HW 3 1.4 # 7, 9, 11, 13 Alex Yeoh

1.5 # 1, 3, 5, 11, 13, 17

1.7 # 2, 5, 8, 16, 17, 19

1.4

7) $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix}$

9) $x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

11) $\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right] \xrightarrow{R_3+2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{array}\right] \xrightarrow{R_3/5} \left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x_3 = 1$ $x_2 + 5(1) = 2$ $x_1 + 2(-3) + 4(1) = -2$

$x_2 = -3$ $x_1 - 6 + 4 = -2$

$x_1 = 0$

$\vec{x} = \begin{bmatrix} x_1 = 0 \\ x_2 = -3 \\ x_3 = 1 \end{bmatrix}$

13) $\left[\begin{array}{cc|c} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4 \\ -2 & 6 & 4 \\ 3 & -5 & 0 \end{array}\right] \xrightarrow{R_2+2R_1} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 3 & -5 & 0 \end{array}\right] \xrightarrow{R_3-3R_1} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & -8 & -12 \end{array}\right] \xrightarrow{R_3+R_2} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & 0 & 0 \end{array}\right]$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 8 & 12 \end{array}\right]$

$8u_2 = 12$ $u_1 + \frac{3}{2} = 4$

$u_2 = \frac{12}{8} = \frac{3}{2}$ $u_1 = 3 - \frac{1}{2} = 2.5$

$\vec{u} = \begin{bmatrix} u_1 = 2.5 \\ u_2 = 1.5 \end{bmatrix}$

Yes, u is in $\mathbb{R}^3$ spanned by columns A

1.5

1) $\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -2 & -7 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 4 & 2 & 7 & 0 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 12 & -9 & 0 \end{array}\right] \xrightarrow{R_3+R_2} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \end{array}\right]$

the system has a non-trivial solution b/c x_3 has to be set free

1.5 continued

5) $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \xrightarrow{R_2+4R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \xrightarrow{R_3+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \end{array}\right]$

free x_3 $\quad 3x_2 + 6x_3 = 0$ $\quad x_1 + 3(-2x_3) + x_3 = 0$

$3x_2 = -6x_3$ $\quad x_1 - 6x_3 + x_3 = 0$

$x_2 = -2x_3$ $\quad x_1 = 5x_3$

$$x = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

11) $\left[\begin{array}{ccccc|c} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

free x_6 $\quad x_5 = -4$ $\quad$ free x_4 $\quad x_3 = -1$ $\quad$ free x_2 $\quad x_1 - 4x_2 - 2(-1) + 3(-4) = -5$

$x_1 - 4x_2 + 2 - 12 = -5$

$x_1 - 4x_2 = 5$

$x_1 = 4x_2 + 5$

$$x = \begin{bmatrix} x_6 \\ 4x_2 + 5 \\ x_2 \\ -1 \\ x_4 \\ -4 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

11) $\left[\begin{array}{cccccc|c} 1 & -4 & -2 & 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccccc|c} 1 & -4 & -2 & 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \end{array}\right]$

free x_6 $\quad x_5 - 4x_6 = 0$ $\quad$ free x_4 $\quad x_3 - x_6 = 0$ $\quad$ free x_2 $\quad x_1 - 4x_2 - 2(x_6) + 3(4x_6) - 5x_6 = 0$

$x_5 = 4x_6$ $\quad x_3 = x_6$ $\quad x_1 = 4x_2 + 2x_6 - 12x_6 + 5x_6$

$x_1 = 4x_2 - 5x_6$

$$x = \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ x_4 \\ 4x_6 \\ x_6 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

1.5 continued

13) $\vec{x} = \begin{bmatrix} 5+4x_3 \\ -2-7x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$ a line through $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ parallel to $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$

17) The solutions are parallel

1.7

2) $\left[\begin{array}{ccc|c} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$

$-3x_3 = 0$ $\quad 5x_2 + 4(0) = 0$ $\quad 2x_1 - 8(0) + 1(0) = 0$

$x_3 = 0$ $\quad x_2 = 0$ $\quad x_1 = 0$

independent, only trivial solutions

5) $\left[\begin{array}{ccc|c} 0 & -8 & 5 & 0 \\ 3 & -7 & 4 & 0 \\ -1 & 5 & 4 & 0 \\ 1 & -3 & 2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -8 & 5 & 0 \\ -1 & 5 & 4 & 0 \\ 3 & -7 & 4 & 0 \end{array}\right] \begin{array}{l} R_3 + R_1 \\ \longrightarrow \\ R_4 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -8 & 5 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \xrightarrow{R_4 - R_3} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -8 & 5 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 0 & -8 & 0 \end{array}\right]$

$-8x_3 = 0$ $\quad 2x_2 + 6(0) = 0$ $\quad x_1 - 3(0) + 2(0) = 0$

$x_3 = 0$ $\quad x_2 = 0$ $\quad x_1 = 0$

independent, only trivial solutions

8) $\left[\begin{array}{cccc|c} 1 & -3 & 3 & -2 & 0 \\ -3 & 7 & -1 & 2 & 0 \\ 0 & 1 & -4 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & -3 & 3 & -2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ -3 & 7 & -1 & 2 & 0 \end{array}\right] \xrightarrow{R_3 + 3R_1} \left[\begin{array}{cccc|c} 1 & -3 & 3 & -2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & -2 & 8 & -4 & 0 \end{array}\right]$

dependent, x_4 must be set free

16) independent, only trivial solutions

17) dependent, contains $\vec{0}$

19) independent, only trivial solutions