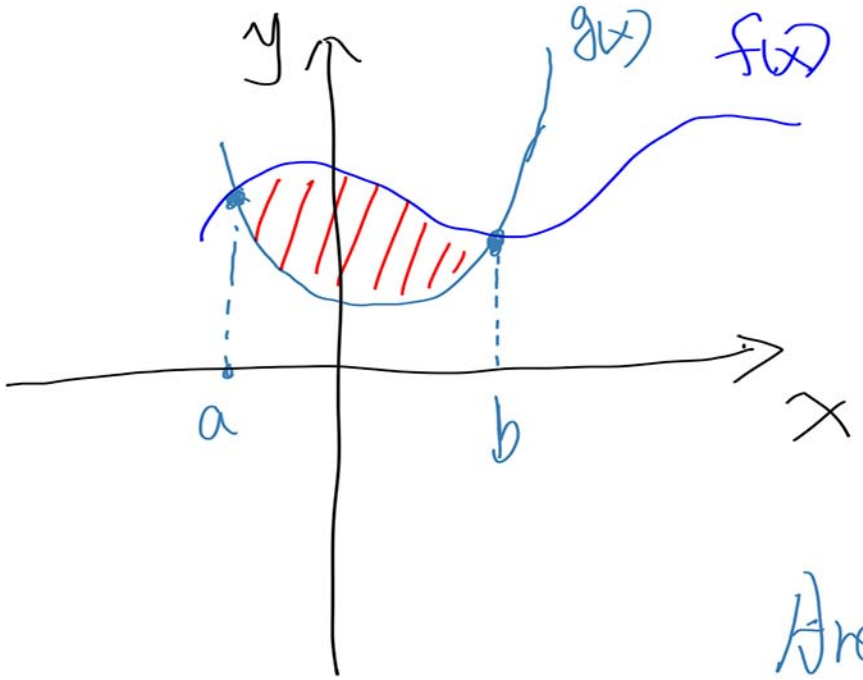
LECTURE NO. 3

2.1 Area Between Curves

Wright State University

Area of a Region Between Two Curves

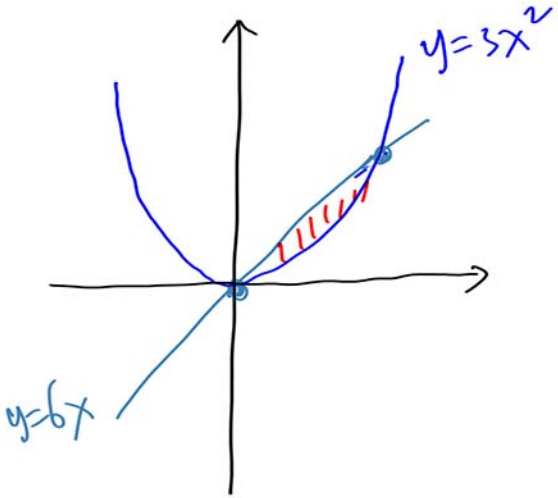


How to find the area of the shaded region?

We use integral to solve it.

$$\text{Area} = \int_a^b f(x) - g(x)\, dx$$

Find the area enclosed by the curves of $y = 3x^2$ and $y = 6x$.



First, we need to find the points of intersection.

set $3x^2 = 6x$ $\quad x^2 = 2x$ $\quad x = 0$ or $x = 2$

$$\text{Area} = \int_0^2 6x - 3x^2 \, dx = 3x^2 - x^3 \Big|_0^2$$

$$= (3(2)^2 - 2^3) - 0 = 4$$

$$\int_0^2 3x^2 - 6x \, dx$$

$$x^3 - 3x^2 \Big|_0^2 = -4 \qquad \text{Area} = |-4| = 4$$

Find the area enclosed by the curves $y = x^3$ and $y = 4x$

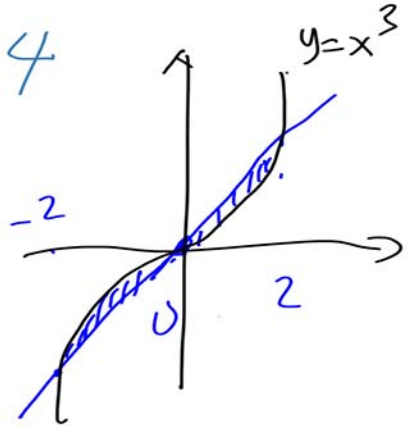
First find points of intersection $x^3 = 4x \quad x^3 - 4x = 0 \quad x(x^2-4) = 0$

$$x = -2,\ 0,\ 2$$

we will set up two integrals.

$$\int_{-2}^{0} x^3 - 4x\,dx = \frac{x^4}{4} - 2x^2 \Big|_{-2}^{0} = (0) - \left(\frac{16}{4} - 8\right) = 4$$

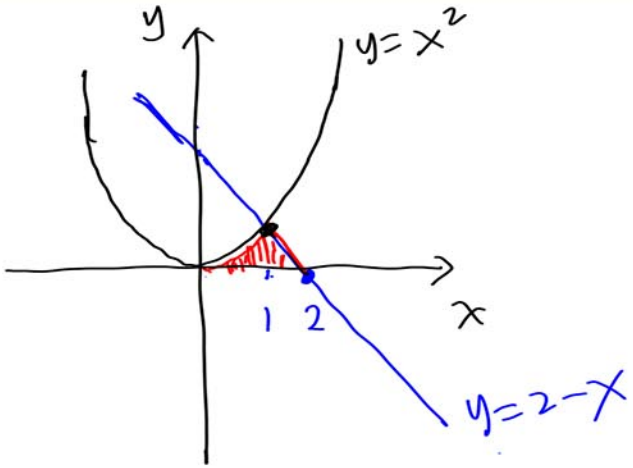
$$\int_{0}^{2} x^3 - 4x\,dx = \frac{x^4}{4} - 2x^2 \Big|_{0}^{2} = \left(\frac{16}{4} - 8\right) - 0 = -4$$



Area = Sum of absolute values of the two integrals above

$$= 4 + |-4| = \boxed{8} \leftarrow \text{FINAL ANSWER.}$$

Find the area enclosed by the curves $y = x^2$, $y = 2 - x$ and x-axis.



set $x^2 = 2 - x$ $\quad x^2 + x - 2 = 0$ $\quad (x-1)(x+2) = 0$

$x = 1$ or ~~$x = -2$~~

$$\text{Area} = \int_0^1 x^2\,dx + \int_1^2 2 - x\,dx$$

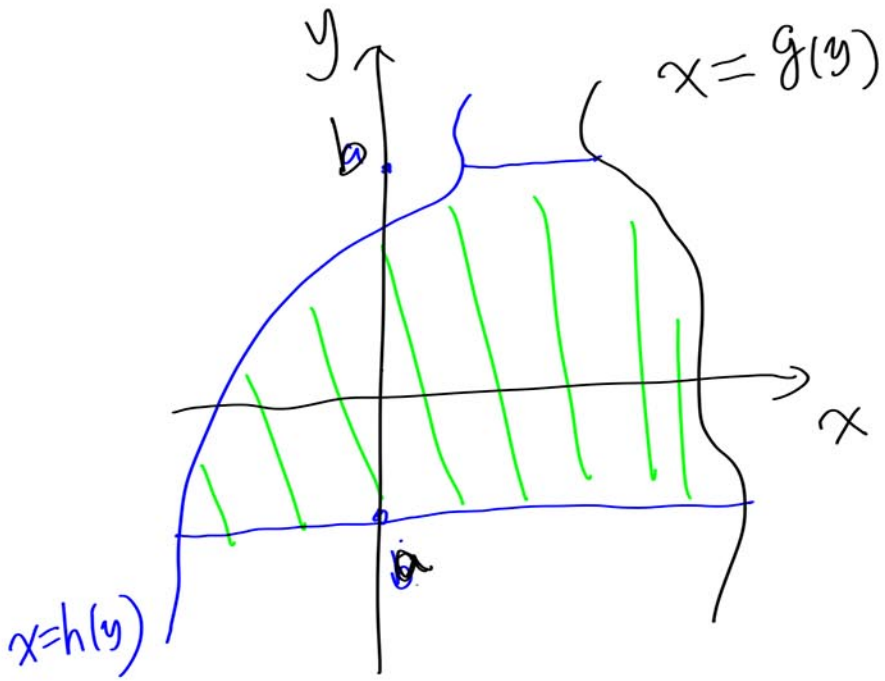
$$= \left(\frac{x^3}{3}\Big|_0^1\right) + \left(2x - \frac{x^2}{2}\Big|_1^2\right)$$

$$= \frac{1}{3} + \left(2 - \left(2 - \frac{1}{2}\right)\right)$$

$$= \frac{1}{3} + \frac{1}{2} = \boxed{\frac{5}{6}}$$

FINAL ANSWER

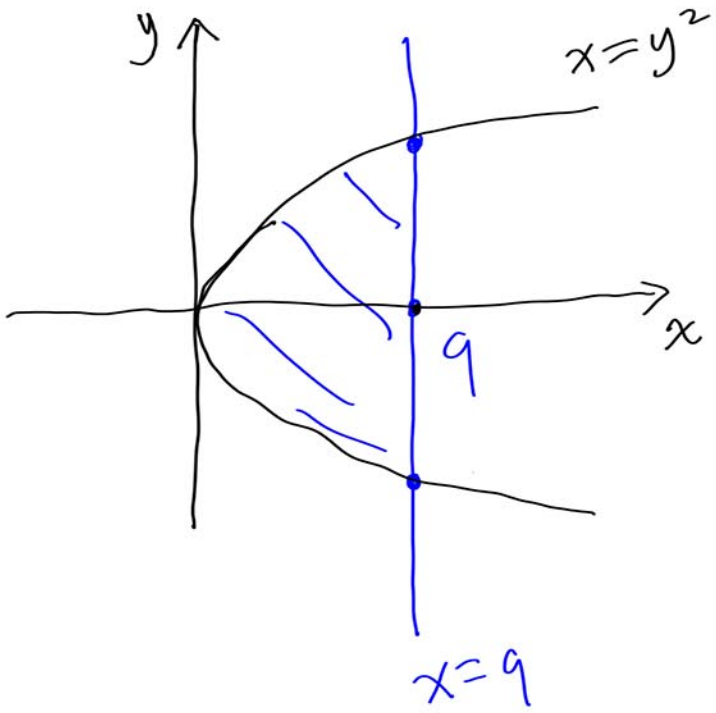
Regions Defined with Respect to y



we may integrate w.r.t y

$$\text{Area} = \int_a^b g(y) - h(y)\, dy$$

Find the area enclosed by the curves $x = y^2$ and $x = 9$



Set $y^2 = 9$ $\quad y = -3$ or $y = 3$

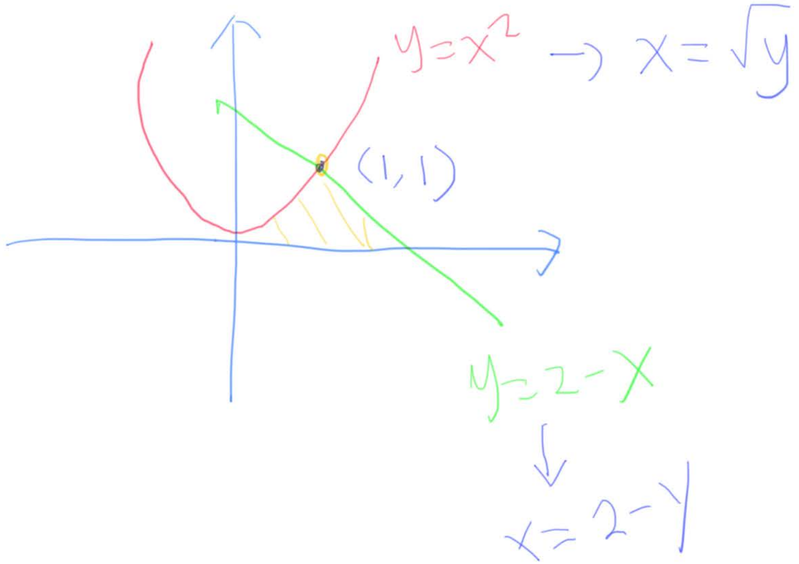
$$\text{Area} = \int_{-3}^{3} 9 - y^2 \, dy$$

$$= 9y - \frac{y^3}{3} \Big|_{-3}^{3}$$

$$= \left(27 - \frac{27}{3}\right) - \left(-27 - \frac{-27}{3}\right)$$

$$= 18 + 18 = 36 \leftarrow \text{FINAL ANSWER.}$$

(Redo a Previous Example using y) Find the area enclosed by the curves $y = x^2$, $y = 2 - x$ and x-axis



$$\text{Area} = \int_0^1 2 - y - \sqrt{y}\, dy$$

$$2y - \frac{y^2}{2} - \frac{2}{3}y^{\frac{3}{2}} \Big|_0^1$$

$$\left(2 - \frac{1}{2} - \frac{2}{3}\right) - 0$$

$\frac{5}{6}$ Same answer.