Cal II lab 11 Solution

#1. point: $x = 2^2 - 2 = 2$ $\quad y = 2^3 + 2 = 10$ $\quad (2, 10)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2+1}{2t-1}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{t=2} = \frac{12+1}{4-1} = \frac{13}{3}$$

$y - 10 = \frac{13}{3}(x-2)$ point-slope equation

(or $y = \frac{13}{3}x + \frac{4}{3}$ slope-intercept form)

#2

$$\text{Arc length} = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(4t^3)^2 + (2t^5)^2}\, dt$$

$$= \int_0^1 \sqrt{16t^6 + 4t^{10}}\, dt$$

$$= \int_0^1 2t^3 \sqrt{4 + t^4}\, dt$$

$u = 4 + t^4 \qquad \frac{du}{dt} = 4t^3 \qquad dt = \frac{du}{4t^3}$

$t: 0 \to 1 \quad u: 4 \to 5$

$$= \int_4^5 2\cancel{t^3} \sqrt{u}\, \frac{du}{4\cancel{t^3}} = \int_4^5 \frac{1}{2} u^{\frac{1}{2}}\, du$$

$$\frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^5 = \frac{5^{\frac{3}{2}}}{3} - \frac{8}{3}$$

#3 Surface Area $= \int_0^4 2\pi y \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$

$= \int_0^4 2\pi (2t) \sqrt{(2t)^2 + 2^2}\, dt$

$= \int_0^4 4\pi t \sqrt{4t^2+4}\, dt$

$= 8\pi \int_0^4 t\sqrt{t^2+1}\, dt$

$u = t^2+1 \quad \frac{du}{dt} = 2t \quad dt = \frac{du}{2t}$

$t: 0 \to 4$

$u: 1 \to 17$

$= \overset{4}{\cancel{8}}\pi \int_1^{17} t\sqrt{u}\, \frac{du}{\cancel{2t}}$

$= 4\pi \int_1^{17} \sqrt{u}\, du$

$= 4\pi \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{17} = \frac{8\pi}{3}\sqrt{17} - \frac{8\pi}{3}$

#4 $r=2 \quad r = 4\cos\theta$

$2 = 4\cos\theta \quad \cos\theta = \frac{1}{2} \quad \theta = \pm\frac{\pi}{3}$

Area $= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (4\cos\theta)^2 - (2)^2\, d\theta$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 8\cos^2\theta - 2\, d\theta \qquad \cos^2\theta = \frac{\cos(2\theta)+1}{2}$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4(\cos(2\theta)+1) - 2\, d\theta$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4\cos 2\theta + 2\, d\theta$

$2\sin 2\theta + 2\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{3}} = 2\sqrt{3} + \frac{4\pi}{3}$

#5 Arc length $= \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$$= \int_0^{\pi} \sqrt{(8+8\cos\theta)^2 + (-8\sin\theta)^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{64 + 128\cos\theta + \underbrace{64\cos^2\theta + 64\sin^2\theta}_{=64}}\, d\theta$$

since $\cos^2\theta + \sin^2\theta = 1$

$$= \int_0^{\pi} \sqrt{128 + 128\cos\theta}\, d\theta$$

$$= \int_0^{\pi} 8\sqrt{2+2\cos\theta}\, d\theta$$

$\square$