20 pts.

ISE 2211 • Exam V • 12 July 2021 NAME SOLUTION

Fact: Joe Tritschler's children are ridic, especially his twins. You would think a task as simple as eating lunch would be pretty easy to accomplish in a timely manner; but alas, it often turns into what is known colloquially as a *goat rope*. (In fact, it sometimes resembles a literal one.) Over the course of a week, the time it takes them to achieve some degree of completion in this task was measured and the results presented below (all in minutes):

| | Twin #1 | Twin #2 |
|---|---|---|
| Sunday | 37 | 18 |
| Monday | 42 | 20 |
| Tuesday | 29 | 31 |
| Wednesday | 24 | 23 |
| Thursday | 65 | 39 |
| Friday | 82 | 30 |
| Saturday | 26 | 28 |

Because Joe Tritschler is an all-around great guy, he has computed sample parameters for you from this data: $\bar{x}_1 = 43.57$, $s_1 = 21.93$, $\bar{x}_2 = 27$, and $s_2 = 7.257$. Use them to test the following hypotheses on the difference in mean lunch-completion time using the *p*-value approach. State your final conclusion with regard to a significance level of $\alpha = 0.05$. What does the data suggest about the difference in mean lunch-completion time between them?

$H_0$: $\mu_1 = \mu_2$

$H_1$: $\mu_1 \neq \mu_2$

-- population variances assumed unequal

$\therefore$ need $v$

$$\frac{s_1^2}{n_1} = \frac{21.93^2}{7} = 68.70 \qquad \frac{s_2^2}{n_2} = \frac{7.257^2}{7} = 7.523$$

$$v = \frac{(68.70 + 7.523)^2}{\frac{68.70^2}{6} + \frac{7.523^2}{6}} = 7.298 \quad (+1)$$

round down to $v = 7$ degrees of freedom (+1)

$$t_0 = \frac{43.57 - 27}{\sqrt{68.70 + 7.523}} = 1.898 \quad (+1)$$

from table: $t_{.05,7} = 1.895$ (+1) ← very close! (+1)

$\therefore \frac{p\text{-value}}{2} \approx 0.05 \rightarrow p \approx 0.1$

(+1) $p \not< \alpha$ $\therefore$ fail to reject $H_0$ !!! (+1)

$\therefore$ no statistically significant difference !!! (+1)

crazy std. dev. of twin #1 no doubt a big factor!

Turns out this is actually a perfect candidate for a *paired t-test*. Test the following hypotheses on the mean difference in lunch-completion time using a 95% confidence interval on $\mu_D$. Note: you will need to compute sample parameters on mean difference; here is the data again for your convenience.

| | Twin #1 | Twin #2 | |
|---|---|---|---|
| Sunday | 37 | 18 | 19 |
| Monday | 42 | 20 | 22 |
| Tuesday | 29 | 31 | -2 |
| Wednesday | 24 | 23 | 1 |
| Thursday | 65 | 39 | 26 |
| Friday | 82 | 30 | 52 |
| Saturday | 26 | 28 | -2 |

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$H_0$: $\mu_D = 0$

$H_1$: $\mu_D \neq 0$

$\bar{d} = 16.57$ min. +1

$S_d = 19.63$ min. +1

$t_{\alpha/2,\, n-1} = t_{.025,\, 6} = 2.447$ +1

$n-1 \downarrow$ pairs

$\mu_D$: $\bar{d} \pm t_{\alpha/2,\, n-1}\, S_d/\sqrt{n}$

$16.57 \pm 2.447 \cdot 19.63/\sqrt{7}$

+2

$-1.585 < \mu_D < 34.73$ (min.)

C.I. does contain zero +1

$\therefore$ fail to reject $H_0$ +1 (again!)

Test the following hypotheses on the difference in population standard deviations of lunch-completion times using the fixed-significance-level approach at $\alpha = 0.05$. What does the data suggest about the difference between the two twins?

$H_0: \sigma_1 = \sigma_2$
$H_1: \sigma_1 > \sigma_2$

$$f_0 = \frac{S_1^2}{S_2^2} = \frac{21.93^2}{7.257^2} = 9.132$$

+1

∴ upper one-sided $H_1$;

∴ critical value is $f_{\alpha, n_1 - 1, n_2 - 1}$

$$= f_{.05, 6, 6} = 4.28$$

+1

$$f_0 \gg f_{.05, 6, 6}$$

+1

∴ reject $H_0$

+1

∴ data suggests more variability w/ #1 than #2

+1