EE 2010 Circuit Analysis

Module 19: Transient Responses: s-domain Notes

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, Michael Richmond and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

From WIKIPEDIA: "In electrical engineering and mechanical engineering, a *transient response* or natural response is the *response* of a system *to a change* from an equilibrium or a steady state. The transient response is not necessarily tied to "on/off" events but to any event that affects the equilibrium of the system. The *impulse response* and *step response* are transient responses to a specific input."

and recall ...

Dynamic elements are passive elements with the capability of energy storage. They are referred to as *dynamic*, as opposed to *instantaneous*, because their present state depends on the present input as well as previous inputs.

Therefore,...

The **transient response** of dynamic systems must (in general) include the effects of both the **input** stimulus and any **stored energy** in any *dynamic element*. We will consider these two contributions separately, with the effects of the **input stimulus** termed the **zero-state response** (since there is assumed to be no stored energy) and the response due to the **stored energy** in any of the *dynamic elements* termed the **zero-input response** (since this response does not result from an input stimulus).

The Transient Response of Dynamic Systems: s-Domain Analysis

There are *two* foundational approaches to determine the transient response for continuous-time systems irrespective of the genesis of the transient:

The s-domain approach:
Models system dynamics as s-domain quantities/sources and results in a direct algebraic analysis. **This approach is extendable to arbitrary input functions and arbitrariiy-high-order systems.**

The t-domain approach:
Relates system dynamics directly to an integro-differential equation representation. While this approach may provide some intuition relating the stored energy in each device – the initial conditions to system behavior, this approach may require repeated circuit analysis and is **not easily extendable to inputs other than step responses nor to general systems with orders higher than one.** Although we shall observe that initial conditions are easily folded into time-domain analyses for low-order systems.

For clarity, we will cover these approaches separately with s-domain approaches covered in this module.

19.1 The Transient Response of Dynamic Systems: s-Domain Analysis, Zero-State Response

We employ s-domain zero-state impedance models,

Zero-State Models

t**-domain**	s**-domain**	s**-domain** Impedance
$v(t) = i(t) \cdot R$	$V(s) = I(s) \cdot R$	$Z_R = R$
$v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau$	$V(s) = \frac{I(s)}{sC}$	$Z_C = \frac{1}{s \cdot C}$
$v(t) = L \frac{di(t)}{dt}$	$V(s) = Ls \cdot I(s)$	$Z_L = s \cdot L$

Table 1: Models for R, C, and L

and s-domain models for common input stimuli,

Laplace transform pairs

Function	**time-domain**	s**-domain**
	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
Unit impulse	$\delta(t)$	1
Delayed impulse	$\delta(t-\tau)$	$e^{-\tau s}$
Unit step	$u(t)$	$\frac{1}{s}$
Delayed unit step	$u(t-\tau)$	$\frac{1}{s} e^{-\tau s}$
Rectangular pulse	$p_T(t) = u(t) - u(t-T)$	$\frac{1}{s}(1 - e^{-Ts})$
Unit ramp	$t \cdot u(t)$	$\frac{1}{s^2}$
nth power	$t^n \cdot u(t)$	$\frac{n!}{s^{n+1}}$
Exponential decay	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$
Exponential approach	$(1 - e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$
Sine	$\sin(\omega t) \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t) \cdot u(t)$	$\frac{s}{s^2+\omega^2}$
Decaying sine	$e^{-\alpha t} \sin(\omega t) \cdot u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
Decaying cosine	$e^{-\alpha t} \cos(\omega t) \cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Table 2: Laplace transform table of common input functions

Procedure for s-domain Analysis of Zero-State Transient Response:

$$v_{in}(t) \overset{\mathcal{L}}{\longleftrightarrow} V_{in}(s) \longrightarrow \boxed{H(s)} \longrightarrow V_{in}(s) \cdot H(s) = V_{out}(s) \overset{\mathcal{L}}{\longleftrightarrow} v_{out}(t)$$

Figure 1: Transient Response Procedure for Zero-State Systems

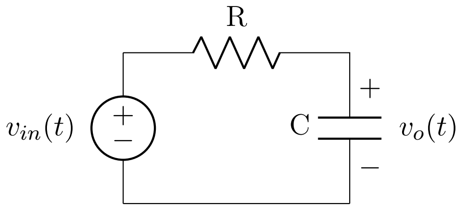
Note: This is exactly the same procedure as for sinusoidal steady-state except we apply the s-domain input function to the transfer function $H(s)$!

1. Replace *all independent sources with symbolic representations such as* $V_{in}(s)$
2. Employ s-domain impedance models
3. Analyze the circuit using appropriate techniques to find the output: $V_{out}(s)$, or whatever output function is dictated by the problem
4. Find the transfer function: $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$, or whatever input-output function is dictated by the problem
5. Find the output $V_{out}(s) = H(s) \cdot V_{in}(s)$, where $V_{in}(s) \overset{\mathcal{L}}{\longleftrightarrow} v_{in}(t)$
6. Find the output $v_{out}(t) \overset{\mathcal{L}}{\longleftrightarrow} V_{out}(s)$ using the inverse Laplace transform solver
$v_{out}(t) = \text{ilaplace}(V_{out}(s))$
7. Plot $v_{out}(t)$ to observe the response of the system to that input

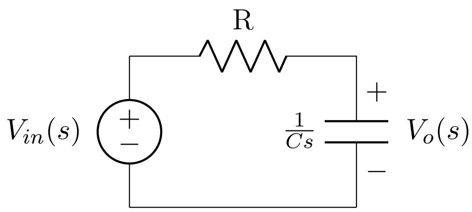
Dynamic Circuits, Zero-State

We begin by considering a change from an equilibrium or a steady state of a circuit containing **exactly one first-order** dynamic element. There are only two of these: an RC and an RL circuit.

Example 1s, Zero-State: Find $v_o(t)$ where $u(t)$ is a unit step function:



We also note that a unit step $u(t)$ has an s-domain representation of $\frac{1}{s}$ so that our example circuit can be redrawn in s-domain as:



with $V_{in}(s) = \frac{1}{s}$,

Comments:

We will notice a pattern of *exponential responses* for impulse and step inputs.

Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

Let's verify with a solver:

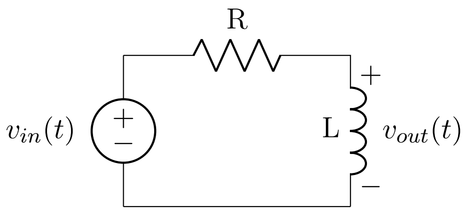
```
%% Example 1
clear all
syms A R C Vin Vout s t
% Voltage divider
Vo = Vin * (1/(C*s))/(R+(1/(C*s)))
H(s) = simplify(Vo/Vin)
% Lets look at the time-domain response
vo(t) = ilaplace(H(s)*A/s)
% ... and were done.
```

which yields....

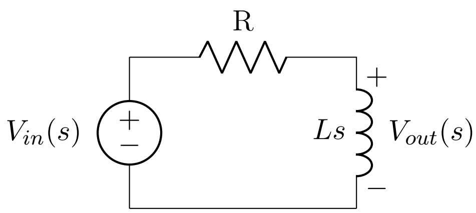
```
Vo = Vin/(C*s*(R + 1/(C*s)))
H(s) = 1/(C*R*s + 1)
vo(t) = A - A*exp(-t/(C*R))
```

QED.

Example 2s, Zero-State: Find $v_o(t)$ where $u(t)$ is a unit step function:



Again, our example circuit can be redrawn in s-domain as:



with $V_{in}(s) = \frac{1}{s}$,

Comments:

We will notice a pattern of *exponential responses* for impulse and step inputs.

Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

Let's verify with a solver:

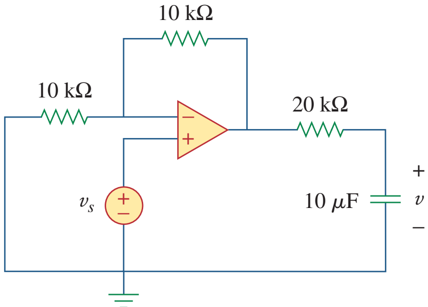
```
%% Example 2
clear all
syms A R L Vin Vout s t
% Voltage divider
Vo = Vin * (L*s)/(R+L*s)
H(s) = simplify(Vo/Vin)
% Lets look at the time-domain response
vo(t) = ilaplace(H(s)*A/s)
% ... and were done.
%
```

which yields....

```
Vo = (L*Vin*s)/(R + L*s)
H(s) = (L*s)/(R + L*s)
vo(t) = A*exp(-(R*t)/L)
```

Problem 7.71, Zero-State, with $v_s = 3u(t)$::

7.71 For the op amp circuit in Fig. 7.136, suppose v_0 = and v_s = 3 V. Find $v(t)$ for $t > 0$.



Labeling the Op-Amp output as V_{out} and writing a node equation at the inverting input, we have:

$$\frac{V_{in}}{10000} + \frac{(V_{in} - V_{out})}{10000} = 0$$

We also then have a voltage-divider to find v as:

$$v = \frac{\frac{V_{out}}{s \cdot 10^{-5}}}{(20000 + \frac{1}{s \cdot 10^{-5}})}$$

Later, we'll use with $V_{source}(s) = \frac{3}{s}$,

Comments:

We will notice a pattern of *exponential responses* for impulse and step inputs.

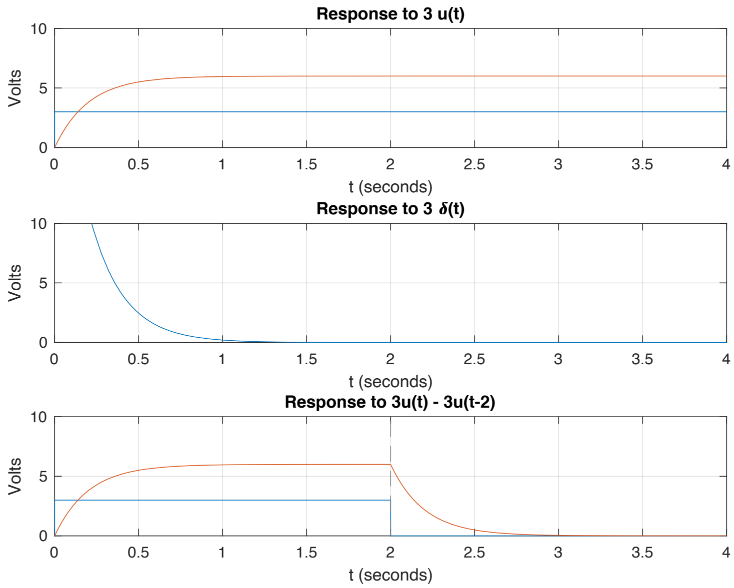
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

Let's verify with input $3u(t) \xleftrightarrow{\mathcal{L}} \frac{3}{s}$ in the solver:

```
%% Problem 7.71
clear all
syms Vin Vout V H s t
[Vin, V] = solve(Vin/10000 + (Vin - Vout)/10000 == 0,...
  V == (Vout/(s*1e-5))/(20000 + 1/(s*1e-5)), Vin, V)
%
H(s) = simplify(V/Vin)
% Define unitstep for plotting
unitstep = @(t) +(t>0);
clf
figure(71)
tmax = 4;
% The response to 3u(t)
v(t) = ilaplace(H(s)*3/s)
subplot(3,1,1)
fplot(@(t) 3*unitstep(t), [0,tmax])
hold on
fplot(v(t), [0,tmax]), grid
axis([0, tmax, 0, 10])
title('Response to 3 u(t)'); xlabel('t (seconds)'); ylabel('Volts')
% The response to an inpulse 3 delta(t)
v(t) = ilaplace(H(s)*3)
subplot(3,1,2)
fplot(v(t), [0,tmax]), grid
axis([0, tmax, 0, 10])
title('Response to 3 \delta(t)'); xlabel('t (seconds)'); ylabel('Volts')
% The response to a rectangular pulse: 3u(t) - 3u(t-2)
v(t) = ilaplace(H(s)*(3/s-3/s*exp(-2*s)))
subplot(3,1,3)
fplot(@(t) 3*unitstep(t) - 3*unitstep(t-2) , [0,tmax])
hold on
fplot(v(t), [0,tmax]), grid
axis([0, tmax, 0, 10])
title('Response to 3u(t) - 3u(t-2)'); xlabel('t (seconds)'); ylabel('Volts')
```

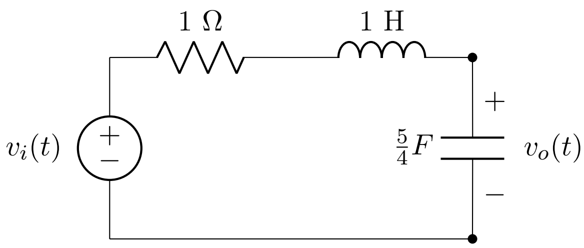
which yields....

```
Vin = Vout/2
V = (100000*Vout)/(s*(100000/s + 20000))
H(s) = 10/(s + 5)
v(t) = 6 - 6*exp(-5*t)
v(t) = 30*exp(-5*t)
v(t) = 30*heaviside(t - 2)*(exp(10 - 5*t)/5 - 1/5) - 6*exp(-5*t) + 6
```



Do these results make sense? Check $v_o(0)$ and $v_o(\infty)$.

Example 4, Zero-State: Find $v_o(t)$ for a unit step input so that $V_{in}(s) = \frac{1}{s}$:



Comments:

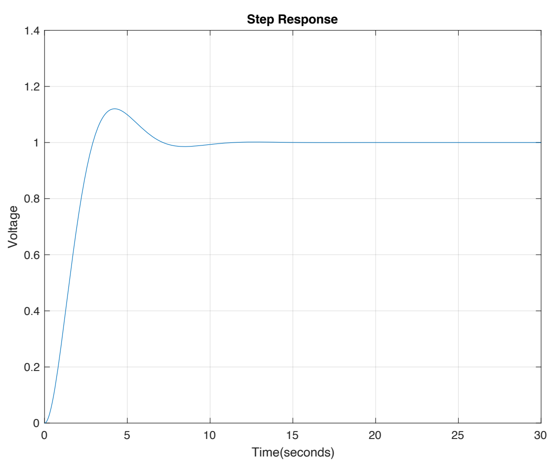
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab:

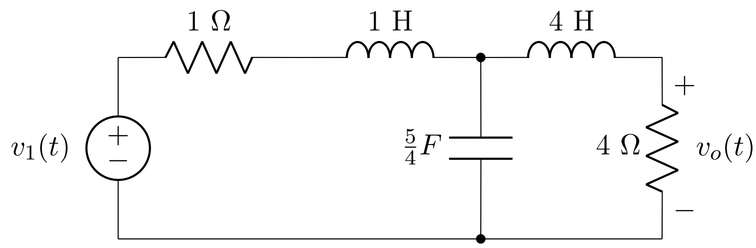
```
%%  Example 4 — a Second—Order Systemr
clear all
% Declare symbolic variables
syms Vout Vin s t
Vout = Vin*(4/(5*s))/(4/(5*s) + s + 1);
% The transfer function from the voltage divider:
H(s) = Vout/Vin
%
figure(4)
% The step response is the inverse Laplace of 1/s * H(s)
vo(t) = ilaplace(H(s)/s)
fplot(vo(t),[0,30]), grid
axis([0, 30, 0, 1.4])
title('Step Response'); xlabel('Time(seconds)'); ylabel('Voltage')
snapnow;
%
```

which yields....

```
H(s) = 0.8000/(s*(s + 0.8000/s + 1))
vo(t) = 1 — exp(—0.5000*t)*(cos(0.7416*t) + 0.6742*sin(0.7416*t))
```



Example 5, Zero-State: A third-order system. Find $v_o(t)$ for a unit step input so that $V_{in}(s) = \frac{1}{s}$:



Comments:

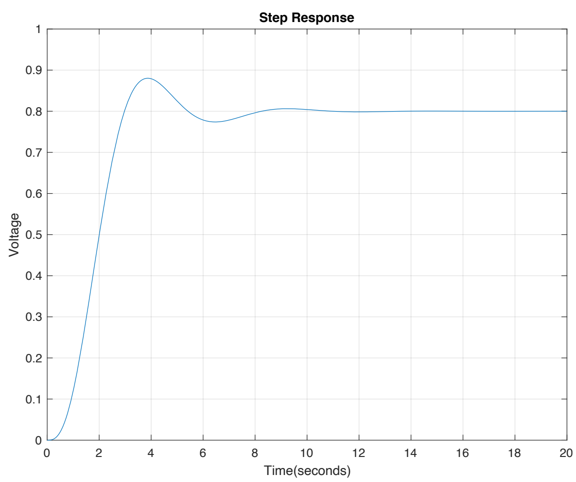
Does this result make sense? Check $v_o(0)$ and $v_o(\infty)$.

We'll verify the particulars in Matlab:

```
%% Example 5 — a Third—Order System
% Declare symbolic variables
syms Vin Va Vout s t
% Node equation directly in solve()
[Va, Vout] = solve(Va*s*0.8 + (Va—Vin)/(1+s) + Va/(4*s+4)==0,...
  Vout == Va*4/(4+4*s),Va,Vout)
% The transfer function
H(s) = simplify(Vout/Vin)
% Find and plot the output
figure(5)
% The step response is the inverse Laplace of 1/s * H(s)
vout(t) = ilaplace(subs(Vout,Vin,1/s))
%
fplot(vout(t),[0,20]), grid
axis([0, 20, 0, 1])
title('Step Response'); xlabel('Time(seconds)'); ylabel('Voltage')
snapnow;
```

which yields....

```
H(s) = 20/((s + 1)*(16*s^2 + 16*s + 25))
vout(t) = 0.8000 — 0.6983*exp(—0.5000*t)*sin(1.1456*t) — 0.8000*exp(—t)
```



Zero-State Homework: Chapter 7 # 42a, 45(with $v_0(0) = 0$), 46, 49, 68, 70, 72, 73

Chapter 8 # 5, 45(with $i_0(0) = 0$), 56, 68

Chapter 16 # 16, 48, 61, 66

7.59, 7.69, 8.39, 8.71, 16.81

7.5 In the circuit of Fig. 7.79, the capacitor voltage just before $t = 0$ is:

(a) 10 V (b) 7 V (c) 6 V
(d) 4 V (e) 0 V

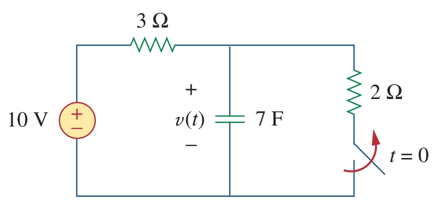


Figure 7.79
For Review Questions 7.5 and 7.6.

7.6 In the circuit in Fig. 7.79, $v(\infty)$ is:

(a) 10 V (b) 7 V (c) 6 V
(d) 4 V (e) 0 V

7.7 For the circuit in Fig. 7.80, the inductor current just before $t = 0$ is:

(a) 8 A (b) 6 A (c) 4 A
(d) 2 A (e) 0 A

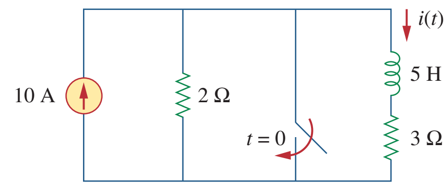


Figure 7.80
For Review Questions 7.7 and 7.8.

7.8 In the circuit of Fig. 7.80, $i(\infty)$ is:

(a) 10 A (b) 6 A (c) 4 A
(d) 2 A (e) 0 A

7.9 If v_s changes from 2 V to 4 V at $t = 0$, we may express v_s as:

(a) $\delta(t)$ V (b) $2u(t)$ V
(c) $2u(-t) + 4u(t)$ V (d) $2 + 2u(t)$ V
(e) $4u(t) - 2$ V

7.10 The pulse in Fig. 7.116(a) can be expressed in terms of singularity functions as:

(a) $2u(t) + 2u(t - 1)$ V (b) $2u(t) - 2u(t - 1)$ V
(c) $2u(t) - 4u(t - 1)$ V (d) $2u(t) + 4u(t - 1)$ V

Answers: 7.1d, 7.2b, 7.3c, 7.4b, 7.5d, 7.6a, 7.7c, 7.8e, 7.9c,d, 7.10b.

Problems

Section 7.2 The Source-Free *RC* Circuit

7.1 In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t}\ \text{V}, \quad t > 0$$
$$i(t) = 8e^{-200t}\ \text{mA}, \quad t > 0$$

(a) Find the values of R and C.
(b) Calculate the time constant τ.
(c) Determine the time required for the voltage to decay half its initial value at $t = 0$.

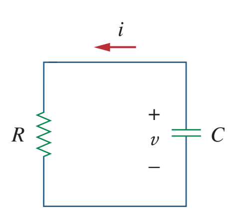


Figure 7.81
For Prob. 7.1.

7.2 Find the time constant for the *RC* circuit in Fig. 7.82.

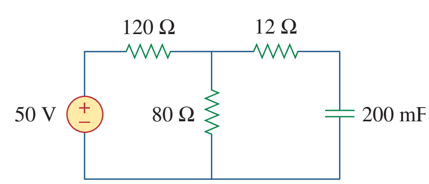


Figure 7.82
For Prob. 7.2.

7.3 Determine the time constant for the circuit in Fig. 7.83.

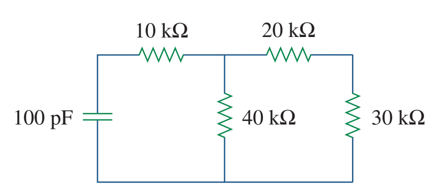


Figure 7.83
For Prob. 7.3.

7.4 The switch in Fig. 7.84 has been in position A for a long time. Assume the switch moves instantaneously from A to B at $t = 0$. Find v for $t > 0$.

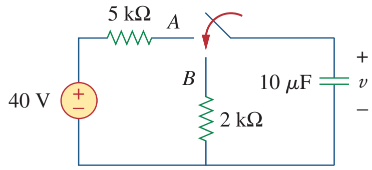


Figure 7.84
For Prob. 7.4.

7.5 Using Fig. 7.85, design a problem to help other students better understand source-free RC circuits.

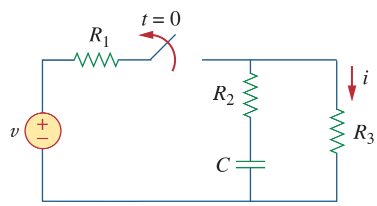


Figure 7.85
For Prob. 7.5.

7.6 The switch in Fig. 7.86 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

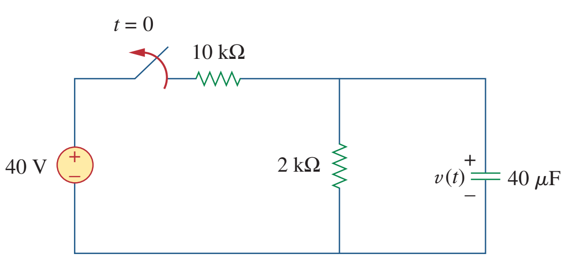


Figure 7.86
For Prob. 7.6.

7.7 Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at $t = 0$, Then at $t = 1$ second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.

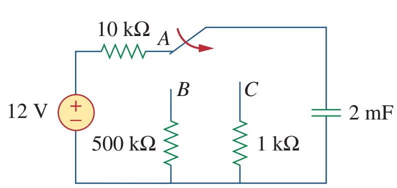


Figure 7.87
For Prob. 7.7.

7.8 For the circuit in Fig. 7.88, if

$$v = 10e^{-4t} \text{ V} \quad \text{and} \quad i = 0.2\, e^{-4t} \text{ A}, \quad t > 0$$

(a) Find R and C.

(b) Determine the time constant.

(c) Calculate the initial energy in the capacitor.

(d) Obtain the time it takes to dissipate 50 percent of the initial energy.

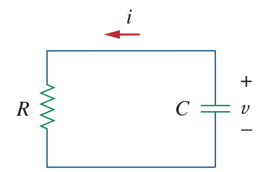


Figure 7.88
For Prob. 7.8.

7.9 The switch in Fig. 7.89 opens at $t = 0$. Find v_o for $t > 0$.

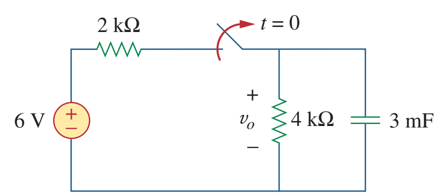


Figure 7.89
For Prob. 7.9.

7.10 For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

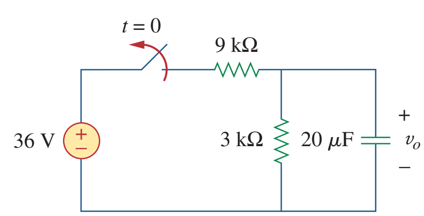


Figure 7.90
For Prob. 7.10.

Section 7.3 The Source-Free *RL* Circuit

7.11 For the circuit in Fig. 7.91, find i_o for $t > 0$.

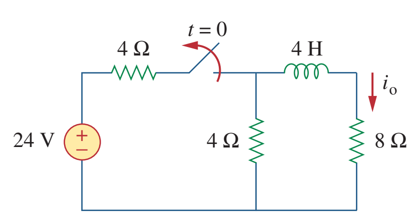


Figure 7.91
For Prob. 7.11.

7.12 Using Fig. 7.92, design a problem to help other students better understand source-free *RL* circuits.

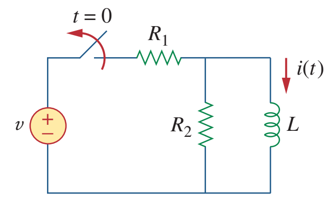


Figure 7.92
For Prob. 7.12.

7.13 In the circuit of Fig. 7.93,

$$v(t) = 80e^{-10^3 t}\ \text{V}, \quad t > 0$$
$$i(t) = 5e^{-10^3 t}\ \text{mA}, \quad t > 0$$

(a) Find R, L, and τ.

(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

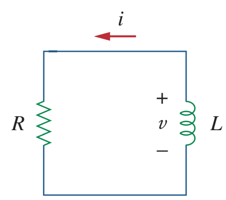


Figure 7.93
For Prob. 7.13.

7.14 Calculate the time constant of the circuit in Fig. 7.94.

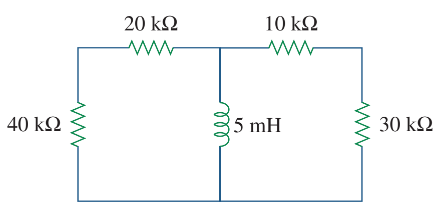


Figure 7.94
For Prob. 7.14.

7.15 Find the time constant for each of the circuits in Fig. 7.95.

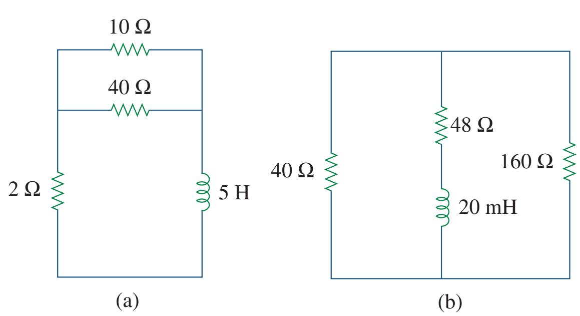


Figure 7.95
For Prob. 7.15.

7.16 Determine the time constant for each of the circuits in Fig. 7.96.

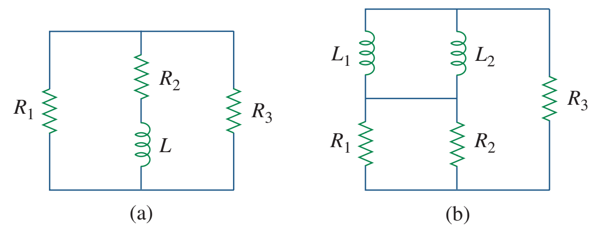


Figure 7.96
For Prob. 7.16.

7.17 Consider the circuit of Fig. 7.97. Find $v_o(t)$ if $i(0) = 6$ A and $v(t) = 0$.

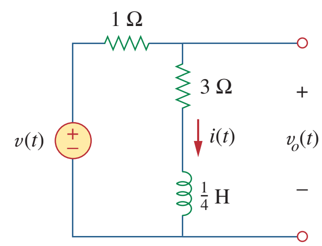


Figure 7.97
For Prob. 7.17.

7.18 For the circuit in Fig. 7.98, determine $v_o(t)$ when $i(0) = 5$ A and $v(t) = 0$.

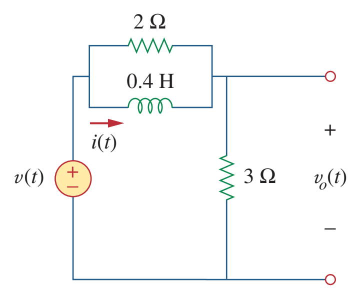


Figure 7.98
For Prob. 7.18.

7.19 In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 6$ A.

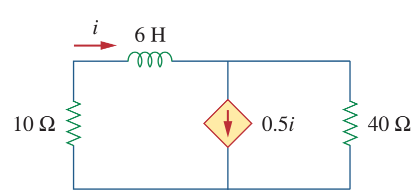


Figure 7.99
For Prob. 7.19.

7.20 For the circuit in Fig. 7.100,

$$v = 90e^{-50t}\ \text{V}$$

and

$$i = 30e^{-50t}\ \text{A}, \qquad t > 0$$

(a) Find L and R.

(b) Determine the time constant.

(c) Calculate the initial energy in the inductor.

(d) What fraction of the initial energy is dissipated in 10 ms?

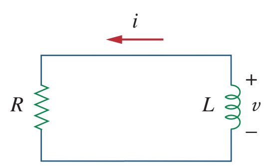


Figure 7.100
For Prob. 7.20.

7.21 In the circuit of Fig. 7.101, find the value of R for which the steady-state energy stored in the inductor will be 1 J.

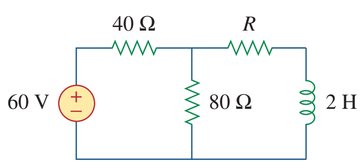


Figure 7.101
For Prob. 7.21.

7.22 Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 7.102 if $i(0) = 10$ A.

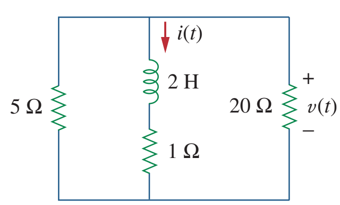


Figure 7.102
For Prob. 7.22.

7.23 Consider the circuit in Fig. 7.103. Given that $v_o(0) = 10$ V, find v_o and v_x for $t > 0$.

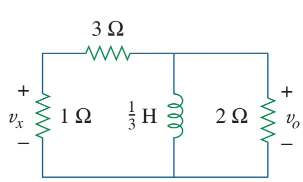


Figure 7.103
For Prob. 7.23.

Section 7.4 Singularity Functions

7.24 Express the following signals in terms of singularity functions.

(a) $v(t) = \begin{cases} 0, & t < 0 \\ -5, & t > 0 \end{cases}$

(b) $i(t) = \begin{cases} 0, & t < 1 \\ -10, & 1 < t < 3 \\ 10, & 3 < t < 5 \\ 0, & t > 5 \end{cases}$

(c) $x(t) = \begin{cases} t - 1, & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 4 - t, & 3 < t < 4 \\ 0, & \text{Otherwise} \end{cases}$

(d) $y(t) = \begin{cases} 2, & t < 0 \\ -5, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$

7.25 Design a problem to help other students better understand singularity functions.

7.26 Express the signals in Fig. 7.104 in terms of singularity functions.

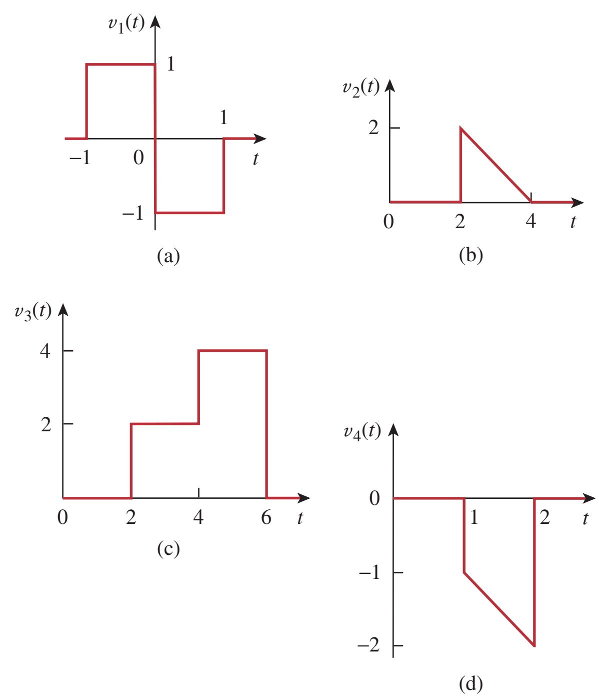


Figure 7.104
For Prob. 7.26.

7.27 Express $v(t)$ in Fig. 7.105 in terms of step functions.

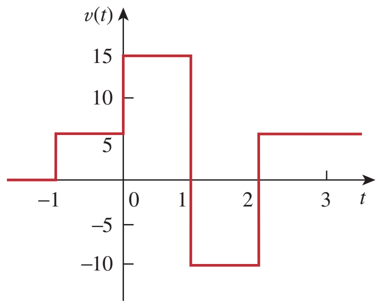


Figure 7.105
For Prob. 7.27.

7.28 Sketch the waveform represented by

$$i(t) = r(t) - r(t-1) - u(t-2) - r(t-2) + r(t-3) + u(t-4)$$

7.29 Sketch the following functions:

(a) $x(t) = 10e^{-t}u(t-1)$,

(b) $y(t) = 10e^{-(t-1)}u(t)$,

(c) $z(t) = \cos 4t\delta(t-1)$

7.30 Evaluate the following integrals involving the impulse functions:

(a) $\displaystyle\int_{-\infty}^{\infty} 4t^2\delta(t-1)\,dt$

(b) $\displaystyle\int_{-\infty}^{\infty} 4t^2 \cos 2\pi t\delta(t-0.5)\,dt$

7.31 Evaluate the following integrals:

(a) $\displaystyle\int_{-\infty}^{\infty} e^{-4t^2}\delta(t-2)\,dt$

(b) $\displaystyle\int_{-\infty}^{\infty} [5\delta(t) + e^{-t}\delta(t) + \cos 2\pi t\delta(t)]\,dt$

7.32 Evaluate the following integrals:

(a) $\displaystyle\int_{1}^{t} u(\lambda)\,d\lambda$

(b) $\displaystyle\int_{0}^{4} r(t-1)\,dt$

(c) $\displaystyle\int_{1}^{5} (t-6)^2\delta(t-2)\,dt$

7.33 The voltage across a 10-mH inductor is $15\delta(t-2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.

7.34 Evaluate the following derivatives:

(a) $\dfrac{d}{dt}[u(t-1)u(t+1)]$

(b) $\dfrac{d}{dt}[r(t-6)u(t-2)]$

(c) $\dfrac{d}{dt}[\sin 4tu(t-3)]$

7.35 Find the solution to the following differential equations:

(a) $\dfrac{dv}{dt} + 2v = 0, \qquad v(0) = -1$ V

(b) $2\dfrac{di}{dt} - 3i = 0, \qquad i(0) = 2$

7.36 Solve for v in the following differential equations, subject to the stated initial condition.

(a) $dv/dt + v = u(t), \qquad v(0) = 0$

(b) $2\,dv/dt - v = 3u(t), \qquad v(0) = -6$

7.37 A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

(a) What is the time constant of the circuit?

(b) What is $v(\infty)$, the final value of v?

(c) If $v(0) = 2$, find $v(t)$ for $t \geq 0$.

7.38 A circuit is described by

$$\frac{di}{dt} + 3i = 2u(t)$$

Find $i(t)$ for $t > 0$ given that $i(0) = 0$.

Section 7.5 Step Response of an *RC* Circuit

7.39 Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.

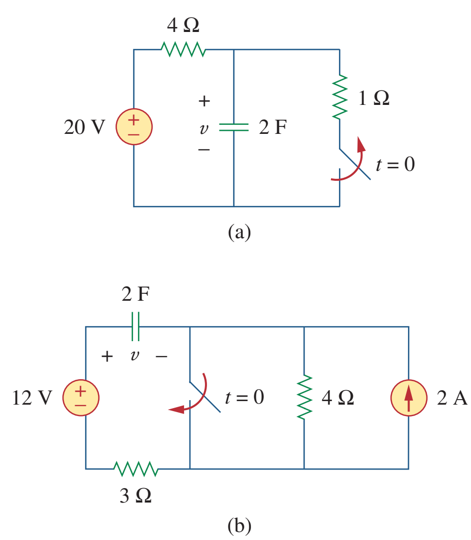


Figure 7.106
For Prob. 7.39.

7.40 Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.

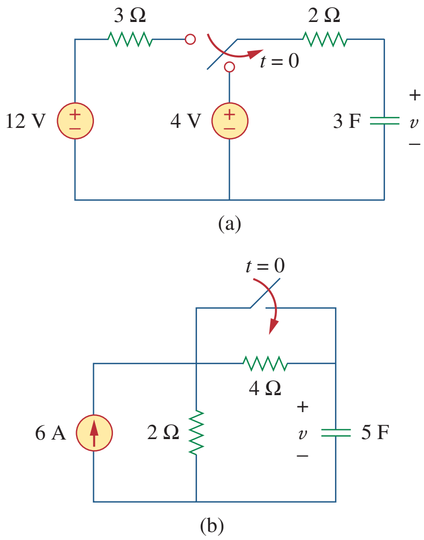


Figure 7.107
For Prob. 7.40.

7.41 e⃞d Using Fig. 7.108, design a problem to help other students better understand the step response of an *RC* circuit.

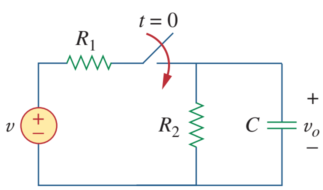


Figure 7.108
For Prob. 7.41.

7.42 (a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

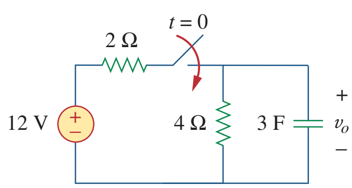


Figure 7.109
For Prob. 7.42.

7.43 Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.

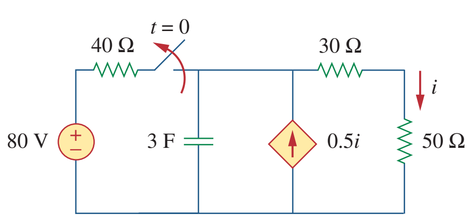


Figure 7.110
For Prob. 7.43.

7.44 The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$, it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

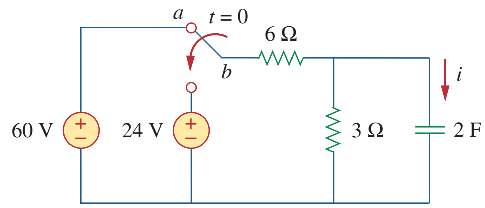


Figure 7.111
For Prob. 7.44.

7.45 Find v_o in the circuit of Fig. 7.112 when $v_s = 30u(t)$ V. Assume that $v_o(0) = 5$ V.

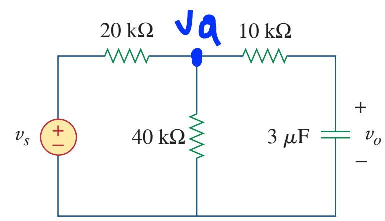


Figure 7.112
For Prob. 7.45.

7.46 For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$. Find $v(t)$.

va:-Is+va/6+(va-vo)/2=0

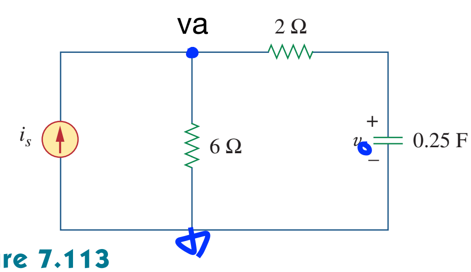


Figure 7.113
For Prob. 7.46.

7.47 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

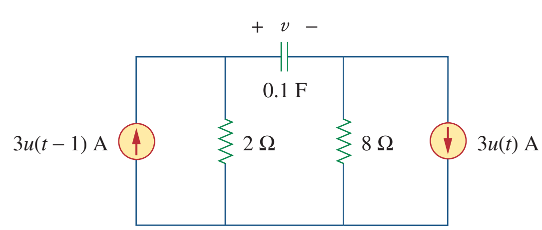


Figure 7.114
For Prob. 7.47.

7.48 Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

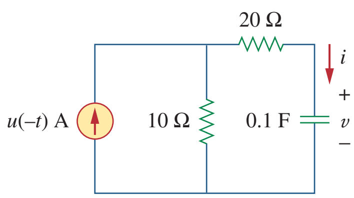


Figure 7.115
For Prob. 7.48.

7.49 If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.

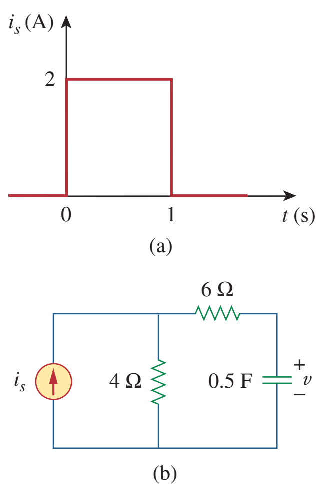


Figure 7.116
For Prob. 7.49 and Review Question 7.10.

***7.50** In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, and $C = 0.25$ mF.

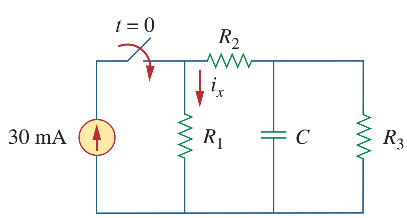


Figure 7.117
For Prob. 7.50.

Section 7.6 Step Response of an *RL* Circuit

7.51 Rather than applying the short-cut technique used in Section 7.6, use KVL to obtain Eq. (7.60).

7.52 Using Fig. 7.118, design a problem to help other students better understand the step response of an *RL* circuit.

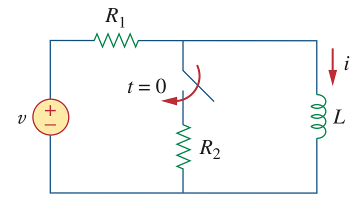


Figure 7.118
For Prob. 7.52.

7.53 Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.

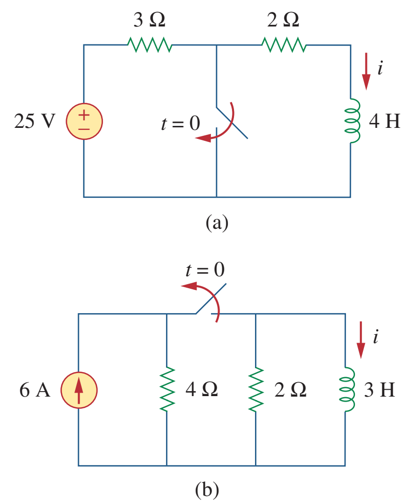


Figure 7.119
For Prob. 7.53.

7.54 Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.

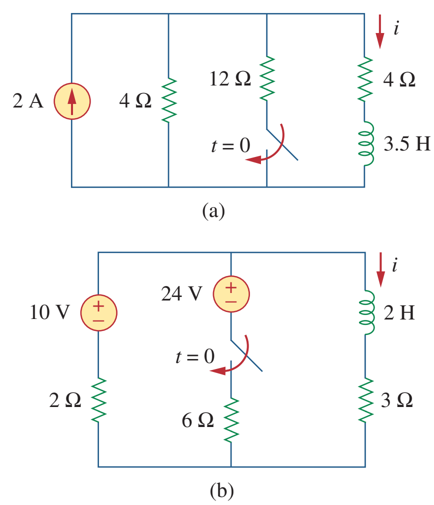


Figure 7.120
For Prob. 7.54.

* An asterisk indicates a challenging problem.

7.55 Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

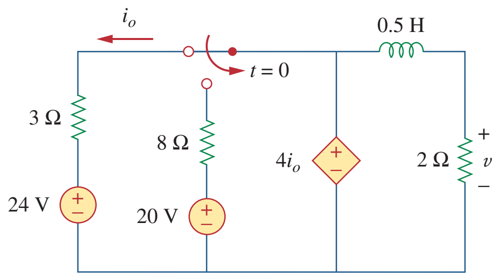


Figure 7.121
For Prob. 7.55.

7.56 For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

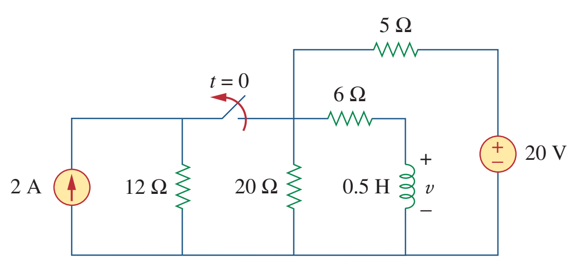


Figure 7.122
For Prob. 7.56.

***7.57** Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

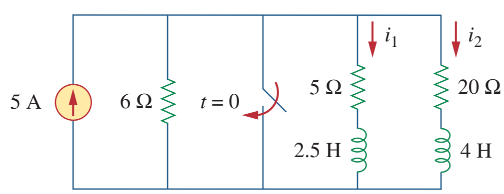


Figure 7.123
For Prob. 7.57.

7.58 Rework Prob. 7.17 if $i(0) = 10$ A and $v(t) = 20u(t)$ V.

7.59 Determine the step response $v_o(t)$ to $v_s = 18u(t)$ in the circuit of Fig. 7.124.

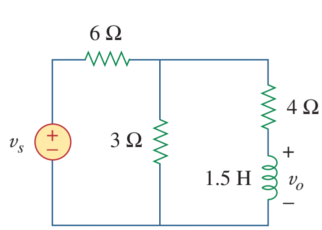


Figure 7.124
For Prob. 7.59.

7.60 Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.125 if the initial current in the inductor is zero.

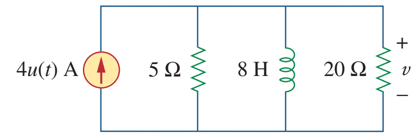


Figure 7.125
For Prob. 7.60.

7.61 In the circuit in Fig. 7.126, i_s changes from 5 A to 10 A at $t = 0$; that is, $i_s = 5u(-t) + 10u(t)$. Find v and i.

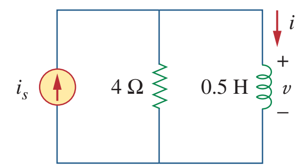


Figure 7.126
For Prob. 7.61.

7.62 For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

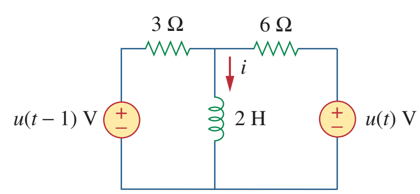


Figure 7.127
For Prob. 7.62.

7.63 Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

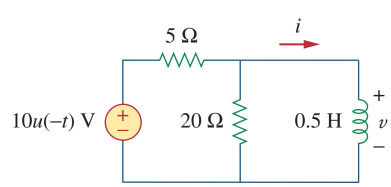


Figure 7.128
For Prob. 7.63.

7.64 Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from $t = 0$ *sec* to $t = \infty$ *sec*. The value of $v_{in}(t)$ is equal to $[40 - 40u(t)]$ volts.

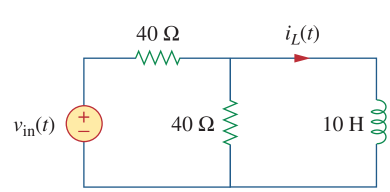


Figure 7.129
For Prob. 7.64.

7.65 If the input pulse in Fig. 7.130(a) is applied to the circuit in Fig. 7.130(b), determine the response $i(t)$.

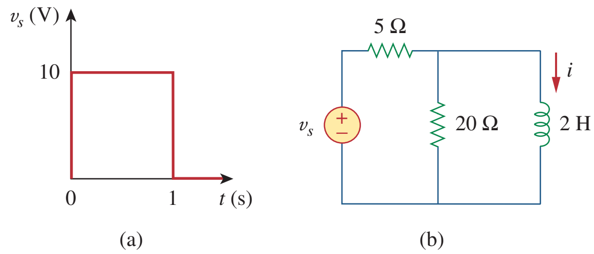


Figure 7.130
For Prob. 7.65.

Section 7.7 First-order Op Amp Circuits

7.66 Using Fig. 7.131, design a problem to help other students better understand first-order op amp circuits.

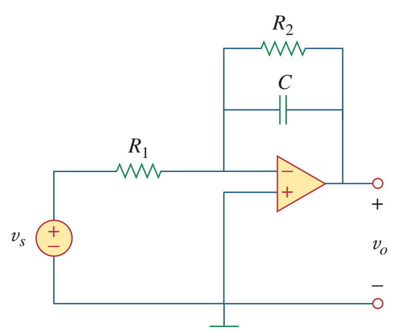


Figure 7.131
For Prob. 7.66.

7.67 If $v(0) = 5$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit in Fig. 7.132. Let $R = 10$ kΩ and $C = 1\ \mu$F.

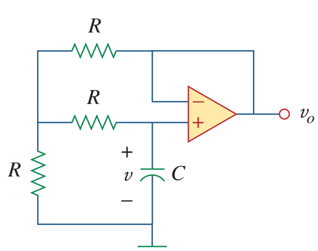


Figure 7.132
For Prob. 7.67.

7.68 Obtain v_o for $t > 0$ in the circuit of Fig. 7.133.

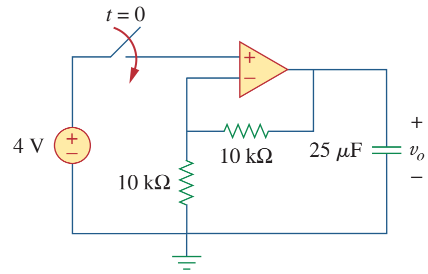


Figure 7.133
For Prob. 7.68.

7.69 For the op amp circuit in Fig. 7.134, find $v_o(t)$ for $t > 0$.

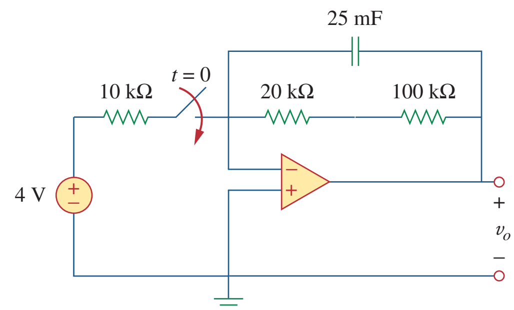


Figure 7.134
For Prob. 7.69.

7.70 Determine v_o for $t > 0$ when $v_s = 20$ mV in the op amp circuit of Fig. 7.135.

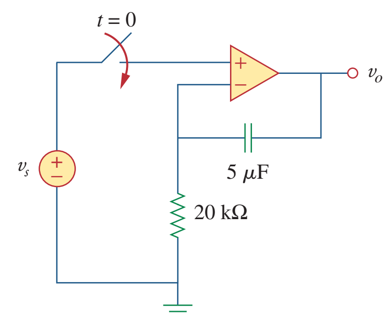


Figure 7.135
For Prob. 7.70.

7.71 For the op amp circuit in Fig. 7.136, suppose $v_0 = 0$ and $v_s = 3$ V. Find $v(t)$ for $t > 0$.

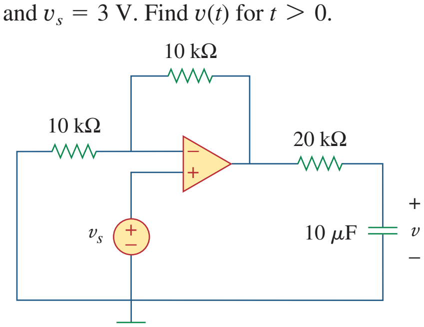


Figure 7.136
For Prob. 7.71.

7.72 Find i_o in the op amp circuit in Fig. 7.137. Assume that $v(0) = -2$ V, $R = 10$ kΩ, and $C = 10$ μF.

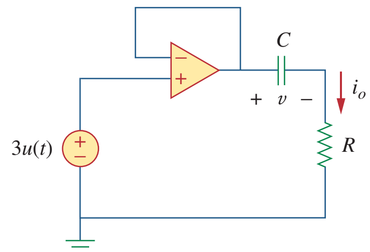


Figure 7.137
For Prob. 7.72.

7.73 For the op amp circuit of Fig. 7.138, let $R_1 = 10$ kΩ, $R_f = 20$ kΩ, $C = 20$ μF, and $v(0) = 1V$. Find v_0.

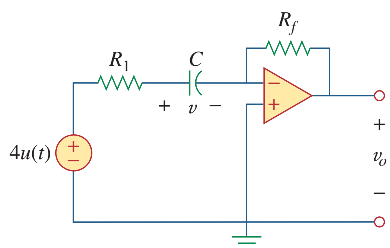


Figure 7.138
For Prob. 7.73.

7.74 Determine $v_o(t)$ for $t > 0$ in the circuit of Fig. 7.139. Let $i_s = 10u(t)$ μA and assume that the capacitor is initially uncharged.

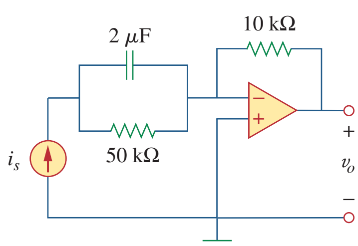


Figure 7.139
For Prob. 7.74.

7.75 In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.

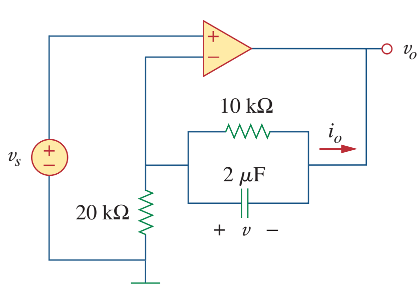


Figure 7.140
For Prob. 7.75.

Section 7.8 Transient Analysis with *PSpice*

7.76 Repeat Prob. 7.49 using *PSpice or MultiSim*.

7.77 The switch in Fig. 7.141 opens at $t = 0$. Use *PSpice or MultiSim* to determine $v(t)$ for $t > 0$.

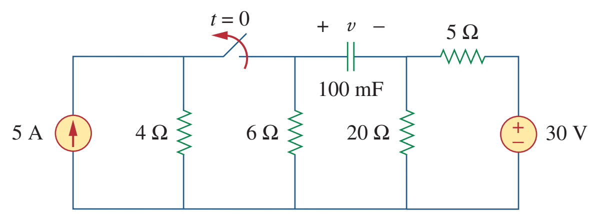


Figure 7.141
For Prob. 7.77.

7.78 The switch in Fig. 7.142 moves from position a to b at $t = 0$. Use *PSpice or MultiSim* to find $i(t)$ for $t > 0$.

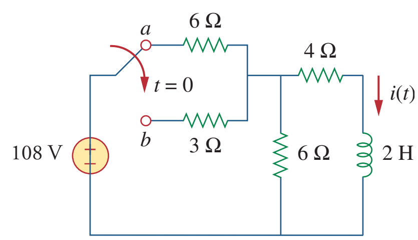


Figure 7.142
For Prob. 7.78.

7.79 In the circuit of Fig. 7.143, the switch has been in position a for a long time but moves instantaneously to position b at $t = 0$. Determine $i_o(t)$.

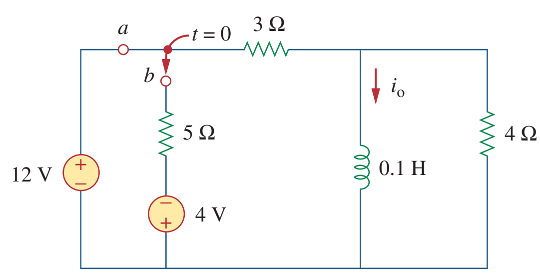


Figure 7.143
For Prob. 7.79.

7.80 In the circuit of Fig. 7.144, assume that the switch has been in position a for a long time, find:

(a) $i_1(0)$, i_2 (0), and $v_o(0)$

(b) $i_L(t)$

(c) $i_1(\infty)$, $i_2(\infty)$, and $v_o(\infty)$.

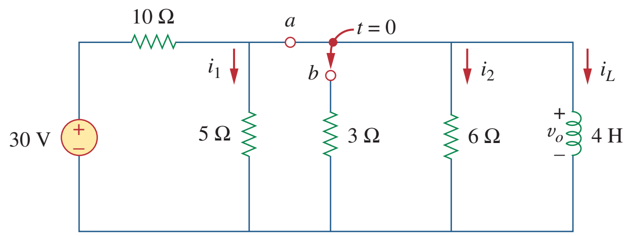


Figure 7.144
For Prob. 7.80.

7.81 Repeat Prob. 7.65 using *PSpice or MultiSim*.

Section 7.9 Applications

7.82 In designing a signal-switching circuit, it was found that a 100-μF capacitor was needed for a time constant of 3 ms. What value resistor is necessary for the circuit?

7.83 An *RC* circuit consists of a series connection of a 120-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

7.84 The resistance of a 160-mH coil is 8 Ω. Find the time required for the current to build up to 60 percent of its final value when voltage is applied to the coil.

7.85 A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is 120 Ω when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?

(b) What is the time interval between light flashes?

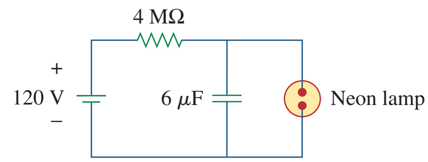


Figure 7.145
For Prob. 7.85.

7.86 Figure 7.146 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

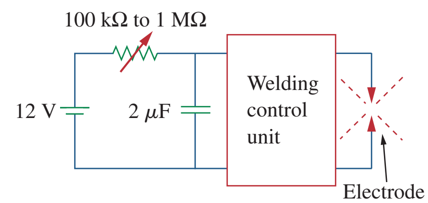


Figure 7.146
For Prob. 7.86.

7.87 A 120-V dc generator energizes a motor whose coil has an inductance of 50 H and a resistance of 100 Ω. A field discharge resistor of 400 Ω is connected in parallel with the motor to avoid damage to the motor, as shown in Fig. 7.147. The system is at steady state. Find the current through the discharge resistor 100 ms after the breaker is tripped.

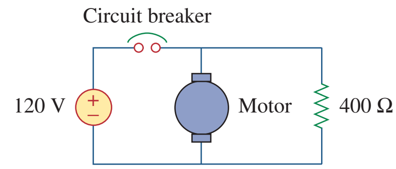


Figure 7.147
For Prob. 7.87.

Comprehensive Problems

7.88 The circuit in Fig. 7.148(a) can be designed as an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant $\tau = RC$ of the circuit and the width T of the input pulse in Fig. 7.148(b). The circuit is a differentiator if $\tau \ll T$, say $\tau < 0.1T$, or an integrator if $\tau \gg T$, say $\tau > 10T$.

(a) What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?

(b) If the output is to be an integrated form of the input, what is the maximum value the pulse width can assume?

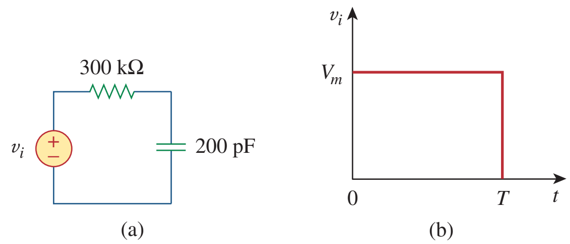


Figure 7.148
For Prob. 7.88.

7.89 An RL circuit may be used as a differentiator if the output is taken across the inductor and $\tau \ll T$ (say $\tau < 0.1T$), where T is the width of the input pulse. If R is fixed at 200 kΩ, determine the maximum value of L required to differentiate a pulse with $T = 10\ \mu$s.

7.90 An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s, while the probe has an internal resistance R_p. If R_p is fixed at 6 MΩ, find R_s and C_s for the circuit to have a time constant of 15 μs.

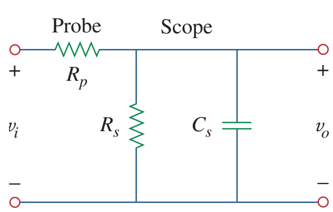


Figure 7.149
For Prob. 7.90.

7.91 The circuit in Fig. 7.150 is used by a biology student to study "frog kick." She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.

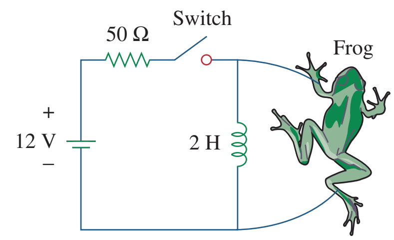


Figure 7.150
For Prob. 7.91.

7.92 To move a spot of a cathode-ray tube across the screen requires a linear increase in the voltage across the deflection plates, as shown in Fig. 7.151. Given that the capacitance of the plates is 4 nF, sketch the current flowing through the plates.

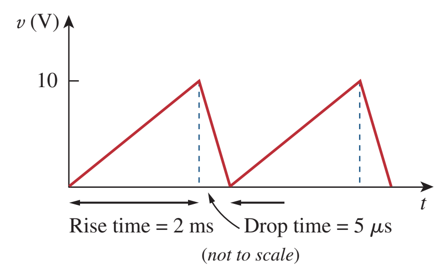


Figure 7.151
For Prob. 7.92.

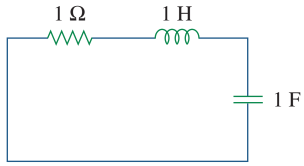


Figure 8.59
For Review Question 8.7.

8.8 Consider the parallel *RLC* circuit in Fig. 8.60. What type of response will it produce?

(a) overdamped

(b) underdamped

(c) critically damped

(d) none of the above

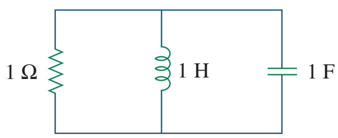


Figure 8.60
For Review Question 8.8.

8.9 Match the circuits in Fig. 8.61 with the following items:

(i) first-order circuit

(ii) second-order series circuit

(iii) second-order parallel circuit

(iv) none of the above

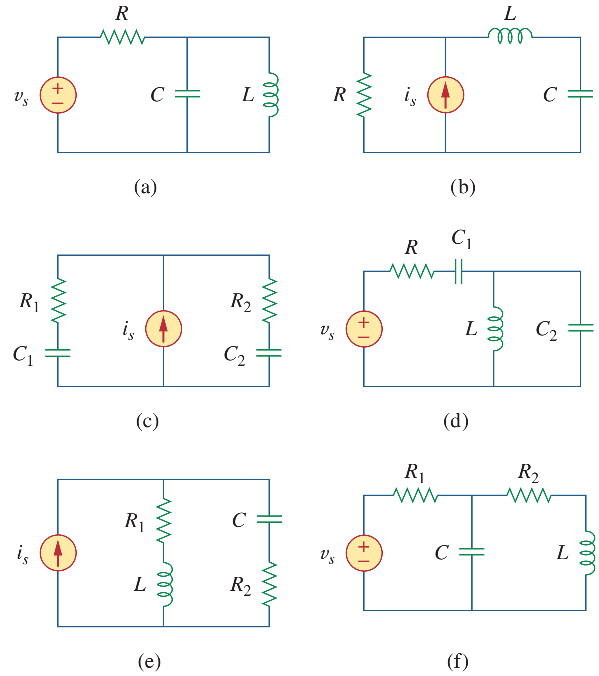


Figure 8.61
For Review Question 8.9.

8.10 In an electric circuit, the dual of resistance is:

(a) conductance (b) inductance

(c) capacitance (d) open circuit

(e) short circuit

Answers: 8.1a, 8.2c, 8.3b, 8.4d, 8.5d, 8.6c, 8.7b, 8.8b, 8.9 (i)-c, (ii)-b, e, (iii)-a, (iv)-d, f, 8.10a.

Problems

Section 8.2 Finding Initial and Final Values

8.1 For the circuit in Fig. 8.62, find:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

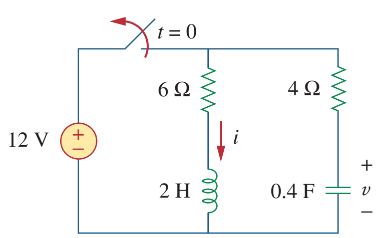


Figure 8.62
For Prob. 8.1.

8.2 e⃞d Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

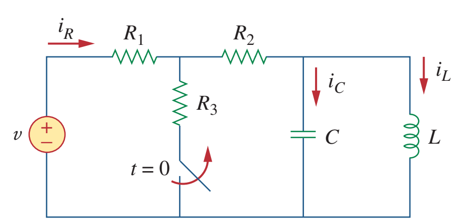


Figure 8.63
For Prob. 8.2.

8.3 Refer to the circuit shown in Fig. 8.64. Calculate:

(a) $i_L(0^+)$, $v_C(0^+)$, and $v_R(0^+)$,

(b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, and $dv_R(0^+)/dt$,

(c) $i_L(\infty)$, $v_C(\infty)$, and $v_R(\infty)$.

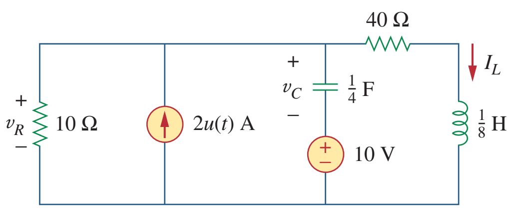


Figure 8.64
For Prob. 8.3.

8.4 In the circuit of Fig. 8.65, find:

(a) $v(0^+)$ and $i(0^+)$,

(b) $dv(0^+)/dt$ and $di(0^+)/dt$,

(c) $v(\infty)$ and $i(\infty)$.

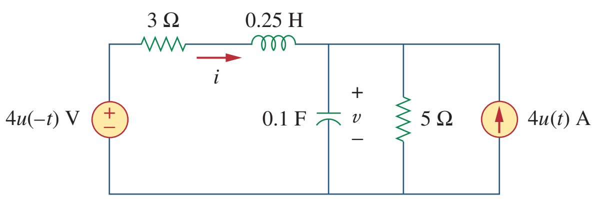


Figure 8.65
For Prob. 8.4.

8.5 Refer to the circuit in Fig. 8.66. Determine:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

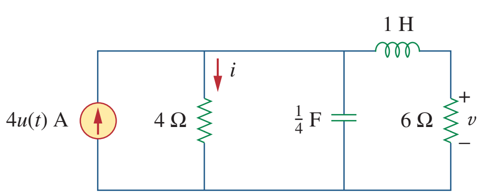


Figure 8.66
For Prob. 8.5.

8.6 In the circuit of Fig. 8.67, find:

(a) $v_R(0^+)$ and $v_L(0^+)$,

(b) $dv_R(0^+)/dt$ and $dv_L(0^+)/dt$,

(c) $v_R(\infty)$ and $v_L(\infty)$.

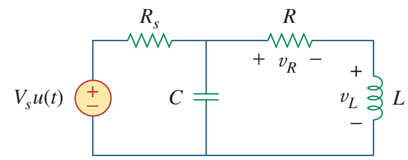


Figure 8.67
For Prob. 8.6.

Section 8.3 Source-Free Series *RLC* Circuit

8.7 A series *RLC* circuit has $R = 20\ \text{k}\Omega$, $L = 0.2$ mH, and $C = 5\ \mu$F. What type of damping is exhibited by the circuit?

8.8 e2d Design a problem to help other students better understand source-free *RLC* circuits.

8.9 The current in an *RLC* circuit is described by

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0$$

If $i(0) = 10$ A and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.

8.10 The differential equation that describes the voltage in an *RLC* network is

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 10$ V/s, obtain $v(t)$.

8.11 The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 0$$

for which the initial conditions are $v(0) = 10$ V and $dv(0)/dt = 0$. Solve for $v(t)$.

8.12 If $R = 50\ \Omega$, $L = 1.5$ H, what value of C will make an *RLC* series circuit:

(a) overdamped,

(b) critically damped,

(c) underdamped?

8.13 For the circuit in Fig. 8.68, calculate the value of R needed to have a critically damped response.

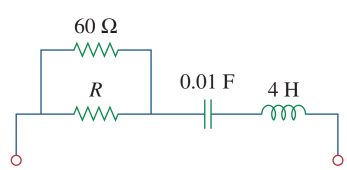


Figure 8.68
For Prob. 8.13.

8.14 The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.

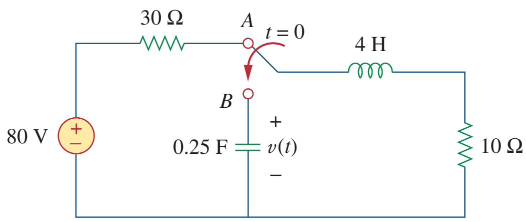


Figure 8.69
For Prob. 8.14.

8.15 The responses of a series RLC circuit are

$$v_C(t) = 30 - 10e^{-20t} + 30e^{-10t} \text{ V}$$
$$i_L(t) = 40e^{-20t} - 60e^{-10t} \text{ mA}$$

where v_C and i_L are the capacitor voltage and inductor current, respectively. Determine the values of R, L, and C.

8.16 Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.70.

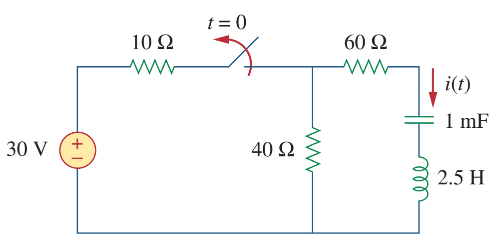


Figure 8.70
For Prob. 8.16.

8.17 In the circuit of Fig. 8.71, the switch instantaneously moves from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.

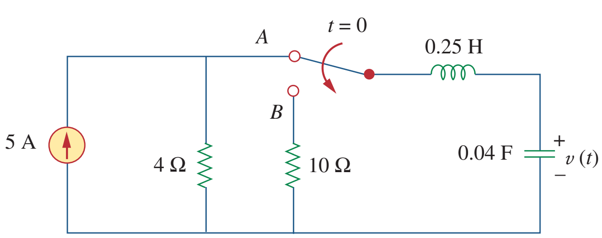


Figure 8.71
For Prob. 8.17.

8.18 Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 8.72. Assume steady-state conditions exist at $t = 0^-$.

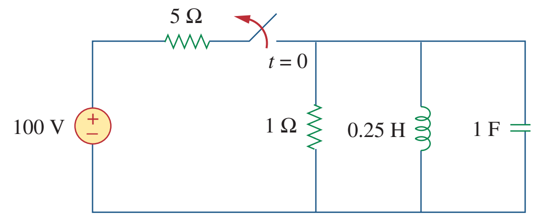


Figure 8.72
For Prob. 8.18.

8.19 Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 8.73.

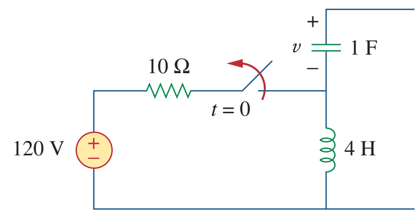


Figure 8.73
For Prob. 8.19.

8.20 The switch in the circuit of Fig. 8.74 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

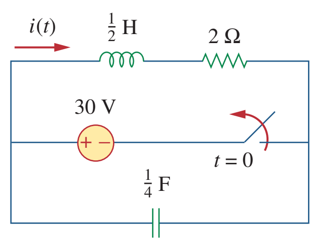


Figure 8.74
For Prob. 8.20.

***8.21** Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.

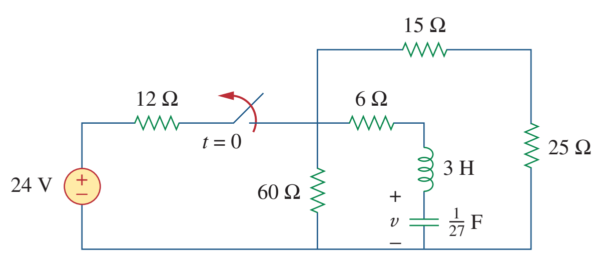


Figure 8.75
For Prob. 8.21.

* An asterisk indicates a challenging problem.

Section 8.4 Source-Free Parallel *RLC* Circuit

8.22 Assuming $R = 2\ \text{k}\Omega$, design a parallel *RLC* circuit that has the characteristic equation

$$s^2 + 100s + 10^6 = 0.$$

8.23 For the network in Fig. 8.76, what value of C is needed to make the response underdamped with unity damping factor ($\alpha = 1$)?

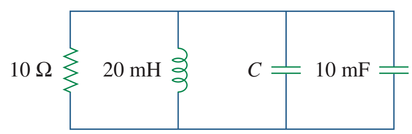


Figure 8.76
For Prob. 8.23.

8.24 The switch in Fig. 8.77 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Determine $i(t)$ for $t > 0$.

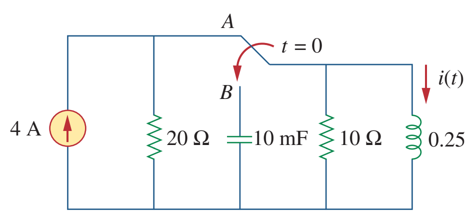


Figure 8.77
For Prob. 8.24.

8.25 Using Fig. 8.78, design a problem to help other students better understand source-free *RLC* circuits.

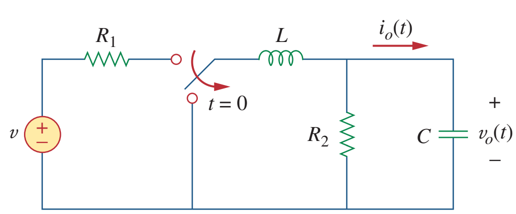


Figure 8.78
For Prob. 8.25.

Section 8.5 Step Response of a Series *RLC* Circuit

8.26 The step response of an *RLC* circuit is given by

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 2$ and $di(0)/dt = 4$, solve for $i(t)$.

8.27 A branch voltage in an *RLC* circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

8.28 A series *RLC* circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5$ H, $R = 4\ \Omega$, and $C = 0.2$ F. Let $i(0) = 1$, $di(0)/dt = 0$.

8.29 Solve the following differential equations subject to the specified initial conditions

(a) $d^2v/dt^2 + 4v = 12$, $v(0) = 0$, $dv(0)/dt = 2$

(b) $d^2i/dt^2 + 5\,di/dt + 4i = 8$, $i(0) = -1$, $di(0)/dt = 0$

(c) $d^2v/dt^2 + 2\,dv/dt + v = 3$, $v(0) = 5$, $dv(0)/dt = 1$

(d) $d^2i/dt^2 + 2\,di/dt + 5i = 10$, $i(0) = 4$, $di(0)/dt = -2$

8.30 The step responses of a series *RLC* circuit are

$$v_C = 40 - 10e^{-2000t} - 10e^{-4000t}\ \text{V}, \qquad t > 0$$
$$i_L(t) = 3e^{-2000t} + 6e^{-4000t}\ \text{mA}, \qquad t > 0$$

(a) Find C. (b) Determine what type of damping is exhibited by the circuit.

8.31 Consider the circuit in Fig. 8.79. Find $v_L(0^+)$ and $v_C(0^+)$.

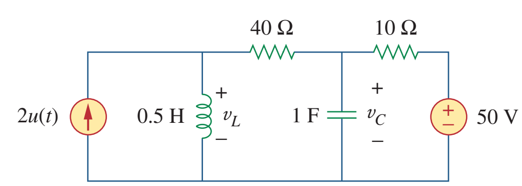


Figure 8.79
For Prob. 8.31.

8.32 For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

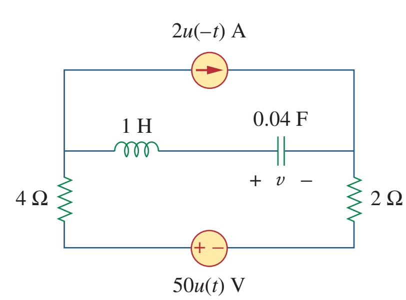


Figure 8.80
For Prob. 8.32.

8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

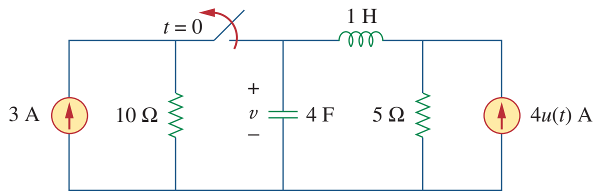


Figure 8.81
For Prob. 8.33.

8.34 Calculate $i(t)$ for $t > 0$ in the circuit of Fig. 8.82.

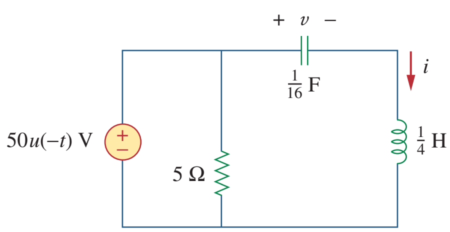


Figure 8.82
For Prob. 8.34.

8.35 Using Fig. 8.83, design a problem to help other students better understand the step response of series *RLC* circuits.

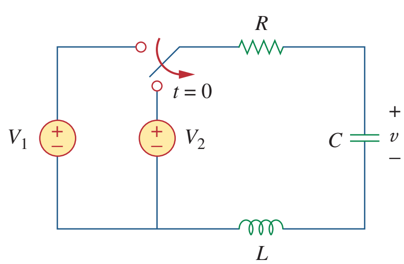


Figure 8.83
For Prob. 8.35.

8.36 Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 8.84.

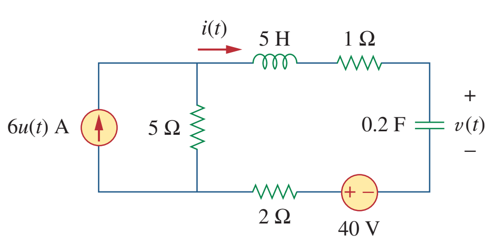


Figure 8.84
For Prob. 8.36.

***8.37** For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

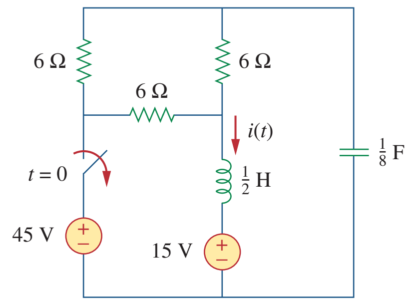


Figure 8.85
For Prob. 8.37.

8.38 Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$.

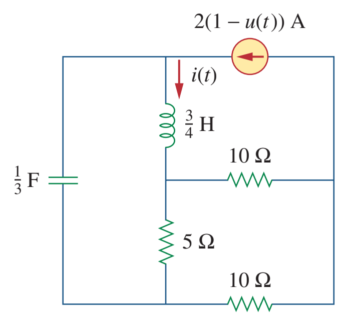


Figure 8.86
For Prob. 8.38.

8.39 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 8.87.

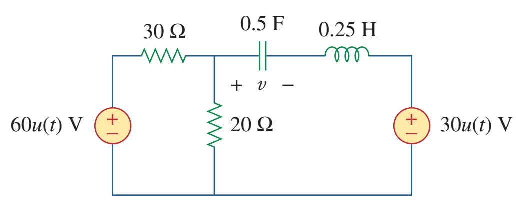


Figure 8.87
For Prob. 8.39.

8.40 The switch in the circuit of Fig. 8.88 is moved from position a to b at $t = 0$. Determine $i(t)$ for $t > 0$.

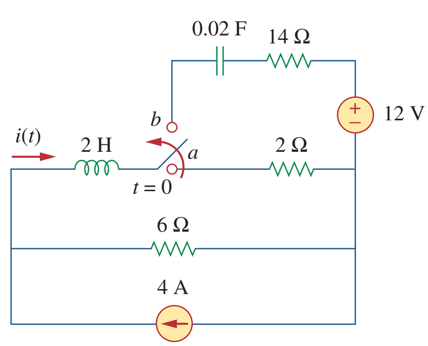


Figure 8.88
For Prob. 8.40.

***8.41** For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

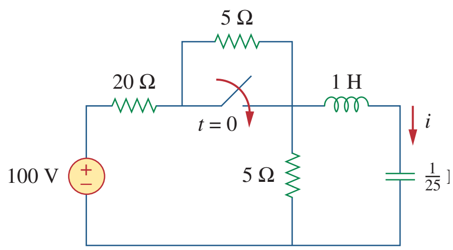


Figure 8.89
For Prob. 8.41.

***8.42** Given the network in Fig. 8.90, find $v(t)$ for $t > 0$.

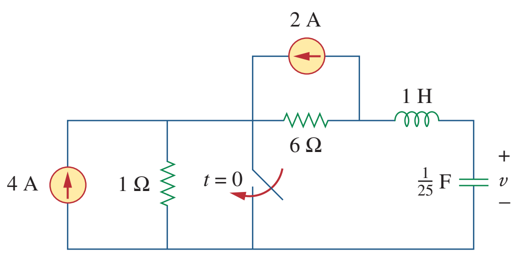


Figure 8.90
For Prob. 8.42.

8.43 The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

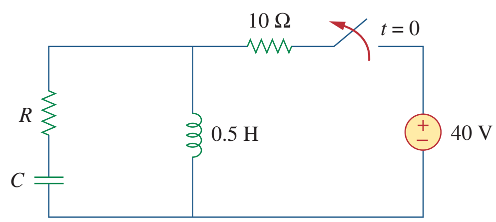


Figure 8.91
For Prob. 8.43.

8.44 A series *RLC* circuit has the following parameters: $R = 1$ kΩ, $L = 1$ H, and $C = 10$ nF. What type of damping does this circuit exhibit?

Section 8.6 Step Response of a Parallel *RLC* Circuit

8.45 In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0$ V and $i(0) = 1$ A.

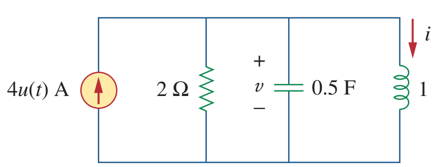


Figure 8.92
For Prob. 8.45.

8.46 Using Fig. 8.93, design a problem to help other students better understand the step response of a parallel *RLC* circuit.

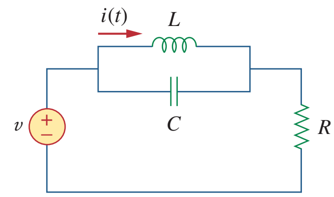


Figure 8.93
For Prob. 8.46.

8.47 Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

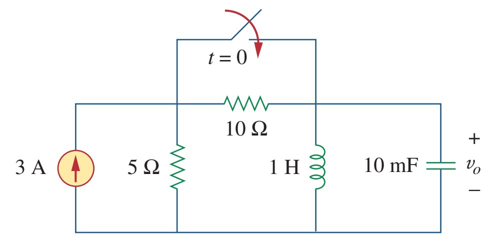


Figure 8.94
For Prob. 8.47.

8.48 Given the circuit in Fig. 8.95, find $i(t)$ and $v(t)$ for $t > 0$.

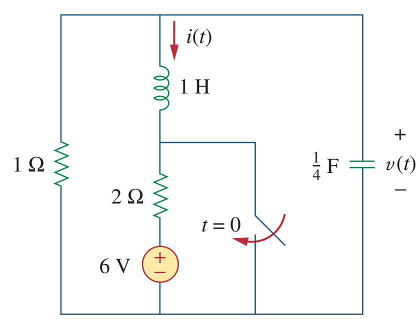


Figure 8.95
For Prob. 8.48.

8.49 Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

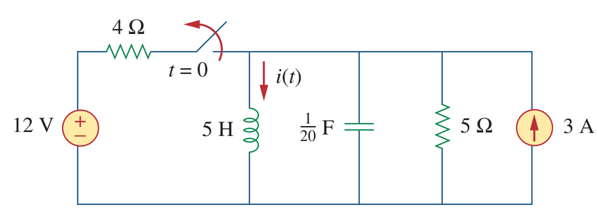


Figure 8.96
For Prob. 8.49.

8.50 For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

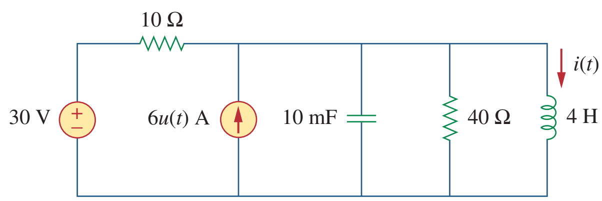


Figure 8.97
For Prob. 8.50.

8.51 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.98.

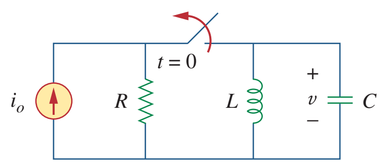


Figure 8.98
For Prob. 8.51.

8.52 The step response of a parallel *RLC* circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2\sin 400t)\ \text{V}, \qquad t \geq 0$$

when the inductor is 50 mH. Find R and C.

Section 8.7 General Second-Order Circuits

8.53 After being open for a day, the switch in the circuit of Fig. 8.99 is closed at $t = 0$. Find the differential equation describing $i(t)$, $t > 0$.

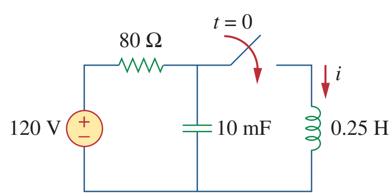


Figure 8.99
For Prob. 8.53.

8.54 Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

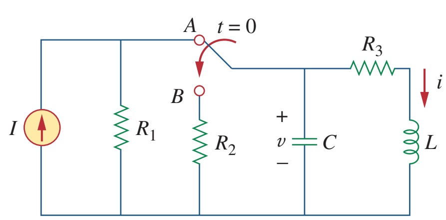


Figure 8.100
For Prob. 8.54.

8.55 For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.

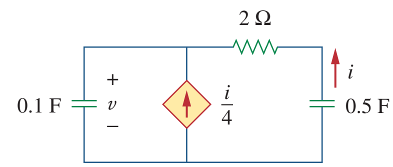


Figure 8.101
For Prob. 8.55.

8.56 In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

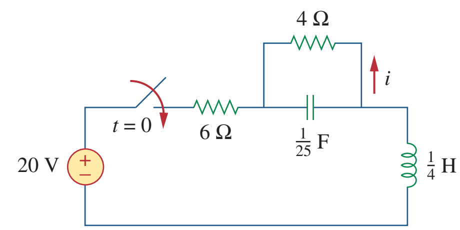


Figure 8.102
For Prob. 8.56.

8.57 If the switch in Fig. 8.103 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine:

(a) the characteristic equation of the circuit,

(b) i_x and v_R for $t > 0$.

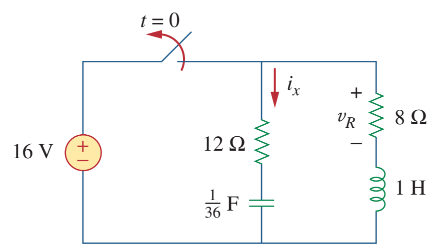


Figure 8.103
For Prob. 8.57.

8.58 In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$,

(b) $v(t)$ for $t \geq 0$.

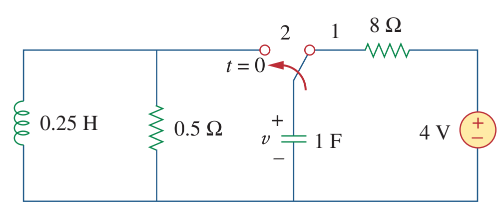


Figure 8.104
For Prob. 8.58.

8.59 The switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at $t = 0$. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

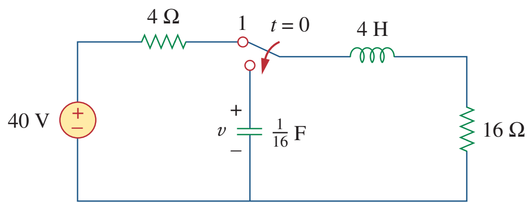


Figure 8.105
For Prob. 8.59.

8.60 Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

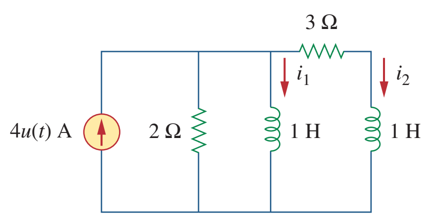


Figure 8.106
For Prob. 8.60.

8.61 For the circuit in Prob. 8.5, find i and v for $t > 0$.

8.62 Find the response $v_R(t)$ for $t > 0$ in the circuit of Fig. 8.107. Let $R = 3\ \Omega$, $L = 2$ H, and $C = 1/18$ F.

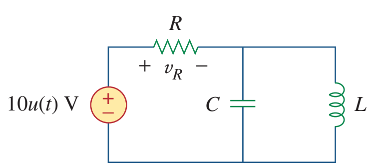


Figure 8.107
For Prob. 8.62.

Section 8.8 Second-Order Op Amp Circuits

8.63 For the op amp circuit in Fig. 8.108, find the differential equation for $i(t)$.

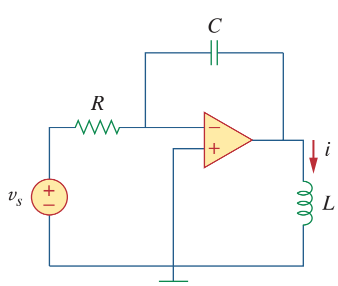


Figure 8.108
For Prob. 8.63.

8.64 Using Fig. 8.109, design a problem to help other students better understand second-order op amp circuits.

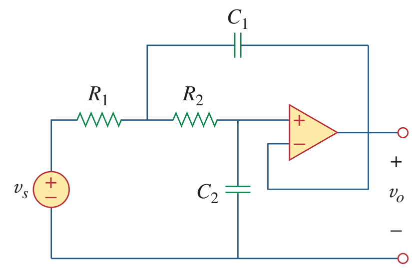


Figure 8.109
For Prob. 8.64.

8.65 Determine the differential equation for the op amp circuit in Fig. 8.110. If $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V, find v_o for $t > 0$. Let $R = 100\ \text{k}\Omega$ and $C = 1\ \mu$F.

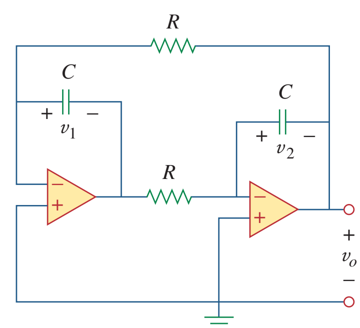


Figure 8.110
For Prob. 8.65.

8.66 Obtain the differential equations for $v_o(t)$ in the op amp circuit of Fig. 8.111.

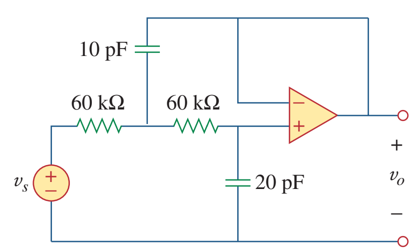


Figure 8.111
For Prob. 8.66.

***8.67** In the op amp circuit of Fig. 8.112, determine $v_o(t)$ for $t > 0$. Let $v_{in} = u(t)$ V, $R_1 = R_2 = 10$ kΩ, $C_1 = C_2 = 100$ μF.

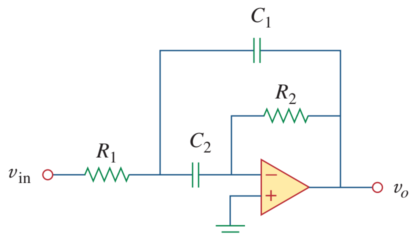


Figure 8.112
For Prob. 8.67.

Section 8.9 *PSpice* Analysis of *RLC* Circuit

8.68 For the step function $v_s = u(t)$, use *PSpice* or *MultiSim* to find the response $v(t)$ for $0 < t < 6$ s in the circuit of Fig. 8.113.

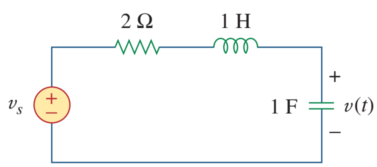


Figure 8.113
For Prob. 8.68.

8.69 Given the source-free circuit in Fig. 8.114, use *PSpice* or *MultiSim* to get $i(t)$ for $0 < t < 20$ s. Take $v(0) = 30$ V and $i(0) = 2$ A.

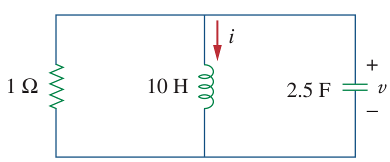


Figure 8.114
For Prob. 8.69.

8.70 For the circuit in Fig. 8.115, use *PSpice* or *MultiSim* to obtain $v(t)$ for $0 < t < 4$ s. Assume that the capacitor voltage and inductor current at $t = 0$ are both zero.

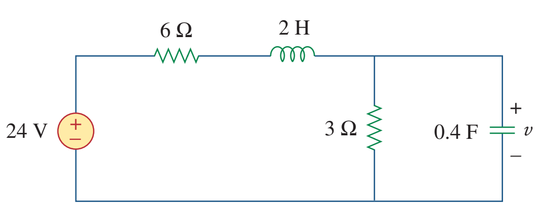


Figure 8.115
For Prob. 8.70.

8.71 Obtain $v(t)$ for $0 < t < 4$ s in the circuit of Fig. 8.116 using *PSpice* or *MultiSim*.

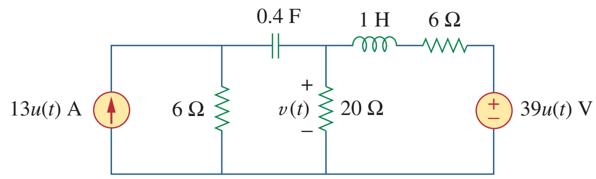


Figure 8.116
For Prob. 8.71.

8.72 The switch in Fig. 8.117 has been in position 1 for a long time. At $t = 0$, it is switched to position 2. Use *PSpice* or *MultiSim* to find $i(t)$ for $0 < t < 0.2$ s.

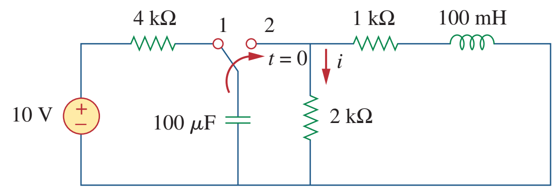


Figure 8.117
For Prob. 8.72.

8.73 Design a problem, to be solved using *PSpice* or *MultiSim,* to help other students better understand source-free *RLC* circuits.

Section 8.10 Duality

8.74 Draw the dual of the circuit shown in Fig. 8.118.

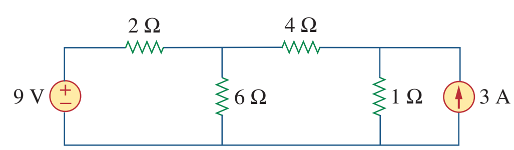


Figure 8.118
For Prob. 8.74.

8.75 Obtain the dual of the circuit in Fig. 8.119.

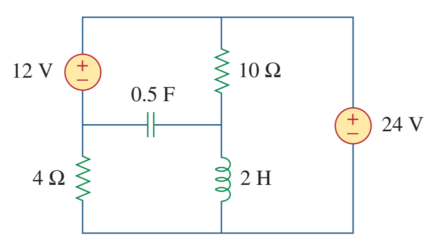


Figure 8.119
For Prob. 8.75.

8.76 Find the dual of the circuit in Fig. 8.120.

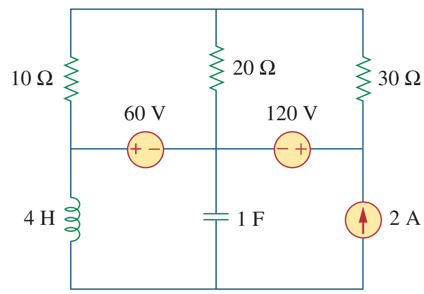


Figure 8.120
For Prob. 8.76.

8.77 Draw the dual of the circuit in Fig. 8.121.

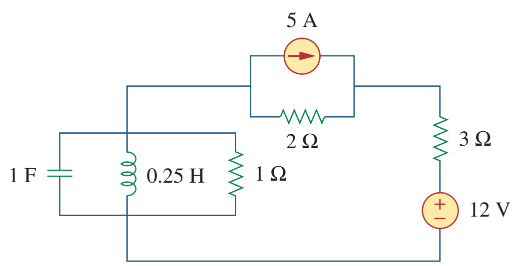


Figure 8.121
For Prob. 8.77.

Section 8.11 Applications

8.78 An automobile airbag igniter is modeled by the circuit in Fig. 8.122. Determine the time it takes the voltage across the igniter to reach its first peak after switching from A to B. Let $R = 3\ \Omega$, $C = 1/30$ F, and $L = 60$ mH.

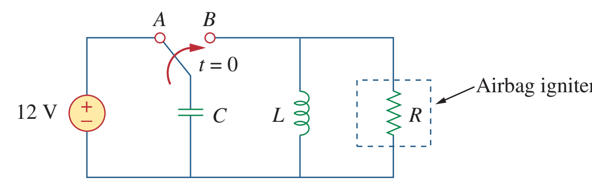


Figure 8.122
For Prob. 8.78.

8.79 A load is modeled as a 250-mH inductor in parallel with a 12-Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.

Comprehensive Problems

8.80 A mechanical system is modeled by a series RLC circuit. It is desired to produce an overdamped response with time constants 0.1 ms and 0.5 ms. If a series 50-kΩ resistor is used, find the values of L and C.

8.81 An oscillogram can be adequately modeled by a second-order system in the form of a parallel RLC circuit. It is desired to give an underdamped voltage across a 200-Ω resistor. If the damping frequency is 4 kHz and the time constant of the envelope is 0.25 s, find the necessary values of L and C.

8.82 The circuit in Fig. 8.123 is the electrical analog of body functions used in medical schools to study convulsions. The analog is as follows:

C_1 = Volume of fluid in a drug

C_2 = Volume of blood stream in a specified region

R_1 = Resistance in the passage of the drug from the input to the blood stream

R_2 = Resistance of the excretion mechanism, such as kidney, etc.

v_0 = Initial concentration of the drug dosage

$v(t)$ = Percentage of the drug in the blood stream

Find $v(t)$ for $t > 0$ given that $C_1 = 0.5\ \mu$F, $C_2 = 5\ \mu$F, $R_1 = 5$ MΩ, $R_2 = 2.5$ MΩ, and $v_0 = 60u(t)$ V.

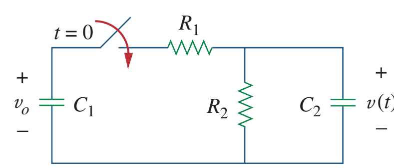


Figure 8.123
For Prob. 8.82.

8.83 Figure 8.124 shows a typical tunnel-diode oscillator circuit. The diode is modeled as a nonlinear resistor with $i_D = f(v_D)$, i.e., the diode current is a nonlinear function of the voltage across the diode. Derive the differential equation for the circuit in terms of v and i_D.

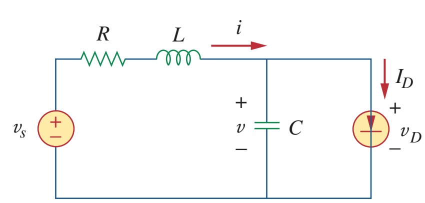


Figure 8.124
For Prob. 8.83.

16.6 If the input to a linear system is $\delta(t)$ and the output is $e^{-2t}u(t)$, the transfer function of the system is:

(a) $\dfrac{1}{s+2}$ (b) $\dfrac{1}{s-2}$ (c) $\dfrac{s}{s+2}$ (d) $\dfrac{s}{s-2}$

(e) None of the above

16.7 If the transfer function of a system is

$$H(s) = \frac{s^2 + s + 2}{s^3 + 4s^2 + 5s + 1}$$

it follows that the input is $X(s) = s^3 + 4s^2 + 5s + 1$, while the output is $Y(s) = s^2 + s + 2$.

(a) True (b) False

16.8 A network has its transfer function as

$$H(s) = \frac{s+1}{(s-2)(s+3)}$$

The network is stable.

(a) True (b) False

16.9 Which of the following equations is called the state equation?

(a) $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bz}$

(b) $\mathbf{y} = \mathbf{Cx} + \mathbf{Dz}$

(c) $\mathbf{H}(s) = \mathbf{Y}(s)/\mathbf{Z}(s)$

(d) $\mathbf{H}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$

16.10 A single-input, single-output system is described by the state model as:

$$\dot{x}_1 = 2x_1 - x_2 + 3z$$
$$\dot{x}_2 = -4x_2 - z$$
$$y = 3x_1 - 2x_2 + z$$

Which of the following matrices is incorrect?

(a) $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}$ (b) $\mathbf{B} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

(c) $\mathbf{C} = [3 \quad -2]$ (d) $\mathbf{D} = \mathbf{0}$

Answers: 16.1b, 16.2d, 16.3c, 16.4b, 16.5b, 16.6a, 16.7b, 16.8b, 16.9a, 16.10d.

Problems

Sections 16.2 and 16.3 Circuit Element Models and Circuit Analysis

16.1 The current in an *RLC* circuit is described by

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0$$

If $i(0) = 2$ and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.

16.2 The differential equation that describes the voltage in an *RLC* network is

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 5$, obtain $v(t)$.

16.3 The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 0$$

for which the initial conditions are $v(0) = 20$ V and $dv(0)/dt = 0$. Solve for $v(t)$.

16.4 If $R = 20\Omega$, $L = 0.6$ H, what value of C will make an *RLC* series circuit:

(a) overdamped?

(b) critically damped?

(c) underdamped?

16.5 The responses of a series *RLC* circuit are

$$v_c(t) = [30 - 10e^{-20t} + 30e^{-10t}]u(t)\text{V}$$
$$i_L(t) = [40e^{-20t} - 60e^{-10t}]u(t)\text{mA}$$

where $v_C(t)$ and $i_L(t)$ are the capacitor voltage and inductor current, respectively. Determine the values of *R, L,* and *C*.

16.6 e2d Design a parallel *RLC* circuit that has the characteristic equation

$$s^2 + 100s + 10^6 = 0.$$

16.7 The step response of an *RLC* circuit is given by

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 6$ and $di(0)/dt = 12$, solve for $i(t)$.

16.8 A branch voltage in an *RLC* circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 48$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

16.9 A series *RLC* circuit is described by

$$L\frac{d^2i(t)}{dt} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 2$$

Find the response when $L = 0.5H$, $R = 4\Omega$, and $C = 0.2\ F$. Let $i(0^-) = 1$ A and $[di(0^-)/dt] = 0$.

16.10 The step responses of a series *RLC* circuit are

$$V_c = 40 - 10e^{-2000t} - 10e^{-4000t}\,\text{V},\ t > 0$$

$$i_L(t) = 3e^{-2000t} + 6e^{-4000t}\,mA,\ \ t > 0$$

(a) Find C.
(b) Determine what type of damping is exhibited by the circuit.

16.11 The step response of a parallel RLC circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2\sin 400t)\text{V},\ \ t \geq 0$$

when the inductor is 50 mH. Find R and C.

16.12 Determine $i(t)$ in the circuit of Fig. 16.35 by means of the Laplace transform.

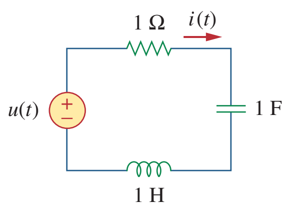


Figure 16.35
For Prob. 16.12.

16.13 Using Fig. 16.36, design a problem to help other students better understand circuit analysis using Laplace transforms.

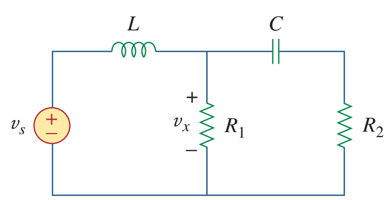


Figure 16.36
For Prob. 16.13.

16.14 Find $i(t)$ for $t > 0$ for the circuit in Fig. 16.37. Assume $i_s(t) = [4u(t) + 2\delta(t)]$mA.

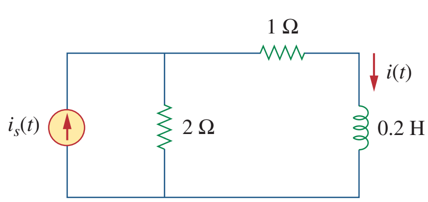


Figure 16.37
For Prob. 16.14.

16.15 For the circuit in Fig. 16.38, calculate the value of R needed to have a critically damped response.

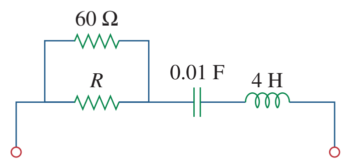


Figure 16.38
For Prob. 16.15.

16.16 The capacitor in the circuit of Fig. 16.39 is initially uncharged. Find $v_0(t)$ for $t > 0$.

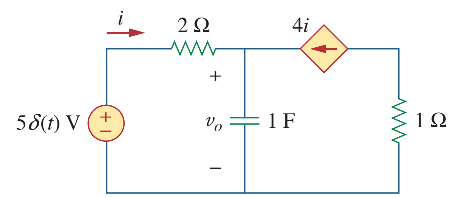


Figure 16.39
For Prob. 16.16.

16.17 If $i_s(t) = e^{-2t}u(t)$ A in the circuit shown in Fig. 16.40, find the value of $i_o(t)$.

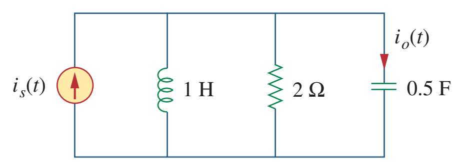


Figure 16.40
For Prob. 16.17.

16.18 Find $v(t)$, $t > 0$ in the circuit of Fig. 16.41. Let $v_s = 20$ V.

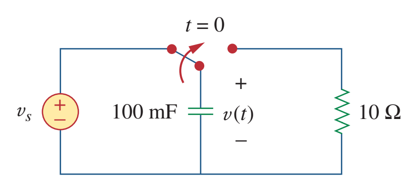


Figure 16.41
For Prob. 16.18.

16.19 The switch in Fig. 16.42 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Find $v(t)$ for $t > 0$.

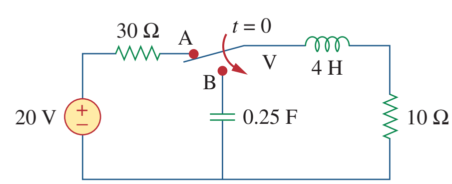


Figure 16.42
For Prob. 16.19.

16.20 Find $i(t)$ for $t > 0$ in the circuit of Fig. 16.43.

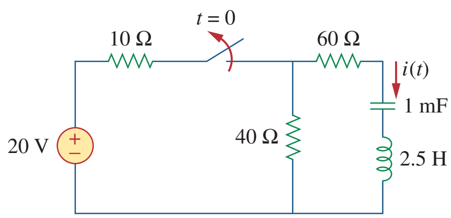


Figure 16.43
For Prob. 16.20.

16.21 In the circuit of Fig. 16.44, the switch moves (make before break switch) from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.

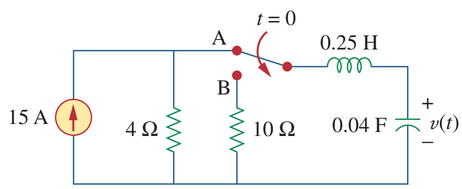


Figure 16.44
For Prob. 16.21.

16.22 Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 16.45. Assume steady-state conditions exist at $t = 0^-$.

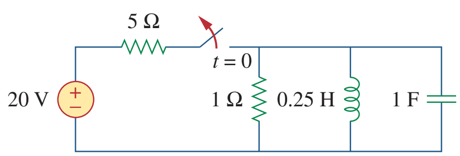


Figure 16.45
For Prob. 16.22.

16.23 Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 16.46.

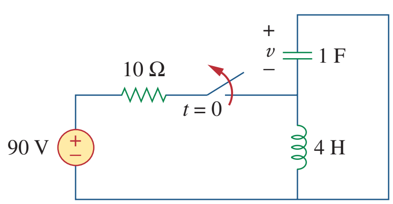


Figure 16.46
For Prob. 16.23.

16.24 The switch in the circuit of Fig. 16.47 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

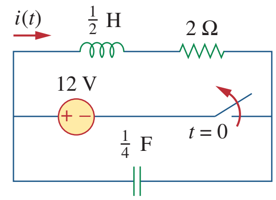


Figure 16.47
For Prob. 16.24.

16.25 Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 16.48.

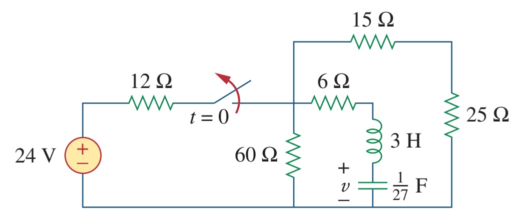


Figure 16.48
For Prob. 16.25.

16.26 The switch in Fig. 16.49 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Determine $i(t)$ for $t > 0$. Also assume that the initial voltage on the capacitor is zero.

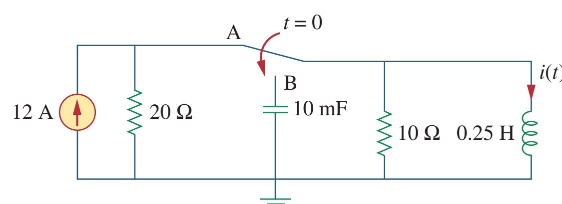


Figure 16.49
For Prob. 16.26.

16.27 Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.50.

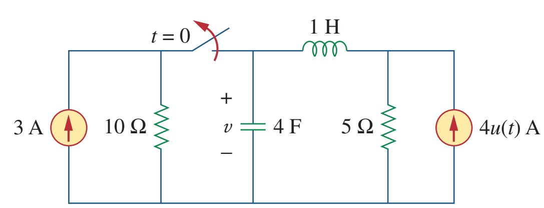


Figure 16.50
For Prob. 16.27.

16.28 For the circuit in Fig. 16.51, find $v(t)$ for $t > 0$.

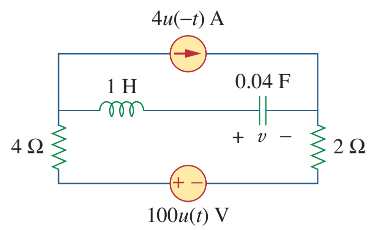


Figure 16.51
For Prob. 16.28.

16.29 Calculate $i(t)$ for $t > 0$ in the circuit in Fig. 16.52.

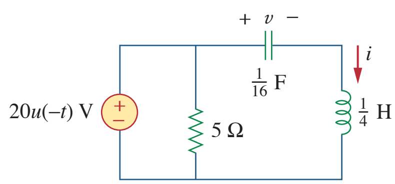


Figure 16.52
For Prob. 16.29.

16.30 Find $v_o(t)$, for all $t > 0$, in the circuit of Fig. 16.53.

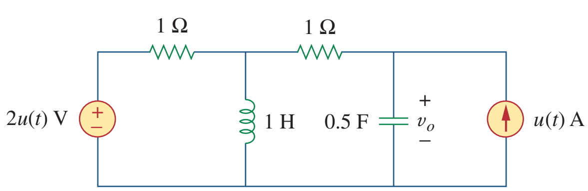


Figure 16.53
For Prob. 16.30.

16.31 Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit in Fig. 16.54.

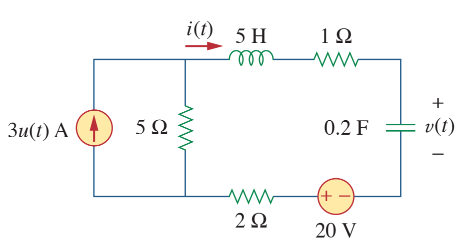


Figure 16.54
For Prob. 16.31.

16.32 For the network in Fig. 16.55, solve for $i(t)$ for $t > 0$.

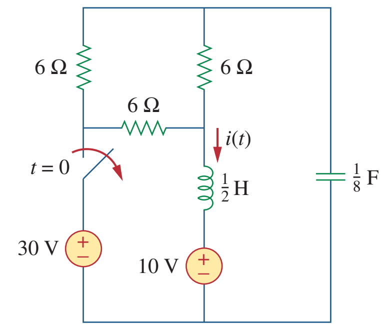


Figure 16.55
For Prob. 16.32.

16.33 Using Fig. 16.56, design a problem to help other students understand how to use Thevenin's theorem (in the s-domain) to aid in circuit analysis.

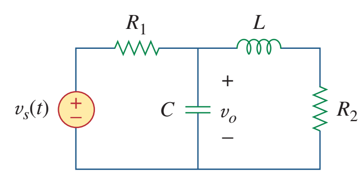


Figure 16.56
For Prob. 16.33.

16.34 Solve for the mesh currents in the circuit of Fig. 16.57. You may leave your results in the s-domain.

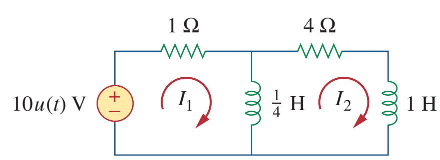


Figure 16.57
For Prob. 16.34.

16.35 Find $v_o(t)$ in the circuit of Fig. 16.58.

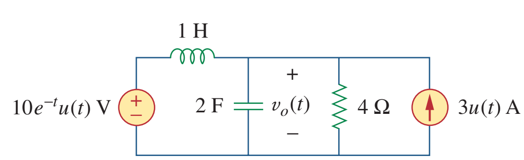


Figure 16.58
For Prob. 16.35.

16.36 Refer to the circuit in Fig. 16.59. Calculate $i(t)$ for $t > 0$.

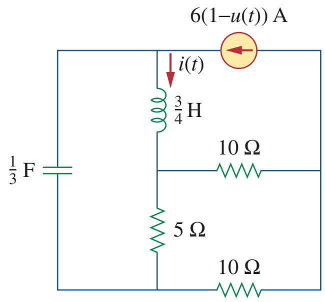


Figure 16.59
For Prob. 16.36.

16.37 Determine v for $t > 0$ in the circuit in Fig. 16.60.

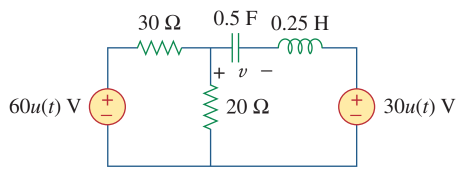


Figure 16.60
For Prob. 16.37.

16.38 The switch in the circuit of Fig. 16.61 is moved from position a to b (a make before break switch) at $t = 0$. Determine $i(t)$ for $t > 0$.

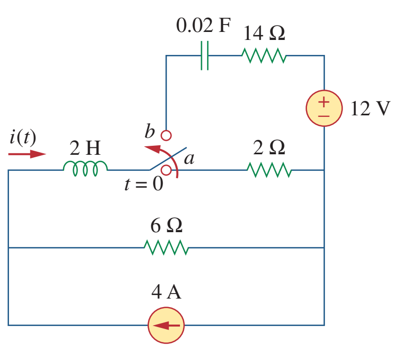


Figure 16.61
For Prob. 16.38.

16.39 For the network in Fig. 16.62, find $i(t)$ for $t > 0$.

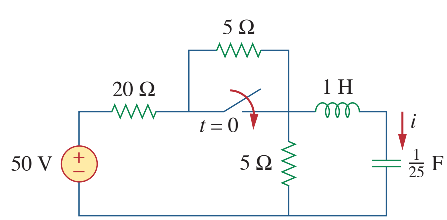


Figure 16.62
For Prob. 16.39.

16.40 In the circuit of Fig. 16.63, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0$ V and $i(0) = 1$ A.

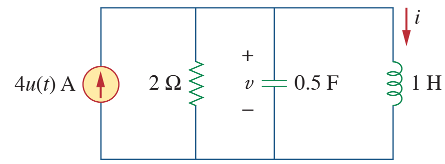


Figure 16.63
For Prob. 16.40.

16.41 Find the output voltage $v_o(t)$ in the circuit of Fig. 16.64.

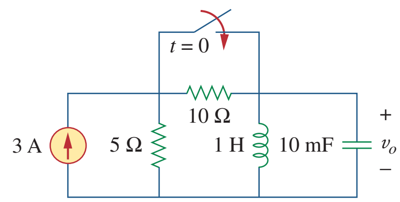


Figure 16.64
For Prob. 16.41.

16.42 Given the circuit in Fig. 16.65, find $i(t)$ and $v(t)$ for $t > 0$.

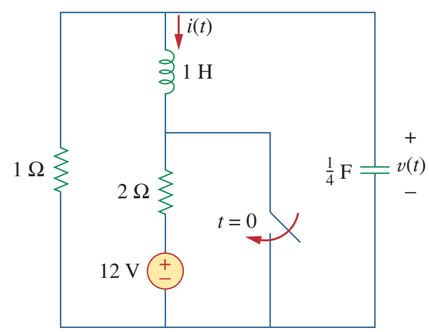


Figure 16.65
For Prob. 16.42.

16.43 Determine $i(t)$ for $t > 0$ in the circuit of Fig. 16.66.

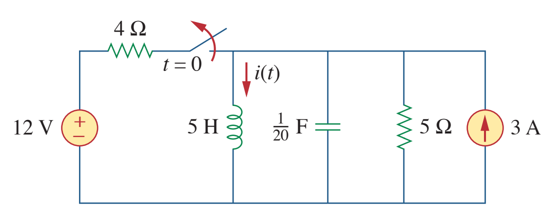


Figure 16.66
For Prob. 16.43.

16.44 For the circuit in Fig. 16.67, find $i(t)$ for $t > 0$.

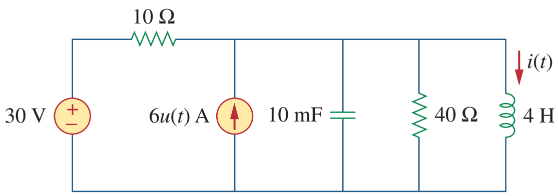


Figure 16.67
For Prob. 16.44.

16.45 Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.68.

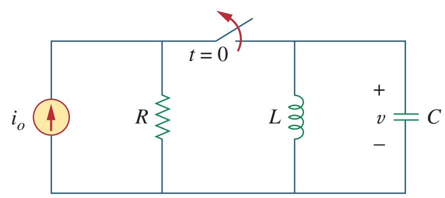


Figure 16.68
For Prob. 16.45.

16.46 Determine $i_o(t)$ in the circuit in Fig. 16.69.

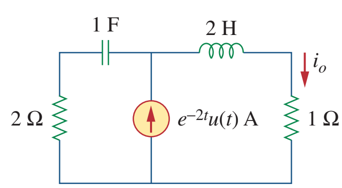


Figure 16.69
For Prob. 16.46.

16.47 Determine $i_o(t)$ in the network shown in Fig. 16.70.

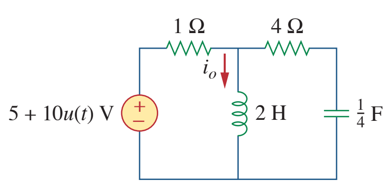


Figure 16.70
For Prob. 16.47.

16.48 Find $V_x(s)$ in the circuit shown in Fig. 16.71.

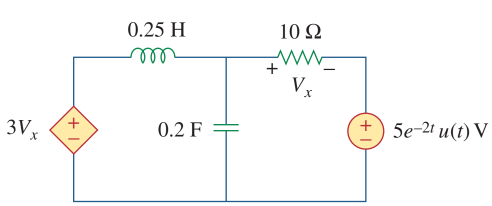


Figure 16.71
For Prob. 16.48.

16.49 Find $i_0(t)$ for $t > 0$ in the circuit in Fig. 16.72.

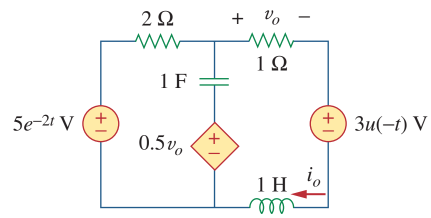


Figure 16.72
For Prob. 16.49.

16.50 For the circuit in Fig. 16.73, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.

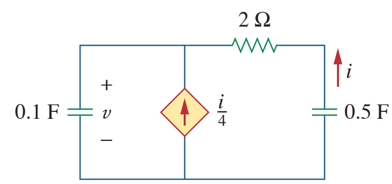


Figure 16.73
For Prob. 16.50.

16.51 In the circuit of Fig. 16.74, find $i(t)$ for $t > 0$.

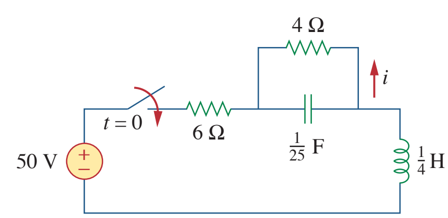


Figure 16.74
For Prob. 16.51.

16.52 If the switch in Fig. 16.75 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine i_x and v_R for $t > 0$.

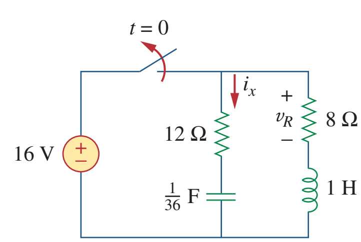


Figure 16.75
For Prob. 16.52.

16.53 In the circuit of Fig. 16.76, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$

(b) $v(t)$ for $t \geq 0$.

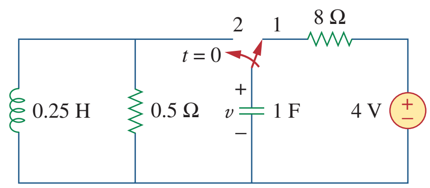


Figure 16.76
For Prob. 16.53.

16.54 The switch in Fig. 16.77 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at $t = 0$. Please note that the switch is a make before break switch; it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine *v(t)*.

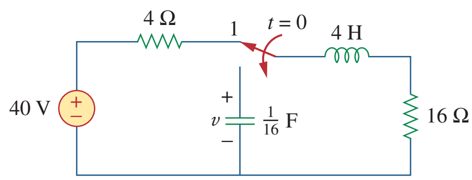


Figure 16.77
For Prob. 16.54.

16.55 Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 16.78.

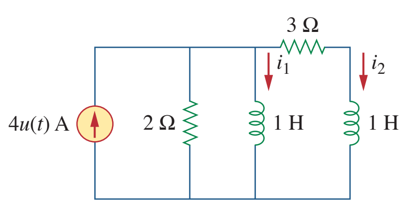


Figure 16.78
For Prob. 16.55.

16.56 Calculate $i_o(t)$ for $t > 0$ in the network of Fig. 16.79.

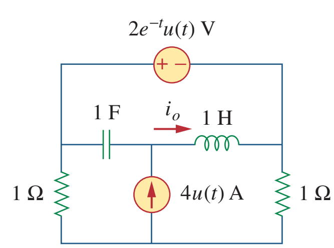


Figure 16.79
For Prob. 16.56.

16.57 (a) Find the Laplace transform of the voltage shown in Fig. 16.80(a). (b) Using that value of $v_s(t)$ in the circuit shown in Fig. 16.80(b), find the value of $v_o(t)$.

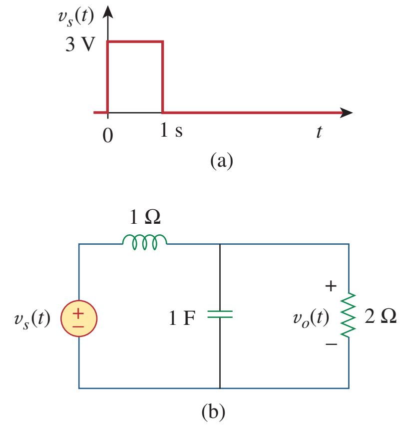


Figure 16.80
For Prob. 16.57.

16.58 Using Fig. 16.81, design a problem to help other students better understand circuit analysis in the *s*-domain with circuits that have dependent sources.

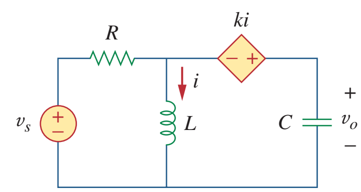


Figure 16.81
For Prob. 16.58.

16.59 Find $v_o(t)$ in the circuit of Fig. 16.82 if $v_x(0) = 2$ V and $i(0) = 1$ A.

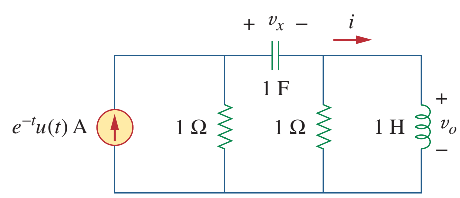


Figure 16.82
For Prob. 16.59.

16.60 Find the response $v_R(t)$ for $t > 0$ in the circuit in Fig. 16.83. Let $R = 3\ \Omega$, $L = 2$ H, and $C = 1/18$ F.

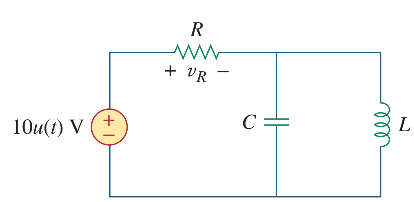


Figure 16.83
For Prob. 16.60.

***16.61** Find the voltage $v_o(t)$ in the circuit of Fig. 16.84 by means of the Laplace transform.

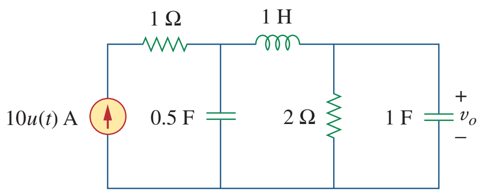


Figure 16.84
For Prob. 16.61.

16.62 Using Fig. 16.85, design a problem to help other students better understand solving for node voltages by working in the s-domain.

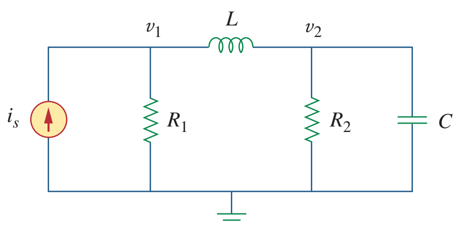


Figure 16.85
For Prob. 16.62.

16.63 Consider the parallel RLC circuit of Fig. 16.86. Find $v(t)$ and $i(t)$ given that $v(0) = 5$ and $i(0) = -2$ A.

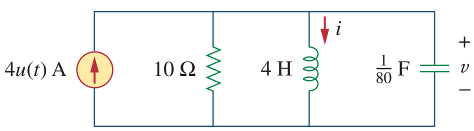


Figure 16.86
For Prob. 16.63.

16.64 The switch in Fig. 16.87 moves from position 1 to position 2 at $t = 0$. Find $v(t)$, for all $t > 0$.

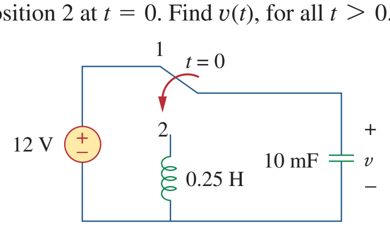


Figure 16.87
For Prob. 16.64.

16.65 For the RLC circuit shown in Fig. 16.88, find the complete response if $v(0) = 2$ V when the switch is closed.

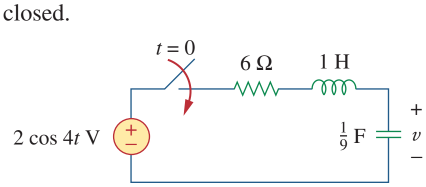


Figure 16.88
For Prob. 16.65.

* An asterisk indicates a challenging problem.

16.66 For the op amp circuit in Fig. 16.89, find $v_0(t)$ for $t > 0$. Take $v_s = 3\,e^{-5t}u(t)$ V.

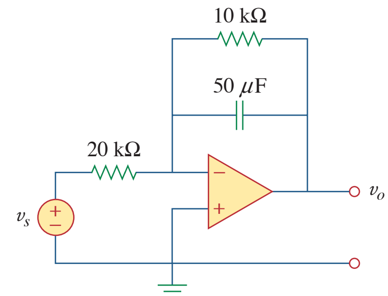


Figure 16.89
For Prob. 16.66.

16.67 Given the op amp circuit in Fig. 16.90, if $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V, find v_0 for $t > 0$. Let $R = 100\text{ k}\Omega$ and $C = 1\ \mu$F.

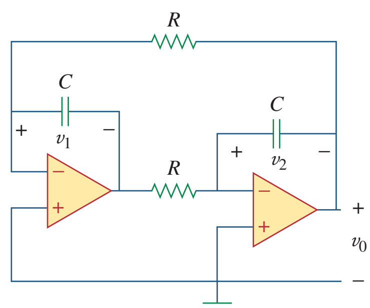


Figure 16.90
For Prob. 16.67.

16.68 Obtain V_0/V_s in the op amp circuit in Fig. 16.91.

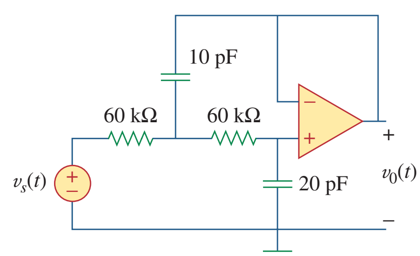


Figure 16.91
For Prob. 16.68.

16.69 Find $I_1(s)$ and $I_2(s)$ in the circuit of Fig. 16.92.

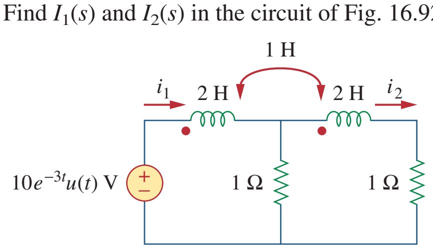


Figure 16.92
For Prob. 16.69.

16.70 Using Fig. 16.93, design a problem to help other students better understand how to do circuit analysis with circuits that have mutually coupled elements by working in the s-domain.

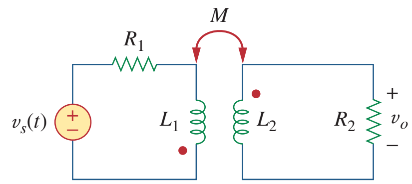


Figure 16.93
For Prob. 16.70.

16.71 For the ideal transformer circuit in Fig. 16.94, determine $i_o(t)$.

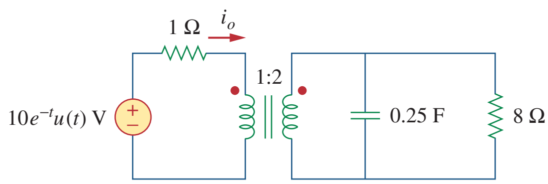


Figure 16.94
For Prob. 16.71.

Section 16.4 Transfer Functions

16.72 The transfer function of a system is

$$H(s) = \frac{s^2}{3s + 1}$$

Find the output when the system has an input of $4e^{-t/3}u(t)$.

16.73 When the input to a system is a unit step function, the response is $10 \cos 2t u(t)$. Obtain the transfer function of the system.

16.74 Design a problem to help other students better understand how to find outputs when given a transfer function and an input.

16.75 When a unit step is applied to a system at $t = 0$, its response is

$$y(t) = \left[4 + \frac{1}{2}e^{-3t} - e^{-2t}(2 \cos 4t + 3 \sin 4t)\right]u(t)$$

What is the transfer function of the system?

16.76 For the circuit in Fig. 16.95, find $H(s) = V_o(s)/V_s(s)$. Assume zero initial conditions.

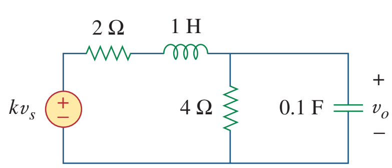


Figure 16.95
For Prob. 16.76.

16.77 Obtain the transfer function $H(s) = V_o/V_s$ for the circuit of Fig. 16.96.

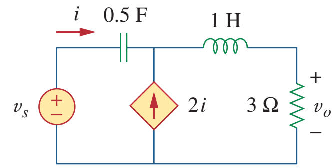


Figure 16.96
For Prob. 16.77.

16.78 The transfer function of a certain circuit is

$$H(s) = \frac{5}{s + 1} - \frac{3}{s + 2} + \frac{6}{s + 4}$$

Find the impulse response of the circuit.

16.79 For the circuit in Fig. 16.97, find:

(a) I_1/V_s (b) I_2/V_x

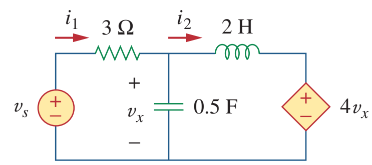


Figure 16.97
For Prob. 16.79.

16.80 Refer to the network in Fig. 16.98. Find the following transfer functions:

(a) $H_1(s) = V_o(s)/V_s(s)$
(b) $H_2(s) = V_o(s)/I_s(s)$
(c) $H_3(s) = I_o(s)/I_s(s)$
(d) $H_4(s) = I_o(s)/V_s(s)$

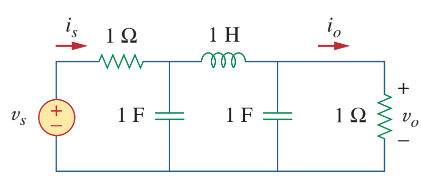


Figure 16.98
For Prob. 16.80.

16.81 For the op-amp circuit in Fig. 16.99, find the transfer function, $T(s) = I(s)/V_s(s)$. Assume all initial conditions are zero.

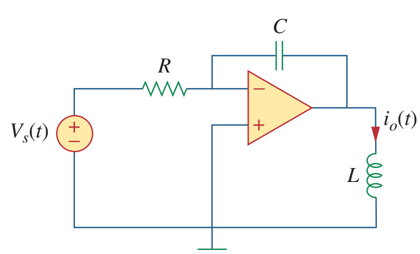


Figure 16.99
For Prob. 16.81.

16.82 Calculate the gain $H(s) = V_o/V_s$ in the op amp circuit of Fig. 16.100.

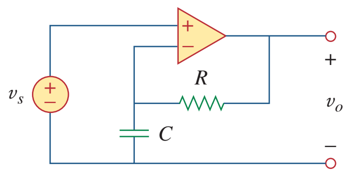


Figure 16.100
For Prob. 16.82.

16.83 Refer to the RL circuit in Fig. 16.101. Find:

(a) the impulse response $h(t)$ of the circuit.

(b) the unit step response of the circuit.

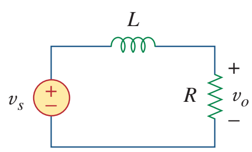


Figure 16.101
For Prob. 16.83.

16.84 A parallel RL circuit has $R = 4\ \Omega$ and $L = 1$ H. The input to the circuit is $i_s(t) = 2e^{-t}u(t)$ A. Find the inductor current $i_L(t)$ for all $t > 0$ and assume that $i_L(0) = -2$ A.

16.85 A circuit has a transfer function

$$H(s) = \frac{s+4}{(s+1)(s+2)^2}$$

Find the impulse response.

Section 16.5 State Variables

16.86 Develop the state equations for Prob. 16.12.

16.87 Develop the state equations for the problem you designed in Prob. 16.13.

16.88 Develop the state equations for the circuit shown in Fig. 16.102.

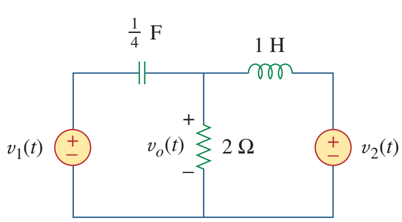


Figure 16.102
For Prob. 16.88.

16.89 Develop the state equations for the circuit shown in Fig. 16.103.

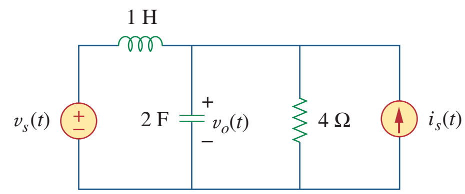


Figure 16.103
For Prob. 16.89.

16.90 Develop the state equations for the circuit shown in Fig. 16.104.

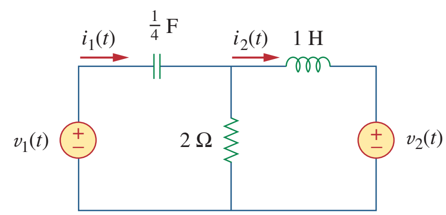


Figure 16.104
For Prob. 16.90.

16.91 Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{6\,dy(t)}{dt} + 7y(t) = z(t)$$

***16.92** Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{7\,dy(t)}{dt} + 9y(t) = \frac{dz(t)}{dt} + z(t)$$

***16.93** Develop the state equations for the following differential equation.

$$\frac{d^3y(t)}{dt^3} + \frac{6\,d^2y(t)}{dt^2} + \frac{11\,dy(t)}{dt} + 6y(t) = z(t)$$

***16.94** Given the following state equation, solve for $y(t)$:

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$\mathbf{y}(t) = [1 \quad 0]x$$

***16.95** Given the following state equation, solve for $y_1(t)$ and $y_2(t)$.

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

Section 16.6 Applications

16.96 Show that the parallel *RLC* circuit shown in Fig. 16.105 is stable.

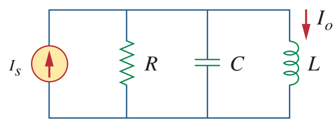


Figure 16.105
For Prob. 16.96.

16.97 A system is formed by cascading two systems as shown in Fig. 16.106. Given that the impulse responses of the systems are

$$h_1(t) = 3e^{-t}u(t), \qquad h_2(t) = e^{-4t}u(t)$$

(a) Obtain the impulse response of the overall system.

(b) Check if the overall system is stable.

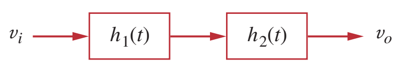


Figure 16.106
For Prob. 16.97.

16.98 Determine whether the op amp circuit in Fig. 16.107 is stable.

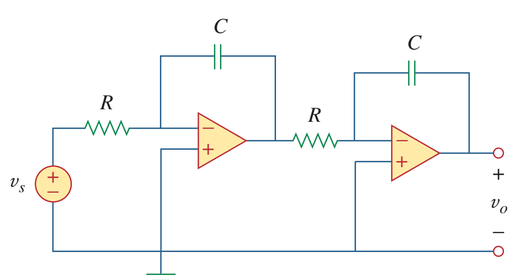


Figure 16.107
For Prob. 16.98.

16.99 It is desired to realize the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{2s}{s^2 + 2s + 6}$$

using the circuit in Fig. 16.108. Choose $R = 1\ \text{k}\Omega$ and find L and C.

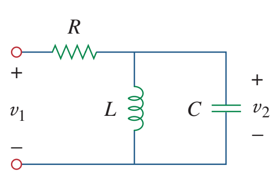


Figure 16.108
For Prob. 16.99.

16.100 Design an op amp circuit, using Fig. 16.109, that will realize the following transfer function:

$$\frac{V_o(s)}{V_i(s)} = -\frac{s + 1000}{2(s + 4000)}$$

Choose $C_1 = 10\ \mu\text{F}$; determine R_1, R_2, and C_2.

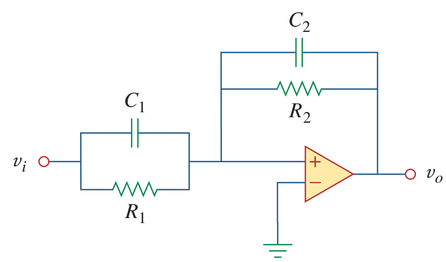


Figure 16.109
For Prob. 16.100.

16.101 Realize the transfer function

$$\frac{V_o(s)}{V_s(s)} = -\frac{s}{s + 10}$$

using the circuit in Fig. 16.110. Let $Y_1 = sC_1$, $Y_2 = 1/R_1$, $Y_3 = sC_2$. Choose $R_1 = 1\ \text{k}\Omega$ and determine C_1 and C_2.

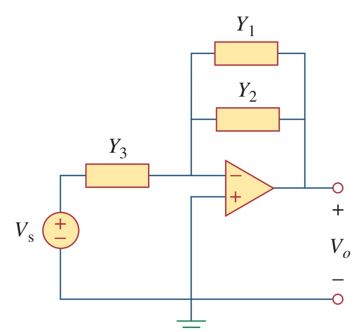


Figure 16.110
For Prob. 16.101.

16.102 Synthesize the transfer function

$$\frac{V_o(s)}{V_{in}(s)} = \frac{10^6}{s^2 + 100s + 10^6}$$

using the topology of Fig. 16.111. Let $Y_1 = 1/R_1$, $Y_2 = 1/R_2$, $Y_3 = sC_1$, $Y_4 = sC_2$. Choose $R_1 = 1\ \text{k}\Omega$ and determine C_1, C_2, and R_2.

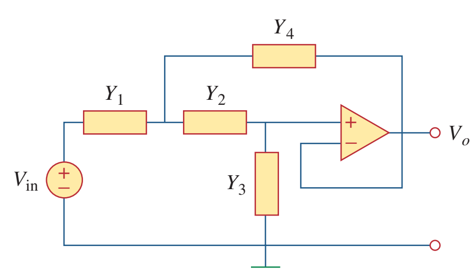


Figure 16.111
For Prob. 16.102.

Comprehensive Problems

16.103 Obtain the transfer function of the op amp circuit in Fig. 16.112 in the form of

$$\frac{V_o(s)}{V_i(s)} = \frac{as}{s^2 + bs + c}$$

where a, b, and c are constants. Determine the constants.

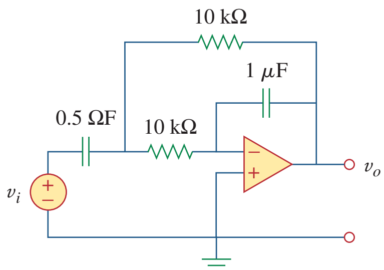


Figure 16.112
For Prob. 16.103.

16.104 A certain network has an input admittance $Y(s)$. The admittance has a pole at $s = -3$, a zero at $s = -1$, and $Y(\infty) = 0.25$ S.

(a) Find $Y(s)$.

(b) An 8-V battery is connected to the network via a switch. If the switch is closed at $t = 0$, find the current $i(t)$ through $Y(s)$ using the Laplace transform.

16.105 A gyrator is a device for simulating an inductor in a network. A basic gyrator circuit is shown in Fig. 16.113. By finding $V_i(s)/I_o(s)$, show that the inductance produced by the gyrator is $L = CR^2$.

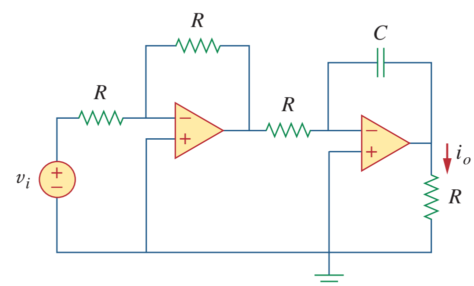


Figure 16.113
For Prob. 16.105.