EE 2010 Circuit Analysis

Module 05: Dynamic Element Models in Steady State: Capacitors and Inductors

These notes are drawn from *Alexander and Sadiku*, 2013, *O'Malley*, 2011, and other sources. They are intended to offer a summary of topics to guide you in focused studies. You should augment this handout with notes taken in class, reading textbook(s), and working additional example problems.

Definition: *Dynamic elements* are passive elements with the capability of energy storage. They are referred to as *dynamic*, as opposed to *instantaneous*, because their present state depends on the present input as well as past inputs. We presently consider the capacitor and the inductor.

Electrical Field Dynamic Element: A *capacitor* consists of two conducting plates separated by an insulator (or dielectric).

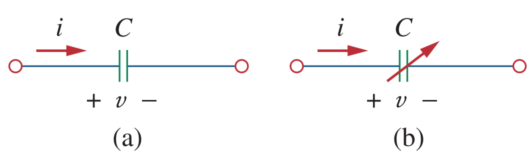


Figure 6.3
Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

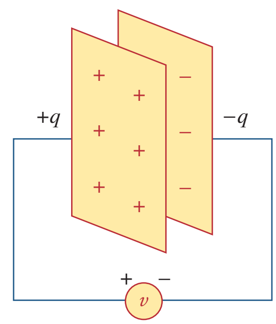


Figure 6.2
A capacitor with applied voltage v.

The voltage-current model for an (ideal) capacitor is:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

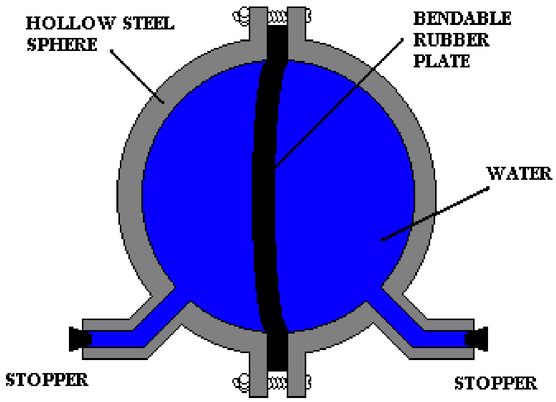
So the capacitor acts as an integrator, or "smoother."

Correspondingly, the current-voltage model is obtained by differentiating the above,

$$i(t) = C\frac{dv(t)}{dt}$$

As a dynamic element, the capacitor is capable of storing energy (in an electric field potential). The energy stored in a capacitor is

$$w(t) = \frac{1}{2}Cv^2(t)$$



CAPACITOR
(water analogy)

Water model:

Deduce series and parallel combinations.

Series and parallel.

It is always true that *impedances in series add:*

$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots$$

and that *impedances in parallel add "inversely":*

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots$$

For the capacitor, since $Z_C = \frac{1}{C \cdot s}$ (see below), the *impedances* of capacitors in series add as

$$\begin{aligned} Z_{eq} &= Z_{C1} + Z_{C2} + Z_{C3} + \cdots \\ &= \frac{1}{C1 \cdot s} + \frac{1}{C2 \cdot s} + \frac{1}{C3 \cdot s} \cdots \\ &= \frac{1}{C_{eq} \cdot s} \end{aligned}$$

where

$$\frac{1}{C_{eq}} = \frac{1}{C1} + \frac{1}{C2} + \frac{1}{C3} \cdots$$

while the *impedances* of capacitors in parallel add as

$$\begin{aligned} \frac{1}{Z_{eq}} &= \frac{1}{Z_{C1}} + \frac{1}{Z_{C2}} + \frac{1}{Z_{C3}} + \cdots \\ &= C1s + C2s + C3s + \cdots \\ &= C_{eq}s \end{aligned}$$

where

$$C_{eq} = C1 + C2 + C3 \cdots$$

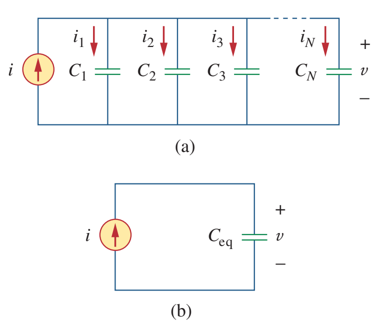


Figure 6.14
(a) Parallel-connected *N* capacitors,
(b) equivalent circuit for the parallel capacitors.

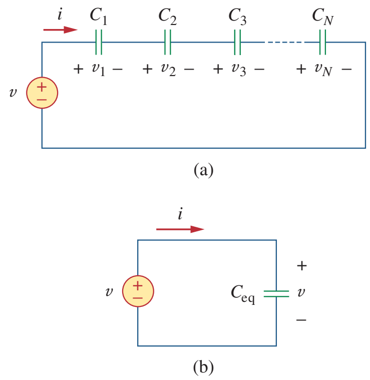


Figure 6.15
(a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.

t-domain	s-domain	f-domain
$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$	$V(s) = \frac{I(s)}{sC}$	$V(f) = \frac{I(f)}{j2\pi fC}$

Table 1: Impedance models for a capacitor of value C Farads

Note: the (s-domain) impedance model for a resistor is still Ohm's law: $V(s) = I(s)Z(s)$

Magnetic Field Dynamic Element: An *inductor* consists of a coil of conducting wire.

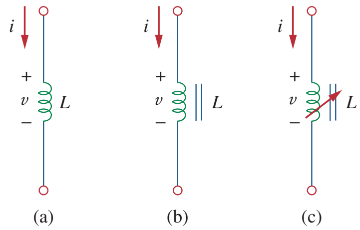


Figure 6.23
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

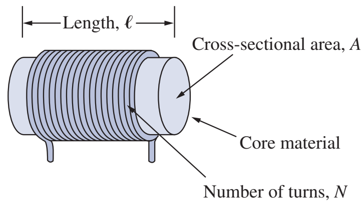


Figure 6.21
Typical form of an inductor.

The voltage-current model for an (ideal) inductor is:

$$v(t) = L\frac{di(t)}{dt}$$

so that inductors act as differentiators. Correspondingly, the current-voltage model is obtained by integrating the above,

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau$$

As a dynamic element, the inductor is capable of storing energy (in a magnetic field potential). The energy stored in an inductor is

$$w(t) = \frac{1}{2}Li^2(t)$$

Note: We will soon use the result:

$$V(s) = \frac{1}{C \cdot s}I(s)$$

which yields the impedance model for a capacitor $Z_C = \frac{1}{C \cdot s}$

Water model: 

Deduce series and parallel combinations.

Series and parallel.

Again, it is always true that *impedances in series add:*

$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots$$

and that *impedances in parallel add "inversely":*

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots$$

For inductors, we have the more intuitive result that the *impedances* of inductors in series add as

$$\begin{aligned} Z_{eq} &= Z_{L1} + Z_{L2} + Z_{L3} + \cdots \\ &= L1s + L2s + L3s + \cdots \\ &= L_{eq}s \end{aligned}$$

where

$$L_{eq} = L1 + L2 + L3 \cdots$$

while the *impedances* of inductors in parallel add as

$$\begin{aligned} \frac{1}{Z_{eq}} &= \frac{1}{Z_{L1}} + \frac{1}{Z_{L2}} + \frac{1}{Z_{L3}} + \cdots \\ &= \frac{1}{L1 \cdot s} + \frac{1}{L2 \cdot s} + \frac{1}{L3 \cdot s} \cdots \\ &= \frac{1}{L_{eq} \cdot s} \end{aligned}$$

where

$$\frac{1}{L_{eq}} = \frac{1}{L1} + \frac{1}{L2} + \frac{1}{L3} \cdots$$

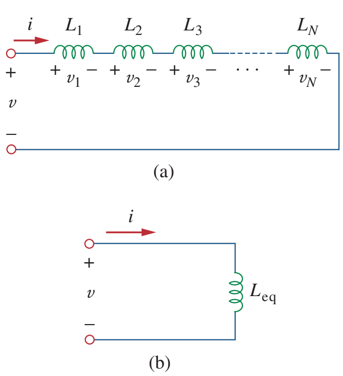


Figure 6.29
(a) A series connection of N inductors, (b) equivalent circuit for the series inductors.

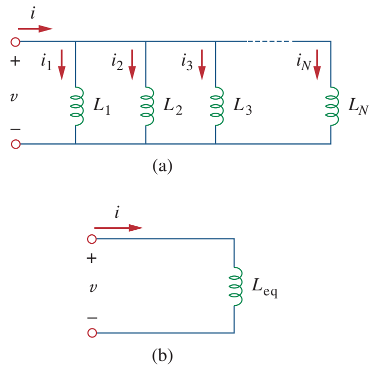


Figure 6.30
(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

t-domain	s-domain	f-domain
$v(t) = L\frac{di(t)}{dt}$	$V(s) = Ls \cdot I(s)$	$V(f) = j2\pi f L \cdot I(f)$

Table 2: Impedance models for an inductor of value L Henrys

Procedure: In our problem solving, we will use...

Steady-State Voltage-Current (Impedance) Models

t**-domain**	s**-domain**	**Steady State @ f Hz**
$v(t) = i(t) \cdot R$	$V(s) = I(s) \cdot R$	$V(f) = I(f) \cdot R$
$v(t) = \frac{1}{C} \int^t i(\tau) d\tau$	$V(s) = \frac{I(s)}{sC}$	$V(f) = \frac{I(f)}{j2\pi f C}$
$v(t) = L \frac{di(t)}{dt}$	$V(s) = Ls \cdot I(s)$	$V(f) = j2\pi f L \cdot I(f)$

Table 3: Steady-State Models for R, C, and L

And more compactly, the s-domain impedance models:

Steady-State Impedance Models

s-domain	**Steady State @ ω R/s**	**Steady State @ f Hz**
$Z_R = R$	$Z_R = R$	$Z_R = R$
$Z_C = \frac{1}{s \cdot C}$	$Z_C = \frac{1}{j\omega \cdot C}$	$Z_C = \frac{1}{j2\pi f \cdot C}$
$Z_L = s \cdot L$	$Z_L = j\omega \cdot L$	$Z_L = j2\pi f \cdot L$

Table 4: Steady-State Models for R, C, and L

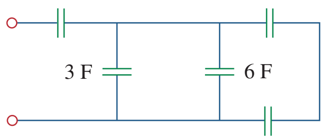
... **AND** approach each *excited* circuit by *replacing all independent sources* with symbolic values V_{in} and I_{in} in the analysis and then use the specified numerical value as the last step, i.e.,

$$\text{ans} = V_{in} F(s)|_{s=\ldots} + I_{in} G(s)|_{s=\ldots}$$

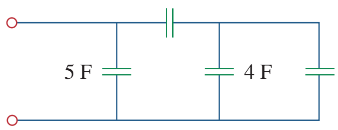
where we'll find the **Transfer Functions** $F(s)$ and $G(s)$ are ratios of polynomials in s.

An *immediate takeaway* of these results is that:
At DC ($\omega = 0$) capacitors present infinite impedance (open-circuit) and inductors present zero impedance (short-circuit).

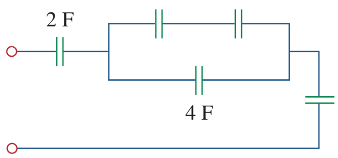
Examples:



(a)



(b)



(c)

Figure 6.51
For Prob. 6.17.

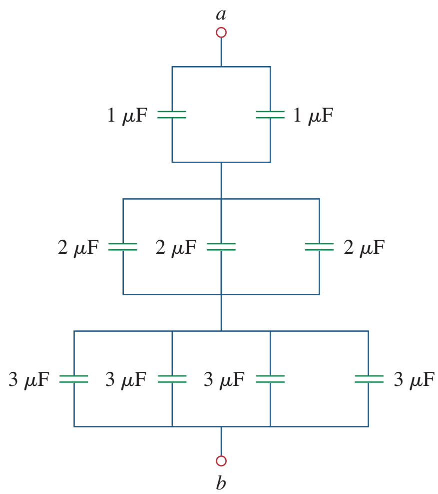


Figure 6.54
For Prob. 6.20.

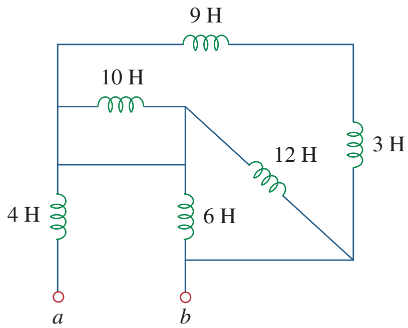
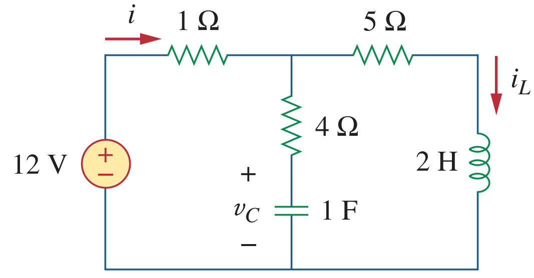


Figure 6.76
For Prob. 6.54.

This one will appear several modules down the road. Conceptually, though, there's nothing new here



(a)

– we see elements in series and elements in parallel.

So what is the equivalent impedance seen by the $12V$ source? By inspection: ...

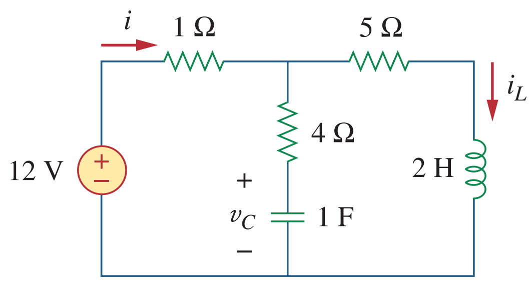
A little help for the case of a general source:

```
%% EE 2010 — Circuit Analysis
clear all
% Series parallel impedance example
% We first define a symbolic (inline) function to calculate the parallel
% combination of two s—domain impedances.
syms s z1 z2
symParllel2 = symfun(z1*z2/(z1+z2), [z1 z2]);
% Then, just reading from the circuit, we have:
Zeq(s) = 1 + symParllel2(4+1/s,5+2*s)
% or a little simpler
Zeq = simplify(Zeq)
% let's look at the steady—state impedance (s = j*2*pi*f) at f=0
double(Zeq(0*1i))
% which matches our intuition
```

which results in

```
>> SeriesParallelExample
Zeq(s) =
((2*s + 5)*(1/s + 4))/(2*s + 1/s + 9) + 1
Zeq(s) =
(10*s^2 + 31*s + 6)/(2*s^2 + 9*s + 1)
ans =
     6
```

Things get **much simpler** for a DC source (i.e., $f = 0$). Find the equivalent at steady-state DC:



(a)

Find the equivalent at steady-state DC:

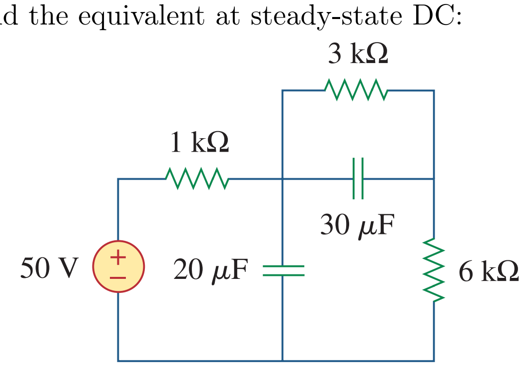


Figure 6.13

Homework: Chapter 6 # 4($v(0) = 0$), 5, 11($v(0) = 0$), 13, 17, 25, 39($i(0) = 0$), 42($i(0) = 0$), 48, 51, 59

2. The voltage across a capacitor is directly proportional to the time integral of the current through it.

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

The voltage across a capacitor cannot change instantly.
3. Capacitors in series and in parallel are combined in the same way as conductances.
4. The voltage across an inductor is directly proportional to the time rate of change of the current through it.

$$v = L\frac{di}{dt}$$

The voltage across the inductor is zero unless the current is changing. Thus, an inductor acts like a short circuit to a dc source.
5. The current through an inductor is directly proportional to the time integral of the voltage across it.

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$

The current through an inductor cannot change instantly.
6. Inductors in series and in parallel are combined in the same way resistors in series and in parallel are combined.
7. At any given time t, the energy stored in a capacitor is $\frac{1}{2}Cv^2$, while the energy stored in an inductor is $\frac{1}{2}Li^2$.
8. Three application circuits, the integrator, the differentiator, and the analog computer, can be realized using resistors, capacitors, and op amps.

Review Questions

6.1 What charge is on a 5-F capacitor when it is connected across a 120-V source?

(a) 600 C (b) 300 C
(c) 24 C (d) 12 C

6.2 Capacitance is measured in:

(a) coulombs (b) joules
(c) henrys (d) farads

6.3 When the total charge in a capacitor is doubled, the energy stored:

(a) remains the same (b) is halved
(c) is doubled (d) is quadrupled

6.4 Can the voltage waveform in Fig. 6.42 be associated with a real capacitor?

(a) Yes (b) No

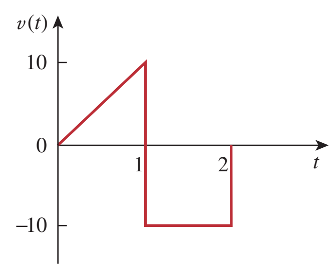


Figure 6.42
For Review Question 6.4.

6.5 The total capacitance of two 40-mF series-connected capacitors in parallel with a 4-mF capacitor is:

(a) 3.8 mF (b) 5 mF (c) 24 mF
(d) 44 mF (e) 84 mF

6.6 In Fig. 6.43, if $i = \cos 4t$ and $v = \sin 4t$, the element is:

(a) a resistor (b) a capacitor (c) an inductor

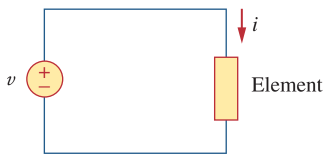


Figure 6.43
For Review Question 6.6.

6.7 A 5-H inductor changes its current by 3 A in 0.2 s. The voltage produced at the terminals of the inductor is:

(a) 75 V (b) 8.888 V
(c) 3 V (d) 1.2 V

6.8 If the current through a 10-mH inductor increases from zero to 2 A, how much energy is stored in the inductor?

(a) 40 mJ (b) 20 mJ
(c) 10 mJ (d) 5 mJ

6.9 Inductors in parallel can be combined just like resistors in parallel.

(a) True (b) False

6.10 For the circuit in Fig. 6.44, the voltage divider formula is:

(a) $v_1 = \frac{L_1 + L_2}{L_1} v_s$ (b) $v_1 = \frac{L_1 + L_2}{L_2} v_s$

(c) $v_1 = \frac{L_2}{L_1 + L_2} v_s$ (d) $v_1 = \frac{L_1}{L_1 + L_2} v_s$

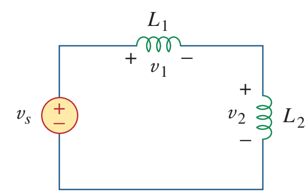


Figure 6.44
For Review Question 6.10.

Answers: 6.1a, 6.2d, 6.3d, 6.4b, 6.5c, 6.6b, 6.7a, 6.8b, 6.9a, 6.10d.

Problems

Section 6.2 Capacitors

6.1 If the voltage across a 7.5-F capacitor is $2te^{-3t}$ V, find the current and the power.

6.2 A 50-μF capacitor has energy $w(t) = 10 \cos^2 377t$ J. Determine the current through the capacitor.

6.3 Design a problem to help other students better understand how capacitors work.

6.4 A current of $4 \sin 4t$ A flows through a 5-F capacitor. Find the voltage $v(t)$ across the capacitor given that $v(0) = 1$ V.

6.5 The voltage across a 4-μF capacitor is shown in Fig. 6.45. Find the current waveform.

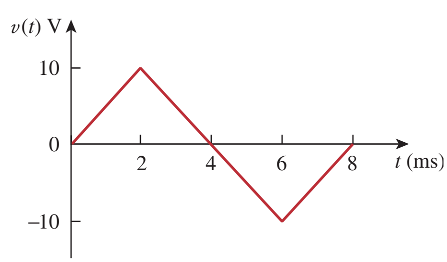


Figure 6.45
For Prob. 6.5.

6.6 The voltage waveform in Fig. 6.46 is applied across a 55-μF capacitor. Draw the current waveform through it.

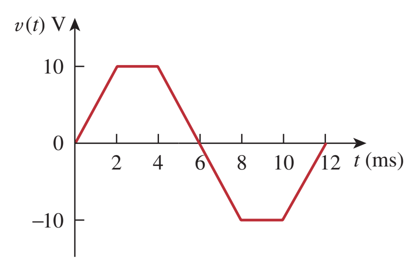


Figure 6.46
For Prob. 6.6.

6.7 At $t = 0$, the voltage across a 25-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current $5t$ mA flows through it.

6.8 A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50 \text{ V}, & t \leq 0 \\ Ae^{-100t} + Be^{-600t} \text{ V}, & t \geq 0 \end{cases}$$

If the capacitor has an initial current of 2 A, find:

(a) the constants A and B,

(b) the energy stored in the capacitor at $t = 0$,

(c) the capacitor current for $t > 0$.

6.9 The current through a 0.5-F capacitor is $6(1 - e^{-t})$ A. Determine the voltage and power at $t = 2$ s. Assume $v(0) = 0$.

6.10 The voltage across a 5-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor.

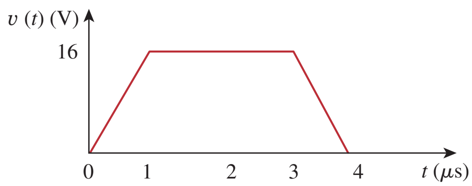


Figure 6.47
For Prob. 6.10.

6.11 A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that $v(0) = 10$ V, sketch the voltage waveform $v(t)$.

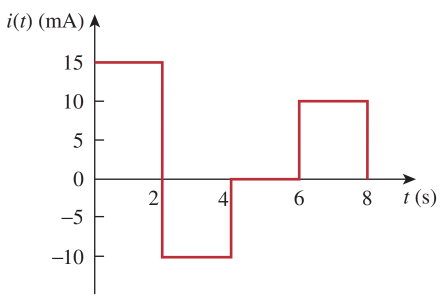


Figure 6.48
For Prob. 6.11.

6.12 A voltage of $30e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

6.13 Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.

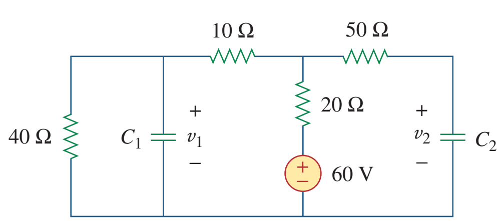


Figure 6.49
For Prob. 6.13.

Section 6.3 Series and Parallel Capacitors

6.14 Series-connected 20-pF and 60-pF capacitors are placed in parallel with series-connected 30-pF and 70-pF capacitors. Determine the equivalent capacitance.

6.15 Two capacitors (25 μF and 75 μF) are connected to a 100-V source. Find the energy stored in each capacitor if they are connected in:

(a) parallel (b) series

6.16 The equivalent capacitance at terminals *a-b* in the circuit of Fig. 6.50 is 30 μF. Calculate the value of C.

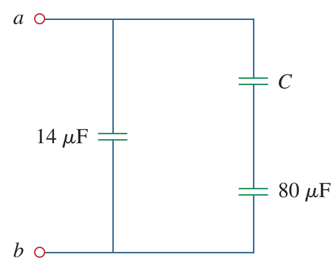


Figure 6.50
For Prob. 6.16.

6.17 Determine the equivalent capacitance for each of the circuits of Fig. 6.51.

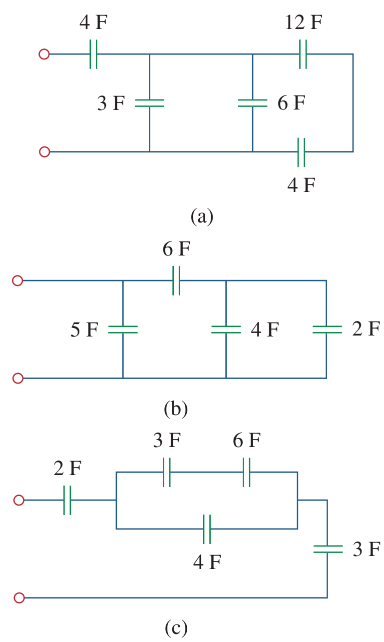


Figure 6.51
For Prob. 6.17.

6.18 Find C_{eq} in the circuit of Fig. 6.52 if all capacitors are 4 μF.

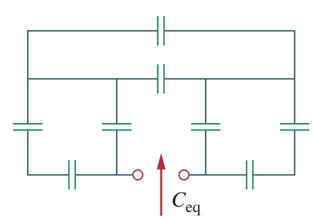


Figure 6.52
For Prob. 6.18.

6.19 Find the equivalent capacitance between terminals a and b in the circuit of Fig. 6.53. All capacitances are in μF.

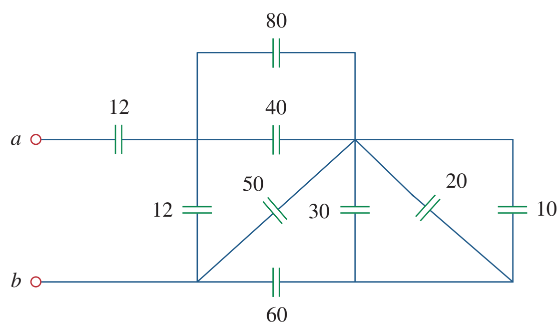


Figure 6.53
For Prob. 6.19.

6.20 Find the equivalent capacitance at terminals a-b of the circuit in Fig. 6.54.

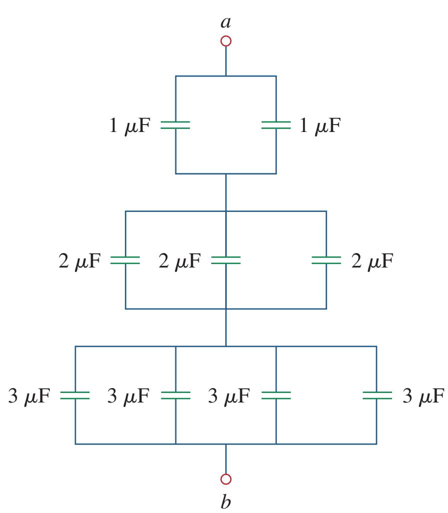


Figure 6.54
For Prob. 6.20.

6.21 Determine the equivalent capacitance at terminals a-b of the circuit in Fig. 6.55.

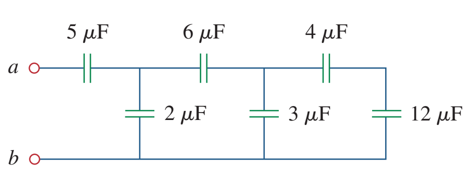


Figure 6.55
For Prob. 6.21.

6.22 Obtain the equivalent capacitance of the circuit in Fig. 6.56.

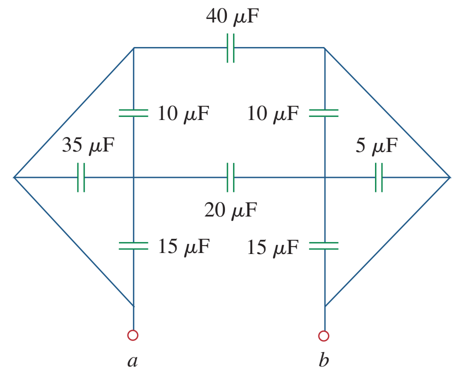


Figure 6.56
For Prob. 6.22.

6.23 Using Fig. 6.57, design a problem that will help other students better understand how capacitors work together when connected in series and in parallel.

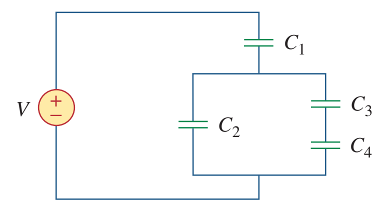


Figure 6.57
For Prob. 6.23.

6.24 For the circuit in Figure 6.58, determine (a) the voltage across each capacitor and (b) the energy stored in each capacitor.

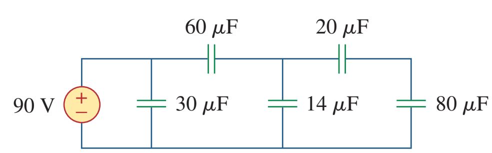


Figure 6.58
For Prob. 6.24.

6.25 (a) Show that the voltage-division rule for two capacitors in series as in Fig. 6.59(a) is

$$v_1 = \frac{C_2}{C_1 + C_2} v_s, \qquad v_2 = \frac{C_1}{C_1 + C_2} v_s$$

assuming that the initial conditions are zero.

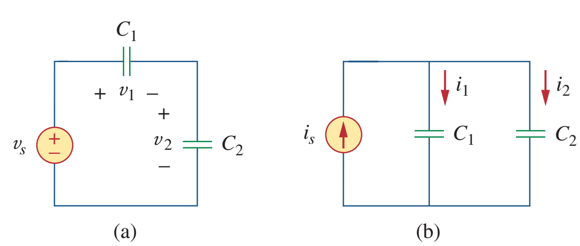


Figure 6.59
For Prob. 6.25.

(b) For two capacitors in parallel as in Fig. 6.59(b), show that the current-division rule is

$$i_1 = \frac{C_1}{C_1 + C_2} i_s, \qquad i_2 = \frac{C_2}{C_1 + C_2} i_s$$

assuming that the initial conditions are zero.

6.26 Three capacitors, $C_1 = 5\ \mu$F, $C_2 = 10\ \mu$F, and $C_3 = 20\ \mu$F, are connected in parallel across a 150-V source. Determine:

(a) the total capacitance,

(b) the charge on each capacitor,

(c) the total energy stored in the parallel combination.

6.27 Given that four 4-μF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

***6.28** Obtain the equivalent capacitance of the network shown in Fig. 6.60.

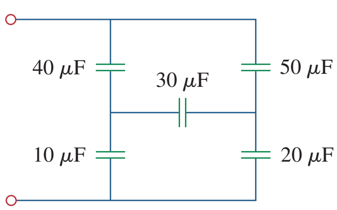


Figure 6.60
For Prob. 6.28.

6.29 Determine C_{eq} for each circuit in Fig. 6.61.

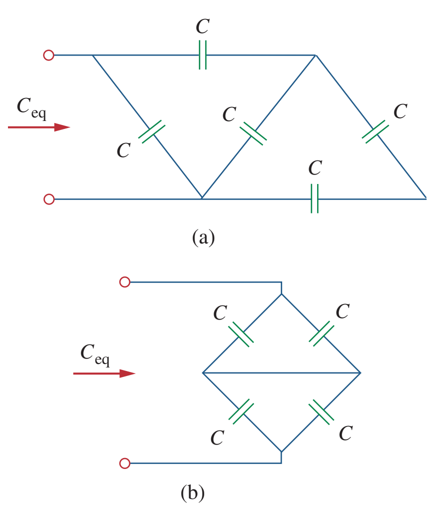


Figure 6.61
For Prob. 6.29.

6.30 Assuming that the capacitors are initially uncharged, find $v_o(t)$ in the circuit of Fig. 6.62.

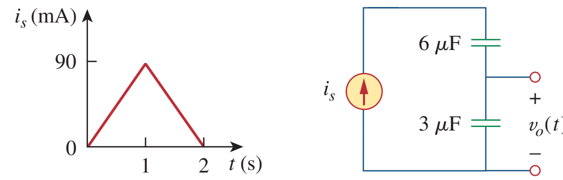


Figure 6.62
For Prob. 6.30.

6.31 If $v(0) = 0$, find $v(t)$, $i_1(t)$, and $i_2(t)$ in the circuit of Fig. 6.63.

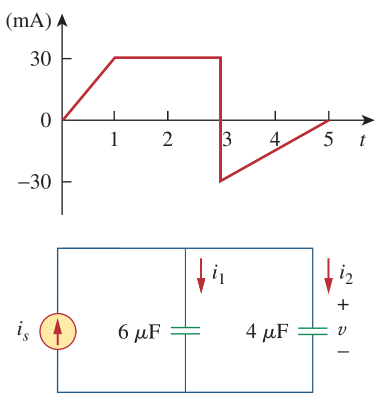


Figure 6.63
For Prob. 6.31.

6.32 In the circuit of Fig. 6.64, let $i_s = 50e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

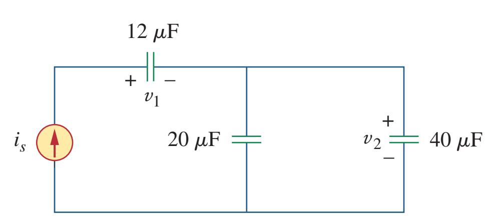


Figure 6.64
For Prob. 6.32.

* An asterisk indicates a challenging problem.

6.33 Obtain the Thevenin equivalent at the terminals, a-b, of the circuit shown in Fig. 6.65. Please note that Thevenin equivalent circuits do not generally exist for circuits involving capacitors and resistors. This is a special case where the Thevenin equivalent circuit does exist.

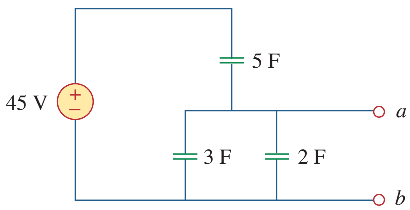


Figure 6.65
For Prob. 6.33.

Section 6.4 Inductors

6.34 The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

6.35 An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

6.36 Design a problem to help other students better understand how inductors work.

6.37 The current through a 12-mH inductor is 4 sin 100t A. Find the voltage, across the inductor for $0 < t < \pi/200$ s, and the energy stored at $t = \frac{\pi}{200}$ s.

6.38 The current through a 40-mH inductor is

$$i(t) = \begin{cases} 0, & t < 0 \\ te^{-2t}\ \text{A}, & t > 0 \end{cases}$$

Find the voltage $v(t)$.

6.39 The voltage across a 200-mH inductor is given by

$$v(t) = 3t^2 + 2t + 4\ \text{V} \qquad \text{for } t > 0.$$

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 1$ A.

6.40 The current through a 5-mH inductor is shown in Fig. 6.66. Determine the voltage across the inductor at $t = 1$, 3, and 5 ms.

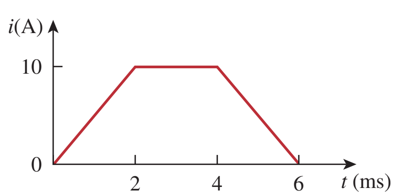


Figure 6.66
For Prob. 6.40.

6.41 The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

6.42 If the voltage waveform in Fig. 6.67 is applied across the terminals of a 5-H inductor, calculate the current through the inductor. Assume $i(0) = -1$ A.

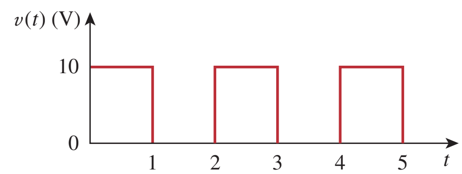


Figure 6.67
For Prob. 6.42.

6.43 The current in an 80-mH inductor increases from 0 to 60 mA. How much energy is stored in the inductor?

*__6.44__ A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage v_L across the inductor. (b) Find the voltage v_R across the resistor. (c) Does $v_R(t) + v_L(t) = 0$? (d) Calculate the energy in the inductor at $t = 0$.

6.45 If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current $i(t)$. Assume $i(0) = 0$.

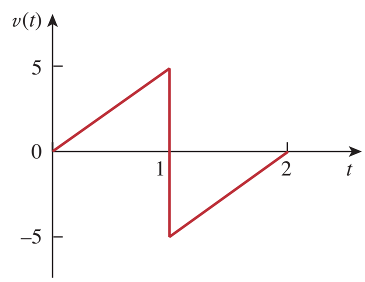


Figure 6.68
For Prob. 6.45.

6.46 Find v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.69 under dc conditions.

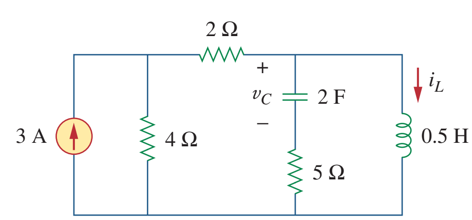


Figure 6.69
For Prob. 6.46.

6.47 For the circuit in Fig. 6.70, calculate the value of R that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

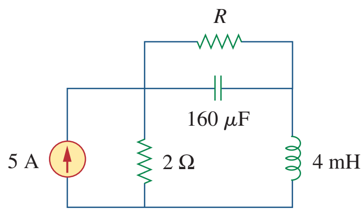


Figure 6.70
For Prob. 6.47.

6.48 Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

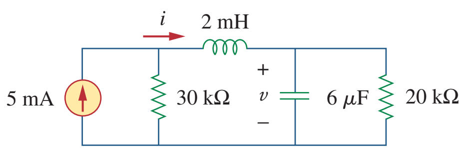


Figure 6.71
For Prob. 6.48.

Section 6.5 Series and Parallel Inductors

6.49 Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 10 mH.

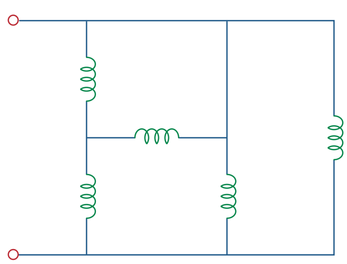


Figure 6.72
For Prob. 6.49.

6.50 An energy-storage network consists of series-connected 16-mH and 14-mH inductors in parallel with series-connected 24-mH and 36-mH inductors. Calculate the equivalent inductance.

6.51 Determine L_{eq} at terminals a-b of the circuit in Fig. 6.73.

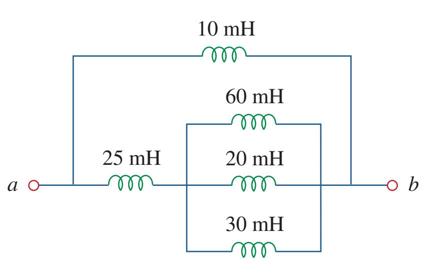


Figure 6.73
For Prob. 6.51.

6.52 Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.

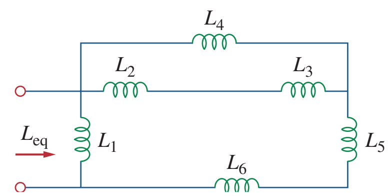


Figure 6.74
For Prob. 6.52.

6.53 Find L_{eq} at the terminals of the circuit in Fig. 6.75.

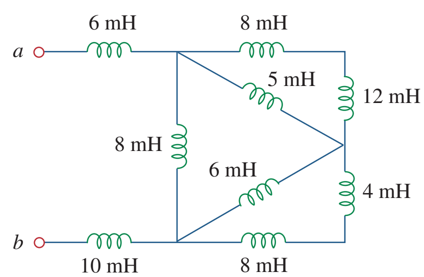


Figure 6.75
For Prob. 6.53.

6.54 Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

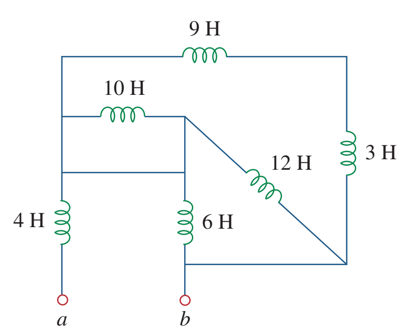


Figure 6.76
For Prob. 6.54.

6.55 Find L_{eq} in each of the circuits in Fig. 6.77.

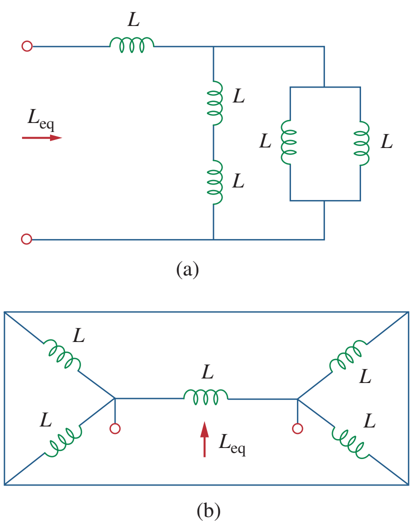


Figure 6.77
For Prob. 6.55.

6.56 Find L_{eq} in the circuit of Fig. 6.78.

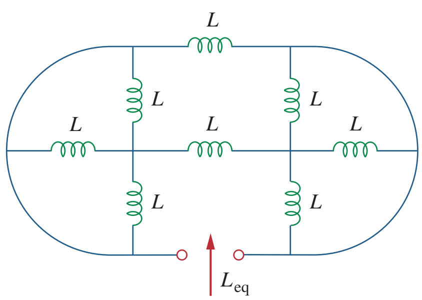


Figure 6.78
For Prob. 6.56.

***6.57** Determine L_{eq} that may be used to represent the inductive network of Fig. 6.79 at the terminals.

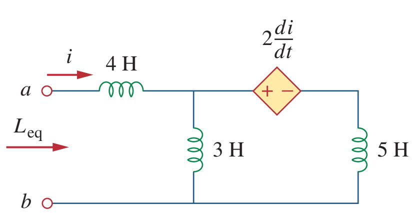


Figure 6.79
For Prob. 6.57.

6.58 The current waveform in Fig. 6.80 flows through a 3-H inductor. Sketch the voltage across the inductor over the interval $0 < t < 6$ s.

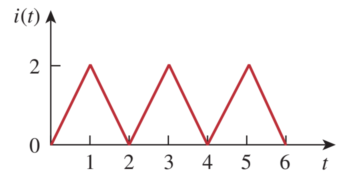


Figure 6.80
For Prob. 6.58.

6.59 (a) For two inductors in series as in Fig. 6.81(a), show that the voltage division principle is

$$v_1 = \frac{L_1}{L_1 + L_2} v_s, \qquad v_2 = \frac{L_2}{L_1 + L_2} v_s$$

assuming that the initial conditions are zero.

(b) For two inductors in parallel as in Fig. 6.81(b), show that the current-division principle is

$$i_1 = \frac{L_2}{L_1 + L_2} i_s, \qquad i_2 = \frac{L_1}{L_1 + L_2} i_s$$

assuming that the initial conditions are zero.

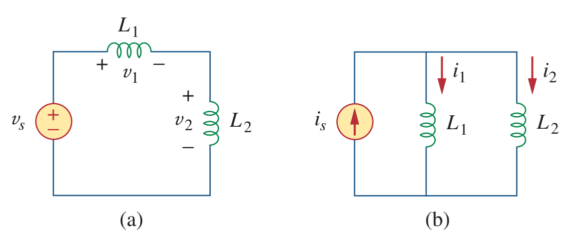


Figure 6.81
For Prob. 6.59.

6.60 In the circuit of Fig. 6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.

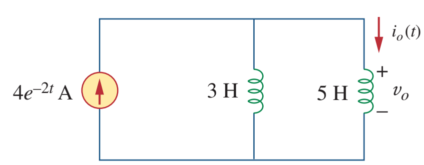


Figure 6.82
For Prob. 6.60.

6.61 Consider the circuit in Fig. 6.83. Find: (a) L_{eq}, $i_1(t)$, and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at $t = 1$ s.

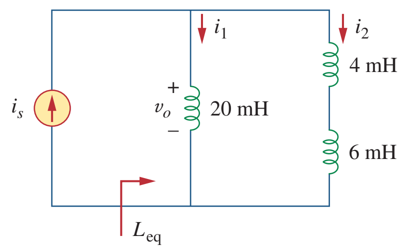


Figure 6.83
For Prob. 6.61.

6.62 Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

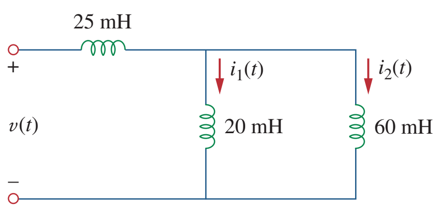


Figure 6.84
For Prob. 6.62.

6.63 In the circuit of Fig. 6.85, sketch v_o.

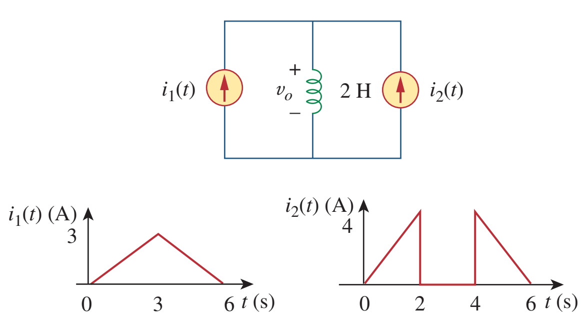


Figure 6.85
For Prob. 6.63.

6.64 The switch in Fig. 6.86 has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

(a) $i(t)$ for $t > 0$,

(b) v *just after* the switch has been moved to position B,

(c) $v(t)$ long after the switch is in position B.

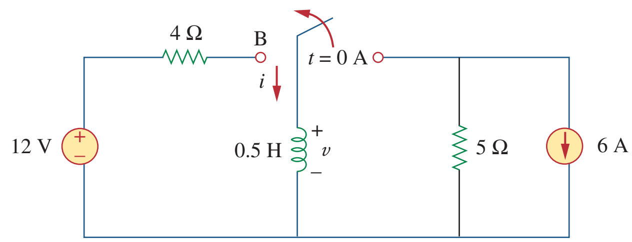


Figure 6.86
For Prob. 6.64.

6.65 The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$, find:

(a) the energy initially stored in each inductor,

(b) the total energy delivered to the black box from $t = 0$ to $t = \infty$,

(c) $i_1(t)$ and $i_2(t)$, $t \geq 0$,

(d) $i(t)$, $t \geq 0$.

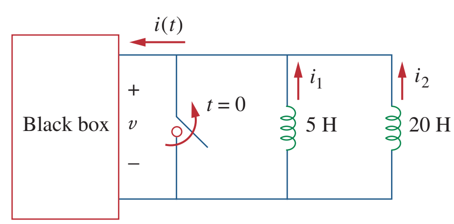


Figure 6.87
For Prob. 6.65.

6.66 The current $i(t)$ through a 20-mH inductor is equal, in magnitude, to the voltage across it for all values of time. If $i(0) = 2$ A, find $i(t)$.

Section 6.6 Applications

6.67 An op amp integrator has $R = 50$ kΩ and $C = 0.04$ μF. If the input voltage is $v_i = 10 \sin 50t$ mV, obtain the output voltage.

6.68 A 10-V dc voltage is applied to an integrator with $R = 50\ \text{k}\Omega$, $C = 100\ \mu\text{F}$ at $t = 0$. How long will it take for the op amp to saturate if the saturation voltages are $+12$ V and -12 V? Assume that the initial capacitor voltage was zero.

6.69 An op amp integrator with $R = 4\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$ has the input waveform shown in Fig. 6.88. Plot the output waveform.

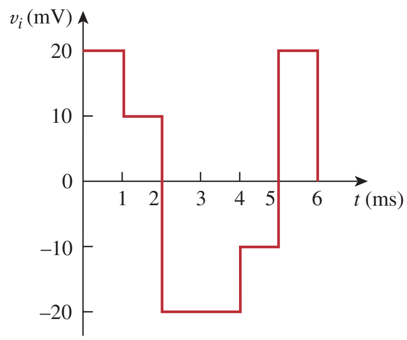


Figure 6.88
For Prob. 6.69.

6.70 Using a single op amp, a capacitor, and resistors of $100\ \text{k}\Omega$ or less, design a circuit to implement

$$v_o = -50 \int_0^t v_i(t)\, dt$$

Assume $v_o = 0$ at $t = 0$.

6.71 Show how you would use a single op amp to generate

$$v_o = -\int_0^t (v_1 + 4v_2 + 10v_3)\, dt$$

If the integrating capacitor is $C = 2\ \mu\text{F}$, obtain the other component values.

6.72 At $t = 1.5$ ms, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at $t = 0$.

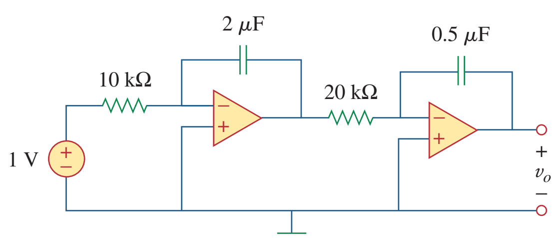


Figure 6.89
For Prob. 6.72.

6.73 Show that the circuit in Fig. 6.90 is a noninverting integrator.

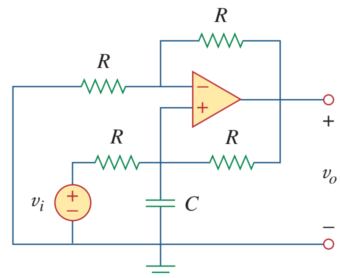


Figure 6.90
For Prob. 6.73.

6.74 The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output.

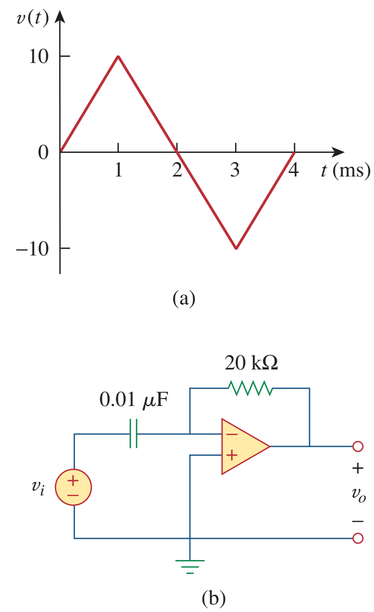
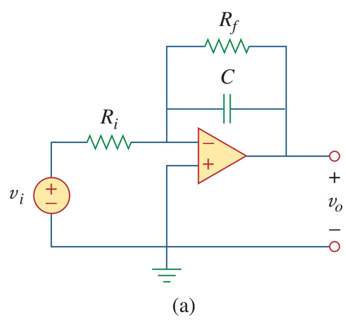


Figure 6.91
For Prob. 6.74.

6.75 An op amp differentiator has $R = 250\ \text{k}\Omega$ and $C = 10\ \mu\text{F}$. The input voltage is a ramp $r(t) = 12t$ mV. Find the output voltage.

6.76 A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of 10 V/s for 10 ms. If the waveform is applied to a differentiator with $R = 50\ \text{k}\Omega$, $C = 10\ \mu\text{F}$, sketch the output voltage waveform.

***6.77** The output v_o of the op amp circuit in Fig. 6.92(a) is shown in Fig. 6.92(b). Let $R_i = R_f = 1\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$. Determine the input voltage waveform and sketch it.



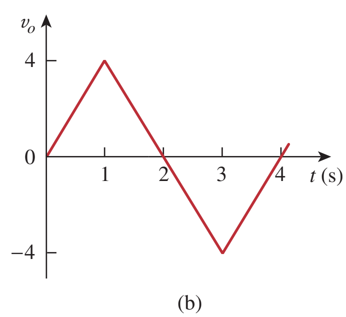


Figure 6.92
For Prob. 6.77.

6.78 Design an analog computer to simulate

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10 \sin 2t$$

where $v_0(0) = 2$ and $v_0'(0) = 0$.

6.79 Design an analog computer circuit to solve the following ordinary differential equation.

$$\frac{dy(t)}{dt} + 4y(t) = f(t)$$

where $y(0) = 1$ V.

6.80 Figure 6.93 presents an analog computer designed to solve a differential equation. Assuming $f(t)$ is known, set up the equation for $f(t)$.

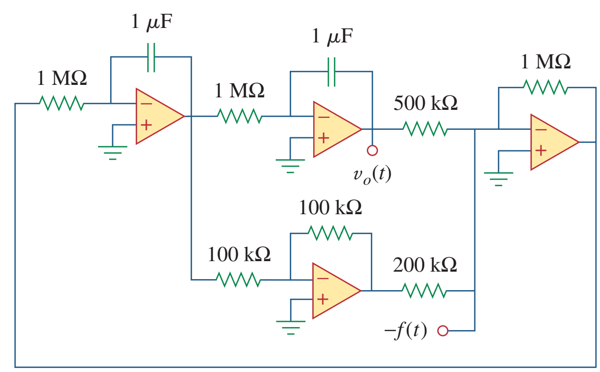


Figure 6.93
For Prob. 6.80.

6.81 Design an analog computer to simulate the following equation:

$$\frac{d^2v}{dt^2} + 5v = -2f(t)$$

6.82 Design an op amp circuit such that

$$v_o = 10v_s + 2\int v_s\,dt$$

where v_s and v_o are the input voltage and output voltage, respectively.

Comprehensive Problems

6.83 Your laboratory has available a large number of 10-μF capacitors rated at 300 V. To design a capacitor bank of 40 μF rated at 600 V, how many 10-μF capacitors are needed and how would you connect them?

6.84 An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 5 \sin^2 \pi t$ mA, $t > 0$, find the power being delivered to the inductor and the energy stored in it at $t = 0.5$ s.

6.85 A square-wave generator produces the voltage waveform shown in Fig. 6.94(a). What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. 6.94(b)? Calculate the value of the component, assuming that it is initially uncharged.

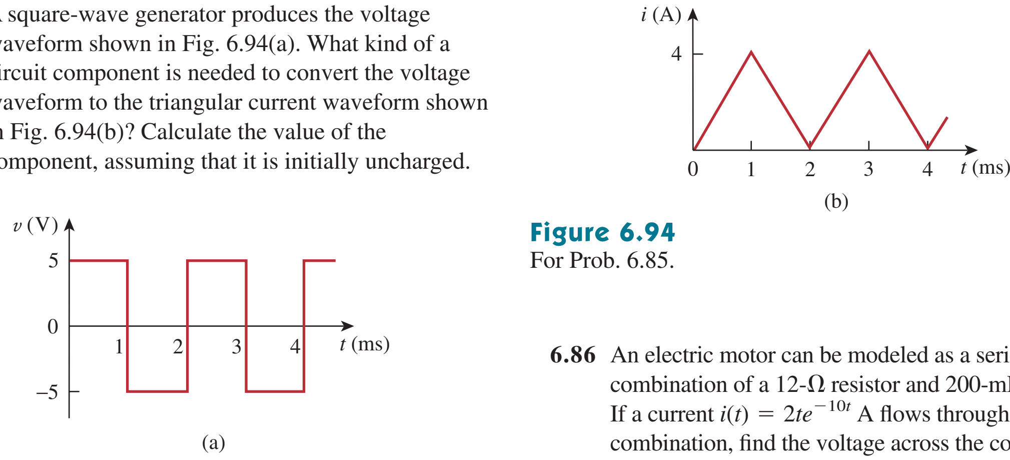


Figure 6.94
For Prob. 6.85.

6.86 An electric motor can be modeled as a series combination of a 12-Ω resistor and 200-mH inductor. If a current $i(t) = 2te^{-10t}$ A flows through the series combination, find the voltage across the combination.