Chapter 10

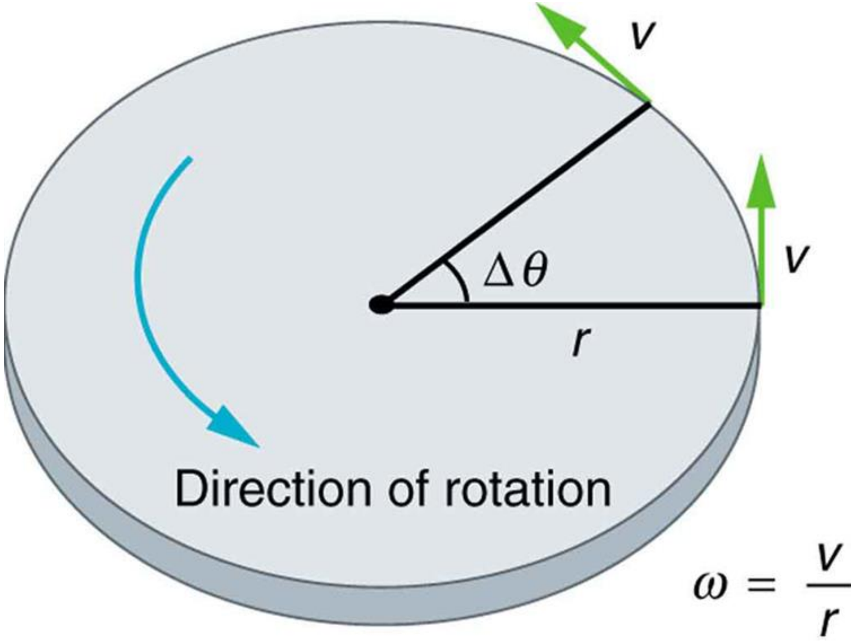
- Angular Quantities
- Constant Angular Acceleration
- Torque
- Rotational Dynamics; Torque and Rotational Inertia
- Rotational Kinetic Energy
- Angular Momentum and Its Conservation
- Vector Nature of Angular Quantities

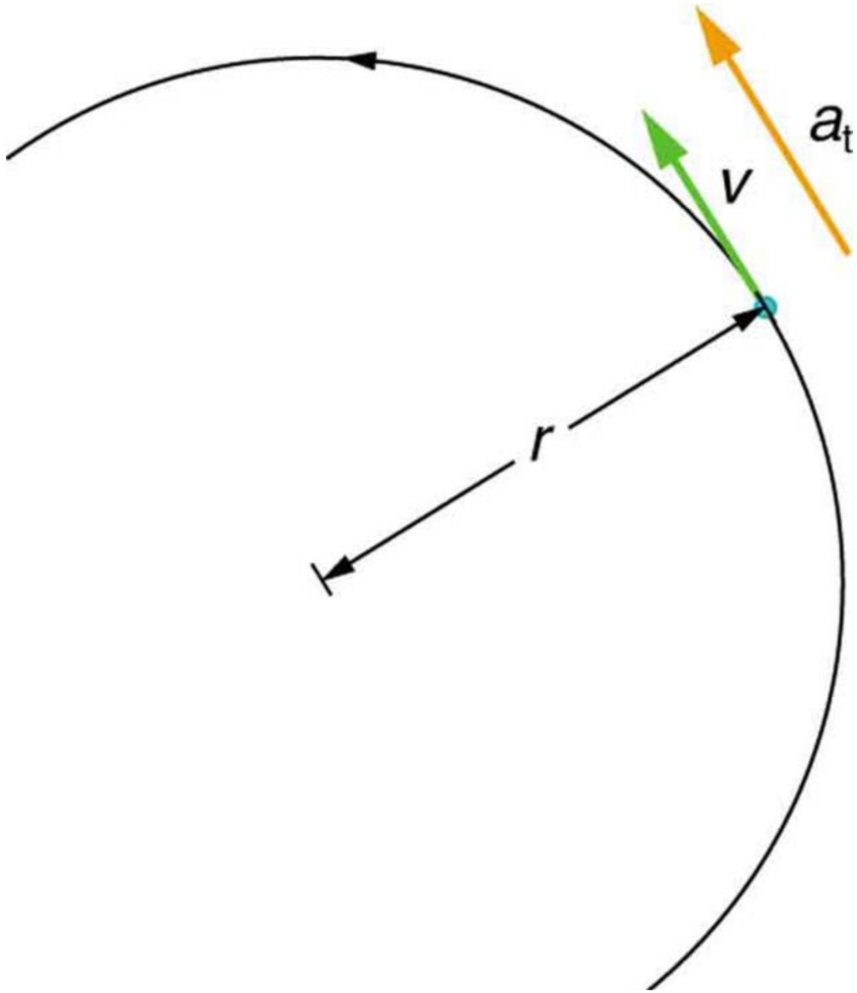
FIGURE 10.2

This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation.

FIGURE 10.3

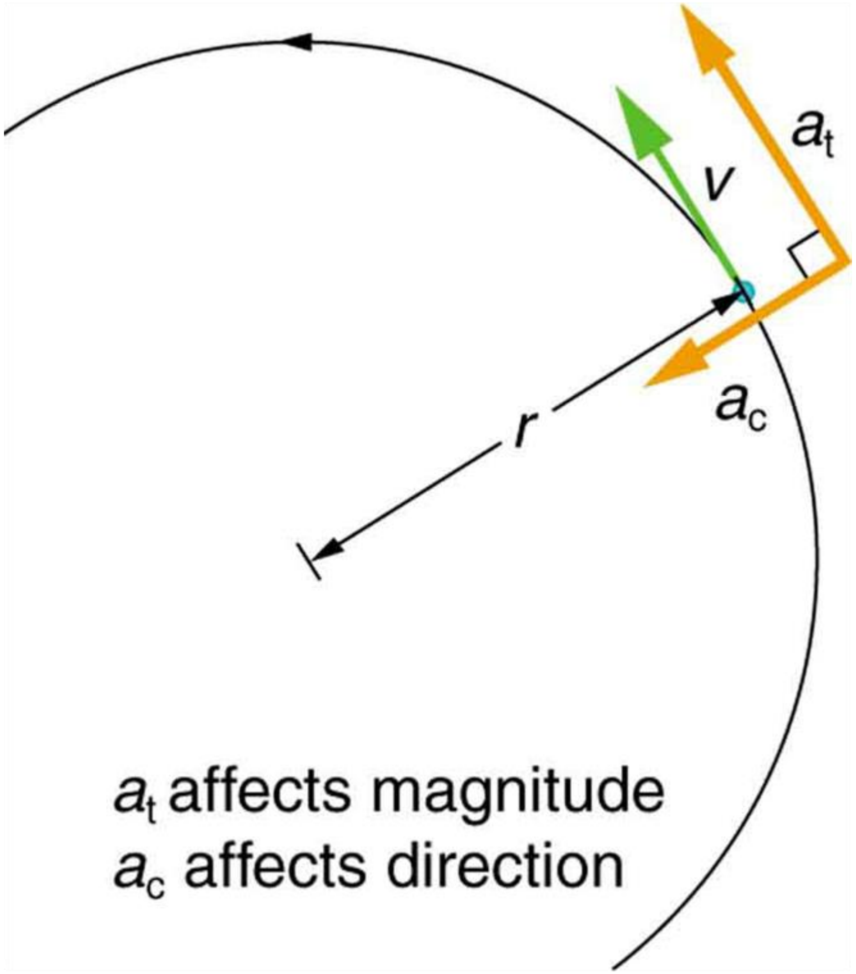
This figure shows uniform circular motion and some of its defined quantities.



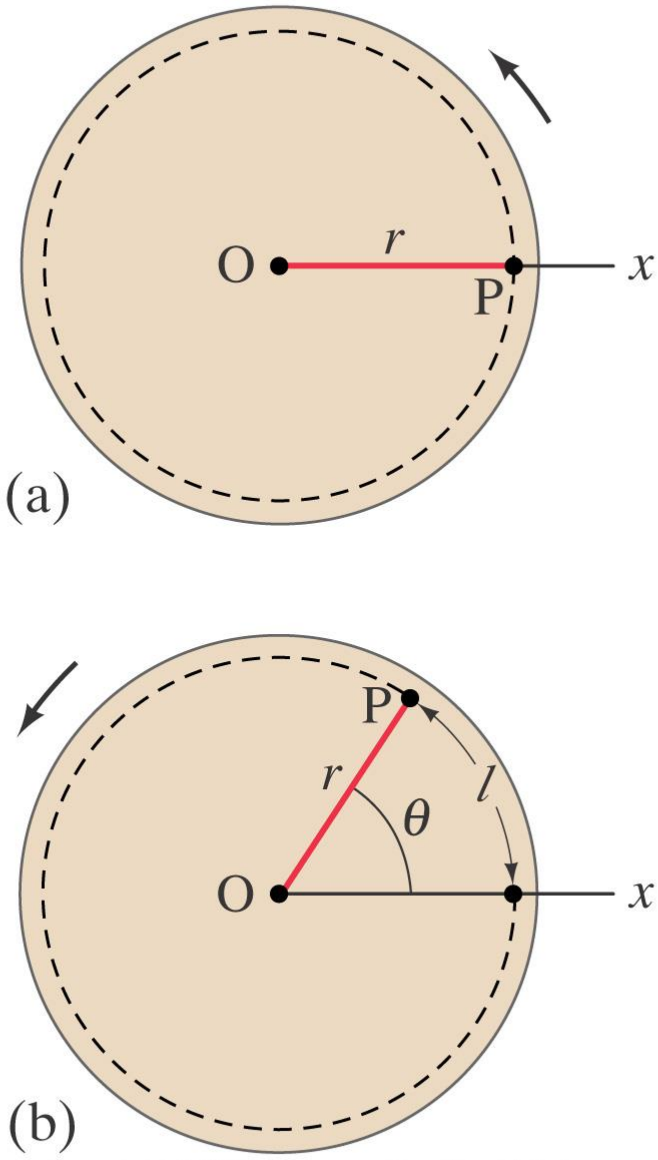


In circular motion, linear acceleration a, occurs as the magnitude of the velocity changes: a is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration a_t .

Centripetal acceleration a_c occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.



Angular Quantities

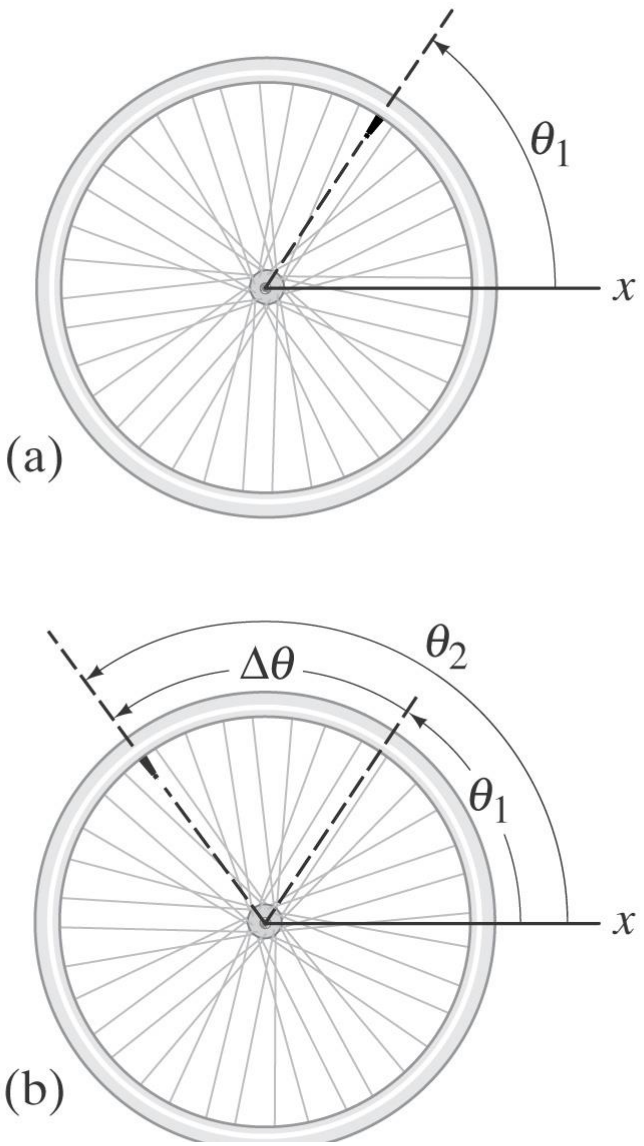


In purely rotational motion, all points on the object move in circles around the axis of rotation (“*O*”). The radius of the circle is *r*. All points on a straight line drawn through the axis move through the same angle in the same time. The angle θ in radians is defined:

$$\theta = \frac{l}{r}$$

where *l* is the arc length.

Angular Quantities



Angular displacement:

$$\Delta\theta = \theta_2 - \theta_1$$

The average angular **velocity** is defined as the total angular **displacement** divided by **time**:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

The **instantaneous** angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Angular Quantities

The angular **acceleration** is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta \omega}{\Delta t}$$

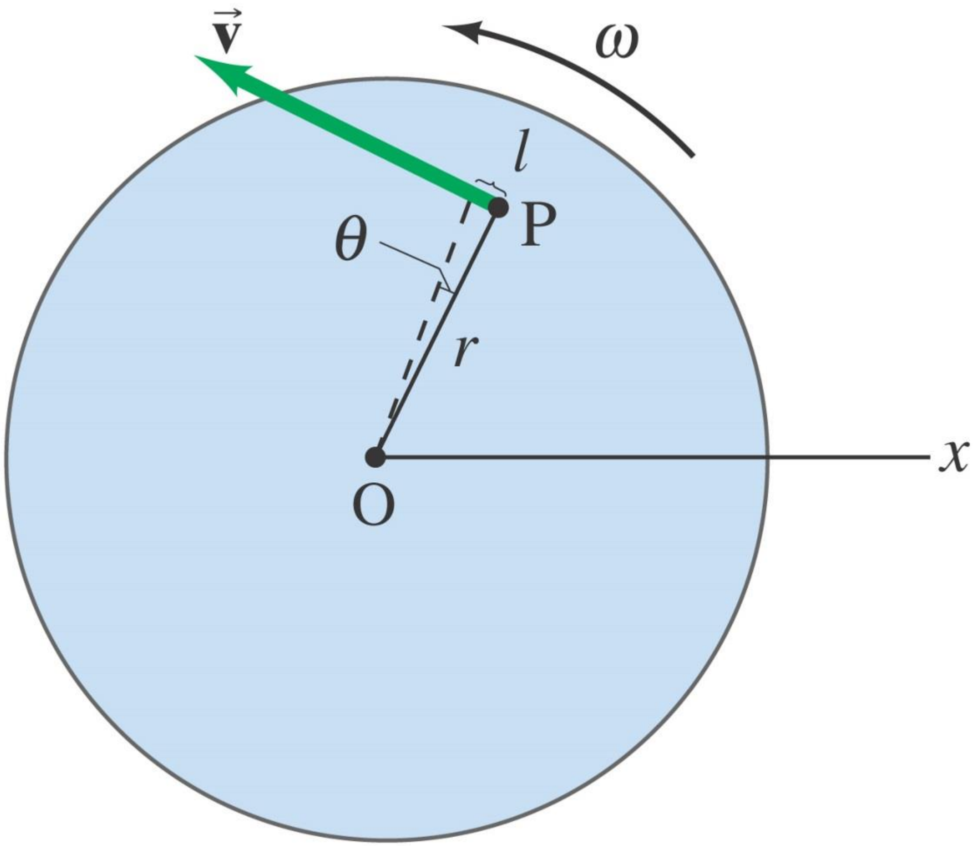
The **instantaneous** acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

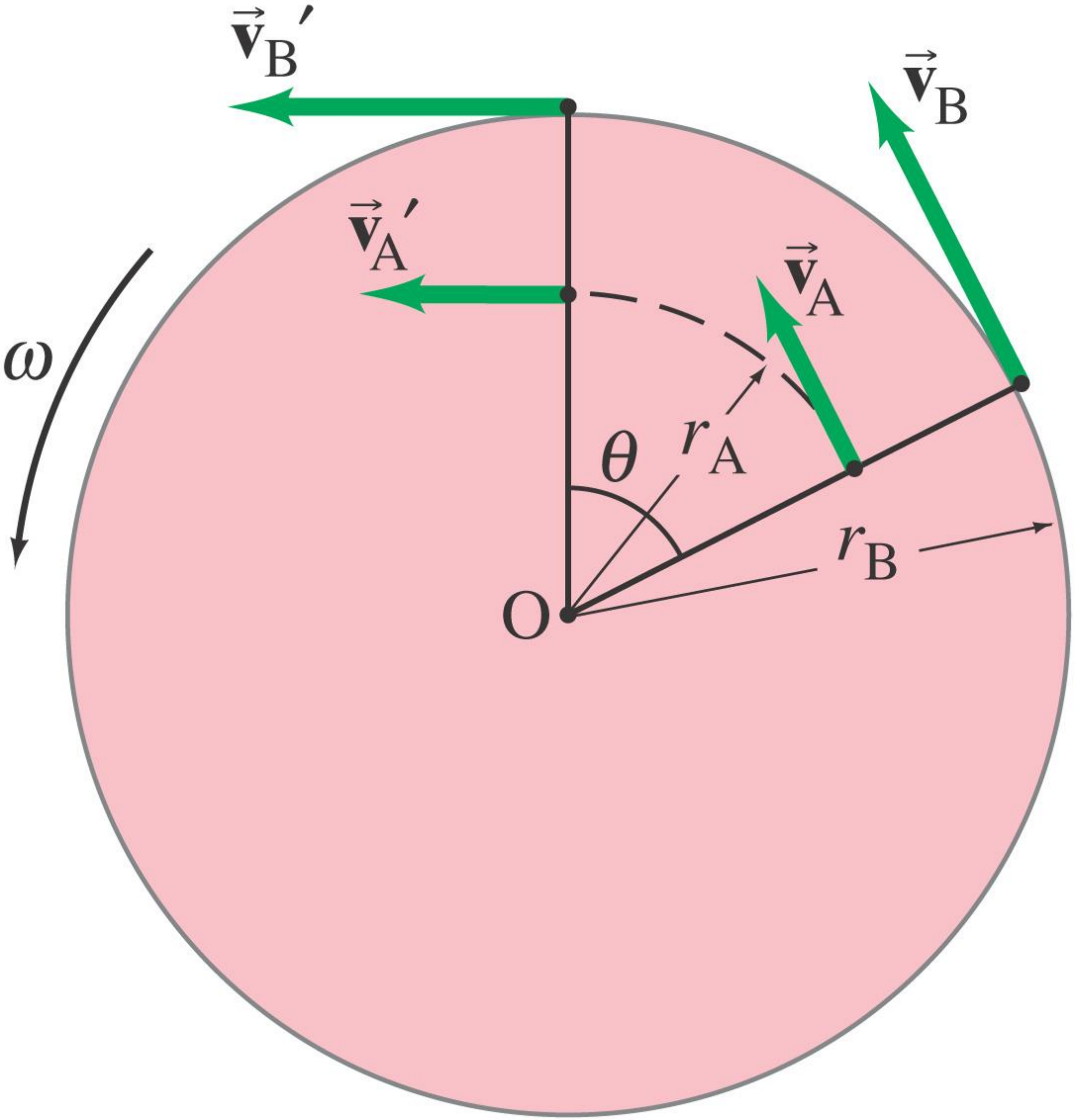
Angular Quantities

Every point on a rotating body has an **angular velocity** ω and a **linear velocity** v.

They are related: $$v = r\omega$$

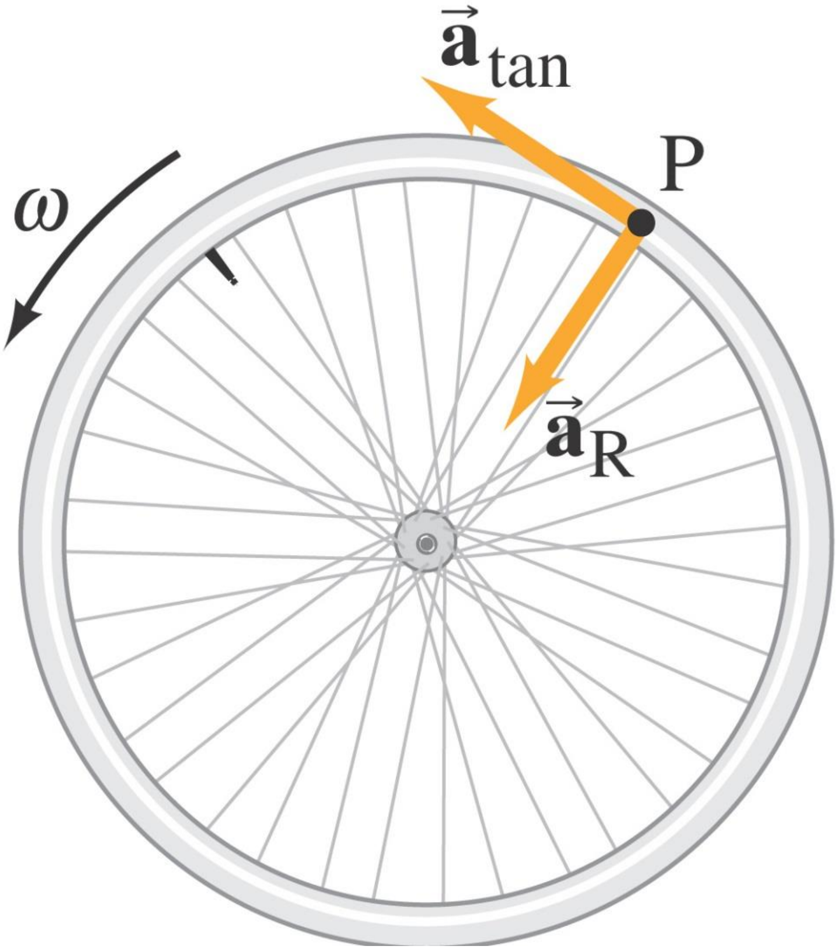


Angular Quantities



Therefore, objects **farther** from the axis of rotation will move **faster**.

Angular Quantities



If the angular velocity of a rotating object **changes**, it has a **tangential** acceleration:

$$a_{\tan} = r\alpha$$

Even if the angular velocity is constant, each point on the object has a **centripetal** acceleration:

$$a_R = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

Angular Quantities

Here is the correspondence between linear and rotational quantities:

Linear and Rotational Quantities

Linear	Type	Rotational	Relation
x	displacement	θ	$x = r\theta$
v	velocity	ω	$v = r\omega$
$a_{\tan}$	acceleration	α	$a_{\tan} = r\alpha$

Angular Quantities

The **frequency** is the number of complete **revolutions** per second:

$$f = \frac{\omega}{2\pi}$$

Frequencies are measured in **hertz**.

$$1\ \text{Hz} = 1\ \text{s}^{-1}$$

The **period** is the time one revolution takes:

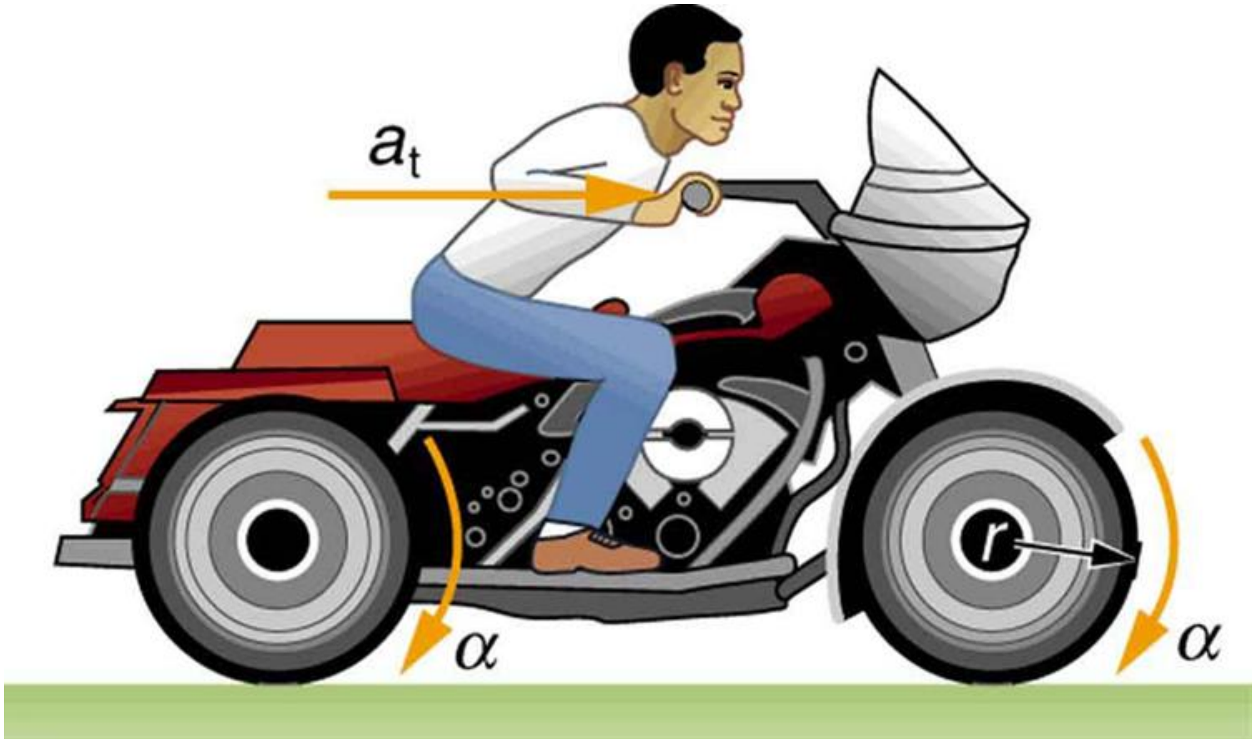
$$T = \frac{1}{f}$$

Constant Angular Acceleration

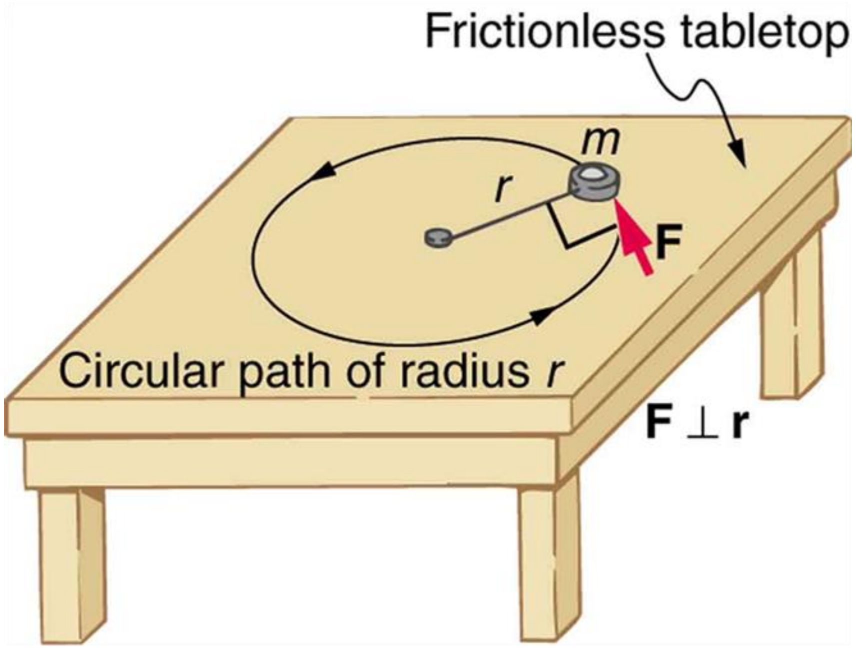
The equations of motion for **constant** angular acceleration are the same as those for **linear** motion, with the substitution of the **angular** quantities for the **linear** ones.

Angular	Linear
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$
$\overline{\omega} = \dfrac{\omega + \omega_0}{2}$	$\overline{v} = \dfrac{v + v_0}{2}$

FIGURE 10.6

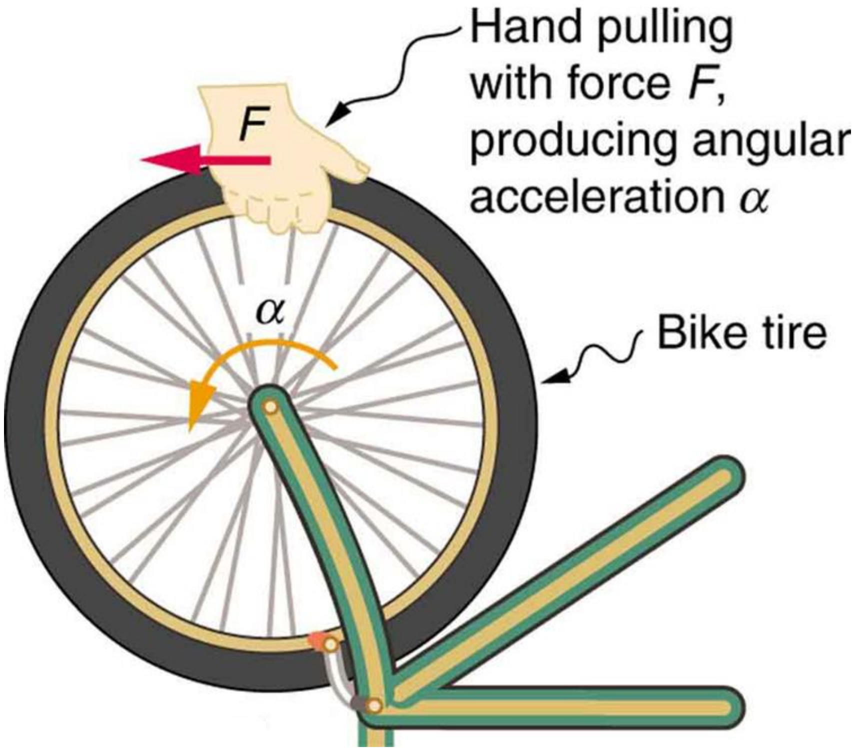


The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.



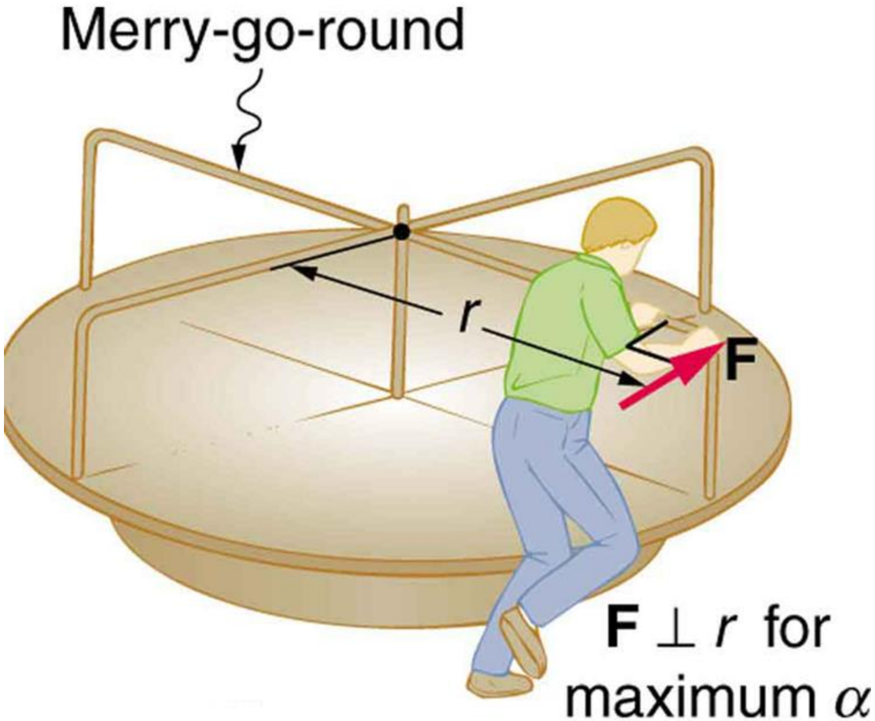
An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r, causing it to accelerate about the pivot point. The force is kept perpendicular to r.

FIGURE 10.10



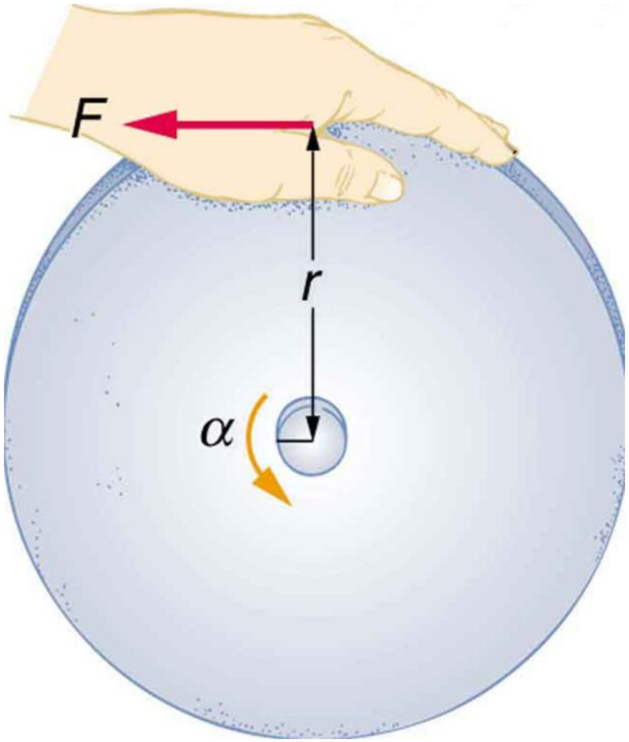
Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

FIGURE 10.13



A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

FIGURE 10.17

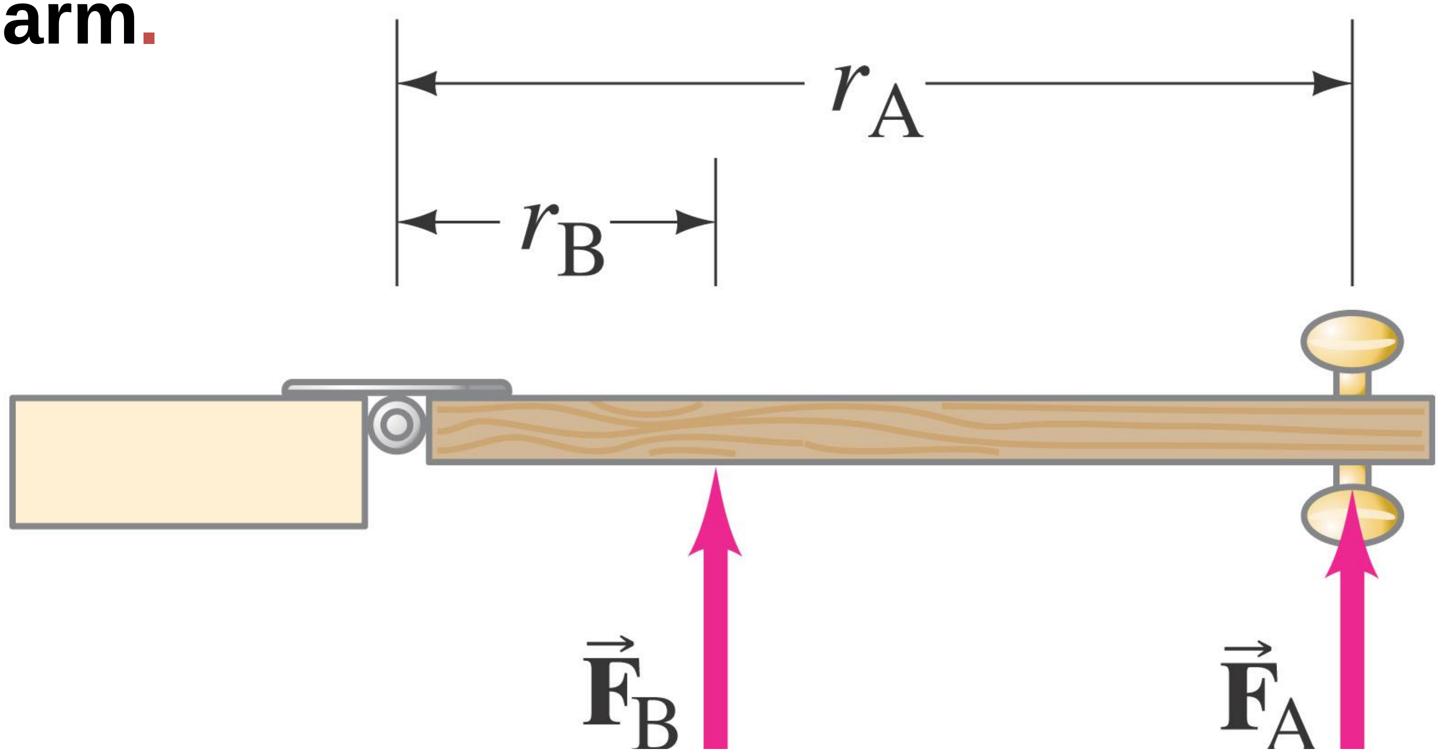


A large grindstone is given a spin by a person grasping its outer edge.

Torque

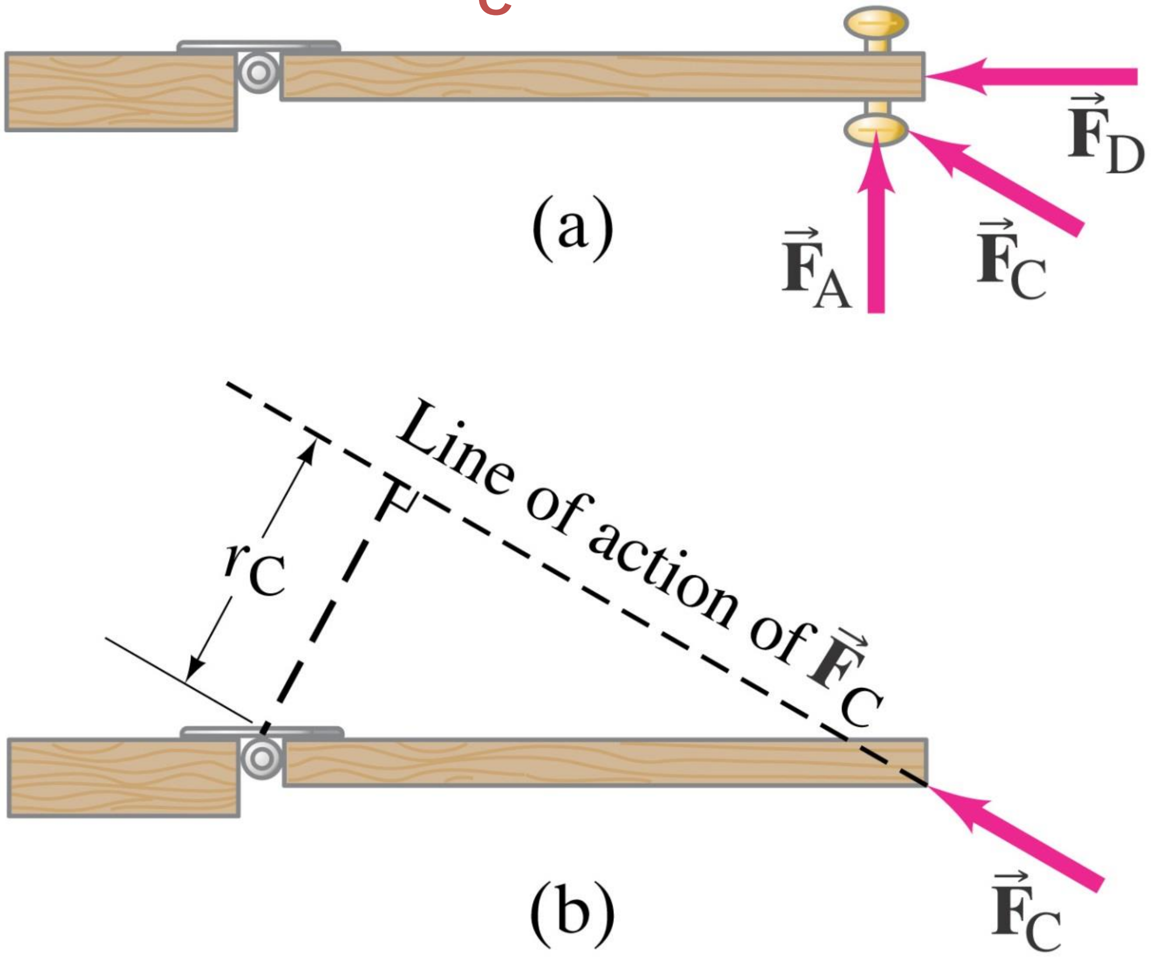
To make an object **start** rotating, a **force** is needed; the **position** and **direction** of the force matter as well.

The **perpendicular** distance from the axis of rotation to the line along which the force acts is called the **lever arm**.

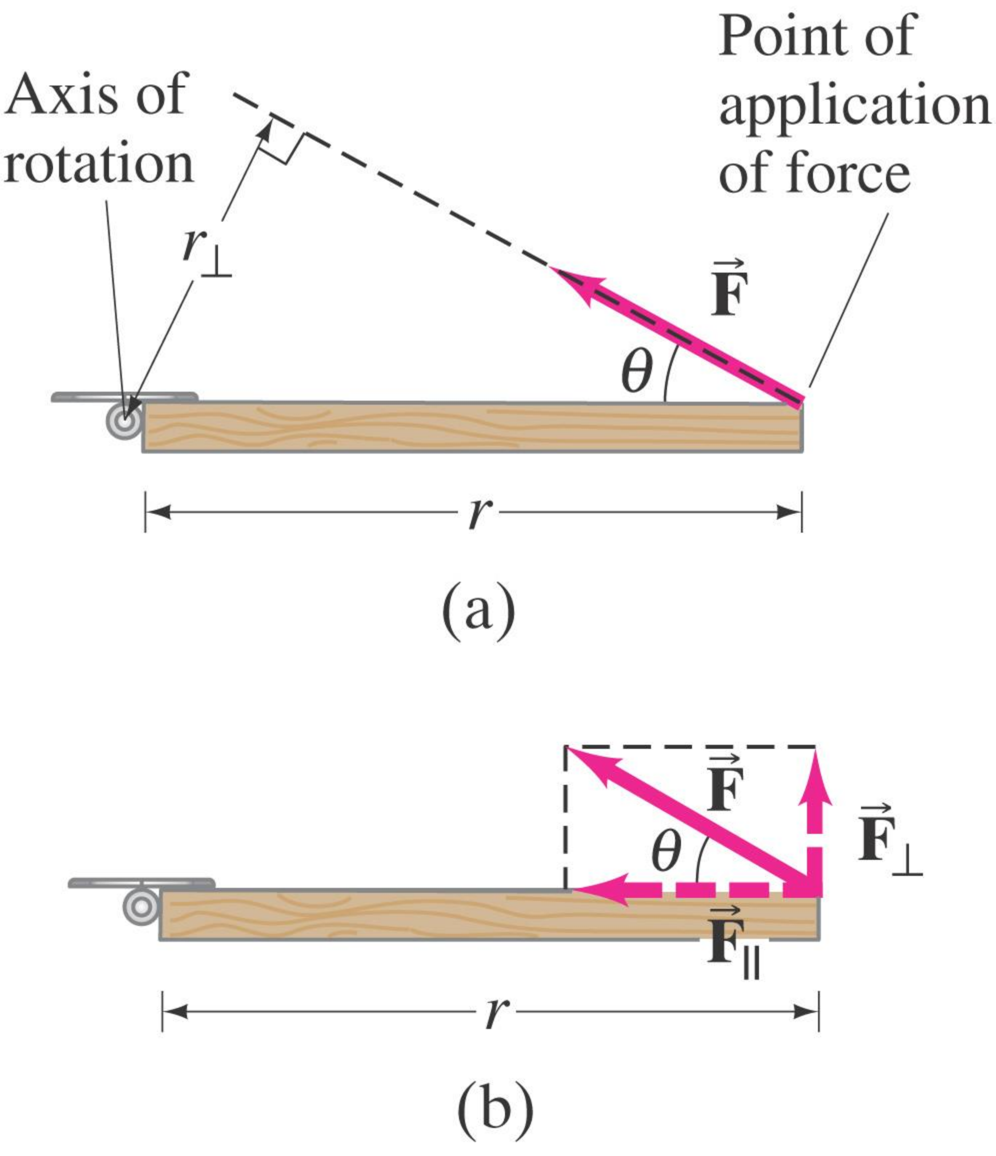


Torque

Here, the lever arm for F_A is the distance from the knob to the hinge; the lever arm for F_D is zero; and the lever arm for F_C is as shown.



Torque

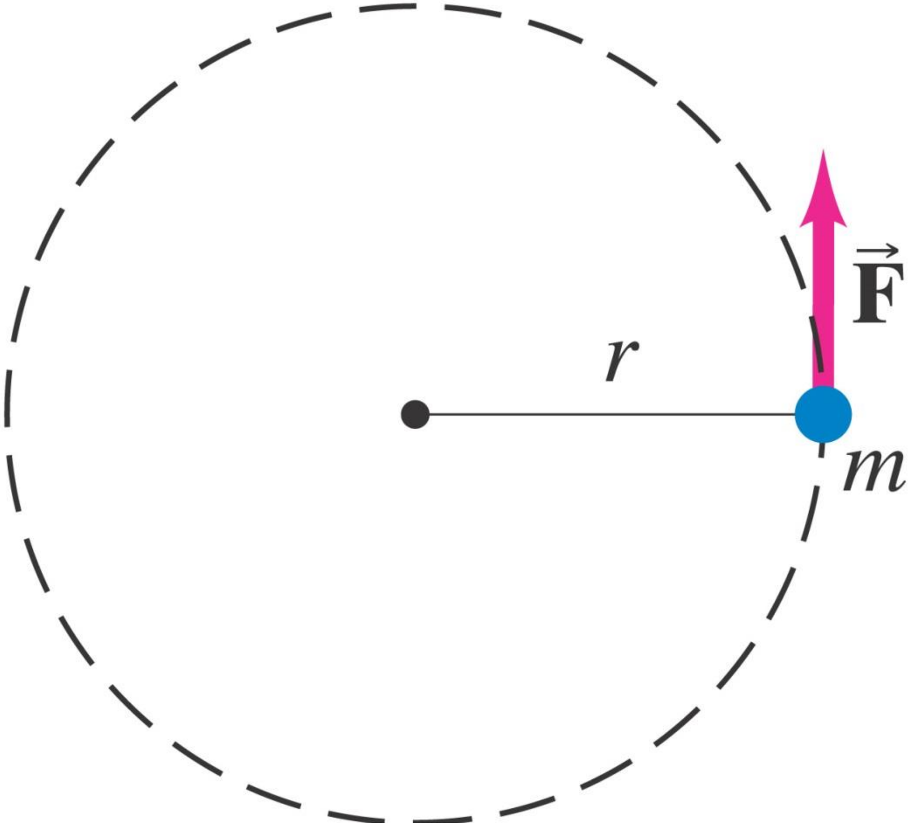


The **torque** is defined as:

$$\tau = r_\perp F$$

Rotational Dynamics; Torque and Rotational Inertia

Knowing that F = ma, we see that $\tau = mr^2\alpha$



This is for a single point mass; what about an **extended** object?

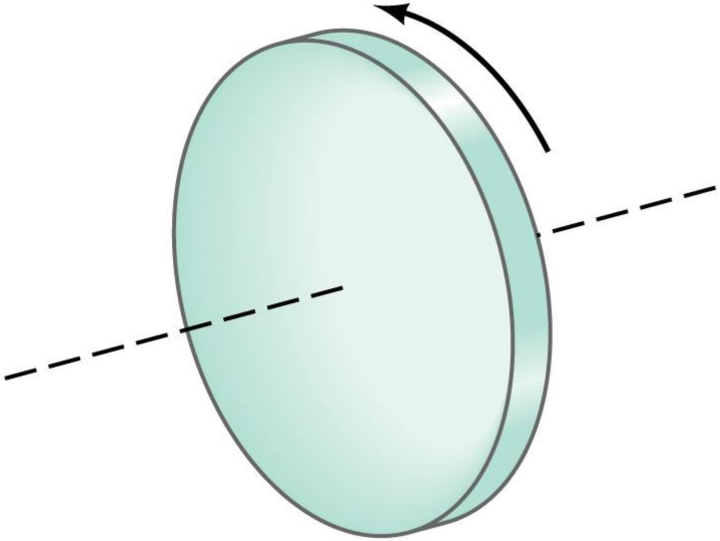
As the angular acceleration is the same for the whole object, we can write:

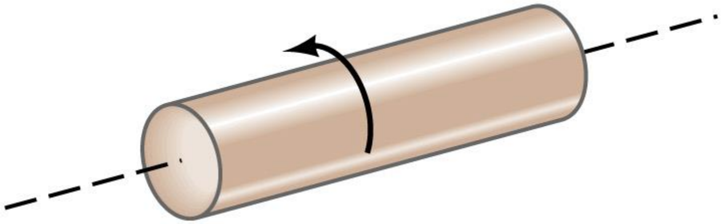
$$\Sigma\tau = \left(\Sigma mr^2\right)\alpha$$

Rotational Dynamics; Torque and Rotational Inertia

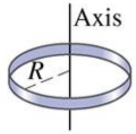
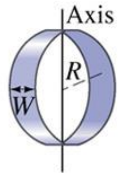
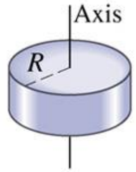
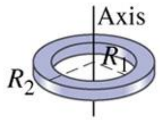
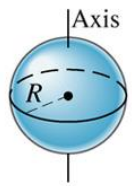
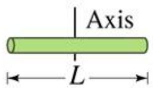
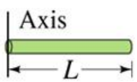
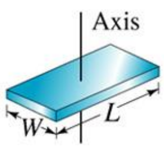
The quantity $I = \Sigma mr^2$ is called the **rotational inertia** of an object.

The **distribution** of mass matters here – these two objects have the same mass, but the one on the left has a greater **rotational inertia**, as so much of its mass is far from the axis of rotation.



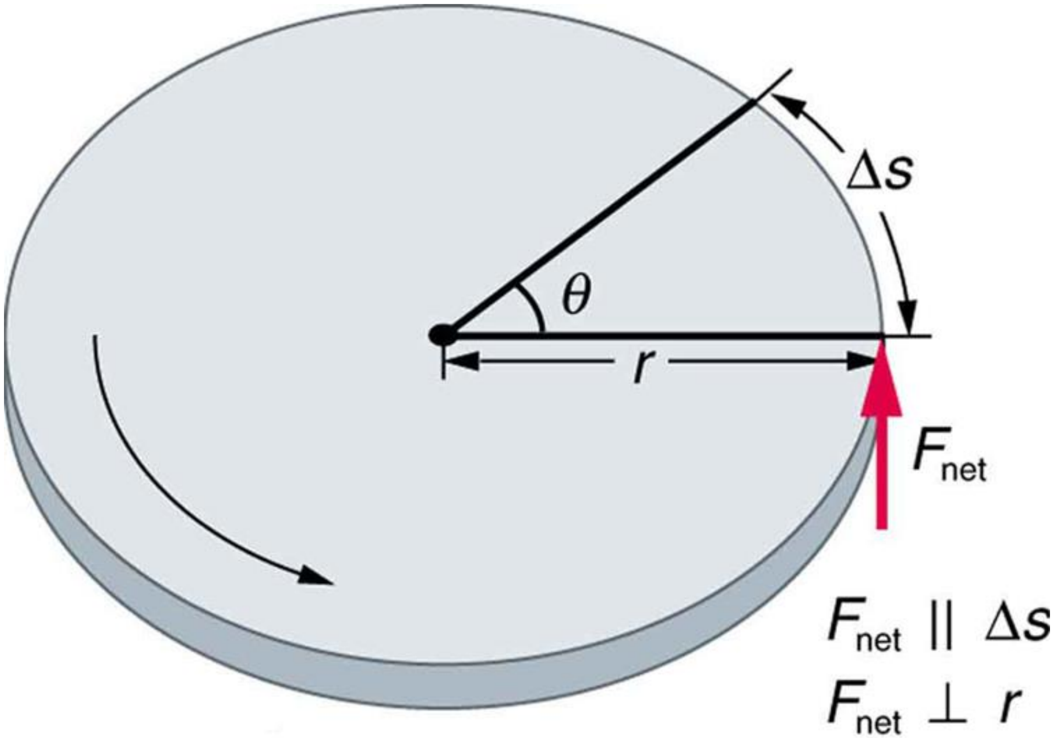


Rotational Dynamics; Torque and Rotational Inertia

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R	Through center		MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center		$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center		$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center		$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end		$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center		$\frac{1}{12}M(L^2 + W^2)$

The **rotational inertia** of an object depends not only on its **mass distribution** but also the location of the **axis** of rotation – compare (f) and (g), for example.

FIGURE 10.15



The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus (net F)Δs . The net work goes into rotational kinetic energy.

Rotational Kinetic Energy

The **kinetic energy** of a rotating object is given by $\text{KE} = \Sigma\left(\frac{1}{2}mv^2\right)$

By substituting the rotational quantities, we find that the rotational kinetic energy can be written:

$$\text{rotational KE} = \frac{1}{2}I\omega^2$$

A object that has both **translational** and **rotational** motion also has both translational and rotational **kinetic energy**:

$$\text{KE} = \frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}I_{\text{CM}}\omega^2$$

Rotational Kinetic Energy

When using **conservation** of energy, both **rotational** and **translational** kinetic energy must be taken into account.

All these objects have the same **potential** energy at the top, but the time it takes them to get down the incline depends on how much **rotational inertia** they have.

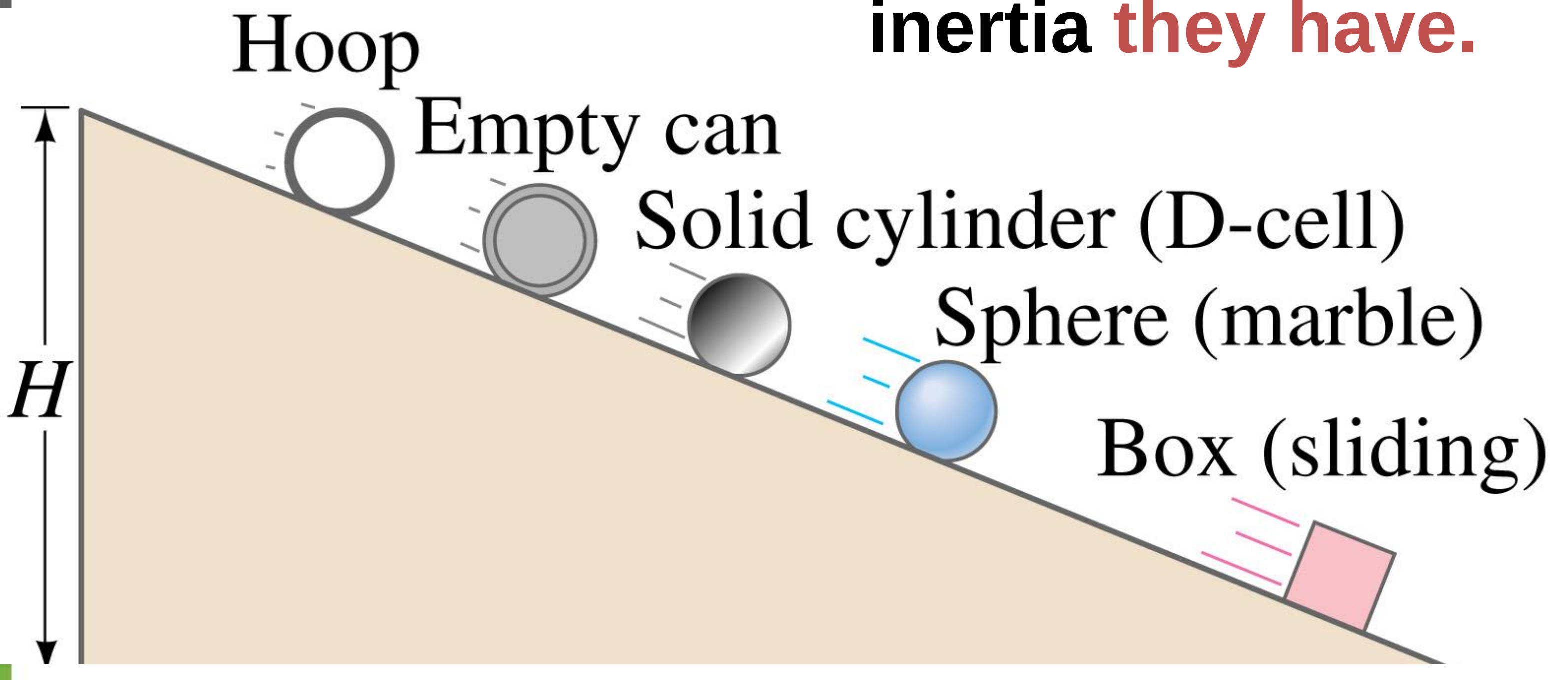
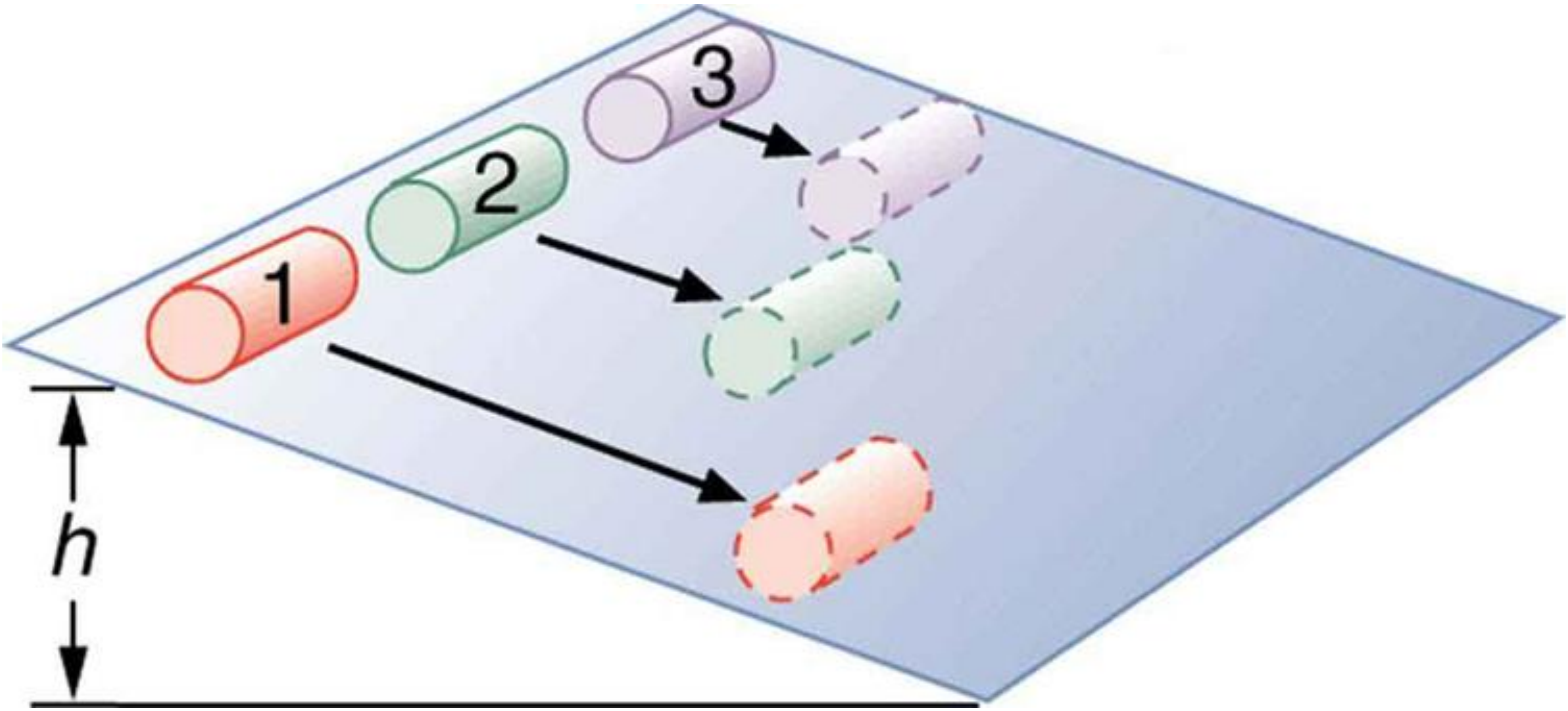
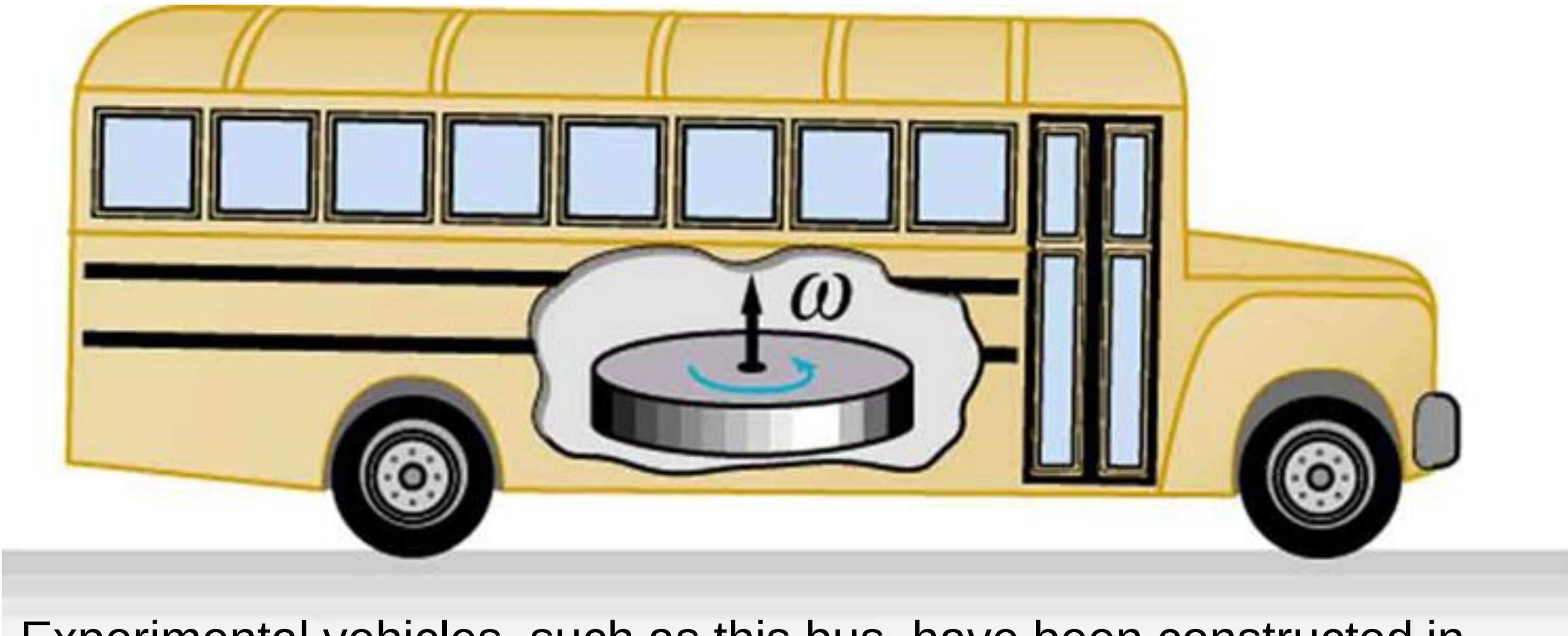


FIGURE 10.19

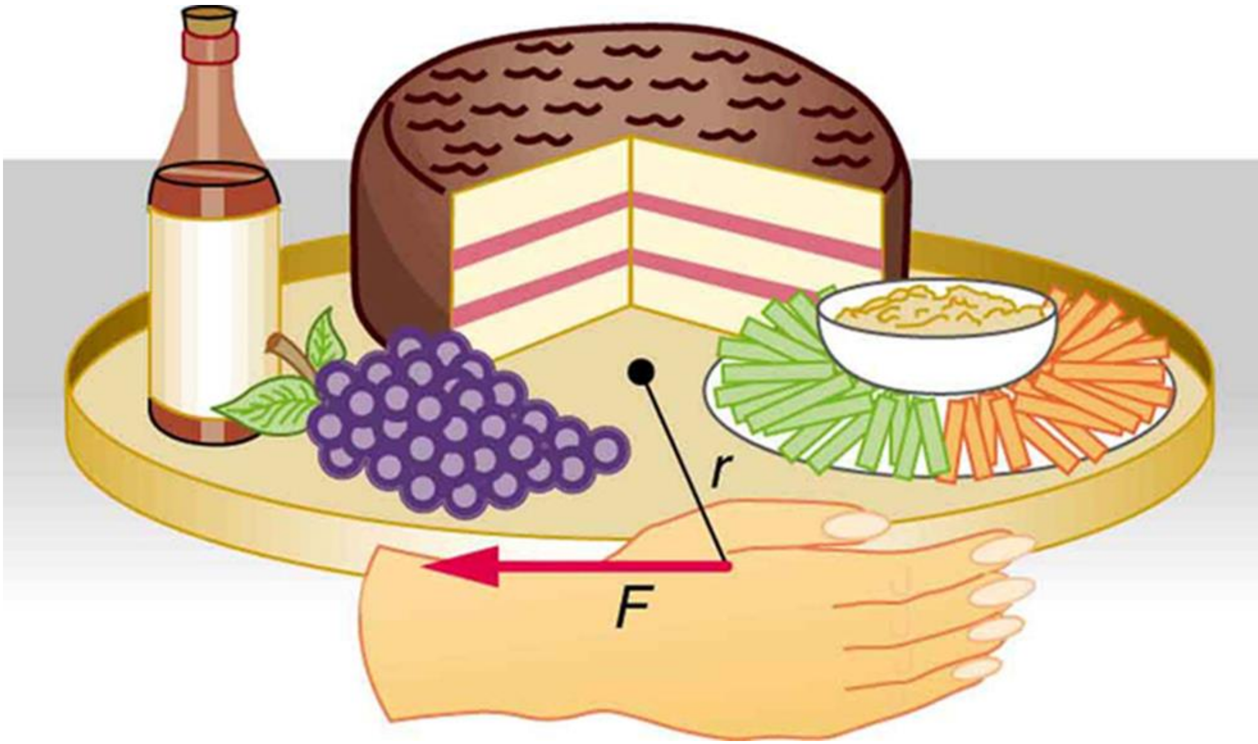


Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

FIGURE 10.16



Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into KE_{rot}. It can also convert translational kinetic energy, when the bus stops, into KE_{rot}. The flywheel’s energy can then be used to accelerate, to go up another hill, or to keep the bus from going against friction.

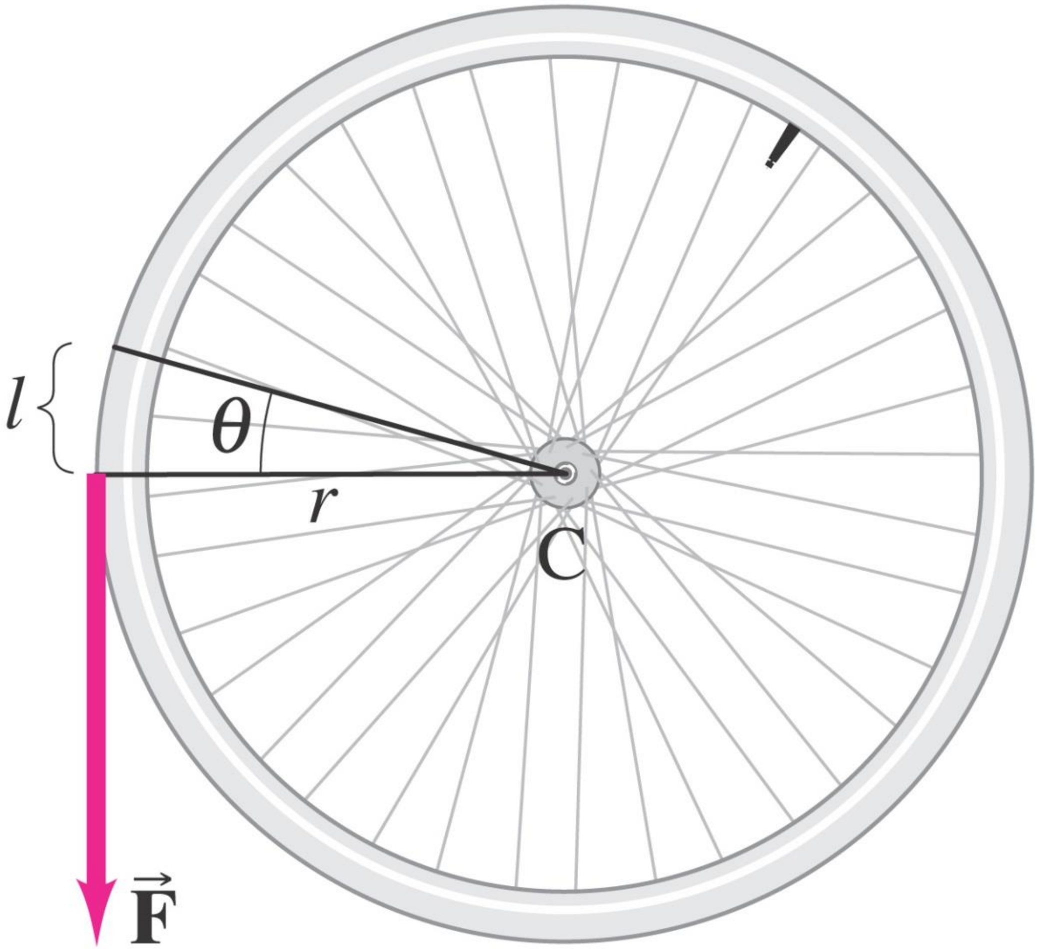


A partygoer exerts a torque on a lazy Susan to make it rotate. The equation net $\tau = \Delta L / \Delta t$ gives the relationship between torque and the angular momentum produced.

Rotational Kinetic Energy

The torque does work as it moves the wheel through an angle *θ*:

$$W = \tau \Delta\theta$$



Angular Momentum and Its Conservation

In analogy with linear momentum, we can define **angular momentum** *L*:

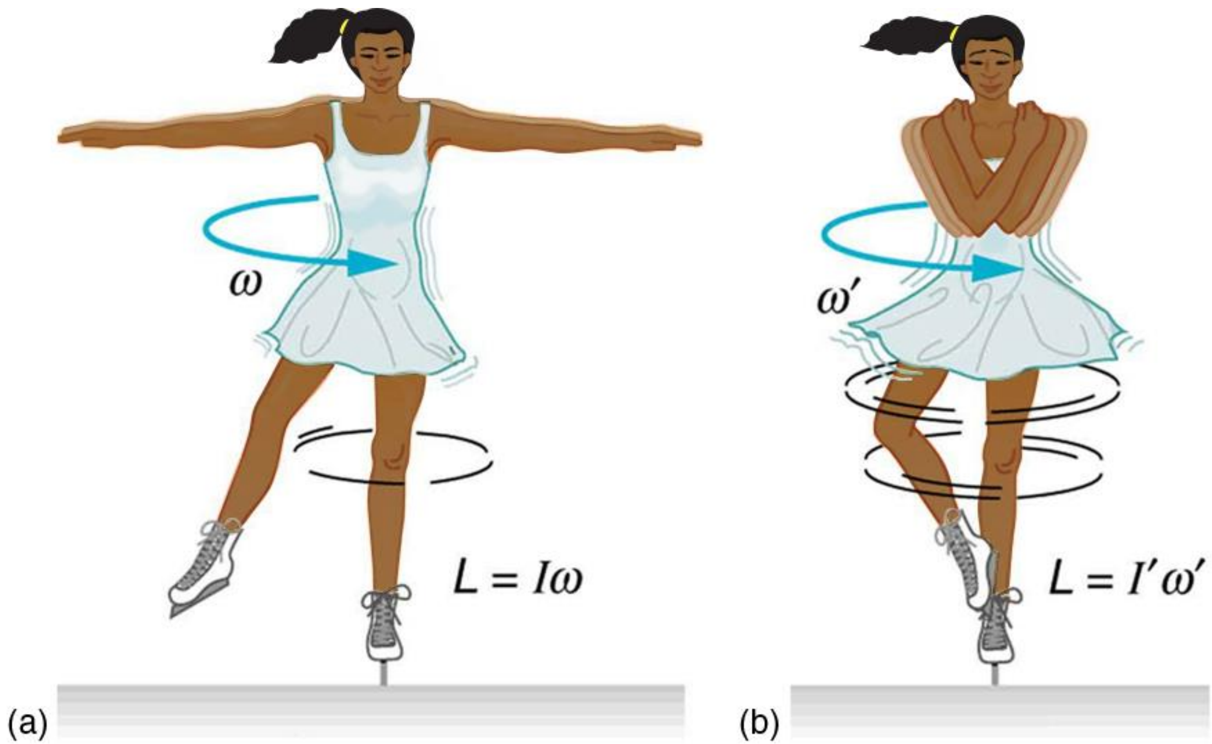
$$L = I\omega$$

We can then write the total **torque** as being the rate of **change** of angular **momentum**.

If the net torque on an object is zero, the total angular momentum is **constant**.

$$I\omega = I_0\omega_0 = \text{constant}$$

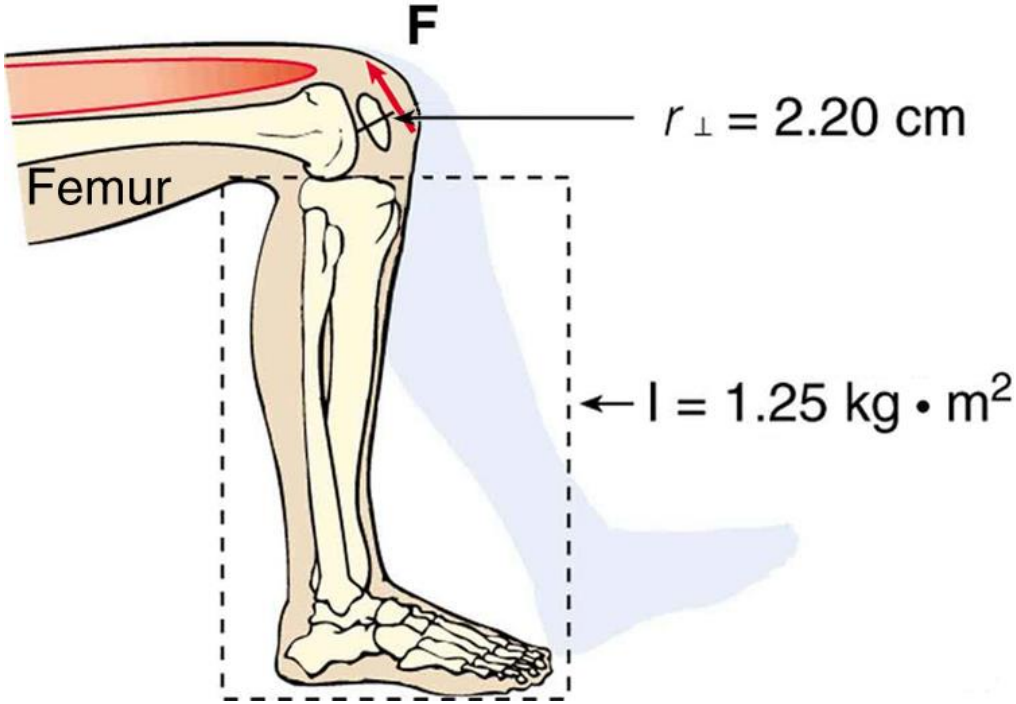
FIGURE 10.23



(a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small.

(b) In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

FIGURE 10.22



The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. **F** is a vector that is perpendicular to *r.*

Vector Nature of Angular Quantities

The **angular velocity** vector points along the **axis of rotation**; its direction is found using a **right hand rule**:

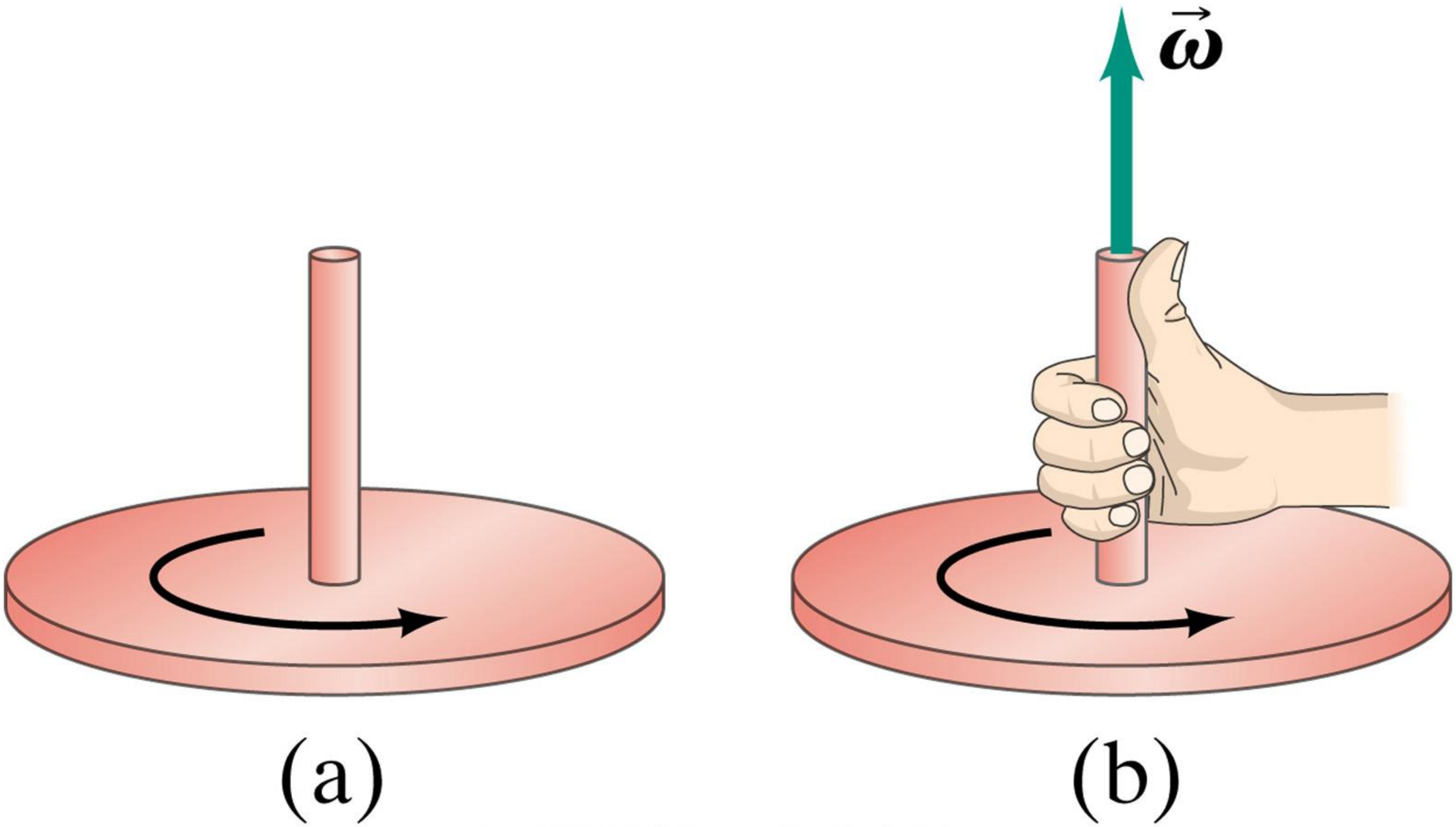


FIGURE 10.28

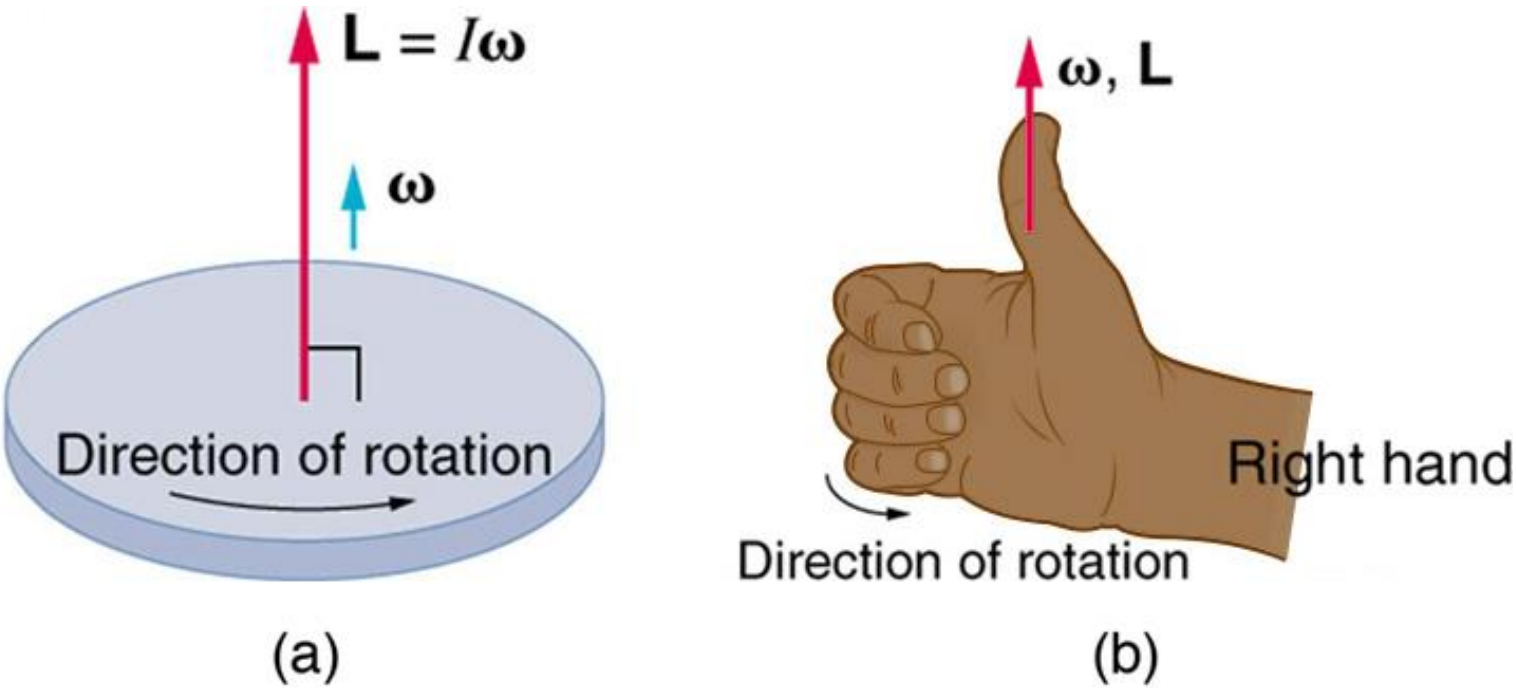
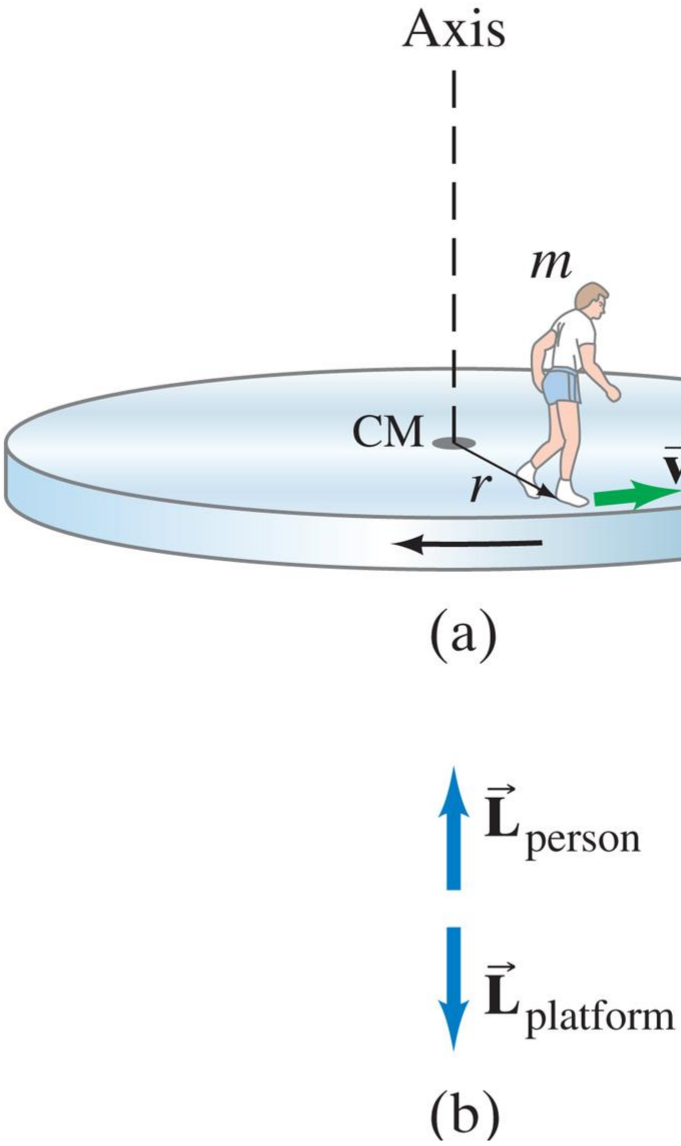


Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity **ω** size and angular momentum **L** are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk’s rotation as shown.

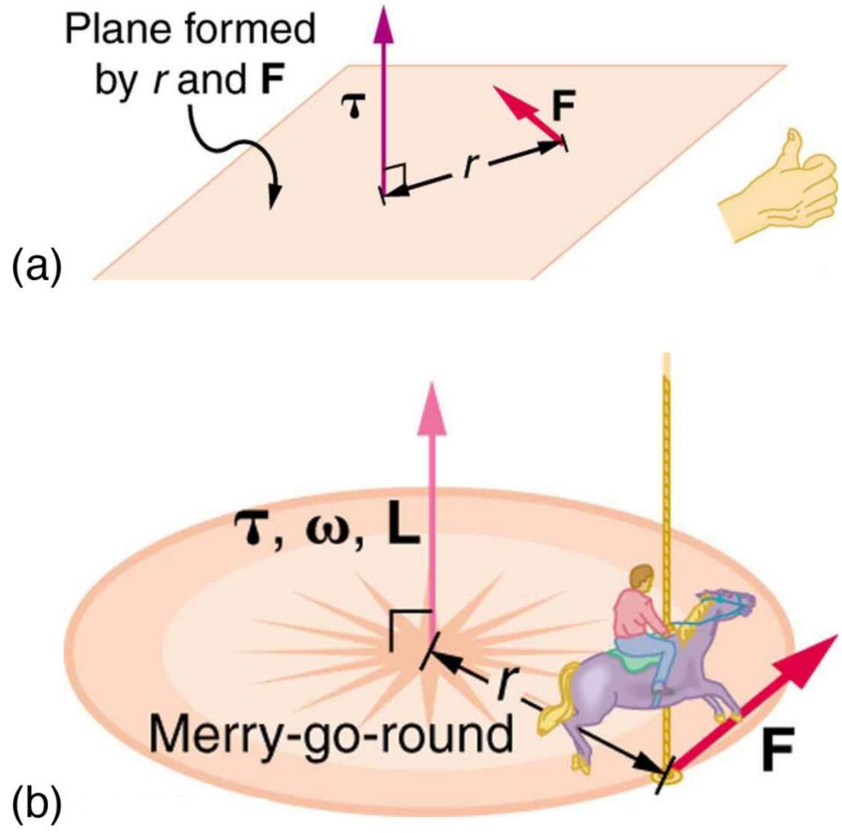
Vector Nature of Angular Quantities



Angular acceleration and angular momentum vectors also point along the axis of rotation.

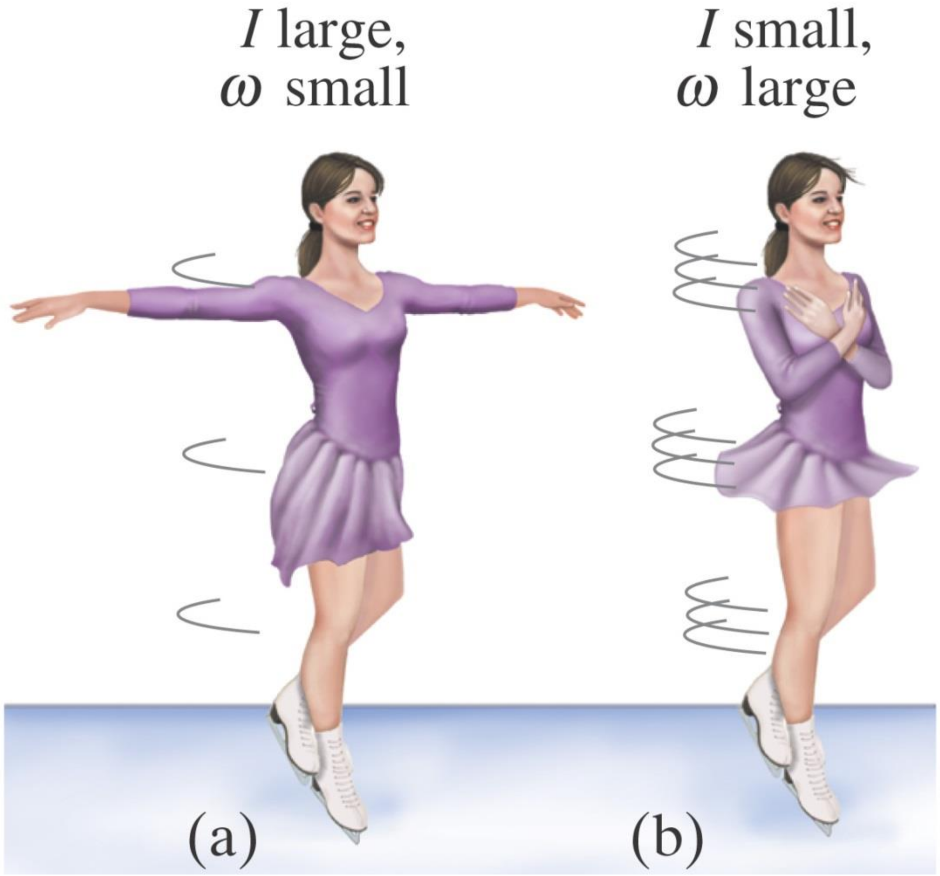
FIGURE 10.29

In figure (a), the torque is perpendicular to the plane formed by *r* and **F** and is the direction your right thumb would point to if you curled your fingers in the direction of **F** . Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.



Angular Momentum and Its Conservation

Therefore, systems that can **change** their rotational inertia through internal forces will also change their **rate** of rotation:



Chapter 10

- Angles are measured in radians; a whole circle is 2π radians.
- Angular velocity is the rate of change of angular position.
- Angular acceleration is the rate of change of angular velocity.
- The angular velocity and acceleration can be related to the linear velocity and acceleration.
- The frequency is the number of full revolutions per second; the period is the inverse of the frequency.

Summary of Chapter 10, cont.

- The equations for rotational motion with constant angular acceleration have the same form as those for linear motion with constant acceleration.
- Torque is the product of force and lever arm.
- The rotational inertia depends not only on the mass of an object but also on the way its mass is distributed around the axis of rotation.
- The angular acceleration is proportional to the torque and inversely proportional to the rotational inertia.

Summary of Chapter 10, cont.

- An object that is rotating has rotational kinetic energy. If it is translating as well, the translational kinetic energy must be added to the rotational to find the total kinetic energy.
- Angular momentum is $L = I\omega$
- If the net torque on an object is zero, its angular momentum does not change.