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4.3 #10, 11, 14, 15, 19

4.4 #1, 3, 5, 7

4.5 #3, 5, 7, 10, 11, 13, 15, 16, 17

4.6 #2, 3, 7, 8, 12, 14, 16

<u>4.3</u>

10) $\left[\begin{array}{ccccc|c} 1 & 0 & -5 & 1 & 4 & 0 \\ -2 & 1 & 6 & -2 & -2 & 0 \\ 0 & 2 & -8 & 1 & 9 & 0 \end{array}\right] \xrightarrow{R_2+2R_1} \left[\begin{array}{ccccc|c} 1 & 0 & -5 & 1 & 4 & 0 \\ 0 & 1 & -4 & 0 & 6 & 0 \\ 0 & 2 & -8 & 1 & 9 & 0 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccccc|c} 1 & 0 & -5 & 1 & 4 & 0 \\ 0 & 1 & -4 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \end{array}\right]$

free x_5 $\quad x_4 - 3x_5 = 0$ $\quad$ free x_3 $\quad x_2 - 4x_3 + 6x_5 = 0$ $\quad x_1 - 5x_3 + 3x_5 + 4x_5 = 0$

$x_4 = 3x_5$ $\quad x_2 = 4x_3 - 6x_5$ $\quad x_1 - 5x_3 + 7x_5 = 0$

$x_1 = 5x_3 - 7x_5$

$$\vec{x} = \begin{bmatrix} 5x_3 - 7x_5 \\ 4x_3 - 6x_5 \\ x_3 \\ 3x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}, \text{ basis} = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

11) $x + 2y + z = 0$

$x = -2y - z$

$y = y$

$z = z$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

14) Col A: $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 5 \\ -2 \end{bmatrix}$

$-9x_5 = 0$ $\quad$ free x_4 $\quad 5x_3 - 7x_4 + 0 = 0$ $\quad$ free x_2 $\quad x_1 + 2x_2 + 4x_4 + 0 = 0$

$x_5 = 0$ $\quad x_3 = \frac{7}{5}x_4$ $\quad x_1 = -2x_2 - 4x_4$

$$\vec{x} = \begin{bmatrix} -2x_2 - 4x_4 \\ x_2 \\ \frac{7}{5}x_4 \\ x_4 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix} \quad \text{Nul} A: \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix}$$

4.3 continued

15) $\begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ -3 & 2 & 1 & -8 & -6 \\ 2 & -3 & 6 & 7 & 9 \end{bmatrix} \xrightarrow[R_4-2R_1]{R_3+3R_1} \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 2 & -8 & -5 & 0 \\ 0 & -3 & 12 & 5 & 5 \end{bmatrix} \xrightarrow[R_4+3R_2]{R_3-2R_2} \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -4 & 8 \end{bmatrix} \xrightarrow{R_4+4R_3} \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

basis: $\{v_1, v_2, v_4\}$

19) $4v_1 + 5v_2 - 3v_3 = 0$

$4v_1 + 5v_2 = 3v_3$ basis: $\{v_1, v_2\}$

$\frac{4}{3}v_1 + \frac{5}{3}v_2 = v_3$

4.4

1) $5\begin{bmatrix} 3 \\ -5 \end{bmatrix} + 3\begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 15-12 \\ -25+18 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

3) $3\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 5 \\ 2 \\ -2 \end{bmatrix} + -1\begin{bmatrix} 4 \\ -7 \\ 0 \end{bmatrix} = \begin{bmatrix} 3-4 \\ -12+7 \\ 9 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ 9 \end{bmatrix}$

5) $b_1 + 2b_2 = -2$ $\quad b_1 + 2(-5) = -2$

$-3b_1 - 5b_2 = 1$ $\quad b_1 = 8$ $\quad [x]_B = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$

$b_2 = -5$

7) $b_1 - 3b_2 + 2b_3 = 8$

$-b_1 + 4b_2 - 2b_3 = -9$

$-3b_1 + 9b_2 + 4b_3 = 6$

$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ -1 & 4 & -2 & -9 \\ -3 & 9 & 4 & 6 \end{array}\right] \xrightarrow[R_3+3R_1]{R_2+R_1} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 10 & 30 \end{array}\right] \xrightarrow{R_3/10} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_1-R_3]{R_1+3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$[x]_B = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$

~~4.5~~

~~3)~~ $\cancel{a\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix}}$ ~~basis:~~ $\cancel{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix}}$ ~~dimensions: 3~~

~~5)~~ $\cancel{a\begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix} + b\begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix} + c\begin{bmatrix} -2 \\ -4 \\ 2 \\ 6 \end{bmatrix}}$ ~~basis:~~ $\cancel{\begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 2 \\ 6 \end{bmatrix}}$ ~~dimensions: 3~~

4.5

3) $\begin{bmatrix}0&0&2\\1&-1&0\\0&1&-3\\1&2&0\end{bmatrix} \xrightarrow{R_1 \Leftrightarrow R_2} \begin{bmatrix}1&-1&0\\0&0&2\\0&1&-3\\1&2&0\end{bmatrix} \xrightarrow{R_4 - R_1} \begin{bmatrix}1&-1&0\\0&0&2\\0&1&-3\\0&3&0\end{bmatrix} \xrightarrow{R_4 \Leftrightarrow R_2} \begin{bmatrix}1&-1&0\\0&3&0\\0&1&-3\\0&0&2\end{bmatrix} \xrightarrow{R_4 \Leftrightarrow R_3} \begin{bmatrix}1&-1&0\\0&3&0\\0&0&2\\0&1&-3\end{bmatrix}$ $R_2/3$, $R_3/2$

$\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\\0&1&-3\end{bmatrix} \xrightarrow[R_4 - R_2 + 3R_3]{R_1 + R_2} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$

basis: $\begin{bmatrix}0\\1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\0\\-3\\0\end{bmatrix}$ dimensions: 3

5) $\begin{bmatrix}1&-4&-2\\2&5&-4\\-1&0&2\\-3&7&6\end{bmatrix} \xrightarrow{\substack{R_2-2R_1\\R_3+R_1\\R_4+3R_1}} \begin{bmatrix}1&-4&-2\\0&13&0\\0&-4&0\\0&-5&0\end{bmatrix} \xrightarrow{R_2/13} \begin{bmatrix}1&-4&-2\\0&1&0\\0&-4&0\\0&-5&0\end{bmatrix} \xrightarrow{\substack{R_3+4R_2\\R_4+5R_2}} \begin{bmatrix}1&-4&-2\\0&1&0\\0&0&0\\0&0&0\end{bmatrix}$

basis: $\begin{bmatrix}1\\2\\-1\\-3\end{bmatrix}, \begin{bmatrix}-4\\5\\0\\7\end{bmatrix}$ dimensions: 2

7) $\left[\begin{array}{ccc|c}1&-3&1&0\\0&1&-2&0\\0&2&-1&0\end{array}\right] \xrightarrow[R_3-2R_1]{R_1+3R_2} \left[\begin{array}{ccc|c}1&0&-5&0\\0&1&-2&0\\0&0&3&0\end{array}\right] \xrightarrow{R_3/3} \left[\begin{array}{ccc|c}1&0&-5&0\\0&1&-2&0\\0&0&1&0\end{array}\right]$

$x_3 = 0$ $\quad R_2 - 0 = 0$ $\quad R_1 - 5(0) = 0$ $\quad \vec{x} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ not a basis, dimensions: 0

$x_2 = 0$ $\quad R_1 = 0$

10) ~~$\begin{bmatrix}2&-4&-3\\5&10&6\end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix}2&-4&-3\\1&18&12\end{bmatrix} \xrightarrow{R_1-2R_2} \begin{bmatrix}0&-40&-27\\1&18&12\end{bmatrix} \xrightarrow{R_1/9} \begin{bmatrix}0&-\frac{40}{9}&-3\\1&18&12\end{bmatrix} \xrightarrow{R_2+4R_1}$~~

$\begin{bmatrix}2&-4&-3\\-5&10&6\end{bmatrix} \xrightarrow{R_1/2} \begin{bmatrix}1&-2&-\frac{3}{2}\\-5&10&6\end{bmatrix} \xrightarrow{R_2+5R_1} \begin{bmatrix}1&-2&-\frac{3}{2}\\0&0&-\frac{3}{2}\end{bmatrix} \xrightarrow{R_2/-\frac{3}{2}} \begin{bmatrix}1&-2&-\frac{3}{2}\\0&0&1\end{bmatrix}$

Two dimensions

11) $\begin{bmatrix}1&3&9&-7\\0&1&4&-3\\2&1&-2&1\end{bmatrix} \xrightarrow{R_3-2R_1} \begin{bmatrix}1&3&9&-7\\0&1&4&-3\\0&-5&-20&15\end{bmatrix} \xrightarrow{R_3+5R_2} \begin{bmatrix}1&3&9&-7\\0&1&4&-3\\0&0&0&0\end{bmatrix}$ 2 dimensions

13) 2 free variables $\therefore$ Nul A = 2 ; 3 pivot points $\therefore$ Col A = 3

15) 2 free variables $\therefore$ Nul A = 2 ; 2 pivot points $\therefore$ Col A = 2

16) $\begin{bmatrix}3&4\\-6&10\end{bmatrix} \xrightarrow{R_2+2R_1} \begin{bmatrix}3&4\\0&18\end{bmatrix}$ No free variables $\therefore$ Nul A = 0 ; 2 pivot points $\therefore$ Col A = 2

17) No free variables $\therefore$ Nul A = 0 ; 3 pivot points $\therefore$ Col A = 3

4.6

2) Rank 3, dim Nul A = 2

$\text{Col } A = \begin{bmatrix}1\\-2\\-3\\3\end{bmatrix}, \begin{bmatrix}4\\-6\\-6\\4\end{bmatrix}, \begin{bmatrix}9\\-10\\-3\\0\end{bmatrix}$ Row $A = (1,-3,0,5,-7), (0,0,2,-3,8), (0,0,0,0,5)$

$5x_5 = 0$ free x_4 $2x_3 - 3x_4 + 0 = 0$ free x_2 $x_1 - 3x_2 + 5x_4 - 0 = 0$

$x_5 = 0$ $x_3 = \frac{3}{2}x_4$ $x_1 = 3x_2 - 5x_4$

$$\vec{x} = \begin{bmatrix}3x_2 - 5x_4\\ x_2\\ \frac{3}{2}x_4\\ x_4\\ 0\end{bmatrix} = x_2\begin{bmatrix}3\\1\\0\\0\\0\end{bmatrix} + x_4\begin{bmatrix}-5\\0\\3/2\\1\\0\end{bmatrix} \quad \text{Nul } A = \begin{bmatrix}3\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-5\\0\\3/2\\1\\0\end{bmatrix}$$

3) Rank 3, dim Nul A = 2

$\text{Col } A = \begin{bmatrix}2\\-2\\4\\-2\end{bmatrix}, \begin{bmatrix}6\\-3\\9\\3\end{bmatrix}, \begin{bmatrix}2\\-3\\5\\-4\end{bmatrix}$ Row $A = (2,-3,6,2,5), (0,0,3,-1,1), (0,0,0,1,3)$

free x_5 $x_4 = -3x_5$ $3x_3 + 3x_5 + x_5 = 0$ free x_2 $2x_1 - 3x_2 + 6(\frac{-4}{3}x_5) + 2(-3x_5) + 5x_5 = 0$

$3x_3 = -4x_5$ $2x_1 - 3x_2 - 8x_5 - 6x_5 + 5x_5 = 0$

$x_3 = \frac{-4}{3}x_5$ $2x_1 - 3x_2 - 9x_5 = 0$

$2x_1 = 3x_2 + 9x_5$

$x_1 = \frac{3}{2}x_2 + \frac{9}{2}x_5$

$$\vec{x} = \begin{bmatrix}\frac{3}{2}x_2 + \frac{9}{2}x_5\\ x_2\\ \frac{-4}{3}x_5\\ -3x_5\\ x_5\end{bmatrix} = x_2\begin{bmatrix}\frac{3}{2}\\1\\0\\0\\0\end{bmatrix} + x_5\begin{bmatrix}\frac{9}{2}\\0\\\frac{-4}{3}\\-3\\1\end{bmatrix} \quad \text{Nul } A = \begin{bmatrix}\frac{3}{2}\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}\frac{9}{2}\\0\\\frac{-4}{3}\\-3\\1\end{bmatrix}$$

7) Yes, no. 4 pivot points $\therefore$ 4 vectors in col A $\therefore$ in $\mathbb{R}^4$; its just a 3-D subspace of $\mathbb{R}^7$, not $\mathbb{R}^3$

8) dim Nul A = 2. No, it is a 4D subspace of $\mathbb{R}^5$

12) 2

14) 3, 3. pivots cannot be > than number of Rows/columns

16) 0