LECTURE NO. 23

7.1 Parametric Equations

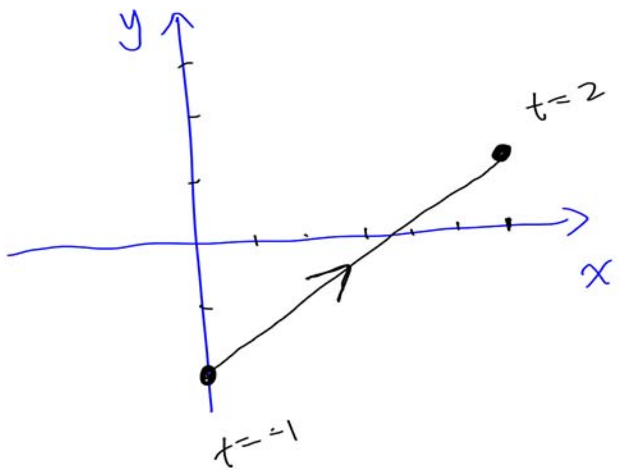
Wright State University

Parametric Equations

- A function $y = f(x)$ with $a \le x \le b$ defines a curve in the 2-dim plane. For examples, $y = 2x^2$ with $0 \le x \le 2$ is part of a parabola.

- Parametric Equations give another way to define a curve in the 2-dim plane, in which each of x and y is a continuous function of the parameter t.

- Example: $x = 2t + 2$, $y = t - 1$, $-1 \le t \le 2$.



$t = -1$

$x = -2+2 = 0$

$y = -1-1 = -2$

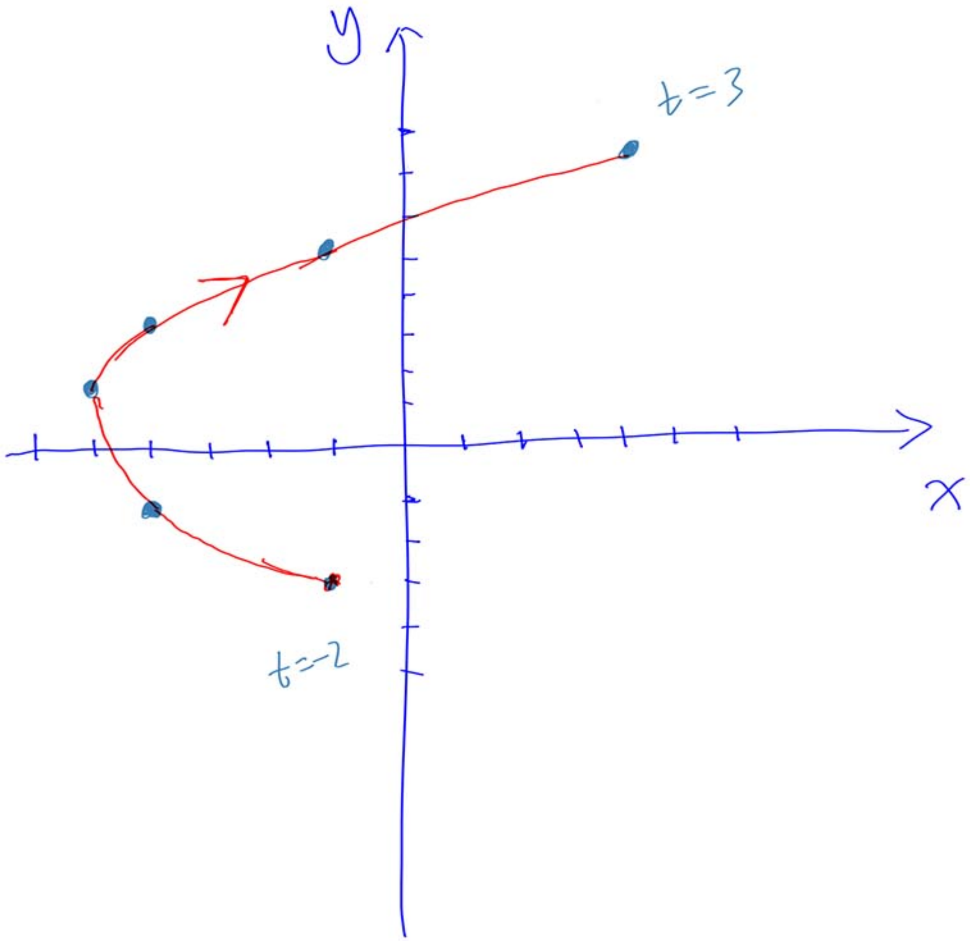
$t = 2$

$x = 4+2 = 6$

$y = 2-1 = 1$

Graph the parametric Equation $x = t^2 - 5$, $y = 2t + 1$, $-2 \leq t \leq 3$

t	x	y
-2	-1	-3
-1	-4	-1
0	-5	1
1	-4	3
2	-1	5
3	4	7

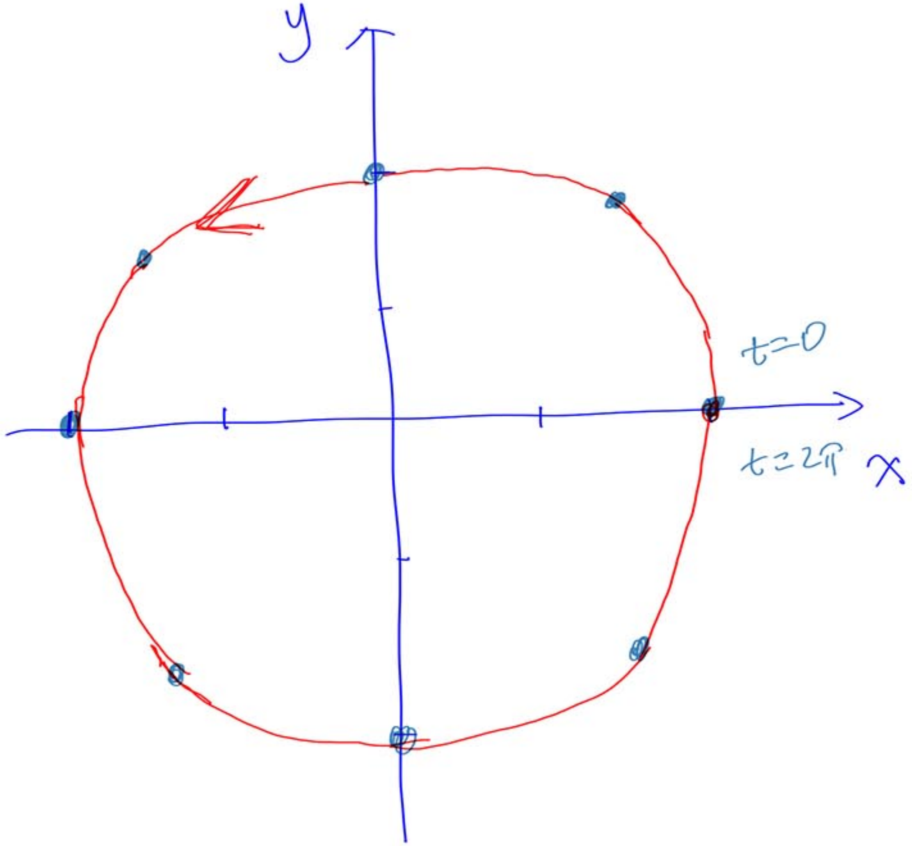


Graph the parametric Equation $x = 2\cos t,\ y = 2\sin t,\ 0 \leq t \leq 2\pi$

(Radian)

t	$x = 2\cos t$	$y = 2\sin t$
0	2	0
$\frac{\pi}{4}$	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{2}$	0	2
$\frac{3\pi}{4}$	$-\sqrt{2}$	$\sqrt{2}$
π	-2	0
$\frac{5\pi}{4}$	$-\sqrt{2}$	$-\sqrt{2}$
$\frac{3\pi}{2}$	0	-2
$\frac{7\pi}{4}$	$\sqrt{2}$	$-\sqrt{2}$

$t = 2\pi$



It may be useful to eliminate the parameter t

$x = t^2 - 3,\ y = 2t + 1,\ -2 \le t \le 3$

$t = -2 \quad y = -4 + 1 = -3$

$t = 3 \quad y = 2 \cdot 3 + 1 = 7$

Solve for t from $y = 2t + 1$

$$y - 1 = 2t$$

$$t = \frac{1}{2}(y-1)$$

now plug in $t = \frac{1}{2}(y-1)$ into the x- equation

$$x = \left[\frac{1}{2}(y-1)\right]^2 - 3$$

$$x = \frac{1}{4}(y-1)^2 - 3$$

a parabola!

Here $-3 \le y \le 7$

Eliminate the parameter t in $x = 2\cos t$, $y = 3\sin t$, $0 \leq t \leq 2\pi$

$$\sin^2 t + \cos^2 t = 1$$

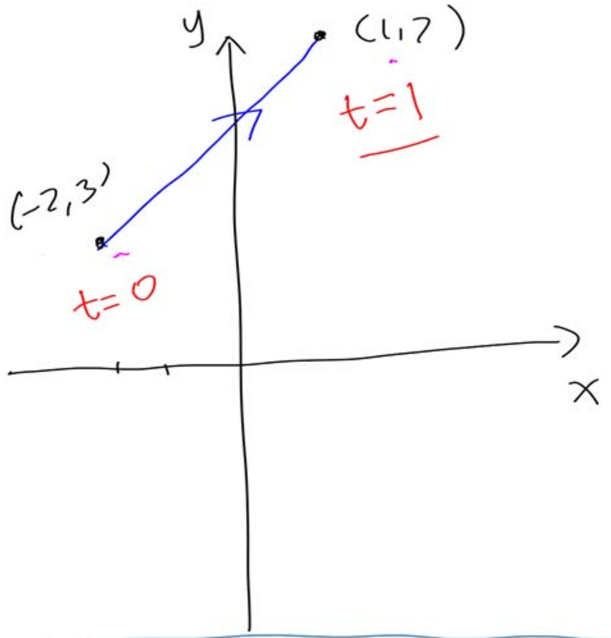
$x = 2\cos t \Rightarrow \cos t = \frac{x}{2}$ $\qquad$ $y = 3\sin t$ $\qquad$ $\sin t = \frac{y}{3}$

$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

<u>ellipse</u> !

Find a parametric equation for the line segment from $(-2,3)$ to $(1,7)$



each x and y is a linear function of t

$$x = mt + b$$
$$y = nt + c$$

Easy way $t=0$ for $(-2,3)$ (starting point)
$t=1$ for $(1,7)$ (ending point)

$b=-2, \quad c=3$

$$x = mt - 2$$
$$y = nt + 3$$

$t=1$

$x = m-2 = 1 \Rightarrow m=3$

$y = n+3 = 7 \Rightarrow n=4$

$x=3t-2, \quad y=4t+3$

Remark

$$x = 3t - 2$$
$$y = 4t + 3$$