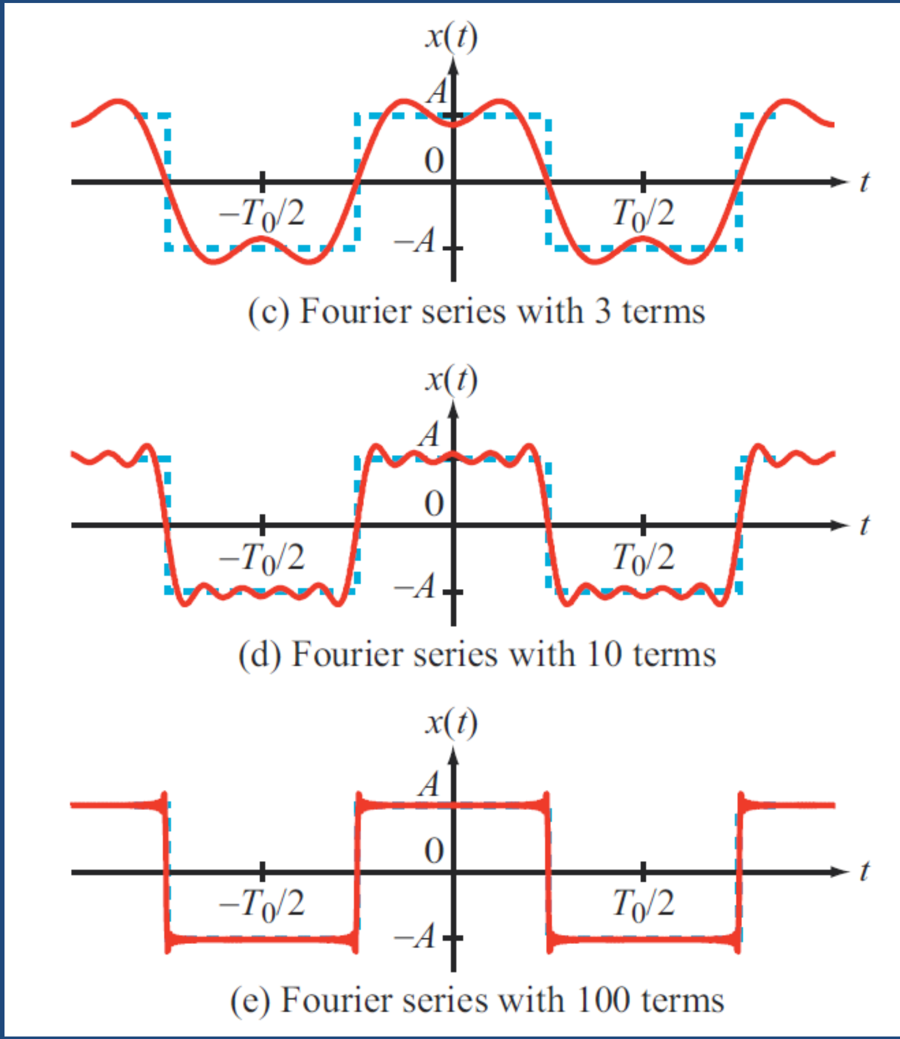
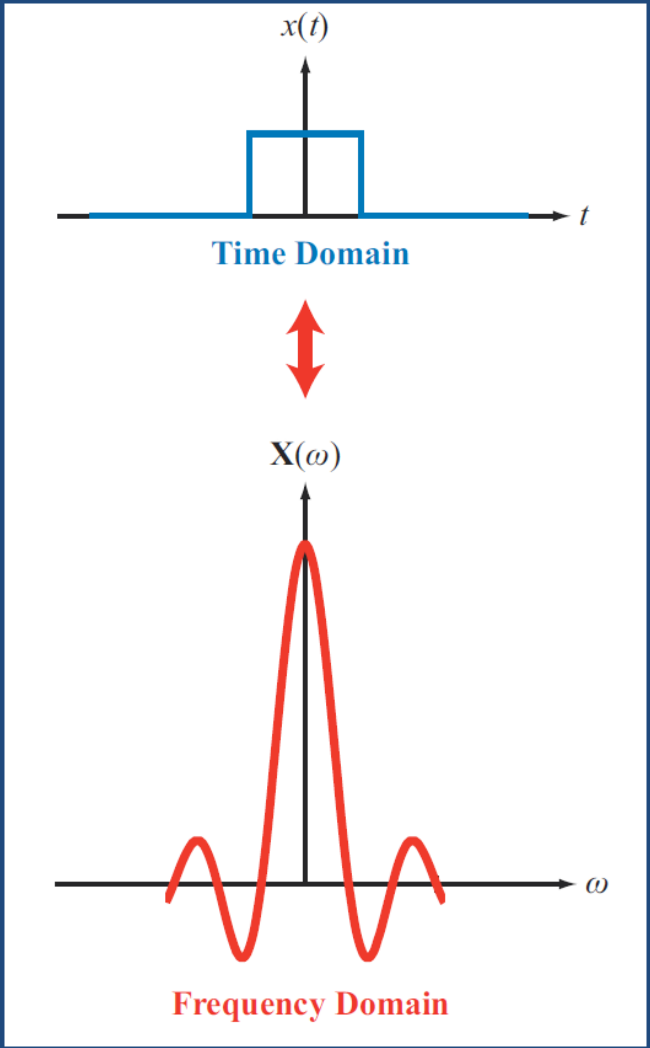




5. FOURIER ANALYSIS TECHNIQUE

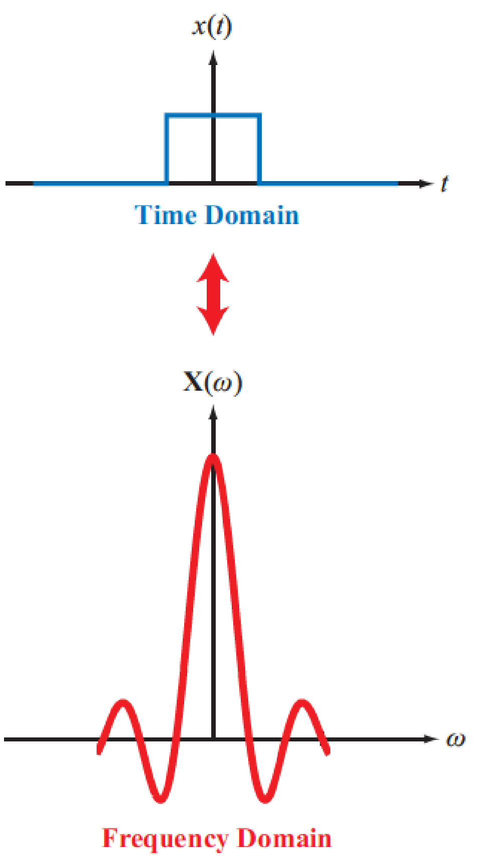
5 Fourier Analysis Techniques

Contents

Objectives

Learn to:

- Apply the phasor-domain technique to analyze systems driven by sinusoidal excitations.
- Express periodic signals in terms of Fourier series.
- Use Fourier series to analyze systems driven by continuous periodic signals.
- Apply Parseval's theorem to compute the power or energy contained in a signal.
- Compute the Fourier transform of nonperiodic signals and use it to analyze the system response to nonperiodic excitations.



Time-domain signals have ***frequency domain spectra***. Because many analysis and design projects are easier to work with in the frequency domain, the ability to easily ***transform signals and systems*** back and forth between the two domains will prove invaluable in succeeding chapters.

Phasor Analysis: Basics

- If an LTI system is described by a differential equation with an eternal sinusoidal input, then phasor analysis is a simple procedure for computing the system response
- The system response is also an eternal sinusoidal signal
- The phasor associated with signal $\upsilon(t) = V_0 \cos(\omega t + \phi)$ is the complex number $\mathbf{V} = V_0 e^{j\phi}$

$$\upsilon(t) = V_0 \cos(\omega t + \phi) \longleftrightarrow \mathbf{V} = V_0 e^{j\phi}$$

- If $\mathbf{X} = |\mathbf{X}| e^{j\phi}$ then $\mathfrak{Re}[|\mathbf{X}| e^{j\phi} e^{j\omega t}] = |\mathbf{X}| \cos(\omega t + \phi)$

Phasor Analysis: Basics

- The effects of differentiation and integration are:

$x(t)$		$\mathbf{X}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j\phi}$
$-A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi\pm\pi)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}=-jA$
$A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi-\pi/2)}$
$-A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi+\pi/2)}$
$\frac{d}{dt}[A\cos(\omega t+\phi)]$	$\leftrightarrow$	$j\omega Ae^{j\phi}$
$\int A\cos(\omega t'+\phi)\,dt'$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j\phi}$

$$i(t)=\Re\mathfrak{e}[\mathbf{I}e^{j\omega t}]$$

$$\frac{di}{dt} \leftrightarrow j\omega\mathbf{I}.$$

$$\int i\,dt' \leftrightarrow \frac{\mathbf{I}}{j\omega}$$

- Use these properties to convert manipulations of sines and cosines into manipulations of complex numbers

Phasor Analysis: Example

- Use phasor analysis to solve the differential equation (a=300 and b=50,000)

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = 10\sin(100t + 60^\circ)$$

- The phasor associated with the input is $\mathbf{X} = 10e^{-j30^\circ}$
- The differential equation becomes algebraic

$$(j\omega)^2\mathbf{Y} + j\omega a\mathbf{Y} + b\mathbf{Y} = 10e^{-j30^\circ}$$

- The phasor associated with the output is then found as

$$\mathbf{Y} = \frac{10e^{-j30^\circ}}{b - \omega^2 + j\omega a} = \frac{10e^{-j30^\circ}}{10^4(4 + j3)} = \frac{10^{-3}e^{-j30^\circ}}{5e^{j36.87^\circ}} = 0.2 \times 10^{-3}e^{-j66.87^\circ}$$

- The response is then $y(t) = 0.2 \times 10^{-3}\cos(100t - 66.87^\circ)$

Fourier Series Expansions of Periodic Signals

- Let x(t) be periodic: $x(t) = x(t + nT_0)$
- Then x(t) can be expressed as a linear combination of sinusoids at harmonic frequencies:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \qquad (5.26a)$$

(sine/cosine representation)

$$= c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n) \qquad (5.26b)$$

(amplitude/phase representation)

Fourier Series Expansions of Periodic Signals

- Let x(t) be periodic: $x(t) = x(t + nT_0)$
- Now x(t) need not be real-valued
- Then x(t) can be expressed as a linear combination of complex exponentials at harmonic frequencies:

$$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t},$$

(exponential representation)

Fourier Series Expansions of Periodic Signals

- Fundamental angular frequency: $\omega_0 = 2\pi / T_0$
- dc or average term: a_0 or c_0
- Fundamental term: $c_1 \cos(\omega_0 t + \phi_1)$
- Harmonics: $c_n \cos(n\omega_0 t + \phi_n)$
- Fundamental has same period as x(t)
- In music: harmonics are known as overtones
- $\{a_n, b_n, c_n, \mathbf{x}_n\}$ are Fourier coefficients

Fourier Series Expansions of Periodic Signals

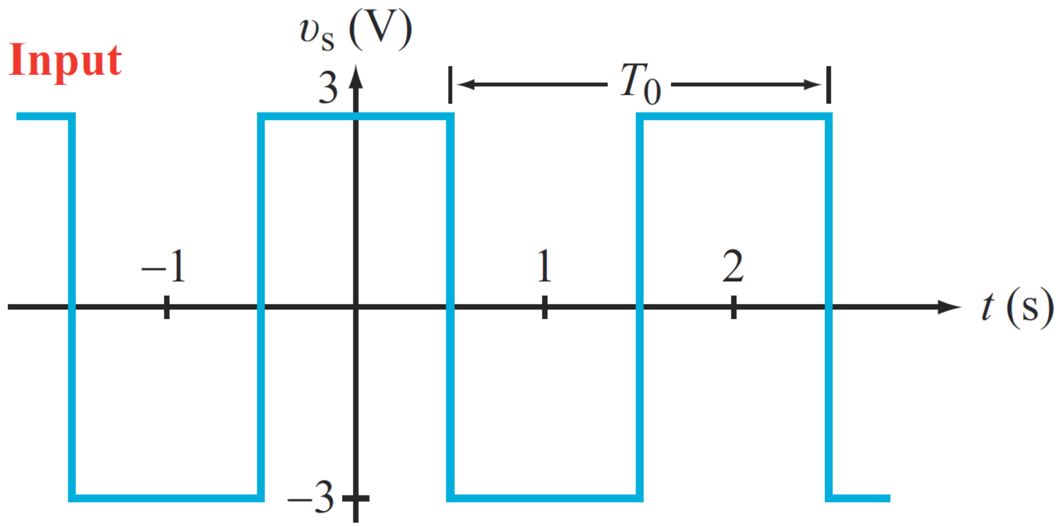
1. A Fourier series is a mathematical version of a ***prism***; it breaks up a signal into different frequencies, just as a prism (or diffraction grating) breaks up light into different colors (which are light at different frequencies).

2. A Fourier series is a mathematical depiction of ***adding overtones*** to a basic note to give a richer and fuller sound. It can also be used as a formula for *synthesis* of sounds and tones.

3. A Fourier series is a representation of $x(t)$ in terms of ***orthogonal functions***.

Fourier Series Example

The Fourier series expansion of the periodic signal

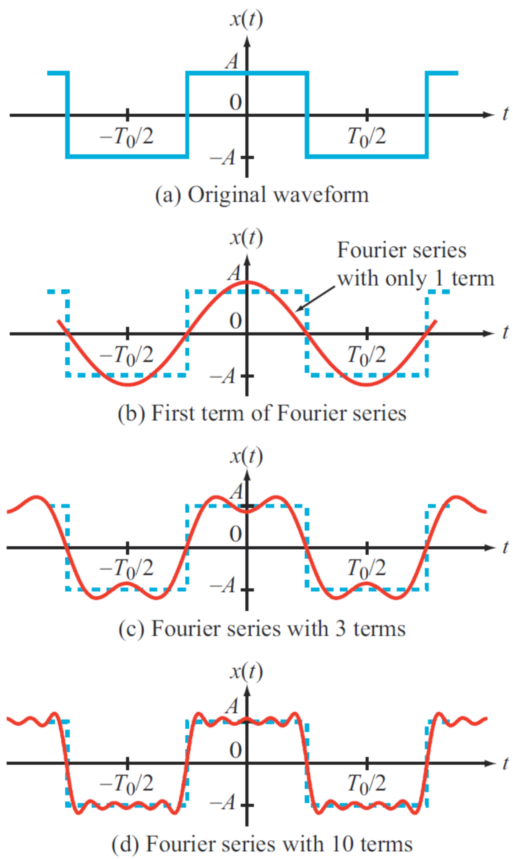


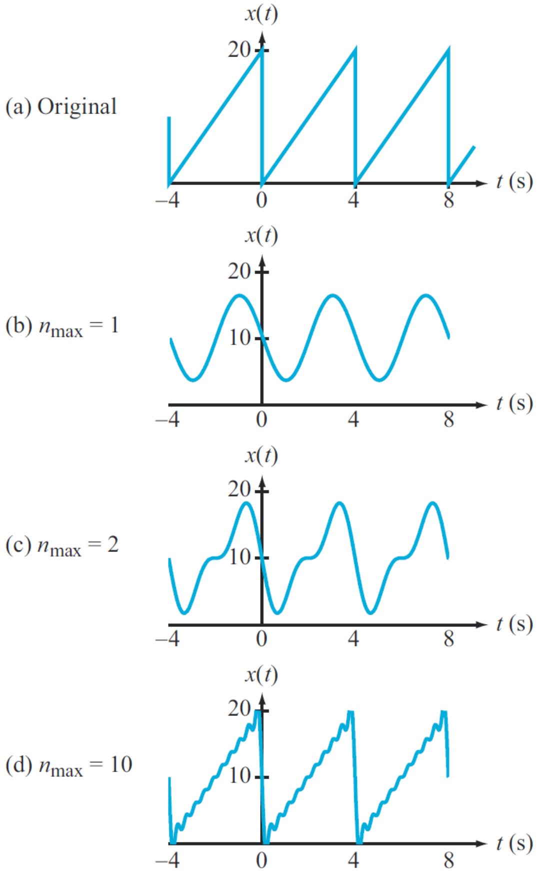
is the infinite series

$$\upsilon_s(t) = \frac{12}{\pi}\left(\cos \omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \cdots\right)$$

Fourier Series Expansions of Periodic Signals

Adding more terms makes the Fourier series resemble more closely the original signal:





Three Forms of Fourier Series Expansion:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]. \tag{5.27}$$

(sine/cosine representation)

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n),$$

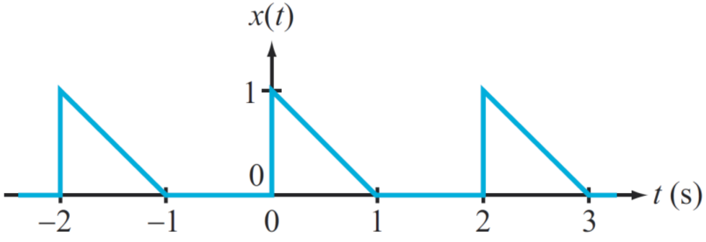
(amplitude/phase representation)

$$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t},$$

(exponential representation)

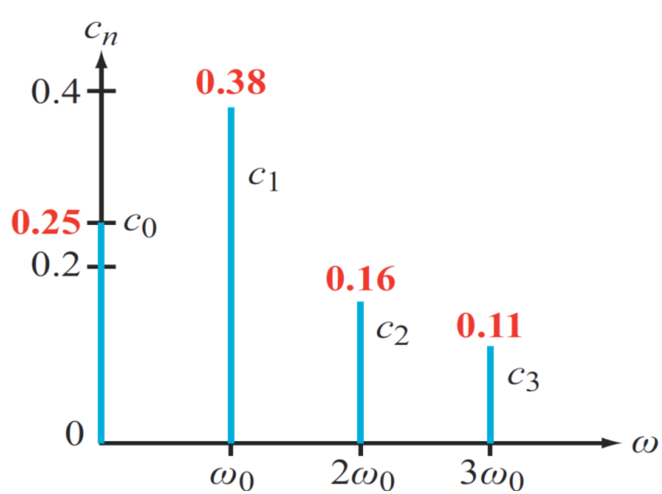
One-Sided Line Spectrum of x(t)

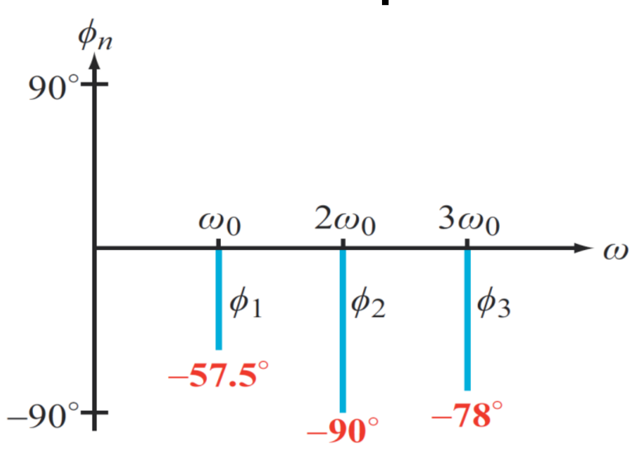
- Plot of amplitudes c_n and phases ϕ_n vs. frequency ω.
- Example: x(t) has period=2.



$$c_n = \begin{cases} \left(\dfrac{4}{n^4\pi^4} + \dfrac{1}{n^2\pi^2}\right)^{1/2} & \text{for } n = \text{odd}, \\ \dfrac{1}{n\pi} & \text{for } n = \text{even} \end{cases}$$

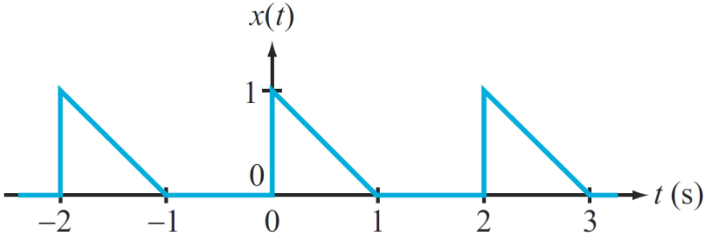
- Amplitude spectrum: Phase spectrum:





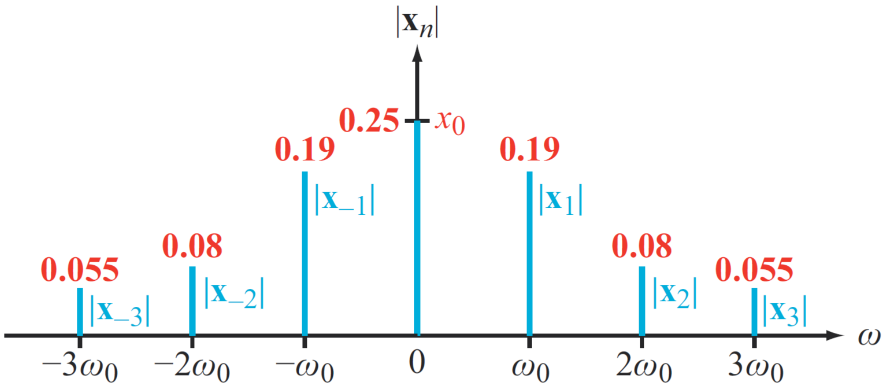
Two-Sided Line Spectrum of x(t)

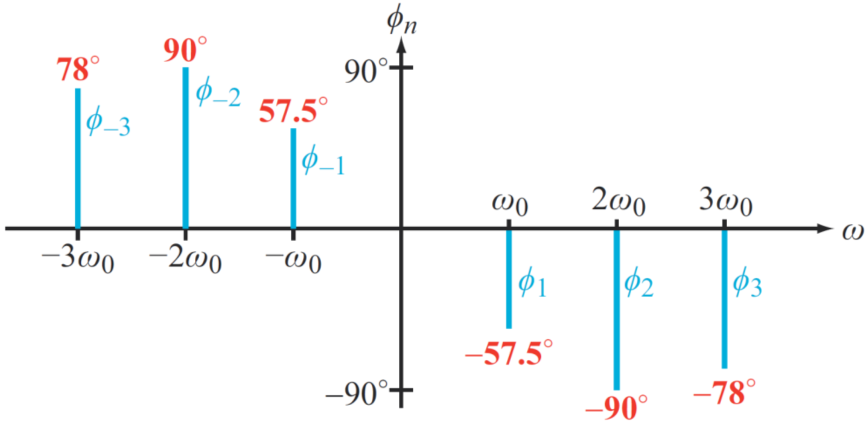
- Plot of magnitudes $|\mathbf{x}_n|$ and phases ϕ_n vs. frequency ω .
- Example: x(t) has period=2.



$$c_n = \begin{cases} \left(\dfrac{4}{n^4\pi^4} + \dfrac{1}{n^2\pi^2}\right)^{1/2} & \text{for } n = \text{odd}, \\ \dfrac{1}{n\pi} & \text{for } n = \text{even} \end{cases}$$

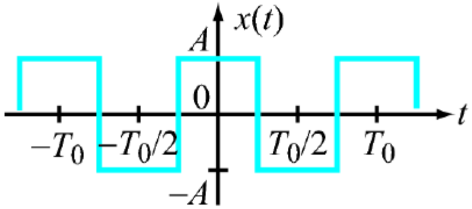
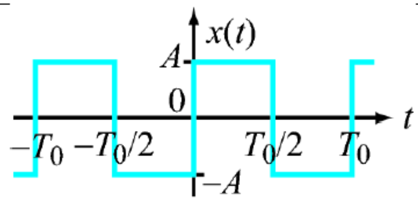
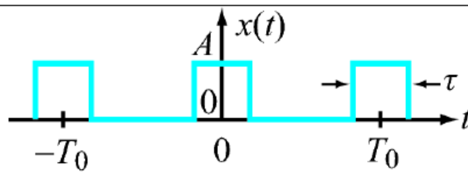
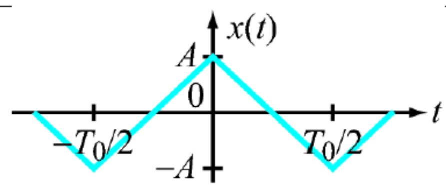
- Magnitude spectrum: Phase spectrum:





Fourier Series Expansions for Some General Forms of Periodic Signals

Table 5-4: Fourier series expressions for a select set of periodic waveforms.

	Waveform	Fourier Series
1. Square Wave		$x(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos\left(\frac{2n\pi t}{T_0}\right)$
2. Time-Shifted Square Wave		$x(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4A}{n\pi} \sin\left(\frac{2n\pi t}{T_0}\right)$
3. Pulse Train		$x(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) \cos\left(\frac{2n\pi t}{T_0}\right)$
4. Triangular Wave		$x(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{8A}{n^2\pi^2} \cos\left(\frac{2n\pi t}{T_0}\right)$

Computing Coefficients of Fourier Series: Sine/Cosine Form of Fourier Series Expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)].$$

(sine/cosine representation) (5.27)

Compute coefficients from a single period of x(t) using the formulae

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt,$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} x(t)\, \cos(n\omega_0 t)\, dt,$$

and $$b_n = \frac{2}{T_0} \int_0^{T_0} x(t)\, \sin(n\omega_0 t)\, dt.$$

Computing Coefficients of Fourier Series: Amplitude/Phase Form of Fourier Series Expansion

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n),$$

(amplitude/phase representation)

Compute coefficients from a single period of x(t) using the formula

$$c_n = \sqrt{a_n^2 + b_n^2}$$

and

$$\phi_n = \begin{cases} -\tan^{-1}\left(\dfrac{b_n}{a_n}\right), & a_n > 0 \\ \pi - \tan^{-1}\left(\dfrac{b_n}{a_n}\right), & a_n < 0 \end{cases}$$

Computing Coefficients of Fourier Series: Exponential Form of Fourier Series Expansion

$$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t},$$

(exponential representation).

Compute coefficients from a single period of x(t) using the formula

$$\mathbf{x}_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt.$$

Note this is much simpler than using the other two representations.

Summary of the Three Fourier Series Representations: Relations Between the Coefficients of the Representations

Table 5-3: Fourier series representations for a real-valued periodic function $x(t)$.

Cosine/Sine	**Amplitude/Phase**	**Complex Exponential**
$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$	$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n)$	$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t}$
$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt$	$c_n e^{j\phi_n} = a_n - jb_n$	$\mathbf{x}_n = \lvert\mathbf{x}_n\rvert e^{j\phi_n};\ \mathbf{x}_{-n} = \mathbf{x}_n^*;\ \phi_{-n} = -\phi_n$
$a_n = \frac{2}{T_0} \int_0^{T_0} x(t)\, \cos n\omega_0 t\, dt$	$c_n = \sqrt{a_n^2 + b_n^2}$	$\lvert\mathbf{x}_n\rvert = c_n/2;\ x_0 = c_0$
$b_n = \frac{2}{T_0} \int_0^{T_0} x(t)\, \sin n\omega_0 t\, dt$	$\phi_n = \begin{cases} -\tan^{-1}(b_n/a_n), & a_n > 0 \\ \pi - \tan^{-1}(b_n/a_n), & a_n < 0 \end{cases}$	$\mathbf{x}_n = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$
$a_0 = c_0 = x_0;\ a_n = c_n \cos\phi_n;\ b_n = -c_n \sin\phi_n;\ \mathbf{x}_n = \frac{1}{2}(a_n - jb_n)$		

Symmetry Simplifies Computation

Even Symmetry: $x(t) = x(-t)$

$$a_0 = \frac{2}{T_0} \int_0^{T_0/2} x(t)\, dt$$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t)\, \cos(n\omega_0 t)\, dt$$

$$b_n = 0$$

$$c_n = |a_n|, \qquad \phi_n = \begin{cases} 0 & \text{if } a_n > 0 \\ 180^\circ & \text{if } a_n < 0 \end{cases}$$

Odd Symmetry: $x(t) = -x(-t)$

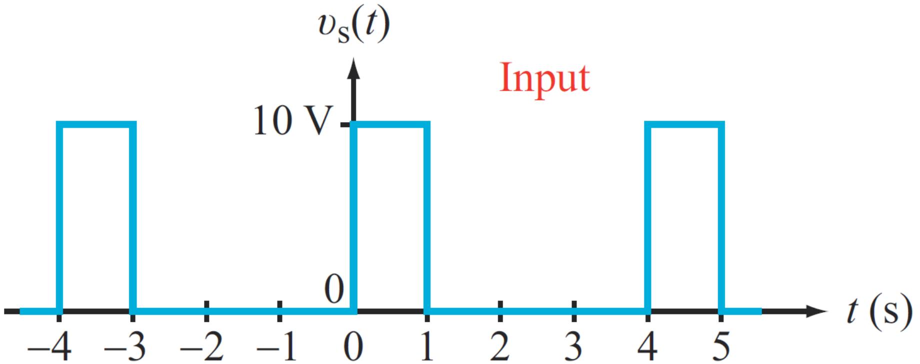
$$a_0 = 0 \qquad a_n = 0$$

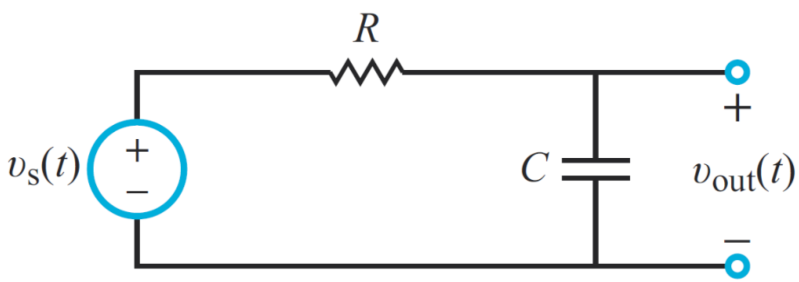
$$b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t)\, \sin(n\omega_0 t)\, dt$$

$$c_n = |b_n| \qquad \phi_n = \begin{cases} -90^\circ & \text{if } b_n > 0 \\ 90^\circ & \text{if } b_n < 0 \end{cases}$$

Circuit Analysis using Fourier Series

- Compute the voltage response of the RC circuit shown to the input square wave shown, using Fourier series. The time constant RC of the circuit is RC=2 s.





Circuit Analysis using Fourier Series

- The input waveform has the Fourier series expansion

$$v_s(t) = a_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \phi_n)$$

$$c_n \angle \phi_n = a_n - jb_n = \frac{10}{n\pi}\left[\sin\frac{n\pi}{2} - j\left(1 - \cos\frac{n\pi}{2}\right)\right]$$

- The RC circuit has the transfer function (RC=2 s)

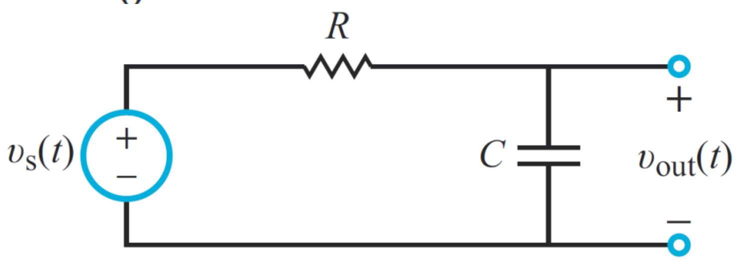
$$\mathbf{H}(\omega) = \frac{1}{\sqrt{1+4\omega^2}}\, e^{-j\tan^{-1}(2\omega)}$$

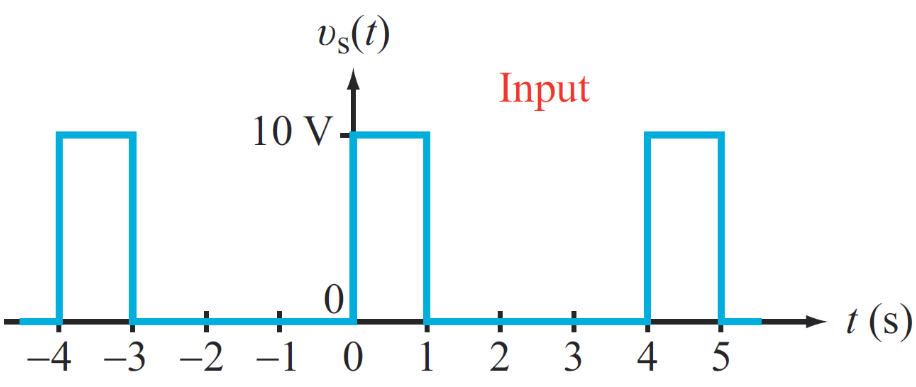
Circuit Analysis using Fourier Series

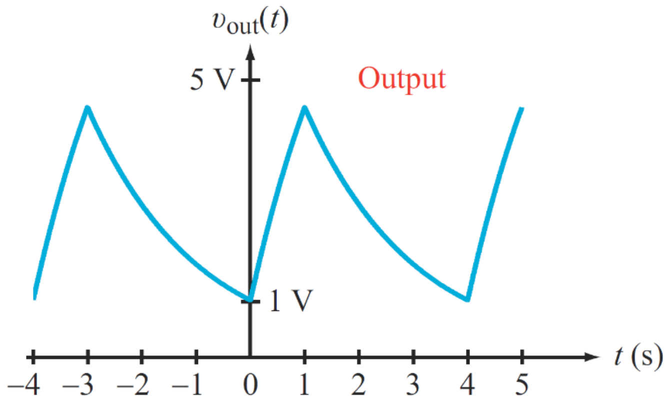
- The output waveform has the Fourier series expansion

$$v_{\text{out}}(t) = 2.5 + \sum_{n=1}^{\infty} \mathfrak{Re}\left\{ c_n \frac{1}{\sqrt{1+4n^2\omega_0^2}} e^{j[n\omega_0 t + \phi_n - \tan^{-1}(2n\omega_0)]} \right\}$$

- Plot of the output waveform







Parseval's Theorem for Fourier Series:

Average Power is the same whether computed in the time domain or frequency domain

$$P_x = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt = a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)/2, \tag{5.58a}$$

$$P_x = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt = c_0^2 + \sum_{n=1}^{\infty} c_n^2/2, \tag{5.58b}$$

$$P_x = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt = \sum_{n=-\infty}^{\infty} |\mathbf{x}_n|^2. \tag{5.58c}$$

Recall that average power of $c_n \cos(n\omega_0 t + \phi_n)$ is $c_n^2/2$

Fourier Transform

- For non-periodic signals
- Obtain from Fourier series as period$\rightarrow\infty$
- Compute forward Fourier transform from x(t):

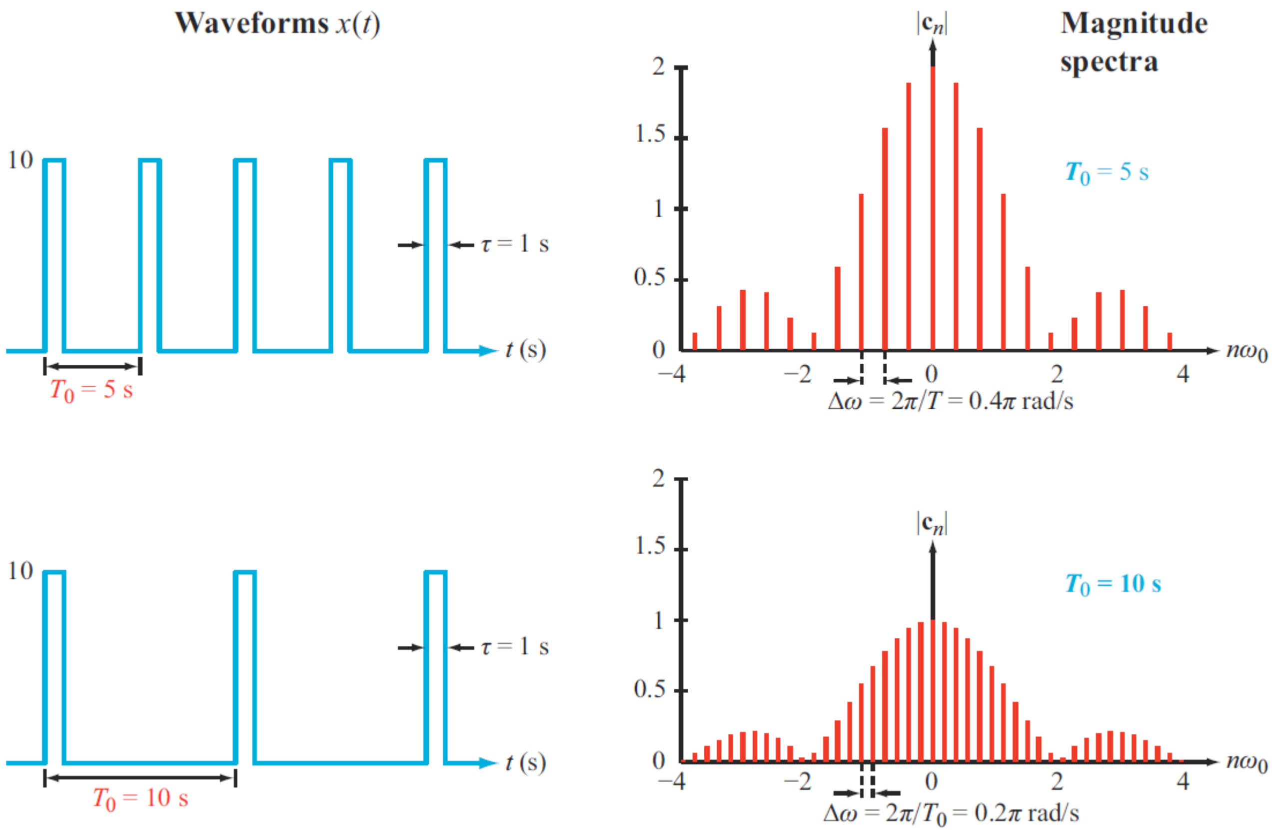
$$\mathbf{X}(\omega) = \mathfrak{F}[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

- Compute inverse Fourier transform: recover x(t):

$$x(t) = \mathfrak{F}^{-1}[\mathbf{X}(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{X}(\omega)\, e^{j\omega t}\, d\omega$$

Fourier Series→Fourier Transform

- Consider a periodic pulse train. Let its period→∞. Then its two-sided magnitude line spectrum looks like:



Fourier Series→Fourier Transform

- Exponential Fourier series: $x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t}$
- Coefficients computed using: $\mathbf{x}_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t')\, e^{-jn\omega_0 t'}\, dt'$
- Spacing between harmonics: $\Delta\omega = (n+1)\omega_0 - n\omega_0 = \omega_0 = \frac{2\pi}{T_0}$
- Insert coefficient formula into exponential series:

$$x(t) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-T_0/2}^{T_0/2} x(t')\, e^{-jn\omega_0 t'}\, dt' \right] e^{jn\omega_0 t}\, \Delta\omega.$$

- Let the period→∞. Bracketed quantity is $\hat{\mathbf{X}}(\omega)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(t')\, e^{-j\omega t'}\, dt' \right] e^{j\omega t}\, d\omega.$$

Important Fourier Transform Pairs

- Impulses in time and frequency:

$$\delta(t) \longleftrightarrow 1. \qquad 1 \longleftrightarrow 2\pi\,\delta(\omega).$$

- Causal exponential signals:

$$Ae^{-at}\,u(t) \longleftrightarrow \frac{A}{a+j\omega} \quad \text{for } a > 0.$$

- Eternal sinusoidal signals:

$$\cos\omega_0 t \longleftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)],$$

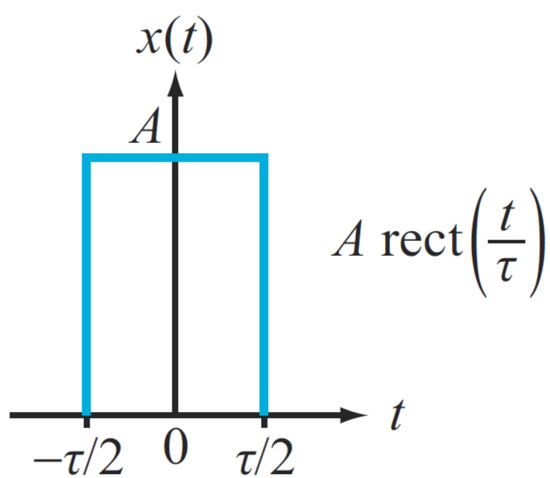
$$\sin\omega_0 t \longleftrightarrow j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)].$$

Fourier Transform of a Pulse

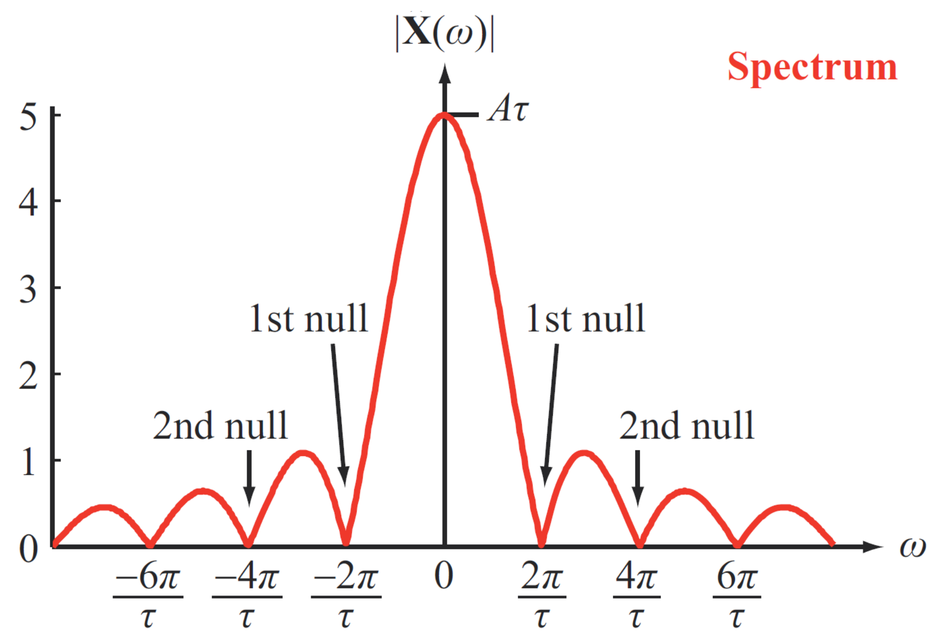
$x(t) = A\,\mathrm{rect}(t/\tau)$ leads to $\mathbf{X}(\omega) = A\tau \dfrac{\sin(\omega\tau/2)}{(\omega\tau/2)} = A\tau\,\mathrm{sinc}\left(\dfrac{\omega\tau}{2}\right)$

The sinc function is defined as $\mathrm{sinc}(\theta) = \dfrac{\sin\theta}{\theta}$ $\quad \mathrm{sinc}(0) = \dfrac{\sin(\theta)}{\theta}\Big|_{\theta=0} = 1$

Signal



Spectrum



Important Fourier Transform Properties

$$K_1 x_1(t) + K_2 x_2(t) \leftrightarrow K_1 \mathbf{X}_1(\omega) + K_2 \mathbf{X}_2(\omega),$$

(linearity property) (5.90)

$$x'(t) \leftrightarrow j\omega \mathbf{X}(\omega).$$

(derivative property)

$$x(at) \leftrightarrow \frac{1}{|a|} \mathbf{X}\left(\frac{\omega}{a}\right), \quad \text{for any } a.$$

(scaling property)

For real-valued x(t), we have:

$$\mathbf{X}(-\omega) = \mathbf{X}^*(\omega).$$

(reversal property)

$$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[\mathbf{X}(\omega - \omega_0) + \mathbf{X}(\omega + \omega_0)].$$

(modulation property) (5.109)

- Modulation property used for radio

Important Fourier Transform Properties

- The following two properties are duals of each other:

$$e^{j\omega_0 t}\, x(t) \quad \longleftrightarrow \quad \mathbf{X}(\omega - \omega_0),$$

(frequency-shift property)

$$x(t - t_0) \quad \longleftrightarrow \quad e^{-j\omega t_0}\, \mathbf{X}(\omega).$$

(time-shift property)

- Parseval's theorem for the Fourier transform: ENERGY is the same when computed in time or in frequency:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\mathbf{X}(\omega)|^2\, d\omega.$$

(Parseval's theorem)

Example of Time Scaling Property

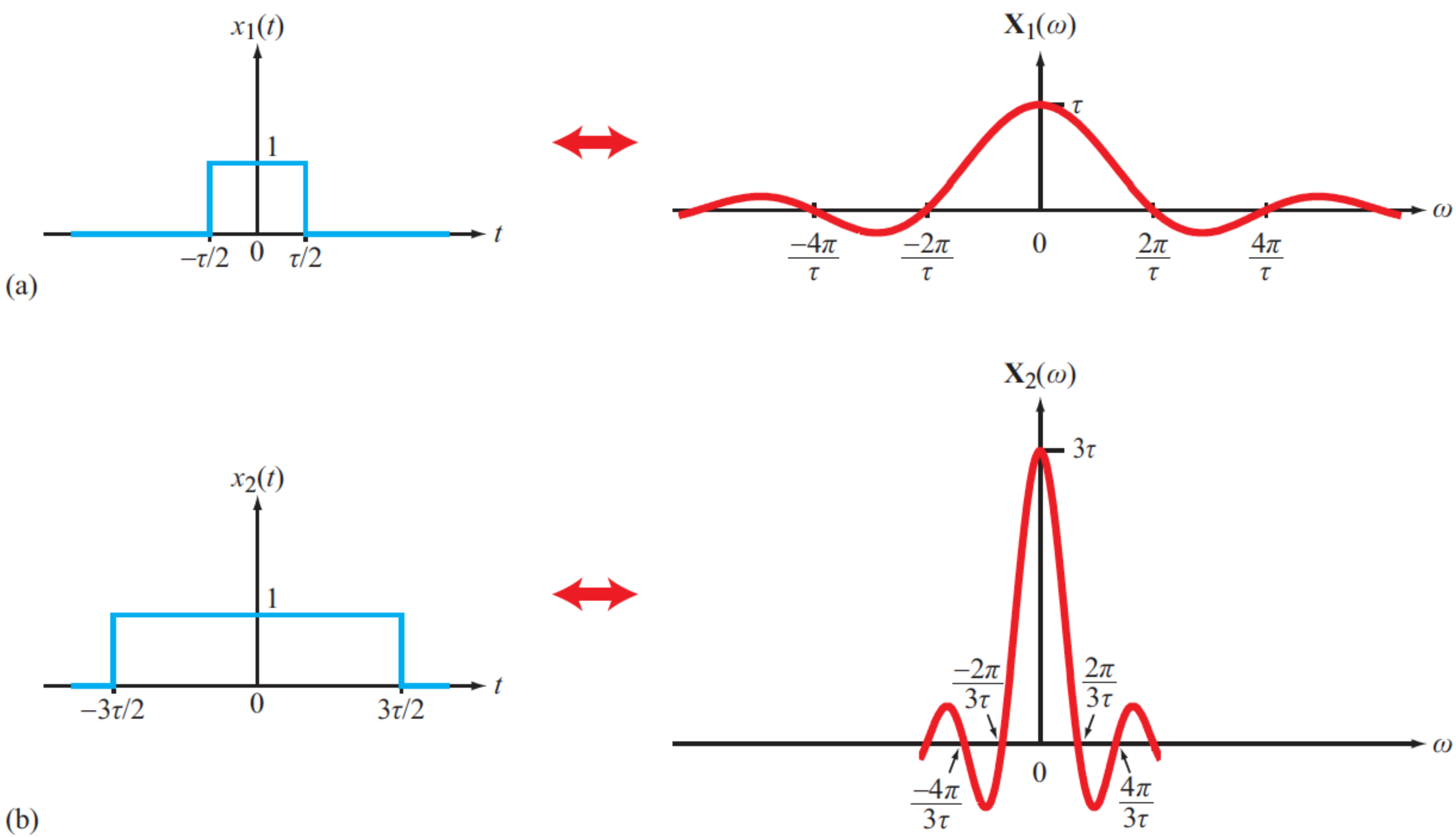


Figure 5-14: Stretching $x_1(t)$ to get $x_2(t)$ entails stretching t by a factor of 3. The corresponding spectrum $\mathbf{X}_2(\omega)$ is a 3-times compressed version of $\mathbf{X}_1(\omega)$.

Conjugate Symmetry Implications

- If x(t) is real-valued, then $\mathbf{X}(\omega)$ has conjugate symmetry:

$$\mathbf{X}(-\omega) = \mathbf{X}^*(\omega).$$

(reversal property)

Consequently, for a *real-valued* $x(t)$, its Fourier transform $\mathbf{X}(\omega)$ exhibits the following symmetry properties:

(a) $|\mathbf{X}(\omega)|$ is an even function ($|\mathbf{X}(\omega)| = |\mathbf{X}(-\omega)|$).

(b) $\arg[\mathbf{X}(\omega)]$ is an odd function ($\arg[\mathbf{X}(\omega)] = -\arg[\mathbf{X}(-\omega)]$).

(c) If $x(t)$ is real and an even function of time, $\mathbf{X}(\omega)$ will be purely real and an even function.

(d) If $x(t)$ is real and an odd function of time, $\mathbf{X}(\omega)$ will be purely imaginary and an odd function.

Example of Parseval's Theorem

- Confirm Parseval's theorem holds for $x(t) = e^{-at}\, u(t)$
- Parseval's theorem: $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{\mathbf{X}}(\omega)|^2\, d\omega$
- Fourier transform pair: $Ae^{-at}\, u(t) \leftrightarrow \frac{A}{a + j\omega}$ for $a > 0$

Energy in time domain:

$$\int_{0}^{\infty} |e^{-at}|^2\, dt = \int_{0}^{\infty} e^{-2at}\, dt = \frac{1}{2a}.$$

Energy in frequency domain:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{a + j\omega} \right|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a^2 + \omega^2}\, d\omega = \frac{1}{2a}$$

Fourier Transform of Periodic Signal

- The Fourier transform of a periodic signal is found by Fourier transforming its Fourier series expansion:

$$x(t) = \sum_{n=-\infty}^{\infty} \mathbf{x}_n e^{jn\omega_0 t}.$$

- The Fourier transform of a complex exponential is

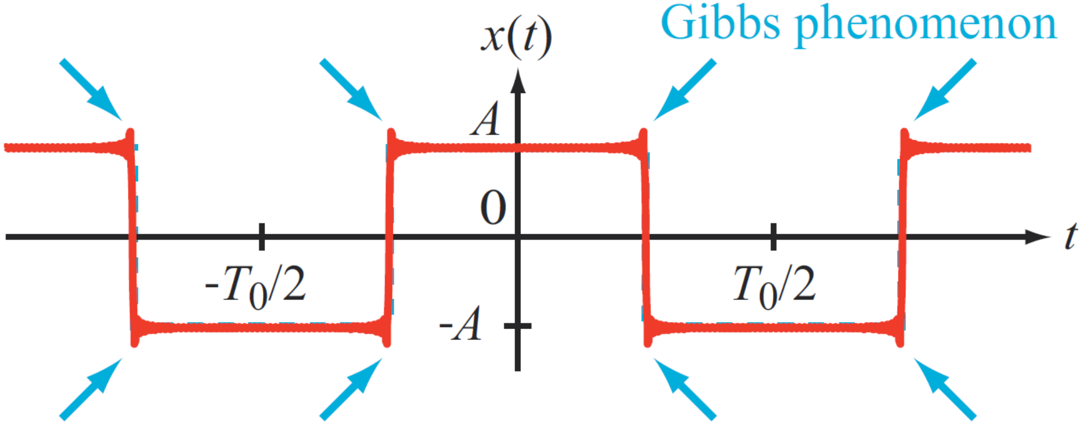
$$\mathcal{F}\left\{e^{j\omega_0 t}\right\} = 2\pi\ \delta(\omega - \omega_0)$$

- So the Fourier transform of the periodic signal is

$$\mathcal{F}\{x(t)\} = \sum_{n=-\infty}^{\infty} \mathbf{x}_n 2\pi\ \delta(\omega - n\omega_0).$$

Gibbs's Phenomenon

- If x(t) is not continuous at t=t_i then the Fourier series expansion of x(t) converges to the midpoint of the discontinuity, which is $\frac{1}{2}[x(t_i^+) + x(t_i^-)]$
- There is an overshoot, called Gibbs phenomenon
- This does not disappear if more terms are used



(b) Fourier series with 100 terms

Phasor vs. Laplace vs. Fourier

- Which of these three methods should be used? This table shows that the input dictates the choice.

Input $x(t)$			
Duration	**Waveform**	**Solution Method**	**Output $y(t)$**
Everlasting	**Sinusoid**	Phasor Domain	Steady State Component (no transient exists)
Everlasting	**Periodic**	Phasor Domain and Fourier Series	Steady State Component (no transient exists)
Causal, $x(t) = 0$, for $t < 0$	**Any**	Laplace Transform (unilateral) (can accommodate non-zero initial conditions)	Complete Solution (transient + steady state)
Everlasting	**Any**	Bilateral Laplace Transform or Fourier Transform	Complete Solution (transient + steady state)