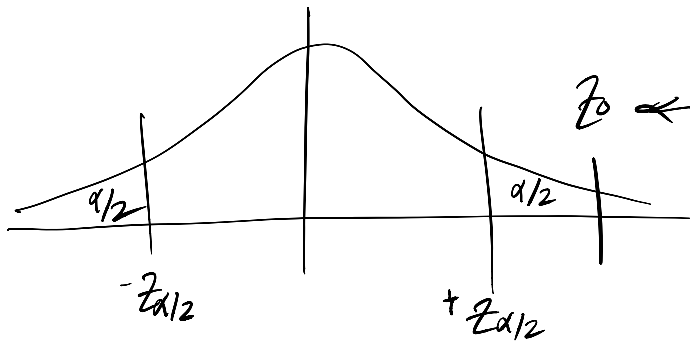
Type-I and Type-II Errors

.. rejecting H_0 when it is in fact true : **Type-I error**



z_0 ← let's say our sampled data lead to this test statistic

.. we would of course reject H_0 ;

$$z_0 > +z_{\alpha/2}$$

.. if it turns out that H_0 is true, we shouldn't have exceeded the critical value

.. probability of this happening : α

.. failing to reject H_0 when H_0 is false : **Type-II Error**

β (refer to book)

Tests on μ, σ Unknown

∴ if $n \geq 30$, plug in s for σ; use Z

per Dave Kender!

∴ if $n < 30$, need new test statistic.

$$T_0 = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

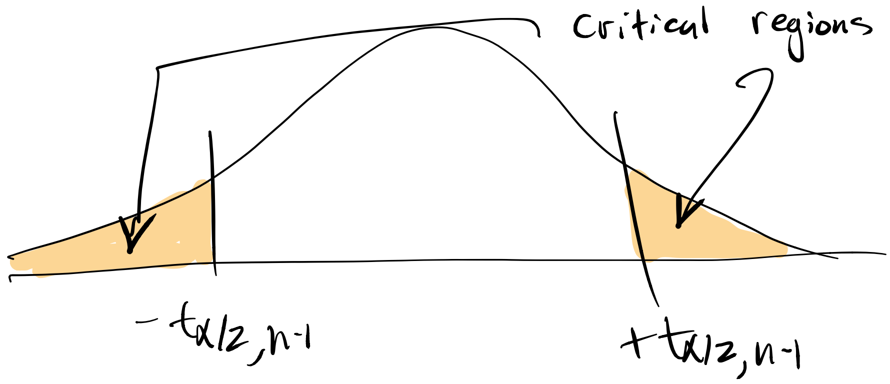
w/ n-1 degrees of freedom

∴ fixed significance level approach (a/k/a fixed-α):

critical values: $\pm t_{\alpha/2, n-1}$

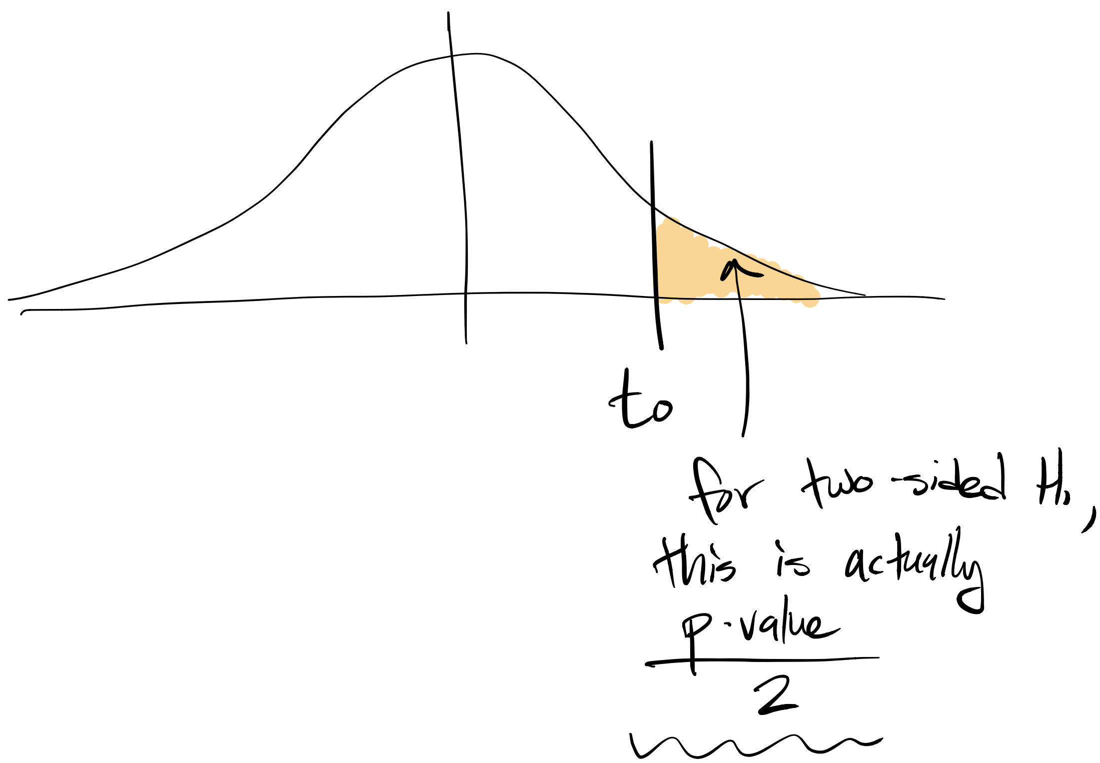
∴ rejection criteria: $t_0 > +t_{\alpha/2, n-1}$

OR $t_0 < -t_{\alpha/2, n-1}$



(reject H_0 if t_0 is in either one)

- p-value approach: this is where the trouble starts!
- recall that p-value represents the probability outside the test statistic, t_0:



∴ our next step was then to determine the probability from the Z table

∴ easy! Z-table gives (cumulative) probabilities given z

∴ the T-table gives us t-values given some α [and degrees of freedom]

∴ $\ddot{\circ}$ we can't find an exact p-value given t_0!

∴ we can get a range for p-value between two levels of significance

∴ enough to reject or fail to reject H_0!

ex: golf club manufacturer

∴ new club design; promises "coefficient of restitution" of 0.82

$\equiv$ "bounce factor"

$e = 0 \longrightarrow$ no bounce, plastic collision
(all kinetic energy absorbed in collision)

$e = 1 \longrightarrow$ perfect bounce; elastic collision, same velocity of separation as velocity of approach

∴ this is tightly controlled in golf clubs!

∴ manufacturer would like to test

$$H_0 : \mu = 0.82$$

$$H_1 : \mu > 0.82$$

∴ why this choice of one-sided H_1?

∴ because if H_0 is rejected, implies it exceeds this value

∴ test using fixed-α and p-value approaches

@ $\alpha = 0.05$

.. fixed-α :

test data :

$n = 15$ golf clubs

$\bar{x} = 0.83725$

$s = 0.02456$

.. test statistic :

hypothesized value of μ, given in H_0

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{0.83725 - 0.82}{0.02456/\sqrt{15}}$$

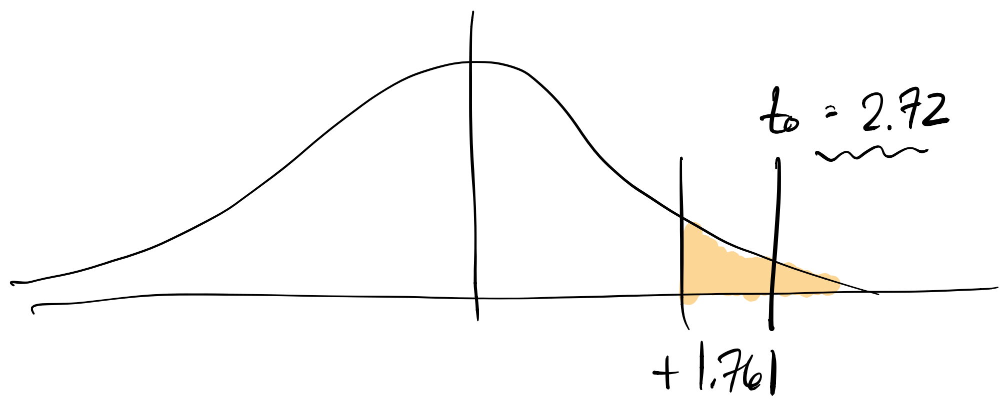
$$t_0 = 2.72$$

uh oh !

2.72 standard deviations from mean <u>ain't</u> good

$\therefore$ critical values: $+t_{\alpha, n-1}$ ← because it's an upper one-sided H_1!

$$+t_{.05,14} = +1.761$$



$$t_0 > +t_{\alpha, n-1}$$

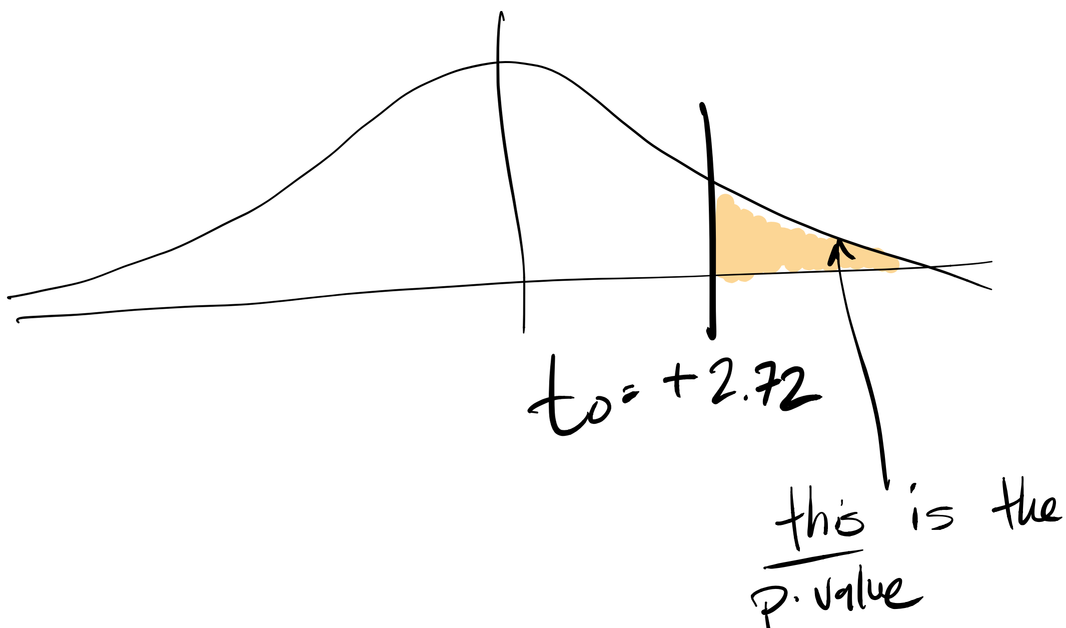
$\therefore$ reject H_0 @ $\alpha = 0.05$

$\therefore$ data suggests $\mu > 0.82$
(can't say proves!)

P-value approach

∴ alway start w/ test statistic ⇓ your data!

$t_0 = 2.72$



∴ because it's a one-sided (upper) H_1, it's the entire p-value; not $\frac{\text{p-value}}{2}$.

$p\text{-value} = P(t > 2.72)$

∵ we <u>cannot</u> get this from t-table!

∵ the <u>best</u> we can do is get as close as possible to 2.72 @ 14 d.o.f.

from table: $t_{.01,14} = 2.624$

$t_{.005,14} = 2.977$

∴ we conclude that

$$0.005 < p\text{-value} < 0.01$$

∵ rejection criterion for p-value:

reject H_0 if $p < \alpha$

∴ this range is less than $\alpha = 0.05$

∴ reject H_0

$\therefore$ <u>if</u> this had been a two-sided H_1

$[H_1 : \mu \neq 0.82]$, then

$$0.005 < \frac{p\text{-value}}{2} < 0.01$$

$$0.01 < p\text{-value} < 0.02$$

[still reject]