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ISE 2211 • Exam VI • 20 November 2019 NAME SOLUTION

1) The relationship between compressive force applied to a strain gage sensor in MPa (x) and its electrical output in microvolts (y) is under investigation. Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model for output voltage vs. force.

Formulae:

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

	σ (Mpa) (x)	V (μV) (y)
1	0	-0.0127
2	100	8.21
3	200	11.96
4	300	14.02
5	400	15.27
6	500	15.65
7	600	15.28
8	700	13.25
9	800	10.35

$\sum y_i x_i = 48075$ (+1)

$\sum x_i = 3600$

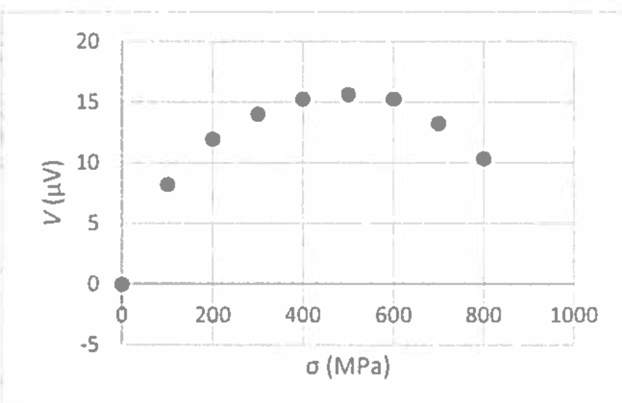
$\sum y_i = 103.9773$ (+1)

$\bar{x} = 400$

$\bar{y} = 11.55303$ (+1)

$\sum x_i^2 = 2040000$

$\sum y_i^2 = 1401.265$ (+1)



$S_{XY} = 48075 - \frac{3600 \cdot 103.9773}{9}$

$S_{XY} = 6484.08$ (+1)

$S_{XX} = 2040000 - \frac{3600^2}{9}$

$S_{XX} = 600,000$ (+1)

$\hat{\beta}_1 = \frac{6484.08}{600,000} = 0.0108068$ (+1) $\mu V / MPa$ (+1)

$\hat{\beta}_0 = 11.55303 - 0.0108068 \cdot 400 = 7.23031$ (+1) μV (+1)

Write an equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

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$\therefore \hat{y} = 7.23031 + 0.0108068\, x$ (+1) (μV) (+1)

Write a 95% confidence interval on the mean voltage at *x* = 650 MPa.

$$\hat{\mu}_{Y|650} = 7.23031 + 0.0108068 \cdot 650$$

$$= \underbrace{14.25473}_{+1} \; (\mu V)$$

$$t_{\alpha/2,\, n-2} = t_{.025,\, 7} = \underline{2.365} \quad +1$$

$$SS_T = 1401.265 - 9 \cdot 11.55303^2 = 200.0124804 \quad +1$$

$$SS_E = 200.0124804 - 0.0108068 \cdot 6484.08$$

$$= 129.9403246 \quad +1$$

$$\hat{\sigma}^2 = \frac{129.9403246}{9-2} = 18.56290352 \quad +1$$

$$\text{C.I.}: \quad 14.25473 \pm 2.365 \sqrt{18.56290352 \left[\frac{1}{9} + \frac{(650-400)^2}{600{,}000} \right]}$$

$$9.527 < \mu_{Y|650} < 18.98 \quad (\mu V)$$

+2

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Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero.

H_0: $\hat{\beta}_1 = 0$

H_1: $\hat{\beta}_1 \neq 0$

$$0.0108068 \pm 2.365\sqrt{18.56/600{,}000}$$

$$-0.002347 < \beta_1 < 0.02396$$

+2

C.I. contains zero; ∴ fail to reject Ho

+1

@ $\alpha = 0.05$

+1

List two theoretical scenarios that would fail to reject H_0. What does your conclusion imply about the relationship output voltage and compressive force?

1.) no relationship

2.) non-linear relationship

+2

-- probably #2! (based on scatter plot)

+1

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Write a 95% confidence interval on the correlation coefficient ρ, if y and x may both be considered random variables. (Ignore the fact that n ≯ 30.)

$$R^2 = 1 - \frac{SS_E}{SS_T}$$

$$= 1 - \frac{129.9}{200.0} = 0.3505 \quad +1$$

not very good!

$$\therefore R = +\sqrt{0.3505} = 0.5920$$

$$Z_{\alpha/2} = 1.960 \quad +1$$

$$\tanh^{-1}(0.5920) = 0.6807 \quad +1$$

$$\rho: \quad \tanh\left(0.6807 \pm \frac{1.960}{\sqrt{6}}\right)$$

$$0.1189 < \rho < 0.9016$$

+2

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