ISE 2211 • Exam VII • August Fools' Day 2019 NAME SOLUTION

It is hypothesized that there is a connection between relative humidity in the atmosphere and the hardness of an automotive finish. Five test samples were finished in each of four different relative humidity environments in random order and subjected to a Vickers hardness test:

RH (%)	Vickers Hardness (VH)					Totals	Averages
	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5		
20	31.1	31.7	32.0	30.9	33.8	159.5	31.9
40	30.1	30.4	29.9	28.0	31.6	150.0	30.0
60	28.2	30.1	29.4	27.3	26.8	141.8	28.36
80	26.2	28.2	25.7	23.9	26.6	130.6	26.12
						581.9	29.095

Use Analysis of Variance (ANOVA) to test the null hypothesis that the treatment means are equal at the $\alpha = 0.01$ level of significance. Fill in the ANOVA table.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0
Treatments	90.39	3	30.13	16.39
Error	29.41	16	1.838	-
Total	119.8	19	-	-

+2 (values in table)

$$\sum_i \sum_j y_{ij}^2 = 17050.17 \quad +1$$

$$SS_T = 17050.17 - \frac{581.9^2}{4 \cdot 5} = 119.8 \quad +1 \qquad d.o.f. = 4 \cdot 5 - 1 = 19 \quad +1$$

$$\sum_i y_{i.}^2 = 85103.85 \quad +1$$

$$SS_{Treatments} = \frac{85103.85}{5} - \frac{581.9^2}{4 \cdot 5} = 90.39 \quad +1 \qquad d.o.f. = 4-1 = 3 \quad +1$$

$$SS_E = 119.8 - 90.39 = 29.41 \quad +1 \qquad d.o.f. = 4(5-1) = 16 \quad +1$$

$$MS_{Treatments} = \frac{90.39}{3} = 30.13 \quad +1$$

$$MS_E = \frac{29.41}{16} = 1.838 \quad +1$$

$$f_0 = \frac{30.13}{1.839} = 16.39 \quad +1$$

$$f_{critical} = f_{.01,3,16} = 5.29 \quad +1$$

$f_0 > f_{critical}$ +1

$\therefore$ reject H_0 +1

treatment means are not equal!

Write a 99% confidence interval on Vickers hardness at the 80% relative humidity level.

$\bar{y}_{4\cdot} = 26.12$ (+1)

$t_{\alpha/2, 9(n-1)} = t_{.005, 16} = \underline{2.921}$ (+1)

μ_4 : $26.12 \pm 2.921\sqrt{\frac{1.838}{5}}$

$24.35 < \mu_4 < 27.89$ (VH) (+1)

(+2)

Use Fisher's Least Significant Difference to determine which, if any, pairs of relative humidities show significant difference at $\alpha = 0.01$.

$$LSD = t_{\alpha/2,\, a(n-1)} \sqrt{\frac{2\, MS_E}{n}}$$

$$= 2.921 \sqrt{\frac{2 \cdot 1.838}{5}}$$

$$= 2.505$$ +1

20 vs. 40 :	$\lvert 30.0 - 31.9 \rvert$	= 1.9	
40 vs. 60 :	$\lvert 28.36 - 30.0 \rvert$	= 1.64	
60 vs. 80 :	$\lvert 26.12 - 28.36 \rvert$	= 2.24	
20 vs. 60 :	$\lvert 31.9 - 28.36 \rvert$	= 3.54	> LSD
40. vs. 80 :	$\lvert 30.0 - 26.12 \rvert$	= 3.88	> LSD
20 vs. 80 :	$\lvert 31.9 - 26.12 \rvert$	= 5.78	> LSD

+1 pairs

+1 differences

+1

Only 20% vs. 60%, 40% vs. 80%, and 20% vs. 80% humidity pairs show significant differences in Vickers hardness @ $\alpha = 0.01$. +1

Formulae:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{N} \quad \textit{with } an - 1 \textit{ degrees of freedom}$$

$$SS_{Treatments} = \sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{N} \quad \textit{with } a - 1 \textit{ degrees of freedom}$$

$$SS_E = SS_T - SS_{Treatments} \quad \textit{with } a(n-1) \textit{ degrees of freedom}$$

$$MS_{Treatments} = \frac{SS_{Treatments}}{a-1}$$

$$MS_E = \frac{SS_E}{a(n-1)}$$

$$f_0 = \frac{MS_{Treatments}}{MS_E}$$

$$f_{critical} = f_{\alpha,a-1,a(n-1)}$$

$$CI \text{ on } \mu_i: \quad \bar{y}_{i.} \pm t_{\alpha/2,a(n-1)}\sqrt{\frac{MS_E}{n}}$$

$$CI \text{ on } \mu_i - \mu_j: \quad \bar{y}_{i.} - \bar{y}_{j.} \pm t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}}$$

$$LSD = t_{\alpha/2,a(n-1)}\sqrt{\frac{2MS_E}{n}}$$