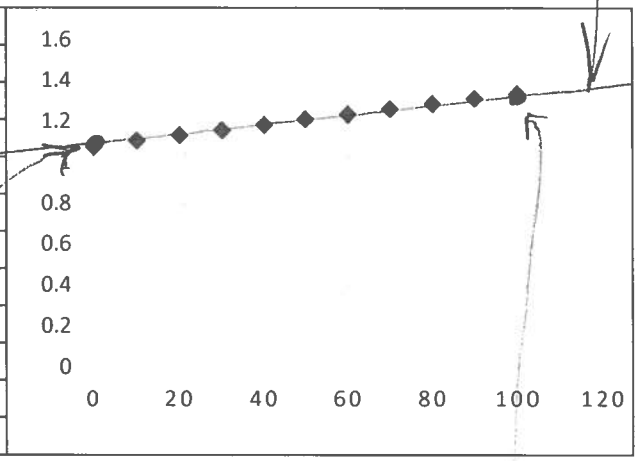
ISE 2211 • Exam VI • 15 November 2021 NAME SOLUTION

A bonded cantilever-type strain-gage sensor was connected to an instrumentation preamplifier and output voltages (y) measured for a series of applied masses in grams (x). The data and scatter plot are presented below, courtesy of Andrew Bleakley (BME 4550L, Fall 2021). +3

Mass (g)	Voltage (V)
0	1.082
10	1.112
20	1.138
30	1.164
40	1.189
50	1.215
60	1.24
70	1.265
80	1.291
90	1.317
100	1.339



Determine least-squares estimated linear regression coefficients for this data. Write the resulting regression equation (with unit!) and draw the line on the scatter plot.

$\sum x_i = 550$

$\sum y_i = 13.352$ } +1

$\bar{x} = 50$

$\bar{y} = 1.2138$ } +1

$\sum x_i^2 = 38500$

$\sum y_i^2 = 16.279$ } +1

$\sum x_i y_i = 695.77$ +1

$S_{xy} = 695.77 - \frac{13.352 \cdot 550}{11}$ +1

$= 28.17$

$S_{xx} = 38500 - \frac{550^2}{11} = 11{,}000$

$\therefore \hat{\beta}_1 = \frac{28.17}{11{,}000} = 0.002561$ +1

$\hat{\beta}_0 = 1.2138 - .002561 \cdot 50$

$= 1.086$ +1

$\hat{y} = 0.002561x + 1.086$ (V) +2

$\hat{y}(100) = 0.2561 + 1.086 = 1.3421$ V

Determine the coefficient of determination for this relationship. Would you say there is a strong linear relationship between applied mass and output voltage for this strain gage system?

$$R^2 = 1 - \frac{SS_E}{SS_T}$$

$$= 1 - \frac{0.0004418}{0.07259}$$

$$R^2 = 0.9939$$

+2

yes, strong linear relationship!

+1

An unknown mass was applied to the end of the cantilever and an output voltage of **1.177 V** recorded. Use your regression line to estimate the value of mass.

$$\hat{y} = \hat{\beta}_1 x + \hat{\beta}_0$$

$$\rightarrow x = \frac{\hat{y} - \hat{\beta}_0}{\hat{\beta}_1} = \frac{1.177 - 1.086}{0.002561}$$

$$x = 35.53 \text{ g}$$ (+3)

Write a ~~95% prediction interval~~ on a 12th measurement @ 120 g.

$$SS_T = 16.279 - 11 \cdot 1.2138^2 = 0.07259$$ (+1)

$$SS_E = 0.07259 - 0.002561 \cdot 28.17 = 0.0004418$$ (+1)

$$\hat{\sigma}^2 = \frac{0.0004418}{11 - 2} = 0.00004909$$ (+1)

$$t_{.025,9} = 2.262$$ (+1)

$$\hat{y}_0 \big|_{120g} = 0.002561 \cdot 120 + 1.086 = 1.393 \text{ V}$$ (+1)

$$y_0: \quad 1.393 \pm 2.262 \sqrt{0.00004909 \left[1 + \frac{1}{11} + \frac{(120 - 50)^2}{11000}\right]}$$

$$1.373 < y_0 < 1.413 \quad [\text{V}]$$