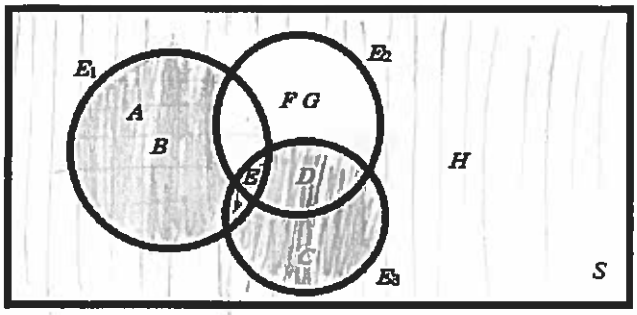
ISE 2211 • Exam I • 6 September 2017 NAME SOLUTION

1) The following Venn diagram applies to outcomes and events in the sample space S, along with associated probabilities.



P(A) = 0.086
P(B) = 0.044
P(C) = 0.172
P(D) = 0.001
P(E) = 0.502
P(F) = 0.058
P(G) = 0.111
P(H) = 0.026

Perform the following set operations. Determine the outcomes associated with each operation and the resulting probability.

$E_1 \cap E_2$

↑ intersection ∴ $\{E\}$ (+1)

$P(E_1 \cap E_2) = P(E) = 0.502$ (+1)

$E_2 \cup E_3$

↑ union ∴ $\{CDEFG\}$ (+1)

$P(E_2 \cup E_3) = 0.172 + 0.001 + 0.502 + 0.058 + 0.111$

$= 0.844$ (+1)

$E_2 \cap E_3'$

$= \{DE'FG\} \cap \{ABFGH\}$ (+1)

$= \{FG\}$ (+1)

$\therefore P(E_2 \cap E_3') = 0.058 + 0.111 = 0.169$ (+1)

$(E_1 \cap E_2') \cup E_3$

Additionally, shade this operation on the Venn diagram. (+1)

$= [\{ABE\} \cap \{ABCH\}] \cup \{CDE\}$ (+1)

$= \{AB\} \cup \{CDE\}$ (+1)

$= \{ABCDE\} \Rightarrow P = 0.086 + 0.044 + 0.172 + 0.001 + 0.502 = 0.805$ (+1)

2) The following numbers are tire pressures in psi (pounds per square inch):

32.0 31.3 21.7 24.3 39.1 31.1 27.4 ordered: 21.7 24.3 27.4 | 31.1 31.3 32.0 | 39.1

Compute the sample mean, sample variance, sample standard deviation, and sample range. Include a unit with each answer.

Hint: $s^2 = \dfrac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$

$\bar{x} = \dfrac{\sum x_i}{n} = 29.56$ psi (+1) (+1)

$s^2 = \dfrac{6312 - \frac{206.9^2}{7}}{6} = 32.75$ psi^2 (+0) (+1)

$s = +\sqrt{s} = 5.722$ psi (+0) (+1)

$x_{max} - x_{min} = 39.1 - 21.7 = 17.4$ psi (+1) (+1)

Draw a histogram that displays the frequency distribution of tire pressures. Choose the number of bins and bin width appropriately. Label all axes.

$\sqrt{n} = 2.65$

use 3 bins (+1)

$$\text{bin width} = \frac{\text{sample range}}{3} = \frac{17.4}{3} = 5.8$$

bin 1: $21.7 + 5.8 = 27.5$

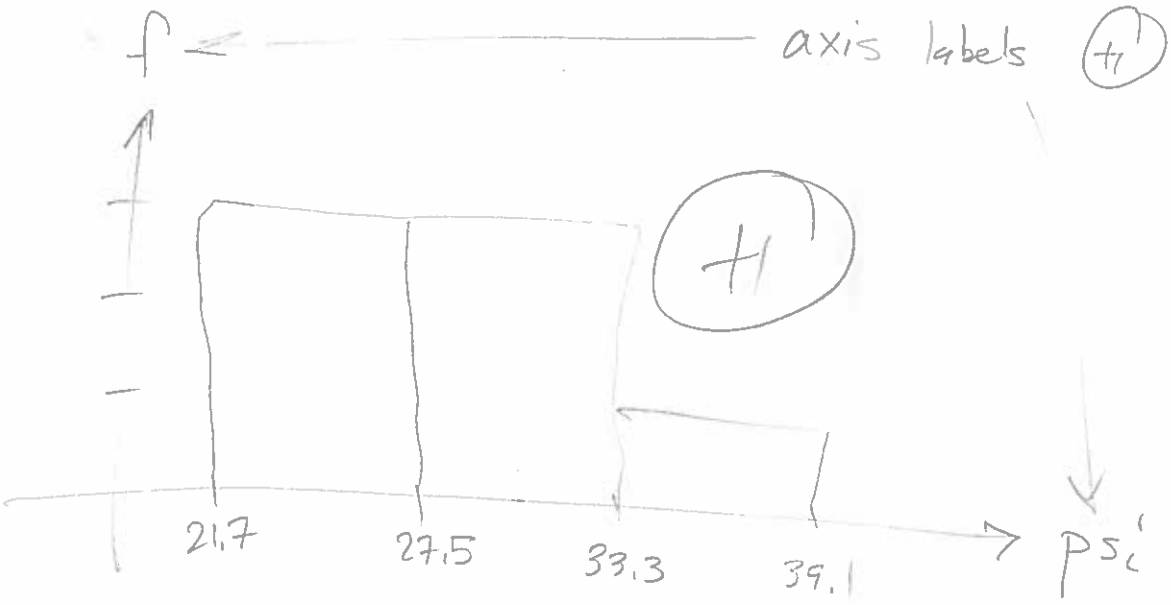
bin 2: $27.5 + 5.8 = 33.3$

bin 3: $33.3 + 5.8 = 39.1$

upper boundaries (+1)

f
3
3
1

(+1)



2) Three woodpeckers live in a forest with <u>fourteen</u> trees acceptable for pecking. Determine the number of configurations of three woodpeckers pecking on 14 trees if it is important <u>which</u> woodpecker is pecking on a tree. Also compute the number of configurations if the identity of the woodpecker is irrelevant.

Formulae:

$$P\binom{n}{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Ordered configs: $P\binom{14}{3} = \frac{14!}{(14-3)!} = \frac{14\times13\times12\times11!}{11!}$

$= 2184$ (+1)

Configurations

Unordered configs (irrelevant woodpeckers):

$\binom{14}{3} = \frac{14!}{3!(14-3)!} = \frac{14\times13\times12\times11!}{3!\,11!}$

$= 364$ (+1)

configurations