29 pts.

ISE 2211 • Exam I • 19 January 2018 NAME SOLUTION

1) Joe Tritschler is having an extremely busy day. His to-do list currently has twelve things, including writing and solving this stats exam and a host of other activities. If he needs to complete six of those things before lunch, determine the number of differently-ordered ways he could do them. Also compute the number of un-ordered ways he could do those six things and state whether Joe Tritschler is likely to lose his mind either way.

Formulae:

$$P\binom{n}{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

differently-ordered ways $\longrightarrow$ permutations $\longrightarrow P\binom{12}{6}$ +1

$$P\binom{12}{6} = \frac{12!}{(12-6)!} = \frac{12\times 11\times 10\times 9\times 8\times 7\times \cancel{6!}}{\cancel{6!}}$$

$= 665280$ ways (!) +1

Un-ordered ways $\longrightarrow$ combinations $\longrightarrow \binom{12}{6}$ +1

$$\binom{12}{6} = \frac{12!}{6!\,(12-6!)} = 924 \text{ ways}$$ +1

+4

2) The following numbers are recorded frequencies in kHz from a crystal-locked oscillator:

67.20 67.29 67.05 67.11 66.98 67.06 67.14 67.23 67.36 67.00

Compute the sample mean, sample variance, sample standard deviation, and sample range. Include a unit with each answer.

Hint: $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$

+1 +1

Sample mean: $\bar{x} = \frac{\sum x}{n} = 67.142 \text{ kHz}$

Sample variance: $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$

$= \frac{45080.6248 - \frac{671.42^2}{10}}{9}$ +1

$= 0.0159 \text{ kHz}^2$

+1 +1

sample std. dev. : $+\sqrt{s^2} = 0.1261 \text{ kHz}$ +1

range : $X_{max.} - X_{min.} = 67.36 - 66.98$

$= 0.38 \text{ kHz}$

+1 +1

+8

Draw a histogram that displays the relative frequency distribution of oscillator frequencies. Choose the number of bins and bin width appropriately. Label all axes.

\# of bins : $\sqrt{10} = 3.16$ choose 3 bins (+1)

[alt. : four bins ok]

range = 0.38 kHz

$\therefore$ bin 1 : $66.98 + \frac{0.38}{3} = 67.107$

bin 2 : $67.36 + \frac{0.38}{3} = 67.233$

bin 3 : $67.74 + \frac{0.38}{3} = 67.36$

(+1)

Ordered frequencies :

bin 1
66.98
67.00
67.05
67.06
(4)

bin 2
67.11
67.14
67.20
67.23
(4)

bin 3
67.29
67.36
(2)

(+1)

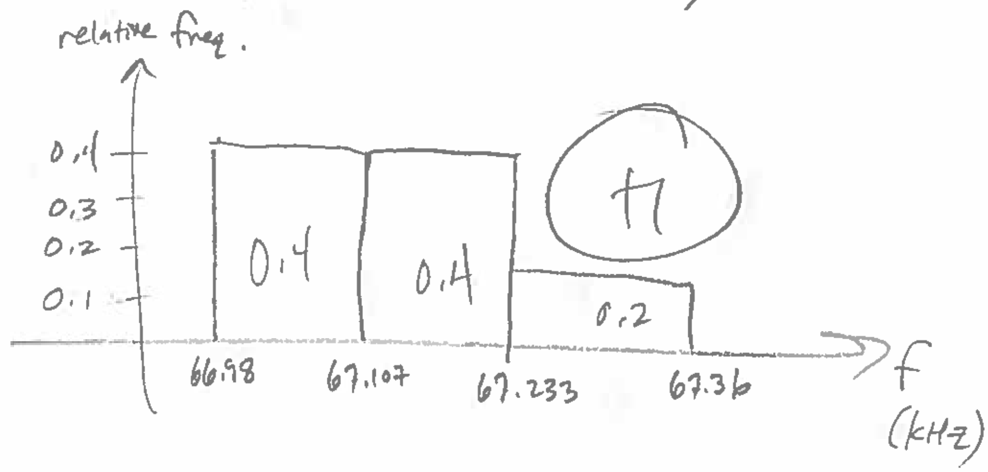
$\therefore$ relative frequencies :

bin 1 $\rightarrow$ 4/10 = 0.4

bin 2 $\rightarrow$ 0.4

bin 3 $\rightarrow$ 0.2

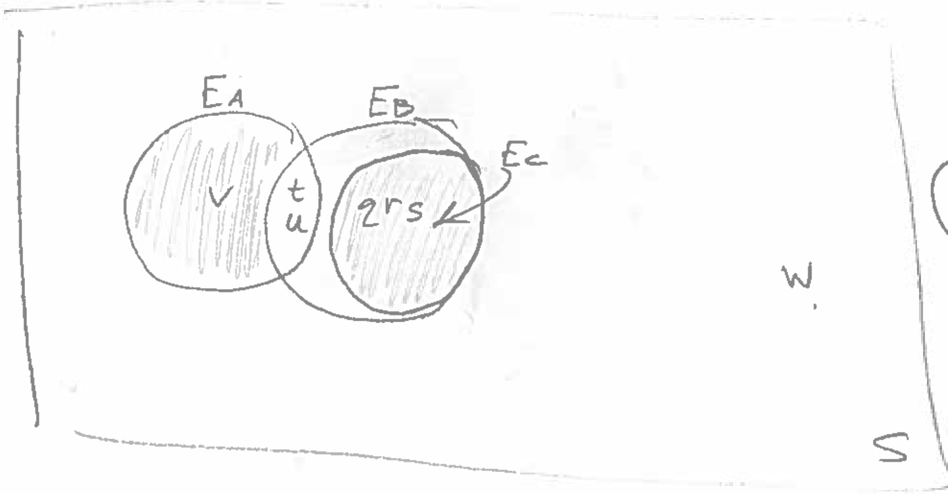
(+1)



+5

3) A sample space contains the outcomes *S{q r s t u v w}* with associated probabilities shown below. Define event E_A {t u v}, E_B {q r s t u}, and E_C {q r s}. Sketch a Venn diagram showing these three events and all outcomes in the sample space.

$P(q) =$	0.434
$P(r) =$	0.235
$P(s) =$	0.101
$P(t) =$	0.139
$P(u) =$	0.004
$P(v) =$	0.027
$P(w) =$	0.060



+2

Perform the following set operations and resulting probability for each.

$E_A \cap E_B$

$= \{tu\}$ +1

$\rightarrow P(t) + P(u) = 0.139 + 0.004 = \underline{\underline{0.143}}$ +1

$E_A \cup E_C'$

$E_c' = \{t\ u\ v\ w\}$ +1

$\therefore \{t\ u\ v\} \cup \{t\ u\ v\ w\} = \{t\ u\ v\ w\}$ +1

$\rightarrow 0.139 + 0.004 + 0.027 + 0.060 = \underline{\underline{0.23}}$ +1

$(E_A \cap E_B') \cup E_C$

Additionally, shade this operation on the Venn diagram. +1

$E_B' = \{v\ w\}$ +1

$\therefore \{t\ u\ v\} \cap \{v\ w\} = \{v\}$ +1

$\{v\} \cup \{q\ r\ s\} = \{q\ r\ s\ v\}$ +1

$\rightarrow 0.434 + 0.235 + 0.101 + 0.027 = \underline{\underline{0.797}}$ +1

+17