Lab 4 Solution.

#1. $$\int_0^{\frac{\pi}{2}} x \sin(2x)\, dx$$

IBP $\quad u = x \qquad dv = \sin(2x)\, dx$

$\qquad du = dx \qquad v = -\dfrac{\cos(2x)}{2}$

$$= -\frac{1}{2} x \cos(2x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\frac{\cos(2x)}{2}\, dx$$

$$= -\frac{1}{2} x \cos(2x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\, dx$$

$$= -\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) \Big|_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{2} \cdot \frac{\pi}{2} \cos(\pi) + \frac{1}{4} \sin \pi - (0 + 0) \qquad \left(\begin{array}{l} \cos \pi = -1 \\ \sin \pi = 0 \end{array} \right)$$

$$= \frac{\pi}{4}.$$

#2 $\int e^{2x} \sin(3x)\, dx$

IBP $\quad u = e^{2x} \quad dv = \sin 3x\, dx$

$\quad du = 2e^{2x} dx \quad v = -\frac{\cos(3x)}{3}$

$$\int e^{2x} \sin(3x)\, dx = -\frac{1}{3} e^{2x} \cos(3x) - \int -\frac{\cos(3x)}{3} \cdot 2e^{2x}\, dx$$

$$\int e^{2x} \sin(3x)\, dx = -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{3} \underbrace{\int e^{2x} \cos(3x)\, dx}_{\text{IBP again}}$$

$\quad u = e^{2x} \quad dv = \cos(3x)\, dx$

$\quad du = 2e^{2x} dx \quad v = \frac{\sin(3x)}{3}$

$$\int e^{2x} \sin(3x)\, dx = -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{3}\left(\frac{1}{3} e^{2x} \sin(3x) - \int \frac{\sin(3x)}{3} \cdot 2e^{2x} dx\right)$$

$$\int e^{3x} \sin(3x)\, dx = -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{9} e^{2x} \sin(3x) - \frac{4}{9} \int e^{2x} \sin(3x)\, dx$$

add $\frac{4}{9} \int e^{2x} \sin(3x)\, dx$ to both sides!

$$\frac{13}{9} \int e^{2x} \sin(3x)\, dx = -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{9} e^{2x} \sin(3x)$$

divide both sides by $\frac{13}{9}$!

$$\int e^{2x} \sin(3x)\, dx = \boxed{-\frac{3}{13} e^{2x} \cos(3x) + \frac{2}{13} e^{2x} \sin(3x) + C}$$

final answer.

#3 $\int \cos^4(2x)\,dx$ $\qquad \cos^2(2x) = \dfrac{1+\cos(4x)}{2}$

$$= \int \left[\frac{1+\cos 4x)}{2}\right]^2 dx$$

$$= \frac{1}{4}\int 1 + 2\cos(4x) + \cos^2(4x)\,dx$$

$$= \frac{1}{4}\int 1+2\cos(4x) + \frac{1+\cos(8x)}{2}\,dx$$

$$= \frac{1}{4}\int \frac{3}{2} + 2\cos(4x) + \frac{1}{2}\cos(8x)\,dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x + \frac{2\sin(4x)}{4} + \frac{1}{2}\frac{\sin(8x)}{8}\right) + C$$

$$= \frac{3}{8}x + \frac{1}{8}\sin(4x) + \frac{1}{64}\sin(8x) + C$$

#4 $\int \tan^3 x \sec^4 x\,dx$

substitution: $u = \tan x$ $\qquad \dfrac{du}{dx} = \sec^2 x \qquad dx = \dfrac{du}{\sec^2 x}$

$$= \int u^3 \sec^4 x \frac{du}{\sec^2 x} = \int u^3 \sec^2 x\,du$$

$\sec^2 x = 1 + \tan^2 x = 1 + u^2$

$$= \int u^3(1+u^2)\,du = \int u^3 + u^5\,du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C$$

#5 $\int \sin^2(4x)\cos^3(4x)\,dx$

$u = \sin(4x) \qquad \frac{du}{dx} = 4\cos(4x) \qquad dx = \frac{du}{4\cos(4x)}$

$$= \int u^2 \cos^3(4x) \cdot \frac{du}{4\cos(4x)}$$

$$= \frac{1}{4}\int u^2 \cos^2(4x)\,du \qquad \left(\cos^2(4x) = 1 - \sin^2(4x) = 1 - u^2\right)$$

$$= \frac{1}{4}\int u^2(1-u^2)\,du$$

$$= \frac{1}{4}\int u^2 - u^4\,du$$

$$= \frac{1}{4}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$$

$$= \frac{1}{4}\left(\frac{\sin^3(4x)}{3} - \frac{\sin^5(4x)}{5}\right) + C$$

$$= \frac{1}{12}\sin^3(4x) - \frac{1}{20}\sin^5(4x) + C.$$