21 pts.

ISE 2211 • Exam III • 1 October 2021 NAME SOLUTION

One day Macy Wallace was making popcorn and, statistics having permanently overtaken her brain, realized that there is statistical significance to the number of un-popped kernels in the bag. She proceeded to pop 27 bags of popcorn, all from the same supplier, all in the same manner and in random order, and got the following data on the number of un-popped kernels:

7	16	4	6	17	17	20	7	24
9	18	19	18	13	22	5	19	8
12	24	10	4	24	26	18	14	22

Because Joe Tritschler is not trying to ruin anyone's life, he computed sample parameters for you. Here they are.

$\bar{x} = 14.93$ $s^2 = 47.30$

Use this information to write an upper 95% confidence bound on the number of un-popped kernels in a bag of popcorn. Be sure to include a unit. Assume population variance is unknown. Sketch the appropriate probability distribution and indicate the location of the bound.

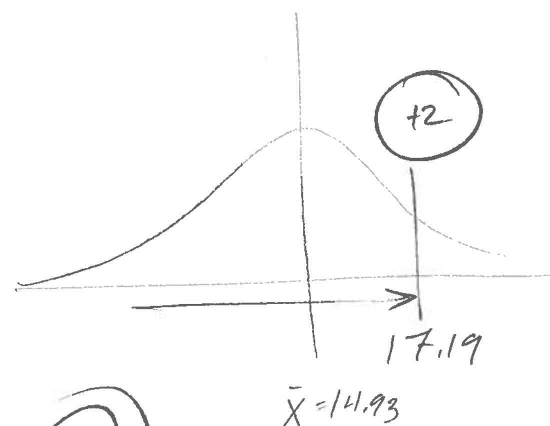
$n < 30$; Use T-distribution.

upper +1

$$\mu : \bar{x} + t_{\alpha, n-1} \, s/\sqrt{n}$$

one-sided +1

+1



$t_{.05, 26} = 1.706$ (table) +2

$$\mu < 14.93 + 1.706 \cdot \sqrt{\frac{47.30}{27}}$$

$\mu < 17.19$ unpopped kernels +2

Write a 95% confidence interval on the variance of un-popped kernels; again, include a unit. Sketch the appropriate probability distribution and indicate the location of the bound.

$$\chi^2_{1-\alpha/2,\,n-1} = \chi^2_{.975,\,26} = 13.84$$

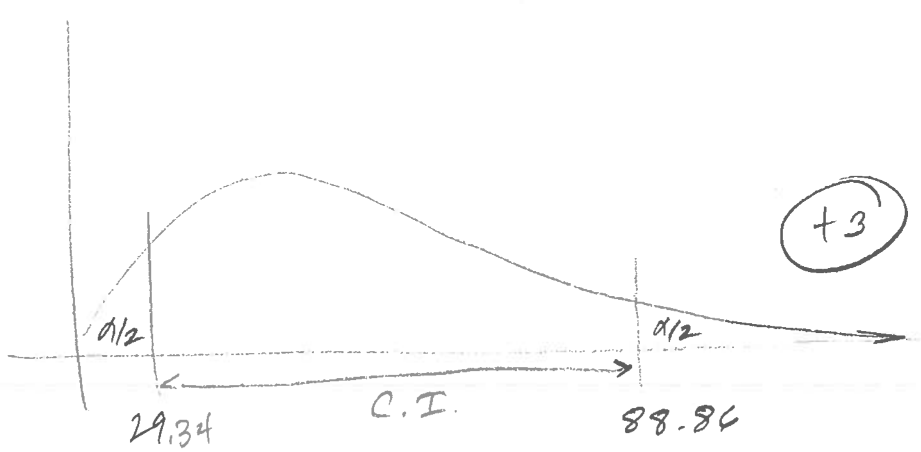
$$\chi^2_{\alpha/2,\,n-1} = \chi^2_{.025,26} = 41.92$$

+2

$$\frac{26 \cdot 47.30}{41.92} \leq \sigma^2 \leq \frac{26 \cdot 47.30}{13.84}$$

$$29.34 \leq \sigma^2 \leq 88.86 \quad (\text{unpopped kernels})^2$$

+3



+3

Macy will not accept a bag of popcorn if the number of un-popped kernels is 20 or greater. Write a 95% confidence interval on the population proportion of unacceptable bags.

$$\hat{p} = x/n = 7/27 \quad +1$$

$$z_{\alpha/2} = z_{.05/2} = 1.960 \quad [\text{bottom row of } t\text{-table}] \quad +1$$

$$p: \quad 7/27 \pm 1.960 \sqrt{\frac{7/27\,(1 - 7/27)}{27}}$$

$$0.09396 < p < 0.4246 \quad +2$$