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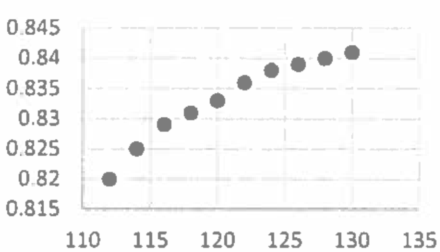
ISE 2211 • Exam VI • 19 July 2018 NAME SOLUTION

1) The electrical resistance of the tungsten filament in an incandescent light bulb increases dramatically with temperature; in fact, all metals have a positive temperature coefficient. This is why incandescent bulbs burn out at turn-on, because their low electrical resistance when cold draws much more current than when they warm up to their operating temperature of a couple thousand degrees Kelvin. It's this huge change in resistance that makes their voltage-to-current relationship inherently non-linear; however, if we know that we will use the bulb over a narrow range of applied voltage, a least-squares linear regression might be useful for approximating the filament's current. The following is data for AC voltage in V_{RMS} (x) vs. current in A_{RMS} (y) for a 100W incandescent light bulb. Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model for filament current. 6

Formulae:

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{XY}}{S_{XX}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

	Voltage (V_{RMS}) (x)	Current (A_{RMS}) (y)
1	112	0.820
2	114	0.825
3	116	0.829
4	118	0.831
5	120	0.833
6	122	0.836
7	124	0.838
8	126	0.839
9	128	0.840
10	130	0.841



$\sum x_i = 1210$

$\sum y_i = 8.332$ +1

$\bar{x} = 121.0$

$\bar{y} = 0.8332$ +1

$\sum x_i^2 = 146740$

$\sum y_i^2 = 6.942658$ +1

ok if this one is later

$\sum x_i y_i = 1008.54$ +1

$S_{XY} = 1008.54 - \frac{1210 \cdot 8.332}{10}$

$= 0.368$

$S_{XX} = 146740 - \frac{1210^2}{10} = 330$

$\therefore \hat{\beta}_1 = 0.368/330 = \underline{\underline{0.00111515}}$ +1

$\hat{\beta}_0 = 0.8332 - 0.00111515 \cdot 121.0 = \underline{\underline{0.698267}}$ +1

Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero.

H_0: $\hat{\beta}_1 = 0$
H_1: $\hat{\beta}_1 \neq 0$

$$\beta_1 : \quad \hat{\beta}_1 \pm t_{\alpha/2,\, n-2} \sqrt{\hat{\sigma}^2 / S_{xx}}$$

$$0.00111515 \pm 2.306 \sqrt{3.1531 \times 10^{-6} / 330}$$

$$0.0008897 < \beta_1 < 0.001341$$

+2

C.I. does not contain zero;

$\therefore$ reject H_0

+2

List two theoretical scenarios that would fail to reject H_0. What does your conclusion imply about the relationship between voltage and current for the incandescent light bulb filament?

1.) no significant relationship
2.) relationship is non-linear

+2

$\therefore$ there is a significant linear relationship between voltage and current over the range of data

+1

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Write an equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$$\hat{y} = 0.698267 + 0.00111515\,x$$ +1

Write a 95% confidence interval on the mean current at x = 117 V_{RMS}.

$$SS_T = \sum y_i^2 - n\bar{y}^2$$

$$= 6.942658 - 10 \cdot 0.8332^2$$

$$= 0.0004356$$ +1

$$SS_E = SS_T - \hat{\beta}_1 S_{XY}$$

$$= 0.0004356 - 0.00111515 \cdot 0.368$$

$$= 0.0000252248$$ +1

$$\hat{\sigma}^2 = \frac{SS_E}{n-2} = \frac{0.0000252248}{8}$$

$$= 0.0000031531$$ +1

$$\hat{\mu}_{Y|117} = 0.698267 + 0.00111515 \cdot 117 = 0.828740$$ +1

$$t_{.025,8} = 2.306$$ +1

$$\mu_{Y|117} : 0.828740 \pm 2.306\sqrt{3.1531 \times 10^{-6}\left[\frac{1}{10} + \frac{(117 - 121.0)^2}{330}\right]}$$

$$0.8287 < \mu_{Y|117} < 0.8303$$ +2

[ARMS]

Write a 95% confidence interval on the correlation coefficient ρ, if y and x may both be considered random variables. (Ignore the fact that n $\ngtr$ 30.) 5

$$R^2 = 1 - \frac{SSE}{SST}$$

$$= 1 - \frac{0.00002522\,48}{0.0004356}$$

$$R^2 = 0.9421 \quad +0$$

$$\rho: \quad \tanh\left(\tanh^{-1} R \pm \frac{Z_{\alpha/2}}{\sqrt{n-3}}\right)$$

$$Z_{.025} = 1.960 \quad +1$$

$$\tanh^{-1}\left(+\sqrt{0.9421}\right) = 2.103 \quad +2$$

$$\tanh\left(2.103 \pm \frac{1.960}{\sqrt{7}}\right)$$

$$0.8769 < \rho < 0.9932 \quad +2$$