ISE 2211 • Exam IV • 29 June 2021 30 pts. NAME SOLUTION

Last time, we measured a batch of twenty 100k potentiometers that were found to have a mean resistance of 104.2 kΩ and a variance of 2.876 (kΩ)². Then we wrote a 95% confidence interval on population mean resistance and found this to be **103.4 < μ < 105.0 kΩ**. Use this confidence interval to test the claim by Clarostat, the manufacturer, that the population mean resistance is 100 kΩ.

H_0: μ = 100 kΩ

H_0: $\mu \neq$ 100 kΩ

∴ the hypothesized value of μ_0 = 100 kΩ is not within the 95% CI on μ, +1

∴ reject Ho +1

(@ $\alpha = 0.05$)

Now test the same hypotheses using the fixed-significance-level approach at α = 0.05. Sketch the appropriate distribution, showing the test statistic, critical values, and critical regions.

$n < 30$, σ unknown

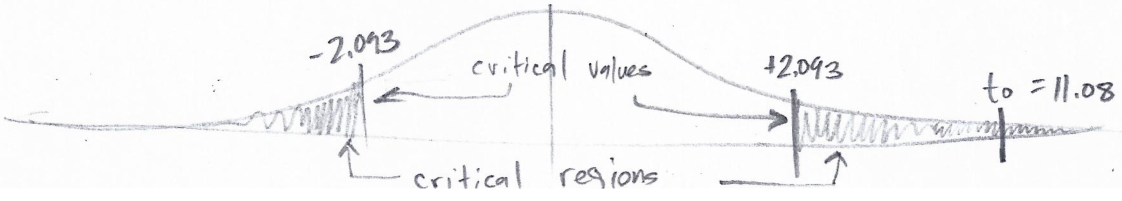
$$\therefore\ t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{104.2 - 100}{\sqrt{2.876}/\sqrt{20}}$$

$= 11.08$ (ouch) +2

$t_{\alpha/2,n-1} = t_{.025,19} = 2.093$ +2 (table)

$t_0 \ggg +t_{\alpha/2,n-1}$ +1

∴ strongly reject Ho +1



+3

Test the following hypotheses on standard deviation using the fixed-significance-level approach at $\alpha = 0.05$. Sketch the appropriate distribution, showing the test statistic, critical value(s), and critical region(s). Don't forget to state your final conclusion in terms of the original problem.

H_0: $\sigma = 2$ kΩ

H_1: $\sigma > 2$ kΩ

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{19 \cdot 2.876}{2^2} = 13.66$$

+2

upper one-sided;

$$\therefore \chi^2_{\alpha, n-1} = \chi^2_{.05, 19} = 30.14$$

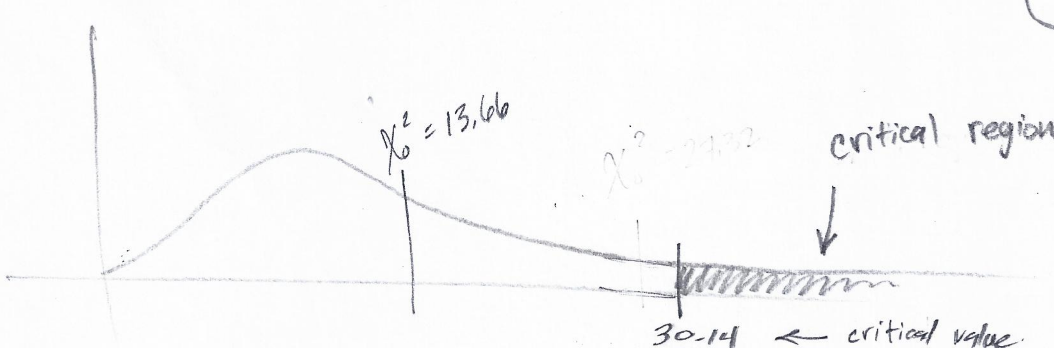
+1 +2 (table)

$$\chi_0^2 \not> \chi^2_{\alpha, n-1}$$ +1

$\therefore$ fail to reject Ho @ $\alpha = 0.05$ +1

data suggests std. dev. does not exceed 2kΩ

+1



+3

In the end, it was decided to cull (i.e., remove from the batch) any pots more than 5% over the target value as being unusable, and this resulted in six out of 20 potentiometers culled. Joe Tritschler finds it appalling to find more than 10% of a batch of parts unusable, and so would like to test the following hypotheses on the proportion of unusable pots using the *p*-value approach. Ignore the "large-sample" requirement in computing the test statistic. Sketch the appropriate distribution, showing the test statistic and region(s) corresponding to the *p*-value. Clearly state your final conclusion with respect to a significance level of $\alpha = 0.05$.

H_0: $p = 10\%$

H_1: $p > 10\%$

$X = 6, \quad n = 20$

$p_0 = 0.10$

$$\therefore Z_0 = \frac{X - np_0}{\sqrt{np_0(1-p_0)}} = \frac{6 - 20 \cdot .10}{\sqrt{20 \cdot .10(.90)}}$$

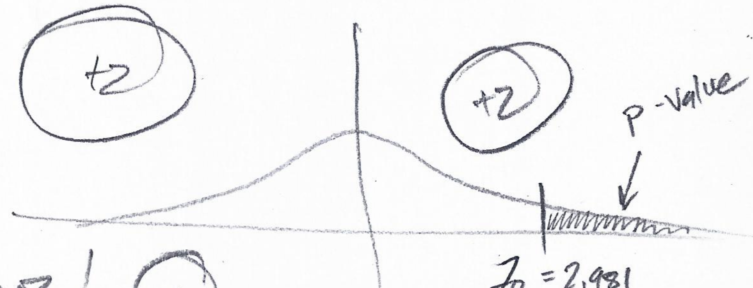
$Z_0 = 2.981$ +2

p-value $= P(Z > 2.981) = P(Z < -2.981)$ due to symmetry!

$\doteq 0.001441$ (table) +2

$P \lll \alpha$

$\therefore$ reject H_0. +2



data suggests proportion of unusable pots is way worse than 10%! +1