Lab 8 Solution

#1 $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$

Integral Test

$$\int_3^{\infty} \frac{1}{x(\ln x)^2}\,dx$$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad dx = x\,du$

$u = \ln 3 \to \infty$

$$\int_{\ln 3}^{\infty} \frac{1}{x\,u^2}\,x\,du$$

$$= \int_{\ln 3}^{\infty} \frac{1}{u^2}\,du = \int_{\ln 3}^{\infty} u^{-2}\,du$$

$$= -\frac{1}{u}\Big|_{\ln 3}^{\infty} = \frac{1}{\ln 3} \leftarrow \text{Convergent!}$$

Therefore the series is Convergent by Integral Test.

#2. $\sum_{n=0}^{\infty} \frac{2^n - 1}{5^n}$

Simple Comparison Test

$$\frac{2^n - 1}{5^n} < \frac{2^n}{5^n}$$

$\sum_{n=0}^{\infty} \frac{2^n}{5^n}$ is a geometric series with $r = \frac{2}{5}$,

so it is Convergent.

Therefore, the original series $\sum_{n=0}^{\infty} \frac{2^n - 1}{5^n}$ is convergent by Simple Comparison Test.

#3 $\sum_{n=1}^{\infty} \frac{2n+1}{3n^2+5}$

Limit Comparison Test

Compare with $\sum_{n=1}^{\infty} \frac{2n}{3n^2} = \underbrace{\frac{2}{3} \sum_{n=1}^{\infty} \frac{1}{n}}$

p-series, $p=1$, divergent.

$$\lim_{n\to\infty} \frac{\frac{2n+1}{3n^2+5}}{\frac{2n}{3n^2}} = \lim_{n\to\infty} \frac{2n+1}{3n^2+5} \cdot \frac{3n^2}{2n} = 1$$

Therefore, the series $\sum_{n=1}^{\infty} \frac{2n+1}{3n^2+5}$ is divergent by Limit Comparison Test.

#4 $\sum_{n=1}^{\infty} \sqrt[n]{5}$

Since $\lim_{n\to\infty} \sqrt[n]{5} = \lim_{n\to\infty} 5^{\frac{1}{n}} = 5^0 = 1 \neq 0$

The series $\sum_{n=1}^{\infty} \sqrt[n]{5}$ is divergent by Test for Divergence.