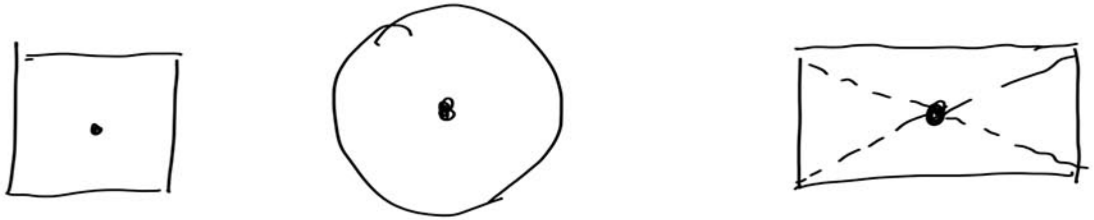
LECTURE NO. 7

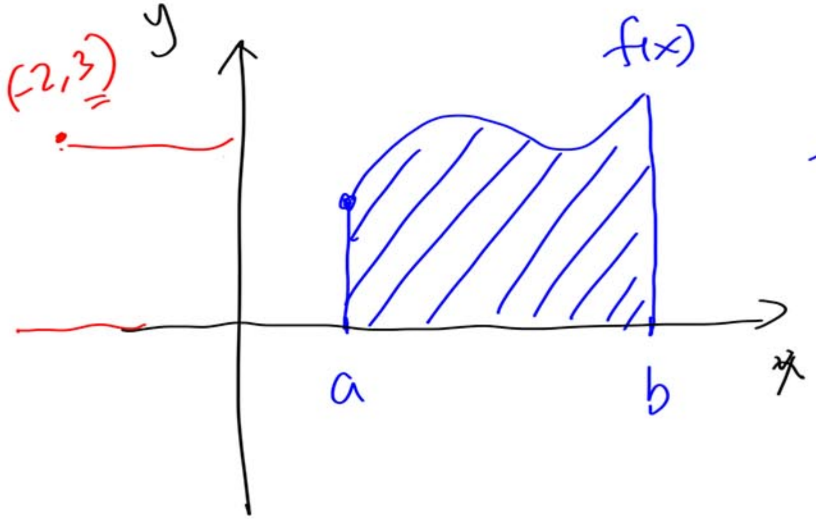
2.6 Moments and Centers of Mass

Wright State University

Center of Mass of a Thin Plate with Regular Shapes and Uniform Density



centroid



Thin plate

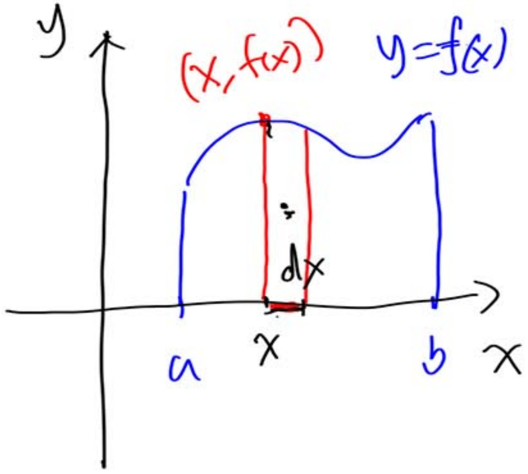
Moment about x-axis = M_x

(Mass · distance to x-axis)

Moment about y-axis = M_y

(Mass · distance to y-axis)

Moments with respect to x-axis and y-axis



density $= \rho$

Total Mass $=$ Area $\cdot \rho = \rho \int_a^b f(x)\,dx = M$

Now we set up integrals for M_x and M_y

Since dx is tiny, we may regard the stripe as a rectangle.

Area $= f(x)\,dx$ Mass $= \rho f(x)\,dx$

Moment about $y = \rho f(x) \cdot dx \cdot x = \rho x f(x)\,dx$

Moment about $x = \rho f(x)\,dx \cdot \frac{1}{2} f(x) = \rho \cdot \frac{1}{2} (f(x))^2\,dx$

$$M_y = \int_a^b \rho\, x f(x)\,dx$$

$$M_y = \rho \int_a^b x f(x)\,dx$$

$$M_x = \int_a^b \rho\, \frac{1}{2} [f(x)]^2\,dx$$

$$M_x = \rho \int_a^b \frac{1}{2} [f(x)]^2\,dx$$

Formula for Center of Mass

- Total Mass $M = \rho \int_a^b f(x)dx$.

$A =$ Area under the curve

$= \int_a^b f(x)\,dx$

- Moment with respect to x-axis $M_x = \rho \int_a^b \frac{1}{2}[f(x)]^2 dx$.

- Moment with respect to y-axis $M_y = \rho \int_a^b xf(x)dx$.

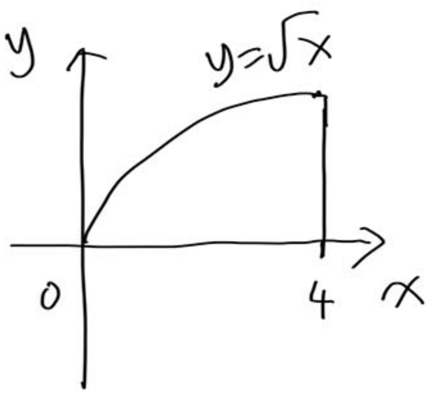
- Coordinates of the center of mass $(\bar{x}, \bar{y}) = (\frac{M_y}{M}, \frac{M_x}{M})$.

Centroid

$$\bar{x} = \frac{\cancel{\rho} \int_a^b x f(x)\,dx}{\cancel{\rho} \int_a^b f(x)\,dx} = \frac{1}{A} \int_a^b x f(x)\,dx$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2\,dx$$

Let R be the region bounded by $y = \sqrt{x}$ and the x-axis on the interval $[0, 4]$. Find the centroid of the region.



$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 = \frac{16}{3}$$

$x^{\frac{1}{2}}$

$$\bar{x} = \frac{1}{A}\int_0^4 x\,f(x)\,dx = \frac{3}{16}\int_0^4 x\sqrt{x}\,dx = \frac{3}{16}\int_0^4 x^{\frac{3}{2}}\,dx$$

$$\bar{x} = \frac{3}{16}\cdot\frac{2}{5} x^{\frac{5}{2}} \Big|_0^4 = \frac{3}{40}\, 4^{\frac{5}{2}} - 0 = \frac{3}{40}\cdot 32 = \frac{12}{5}$$

$$\bar{y} = \frac{1}{A}\int_0^4 \frac{1}{2}[f(x)]^2\,dx = \frac{3}{16}\cdot\frac{1}{2}\int_0^4 (\sqrt{x})^2\,dx = \frac{3}{32}\int_0^4 x\,dx$$

$$\bar{y} = \frac{3}{32}\,\frac{x^2}{2}\Big|_0^4 = \frac{3}{32}\cdot 8 - 0 = \frac{3}{4}$$

Centroid

$\left(\frac{12}{5}, \frac{3}{4}\right)$