Discrete Random Variables

<u>random variable</u>: each outcome in the sample space of a random experiment is a <u>real number</u>.

.. denoted by an upper-case variable if <u>symbolic</u> (usually X).

.. an actual outcome is then lower case ($x = 4.2$ kg)

discrete random variable: finite or "countably-infinite" range of real numbers

ex: # of defective parts, transmitted bits, etc..

probability distribution: describes probabilities associated with possible values of X

.. can be an equation; for discrete, often a <u>list</u> of possible outcomes and associated probabilities ex: three cell phone cameras tested for flash recharge time; it is known that they pass <u>80% of the time</u>.

∴ write a probability distribution for three independent cameras passing / failing

First: determine the sample space of the experiment

let's let p : pass   f : fail

ppp   fff   pfp   etc..

↑ all pass   ↑ all fail

∴ this is basically a <u>factorial</u> experiment w/

2^k outcomes, where k = 3 cameras

↑ either pass or fail

$2^3 = 8$

∴ let X represent the # of <u>passing cameras</u>

#1	#2	#3	X	P
p	p	p	3	$0.8 \cdot 0.8 \cdot 0.8 = 0.512$
p	p	f	2	$0.8 \cdot 0.8 \cdot 0.2 = 0.128$
p	f	p	2	0.128
p	f	f	1	0.032
f	p	p	2	0.128
f	p	f	1	0.032
f	f	p	1	0.032
f	f	f	0	0.008

∴ next: associate these outcomes with values of X

∴ now we need probabilities associated w/ each outcome

∴ because they're independent, we use multiplication rule

$$P\{ppp\} = 0.8 \cdot 0.8 \cdot 0.8 = 0.512$$

or 51.2%

known: they pass 80% of the time

this is - finally! - useful info!!

decision-making

$$P\{ppf\} = 0.8 \cdot 0.8 (1 - 0.8) = 0.128$$

↓ p

or 12.8%

order of two passes irrelevant!

$$P\{pfp\} = 0.8 \cdot 0.2 \cdot 0.8 = 0.128$$

$P\{p\,f\,f\} = P\{f\,p\,f\} = P\{f\,f\,p\}$

$= 0.2 \cdot 0.2 \cdot 0.8 = \underline{0.032}$

$P\{f\,f\,f\} = 0.2 \cdot 0.2 \cdot 0.2$

$= \underline{0.008}$

- this is not yet a probability distribution!
  - by definition, that is a list of probabilities associated w/ values of X.
  - what are the possible values of X?

$\{0 \quad 1 \quad 2 \quad 3\}$ (passing cameras)

$P(0) = P\{fff\} = \underline{0.008}$

$P(1) = P\{pff \;\; fpf \;\; ffp\} = 0.032 + 0.032 + 0.032 = \underline{0.096}$

exactly one passing camera

$P(2) = 0.128 + 0.128 + 0.128 = \underline{0.384}$

$P(3) = P\{ppp\} = \underline{0.512}$

this is a probability distribution

∴ note: these probabilities add up to 1 or 100%!

that describes probabilities

probability mass function

1.) $f(x_i) = P(X = x_i)$

2.) $f_n(x_i) \geq 0$

* 3.) $\sum_{i=1}^{T} f(x_i) = 1$ or 100%