ISE 2211 • Exam II • 3 June 2021 NAME SOLUTION

A 1960 episode of *Alfred Hitchcock Presents* called "The Man from the South" has the following premise, reprised in the Quentin Tarantino-directed segment of the 1995 film *Four Rooms*:

> *While vacationing at a resort in Jamaica, the narrator encounters an elderly South American man named Carlos. They are soon joined by a young American naval cadet, who boasts about the reliability of his cigarette lighter. Carlos offers to bet his Cadillac against the American's left little finger that the American cannot ignite the lighter ten times in a row. The American accepts, with the narrator agreeing to act as referee and hold the car key, and they adjourn to Carlos' room.*
>
> *After Carlos has a maid bring in the necessary supplies, he ties the American's left wrist to the table and the challenge begins. After the eighth successful strike, a woman bursts into the room and forces Carlos to drop the knife he has held ready to sever the American's finger. She explains that Carlos is mentally disturbed, having played this game so often in their home country that they had to flee in order to keep the authorities from committing him to a psychiatric hospital. He has taken 47 fingers and lost 11 cars, but no longer has anything of his own to bet with; she won it all from him long ago, including the car he claimed to own. As the narrator offers the key to her, she reaches out to take it with a hand that has only its thumb and one finger still attached.*
> *[Source: https://en.wikipedia.org/wiki/Man_from_the_South]*

If the probability of successfully lighting the cigarette lighter is fixed at 79.2% with a binomial distribution, compute the probability of ten out of ten successful ignitions.

Formulae:

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

$\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$\left.\begin{aligned} n &= 10 \\ x &= 10 \\ p &= .792 \end{aligned}\right\} +1$$

$$\binom{10}{10} = \frac{10!}{10!\,(10-10)!} = 1$$

$$\therefore f(10) = 1 \cdot .792^{10} (1-.792)^{10-10}$$

$$= .792^{10}$$ ← intuitively obvious; multiplication rule!

$$= 0.09711$$

or 9.711 %

+3

Spoiler alert!!! Compute the probability of the very first ignition being a failure, as per *Four Rooms*.

-- assume $n=1$ because trials would not continue after the amputation! (+1)

$\therefore X = 0$

$$\binom{1}{0} = \frac{1!}{0!(1-0)!} = 1$$

again, obvious! one failure!

$$f(0) = 1 \cdot .792^0 (1-.792)^{1-0} = (1-.792)^1 = 0.208 \text{ or } 20.8\%$$ (+3)

Now compute the probability of the *very last ignition* being a failure.

-- implies nine successes and one failure! (+1)

-- the trick here is that, unlike a true binomial distribution, there is only one way this can happen; nine ignitions and then one failure

NOT $\binom{10}{9} = 10$ ways

$$\therefore (.792)^9 (1-.792)^1 = 0.02550 \text{ or } 2.55\%$$ (+3)

Compute the expected number and variance of successful ignitions, including a unit with each answer.

$$\mu = np = 10 \cdot .792 = 7.92 \text{ successful ignitions}$$

(+2)

$$\sigma^2 = np(1-p) = 7.92(1-.792) = 1.647 \text{ (successful ignitions)}^2$$

(+2)

Now suppose that, for some engineering reason, the *time* between ignition failures may be modelled as a continuous Poisson process with a mean value of eight minutes between failures. Determine the probability of a failure within the next 60 seconds while the premise of The Man from the South is taking place.

Formulae:

$f(x) = \lambda e^{-\lambda x}$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mu = \frac{1}{\lambda}$

$\sigma^2 = \frac{1}{\lambda^2}$

Hint: λ must be of the form "events per interval."

choose interval = 1 minute

then $\lambda = \frac{1 \text{ failure}}{8 \text{ min.}} = .125$ failures/min.

+2

∴ one failure within 60 s. is equivalent to the time to the next failure being < 1 min.

∴ $F(1) = 1 - e^{-.125 \cdot 1}$

cumulative probability

$= .1175$ or 11.75%

+3