

H/W 1 1.1 7, 8, 9, 10  
1.2 2, 11, 13, 15, 16

Alex Yeoh

1.1

$$7) \left[ \begin{array}{ccc|c} 1 & 7 & 3 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array} \right] \text{ no solution}$$

$$0x_1 + 0x_2 + 0x_3 = 1$$

$$8) \left[ \begin{array}{ccc|c} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \begin{cases} 0x_1 + 0x_2 + 2x_3 = 0 \\ x_3 = 0 \end{cases} \left\{ \begin{array}{l} 0x_1 + 1x_2 + 7(0) = 0 \\ x_2 = 0 \end{array} \right\} \left\{ \begin{array}{l} 1x_1 + (-4)(0) + 9(0) = 0 \\ x_1 = 0 \end{array} \right.$$

$$\vec{x} = \begin{pmatrix} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{pmatrix}$$

$$9) \left[ \begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{array} \right] \begin{cases} 2x_4 = 4 \\ x_4 = 2 \end{cases} \left\{ \begin{array}{l} 1x_3 + (-3)(2) = -1 \\ x_3 - 6 = -1 \\ x_3 = 5 \end{array} \right\} \left\{ \begin{array}{l} 1x_2 + (-3)(5) = -7 \\ x_2 - 15 = -7 \\ x_2 = 8 \end{array} \right\} \left\{ \begin{array}{l} 1x_1 + (-1)(8) = -4 \\ x_1 - 8 = -4 \\ x_1 = 4 \end{array} \right.$$

$$\vec{x} = \begin{pmatrix} x_1 = 4 \\ x_2 = 8 \\ x_3 = 5 \\ x_4 = 2 \end{pmatrix}$$

$$10) \left[ \begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \begin{cases} 1x_4 = 3 \\ x_4 = 3 \end{cases} \left\{ \begin{array}{l} 1x_3 = 6 \\ x_3 = 6 \end{array} \right\} \left\{ \begin{array}{l} 1x_2 + (-4)(3) = 7 \\ x_2 - 12 = 7 \\ x_2 = 19 \end{array} \right\} \left\{ \begin{array}{l} 1x_1 + (-2)(19) + 3(3) = -2 \\ x_1 - 38 + 9 = -2 \\ x_1 - 32 = -2 \\ x_1 = 30 \end{array} \right.$$

$$\vec{x} = \begin{pmatrix} x_1 = 30 \\ x_2 = 19 \\ x_3 = 6 \\ x_4 = 3 \end{pmatrix}$$

1.2

2, 11, 13, 15, 16

2 a) reduced echelon form

b) echelon form

c) echelon form

d) echelon form

$$11) \left[ \begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array} \right] \xrightarrow[R_3+3R_1]{R_2+2R_1} \left[ \begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -6 & 8 & -4 & 0 \\ -6 & 8 & -4 & 0 \end{array} \right] \xrightarrow{R_2+2R_1} \left[ \begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

free  $x_3$  and  $x_2$   
 $3x_1 - 4x_2 + 2x_3 = 0$   
 $3x_1 = 4x_2 - 2x_3$   
 $x_1 = \frac{4x_2 - 2x_3}{3}$

$$\vec{x} = \begin{pmatrix} \frac{4x_2 - 2x_3}{3} \\ x_2 \\ x_3 \end{pmatrix}$$

$$13) \left[ \begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{} \left[ \begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \end{array} \right]$$

Free  $x_5$   $\left\{ \begin{array}{l} \text{Free } x_3 \\ x_4 + 9x_5 = 4 \\ x_4 = 4 - 9x_5 \end{array} \right.$   $\left\{ \begin{array}{l} \text{Free } x_3 \\ x_2 - 4x_5 = 1 \\ x_2 = 1 + 4x_5 \end{array} \right.$

---


$$(x_1 + 3)(1 - 4x_5) + (-1)(4 - 9x_5) = -2$$

$$x_1 - 3 + 12x_5 - 4 + 9x_5 = -2$$

$$x_1 - 7 + 21x_5 = -2$$

$$x_1 = 5 - 21x_5$$

15a) Consistent, unique

b) inconsistent

16 a) Consistent, unique

b) Consistent, not unique