LECTURE NO. 1

1.6 Integrals Involving Exponential and Logarithmic Functions

Wright State University

Integration Formulas

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x} = \ln|x| + C$
- $\int e^x dx = e^x + C$, $\int a^x dx = \frac{a^x}{\ln a} + C$
- $\int \sin x dx = -\cos x + C$, $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$, $\int \csc^2 x dx = -\cot x + C$
- $\int \sec x \tan x dx = \sec x + C$, $\int \csc x \cot x dx = -\csc x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$, $\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$

Two Integration Techniques

- Use these formulas to integrate "term by term".
- $\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$
- $\int k \cdot f(x) dx = k \cdot \int f(x) dx$
- The second technique we learned is "Substitution".

Substitution $\quad u = 2x \quad \frac{du}{dx} = 2 \quad$ Solve for dx: $\quad dx = \frac{du}{2}$

$$\int e^u \frac{du}{2} = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \underbrace{\frac{1}{2}e^{2x} + C}$$

FINAL ANSWER

In general, $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$

(k is a constant.)

$$\int e^{5x}\,dx = \frac{e^{5x}}{5} + C \qquad \int e^{\frac{x}{2}}\,dx = \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + C = 2e^{\frac{x}{2}} + C$$

Substitution $u = 2x^3$ $\frac{du}{dx} = 6x^2$ Solve for dx: $dx = \frac{du}{6x^2}$

$$\int x^2 e^u \cdot \frac{du}{6x^2} = \int e^u \cdot \frac{du}{6} = \frac{1}{6}\int e^u\, du$$

$\downarrow$ Take antiderivative!

$$= \frac{1}{6} e^u + c$$

$$= \frac{1}{6} e^{2x^3} + c$$

FINAL ANSWER.

$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

Substitution $u = \frac{1}{x}$ $\quad \frac{du}{dx} = -x^{-2} = -\frac{1}{x^2}$ $\quad$ Solve for dx: $dx = -x^2 du$

$$\left(dx = \frac{du}{-\frac{1}{x^2}} = -x^2 du \right)$$

$x: 1 \longrightarrow 2$

$u = \frac{1}{x}: 1 \longrightarrow \frac{1}{2}$

$$\int_1^{\frac{1}{2}} \frac{e^u}{\cancel{x^2}} \cdot (-\cancel{x^2} du)$$

$$= -\int_1^{\frac{1}{2}} e^u du = -e^u \Big|_1^{\frac{1}{2}} = -e^{\frac{1}{2}} - (-e^1)$$

$$= -e^{\frac{1}{2}} + e$$

FINAL ANSWER

$$-e^u = -e^{\frac{1}{x}} \Big|_1^2 = -e^{\frac{1}{2}} - (-e)$$

$$\frac{2}{3x-1} = 2\underbrace{(3x-1)}_{\text{inner}}^{-1} \qquad u = 3x-1 \qquad \frac{du}{dx} = 3 \qquad dx = \frac{du}{3}$$

$$\int \frac{2}{u}\frac{du}{3} = \frac{2}{3}\int \frac{1}{u}\,du = \frac{2}{3}\,\mathrm{Ln}|u| + C$$

$$= \frac{2}{3}\,\mathrm{Ln}|3x-1| + C$$

FINAL ANSWER

$u = 1 + \cos x \quad \frac{du}{dx} = -\sin x \quad dx = \frac{du}{-\sin x}$

$x: 0 \to \frac{\pi}{2}$

$u = 1+\cos x: 2 \to 1$

$$\int_2^1 \frac{\sin x}{u} \frac{du}{-\sin x} = -\int_2^1 \frac{1}{u} du = \int_1^2 \frac{1}{u} du$$

$$= \ln|u| \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$$