ISE 2211 • Exam VI • 13 April 2020 NAME SOLUTION

Data suggests that there may be a relationship between the weekly amount of rainfall measured at WPAFB in inches (x) and the number of minutes one has to wait in line at Kroger Pickup (y).

Week #	Rainfall, in. (x)	Wait time at Kroger Pickup, min. (y)
1	0.25	1
2	1.6	4
3	0.0	12
4	2.7	61
5	4.4	92

Determine least-squares estimates for slope (β_1) and intercept (β_0) of the simple linear regression model for Kroger Pickup wait time vs. weekly rainfall.

$$\hat{\beta}_1 = \frac{\sum y_i x_i - \frac{(\sum y_i)(\sum x_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{S_{xy}}{S_{xx}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$\sum y_i x_i = 576.15$ (+1)

$\sum x_i = 8.95$, $\sum y_i = 170$ (+1)

$\sum x_i^2 = 29.2725$, $\sum y_i^2 = 12346$ (+1)

$\bar{y} = 34$, $\bar{x} = 1.79$ (+1)

$S_{xy} = 576.15 - \frac{170 \cdot 8.95}{5}$

$= 271.85$ (+1)

$S_{xx} = 29.2725 - \frac{8.95^2}{5}$

$= 13.252$ (+1)

$\hat{\beta}_1 = \frac{271.85}{13.252} = 20.51$ (+1)

$\hat{\beta}_0 = 34 - 20.51 \cdot 1.79 = -2.720$ (+1)

Write the equation for the estimated regression line ($\hat{y}$) with your actual numbers for $\hat{\beta}_0$ and $\hat{\beta}_1$.

$\hat{y} = 20.51x - 2.720$ (+1)

Write a 95% prediction interval on the mean wait time at $x = 1$ inch of rainfall.

$$SS_T = 12346 - 5.34^2 = 6566 \quad (+1)$$

$$SS_E = 6566 - 20.51 \cdot 271.85 = 990.4 \quad (+1)$$

$$\hat{\sigma}^2 = \frac{990.4}{3} = 330.1 \quad (+1)$$

$$\hat{y}_0\big|_{lin.} = 20.51 \cdot 1 - 2.720 = 17.79 \text{ min.} \quad (+1)$$

$$t_{\alpha/2, n-2} = t_{.025, 3} = 3.182 \quad (+1)$$

$$Y_0: \quad 17.79 \pm 3.182\sqrt{330.1\left[1 + \frac{1}{6} + \frac{(1 - 1.79)^2}{13.252}\right]}$$

$$17.79 \pm 64.56$$

$$-46.77 < Y_0 < 82.35 \text{ min.} \quad (+2)$$

Write a 95% confidence interval on the value of slope and use it to test the following hypotheses that the slope is zero. Include a unit with the C.I.

H_0: $\hat{\beta}_1 = 0$

H_1: $\hat{\beta}_1 \neq 0$

β_1: $20.51 \pm 3.182\sqrt{330.1/13.252}$

20.51 ± 15.88

$4.63 < \beta_1 < 36.39$ min./in. +1

+2

C.I. does not include zero

∴ reject H_0 +1

Write a 95% confidence interval on the correlation coefficient ρ, if y and x may both be considered random variables. (Ignore the fact that $n \not> 30$.)

$$R^2 = 1 - \frac{990.1}{6566} = 0.8492$$ (+1)

$$z_{.025} = 1.960$$ (+1)

$$\rho: \quad \tanh\left(\tanh^{-1}\sqrt{0.8492} \pm \frac{1.960}{\sqrt{2}}\right)$$

$$= 1.599$$

(+1)

$$\tanh(0.2131) < \rho < \tanh(2.985)$$

$$0.2099 < \rho < 0.9949$$

(+2)