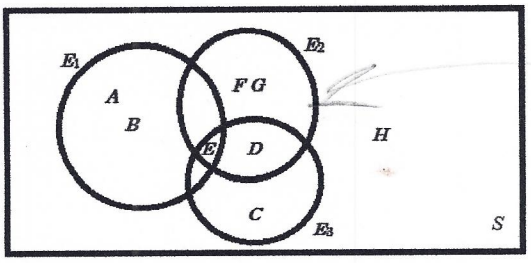
28 pts.

ISE 2211 • Exam I • 2 September 2020 NAME SOLUTION

1) The following Venn diagram applies to outcomes and events in the sample space *S*, with associated probabilities, where *S* { A B C D E F G H }.



P(*A*) = 0.086
P(*B*) = 0.044
P(*C*) = 0.172
P(*D*) = 0.001
P(*E*) = 0.502
P(*F*) = 0.058
P(*G*) = 0.111
P(*H*) = 0.026

a) Is *S* a *discrete* or *continuous* sample space? +1

b) Is the set of events E_1, E_2, and E_3 *exhaustive*? Why or why not?

no; H is not in an event +2

c) Compute the probability of $E_1 \cup E_2 \cup E_3$ using the addition rule. Perform all the intermediate set operations first and check that this matches what is shown in the Venn diagram before plugging in numbers at the very end.

$P(E_1) = P\{ABE\}$ +1

$P(E_2) = P\{DEFG\}$ +1

$P(E_3) = P\{CDE\}$ +1

$P(E_1 \cap E_2) = P\{E\}$ +1

$P(E_2 \cap E_3) = P\{DE\}$ +1

$P(E_1 \cap E_3) = P\{E\}$ +1

$P(E_1 \cap E_2 \cap E_3) = \{E\}$ +1

Addition rule:

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$ +1

$$= P[\{ABE\} + \{DFG\} + \{CDE\} - \{E\} - \{DE\} - \{E\} + \{E\}]$$ +1

$= P\{ABEDFGC\}$ +1 yes $= 0.086 + 0.044 + 0.172 + 0.001 + 0.502 + 0.058 + 0.111 = 0.974$ +1

2) One of my esteemed colleagues suspected trouble with his domestic line voltage last summer, and recorded these actual voltages (in V_{RMS}) using a Fluke 77-series multimeter:

{125.2 125.4 124.3 123.5 126.6 124.2 126.6 124.2 123.5 122.1 122.8 121.5 121.3 121.7 122.2}

Compute the sample mean, sample variance, sample standard deviation, and sample range. Include a unit with each answer.

Hint: $s^2 = \dfrac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$

$$\bar{X} = \frac{\sum x_i}{n} = \frac{1855.1}{15} = 123.7 \; V_{RMS}$$

(+1) (+1)

$$s^2 = \frac{229469.5 - \frac{1855.1^2}{15}}{14} = 3.079 \; (V_{RMS})^2$$

(+2) (+1)

$$s = +\sqrt{3.079} = 1.755 \; V_{RMS}$$

(+1) (+1)

$$r = 126.6 - 121.3 = 5.3 \; V_{RMS}$$

(+1) (+1)

3) Electronic printed circuit boards (PCBs) are generally constructed using preprogrammed "robots" that populate (pick up and place) various components prior to soldering them in place. Consider the following simple PCB:

Three 10-kΩ resistors = n_1

Seven 1-kΩ resistors = n_2

Four Green LEDs = n_3

Four 2N3904 transistors = n_4

How many ways can this PCB be properly assembled, if it doesn't matter which type of component is populated first?

$n = 3 + 7 + 4 + 4 = 18$

$\therefore \# = \frac{18!}{3!\,7!\,4!\,4!} = 3\,675\,672\,00$ (+3)

If all ten of the resistors must be placed on the board first, how many ways are there to select the remaining parts?

$n = 4 + 4 = 8$ remaining parts (LEDs and transistors)

$\therefore \# = \frac{8!}{4!\,4!} = 70$ (+3)

Formulae:

$$P\binom{n}{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$\frac{n!}{n_1!\,n_2!\,n_3!\,\ldots\,n_r!}$$