ISE 2211 • Exam III • 18 June 2020 NAME SOLUTION

Include a unit with each answer. Note that this data has not been altered in any way to make "nice" results!

Last week, one of your exam problems involved domestic line voltage measurements performed by one of my esteemed colleagues. Here are actual measurements, performed using a Fluke 77-series multimeter:

{125.2 125.4 124.3 123.5 126.6 124.2 126.6 124.2 123.5 122.1 122.8 121.5 121.3 121.7 122.2}

...and because Joe is a *really nice guy*, he calculated the following sample parameters for you (actually he calculated them for you to make grading the exams easier, because if you make a bonehead mistake here he'd have to figure out if that's why the rest of your exam is also jacked, or if you botched that too):

$\bar{x}$ = 123.7 V and s = 1.755 V.

Write a 95% confidence interval on the mean value of line voltage, assuming unknown population standard deviation.

$n = 15$, unknown pop. variance;

use t distribution

$\mu: \ \bar{x} \pm t_{\alpha/2, n-1} \, s/\sqrt{n}$ 95% C.I. $\rightarrow \alpha = 0.05$

$t_{0.05/2, 15-1} = t_{.025, 14} = 2.145$ (table)

$123.7 \pm 2.145 \cdot 1.755/\sqrt{15}$

$122.7 \leq \mu \leq 124.7$ (V)

+5

Write a 95% prediction interval on the next value of line voltage.

$$X_{16} : \bar{x} \pm t_{\alpha/2, n-1} \, s \sqrt{1 + \frac{1}{n}}$$

$$123.7 \pm 2.145 \cdot 1.755 \sqrt{1 + \frac{1}{15}}$$

$$119.8 \leq \mu \leq 127.6 \quad (V)$$

+3

Write a 95% confidence interval on the variance of line voltage.

$$\chi^2_{\alpha/2, n-1} = \chi^2_{0.025, 14} = 26.12$$

$$\chi^2_{1-\alpha/2, n-1} = \chi^2_{0.975, 14} = 5.63$$

$$\frac{14 \cdot 1.755^2}{26.12} \leq \sigma^2 \leq \frac{14 \cdot 1.755^2}{5.63}$$

$$1.651 \leq \sigma^2 \leq 7.659 \quad (V^2)$$

+5

If any voltage over 126 V is considered out-of-spec, write an upper 95% confidence bound on the proportion of out-of-spec line voltage measurements. [Ignore the "large sample" requirement and compute it anyway.]

$$\hat{p} = x/n = 2/15$$

upper bound: $$p \le \hat{p} + Z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$Z_\alpha = Z_{.05} = 1.645$ (bottom row of t-table)

$$p \le 2/15 + 1.645 \sqrt{\frac{2/15\,(1-2/15)}{15}}$$

$p \le 0.2777$ or 27.78%

+3

~~If the proportion of out-of-spec line voltages must be known within ±1%, determine the sample size necessary to achieve this.~~