$$\boxed{\text{Show all of your work for full credit.}}$$

Name (print): _____

1. (a) (5 points) Write down the corresponding augmented matrix of the given system:

$$\begin{cases} -3x_3 + 7x_4 &= 0 \\ 7x_1 + x_2 &= 2 \\ x_1 + x_2 + x_3 + x_4 &= 1 \end{cases}$$

   (b) (15 points) Find the solutions in the vector form $\mathbf{x}$. Use back-substitution to select the free variables. Provide a geometric description of the solution set.

2. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$.

   (a) (15 points) Determine if the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. Justify your answer.

   (b) (5 points) Is $\mathbf{v}_3$ in the Span$\{\mathbf{v}_1, \mathbf{v}_2\}$? Justify your answer.

3. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$, and let $T : R^3 \to R^2$ be a linear transformation that maps $\mathbf{x}$ into $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3$.

   (a) (5 points) Find a matrix A such that $T(\mathbf{x})$ is $A\mathbf{x}$ for each $\mathbf{x}$.

   (b) (5 points) What is the range of T?

   (c) (10 points) Find a vector $\mathbf{x} \in R^3$ such that $T(\mathbf{x}) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Use back-substitution to select the free variables.

4. (a) (5 points) Find the matrix A such that $A\mathbf{x} = \begin{bmatrix} -2x_1 - 7x_2 - 9x_3 \\ 2x_1 + 5x_2 + 6x_3 \\ x_1 + 3x_2 + 4x_3 \end{bmatrix}$ for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

   (b) (10 points) Define a linear transformation T on R^3 by $T(\mathbf{x}) = A\mathbf{x}$. Find the image of $\mathbf{x} = (1, 2, 3)$. Find the range of T.

   (c) (10 points) Is T invertible? Justify your answer.

5. (a) (10 points) Calculate the determinant of

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}.$$

   (b) (5 points) Is A invertible? What is the determinant of A^{-1} if A is invertible?