Lab 2 (cal II) Solution

#1 Area $y = 2x - 1$ $y = x^2 - 3x + 3$

$2x - 1 = x^2 - 3x + 3$ $x^2 - 5x + 4 = 0$ $(x-1)(x-4) = 0$ $x = 1, 4.$

$$\int_1^4 (2x-1) - (x^2 - 3x + 3)\,dx$$

$$= \int_1^4 5x - 4 - x^2\,dx = \frac{5}{2}x^2 - 4x - \frac{x^3}{3} \Big|_1^4$$

$$\left(40 - 16 - \frac{64}{3}\right) - \left(\frac{5}{2} - 4 - \frac{1}{3}\right) = \frac{9}{2}$$

#2 Area $x = 3y + 1$ $x = y^2 - y - 4$

$3y + 1 = y^2 - y - 4$ $y^2 - 4y - 5 = 0$ $(y-5)(y+1) = 0$ $y = 5, y = -1$

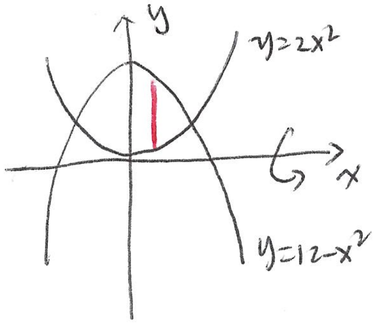
$$\int_{-1}^5 (3y+1) - (y^2 - y - 4)\,dy$$

$$= \int_{-1}^5 4y + 5 - y^2\,dy = \left(2y^2 + 5y - \frac{y^3}{3}\right) \Big|_{-1}^5$$

$$= \left(50 + 25 - \frac{125}{3}\right) - \left(2 - 5 + \frac{1}{3}\right)$$

$$= 36$$

#3 Volume $y = 12 - x^2$, $y = 2x^2$, around x-axis



Washer Method

$OR = 12 - x^2 \qquad IR = 2x^2$

$$V = \int_{-2}^{2} \pi(12-x^2)^2 - \pi(2x^2)^2\, dx$$

$$= \pi\int_{-2}^{2} 144 - 24x^2 + x^4 - 4x^4\, dx$$

$$= \pi\int_{-2}^{2} 144 - 24x^2 - 3x^4\, dx$$

$$= \pi\left(144x - 8x^3 - \frac{3}{5}x^5\right)\Big|_{-2}^{2}$$

$$= \pi\left(288 - 64 - \frac{96}{5}\right) - \pi\left(-288 + 64 + \frac{96}{5}\right)$$

$$= \pi \cdot \frac{2048}{5}$$

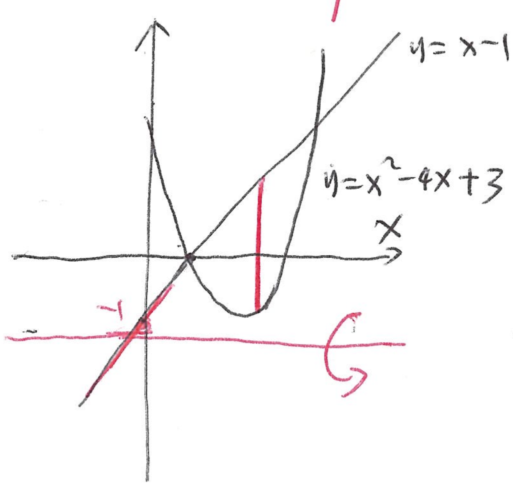
$2x^2 = 12 - x^2$

$3x^2 = 12 \quad x^2 = 4$

$x = \pm 2$

#4 Volume $x - y = 1$ $\quad y = x^2 - 4x + 3$, rotate around $y = -1$

Solve for y $\quad y = x - 1$



Washer Method

$OR = (x-1) - (-1) = x$

$IR = (x^2 - 4x + 3) - (-1) = x^2 - 4x + 4$

$$V = \int_{1}^{4} \pi(x)^2 - \pi(x^2 - 4x + 4)^2\, dx$$

FOIL

$$= \pi\int_{1}^{4} x^2 - (x^4 + 16x^2 + 16 - 8x^3 + 8x^2 - 32x)\, dx$$

$$= \pi\int_{1}^{4} -x^4 + 8x^3 - 23x^2 + 32x - 16\, dx$$

$$= \pi\left(-\frac{x^5}{5} + 2x^4 - \frac{23}{3}x^3 + 16x^2 - 16x\right)\Big|_{1}^{4}$$

$$= \pi\left(-\frac{4^5}{5} + 2\cdot 4^4 - \frac{23}{3}\cdot 4^3 + 16\cdot 16 - 16\cdot 4\right) - \pi\left(-\frac{1}{5} + 2 - \frac{23}{3} + 16 - 16\right)$$

$$= \pi \cdot \frac{72}{5}$$

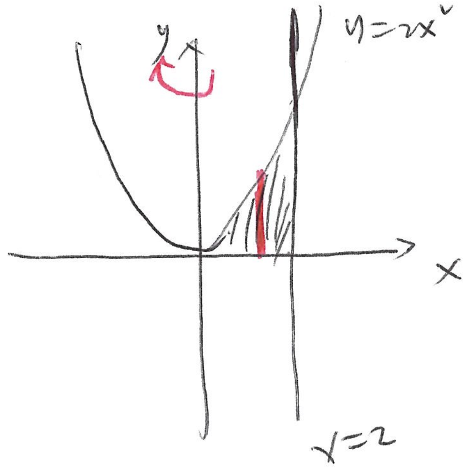
$x - 1 = x^2 - 4x + 3$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1 \quad x = 4$

#5 Volume $y=2x^2$, $x=2$, $y=0$ around y-axis.



Shell Method

Radius $= x$ height $= 2x^2$

$$V = \int_0^2 2\pi x \cdot 2x^2 \, dx$$

$$= \int_0^2 4\pi \cdot x^3 \, dx$$

$$= \pi x^4 \Big|_0^2$$

$$= 16\pi$$