ISE 2211 • Exam V • 29 March 2019 NAME SOLUTION

Joe Tritschler has seriously **had it** with these unscrupulous snack manufacturers and is about to blow his top over a recent, infuriating experience with two boxes of Devil Dogs from the diabolical Drake's Cakes corporation. It seems quality control over the net weight of their snack cakes *isn't* taken seriously and Joe Tritschler decided to perform a serious investigation to determine if there is a significant difference between the two boxes. (Sabotage? Inside job? Government conspiracy??!!!! Think of the poor, innocent child for whom a box of Devil Dogs is the most important thing in their life – that child shouldn't have to wonder if they got shortchanged!!!!) Samples of $n_1 = 8$ and $n_2 = 6$ Devil Dogs were measured with the following results: $\bar{x}_1 = 12.69$ oz., $s_1 = 0.4321$ oz., $\bar{x}_2 = 13.15$ oz. and $s_2 = 0.2248$ oz. Test the following hypotheses and state whether you would reject or fail to reject the null hypothesis that the two different boxes of Devil Dogs have equal variances in net weight at the $\alpha = 0.05$ fixed level of significance. <u>Sketch</u> the appropriate distribution, indicating the test statistic and critical values.

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

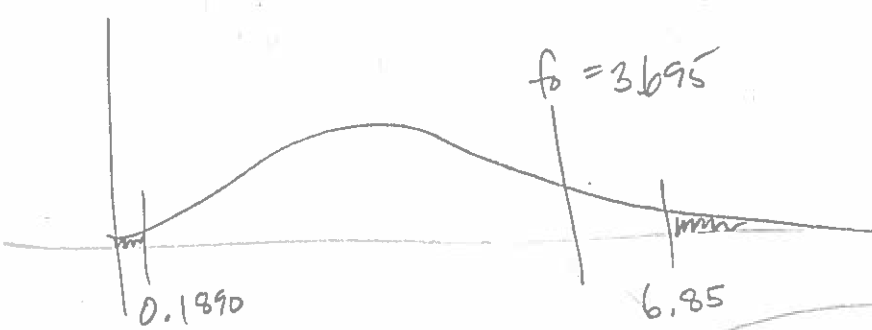
$$f_0 = \frac{s_1^2}{s_2^2} = \frac{0.4321^2}{0.2248^2}$$

$$f_0 = 3.695 \quad +1$$

$$f_{\alpha/2, n_1-1, n_2-1} = f_{.025,7,5} = 6.85 \quad +1$$

$$f_{1-\alpha/2, n_1-1, n_2-1} = \frac{1}{f_{\alpha/2, n_2-1, n_1-1}} = \frac{1}{f_{.025,5,7}} = \frac{1}{5.29} \quad +1$$

$$= 0.1890$$



+1

$f_0 \not> f_{upper}$ ∴ fail to reject H_0

Write a 95% C.I. on the ratio of population variances and verify that it draws the same conclusion as the fixed-α hypothesis test above. Include a unit with the C.I.

$$\frac{s_1^2}{s_2^2} f_{1-\alpha/2,\, n_2-1,\, n_1-1} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2,\, n_2-1,\, n_1-1}$$

$$f_{\alpha/2,\, n_2-1,\, n_1-1} = f_{.025,\, 5,\, 7} = 5.29 \quad +1$$

$$f_{1-\alpha/2,\, n_2-1,\, n_1-1} = \frac{1}{f_{\alpha/2,\, n_1-1,\, n_2-1}} = \frac{1}{f_{.025,\, 7,\, 5}} = \frac{1}{6.85}$$

$$= 0.1460 \quad +1$$

$$3.695 \cdot .1460 < \frac{\sigma_1^2}{\sigma_2^2} < 3.695 \cdot 5.29$$

$$0.5394 < \frac{\sigma_1^2}{\sigma_2^2} < 19.55 \quad \left(\frac{\sigma z^2}{\sigma z^2}\right)$$

+1   +1 (unit)

includes 1 ; $\therefore$ fail to reject   +1

Determine a range of *p*-value for the test on difference in population variances and verify that it draws the same conclusion relative to $\alpha = 0.05$.

$f_{.01,7,5} = 10.46$

$f_{.025,7,5} = 6.85$

$f_{.05,7,5} = 4.88$

$f_{.10,7,5} = 3.37$

$f_{.25,7,5} = 1.89$

+1

f_0 is in here!

$\therefore \; .05 < \frac{P}{2} < .10$ +1

$.10 < P < .20$ +1

$P > 0.05$

$\therefore$ fail to reject H_0

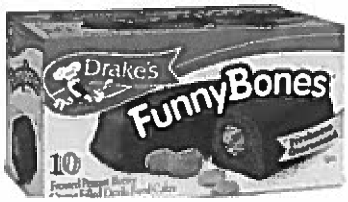
+1

2) Samples of $n_1 = 32$ Devil Dogs and $n_2 = 30$ Funny Bones were *heroically* and *altruistically* consumed by Joe Tritschler -- in the name of science and for the benefit of mankind! -- in order to gather imperative data on the adequacy of their filling, with the following results: $x_1 = 7$ inadequately-filled Devil Dogs and $x_2 = 13$ inadequately-filled Funny Bones. Test the following hypotheses on the proportion of inadequately-filled snack cakes using the p-value approach $\alpha = 0.05$:

H_0: $p_1 = p_2$

H_1: $p_1 \neq p_2$

Does the data conclusively suggest that either type has a higher proportion of inadequately-filled snack cakes? What next course of action should Joe take?



$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{7}{32} = 0.21875$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{13}{30} = 0.4333$$

+1

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{7+13}{32+30}$$

$$= 0.3226 \quad +1$$

$$Z_0 = \frac{0.21875 - 0.4333}{\sqrt{.3226(1-.3226)\left[\frac{1}{32} + \frac{1}{30}\right]}}$$

$$= -1.806 \quad +1$$

table +1

$$p\text{-value} = 2 \cdot \left[P(Z < -1.81)\right] = 2(0.035148)$$

$$= 0.0703 \quad +1$$

$p > 0.05$; ∴ fail to reject H_0