

1. [30 pts] Below is a behavioral VHDL code.

```

entity priority_4in is
    port ( inputs : in std_logic_vector(3 downto 0);
           result : out std_logic_vector(1 downto 0);
           valid : out std_logic; );
end priority_4in;

architecture if_statement of priority_4in is
begin
    process(inputs)
    begin
        if (inputs(3) = '1') then
            result <= "11";
            valid <= '1';
        elsif (inputs(2) = '1') then
            result <= "10";
            valid <= '1';
        elsif (inputs(1) = '1') then
            result <= "01";
            valid <= '1';
        elsif (inputs(0) = '1') then
            result <= "00";
            valid <= '1';
        else
            result <= "00";
            valid <= '0';
        end if;
    end process;
end if_statement;

```

- a) [15 pts] Construct the truth table and derive the optimized Boolean expressions for the outputs.

| Input(3) | Input(2) | Input(1) | Input(0) | Result(1) | Result(0) | valid |
|----------|----------|----------|----------|-----------|-----------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$$\text{Result}(1) = \text{input}(3) + \text{input}(2)$$

$$\text{Result}(0) = \text{input}(3) + (\text{input}(1) \text{ input}(2)')$$

$$\text{Valid} = \text{input}(3) + \text{input}(2) + \text{input}(1) + \text{input}(0)$$

b) [15 pt] Use **case** statement to complete the VHDL code.

```
entity priority_4in is
  port (
    inputs : in std_logic_vector(3 downto 0);
    result : out std_logic_vector(1 downto 0);
    valid : out std_logic;
  );
end priority_4in;
```

```
architecture case_statement of priority_4in is
begin
```

```
  process(inputs)
  begin
    --- use case statement
```

```
    case (inputs) is
      when "1--" =>
        result <= "11";
        valid <= '1';
```

```
      when "01--" =>
        result <= "10";
        valid <= '1';
```

```
      when "001-" =>
        result <= "01";
        valid <= '1';
```

```
      when "0001" =>
        result <= "00";
        valid <= '1';
```

```
      when others =>
        result <= "00";
        valid <= '0';
```

```
    end case;
```

```
  end process;
end case_statement;
```

2. [30 pts] A function takes a binary vector of any size as input and returns an integer representing the **maximum consecutive 1's** in the input vector. It iterates through each element of the input vector, and finally, it returns **max_count**, which represents the **maximum consecutive 1's** in the input vector. The following VHDL defines an entity named **MaxConsecutiveOnesEntity** with an 8-bit input vector (**input_vector**) and an output representing the **maximum consecutive 1's** (**max_count_output**). In the architecture, a process is used to call the **max_consecutive_ones** function with the **input_vector** and assign the result to **max_count_output**.

For example: If the **input_vector** is “00111100” then the output, **max_count_output** = 4. If the input DATA is “11100010” or “10010111” then the output, **max_count_output** = 3.

```

entity MaxConsecutiveOnesEntity is
  port (
    input_vector : in std_logic_vector(7 downto 0); -- Example: 8-bit input vector
    max_count_output : out integer -- Output representing max consecutive 1's
  );
end entity MaxConsecutiveOnesEntity;

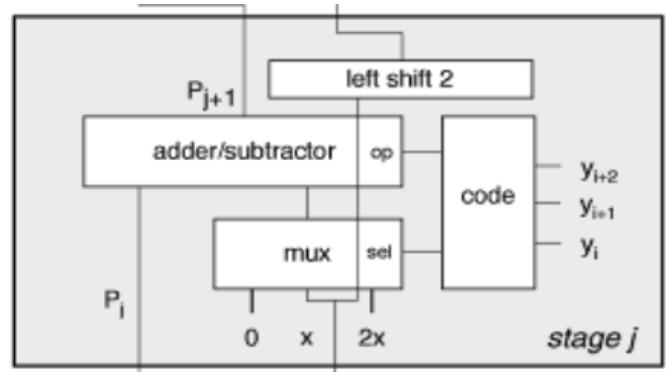
architecture Behavioral of MaxConsecutiveOnesEntity is
  function max_consecutive_ones (in_vector : std_logic_vector) return integer is
    variable max_count : integer := 0; -- you may use "positive"
    variable current_count : integer := 0;
  begin
    for i in in_vector'range loop
      if in_vector(i) = '1' then
        current_count := current_count + 1;
        if current_count > max_count then
          max_count := current_count;
        end if;
      else
        current_count := 0;
      end if;
    end loop;
    return max_count;
  end function max_consecutive_ones;

begin
  process(input_vector)
    begin
      max_count_output <= max_consecutive_ones(input_vector);
    end process;
  end architecture Behavioral;

```

3. [30 pts] Below is a Booth action and architecture stage for Booth multiplication.

| y_i | y_{i-1} | y_{i-2} | increment |
|-------|-----------|-----------|-----------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | x |
| 0 | 1 | 0 | x |
| 0 | 1 | 1 | 2x |
| 1 | 0 | 0 | -2x |
| 1 | 0 | 1 | -x |
| 1 | 1 | 0 | -x |
| 1 | 1 | 1 | 0 |



- a) [15 pts] Use 7-bit 2's complement number system, Booth encoding and algorithm to explain the Booth multiplication of $x = 1001101$ and $y = 1011011$. **Note: Show your bit operation step-by-step and find ALL partial products** (in 2's complement).

$$X = 1001101 \quad 2X = 10011010$$

$$-X = 0110011 \quad -2X = 01100110$$

$$Y \Rightarrow \frac{10110110}{\overline{y_{P_4} y_{P_3} y_{P_2}} y_{P_1}}$$

Result = $7 + 7 - 1 = 13$ bit

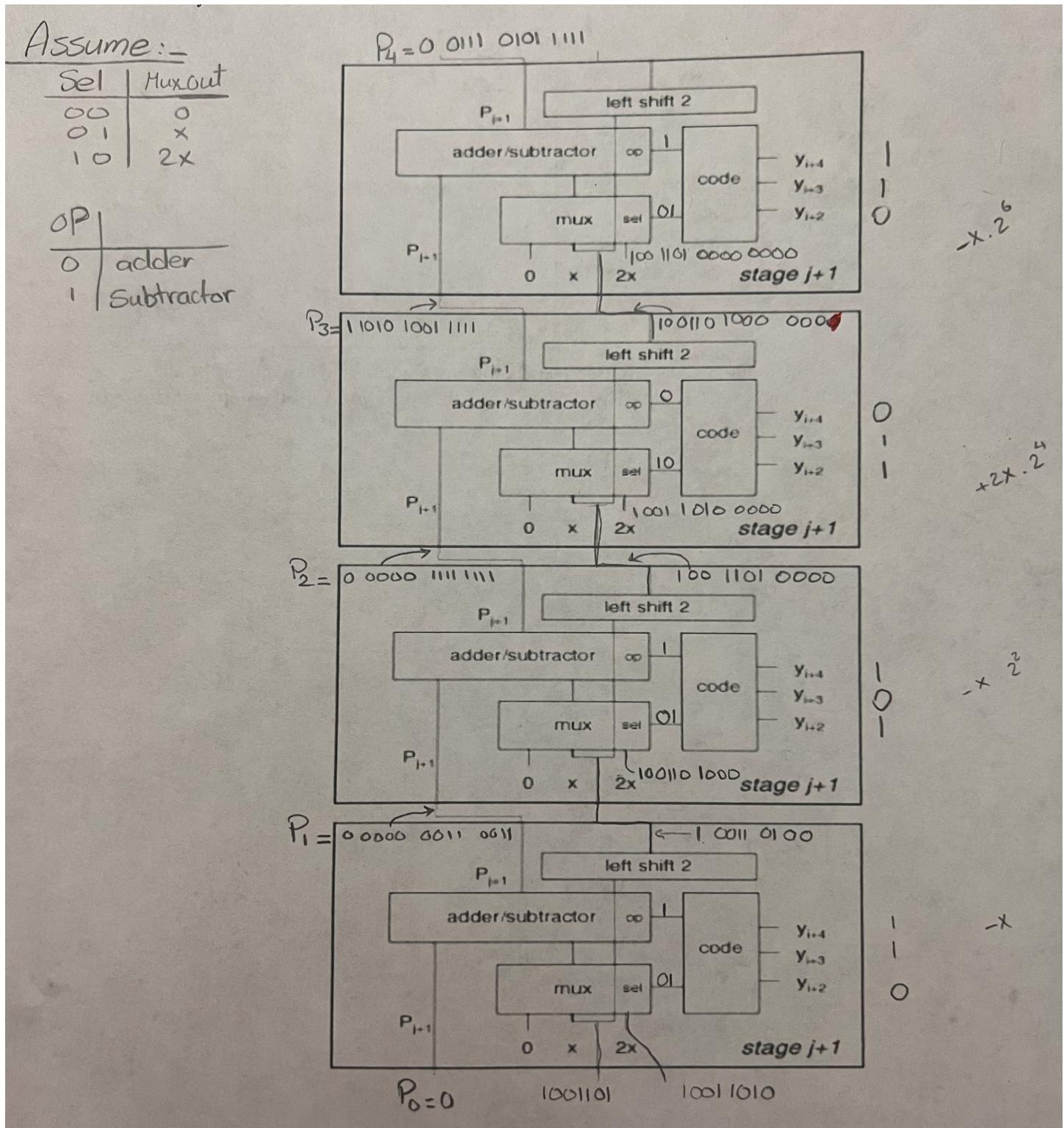
$$y_{P_1} = 110 \Rightarrow P_1 = P_0 - X = \begin{array}{r} 00000 \\ 0000000110011 \\ \hline 0000000110011 \end{array}$$

$$y_{P_2} = 10 \Rightarrow P_2 = P_1 - X \cdot 2^2 = \begin{array}{r} 0000000110011 \\ 0000011001100 \\ \hline 0000011111111 \end{array}$$

$$y_{P_3} = 011 \Rightarrow P_3 = P_2 + 2X \cdot 2^4 = \begin{array}{r} 0000011111111 \\ 1100110100000 \\ \hline 1101010011111 \end{array}$$

$$y_{P_4} = 110 \Rightarrow P_4 = P_3 - X \cdot 2^6 = \begin{array}{r} 1101010011111 \\ 0110011000000 \\ \hline *0011101011111 \end{array}$$

- b) [15 pts] Draw Booth multiplier architecture to implement the Booth multiplication of $x = 1001101$ and $y = 1011011$. Show how data (in 2's complement) is propagated and your **bit operation** through every architecture stage. Include (y_i y_{i+1} y_{i+2}) bits, your “mux” output bits, “adder/subtractor” output bits, and “left shift 2” output bits in your architecture.



4. [40 pts]

A) [20 pts] Consider two fixed-number numbers A and B where, A = -3.75 and B = 2.75. **Use 2's complement number, I3.Q2 format, for both A and B.** Calculate the following calculation results.

a) [4 pts] What is your **I3.Q2 format** result for A?

$$3.75 = 011.11$$

$$A = -3.75 = 2\text{'s } (011.11) = 100.01$$

b) [4 pts] What is your **I3.Q2 format** result for B?

$$B = 2.75 = 010.11$$

c) [4 pts] What is your **I3.Q2 format** result for -B?

$$-B = 2\text{'s } (010.11) = 101.01$$

d) [4 pts] Calculate A + B and find your final **I4.Q2 format** result. Note: show your bit operation.

$$\begin{array}{r} 100.01 \\ +010.11 \\ \hline 1111.00 \end{array}$$

e) [4 pts] Calculate A – B and find your final **I4.Q2 format** result. Note: show your bit operation.

$$\begin{array}{r} 100.01 \\ +101.01 \\ \hline 1001.10 \end{array}$$

B) [20 pts]

Note: In the following (h) ~ (k), you must use the exact multiplicand and multiplier in your calculation. Also, you cannot switch both. A = -3.75 and B = 2.75.

f) [10 pts] Calculate A x B and find your final **I5.Q5 format** result. Note: show your bit operation.

$$\begin{array}{r} 10001 \\ 01011 \\ \hline 111110001 \\ 11110001 \\ 0000000 \\ 110001 \\ 00000 \\ \hline 101010110110 \end{array}$$

I5.Q5 format result = **10101.10110**

g) [10 pts] Calculate A x (-B) and find your final **I5.Q5 format** result. Note: show your bit operation.

$$\begin{array}{r} 10001 \\ 10101 \\ \hline 111110001 \\ 00000000 \\ 1110001 \\ 000000 \\ 01111 \\ \hline 100101001010 \end{array}$$

I5.Q5 format result = **01010.01010**

5. [40 pts (including Bonus 20 pts)] A **counter** that counts from **0 to 31** in integer has input ports RSTn, CLK, EN, PL, and DATA. The **output port COUNT receives the value of the counter**. RSTn serves as the asynchronous reset, while CLK is the clock input signal. The asynchronous reset **RSTn** is prioritized, activated by verifying if RSTn is '0', and initializing the counter to 0 before considering the rising edge trigger of the clock. If CLK is **rising edge-triggered**, the synchronous behavior of either the **parallel load** or **count model** of the counter is modeled.

Note: 1) Parallel load (PL) takes precedence over the count enable (EN). PL = '1' is checked first, and EN = '1' is only checked when PL = '1' is not TRUE.

2) When PL is '1', the counter performs the **parallel load**, loaded with the DATA value.

3) When PL = '0' and EN = '1', the counter starts the count mode. The counter is triggered to increment its count on the rising edge of the clock signal (**CLK**). **It's important to note that after the counter reaches the value (31), it resets to the initial value (0).**

4) When PL = '0' and EN = '0', the counter holds its previous value.

5) COUNT is declared as an output port; however, VHDL does not allow the output port signal of COUNT to be read. Therefore, a temporary signal **COUNT_VALUE** is declared and used in the process. Finally, COUNT_VALUE is assigned to the output port COUNT outside the process.

```
entity FiveBitCounter is
port (
```

```
    RSTn : in std_logic;
    CLK : in std_logic;
    EN : in std_logic;
    PL : in std_logic;
    DATA : in integer;
    COUNT : out integer;
);
```

```
end entity FiveBitCounter;
```

```
architecture Behavioral of FiveBitCounter is
variable COUNT_VALUE : integer;
```

```
begin
```

```
    process(CLK, RSTn)
```

```
    begin
```

```
        if RSTn = '0' then
```

```
            COUNT_VALUE := 0;
```

```
        elsif (CLK'EVENT and CLK = '1') then
```

```
            if PL = '1' then
```

```
                COUNT_VALUE := DATA; -- Parallel load
```

```
            elsif EN = '1' then
```

```
                if COUNT_VALUE = 31 then
```

```
                    COUNT_VALUE := 0; -- Reset counter to 0 when it reaches 31
```

```
                else
```

```
                    COUNT_VALUE := COUNT_VALUE + 1; -- Count mode
```

```
                end if;
```

```
            else
```

```
                --Optional
```

```
                COUNT_VALUE := COUNT_VALUE --Optional
```

```
            end if;
```

```
        end if;
```

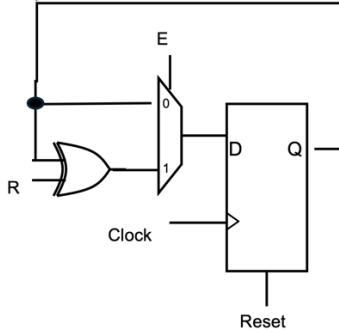
```
    end process;
```

```
    COUNT <= COUNT_VALUE;
```

```
end architecture Behavioral;
```

6. [50 pts]

a) [20 pts] A D flip-flop with an **asynchronous reset** input and an **enable** input is given below. Complete the following entity and architecture pair of the D flip-flop. The DFF has an **asynchronous reset** “Reset”, i.e., the output $Q = 0$ when Reset = 1. The **Reset** has a higher priority than the **clock**. And, the **clock** has a higher priority than the **enable ‘E’**. The **enable ‘E’** controls the multiplexer input selection, i.e., E = 0 selects the flip-flop’s output, Q ; E =1 selects R XOR Q; The DFF loads new data only when Reset = 0 and the clock in **rising edge trigger**.



```
entity rege is
  port (R, Reset, Clock, E: in bit;
        Q: out bit);
end entity rege;
```

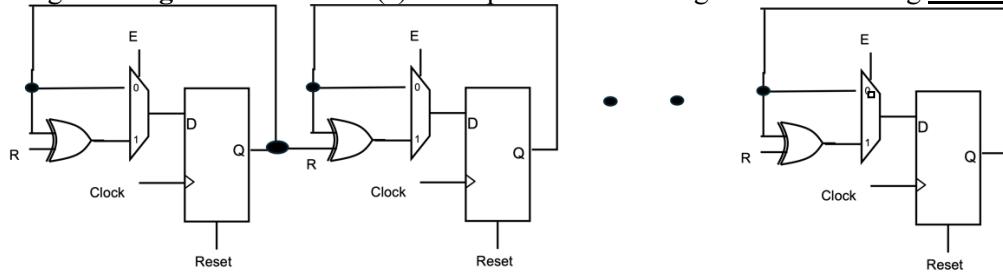
ARCHITECTURE Behavior OF rege IS
BEGIN

```
  PROCESS (Reset, Clock)
  BEGIN
```

```
    IF Reset = '1' THEN
      Q <= '0';
    ELSIF (Clock'EVENT AND Clock= '1') THEN
      IF E = '1' THEN
        Q<=R XOR Q;
      ELSE
        Q<=Q;
      END IF;
    END IF;
```

```
  END PROCESS ;
END Behavior ;
```

b) [15 pts] Use "generate" statements to write an entity and architecture pair of VHDL description for a 16-bit wide using 1-bit **rege** as described in (a). Complete the following 16-bit wide using **3 GENERATE** statements.



```
entity rege_16 is
    port (D, Reset, E, Clock: in bit; Y: out bit);
end entity rege_16;
```

architecture reg16 of **rege_16** is

-- component declaration

component **rege**

port (R, Reset, Clock, E: in bit;

Q: out bit);

end component rege;

b.1) [5 pts] --- signal declaration **Note: Write your signals on the above 16-bit wide shift registers logic.**

```
signal m: bit_vector(1 to 15);
```

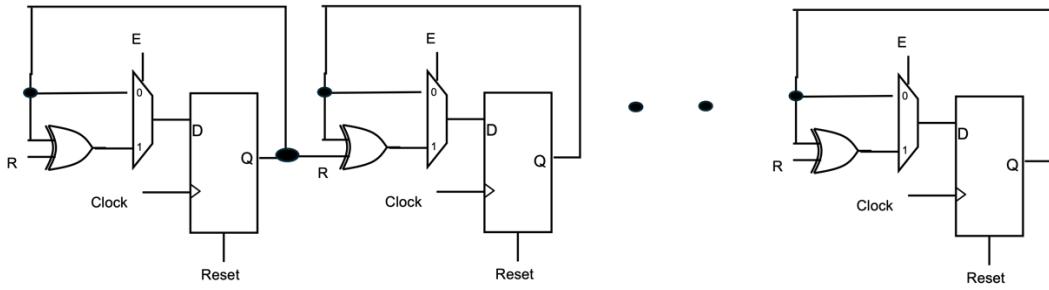
b.2) [10 pts] --- Use **3 GENERATE** statements

--- Use port map by **POSITION**.

begin

```
shift_reg: for i in 1 to 16 generate begin
    dff_left: if i=1 generate
        dff1: rege
            port map (D, Reset, Clock, E, m(i));
        end generate dff_left;
    dff_others: if (i>1 and i<16) generate
        dff2: rege
            port map (m(i-1), Reset, Clock, E, m(i));
        end generate dff_other;
    dff_right: if i=16 generate
        dff3: rege
            port map (m(i-1), Reset, Clock, E, Y);
    end generate dff_right;
```

end architecture;



c) [15 pts] Complete the 16-bit wide shift registers using only one GENERATE statement.

```
entity rege_16 is
  port (D, Reset, Clock, E: in bit;
        Q: out bit);
end entity rege_16;
```

architecture reg16 of rege_16 is

```
-- component declaration
component rege
  port (R, Reset, Clock, E: in bit;
        Q: out bit);
end component rege;
```

c.1) [5 pts] --- signal declaration (Note: Write your signals on the above 16-bit wide shift registers logic.)

```
signal m: bit_vector(0 to 16);
```

c.2) [10 pts] --- Use one GENERATE statement
--- Use port map by POSITION.

```
begin
  m(0) <= D;
  Y <= m(16);
  dff: for i in 1 to 16 generate
    dff1: rege
      port map (m(i-1), Reset, Clock, E, m(i));
    end generate dff;
```

end architecture;