

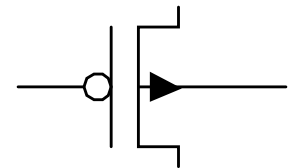
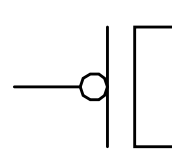
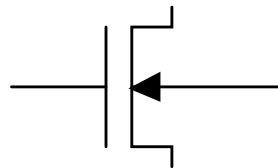
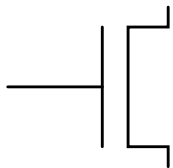
# CMOS Transistor Theory Outline

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- ☐ Introduction
- ☐ MOS Capacitor
- ☐ nMOS I-V Characteristics
- ☐ pMOS I-V Characteristics
- ☐ Gate and Diffusion Capacitance

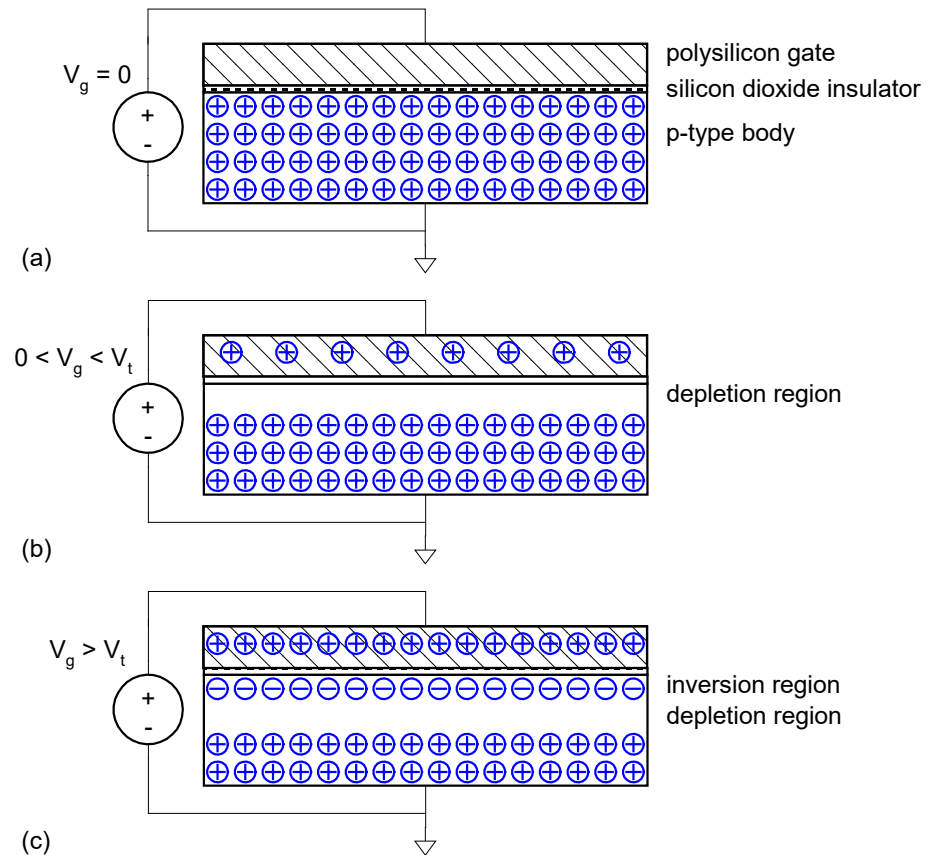
# Introduction

- ❑ So far, we have treated transistors as ideal switches
- ❑ An ON transistor passes a finite amount of current
  - Depends on terminal voltages
  - Derive current-voltage (I-V) relationships
- ❑ Transistor gate, source, drain all have capacitance
  - $I = C (\Delta V / \Delta t) = \Delta Q / \Delta t \rightarrow \Delta t = (C / I) \Delta V$
  - Capacitance and current determine speed



# MOS Capacitor

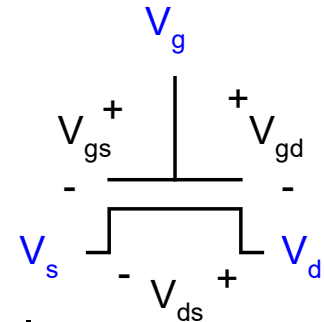
- ❑ Gate and body form MOS capacitor
- ❑ Operating **modes**
  - Accumulation
  - Depletion
  - Inversion



# NMOS Terminal Voltages

- ❑ Mode of operation depends on  $V_g$ ,  $V_d$ ,  $V_s$

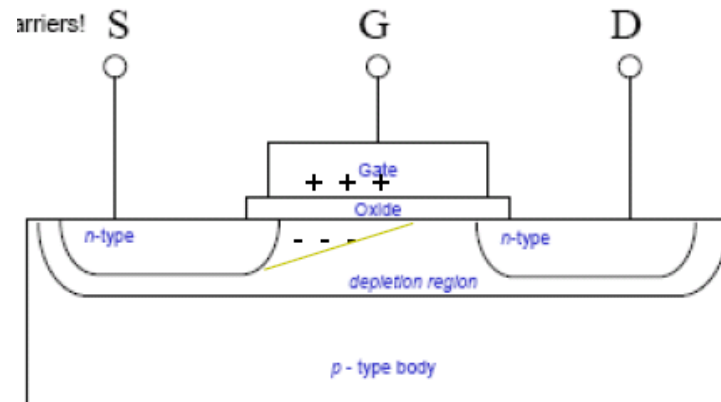
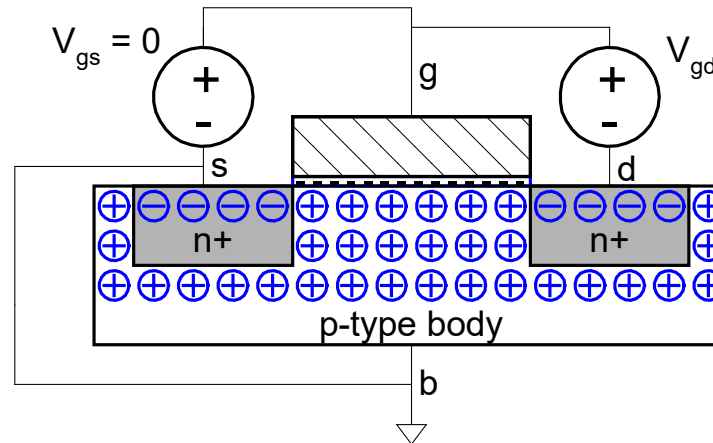
- $V_{gs} = V_g - V_s$
- $V_{gd} = V_g - V_d$
- $V_{ds} = V_d - V_s = V_{gs} - V_{gd}$



- ❑ Source and drain are **symmetric diffusion** terminals
  - By convention, source is terminal at lower voltage for nMOS
  - Hence  $V_{ds} \geq 0$
- ❑ nMOS body is grounded. First assume source is 0 too.
- ❑ Three **regions** of operation
  - *Cutoff*
  - *Linear*
  - *Saturation*

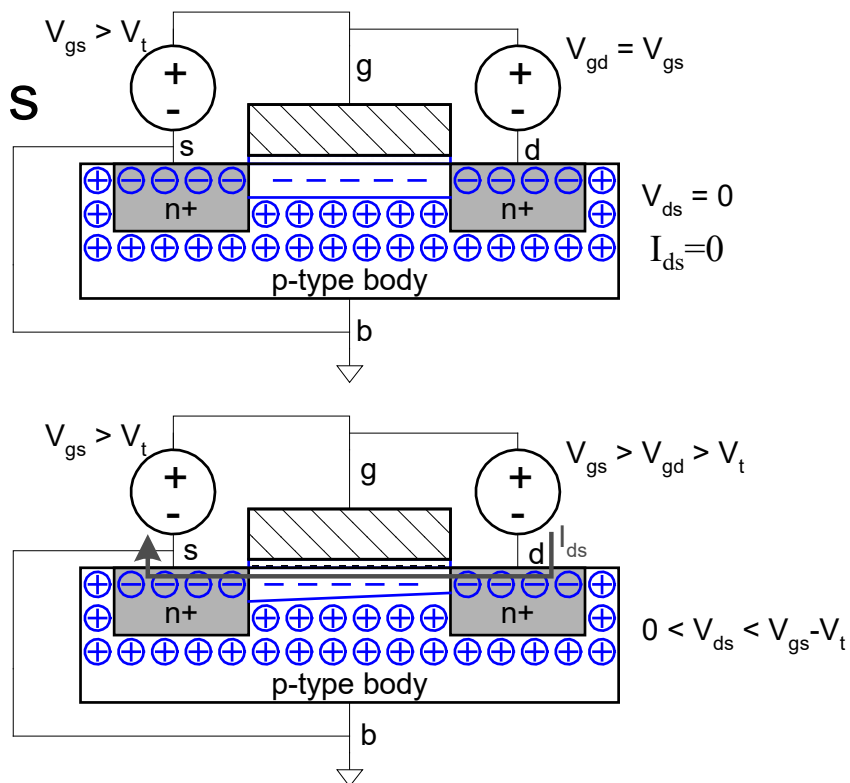
# nMOS Cutoff Region

- ❑  $V_{gs} < V_t$
- ❑ No channel
- ❑  $I_{ds} = 0$



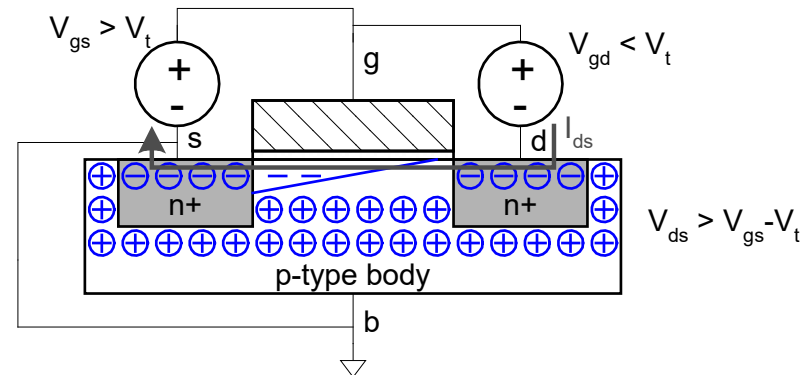
# nMOS Linear (Resistive) Region

- ❑  $V_{gs} > V_{th}$ , channel forms, but no current if  $V_{ds} = 0$
- ❑ Until  $V_{ds} > 0$ 
  - Current flows from d to s
  - $e^-$  from s to d
- ❑  $I_{ds}$  increases with  $V_{ds}$
- ❑ Similar to linear resistor



# nMOS Saturation

- ❑ As  $V_{ds}$  increased,  $V_{gd}$  is getting smaller, the channel of drain side is getting narrower
- ❑ When  $V_{gd} \leq V_t$ , channel pinches off, the channel is no longer inverted near the drain. Conduction is brought about by the drift of electrons under the influence of the positive drain voltage.
- ❑  $I_{ds}$  is independent of  $V_{ds}$ , only controlled by  $V_{gs}$  for **long channel**
- ❑ We say current saturates
- ❑ Similar to current source



# nMOS I-V Characteristics

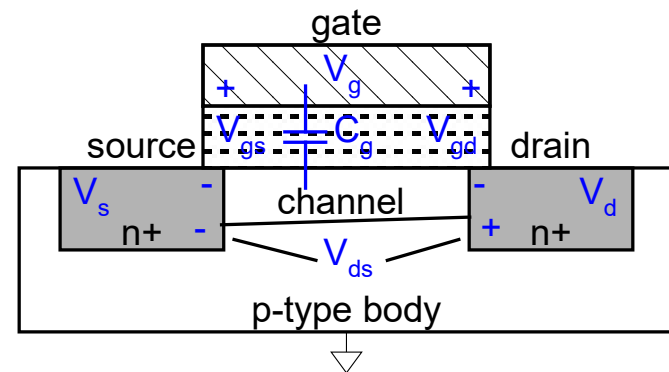
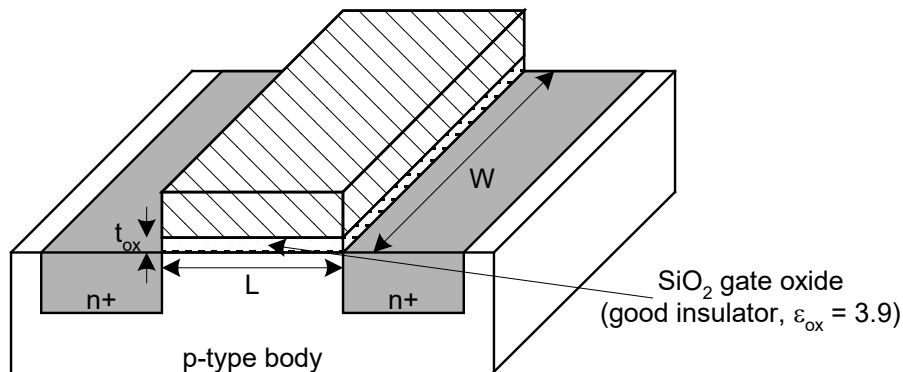
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- ❑ In Linear region,  $I_{ds}$  depends on
  - How much charge is in the channel?
  - How fast is the charge moving?



# N Channel Charge

- ❑ MOS structure looks like parallel plate capacitor while operating in inversion
  - Gate – oxide – channel
- ❑ Charge on each plate of the capacitor is  $Q_{\text{channel}} = CV$
- ❑  $C = C_g = \epsilon_{\text{ox}} WL/t_{\text{ox}} = C_{\text{ox}} WL$



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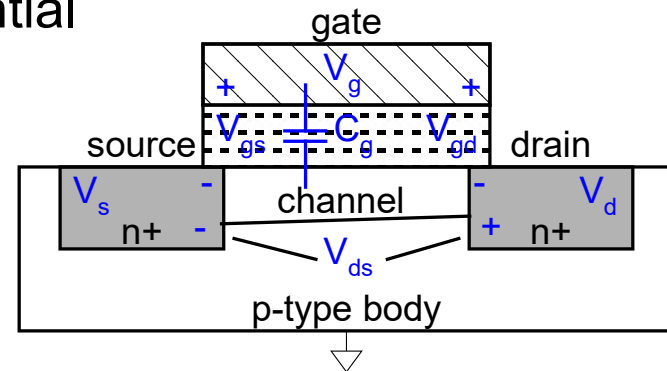
Permittivity  $\epsilon_{\text{ox}} = 3.9\epsilon_0$ ,  $\epsilon_0$  is the permittivity of free space, equals  $8.85 \times 10^{-14} \text{F/cm}$

$C_{\text{ox}} = \epsilon_{\text{ox}} / t_{\text{ox}}$ , the capacitance per unit area of the gate oxide.

$t_{\text{ox}}$  is the thickness of the gate oxide

# N Channel Charge

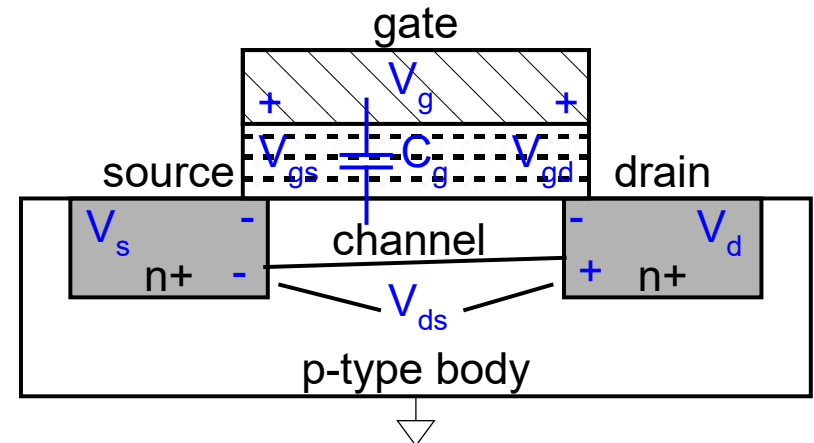
- ❑ MOS structure looks like parallel plate capacitor while operating in inversion
  - Gate – oxide – channel
- ❑ Charge on each plate of the plate is  $Q_{\text{channel}} = CV$
- ❑  $C = C_g = \epsilon_{\text{ox}} WL/t_{\text{ox}} = C_{\text{ox}} WL$
- ❑  $V = V_{gc} - V_t = (V_{gs} - V_{ds}/2) - V_t = V_{gs} - V_t - V_{ds}/2$ 
  - Average gate to channel potential
$$V_{gc} = (V_{gs} + V_{gd})/2$$
$$= (V_{gs} + V_{gs} - V_{ds})/2$$
$$= V_{gs} - V_{ds}/2$$



# Carrier velocity

- ❑ Charge is carried by e- for nMOS and hole for pMOS
- ❑ Carrier velocity  $v$  proportional to lateral E-field between source and drain
- ❑  $v = \mu E$      $\mu$  called mobility
- ❑  $E = V_{ds}/L$
- ❑ Time for carrier to cross channel:

$$t = \frac{L}{v} = \frac{L}{\mu E} = \frac{L}{\mu V_{ds}/L} = \frac{L^2}{\mu V_{ds}}$$



# nMOS Linear I-V

□ Now we know

- How much charge  $Q_{\text{channel}}$  is in the channel
- How much time ( $t$ ) each carrier takes to cross

$$\begin{aligned} I_{ds} &= \frac{Q_{\text{channel}}}{t} \longrightarrow = C_{ox}(WL) \frac{V_{gs} - V_t - V_{ds}/2}{L^2 / \mu V_{ds}} \\ &= \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} \\ &= \beta \left( V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} \end{aligned} \quad \beta = \mu C_{ox} \frac{W}{L}$$

# nMOS Saturation I-V

- When  $V_{gd} = V_t$ , channel pinches off near drain that  $V_{ds}$  is called **drain saturation voltage**,  $V_{dsat}$

- Calculate  $V_{dsat}$

$$\begin{aligned} V_{dsat} &= V_d - V_s = V_d - V_g + V_g - V_s = (V_g - V_s) - (V_g - V_d) \\ &= V_{gs} - V_{gd} = V_{gs} - V_t \end{aligned}$$

- Now drain voltage no longer increases current

$$\begin{aligned} I_{ds} &= \beta \left( V_{gs} - V_t - \frac{V_{dsat}}{2} \right) V_{dsat} \\ &= \frac{\beta}{2} (V_{gs} - V_t)^2 \end{aligned}$$

# nMOS I-V Summary

□ Shockley 1<sup>st</sup> order transistor models

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_t \quad \text{cutoff} \\ \beta \left( V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} & \begin{matrix} V_{gs} > V_t \\ V_{ds} < V_{dsat} \end{matrix} \quad \text{linear} \\ \frac{\beta}{2} (V_{gs} - V_t)^2 & \begin{matrix} V_{gs} > V_t \\ V_{ds} > V_{dsat} \end{matrix} \quad \text{saturation} \end{cases}$$

$$\beta = \mu C_{ox} \frac{W}{L}$$

$$V_{dsat} = V_{gs} - V_t$$

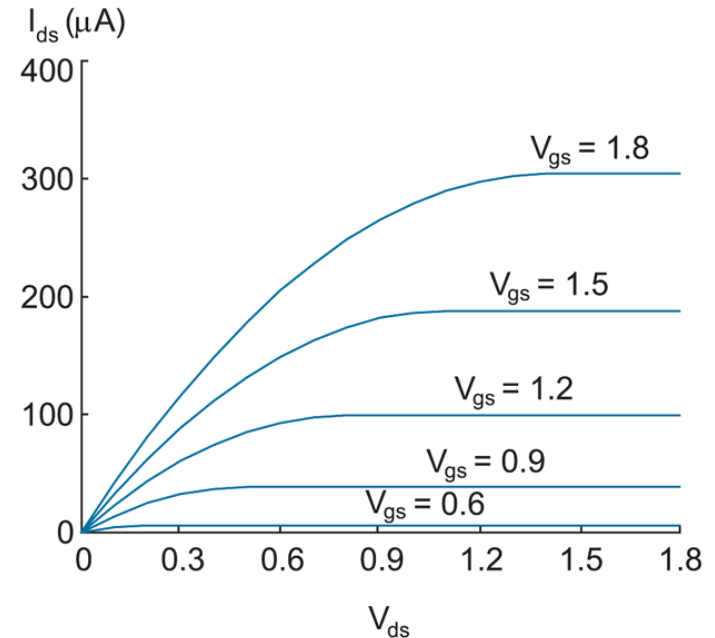
# Example

## ❑ Example: nMOS transistor in a 180 nm process

- $t_{ox} = 40 \text{ \AA}$
- $\mu_n = 180 \text{ cm}^2/\text{V}^*\text{s}$
- $V_{tn0} = 0.4 \text{ V}$

## ❑ Plot $I_{ds}$ vs. $V_{ds}$

- $V_{gs} = 0, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8 \text{ V}$
- Use  $W/L = (4 \lambda) / (2 \lambda)$



**FIG 2.7** I-V characteristics of ideal nMOS transistor

$$\beta = \mu C_{ox} \frac{W}{L} = (180 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}) \left( \frac{3.9 \times 8.85 \cdot 10^{-14} \frac{\text{F}}{\text{cm}}}{40 \cdot 10^{-8} \text{cm}} \right) \left( \frac{W}{L} \right) = 155 \frac{W}{L} \frac{\mu\text{A}}{\text{V}^2}$$



# pMOS I-V

- ❑ All dopings and voltages are inverted for pMOS
- ❑ Mobility  $\mu_p$  is determined by holes
  - Typically 2-3x lower than that of electrons  $\mu_n$
  - 90 cm<sup>2</sup>/V\*s in 180  $\mu\text{m}$  process
  - $V_{tp} = -0.4 \text{ V}$
- ❑ Thus pMOS must be wider to provide same current
  - In this class, assume  $\mu_n / \mu_p = 2$

# pMOS I-V Summary (1)

□ Shockley 1<sup>st</sup> order transistor models

$$I_{ds} = \begin{cases} 0 & V_{gs} > V_{thp} \quad \text{cutoff} \\ \beta \left( V_{gs} - V_{thp} - \frac{V_{ds}}{2} \right) V_{ds} & V_{gs} < V_{thp} \text{ and } V_{ds} > V_{dsat} \quad \text{linear} \\ -\frac{\beta}{2} (V_{gs} - V_{thp})^2 & V_{gs} < V_{thp} \text{ and } V_{ds} < V_{dsat} \quad \text{saturation} \end{cases}$$

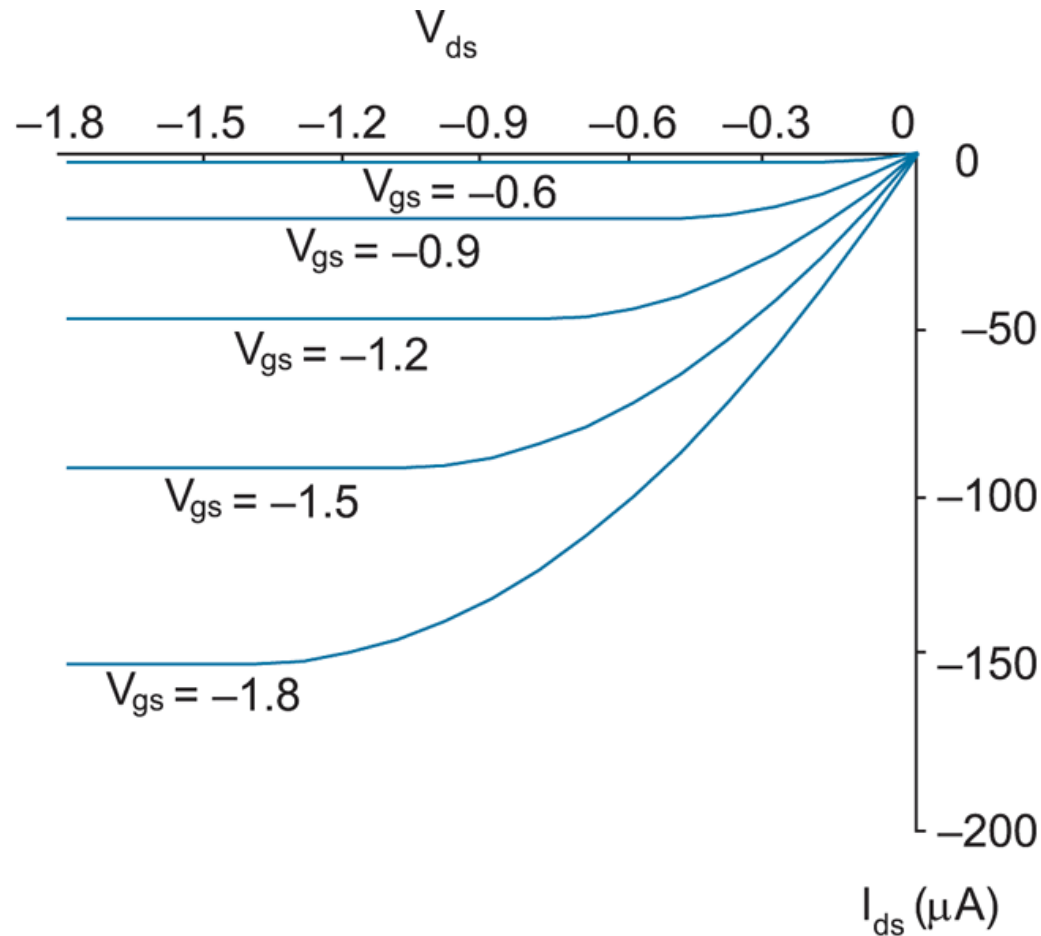
$$V_{dsat} = V_{gs} - V_{thp}$$

# pMOS I-V Summary (2)

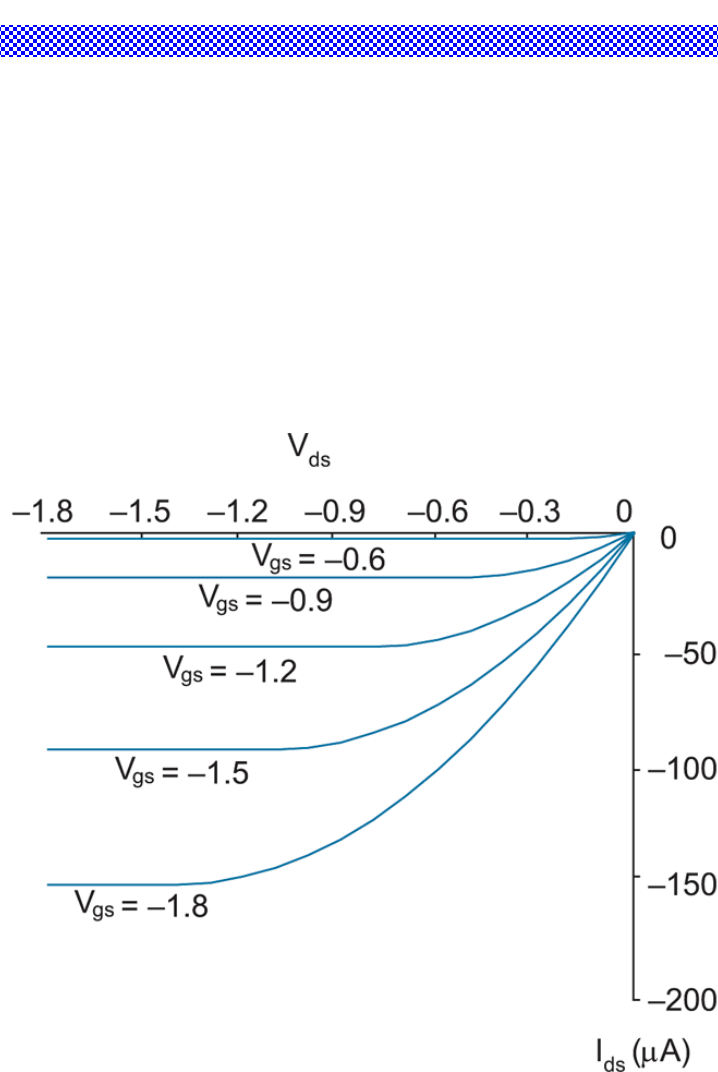
□ Shockley 1<sup>st</sup> order transistor models

$$I_{sd} = \begin{cases} 0 & V_{sg} < |V_{tp}| \quad \text{cutoff} \\ \beta_p \left( V_{sg} - |V_{tp}| - \frac{V_{sd}}{2} \right) V_{sd} & V_{sd} < V_{ssat} \quad \text{linear} \\ \frac{\beta_p}{2} (V_{sg} - |V_{tp}|)^2 & V_{sd} > V_{ssat} \quad \text{saturation} \end{cases}$$
$$V_{ssat} = V_{sg} - |V_{tp}|$$

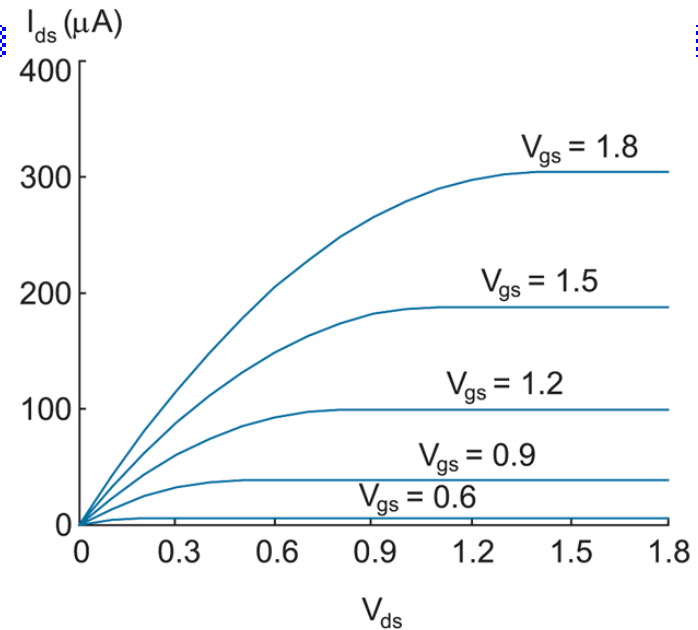
# pMOS I-V Characteristic



# MOS I-V Characteristic



**FIG 2.8** I-V characteristics of ideal pMOS transistor

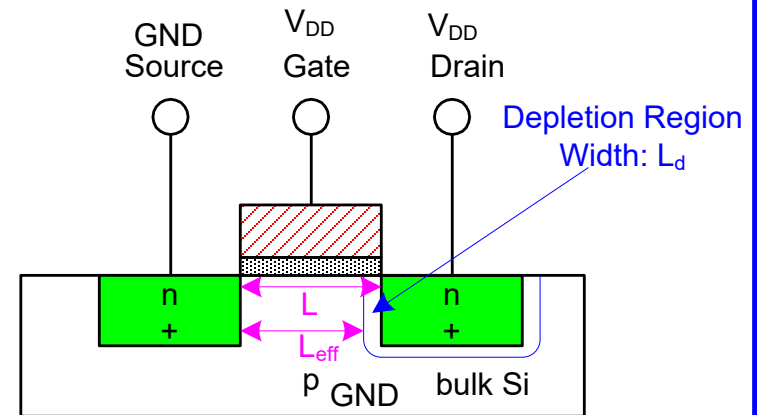


**FIG 2.7** I-V characteristics of ideal nMOS transistor

# Channel Length Modulation

- ❑ Ideally  $I_{ds}$  is independent of  $V_{ds}$  under saturation—long channel
- ❑  $L_{eff} = L - L_d$ , where  $L_{eff}$  is the effective channel length,  $L$  is the drawn channel length,  $L_d$  is the depletion region formed by the reverse-biased p-n junction between the drain and body.  $L_d$  is proportional to  $V_{db}$
- ❑ In the saturation 
$$I_{ds} = \frac{\beta}{2} (V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

$\lambda$  is called **channel modulation factor**, which is inversely dependent on channel length.



# Body Effect

□ The threshold voltage is modeled as

$$V_t = V_{t0} + \gamma \left( \sqrt{\phi_s + V_{sb}} - \sqrt{\phi_s} \right)$$

where  $V_{t0}$  is the threshold voltage when  $V_{sb} = 0$ ,  $\phi_s$  is the surface potential at threshold,  $\gamma$  is the body effect coefficient

$$\phi_s = 2v_T \ln \frac{N_A}{n_i}; \quad \gamma = \frac{t_{ox}}{\epsilon_{ox}} \sqrt{\frac{2q\epsilon_{si}N_A}{C_{ox}}}$$

where  $v_T$  is the thermal voltage,  $N_A$  is the doping level,  $n_i$  is the silicon intrinsic level

# Example

- ❑ 180nm process with  $v_{tn0}=0.4V$ ,  $N_A=8 \cdot 10^{17} \text{ cm}^{-3}$ . The body is tied to ground and  $V_{sb}=1.1V$ .

At room temperature,  $v_T = kT/q = 26\text{mV}$  and  $n_i = 1.45 \cdot 10^{10} \text{ cm}^{-3}$

$$\phi_s = 2(.026V) \ln \frac{8 \cdot 10^{17} \text{ cm}^{-3}}{1.45 \cdot 10^{10} \text{ cm}^{-3}} = 0.93V$$

$$\gamma = \frac{40 \cdot 10^{-8} \text{ cm}^{-3}}{3.9 \cdot 10^{-14} \frac{F}{\text{cm}}} \sqrt{2(1.6 \cdot 10^{-19} \text{ C}) \left( 11.7 \cdot 8.85 \cdot 10^{-14} \frac{F}{\text{cm}} \right) (8 \cdot 10^{17} \text{ cm}^{-3})}$$

$$= 0.60V^{\frac{1}{2}}$$

$$V_t = 0.4V + \gamma \left( \sqrt{\phi_s + 1.1V} - \sqrt{\phi_s} \right) = 0.68V$$

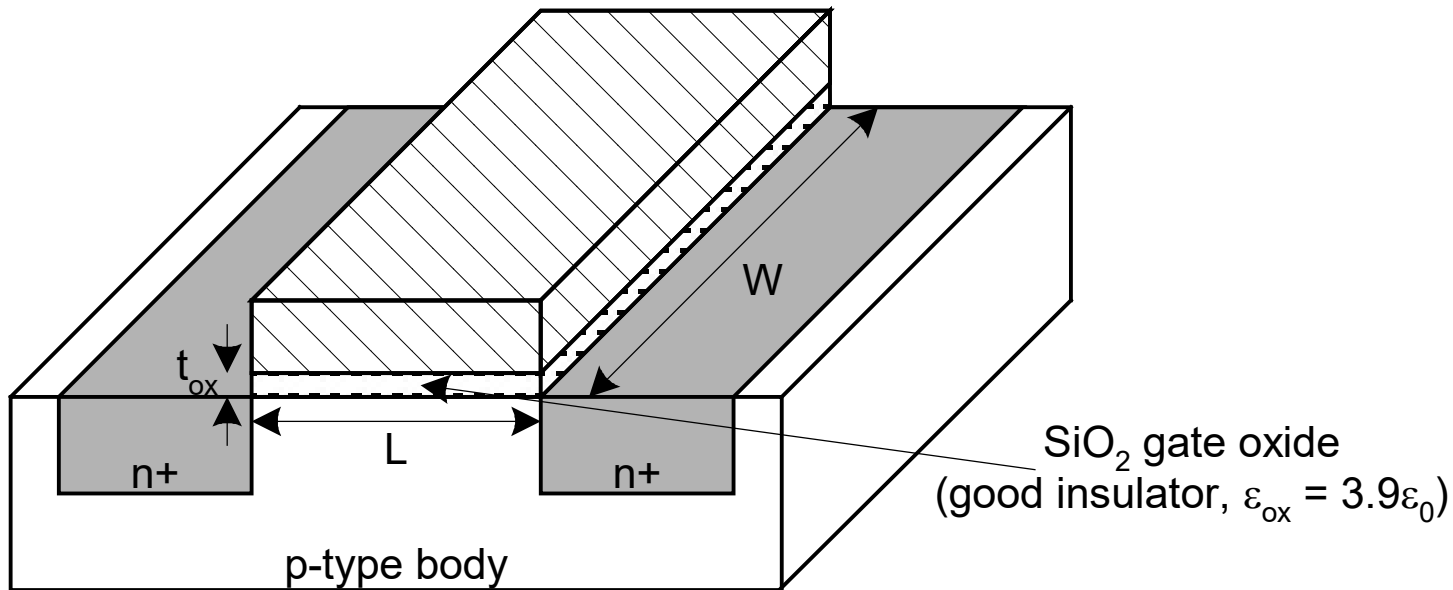


# Capacitance

- ❑ Any two conductors separated by an insulator have capacitance
- ❑ Gate to channel capacitor ( $C_g$ ) is very important
  - Creates channel charge necessary for operation
- ❑ Source and drain have capacitance to body
  - Across reverse-biased diodes
  - Called **diffusion capacitance** because it is associated with source/drain diffusion

# Gate Capacitance

- Approximate channel as connected to source
- $C_g = \epsilon_{ox} WL/t_{ox} = C_{ox} WL = C_{permicron} W = C_0$
- $C_{permicron}$  is typically about 2 fF/ $\mu\text{m}$



$$C_0 = WLC_{ox} = C_{permicron}(W)$$

**Table 2.1** Approximation of intrinsic MOS gate capacitance

Parameter	Cutoff	Linear	Saturation
$C_{gb}$	$C_0$	0	0
$C_{gs}$	0	$C_0/2$	$2/3 C_0$
$C_{gd}$	0	$C_0/2$	0
$C_g = C_{gs} + C_{gd} + C_{gb}$	$C_0$	$C_0$	$2/3 C_0$

# Diffusion Capacitance

- ❑  $C_{sb}$ ,  $C_{db}$
- ❑ Undesirable, also called *parasitic* capacitance
- ❑ Capacitance depends on area and perimeter
  - Use small diffusion nodes
  - Comparable to  $C_g$  for contacted diffusion cap
  - $\frac{1}{2} C_g$  for uncontacted
  - Varies with process

