Homework II

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The methods used to solve the model are fixed-grid VFI on 250 grid points, accelerated VFI on 250 grid points, multigrid VFI on 10,000 grid points, and projection by Chebyshev polynomial. Projection by finite elements was attempted, but with incoherent results. However, the algorithm still converged to incoherent results so its runtime can be compared to the other methods. The time taken to approximate the policy functions for capital, consumption, and labor, as well as the functions for wages and the return on capital, are below. Note that the acceleration rate employed is 0.05 rather than 0.1, due to the lack of performance of the machine used to execute the programs. The fastest method by far is projection by Chebyshev, with

Table 1: Run Times

Method	Second
VFI Fixed Grid	1018.46
VFI Accelerator	56.34
VFI Accelerator and Multigrid	6692.69
Chebyshev Projection	0.78
Finite Element Projection	1430.41

the next fastest method (accelerated VFI on 250 grid points) two orders of magnitude behind. The slowest, accelerated VFI with multigrid on 10,000 grid points, is another two orders of magnitude behind accelerated VFI on 250 grid points.

Accuracies of the above solutions (excluding projected finite elements) are compared by examining the policy functions for next period capital, and the impulse response from a positive TFP shock that lasts for 33 periods, close the the expected duration of such a shock given the transition matrix.

In the interest of disclosure, despite the fact that the projected Chebyshev results match the VFI results very closely for the high and medium TFP states, the results of Chebyshev approximation are very divergent from the VFI approximation for the low TFP state. Moreover, the Chebyshev approximation for this state is almost certainly incorrect, since it implies a downward-sloping consumption policy. This is also almost certainly due to a programming bug, although it is difficult to imagine what kind of programming bug would have such asymmetric implications for the results, to only affect the functions in one TFP state.

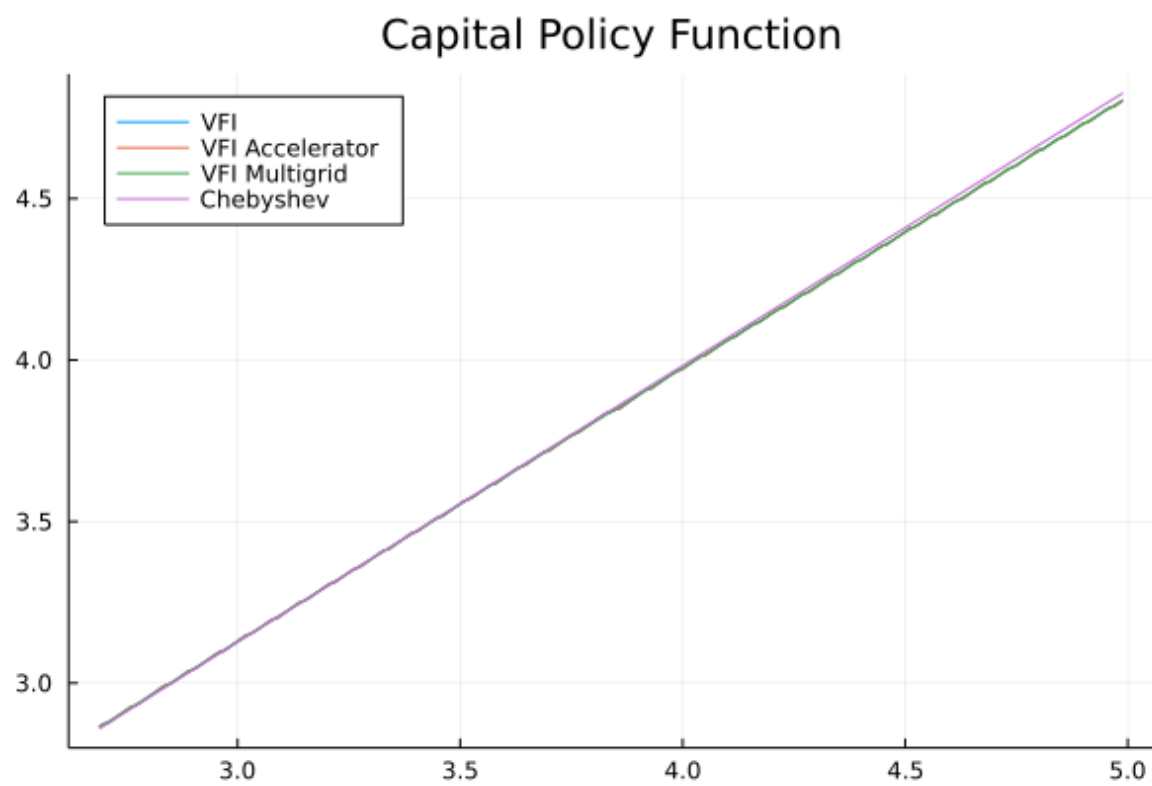The policy function and impulses response plots are below.
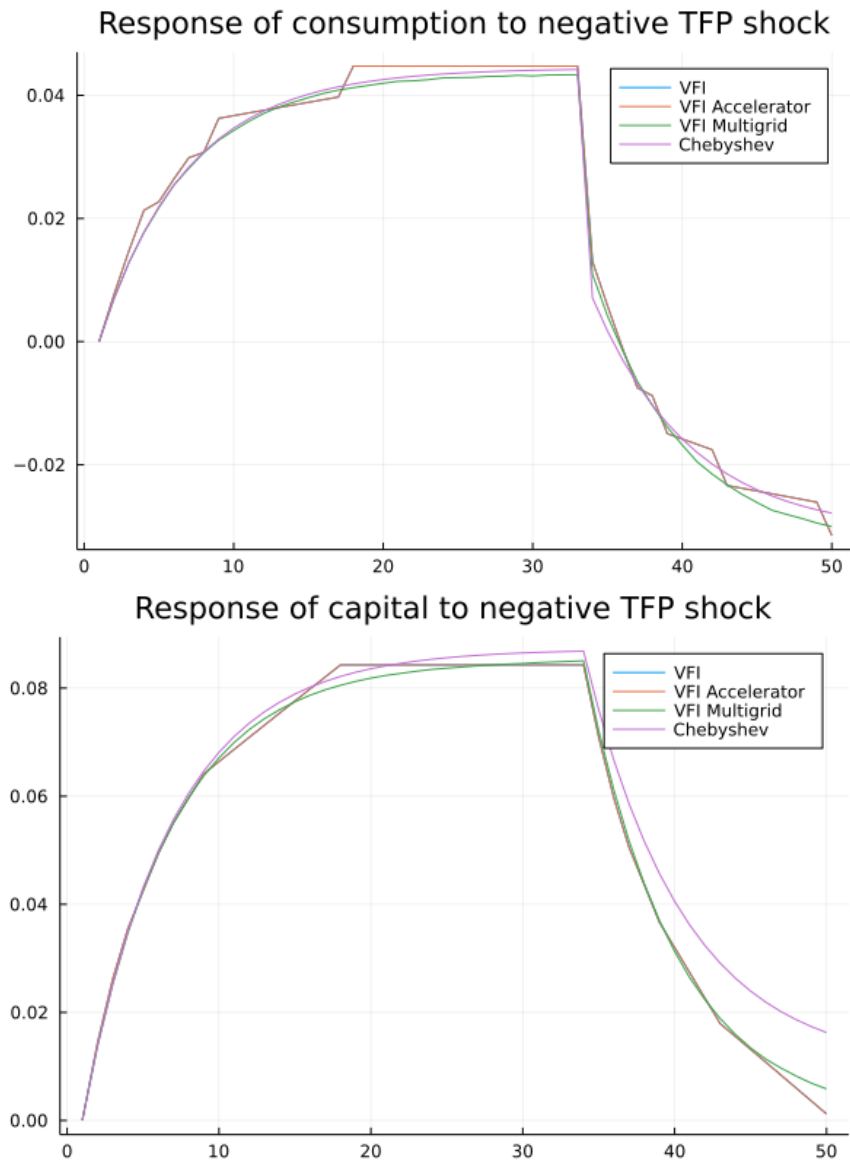
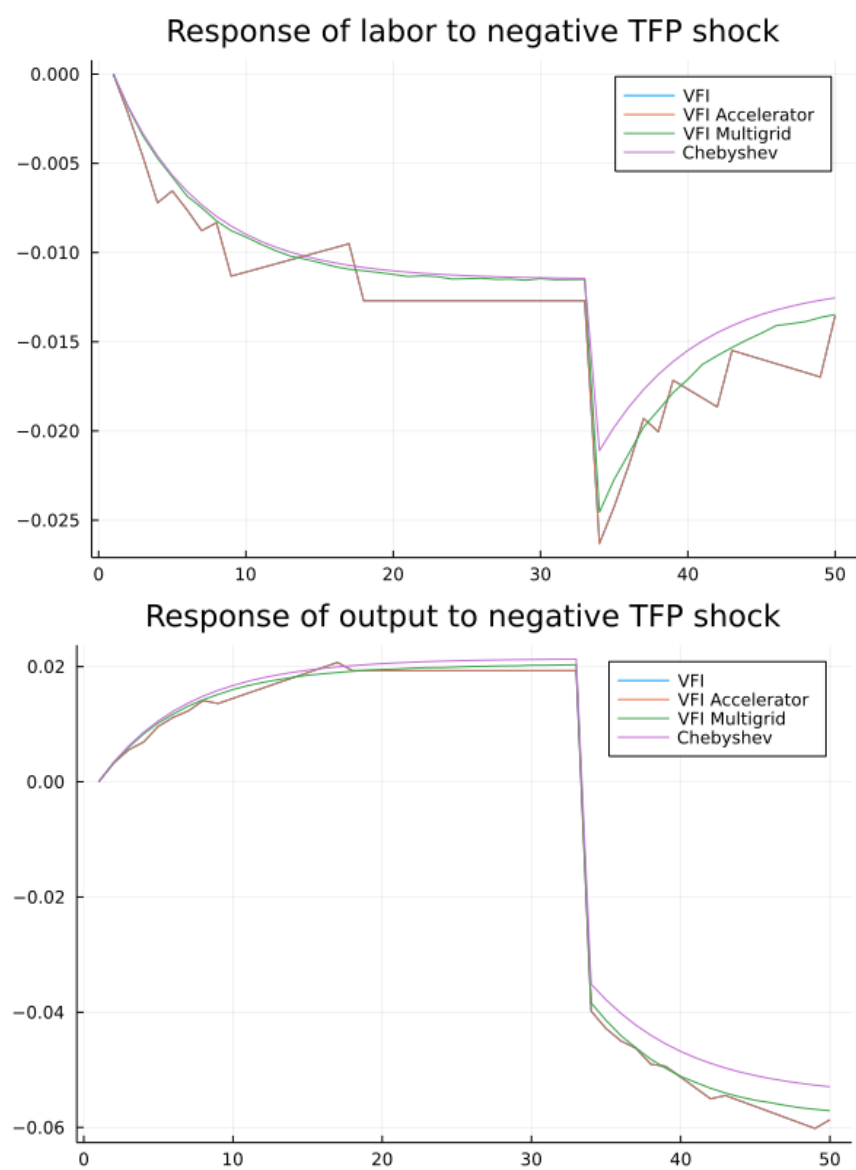Figure 1: High TFP state
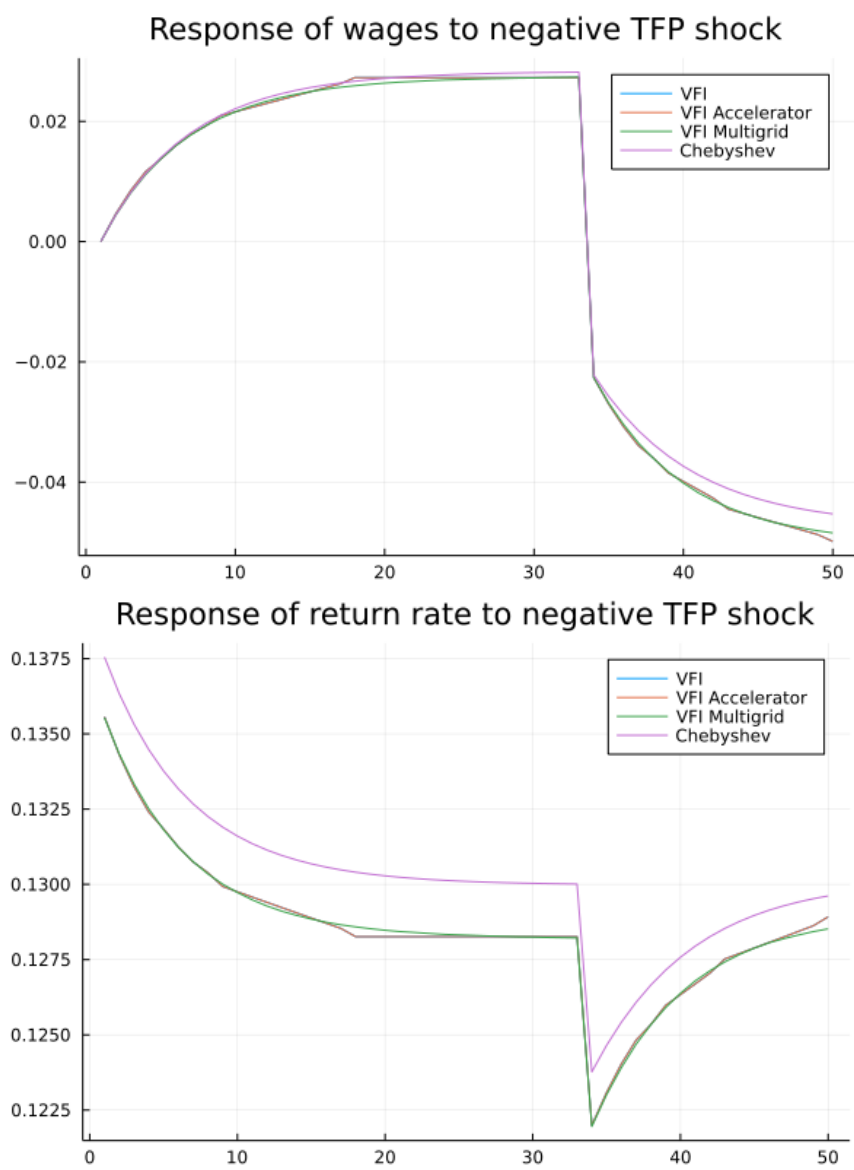
Figure 2

Figure 3

Figure 4

Up until the reversion of TFP back to the medium state at period 33, all methods are quantitatively similar. After the reversion, the Chebyshev response implies less of a capital decline than the other methods, with the effect that wages, output, return rates, and labor decline less upon TFP reversion according to the Chebyshev repsonse than the VFI responses. This is because the Chebyshev inaccuracy for $z = -0.05$ has no influence during the first half of the time path, but begins to have an influence after TFP reverts to the median level since TFP has a nonzero probability of entering the low state from the medium state. Thus, all methods are accurate relative to one another excepting the aberration of Chebyshev in the low TFP state. As a second note, the increase in grid points by a factor of 40 from VFI and VFI Accelerator to VFI multigrid dramatically increases the smoothness of the VFI-approximated functions; this is evident in all of the response functions.