Replication Exercise

Alexander Sawyer

December 14, 2021

1 Summary of Guvenen 2007

The model in the paper is a partial equilibrium Bewley model wherein households face an earnings process that departs from the canonical model in two respects: households are ex-ante heterogeneous in the earnings process they face, and households do not initially know the parameters that govern their idiosyncratic process but only learn them over time. In particular, household i for the first T periods faces the finite-horizon problem

$$\begin{aligned} V_t^i(\omega_t^i, y_t^i, \hat{x}_t^i) &= \max_{c_t^i, \omega_t^i} \left\{ U(c_t^i) + \delta E\left[V_{t+1}^i\left(\omega_{t+1}^i, y_{t+1}^i, \hat{x}_{t+1}^i\right)\right]\right\} \\ s.t. \qquad & c_t^i + P^b \omega_{t+1}^i = \omega_t^i + y_t^i; \qquad \omega_{t+1} \geq \underline{\omega}_{t+1} \\ & y_t^i = \alpha^i + \beta^i t + z_t + \varepsilon_t; \quad z_t = \rho z_{t-1} + \eta_t, \end{aligned}$$

where η and ε are idd and normal and α^i and β^i are the unknown idiosyncratic parameters. For $t > T$ the household is retired and solves

$$\begin{aligned} V_t^i(\omega_t^i, y^i) &= \max_{c_t^i, \omega_t^i} \left\{ U(c_t^i) + \delta V_{t+1}^i(\omega_{t+1}^i, y^i)\right\} \\ s.t. \qquad & c_t^i + P^b \omega_{t+1}^i = \omega_t^i + d^{y_t^i}; \qquad \omega_{t+1} \geq \underline{\omega}_{t+1}; \\ & y^i = \Phi(y_T^i), \end{aligned}$$

where Φ is a social security system. Hidden states $x = [\alpha \quad \beta \quad z]$ are updated through the Kalman filter, for which the measurement equation is the log-income equation above. The population distribution for (α^i, β^i) is

$$x \sim N\left(\begin{bmatrix} \overline{\alpha} \\ \overline{\beta} \end{bmatrix}, \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha,\beta} & \sigma_\beta^2 \end{bmatrix} \right)$$

This is also the prior distribution for the Kalman filtering problem for the households, with the exception that the prior variance term σ_β^2 for β is replaced by $(1-\lambda)\sigma_\beta^2$, where λ signifies the level of additional knowledge households have regarding their learning rate over and above knowledge of the population distribution.

For this exercise I am replicated the learning process for households and the grid construction algorithm for the household problem. In particular, I am aiming to replicate figures 4 and 5 in Guvenen (2007), which depict the rate of learning for each parameter α and β.

2 Replication Results

The parameter-set from Guvenen (2007) that were used in this exercise are below. Since I was not able to manage solving the household problem, preference parameters are omitted.

Table 1: Parameters

Parameter	Value
σ_α^2	0.022
σ_β^2	0.00038
$\sigma_{\alpha\beta}$	0.002
λ	0
$\overline{\beta}$	0.0009
$\overline{\alpha}$	1.5
ρ	0.821
σ_η^2	0.029
σ_ε^2	0.047
T	40

The first two figures are the figures targeted from Guvenen for replication, and then following are the replicating figures.

These figures measure the period-by-period improvement in the precision of the household's forecast of each parameter for each age. The purpose of these figures is to illustrate that households learn much quicker about the intercept α than the learning rate β, even as β is more informative about lifetime earnings, which are growing exponentially in β. According to Guvenen, the learning-rate differential is due to the fact that β is uninfluential for earnings early in life, but is more influential as the household ages.

With the exception of a few periods of quick learning early in life, the replicated results match the original results qualitatively. Quantitatively, the results are similar except that the precision-improvement rate is a few percentage points faster in the replicated results than the original results, for both parameters at all ages. The replication of figure 5 from Guvenen matches the original results more precisely. Note that λ is kept at 0 for all replicated figures shown; too high a value of λ induces instability in the program used to solve the grid construction and filtering problem.

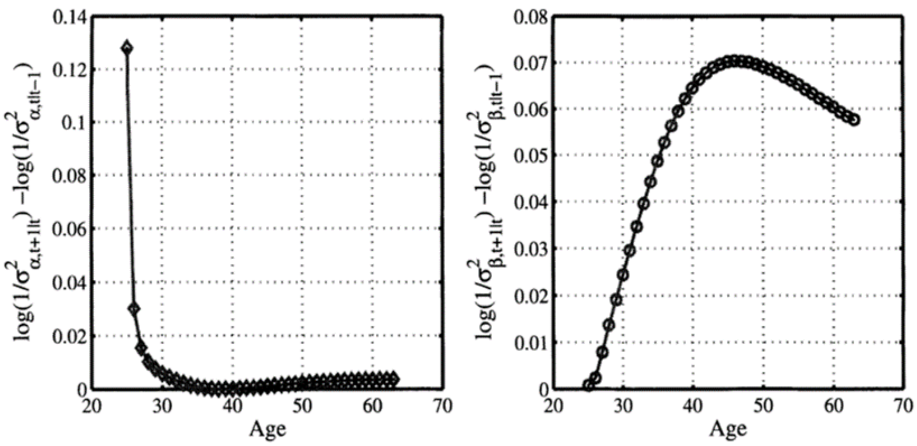


FIGURE 4. CHANGE IN THE PRECISION OF BELIEFS ABOUT α AND β

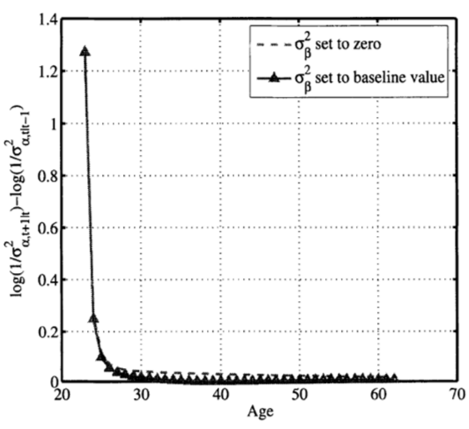
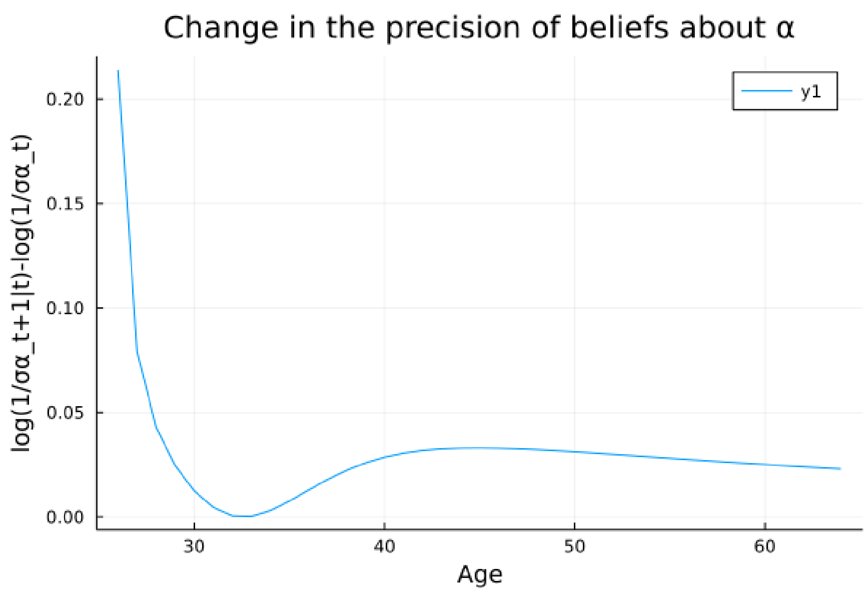
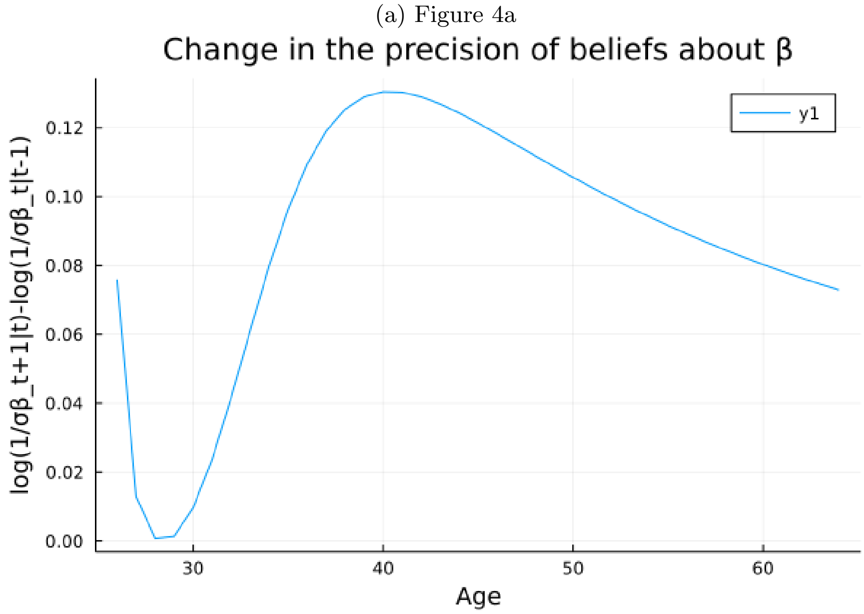


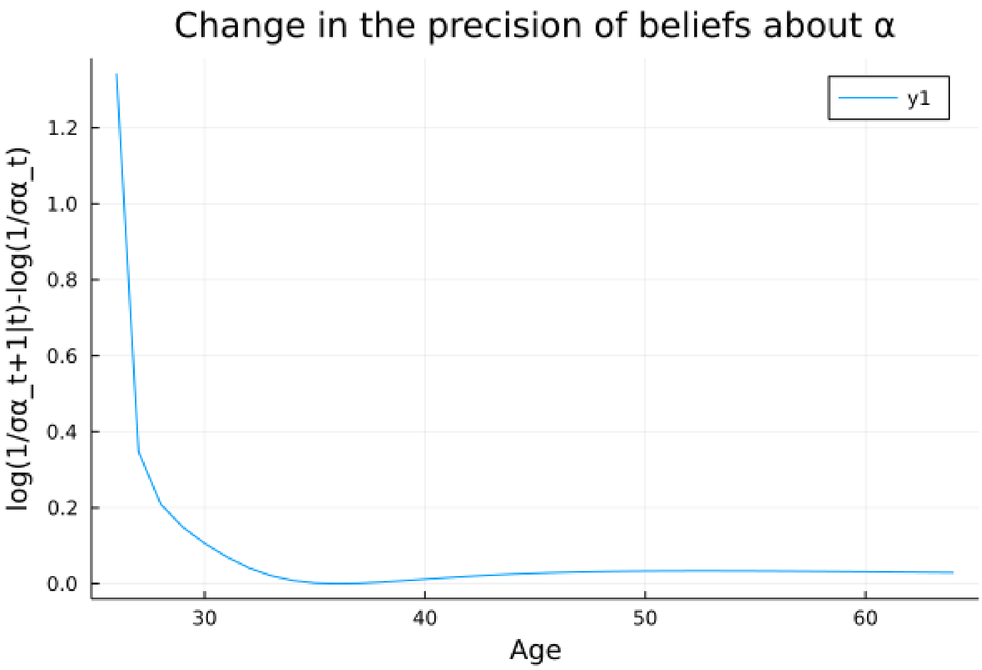
FIGURE 5. CHANGE IN THE PRECISION OF BELIEFS ABOUT α WHEN $\sigma^2_{\alpha,0}$ IS SET TO 10 TIMES ITS BASELINE VALUE



(a) Figure 4a



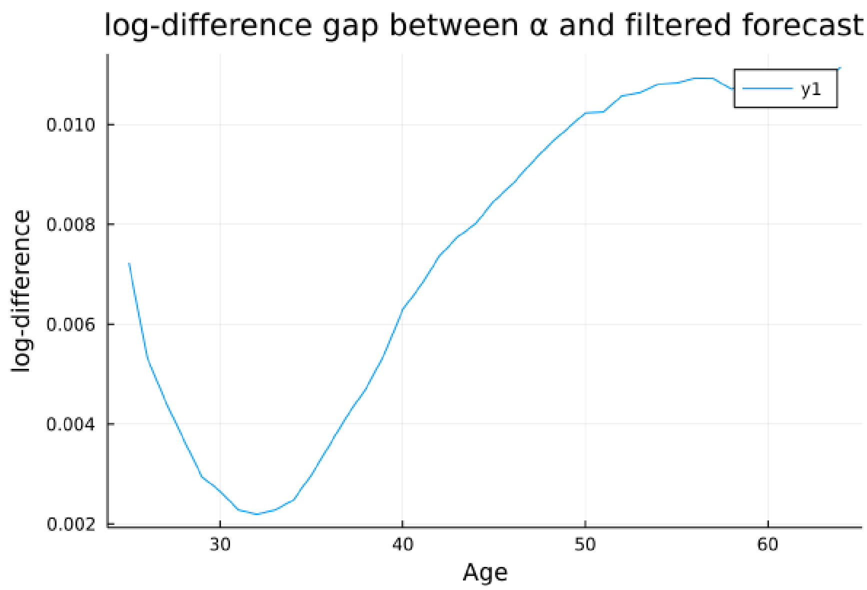
(b) Figure 4b

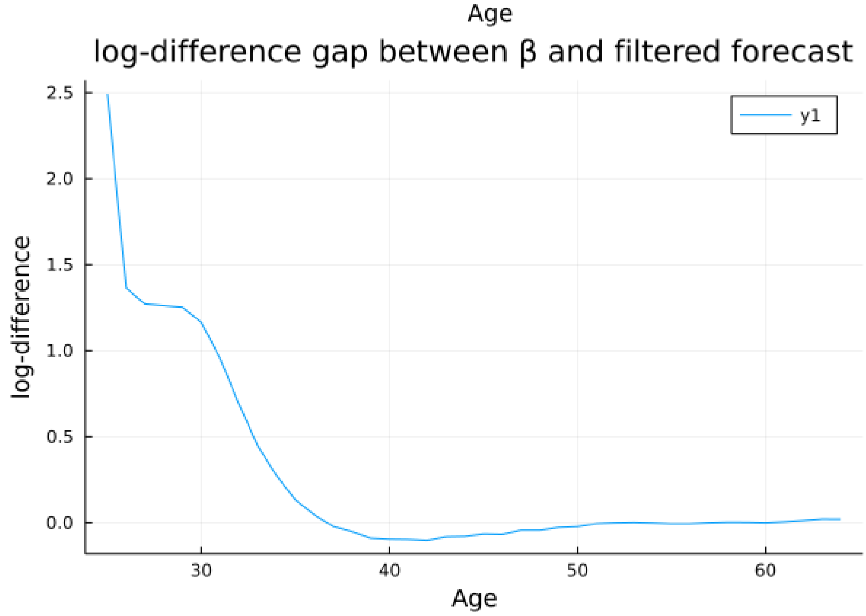


(a) Figure 5 Replication

The subsequent final figures further demonstrate that the main economic point here, that households have much more uncertainty regarding β and resolve this uncertainty more slowly, can be made with the replicated results. Each graph shows the log of the absolute value of the ratio of the of the actual parameter value to the filtered expectation of the parameter value, averaged over simulated household paths. The interpretation is the percent deviation of the filtered expected value of the parameter from its true value.

The households' forecasts of their true α is fairly consistently close to the actual level of over the entire working-life. In contrast, households' forecasted values of β are initially far from their respective actual values, but converges quickly during the households' 30's. The hump-shape in the precision improvement rate with respect to β manifests as a two-plateau shape in this figure.


log-difference gap between α and filtered forecast
log-difference
0.002
0.004
0.006
0.008
0.010
30
40
50
60
Age
y1


log-difference gap between β and filtered forecast
log-difference
0.0
0.5
1.0
1.5
2.0
2.5
30
40
50
60
Age
y1