E.G. PIEPER'S METHOD (USING DATA OF Q I OF PROBLEM SHEET)

$$T_{base}^{tool} = T_0^n = \begin{bmatrix} 0.884 & 0.306 & 0.354 & 0.789 \\ 0.177 & -0.919 & 0.354 & 0.789 \\ 0.433 & -0.25 & -0.866 & 0.067 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(\text{or} \quad \omega(q) = [0.789, 0.789, 0.067, 0.354, 0.354, -0.866, 30^\circ] \right)$$

$$4.51 \Rightarrow T_0^{n'} = (P_{n'\,org})^0 = \begin{bmatrix} 0.789 \\ 0.789 \\ 0.067 \end{bmatrix} - 0.5 \begin{bmatrix} 0.354 \\ 0.354 \\ -0.866 \end{bmatrix}$$

$$= \begin{bmatrix} 0.612 \\ 0.612 \\ 0.5 \end{bmatrix}$$

BASE JOINT $\theta_1 = \text{atan2}(0.612, 0.612)$ FROM (4.53)

$* \Rightarrow \theta_1 = +45^\circ \; *$

$b_1 = C_1 P_{x n'} + S_1 P_{y n'}$ FROM (4.54)

$\Rightarrow b_1 = 0.866$

$b_2 = d_1 - P_{3 n'}$ FROM (4.55)

$\Rightarrow b_2 = 1.5.$

ELBOW JOINT $\theta_3 = \pm \cos^{-1}\left[\dfrac{b_1^2 + b_2^2 - a_1^2 - a_2^2}{2 a_1 a_2} \right]$ FROM (4.61)

$\Rightarrow \theta_3 = \pm \cos^{-1}(0.5) = \pm 60^\circ$

CHOOSE ELBOW UP $\Rightarrow * \; \theta_3 = +60^\circ \; *$

$(4.68) \Rightarrow \alpha = a_2 + a_3 C_3 = 1.5$

$(4.69) \Rightarrow \beta = -0.866$

$(4.70) \Rightarrow \gamma = +0.866$

$(4.71) \Rightarrow \delta = 1.5$

SHOULDER JOINT $* \; \theta_2 = \text{atan2}(1.5, 2.60) = 30^\circ \; *$ FROM (4.75)

$$T_{Base}^{Wrist} = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & C_1(a_2 C_2 + a_3 C_{23}) \\ S_1 C_{23} & \ddots & \vdots \\ -S_{23} & & \\ 0 & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.7071 & -7071 & 0.612 \\ 0 & -0.7071 & 0.7071 & 0.612 \\ -1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(NOTE POSITION PART ALREADY KNOWN)

$$\left(T_{Base}^{Wrist}\right)^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0.5 \\ -0.7071 & -0.7071 & 0 & 0.866 \\ -0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{Wrist}^{n'} = \left(T_{Base}^{Wrist}\right)^{-1} \left(T_0^{n'}\right)$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0.5 \\ -0.7071 & -0.7071 & 0 & 0.866 \\ -0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.884 & 0.306 & 0.354 & 0.612 \\ 0.177 & -0.919 & 0.354 & 0.612 \\ 0.433 & -0.25 & -0.866 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.433 & 0.25 & 0.866 & 0 \\ -0.75 & 0.433 & -0.5 & 0 \\ -0.5 & -0.866 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PITCH JOINT $(4.78) \Rightarrow \theta_4 = \text{atan2}(-0.866, -0.5)$

$= 240^\circ \; (\text{or } -120^\circ)$

ROLL JOINT $(4.79) \Rightarrow \theta_5 = \text{atan2}(0.5, 0.866)$

$= 30^\circ.$

PROB SHEET SECN 4

ALTERNATIVE NUMBERS FOR Q I

$$\begin{bmatrix} 0.6597 & 0.0474 & 0.75 & 1.8238 \\ -0.4356 & -0.7892 & 0.433 & 1.0530 \\ 0.6123 & -0.6123 & -0.5 & 2.7159 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(\theta_1 = 30^\circ, \theta_2 = -45^\circ, \theta_3 = 30^\circ, \theta_4 = -45^\circ, \theta_5 = 45^\circ)$

E.G. ALPHA II – NO PARTITIONING

$d_1 = 2.0\,m;\quad a_2 = a_3 = 1\,m;\quad d_5 = 0.5\,m$

Assume $\theta_3 > 0$.

$[\theta_1 = 30^\circ, \theta_2 = -30^\circ, \theta_3 = 30^\circ, \theta_4 = 45^\circ; \theta_5 = 90^\circ;$
$\theta_{23} = 0^\circ, \theta_{234} = 45^\circ;$
$C_1 = 0.866, S_1 = 0.5, C_2 = 0.866, S_2 = -0.5, C_3 = 0.866,$
$S_3 = 0.5, C_4 = 0.7071, S_4 = 0.7071, C_5 = 0,$
$S_5 = 1, C_{23} = 1, S_{23} = 0, S_{234} = 0.7071, C_{234} = 0.7071]$

$$T_{Base}^{Tool} = \begin{bmatrix} 0.5 & -0.6123 & -0.6123 & 1.3098 \\ -0.866 & -0.3535 & -0.3535 & 0.7562 \\ 0 & 0.7071 & -0.7071 & 2.1465 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Or the tool configuration vector may be given

$$w(q) = \begin{bmatrix} (P_{tool-org})^{base} \\ a^{base} \\ \theta_n \end{bmatrix} = \begin{bmatrix} 1.3098 \\ 0.7562 \\ 2.1465 \\ -0.6123 \\ -0.3535 \\ -0.7071 \\ 90^\circ \end{bmatrix}$$

Apply inverse kinematic equations:

θ_1 : BASE JOINT ANGLE

4.104 $\Rightarrow \theta_1 = \operatorname{atan2}(0.7562, 1.3098)$

$\left(\tan^{-1}\left(\frac{0.7562}{1.3098}\right) = 30^\circ\right)$

$\theta_1 \approx 30^\circ$

θ_{234} : GLOBAL TOOL PITCH ANGLE

4.105 $\Rightarrow \theta_{234} = \operatorname{atan2}((-0.866)(-0.6123) - (0.5)(-0.3535),\ 0.7071)$

$= \operatorname{atan2}(0.7071, 0.7071)$

$= 45^\circ$

θ_3 : ELBOW JOINT ANGLE

(4.108) $\Rightarrow$

$b_1 = C_1 P_x + S_1 P_y + d_5 S_{234}$

$= (0.866)(1.3098) + (0.5)(0.7562) + (0.5)(0.7071)$

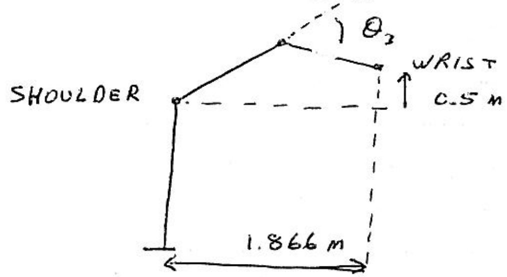
$\Rightarrow b_1 \approx 1.866\,m$

(4.109) $\Rightarrow$

$b_2 = d_1 - d_5 C_{234} - P_3$

$= 2 - (0.5)(0.7071) - 2.1465$

$= -0.5\,m$



SIDE VIEW SOMETHING LIKE THIS.

(4.111) $\Rightarrow \theta_3 = \pm\cos^{-1}\left[\frac{(1.866)^2 + (0.5)^2 - 1 - 1}{(2).(1)(1)}\right]$

$= \pm\cos^{-1}\left(\frac{1.732}{2}\right)$

$\Rightarrow \theta_3 = \pm 30^\circ$

We choose $\theta_3 > 0$ – ELBOW UP.

θ_2 : SHOULDER ANGLE

(4.112) $\Rightarrow$

$\theta_2 = \operatorname{atan2}(-(0.5)(1.866) + (1 + 0.866)(-0.5),\ (1.866)(1.866) + (0.5)(-0.5))$

$= \operatorname{atan2}(-1.866, 3.232)$

$\left(\tan^{-1}\left(\frac{1.866}{3.232}\right) = 30^\circ,\ \text{4th quad} \Rightarrow \theta_2 = -30^\circ\right)$

$\theta_2 = -30^\circ$.

θ_4: PITCH JOINT $\measuredangle$

$(4.113) \Rightarrow \quad \theta_4 = \theta_{234} - \theta_2 - \theta_3$

$$= 45^\circ - (-30^\circ) - 30^\circ$$

$$= 45^\circ$$

θ_5 TOOL ROLL JOINT

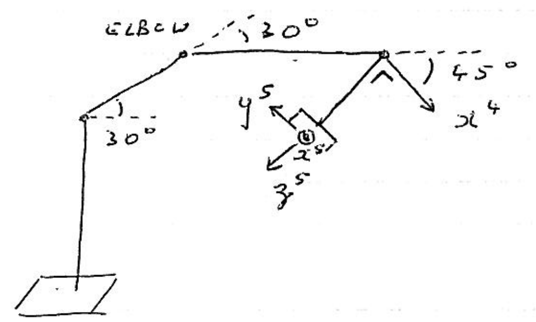
$(4.115) \Rightarrow \quad S_5 = (0.5)(0.5) - (0.866)(-0.866)$

$$\approx 1$$

$(4.116) \Rightarrow \quad C_5 = (0.5)(-0.6123) - (0.866)(-0.3535)$

$$\approx 0$$

$(4.117) \Rightarrow \quad \theta_5 = \operatorname{atan2}(1, 0) = 90^\circ$



EE4009 Q6

SUM '09

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & p_x \\ R_{21} & R_{22} & R_{23} & p_y \\ R_{31} & R_{32} & R_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1 C_1 + a_2 C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1 S_1 + a_2 S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{1}$ VERTICAL EXTENSION $\quad q_3 = d_1 - d_4 - p_z$

$\boxed{3}$ ELBOW JOINT

$$p_x = a_1 C_1 + a_2 C_{1-2}$$
$$p_y = a_1 S_1 + a_2 S_{1-2}$$
$$p_x^2 + p_y^2 = a_1^2 + 2 a_1 a_2 C_2 + a_2^2$$
$$\Rightarrow \theta_2 = \pm \cos^{-1}\left[\frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}\right]$$

CHOOSE LEFT-HANDED $\theta_2 > 0$

$\boxed{5}$ BASE JOINT

$$\alpha C_1 + \beta S_1 = p_x$$
$$\gamma C_1 + \delta S_1 = p_y$$

where

$$\alpha = a_1 + a_2 C_2$$
$$\beta = a_2 S_2$$
$$\gamma = -a_2 S_2$$
$$\delta = a_1 + a_2 C_2$$

$$\begin{bmatrix} C_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}^{-1} \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \frac{1}{DET}\begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix}\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\Rightarrow \theta_1 = \text{atan2}(S_1, C_1) = \text{atan2}(-\gamma p_x + \alpha p_y, \delta p_x - \beta p_y)$$

$$= \text{atan2}\left(a_2 s_2 p_x + (a_1 + a_2 c_2) p_y, (a_1 + a_2 c_2) p_x - a_2 s_2 p_y\right)$$

NOTE: -

$$DET = \alpha\delta - \gamma\beta = (a_1 + a_2 C_2)^2 + (a_2 S_2)^2 > 0.$$

$\boxed{1}$ TOOL ROLL ANGLE $\quad \theta_4 = \theta_1 - \theta_2 - \text{atan2}(R_{21}, R_{11})$.

EE4009 SUM 09

Q6 (b)

$\theta_2 = 130^\circ \Rightarrow S_2 = 0.7660, \quad C_2 = -0.6428$

$\Rightarrow \alpha = 0.8572 = \delta, \ \beta = 0.7660 = -\gamma.$

$\theta_1 = \text{atan2}[1 \times 0.766 \times 1.09202, + (0.8572)(-0.35945),$
$(0.8572)(1.09202) - (0.766)(-0.35935),$
$= \text{atan2}(0.5285, 1.2112)$

$\boxed{6}$ $= 23.57^\circ.$

$\boxed{2}$ $\theta_3 = d_1 - d_4 - P_z = 2.2 - 0.6 - 1 = 0.6 \text{ m}$

$\theta_4 = \theta_1 - \theta_2 - \text{atan2}(R_{21}, R_{11})$
$= 23.57^\circ - 130^\circ - \text{atan2}(0.76604, -0.64279)$
$= -106.43^\circ - (+130^\circ)$

$\boxed{2}$ $= -236.43^\circ$

$\equiv +123.57^\circ$