EE4004
Digital Modulation Systems

Bandpass Digital Transmission

Emile Baudot

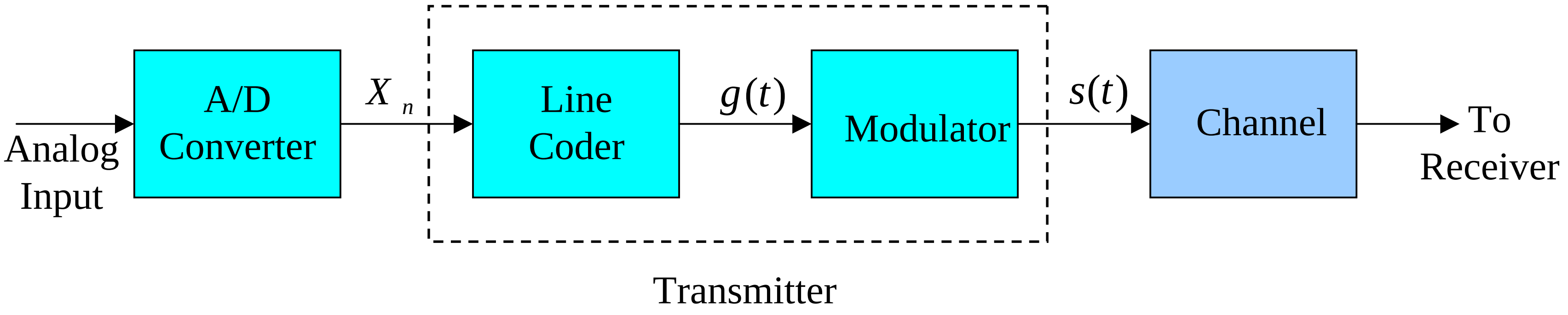
- Baudot Code – 1870   5 bits -> 32 characters

Bit Rate & Baud Rate

- Bit Rate defines rate at which information is transmitted (in bits/second   bps)
- Symbol (or Baud or Signalling) Rate defines the number of symbols/second. Each symbol can represent n bits and have M symbol states where $M=2^n$ This is called M-ary signalling.

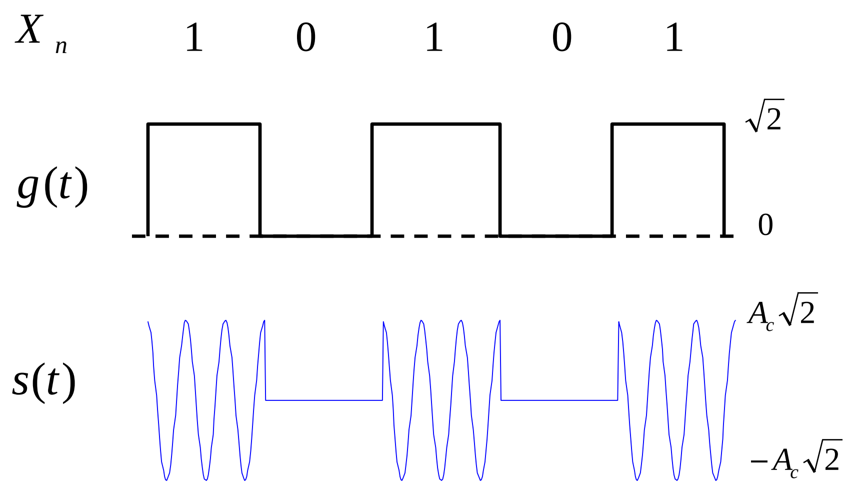
Bandpass Communication System

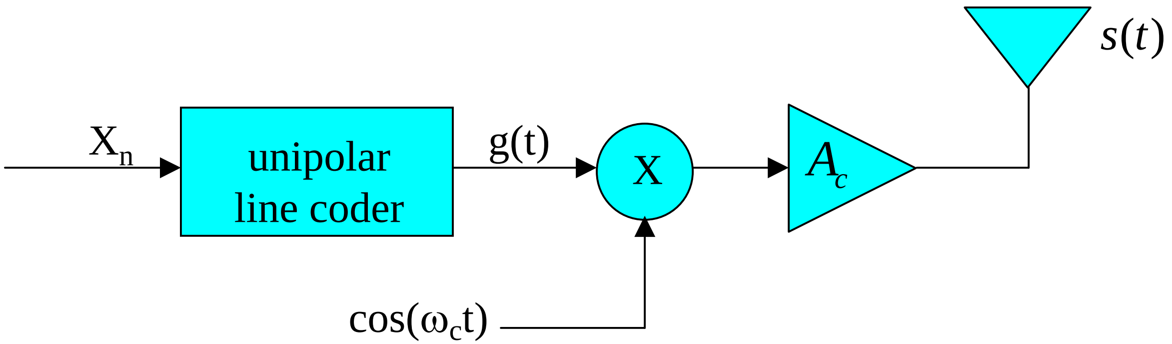
- In a ***bandpass digital communication system***, the bit stream X_n is first converted to a baseband line code g(t) by a line coder and is then converted to a bandpass signal s(t) by a modulator.



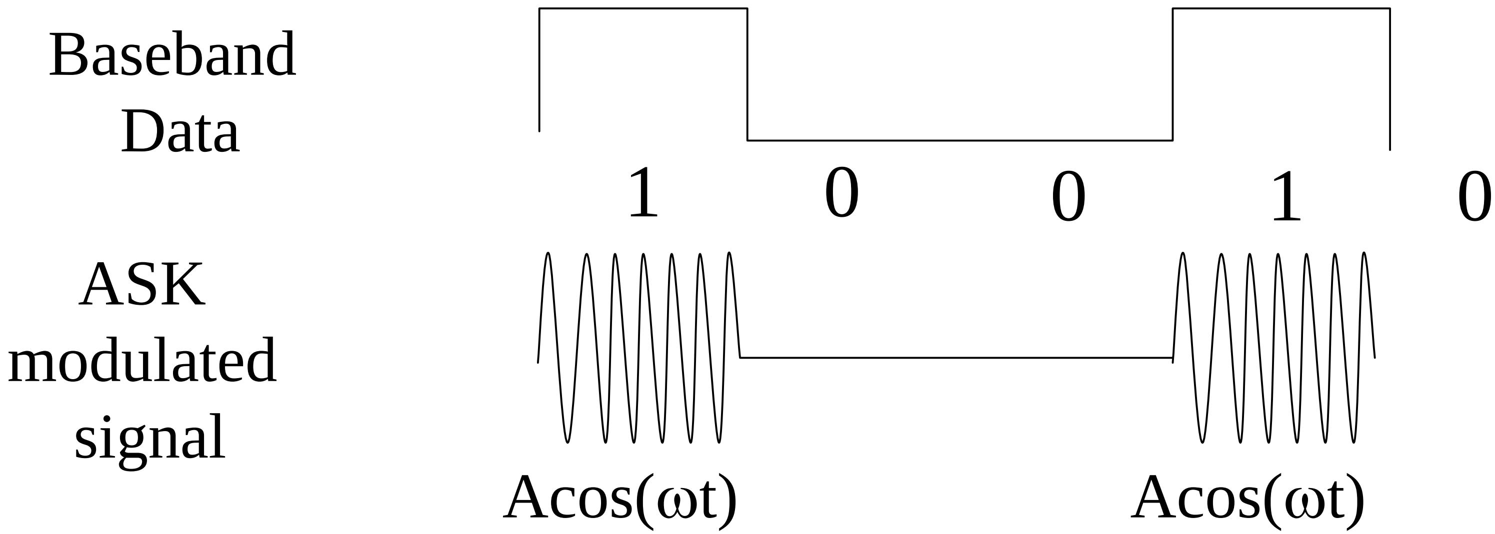
On-Off Keying (OOK)
{Amplitude Shift Keying (ASK}

- On off keying is DSB-SC with the message signal being a ***unipolar*** line code.





Amplitude Shift Keying (ASK)



- Pulse shaping can be employed to remove spectral spreading
- ASK demonstrates poor performance, as it is heavily affected by noise, fading, and interference

ASK Essentials

- ***Bandwidth*** (null-to-null):
  - 2X the BW of unipolar line code.
  - Rectangular (NRZ) pulse shape:

    $$B = 2R_b$$

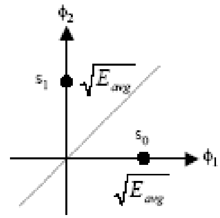
  - RC-rolloff pulse shape:

    $$B = R_b(1+\alpha)$$

- Bit Error Rate (BER):
  - Same as ***unipolar*** line code:

    $$P_b \approx Q\left(\sqrt{\frac{E_b}{N_o}}\right)$$

7.3 Binary Orthogonal Modulation



In binary orthogonal modulation there are two signals; one signal is projected on to each of two orthogonal basis functions. Using the total probability theorem and assuming equally likely symbols, the probability of error is

$$\begin{aligned} P(E) &= P(E|0)\Pr\{s_0 \text{ was sent}\} \\ &\quad + P(E|1)\Pr\{s_1 \text{ was sent}\} \\ &= \frac{1}{2}P(E|0) + \frac{1}{2}P(E|1) \end{aligned} \quad (7.19)$$

Assume s_0 was sent. Then $r[n] = s_0[n] + w[n]$ so that

$$\begin{aligned} \hat{a}_1 &= \sqrt{E_{avg}} + N_1 \\ \hat{a}_2 &= N_2 \end{aligned}$$

where N_1 and N_2 are independent Gaussian random variables with zero mean and common variance $\sigma^2 = N_0/2$. The conditional error probability is

$$\begin{aligned} P(E|0) &= \Pr\{\hat{a}_2 > \hat{a}_1\} \\ &= \Pr\left\{N_2 > N_1 + \sqrt{E_{avg}}\right\} \\ &= \Pr\left\{N_2 - N_1 > \sqrt{E_{avg}}\right\} \\ &= Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right) \end{aligned}$$

since $N_2 - N_1$ is a Gaussian random variable with zero mean and variance $2\sigma^2 = N_0$ (do you see why?).

Now assume a s_1 was sent. Then $r[n] = s_1[n] + w[n]$ so that

$$\begin{aligned} \hat{a}_1 &= N_1 \\ \hat{a}_2 &= \sqrt{E_{avg}} + N_2 \end{aligned}$$

where N_1 and N_2 are independent Gaussian random variables with zero mean and common variance $\sigma^2 = N_0/2$. The conditional error probability is

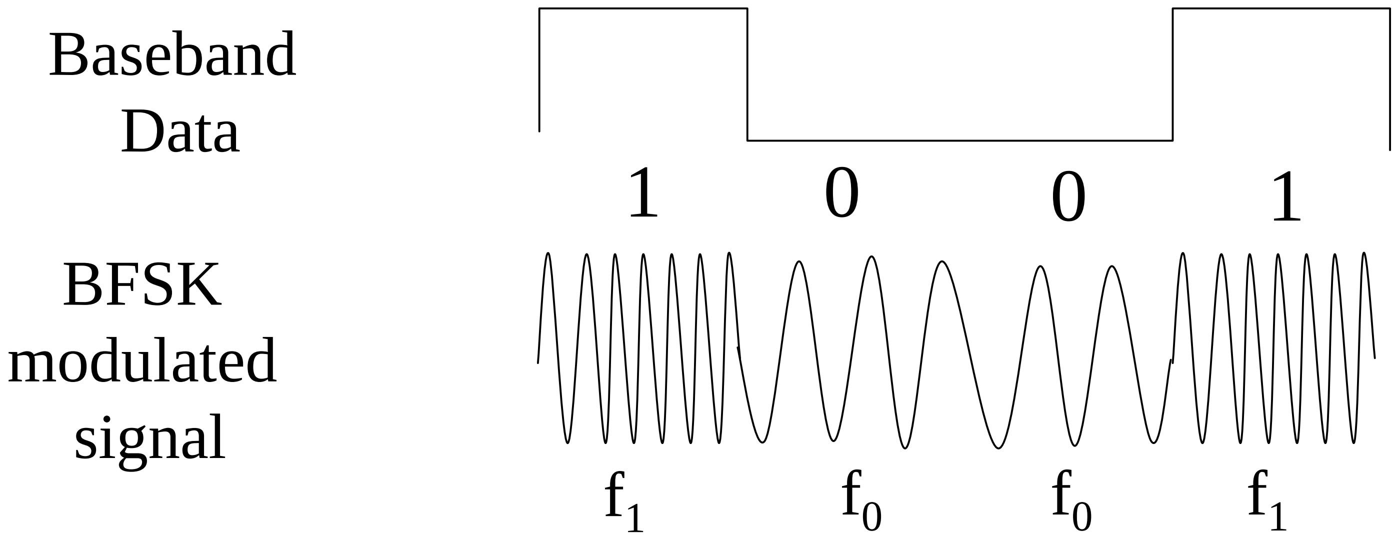
$$\begin{aligned} P(E|1) &= \Pr\{\hat{a}_1 > \hat{a}_2\} \\ &= \Pr\left\{N_1 > N_2 + \sqrt{E_{avg}}\right\} \\ &= \Pr\left\{N_1 - N_2 > \sqrt{E_{avg}}\right\} \\ &= Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right) \end{aligned}$$

Applying (7.19) we see that the total probability of error is

$$P(E) = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right) \quad (7.20)$$

Important Observations Again we make the same two important observations:

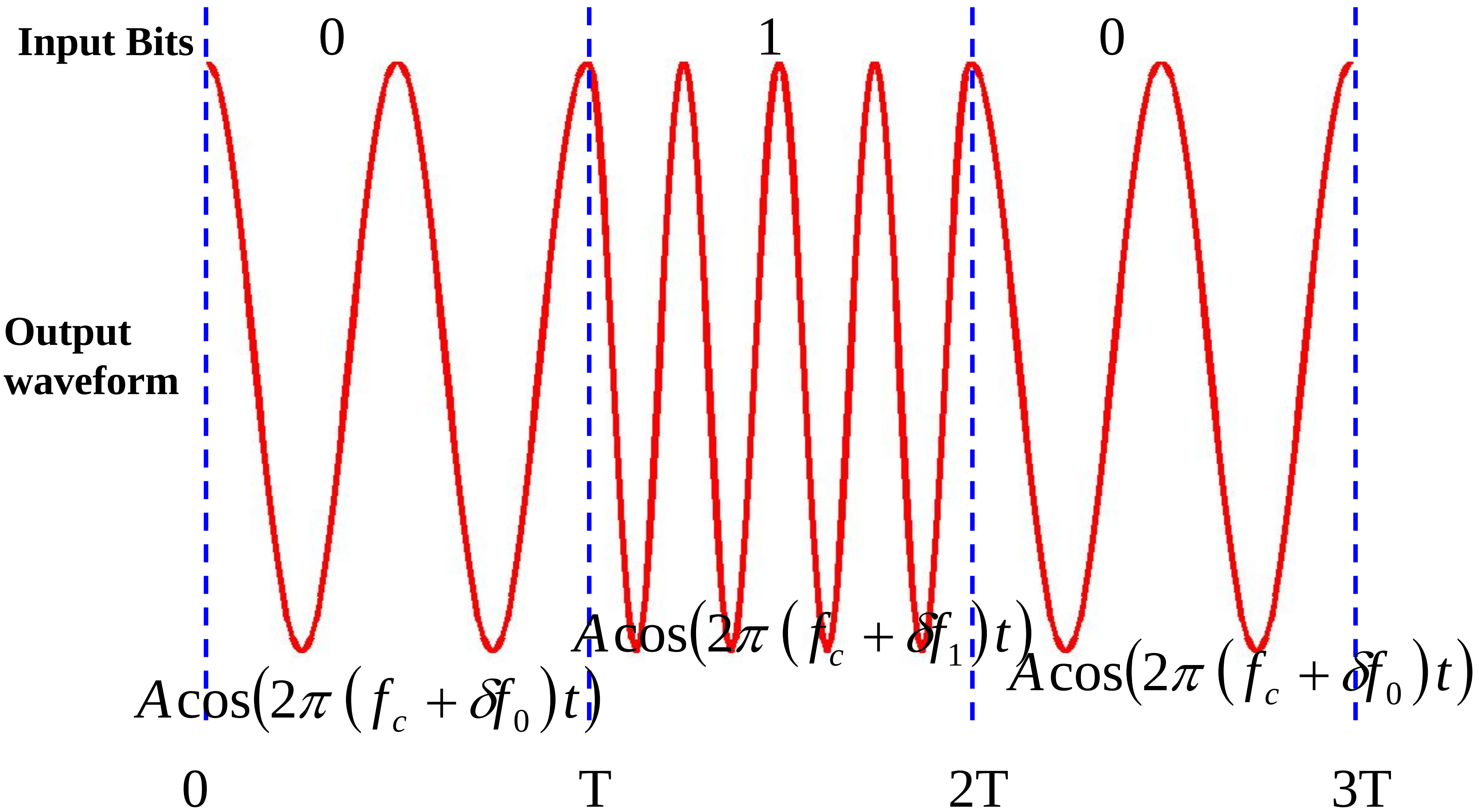
Frequency Shift Keying (FSK)



where $f_0 = A\cos(\omega_c - \Delta\omega)t$ and $f_1 = A\cos(\omega_c + \Delta\omega)t$

- Example: The ITU-T V.21 modem standard uses FSK
- Bandwidth of FSK is dependent on the spacing of the two symbols. A frequency spacing of 0.5 times the symbol period is frequently used.
- FSK can be expanded to a M-ary scheme, employing multiple frequencies as different states

FSK Output Waveform



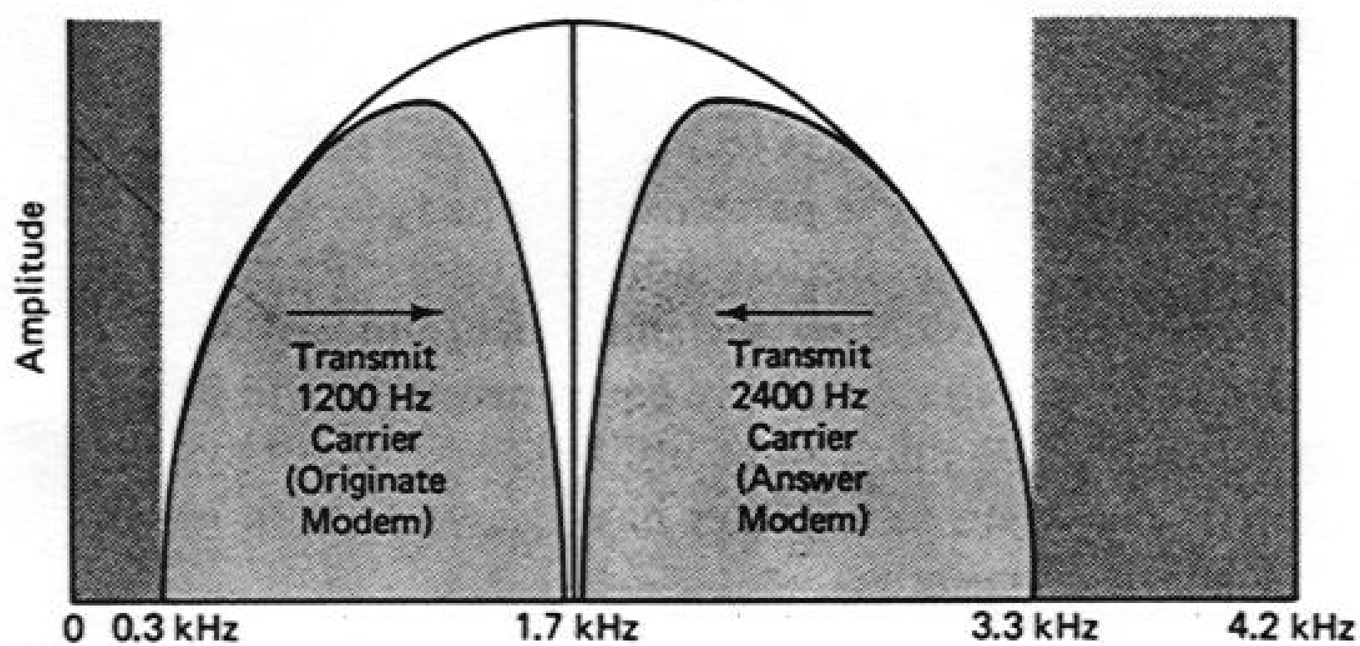
Frequency Shift Keying Modulation

- The input bits are represented by the frequency offsets
- Binary Frequency Shift Keying – BFSK

Input Bits	Output Waveform
0	$s_0(t) = A\cos(2\pi\left(f_c + \delta f_0\right)t),\quad iT \le t < (i+1)T$
1	$s_1(t) = A\cos(2\pi\left(f_c + \delta f_1\right)t),\quad iT \le t < (i+1)T$

Duplex Modem

- Full-Duplex modem, spit channel
- Frequency from 0.3 to 3.3kHz divided in half
- Two signals, centered on 1200Hz and 2400Hz (Frequency Division Multiplexing)



- **Very Old Standards;**
- **Bell 103 – 300 bps**
- **Bell 212A – 1200 bps**
  - **Synchronous (no start/stop bit)**
  - **PSK at 600 baud**
  - **Multibit – 4 phases – 2 bits/baud**
  - **Full Duplex (frequency Division)**
  - **600 baud: time=1/600=0.0017 secs/baud**

Modem Standards

- V.32 – 9600 bps
- Synchronous
- Full duplex
- FSK at 2400 baud
- Multibit – 16 “tones” = 4 bits/baud
- 4 X 2400 = 9600

Modem Standards

- V.32bis (french for 2nd?)
- Synchronous
- Full duplex
- FSK at 2400 baud
- Multibit – 64 “tones” = 6 bits/baud
- 6 X 2400 = 14,400

<u>THE MODEM:</u>

(Modulate at the Sender side, Demodulate at the Receiver side)

1. <u>Time sharing</u> Terminal modems.

2. <u>High speed</u> :
   - Asynchronous modem,
   - Synchronous modem

3. <u>Very high speed</u> modems.

<u>Time Sharing</u> Terminal modems.

The time sharing modems have 2-way full duplex communication. The modem has 2 channels numbered 1 & 2; each channel has 2 frequencies for binary 1 and 0.
Frequencies for the 2 channels are different to provide full duplex communications.

Frequencies used:	Channel 1	binary 0	1180 Hz
		binary 1	980 Hz
	Channel 2	binary 0	1850 Hz
		binary 1	1650 Hz

Identical modems are used at both ends of the connection.

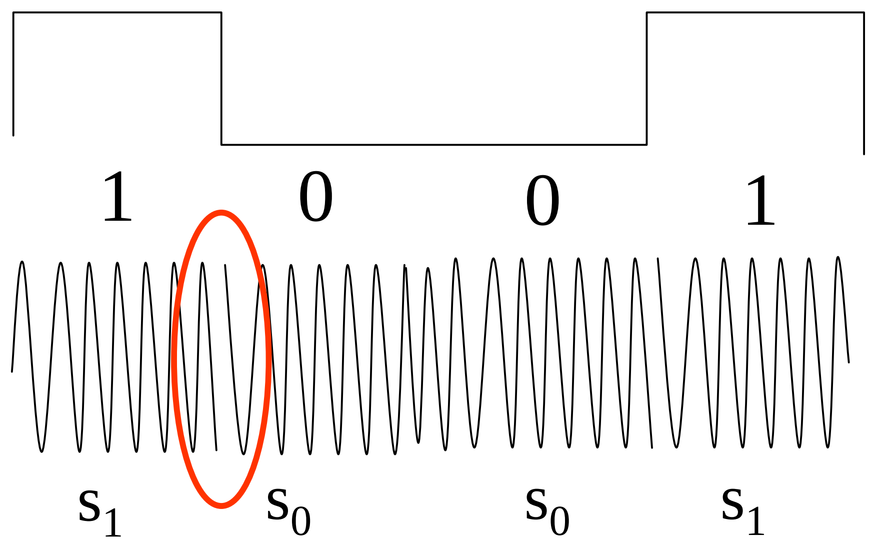
Terminal — **Modem** — **Telephone network** — **Modem** — **Computer**

To decide which channel is used for receiving and which for sending there is internationally agreed sequence.

Phase Shift Keying (PSK)

Baseband Data

BPSK modulated signal

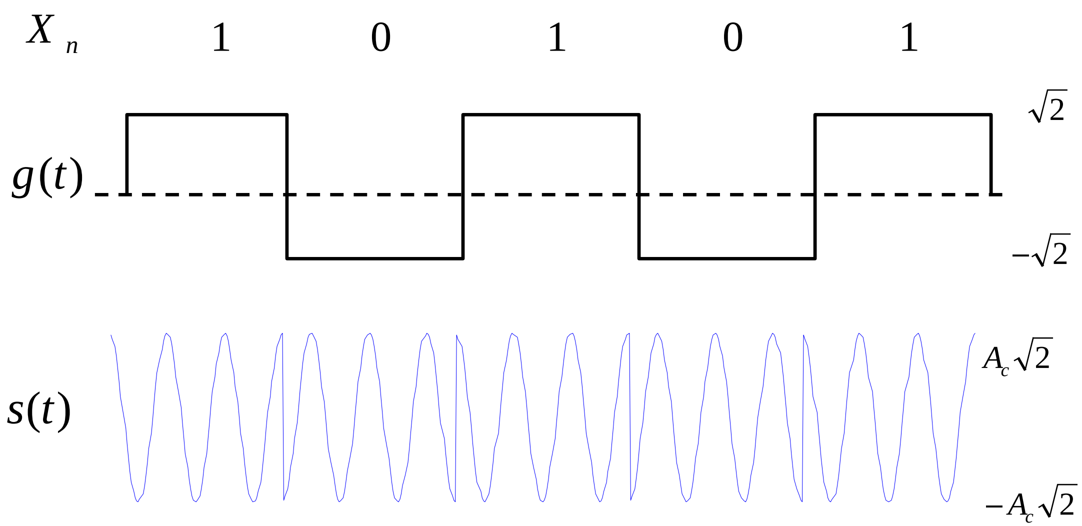


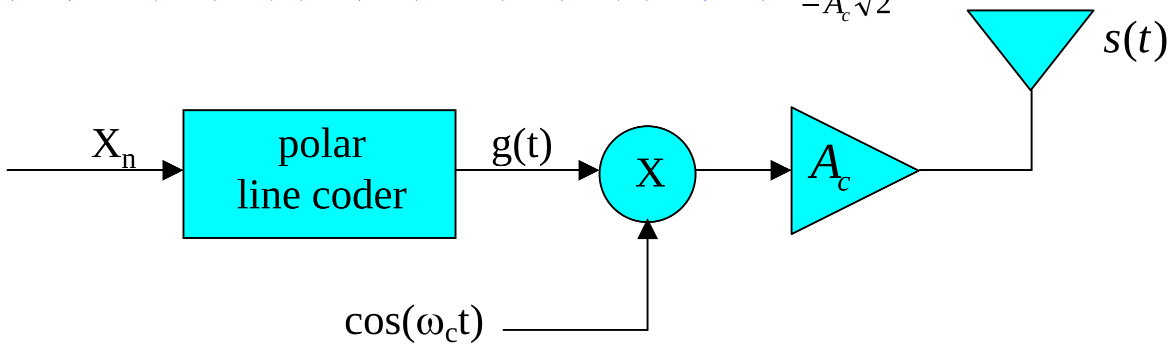
where $s_0 = -A\cos(\omega_c t)$ and $s_1 = A\cos(\omega_c t)$

- Major drawback – rapid amplitude change between symbols due to phase discontinuity, which requires infinite bandwidth. Binary Phase Shift Keying (BPSK) demonstrates better BER performance than ASK and BFSK

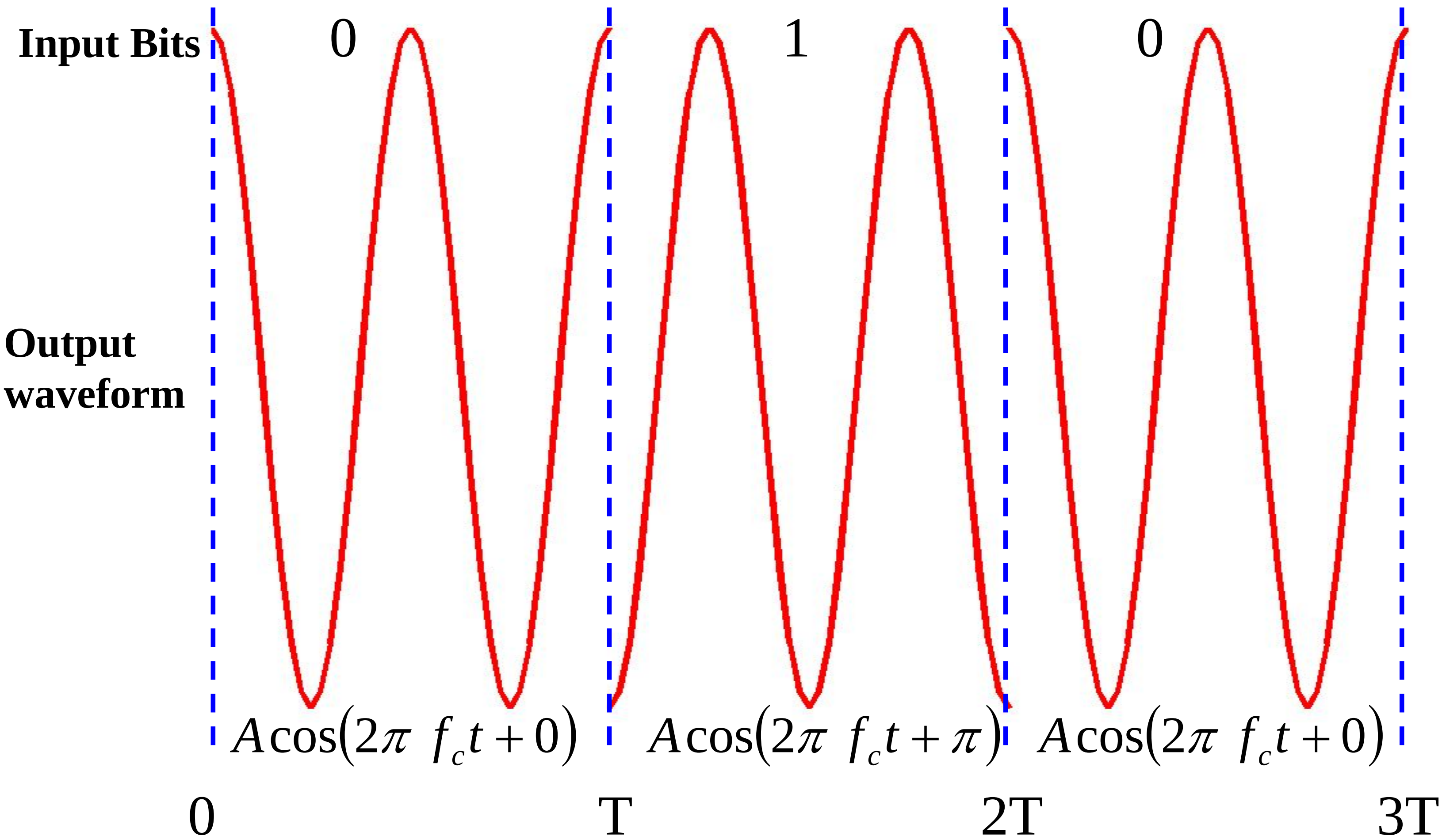
Binary Phase Shift Keying (BPSK)

- BPSK keying is DSB-SC with the message signal being a ***polar*** line code.





BPSK Output Waveform



Phase Shift Keying Modulation

- The input bits are represented by the phase change
- Binary Phase Shift Keying – BPSK

Input Bits	Output Waveform
0	$s_0(t) = A\cos(2\pi f_c t + 0),$ $iT \le t < (i+1)T$
1	$s_1(t) = A\cos(2\pi f_c t + \pi),$ $iT \le t < (i+1)T$

BPSK Essentials

- Bandwidth (null-to-null):
  - 2X the BW of polar line code.
  - Rectangular (NRZ) pulse shape:

$$B = 2R_b$$

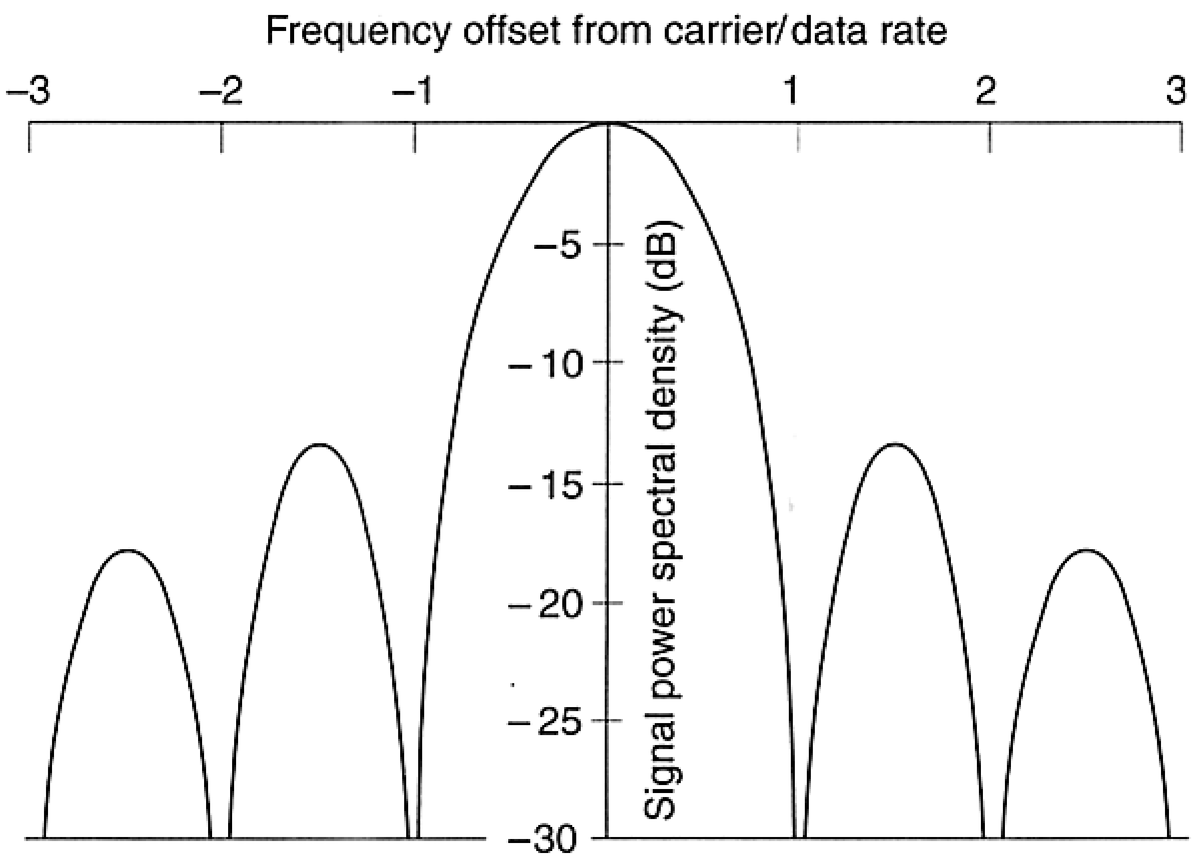
  - RC-rolloff pulse shape:

$$B = R_b(1+\alpha)$$

- Bit Error Rate (BER):
  - Same as polar line code:

$$P_b \approx Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

BPSK spectra (rectangular pulses)



$$G_{PSK}(f) = \sigma^2 r \,\mathrm{sinc}^2 \frac{f}{r_b}$$

Constellation diagrams

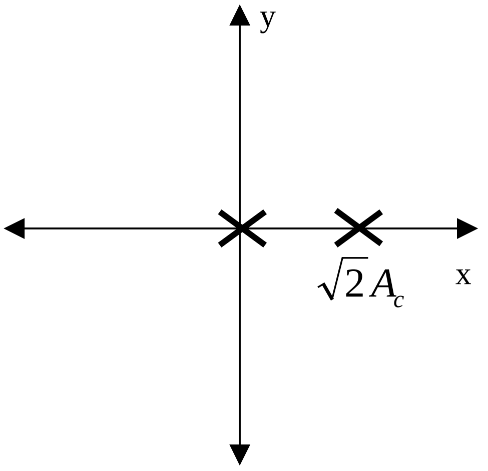
= graphical representation of the complex envelope of each possible symbol state

- The x-axis represents the in-phase component and the y-axis the quadrature component of the complex envelope

- The distance between signals on a constellation diagram relates to how different the modulation waveforms are and how easily a receiver can differentiate between them.

Constellation Diagrams

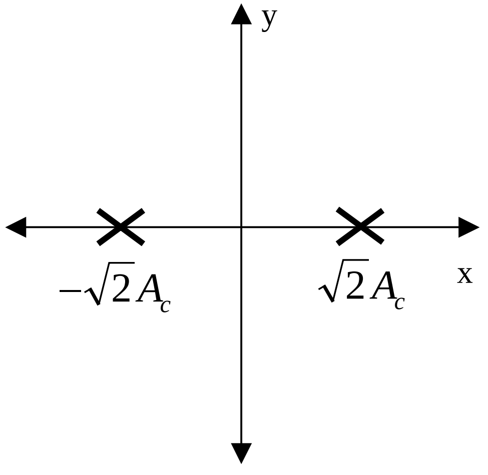
- ***Signal constellation diagrams*** graphically display all the possible signal vectors.



Example signal constellation diagram for ASK signal.

Constellation Diagram for BPSK

- The signal constellation diagram for BPSK is:



Example signal constellation diagram for BPSK signal.

Example: BPSK Constellation Diagram

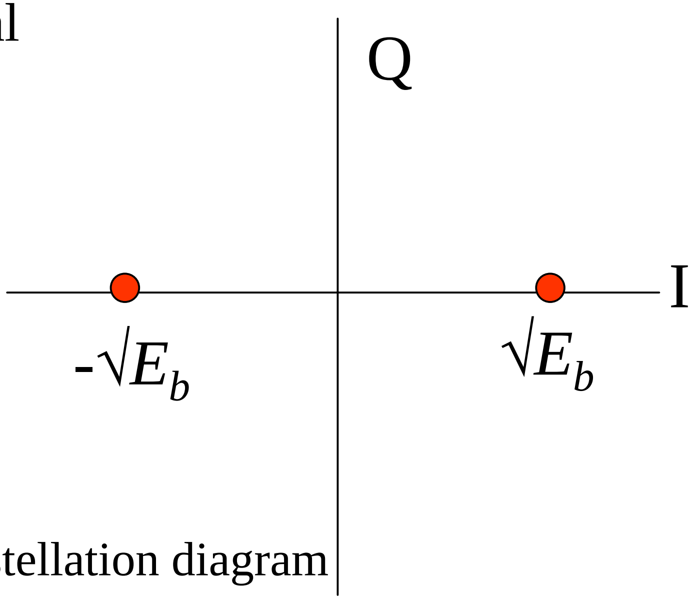
$$S_{BPSK} = \left\{ \left[s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \right], \left[s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t); \right] \right\}; \quad 0 \le t \le T_b$$

$E_b =$ energy per bit; $T_b =$ bit period

For this signal set, there is a single basic signal

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t); \quad 0 \le t \le T_b$$

$$S_{BPSK} = \left\{ \left[\sqrt{E_b}\phi_1(t) \right], \left[-\sqrt{E_b}\phi_1(t) \right] \right\}$$



Constellation diagram

Differential Modulation

- In the transmitter, each symbol is modulated relative to the previous symbol and modulating signal, for instance in BPSK 0 = no change in phase, 1 = +180^0
- In the receiver, the current symbol is demodulated using the *previous symbol as a reference*. The previous symbol serves as an estimate of the channel. A no-change condition causes the modulated signal to remain at the same 0 or 1 state of the previous symbol.

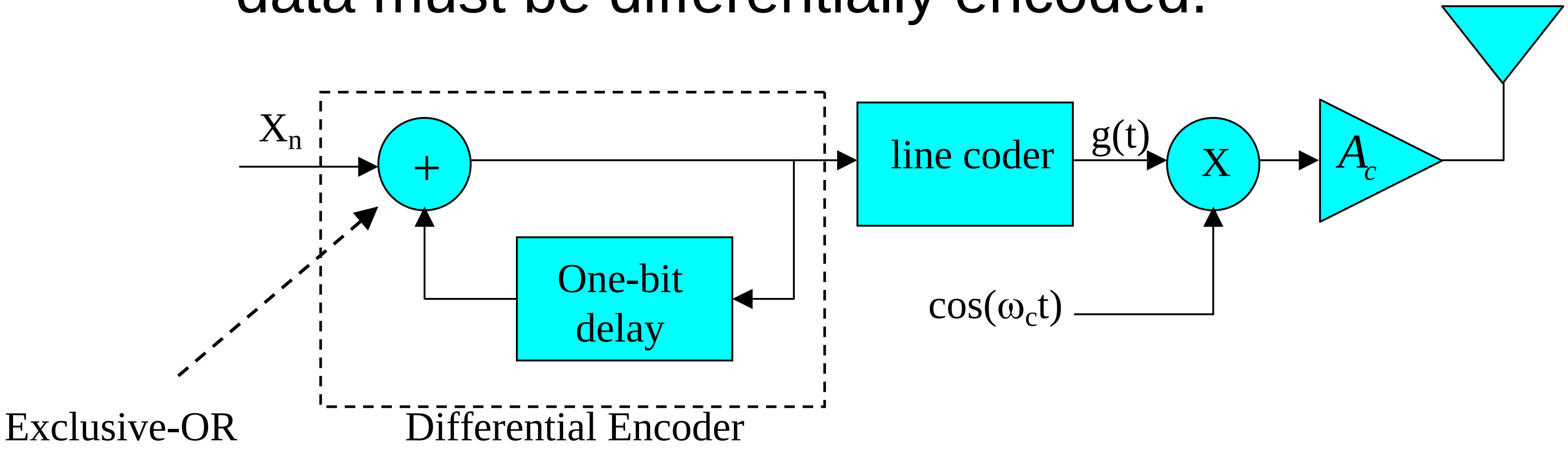
DPSK

- Differential modulation is theoretically 3dB poorer than coherent. This is because the differential system has 2 sources of error: a corrupted symbol, and a corrupted reference (the previous symbol)

- DPSK = Differential phase-shift keying: In the transmitter, each symbol is modulated relative to (a) the phase of the immediately preceding signal element and (b) the data being transmitted.

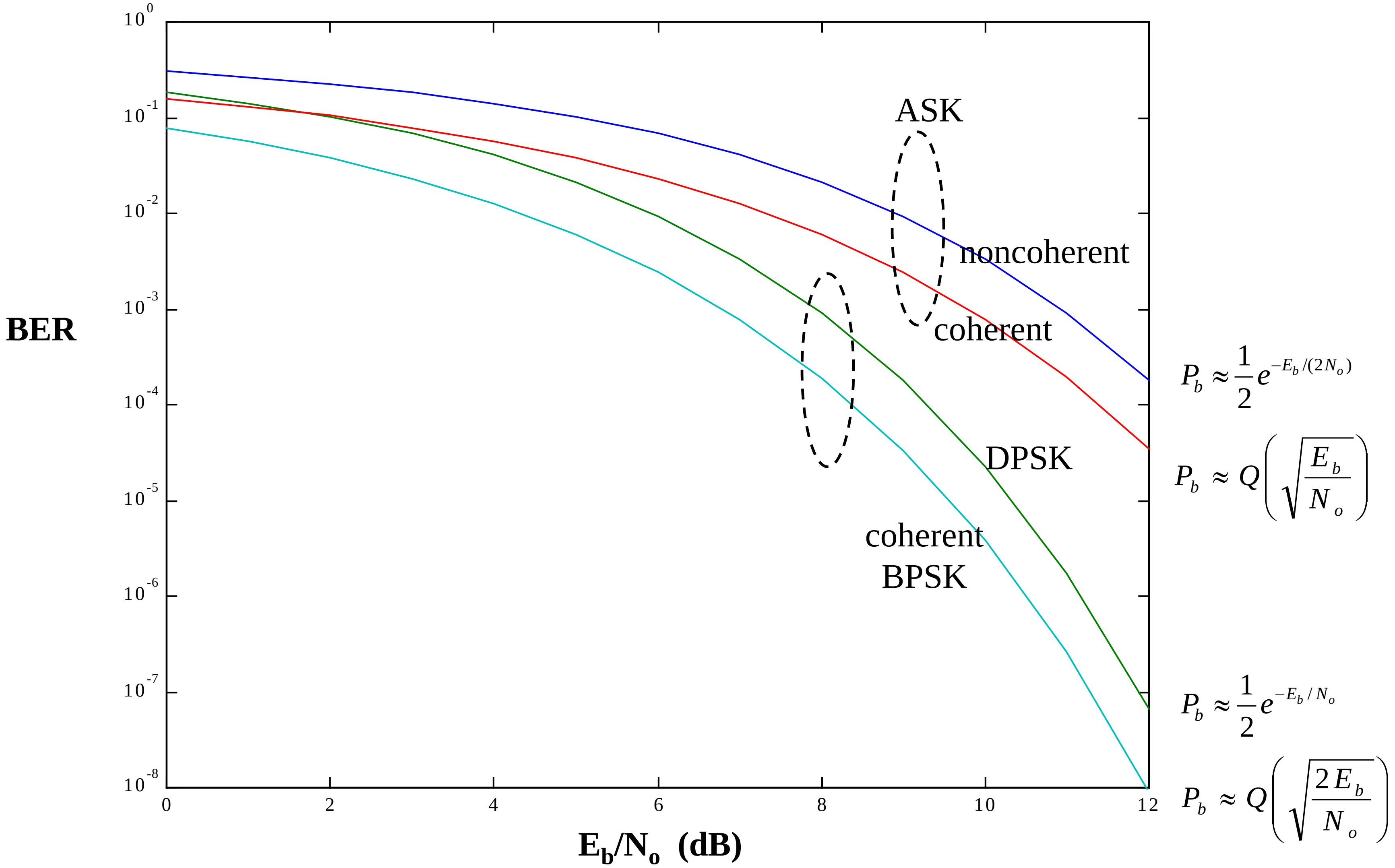
Differential Phase Shift Keying

- In order to use the partially coherent receiver, the data must be differentially encoded.



- Encoder follows the rule: “Change the phase if the input data is a one, otherwise keep the phase the same.”
- Decoder estimates a one was sent if a phase change is detected, otherwise estimates a zero was transmitted.
- Differential encoding used with a partially coherent receiver is called ***differential phase shift keying*** **(DPSK)**.

Performance Comparison ASK/PSK



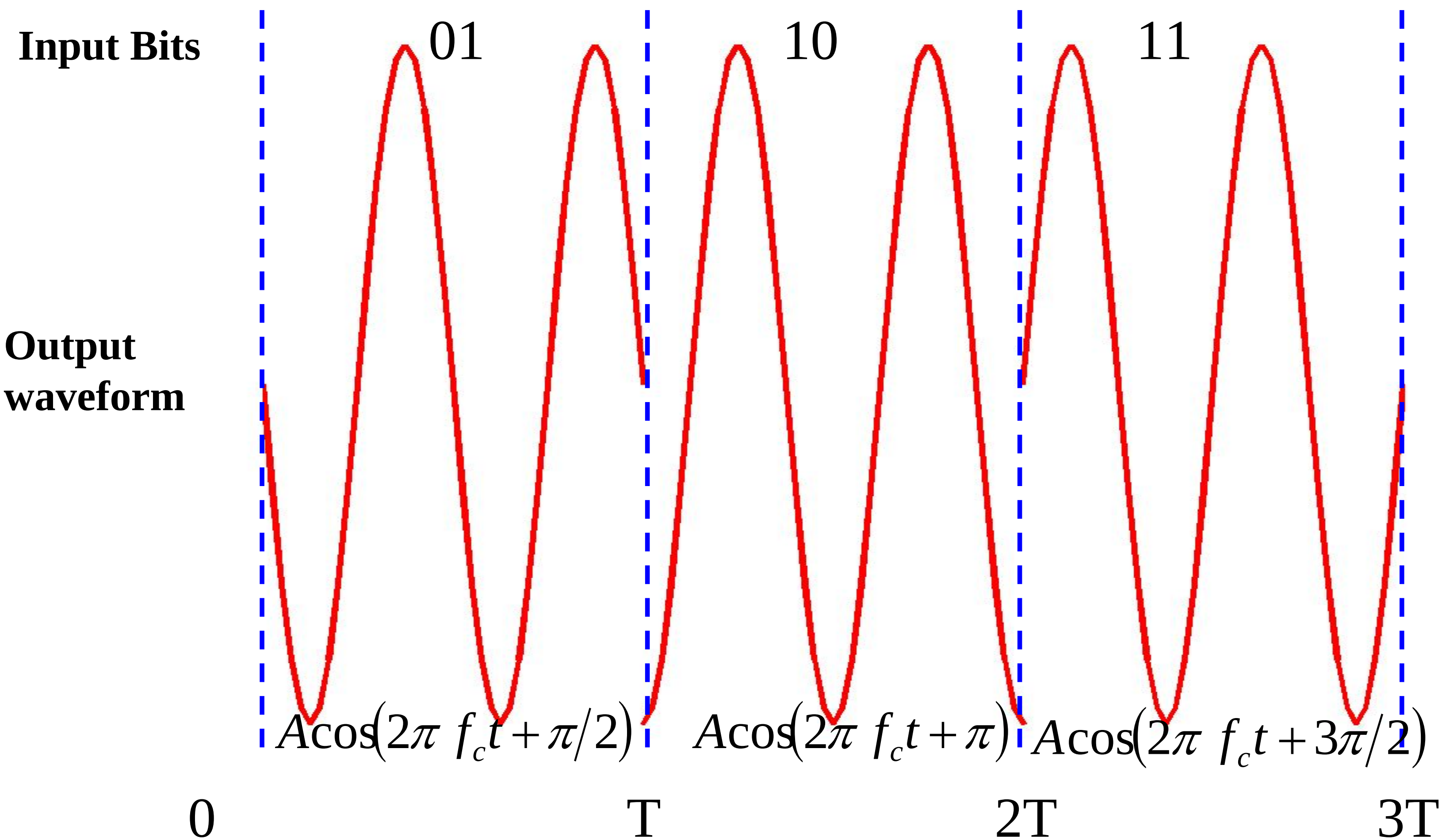
Beyond Binary
QPSK

- Quadrature Phase Shift Keying (QPSK) can be interpreted as two independent BPSK systems (one on the I-channel and one on Q), and thus the same performance but twice the bandwidth efficiency

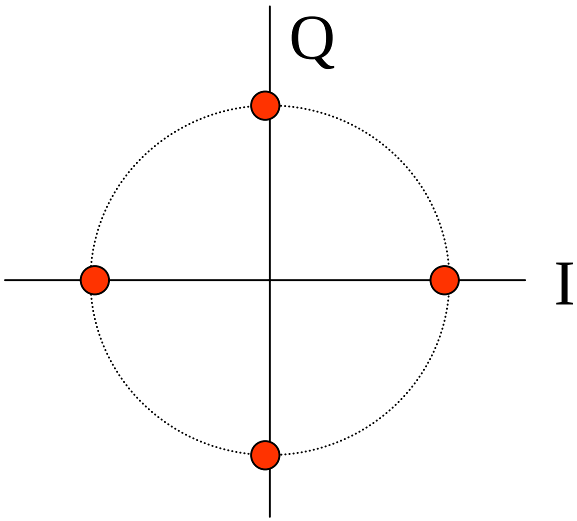
- Large envelope variations can occur due to abrupt phase transitions, thus requiring highly linear amplification.

QPSK Output Waveform

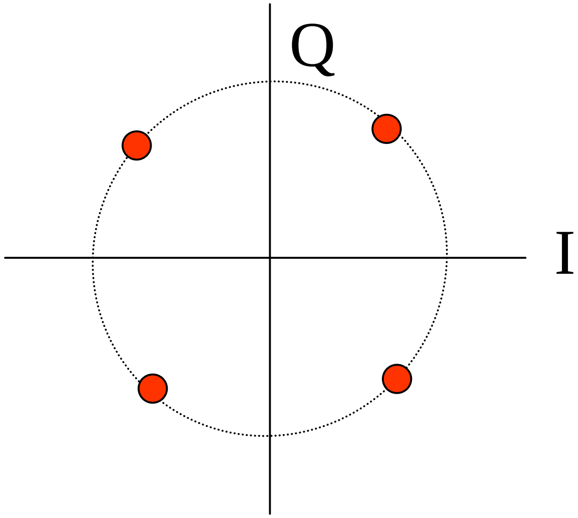
(3 of 4 states)



QPSK Constellation Diagram



Carrier phases
{0, π/2, π, 3π/2}



Carrier phases
{π/4, 3π/4, 5π/4, 7π/4}

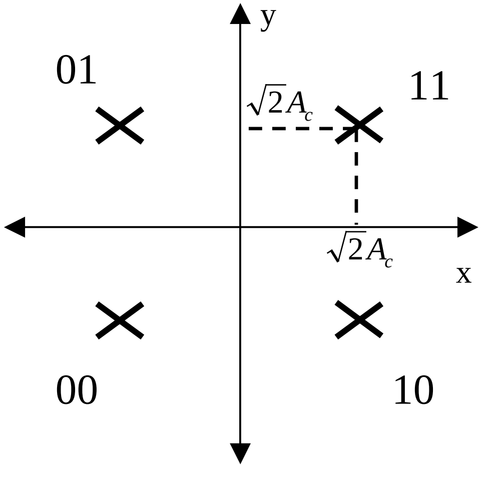
- Quadrature Phase Shift Keying has twice the bandwidth efficiency of BPSK since 2 bits are transmitted in a single modulation symbol

QPSK Modulation

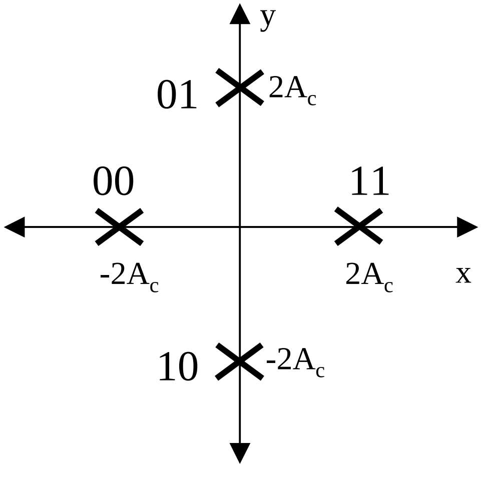
Input Bits	Output Waveform
00	$s_0(t) = A\cos(2\pi\ f_c t + 0), iT \le t < (i+1)T$
01	$s_1(t) = A\cos(2\pi\ f_c t + \pi/2), iT \le t < (i+1)T$
10	$s_2(t) = A\cos(2\pi\ f_c t + \pi), iT \le t < (i+1)T$
11	$s_3(t) = A\cos(2\pi\ f_c t + 3\pi/2), iT \le t < (i+1)T$

Constellation Diagram (2) QPSK

- The signal constellation diagram for QPSK:
  - QPSK also called “Quadriphase shift keying”

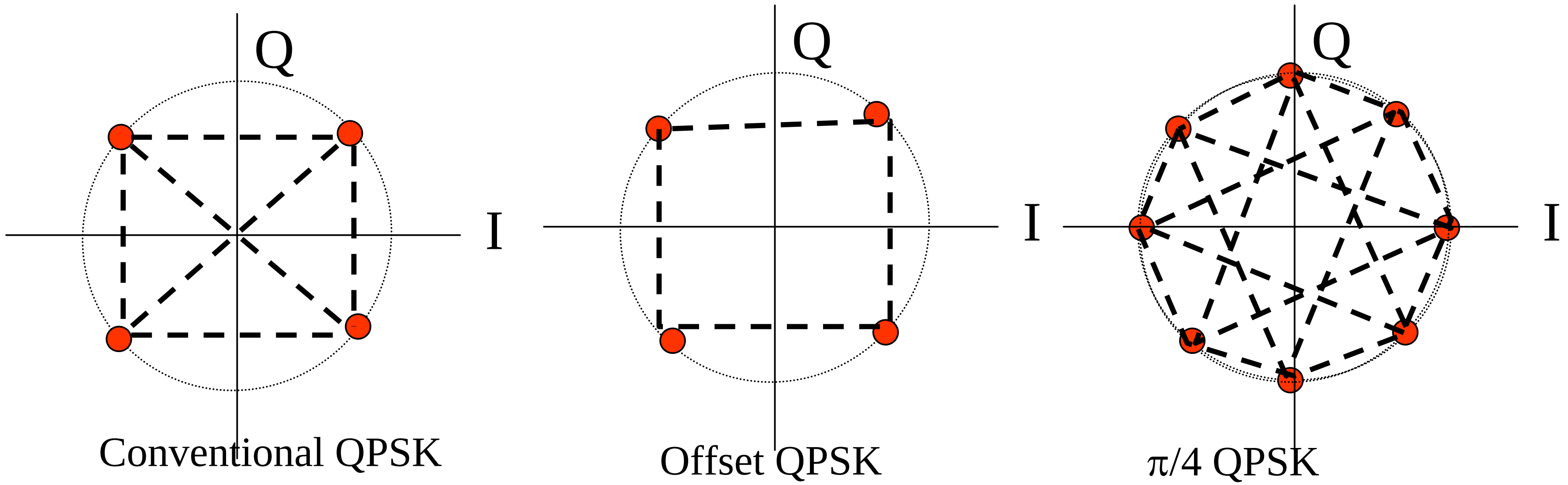


Example signal constellation diagram for QPSK signal.



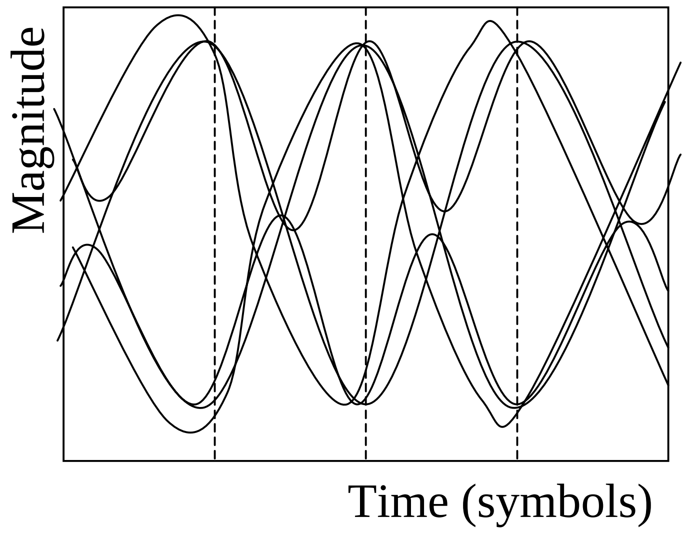
An equivalent constellation, with everything rotated by 45 degrees.

Types of QPSK



- Conventional QPSK has transitions through zero (i.e. 180^0 phase transition). Highly linear amplifiers required.
- In Offset QPSK, the phase transitions are limited to 90^0, the transitions on the I and Q channels are staggered.
- In π/4 QPSK the set of constellation points are toggled each symbol, so transitions through zero cannot occur. This scheme produces the lowest envelope variations.
- All QPSK schemes require linear power amplifiers

Eye Diagram



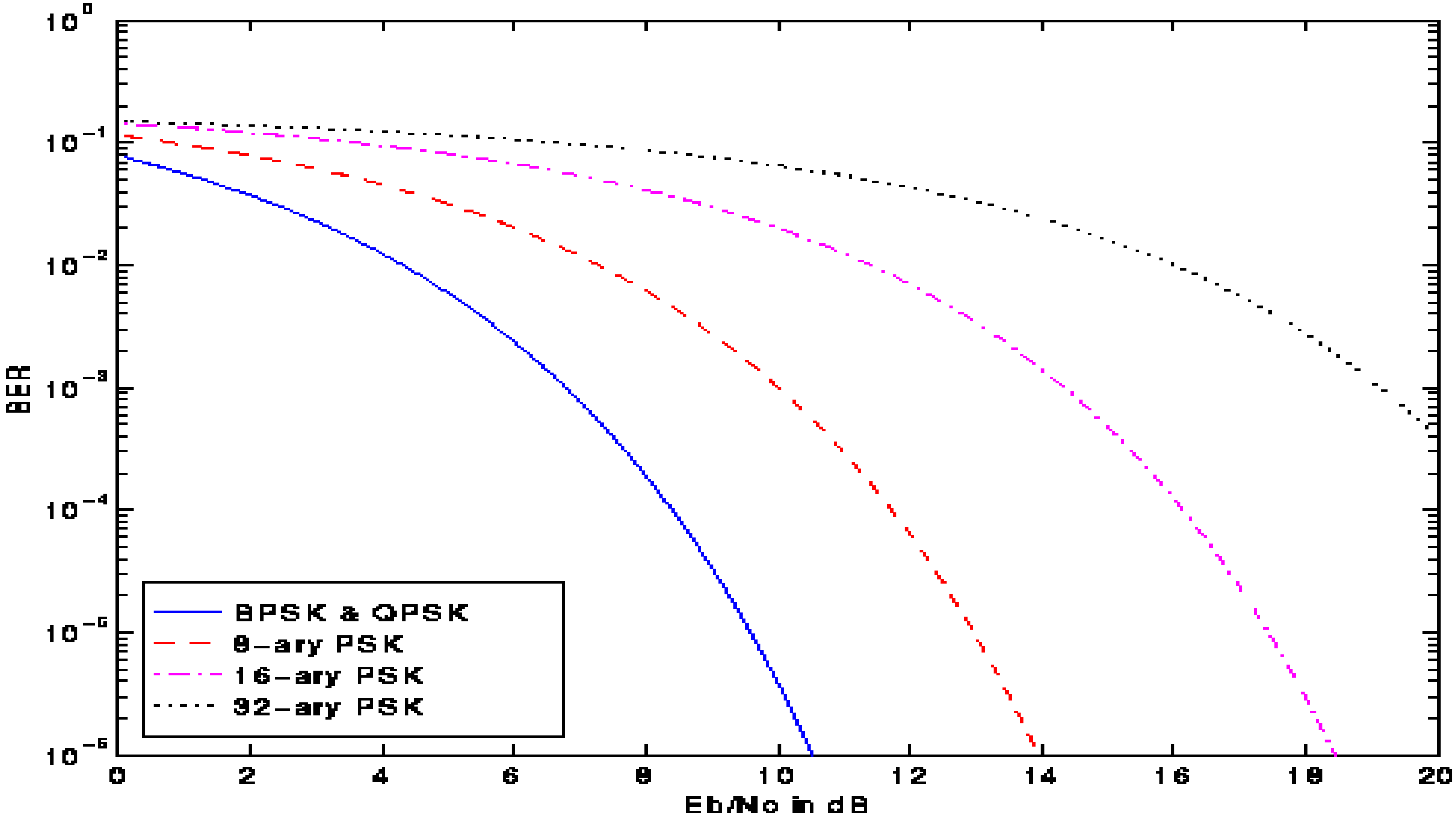
- Eye pattern is an oscilloscope display in which digital data signal from a receiver is repetitively superimposed on itself many times (sampled and applied to the vertical input, while the data rate is used to trigger the horizontal sweep).
- It is so called because the pattern looks like a series of eyes between a pair of rails.
- If the “eye” is not open at the sample point, errors will occur due to signal corruption.

Phase Shift Keying Modulation

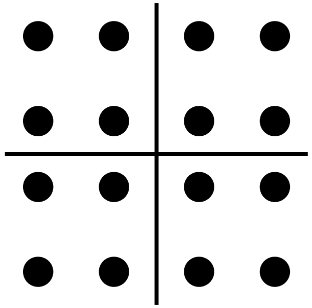
- 8-PSK
  - Three bits are the input to the modulation block
  - Output waveform is one of eight possible waveforms
  - The phases for each waveform are 0, $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$,
- The idea applies for *n*-PSK modulation
- Question: can I have 10192-PSK?

Comparison of PSK Performance

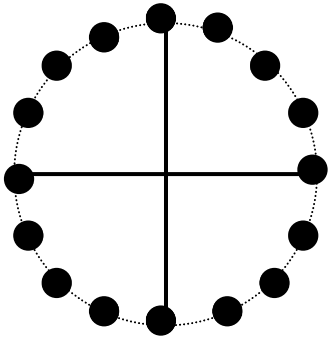
- QPSK and BPSK have identical performance.
- Performance severely degrades as M > 4.



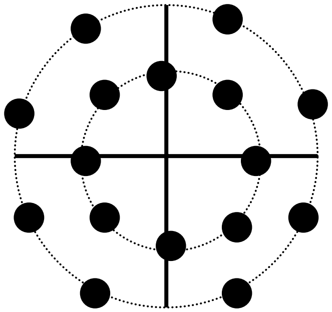
Multi-level (M-ary) Phase and Amplitude Modulation



16 QAM



16 PSK

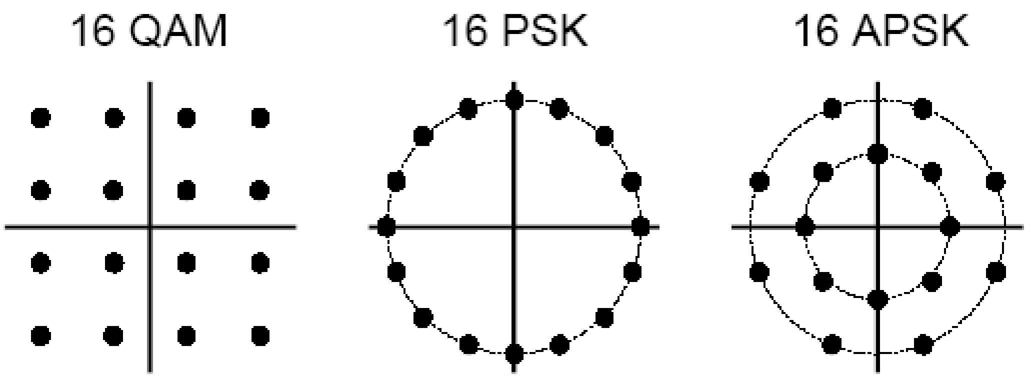


16 APSK

- Amplitude and phase shift keying can be combined to transmit *several bits per symbol*.
  - More bandwidth-efficient, but more susceptible to noise.
- For M=4, 16QAM has the largest distance between points, but requires very linear amplification. 16PSK has less stringent linearity requirements, but has less spacing between constellation points, and is therefore more affected by noise.

Multi-level (M-ary) Phase and Amplitude Modulation

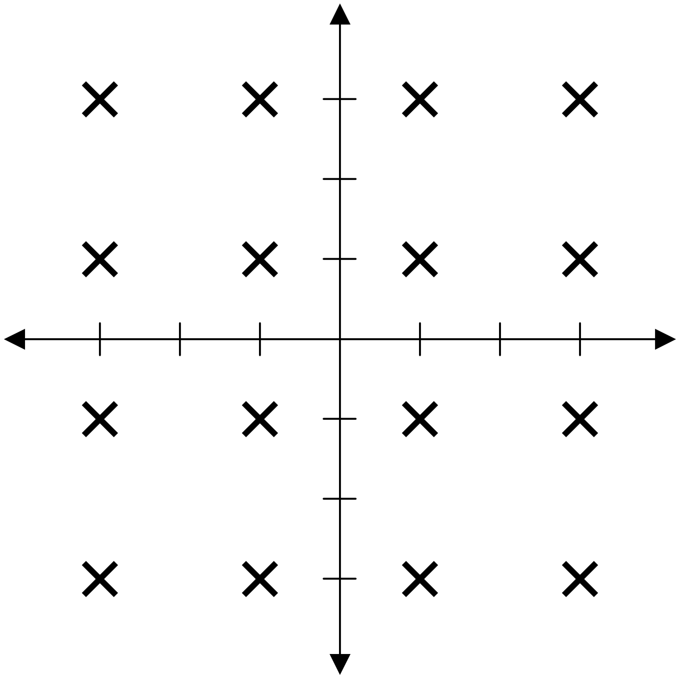
- Amplitude and phase shift keying can be combined to transmit several bits per symbol (in this case M=4). These modulation schemes are often refered to as *linear*, as they require linear amplification.
- 16QAM has the largest distance between points, but requires very linear amplification. 16PSK has less stringent linearity requirements, but has less spacing between constellation points, and is therefore more affected by noise.
- M-ary schemes are more bandwidth efficient, but more susceptible to noise.



Quadrature Amplitude Modulation

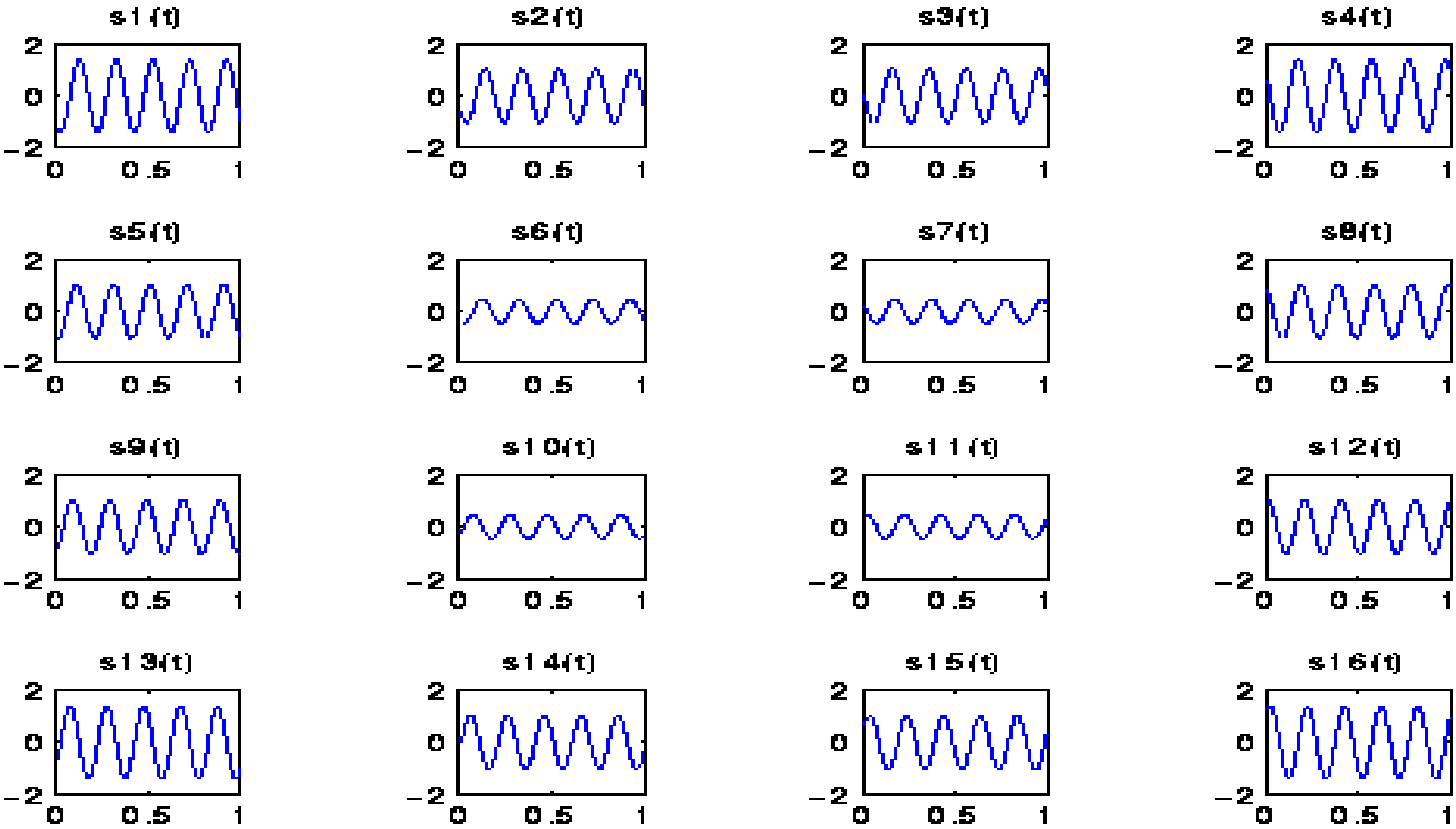
- In ***quadrature amplitude modulation*** **(QAM)** both the amplitude and phase can be varied simultaneously.
- QAM allows the signal vectors to be placed anywhere on the plane of the signal constellation diagram.
- A QAM system is specified by its signal vectors.
  - It is usually defined graphically by its constellation diagram.
- The usual convention is to place the signal points so that they are equally spaced.
- QAM is used by high-speed wireline modems.
  - Allows data rates of 9600 bps and above over ordinary telephone lines.
  - 9,600 bps modem uses 16QAM or 32QAM
  - 14.4 kbps uses 128QAM
  - 28.8 kbps to 56 kbps uses 512QAM

Example: 16 QAM

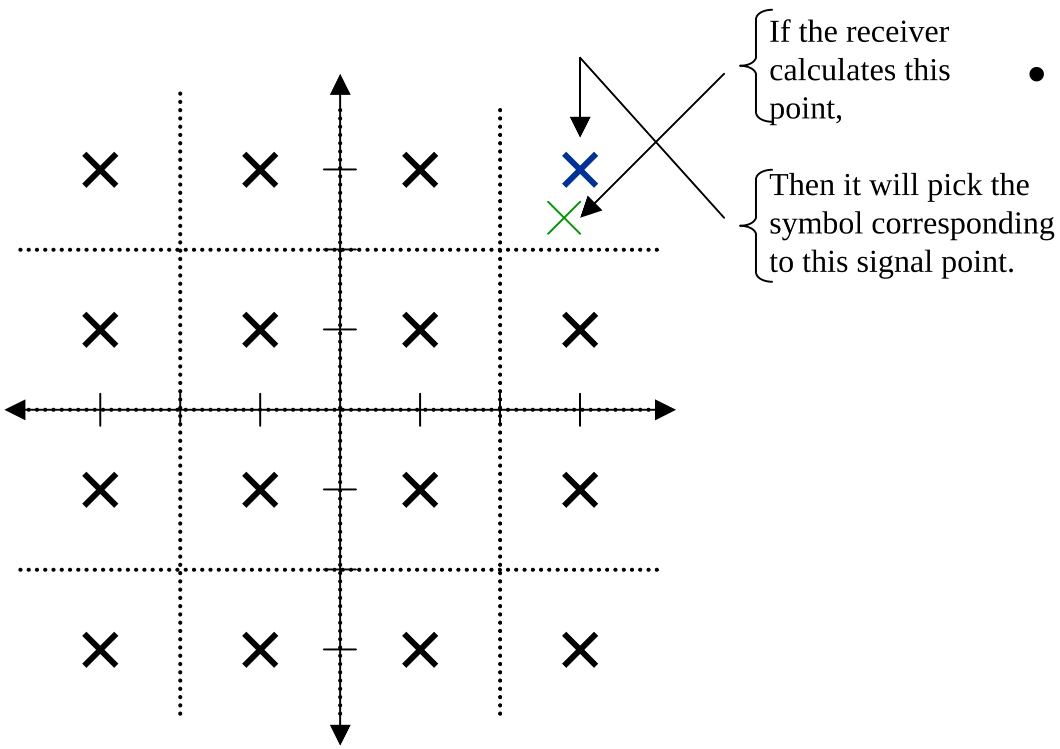


- In 16 QAM there are 16 signal points in the diagram.
- The signal set is most compactly described by the set of signal points or the signal constellation diagram itself.
- This constellation is used by the V.22bis and V.32 9,600 bps modem standards.

16 QAM Signal Waveforms



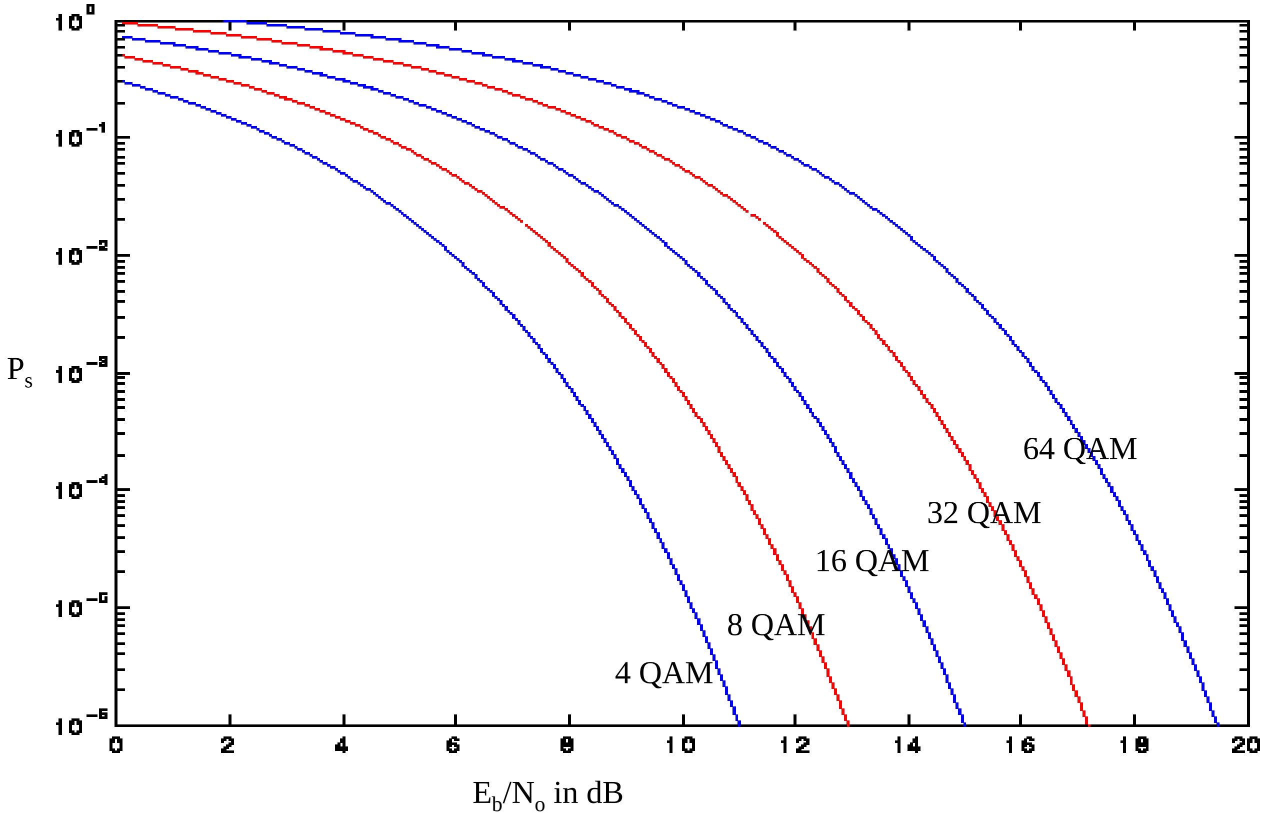
Example Decision Regions



- Recall the case of ***16 QAM***.
- We can partition the signal constellation diagram into ***decision regions***.

Comparison of QAM Performance

- Here the symbol error probability is plotted.
  - Bit error probability is more difficult to obtain and depends on the manner that bits are mapped to symbols.



GMSK

- Gaussian Minimum Shift Keying (GMSK) is a form of continuous-phase FSK in which the phase change is changed between symbols to provide a constant envelope. Consequently it is a popular alternative to QPSK
- The RF bandwidth is controlled by the Gaussian low-pass filter bandwidth. The degree of filtering is expressed by multiplying the filter 3dB bandwidth (B) by the bit period of the transmission (T), i.e. by BT
- GMSK allows efficient class C non-linear amplifiers to be used

Digital Modulation Formats for Operational Systems

Modulation format	Application
MSK, GMSK	GSM, CDPD
BPSK	Deep space telemetry, cable modems
QPSK, $\pi/4$ DQPSK	Satellite, CDMA, NADC, TETRA, PHS, PDC, LMDS, DVB-S, cable (return path), cable modems, TFTS
OQPSK	CDMA, satellite
FSK, GFSK	DECT, paging, RAM mobile data, AMPS, CT2, ERMES, land mobile, public safety
8, 16 VSB	North American digital TV (ATV), broadcast, cable
8PSK	Satellite, aircraft, telemetry pilots for monitoring broadband video systems
16 QAM	Microwave digital radio, modems, DVB-C, DVB-T
32 QAM	Terrestrial microwave, DVB-T
64 QAM	DVB-C, modems, broadband set top boxes, MMDS
256 QAM	Modems, DVB-C (Europe), Digital Video (US)

Bandwidth Efficiency

$$\frac{f_b}{W} = \log_2\left(1 + \frac{E_b f_b}{\eta W}\right)$$

$f_b =$ capacity (bits per second)

$W =$ bandwidth of the modulating baseband signal (Hz)

$E_b =$ energy per bit

$\eta =$ noise power density (watts/Hz)

Thus

$E_b f_b =$ total signal power

$\eta W =$ total noise power

$\frac{f_b}{W} =$ bandwidth use efficiency

= bits per second per Hz

Comparison of Modulation Types

Modulation Format	Bandwidth efficiency C/B	Log2(C/B)	Error-free Eb/No
16 PSK	4	2	18dB
16 QAM	4	2	15dB
8 PSK	3	1.6	14.5dB
4 PSK	2	1	10dB
4 QAM	2	1	10dB
BFSK	1	0	13dB
BPSK	1	0	10.5dB

Spectral Efficiencies - Examples

- GSM Digital Cellular
  - Data Rate = 270kb/s; Bandwidth = 200kHz
  - Bandwidth efficiency = 270/200 = 1.35bits/sec/Hz
- IS North American Digital Cellular
  - Data Rate = 48kb/s; Bandwidth = 30kHz
  - Bandwidth efficiency = 48/30 = 1.6bits/sec/Hz

Modulation Summary

- Phase Shift Keying (PSK) is often used as it provides efficient use of RF spectrum. $\pi/4$ QPSK (Quadrature PSK) reduces the envelope variation of the signal.
- High level M-array schemes (such as 64-QAM) are very bandwidth-efficient but more susceptible to noise and require linear amplification
- Constant envelope schemes (such as GMSK) allow for non-linear power-efficient amplifiers
- Coherent reception provides better performance but requires a more complex receiver