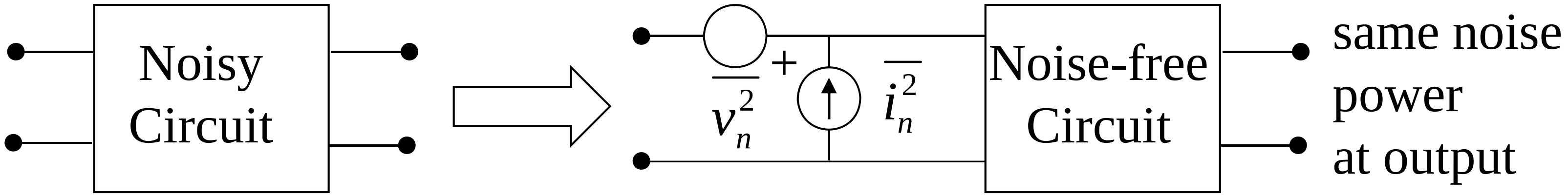
EE4011: RF IC Design

Noise Factor/Figure

Input Referred Noise

A real circuit has many noise sources which all contribute to the noise power at the output of the circuit. In many cases it is helpful to think of the circuit itself as noise free and to model the effect of noise by means of an equivalent input noise voltage and an equivalent input noise current which would generate the same noise output power as the noisy circuit i.e.

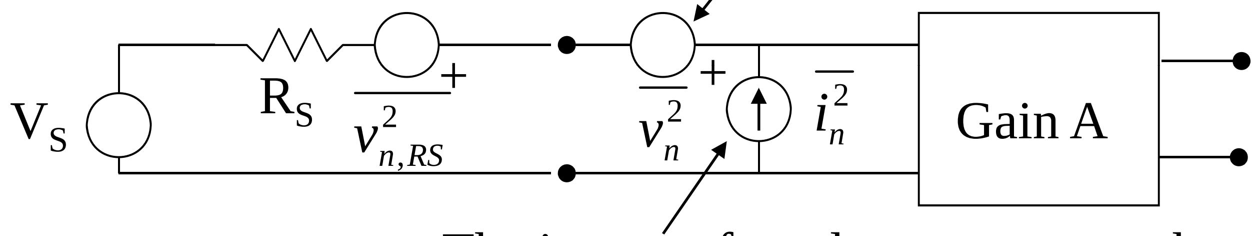


For input referred noise, the input noise voltage and the input noise current are usually correlated and must be treated together in any analysis. Their values depend on the noise sources in the circuit but also on the impedance of the source driving the circuit – if the source impedance changes the equivalent input noise sources will also change. The input referred noise sources are a measure of the noise performance of the circuit. For instance if an amplifier has an input referred noise voltage of 1mV r.m.s. then it will not be able to amplify signals smaller than 1mV r.m.s. as these will be “lost” in the noise.

Note on direction of correlated noise sources

When determining the noise power at the output of a circuit with many independent noise sources, the mean square noise voltage at the output due to each source is calculated independently and then all the mean square voltages are added to give the total mean square noise voltage at the output. Because noise analysis is concerned with voltages squared, the direction of the noise sources doesn't matter for independent sources. However, if some sources are correlated, the directions in which these are drawn effect the final result and are usually drawn so that their effect is additive.

The voltage source has its positive terminal connected to the amplifier side of the circuit so that it gives rise to noise current flowing *into* the amplifier.

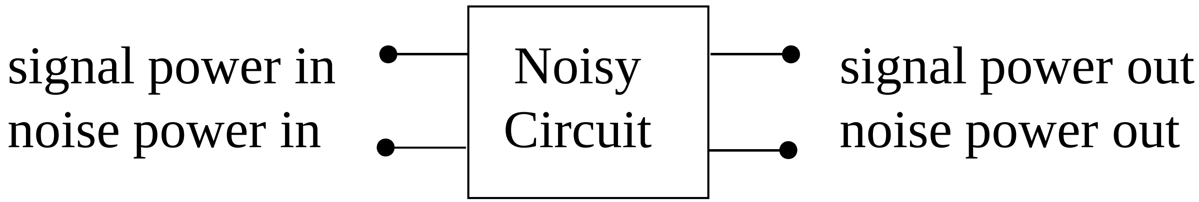


The input referred current source has the direction shown so that noise current flows *into* the amplifier.

The input referred voltage source and the input referred current source are correlated - with the directions shown their effect on output noise is additive.

Noise Factor/Noise Figure

These are two terms which are also used to indicate the noise performance of an individual device or a full circuit. The noise factor/noise figure indicate how much a device or circuit degrades the Signal-to-Noise Ratio (SNR).



SNR_{in} = (signal power in)/(noise power in)

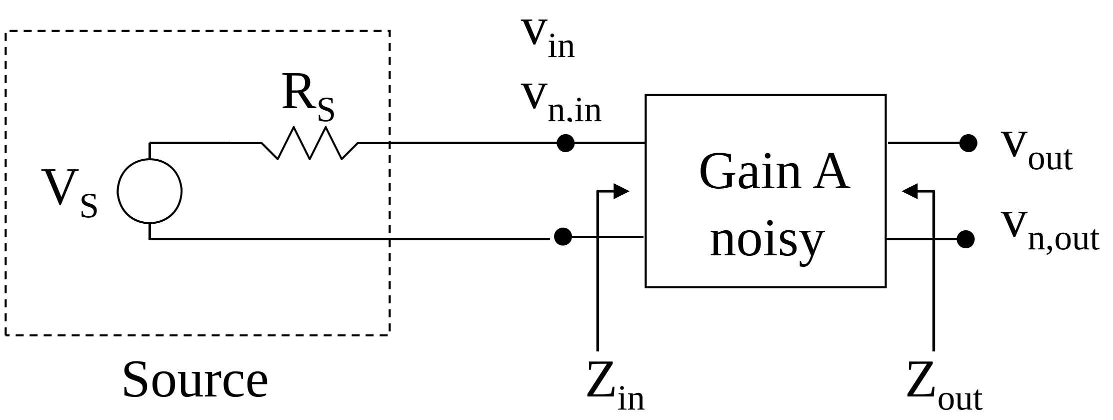
SNR_{out} = (signal power out)/(noise power out)

$$\text{Noise Factor, F} = \frac{SNR_{in}}{SNR_{out}} \quad (\geq 1) \qquad \text{Noise Figure, NF} = 10\log_{10}\left(\frac{SNR_{in}}{SNR_{out}}\right) \quad (\text{in dB}, \geq 0)$$

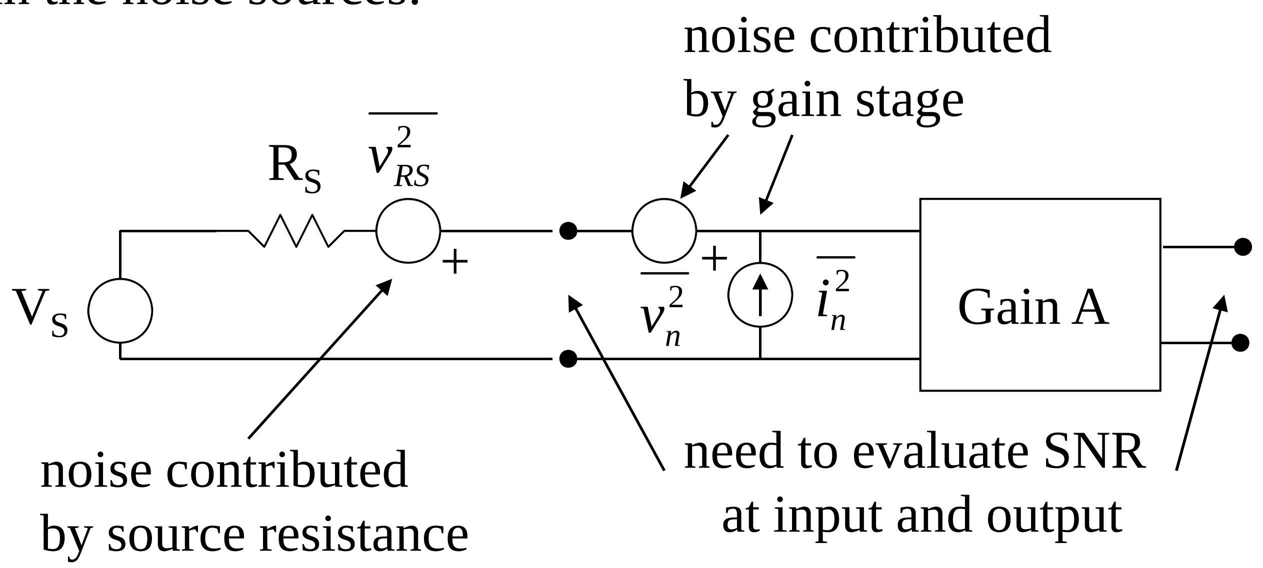
Depending on the text these definitions are often reversed. Either way in one case the ratio is a “number”, while the other way the ratio is expressed in dB.

Note that as the signal moves from input to output the SNR usually gets *worse*.

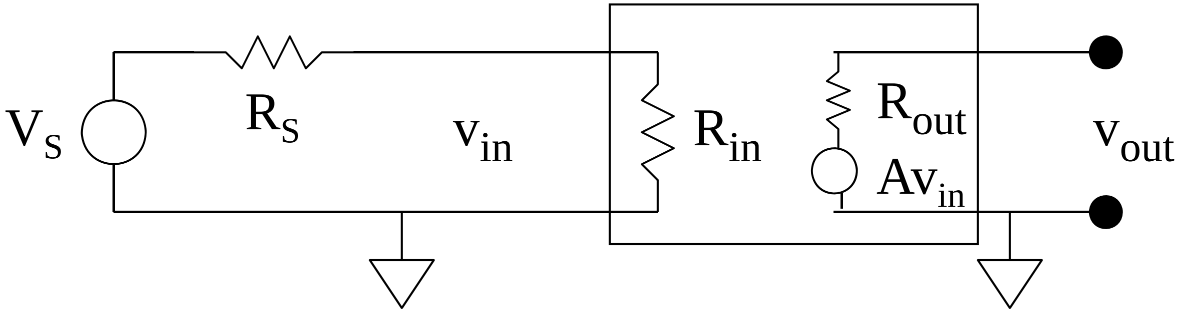
Noise Factor of a Gain Stage



Add in the noise sources:



Noise Factor of Single Gain Stage - 1



The input voltage at the amplifier is:

$$v_{in} = \frac{R_{in}}{R_{in} + R_S} V_S = \alpha V_S \qquad \alpha = \frac{R_{in}}{R_{in} + R_S}$$

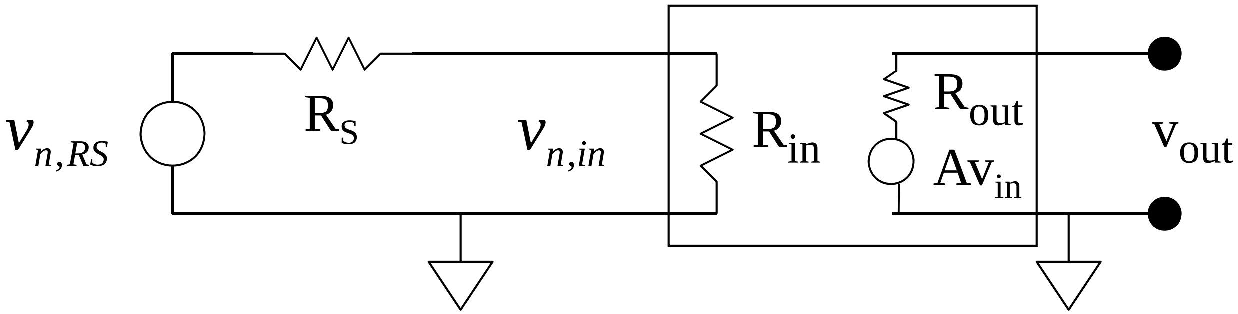
If A is the voltage gain of the amplifier the output voltage is:

$$v_{out} = Av_{in} = \alpha\, AV_S$$

Also:

$$v_{out}^2 = A^2 v_{in}^2 = \alpha^2\, A^2\, V_S^2$$

Noise Factor of Single Gain Stage - 2



The input noise voltage due to the ***source resistance alone*** is:

$$v_{n,in} = \frac{R_{in}}{R_{in} + R_S} v_{n,RS} = \alpha\, v_{n,RS}$$

The signal to noise ratio at the input is:

$$SNR_{in} = \frac{v_{in}^2}{\overline{v_{n,in}^2}} = \frac{\alpha^2 V_S^2}{\alpha^2\, \overline{v_{n,RS}^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} = \frac{V_S^2}{4kTR_S \Delta f}$$

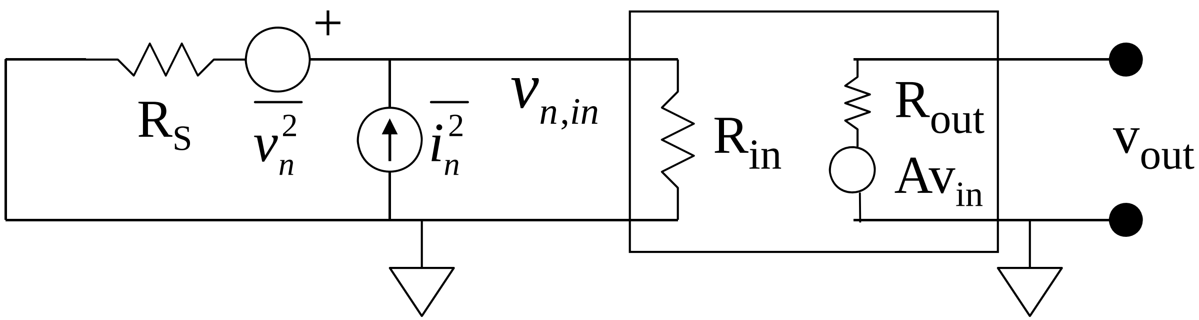
Notes: Power is proportional to voltage squared.

For a noise source the power is proportional to the mean square voltage.

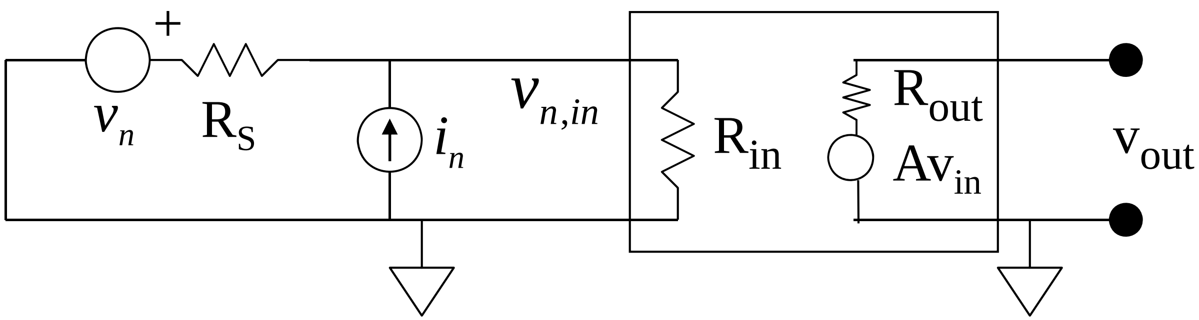
For SNR_{in} only the noise from the source resistance is considered.

Noise Factor of Single Gain Stage - 3

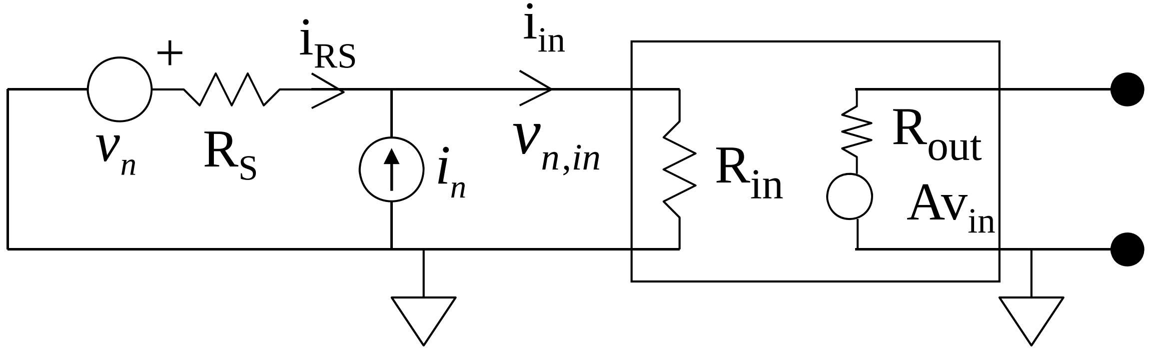
Now consider the noise due to the ***input referred noise sources*** of the amplifier – because they are correlated they must be analysed *together*.



Slightly easier to understand if v_n and R_S are swapped around and the equivalent input sources are denoted as normal voltage and current sources:



Noise Factor of Single Gain Stage - 4



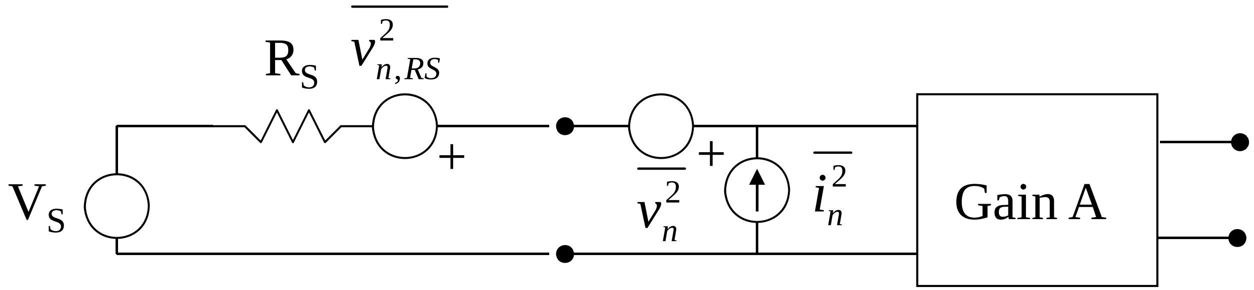
$$i_{in} = \frac{v_{n,in}}{R_{in}} \qquad i_{in} = i_n + i_{RS} \Rightarrow i_{RS} = i_{in} - i_n = \frac{v_{n,in}}{R_{in}} - i_n$$

$$v_{n,in} = v_n - R_S i_{RS} = v_n - R_S\left(\frac{v_{n,in}}{R_{in}} - i_n\right)$$

$$\Rightarrow v_{n,in}\left(1 + \frac{R_S}{R_{in}}\right) = v_n + R_S i_n \Rightarrow v_{n,in}\frac{R_{in} + R_S}{R_{in}} = v_n + R_S i_n$$

$$\Rightarrow v_{n,in} = \frac{R_{in}}{R_{in} + R_S}\left(v_n + R_S i_n\right) = \alpha\left(v_n + R_S i_n\right) \Rightarrow \overline{v_{n,in}^2} = \alpha^2\overline{\left(v_n + R_S i_n\right)^2}$$

Noise Factor of Single Gain Stage - 5



The mean square input noise voltage contributed by the source resistance alone is:

$$\overline{v_{n,in}^2} = \alpha^2 \overline{v_{n,RS}^2}$$

The mean square input noise voltage contributed by the input referred amplifier noise sources is:

$$\overline{v_{n,in}^2} = \alpha^2 \overline{(v_n + R_S i_n)^2}$$

The total input noise is the sum of these mean square values:

$$\left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 \overline{v_{n,RS}^2} + \alpha^2 \overline{(v_n + R_S i_n)^2}$$

So the output noise is:

$$\overline{v_{n,out}^2} = A^2 \left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{(v_n + R_S i_n)^2}$$

The amplifier amplifies noise just the same as the desired signal.

Noise Factor of Single Gain Stage - 6

The signal to noise ratio at the output is:

$$SNR_{out} = \frac{\overline{v_{out}^2}}{\overline{v_{n,out}^2}} = \frac{\alpha^2 A^2 V_S^2}{\alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{(v_n + R_S i_n)^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2} + \overline{(v_n + R_S i_n)^2}}$$

Recall: $SNR_{in} = \dfrac{V_S^2}{\overline{v_{n,RS}^2}}$

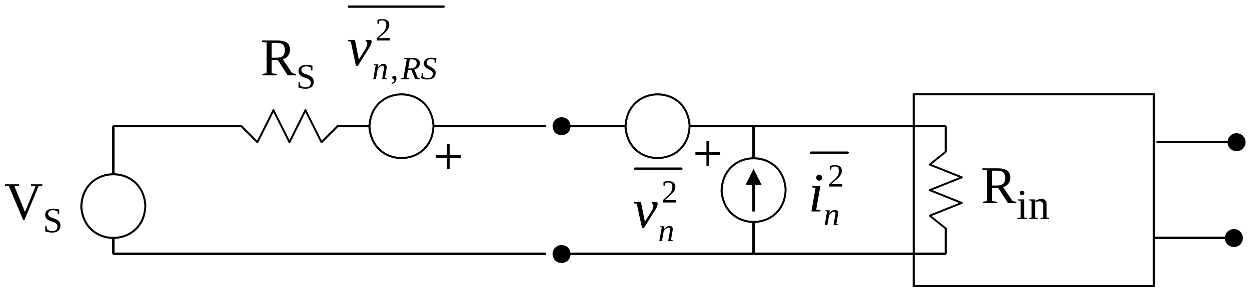
The noise factor of the gain stage is:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} \frac{\overline{v_{n,RS}^2} + \overline{(v_n + R_S i_n)^2}}{V_S^2} = \frac{\overline{v_{n,RS}^2} + \overline{(v_n + R_S i_n)^2}}{\overline{v_{n,RS}^2}}$$

$$F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{\overline{v_{n,RS}^2}} = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S \Delta f} = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S} \text{ for } \Delta f = 1Hz$$

The noise factor depends on the input referred noise sources of the amplifier AND the source resistance.

Some observations



The input noise due to the noise of the source resistance is:

$$v_{n,in} = \alpha\, v_{n,RS} = \frac{R_{in}}{R_{in} + R_S} v_{n,RS}$$

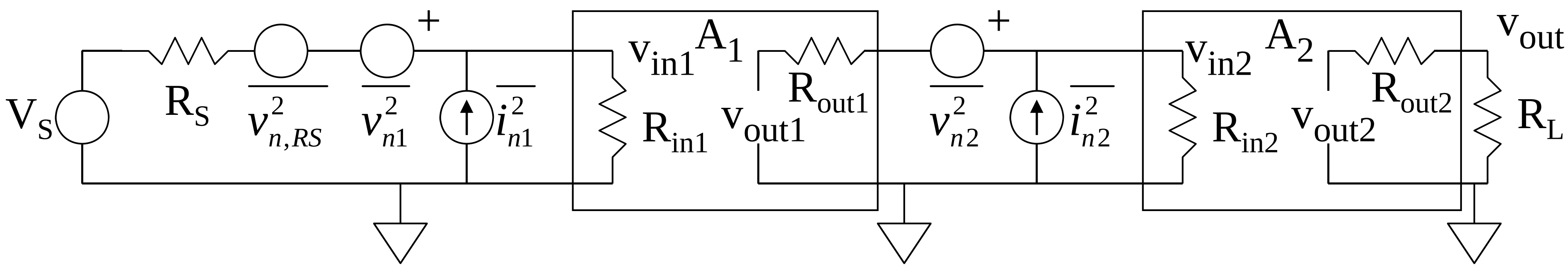
The input noise due to the input referred noise sources is:

$$v_{n,in} = \alpha\left(v_n + R_S i_n\right) = \frac{R_{in}}{R_{in} + R_S}\left(v_n + R_S i_n\right) = \frac{R_{in}}{R_{in} + R_S} v_n + \left(R_S \parallel R_{in}\right) i_n$$

Input noise voltage sources are scaled by the voltage divider network formed by R_{in} and R_S .

Input noise current sources develop a corresponding noise voltage by flowing through the parallel combination of R_{in} and R_S .

Two Cascaded Stages

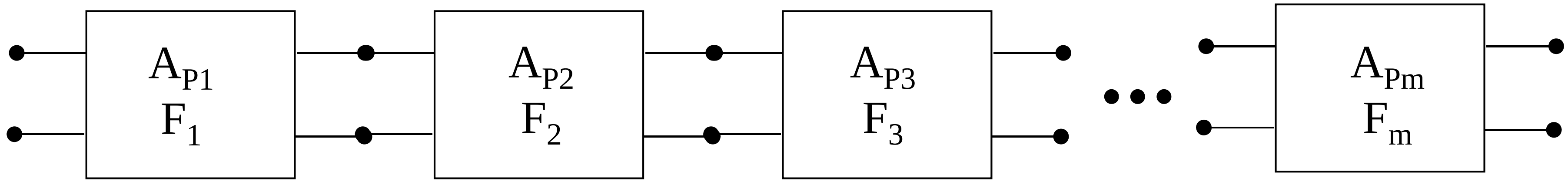


The same approach as before can be used to derive the noise factor of the circuit although, because there are many terms to keep track of, the equations become tedious. The final noise factor is:

$$F = \frac{4kTR_S + \overline{\left(v_{n1} + R_S i_{n1}\right)^2}}{4kTR_S} + \frac{\overline{\left(v_{n2} + R_{out1} i_{n2}\right)^2}}{A_{v1}^2}\left(\frac{R_S + R_{in1}}{R_{in1}}\right)^2 \frac{1}{4kTR_S}$$

Here, A_{v1} is the “unloaded gain” of stage 1.

Noise Factor for Multiple Stages



The Friis equation is used to determine the overall noise factor of a system with multiple stages (m stages):

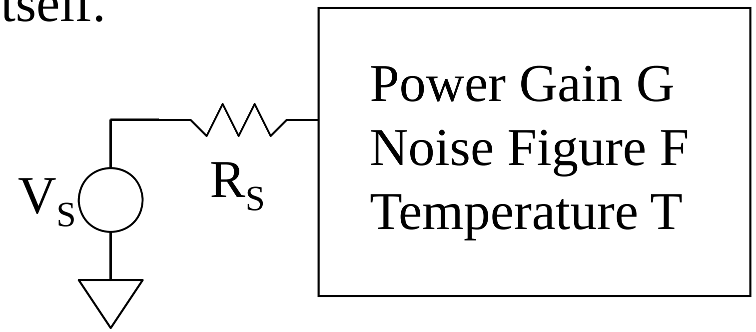
$$F_{tot} = 1 + (F_1 - 1) + \frac{F_2 - 1}{A_{P1}} + \frac{F_3 - 1}{A_{P1} A_{P2}} + \cdots + \frac{F_m - 1}{A_{P1} \cdots A_{P(m-1)}} \quad \text{for } \Delta f = 1Hz$$

Here, the noise factor F of each stage after the first is calculated assuming the "source resistance" driving that stage is the o/p resistance of the previous stage. The gain term for each stage is the "available gain" for that stage.

Note, all our equations for noise in this section have been derived assuming purely resistive source, input and output elements. In practice, these elements can be general impedances with resistive and inductive components. The noise equations used in RF thus include the resistive and reactive elements or are written in terms of impedance or reflection coefficients.

Noise Temperature - 1

The term noise temperature is sometimes used to denote the noise performance of a system. This is defined as the temperature, T_N, at which the source resistance should be held, so that the noise at the output contributed by the source resistance would be the same as the noise at the output contributed by the circuit itself.



$$F = \frac{SNR_i}{SNR_o} = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} \Rightarrow N_o = FGN_i$$

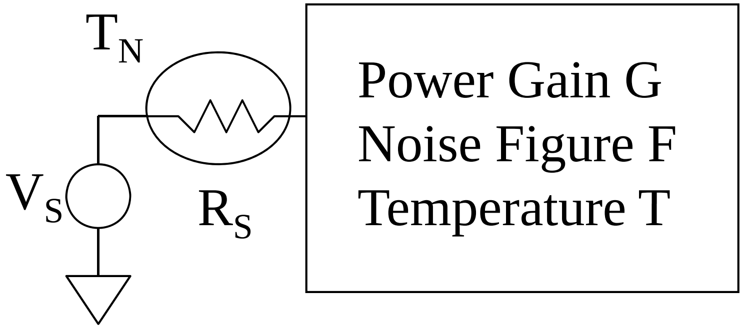
The noise at the output contributed by the source resistance alone is

$$N_{o,RS} = GN_i$$

The noise at the output contributed by the circuit alone is

$$N_{o,circuit} = N_o - N_{o,RS} = FGN_i - GN_i = GN_i(F-1)$$

Noise Temperature - 2



If the source resistor was at temperature T_N, then the noise at the output contributed by the source resistor alone would be:

$$N_{o,RS(T_N)} = GN_{i(T_N)}$$

Equating this to the noise output (at the normal operating temperature) contributed by the circuit alone:

$$GN_{i(T_N)} = GN_i(F-1) \Rightarrow N_{i(T_N)} = N_i(F-1)$$

Putting in the formula for the thermal noise due to the source resistor at T_N and T:

$$4kT_N R_S \Delta f = 4kTR_S \Delta f\,(F-1) \Rightarrow T_N = T(F-1)$$

In systems where F is very close to 1, the noise temperature T_N is a popular alternative way of specifying the noise performance.