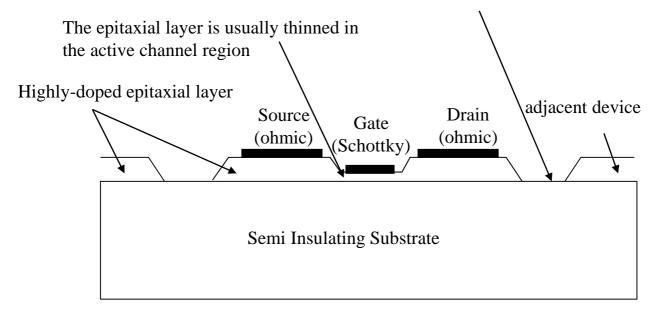
### EE4011 Summer 2013 RFIC Design / Dr Kevin McCarthy

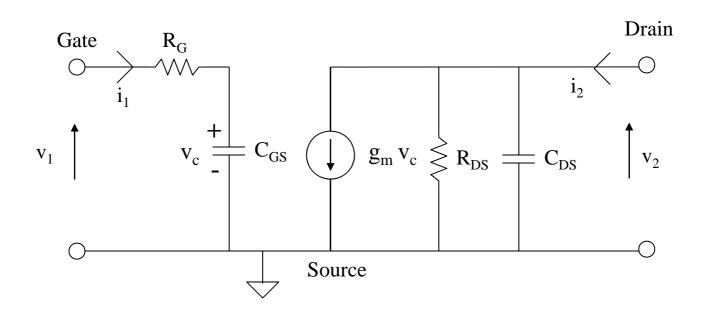
### Q1 (a) MESA-isolated MESFET – 4 marks

By removing the highly-doped epitaxial layer between the devices, the MESA-etch process provides isolation between the different devices.



### Q1 (b) Small-signal model and y-parameters - 8 marks

Simplified MESFET small-signal circuit, ignoring  $C_{GD}$ 



#### Q1 (b) Small-signal model and y-parameters

Definition of y-parameters

$$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$$
  $y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$   $y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$   $y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0}$ 

Analysing the small-signal circuit when  $v_2=0$ 

$$i_{1} = \frac{v_{1}}{R_{G} + 1/j\omega C_{GS}} = v_{1} \frac{j\omega C_{GS}}{1 + j\omega R_{G}C_{GS}}$$

$$\Rightarrow y_{11} = \frac{i_{1}}{v_{1}}\Big|_{v_{2}=0} = \frac{j\omega C_{GS}}{1 + j\omega R_{G}C_{GS}}$$

$$v_{c} = v_{1} \frac{1/j\omega C_{GS}}{R_{G} + 1/j\omega C_{GS}} = v_{1} \frac{1}{1 + j\omega R_{G}C_{GS}}$$

$$i_{2} = g_{m}v_{c} = v_{1} \frac{g_{m}}{1 + j\omega R_{G}C_{GS}}$$
Note: there is no current in R<sub>DS</sub> or C<sub>DS</sub> because v<sub>2</sub>=0
$$\Rightarrow y_{21} = \frac{i_{2}}{v_{1}}\Big|_{v_{2}=0} = \frac{g_{m}}{1 + j\omega R_{G}C_{GS}}$$

Analysing the small-signal circuit when  $v_1=0$ 

$$\begin{aligned} v_1 &= 0 \Rightarrow i_1 = 0 \Rightarrow y_{12} = \frac{i_1}{v_2} \bigg|_{v_1 = 0} = 0 \\ i_1 &= 0 \Rightarrow v_c = 0 \Rightarrow i_2 = v_2 \bigg( \frac{1}{R_{DS}} + j\omega C_{DS} \bigg) \end{aligned} \quad \begin{array}{l} \text{Note: no current in the transconductance element when } v_c = 0 \\ \Rightarrow y_{22} &= \frac{i_2}{v_2} \bigg|_{v_1 = 0} = \frac{1}{R_{DS}} + j\omega C_{DS} \end{aligned}$$

Q1 (c) Determining the small-signal values – 5 marks

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \Rightarrow \frac{1}{y_{11}} = \frac{1 + j\omega R_G C_{GS}}{j\omega C_{GS}} = -j\frac{1}{\omega C_{GS}} + R_G$$

$$\Rightarrow R_G = \mathbf{Real} \left(\frac{1}{y_{11}}\right) \quad C_{GS} = -\frac{1}{\omega \mathbf{Imag}} \left(\frac{1}{y_{11}}\right)$$

$$y_{12} = 0$$

$$y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}} \Rightarrow \frac{1}{y_{21}} = \frac{1 + j\omega R_G C_{GS}}{g_m} = \frac{1}{g_m} + j\frac{\omega R_G C_{GS}}{g_m}$$

$$\Rightarrow g_m = \frac{1}{\mathbf{Real}} \left(\frac{1}{y_{21}}\right)$$

$$y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

$$\Rightarrow R_{DS} = \frac{1}{\mathbf{Real}} (y_{22}) \quad C_{DS} = \frac{\mathbf{Imag}(y_{22})}{\omega}$$

$$\varpi = 2\pi f$$

Doing the calculations gives

$$R_G = 7.5\Omega$$
,  $C_{GS} = 2.5pF$ ,  $g_m = 0.1S$ ,  $R_{DS} = 100\Omega$ ,  $C_{DS} = 0.5pF$ 

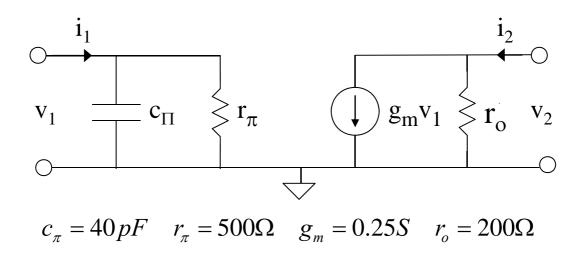
Q1 (d) Calculating the gate resistance – 3 marks

The effective gate resistance of a long single stripe contacted at one side is

$$R_{G,1} = \frac{1}{3} \frac{W}{L} R_{SQ}$$

The effective gate resistance of N parallel stripes each with the original gate length, L, and with the same total width W as the original device and with the gate stripes contacted on both sides is:

$$R_{G,N} = \frac{1}{12N^2} \frac{W}{L} R_{SQ} = \frac{1}{4N^2} R_{G,1} = \frac{1}{4 \times 5^2} 10 = 0.1\Omega$$



Q2 (a) Expressions for  $i_1$  and  $i_2$  in terms of  $v_1$  and  $v_2 - 2$  marks

$$i_1 = v_1 \left( \frac{1}{r_{\pi}} + j\omega c_{\pi} \right) \quad i_2 = g_m v_1 + \frac{v_2}{r_o}$$

Q2 (b) z-parameters – 8 marks

$$z_{11} = \frac{v_1}{i_1}\bigg|_{i_2=0}$$
  $z_{21} = \frac{v_2}{i_1}\bigg|_{i_2=0}$   $z_{12} = \frac{v_1}{i_2}\bigg|_{i_1=0}$   $z_{22} = \frac{v_2}{i_2}\bigg|_{i_1=0}$ 

if 
$$i_1 = 0$$
 then  $v_1 = 0$  and  $i_2 = \frac{v_2}{r_o}$ 

$$z_{12} = \frac{v_1}{i_2} \bigg|_{i_1 = 0} = 0$$

$$z_{22} = \frac{v_2}{i_2}\Big|_{i_1=0} = \frac{v_2}{1} \frac{r_o}{v_2} = r_o$$

$$if \ i_{2} = 0 \ then \ g_{m}v_{1} + \frac{v_{2}}{r_{o}} = 0 \Rightarrow v_{2} = -g_{m}r_{o}v_{1}$$

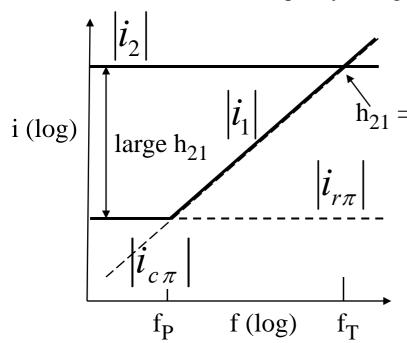
$$z_{11} = \frac{v_{1}}{i_{1}}\Big|_{i_{2}=0} = \frac{v_{1}}{v_{1}\left(\frac{1}{r_{\pi}} + j\omega c_{\pi}\right)} = \frac{r_{\pi}}{1 + j\omega c_{\pi}r_{\pi}}$$

$$z_{21} = \frac{v_{2}}{i_{1}}\Big|_{i_{2}=0} = \frac{-g_{m}r_{o}v_{1}}{v_{1}\left(\frac{1}{r_{\pi}} + j\omega c_{\pi}\right)} = -\frac{g_{m}r_{o}r_{\pi}}{1 + j\omega c_{\pi}r_{\pi}}$$

Putting in the small-signal element values and doing the calculations gives:

$$\omega = 2\pi f$$
  $f = 0.5GHz$ 
 $z_{11} = 7.96 \angle -89^{\circ} \Omega$ 
 $z_{12} = 0 \Omega$ 
 $z_{21} = 397.8 \angle 91^{\circ} \Omega$ 
 $z_{22} = 200 \Omega$ 

Q2(c) Sketch of currents vs. frequency during  $h_{21}$  measurement – 4 marks



Note: for  $h_{21}$  measurement,  $v_2$  is set to 0 and  $i_1$  and  $i_2$  are measured vs. frequency for an ac signal applied to port 1.

 $Q2(d)(i) h_{21}$  at low frequencies – 2 marks

$$i_{1} = v_{1} \left( \frac{1}{r_{\pi}} + j\omega c_{\pi} \right) \quad i_{2} = g_{m}v_{1} + \frac{v_{2}}{r_{o}}$$

$$h_{21} = \frac{i_{2}}{i_{1}} \Big|_{v_{2}=0} = \frac{g_{m}v_{1}}{v_{1} \left( \frac{1}{r_{\pi}} + j\omega c_{\pi} \right)} = \frac{g_{m}r_{\pi}}{1 + j\omega c_{\pi}r_{\pi}}$$

At low frequencies  $h_{21} \approx g_m r_{\pi} = 125$ 

Q2(d)(ii) The input-circuit pole frequency – 2 marks

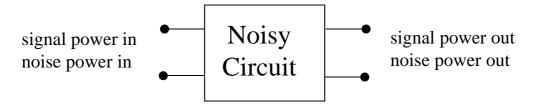
$$\begin{aligned} i_{r\pi} &= \frac{v_1}{r_{\pi}} \quad i_{c\pi} = v_1 j \omega c_{\pi} \\ \left| i_{r\pi} \right| &= \left| i_{c\pi} \right| \Rightarrow \frac{1}{r_{\pi}} = 2\pi f c_{\pi} \Rightarrow f = \frac{1}{2\pi r_{\pi} c_{\pi}} = 7.96 MHz \end{aligned}$$

Q2(d)(iii) The cut-off frequency – 2 marks

For high frequencies 
$$h_{21} \approx \frac{g_m r_\pi}{j\omega c_\pi r_\pi} = \frac{g_m}{j\omega c_\pi} \Rightarrow |h_{21}| \approx \frac{g_m}{\omega c_\pi}$$

$$|h_{21}| = 1 \Rightarrow f = f_T = \frac{g_m}{2\pi c_\pi} = 0.99GHz$$

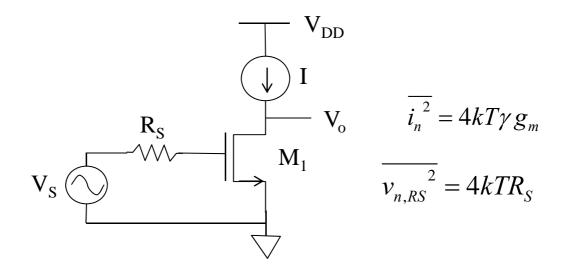
### Q3(a) Noise Factor – 2 marks



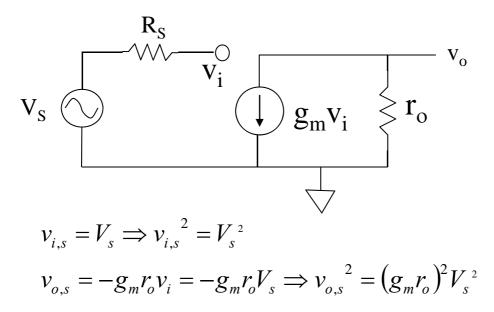
SNR<sub>in</sub> = (signal power in)/(noise power in) SNR<sub>out</sub> = (signal power out)/(noise power out)

Noise Factor, 
$$F = \frac{SNR_{in}}{SNR_{out}}$$
 (usually  $\geq 1$ )

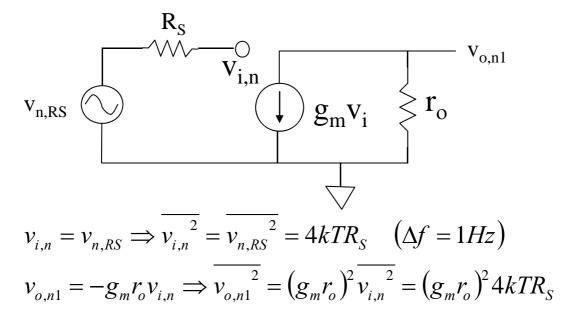
### Q3(b) Noise Factor of common-source amplifier – 14 marks



First analyse the circuit with just the signal source and no noise sources



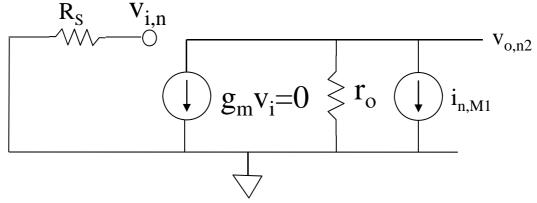
Next analyse the circuit with just the noise from the source resistance



The signal to noise ratio at the input considers only the noise from the source resistance:

$$SNR_i = \frac{{v_{i,s}}^2}{{v_{i,n}}^2} = \frac{{V_s}^2}{4kTR_S}$$

Now analyse the circuit with the noise from the transistor on its own



 $v_{i,n} = 0$  (ignoring the noise from the source resistance)

$$v_{o,n2} = -r_o i_{n,M1} \Rightarrow \overline{v_{o,n2}^2} = r_o^2 \overline{i_{n,M1}^2} = r_o^2 4kT\gamma g_m \quad (\Delta f = 1Hz)$$

The total noise power at the output can now be determined by adding the contributions at the output from the source resistance and from the transistor

$$\overline{v_{o,n,Total}^{2}} = \overline{v_{o,n1}^{2}} + \overline{v_{o,n2}^{2}} = (g_{m}r_{o})^{2}kTR_{S} + r_{o}^{2}4kT\gamma g_{m}$$

The signal to noise ratio at the output is:

$$SNR_o = \frac{v_{o,s}^2}{v_{o,n,Total}^2} = \frac{(g_m r_o)^2 V_s^2}{(g_m r_o)^2 4kTR_S + r_o^2 4kT\gamma g_m}$$

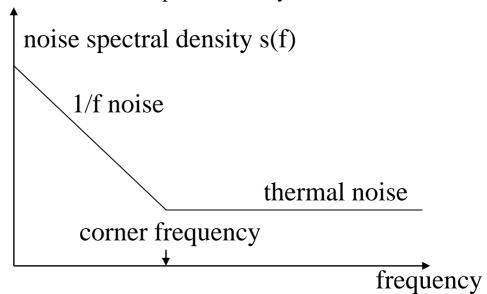
The noise factor is then:

$$F = \frac{SNR_i}{SNR_o} = \frac{V_s^2}{4kTR_S} \frac{(g_m r_o)^2 4kTR_S + r_o^2 4kT\gamma g_m}{(g_m r_o)^2 V_s^2}$$
$$= 1 + \frac{\gamma}{g_m R_S}$$

Q3(c) Noise Figure Calculation – 2 marks

$$R_S = 50\Omega$$
  $\gamma = \frac{2}{3}$   $g_m = 0.01S$   $F = 1 + \frac{\gamma}{g_m R_S} = 2.33 = 3.7 dB$ 

Q3(d) Typical MOSFET noise spectral density – 2 marks



Q4(a) Transistor Stability Considerations – 2 marks

$$s_{11} = 0.65 \angle -160^{\circ} \quad s_{12} = 0.1 \angle 30^{\circ} \quad s_{21} = 4.0 \angle 90^{\circ} \quad s_{22} = 0.4 \angle -60^{\circ}$$

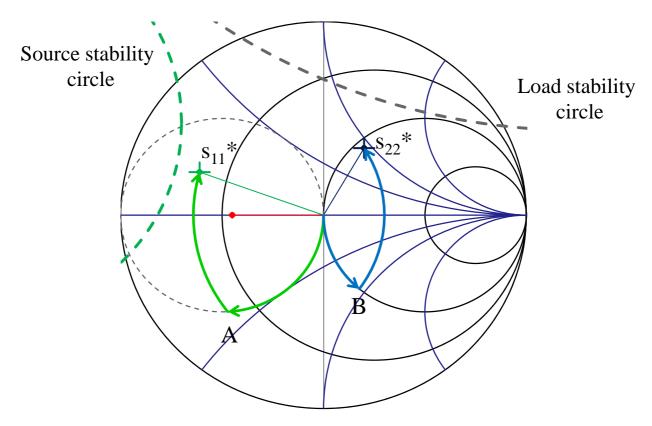
$$\Delta = s_{11} s_{22} - s_{12} s_{21} = 0.18 \angle -90^{\circ} \quad K = \frac{1 - \left| s_{11} \right|^2 - \left| s_{22} \right|^2 + \left| \Delta \right|^2}{2 \left| s_{12} s_{21} \right|} = 0.56$$

K <1 so the device is only *conditionally stable* and the stability will depend on the source and load reflection coefficients.

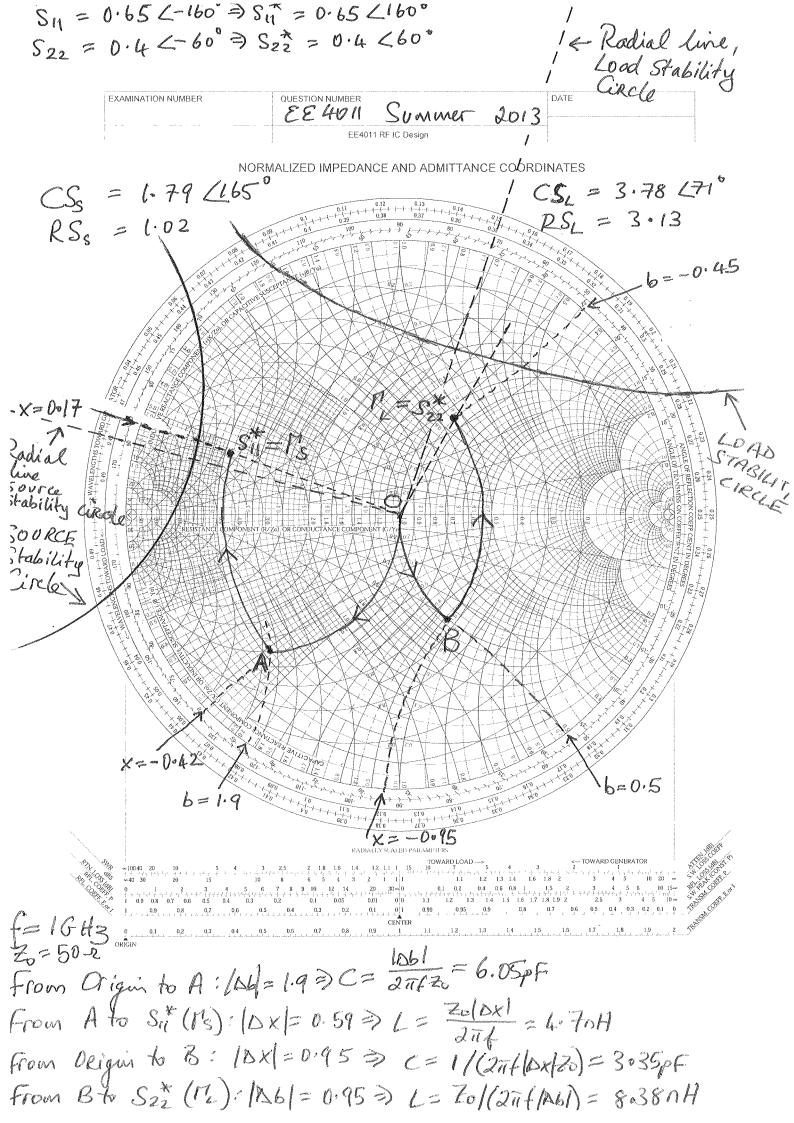
Q4(b) Calculations for Source and Load Stability Circles and display on Smith Chart – 4 marks

$$CS_{S} = \frac{s_{11}^{*} - \Delta^{*} s_{22}}{|s_{11}|^{2} - |\Delta|^{2}} = 1.79 \angle 165^{\circ} \quad RS_{S} = \frac{|s_{12} s_{21}|}{|s_{11}|^{2} - |\Delta|^{2}} = 1.02$$

$$CS_{L} = \frac{s_{22}^{*} - \Delta^{*} s_{11}}{|s_{22}|^{2} - |\Delta|^{2}} = 3.78 \angle 71^{\circ} \quad RS_{L} = \frac{|s_{12} s_{21}|}{|s_{22}|^{2} - |\Delta|^{2}} = 3.13$$



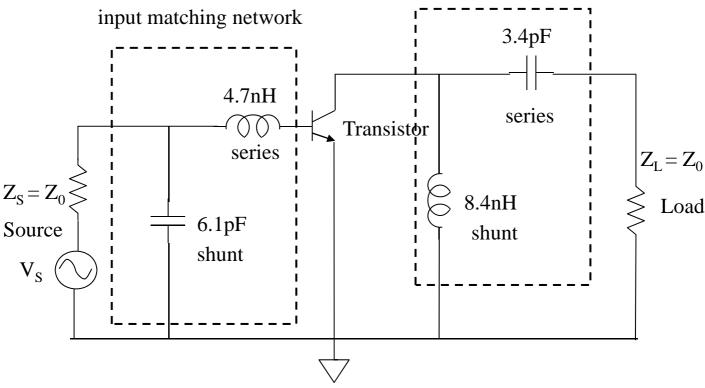
In this case the stability circles don't effect the placement of the source and load reflection coefficients at  $s_{11}^*$  and  $s_{22}^*$  for maximum unilateral transducer gain  $_{10}$  because  $s_{11}^*$  and  $s_{22}^*$  are on the stable sides of the stability circles.



### Q4(c) Input and Output Matching Networks – 10 marks

Using the Smith Chart to design input and output matching networks for maximum unilateral transducer gain.

output matching network



(The matching calculations below are rounded to one decimal place) Input Matching Element Values – need to get from origin to  $s_{11}^{*}$ 

Moving from  $Z_0$  ( $\Gamma$ =0) to point A: Clockwise on conductance circle – shunt capacitor

susceptance at 
$$Z_0$$
:  $b = 0$  susceptance at A:  $b = 1.9$   $C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|1.9|}{2\pi \times 1 \times 10^9 \times 50} = 6.1 pF$ 

Moving from A to  $\Gamma_S(s_{11}^*)$ : Clockwise on resistance circle – series inductor

reactance at A: 
$$x = -0.42$$
 reactance at  $\Gamma_S$ :  $x = 0.17$   $L = \frac{Z_0 |\Delta x|}{2\pi f} = \frac{50 \times |0.59|}{2\pi \times 1 \times 10^9} = 4.7 nH$ 

Output Matching Element Values – need to get from origin to s<sub>22</sub>\*

Moving from  $Z_0$  ( $\Gamma$ =0) to point B:

Anti-clockwise on resistance circle – series capacitor

reactance at  $Z_0$ : x = 0 reactance at B: x = -0.95

$$C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 1 \times 10^9 \times |-0.95| \times 50} = 3.4 \, pF$$

Moving from B to  $\Gamma_L$ :

Anti-clockwise on conductance circle – shunt inductor

susceptance at B: b = 0.5susceptance at  $\Gamma_L$ : b = -0.45

$$L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 1 \times 10^9 \times |0.95|} = 8.4nH$$

Q4(d)(i) Maximum Unilateral Transducer Gain – 2 marks

$$G_{TU,\text{max}} = \frac{1}{1 - |s_{11}|^2} |s_{21}|^2 \frac{1}{1 - |s_{22}|^2} = 33 = 15.2dB$$

Q4(d)(ii) Unilateral Figure of Merit and comment – 2 marks

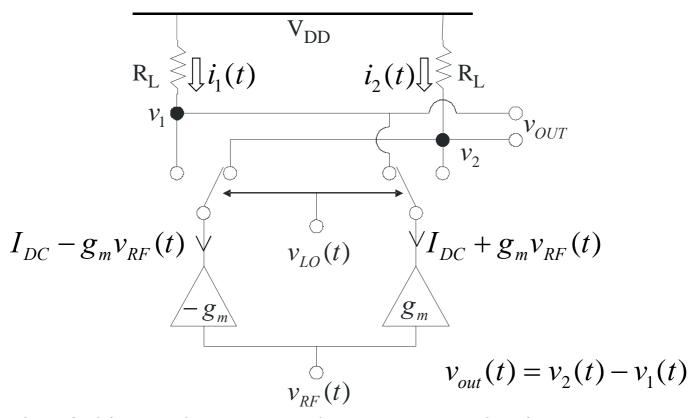
$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{(1 - |s_{11}|^2)(1 - |s_{22}|^2)} = 0.214$$

$$\frac{1}{(1+M)^2} < \frac{G_T}{G_{TU,\text{max}}} < \frac{1}{(1-M)^2}$$

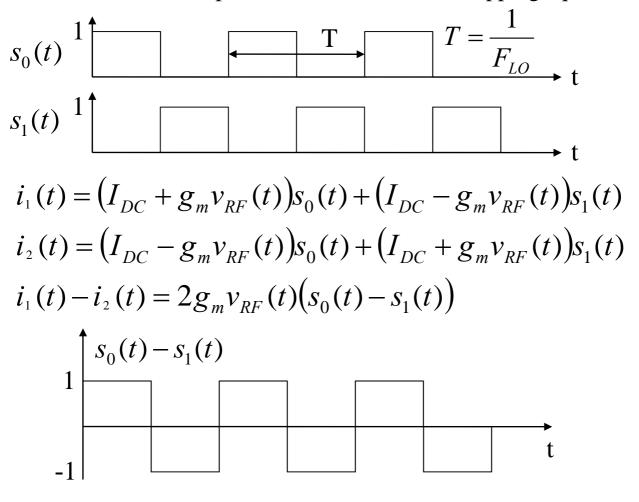
$$\frac{1}{(1+M)^2} = 0.68 = -1.7dB \quad \frac{1}{(1-M)^2} = 1.67 = 2.1dB$$

These calculations indicate that the error in calculating gain with the unilateral approximation is as high as 2dB and is not acceptable.

# Question 5(a) Double-Balanced Mixer Derivation – 12 marks



The LO drive can be represented as two non-overlapping square waves



$$\begin{aligned} v_{out}(t) &= v_{2}(t) - v_{1}(t) = \left(V_{DD} - i_{2}(t)R_{L}\right) - \left(V_{DD} - i_{1}(t)R_{L}\right) \\ &= R_{L}\left(i_{1}(t) - i_{2}(t)\right) \\ &= 2g_{m}R_{L}v_{RF}(t)\left(s_{0}(t) - s_{1}(t)\right) \\ &= 2g_{m}R_{L}v_{RF}(t)\frac{4}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \\ &= \frac{8g_{m}R_{L}}{\pi}v_{RF}(t)\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \end{aligned}$$

If the RF waveform is of the form:

$$V_{RF}(t)\cos(\varpi_{RF}t)$$

Then, use of cos(A)sin(B) expressions leads to

$$v_{out}(t) = \frac{8g_m R_L v_{RF}(t)}{\pi} \left[ \sin(\varpi_{LO} t) + \frac{1}{3} \sin(3\varpi_{LO} t) + \cdots \right]$$

$$= \frac{8g_m R_L V_{RF} \cos(\varpi_{RF} t)}{\pi} \left[ \sin(\varpi_{LO} t) + \frac{1}{3} \sin(3\varpi_{LO} t) + \cdots \right]$$

$$= \frac{4g_m R_L V_{RF}}{\pi} \left[ \frac{\sin((\varpi_{RF} + \varpi_{LO})t) - \sin((\varpi_{RF} - \varpi_{LO})t)}{\sin((\varpi_{RF} - \varpi_{LO})t) - \frac{1}{3} \sin((\varpi_{RF} - 3\varpi_{LO})t) + \cdots \right]$$

In this expression for the output voltage, there are no terms at DC or at the LO or RF frequencies so these have all been eliminated. The largest two terms are the LO and RF sum and difference frequencies as desired and then the higher order terms are the sum and difference between the RF signal and the odd harmonics of the LO.

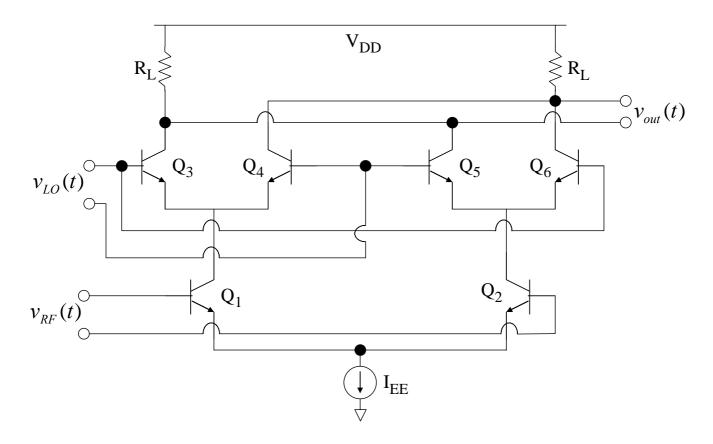
## Question 5(b) – Voltage Conversion Gain – 4 marks

$$V_{T} = \frac{kT}{q} = 25.8 mV$$
 $g_{m} = \frac{I_{C}}{V_{T}} = 9.675 mS$ 
 $A_{CF} = \frac{4g_{m}R_{L}}{\pi} = 24.65$ 

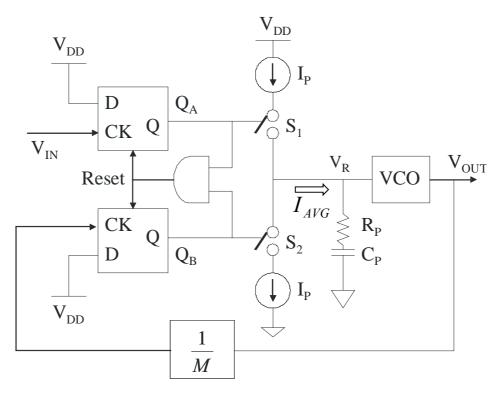
Note: If the total tail current is 0.5 mA under small-signal conditions there will be 0.25 mA flowing through each transistor in the diff pairs so  $I_C$ =0.25 mA for the  $g_m$  calculation.

Question 5(c) – Gilbert Cell 4 marks

### A Gilbert Cell Double Balanced Mixer



## Question 6



## Q6(a) - 10 marks

If the  $V_{IN}$  and  $V_{OUT}$  waveforms have the same frequency but a phase offset  $\Delta \phi$  then the average current supplied to the loop filter is

$$I_{AVG} = \frac{\Delta \varphi}{2\pi} I_P$$

The impedance of the loop filter to ground is given by:

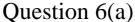
$$Z = R_P + \frac{1}{C_P s}$$

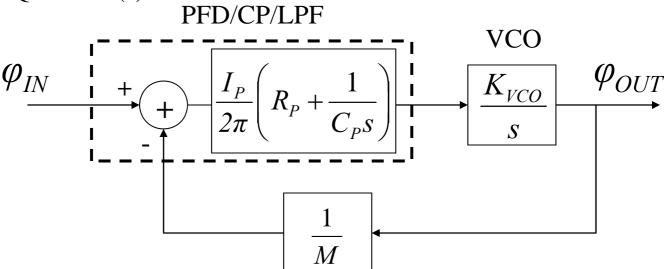
The transfer function of the PFD/CP/LPF can then be approximated by:

$$PFD \_CP \_LPF(s) = \frac{I_P}{2\pi} \left( R_P + \frac{1}{C_P s} \right)$$

The transfer function of the VCO is given by:

$$VCO(s) = \frac{K_{VCO}}{s}$$





Open Loop Response

$$H(s) = \frac{I_P K_{VCO}}{2\pi s} \left( R_P + \frac{1}{C_P s} \right)$$

**Closed Loop Response** 

$$\begin{split} \varphi_{OUT}(s) &= \left(\varphi_{IN}(s) - \frac{\varphi_{OUT}(s)}{M}\right) H(s) \\ \Rightarrow & H_{Closed}(s) = \frac{H(s)}{1 + \frac{H(s)}{M}} = \frac{\frac{I_P K_{VCO}}{2\pi s} \left(R_P + \frac{1}{C_P s}\right)}{1 + \frac{I_P K_{VCO}}{2\pi M s} \left(R_P + \frac{1}{C_P s}\right)} \\ &= \frac{\frac{I_P K_{VCO}}{2\pi C_P} \left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} \end{split}$$

Question 6(b)

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P} (R_P C_P s + 1)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} \equiv \frac{\frac{I_P K_{VCO}}{2\pi C_P} (R_P C_P s + 1)}{s^2 + 2\varsigma \omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \varsigma = \frac{R_P}{2} \sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\varsigma \omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

Using

$$I_p$$
=3mA,  $C_p$ =50pF,  $R_p$ =20kΩ,  $K_{VCO}$ =200MHz/V, M=250 Note:  $K_{VCO}$ =200MHz/V = 1.26 x 10<sup>9</sup> rad/s/V

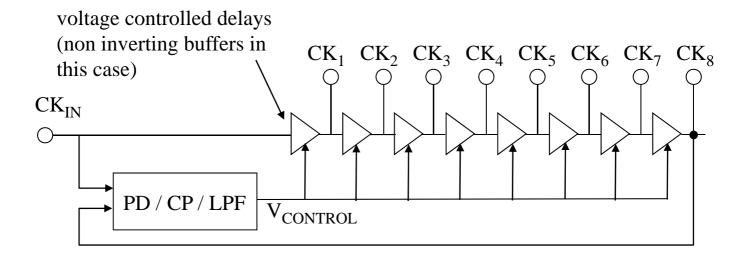
(i) The natural frequency – 2 marks

$$\omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} = 2.19 \times 10^6 \text{ rad/s} = 349 kHz$$

(ii) The damping factor – 2 marks

$$\varsigma = \frac{R_P}{2} \sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} = 1.1$$

## Question 6(c) A Delay Locked Loop (DLL) – 6 marks



The DLL uses a phase-detector, charge pump and low pass filter similar to a Phase Locked Loop but the output voltage from the low pass filter drives a set of voltage-controlled delay blocks instead of a VCO. Assuming that the delay circuits are identical, then the same time delay is created between each of the clock phases. Voltage controlled delays can be formed from CMOS inverters where the current flowing in the inverters is controlled by an applied voltage in a voltage-controlled current-source type configuration.

Q7 – Essay type question based on Continuous Assessment