Exam Solutions UE4002 Summer 2011

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an NMOS in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n = \lambda_p = 0$.
In each question, capacitances other than those shown may be ignored.

Question 1

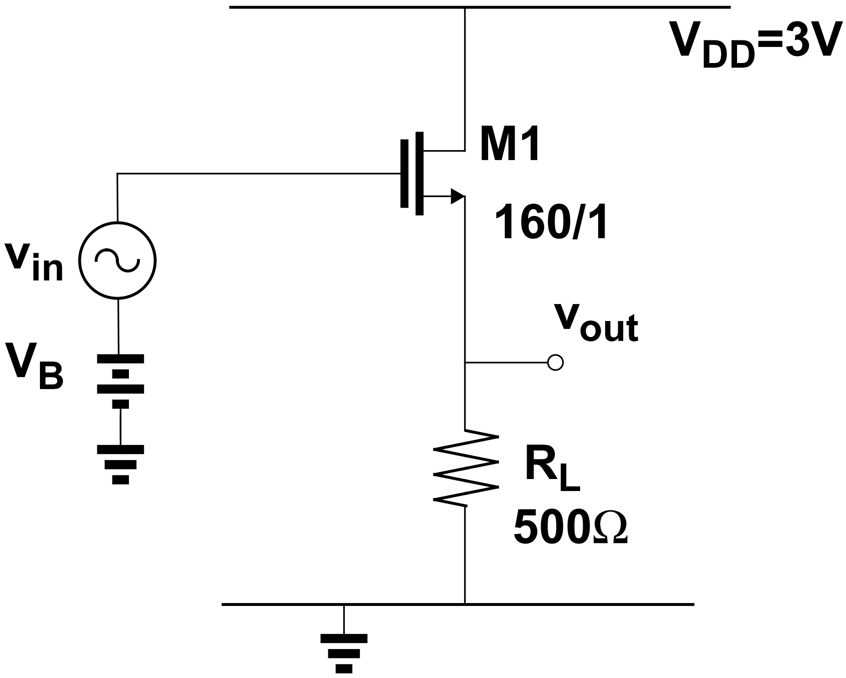
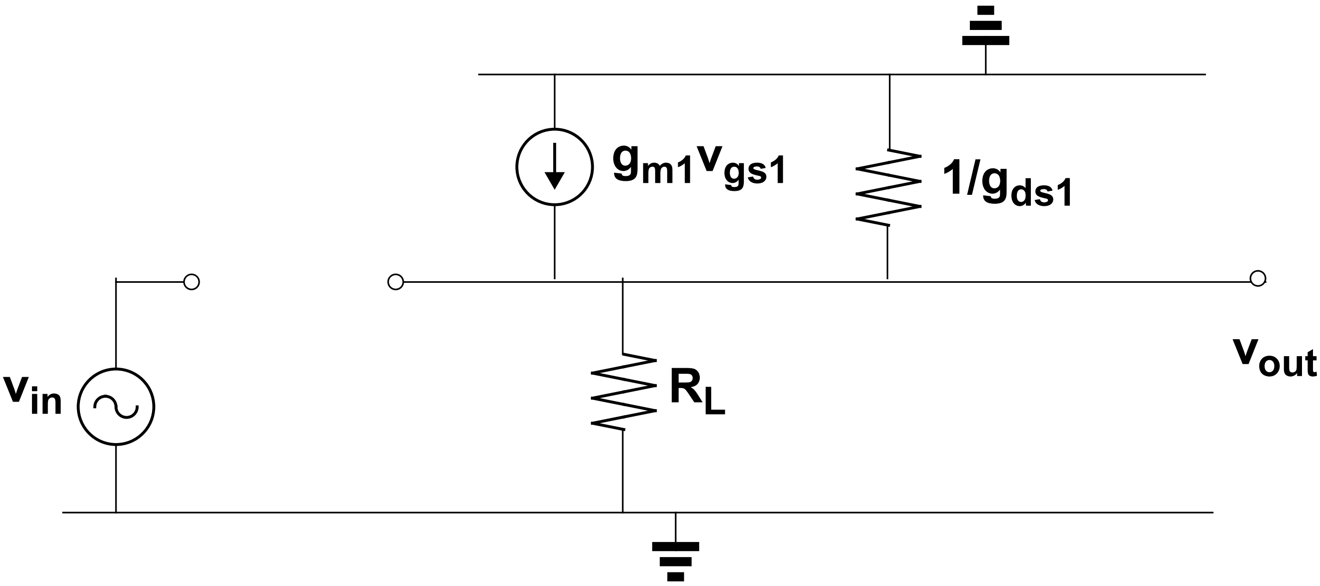


Figure 1

Figure 1 shows an NMOS source follower driving a resistive load. Assume M1 is in saturation. Resistor value and transistor dimensions in µm and are as shown in Figure 1. Take $K_n' = 200\mu A/V^2$. Assume $g_{ds1} << 1/R_L$.

(i) Draw the small-signal equivalent circuit for the source follower stage shown in Figure 1.
(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}).
(iii) Calculate the small-signal voltage gain (v_{out}/v_{in}). Take $I_{D1} = 250\mu A$.
(iv) What is the minimum required value of I_{D1} so that the small-signal voltage loss between the input and output is less than 20%?

(i) Draw the small-signal equivalent circuit for the source follower stage shown in Figure 1.



(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in})

$$v_{gs1} = v_{in} - v_{out}$$

KCL at output node

$$g_{m1}(v_{in} - v_{out}) - (v_{out}g_{ds1}) - \left(\frac{v_{out}}{R_L}\right) = 0$$

$$g_{m1}v_{in} = \left(g_{m1} + g_{ds1} + \frac{1}{R_L}\right)v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{ds1} + \frac{1}{R_L}} \approx \frac{g_{m1}}{g_{m1} + \frac{1}{R_L}}$$

(iii) Calculate the small-signal voltage gain (v_{out}/v_{in}).
Take I_{D1}=250μA

$$g_{m1} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{160}{1} \times 250\mu A} = 4mA/V$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + \frac{1}{R_L}} = \frac{4mA/V}{4mA/V + \frac{1}{500}} = \underline{\underline{0.66}} = -3.5dB$$

(iv) What is the minimum required value of I_{D1} so that the small-signal voltage loss between the input and output is less than 20%??

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + \frac{1}{R_L}} > 0.8$$

$$g_{m1} > 0.8\left(g_{m1} + \frac{1}{R_L}\right)$$

$$g_{m1} > \frac{0.8\left(\frac{1}{R_L}\right)}{0.2} = \frac{0.8\left(\frac{1}{500}\right)}{0.2} = 8mA/V$$

g_{m1} must increase by 2x so I_{D1} must increase by 4x

$$\underline{\underline{I_{D1} = 1mA}}$$

Question 2

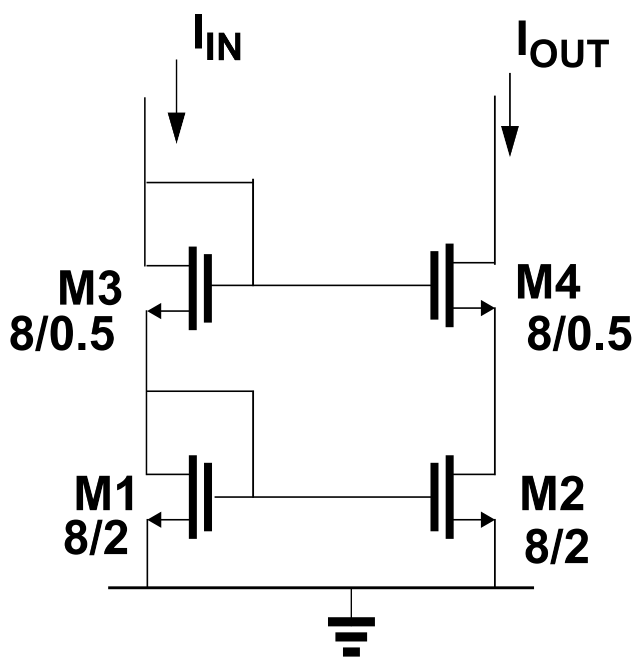


Figure 2

Figure 2 shows a cascoded current mirror.
Assume $I_{IN}=I_{OUT}=100\mu A$, $K_n'=200\mu A/V^2$, $V_{tn}=750mV$, $\lambda_n = 0.04V^{-1}/L$, L in μm.
Transistor dimensions in μm are shown in Figure 2.

(i) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?
(ii) Derive an expression for the small-signal output resistance.
Assume $g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}$.
(iii) What is the change in output current if the voltage at the output node varies by 10mV? Assume all transistors are in saturation.
(iv) It is desired to increase the mirroring ratio by increasing the width of M2 only. What is the largest value of output current such that M2 remains in saturation?

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(i) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?

The minimum voltage at the output is given by the voltage at the drain of M2 plus the required V_{DS} across M4 for it to be in saturation i.e. V_{GS4}-V_t

For M1, M2

$$|V_{GS}-V_t| = \sqrt{\frac{2I_{D1}}{K_n'\frac{W}{L}}} = \sqrt{\frac{2\cdot 100\mu A}{200\mu A/V^2\frac{8}{2}}} = 500mV \Rightarrow V_{GS2} = 1.25V$$

For M3, M4

$$|V_{GS}-V_t| = \sqrt{\frac{2I_{D3}}{K_n'\frac{W}{L}}} = \sqrt{\frac{2\cdot 100\mu A}{200\mu A/V^2\frac{8}{0.5}}} = 250mV \Rightarrow V_{GS4} = 1V$$

$$V_{OUTmin} = V_{D2} + (V_{GS4} - V_t)$$

$$V_{OUTmin} = V_{GS1} + V_{GS3} - V_{GS2} + (V_{GS4} - V_t)$$

$$V_{OUTmin} = 1.25 + 1 - 1 + |0.25| = 1.5V$$

(ii) Derive an expression for the small-signal output resistance.
Assume g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}.

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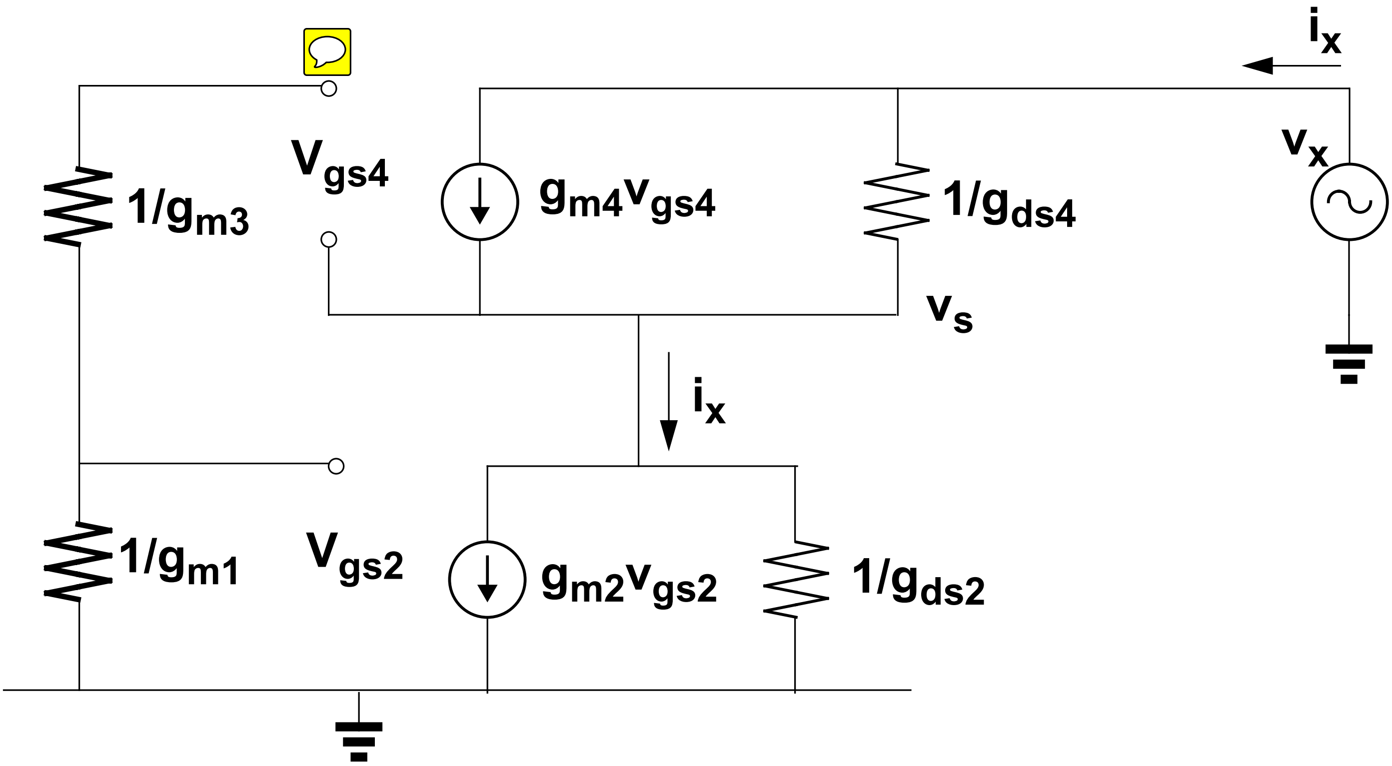


Note: $v_{gs2} = 0 \Rightarrow g_{m2}v_{gs2} = 0$

$$i_x = g_{m4}v_{gs4} + (v_x - v_s)g_{ds4}$$

Since $v_{gs4} = -v_s$ and $v_s = \dfrac{i_x}{g_{ds2}}$

$$i_x = -(g_{m4})\frac{i_x}{g_{ds2}} + \left(v_x - \frac{i_x}{g_{ds2}}\right)g_{ds4}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1 + \dfrac{g_{m4}}{g_{ds2}} + \dfrac{g_{ds4}}{g_{ds2}}}{g_{ds4}}$$

Since g_{m2},g_{m4} >> g_{ds2},g_{ds4} this can be reduced to

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

(iii) What is the change in current if the voltage at the output node varies by 10mV?
Assume all transistors are in saturation.

$$i_{out} = \frac{v_{out}}{r_{out}}$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

$$g_{m4} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 100\mu A}{0.25} = 800\mu A/V$$

$$g_{ds4} = \lambda I_{D4} = \frac{0.04V^{-1}}{0.5} \times 100\mu A = 8\mu A/V$$

$$g_{ds2} = \lambda I_{D2} = \frac{0.04V^{-1}}{2} \times 100\mu A = 2\mu A/V$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}} = \frac{800\mu A/V}{8\mu A/V} \frac{1}{2\mu A/V} = 50M\Omega$$

$$i_{out} = \frac{v_{out}}{r_{out}} = \frac{10mV}{50M\Omega} = 0.2nA$$

(iv) It is desired to increase the mirroring ratio by increasing the width of M2 only. What is the largest value of output current such that M2 remains in saturation?

If the current of M2 is increased by increasing the width of M2 only, then its V_{GS}-Vt will remain the same.
However V_{GS4} will increase, reducing V_{D4}, until M2 goes out of saturation
This determines V_{GS4max} which determines the max. current.

$$V_{S4min} = V_{D2min} = V_{GS2} - V_t = 0.5V$$

$$V_{GS4max} = V_{G4} - V_{S4min} = 2.25 - 0.5V = 1.75V$$

$$I_{D4max} = \frac{K'_n}{2} \frac{W}{L} (V_{GS4} - V_t)^2 = \frac{200\mu A/V^2}{2} \frac{16}{1} (1.75 - 0.75)^2 = 1.6mA$$

Question 3

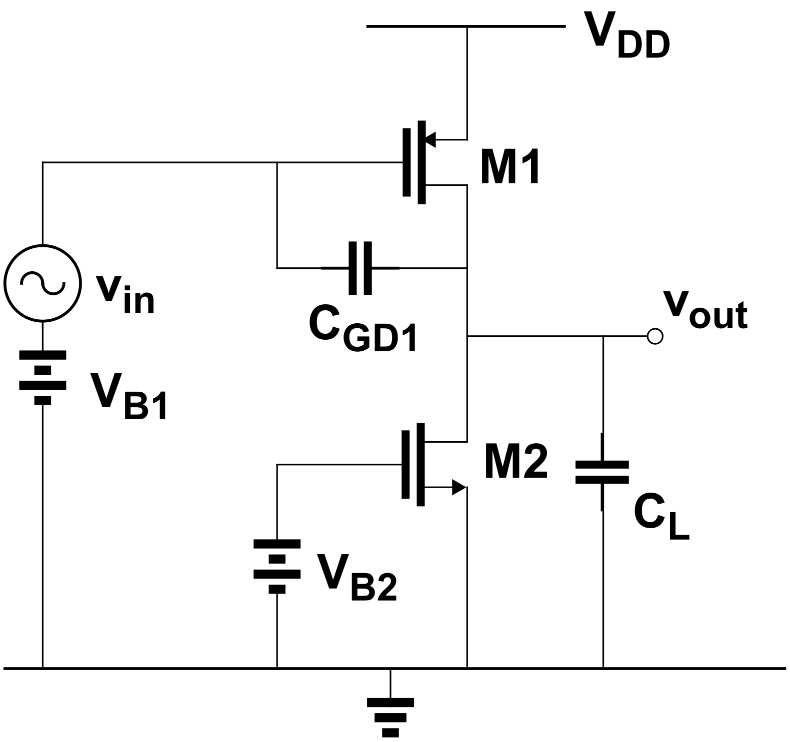
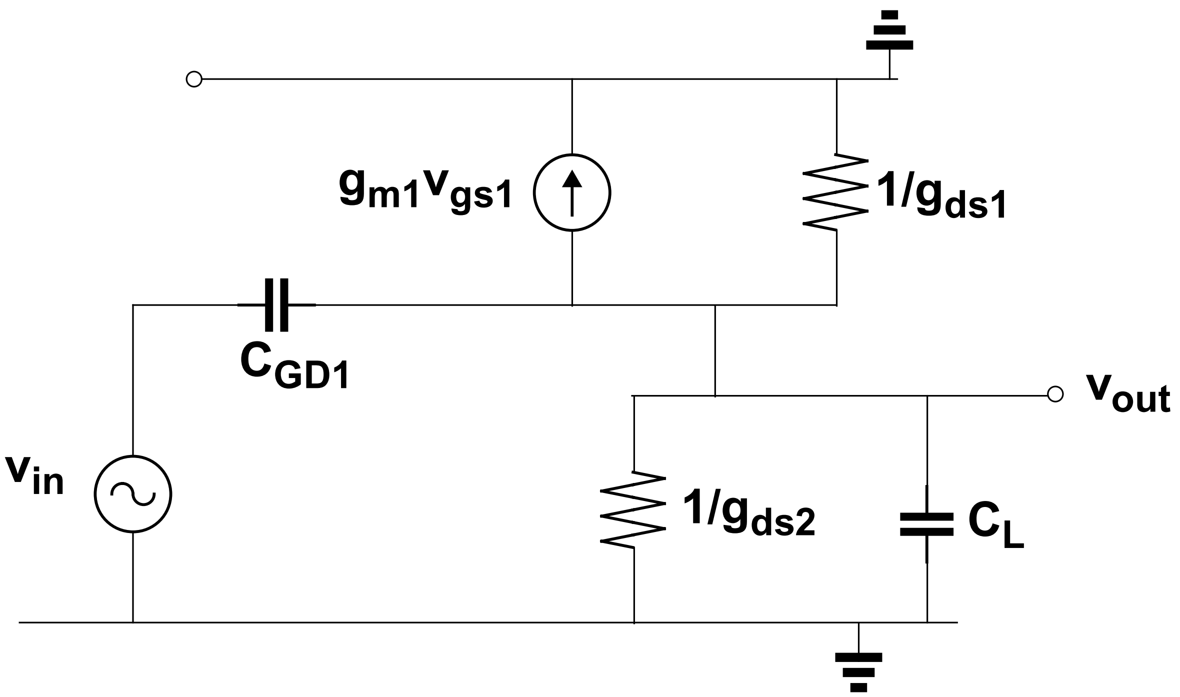


Figure 3

For the questions below you may assume $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$ and that all devices are biased in saturation.

(i) Figure 3 shows a gain stage with an active load. Draw the small-signal model for this circuit.
(ii) Derive an expression for the high-frequency transfer function of the circuit.
(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the break frequencies (i.e. pole and/or zero frequencies) if
V_{DD}= 3V, V_{B1}= 2V, $|V_{tp}|$ = 0.75V, $|I_{D1}|$ = 200μA, $\lambda_p = \lambda_n = 0.04V^{-1}$, C_{GD1}= 0.1pF, C_L= 1.5pF.
(iv) Draw a Bode diagram of the gain response.
Indicate the values of gain at d.c. and at frequencies well above the pole and/or zero frequencies.

(i) Figure 3 shows a gain stage with a diode-connected load. Draw the small-signal model for this circuit.



(ii) Ignoring all capacitances except C_{GD1}, C_{GS2} and C_L derive an expression for the high-frequency transfer function.

KCL at output node

$$g_{m1}v_{gs1} + v_{out}g_{ds1} + v_{out}g_{ds2} + v_{out}s(C_L) + (v_{out} - v_{in})sC_{GD1} = 0$$

$$g_{m1}v_{in1} + v_{out}g_{ds1} + v_{out}g_{ds2} + v_{out}s(C_L) + (v_{out} - v_{in})sC_{GD1} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} - sC_{GD1}}{g_{ds1} + g_{ds2} + s(C_L + C_{GD1})}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}\left(1 - \frac{sC_{GD1}}{g_{m1}}\right)}{g_{ds1} + g_{ds2}\left(1 + \frac{s(C_L + C_{GD1})}{g_{ds1} + g_{ds2}}\right)}$$

(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the break frequencies (i.e. pole and/or zero frequencies) if V_{DD}= 3V,V_{B1}= 2V, $|V_{tp}|$ = 0.75V, $|I_{D1}|$ = 200μA, $\lambda_p = \lambda_n = 0.04V^{-1}$, C_{GD1}= 0.1pF, C_L= 1.5pF.

$$g_{m1} = \frac{2I_{D1}}{(|V_{GS1}|-|V_{tp}|)} = \frac{2 \times 200\mu A}{1-0.75} = 1600\mu A/V$$

$$g_{ds1} = \lambda_n I_D = 0.04V^{-1}200\mu A = 8\mu A/V$$

$$g_{ds2} = \lambda_n I_D = 0.04V^{-1}200\mu A = 8\mu A/V$$

Low-frequency gain given by

$$Gain = -\left(\frac{g_{m1}}{g_{ds1}+g_{ds2}}\right) = -\frac{1600\mu A/V}{16\mu A/V} = -100 = 40dB$$

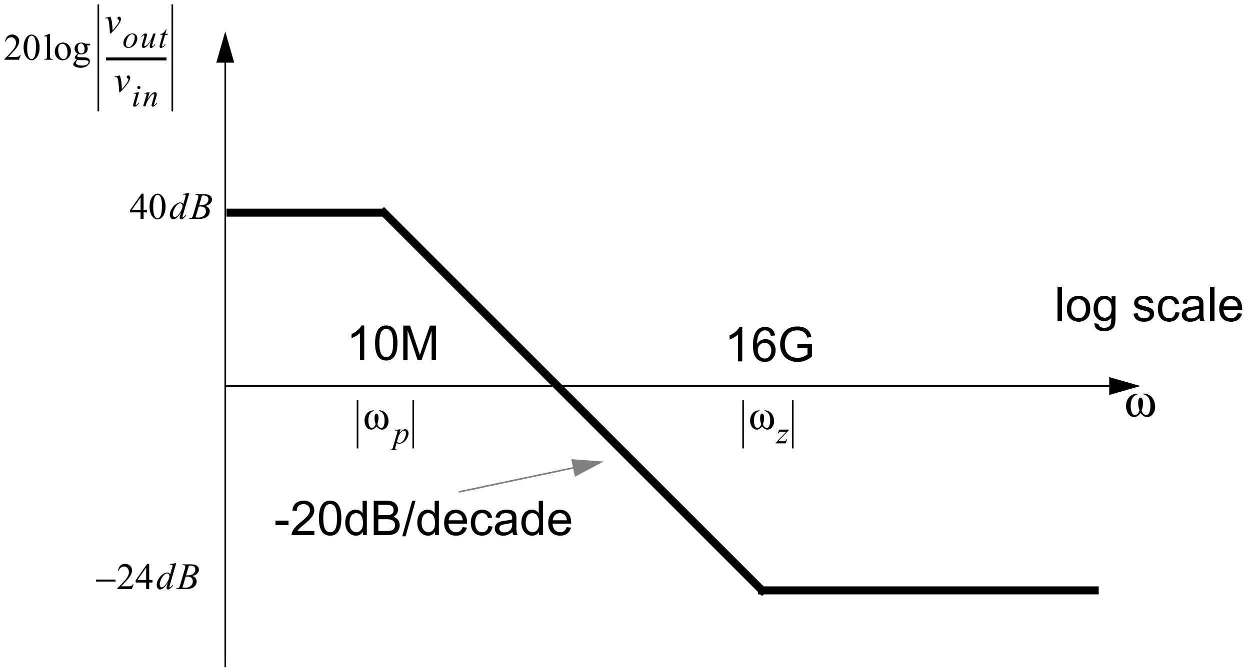
Zero frequency given by

$$|\omega_z| = \frac{g_{m1}}{C_{GD1}} = \frac{1600\mu A/V}{0.1pF} = \underline{\underline{16Grad/s}}$$

Pole frequency given by

$$|\omega_p| = \frac{g_{ds1}+g_{ds2}}{C_L+C_{GD1}} = \frac{16\mu A/V}{1.5pF+0.1pF} = \underline{\underline{10Mrad/s}}$$

(iv) Draw a Bode diagram of the gain response. Indicate the values of gain at d.c. and at frequencies well above the pole and zero frequencies.



Zero frequency is $20\log\left|\frac{16G}{10M}\right| = 3.2$ decades down

Therefore gain is $3.2 \times 20dB = 64dB$ down =>

$$40dB - 64dB = -24dB$$

Question 4

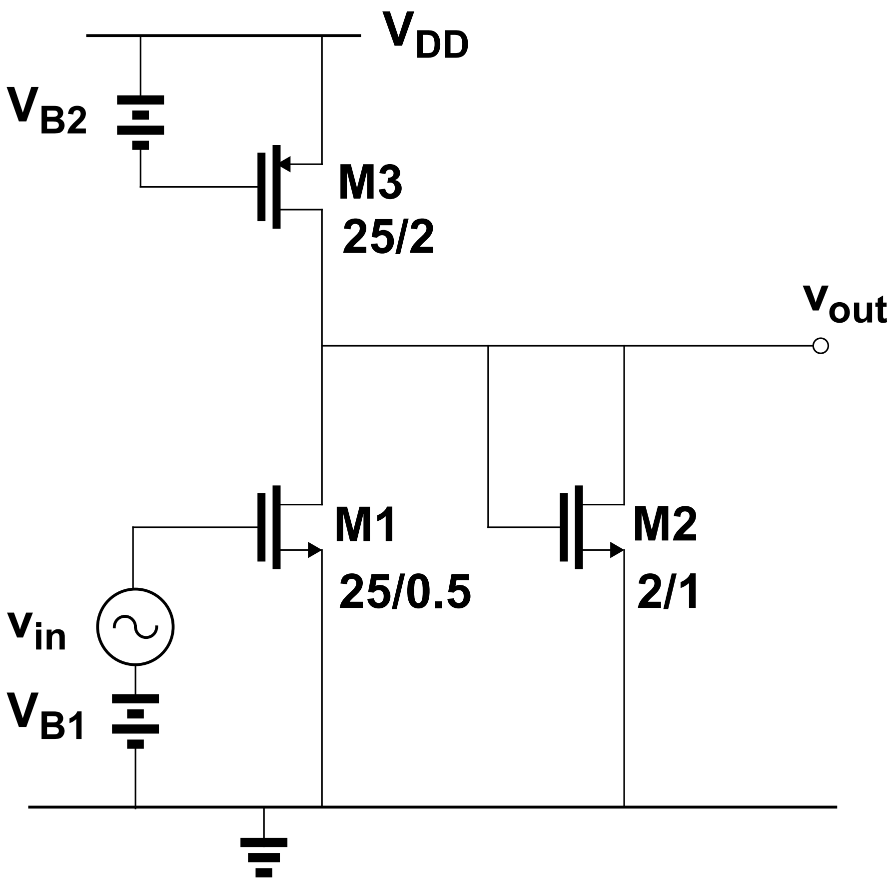


Figure 4

For the questions below you may assume g_{m1}, g_{m2}, g_{m3} >> g_{ds1}, g_{ds2}, g_{ds3}, and that all transistors are biased in saturation.
Transistor dimensions μm are as shown in Figure 4.
Only thermal noise sources need be considered.
For calculations take Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the small-signal voltage gain (v_{out}/v_{in}) in terms of the transistor small-signal parameters?

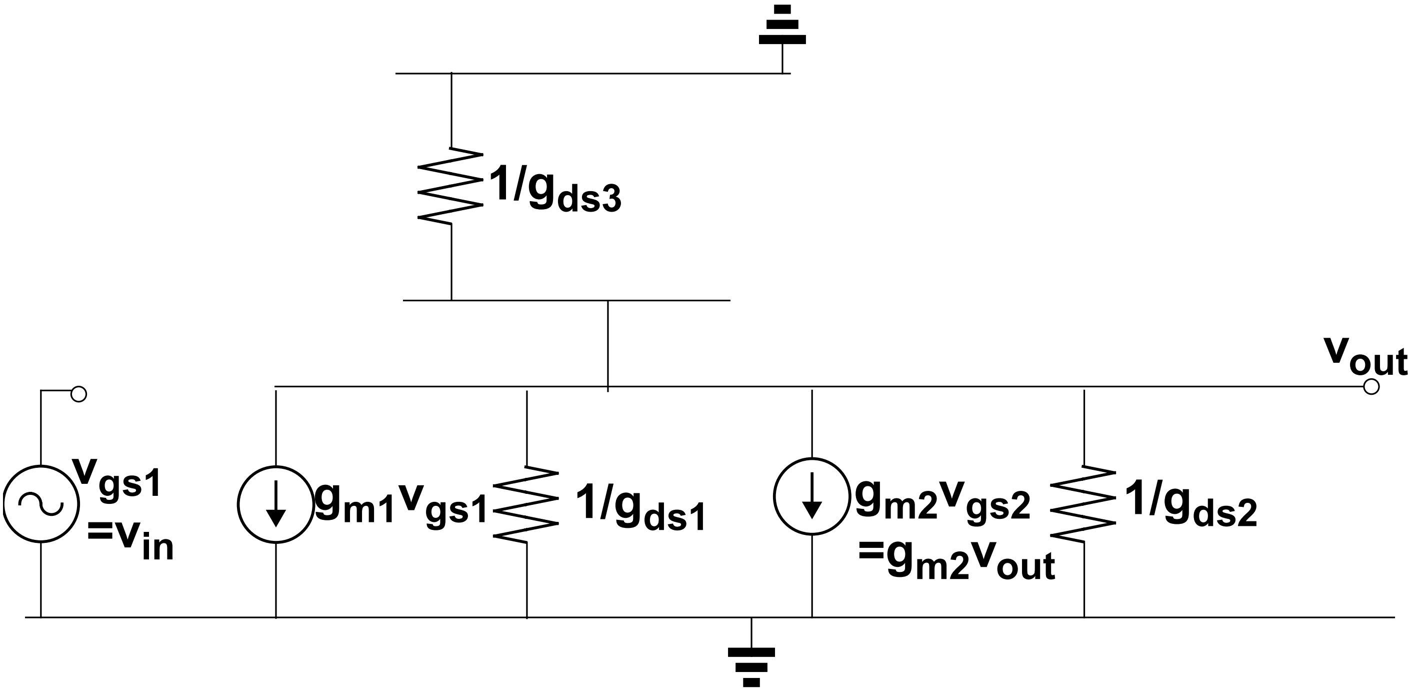
(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4?
The answer should be in terms of the transistor small-signal parameters, Boltzmann's constant k and temperature T.

(iii) Calculate the input-referred thermal noise voltage density of the circuit if K_n'=200μA/V^2, K_p'=50μA/V^2, I_{D1}=50μA, $|I_{D3}|$=100μA.

(iv) If the output signal is 100mVrms, what is the signal-to-noise ratio at the output if the noise is measured over a bandwidth of 10MHz?

Solution

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters of M1 and M2?

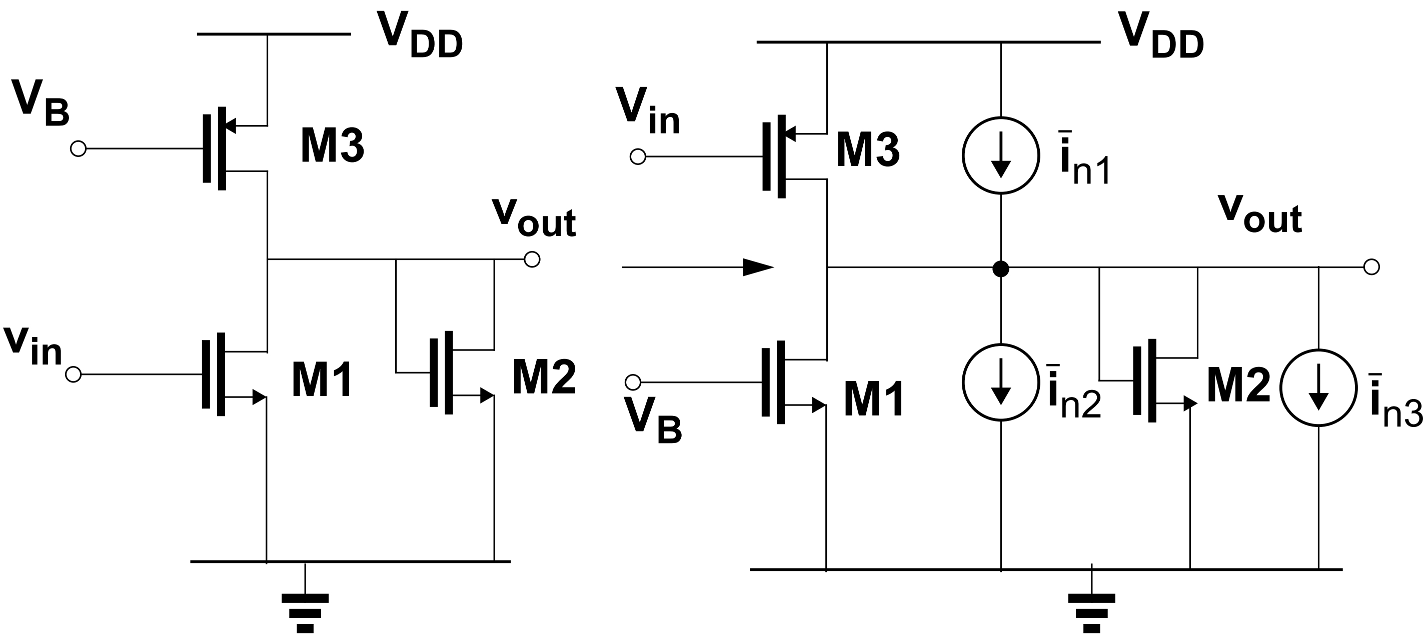


KCL at output node:

$$g_{m1}v_{in} + v_{out}g_{m2} + v_{out}g_{ds1} + v_{out}g_{ds2} + v_{out}g_{ds3} = 0$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_{m1}}{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3}} \approx -\frac{g_{m1}}{g_{m2}}$$

(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4? The answer should be in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T.?



Noise current density of MOS:

$$\overline{i_n^2} = 4kT\left(\frac{2}{3}g_m\right) \qquad A^2/Hz$$

$$\overline{i_n} = \sqrt{4kT\left(\frac{2}{3}g_m\right)} \qquad A/\sqrt{Hz}$$

Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right) + 4kT\left(\frac{2}{3}g_{m3}\right)}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_{m1}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right) + 4kT\left(\frac{2}{3}g_{m3}\right)}}{g_{m1}} \; V/\sqrt{Hz}$$

(iii) Calculate the input-referred thermal noise voltage density of the circuit if $K_n' = 200\mu A/V^2$, $K_p' = 50\mu A/V^2$, $I_{D1} = 50\mu A$, $|I_{D3}| = 100\mu A$.

Current from M3 splits between M1 and M2 => 50µA in M2

$$g_{m1} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{25}{0.5} \times 50\mu A} = 1000\mu A/V$$

$$g_{m2} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{2}{1} \times 50\mu A} = 200\mu A/V$$

$$g_{m3} = \sqrt{2K_p'\frac{W}{L}I_D} = \sqrt{2 \times 50\mu A/V \times \frac{25}{2} \times 100\mu A} = 353\mu A/V$$

$$\overline{v_{ni}} = \frac{\sqrt{4kT\left(\frac{2}{3}(g_{m1} + g_{m2} + g_{m3})\right)}}{g_{m1}}$$

$$\overline{v_{ni}} = \frac{\sqrt{(4 \cdot 1.38\times10^{-23} \cdot 300)\left(\frac{2}{3}\right)(1000\mu A/V + 200\mu A/V + 353\mu A/V)}}{1000\mu A/V}$$

$$\overline{v_{ni}} = 4.14nV/\sqrt{Hz}$$

(iv) If the output signal is 100mVrms, what is the signal-to-noise ratio in dB at the output if the noise is measured over a bandwidth of 10MHz?

Signal-to-Noise ratio at the output is given by the output signal divided by the product of the input-referred noise density, the gain and the sqrt of the bandwidth

$$\frac{S}{N} = \frac{v_{out}}{\overline{v_{nitot}} \cdot \left(\frac{g_{m1}}{g_{m2}}\right) \cdot \sqrt{BW}} = \frac{100mVrms}{4.0nV/\sqrt{Hz} \cdot \left(\frac{1000\mu A/V}{200\mu A/V}\right) \cdot \sqrt{10MHz}}$$

$$\frac{S}{N} = 1528 = 63.7dB$$