OLLSCOIL NA hÉIREANN, CORCAIGH

THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH

UNIVERSITY COLLEGE, CORK

AUTUMN EXAMINATIONS, 2011

B.E. DEGREE (ELECTRICAL & ELECTRONIC)

TELECOMMUNICATIONS

EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of log tables and a departmentally approved non-programmable calculator is permitted.

1. (a) Describe the operation of time division multiplexing in a fixed telephone network and illustrate the format and timing of data on a trunk line (E1) in Europe.

[*5 marks*]

(b) Describe the operation of statistical multiplexing in a data communications system. In your discussion, include a definition of multiplexing gain and indicate the expected ranges of multiplexing gain for various data sources.

[*5 marks*]

(c) Illustrate the format of an ATM cell and briefly describe the function of each field of the cell.

[*5 marks*]

(d) Illustrate the error handling procedure for an ATM packet and briefly describe the operation of the procedure.

[*5 marks*]

2. (a) Draw two protocol stacks to compare the functions of the OSI and TCP/IP (internet) systems for wide area networks. Clearly label each layer in the diagrams you draw, align them to allow a direct comparison of the layer functions and briefly describe these functions.

[9 *marks*]

(b) Briefly describe the following topics in TCP/IP:

(i) IP Addressing. [4 *marks*]

(ii) The Domain Name System. [*3 marks*]

(iii) The format of an IP packet. [*4 marks*]

3. (a) Illustrate the architecture of a UMTS Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) For mobile telephone networks briefly describe the following, noting any differences between 2nd and 3rd generation systems (2G and 3G) where they exist:

(i) The types of signal fading. [4 marks]

(ii) The main power control algorithms. [4 marks]

(iii) The hand-off algorithms when a user moves between adjacent cells.

[4 marks]

4. Given that the 3×3 channel matrix $\left[P\left(Y_1|X\right)\right]$ for the 3-ary uniform channel with 3 input symbols, denoted x_i, $1 \le i \le 3$ and 3 output symbols, denoted y_j, $1 \le j \le 3$, is given by: -

$$\left[P\left(Y_1|X\right)\right] = \begin{bmatrix} 1-p & \alpha & \alpha \\ \alpha & 1-p & \alpha \\ \alpha & \alpha & 1-p \end{bmatrix}$$
$$= \underbrace{\begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} 1-3p/2 & 0 & 0 \\ 0 & 1-3p/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{D} \underbrace{\left(\frac{1}{3}\begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}\right)}_{F^{-1}}$$

where $\alpha = p/2$, D is a diagonal matrix and the columns of F are eigenvectors of $\left[P\left(Y_1|X\right)\right]$, show that if n such 3-ary uniform channels are connected in series (i.e. the outputs of 3-ary uniform channel i become the inputs of 3-ary uniform channel $i+1$, $1 \le i \le n-1$), then: -

(a) The composite channel matrix $\left[P\left(Y_n|X\right)\right]$ is given by: -

$$\left[P\left(Y_n|X\right)\right] = \frac{1}{3}\begin{bmatrix} 1+2q & 1-q & 1-q \\ 1-q & 1+2q & 1-q \\ 1-q & 1-q & 1+2q \end{bmatrix}$$

where $q = \left(1-3p/2\right)^n$. [8 *marks*]

(b) Given that the input symbols x_i, $1 \le i \le 3$ are equiprobable and the maximum entropy of an m symbol source is given by $\log_2[m]$, the composite channel capacity C_s^c is given by: -

$$C_s^c = \log_2\left[3\left(\frac{1+2q}{3}\right)^{\frac{1+2q}{3}}\left(\frac{1-q}{3}\right)^{2\left(\frac{1-q}{3}\right)}\right]$$

where $q = \left(1-3p/2\right)^n$. [8 *marks*]

(c) If p is sufficiently small such that its square and higher powers can be neglected, show that the composite channel capacity C_s^c is approximately that of a single 3-ary uniform channel with probability of error free transmission (i.e. probability of receiving y_j having sent x_j) equal to $1 - n \times p$. Note that: -

$$\left(x+y\right)^n = \sum_{i=0}^{n}\binom{n}{i}x^i y^{n-i} \quad \text{and} \quad \binom{n}{i} = \frac{n!}{i!\left(n-i\right)!}.$$

[4 *marks*]

5. A baseband digital communications system uses rectangular wave signaling with A_1 volts representing logic 1 and A_2 volts representing logic 0 (where $A_2 < A_1$). The receiver takes a single sample of the received signal during the bit signaling time and compares this sample with a decision threshold T. The communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n(v) = \frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}},$$

P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1 and, to minimize the resulting overall probability of error P_e, the threshold T is given by: -

$$T = \frac{A_1 + A_2}{2} + \frac{\sigma^2}{A_1 - A_2} \ln\left[\frac{P_0}{P_1}\right].$$

(a) Show that, if $P_0 > P_1$, then the average probability of error, denoted P_e, is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{T - A_2}{\sqrt{2\sigma^2}}\right] + (1 - P_0) erf\left[\frac{A_1 - T}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

[10 *marks*]

(b) Consider a system for which $A_1 = 2.5V$, $A_2 = -2.5V$ and $\sigma^2 = 0.45W$. Using the table of values of $erf[x]$ provided : -

(i) Prove that, if the threshold remains fixed at $T = 0V$, then P_e is independent of P_0 and calculate its value in this case. [4 marks]

(ii) When the optimum threshold is employed in each case, calculate the value of P_e when $P_0 = 0.65$ and when $P_0 = 0.75$. [6 marks]

6. (a) Given the Schwarz inequality, which states: -

$$\left|\int_{-\infty}^{\infty} f_1(\omega) f_2(\omega) d\omega\right|^2 \le \int_{-\infty}^{\infty} |f_1(\omega)|^2 d\omega \int_{-\infty}^{\infty} |f_2(\omega)|^2 d\omega,$$

or otherwise, show that the signal to noise ratio (SNR) at the output of a linear filter subject to an input signal $s(t)$, with Fourier transform $S(\omega)$, and input coloured noise with power spectral density $S_{nn}(\omega)$ satisfies: -

$$\left(\frac{S}{N}\right)_o \le \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{S_{nn}(\omega)} d\omega$$

and state the equation for the matched filter for this coloured noise. [10 *marks*]

(b) Hence, or otherwise, deduce the maximum attainable SNR if the input noise is additive white Gaussian (AWGN) with a power spectral density of $\eta/2$ W/Hz. [3 *marks*]

(c) The matched filter for an input signal $s(t)$, with Fourier transform $S(\omega)$, and additive white Gaussian (AWGN) with a power spectral density $\eta/2$ W/Hz is implemented in a receiver. The actual noise, however, is coloured with a power spectral density of $S_{nn}(\omega)$ (i.e. the designer designed for white noise but, in practice, coloured noise affected the receiver). Show that, in this case, the SNR, $\left(\frac{S}{N}\right)_o$, is given by: -

$$\left(\frac{S}{N}\right)_o = \frac{2\pi \ E^2}{\int_{-\infty}^{\infty} S_{nn}(\omega)|S(\omega)|^2 d\omega}$$

where E denotes the energy content of $s(t)$ (assuming a 1Ω reference resistor). [7 *marks*]

7. Given the following table of field elements of $GF(2^5)$: -

0	$\alpha^7 = \alpha^4 + \alpha^2$	$\alpha^{15} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$	$\alpha^{23} = \alpha^3 + \alpha^2 + \alpha + 1$
1	$\alpha^8 = \alpha^3 + \alpha^2 + 1$	$\alpha^{16} = \alpha^4 + \alpha^3 + \alpha + 1$	$\alpha^{24} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha$
α	$\alpha^9 = \alpha^4 + \alpha^3 + \alpha$	$\alpha^{17} = \alpha^4 + \alpha + 1$	$\alpha^{25} = \alpha^4 + \alpha^3 + 1$
α^2	$\alpha^{10} = \alpha^4 + 1$	$\alpha^{18} = \alpha + 1$	$\alpha^{26} = \alpha^4 + \alpha^2 + \alpha + 1$
α^3	$\alpha^{11} = \alpha^2 + \alpha + 1$	$\alpha^{19} = \alpha^2 + \alpha$	$\alpha^{27} = \alpha^3 + \alpha + 1$
α^4	$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha$	$\alpha^{20} = \alpha^3 + \alpha^2$	$\alpha^{28} = \alpha^4 + \alpha^2 + \alpha$
$\alpha^5 = \alpha^2 + 1$	$\alpha^{13} = \alpha^4 + \alpha^3 + \alpha^2$	$\alpha^{21} = \alpha^4 + \alpha^3$	$\alpha^{29} = \alpha^3 + 1$
$\alpha^6 = \alpha^3 + \alpha$	$\alpha^{14} = \alpha^4 + \alpha^3 + \alpha^2 + 1$	$\alpha^{22} = \alpha^4 + \alpha^2 + 1$	$\alpha^{30} = \alpha^4 + \alpha$

(a) Show that the generator polynomial for the $(31,21)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^3 + 1.$$

[10 *marks*]

(b) Show that the codeword, $c(x)$, representing the user data polynomial, $i(x) = x^{15} + x^{12} + x^7 + 1$, is given by : -

$$c(x) = x^{25} + x^{24} + x^{23} + x^{22} + x^{16} + x^{15} + x^{13} + x^9 + x^8 + x^7 + x^6 + x^5 + x^3 + 1.$$

[2 *marks*]

(c) Due to the presence of errors, represented by the polynomial, $e(x)$, where: -

$$e(x) = x^{18} + x^6,$$

affecting the transmission of the codeword $c(x)$ in (b) above, the received polynomial, $v(x) = c(x) + e(x)$, does not equal $c(x)$.

(i) Show that the error location polynomial is given by: -

$$x^2 + S_1 x + \frac{S_1^3 + S_3}{S_1} = 0.$$

[4 *marks*]

(ii) Show how the syndrome decoding method determines the correct error polynomial, $e(x)$, in this case.

[4 *marks*]

Table of values of $erf(x)$

x	$erf(x)$		x	$erf(x)$
2.5	0.999593		2.63	0.9998
2.505	0.999604		2.635	0.999806
2.51	0.999614		2.64	0.999811
2.515	0.999625		2.645	0.999816
2.52	0.999635		2.65	0.999822
2.525	0.999644		2.655	0.999826
2.53	0.999654		2.66	0.999831
2.535	0.999663		2.665	0.999836
2.54	0.999672		2.67	0.999841
2.545	0.999681		2.675	0.999845
2.55	0.999689		2.68	0.999849
2.555	0.999698		2.685	0.999854
2.56	0.999706		2.69	0.999858
2.565	0.999714		2.695	0.999862
2.57	0.999722		2.7	0.999866
2.575	0.999729		2.705	0.999869
2.58	0.999736		2.71	0.999873
2.585	0.999744		2.715	0.999877
2.59	0.999751		2.72	0.99988
2.595	0.999757		2.725	0.999884
2.6	0.999764		2.73	0.999887
2.605	0.99977		2.735	0.99989
2.61	0.999777		2.74	0.999893
2.615	0.999783		2.745	0.999896
2.62	0.999789		2.75	0.999899
2.625	0.999795			