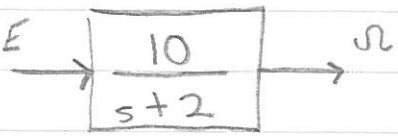
25/3/09 Control

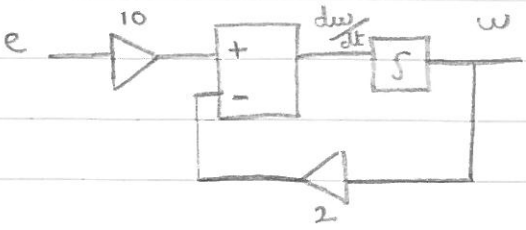
Q5(b);

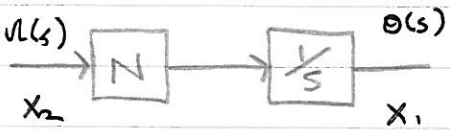


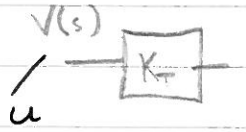
$\frac{\Omega}{E} = \frac{10}{s+2}$

$(s+2)\Omega = 10E$

$\frac{dw}{dt} = 10e - 2w$







$\frac{dx_1}{dt} = Nx_2$

$\frac{dx_2}{dt} = 10e - 2x_2$

$e \rightarrow K_T u - K_s x_1$

$\frac{d}{dt}x_2 = 10K_T u - 10K_s x_1 - 2x_2$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & N \\ -10K_s & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10K_T \end{bmatrix} u$$

$$\underline{Y} = \begin{bmatrix} V_\theta \\ V_w \end{bmatrix} = \begin{bmatrix} K_\theta & 0 \\ 0 & K_w \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(ii) $\mathcal{C}_x = [B \mid AB]$

$$\begin{bmatrix} 0 & N \\ -10K_s & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 10K_T \end{bmatrix} = \begin{bmatrix} 10NK_T \\ -20K_T \end{bmatrix}$$

$$\mathcal{C}_x = \begin{bmatrix} 0 & 10NK_T \\ 10K_T & -20K_T \end{bmatrix}$$

$\det \mathcal{C} = 100NK_T^2$

Here $n = 2$ (second order)
For controllability rank $\mathcal{C}_x = n$
Here $\mathcal{C} = -100NK_T^2 \neq 0$ if N and K_T are non-zero
Hence rank $\mathcal{C}_e = n$

(iii) Put in numbers

$$\frac{d}{dt}\underline{x} = \begin{bmatrix} 0 & 10 \\ -900 & -2 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 200 \end{bmatrix} u$$

$$v(t) = R(t) - k_1 V_\omega - k_2 V_\theta$$

$K_\omega X_2 \qquad K_\theta X_1$

$$u = R - k_1' X_1 - k_2' X_2$$

$$\det(sI - A + BK) = C_{des}(s)$$

$$\xi = 0.5 \qquad T_{s2\%} = 0.02$$

$$\frac{4}{\xi \omega_N} = 0.02$$

$$\omega_N = 400 \text{ rad/s}$$

$$s^2 + 2\xi\omega_N + \omega_N^2$$

$$\det(sI - A + BK) = s^2 + 400s + 160{,}000$$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 10 \\ -900 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 200 \end{pmatrix} \begin{pmatrix} k_1' & k_2' \end{pmatrix} \right|$$

$$\begin{vmatrix} s & -10 \\ 900 + 200k_1' & s + 2 + 200k_2' \end{vmatrix}$$

$$= s^2 + 2s + 200k_2's + 9000 + 2000k_1'$$
$$= s^2 + (2 + 200k_2')s + 9000 + 2000k_1'$$

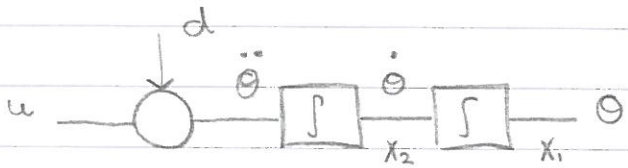
$2 + 200k_2' = 400$ $\qquad$ $9000 + 2000k_1' = 160{,}000$

$k_2' = 1.99$ $\qquad$ $k_1' = 75.5$

$$v = r(t) - \underbrace{\frac{75.5}{1.5}V_\theta}_{X_1} - \underbrace{\frac{1.99}{0.2}V_\omega}_{X_2}$$

$$= r(t) - 50.3V_\theta - 9.95V_\omega$$

6(b).



$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} 0 \\ 1 \end{bmatrix}d$$

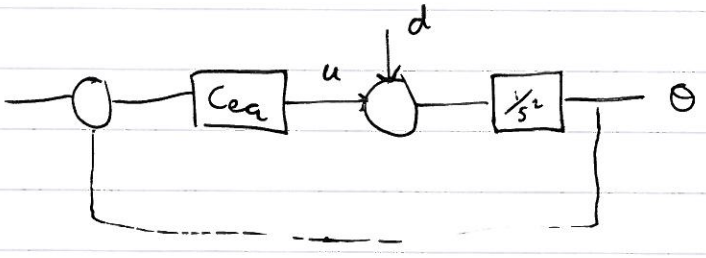
$u = -2x_2 - 2x_1$ $\qquad$ $\theta = [1\ 0]\underline{x}$

$$\frac{d}{dt}\underline{x} = \overbrace{\begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}}^{A-BK}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}d \qquad BK = \begin{bmatrix} 0 \\ 1 \end{bmatrix}[2\ 2] = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$G = \frac{\theta}{D} = C(sI - A + BK)^{-1}E$$

$$= \frac{1}{s^2 + 2s + 2}$$

(ii)



4th ordr sys work out Ceq