Solutions UE4002 Summer 2005

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an nmos in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n = \lambda_p = 0$.
In each question, capacitances other than those mentioned may be ignored.

Question 1

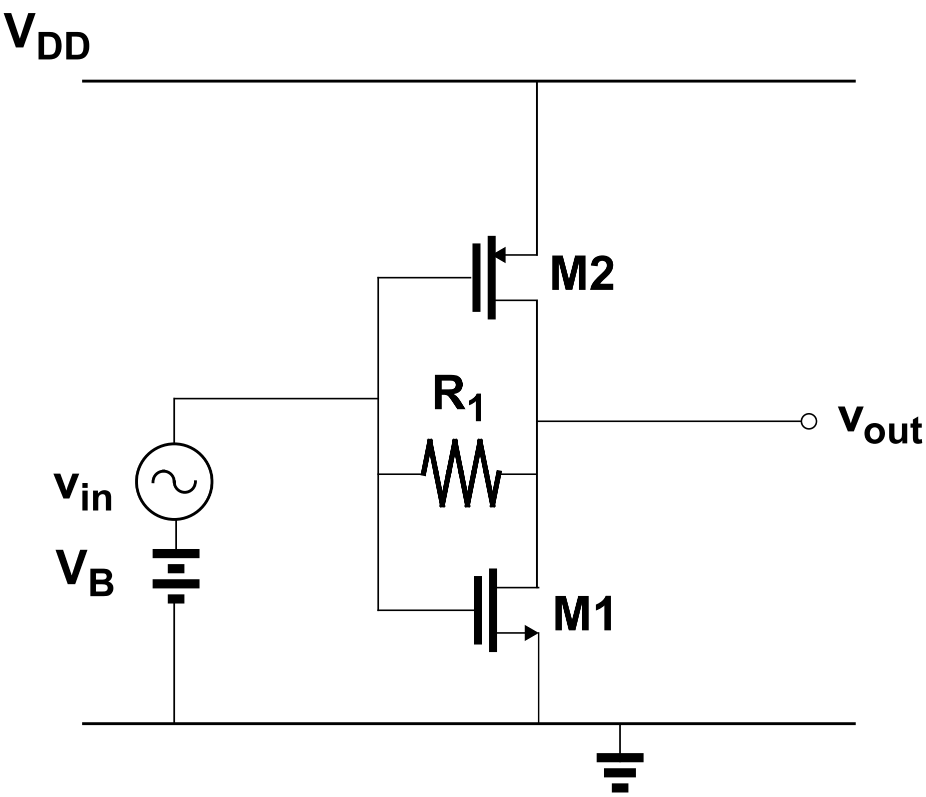

Figure 1

(i) Draw the small-signal equivalent circuit for the CMOS inverter stage shown in Figure 1.
(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal transistor parameters and R_1.
(iii) Calculate the small-signal gain if
$V_B = 1.5V$, $V_{tn} = 0.7V$, $V_{tp} = -0.7V$, $\lambda_n = \lambda_p = 0.04V^{-1}$, $R_1 = 5k\Omega$, $V_{DD} = 3V$.
Assume both transistors are in saturation with a drain current of 200µA.
(iv) What is the value of the gain if R_1 is increased to 10kΩ?
What is the value of the gain if R_1 is increased to infinity?

(i) Draw the small-signal equivalent circuit for the CMOS inverter stage shown in Figure 1.

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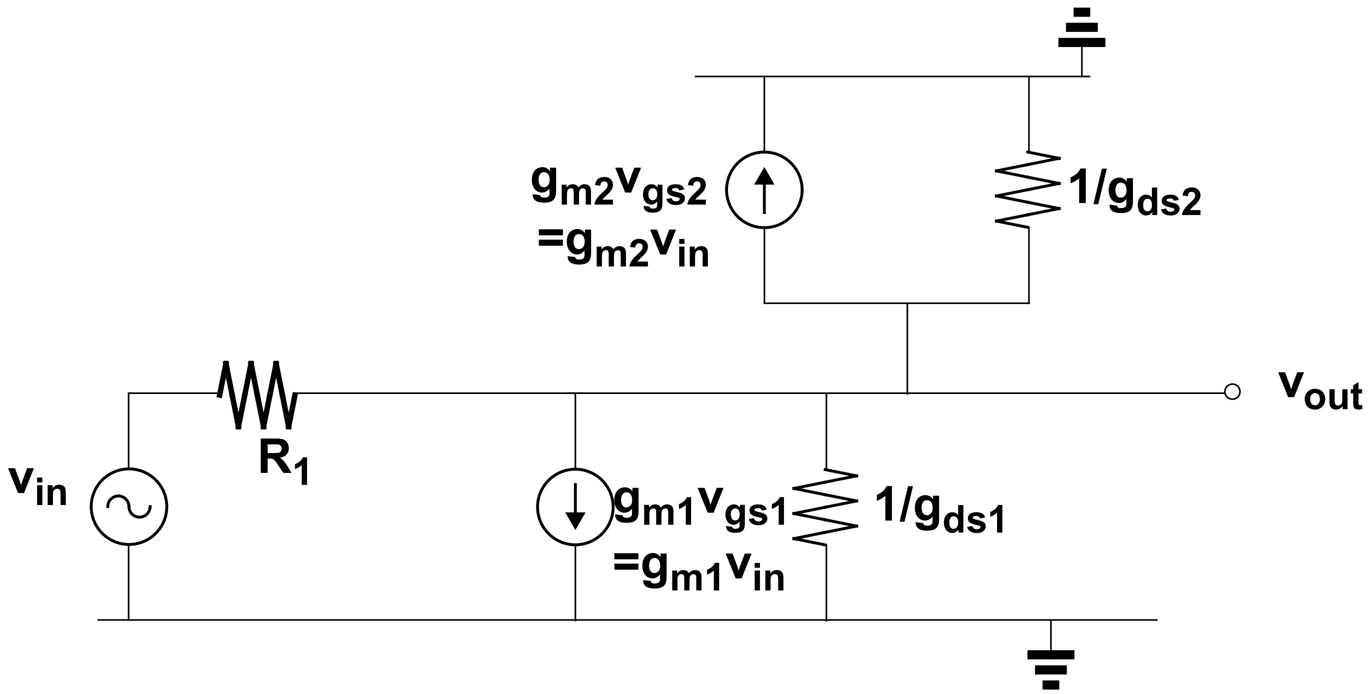

(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal transistor parameters and R_1.

KCL at output node

$$\frac{(v_{out} - v_{in})}{R_1} + g_{m1}v_{in} + g_{m2}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\left(g_{m1} + g_{m2} - \frac{1}{R_1}\right)v_{in} = -\left(g_{ds1} + g_{ds2} + \frac{1}{R_1}\right)v_{out}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2} - \frac{1}{R_1}}{g_{ds1} + g_{ds2} + \frac{1}{R_1}}$$

(iii) Calculate the small-signal gain if
$V_B = 1.5V$, $V_{tn} = 0.7V$, $V_{tp} = -0.7V$, $\lambda_n = \lambda_p = 0.04V^{-1}$, $R_1 = 5k\Omega$, $V_{DD} = 3V$.
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$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2} - \frac{1}{R_1}}{g_{ds1} + g_{ds2} + \frac{1}{R_1}}$$

$$g_{m1} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 200\mu A}{1.5 - 0.7} = 500\mu A/V$$

$$g_{m2} = \frac{2I_D}{(|V_{GS2}| - |V_{tp}|)} = \frac{2 \times 200\mu A}{1.5 - 0.7} = 500\mu A/V$$

$$g_{ds1} = \lambda I_D = 0.04V^{-1} \times 200\mu A = 8\mu A/V$$

$$g_{ds2} = \lambda I_D = 0.04V^{-1} \times 200\mu A = 8\mu A/V$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2} - \frac{1}{R_1}}{g_{ds1} + g_{ds2} + \frac{1}{R_1}} = \frac{500\mu A/V + 500\mu A/V - \frac{1}{5k\Omega}}{8\mu A/V + 8\mu A/V + \frac{1}{5k\Omega}} = 3.7$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 11.4dB$$

(iv) What is the value of the gain if R_1 is increased to 10kΩ?
What is the value of the gain if R_1 is increased to infinity?

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2} - \frac{1}{R_1}}{g_{ds1} + g_{ds2} + \frac{1}{R_1}} = \frac{500\mu A/V + 500\mu A/V - \frac{1}{10k\Omega}}{8\mu A/V + 8\mu A/V + \frac{1}{10k\Omega}} = 7.6 = 17.6dB$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2}} = \frac{500\mu A/V + 500\mu A/V}{8\mu A/V + 8\mu A/V} = 62.5 = 35.9dB$$

Question 2

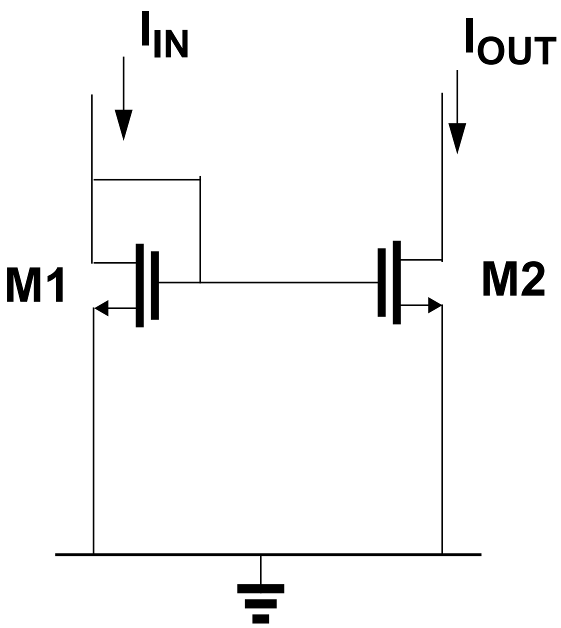


Figure 2a

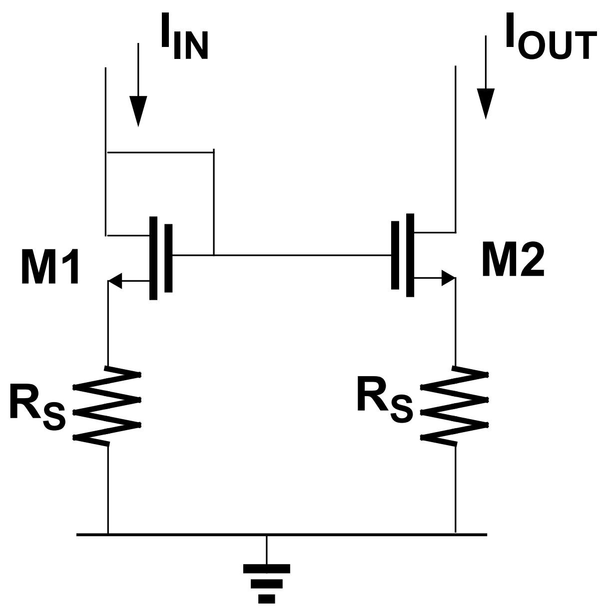


Figure 2b

For the current mirrors in Figure 1a and Figure 1b
$I_{IN} = I_{OUT} = 100\mu A$, $\lambda_n = 0.04V^{-1}$, $V_{GS1} = V_{GS2} = 1.2V$, $V_{tn} = 0.7V$, $R_S = 10k\Omega$.
Assume all devices are in saturation and that $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.

(i) What is the small-signal output resistance of the current mirror shown in Figure 2a in terms of the small-signal parameters of M2?

(ii) What is the variation in output current of the current mirror shown in Figure 2a if the voltage at the output node varies by 10mV?

(iii) Derive an expression for the output resistance of the current mirror shown in Figure 2b in terms of the small-signal parameters of M2 and the resistance R_S?

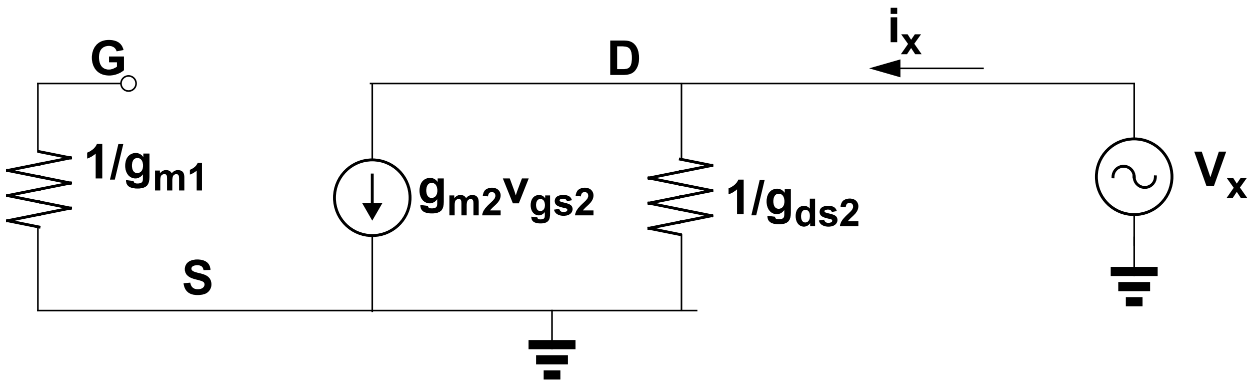
(iv) What is the variation in output current of the current mirror shown in Figure 2b if the voltage at the output node varies by 10mV?

(i) What is the small-signal output resistance of the current mirror shown in Figure 2a in terms of the small-signal parameters of M2?

The small signal output impedance is simply the inverse of the output conductance of M1

$$r_{out} = \frac{1}{g_{ds2}}$$

Alternatively draw the small signal model, apply a test voltage at the output and calculate the current back into the circuit, recognising the gate of M2 is at ac ground



Apply a test voltage at the output

$$i_x = g_{m2}v_{gs2} + v_x g_{ds2}$$

$$v_{gs2} = 0$$

$$i_x = v_x g_{ds2}$$

$$r_{in} = \frac{v_x}{i_x} = \frac{1}{g_{ds2}}$$

(ii) What is the variation in output current of the current mirror shown in Figure 2a if the voltage at the output node varies by 10mV?

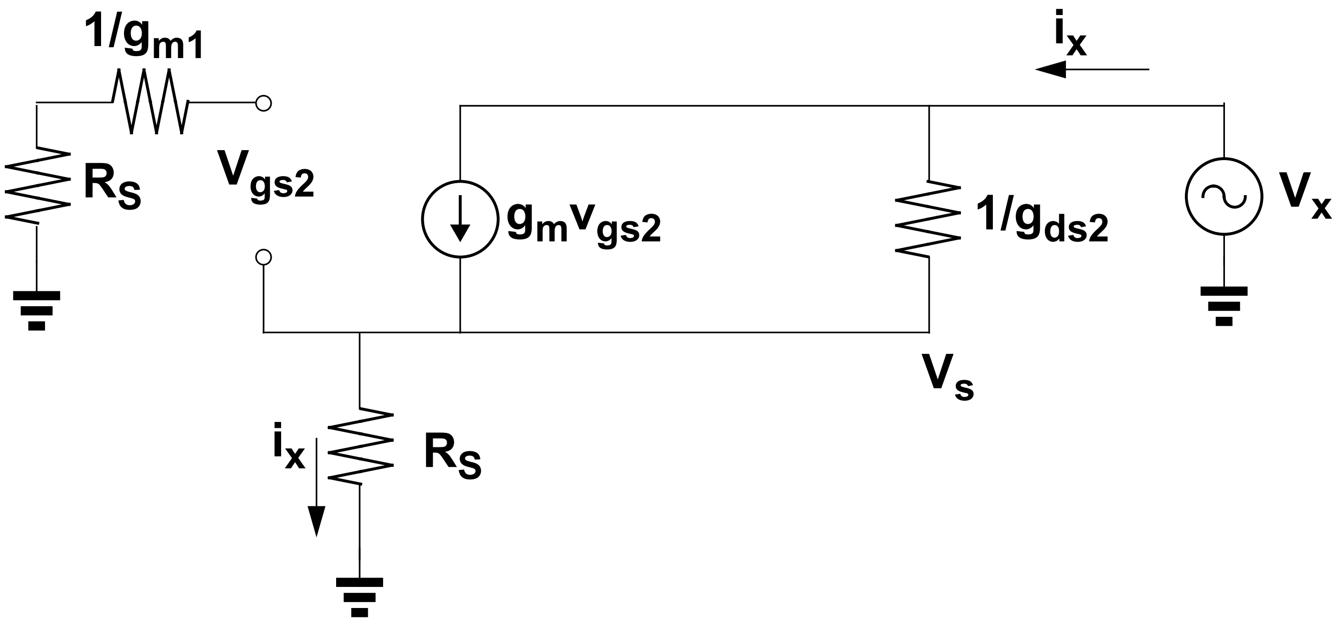
$$i_{out} = v_{out} g_{ds2}$$

$$g_{ds2} = \lambda I_D = 0.04V^{-1} \times 100\mu A = 4\mu A/V$$

$$i_{out} = v_{out} g_{ds2} = 10mV \times 4\mu A/V = 40nA$$

(iii) Derive an expression for the output resistance of the current mirror shown in Figure 2b in terms of the small-signal parameters of M2 and the resistance R_S?

Draw the small signal model, apply a test voltage at the output and calculate the current back into the circuit, recognising the gate of M2 is at ac ground



Connect input to ground, apply test voltage at output, derive current

$$i_x = g_{m2}v_{gs2} + (v_x - v_s)g_{ds2}$$

$$= -g_{m2}v_s + v_x g_{ds2} - v_s g_{ds2} \qquad v_{gs} = -v_{s2}$$

$$= g_{m2}i_x R_S + v_x g_{ds2} - i_x R_S g_{ds2} \qquad v_s = i_x R_S$$

$$i_x(1 + g_{m2}R_S + g_{ds2}R_S) = v_x g_{ds2}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1}{g_{ds2}}(1 + g_{m2}R_S + g_{ds2}R_S) \approx \underline{\underline{\frac{g_{m2}}{g_{ds2}}R_S + \frac{1}{g_{ds2}}}}$$

(iv) What is the variation in output current of the current mirror shown in Figure 2b if the voltage at the output node varies by 10mV?

$$i_{out} = \frac{v_{out}}{r_{out}}$$

$$r_{out} = \frac{g_{m2}}{g_{ds2}} R_S + \frac{1}{g_{ds2}}$$

$$g_{m2} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 100\mu A}{1.2 - 0.7} = 400\mu A/V$$

$$g_{ds2} = \lambda I_D = 0.04 V^{-1} \times 100\mu A = 4\mu A/V$$

$$r_{out} = \frac{g_{m2}}{g_{ds2}} R_S + \frac{1}{g_{ds2}} = \frac{400\mu A/V}{4\mu A/V} 10k\Omega + \frac{1}{4\mu A/V} = 1.25 M\Omega$$

$$i_{out} = \frac{v_{out}}{r_{out}} = \frac{10mV}{1.25 M\Omega} = 8nA$$

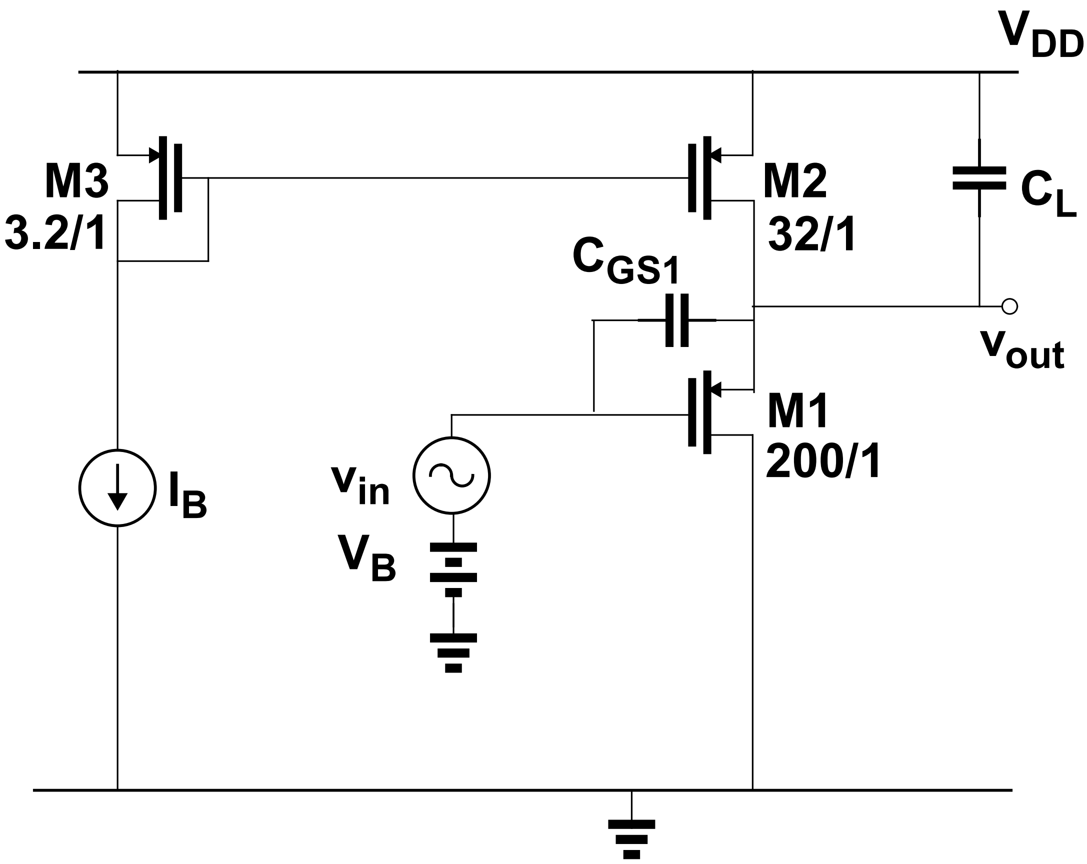


Figure 3

Figure 3 shows a pmos source follower. Assume all transistors are in saturation and g_{m1},g_{m2}>> g_{ds1},g_{ds2}.

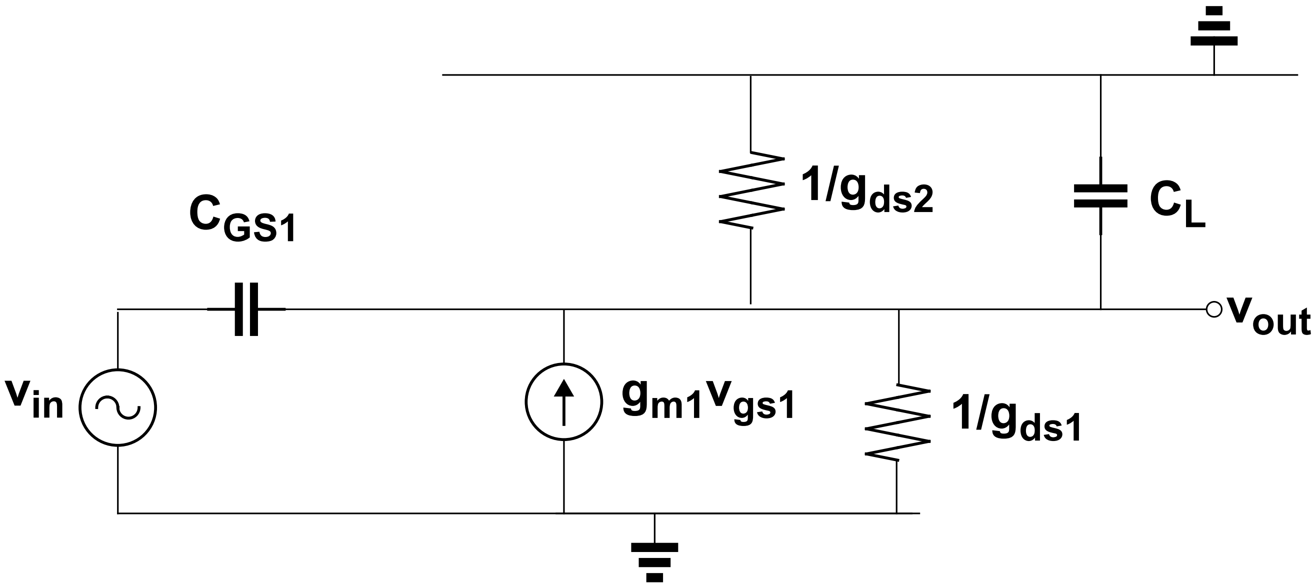
(i) Draw the small-signal equivalent circuit for the source follower stage shown in Figure 3.

(ii) Ignoring all capacitances except C_{GS1} and C_L derive an expression for the high frequency transfer function.

(iii) Calculate the dc gain in dB, and the pole and zero frequencies, if K_p'=50μA/V^2, I_B=20μA, C_{GS1}=1pF, C_L=9pF. W/L in μm for each transistor is as indicated in Figure 3.

(iv) Draw a Bode diagram of the gain. On the diagram indicate the pole and zero frequencies, the value of the dc gain in dB, and the value of the gain at frequencies well above the pole and zero frequencies.

(i) Draw the small-signal equivalent circuit for the source follower stage shown in Figure 3.



(ii) Ignoring all capacitances except C_{GS1} and C_L derive an expression for the high frequency transfer function.

$$v_{gs1} = v_{in} - v_{out}$$

KCL at output node

$$(v_{in} - v_{out})sC_{gs1} + g_{m1}(v_{in} - v_{out}) - (v_{out}g_{ds1}) - (v_{out}g_{ds2}) - v_{out}sC_L = 0$$

$$(g_{m1} + sC_{gs1})v_{in} = (g_{m1} + g_{ds1} + g_{ds2} + sC_{gs1} + sC_L)v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1} + sC_{gs1}}{g_{m1} + g_{ds1} + g_{ds2} + sC_{gs1} + sC_L}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{ds1} + g_{ds2}} \frac{\left(1 + \frac{sC_{gs1}}{g_{m1}}\right)}{\left(1 + \frac{s(C_{gs1} + C_L)}{g_{m1} + g_{ds1} + g_{ds2}}\right)}$$

(iii) Calculate the dc gain in dB, and the pole and zero frequencies, if $K_p'=50\mu A/V^2$, $I_B=20\mu A$, $C_{GS1}=1pF$, $C_L=9pF$. W/L in μm for each transistor is as indicated in Figure 3..

DC gain given by

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1}+g_{ds1}+g_{ds2}} \approx 1 = 0dB$$

Pole frequency given by

$$|\omega_p| = \frac{g_{m1}+g_{ds1}+g_{ds2}}{(C_{gs1}+C_L)} \approx \frac{g_{m1}}{(C_{gs1}+C_L)}$$

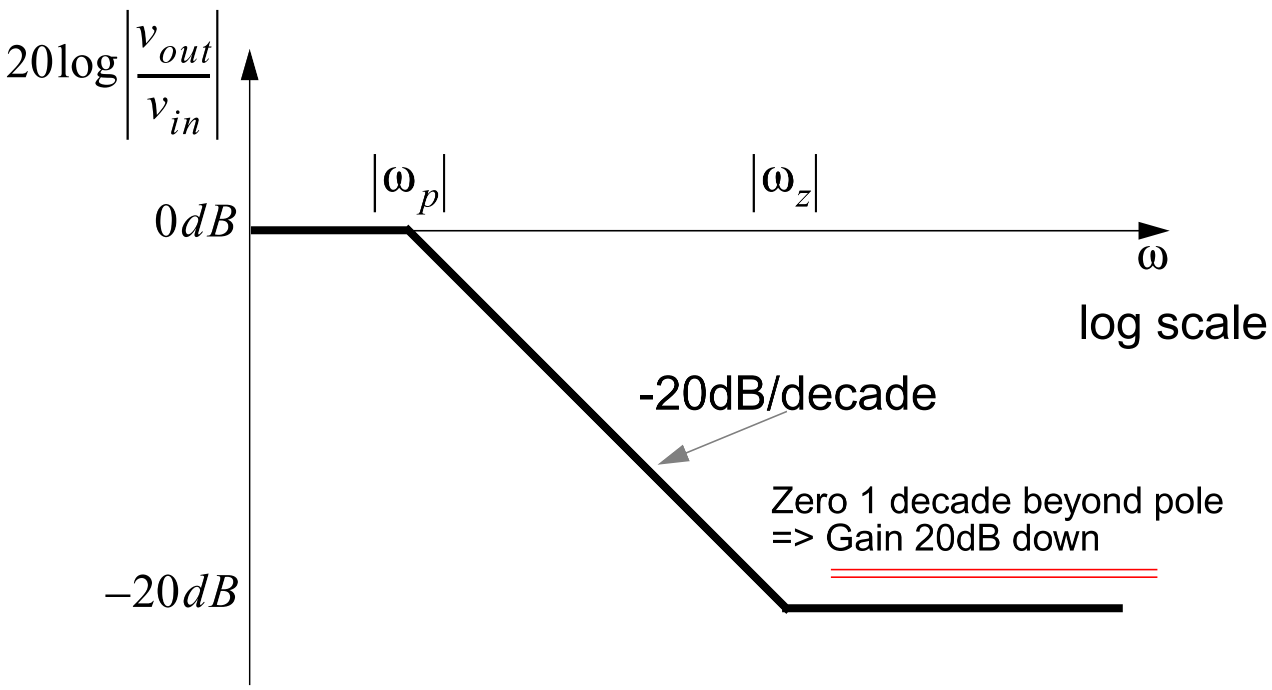
$$g_{m1} = \sqrt{2K_p'\frac{W}{L}I_D} = \sqrt{2\times 50\mu A/V\times\frac{200}{1}\times 200\mu A} = 2000A/V$$

$$|\omega_p| \approx \frac{2000\mu A/V}{1pF+9pF} = 200Mrad/s$$

Zero frequency given by

$$|\omega_z| = \frac{g_{m1}}{C_{gs1}} = \frac{2000\mu A/V}{1pF} = 2Grad/s$$

(iv) Draw a Bode diagram of the gain. On the diagram indicate the pole and zero frequencies, the value of the dc gain in dB, and the value of the gain at frequencies well above the pole and zero frequencies..



Question 4

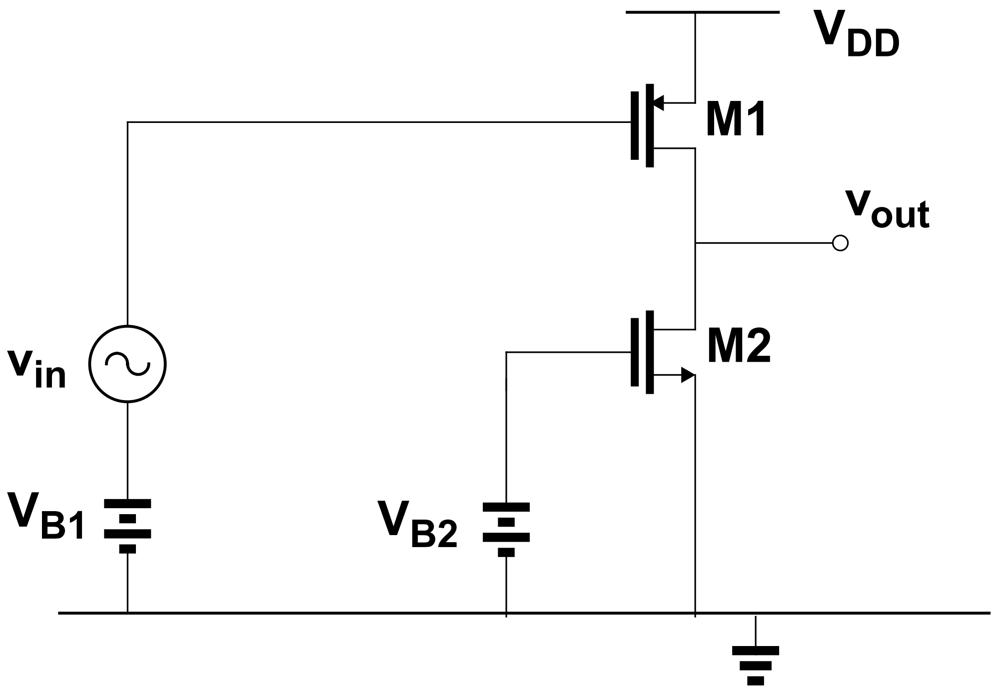


Figure 4

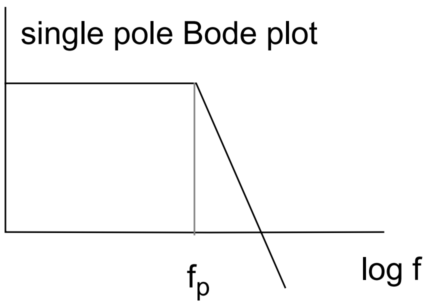
Assume M1 and M2 are operating in saturation. Only thermal noise sources need be considered.

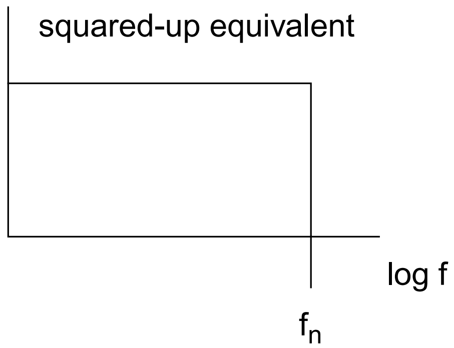
(i) What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 4?

(ii) What is the input-referred thermal noise voltage in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T?

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:





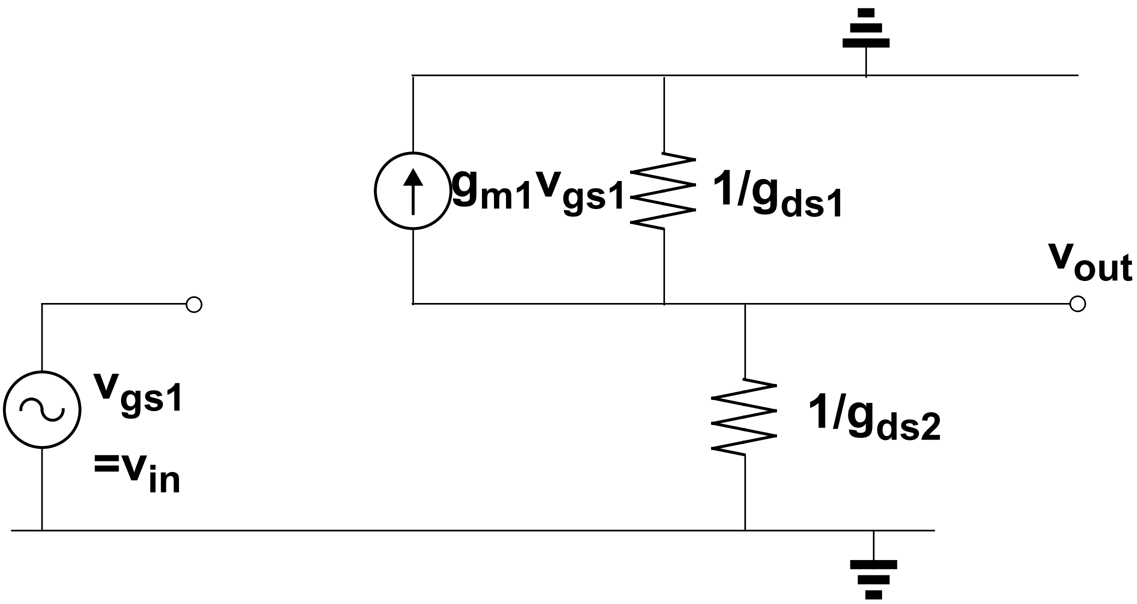
For the area underneath the curves to be the same then $f_n=(\pi/2)*f_p$

(iv) It is desired to limit the bandwidth such that a signal-to noise ratio of 40dB is achieved at the output, when the input is a 1mVrms sine wave with a frequency much lower than the frequency of the pole at the output node. Using the result of (iii) calculate the minimum value of C_L required.
For this calculation take V_{B1} = 2.0V, V_{B2} = 1.0V, V_{DD} = 3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n = \lambda_p$= 0.04V^{-1}.
The drain current of M1 is 100μA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

Solution

(i) What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 4?

Write the result directly i.e. g_{m1} divided by the conductance at the output node or use the small-signal model.

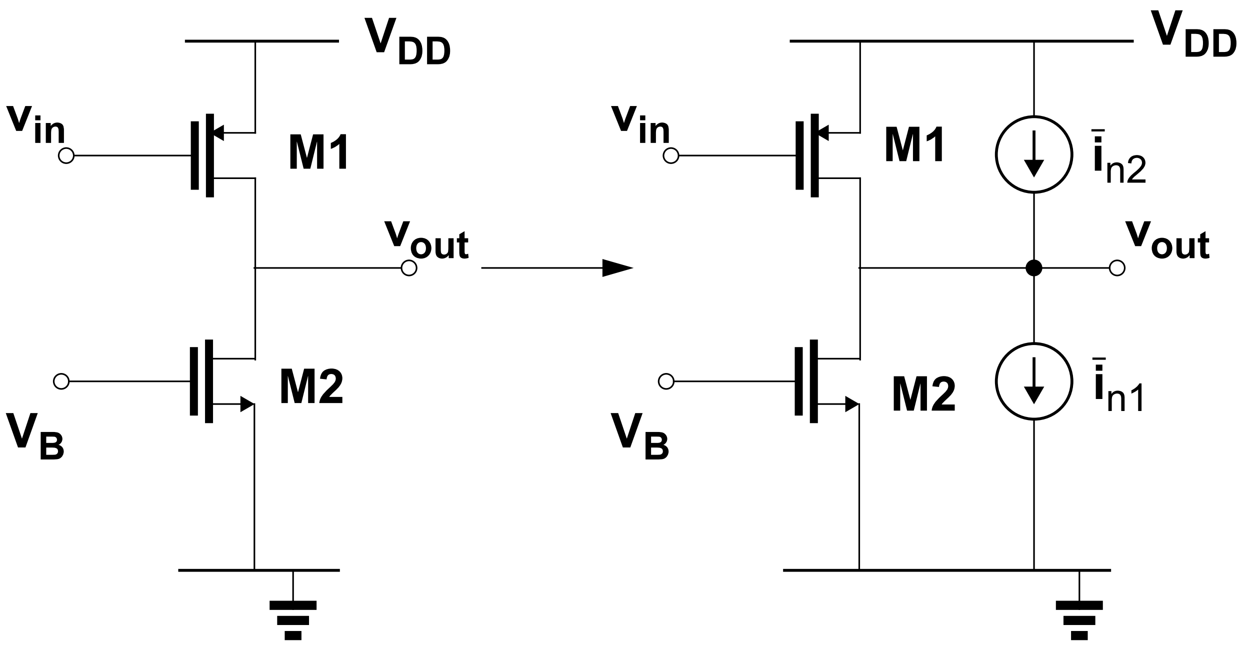


Current at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

(ii) What is the input-referred thermal noise voltage in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T?



Noise current of MOS: $\overline{i_n^2} = 4kT\left(\frac{2}{3}g_m\right)$

Noise sources uncorrelated => total noise is the sum of squares

$$\overline{i_{nt}^2} = i_{n1}^2 + i_{n2}^2 \quad \text{or} \quad \overline{i_{nt}} = \sqrt{i_{n1} + i_{n2}} \quad \text{rms value}$$

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \quad \text{rms noise at input}$$

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \sqrt{4kT \cdot \frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)}$$

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

To get voltage noise at output multiply input-referred noise by gain of circuit

$$\overline{v_{no}} = \overline{v_{ni}}\frac{g_{m1}}{g_{ds1}+g_{ds2}}$$

$$\overline{v_{no}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right)+4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{ds1}+g_{ds2}}$$

To get total noise voltage at output need to integrate this over all frequencies

The circuit is first-order circuit with a pole at

$$\omega_o = -\frac{g_{ds1}+g_{ds2}}{C_L}$$

$$\overline{v^2_{nototal}} = \int_0^\infty\left(\frac{4kT\left(\frac{2}{3}g_{m1}\right)+4kT\left(\frac{2}{3}g_{m2}\right)}{(g_{ds1}+g_{ds2})^2}\cdot\frac{1}{1+\frac{C_L^2}{(g_{ds1}+g_{ds2})^2}\cdot(2\pi f)^2}\right)df$$

This is equal to multiplying by the noise bandwidth

$$v^2_{nototal} = v^2_{no}\cdot\frac{\pi}{2}\cdot f_o = \frac{4kT\left(\frac{2}{3}g_{m1}\right)+4kT\left(\frac{2}{3}g_{m2}\right)}{(g_{ds1}+g_{ds2})^2}\cdot\frac{\pi}{2}\cdot\frac{g_{ds1}+g_{ds2}}{2\pi C_L}$$

$$= \left(\frac{\frac{2}{3}(g_{m1}+g_{m2})}{g_{ds1}+g_{ds2}}\cdot\frac{kT}{C_L}\right)$$

(iv) It is desired to limit the bandwidth such that a signal-to noise ratio of 40dB is achieved at the output, when the input is a 1mVrms sine wave with a frequency much lower than the frequency of the pole at the output node. Using the result of (iii) calculate the minimum value of C_L required.
For this calculation take V_{B1} = 2.0V, V_{B2} = 1.0V, V_{DD} = 3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n = \lambda_p$= 0.04V^{-1}.
The drain current of M1 is 100µA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

g_m given by

$$g_m = \frac{2I_D}{(|V_{GS}|-|V_t|)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V \qquad g_{m2} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V$$

$$g_{ds1} = \lambda_n I_D = 0.04V^{-1}100\mu A = 4\mu A/V$$

$$g_{ds2} = \lambda_n I_D = 0.04V^{-1}100\mu A = 4\mu A/V$$

Output signal

$$v_{out} = -\left(\frac{g_{m1}}{g_{ds1}+g_{ds2}}\right)v_{in} = -\frac{800\mu A/V}{8\mu A/V} \cdot 1mV_{rms} = 100mV_{rms}$$

SNR of 40dB => Total noise at output = 1mVrms:

This requires C_L:

$$C_L = \frac{\frac{2}{3}(g_{m1}+g_{m2})}{g_{ds1}+g_{ds2}} \cdot \frac{kT}{v_{nototal}^2}$$

$$C_L = \frac{2}{3}\left(\frac{800\mu A/V + 800\mu A/V}{4\mu A/V + 4\mu A/V}\right) \cdot \frac{1.38\times10^{-23}300}{1mV^2} = \underline{\underline{0.55pF}}$$