Solutions UE4010 Summer 2005 Part A

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an nmos in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{tn})^2(1+\lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n=\lambda_p=0$.
In each question, capacitances other than those mentioned may be ignored.

Question 1

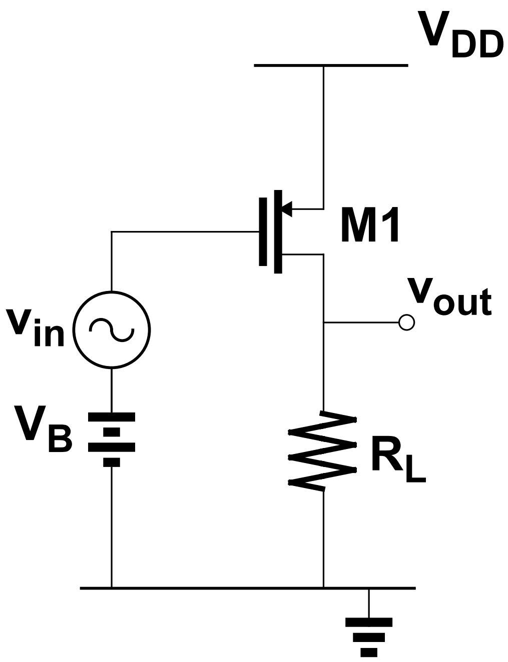


Figure 1a

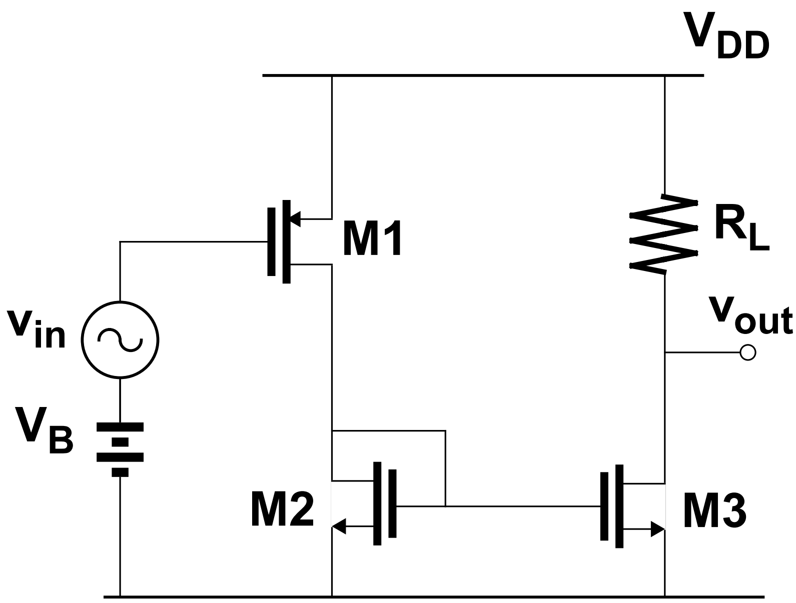
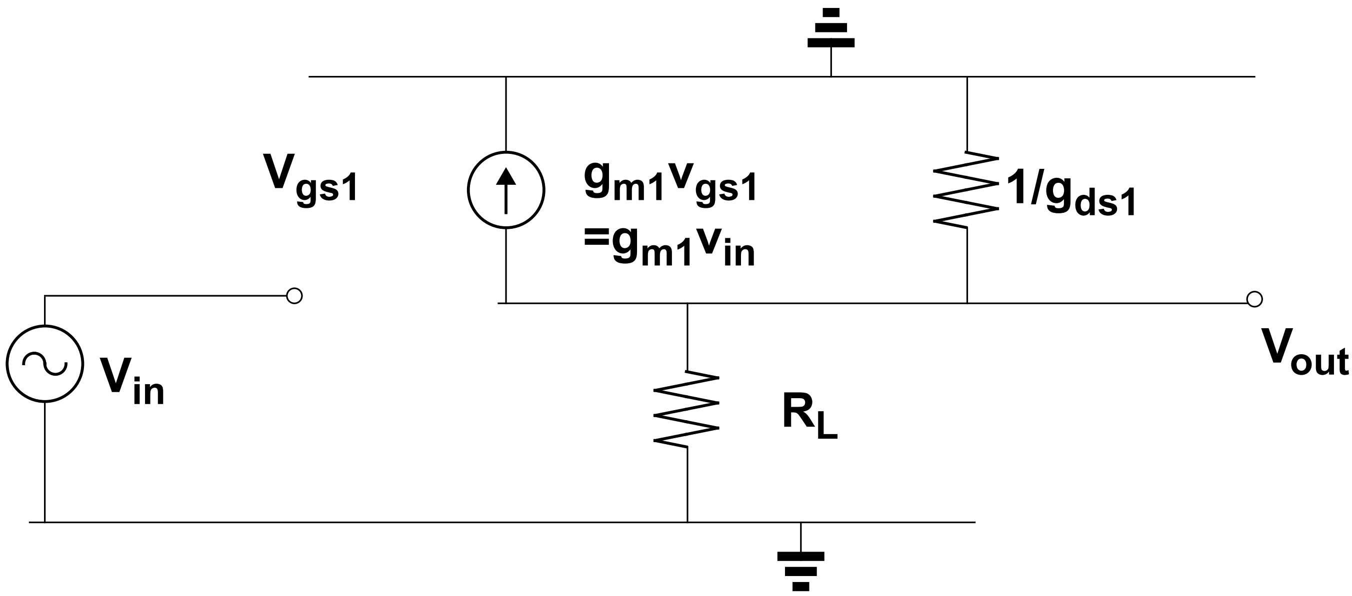


Figure 1b

(i) Draw the small-signal equivalent circuit for the gain stage shown in Figure 1a.
(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters and R_L.
(iii) What is the largest value of gain in dB that can be achieved by increasing R_L?
Assume V_{DD}=3V, V_B = 1.7V, V_{tp}= -0.8V. K_p'=50µA/V^2, W_1=20µm, L_1=1µm, R_L<<$1/g_{ds1}$.
(iv) In Figure 1b, M3 has twice the width of M2 but the same length. Give an expression for the small-signal voltage gain (v_{out}/v_{in}) of this circuit in terms of the small-signal parameters and R_L .

Solution

(i) Draw the small-signal equivalent circuit for the gain stage shown in Figure 1a.



(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters and R_L.

Current at output node

$$g_{m1}v_{gs} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + \frac{1}{R_L}} \cong -g_{m1}R_L$$

(iii) What is the largest value of gain in dB that can be achieved by increasing R_L?
Assume V_{DD}=3V, V_B = 1.7V, V_{tp}= -0.8V. K_p'=50μA/V^2, W_1=20μm, L_1=1μm, R_L<<1/g_{ds1}.

R_L can be increased until the voltage at V_{out} forces M1 out of saturation.
For M1 to be still in saturation

$$V_{DD} - V_{OUT} \geq |V_{GS} - V_t|$$

$$V_{OUTmax} = V_{DD} - ||V_{GS}| - |V_{tp}|| = 3 - (1.3 - 0.8) = 2.5V$$

$$I_D R_{Lmax} = V_{OUTmax}$$

$$R_{Lmax} = \frac{V_{OUTmax}}{I_{D1}}$$

$$I_{D1} = \frac{K_p'}{2}\frac{W}{L}(|V_{GS1}| - |V_{tp}|)^2 = \frac{50\mu A/V^2}{2}\frac{20}{1}(1.3 - 0.8)^2 = 125\mu A$$

$$R_{Lmax} = \frac{2.5V}{125\mu A} = 20k\Omega$$

$$\left(\frac{v_{out}}{v_{in}}\right)_{max} = -g_m R_{Lmax}$$

$$g_{m1} = \frac{2I_D}{(|V_{GS1}| - |V_{tp}|)} = \frac{2 \times 125\mu A}{1.3 - 0.8} = 500\mu A/V$$

$$\left(\frac{v_{out}}{v_{in}}\right)_{max} = -g_m R_L = -500\mu A/V \times 20k\Omega = -10 = \underline{\underline{20dB}}$$

(iv) In Figure 1b, M3 has twice the width of M2 but the same length. Give an expression for the small-signal voltage gain (v_{out}/v_{in}) of this circuit in terms of the small-signal parameters and R_L .

The gain of the original circuit was gm times the resistance at the output node. In Figure 1b the current out of M1 is mirrored with 1:2 ratio to the output, so that the gain becomes gm times the mirroring ratio times the resistance at the output node.

$$\frac{v_{out}}{v_{in}} \cong -g_{m1} \times 2 \times R_L = \underline{\underline{2g_{m1}R_L}}$$

.Question 2

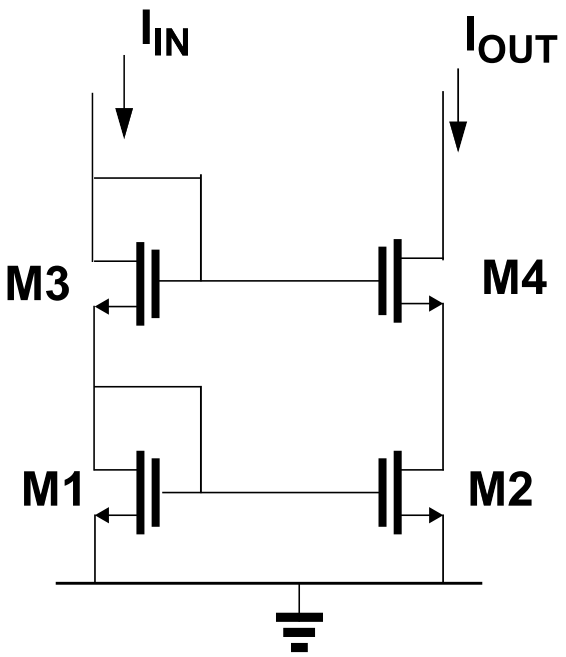


Figure 2

Figure 2 shows a cascoded current mirror.

Assume $I_{IN}=I_{OUT}=100\mu A$, $K_n'=200\mu A/V^2$, $V_{tn}=750mV$, $\lambda_n = 0.04V^{-1}$.

All transistors have W/L=16/1.

(i) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?

(ii) Derive an expression for the small-signal output resistance.
Assume $g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}$.

(iii) What is the change in current if the voltage at the output node varies by 10mV?
Assume all transistors are in saturation.

(iv) It is desired to increase the mirroring ratio by increasing the width of M2 only. What is the largest value of output current such that M2 remains in saturation?

(i) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?

The minimum voltage at the output is given by the voltage at the drain of M2 plus the required V_{DS} across M4 for it to be in saturation i.e. V_{GS4}-V_t

For all transistors

$$|V_{GS} - V_t| = \sqrt{\frac{2I_{D1}}{K_n' \frac{W}{L}}} = \sqrt{\frac{2 \cdot 100\mu A}{200\mu A/V^2 \frac{16}{1}}} = 250mV \Rightarrow V_{GS} = 1V$$

$$V_{OUTmin} = V_{D1} + |V_{GS4} - V_t|$$

$$V_{OUTmin} = V_{GS1} + V_{GS3} - V_{GS2} + |V_{GS4} - V_t| = \underline{\underline{1.25V}}$$

(ii) Derive an expression for the small-signal output resistance.
Assume $g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}$.

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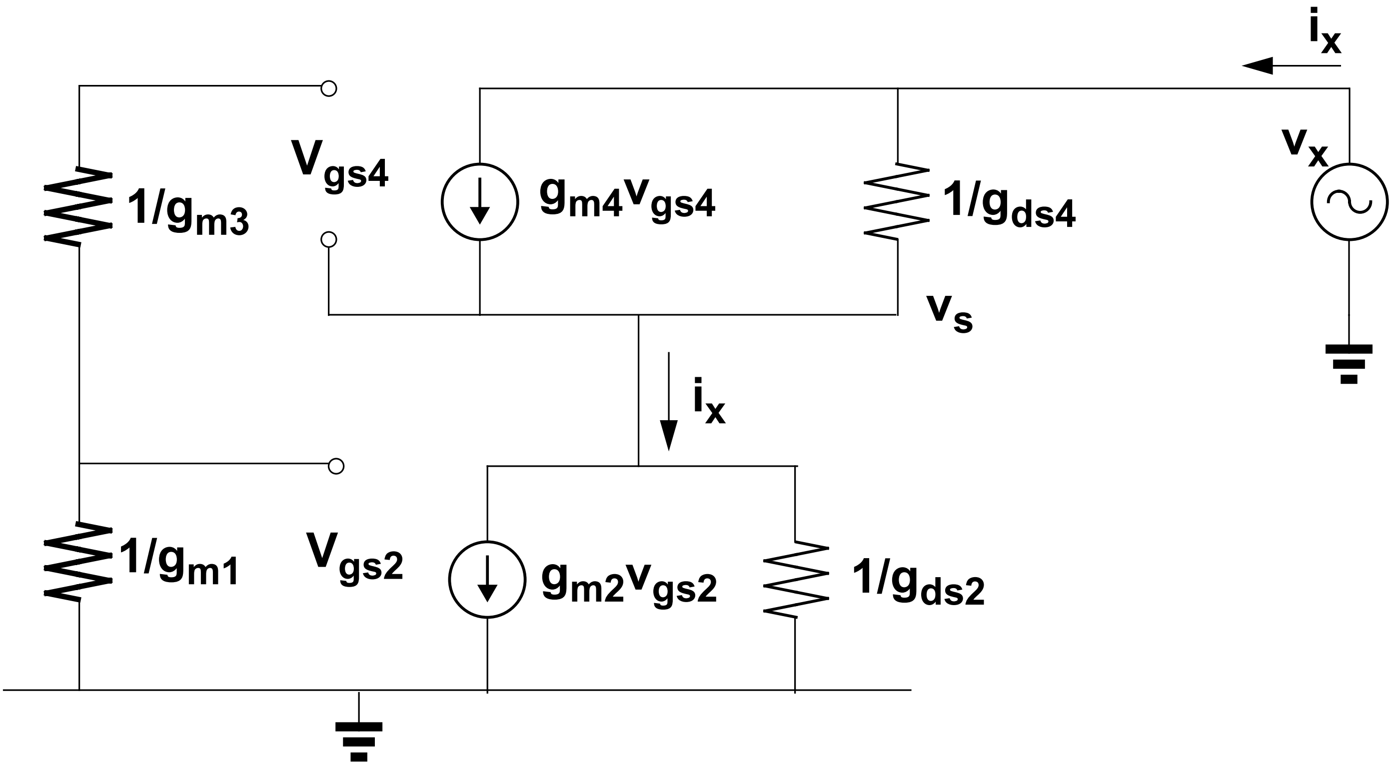


Note: $v_{gs2} = 0 \Rightarrow g_{m2}v_{gs2} = 0$

$$i_x = g_{m4}v_{gs4} + (v_x - v_s)g_{ds4}$$

Since $v_{gs4} = -v_s$ and $v_s = \frac{i_x}{g_{ds2}}$

$$i_x = -(g_{m4})\frac{i_x}{g_{ds2}} + \left(v_x - \frac{i_x}{g_{ds2}}\right)g_{ds4}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1 + \frac{g_{m4}}{g_{ds2}} + \frac{g_{ds4}}{g_{ds2}}}{g_{ds4}}$$

Since $g_{m2}, g_{m4} >> g_{ds2}, g_{ds4}$ this can be reduced to

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

(iii) What is the change in current if the voltage at the output node varies by 10mV? Assume all transistors are in saturation.

$$i_{out} = \frac{v_{out}}{r_{out}}$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

$$g_{m4} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 100\mu A}{0.25} = 800\mu A/V$$

$$g_{ds2} = \lambda I_{D2} = 0.04V^{-1} \times 100\mu A = 4\mu A/V$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}} = \frac{800\mu A/V}{4\mu A/V} \frac{1}{4\mu A/V} = 50M\Omega$$

$$i_{out} = \frac{v_{out}}{r_{out}} = \frac{10mV}{50M\Omega} = 0.2nA$$

(iv) It is desired to increase the mirroring ratio by increasing the width of M2 only. What is the largest value of output current such that M2 remains in saturation?

If the current of M2 is increased by increasing the width of M2 only, then its V_{GS}-Vt will remain the same.
However V_{GS4} will increase, reducing V_{D4}, until M2 goes out of saturation
This determines V_{GS4max} which determines the max. current.

$$V_{S4min} = V_{D2min} = V_{GS2} - V_t = 250mV$$

$$V_{GS4max} = V_{G4} - V_{S4min} = 2 - 0.25V = 1.75V$$

$$I_{D4max} = \frac{K'_n}{2}\frac{W}{L}(V_{GS4} - V_t)^2 = \frac{200\mu A/V^2}{2}\frac{16}{1}(1.75 - 0.75)^2 = 1.6mA$$

Question 3

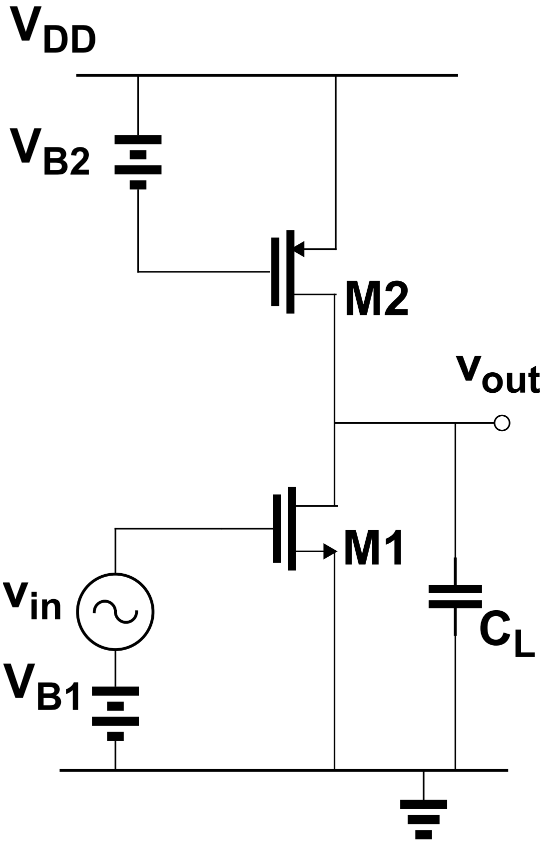


Figure 3a

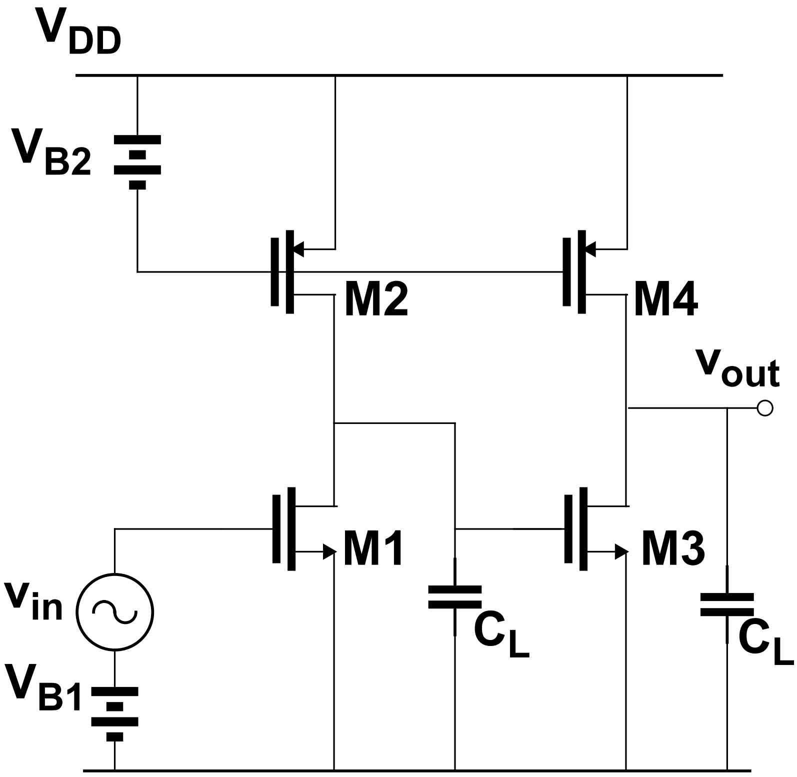
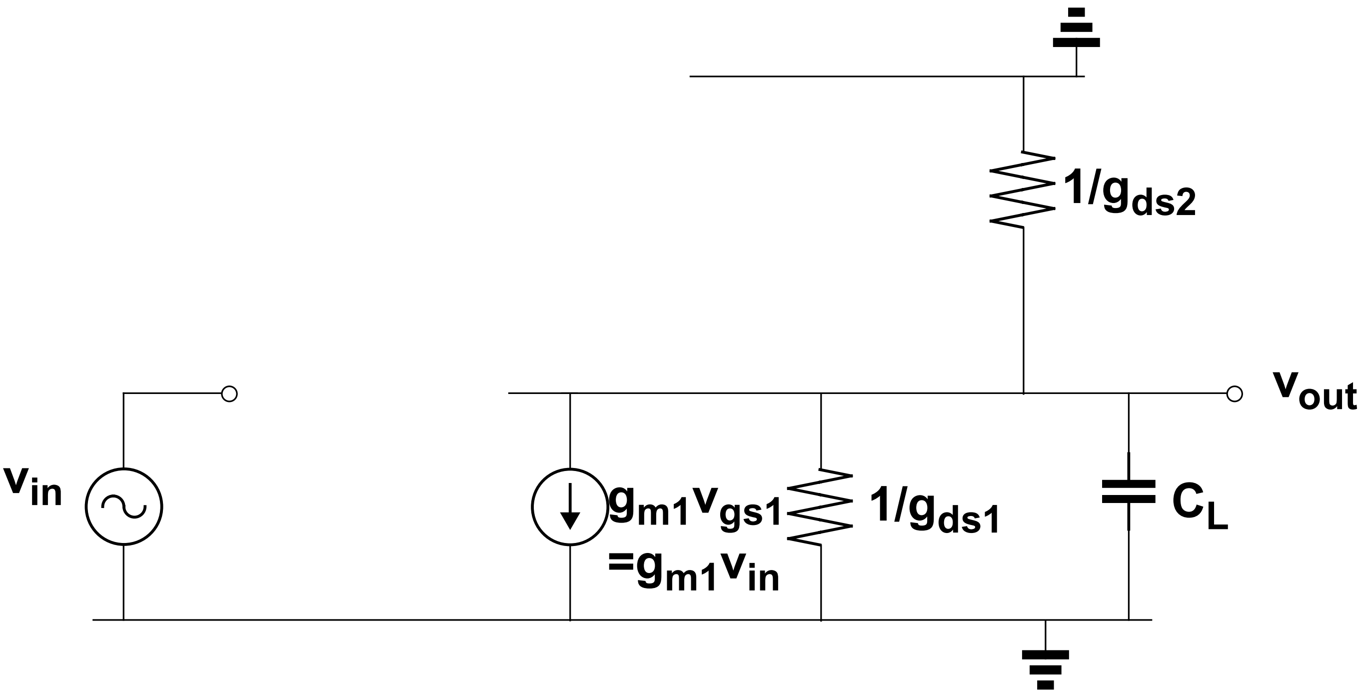


Figure 3b

(i) Draw the small-signal equivalent circuit for the gain stage shown in Figure 3a.
(ii) Ignoring all capacitances except C_L, derive an expression for the high frequency transfer function (v_{out}/v_{in}).
(iii) Draw a Bode diagram of the gain. Indicate the values of the dc gain in dB, the pole frequency and the high-frequency roll-off if
V_{B1}=1.1V, V_{tn}=0.7V, V_{tp}=-0.7V, $\lambda_n=\lambda_p$=0.02V^{-1}, C_L=1pF.
Assume both transistors are in saturation with a drain current of 200μA.
(iv) The circuit shown in Figure 3a is cascaded with an identical stage with an identical load capacitance as shown in Figure 3b . Draw the Bode diagram of the gain of the cascaded circuit, indicating the values of the dc gain in dB, the pole frequency and the high-frequency roll-off.

(i) Draw the small-signal equivalent circuit for the gain stage shown in Figure 3a.



(ii) Ignoring all capacitances except C_L, derive an expression for the high frequency transfer function (v_{out}/v_{in}).

KCL at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} + v_{out}sC_L = 0$$

$$g_{m1}v_{in} = -(g_{ds1} + g_{ds2} + sC_L)v_{out}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2} + sC_L}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}\left(1 + \frac{sC_L}{g_{ds1} + g_{ds2}}\right)}$$

(iii) Draw a Bode diagram of the gain. Indicate the values of the dc gain in dB, the pole frequency and the high-frequency roll-off if
V_{B1}=1.1V, V_{tn}=0.7V, V_{tp}=-0.7V, λ_n=λ_p=0.02V^{-1}, C_L=1pF.
Assume both transistors are in saturation with a drain current of 200µA.

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}\left(1 + \frac{sC_L}{g_{ds1} + g_{ds2}}\right)}$$

DC Gain given by

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

$$g_{m1} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 200\mu A}{1.1 - 0.7} = 1mA/V$$

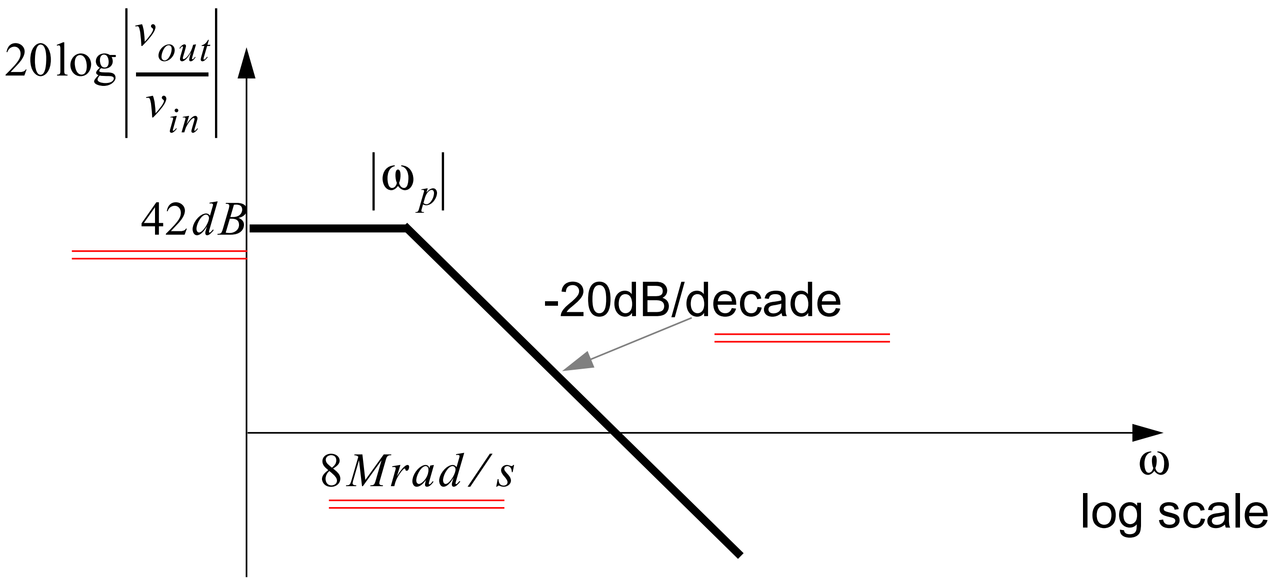
$$g_{ds1} = \lambda I_D = 0.02V^{-1} \times 200\mu A = 4\mu A/V$$

$$g_{ds2} = \lambda I_D = 0.02V^{-1} \times 200\mu A = 4\mu A/V$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}} = \frac{1mA/V}{4\mu A/V + 4\mu A/V} = 125 \Rightarrow 42dB$$

Pole frequency given by

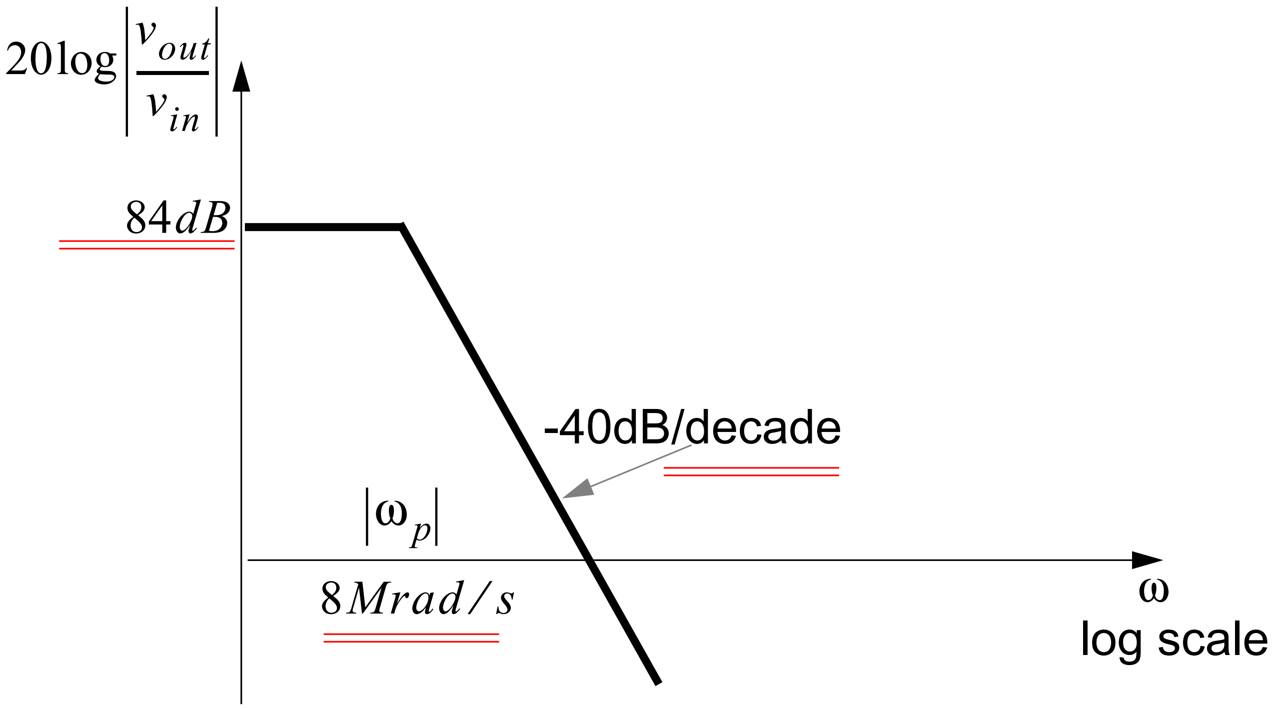
$$|\omega_p| = \frac{g_{ds1} + g_{ds2}}{C_L} = \frac{4\mu A/V + 4\mu A/V}{1pF} = 8Mrad/s$$



(iv) The circuit shown in Figure 3a is cascaded with an identical stage with an identical load capacitance as shown in Figure 3b . Draw the Bode diagram of the gain of the cascaded circuit, indicating the values of the dc gain in dB, the pole frequency and the high-frequency roll-off.

Gain given by

$$\frac{v_{out}}{v_{in}} = \left(-\frac{g_{m1}}{g_{ds1}+g_{ds2}}\right)^2 = 125^2 \Rightarrow 84dB$$



Question 4

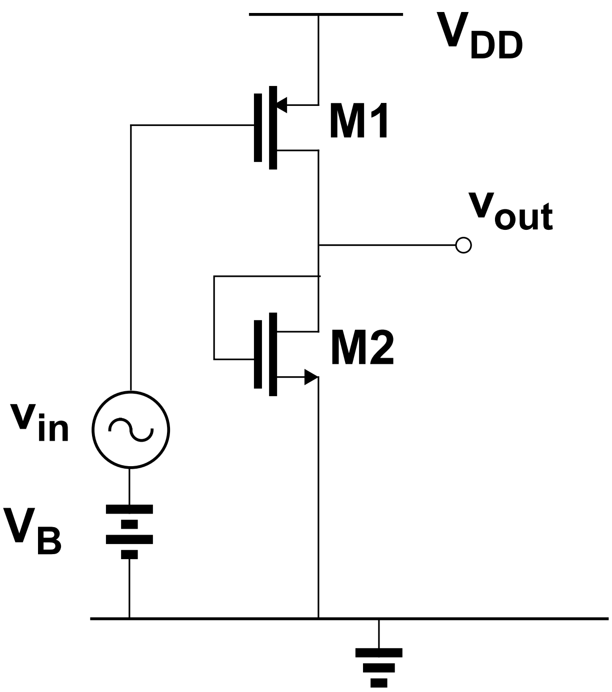


Figure 4

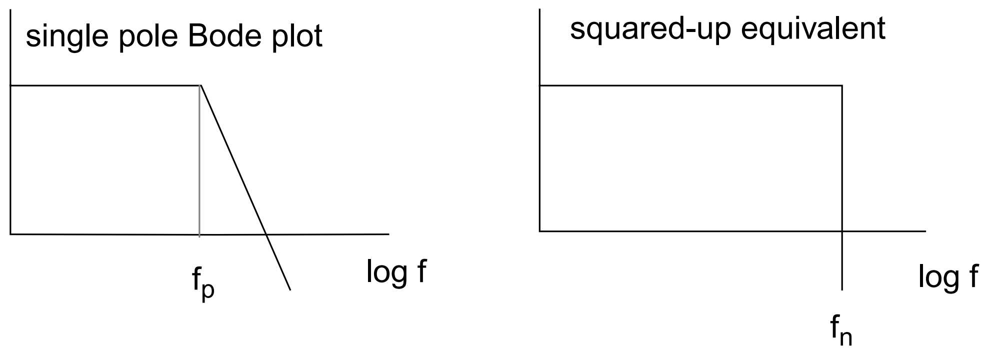
Assume M1 and M2 are operating in saturation.

(i) What is the low-frequency small signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 4? Assume that $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.

(ii) What is the input-referred thermal noise voltage in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T?

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:

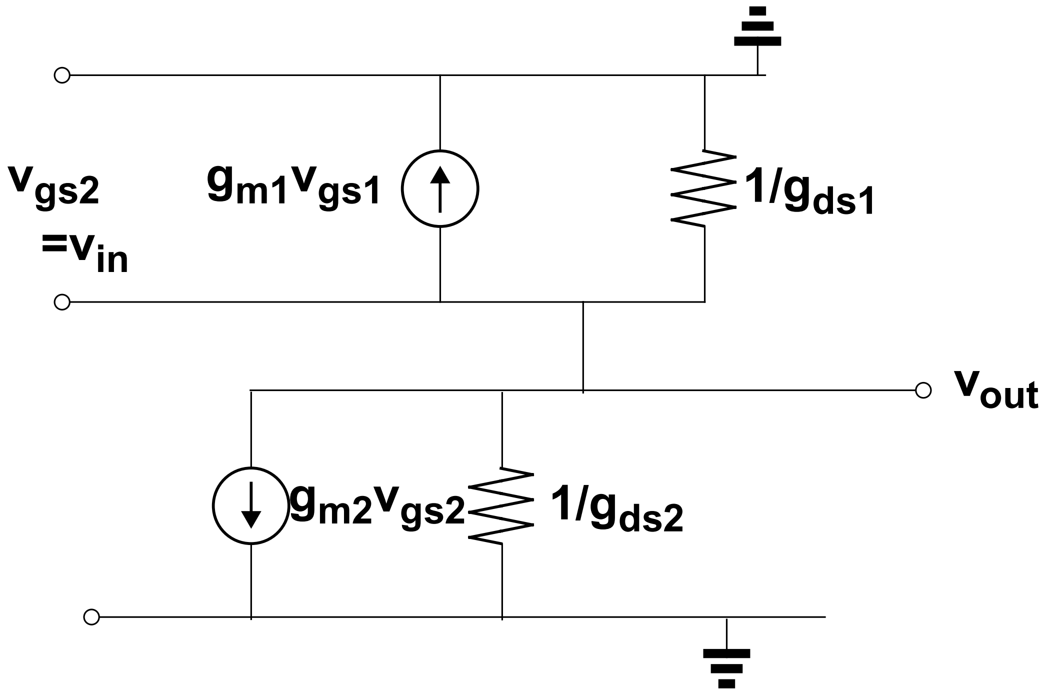


For the area underneath the curves to be the same then $f_n=(\pi/2)*f_p$

(iv) It is desired to limit the bandwidth such that a signal-to noise ratio of 60dB is achieved at the output, when the input is a 1mVrms sine wave with a frequency much lower than the frequency of the pole at the output node. Using the result of (iii) calculate the minimum value of C_L required.
For this calculation take $|V_{GS1}|$ = 1V, V_{GS2} = 2.8V, V_{tn} = 0.8V, V_{tp} = -0.8V.
The drain current of M1 is 100µA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K

Solution

(i) What is the low-frequency small signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 4? Assume that $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.



Assume that $g_{m1} >> g_{ds1}, g_{ds2}$ and that $g_{m2} >> g_{ds1}, g_{ds2}$

Current at output node

$$g_{m1}v_{gs1} + g_{m2}v_{gs2} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

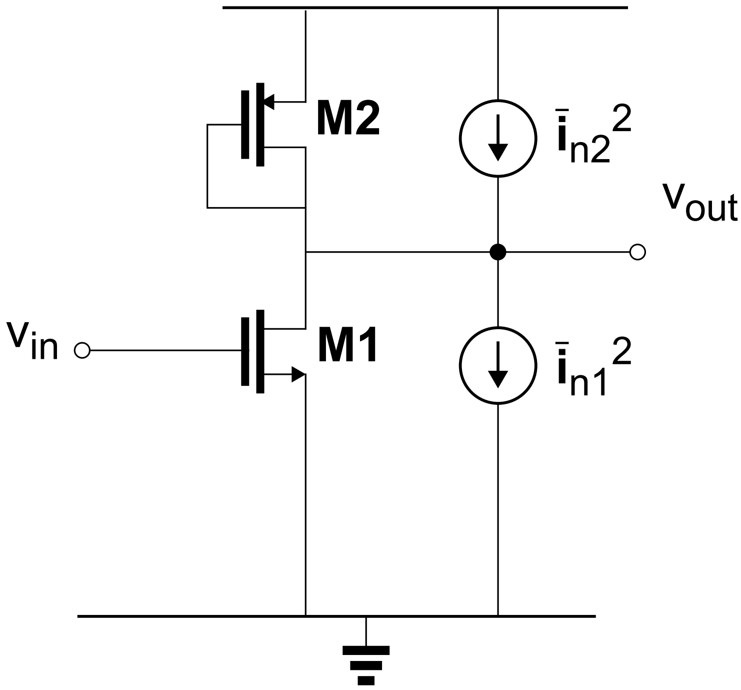
$$g_{m1}v_{in} + g_{m2}v_{out} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + g_{ds1} + g_{ds2}} \cong -\frac{g_{m1}}{g_{m2}}$$

Alternatively recognise that the current of the current-source $g_{m2}v_{gs2}$ is determined by voltage across its terminals i.e. is equivalent to a resistance $1/g_{m2}$.
Since $1/g_{m2} << 1/g_{ds2}$, $1/g_{m2} << 1/g_{ds1}$, can write directly

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

(ii) What is the input-referred thermal noise voltage in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T?



Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

Noise voltage at output given by input referred noise multiplied by gain

$$\overline{v_{no}} = \overline{v_{ni}}\frac{g_{m1}}{g_{m2}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}}\frac{g_{m1}}{g_{m2}}$$

$$= \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m2}}$$

Capacitor C_L connected between the output node and ground
=> pole at output node given by

$$|f_p| = \frac{g_{m2}}{2\pi C_L}$$

Total integrated thermal noise power at the output node is given by the product of the thermal noise power and the squared-up equivalent of the first order filter function

$$\overline{v^2_{nototal}} = \overline{v^2_{no}}\frac{\pi}{2}f_p$$

$$\overline{v^2_{nototal}} = \frac{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}{g^2_{m2}} \cdot \frac{\pi}{2} \cdot \frac{g_{m2}}{2\pi C_L}$$

$$\overline{v^2_{nototal}} = \frac{\frac{2}{3}(g_{m1} + g_{m2})}{g_{m2}} \cdot \frac{kT}{C_L}$$

(iv) It is desired to limit the bandwidth such that a signal-to noise ratio of 60dB is achieved at the output, when the input is a 1mVrms sine wave with a frequency much lower than the frequency of the pole at the output node. Using the result of (iii) calculate the minimum value of C_L required.
For this calculation take $|V_{GS1}| = 1V$, $V_{GS2} = 2.8V$, $V_{tn} = 0.8V$, $V_{tp} = -0.8V$.
The drain current of M1 is 100µA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K

g_m given by

$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.8V} = 1mA/V$$

$$g_{m2} = \frac{2 \cdot 100\mu A}{2.8V - 0.8V} = 100\mu A/V$$

Output signal

$$v_{out} = -\frac{g_{m1}}{g_{m2}} v_{in} = -10 \cdot 10mV_{rms} = 100mV_{rms}$$

SNR of 60dB => Total noise at output = 0.1mVrms:

This requires C_L:

$$C_L = \frac{\frac{2}{3}(g_{m1} + g_{m2})}{g_{m2}} \cdot \frac{kT}{v_{nototal}^2}$$

$$C_L = \frac{\frac{2}{3}(1mA/V + 100\mu A/V)}{100\mu A/V} \cdot \frac{1.38\times10^{-23}300}{0.1mV^2} = 3pF$$