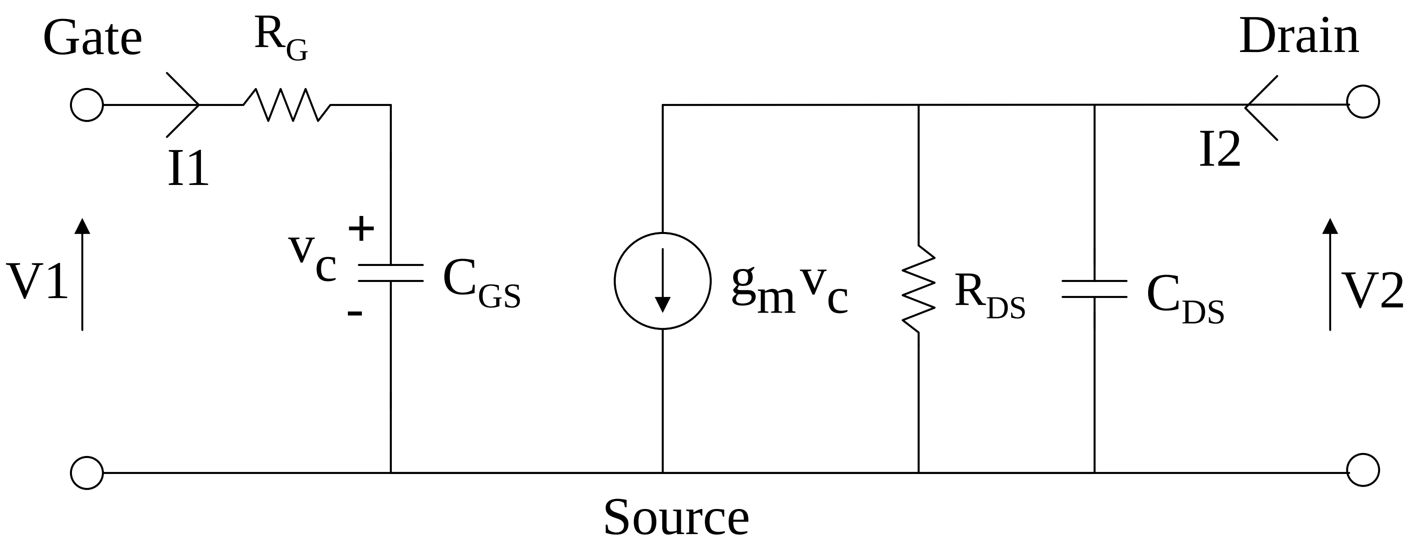
EE4011 RF IC Design Summer 2007
Question 1(a)

Simplified MESFET small-signal circuit, ignoring C_{GD}



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the above formulas to the equivalent circuit and simplifying the resulting expressions leads to the final y-parameter formulas:

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \qquad y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

$$y_{12} = 0 \qquad y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

10 marks

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Question 1(b)

Applying the formulas of the previous page to the case where

f=1GHz R_G = 6Ω, C_{GS} = 0.8pF, g_m = 0.15S, R_{DS} = 50Ω, C_{DS} = 0.3pF

gives

$$y_{11} = 0.0075\angle 87.4°$$
$$y_{12} = 0$$
$$y_{21} = 0.1498 \angle -2.6°$$
$$y_{22} = 0.0202 \angle 8.05°$$

4 marks

Question 1(c)

(i) Input reflection coefficient

$$Z_{11} = \frac{1}{Y_{11}} \quad \Gamma_{IN} = \frac{Z_{11} - Z_0}{Z_{11} + Z_0} \qquad \Gamma_{IN} = 0.97\angle -41°$$

3 marks

(i) Output reflection coefficient

$$Z_{22} = \frac{1}{Y_{22}} \quad \Gamma_{OUT} = \frac{Z_{22} - Z_0}{Z_{22} + Z_0} \qquad \Gamma_{OUT} = 0.07\angle -94°$$

3 marks

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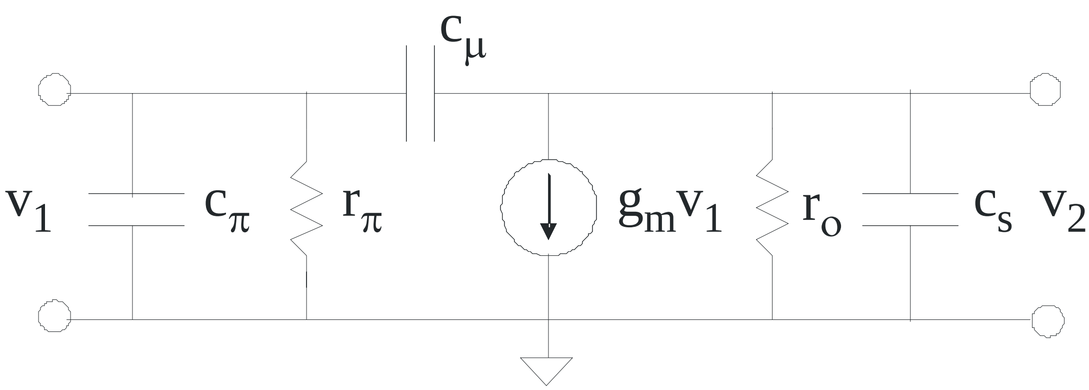
Question 2(a)



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the y-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$y_{11} = \frac{1}{r_\pi} + j\omega\left(c_\pi + c_\mu\right)$$

$$y_{12} = -j\omega c_\mu$$

$$y_{21} = g_m - j\omega c_\mu$$

$$y_{22} = \frac{1}{r_o} + j\omega\left(c_s + c_\mu\right)$$

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Question 2(a) continued

The expressions on the previous page have to be manipulated to express the circuit element values in terms of the y-parameters. The final result of this manipulation is as follows:

$$g_m = \mathbf{Re}(y_{21}) = 0.2S$$

$$r_\pi = \frac{1}{\mathbf{Re}(y_{11})} = 600\Omega$$

$$r_o = \frac{1}{\mathbf{Re}(y_{22})} = 5k\Omega$$

$$c_\mu = \frac{-\mathbf{Im}(y_{12})}{2\pi f} = 0.5pF$$

$$c_\pi = \frac{\mathbf{Im}(y_{11})}{2\pi f} - c_\mu = 3pF$$

$$c_s = \frac{\mathbf{Im}(y_{22})}{2\pi f} - c_\mu = 1pF$$

16 marks

Question 2(b)

$$f_T = \frac{g_m}{2\pi(c_\pi + c_\mu)} = 9.1GHz$$

4 marks

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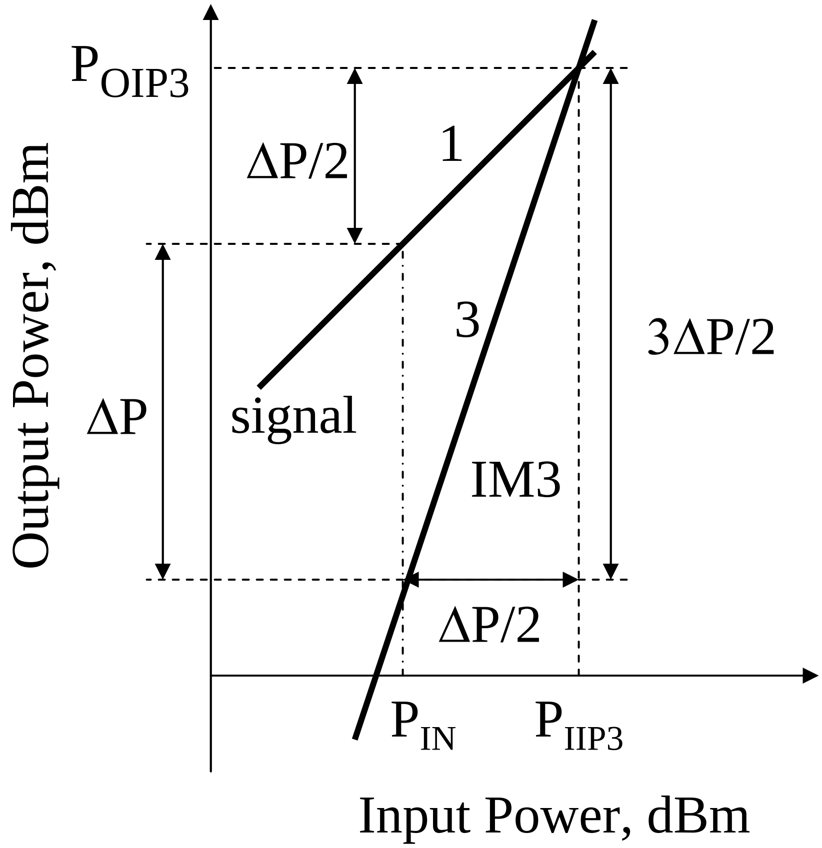
Question 3(a)



By applying two signals with input power (P_{in}) and measuring the associated output signal power ($P_{sig,out}$) and at the IM3 frequencies ($P_{IM3,out}$) it is apparent from the graph that:

$$P_{IIP3} = P_{in} + \frac{P_{sig,out} - P_{IM3,out}}{2}$$

5 marks

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Question 3(b)

The sensitivity of system is defined as the minimum input signal power which is required to give a specified minimum signal-to-noise ratio at the output.

For a given output SNR the input power can be found from the noise figure:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig} / P_{RS}}{SNR_{out}} \Rightarrow P_{sig} = P_{RS} \cdot F \cdot SNR_{out}$$

(per unit bandwidth)

Assuming the system bandwith is B:

$$P_{sig} = P_{RS} \cdot F \cdot SNR_{out} \cdot B$$

Turning the quantities into logs and setting the output SNR to the minimum required value and the input signal power to the minimum value needed to give the required output SNR:

$$P_{\min} = P_{RS}\Big|_{dBm/Hz} + NF + SNR_{\min}\Big|_{dB} + 10\log_{10} B$$

where $NF = 10\log_{10} F$

If the input is conjugate matched to the source the noise power delivered to the input will be:

$$P_{RS} = \frac{\overline{v_n^2}}{4R_S} = \frac{4kTR_S}{4R_S} = kT = -174\, dBm / Hz$$

$$P_{\text{in,min}} = \underbrace{-174\,\text{dBm/Hz} + NF + 10\log_{10} B}_{\text{noise floor}} + SNR_{\min}\Big|_{dB}$$

$$\text{P}_{\text{Floor}} = -174\,\text{dBm/Hz} + NF + 10\log_{10} B$$

8 marks

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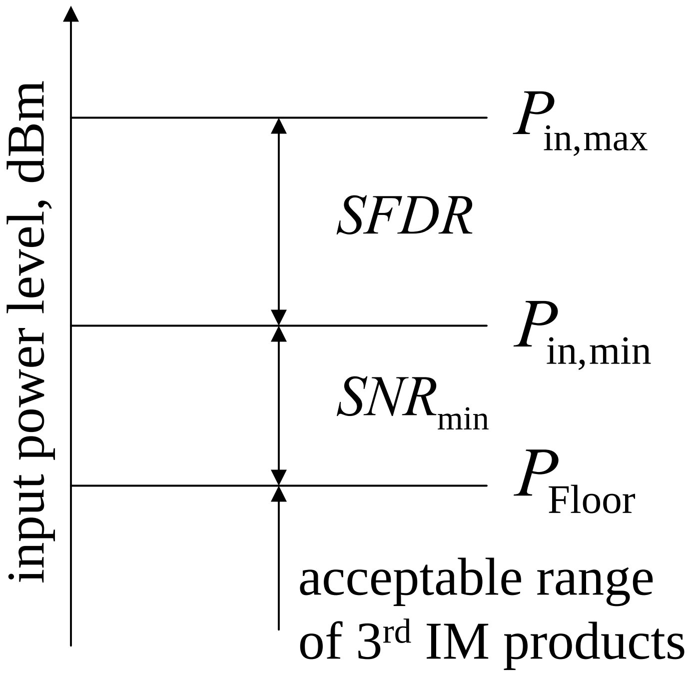
Question 3(c)



$$P_{in,\min} = P_{Floor} + SNR_{\min}$$

$$P_{in,\max} = \frac{2P_{IIP3} + P_{Floor}}{3}$$

The minimum acceptable power in dB is the noise floor plus the required minimum output SNR. As the input power level increase, two or more signals will give IM3 products. The maximum acceptable input power level is considered to be the input power level at which the IM3 products are as high as the noise floor. The range of power between the minimum level and the maximum level is known as the spurious free dynamic range (SFDR).

$$SFDR = P_{in,\max} - P_{in,\min} = \frac{2P_{IIP3} + P_{Floor}}{3} - \left(P_{Floor} + SNR_{\min}\right)$$

$$= \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min}$$

Using NF = 12 dB, P_{IIP3} = -10dBm, B = 1MHz, T = 300K,
SRN_{min} = 12dB gives:

$$\mathrm{P_{Floor}} = -174 + 12 + 10\log_{10}(1\times 10^{6}) = -102\,\mathrm{dBm}$$

$$SFDR = \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min} = \frac{2(-10+102)}{3} - 12 \approx 49.3\,\mathrm{dB}$$

7 marks

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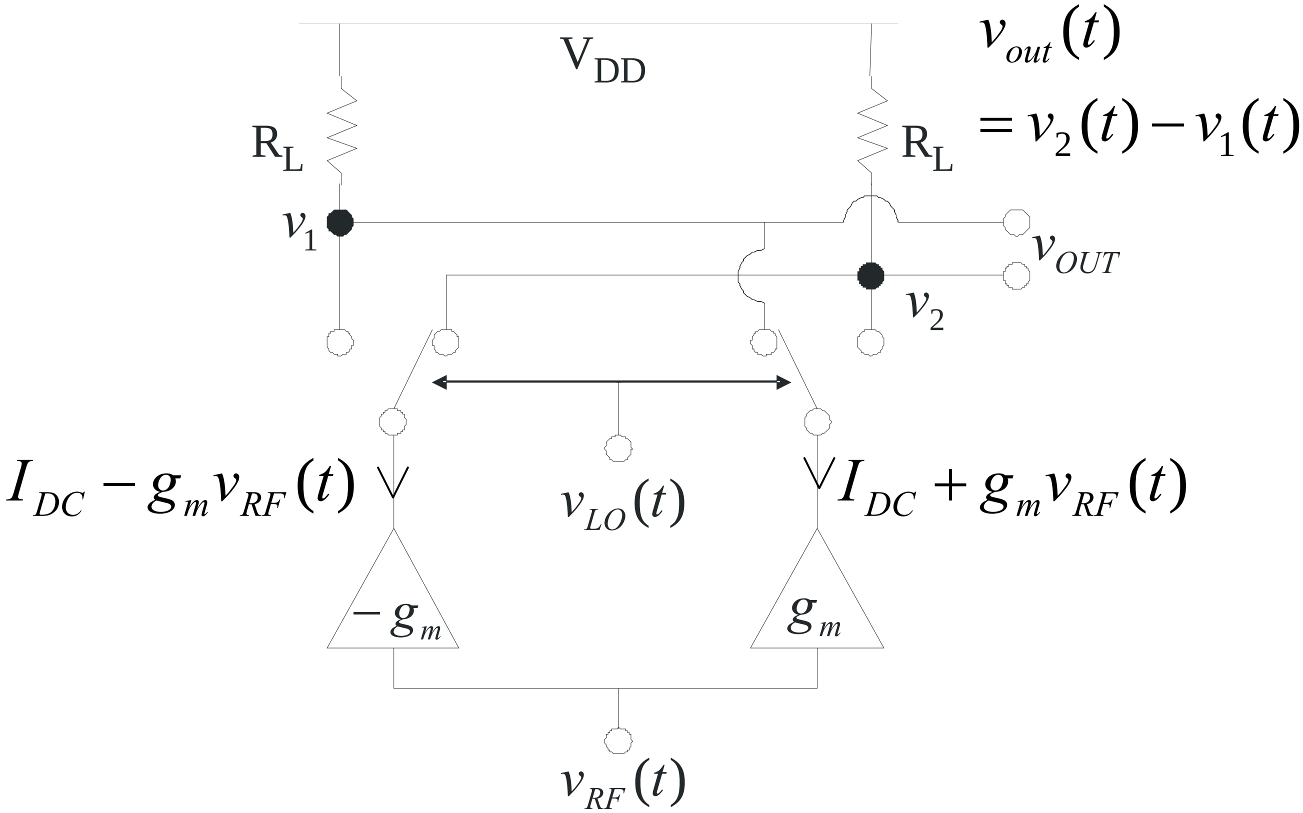
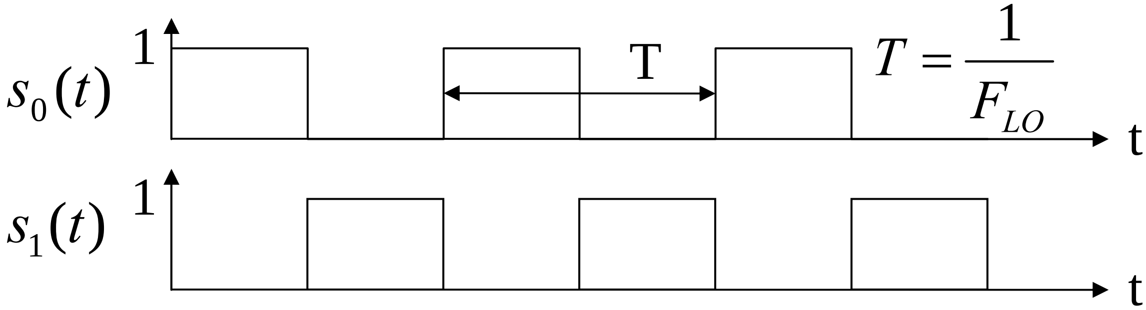
Question 6(a)



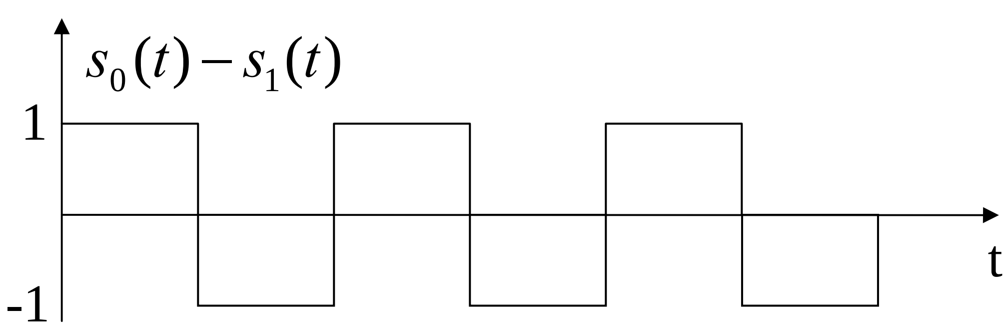
The LO drive can be represented as two non-overlapping square waves



$$i_1(t) = \left(I_{DC} + g_m v_{RF}(t)\right)s_0(t) + \left(I_{DC} - g_m v_{RF}(t)\right)s_1(t)$$

$$i_2(t) = \left(I_{DC} - g_m v_{RF}(t)\right)s_0(t) + \left(I_{DC} + g_m v_{RF}(t)\right)s_1(t)$$

$$i_1(t) - i_2(t) = 2 g_m v_{RF}(t)\left(s_0(t) - s_1(t)\right)$$



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Question 6(a) continued

$$
\begin{aligned}
v_{out}(t) &= v_2(t) - v_1(t) = \left(V_{DD} - i_2(t)\right)R_L - \left(V_{DD} - i_1(t)\right)R_L \\
&= R_L\left(i_1(t) - i_2(t)\right) \\
&= 2g_m R_L v_{RF}(t)\left(s_0(t) - s_1(t)\right) \\
&= 2g_m R_L v_{RF}(t)\frac{4}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \\
&= \frac{8g_m R_L}{\pi} v_{RF}(t)\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]
\end{aligned}
$$

If the RF waveform is of the form:

$$V_{RF}(t)\cos(\varpi_{RF}t)$$

Then, use of cos(A)sin(B) expressions leads to

$$
\begin{aligned}
v_{out}(t) &= \frac{8g_m R_L v_{RF}(t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \\
&= \frac{8g_m R_L V_{RF}\cos(\varpi_{RF}t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \\
&= \frac{4g_m R_L V_{RF}}{\pi}\begin{bmatrix}\sin\left((\varpi_{RF} + \varpi_{LO})t\right) - \sin\left((\varpi_{RF} - \varpi_{LO})t\right) \\ + \frac{1}{3}\sin\left((\varpi_{RF} + 3\varpi_{LO})t\right) - \frac{1}{3}\sin\left((\varpi_{RF} - 3\varpi_{LO})t\right) + \cdots\end{bmatrix}
\end{aligned}
$$

In this expression for the output voltage, there are no terms at DC or at the LO or RF frequencies so these have all been eliminated. The largest two terms are the LO and RF sum and difference frequencies as desired.

12 marks

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Question 6(b)

Voltage conversion gain

$$V_T = \frac{kT}{q} = 25.8mV$$

$$g_m = \frac{I_C}{V_T} = 19.35mS$$

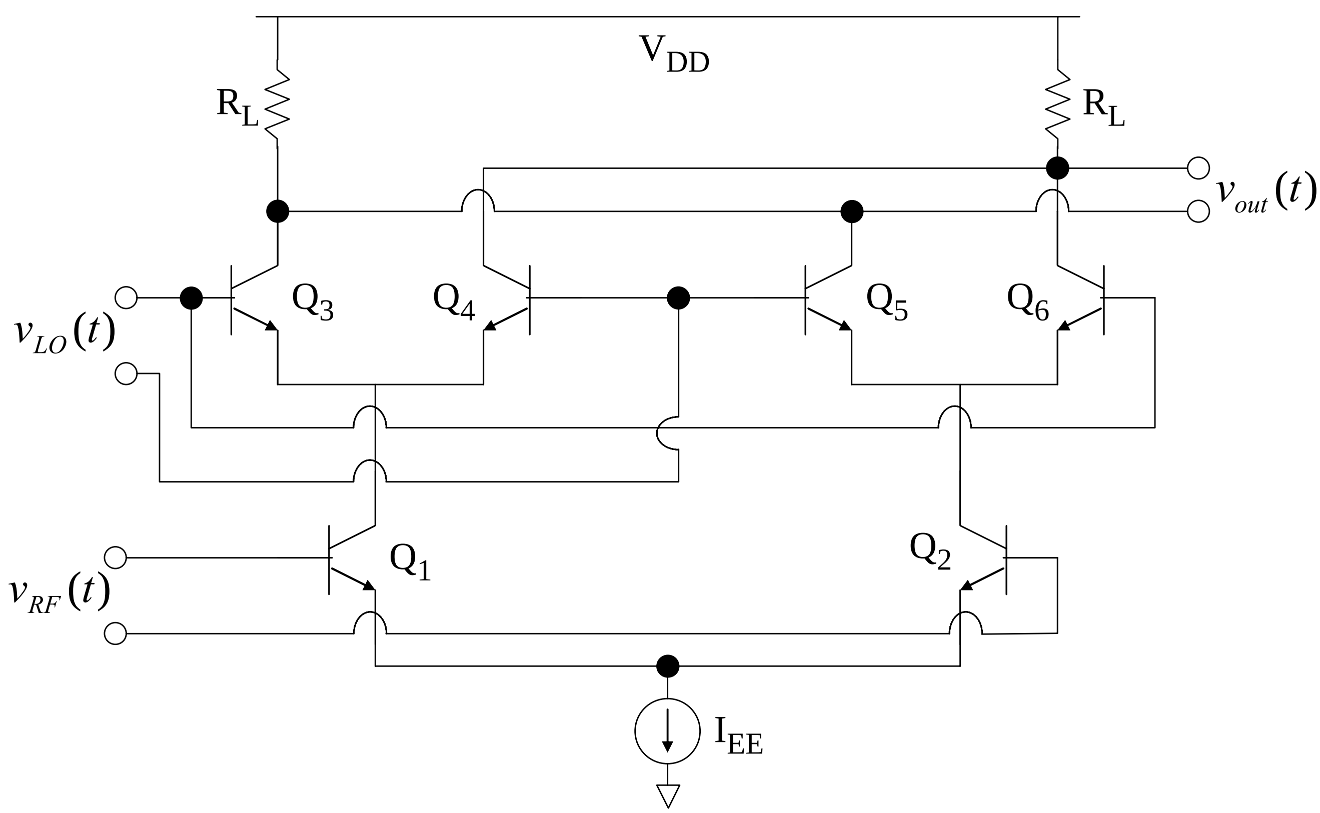
$$A_{CF} = \frac{4g_m R_L}{\pi} = 24.65$$

Note: If the total tail current is 1mA under small-signal conditions there will be 0.5mA flowing through each transistor in the diff pairs so I_C=0.5mA for the g_m calculation.

4 marks

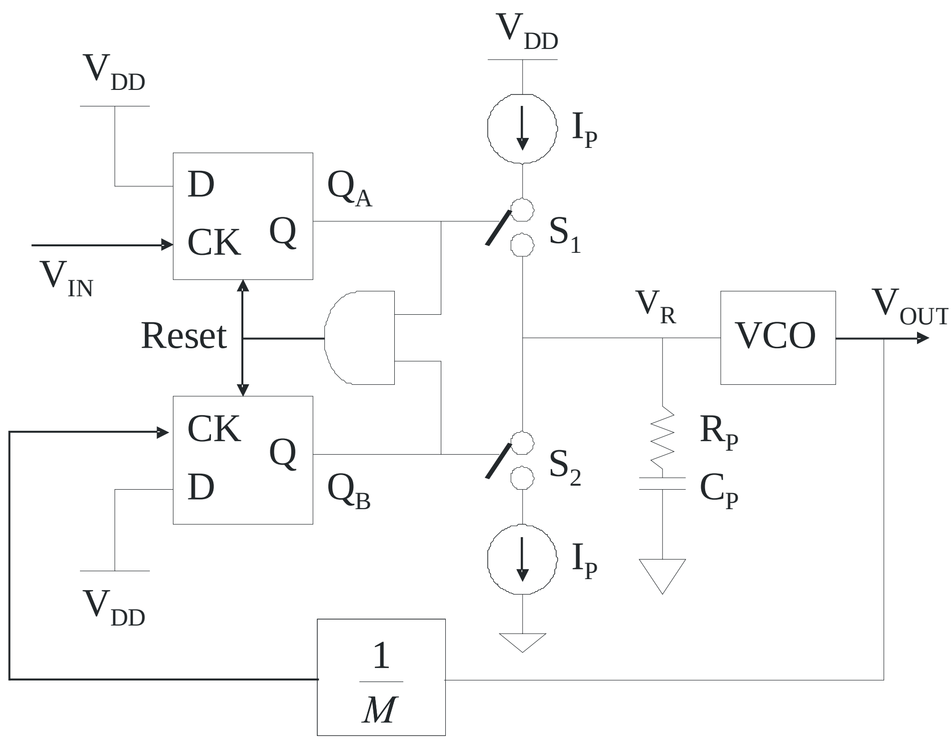
Question 6(c)

A Gilbert Cell Double Balanced Mixer

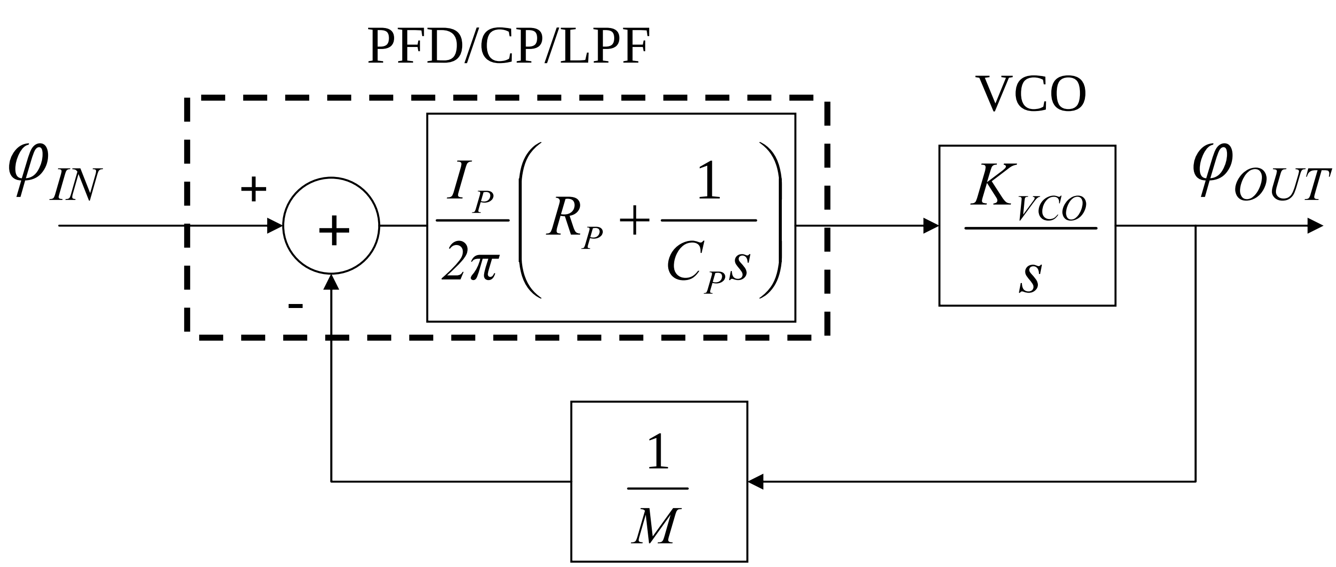


4 marks

EE4011 RF IC Design Summer 2007
Question 7(a)



The individual blocks can be replaced by their transfer functions as follows:



(The PD/CP/LPF transfer function above can be derived by applying the average current method)

(i) Open Loop Response

The open loop transfer function is just the product of the individual transfer functions in the forward path

$$H(s)=\frac{I_P K_{VCO}}{2\pi s}\left(R_P+\frac{1}{C_P s}\right)$$

6 marks

(ii) Closed Loop Response

$$\varphi_{OUT}(s)=\left(\varphi_{IN}(s)-\frac{\varphi_{OUT}(s)}{M}\right)H(s)$$

$$\Rightarrow H_{Closed}(s)=\frac{H(s)}{1+\frac{H(s)}{M}}=\frac{\frac{I_P K_{VCO}}{2\pi s}\left(R_P+\frac{1}{C_P s}\right)}{1+\frac{I_P K_{VCO}}{2\pi M\, s}\left(R_P+\frac{1}{C_P s}\right)}$$

$$=\frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s+1\right)}{s^2+\frac{I_P K_{VCO}}{2\pi M}R_P s+\frac{I_P K_{VCO}}{2\pi C_P M}}$$

4 marks

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Question 7(b)

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} \equiv \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\varsigma\omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

Using

I_P=1mA, C_P=100pF, R_P=10kΩ, K_{VCO}=100MHz/V, M=1000

Note: K_{VCO}=100MHz/V = 6.28 x 10^8 rad/s/V

(i) The natural frequency

$$\omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} = 1\times 10^6 \text{ rad/s} = 159kHz$$

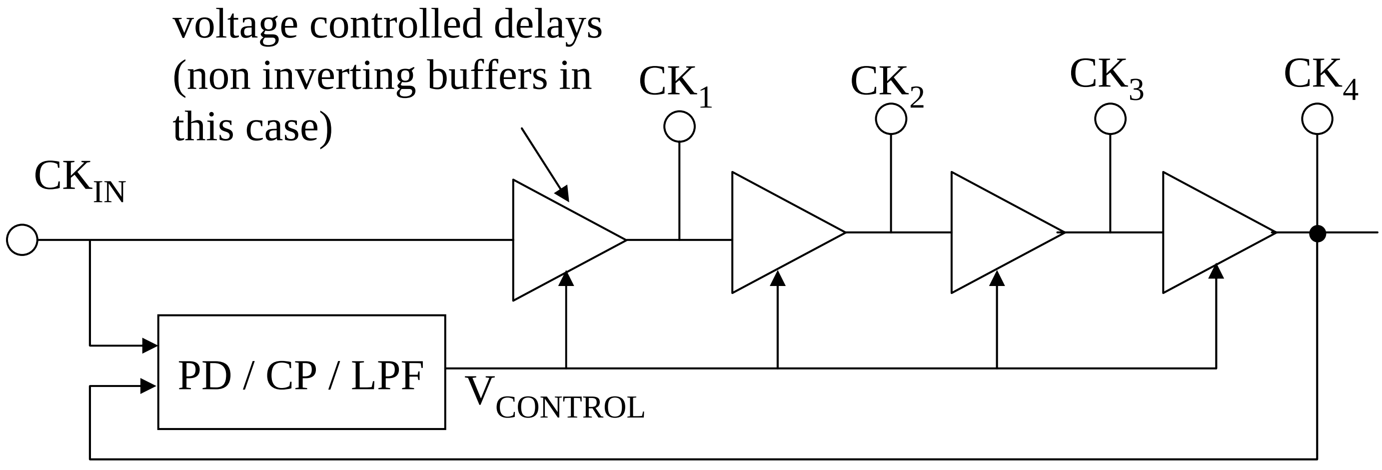
2 marks

(ii) The damping factor

$$\varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} = 0.5$$

2 marks

Question 7(c) A Delay Locked Loop



The DLL uses a voltage controlled delay line (VCDL) instead of a Voltage Controlled Oscillator (VCO)

6 marks