Summer '08

2

$$T_b^{c_1} = \begin{bmatrix} -1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{c_1}^{b} = \begin{bmatrix} -1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$f_0 = 2\,cm$

$$T_{c_1}^{b} A \Rightarrow \begin{bmatrix} -1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_0 - 20 \\ -y_0 + 30 \\ z_0 \\ 1 \end{bmatrix}$$

$$\left(\frac{2}{2 - z_0}\right)(x_0 - 20) = 0.5769$$
$$\left(\frac{2}{2 - z_0}\right)(30 - y_0) = 0.5769$$

$$T_b^{c_2} = \begin{bmatrix} -1 & 0 & 0 & (x_0 + 20) \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{c_2}^{b} = \begin{bmatrix} -1 & 0 & 0 & (x_0 + 20) \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{c_2}^{b} A \Rightarrow \begin{bmatrix} -1 & 0 & 0 & (x_0 + 20) \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_0 \\ 30 - y_0 \\ z_0 \\ 1 \end{bmatrix}$$

$$\left(\frac{2}{2 - z_0}\right)(x_0) = -0.1923$$
$$\left(\frac{2}{2 - z_0}\right)(30 - y_0) = 0.5769$$

$$2x_0 = -0.3846 + z_0(0.1923)$$
$$2x_0 = 40 + 1.1538 - 0.5769 z_0$$
$$-0.5769 z_0 + 41.1538 = 0.1923 z_0 - 0.3846$$
$$z_0 = 54\,cm$$
$$x_0 = 5\,cm$$
$$y_0 = y_0 = 45\,cm$$

Summer '07

$$T_b^c = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & -1 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_c^b = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & -1 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_c^b A = \begin{bmatrix} 30-x_0 \\ y_0-40 \\ z_0-5 \\ 1 \end{bmatrix} \quad T_c^b B = \begin{bmatrix} 50-x_0 \\ y_0-60 \\ z_0-5 \\ 1 \end{bmatrix} \quad T_c^b A' = \begin{bmatrix} 30-x_0 \\ y_0-40 \\ z_0-10 \\ 1 \end{bmatrix} \quad T_c^b B' = \begin{bmatrix} 50-x_0 \\ y_0-60 \\ z_0-10 \\ 1 \end{bmatrix}$$

$$\left(\frac{f_0}{f_0+5-z_0}\right)(30-x_0) = -0.5 \qquad \left(\frac{f_0}{f_0+5-z_0}\right)(50-x_0) = -1.5$$
$$\left(\frac{f_0}{f_0+5-z_0}\right)(y_0-40) = -2 \qquad \left(\frac{f_0}{f_0+5-z_0}\right)(y_0-60) = -1$$

$$\left(\frac{f_0}{f_0+10-z_0}\right)(30-x_0) = -0.5714 \qquad \left(\frac{f_0}{f_0+10-z_0}\right)(50-x_0) = -1.7143$$
$$\left(\frac{f_0}{f_0+10-z_0}\right)(y_0-40) = -2.2857 \qquad \left(\frac{f_0}{f_0+10-z_0}\right)(y_0-60) = -1.1429$$

$$30-x_0 = \frac{1}{f_0}\left(\frac{z_0}{2} - 2.5 - \frac{f_0}{2}\right)$$
$$30-x_0 = \frac{1}{f_0}(0.5714 z_0 - 5.714 - 0.5714 f_0)$$
$$0.5714 z_0 - 5.714 - 0.5714 f_0 = 0.5 z_0 - 2.5 - 0.5 f_0$$
$$0.0714 z_0 - 3.214 = +0.0714 f_0$$
$$z_0 = f_0 + 45$$

$$50-x_0 = \frac{1}{f}(1.5 z_0 - 7.5 - 1.5 f_0)$$
$$50-x_0 = \frac{1}{f}(1.5 f_0 + 67.5 - 7.5 - 1.5 f_0) = \frac{1}{f_0}(60)$$
$$30-x_0 = \frac{1}{f}\left(\frac{f_0}{2} + 22.5 - 2.5 - \frac{f_0}{2}\right) = \frac{1}{f_0}(20)$$
$$\frac{1}{f_0}(20) = \frac{1}{f_0}(60) - 20 \Rightarrow 20 f_0 = 40 \quad f_0 = 2 \quad z_0 = 47$$

$f_0 = 2$ $\qquad x_0 = 30 - \frac{1}{f_0}(20) = 20$

$z_0 = 47$ $\qquad y_0 = 80$

$y_0 = 80$

$x_0 = 20$

Summer '06

$$T_b^c = \begin{bmatrix} \sin\theta & \cos\theta & 0 & 25 \\ \cos\theta & -\sin\theta & 0 & 30 \\ 0 & 0 & -1 & z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_b^{ic} = \begin{bmatrix} \sin\theta & \cos\theta & 0 & 25 \\ \cos\theta & -\sin\theta & 0 & 30 \\ 0 & 0 & 0 & z_o - i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 20 \\ 20 \\ 0 \\ 1 \end{bmatrix} \qquad A^i = \begin{bmatrix} 0.258698 \\ -0.086233 \\ 0 \\ 1 \end{bmatrix} \qquad A^{ic} = \begin{bmatrix} 0.265165 \\ -0.088388 \\ 0 \\ 1 \end{bmatrix} \qquad f = 1$$

$$T_c^b = \begin{bmatrix} \sin\theta & \cos\theta & 0 & -30\cos\theta - 25\sin\theta \\ \cos\theta & -\sin\theta & 0 & -25\cos\theta + 30\sin\theta \\ 0 & 0 & -1 & z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_c^{ib} = \begin{bmatrix} \sin\theta & \cos\theta & 0 & -30\cos\theta - 25\sin\theta \\ \cos\theta & -\sin\theta & 0 & -25\cos\theta + 30\sin\theta \\ 0 & 0 & 0 & z_o - i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_c^b A = \begin{bmatrix} -10\cos\theta - 5\sin\theta \\ -5\cos\theta + 10\sin\theta \\ 0 \\ 1 \end{bmatrix} \qquad T_c^{ib} A = \begin{bmatrix} -10\cos\theta - 5\sin\theta \\ -5\cos\theta + 10\sin\theta \\ 0 \\ 1 \end{bmatrix}$$

$$\left(\frac{1}{1-z_o}\right)(-10\cos\theta - 5\sin\theta) = 0.258698$$
$$\left(\frac{1}{1-z_o}\right)(-5\cos\theta + 10\sin\theta) = -0.086233$$
$$\left(-\frac{1}{z_o}\right)(-10\cos\theta - 5\sin\theta) = 0.265165$$
$$\left(-\frac{1}{z_o}\right)(-5\cos\theta + 10\sin\theta) = -0.088388$$

$$\Rightarrow \quad 0.088388\, z_o = -5\cos\theta + 10\sin\theta$$
$$0.086233\, z_o = -5\cos\theta + 10\sin\theta + 0.086233$$
$$\Rightarrow 0.002155\, z_o = 0.086233$$
$$z_o = 40$$

$$\Rightarrow \quad +10\cos\theta + 5\sin\theta = 10.6066$$
$$-10\cos\theta + 20\sin\theta = 7.07104$$
$$25\sin\theta = 17.67764$$
$$\theta = 45^\circ$$

Summer '05

$$T_c^b = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & -1 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 30 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad N = \begin{bmatrix} 50 \\ 0 \\ 20 \\ 1 \end{bmatrix} \quad M^i = \begin{bmatrix} -0.5263 \\ -1.5789 \\ 0 \\ 1 \end{bmatrix}$$

$$N^i = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

$$T_c^b M = \begin{bmatrix} 30 - x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} \quad T_c^b N = \begin{bmatrix} 50 - x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$$

$$\left(\frac{f}{f - z_0}\right)(30 - x_0) = -0.5263$$
$$\left(\frac{f}{f - z_0}\right) y_0 = -1.5789$$
$$\left(\frac{f}{f + 20 - z_0}\right)(50 - x_0) = -2$$
$$\left(\frac{f}{f + 20 - z_0}\right) y_0 = -2$$

$$y_0 = \frac{1}{f}(1.5789 z_0 - 1.5789 f) = \frac{1}{f}(2 z_0 - 2f - 40)$$

$$-0.4211 z_0 = -0.4211 f - 40$$

$$z_0 = f + 95$$

$$\left(\frac{f}{-95}\right) y_0 = -1.5789 \Rightarrow f y_0 = 150$$

$$\left(\frac{f}{-75}\right) y_0 = -2$$

$$\left(\frac{f}{-95}\right)(30 - x_0) = -0.5263 \Rightarrow 50 - x_0 = 50\tfrac{1}{f} + 20$$

$$\left(\frac{f}{-75}\right)(50 - x_0) = -2 \Rightarrow 50 - x_0 = +150\tfrac{1}{f}$$

$$20f = 100 \qquad f = 5$$

$$x_0 = 20$$
$$y_0 = 30$$
$$z_0 = 100$$

Summer '04

$$T_b^c = \begin{bmatrix} 0 & -1 & 0 & x_0 \\ -1 & 0 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_c^b$$

$$M = \begin{bmatrix} 20 \\ 10 \\ 0 \\ 1 \end{bmatrix} \Rightarrow N = \begin{bmatrix} 35 \\ 25 \\ 5 \\ 1 \end{bmatrix} \qquad M^i = \begin{bmatrix} -3.3333 \\ -1.1111 \\ 0 \\ 1 \end{bmatrix} \qquad N^i = \begin{bmatrix} -1.875 \\ 0.625 \\ 0 \\ 1 \end{bmatrix}$$

$$T_c^b M = \begin{bmatrix} x_0 - 10 \\ y_0 - 20 \\ z_0 \\ 1 \end{bmatrix} \qquad T_c^b N = \begin{bmatrix} x_0 - 25 \\ y_0 - 35 \\ z_0 - 5 \\ 1 \end{bmatrix}$$

$$\left(\frac{5}{5-z_0}\right)(x_0 - 10) = -3.3333$$
$$\left(\frac{5}{5-z_0}\right)(y_0 - 20) = -1.1111$$
$$\left(\frac{5}{10-z_0}\right)(x_0 - 25) = -1.875$$
$$\left(\frac{5}{10-z_0}\right)(y_0 - 35) = 0.625$$

$$x_0 - 10 = 0.2\left(\frac{10}{3}z_0 - \frac{50}{3}\right) = \frac{2}{3}z_0 - \frac{10}{3}$$
$$x_0 - 25 = 0.2(1.875z_0 - 18.75) = 0.375z_0 - 3.75$$
$$0.375z_0 - 3.75 = \frac{2}{3}z_0 - \frac{10}{3} - 15$$
$$\frac{7}{24}z_0 = \frac{175}{12} \Rightarrow z_0 = 50$$
$$x_0 = 40$$
$$y_0 = 30$$