EE4011 RFIC Design

Non-linear Effects in RF Systems: Gain Compression

A linear system (1)

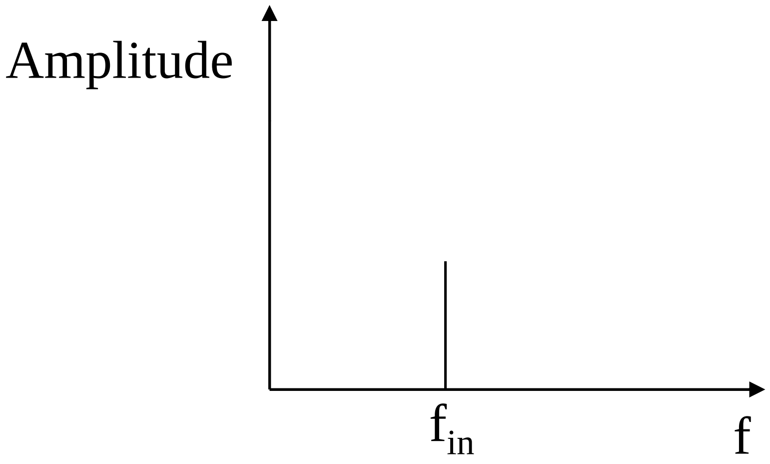
A linear system changes the amplitude and phase of the input signal but does not create any new frequencies. For instance an input signal

$$x(t) = A\cos(\omega t)$$

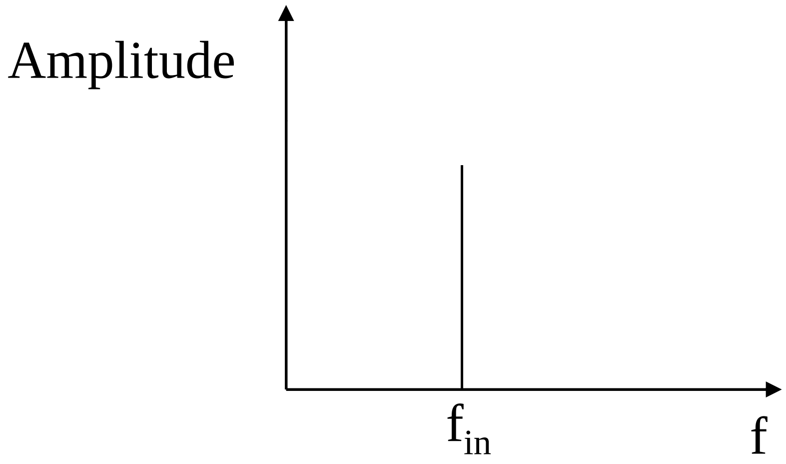
will give rise to the output

$$y(t) = B\cos(\omega t - \varphi)$$

Where B/A is the voltage gain of the circuit and ϕ is the phase shift.



Input Spectrum



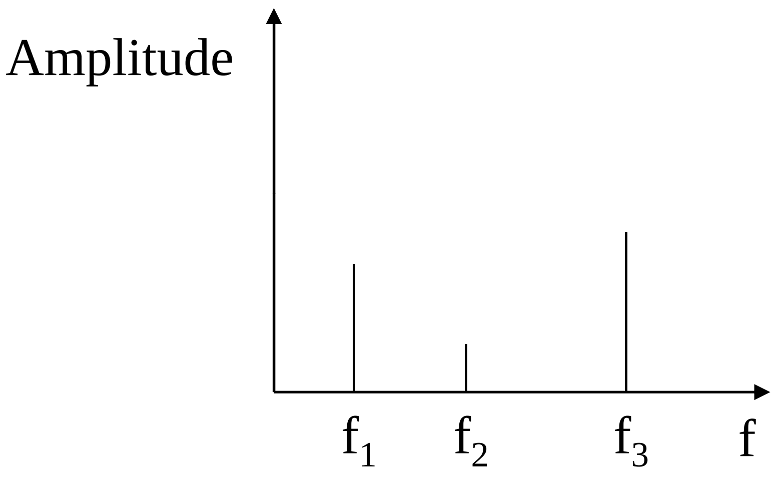
Output Spectrum

A linear system (2)

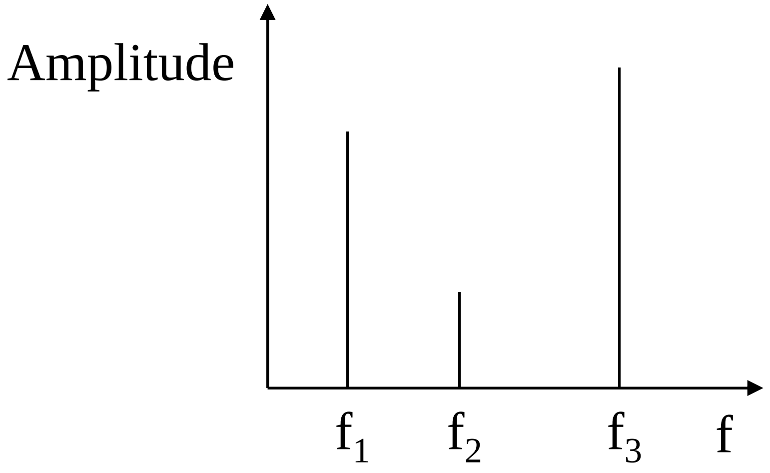
In a linear system, if the input consists of several frequency components, the output contains the same frequencies:

$$x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t) + A_3 \cos(\omega_3 t)$$

$$y(t) = B_1 \cos(\omega_1 t - \varphi_1) + B_2 \cos(\omega_2 t - \varphi_2) + B_3 \cos(\omega_3 t - \varphi_3)$$



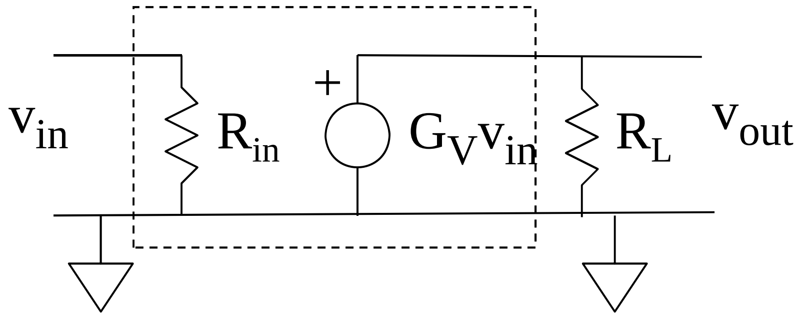
Input Spectrum



Output Spectrum

Voltage Gain in a Linear System

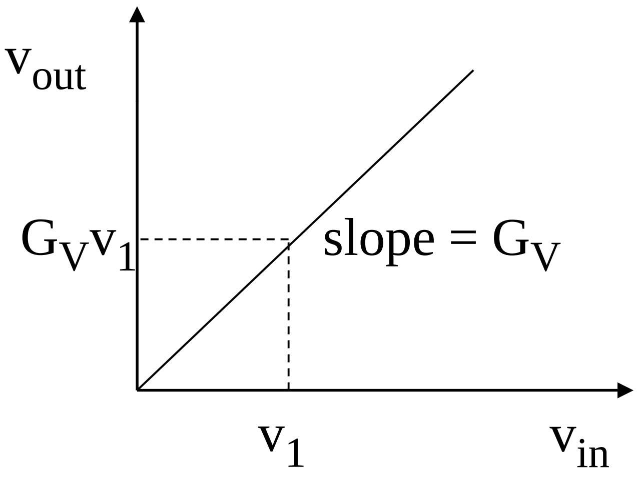
Looking at a very simple linear system with a voltage gain G_V, which has an input resistance R_{in}, and is driving a load of resistance R_L.



The output voltage is given by:

$$v_{out} = G_V v_{in}$$

Therefore a plot of v_{out} vs. v_{in} will give a straight line with slope G_V.



Power Gain in a Linear System

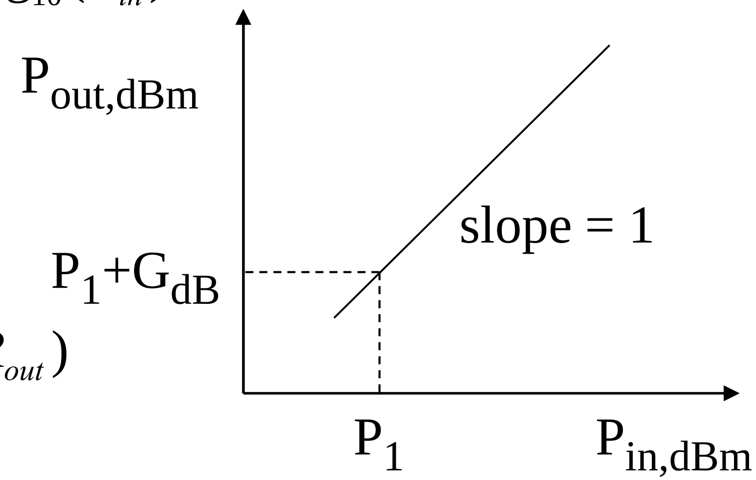
Assuming the voltages are r.m.s., the power delivered to the input, the power delivered to the load and the power gain can be calculated as follows and expressed in dB notation:

$$P_{in} = \frac{v_{in}^2}{R_{in}} \quad P_{in,dB} = 10\log_{10}(P_{in}) = 20\log_{10}(v_{in}) - 10\log_{10}(R_{in})$$

$$P_{out} = \frac{v_{out}^2}{R_{out}} = \frac{G_V^2 v_{in}^2}{R_{out}} \quad P_{out,dB} = 10\log_{10}(P_{out})$$

$$\begin{aligned} P_{out,dB} &= 20\log_{10}(G_V) + 20\log_{10}(v_{in}) - 10\log_{10}(R_{out}) \\ &= 20\log_{10}(G_V) + P_{in,dB} + 10\log_{10}(R_{in}) - 10\log_{10}(R_{out}) \\ &= G_{dB} + P_{in,dB} + 10\log_{10}(R_{in}/R_{out}) \\ &= G_{dB} + P_{in,dB} \quad \text{assuming} \quad R_{in} = R_{out} \end{aligned}$$

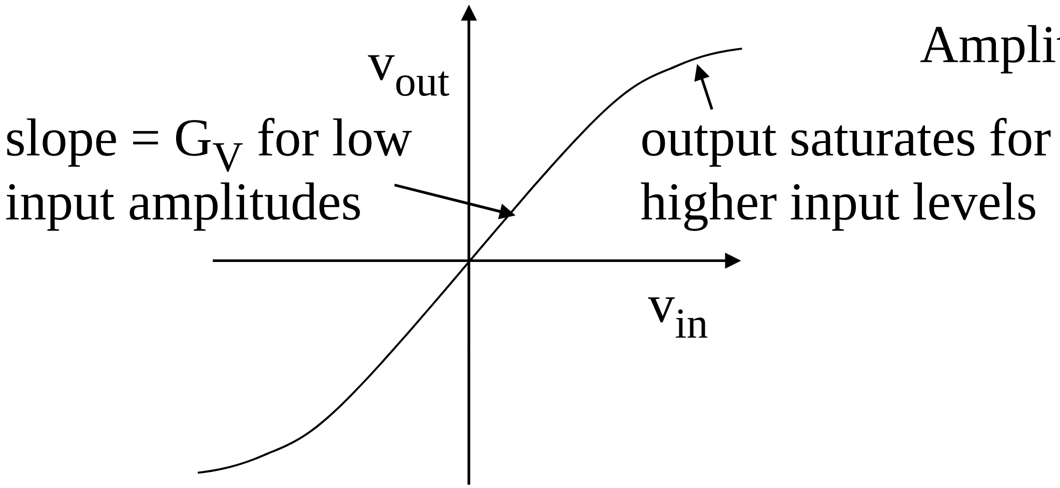
$$G_{dB} = 20\log_{10}(G_V)$$

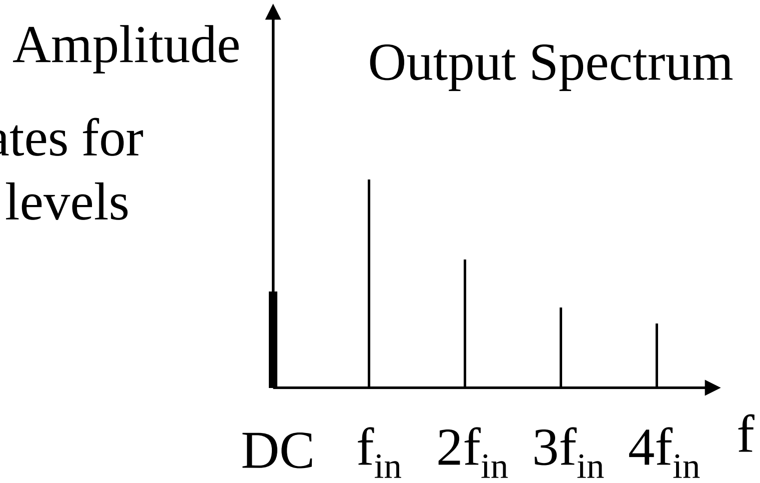


Therefore in a linear system, a plot of output power vs. input power (in dBm, dBW, etc) will give a straight line with a slope of 1, where the output power is always G_{dB} higher than the input power.

A real system (1)

A real system always has some non-linearities causing the input/output relationship to deviate from a straight line, especially for higher input levels. These non-linearities also cause harmonics of the input frequency to be generated giving rise to harmonic distortion and may also introduce a DC offset.



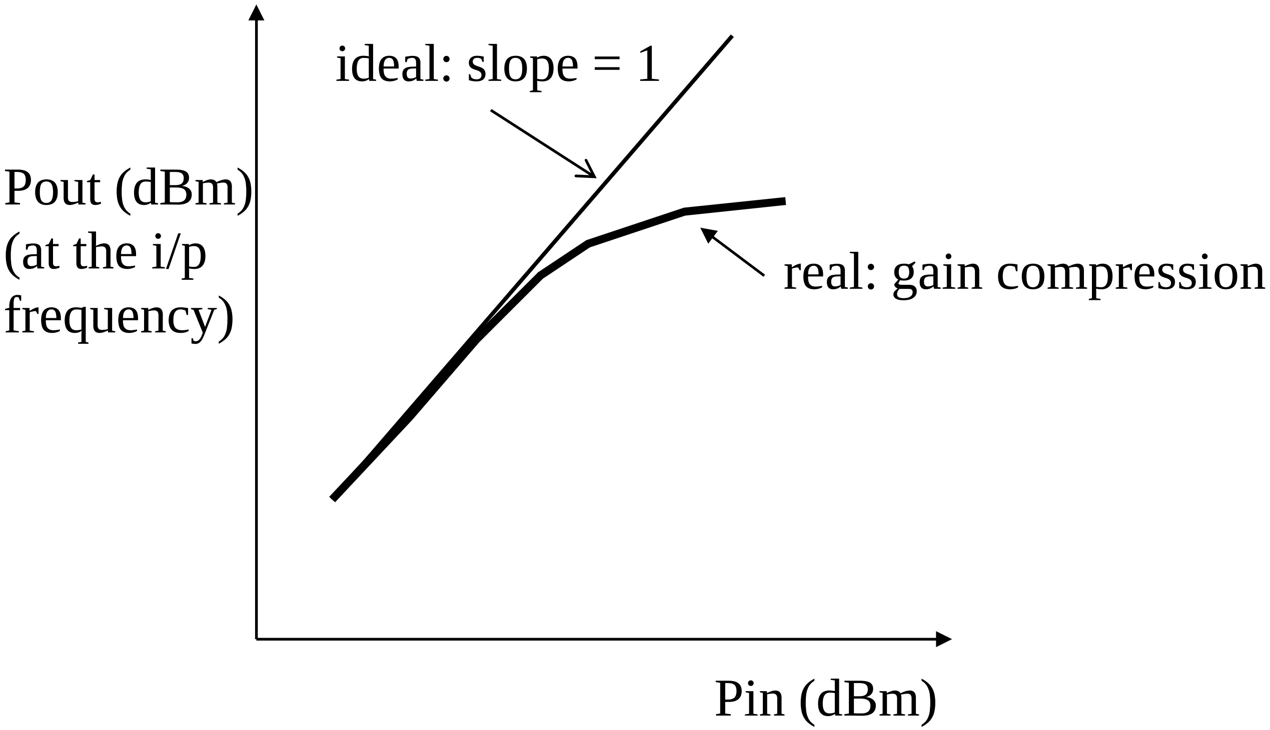


The o/p of a non-linear system in response to a single-frequency input could be written as:

$$x(t) = A\cos(\omega t) \qquad y(t) = \sum_{n=0}^{\infty} B_n \cos(n\omega t + \varphi_n)$$

A real system (2)

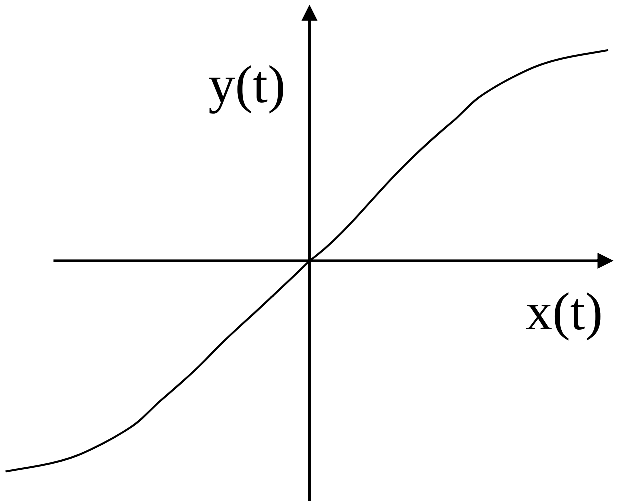
In a real system, for low input power, as the input power is increased, the output power increases in a linear fashion with a slope of 1 when the power is plotted as dB. However the o/p deviates from this behaviour for larger input levels.
The following is an example of a real “compressive” system i.e. is one where the output power drops below the ideal value as the input power is increased.



Systems with odd symmetry

In general for a non-linear system:

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots = \sum \alpha_j x^j(t)$$



The graph on the left illustrates a special type of system that has odd symmetry i.e.

$$y(-x(t)) = -y(x(t))$$

This can only happen if the coefficients with an even subscript in the formula for y(t) are zero i.e.

$$\alpha_j = 0 \quad \text{for even } j$$

A system possessing odd symmetry is often referred to as differential or balanced.

Some Useful Trigonometric Relationships

$$\cos(-A) = \cos(A)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$\cos^2 A = \frac{1}{2}\left(1 + \cos(2A)\right)$$

$$\cos A \cos^2 B = \frac{1}{2}\cos A + \frac{1}{4}\cos(2B+A) + \frac{1}{4}\cos(2B-A)$$

$$\cos^2 A \cos B = \frac{1}{2}\cos B + \frac{1}{4}\cos(2A+B) + \frac{1}{4}\cos(2A-B)$$

$$\cos^3 A = \frac{3}{4}\cos A + \frac{1}{4}\cos(3A)$$

A 3rd Order System (1)

$$x(t) = A\cos(\varpi t) \qquad y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$\begin{aligned}
&y(t) \\
&= \alpha_1 A\cos(\varpi t) + \alpha_2 A^2 \cos^2(\varpi t) + \alpha_3 A^3 \cos^3(\varpi t) \\
&= \alpha_1 A\cos(\varpi t) + \alpha_2 A^2 \frac{1}{2}\big(1+\cos(2\varpi t)\big) + \alpha_3 A^3 \frac{1}{2}\big(1+\cos(2\varpi t)\big)\cos(\varpi t) \\
&= \alpha_1 A\cos(\varpi t) + \alpha_2 A^2 \frac{1}{2} + \alpha_2 A^2 \frac{1}{2}\cos(2\varpi t) + \alpha_3 A^3 \frac{1}{2}\cos(\varpi t) + \alpha_3 A^3 \frac{1}{2}\cos(2\varpi t)\cos(\varpi t) \\
&= \alpha_1 A\cos(\varpi t) + \alpha_2 A^2 \frac{1}{2} + \alpha_2 A^2 \frac{1}{2}\cos(2\varpi t) + \alpha_3 A^3 \frac{1}{2}\cos(\varpi t) + \alpha_3 A^3 \frac{1}{4}\big(\cos(3\varpi t) + \cos(\varpi t)\big) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos(\varpi t) + \frac{\alpha_2 A^2}{2}\cos(2\varpi t) + \frac{\alpha_3 A^3}{4}\cos(3\varpi t)
\end{aligned}$$

The output has a DC offset, the fundamental (input) frequency, and frequencies at the second-harmonic and the third harmonic.

3^{rd} Order System (2)

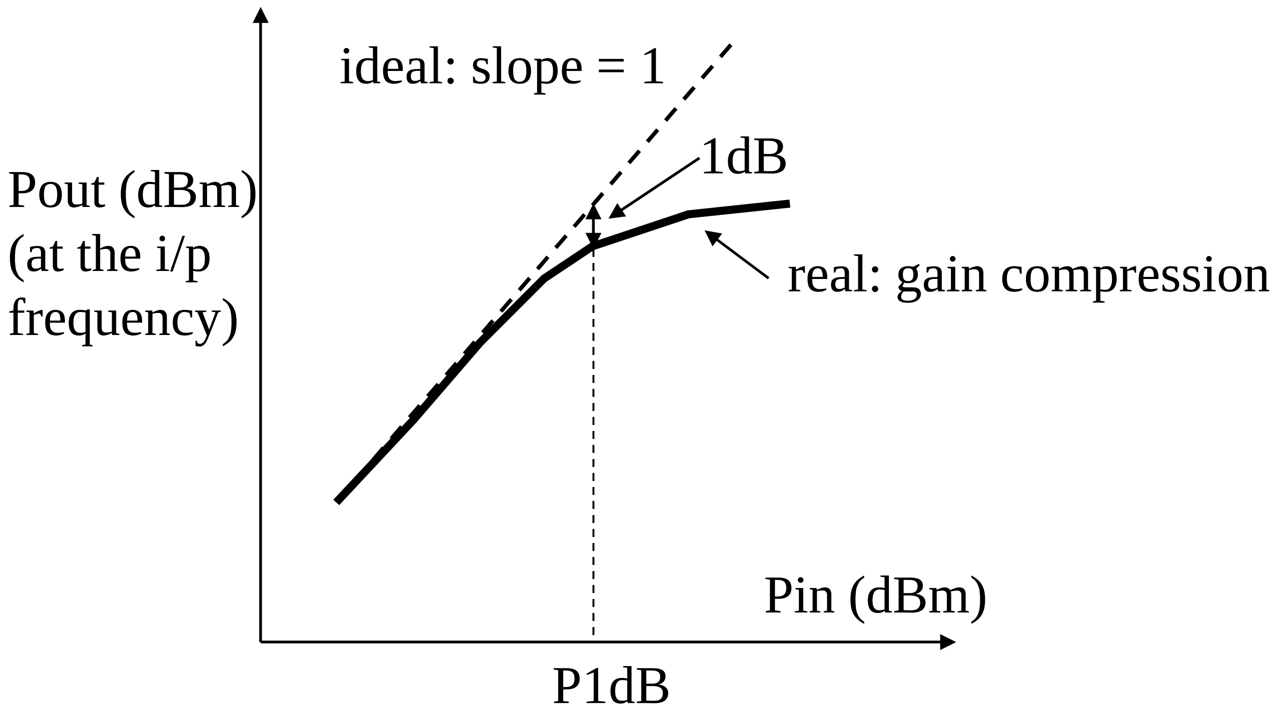
$$y(t) = \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4} \right) \cos(\varpi t) + \frac{\alpha_2 A^2}{2} \cos(2\varpi t) + \frac{\alpha_3 A^3}{4} \cos(3\varpi t)$$

If A is small, then $A^3 << A$ and the coefficient of the fundamental is just $\alpha_1 A$. In that case the amplitude of harmonic n is proportional to A^n .
Many RF systems are designed with a balanced (differential) topology that ensures that no second-order harmonics are generated i.e. $a_2=0$ by design. In that case the first troublesome harmonic is the 3rd harmonic and therefore formulas for 3rd harmonics are encountered frequently in RF design.
Looking at the output at the fundamental frequency only:

$$y_1(t) = \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4} \right) \cos(\varpi t) = \alpha_1 A \left(1 + \frac{3\alpha_3 A^2}{4\alpha_1} \right)$$

Usually $\alpha_1>0$. In a compressive system $\alpha_3<0$ and the gain is reduced from its ideal value. This reduction from the ideal increases as A is increased i.e. the gain depends on the input amplitude and is lower for higher input amplitudes giving rise to the so called compressive characteristic.

1dB Compression Point: P1dB



A common figure of merit for RF systems is the 1dB compression point (P1dB). This is the input power level at which the output power deviates from the ideal power transfer characteristic by 1dB.

This is regarded as the maximum input power that the system can handle without suffering too much loss of gain.

P1dB for a simple system (1)

The output at the fundamental (i.e. the input) frequency for a balanced system is:

$$y_1(t) = \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4} \right) \cos(\varpi t) = A_{OUT} \cos(\varpi t)$$

The voltage gain of the circuit considering the fundamental is:

$$G_V = \frac{y_1(t)}{x(t)} = \frac{A_{OUT} \cos(\varpi t)}{A \cos(\varpi t)} = \frac{\alpha_1 A + \frac{3\alpha_3 A^3}{4}}{A} = \alpha_1 + \frac{3\alpha_3 A^2}{4}$$

Converting to dB:

$$G_{dB} = 20 \log_{10}(G_V) = 20 \log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4} \right)$$

For small A (amplitude) the A^2 term is very small and the gain is the “ideal gain” for small input amplitudes:

$$G_{dB,small} = 20 \log_{10}(\alpha_1)$$

P1dB for a simple system (2)

The power gain in dB is:

$$G_{dB} = 20\log_{10}(G_V) = 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right)$$

For a compressive gain stage α_3 has the opposite sign to α_1 i.e. if $\alpha_1 > 0$ then $\alpha_3 < 0$

For small input signals (small A) the power gain is approximately:

$$G_{dB,small} = 20\log_{10}(\alpha_1)$$

At the 1dB point the gain is 1dB smaller than this ideal value i.e.

$$G_{dB,P1dB} = G_{dB,small} - 1 = 20\log_{10}(\alpha_1) - 1$$

The amplitude A corresponding to P1dB can be found by equating the last expression to the first expression above:

$$\text{At} \quad \text{P1dB}: \quad G_{dB} = G_{dB,P1dB} \Rightarrow 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) = 20\log_{10}(\alpha_1) - 1$$

P1dB for a simple system (3)

$$20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) = 20\log_{10}(\alpha_1) - 1$$

$$\Rightarrow 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) - 20\log_{10}(\alpha_1) = -1$$

$$\Rightarrow \log_{10}\left(\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) / \alpha_1\right) = -0.05$$

$$\Rightarrow 1 + \frac{3\alpha_3 A^2}{4\alpha_1} = 10^{-0.05}$$

Note α_1 and α_2 have opposite signs so: $\frac{\alpha_1}{\alpha_3} = -\left|\frac{\alpha_1}{\alpha_3}\right|$

$$\Rightarrow A^2 = \left(10^{-0.05} - 1\right)\frac{4}{3}\frac{\alpha_1}{\alpha_3} = -0.145\,\frac{\alpha_1}{\alpha_3} = 0.145\left|\frac{\alpha_1}{\alpha_3}\right|$$

$$\Rightarrow A = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

If α_1 and α_3 are known, the input amplitude at the 1dB compression point can be determined – alternatively α_1/α_3 can be determined by measuring P1dB