Chapter 9

Introduction to AC Machines

Introduction

- r Primary AC motor drives
  - x Induction motors (asynchronous)
    - x Squirrel cage – brushless
    - x Wound rotor - brushed
  - x Synchronous Motors
    - x Permanent Magnet – brushless
    - x Wound rotor – brushed

DFI - Doubly Fed Induction Motor, ~~eg.~~ ~~Prius~~

→ Prius

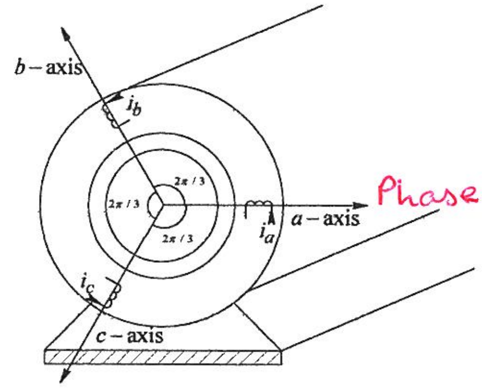
→ Synchronous Generator

x These machines have similar stators but different rotor constructions.

Introduction

Stator windings produce a sinusoidal field distribution in the airgap.

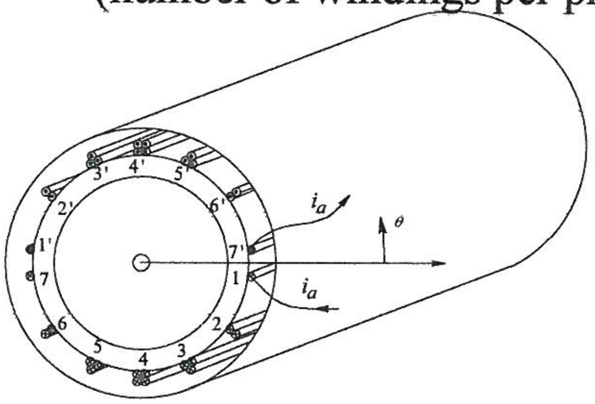
These magnetic field distributions are displaced by 120 ° w.r.t. each other.

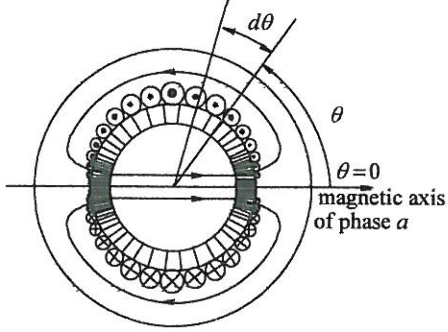


Phase 'a' magnetic axis

Sinusoidally-distributed Stator Windings

(number of windings per phase vary with angle)





Conductor density $n_s(\theta) = \hat{n}_s \sin\theta$ = number of semiconductors per radian, $0 < \theta < \pi$

Total

$$N_s = \int_0^{\pi} n_s(\theta)\, d\theta = \int_0^{\pi} \hat{n}_s \sin(\theta)\, d\theta = 2\hat{n}_s$$

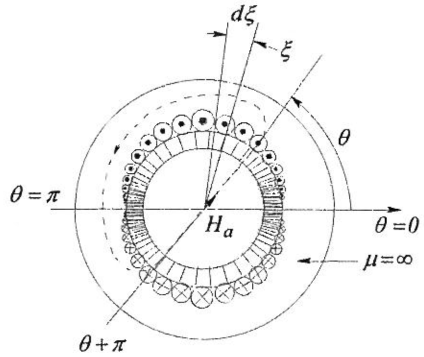
$$\Rightarrow n_s(\theta) = \frac{N_{sp}}{2}\sin(\theta) \qquad 0 < \theta < \pi$$

N_{sp} is the number of conductors/phase/pole.

I.M. – typically 4 pole; P.M. - multipole

$N_{sp} = \frac{N_s}{2}$ in 2-pole machine

Air-gap Field Distribution



Apply Ampere's Law.

$\sum H \cdot dl = Ni = i\int_{\theta}^{\theta+\pi} n_s(\xi)\, d\xi$

RHR gives direction.

From symmetry

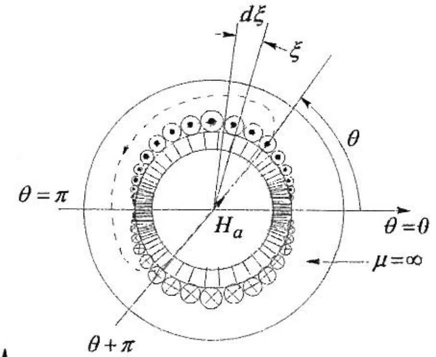
$$H_a(\theta) = -H_a(\theta+\pi)$$

(negative sign because line of integration points inwards at $\theta+\pi$)

$$\sum H.dl = \cancel{H_m l_m}^{0} + 2H_a(\theta)l_g = \int_0^{\pi} n_s(\theta+\xi)\, i_a\, d\xi$$

$$2H_a(\theta)\ell_g = \frac{N_s}{2} i_a \int_0^{\pi} \sin(\theta+\xi)\, d\xi = N_s\, i_a \cos(\theta)$$

Air-gap Field Distribution



Radial magnetic field strength

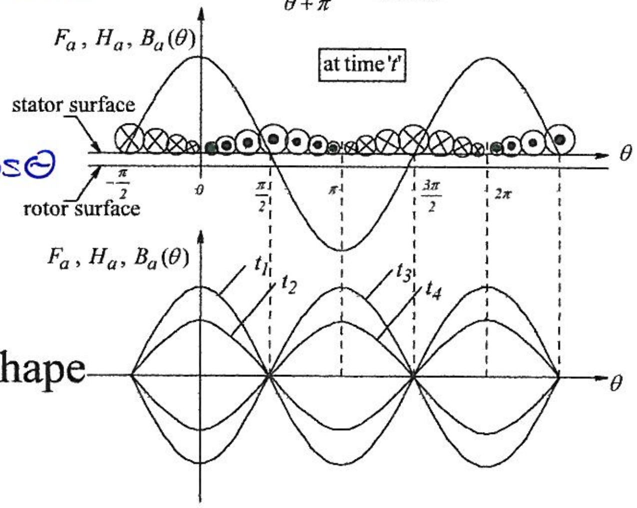
$H_a(\theta) = \frac{N_s}{2l_g} \cdot i_a \cdot \cos\theta$

Radial magnetic flux density

$B_a(\theta) = \mu_0 H_a(\theta) = \left(\frac{\mu_0 N_s}{2l_g}\right) i_a \cos\theta$

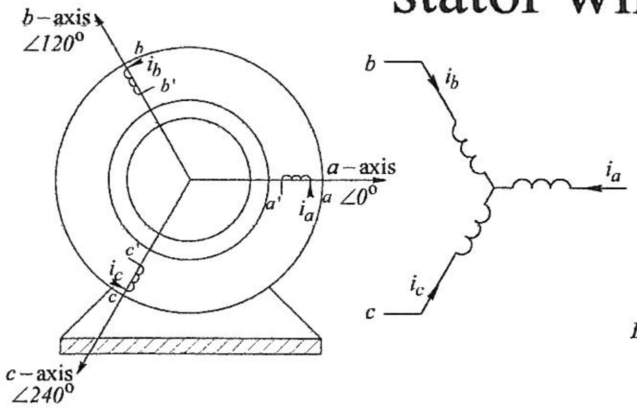
Radial magnetomotive force

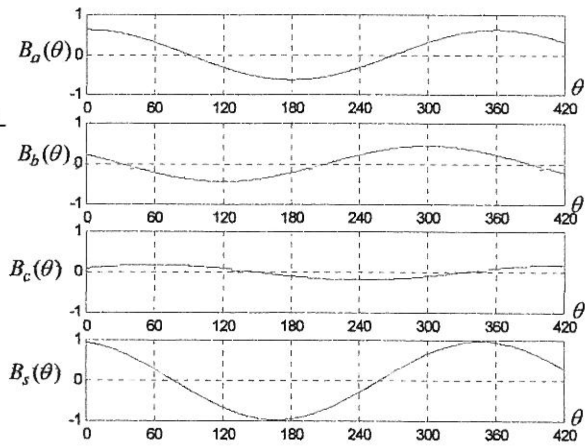
$F_a(\theta) = l_g H_a(\theta) = \frac{N_s}{2} i_a \cos\theta$



Field quantities have different magnitudes and units but same shape

Three-phase sinusoidally-distributed stator windings





r Example 9-3: 2-pole, l_g = 1mm, i_a = 10 A, i_b = -7 A, i_c = -3 A, N_{sp} = 50.

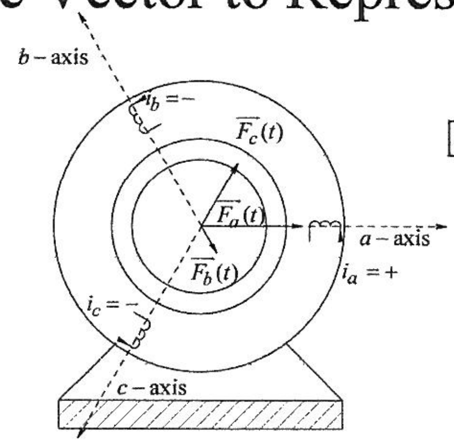
$$B_a(\theta) = \frac{\mu_o N_s i_a}{2\ell_g}\cos\theta = 0.628\cos\theta \; Wb/m^2$$

$$B_b(\theta) = -0.440\times\cos\left(\theta - 120^0\right)\; Wb/m^2 \qquad B_c(\theta) = -0.188\times\cos\left(\theta - 240^0\right)\; Wb/m^2$$

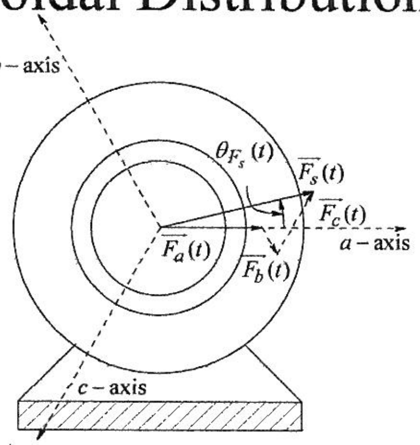
$$B_s(\theta) = B_a(\theta) + B_b(\theta) + B_c(\theta) = 0.967\cos(\theta - 13.03^\circ)$$

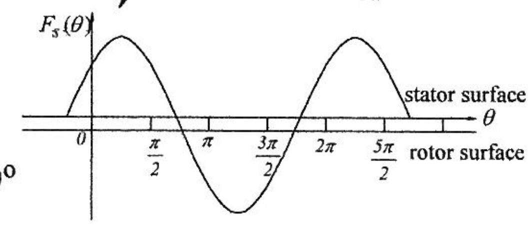
= combined stator-produced flux density

Space Vector to Represent Sinusoidal Distributions



At time 't'





r Complex number representation

$$F_a(\theta, t) = \frac{N_s}{2} i_a(t)\cos(\theta) \Leftrightarrow \vec{F_a}(t) = \frac{N_s}{2} i_a(t)\angle 0^0$$

Similarly, $\vec{F_b}(t) = \frac{N_s}{2} i_b(t)\angle 120^\circ$; $\vec{F_c}(t) = \frac{N_s}{2} i_c(t)\angle 240^\circ$

And $\vec{F_s} = \vec{F_a} + \vec{F_b} + \vec{F_c} = \hat{F}_s \angle\theta_{F_s}$ = resultant stator space vector for magnetomotive force

r Similar expressions for *B* and *H*

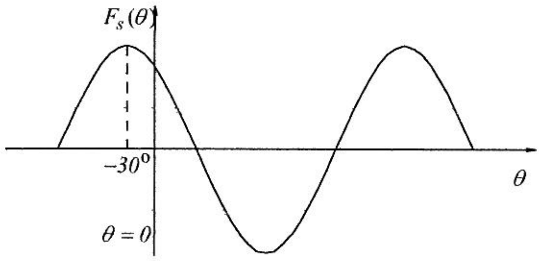
Example

Three-phase, sinusoidally-distributed stator with $\frac{N_s}{2} = 50$ turns
At time t, $i_a = 10A$, $i_b = -10A$ and $i_c = 0A$
Find $\vec{F}_s$

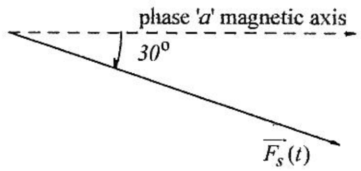
$$\vec{F}_s(t) = \frac{N_s}{2}\left(i_a\angle 0^0 + i_b\angle 120^0 + i_c\angle 240^0\right)$$

$$= 50\left\{10 + (-10)\left[\cos 120^0 + j\sin 120^0\right] + (0)\left[\cos 240^0 + j\sin 240^0\right]\right\}$$

$$\vec{F}_s(t) = 50 \times 17.32\angle -30^\circ = 866\angle -30^\circ\ A\cdot\text{turns}$$



$\boxed{\text{at } t}$



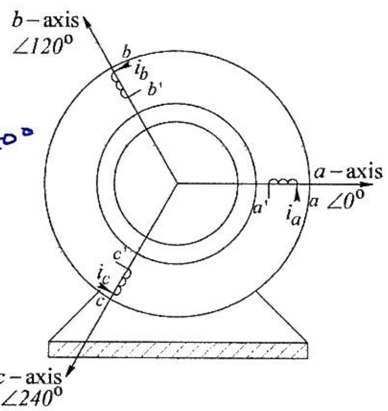
Space Vectors Representation of Combined Phase Currents and Voltages

- Mathematical concept

At time t

$\vec{i}_s = i_a(t)\angle 0^\circ + i_b(t)\angle 120^\circ + i_c(t)\angle 240^\circ$
$= \hat{I}_s(t)\angle 0^\circ$

= *stator current space vector*



$$\vec{v}_s(t) = v_a(t)\angle 0^0 + v_b(t)\angle 120^0 + v_c(t)\angle 240^0$$
$$= \hat{V}_s(t)\angle\theta_{v_s}(t)$$
= *stator voltage space vector*

Physical interpretation of $\vec{i_s}(t)$

$$\frac{N_s}{2}\vec{i_s}(t) = \underbrace{\frac{N_s}{2} i_a(t)\angle 0^0}_{\vec{F_a}(t)} + \underbrace{\frac{N_s}{2} i_b(t)\angle 120^0}_{\vec{F_b}(t)} + \underbrace{\frac{N_s}{2} i_c(t)\angle 240^0}_{\vec{F_c}(t)} = \vec{F_s}(t)$$

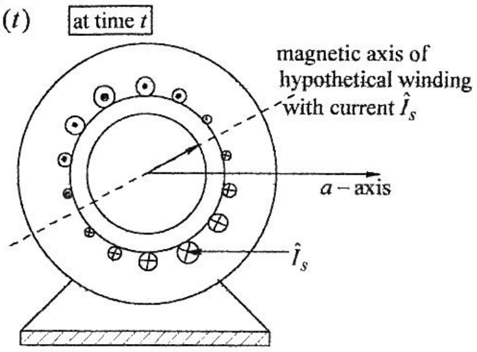
$\boxed{\text{at time } t}$

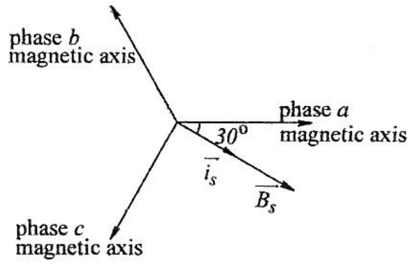
$$\vec{i_s}(t) = \frac{\vec{F_s}(t)}{N_s/2} \quad \Rightarrow \quad \hat{I}_s(t) = \frac{\hat{F}_s(t)}{N_s/2}$$

and $\theta_{i_s}(t) = \theta_{F_s}(t)$

$\vec{F_s}(t)$ and $\vec{i_s}(t)$ are collinear

$$\vec{B_s}(t) = \frac{N_s \mu_0}{2 l_g} \vec{i_s}(t)$$





- Magnetic field is produced by combined effect of i_a, i_b and i_c but could equivalently be produced by hypothetical winding current $\vec{i_s}(t)$ at θ_{i_s}
- helps in obtaining expression for torque

Space Vector Components:

Finding Phase Currents from Current Space Vector

$$\mathrm{Re}\left[\vec{i_s}\angle 0^0\right] = i_a + \underbrace{\mathrm{Re}\left[i_b\angle 120^0\right]}_{-\frac{1}{2}i_b} + \underbrace{\mathrm{Re}\left[i_b\angle 240^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_a$$

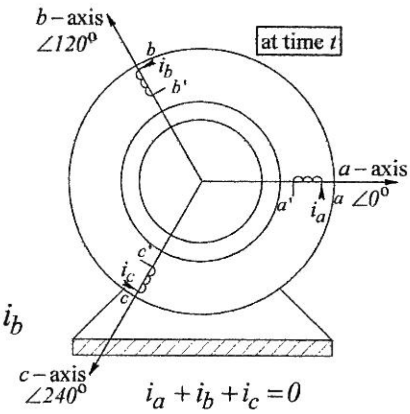
$$\Rightarrow i_a(t) = \frac{2}{3}\mathrm{Re}(\vec{i_s}\angle 0^\circ) = \frac{2}{3}\hat{I}_s\cos\theta_{i_s}$$

$$\mathrm{Re}\left[\vec{i_s}\angle -120^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -120^0\right]}_{-\frac{1}{2}i_a} + i_b + \underbrace{\mathrm{Re}\left[i_c\angle 120^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_b$$

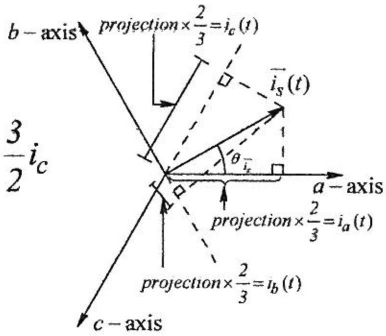
$$\Rightarrow i_b(t) = \frac{2}{3}\mathrm{Re}(\vec{i_s}\angle -120^\circ) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 120^\circ\right)$$

$$\mathrm{Re}\left[\vec{i_s}\angle -240^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -240^0\right]}_{-\frac{1}{2}i_a} + \underbrace{\mathrm{Re}\left[i_b\angle -240^0\right]}_{-\frac{1}{2}i_b} + i_c = \frac{3}{2}i_c$$

$$\Rightarrow i_c(t) = \frac{2}{3}\mathrm{Re}(\vec{i_s}\angle -240^\circ) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 240^\circ\right)$$

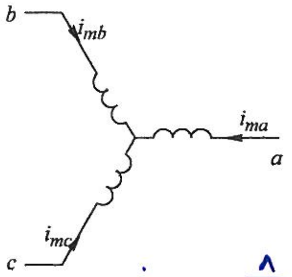


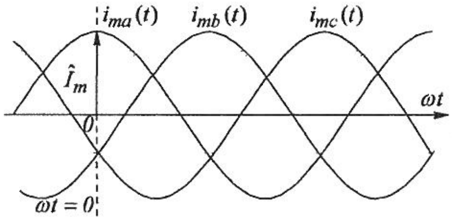
$i_a + i_b + i_c = 0$

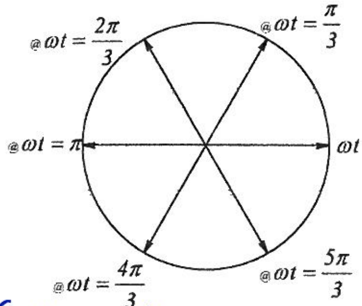


Balanced Sinusoidal Steady-State Excitation

(Rotor Open-Circuited – neglect stator winding resistance and leakage inductance)







$i_{ma} = \hat{I}_m \cos \omega t \qquad i_{mb} = \hat{I}_m \cos(\omega t - 120^\circ)$

$i_{mc} = \hat{I}_m \cos(\omega t - 240^\circ)$

$\overrightarrow{i_{ms}}(t) = \hat{I}_m \left[\cos \omega t \angle 0^\circ + \cos(\omega t - 120^\circ)\angle 120^\circ + \cos(\omega t - 240^\circ)\angle 240^\circ\right]$

$\Rightarrow \overrightarrow{i_{ms}}(t) = \hat{I}_{ms}\angle \omega t \quad$ where $\hat{I}_{ms} = \frac{3}{2}\hat{I}_m$

- Rotating MMF $\overrightarrow{F_{ms}}(t) = \frac{N_s}{2}\vec{i_s}(t) = \hat{F}_{ms}\angle\omega t \quad$ where $\hat{F}_{ms} = \frac{3}{2}\frac{N_s}{2}\hat{I}_m = \frac{N_s}{2}\hat{I}_{ms}$

  & Flux density $\overrightarrow{B_{ms}}(t) = \left(\frac{\mu_o}{\ell_g}\right)\frac{N_s}{2}\overrightarrow{i_{ms}}(t)$

- Constant amplitude

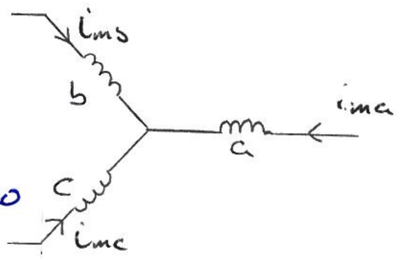
Question:

Derive an expression for the magnetizing current-space vector in an AC machine with a 3φ balanced sinusoidal s.s. excitation & sinusoidally distributed windings. Neglect the stator winding resistance & leakage inductance, and assume an open-circuited rotor.

$\overrightarrow{i_{ms}}(t) = i_a(t)\angle 0^\circ + i_b(t)\angle 120^\circ + i_c(t)\angle 240^\circ$

$= i_a(\cos 0 + j\sin 0) + i_b(\cos 120 + j\sin 120) + i_c(\cos 240 + j\sin 240)$

$= i_a(1 + j0) + i_b\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$



$\Rightarrow \overrightarrow{i_{ms}}(t) = \underbrace{\left[i_a - \frac{1}{2}(i_b + i_c)\right]}_{\text{Real}} + j\underbrace{\left[\frac{\sqrt{3}}{2}(i_b - i_c)\right]}_{\text{Imag.}}$

3-ϕ balanced

$$\Rightarrow i_a(t) = I_m \cos(\omega t)$$

$$i_b(t) = I_m \cos(\omega t - 120) = I_m [\cos\omega t \cos 120 + \sin\omega t \sin 120]$$

$$= I_m(-\tfrac{1}{2}\cos\omega t + \tfrac{\sqrt{3}}{2}\sin\omega t)$$

$$i_c(t) = I_m \cos(\omega t - 240)$$

$$= I_m(-\tfrac{1}{2}\cos\omega t - \tfrac{\sqrt{3}}{2}\sin\omega t)$$

$$\Rightarrow \text{Real}[\vec{i}_s(t)] = i_a - \tfrac{1}{2}(i_b + i_c)$$

$$= I_m [\cos\omega t - \tfrac{1}{2}(-\tfrac{1}{2}\cos\omega t + \tfrac{\sqrt{3}}{2}\sin\omega t - \tfrac{1}{2}\cos\omega t - \tfrac{\sqrt{3}}{2}\sin\omega t)]$$

$$= \tfrac{3}{2} I_m \cos\omega t$$

$$\text{Imag.}[\vec{i}_s(t)] = \tfrac{\sqrt{3}}{2}(i_b - i_c)$$

$$= \tfrac{\sqrt{3}}{2} I_m [-\tfrac{1}{2}\cos\omega t + \tfrac{\sqrt{3}}{2}\sin\omega t + \tfrac{1}{2}\cos\omega t + \tfrac{\sqrt{3}}{2}\sin\omega t]$$

$$= \tfrac{3}{2} I_m \sin\omega t$$

$$\Rightarrow \vec{i}_s(t) = \tfrac{3}{2} I_m (\cos\omega t + j\sin\omega t)$$

$$= \tfrac{3}{2} I_m \angle \omega t$$

Sinusoidal Distribution

$\Rightarrow$ Physical Vectors

$$\bar{H}(\theta) = \frac{N_{sp}}{l_g} i \cos\theta$$

$$\bar{B}(\theta) = \mu H(\theta) = \mu \frac{N_{sp}}{l_g} i \cos\theta$$

$$\bar{F}(\theta) = l_g H(\theta) = N_{sp} i \cos\theta$$

Space Vectors (Physical)

$$\bar{F}_s(t) = \bar{F}_a(t) + \bar{F}_b(t) + \bar{F}_c(t) = \hat{F}_s \angle \theta_{Fs} = N_{sp} \hat{I}_s \angle \theta_{Is}$$

$$\bar{H}_s(t) = \bar{H}_a(t) + \bar{H}_b(t) + \bar{H}_c(t) = \hat{H}_s \angle \theta_{Hs}$$

$$\bar{B}_s(t) = \bar{B}_a(t) + \bar{B}_b(t) + \bar{B}_c(t) = \hat{B}_s \angle \theta_{Bs}$$

Space Vectors (Math)

$$\bar{i}_s(t) = i_a(t)\angle 0^\circ + i_b(t)\angle 120^\circ + i_c(t)\angle 240^\circ = \hat{I}_s \angle \theta_{is}$$

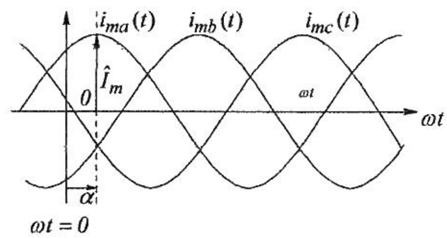
$$\bar{v}_s(t) = v_a(t)\angle 0^\circ + v_b(t)\angle 120^\circ + v_c(t)\angle 240^\circ = \hat{V}_s \angle \theta_{vs}$$

Blank

Relation Between Space Vectors and Phasors

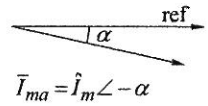
- Time domain

$$i_a(t) = \hat{I}_m \cos(\omega t - \alpha)$$



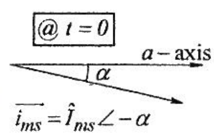
- Phasor

$$\vec{I}_a = \hat{I}_m \angle -\alpha$$



- Space Vector

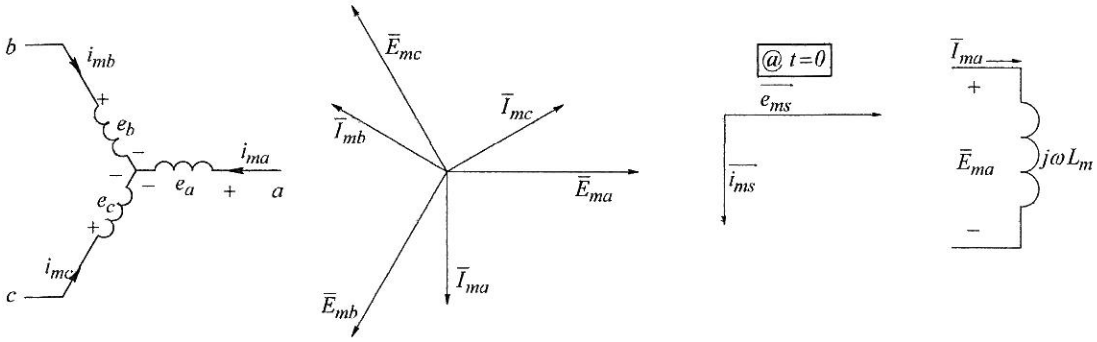
$$\vec{i}_{ms}\big|_{t=0} = \hat{I}_{ms} \angle -\alpha; \quad \hat{I}_{ms} = \frac{3}{2}\hat{I}_m$$



- Space Vector ⇔ phasor

$$\vec{i}_{ms}\big|_{t=0} \Longleftrightarrow \frac{3}{2}\bar{I}_{ma}$$

Voltages in the stator windings



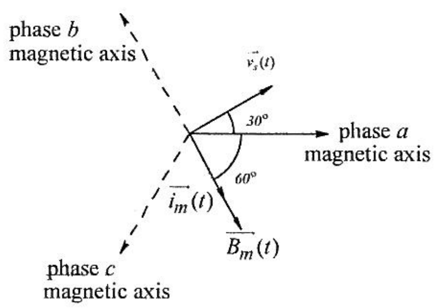
Where the three phase magnetizing inductance (2 pole), $L_m = \frac{3}{2}\frac{\pi\mu_o r l}{l_g}\left(\frac{N_s}{2}\right)^2$

Example

$$v_a(t) = 120\sqrt{2}\cos\omega t$$

$$v_b(t) = 120\sqrt{2}\cos(\omega t - 120°)$$

$$v_c(t) = 120\sqrt{2}\cos(\omega t - 240°)$$



$$\vec{v}_s = \frac{3}{2} \times 120\sqrt{2}\angle 30^0 = 254.56\angle 30^0 \ V$$

$$\overrightarrow{i_{ms}} = \frac{\vec{v}_s}{j\omega L_m} = \frac{254.56\angle(30^0 - 90^0)}{2\pi \times 60 \times 0.777} = 0.869\angle -60^0 \ A$$

$$\overrightarrow{B_{ms}} = \frac{\mu_o N_s \vec{i}_{ms}}{2\ell_g} = \frac{4\pi \times 10^{-7} \times 50 \times 0.869\angle -60^0}{10^{-3}} = 0.055\angle -60^0 \ Wb/m^2$$

EE4001 AC Machines

Derivation of per-phase magnetizing inductance

With only one phase, for example phase a excited by a magnetizing current i_a, the flux distribution in a 2-pole machine is

$$B_a(\theta)=\mu_0 \frac{N_{SP}}{l_g} i_{ma} \cos\theta$$

The energy density at an angle θ is

$$w_{ma}(\theta)=\frac{1}{2}\frac{B_a(\theta)^2}{\mu_0}$$

The differential energy stored at an angle θ (with respect to the a-axis) in a differential angle dθ is

$$\mathrm{d}W_{ma}(\theta)=w_{ma}(\theta)\cdot \mathrm{d}(\text{volume})$$

$$=\frac{1}{2}\frac{\left(\mu_0 \frac{N_{SP}}{l_g} i_{ma} \cos\theta\right)^2}{\mu_0}.r\mathrm{d}\theta.l.l_g$$

$$=\frac{\mu_0 N_{SP} r l}{2 l_g} i_{ma}^{\ 2}.\cos^2\theta \mathrm{d}\theta$$

Integrating both sides with respect to θ from 0 to 2π gives the total energy stored in the airgap.

$$W_{ma}=\frac{\mu_0 N_{SP} r l}{2 l_g} i_{ma}^{\ 2}.\int_0^{2\pi}\cos^2\theta \mathrm{d}\theta$$

$$=\frac{1}{2}\left(\frac{\pi\mu_0 N_{SP}^{\ 2} r l}{l_g}\right) i_{ma}^{\ 2}$$

The energy stored in an inductor is given by

$$E=\frac{1}{2}L_{a-a}\, i_{ma}^{\ 2}$$

Therefore, the per-phase self inductance is

$$L_{ph}=L_{a-a}=\frac{\pi\mu_0 N_{SP}^{\ 2} r l}{l_g}$$

The magnetizing inductance is easily derived based on an understanding of the relationships between the self and mutual inductances.

(i) Suppose the winding of phase b is aligned with the winding of phase a, i.e. they have aligned magnetic axes.

$$\theta_{mb}=0°$$

$$L_{a-b}=L_{a-a}$$

Thus, any flux generated by phase a is perfectly coupled to phase b. So the mutual inductance between then is equal to the self inductance of either winding.

(ii) Suppose phase b is 180° out of phase with phase a.

$$\theta_{mb}=180°$$

$$L_{a-b}=-L_{a-a}$$

Thus, any flux generated by phase a is perfectly coupled to phase b but has opposite polarity. So the mutual inductance between then is equal and opposite to the self inductance of either winding.

(iii) Suppose phase b is 90° out of phase with phase a,

$$\theta_{mb} = 90°$$

$$L_{a-b} = 0$$

In this case there is no magnetic flux linkage due to orthogonality of the windings and so the mutual inductance is zero.

The relationship between the mutual and self inductance above is simply a function of $\cos\theta_{mb}$. In the three-phase machine, the phase windings are physically displaced w.r.t. each other by 120° or $\cos\theta_{mb} = -\frac{1}{2}$. Thus the mutual inductances from phases b and c w.r.t. phase a are given by

$$L_{mut} = -\frac{1}{2}L_{ph} = -\frac{1}{2}\frac{\pi\mu_0 N_{SP}^2 rl}{l_g} \tag{1}$$

In a three-phase ac machine the back emf of the three phases are given by

$$\begin{bmatrix} e_{ma} \\ e_{mb} \\ e_{mc} \end{bmatrix} = \begin{bmatrix} L_{a-a} & L_{a-b} & L_{a-c} \\ L_{a-b} & L_{b-b} & L_{b-c} \\ L_{a-c} & L_{b-c} & L_{c-c} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_{ma} \\ i_{mb} \\ i_{mc} \end{bmatrix}$$

Thus, the back emf for phase a is as follows

$$e_{ma} = L_{a-a}\frac{\mathrm{d}}{\mathrm{d}t}i_{ma} + L_{a-b}\frac{\mathrm{d}}{\mathrm{d}t}i_{mb} + L_{a-c}\frac{\mathrm{d}}{\mathrm{d}t}i_{mc}$$

or

$$e_{ma} = L_{ph}\frac{\mathrm{d}}{\mathrm{d}t}i_{ma} + L_{mut}\frac{\mathrm{d}}{\mathrm{d}t}i_{mb} + L_{mut}\frac{\mathrm{d}}{\mathrm{d}t}i_{mc} \tag{2}$$

Substituting (1) into (2) gives

$$e_{ma} = L_{ph}\left(\frac{\mathrm{d}}{\mathrm{d}t}i_{ma} - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}i_{mb} - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}i_{mc}\right) \tag{3}$$

We already know that in a balanced three-phase machine

$$i_{ma} + i_{mb} + i_{mc} = 0 \quad \text{or} \quad i_{ma} = -i_{mb} - i_{mc} \tag{4}$$

Substituting (4) into (3) gives

$$e_{ma} = \frac{3}{2}L_{ph}\frac{\mathrm{d}}{\mathrm{d}t}i_{ma} = L_m\frac{\mathrm{d}}{\mathrm{d}t}i_{ma}$$

where

$$L_m = \frac{3}{2}L_{ph} = \frac{3}{2}\frac{\pi\mu_0 N_{SP}^2 rl}{l_g}$$

is the per-phase **magnetizing** inductance.