Tutorial Questions for EE4008

1. (a) A First Order Lowpass IIR Digital Filter has a transfer function

$$H_{LP}(z) = G\frac{1 + z^{-1}}{1 - \alpha z^{-1}}$$

Determine the Gain factor G in terms of α.

(b) Derive an expression for α in terms of the 3dB cutoff frequency ω_c.

(c) Determine the transfer function $H_{LP}(z)$, of a first-order Low Pass filter with a 3-dB cutoff Frequency of 0.65π. Determine the constant coefficient difference equation that implements the filter in the time domain. Plot the magnitude response of the filter for $0 \leq \omega \leq \pi$, clearly identifying the 3dB cutoff frequency ω_c.

(d) A comb filter with a transfer function $G(z)$ is formed by taking the transfer function $H_{LP}(z)$ and replacing each delay by M delays, such that:

$$G(z) = H_{LP}(z^M)$$

Draw the pole/zero plot for $M = 2$ and sketch the magnitude response for the same value of M, for $0 \leq \omega \leq \pi$.

2. (a) A First Order Highpass IIR Digital Filter has a transfer function

$$H_{HP}(z) = G\frac{1 - z^{-1}}{1 - \alpha z^{-1}}$$

Determine the Gain factor G in terms of α.

(b) Derive an expression for α in terms of the 3dB cutoff frequency ω_c.

(c) Determine the transfer function $H_{HP}(z)$, of a first-order High Pass filter with a 3-dB cutoff Frequency of $F_c = 10$, when $F_s = 100$Hz. Determine the constant coefficient difference equation that implements the filter in the time domain. Plot the squared magnitude response of the filter.

3. (a) A Second order Bandpass IIR Digital Filter has a transfer function

$$H_{BP}(z) = \frac{K(1 - z^{-2})}{1 - \beta(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

Determine the Gain factor K in terms of α and the value β in terms of the centre frequency ω_0.

(b) $H_{BP}(z)$ has a pair of complex conjugate poles at $z = re^{\pm j\phi}$ such that

$$H_{BP}(z) = \frac{K(1 - z^{-2})}{(1 - re^{j\phi}z^{-1})(1 - re^{-j\phi}z^{-1})}$$

Show that

$$r = \sqrt{\alpha}$$

and

$$\phi = \cos^{-1}\left(\frac{\beta(1 + \alpha)}{2\sqrt{\alpha}}\right)$$

(c) The 3-dB Bandwidth of the 2nd order Bandpass filter is given by:

$$\Delta\omega_{3db} = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$$

Determine the transfer function $H(z)$, of a second-order bandpass filter with a centre frequency of 50Hz and a 3-dB bandwidth of 10Hz when the sampling frequency is 256Hz. Determine the constant coefficient difference equation that implements the filter in the time domain. Draw the pole/zero plot of $H(z)$ and determine the values of the polar co-ordinates r and ϕ. Sketch the magnitude response of the filter, clearly identifying the centre frequency of 50Hz and the 3-dB Bandwidth.

4. (a) A second order bandstop IIR digital filter has a transfer function

$$H_{BS}(z) = \frac{K(1 - 2\beta z^{-1} + z^{-2})}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$

Determine the gain factor K in terms of α and the value of β in terms of the centre frequency of the Bandstop filter ω_0.

(b) The 3-dB bandwidth of the second order bandstop filter is given by:

$$\Delta\omega_{3db} = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$$

Determine the transfer function $H(z)$, of a second order bandstop filter to eliminate 50Hz noise in a system with a sampling frequency of 250Hz. The 3dB bandwidth should be 2Hz. Determine the constant coefficient difference equation that implements the filter $H(z)$ in the time domain. Express $H(z)$ in terms of polar co-ordinates and draw the pole/zero plot of $H(z)$. Sketch the magnitude response $|H(z)|$ of the filter, clearly identifying the bandstop frequency of 50Hz and the 3-dB Bandwidth.

5. Determine the system function $H(z)$ of a digital lowpass filter using the bilinear transformation of a Butterworth filter. Sketch the magnitude response of the filter. The digital filter specifications are:

passband edge frequency	ω_p	0.11π rad/s
stopband edge frequency	ω_s	0.22π rad/s
passband ripple	A_{max}	3dB
minimum stopband attenuation	A_{min}	15dB

6. Determine the system function $H(z)$ of a digital lowpass filter using the bilinear transformation of a Butterworth filter. The digital filter specifications are:

passband edge frequency	ω_p	0.12π rad/s
stopband edge frequency	ω_s	0.32π rad/s
passband ripple	A_{max}	3dB
minimum stopband attenuation	A_{min}	20dB

Note for $A_{max} = 3$dB then $\Omega_c = \Omega_p$

Draw the pole-zero plots of the filter designed and use this to estimate and sketch the magnitude response of the filter, for $0 \leq \omega \leq \pi$.

Appendix of Equations

- Butterworth Filter Order given by:

$$n = \left\lceil \frac{\log_{10}\left[\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}\right]}{2\log_{10}\left[\frac{\Omega_s}{\Omega_p}\right]} \right\rceil$$

- Butterworth 3-dB cutoff frequency given by:

$$\Omega_c = \Omega_p 10^{-\left[\frac{\log_{10}\left[10^{0.1A_{max}}-1\right]}{2n}\right]}$$

- Table of Butterworth Polynomials:

n	
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.76536s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$