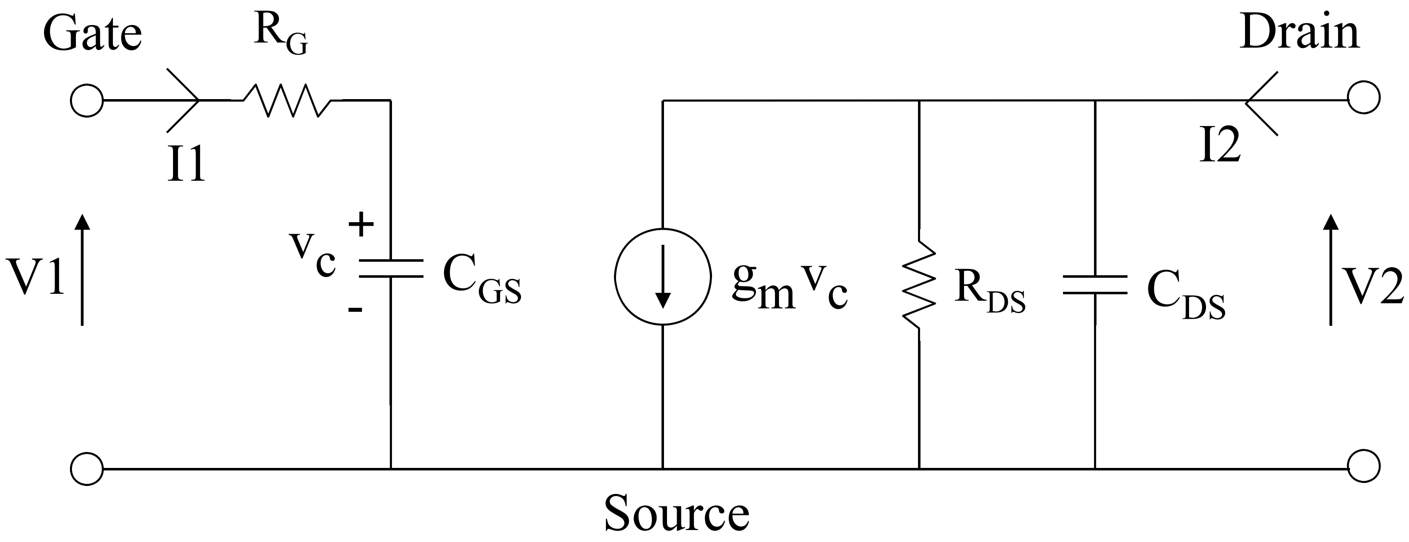
EE4011 RF IC Design Summer 2009
Question 1(a)

Simplified MESFET small-signal circuit, ignoring C_{GD}



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the above formulas to the equivalent circuit and simplifying the resulting expressions leads to the final y-parameter formulas:

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \qquad y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

$$y_{12} = 0 \qquad y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

10 marks

Question 1(b)

The previous expressions for the y-parameters can be re-arranged to allow the small-signal element values to be determined from the y-parameters (assuming y_{12}=0). Putting in the values of the y-parameters at 1.5GHz gives:

$y_{11} = 0.0071 \angle 87.98°$
$y_{12} = 0$
$y_{21} = 0.1999 \angle -2.02°$
$y_{22} = 0.0173 \angle 15.79°$

$$R_G = \mathcal{Re}\left\{\frac{1}{y_{11}}\right\} = 5\Omega$$

$$C_{GS} = -\frac{1}{\omega \mathcal{Im}\left\{\frac{1}{y_{11}}\right\}} = 0.75pF$$

$$g_m = \frac{1}{\mathcal{Re}\left\{\frac{1}{y_{21}}\right\}} = 0.2S$$

$$R_{DS} = \frac{1}{\mathcal{Re}\{y_{22}\}} = 60\Omega$$

$$C_{DS} = \frac{\mathcal{Im}\{y_{22}\}}{\omega} - 0.5pF$$

$$f_T = \frac{g_m}{2\pi C_{GS}} = 42.44GHz$$

10 marks

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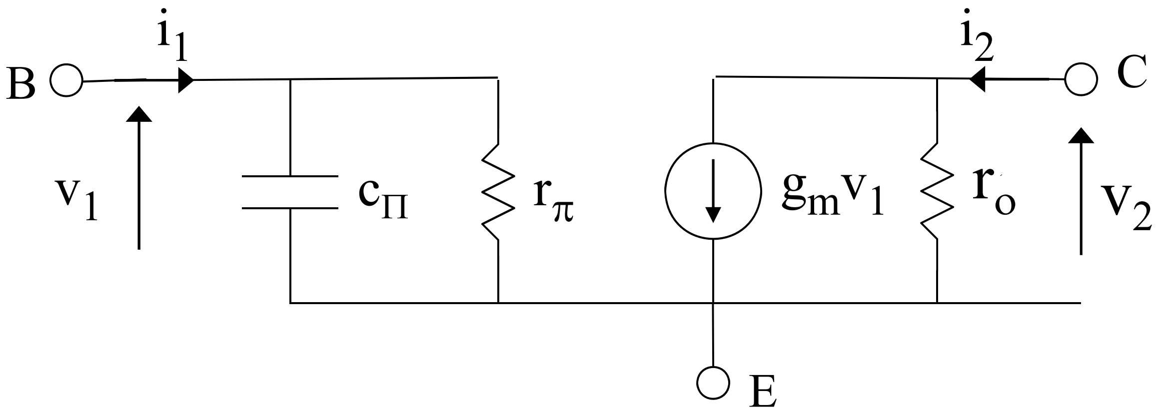
Question 2(a)

A suitable small-signal model for a BJT, accounting only for the base-emitter capacitances is:



Applying the z-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = \frac{r_\pi}{1 + j\varpi r_\pi c_\pi}$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = -\frac{g_m r_\pi r_o}{1 + j\varpi r_\pi c_\pi}$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = 0$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = r_o$$

10 marks

Question 2(b)

Calculate the z-parameters under the following conditions

T=300K, f=1GHz, V_{BE} = 0.8V, V_{CE} = 3.0V, $I_S = 1\times10^{-15}$A, V_A = 10V, β = 100, C_{JE} = 0.3pF, M_{JE} = 0.5, V_{JE} = 1.0V and τ_F = 0.1ns.

$$V_T = \frac{kT}{q} = 25.9mV \quad I_C = I_S \exp\left(\frac{qV_{BE}}{kT}\right)\left(1+\frac{V_{CE}}{V_A}\right) = 35.4mA$$

$$g_m = \frac{I_C}{V_T} = 1.37S \quad r_o \approx \frac{V_A}{I_C} = 283\,\Omega$$

$$c_\pi = \frac{C_{JE}}{\left(1-\frac{V_{BE}}{V_{JE}}\right)^{M_{JE}}} + g_m\tau_F = 0.67\times10^{-12} + 1.37\times10^{-10} \approx 1.37\times10^{-10}F$$

$$r_\pi = \frac{\beta}{g_m} = 73\,\Omega$$

Inserting these values into the previous formulas for the z-parameters at 1GHz gives:

$z_{11} = 1.16 \angle -89.1°$
$z_{12} = 0$
$z_{21} = 448 \angle 90.9°$
$z_{22} = 283 \angle 0°$

10 marks

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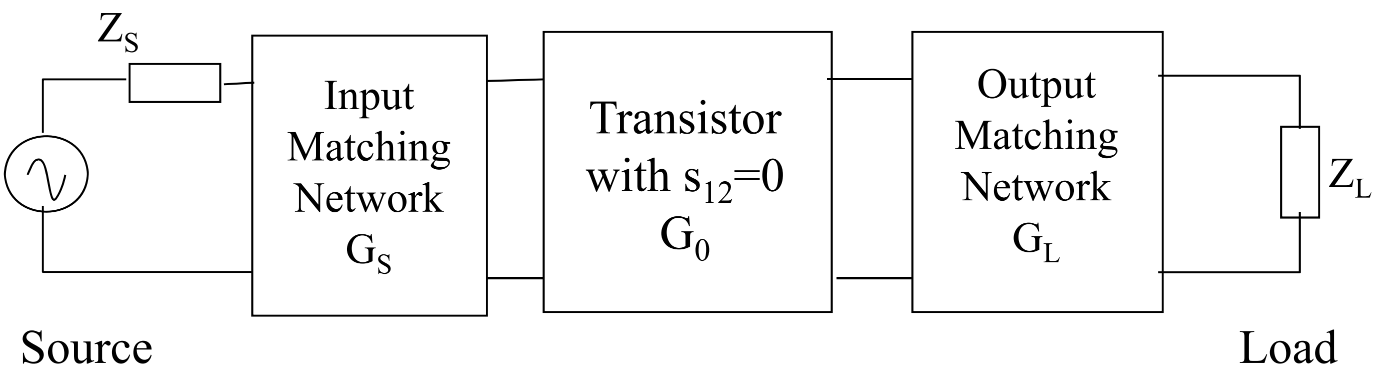
Question 3(a)

(i) The operating power gain (also just called the power gain) is the ratio of the power delivered to the load to the power delivered to the network by the source.
(ii) The transducer power gain is the ratio of the power delivered to the load to the power *available* from the source.
(iii) The available power gain is the ratio of the power *available* from the network to the power *available* from the source.

3 marks

(b) Maximum unilateral gain



To obtain the maximum unilateral gain, input and output matching networks are designed to provide conjugate matching to the transistor input and output ports as follows:

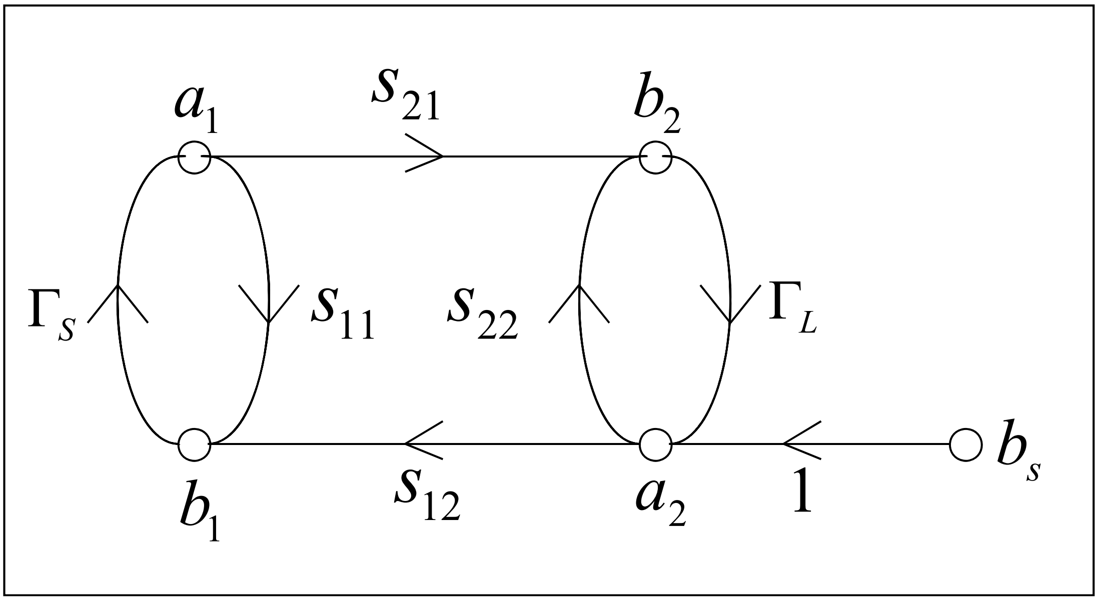
$$\Gamma_S = s_{11}^* \quad \Gamma_L = s_{22}^*$$

If the transistor is not already unilateral then an extra network can be added to force s_{12}=0.

3 marks

Question 3(c)

(c) Output reflection coefficient



$$\Gamma_{OUT} = \frac{b_2}{a_2}$$

Following the signal flow diagram from left to right:

$$a_1 = \Gamma_S b_1$$

$$b_1 = s_{11}a_1 + s_{12}a_2 = s_{11}\Gamma_S b_1 + s_{12}a_2 \Rightarrow b_1 = \frac{s_{12}a_2}{1 - s_{11}\Gamma_S} \Rightarrow a_1 = \frac{\Gamma_S s_{12} a_2}{1 - s_{11}\Gamma_S}$$

$$b_2 = s_{21}a_1 + s_{22}a_2 = \frac{s_{21}\Gamma_S s_{12} a_2}{1 - s_{11}\Gamma_S} + s_{22}a_2 = \frac{\left(s_{21}s_{12}\Gamma_S + s_{22} - s_{11}s_{22}\Gamma_S\right)a_2}{1 - s_{11}\Gamma_S}$$

$$b_2 = \frac{\left(s_{21}s_{12}\Gamma_S + s_{22}\left(1 - s_{11}\Gamma_S\right)\right)a_2}{1 - s_{11}\Gamma_S}$$

$$\Rightarrow \Gamma_{OUT} = \frac{b_2}{a_2} = s_{22} + \frac{s_{21}s_{12}\Gamma_S}{1 - s_{11}\Gamma_S}$$

12 marks

(d)

$s_{11} = 0.863 \angle -79.1°$
$s_{12} = 0.072 \angle 36.5°$
$s_{21} = 3.434 \angle 106.2°$
$s_{22} = 0.627 \angle -58.3°$
$\Gamma_S = 0.1 \angle 0°$

$\Gamma_{OUT} = 0.603 \angle -59°$

2 marks

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Question 4

$$s_{11} = 0.707\angle -155^\circ \quad s_{12} = 0 \quad s_{21} = 5.00\angle 180^\circ \quad s_{22} = 0.51\angle -20^\circ$$

$$Z_0 = 50\,\Omega \qquad F_{\min} = 3\,dB \quad \Gamma_{opt} = 0.45\angle 180^\circ \quad R_N = 4\,\Omega$$

(a) (i)

$$G_{S,\max} = \frac{1}{1-|s_{11}|^2} = \frac{1}{1-|0.707|^2} = 2\,(ratio) = 3dB$$

$$G_0 = |5|^2 = 25\,(ratio) = 14dB$$

$$G_{L,\max} \frac{1}{1-|s_{22}|^2} = \frac{1}{1-|0.51|^2} = 1.35\,(ratio) = 1.3dB$$

maximum gain:

$$G_{TU,\max,dB} = G_{S,\max,dB} + G_{0,dB} + G_{L,\max,dB} = 3dB + 14dB + 1.3dB = 18.3dB$$

2 marks

(a)(ii)

The noise circle for $F_i = 3.1$dB

$$F_{dB} = 10 * \log_{10}(F_{ratio}) \Rightarrow F_{ratio} = 10^{\frac{F_{dB}}{10}}$$

$$F_{\min,ratio} = 10^{\frac{3}{10}} = 1.9953 \qquad F_{i,ratio} = 10^{\frac{3.1}{10}} = 2.0417$$

$$\Gamma_{opt} = 0.45\angle 180^\circ = -0.45$$ Caution – see below!

$$N_i = \frac{F_{i,ratio} - F_{\min,ratio}}{4R_N / Z_0}\left|1+\Gamma_{opt}\right|^2 = \frac{2.0417 - 1.9953}{4 \times 4/50}\left|1-0.45\right|^2 = 0.0439$$

$$C_{Fi} = \frac{\Gamma_{opt}}{N_i + 1} = \frac{-0.45}{0.0439+1} = -0.43 = 0.43\angle 180^\circ$$

$$R_{Fi} = \frac{\sqrt{N_i\left(N_i + 1 - \left|\Gamma_{opt}\right|^2\right)}}{(N_i+1)} = \frac{\sqrt{0.0439\left(0.0439+1-|0.45|^2\right)}}{(0.0439+1)} = 0.18$$

3 marks

The centre of the 3.1dB noise circle is at Γ = -0.43 and it has a radius of 0.18.

Caution – in this example Γ_{OPT} was a real number making it easier to put into the formulas but in most cases it will be a complex number so be careful when evaluating the noise circle formulas.

Question 4(b) 15 marks

The maximum source gain is 3dB. But the noise cannot exceed 3.1dB and it is seen on the Smith Chart that the point s_{11}* is outside the 3.1dB noise circle and so would give too much noise. Therefore, some source noise circles will have to be drawn to try to find a source reflection coefficient that will give the highest gain, without exceeding 3.1dB noise.

Because the maximum source gain is 3dB, pick a value lower than this and draw the circle for this – e.g. pick a source gain of 2.5dB and draw the circle.

The source gain circle for $G_{S,dB} = 2.5$

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{2.5}{10}} = 1.778$$

$$g_s = \frac{G_S}{G_{S,\max}} = \frac{1.778}{2} = 0.889$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.889\times|0.707|}{1-|0.707|^2(1-0.889)} = 0.665$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.889}\left(1-|0.707|^2\right)}{1-|0.707|^2(1-0.889)} = 0.18$$

The centre of the 2.5dB source gain circle is a distance 0.665 along the line joining the origin and the point s_{11}^* and its radius is 0.18. When this circle is drawn it overlaps the 3.1dB noise circle by a small amount. This indicates that the gain can be increased a little while still satisfying the noise criterion. Therefore, plot a gain circle for a slightly higher gain e.g. 2.6dB.

The source gain circle for $G_{S,dB} = 2.6$

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{2.6}{10}} = 1.820$$

$$g_s = \frac{G_S}{G_{S,\max}} = \frac{1.820}{2} = 0.910$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.910\times|0.707|}{1-|0.707|^2(1-0.910)} = 0.674$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.910}\left(1-|0.707|^2\right)}{1-|0.707|^2(1-0.910)} = 0.16$$

The centre of the 2.6dB source gain circle is a distance 0.674 along the line joining the origin and the point s_{11}^* and its radius is 0.16. When this is drawn it is seen to just touch the 3.1dB noise circle. Therefore, this point will give a noise of 3.1dB and the highest possible source gain of 2.6dB. Take this point as the source reflection coefficient.

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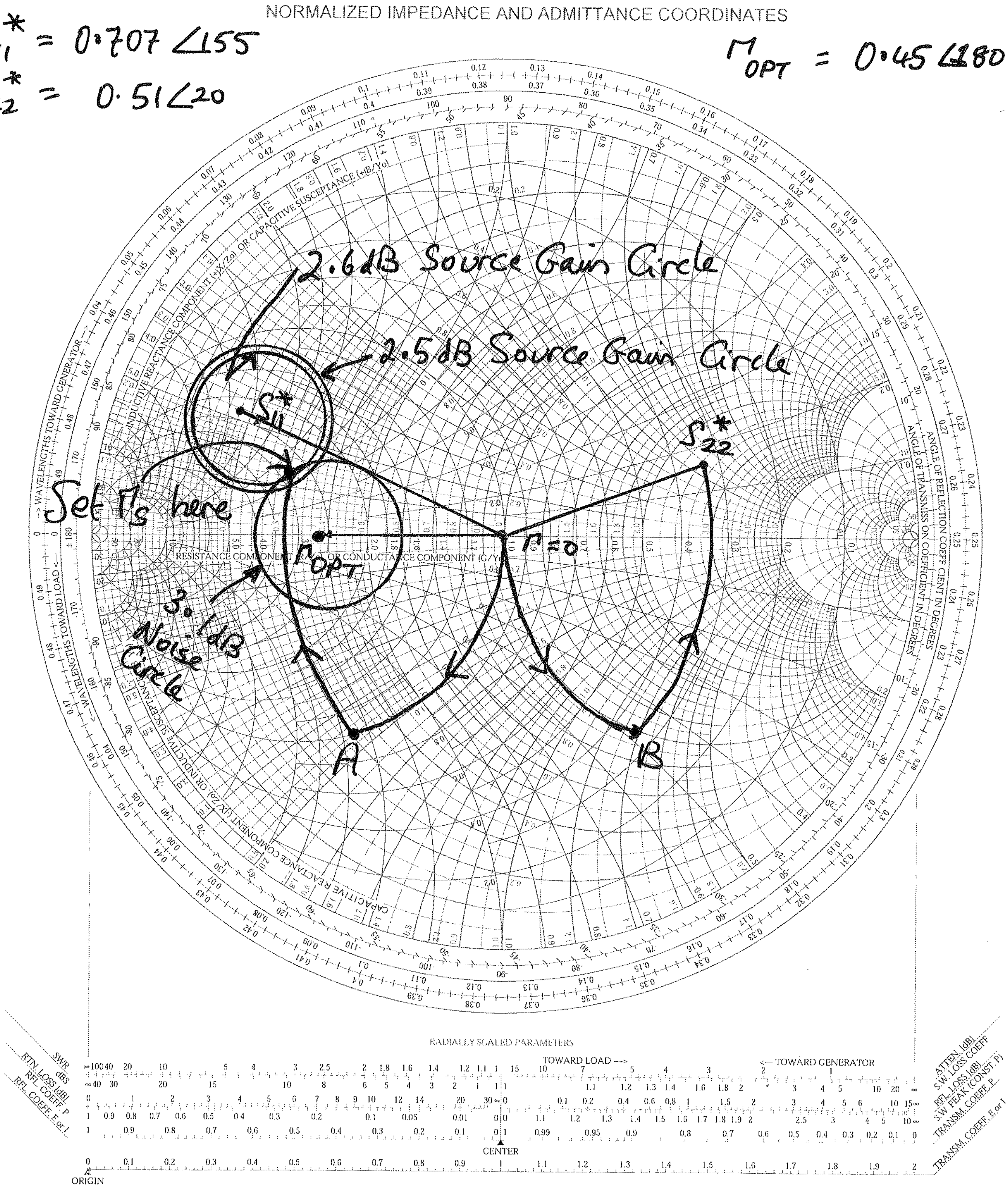
$S_{11}^* = 0.707\angle 155$

$S_{22}^* = 0.51\angle 20$

$\Gamma_{OPT} = 0.45\angle 180°$



At the origin of the Smith Chart: $x = 0, b = 0$

At point A: $x = -0.46, b = 1.54$ At point B: $x = -1.4, b = 0.48$

At Γ_S : $x = 0.13, b = -1.2$ At S_{22}^* : $x = +1.2, b = -0.16$

Question 4(b) continued

Design of the input and output matching networks. The load reflection coefficient can be set to s_{22}* because this does not effect noise and can be set for maximum gain.

Moving from Z_0 (Γ=0) to point A:
Clockwise on conductance circle – shunt capacitor

susceptance at Z_0: b = 0
susceptance at A: b = 1.54

$$C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|1.54|}{2\pi \times 1 \times 10^9 \times 50} = 4.91 pF$$

Moving from A to Γ_S:
Clockwise on resistance circle – series inductor

reactance at A: x = -0.46
reactance at Γ_S: x = 0.13

$$L = \frac{Z_0 |\Delta x|}{2\pi f} = \frac{50 \times |0.59|}{2\pi \times 1 \times 10^9} = 4.69 nH$$

Output Matching Element Values

Moving from Z_0 (Γ=0) to point B:
Anti-clockwise on resistance circle – series capacitor

reactance at Z_0: x = 0
reactance at B: x = -1.41

$$C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 1 \times 10^9 \times |-1.41| \times 50} = 2.25 pF$$

Moving from B to s_{22}^*:
Anti-clockwise on conductance circle – shunt inductor

susceptance at B: b = 0.47
susceptance at s_{22}^*: b = -0.16

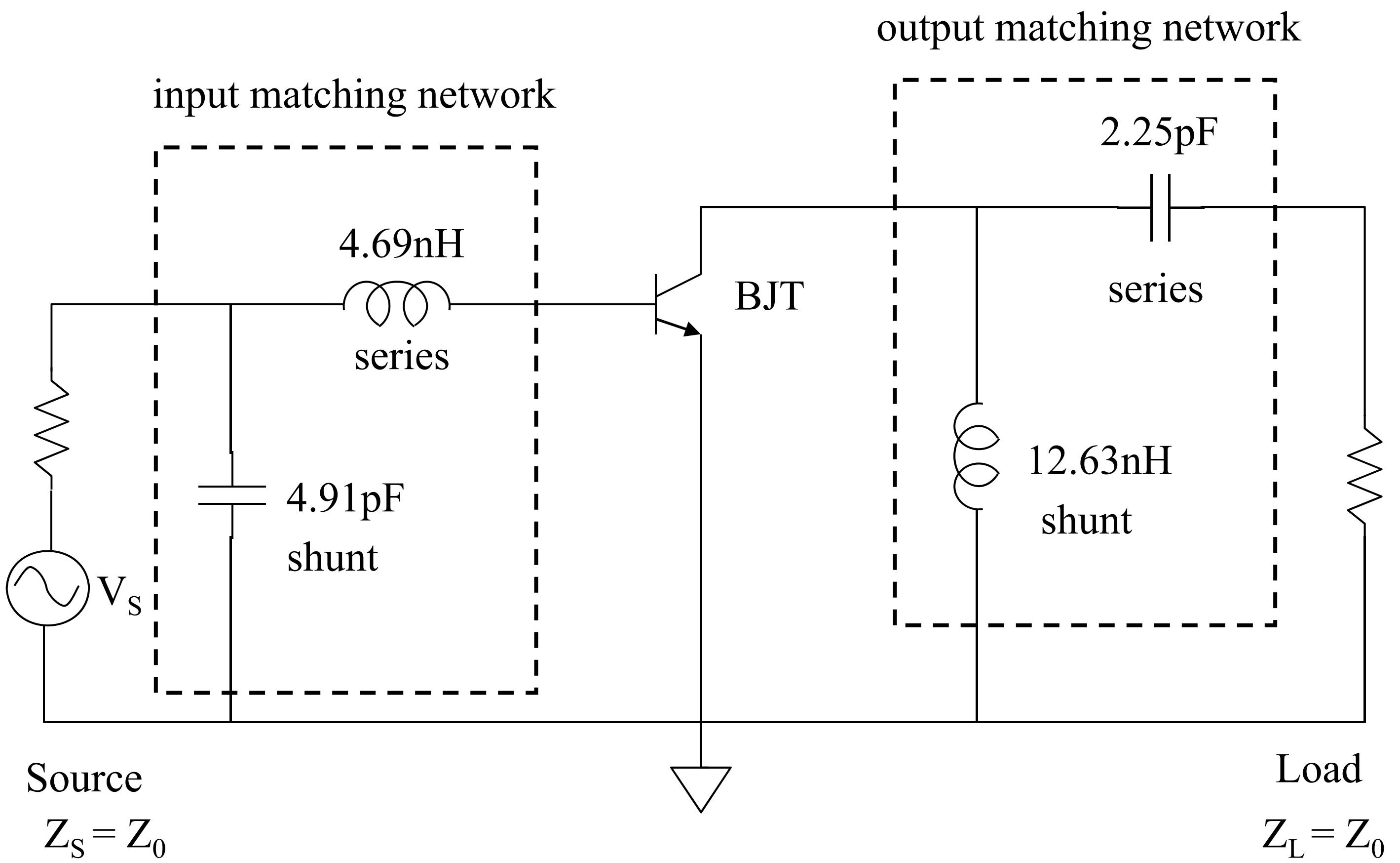
$$L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 1 \times 10^9 \times |-0.63|} = 12.63 nH$$

The maximum gain that can be achieved while meeting the noise specification is

$$G_{MAX} = G_S + G_0 + G_{L,\max} = 2.6 + 14 + 1.3 = 17.9 dB$$

The matching elements are shown on the next page.

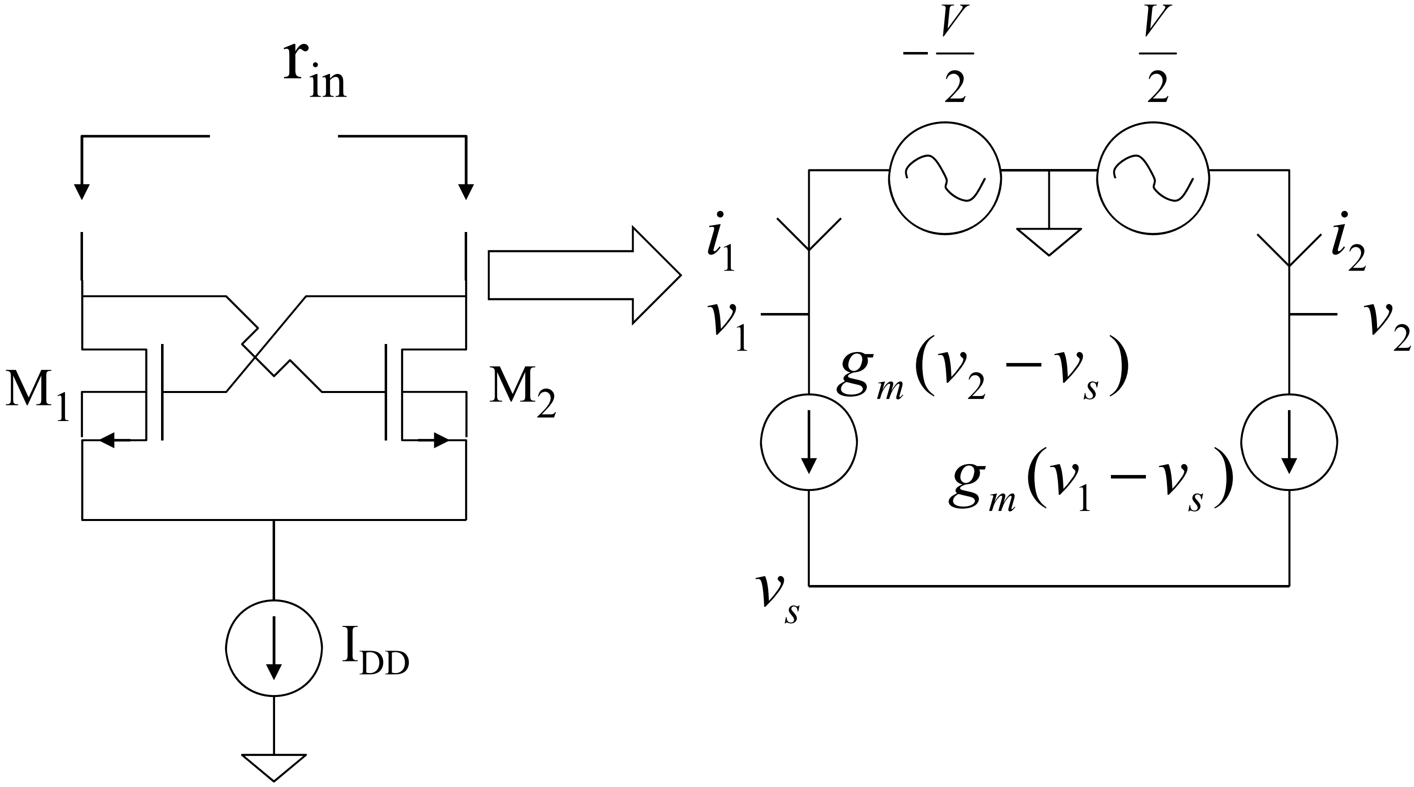
Question 4(b) continued



EE4011 Summer 2009
Question 5(a)

Small-signal analysis to determine input resistance of cross-coupled MOS pair:

assume: $g_{m1} = g_{m2} = g_m$



$$i_1 = -i_2 \Rightarrow g_m(v_2 - v_s) = -g_m(v_1 - v_s)$$

$$\Rightarrow g_m(v_1 + v_2) = 2g_m v_s$$

$$\Rightarrow v_s = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}\left(\frac{V}{2} - \frac{V}{2}\right) = 0 \qquad r_{in} = \frac{v_{tot}}{i_2} = \frac{\frac{V}{2} - \left(-\frac{V}{2}\right)}{g_m\left(-\frac{V}{2}\right)} = -\frac{2}{g_m}$$

$$\Rightarrow i_2 = g_m v_1 = g_m\left(-\frac{V}{2}\right)$$

The MOSFETs sustain oscillation by choosing gm so that the effect of any parasitic resistances in the circuit (especially in the inductors) are cancelled out.

10 marks

Question 5(b)

Ibias = 5mA, L=0.25μm T_{OX}=5nm, μ=400cm^2/Vs,
ε_r = 3.9 (dielectric constant of SiO_2) R_{IND} = 5Ω

Determine W – the total resistance of the inductors in 10Ω and this has to be cancelled out with the MOSFETs by choosing r_{in}=-10Ω

$$g_m = -\frac{2}{r_{in}} = -\frac{2}{-10} = 0.2$$

For MOSFETs in saturation:

$$g_m = \sqrt{2\frac{W}{L}\mu C'_{OX} I_{DS}} \Rightarrow$$

$$W = \frac{{g_m}^2 L}{2\mu C'_{OX} I_{DS}} = \frac{{g_m}^2 T_{OX} L}{2\mu\varepsilon_0\varepsilon_r I_{DS}}$$

For each MOSFET I_{DS}=Ibias/2 and putting in the values gives:

$$W = 7.2mm$$

5 marks

Question 5(c)

Diode Parameters: M_J = 0.5, V_J = 0.8V and C_{J0} = 1.5pF
L=4nH, Vout (DC) = 1.5V, Vbias = 0V, C_{par} = 1pF

The diode capacitance is:

$$C_D = \frac{C_{J0}}{(1 - V_D / V_J)^{MJ}} = \frac{1.5\times10^{-12}}{(1-(-1.5)/0.8)^{0.5}} = 0.885\,pF$$

The total capacitance at each output node is:

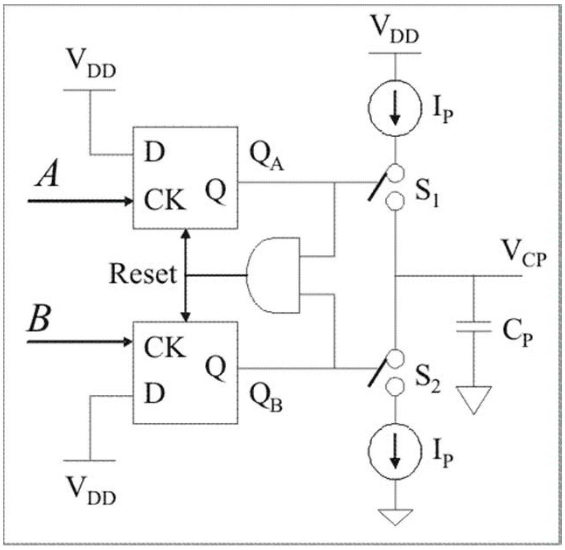
$$C_{TOT} = C_{PAR} + C_D = 1pF + 0.885\,pF = 1.885\,pF$$

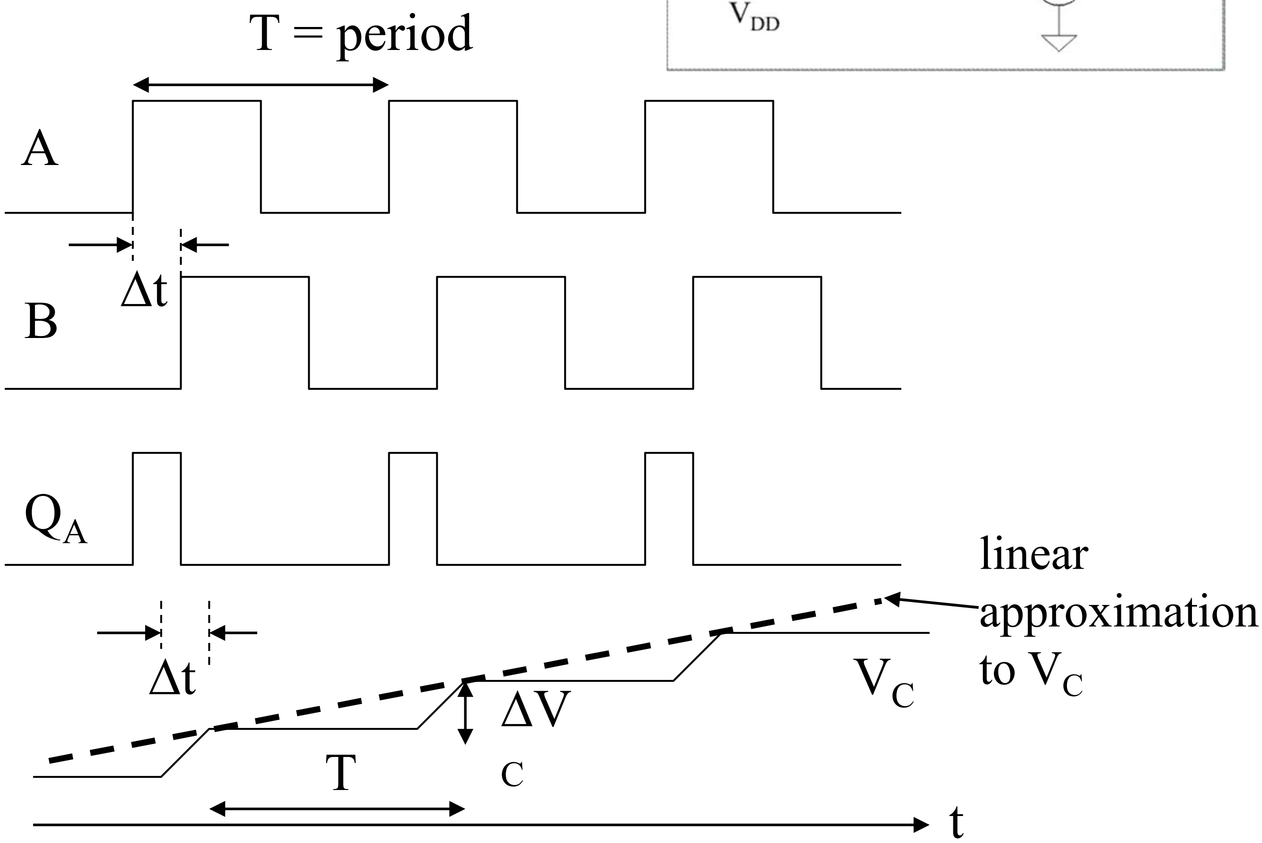
The oscillation frequency is

$$F_{OSC} = \frac{1}{2\pi\sqrt{LC_{TOT}}} = 1.833GHz$$

5 marks

Question 6(a)





$$\Delta t = \frac{\Delta\varphi}{2\pi}T \qquad \Delta V_C = \frac{I_P}{C_P}\Delta t = \frac{I_P}{C_P}\frac{\Delta\varphi}{2\pi}T$$

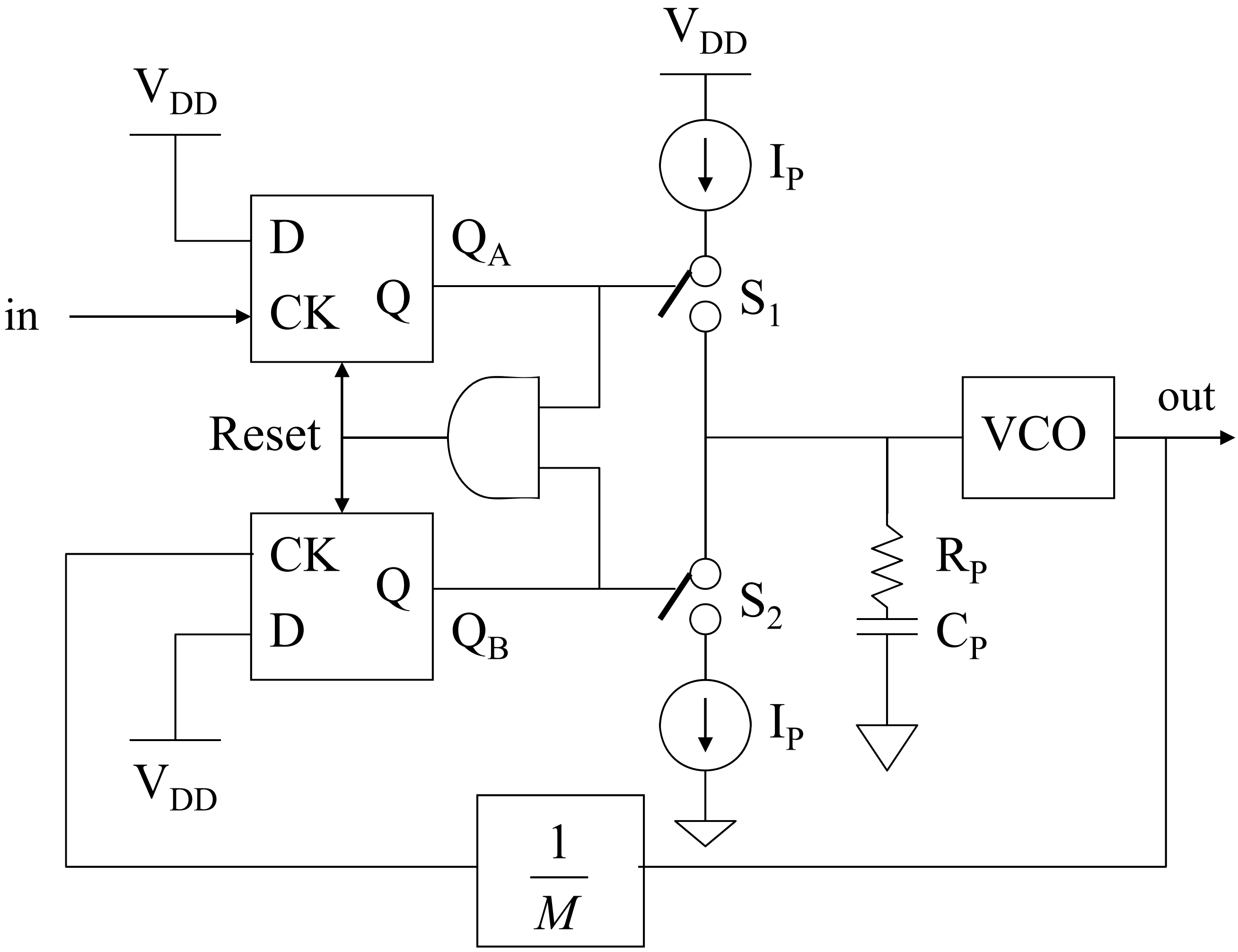
$$slope = \frac{dV_C}{dt} \approx \frac{\Delta V_C}{T} = \frac{I_P}{C_P}\frac{\Delta\varphi}{2\pi}T\frac{1}{T} = \frac{I_P}{2\pi C_P}\Delta\varphi$$

$$\Rightarrow \frac{dV_C}{dt} = \frac{I_P}{2\pi C_P}\Delta\varphi \Rightarrow V_C = \frac{I_P}{2\pi C_P}\int \Delta\varphi$$

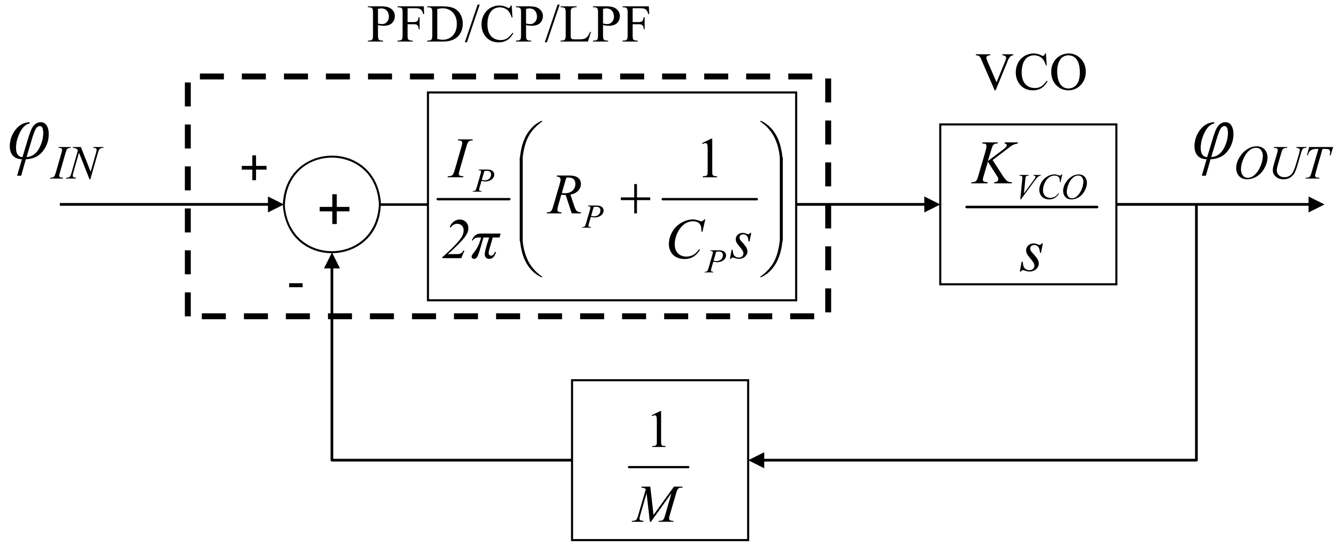
$$\Rightarrow V_C(s) = \frac{I_P}{2\pi C_P}\frac{1}{s}\Delta\varphi(s)$$

5 marks

Question 6(b) Type II PLL with integer divider:



In terms of transfer functions:



Question 6(b) continued

The open look transfer function of the Type II PLL is:

$$H(s) = \frac{I_P K_{VCO}}{2\pi s}\left(R_P + \frac{1}{C_P s}\right)$$

The closed loop transfer function is:

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\varsigma\omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

6 marks

Question 6(c)

Putting the values given into the formulas above:

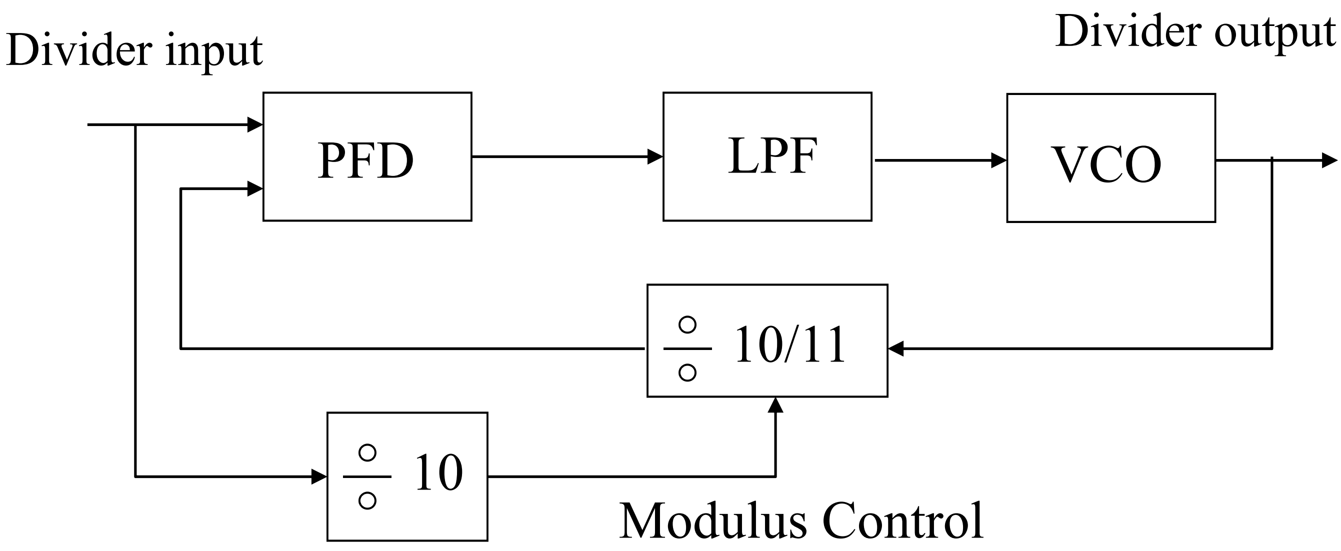
(i) $w_n = 1.41 \times 10^6$ rad/s

2 marks

(ii) $\zeta = 0.707$

2 marks

Question 6(d)



A Type II PLL with $F_{OUT} = 10.1F_{IN}$ can be made with the same overall architecture as shown earlier but with the divided replaced by the above.

The dual-modulus counter normally divides by 10 (with the control line low). When the modulus control line is high it divides by 11. This only happens for every tenth pulse of F_{IN} because of the divide-by-10 counter. The divide ratio is thus 10.1 so $F_{OUT}=10.1F_{IN}$.

Question 7

This is an essay-type question based on a continuous assessment assignment carried out during the year. The resulting short essay is expected to recount as much technical detail as possible regarding the GPS system studied in the assignment.