21/4/09

Summer 07

Q 1 (a). $m(t) = K_P\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)\,d\tau + T_D \frac{de(t)}{dt}\right)$

$$M(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right)E(s)$$

$$C(s) = \frac{M(s)}{E(s)} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right)$$

$$= \frac{K_P T_D\left(s^2 + \frac{1}{T_D}s + \frac{1}{T_I T_D}\right)}{s}$$

$$= \frac{T_D K_P\left(s^2 + \frac{1}{T_D}s + \frac{1}{4T_D^2}\right)}{s}$$

$$= \frac{K_c(s+\xi)^2}{s} \qquad K_c = T_D K_P \qquad \xi = \frac{1}{2T_D}$$

Matched pole zero

Pole @ $s = 0 \xrightarrow{e^{sT}}$ Pole @ $z = 1$

Zero @ $s = -\xi \xrightarrow{e^{sT}}$ Zero @ $z = e^{-\xi T}$

$$D(z) = \frac{K_d\left(z - e^{-\xi T}\right)^2}{z(z-1)}$$

↑ TOLD TO ADD THIS IN QUESTION

$$\lim_{s\to 0} sC(s) = \lim_{z\to 1}(z-1)D(z)$$

$$\lim_{s\to 0} K_c(s+\xi)^2 = \lim_{z\to 1}\frac{K_d\left(z - e^{-\xi T}\right)^2}{z}$$

$$\Rightarrow K_c\xi^2 = K_d\left(1 - e^{-\xi T}\right)^2$$

$$\Rightarrow K_d = \frac{K_c\xi^2}{\left(1 - e^{-\xi T}\right)^2}$$

$$D(z) = \frac{M(z)}{E(z)} = \frac{K_d\left(z^2 - 2z e^{-\xi T} + e^{-2\xi T}\right)}{z^2 - z}$$

$$(z^2 - z)M(z) = K_d(z^2 - 2ze^{-\xi T} + e^{-2\xi T})E(z)$$
$$(1 - z^{-1})M(z) = K_d(1 - 2z^{-1}e^{-\xi T} + e^{-2\xi T}z^{-2})E(z)$$

Take inverse Z Transforms

$$m(k) - m(k-1) = K_d e(k) - 2K_d e^{-\xi T} e(k-1) + K_d e^{-2\xi T} e(k-2)$$
$$\Rightarrow m(k) = m(k-1) + \underbrace{K_d}_{\alpha} e(k) \underbrace{- 2K_d e^{-\xi T}}_{\beta} e(k-1) + \underbrace{K_d e^{-2\xi T}}_{\gamma} e(k-2)$$

(b). $m(k) = Ke(k-1) + 0.8m(k-1)$

$$M(z) = Kz^{-1}E(z) + 0.8z^{-1}M(z)$$
$$(1 - 0.8z^{-1})M(z) = Kz^{-1}E(z)$$
$$D(z) = \frac{M(z)}{E(z)} = \frac{Kz^{-1}}{1 - 0.8z^{-1}} = \frac{K}{z - 0.8}$$

$$G(z) = Z\left\{\frac{1 - e^{-sT}}{s} \cdot \frac{2}{1+2s} \cdot 2\right\}$$

$$= 2(1 - z^{-1})Z\left\{\frac{1}{s(s+0.5)}\right\}$$

From tables:

$$Z\left\{\frac{1}{s(s+a)}\right\} = \frac{1}{a}\frac{(1 - e^{-aT})z^{-1}}{(1 - z^{-1})(1 - e^{-aT}z^{-1})} \qquad \begin{array}{l} a = 0.5 \\ T = 1 \end{array}$$

$$G(z) = 2\cancel{(1 - z^{-1})} \cdot \frac{1}{0.5}\frac{(1 - e^{-0.5})z^{-1}}{\cancel{(1 - z^{-1})}(1 - e^{-0.5}z^{-1})}$$

$$G(z) = \frac{1.57z^{-1}}{1 - 0.61z^{-1}} = \frac{1.57}{z - 0.606}$$

$D(z)$ pole @ 0.8

$G(z)$ pole @ 0.606

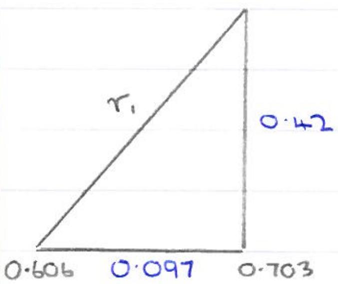
$$\alpha = \frac{\text{sum of poles} - \text{sum of zeros}}{\text{no. of poles} - \text{no. of zeros}}$$

$$\alpha = \frac{0.606 + 0.8}{2} = 0.703$$

Peak overshoot = 30%

$\Rightarrow \xi = 0.35$ (from graph)

$\Rightarrow \Delta = 0.703 + j0.42$



$r_1^2 = 0.097^2 + 0.42^2$

$r_1 = 0.43$

$r_2 = 0.43$

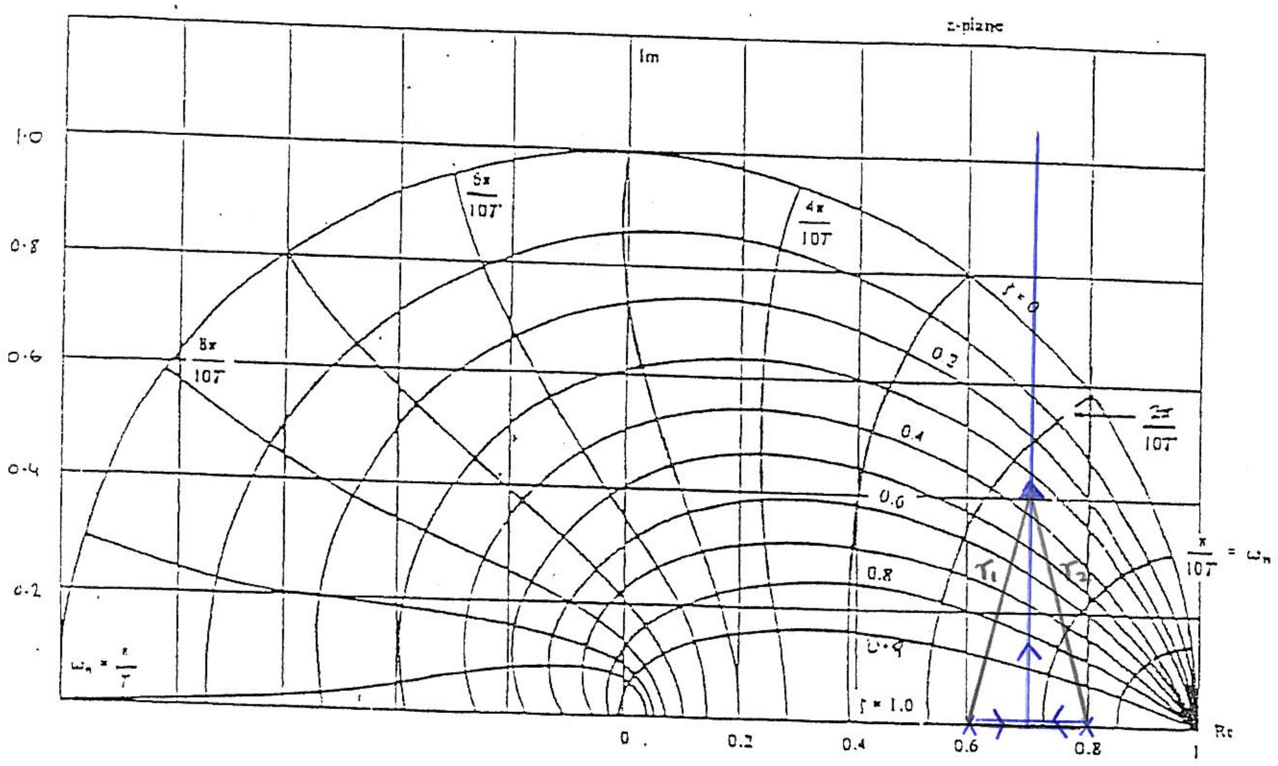
$|D(z)G(z)|_{z=0.7+j0.42} = 1$

$\frac{1.57K}{r_1 r_2} = 1$

$\frac{1.57K}{0.43^2} = 1$

$K = 0.12$

As $K\uparrow$ $\omega_n\uparrow$ $\xi\downarrow$



Z Plane Design Template

<u>*Please submit with your script*</u>

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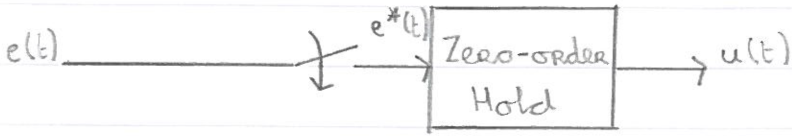
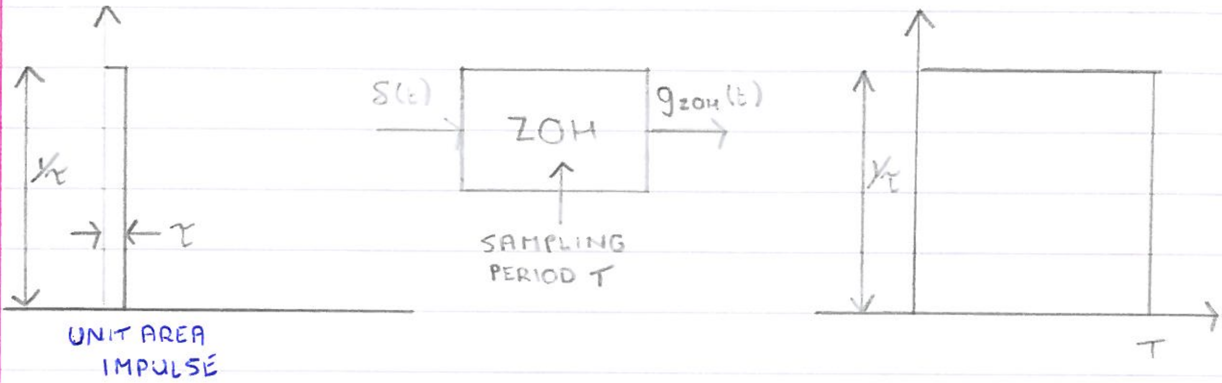
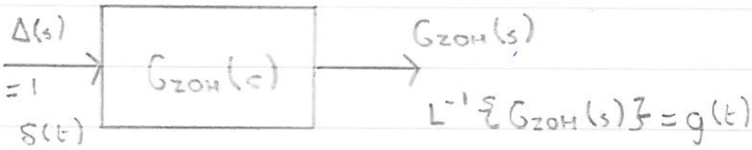
Q2(a).



Consider the impulse response of the ZOH





The transfer function $G_{ZOH}(s)$ is obtained by the Laplace transform of the impulse response:

$$G_{ZOH}(s) = L\{g_{ZOH}(t)\}$$

$$= \int_0^\infty g_{ZOH}(t)\, e^{-st}\, dt$$

$$= \int_0^T 1/\tau \, e^{-st}\, dt$$

$$= \frac{-1}{s\tau}\left[e^{-st}\right]_0^T$$

$$= \frac{1-e^{-sT}}{s\tau} \qquad \text{QED}$$

The frequency response is determined as $G(j\omega)$

$$G_{ZOH}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega\tau}$$

Using the identity $e^{-j\theta} = \cos\theta - j\sin\theta$

$$G_{ZOH}(j\omega) = \frac{1-\cos\omega T + j\sin\omega T}{j\omega\tau} = |G_{ZOH}(j\omega)| \angle \arg(G_{ZOH}(j\omega))$$

$$G_{ZOH}(j\omega) = \frac{\sin\omega T + j(\cos\omega T - 1)}{\omega\tau}$$

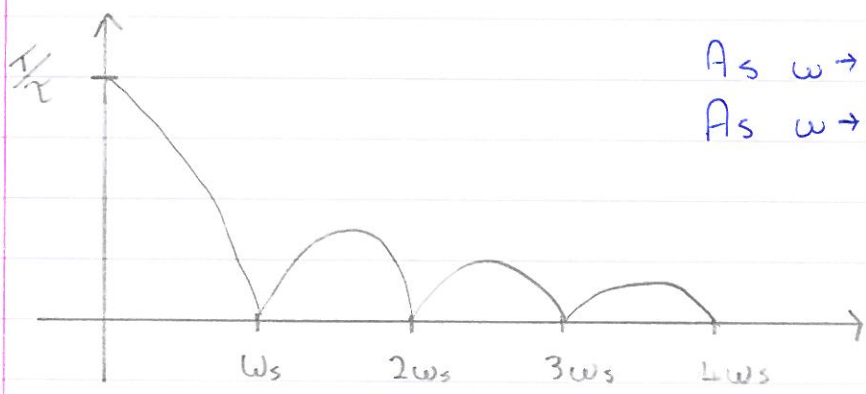
$$|G_{ZOH}(j\omega)| = \frac{\sqrt{\sin^2\omega T + (\cos\omega T - 1)^2}}{\omega\tau} = \frac{\sqrt{\sin^2\omega T + \cos^2\omega T - 2\cos\omega T + 1}}{\omega\tau}$$

$$|G_{ZOH}(j\omega)| = \frac{\sqrt{2(1-\cos\omega T)}}{\omega\tau}$$

Using the identity $1-\cos\theta = 2\sin^2(\theta/2)$

$1-\cos\omega T = 2\sin^2(\omega T/2)$

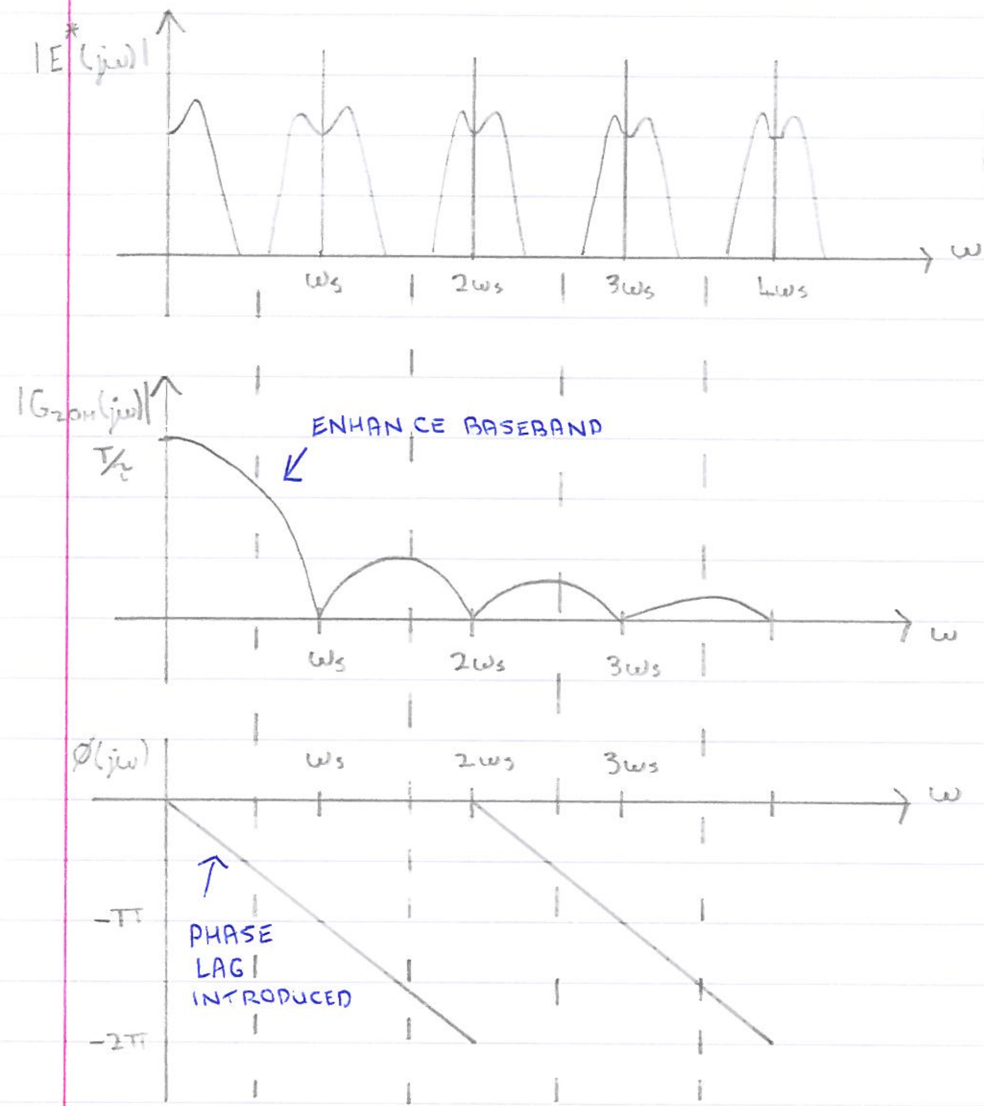
$$|G_{ZOH}(j\omega)| = \frac{\sqrt{4\sin^2 \omega T/2}}{\omega\tau} = \frac{2\sin \omega T/2}{\omega\tau} = \frac{T}{\tau}\,\frac{\sin \omega T/2}{\omega T/2}$$



As $\omega \to 0$ $|G_{ZOH}(j\omega)| = T/\tau$

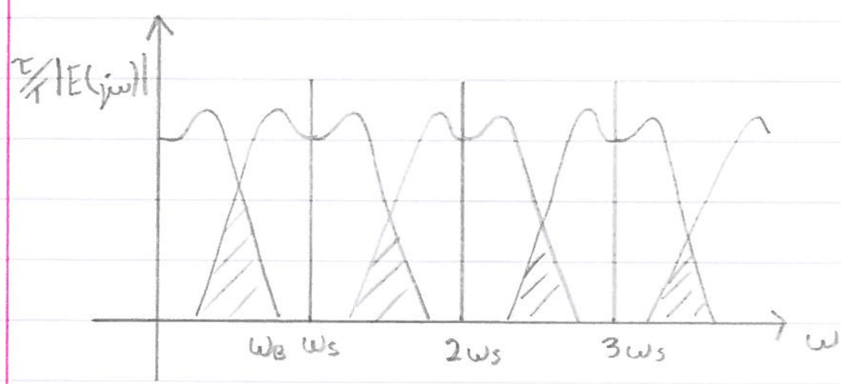
As $\omega \to N\omega_s$ $|G_{ZOH}(j\omega)| \to 0$

$\phi(j\omega) = -\omega/\omega_s\,\pi$ Radians



INFINITE NUMBERS OF REPLICATIONS OF BASEBAND AT INTEGER MULTIPLES OF ω_s

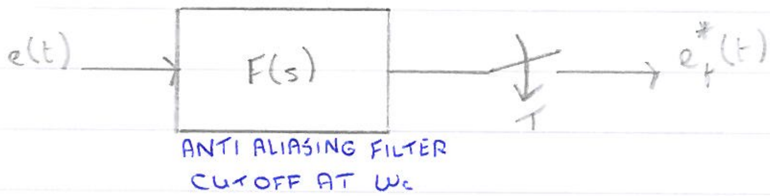
If $\omega_s > 2\omega_B$ the spectra are distinct $\Rightarrow$ the baseband can easily be extracted by low pass filtering
If $\omega_s < 2\omega_B$, aliasing occurs and we cannot simply reconstruct the baseband from the sampled signal

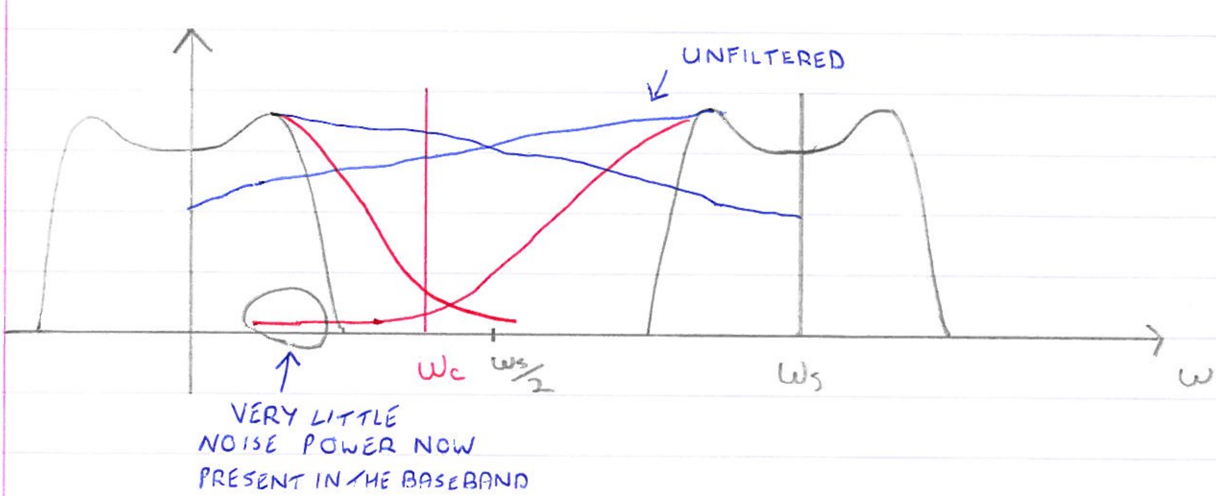


Shannon's Sampling Theorem

For a continuous time signal $e(t)$ with $|E(j\omega)| = 0$ for $|\omega| \geq \omega_B$, then the sampling frequency should be chosen as $\omega_s \geq 2\omega_B$ to ensure that aliasing does not occur.

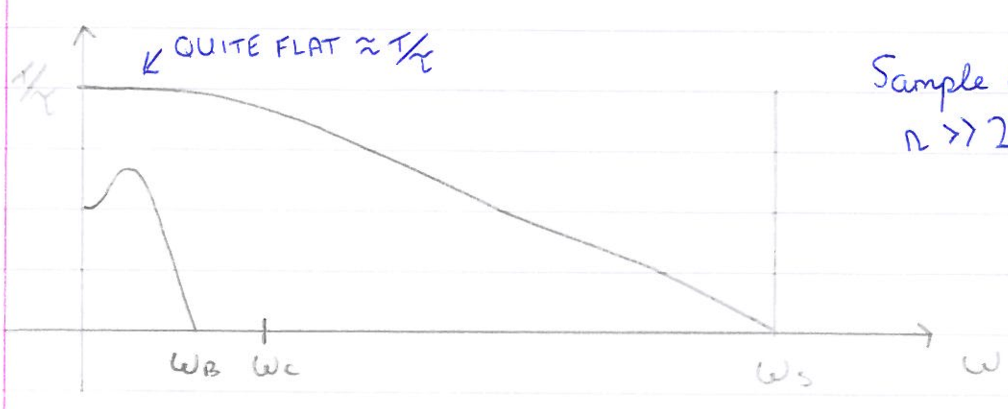
In practice, there is not a finite spectrum to $e(t)$ due to noise. This can introduce high frequency noise into the frequency range of interest (baseband). It is essential to prefilter the signal $e(t)$ before sampling to avoid large aliasing errors



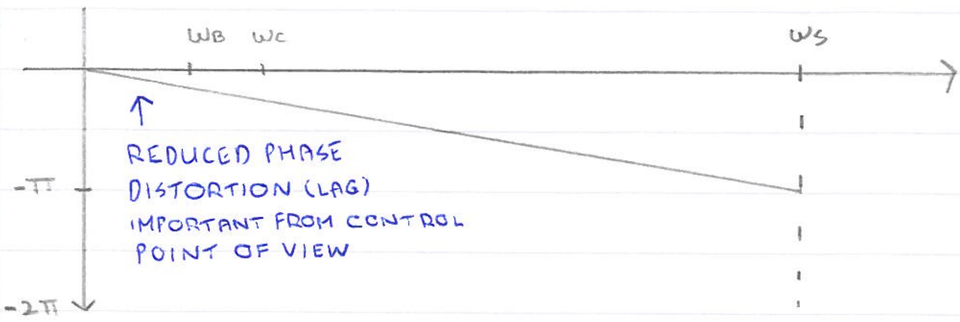


We don't want to distort the baseband signal
$\Rightarrow \omega_c > \omega_B$

Benefits of oversampling

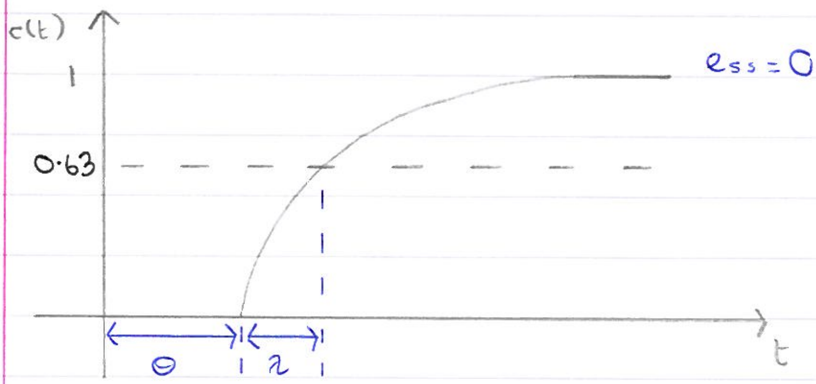


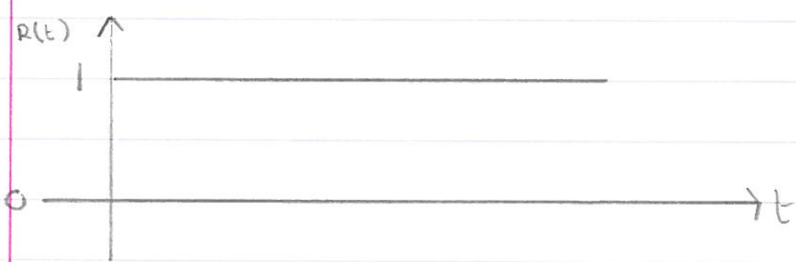
Sample at $\omega_s = n\omega_c$
$n >> 2$



(b) Dahlins Method

First specify a desired step-response for the continuous-time signal $c(t)$





First order response
- time constant λ
- time delay θ

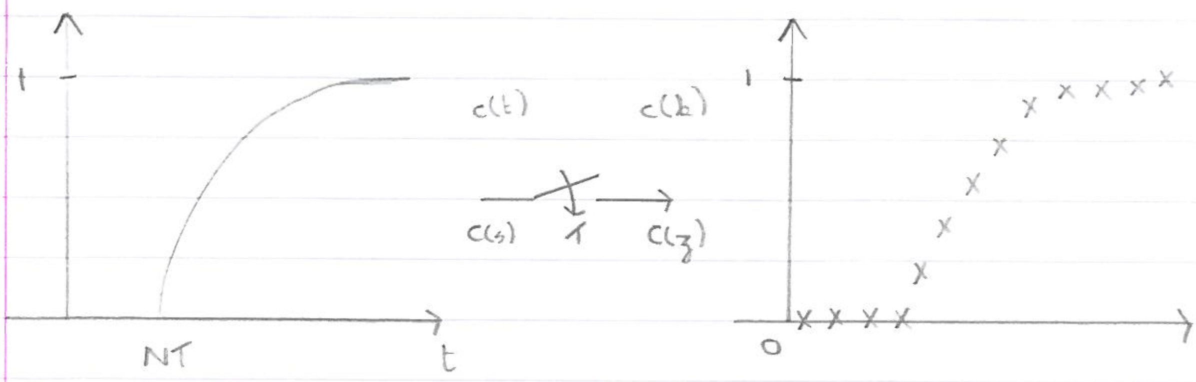
$$c(t) = \begin{cases} 0 & \text{for } t < \theta \\ 1 - e^{-\frac{t-\theta}{\lambda}} & \text{for } t \geq \theta \end{cases}$$

$$C(s) = L\{c(t)\} = \frac{e^{-\theta s}}{s(1+\lambda s)}$$

$$\theta \approx NT \quad , \quad \theta > NT$$

$$\Rightarrow C(s) = \frac{e^{-NTs}}{s(1+\lambda s)}$$

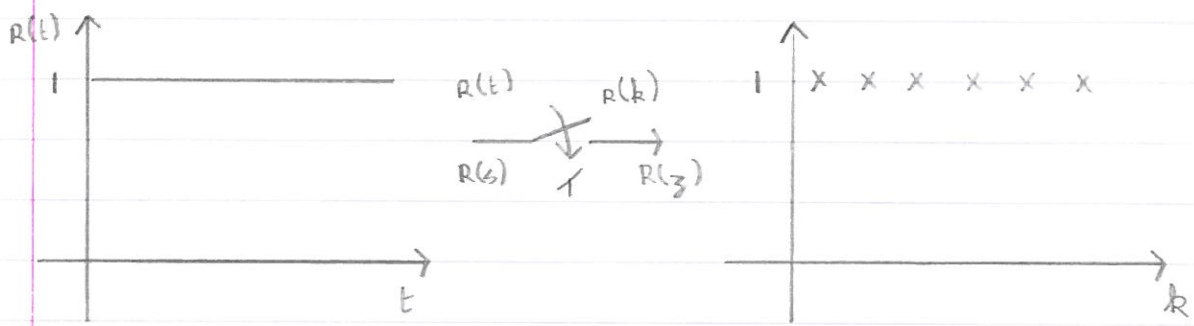
Consider that the signal c(t) was sampled with sample-time T



From the Z transform tables:

$$C(z) = \frac{\left(1-e^{-T/\tau}\right) z^{-(N+1)}}{\left(1-z^{-1}\right)\left(1-e^{-T/\tau} z^{-1}\right)}$$ ⟵ 1 EXTRA DELAY DUE TO SAMPLING

This was the response to a unit step setpoint signal



$$R(z) = \frac{1}{1-z^{-1}}$$

$$\frac{C(z)}{R(z)} = \frac{\left(1-e^{-T/\tau}\right) z^{-(N+1)}}{1-e^{-T/\tau} z^{-1}}$$

$$D(z) = \frac{1}{G(z)} \frac{\frac{C(z)}{R(z)}}{1-\frac{C(z)}{R(z)}} = \frac{1}{G(z)} \frac{\frac{\left(1-e^{-T/\tau}\right) z^{-(N+1)}}{1-e^{-T/\tau} z^{-1}}}{1-\frac{\left(1-e^{-T/\tau}\right) z^{-(N+1)}}{1-e^{-T/\tau} z^{-1}}}$$

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s} \cdot \frac{K e^{-NTs}}{1+s\tau}\right\}$$

$$G(z) = \left(1-z^{-1}\right) z^{-N} \mathcal{Z}\left\{\frac{K}{s(1+s\tau)}\right\}$$

$$G(z) = \frac{K(1-e^{-T/\tau})z^{-(N+1)}}{1-e^{-T/\tau}z^{-1}}$$

$$D(z) = \frac{1-e^{-T/\tau}z^{-1}}{K(1-e^{-T/\tau})\cancel{z^{-(N+1)}}} \cdot \frac{(1-e^{-T/\tau})\cancel{z^{-(N+1)}}}{1-e^{-T/\tau}z^{-1}-(1-e^{-T/\tau})z^{-(N+1)}}$$

$$= \frac{1-e^{-T/\tau}}{K(1-e^{-T/\tau})} \cdot \frac{1-e^{-T/\tau}z^{-1}}{1-e^{-T/\tau}z^{-1}-(1-e^{-T/\tau})z^{-N-1}}$$

$$= \underbrace{\frac{1}{K}}_{K_d} \cdot \frac{1\overbrace{-e^{-T/\tau}}^{\gamma}z^{-1}}{1\underbrace{-e^{-T/\tau}}_{\alpha}z^{-1}\underbrace{-(1-e^{-T/\tau})}_{\beta}z^{-N-1}}$$

Integral action

$\Rightarrow$ pole at $s=0$ or $z=1$

$1-e^{-T/\tau}z^{-1}-(1-e^{-T/\tau})z^{-(N+1)}$

$z=1$

$1-e^{-T/\tau}-1+e^{-T/\tau}=0$

$\Rightarrow$ $z=1$ is a root of the denominator of $D(z)$

$\Rightarrow$ pole of $D(z)$

$\Rightarrow$ integral action.

Q 3 (a). $$G(z) = \frac{\gamma z^{-2}}{1+\alpha z^{-1}+\beta z^{-2}} = \frac{\gamma}{z^2+\alpha z+\beta} = \frac{B(z)}{A(z)}$$

Diophantine pole placement equation

$A_{cl}(z) = A(z)Q(z) + B(z)S(z)$

$Q(z) = z + q_1$

$S(z) = s_0 z + s_1$

$A_{cl}(z) = (z^2+\alpha z+\beta)(z+q_1) + \gamma(s_0 z + s_1)$

$= z^3 + q_1 z^2 + \alpha z^2 + \alpha q_1 z + \beta z + \beta q_1 + \gamma s_0 z + \gamma s_1$

$A_{cl}(z) = z^3 + (q_1 + \alpha)z^2 + (\alpha q_1 + \beta + \gamma s_0)z + (\beta q_1 + \gamma s_1)$

The desired closed loop characteristic equation for a 3^{rd} order process is: (2nd order open loop + 1st order controller)

$A_{cl}(z) = z^3 + c_1 z^2 + c_2 z + c_3$

Comparing similar powers of z

$q_1 + \alpha = c_1 \Rightarrow q_1 = c_1 - \alpha$

$\alpha q_1 + \beta + \gamma s_0 = c_2 \Rightarrow \alpha q_1 + \gamma s_0 = c_2 - \beta$

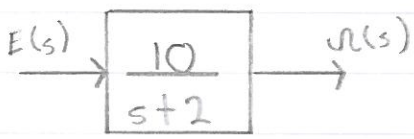
$\beta q_1 + \gamma s_1 = c_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ \alpha & \gamma & 0 \\ \beta & 0 & \gamma \end{bmatrix} \begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} c_1 - \alpha \\ c_2 - \beta \\ c_3 \end{bmatrix}$$

$A_{cl} = \underbrace{(z - p_1)}_{\text{FAST}}\underbrace{(z - p_2)(z - p_3)}_{\text{2ND ORDER DOMINANCE}}$

$T(z) = t_0(z - p_1)$ where $t_0 = \lim_{z \to 1} \frac{(z - p_2)(z - p_3)}{\gamma}$

Q5(b)i

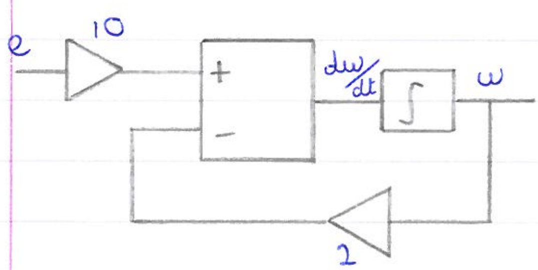


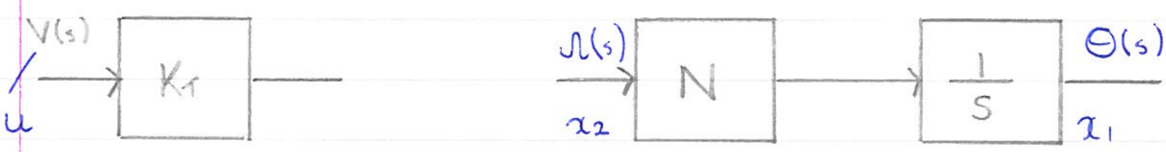
$\frac{\Omega(s)}{E(s)} = \frac{10}{s+2}$

$(s+2)\Omega(s) = 10E(s)$

$s\Omega(s) = 10E(s) - 2\Omega(s)$

$\frac{d\omega}{dt} = 10e(t) - 2\omega(t)$





$dx_1/dt = Nx_2$

$dx_2/dt = 10e - 2x_2$

$= 10(K_T u - K_s x_1) - 2x_2$

$= 10K_T u - 10K_s x_1 - 2x_2$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & N \\ -10K_s & -2 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 10K_T \end{bmatrix}} u$$

$$\underline{y} = \begin{bmatrix} V_\theta \\ V_\omega \end{bmatrix} = \overset{C}{\begin{bmatrix} K_\theta & 0 \\ 0 & K_\omega \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(ii) $C_x = [B \,\vdots\, AB]$

$$AB = \begin{bmatrix} 0 & N \\ -10K_s & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 10K_T \end{bmatrix} = \begin{bmatrix} 10NK_T \\ -20K_T \end{bmatrix}$$

$$C_x = \begin{bmatrix} 0 & 10NK_T \\ 10K_T & -20K_T \end{bmatrix}$$

$\det(C_x) = 0(-20K_T) - 10NK_T(10K_T) = -100NK_T^2$

For controllability rank$(C_x) = N$ (process order)

$\Rightarrow \det(C_x) \neq 0$

$\Rightarrow -100NK_T^2 \neq 0$ if N and K_T are non-zero

$\Rightarrow$ controllable.

(iii) $N = 10$, $K_T = 20 \text{ Nm V}^{-1}$, $K_s = 90 \text{ Nm rad}^{-1}$

$$\frac{d}{dt}\underline{x}(t) = \begin{bmatrix} 0 & 10 \\ -900 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 200 \end{bmatrix} u(t)$$

$v(t) = r(t) - k_1 V_\omega - k_2 V_\theta$

$= r(t) - k_1 K_\omega x_2 - k_2 K_\theta x_1$

$= r(t) - k_1' x_2 - k_2' x_1$

$\xi = 0.5$; $T_{s2\%} = 0.02$

$\frac{4}{\xi \omega_n} = 0.02$

$\omega_n = 400 \text{ rad/s}$

$\Rightarrow s^2 + 2\xi\omega_n + \omega_n^2 = s^2 + 400s + 160{,}000$

$C_{des}(s) = \det(sI - A + BK)$

$\det(sI - A + BK) = \left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 10 \\ -900 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 200 \end{pmatrix} (k_2' \; k_1') \right|$

$= \begin{vmatrix} s & -10 \\ 900 + 200k_2' & s + 2 + 200k_1' \end{vmatrix}$

$= s(s + 2 + 200k_1') + 10(900 + 200k_2')$

$= s^2 + (2 + 200k_1')s + 9000 + 2000k_2'$

$2 + 200k_1' = 400$ $\qquad$ $9000 + 2000k_2' = 160{,}000$

$k_1' = 1.99$ $\qquad$ $k_2' = 75.5$

$k_1' = k_1 K_\omega$ $\qquad$ $k_2' = k_2 K_\theta$

$1.99 = k_1(0.2)$ $\qquad$ $75.5 = k_2(1.5)$

$k_1 = 9.95$ $\qquad$ $k_2 = 50.3$

$\Rightarrow v(t) = r(t) - 9.95 v_\omega - 50.3 v_\theta$

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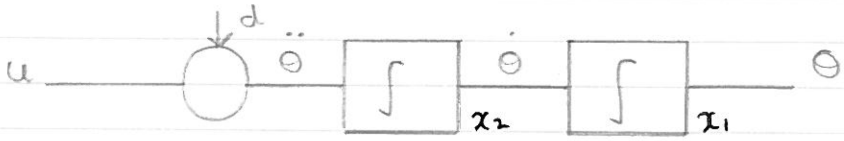
Q 6 (b). $\frac{d^2\theta(t)}{dt^2} = u(t) + d(t)$



$\frac{d}{dt} x_1 = x_2$

$\frac{d}{dt} x_2 = u(t) + d(t)$

$\underline{\dot{x}}(t) = A\underline{x}(t) + Bu(t) + Ed(t)$

$\underline{y}(t) = C\underline{x}(t)$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}u + \overset{E}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}d \qquad \theta = \overset{C}{[1\ 0]}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} u(t) &= -2\frac{d\theta}{dt} - 2\theta(t) \\ &= -2x_2 - 2x_1 \\ &= \underset{K}{[-2\ \ -2]}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

$\underline{\dot{x}}(t) = (A - BK)\underline{x}(t) + Ed(t)$

$$A - BK = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}(+2\ \ +2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ +2 & +2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix}$$

$G(s) = C(sI - A + BK)^{-1}E$

$$sI - A + BK = sI - (A - BK) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s+2 \end{pmatrix}$$

$$(sI - A + BK)^{-1} = \frac{1}{s(s+2)+2}\begin{pmatrix} s+2 & 1 \\ -2 & s \end{pmatrix} = \frac{1}{s^2+2s+2}\begin{pmatrix} s+2 & 1 \\ -2 & s \end{pmatrix}$$

$$(1\ \ 0)\begin{pmatrix} s+2 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (s+2\ \ 1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$\Rightarrow G(s) = \frac{1}{s^2+2s+2}$ no closed loop zeros in TF

(ii) Full order Luenberger Observer

$\frac{d}{dt}\underline{\hat{x}} = (A - GC)\underline{\hat{x}} + B\underline{u} + G\underline{y}$

$$G(s) = \frac{1 \leftarrow b_0}{s^2 + \underset{\substack{\uparrow \\ e_1}}{2}s + \underset{\substack{\uparrow \\ e_0}}{2}}$$

$$\dot{\underline{x}}(t) = \overset{A}{\begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}} \underline{x}(t) + \overset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u(t)$$

$$y(t) = \overset{C}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \underline{x}(t)$$

$$A - GC = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} - \overset{G}{\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} - \begin{bmatrix} g_1 & 0 \\ g_2 & 0 \end{bmatrix} = \begin{bmatrix} -g_1 & 1 \\ -2-g_2 & -2 \end{bmatrix}$$

$$\det(sI - F) = 0$$

$$\det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -g_1 & 1 \\ -2-g_2 & -2 \end{pmatrix} \right] = 0$$

$$\det \begin{pmatrix} s+g_1 & -1 \\ 2+g_2 & s+2 \end{pmatrix} = 0$$

$$(s+g_1)(s+2) + (2+g_2) = 0$$
$$s^2 + 2s + g_1 s + 2g_1 + (2+g_2) = 0$$
$$s^2 + (2+g_1)s + (2g_1 + 2 + g_2) = 0$$

$$(s+10)^2 = s^2 + 20s + 100$$

$2 + g_1 = 20 \qquad 2g_1 + 2 + g_2 = 100$

$g_1 = 18 \qquad g_2 = 62$

$$\frac{d}{dt}\hat{\underline{x}} = \begin{bmatrix} -18 & 1 \\ -64 & -2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 18 \\ 62 \end{bmatrix} y$$

$$C_{eq}(s) = K(sI - A + GC + BK)^{-1} G$$

$$(sI - A + GC + BK) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -18 & 1 \\ -64 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} s+18 & -1 \\ 64 & s+2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -2 & -2 \end{pmatrix} = \begin{pmatrix} s+18 & -1 \\ 62 & s \end{pmatrix}$$

$$\begin{pmatrix} s+18 & -1 \\ 62 & s \end{pmatrix}^{-1} = \frac{1}{s^2+18s+62} \begin{pmatrix} s & 1 \\ -62 & s+18 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -2 \end{pmatrix} \begin{pmatrix} s & 1 \\ -62 & s+18 \end{pmatrix} = \begin{pmatrix} 124-2s & -2s-38 \end{pmatrix}$$

$$\begin{pmatrix} 124-2s & -2s-38 \end{pmatrix} \begin{pmatrix} 18 \\ 62 \end{pmatrix} = 2232 - 36s - 124s - 2356 = -124 - 160s$$

$$\Rightarrow C_{eq}(s) = \frac{-124-160s}{s^2+18s+62}$$