Perspective Transformations

Note 1

Homogenous transformations

$$\begin{bmatrix} x \\ y \\ z \\ s \end{bmatrix} \qquad \text{E.G. if } s=2 \quad \begin{bmatrix} x \\ y \\ z \\ 2 \end{bmatrix} \rightarrow \begin{array}{l} x, y, z \text{ coordinates} \\ \text{have been scaled} \\ \text{by 2.} \end{array}$$

$$\Rightarrow \text{Must divide by } s=2 \text{ to get physical values.}$$

$$\begin{bmatrix} 3\times 3 & \vdots & 3\times 1 \\ \text{Rot} & \vdots & t \\ \hline 1\times 3 & \vdots & 1\times 1 \\ \text{Perspective} & \vdots & \text{scaling} \end{bmatrix} \Rightarrow \begin{bmatrix} R & t \\ P & S \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} R\begin{bmatrix} x\\y\\z \end{bmatrix} + t \\ \text{Per}\begin{bmatrix} x\\y\\z \end{bmatrix} + s \end{bmatrix} \quad \text{e.g.} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x\\y\\z\\1 \end{bmatrix} = \begin{bmatrix} x\\y\\z\\2 \end{bmatrix}$$

$$= \begin{bmatrix} x/2 \\ y/2 \\ z/2 \\ 1 \end{bmatrix}$$

E.G. 2

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x\\y\\z\\1 \end{bmatrix} = \begin{bmatrix} x+1\\y\\z\\z+1 \end{bmatrix} = \begin{bmatrix} \frac{x+1}{z+1} \\ \frac{\cdot y}{z+1} \\ \frac{z}{z+1} \\ 1 \end{bmatrix}$$