Chapter 11

Induction Motors: Balanced, Sinusoidal Steady State Operation

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Squirrel-cage Induction Motors

- Widely used in industry
  - Power to Kinsale approx. 5 MW
  - Local drug plant approx. 4.5 MW
    - Heating, lighting, computers, etc - 0.8 MW
    - Induction motors - 4 MW
      - 1500 line fed, 150 ASDs

C11 - Induction motors under balanced sinusoidal steady state (line fed, constant speed)

C12 - Adjustable speed drives (volts/hertz control for energy efficiency and speed control)

C13 - Servo drives - field oriented control for fast dynamics

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Line-fed Induction Motors (Squirrel-cage)

- Structure
- Principle of operation
  -Torque derivation and transformer model
- Equivalent circuits
  -Determining parameters from tests and spec. sheet.
- Performance characteristics
  -Determining full-load and partial load values.

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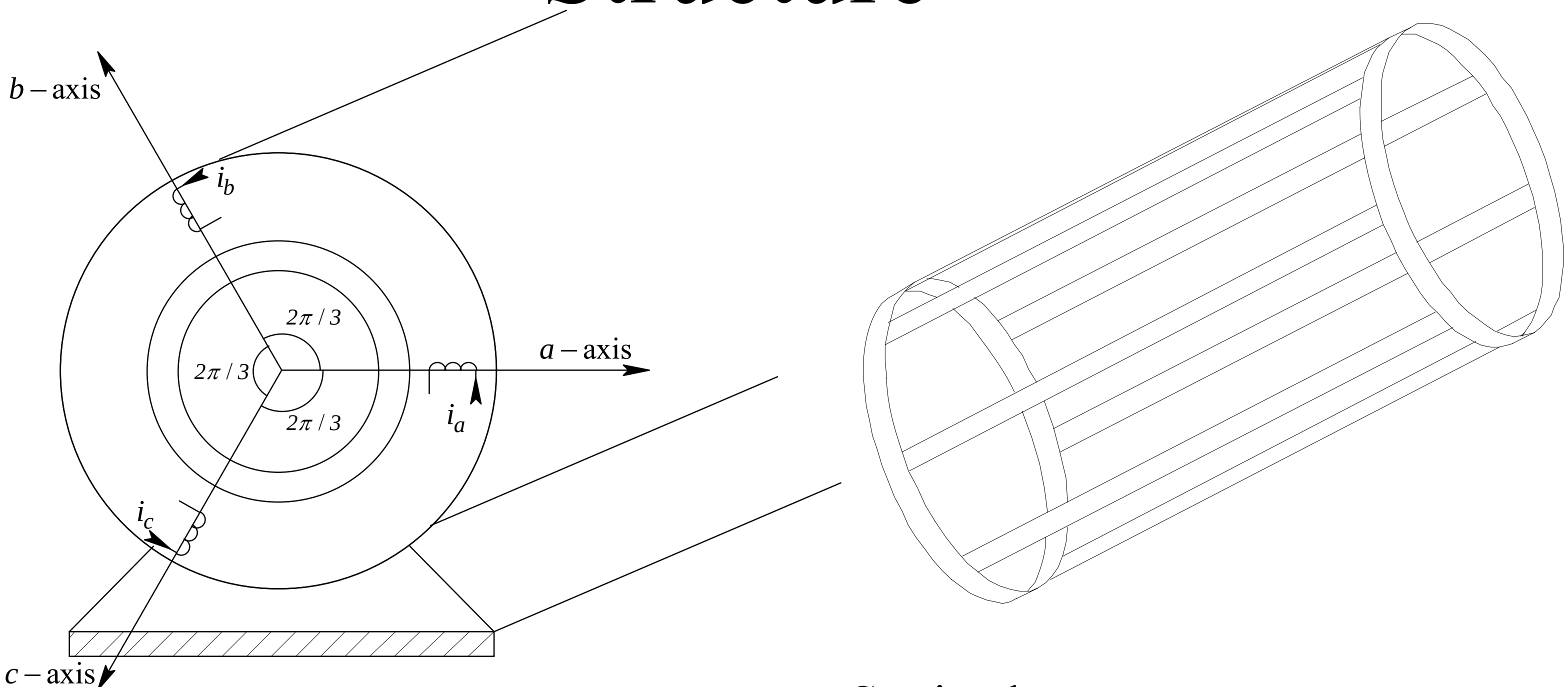
Structure



Simple representation of three phase stator windings

Squirrel-cage rotor
-Stacked laminations
-Conducting bars shorted at each end

Copper is the common rotor conductor material. Aluminium is also used due to being lighter and cheaper than Cu. Al has a higher resistance, especially with higher temperatures.

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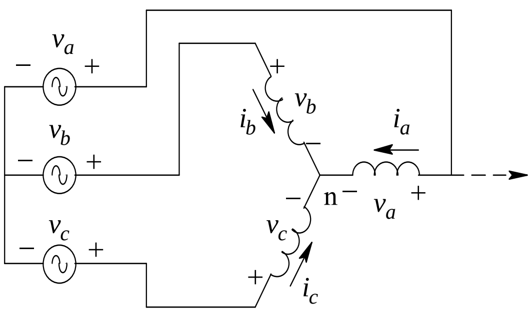
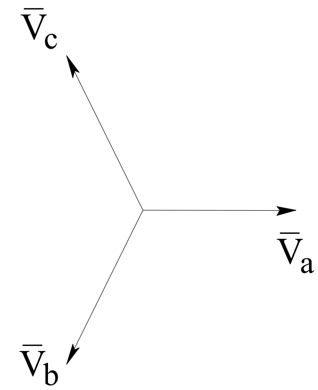
Stator Representation





- Assumptions : R_s, $L_{s,leakage} = 0$

$$v_a(t) = \sqrt{2}\, E \cos(2\pi f t)$$

$$v_b(t) = \sqrt{2}\, E \cos(2\pi f t - \frac{2\pi}{3})$$

$$v_c(t) = \sqrt{2}\, E \cos(2\pi f t - \frac{4\pi}{3})$$

$$\omega_{syn} = \omega = 2\pi f \qquad \text{(for a 2-pole machine)}$$

$$\omega_{syn} = \frac{2}{p}\omega = \frac{2}{p}(2\pi f) \qquad \text{(for a p-pole machine)}$$

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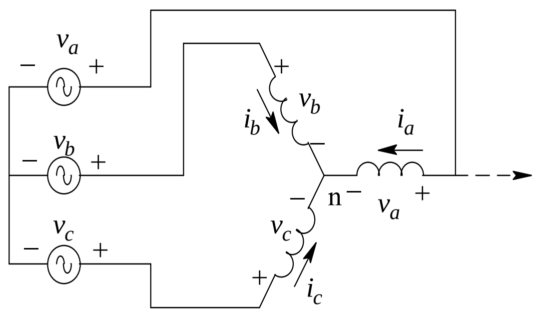
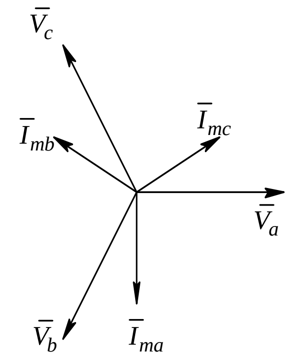
Electrically Open-circuited Rotor





- Only magnetizing currents are present because rotor is inert i.e. rotor bars are not shorted – no rotor current.
- Magnetizing currents set up rotating flux

$$\hat{I}_m = \frac{\hat{V}}{\omega L_m}$$

$$i_{ma}(t) = \hat{I}_m \cos(\omega t - \pi/2), \text{ etc.}$$

$$\vec{i}_{ms}(t) = \frac{3}{2}\hat{I}_m \angle(\omega t - \frac{\pi}{2})$$

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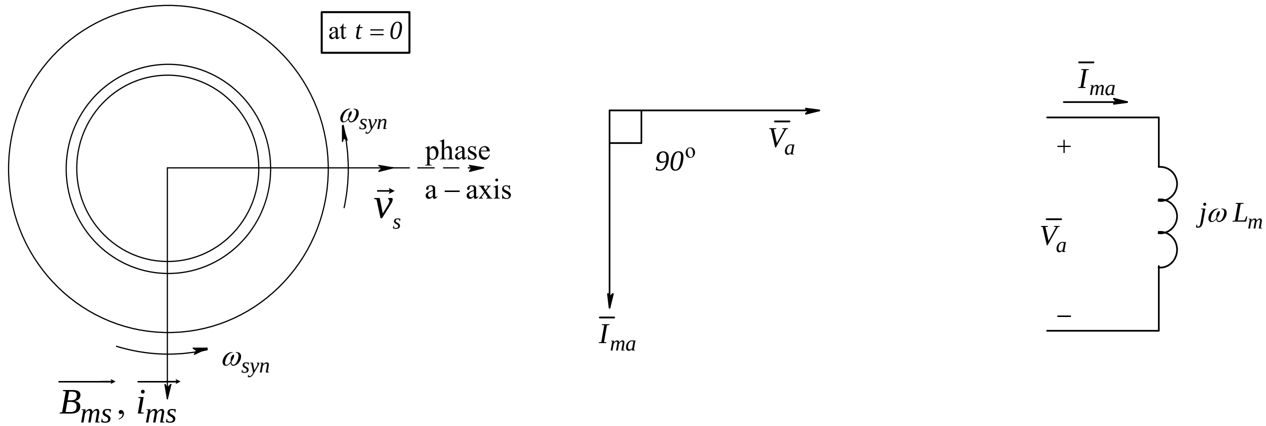
Electrically Open-circuited Rotor Fields



- $\vec{B}_{ms}$ is a constant magnitude, rotating flux

$$\therefore\ \vec{B}_{ms}(t) = \frac{\mu_0 N_{sp}}{\ell_g} \hat{I}_{ms} \angle(\omega t - \pi/2)$$

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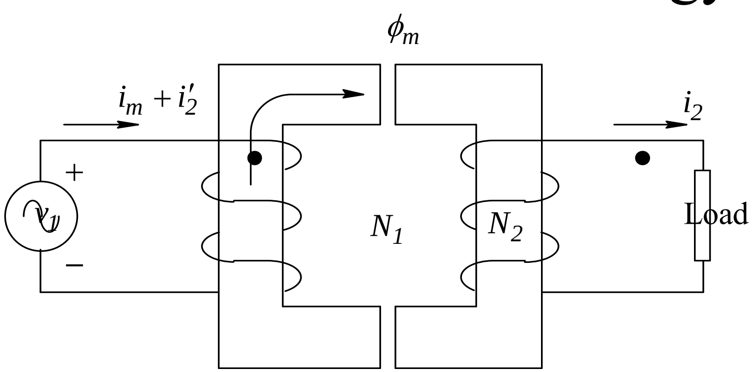
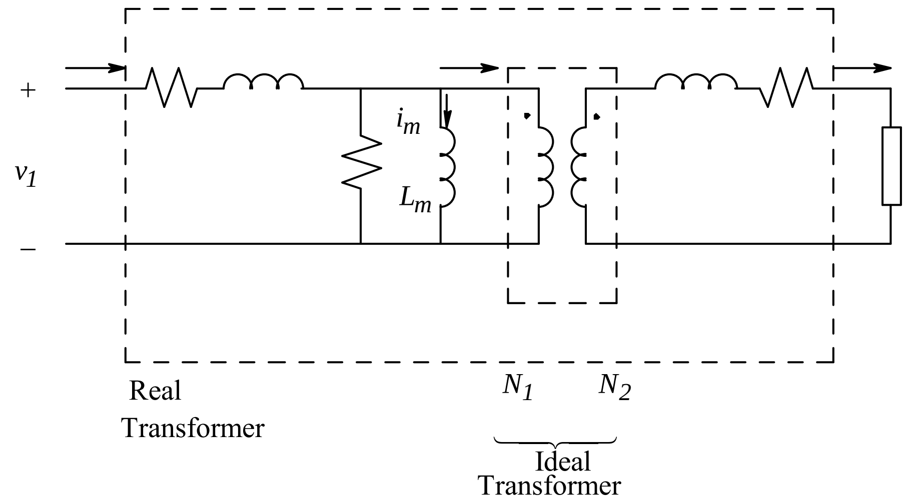
Short-circuiting the Rotor (R_s, $L_{s,leakage} = 0$)

- Transformer Analogy





- Assuming no resistances or leakage inductance in the stator windings, the stator voltages completely determine the motor flux regardless of any rotor currents

- Current i_m and flux Φ_m is unaffected by the load

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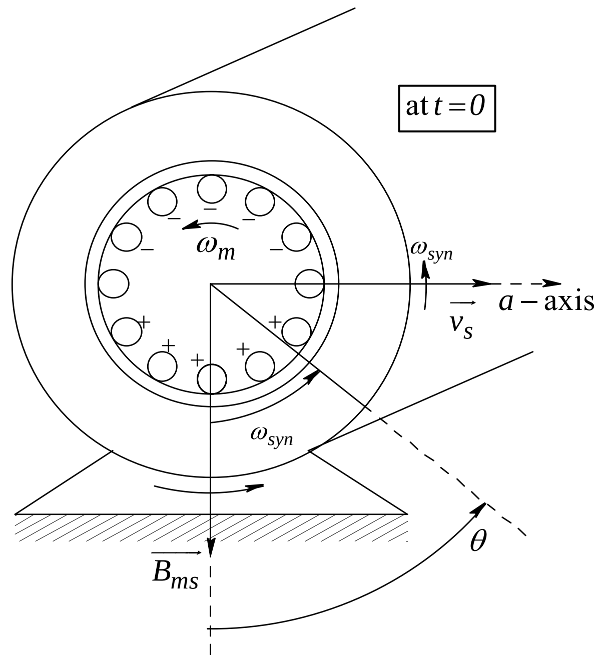
Induced Voltages on Rotor



Flux rotating at speed ω_{syn}
Rotor rotating at speed ω_m
Rotor conductors cutting flux at speed:
$\omega_{syn} - \omega_m = \omega_{slip}$ (slip speed)

Cutting flux generates voltage across rotor conductors:

$$e_{bar}(\theta) = N\frac{d\phi}{dt} = \frac{d\phi}{dt} \text{ for } N = 1$$

$$\therefore e_{bar}(\theta) = B_{ms}(\theta)\, l r \omega_{slip}$$

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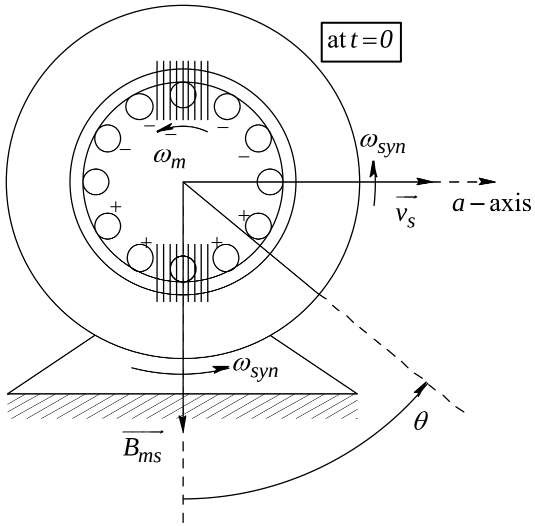
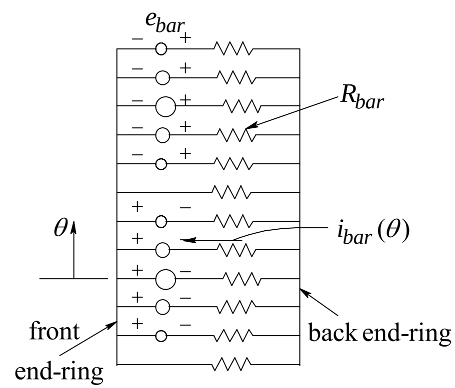
Induced Currents in Rotor





- Rotor conductors (bars) shorted together by end rings
- Because of symmetry of induced bar voltages, end rings are at same potential, therefore bar voltage is dropped across bar resistance (assuming $L_{r,l} = 0$) generating currents by Ohms Law

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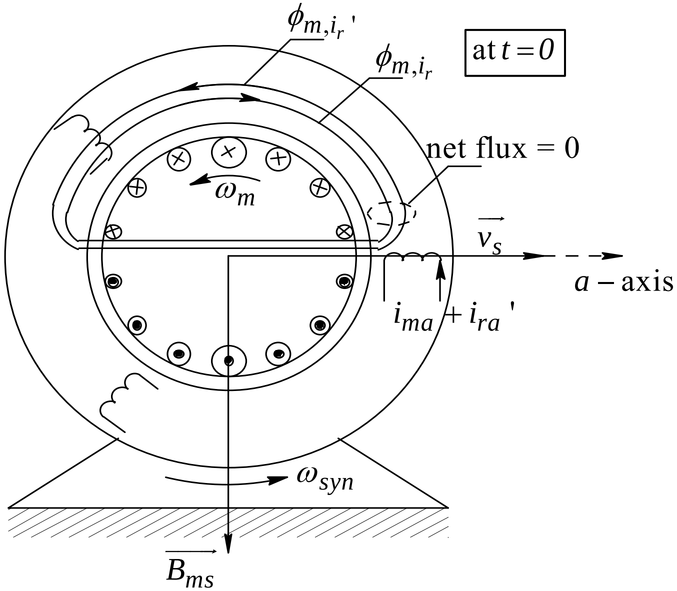
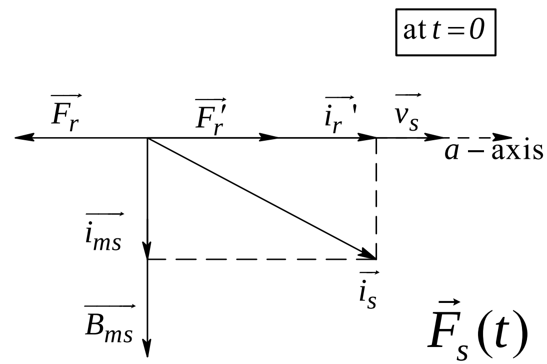
Rotor MMF – Reflected Rotor MMF – Reflected Rotor Current





$$\vec{F}_s(t) = \vec{F}_{ms}(t) + \vec{F}_r'(t)$$

$$\vec{i}_s(t) = \vec{i}_{ms}(t) + \vec{i}_r'(t)$$

$$\hat{I}_r' = k_i \hat{B}_{ms}\omega_{slip}$$

- $\vec{F}_r$ produced by rotor currents
- $\vec{F}_r'$ produced by additional stator currents to keep total flux unchanged (transformer analogy)
- These currents are viewed as a current space vector $\vec{i}_r'$
- Total stator current is magnetizing current plus this reflected rotor current

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Slip frequency (f_{slip}) in the rotor circuit

$$f_{syn} = \frac{f}{p/2} \text{ where } p \text{ is the number of poles.}$$

$$f_{slip} + f_m = f_{syn}$$

$$slip = s = \frac{f_{slip}}{f_{syn}} = \frac{f_{syn} - f_m}{f_{syn}}$$

$$motoring: \; f_{syn} > f_m \Rightarrow f_{slip}, s \text{ are positive}$$

$$generating: \; f_{syn} < f_m \Rightarrow f_{slip}, s \text{ are negative}$$

- Slip is rotor speed normalized to synchronous speed
- Slip generally small (< 3%), therefore rotor current frequency is very low

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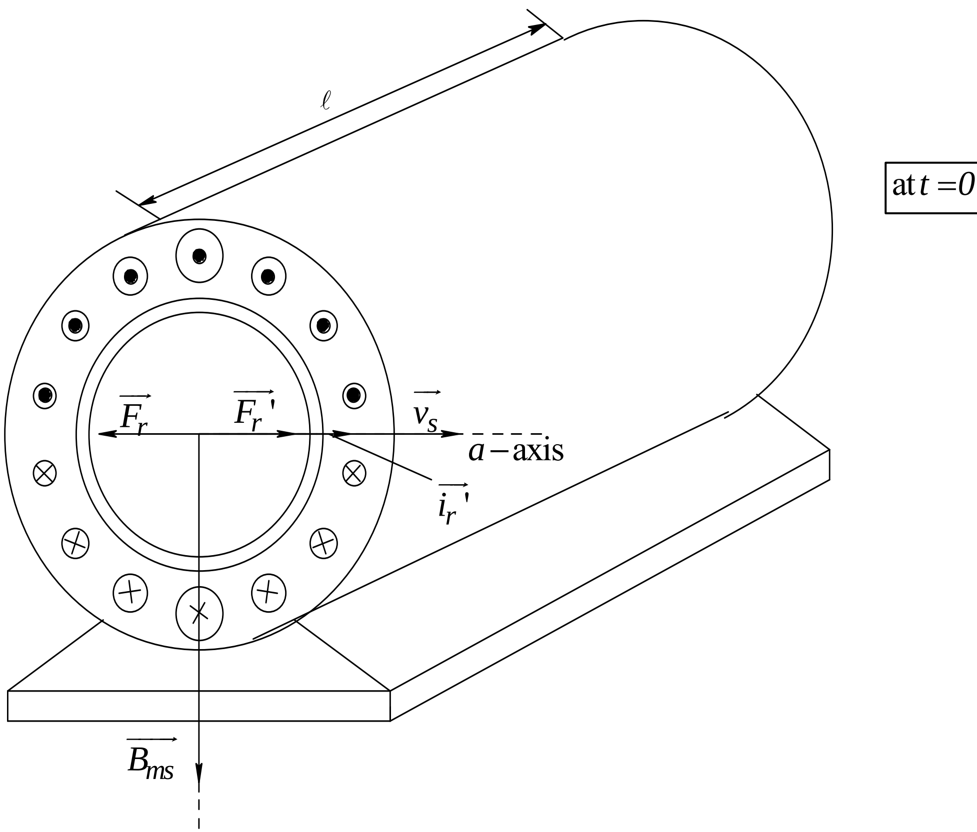
Electromagnetic Torque Production



- Current/mmf induced in stator as shown to oppose rotor current/mmf by transformer action.

- Let's derive the C.W. torque on the stator reflected currents due to the rotating field. This is equal and opposite to the C.C.W. torque between the rotor and the flux

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Electromagnetic Torque Production

Differential torque at angle ξ is given by

$$\begin{aligned} \mathrm{d}T_{em}(\xi) &= r.l.B_{ms}(\xi)\hat{I}_r' n_{\mathrm{sp}}(\xi) \\ &= r.l.\hat{B}_{ms}\sin\xi\,\hat{I}_r' N_{\mathrm{sp}}\sin\xi\,\mathrm{d}\xi \end{aligned}$$

The total torque developed by the interaction of the magnetic flux and the reflected rotor current is

We know that $\int_0^{2\pi}\sin^2\xi\,\mathrm{d}\xi = \pi$

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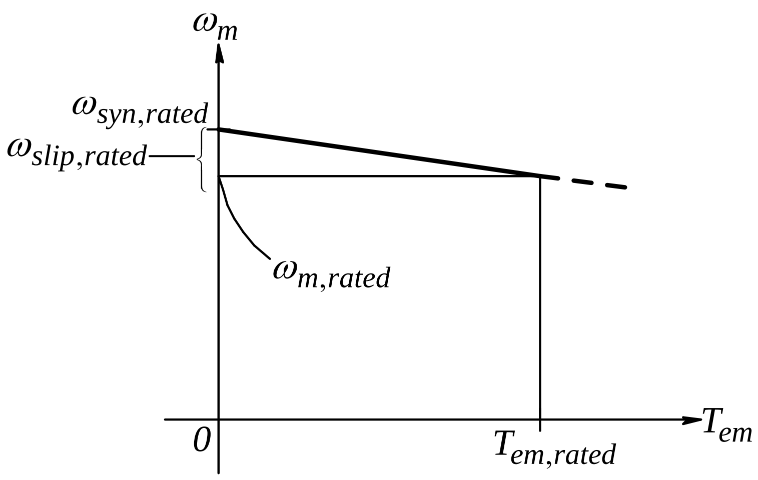
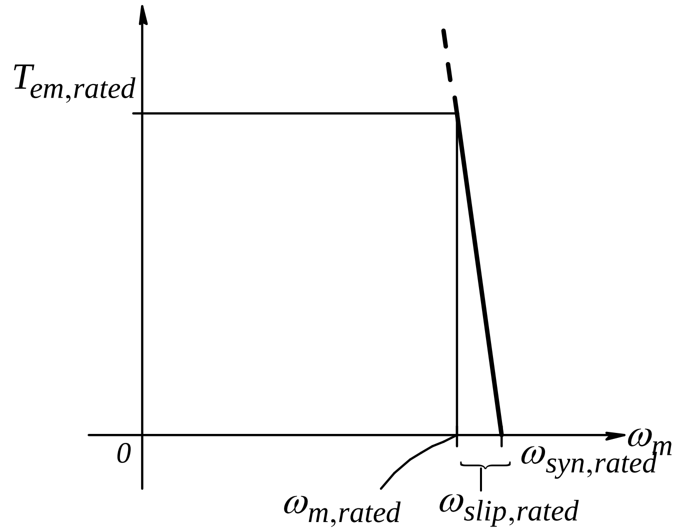
Torque – Speed Characteristics (slip small ; ω_m near ω_{syn}) – neglecting leakage





- Linear relationship between torque and slip for given flux
- Machine constant – analogous to dc motor.
- Control of flux and current vectors results in fast dynamic control

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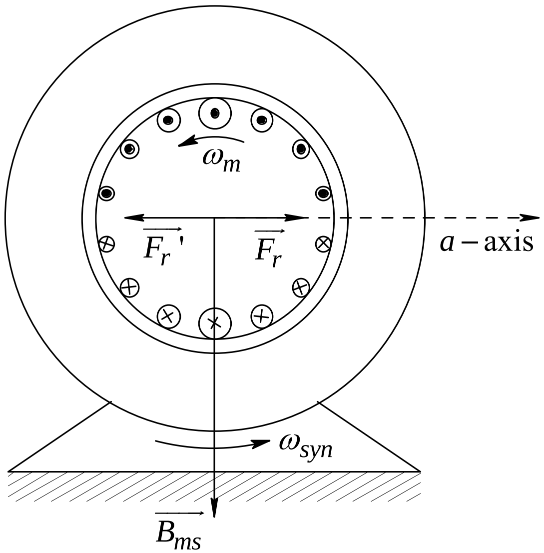
Generator (Regenerative Braking) Mode



- For generation or for braking – in either case rotor speed exceeds synchronous speed, $\omega_m > \omega_{syn}$
- $\omega_{slip} < 0$
  - Bar voltage polarities reversed
  - Rotor currents and mmf ($\vec{F}_r$) reversed
  - Reflected rotor currents and mmf ($\vec{F}_r{}'$) reversed
  - Torque reversed

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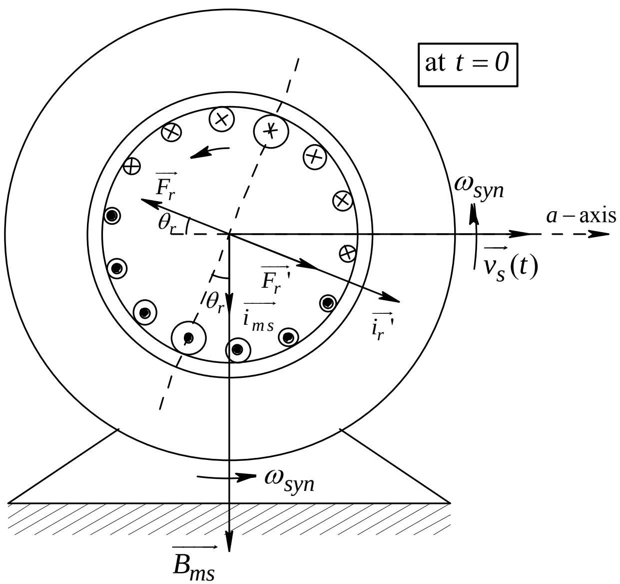
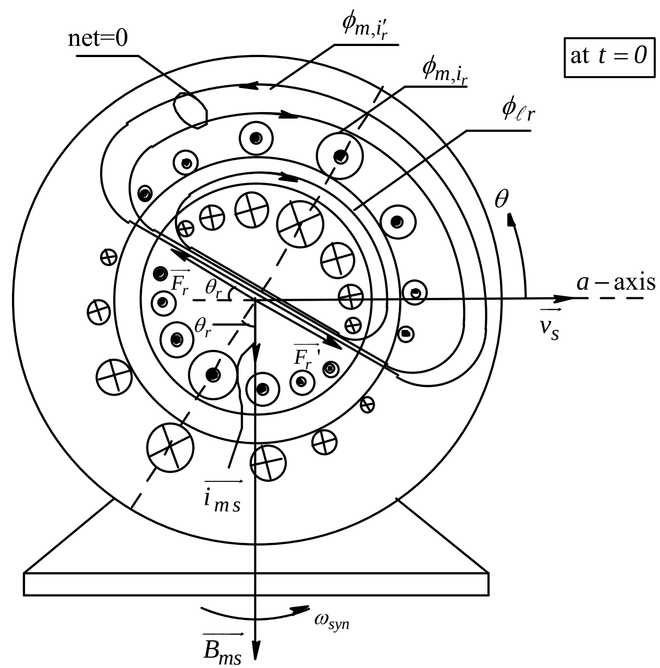
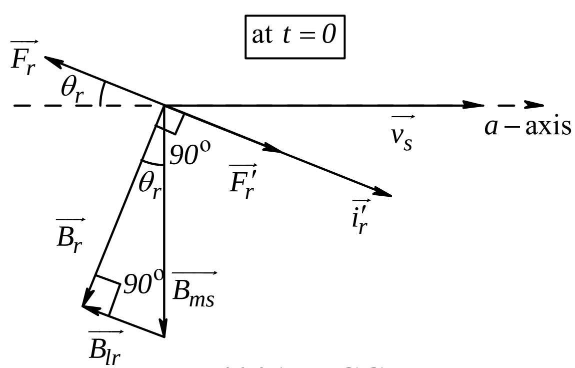
Rotor Leakage Inductance







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Rotor Leakage Inductance (cont…)

- Rotor leakage inductance is often neglected when motor is operating near synchronous speed (below the rated torque)
- Effect of rotor leakage inductance is to reduce T_{em} at high slip
- Revised torque equation -

$$T_{em} = \pi N_{sp} r \ell \hat{B}_r \hat{I}_r' = \pi N_{sp} r \ell \hat{B}_{ms} \hat{I}' \sin\left(\pi/2 - \theta_r \right)$$

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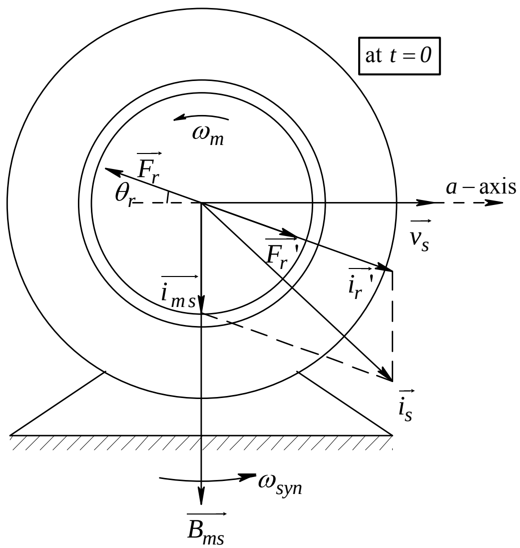
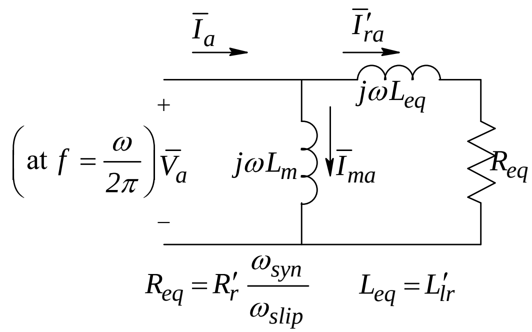
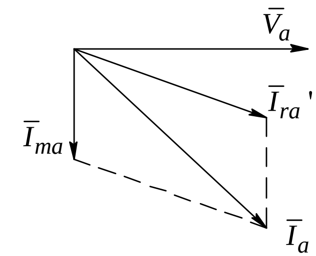
Per-Phase Equivalent Circuit including rotor leakage



Space Vectors



Equivalent Circuit



Phasor Diagram

- ❑ Includes rotor leakage inductance
- ❑ Does not include stator leakage inductance or resistance
- ❑ R_{eq} depends on slip

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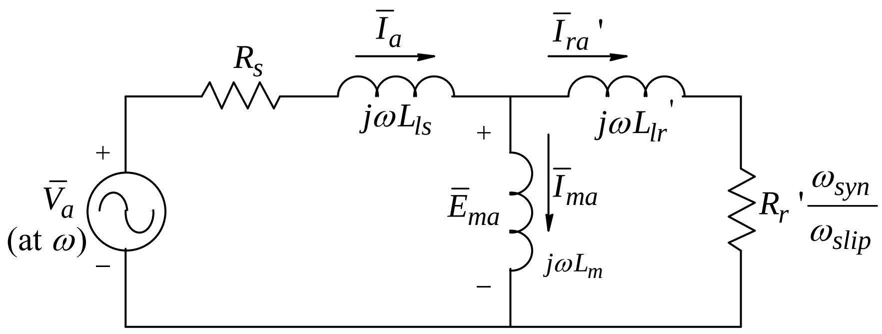
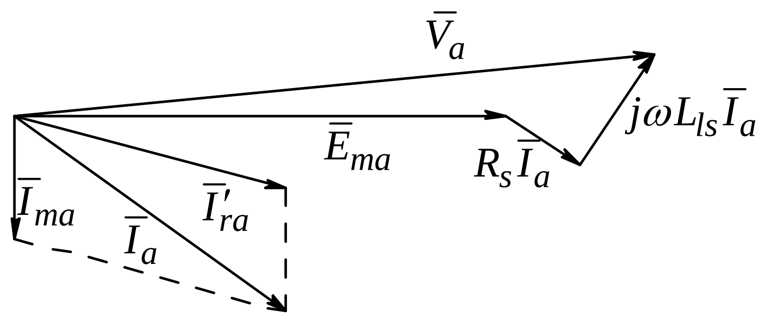
Stator Winding Resistance and Leakage Inductance



Equivalent Circuit



Phasor Diagram

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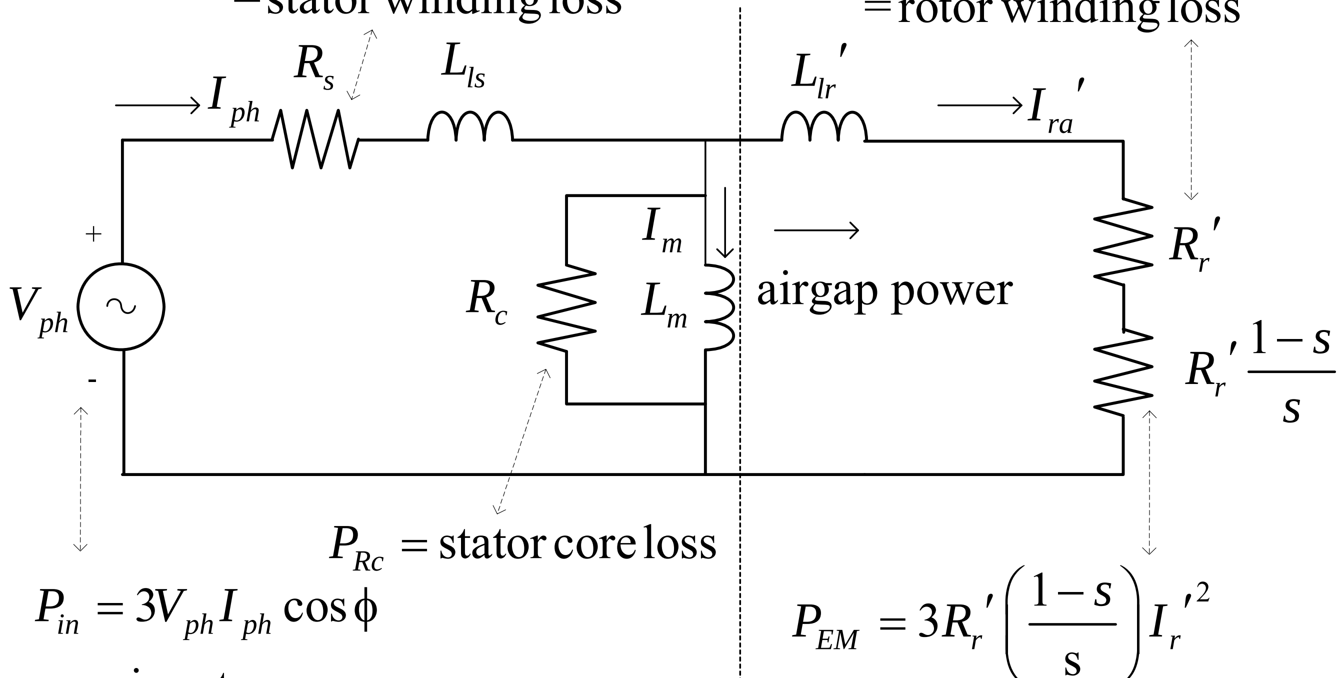
Per-phase IM Eq. Cct Model

$P_{Rs} = 3R_s I_{ph}{}^2$

$= \text{stator winding loss}$

$P_{Rs} = 3R_r' I_r'^2$

$= \text{rotor winding loss}$



$P_{Rc} = \text{stator core loss}$

$P_{in} = 3V_{ph} I_{ph} \cos\phi$

$= \text{input power}$

$$P_{EM} = 3R_r' \left(\frac{1-s}{s} \right) I_r'^2$$

$= \text{mechanical power}$

$= \text{useful output power}$

$+ \text{friction and windage}$

Exit

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Motor Tests to Characterize Machines

- DC Resistance Test: $R_s = \frac{R_{phase-phase}}{2}$ for star winding
- No Load Test: $L_m + L_{ls}, P_{cfw}$
- Blocked Rotor Test: $R_r',\ L_{ls},\ L_{lr}',\ L_m$

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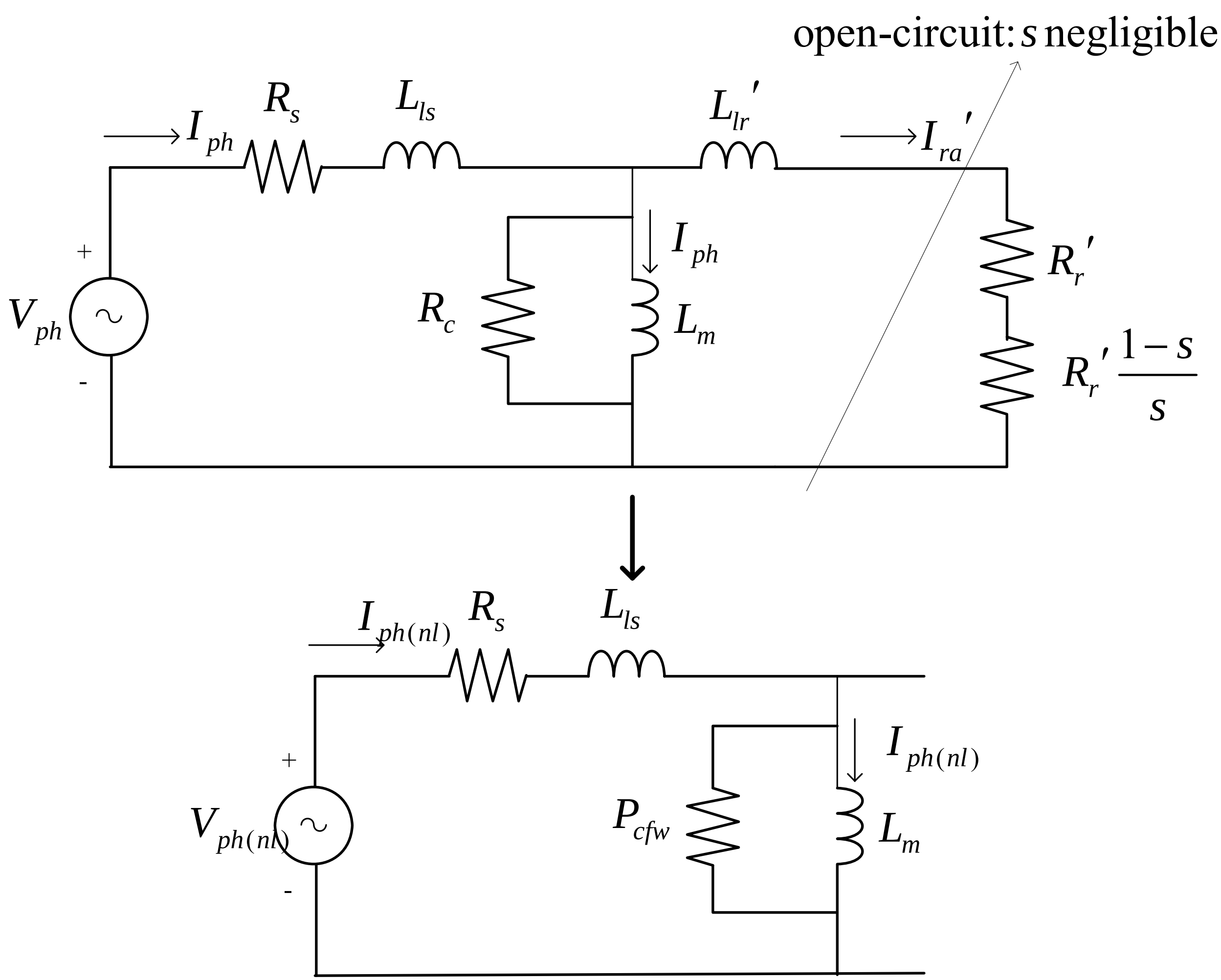
No Load Test



- Rotor spins as $f_m = f_{syn}$, $s=0$ ideally.
- Under no load conditions the equivalent circuit is dominated by the magnetizing inductance.

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No-load Test

- Measurements: three-phase P_{nl}(or power factor), $V_{ph(nl)}$, $I_{ph(nl)}$
- Core, friction and windage losses are required to spin the rotor at no-load and are given by

- The lumped inductance is determined from the per-phase reactive power

$$Q_{ph(nl)} = \sqrt{\left(V_{ph(nl)} I_{ph(nl)}\right)^2 - \left(\frac{P_{nl}}{3}\right)^2} = 2\pi f \left(L_m + L_{ls}\right) I_{ph(nl)}{}^2$$

$$\Rightarrow L_m + L_{ls} = \frac{Q_{ph(nl)}}{2\pi f \, I_{ph(nl)}{}^2}$$

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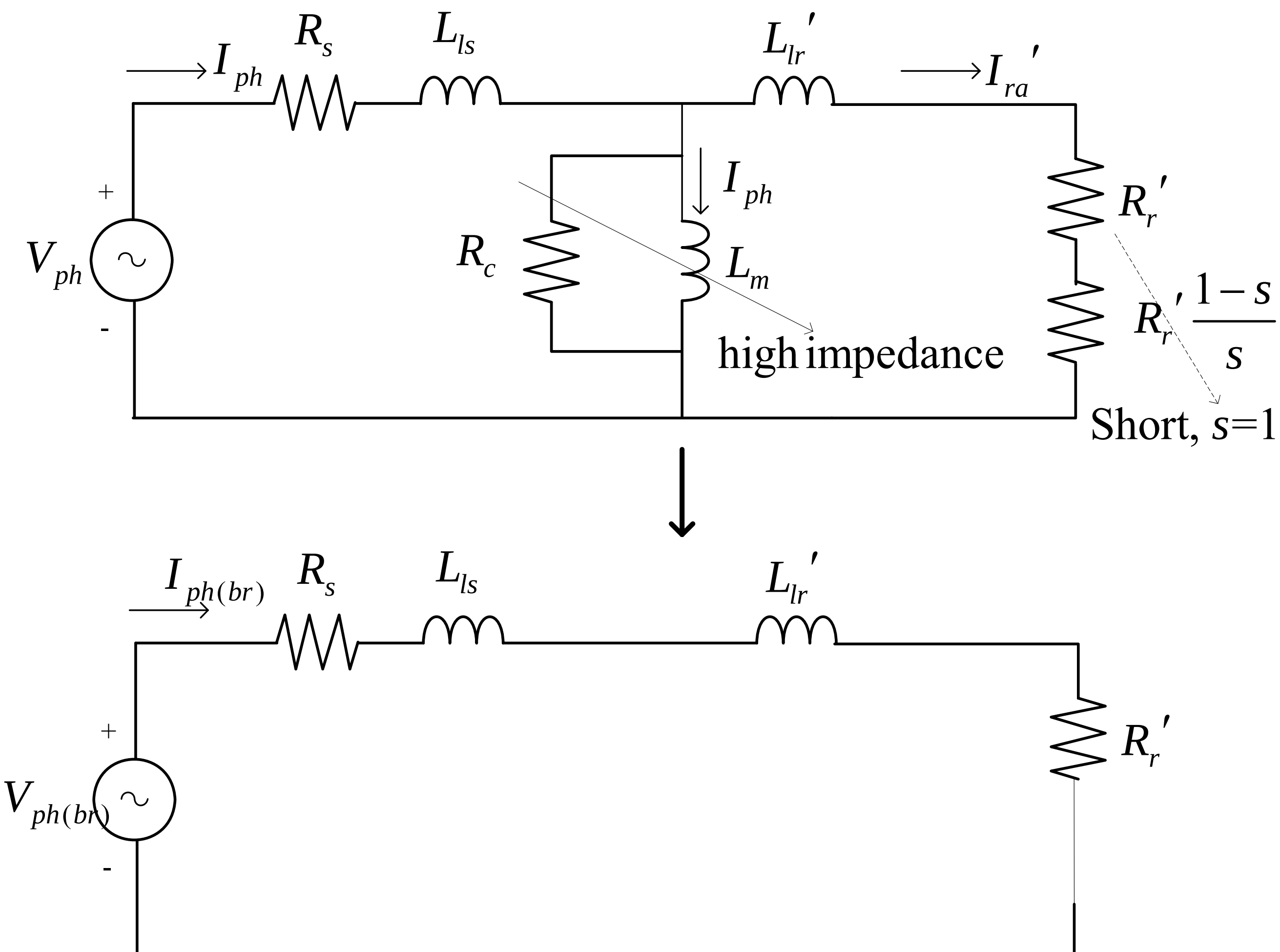
Blocked Rotor Test



- Rotor blocked from spinning f_m=0, s=1.
- Magnetizing branch much larger than rotor impedance and is neglected.

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Blocked Rotor Test

- Measurements: three-phase P_{br}(or power factor), $V_{ph(br)}$, $I_{ph(br)}$.
- Rotor resistance is derived as follows:

- Knowing the resistances the lumped inductances can be determined as follows:

$$Z_{ph(br)} = \sqrt{(R_s + R_r')^2 + [2\pi f\,(L_{ls} + L_{lr}')]^2}$$

$$\Rightarrow L_{ls} + L_{lr}' = \frac{\sqrt{Z_{ph(br)}{}^2 - (R_s + R_r')^2}}{2\pi f}$$

- Split stator/rotor leakage 40/60 or 50/50 depending on the motor class.
- Thus, all parameters now known.

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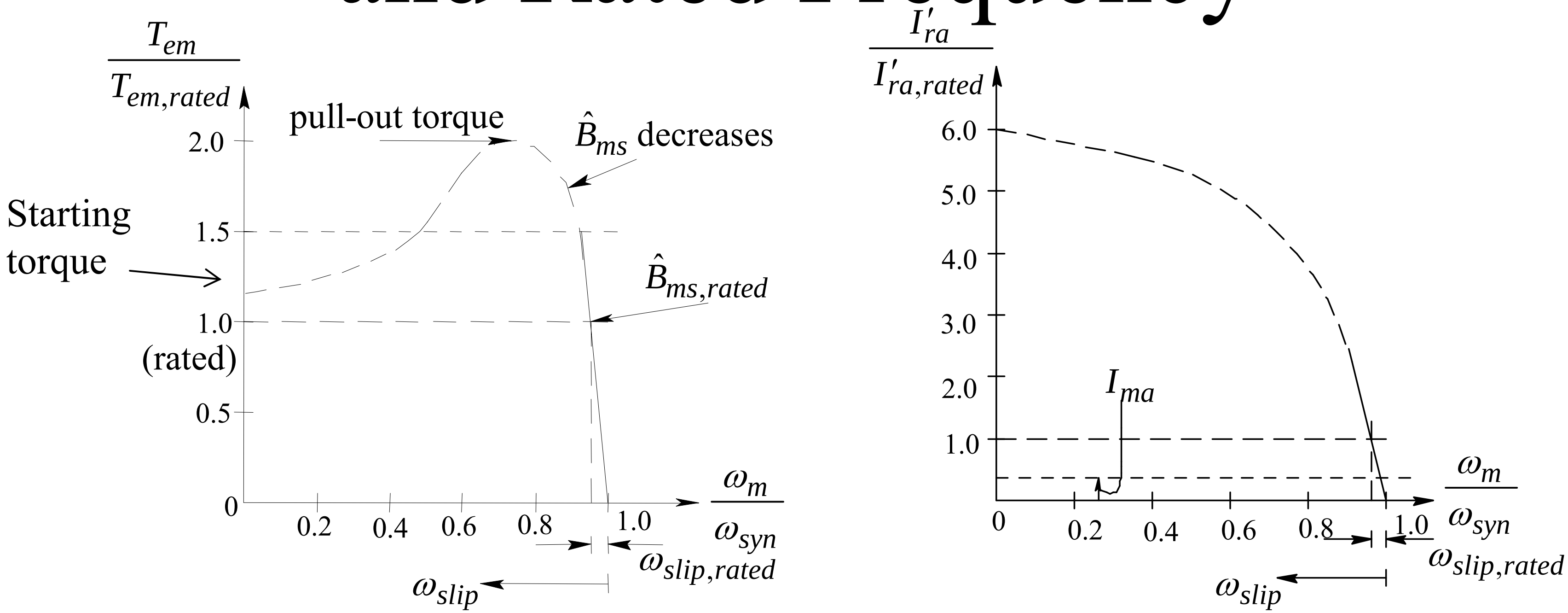
Characteristics at Rated Voltage and Rated Frequency



- Nearly linear near ω_{syn}
- At higher slip (ω_m smaller) leakage inductances and stator resistance reduce torque $T_{em} = k_t \hat{B}_{ms} \hat{I}' \sin\left(\pi/2 - \theta_r\right)$
- High currents at low speeds (start-up condition)

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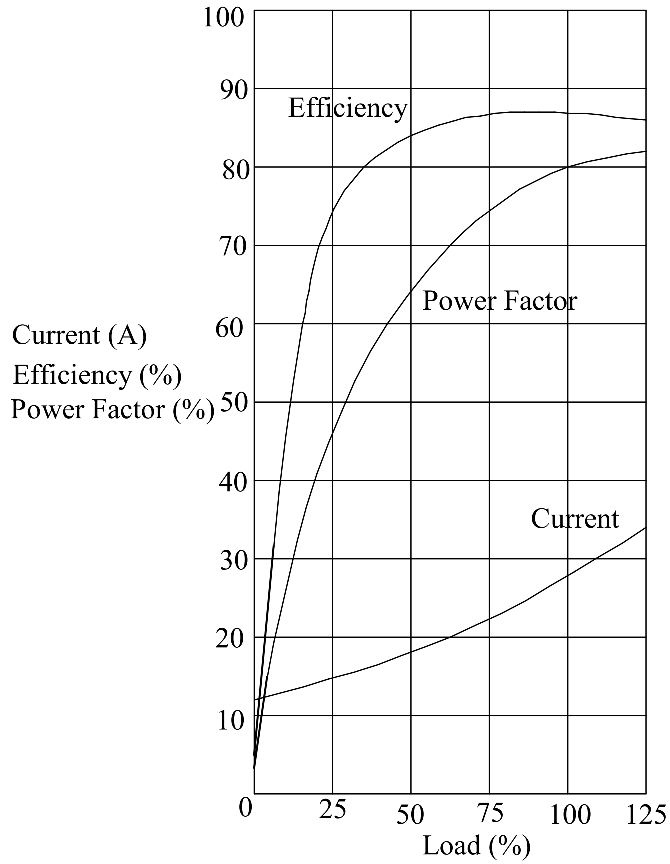
Motor Currents, Efficiency, Power Factor As a Function of Load



Typical for design B 10 kW, 4 pole, three-phase induction motor

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Useful Equations (lumping core and friction and windage*)

$$P_{EM} = \text{electromechanical power}$$
$$= \text{useful output power} + \text{core friction and windage} = T_{\mathrm{EM}}\omega_m$$
$$= P_m + P_{\mathrm{fw}} = T_{\mathrm{m}}\omega_m + T_{\mathrm{cfw}}\omega_m$$
$$= 3R_r'\left(\frac{1-s}{\mathrm{s}}\right)I_r'^2$$

$$P_{AG} = \text{airgap power}$$
$$= \text{electromechanical power} + \text{rotor loss}$$
$$= P_{EM} + P_{\mathrm{Rr}} = T_{\mathrm{EM}}\omega_{syn}$$
$$= 3R_r'\left(\frac{1-s}{\mathrm{s}}\right)I_r'^2 + 3R_r'I_r'^2 = \frac{3R_r'I_r'^2}{s}$$

*Core loss is a stator loss and should be moved there if sufficient information exists.

Exit

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Useful Equations-II

$$P_{in} = \text{input power}$$
$$= \text{airgap power} + \text{stator cu and core loss} = T_{\mathrm{EM}}\omega_{syn}$$
$$= P_{AG} + P_{\mathrm{Rs}}$$
$$= \frac{3R_r' I_r'^2}{s} + 3R_s I_s^2$$

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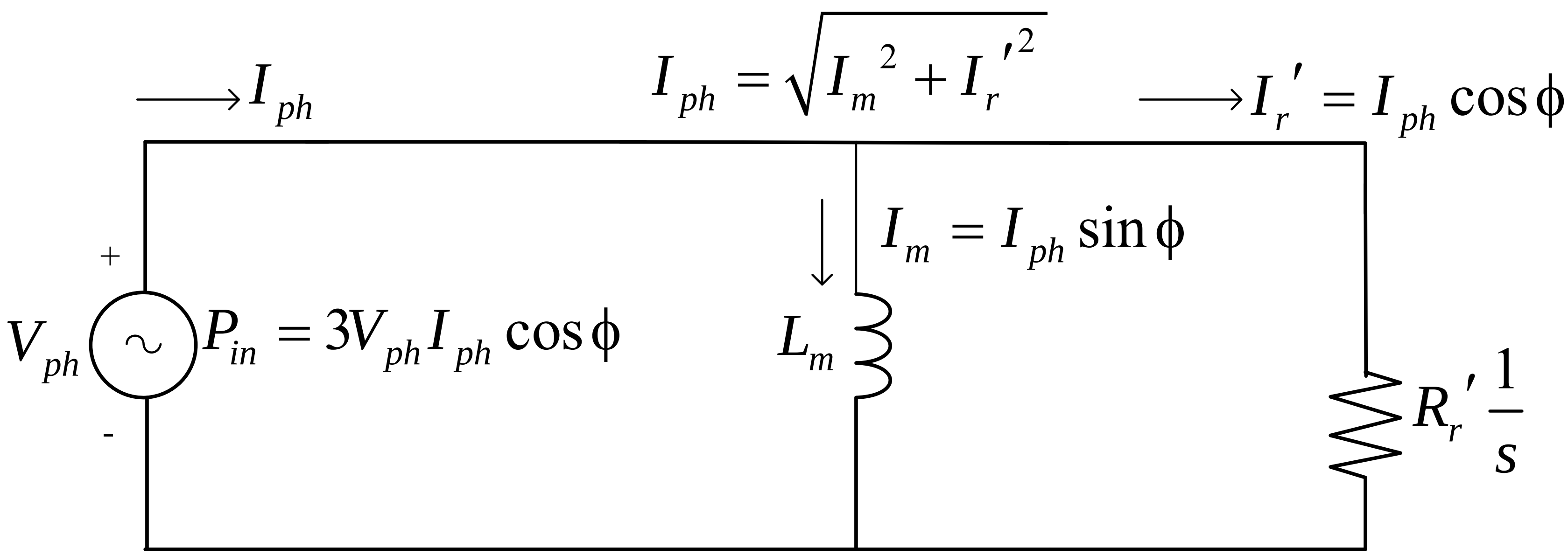
Reduced Eq. Cct for Operation Close to Rated



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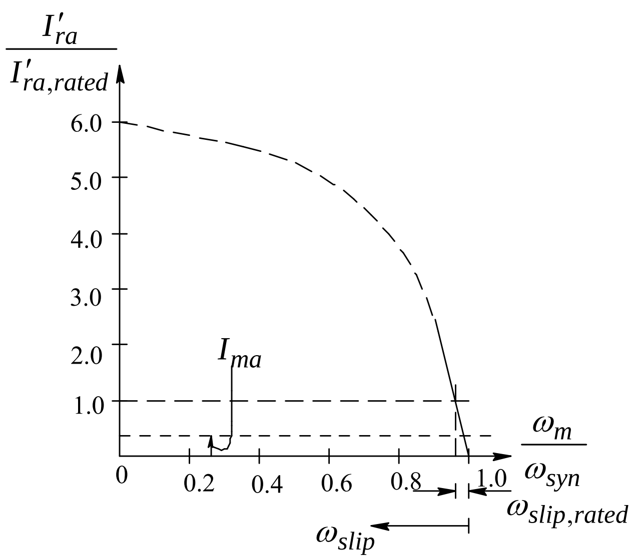
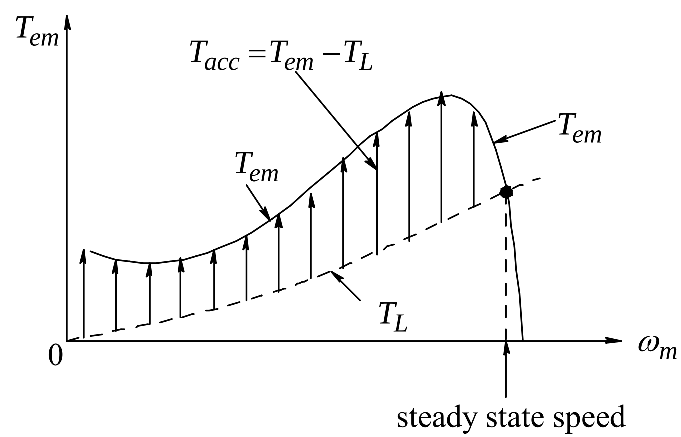
Line Start

- When started directly off the line, induction motor draws a very large current (approx. 8 x rated)
- At the same time the torque available to accelerate the motor/load is limited
- Motor can quickly overheat – Solution: Reduced voltage soft start



Current vs. Speed



Accelerating Torque

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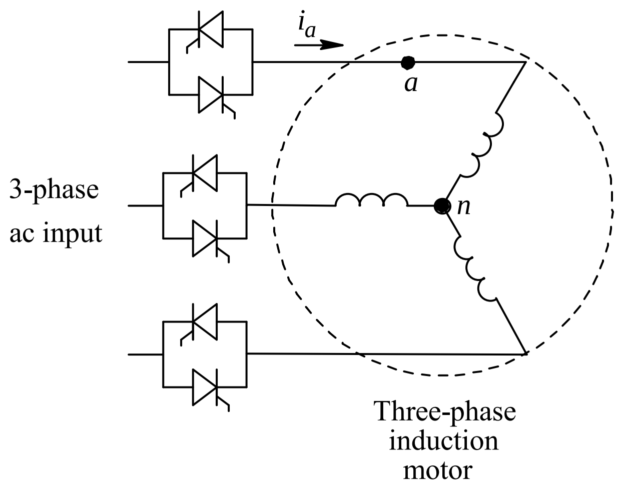
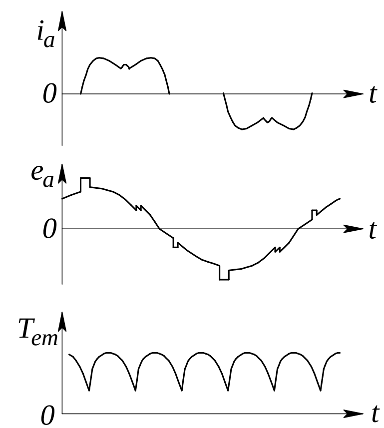
Reduced Voltage Starting (Soft Start) Energy Savings in Lightly – Loaded Machines





- Circuit applies reduced voltage to motor during start-up to avoid large currents and over heating
- Circuit also used to reduce voltage to motor under light load steady state conditions. This improves efficiency

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