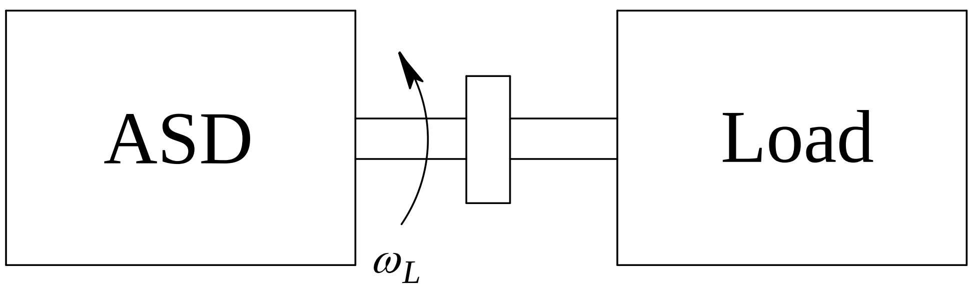
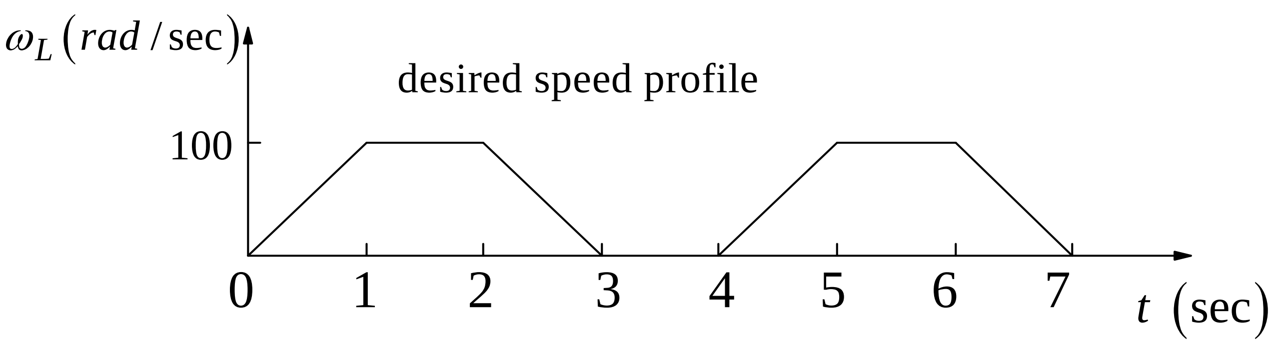
Chapter 2

Understanding Mechanical System Requirements

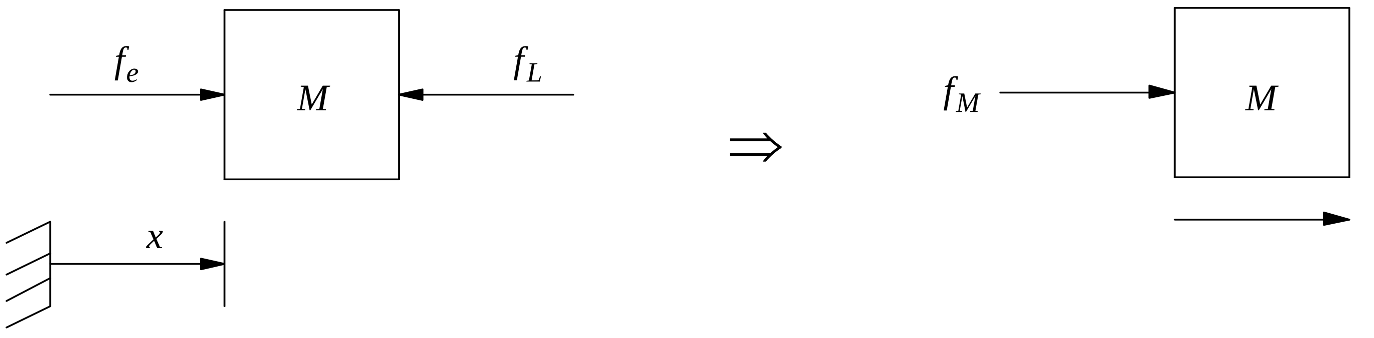
Motivation

- How can the ASD accelerate and decelerate the load to give desired speed profile





Systems With Linear Motion



- Figure on left includes load force, f_L , that must be overcome
- Figure on right shows only the force, f_M , available to accelerate the mass, M

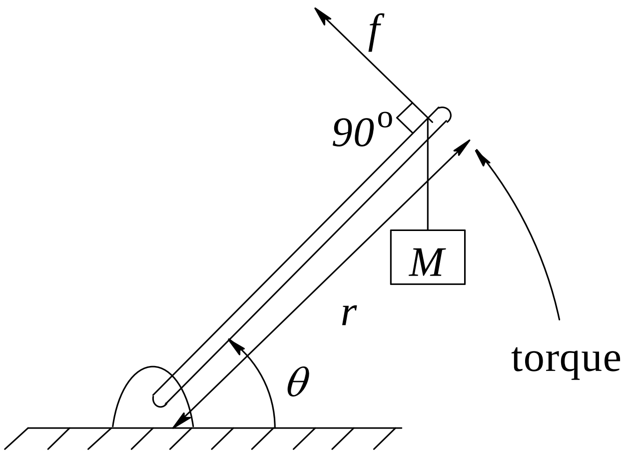
Acceleration

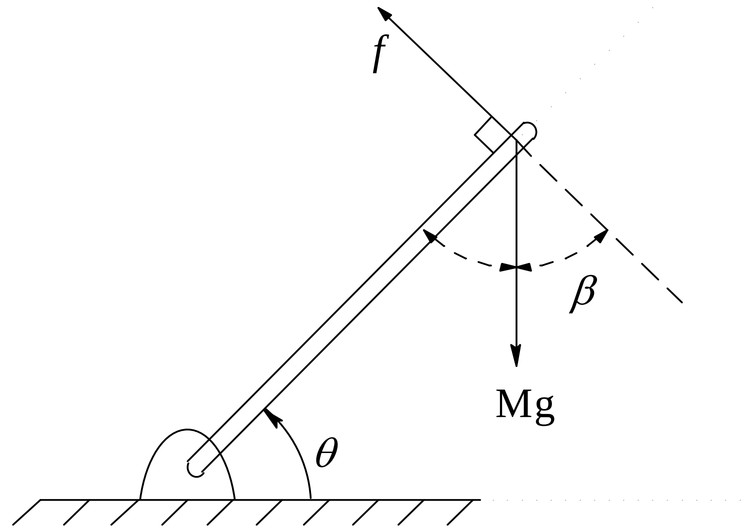
Power Input

Kinetic energy

$(Newton's\ 2nd\ Law)$

Rotating Systems

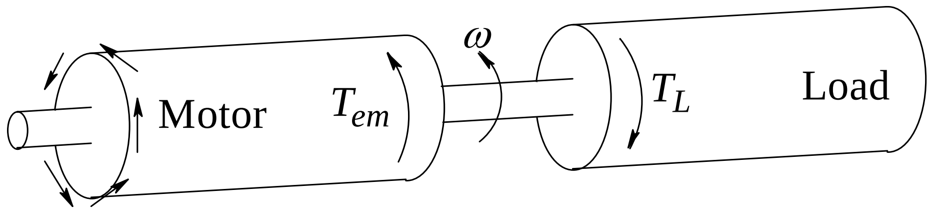




- Torque:

- Example: what torque is needed to hold *M* motionless

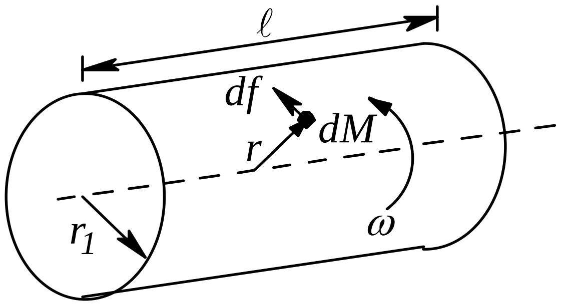
❑ Torque in an electric drive

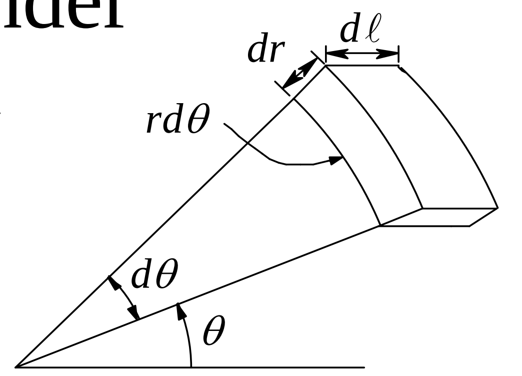


- T_{em} electromagnetic torque produced by motor
- T_{em} is opposed by load torque, T_L
- The difference, $T_{em} - T_L = T_J$, will accelerate the system
- Newton's 2nd Law for Rotational Systems

where J is the moment of inertia

Calculation of Moment of Inertia J of a Uniform Cylinder



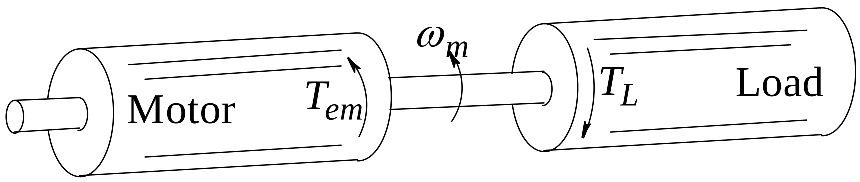


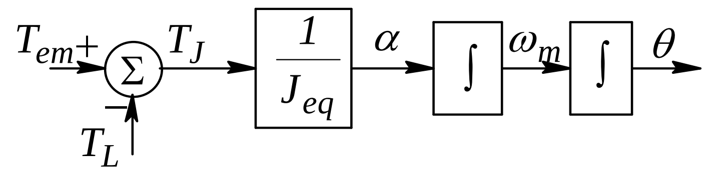
$$\Rightarrow dT = r^2\, dM \frac{d}{dt}\omega = \rho\,(r^3 dr\, d\theta\, d\ell)\frac{d}{dt}\omega$$

$$T = \rho\,\left(\int_0^{r_1} r^3 dr \int_0^{2\pi} d\theta \int_0^{\ell} d\ell\right)\frac{d}{dt}\omega = \underbrace{\left(\frac{\pi}{2}\rho\,\ell\, r_1^4\right)}_{J}\frac{d}{dt}\omega$$

$$J_{solid} = \frac{\pi}{2}\rho\,\ell\, r_1^4 = \frac{1}{2} M\, r_1^2$$

Accelaration, Speed and Position, Power and Energy





acceleration, $\alpha =$

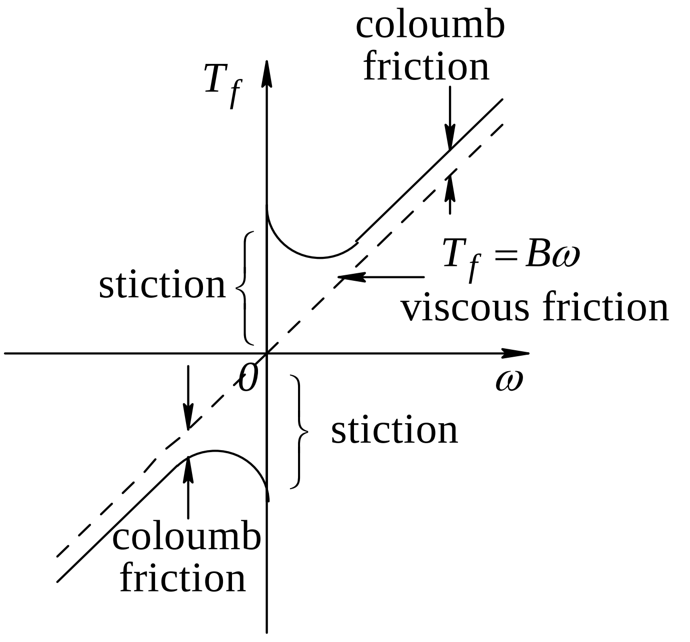
$\Rightarrow$ speed, $\omega_m(t) =$

$\Rightarrow$ position, $\theta(t) =$

Power $P_{em} =$; $P_L =$

Kinetic Energy $W =$

Frictional Torque



- Stiction: static component
- Coulomb friction: dynamic component constant magnitude)
- Viscous friction: speed dependent
- In general, friction is non-linear

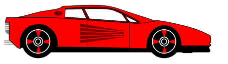
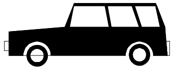
❑ Example: Aerodynamic drag

Drag power at different speeds

$f_L =$ $\qquad$ (C_w: drag coefficient)

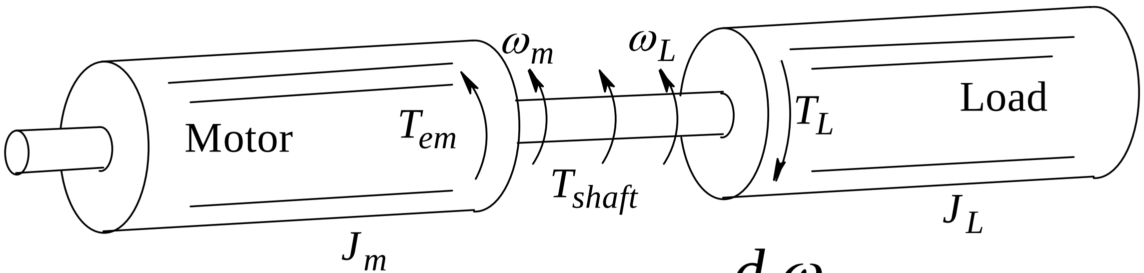
$p = f_L \cdot u$

$\therefore$ power α speed^3

Speed (km/h)	Power (W)	
	 $C_w = 0.3$	 $C_w = 0.5$
50	0.86 kW	1.44 kW
100	6.9 kW	11.5 kW
150	23.3 kW	38.8 kW

❑ Example: Rolling Resistance occurs through the work of deformation on wheel and road surface.

Torsional Resonances



At motor end $$T_{shaft} = T_{em} - J_m \frac{d\,\omega_m}{dt}$$

At load end $$T_{shaft} = T_L + J_L \frac{d\,\omega_L}{dt}$$

$$(\theta_m - \theta_L) = \frac{T_{shaft}}{K}$$

θ_m and θ_L : angular rotation at the two ends of the shaft

- If $K \to \infty$, $\theta_m = \theta_L$

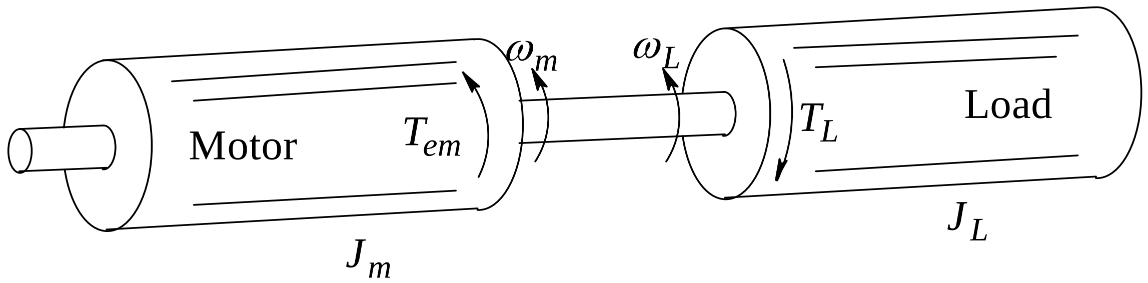
  (J_M and J_L can be treated as one inertial mass)

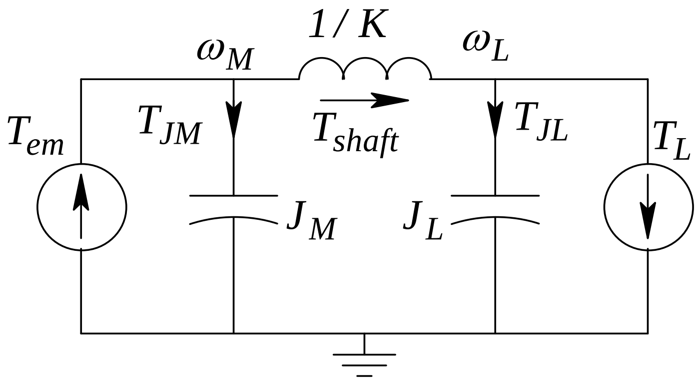
- Finite K may lead to resonances

Mechanical - Electrical Analogy

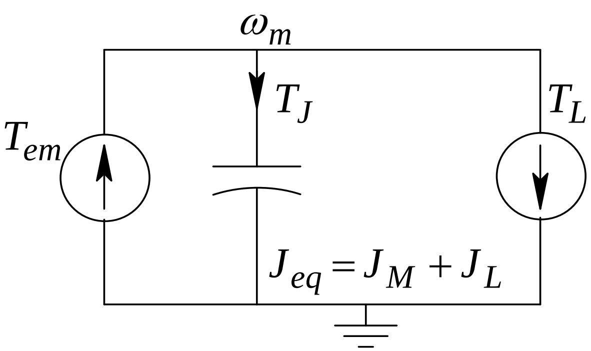
Mechanical	Electrical
Torque	Current
Angular Velocity	Voltage
Angular Displacement	Flux Linkage
Moment of Inertia	Capacitance
Spring Constant	1/Inductance
Damping Coefficient	1/Resistance
Coupling Ratio	Transformer ratio

Electrical Analogy of Motor & Load





Finite shaft stiffness



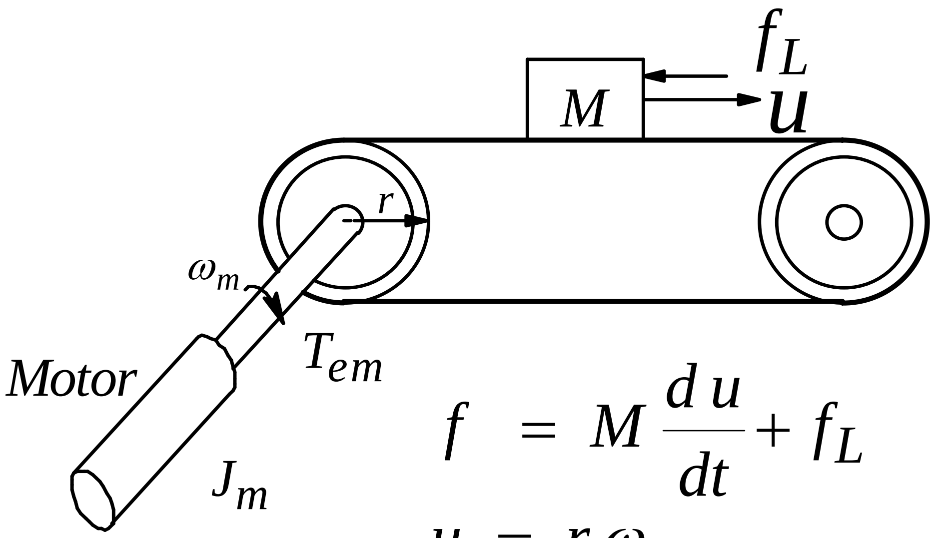
Infinite shaft stiffness

Coupling Mechanisms

- Required when
  - a (rotary) motor is driving a load which requires linear (translational) motion
  - motors prefer higher rotational speed than that required by the load
    -
  - the axis of rotation needs to be changed
- Disadvantages of gearing:

- Types
  - Conveyor belts (belt and pulley)
  - Rack and pinion or a lead-screw type of arrangement
  - Gear mechanisms

Conversion between Linear and Rotary Systems



J_m = motor inertia

M = mass of load

r = pulley radius

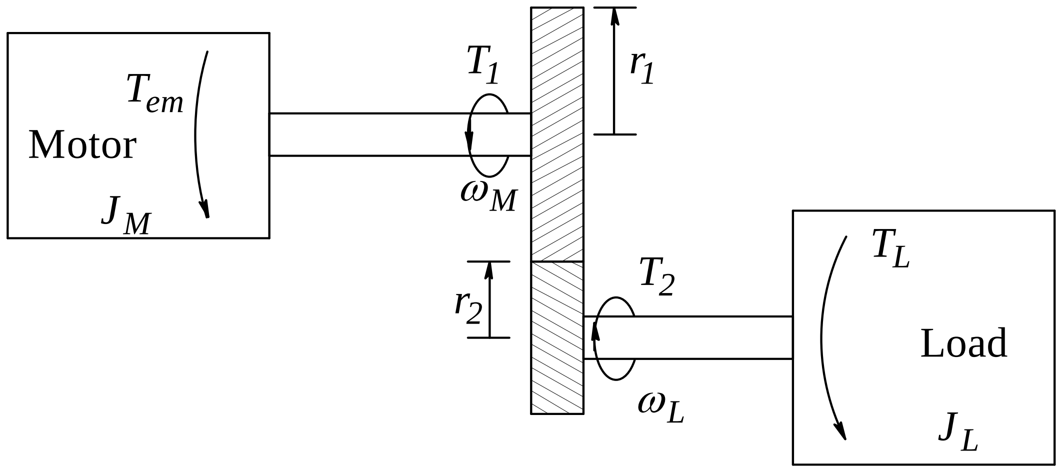
$$f = M\frac{du}{dt} + f_L$$

$$u = r\,\omega_m$$

$$T = r\,f = r^2 M\frac{d\omega_m}{dt} + r f_L$$

$$T_{em} = \underbrace{J_m\frac{d\,\omega_m}{dt}}_{\text{required to accelerate motor}} + \underbrace{r^2 M\frac{d\,\omega}{dt} + r\,f_L}_{\text{due to load}}$$

Gears



- Basic relationships: radius, speed, torque

  Equal speeds at gear surfaces

  Power transferred across gears

$$\Rightarrow \frac{r_1}{r_2} = \frac{\omega_L}{\omega_M} = \frac{T_1}{T_2} \qquad \& \quad \underbrace{\left(T_{em} - J_M \frac{d\,\omega_M}{dt}\right)}_{T_1} \frac{\omega_M}{\omega_L} = \underbrace{\left(T_L + J_L \frac{d\,\omega_L}{dt}\right)}_{T_2}$$

- Geared up: speed increased, torque decreased $\omega_L > \omega_M\,;\ T_2 < T_1;\ r_2 < r_1$
- Geared down: speed decreased, torque increased

$$\omega_L < \omega_M\,;\ T_2 > T_1;\ r_2 > r_1$$

Gears (cont'd)

- Equivalent Inertia

$$T_{em} = \underbrace{\left[J_m + J_L \left(\frac{\omega_L}{\omega_m} \right)^2 \right]}_{J_{eq}} \frac{d\,\omega_m}{dt} + \left(\frac{\omega_L}{\omega_m} \right) T_L$$

$$\Rightarrow J_{eq} = J_m + J_L \left(\frac{\omega_L}{\omega_m} \right)^2 = J_m + J_L \left(\frac{r_1}{r_2} \right)^2$$

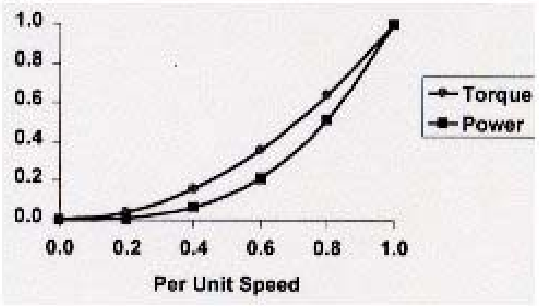
- Optimum gear ratio (to minimize T_{em})

$$J_m = \left(\frac{r_1}{r_2} \right)^2_{opt.} \cdot J_L \qquad \Rightarrow \qquad \left(\frac{r_1}{r_2} \right)_{opt.} = \sqrt{\frac{J_m}{J_L}}$$

and $(T_{em})_{opt.} = 2\,J_m \dfrac{d\,\omega_m}{dt} = 2\,J_m \left(\dfrac{r_2}{r_1} \right)_{opt.} \dfrac{d\omega_L}{dt}$

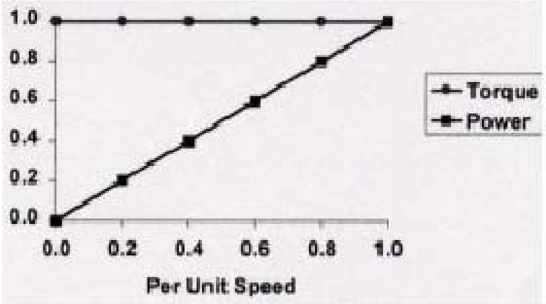
Types of Loads

Centrifugal loads



Fan

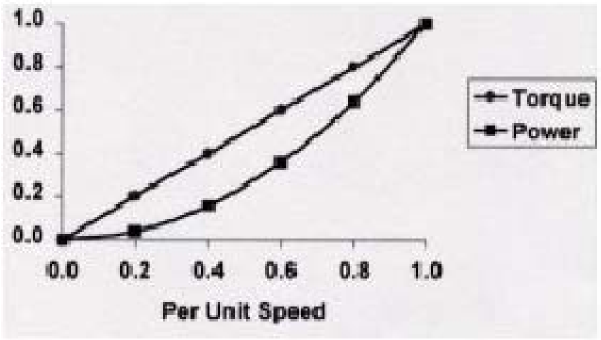
Constant Torque loads

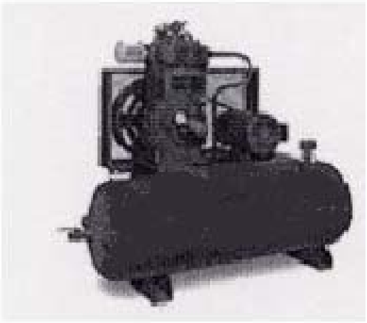


Hoist

Types of Loads

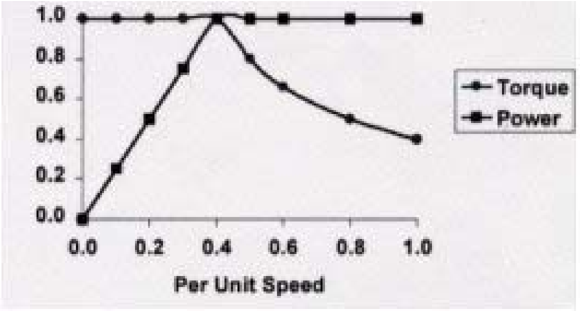
Squared power loads





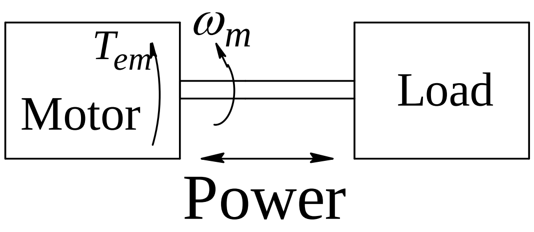
Compressor

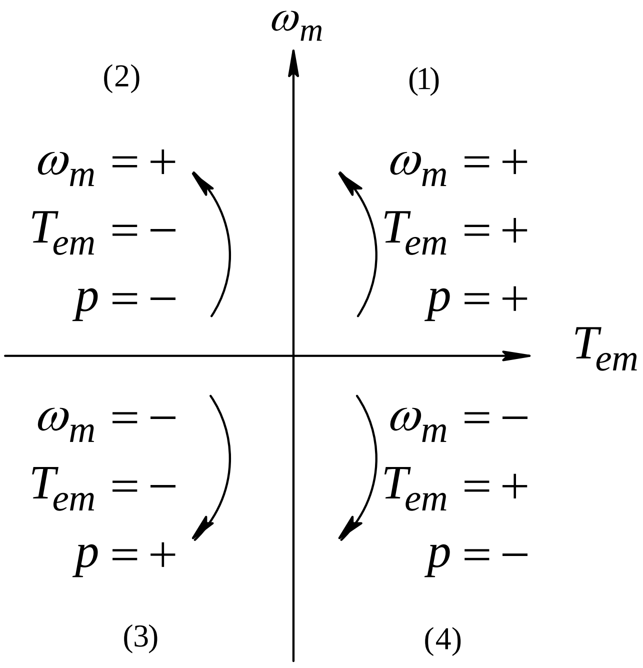
Constant power loads



Winder

Four-Quadrant Operation





Dynamic Operation

- How the operating point changes with time
- Important for High Performance Drives
- Speed change: rapid and without any oscillations
- Requires good controller design

Summary

- What are the MKS units for force, torque, linear velocity, angular velocity, speed, and power?
- What is the relationship between force, torque, and power?
- Show that torque is the fundamental variable in controlling speed and position.
- What is the kinetic energy stored in a moving mass and a rotating inertia?
- What is the mechanism for torsional resonances?
- What are the various types of coupling mechanisms?
- What is the optimum gear ratio to minimize the torque required from the drive to accelerate a load?
- What are the torque-speed and the power-speed profiles for various types of loads?