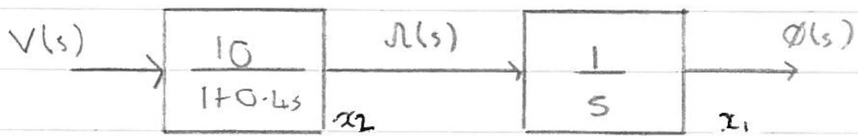
Autumn 05

Q5(a)



$$\frac{\Omega(s)}{V(s)} = \frac{10}{1+0.4s}$$

$$\Omega(s) + 0.4s\,\Omega(s) = 10V(s)$$

$$\omega(t) + 0.4\frac{d\omega}{dt} = 10v(t)$$

$$0.4\frac{d\omega}{dt} = 10v(t) - \omega(t)$$

$$\frac{d\omega}{dt} = 25v(t) - 2.5\omega(t)$$

$$\frac{d}{dt}x_2 = 25v - 2.5x_2$$

$$\frac{d}{dt}x_1 = x_2$$

$$\varnothing(t) = x_1$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ 0 & -2.5 \end{bmatrix}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 25 \end{bmatrix}} v$$

$$\varnothing(t) = \overset{C}{\begin{bmatrix} 1 & 0 \end{bmatrix}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Peak overshoot 5%

$\Rightarrow \xi = 0.7$ from graph

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$0.2\,\omega_n\sqrt{1-0.7^2} = \pi$$

$$\Rightarrow \omega_n = 22 \text{ rad/s}$$

$$C_{des}(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$= s^2 + 2(0.7)(22)s + 22^2$$

$$= s^2 + 30.8s + 484$$

$$\det(sI - A + BK) = C_{des}(s)$$

$$\det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & -2.5 \end{pmatrix} + \begin{pmatrix} 0 \\ 25 \end{pmatrix}(k_1 \ k_2)\right]$$

$$\det\left[\begin{pmatrix} s & -1 \\ 0 & s+2.5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 25k_1 & 25k_2 \end{pmatrix}\right]$$

$$\det\begin{pmatrix} s & -1 \\ 25k_1 & s+2.5+25k_2 \end{pmatrix} = s^2 + (2.5+25k_2)s + 25k_1$$

$$2.5 + 25k_2 = 30.8 \qquad 25k_1 = 484$$

$$k_2 = 1.132 \qquad k_1 = 19.36$$

$$u(t) = -[k_1 \ k_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow v(t) = -[19.36 \ \ 1.132]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$v(t) = -19.36\,\phi(t) - 1.132\,\omega(t)$$

(b). Seperation Principle

- We can design K for regulator to place N closed loop poles assuming states are available
- Then we design G for our estimator to provide these states with desired error dynamics

$$|sI_N - A + BK||sI_N - A + GC| = 0$$

MINIMISE EFFECT OF ESTIMATOR BY HAVING FAST POLES

From Regulator

$$C_{des}(s) = s^2 + 30.8s + 484$$

$$s^2 + 30.8s + 484 = 0$$

$$s = \frac{-30.8 \pm \sqrt{30.8^2 - 484}}{2}$$

$$s = \frac{-30.8 \pm 21.55}{2}$$

$$s = -26.175 \;,\; -4.625$$

Place estimation poles at $5(-26.175)$

$\Rightarrow s = -130$ twice

$$C_{des}(s) = (s+130)^2$$
$$= s^2 + 260s + 16900$$

Full order Luenberger observer

$$\frac{d}{dt}\underline{\hat{x}} = (A - GC)\underline{\hat{x}} + B\underline{u} + G\underline{y}$$

$$F = A - GC$$
$$= \begin{pmatrix} 0 & 1 \\ 0 & -2.5 \end{pmatrix} - \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}(1 \;\; 0)$$
$$= \begin{pmatrix} 0 & 1 \\ 0 & -2.5 \end{pmatrix} - \begin{pmatrix} g_1 & 0 \\ g_2 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} -g_1 & 1 \\ -g_2 & -2.5 \end{pmatrix}$$

$$\det(sI - F) = 0$$
$$\det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -g_1 & 1 \\ -g_2 & -2.5 \end{pmatrix}\right] = 0$$
$$\det\begin{bmatrix} s+g_1 & -1 \\ g_2 & s+2.5 \end{bmatrix} = 0$$

$$(s+g_1)(s+2.5) + g_2 = 0$$
$$s^2 + (g_1 + 2.5)s + 2.5g_1 + g_2 = 0$$

$g_1 + 2.5 = 260$ $\qquad$ $2.5g_1 + g_2 = 16900$

$g_1 = 257.5$ $\qquad$ $g_2 = 16256.25$

$$\frac{d}{dt}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} = \begin{bmatrix}-257.5 & 1\\ -16256.25 & -2.5\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} + \begin{bmatrix}0\\ 25\end{bmatrix} v + \begin{bmatrix}257.5\\ 16256.25\end{bmatrix}\phi$$