Fourth Year Electrical Engineering

EE4010

Electrical and Electronic Power Supply Systems

Symmetrical Components Worked Examples

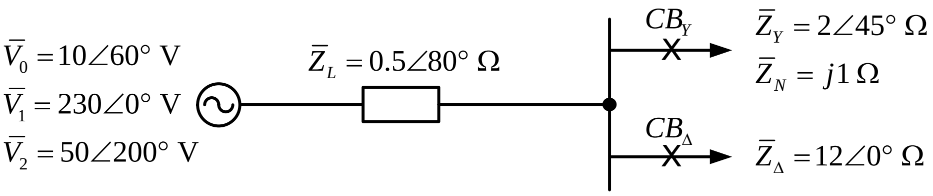
Example 1 (Symmetrical Components)

A three-phase, impedance load consists of a balanced delta-connected load in parallel with a balanced star-connected load. The impedance in each leg of the delta load is $12\angle 0°\ \Omega$ and the impedance in each leg of the star-connected load is $2\angle 45°\ \Omega$. The load star point is grounded via a neutral impedance, $\overline{Z}_n = j1\,\Omega$. Unbalanced line-to-ground voltages of $\overline{V}_{ag}$, $\overline{V}_{bg}$ and $\overline{V}_{cg}$ whose sequence components are given by $\overline{V}_0 = 10\angle 60°$ V, $\overline{V}_1 = 230\angle 0°$ V, and $\overline{V}_2 = 50\angle 200°$ V are applied to the load via a three-phase cable with a per-phase impedance of $0.5\angle 80°\ \Omega$. Draw the zero, positive and negative sequence networks corresponding to this system.

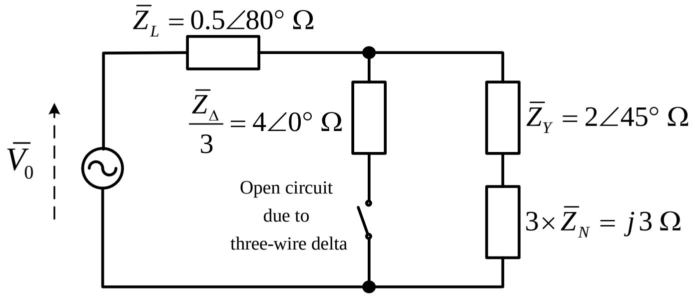
Calculate the current in Phase *a* of the system. If the three-phase circuit breaker CB_Δ in series with the delta-connected load trips, calculate the current in Phase *a* of the resultant system.

Solution 1

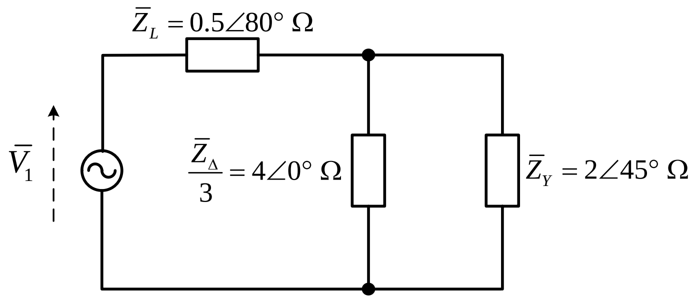
A single-line diagram of the complete system is illustrated below.



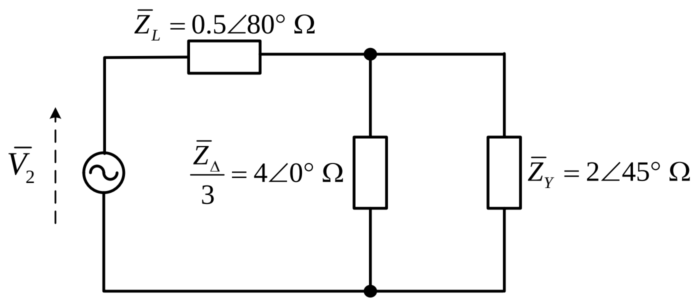
The zero, positive and negative sequence networks for this system with the delta-connected load are shown below.



Zero sequence network



Positive sequence network



Negative sequence network

Since the two loads are balanced, $\bar{Z}_0 = \bar{Z}_1 = \bar{Z}_2 = \bar{Z}_{phase}$ and the mutual impedance terms in the sequence impedance matrix $\bar{Z}_s$ are uniformly zero. Note that the delta-connected load is converted to an equivalent star so that

$$\bar{Z}_{eq\ \Delta} = \frac{\bar{Z}_\Delta}{3} = \frac{12\angle 0^\circ}{3} = 4\angle 0^\circ\ \Omega$$

Also, the neutral impedance in the star-connected load appears only in the corresponding zero sequence network and is given by

$$\bar{Z}_{eq\ N} = 3\bar{Z}_N = 3\times j1 = j3\ \Omega$$

Case (i). CB_Δ *closed.*

In this case, the zero sequence current is

$$\bar{I}_0 = \frac{\bar{V}_0}{\bar{Z}_L + \bar{Z}_Y + 3\bar{Z}_N} = \frac{10\angle 60^\circ}{0.5\angle 80^\circ + 2.0\angle 45^\circ + 3\angle 90^\circ} = 1.949\angle -12.99^\circ\ \text{A}\ .$$

The positive sequence current is

$$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_L + \left(\dfrac{\bar{Z}_Y \times \bar{Z}_{eq\ \Delta}}{\bar{Z}_Y + \bar{Z}_{eq\ \Delta}}\right)} = \frac{230\angle 0^\circ}{0.5\angle 80^\circ + \left(\dfrac{2.0\angle 45^\circ \times 4\angle 0^\circ}{2.0\angle 45^\circ + 4\angle 0^\circ}\right)} = 128.20\angle -42.62^\circ\ \text{A}$$

The negative sequence current is

$$\bar{I}_2 = \frac{\bar{V}_2}{\bar{Z}_L + \left(\dfrac{\bar{Z}_Y \times \bar{Z}_{eq\ \Delta}}{\bar{Z}_Y + \bar{Z}_{eq\ \Delta}}\right)} = \frac{50\angle 200^\circ}{0.5\angle 80^\circ + \left(\dfrac{2.0\angle 45^\circ \times 4\angle 0^\circ}{2.0\angle 45^\circ + 4\angle 0^\circ}\right)} = 27.87\angle 157.38^\circ\ \text{A}$$

Hence, the actual current in Phase *a* of the load is

$$\bar{I}_a = \bar{I}_0 + \bar{I}_1 + \bar{I}_2 = 104.10\angle -47.34^\circ\ \text{A}\ .$$

Case (i). CB_Δ *open.*

In this case, the zero sequence network is unchanged so that again

$$\bar{I}_0 = \frac{\bar{V}_0}{\bar{Z}_L + \bar{Z}_Y + 3\bar{Z}_N} = \frac{10\angle 60^\circ}{0.5\angle 80^\circ + 2.0\angle 45^\circ + 3\angle 90^\circ} = 1.949\angle -12.99^\circ\ \text{A}\ .$$

The positive sequence network now consists only of the transmission line and the star-connected load so that

$$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_L + \bar{Z}_Y} = \frac{230\angle 0^\circ}{0.5\angle 80^\circ + 2.0\angle 45^\circ} = 94.78\angle -51.79^\circ\ \text{A}$$

The negative sequence network also consists only of the transmission line and star-connected load and thus the negative sequence current is

$$\overline{I}_2 = \frac{\overline{V}_2}{\overline{Z}_L + \overline{Z}_Y} = \frac{50\angle 200°}{0.5\angle 80° + 2.0\angle 45°} = 20.61\angle 148.20° \text{ A}$$

Hence, the actual current in Phase *a* of the load in this case is

$$\overline{I}_a = \overline{I}_0 + \overline{I}_1 + \overline{I}_2 = 77.16\angle -56.12° \text{ A} .$$

Note that in this solution all of the phase angles are measured with respect to the positive sequence component of the applied voltage in Phase *a* of the three-phase source. Show that the phase angle of the actual voltage in Phase *a* of the system with respect to this reference is $-2.57°$.

Exercises

1. Find the symmetrical components of a system of unbalanced three-phase currents given by $\overline{I}_a = 200\angle 0°$ A, $\overline{I}_b = 100\angle -90°$ A and $\overline{I}_c = 50\angle 45°$A .

   [$\overline{I}_0 = 81.3\angle -15.4°$ A, $\overline{I}_1 = 99.8\angle 0.3°$ A and $\overline{I}_2 = 30.2\angle 44.0°$ A]

2. A three-phase, three-wire transmission system of positive phase sequence *abc* has positive and negative sequence components of current given by $1846 \angle -60°$ A and $695 \angle 60°$ A respectively. Calculate the line current in Phase *a* in magnitude and phase.

   [$\overline{I}_a = 1615\angle -38°$ A]

3. Show that an unsymmetrical three-phase system of voltages or currents can be represented by three symmetrical systems and find expressions for the symmetrical components.

   Under fault conditions in a three-phase system, the following currents were recorded: $\overline{I}_a = 3000 \angle 0°$ A, $\overline{I}_b = 2000 \angle 270°$ A, $\overline{I}_c = 1000 \angle 120°$ A . Calculate the sequence components of the unbalanced set of currents.

   [$\overline{I}_0 = 915\angle -24.4°$ A, $\overline{I}_1 = 1940\angle 9.9°$ A and $\overline{I}_2 = 260\angle 9.9°$ A]

Example 2 (Symmetrical Components)

A symmetrical three-phase, three-wire voltage source supplies an unsymmetrical, three-phase, three-wire, star-connected load. The red, yellow and blue phases of the load consist of a resistance of $60\ \Omega$, a resistance of $30\ \Omega$ and a loss-less inductive reactance of $j60\ \Omega$ respectively. The supply line voltage is 400 V. Calculate the current in the $60\ \Omega$ resistor.

Solution 2

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(* Solution to Worked Example Question 2*)
(* Define phase self impedances *)
(* Note: Phase mutual impedances are zero *)
Za = 60;
Zb = 30;
Zc = ⅈ 60;

(* Define the a operator *)
a = Cos[120 Degree] + ⅈ Sin[120 Degree] ;

(* Calculate the components of the sequence impedance matrix, Eqns. 3.58 through 3.63 *)
Z0 = 1/3 (Za + Zb + Zc);
Z01 = 1/3 (Za + a^2 Zb + a Zc);
Z02 = 1/3 (Za + a Zb + a^2 Zc);
Print["Z0 = ", N[Abs[Z0]], " at a phase angle of ", N[Arg[Z0]/Degree], " degrees"]
Print["Z01 = ", N[Abs[Z01]], " at a phase angle of ", N[Arg[Z01]/Degree], " degrees"]
Print["Z02 = ", N[Abs[Z02]], " at a phase angle of ", N[Arg[Z02]/Degree], " degrees"]

(* Define the applied phase voltage *)
Vline = 400;
Vphase = Vline/Sqrt[3];

(* Calculate the sequence components of current *)
(* Note: Io=0 due to the three-wire load *)
I1 = Vphase (Z0/(Z0^2 - Z01 Z02));
I2 = -I1 (Z02/Z0);

(* Calculate the sequence currents and the actual red phase current *)
Ia = N[I1 + I2];
Print["I1 = ", N[Abs[I1]], " at a phase angle of ", N[Arg[I1]/Degree], " degrees"]
Print["I2 = ", N[Abs[I2]], " at a phase angle of ", N[Arg[I2]/Degree], " degrees"]
Print["Ia = (", N[Re[Ia]], " + j ", N[Im[Ia]], ") A"]
Print["Ia = ", N[Abs[Ia]], " at a phase angle of ", N[Arg[Ia]/Degree], " degrees"]
Z0 = 36.0555 at a phase angle of 33.6901 degrees
Z01 = 18.804 at a phase angle of -97.0887 degrees
Z02 = 32.3483 at a phase angle of -2.37366 degrees
I1 = 4.38854 at a phase angle of -37.875 degrees
I2 = 3.93731 at a phase angle of 106.061 degrees
Ia = (2.37479 + j 1.08932 ) A
Ia = 2.6127 at a phase angle of 24.641 degrees
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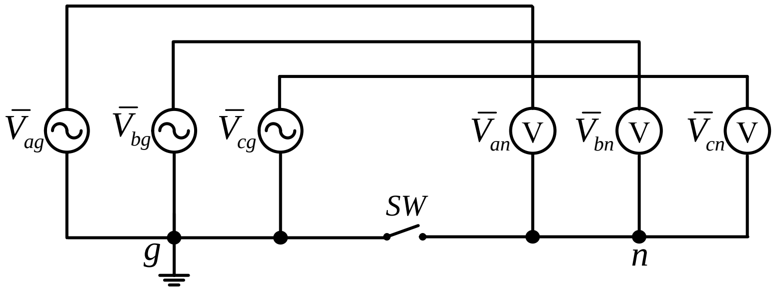
Example 3 (Symmetrical Components)

Three ac voltmeters are connected in star to a balanced 400V, three-phase, star-connected, four-wire supply. The star point *n* of the three voltmeters is connected to the star point *g* of the ac supply via a switch *SW* as illustrated in the diagram below. The internal resistances of the voltmeters are $10\ \text{k}\Omega$, $10\ \text{k}\Omega$, and $5\ \text{k}\Omega$, respectively. Determine the readings on the three voltmeters if (i) the star point of the voltmeters is connected to the star point of the balanced supply and (ii) the star point of the voltmeters is floating relative to the star point of the balanced supply



Solution 3

Case (i)

This is a trivial case since the balanced phase voltages given by

$$V_{ag} = V_{bg} = V_{cg} = V_{phase} = \frac{V_{LL}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\text{ V}$$

are impressed across each voltmeter equally thus giving

$$V_{an} = V_{bn} = V_{cn} = 231\text{ V}.$$

Case (ii)

In this case, the star point of the voltmeters is floating with a voltage $\bar{V}_{ng}$ with respect to the source star point so that Kirchhoff's voltage law gives

$$\begin{aligned}
\bar{V}_{ag} &= \bar{V}_{an} + \bar{V}_{ng} = \bar{I}_a R_a + \bar{V}_{ng} \\
\bar{V}_{bg} &= \bar{V}_{bn} + \bar{V}_{ng} = \bar{I}_b R_b + \bar{V}_{ng} \\
\bar{V}_{cg} &= \bar{V}_{cn} + \bar{V}_{ng} = \bar{I}_c R_c + \bar{V}_{ng}
\end{aligned}$$

where R_a, R_b and R_c are the resistances of the three voltmeters. Now, since the source voltages are balanced, adding the above equations gives

$$\bar{I}_a R_a + \bar{I}_b R_b + \bar{I}_c R_c + 3\bar{V}_{ng} = \left(\bar{V}_{ag} + \bar{V}_{bg} + \bar{V}_{cg}\right) = 0.$$

Now, since the voltmeter star point is floating, $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$, and voltmeters *a* and *b* have identical resistances, $R_a = R_b = R$, we get that

$$\begin{aligned}
&\left(\bar{I}_a + \bar{I}_b\right)R + \bar{I}_c R_c + 3\bar{V}_{ng} = 0 \\
&\left(-\bar{I}_c\right)R + \bar{I}_c R_c + 3\bar{V}_{ng} = 0 \\
&\bar{V}_{ng} = \frac{\bar{I}_c\left(R - R_c\right)}{3}.
\end{aligned}$$

Also, since

$$\begin{aligned}\bar{V}_{cg} &= \bar{I}_c R_c + \bar{V}_{ng} \\ &= \left(\frac{3\bar{V}_{ng}}{R - R_c}\right) R_c + \bar{V}_{ng} \\ &= \left(\frac{R + 2R_c}{R - R_c}\right) \bar{V}_{ng}\end{aligned}$$

so that

$$\bar{V}_{ng} = \left(\frac{R - R_c}{R + 2R_c}\right) \bar{V}_{cg}$$

Thus, general expressions for the three voltmeter readings can be derived from

$$\bar{V}_{an} = \bar{V}_{ag} - \bar{V}_{ng} = \bar{V}_{ag} - \left(\frac{R - R_c}{R + 2R_c}\right) \bar{V}_{cg} = V_{phase}\angle 0° - \left(\frac{R - R_c}{R + 2R_c}\right) V_{phase}\angle +120°$$

$$\bar{V}_{bn} = \bar{V}_{bg} - \bar{V}_{ng} = \bar{V}_{bg} - \left(\frac{R - R_c}{R + 2R_c}\right) \bar{V}_{cg} = V_{phase}\angle -120° - \left(\frac{R - R_c}{R + 2R_c}\right) V_{phase}\angle +120°$$

$$\bar{V}_{cn} = \bar{V}_{cg} - \bar{V}_{ng} = \bar{V}_{cg} - \left(\frac{R - R_c}{R + 2R_c}\right) \bar{V}_{cg} = \left(1 - \frac{R - R_c}{R + 2R_c}\right) V_{phase}\angle +120°.$$

Thus, using the values specified,

$$\begin{aligned}\bar{V}_{an} &= \frac{400}{\sqrt{3}}\angle 0° - \left(\frac{10 - 5}{10 + 2\times 5}\right) \frac{400}{\sqrt{3}}\angle +120° = 265\angle -10.9° \text{ V} \\ \bar{V}_{an} &= \frac{400}{\sqrt{3}}\angle -120° - \left(\frac{10 - 5}{10 + 2\times 5}\right) \frac{400}{\sqrt{3}}\angle +120° = 265\angle -109.1° \text{ V} \\ \bar{V}_{an} &= \frac{400}{\sqrt{3}}\angle +120° - \left(\frac{10 - 5}{10 + 2\times 5}\right) \frac{400}{\sqrt{3}}\angle +120° = 173\angle 120° \text{ V}\end{aligned}$$

Clearly, the voltmeter readings are unbalanced due to the presence of the finite neutral point voltage of the floating three-phase voltmeter connection.

Note from the above equations that if the voltmeter internal resistances are balanced then $R_c = R$ and it is clear that the voltmeters will correctly read the corresponding phase voltages.