APPENDIX 6A Proving that $\frac{\hat{I}_{s,3}}{\hat{I}_{L,2}} = \frac{1}{2}$

The output of the voltage regulator G_v in Fig. 6-6 in steady state is

$$\text{output of } G_v = \hat{I}_L - \hat{I}_{L,2} \cos 2\omega t \qquad (6A\text{-}1)$$

in which $\hat{I}_{L,2}$, by a proper controller design is much smaller than $\hat{I}_L$, for example less than 1.5 percent. The above expression is multiplied by $|\sin \omega t|$ to establish the reference for the inductor current. The second-harmonic distortion in Eq. 6A-1 results in a third-harmonic distortion in the input ac current. This can be proven by multiplying the second-harmonic component in Eq. 6A-1 with $\sin \omega t$, in order to see the distortion in the input ac current, as follows:

$$(-\hat{I}_{L,2} \cos 2\omega t) \sin \omega t = \frac{1}{2} \hat{I}_{L,2} \sin \omega t - \underbrace{\frac{1}{2} \hat{I}_{L,2}}_{\hat{I}_{s,3}} \sin 3\omega t \qquad (6A\text{-}2)$$

In Eq. 6A-2, the fundamental-frequency component, due to the second-harmonic distortion, is compensated by the voltage-loop controller. However, the second-harmonic distortion with a peak $\hat{I}_{L,2}$ results in a third-harmonic distortion with one-half the amplitude. Therefore,

$$\frac{\hat{I}_{s,3}}{\hat{I}_{L,2}} = \frac{1}{2} \qquad (6A\text{-}3)$$

APPENDIX 6B Proving that $\frac{\tilde{v}_d(s)}{\hat{I}_{L\sim}(s)} = \frac{1}{2}\frac{\hat{V}_s}{V_d}\frac{R}{1+sRC}$

In designing the controller, the output of $G_v(s)$ in Fig. 6-6 under dynamic conditions has a strong dc component plus a low-frequency (less than 15 Hz) perturbation term:

$$\text{Output of the voltage regulator } = \hat{I}_L + i_{L,\sim}(t) \tag{6B-1}$$

The perturbation term can be expressed as

$$\tilde{i}_{L,\sim}(t) = \hat{i}_{L,\sim}\sin(\omega_\sim t + \phi) \tag{6B-2}$$

where, $\omega_\sim$ is the perturbation frequency below 15 Hz (assumed upper limit on the bandwidth of the voltage loop). It has an arbitrary phase angle ϕ. Substituting Eq. 6B-2 into Eq. 6B-1 and multiplying with $|\sin\omega t|$, the inductor current is

$$\overline{i_L}(t) = [\hat{I}_L + \hat{i}_{L,\sim}\sin(\omega_\sim t + \phi)]\,|\sin\omega t| \tag{6B-3}$$

In the circuit of Fig. 6-2, assuming that the voltage drop across L_d is negligible,

$$|v_s(t)|\,\overline{i_L}(t) = V_d\,\overline{i_d} \tag{6B-4}$$

Substituting into Eq. 6B-4 $|v_s(t)| = \hat{V}_s|\sin\omega t|$ and the inductor current from Eq. 6B-3,

$$\hat{V}_s|\sin\omega t|[\hat{I}_L + \hat{i}_{L,\sim}\sin(\omega_\sim t + \phi)]\,|\sin\omega t| = V_d\,\overline{i_d}(t) \tag{6B-5}$$

Noting that $|\sin\omega t|^2 = \sin^2\omega t$,

$$\overline{i_d}(t) = \frac{\hat{V}_s\hat{I}_L}{V_d}\sin^2\omega t + \frac{\hat{V}_s}{V_d}\underbrace{\hat{i}_{L,\sim}\sin(\omega_\sim t + \phi)}_{=i_{L,\sim}(t)}\sin^2\omega t \tag{6B-6}$$

Substituting $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$ in Eq. 6B-6,

$$\overline{i_d}(t) = \frac{\hat{V}_s\hat{I}_L}{2V_d}(1-\cos 2\omega t) + \underbrace{\frac{\hat{V}_s}{2V_d}i_{L,\sim}(t)}_{only\ pert.\ freq.\ term} - \frac{\hat{V}_s}{2V_d}i_{L,\sim}(t)\cos 2\omega t \tag{6B-7}$$

Equating the perturbation-frequency term on the right-side of Eq. 6B-7 to the low-frequency perturbation in the output current,

$$\frac{\tilde{\bar{i}}_d(s)}{i_{L,\sim}(s)} = \frac{1}{2}\frac{\hat{V}_s}{V_d} \tag{6B-8}$$

The transfer function of the power stage in Fig. 6-6b at these low perturbation frequencies (ignoring the capacitor ESR) is:

$$\frac{\tilde{v}_d(s)}{\hat{I}_{L\sim}(s)} = \frac{\tilde{\bar{i}}_d(s)}{i_{L,\sim}(s)}\frac{\tilde{v}_d(s)}{\tilde{\bar{i}}_d(s)} = \frac{1}{2}\frac{\hat{V}_s}{V_d}\frac{R}{1+sRC} \tag{6B-9}$$