Solutions UE4002 Autumn 2007

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an NMOS in saturation:

$$I_D = \frac{K_n^{'}}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n = \lambda_p = 0$.
In each question, capacitances other than those mentioned may be ignored.

Question 1

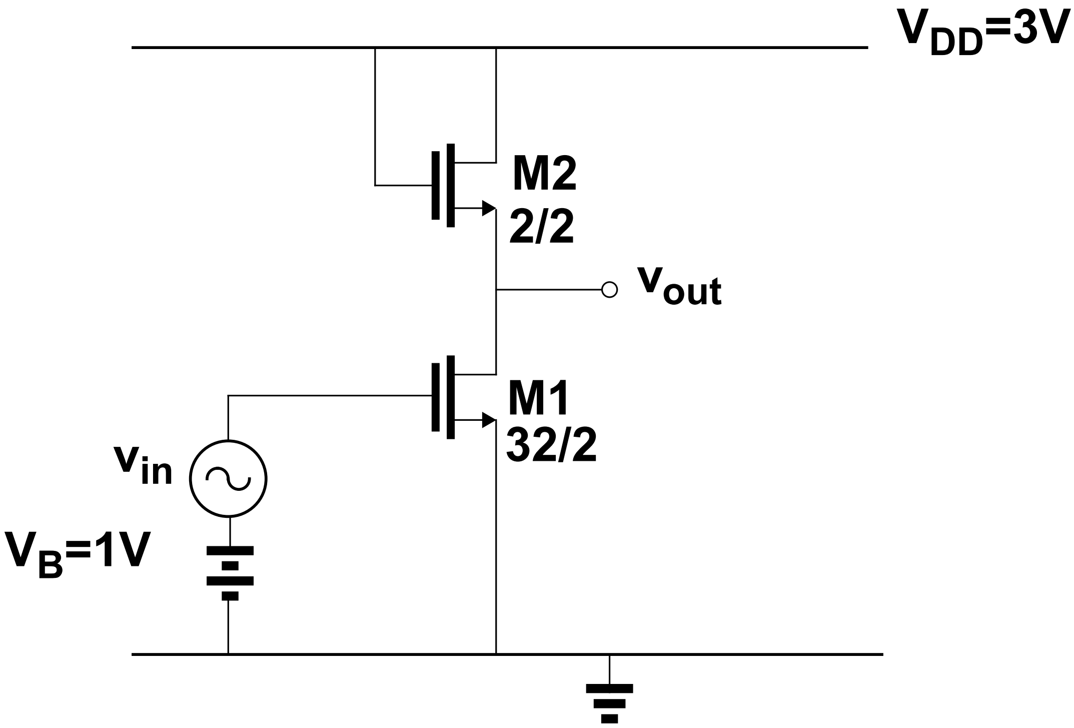

Figure 1

Figure 1 shows a common-source stage with an NMOS diode load.
Biasing and transistor dimensions are as shown in Figure 1.
Take $K_n^{'} = 200\mu A/V^2$, $V_{tn} = 0.75V$.

(i) Draw the small-signal equivalent circuit for the circuit shown in Figure 1.
(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal transistor parameters of M1 and M2.
(iii) Calculate the small-signal voltage gain in dB assuming M1 is in saturation.
Assume $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.
(iv) The gain of the circuit is increased by changing only the W/L ratio of M1.
What is the maximum value of W/L such that M1 is still in saturation?
What is the small-signal voltage gain in dB with this value of W/L?

(i) Draw the small-signal equivalent circuit for the circuit shown in Figure 1.

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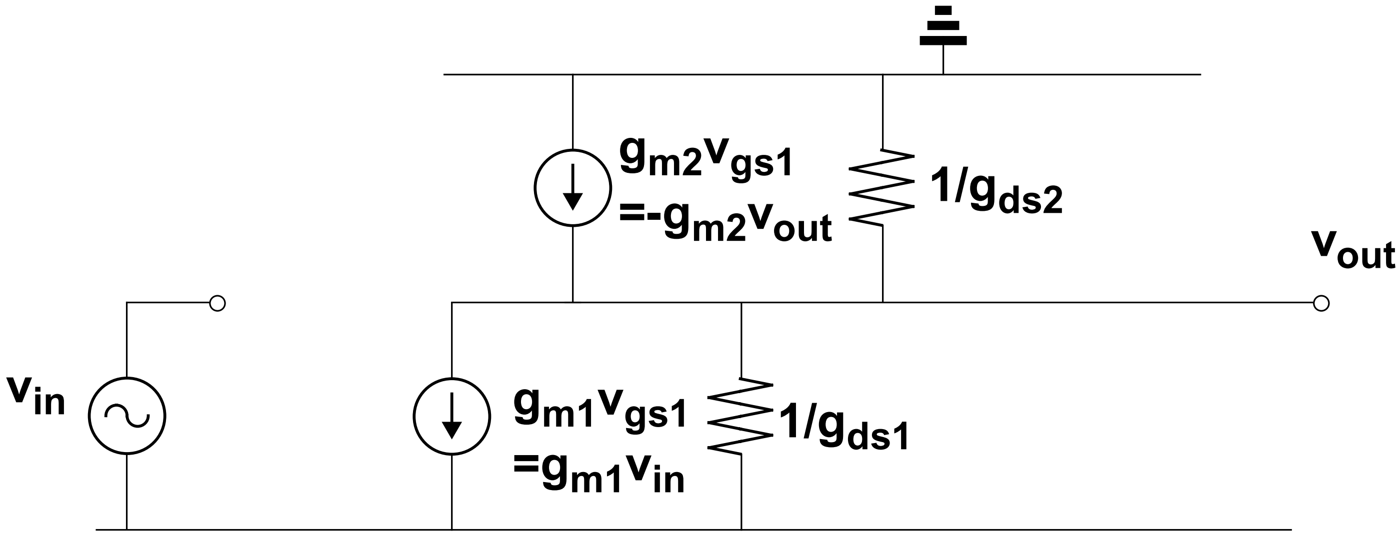

(ii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal transistor parameters of M1 and M2.

$$g_{m1}v_{in} + v_{out}g_{ds1} + g_{m2}v_{out} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + g_{ds1} + g_{ds2}} \approx -\frac{g_{m1}}{g_{m2}}$$

Alternatively recognise that M2 is equivalent to a small-signal resistance $1/g_{m2}$ and write result directly

(iii) Calculate the small-signal voltage gain in dB assuming M1 is in saturation. Assume $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$

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$$g_{m1} = \sqrt{2K_n' \frac{W_1}{L_1} I_{D1}}$$

$$g_{m2} = \sqrt{2K_n' \frac{W_2}{L_2} I_{D2}}$$

$I_{D1} = I_{D2} \Rightarrow$ $$\frac{v_{out}}{v_{in}} \approx -\frac{g_{m1}}{g_{m2}} = -\frac{\frac{W_1}{L_1}}{\frac{W_2}{L_2}} = -\frac{16}{1}$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 24dB$$

(iv) The gain of the circuit is increased by changing the W/L ratio of M1. What is the maximum value of W/L such that M1 is still in saturation.? What is the small-signal voltage gain in dB with this value of W/L.

For M1: $$V_{GS1} - V_{tn} = 1V - 0.75V = 0.25V$$

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS1} - V_{tn})^2 = \frac{200\mu A/V^2}{2} \cdot \frac{16}{1} \cdot (1-0.75)^2 = 100\mu A$$

For M2: $$V_{GS2} - V_{tn} = \sqrt{\frac{2I_{D1}}{K_n'\frac{W}{L}}} = \sqrt{\frac{2 \cdot 100\mu A}{200\mu A/V^2 \frac{1}{1}}} = 1V$$

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS1} - V_{tn})^2$$

If W/L only increases, then I_{D1} increases in the same proportion

This increases the voltage drop across M2 (i.e. V_{GS2})

M1 remains in saturation as longs as V_{DS1}>= 0.25V, i.e as longs as

$$V_{DD} - V_{GS2} \geq 0.25V$$

$$V_{DD} - (V_{GS2} - V_{tn} + V_{tn}) > 0.25V$$

$$V_{GS2} - V_{tn} \leq V_{DD} - V_{tn} - 0.25V$$

$$V_{GS2} - V_{tn} \leq 2V$$

Original V_{GS2}-V_{tn}=1V

i.e. M1 remains in saturation until V_{GS2}-V_{tn} doubles ie until I_{D2} (=I_{D1}) quadruples

ie until W1/L1 quadruples

W1/L1=64/1

Gain then given by

$$\frac{v_{out}}{v_{in}} \approx -\frac{g_{m1}}{g_{m2}} = -\frac{\frac{W_1}{L_1}}{\frac{W_2}{L_2}} = -\frac{64}{1}$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 36dB$$

Question 2

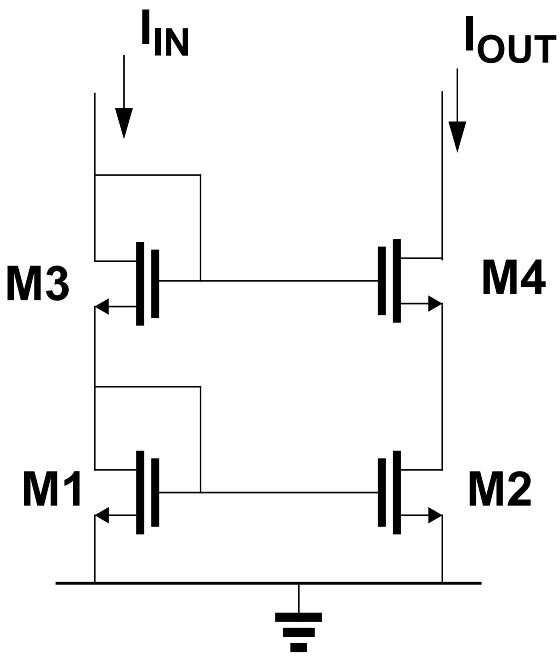


Figure 2

Figure 2 shows a cascoded current mirror.
Assume $I_{IN}=I_{OUT}=100\mu A$, $K_n'=200\mu A/V^2$, $V_{tn}=750mV$, $\lambda_n = 0.04V^{-1}$.
All transistors have W/L=16/1.

(i) It is required to measure the small-signal output resistance of the current mirror (i.e. the small-signal resistance looking into the drain of M4). Draw a small-signal model showing how this can be done.

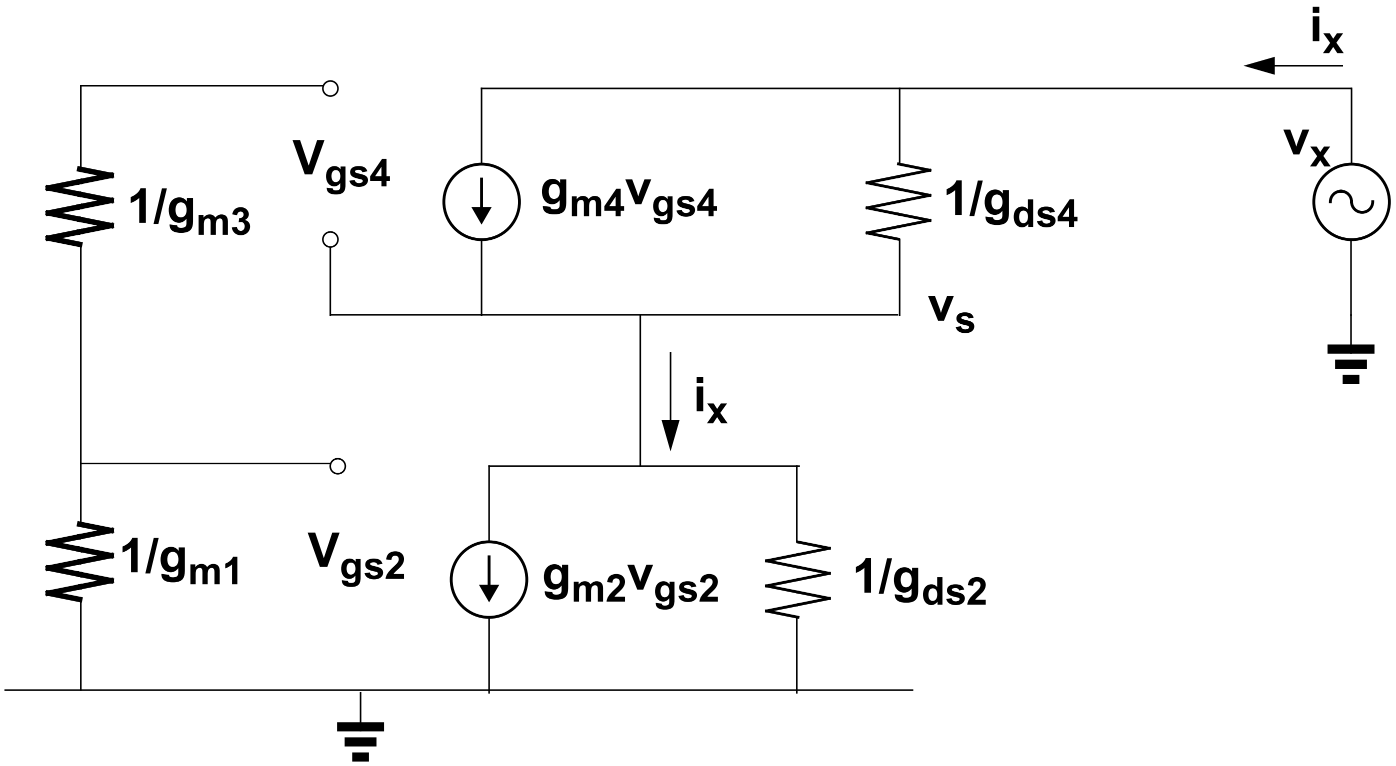
(ii) Derive an expression for the small-signal output resistance.
Show by assuming $g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}$ that this approximates to

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

(iii) What is the change in current if the voltage at the output node varies by 10mV?
Assume all transistors are in saturation.

(iv) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?

(i) It is required to measure the small-signal output resistance of the current mirror (i.e. the small-signal resistance looking into the drain of M4). Draw a small signal model showing how this can be done



(ii) Derive an expression for the small-signal output resistance.
Assume $g_{m1}, g_{m2}, g_{m3}, g_{m4} >> g_{ds1}, g_{ds2}, g_{ds3}, g_{ds4}$.

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Note: $v_{gs2} = 0 \Rightarrow g_{m2} v_{gs2} = 0$

$$i_x = g_{m4} v_{gs4} + (v_x - v_s) g_{ds4}$$

Since $v_{gs4} = -v_s$ and $v_s = \dfrac{i_x}{g_{ds2}}$

$$i_x = -(g_{m4}) \frac{i_x}{g_{ds2}} + \left(v_x - \frac{i_x}{g_{ds2}} \right) g_{ds4}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1 + \dfrac{g_{m4}}{g_{ds2}} + \dfrac{g_{ds4}}{g_{ds2}}}{g_{ds4}}$$

Since $g_{m2}, g_{m4} >> g_{ds2}, g_{ds4}$ this can be reduced to

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

(iii) What is the change in current if the voltage at the output node varies by 10mV? Assume all transistors are in saturation.

$$i_{out} = \frac{v_{out}}{r_{out}}$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}}$$

$$g_{m4} = \frac{2I_D}{(V_{GS1} - V_{tn})} = \frac{2 \times 100\mu A}{0.25} = 800\mu A/V$$

$$g_{ds2} = \lambda I_{D2} = 0.04V^{-1} \times 100\mu A = 4\mu A/V$$

$$r_{out} = \frac{g_{m4}}{g_{ds4}} \cdot \frac{1}{g_{ds2}} = \frac{800\mu A/V}{4\mu A/V} \frac{1}{4\mu A/V} = 50M\Omega$$

$$i_{out} = \frac{v_{out}}{r_{out}} = \frac{10mV}{50M\Omega} = \underline{\underline{0.2nA}}$$

(iv) What is the minimum voltage at the output node, i.e. the drain of M4, such that all transistors are biased in saturation?

The minimum voltage at the output is given by the voltage at the drain of M2 plus the required V_{DS} across M4 for it to be in saturation i.e. V_{GS4}-V_t

For all transistors

$$|V_{GS} - V_t| = \sqrt{\frac{2I_{D1}}{K'_n \frac{W}{L}}} = \sqrt{\frac{2 \cdot 100\mu A}{200\mu A/V^2 \frac{16}{1}}} = 250mV \Rightarrow V_{GS} = 1V$$

$$V_{OUTmin} = V_{D1} + |V_{GS4} - V_t|$$

$$V_{OUTmin} = V_{GS1} + V_{GS3} - V_{GS2} + |V_{GS4} - V_t| = \underline{\underline{1.25V}}$$

Question 3

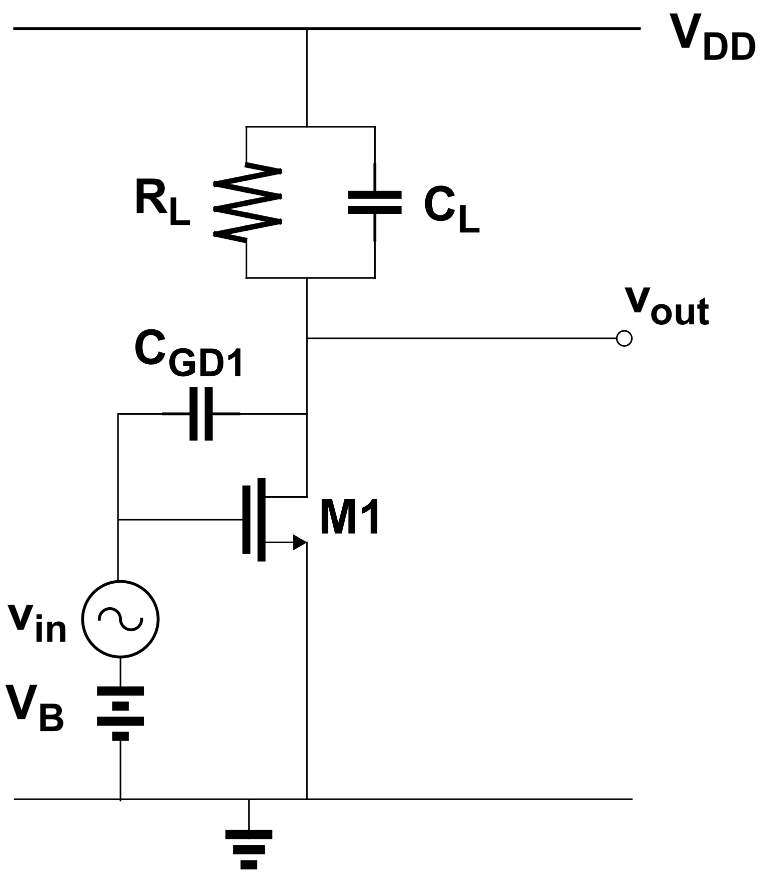
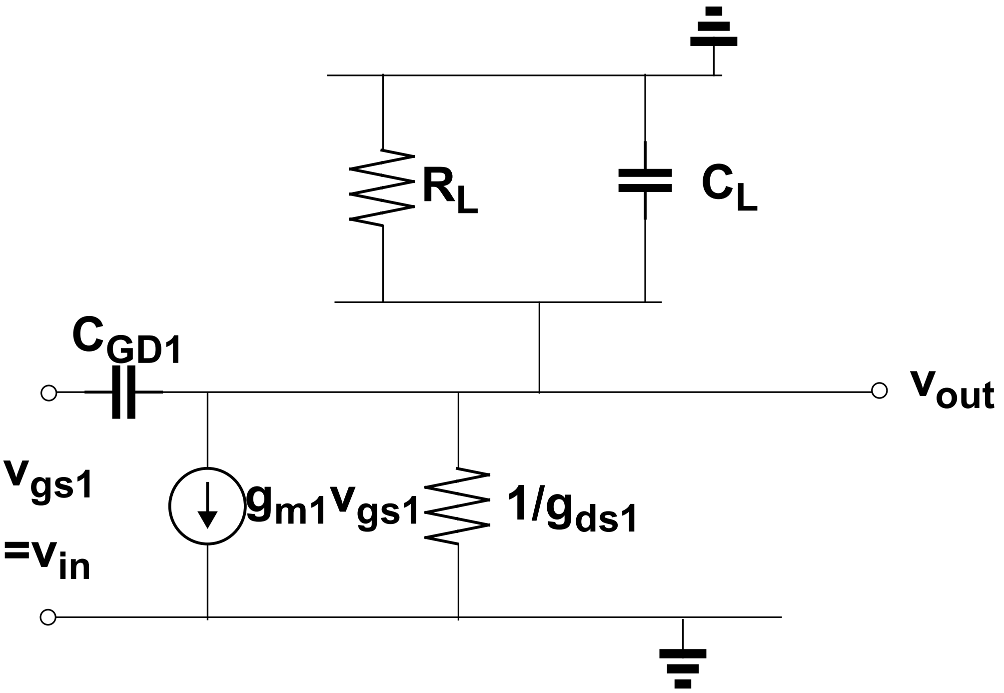


Figure 3

For the questions below you may assume $g_{ds1} << 1/R_L$ and that M1 is biased in saturation.

(i) Figure 3 shows a gain stage with an RC load. Draw the small-signal model for this circuit.
(ii) Ignoring all capacitances except C_{GD1} and C_L, derive an expression for the high-frequency transfer function.
(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the break frequencies (i.e. pole and/or zero frequencies) if V_B=1V, V_{tn}=0.75V, I_{D1}=250μA, C_{GD1}=0.1pF, C_L=4.9pF, R_L=10kΩ.
(iv) Draw a Bode diagram of the gain response. Indicate the values of gain at d.c. and at frequencies well above the pole and/or zero frequencies.

(i) Figure 2 shows a gain stage with an RC load. Draw the small-signal model for this circuit.



(ii) Ignoring all capacitances except C_{GD1} and C_L, derive an expression for the high-frequency transfer function.

KCL at output node:

$$(v_{out} - v_{in})sC_{GD1} + g_m v_{in} + v_{out} g_{ds} + v_{out}/R_L + v_{out} sC_L = 0$$

$$v_{in}(g_m - sC_{GD1}) + v_{out}(g_{ds} + 1/R_L + s(C_{GD1} + C_L)) = 0$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_m - sC_{GD1}}{g_{ds} + 1/R_L + s(C_{GD1} + C_L)}$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_m}{g_{ds} + 1/R_L}\left(\frac{1 - s\dfrac{C_{GD1}}{g_m}}{1 + \dfrac{s(C_{GD1} + C_L)}{g_{ds} + 1/R_L}}\right)$$

(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the pole and zero frequencies if V_B=1V,V_{GS2}=1.75V, V_{tn}=$|V_{tp}|$=0.75V, I_{D1}=250μA, C_{GD1}=0.1pF, R_L=4.9pF.

$$g_{m1} = \frac{2I_{D1}}{(V_{GS1}-V_{tn})} = \frac{2\times 250\mu A}{1-0.75} = 2000\mu A/V$$

Low-frequency gain given by

$$\frac{v_{out}}{v_{in}} \cong -\frac{g_{m1}}{g_{ds1}+1/R_L} \approx -g_{m1}R_L = -2000\mu A/V\times 10k = -20 \Rightarrow 26dB$$

Zero frequency given by

$$|\omega_z| = \frac{g_{m1}}{C_{GD1}} = \frac{2000\mu A/V}{0.1pF} = 20Grad/s$$

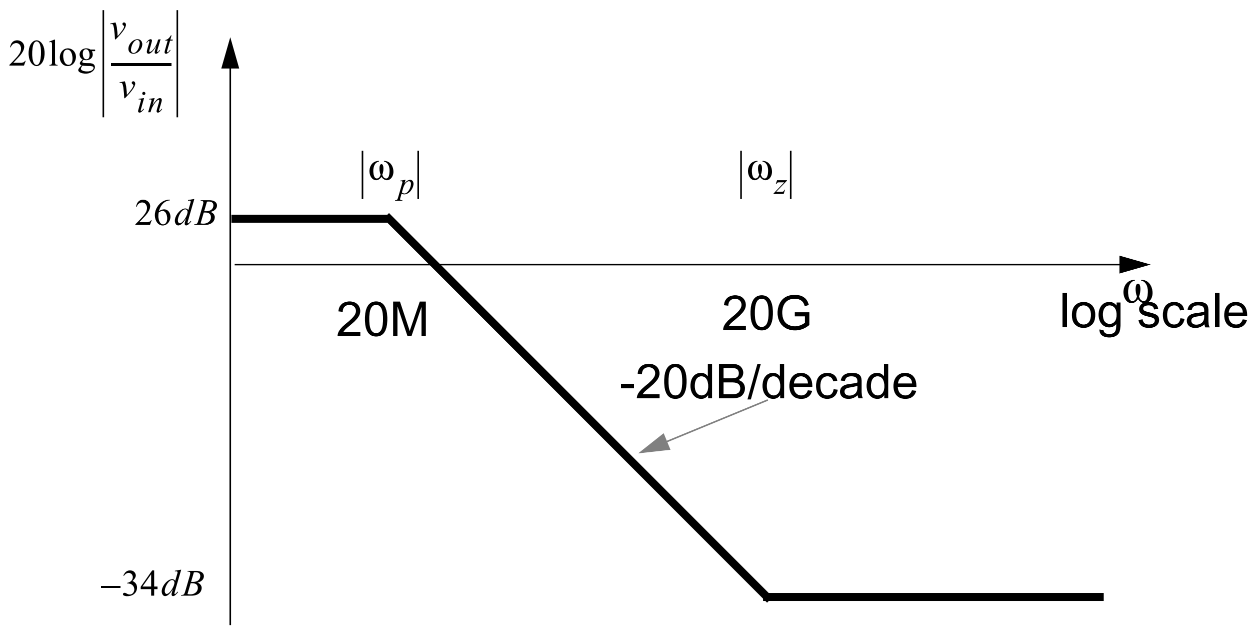
Pole frequency given by

$$|\omega_p| = \frac{g_{ds}+\frac{1}{R_L}}{C_L+C_{GD1}} \approx \frac{\frac{1}{R_L}}{C_L+C_{GD1}}$$

$$|\omega_p| = \frac{\frac{1}{10k\Omega}}{4.9pF+0.1pF} \approx \frac{1}{10k\Omega\times 2pF} = 20Mrad/s$$

(iv) Draw a Bode diagram of the gain response. Indicate the values of gain at d.c. and at frequencies well above the pole and zero frequencies.

Zero is 3 decades down, so gain at high freqecies = -34dB



Question 4

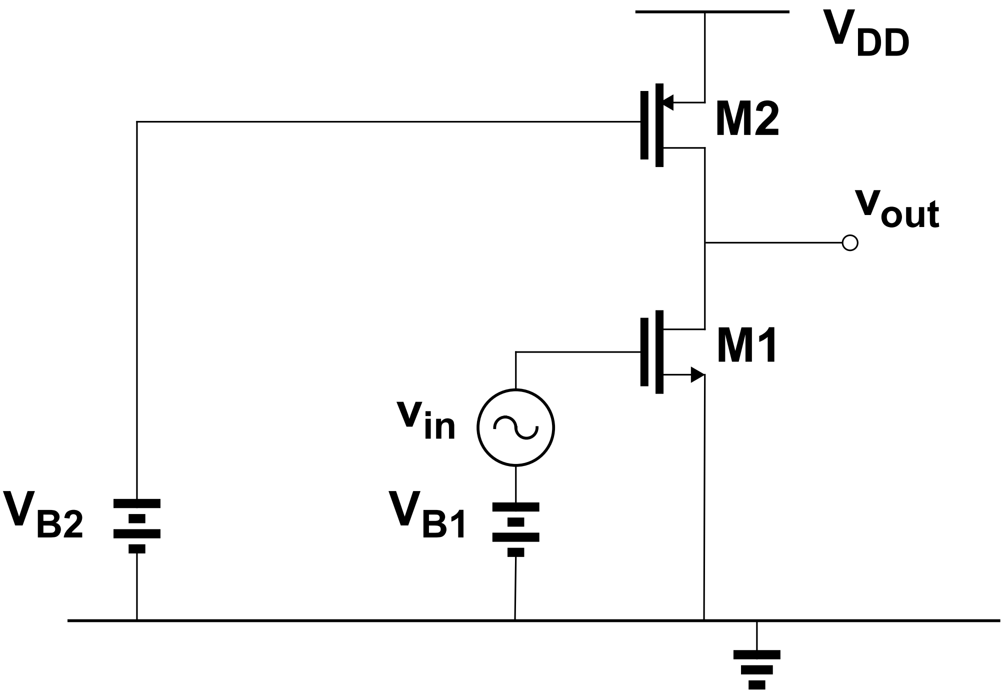


Figure 4

Assume M1 and M2 are operating in saturation. Only thermal noise sources need be considered.

For calculations take Boltzmann's constant $k=1.38X10^{-23}J/^oK$, temperature $T=300^oK$.

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters of M1 and M2?

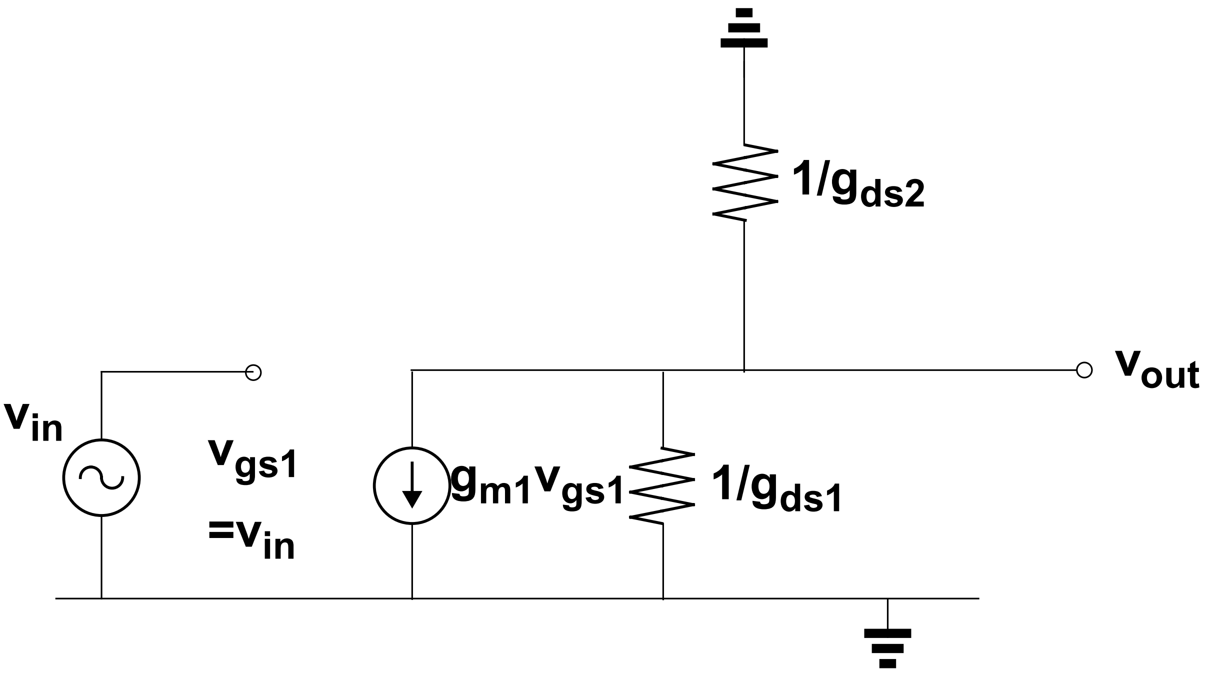
(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4?
The answer should be in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T.

(iii) Calculate the input-referred thermal noise voltage density of the circuit if
V_{B1}=1.0V, V_{B2}=1.25V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, I_{D1}=200µA, $\lambda_n=\lambda_p=0.04V^{-1}$.

(iv) Calculate the total noise voltage at the output over a bandwidth of 1MHz.
If the input signal v_{in} is a 1mV_{rms} sine wave in this bandwidth, calculate the signal-to-noise ratio in dB at the output over a bandwidth of 1MHz.

Solution

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters of M1 and M2?

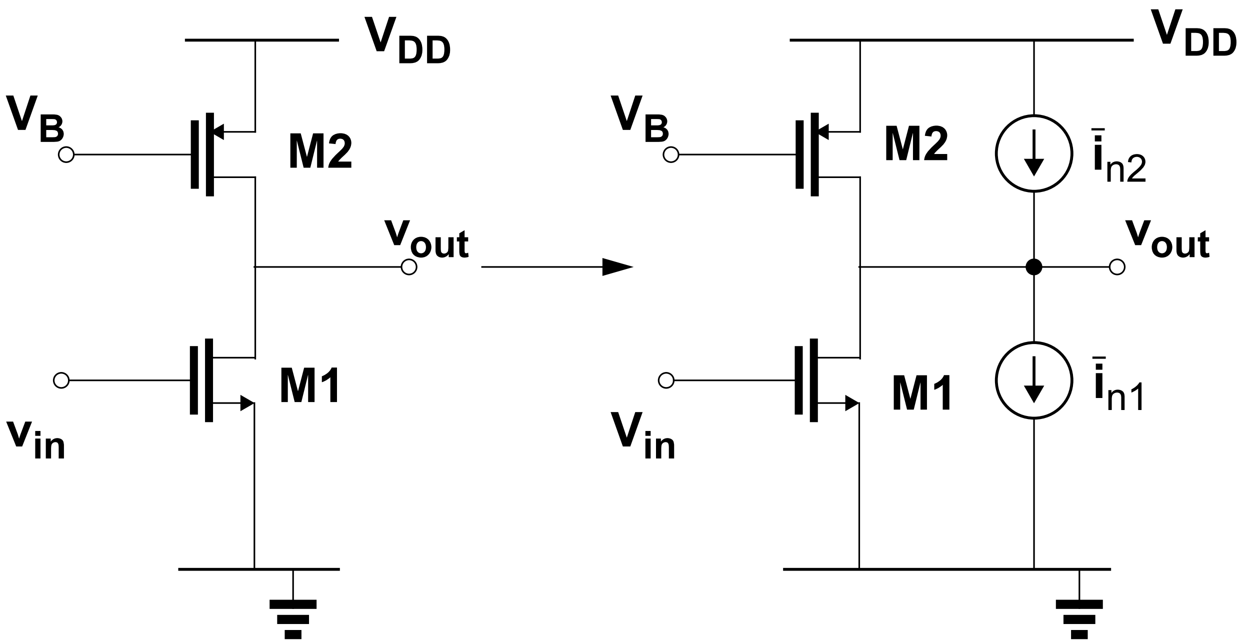


Current at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4?
Answers should be in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T.



Noise current of MOS: $$\overline{i_n^2} = 4kT\left(\frac{2}{3}g_m\right)$$

Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_{m1}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) Calculate the input-referred thermal noise voltage density of the circuit if
V_{B1}=1.0V, V_{B2}=1.25V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, I_{D1}=200µA, $\lambda_n=\lambda_p=0.04V^{-1}$.

g_m given by

$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$

$$g_{m1} = \frac{2 \cdot 200\mu A}{1V - 0.75V} = 1600\mu A/V \qquad g_{m2} = \frac{2 \cdot 200\mu A}{1.75V - 0.75V} = 400\mu A/V$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\sqrt{4kT\left(\frac{2}{3}(g_{m1} + g_{m2})\right)}}{g_{m1}}$$

$$\overline{v_{nitot}} = \frac{\sqrt{(4 \cdot 1.38\times10^{-23} \cdot 300)\left(\frac{2}{3}\right)(1600\mu A/V + 400\mu A/V)}}{1600\mu A/V} = 2.94nV/\sqrt{Hz}$$

(iv) Calculate the total noise voltage at the output over a bandwidth of 1MHz.
If the input signal v_{in} is a $1mV_{rms}$ sine wave in this bandwidth, calculate the signal-to-noise ratio in dB at the output over a bandwidth of 1MHz.

$$g_{ds1} = \lambda_n I_D = 0.04V^{-1} 200\mu A = 8\mu A/V$$

$$g_{ds2} = \lambda_n I_D = 0.04V^{-1} 200\mu A = 8\mu A/V$$

Gain of stage

$$Gain = -\left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) = -\frac{1600\mu A/V}{16\mu A/V} = -100$$

Total noise at output given by

$$\overline{v_{notot}} = \overline{v_{nitot}} \cdot \left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) \cdot \sqrt{BW} = 2.94nV/\sqrt{Hz} \cdot 100 \cdot \sqrt{1MHz} = \underline{\underline{294\mu V_{rms}}}$$

Output signal

$$v_{out} = -\left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) v_{in} = -\frac{1600\mu A/V}{16\mu A/V} \cdot 1mV_{rms} = 100mV_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{100mV}{294\mu V_{rms}} = 341$$

or $\underline{\underline{\text{50.6 dB}}}$