IIR Tutorial Answers for EE4008

4. (a)

$$
\begin{aligned}
|H_{BS}(\omega)|^2 &= \left(\frac{K(1-2\beta e^{-j\omega}+e^{-2j\omega})}{1-\beta(1+\alpha)e^{-j\omega}+\alpha e^{-2j\omega}}\right)\left(\frac{K(1-2\beta e^{j\omega}+e^{2j\omega})}{1-\beta(1+\alpha)e^{j\omega}+\alpha e^{2j\omega}}\right)\\
&= \frac{K^2(1-2\beta e^{-j\omega}+e^{-2j\omega}-2\beta e^{j\omega}+4\beta^2-2\beta e^{-j\omega}+e^{2j\omega}-2\beta e^{j\omega}+1)}{1-\beta(1+\alpha)e^{-j\omega}+\alpha e^{-2j\omega}-\beta(1+\alpha)e^{j\omega}+\beta^2(1+\alpha)^2-\alpha\beta(1+\alpha)e^{-j\omega}+\alpha e^{2j\omega}-\alpha\beta(1+\alpha)e^{j\omega}+\alpha^2}\\
&= \frac{K^2(2+4\beta^2-4\beta e^{-j\omega}-4\beta e^{j\omega}+e^{-2j\omega}+e^{2j\omega})}{1+\beta^2(1+\alpha)^2+\alpha^2-2\beta(1+\alpha)\cos\omega-2\alpha\beta(1+\alpha)\cos\omega+2\alpha\cos 2\omega}\\
&= \frac{2K^2(1+2\beta^2-4\beta\cos\omega+\cos 2\omega)}{1+\beta^2(1+\alpha)^2+\alpha^2-2\beta(1+\alpha)^2\cos\omega+2\alpha\cos 2\omega}\\
&= \frac{4(\beta^2-2\beta\cos\omega+\cos^2\omega)}{1+\beta^2(1+\alpha)^2+\alpha^2-2\beta(1+\alpha)^2\cos\omega+4\alpha\cos^2\omega-2\alpha}\\
&= \frac{4K^2(\beta-\cos\omega)^2}{(1+\alpha)^2(\beta^2-2\beta\cos\omega+\cos^2\omega)+1+\alpha^2+4\alpha\cos^2\omega-2\alpha-(1+\alpha)^2\cos^2\omega}\\
&= \frac{4K^2(\beta-\cos\omega)^2}{(1+\alpha)^2(\beta-\cos\omega)^2+1+\alpha^2-2\alpha-\cos^2\omega-\alpha^2\cos^2\omega+2\alpha\cos^2\omega}\\
&= \frac{4K^2(\beta-\cos\omega)^2}{(1+\alpha)^2(\beta-\cos\omega)^2+(1-\alpha)^2-\cos^2\omega(1-\alpha)^2}\\
&= \frac{4K^2(\beta-\cos\omega)^2}{(1+\alpha)^2(\beta-\cos\omega)^2+(1-\alpha)^2\sin^2\omega}
\end{aligned}
$$

Note $\cos^2\omega = \frac{1}{2}(1+\cos 2\omega)$

- $|H_{BS}(\omega)|^2$ Goes to zero at $\omega = \omega_0$ where $\beta = \cos\omega_0$
- $|H_{BS}(\omega)|^2$ has a maximum value of $\frac{4K^2}{(1+\alpha)^2}$ at $\omega = 0$ or at $\omega = \pi$.
- Squared Magnitude response at $\omega = 0, \pi$ is 1.

$$
\begin{aligned}
|H_{BS}(0)|^2 = |H_{BS}(\pi)|^2 &= \frac{4K^2}{(1+\alpha)^2} = 1\\
K &= \frac{1+\alpha}{2}
\end{aligned}
$$