OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2006

B.E. DEGREE (ELECTRICAL)

CONTROL ENGINEERING
EE4002

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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of a Casio fx570w or fx570ms calculator is permitted.

1.

(a) Derive Tustins's transformation.

[5 marks]

(b) A closed-loop speed control scheme for a DC motor is shown below.

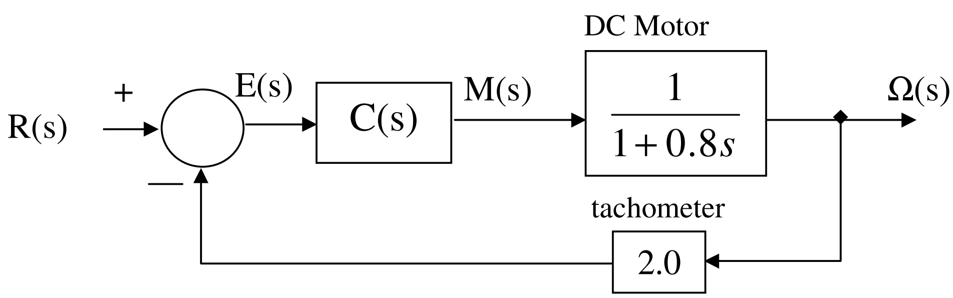

Fig. 1.1: Closed-loop motor speed control

The following PI controller is proposed:

$$m(t) = K_p\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau\right).$$

This has been designed in the continuous domain to achieve a closed loop damping $\xi = 0.6$ and a natural frequency ω_n =1.7rad/s. This specification is met when, $K_p = 0.81$ *and* $T_I = 2.5$ *s.*

It was decided to implement this PI control-law on a micro-controller, with sample time T. Tustin's approximation was used to convert the continuous algorithm designed above to a discrete-time PI control algorithm.

i) Show that the transfer function of the equivalent digital controller is,

$$D(z) = K_d \frac{z-\gamma}{z-1}.$$

Where the digital controller parameters are related to the continuous controller parameters as follows:

$$K_d = \left(1 + \frac{T}{2T_I}\right)K_p \quad \text{and} \quad \gamma = \frac{1 - \frac{T}{2T_I}}{1 + \frac{T}{2T_I}}.$$

ii) The sample time is T=1 second, and a zero-order hold is used. Sketch the root locus diagram for the system under ***digital*** PI speed control. Show that the chosen controller parameters T_I and K_p, will not achieve the design specifications, when directly used within the digital speed controller.

iii) Use root locus design to redesign the ***digital*** PI controller in the Z domain, to achieve a closed loop damping $\xi = 0.6$ and a natural frequency ω_n =1.7rad/s.

Compare your new controller parameters with the T_I and K_p designed in the continuous domain.

[20 marks]

2.

(a) Consider the following closed-loop discrete-time system.

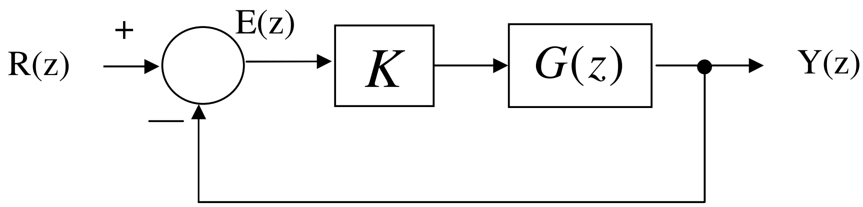

Fig. 2.1 Closed-loop digital control system

The following unit step response has been obtained for the open-loop process G(z).

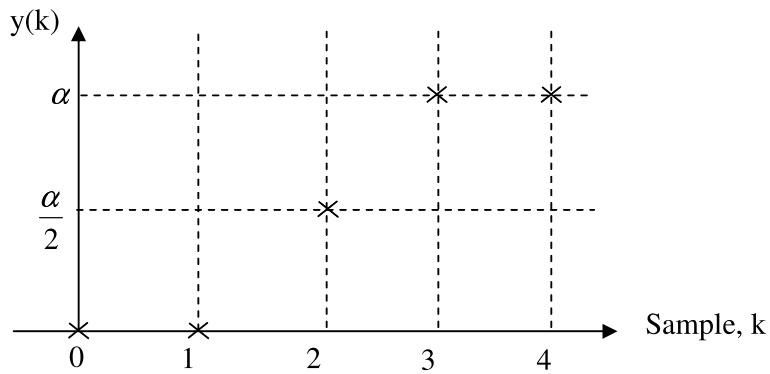

Fig. 2.2 Discrete unit step response for the open-loop process G(z)

Show how the steady-state error depends on the controller gain *K* if, $\lim_{k \to \infty} r(k) = r_\infty$.

The controller gain is now set as *K*=1. Use the difference equation method to sketch the unit step response *y(k)* for the closed loop process.

[8 marks]

(b) Consider in Fig. 2.3 the block diagram for a sample and hold. Briefly explain (without proof) the effect of varying the sampling frequency on the spectrum of the reconstructed signal u(t).

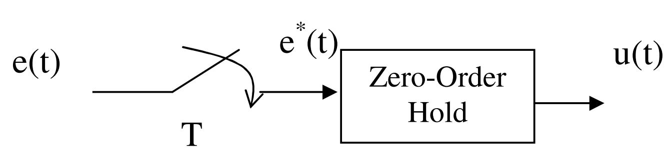

Fig. 2.3 sample and hold

Give Shannon's sampling theorem and comment on the benefits of over-sampling, in particular focussing on control applications.

[8 Marks]

(c) Consider the following general first-order system, within a closed-loop digital control scheme. The sampling time is T and a zero-order hold is assumed.

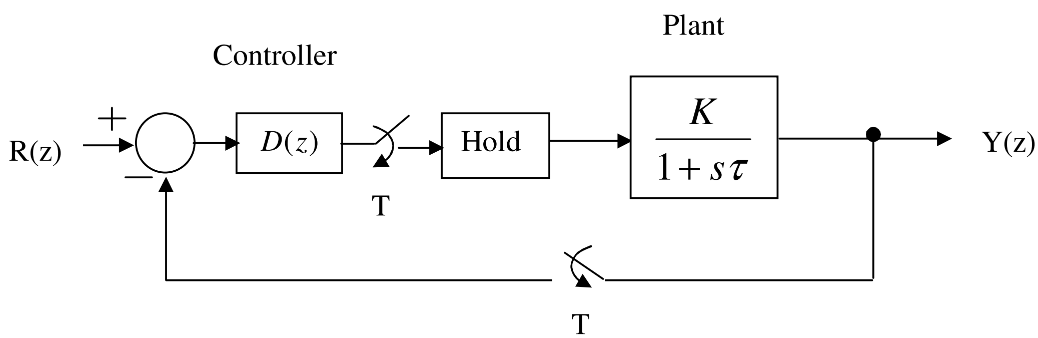

Fig. 2.4 Digital closed-loop control of a first order plant

Derive the following Dahlin's controller for the general first order process, from a basic prescription of the shape of the desired closed-loop step response.

$$D(z) = K_d \frac{z-\gamma}{z-1}.$$

What other popular controller is this identical to?

[9 marks]

3.

(a)

A certain process is known to have an open-loop transfer function of the following structure:

$$G(z) = \frac{\gamma z^{-2}}{1 + \alpha z^{-1} + \beta z^{-2}}.$$

Give the design equations for a Diophantine pole-placement adaptive controller based on estimates of the parameters of this model, provided by a recursive least-squares algorithm. Define the controller polynomials and the desired characteristic equation for this process.

Clearly show the development of the Sylvester matrix used to solve the Diophantine pole-placement design equation.

[10 marks]

(b)

Derive in full, the following least-squares algorithm, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains the sampled process output, up to the current kth sample, *y(k).*

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1} \Phi(k)^T \underline{Y}(k)$$

If a square matrix *P(k)* is now defined as $P(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1}$, derive the following update equation to obtain *P(k+1)* from process data up to the (k+1)th sample,

$$P(k+1) = \left(P(k)^{-1} + \underline{\psi}(k+1)\underline{\psi}(k+1)^T\right)^{-1},$$

where vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the (k+1)th sample.

Show by application of Householder's Matrix Inversion Lemma,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1},$$

that the following update equation for the model parameter vector can be obtained:

$$\underline{\hat{\theta}}(k+1) = \left[P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^T(k+1)P(k)}{1 + \underline{\psi}^T(k+1)P(k)\underline{\psi}(k+1)}\right]\left[\Phi(k)^T \underline{Y}(k) + \underline{\psi}(k+1)\, y(k+1)\right].$$

[15 marks]

4.

(a) A certain mechatronic system can be modeled by the following differential equation, where u(t) is the input voltage, and θ(t) is the resulting angle of rotation.

$$\frac{d^2\theta(t)}{dt^2} + 7\frac{d\theta(t)}{dt} + 12\theta(t) = \frac{du(t)}{dt} + u(t)$$

i) Show how this system could be represented as a simulation diagram (eg. Simulink diagram), using only two integrators, a variety of gains and summers.

[4 marks]

ii) Use this simulation diagram to derive the control-canonical state-space model of this process.

[4 marks]

iii) If the initial conditions are θ(0)=1 and $\frac{d\theta(0)}{dt} = 0$, determine an expression for the zero-input responses of the *states of your model.*

[7 marks]

(b) Consider the following second-order SISO process,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

i) Determine the transfer function of this process, G(s)=Y(s)/U(s).

[3 marks]

ii) Is this system controllable?

[3 marks]

iii) Determine the transformation $\underline{z}$=T$\underline{x}$, which would transform this system into the control-canonical form

[4 marks]

5.

(a) Consider the following Nth order open-loop process with a single input *u(t),* a single output *y(t)* and a single unmeasured disturbance *d(t)*,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + Bu(t) + Ed(t)$$
$$y(t) = C\underline{x}(t).$$

If there is no measurement of the disturbance, but it is known that, $\lim_{t\to\infty} d(t) = d_\infty$, show that the steady state estimation error vector, for a Luenberger observer is:

$$\underline{e}_{ss} = \lim_{t\to\infty}\left(\underline{x}(t) - \underline{\hat{x}}(t)\right) = -\left(A - GC\right)^{-1} Ed_\infty,$$

where G is the Luenberger observer gain matrix.

[6 marks]

(b) Consider the following ball-on-beam apparatus consisting of a rigid beam, free to rotate in one plane about its central pivot. A servo-motor is used to rotate the beam. There are two parallel guide rails, on which a steel ball sits.

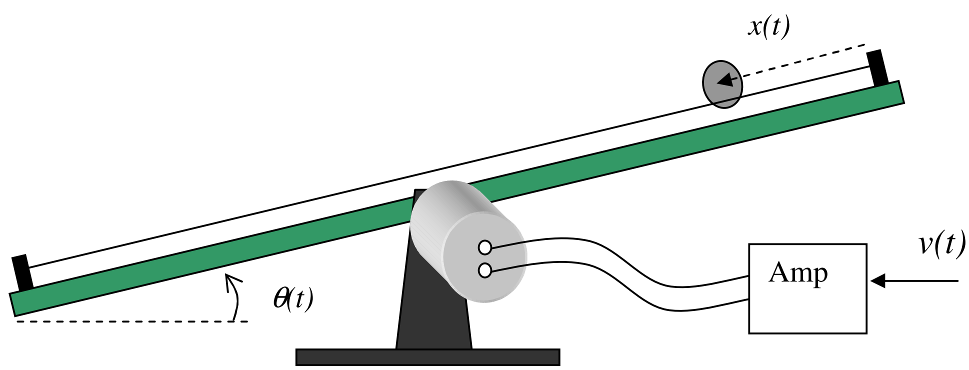

Fig.5.1: Ball-on-Beam Apparatus

Two sensors are available. The first is a simple rotary potentiometer that is used to provide a measure of the beam angle $\theta(t)$. The second sensor provides a measurement of the ball position *x(t),* using the wire guide rails as a linear potentiometer.

The servo-motor dynamics are so fast that the rotation of the beam can be described by the following first-order differential equation:

$$\frac{d\theta(t)}{dt} = Kv(t).$$

The gains of the linear and rotary potentiometers are K_x and K_θ. respectively

If the moment of inertia, about the axis of rotation, of the ball of mass m and radius r, is $J={}^2/_5\, mr^2$, basic rotational mechanics yields the following expression for the linear acceleration:

$$\frac{d^2x}{dt^2} = 7\theta(t).$$

(i) Assume first that all the states of this third order model are available and that the gain K=2Vrad^{-1}s. Design a state-space position controller, that will meet the following specifications.

- Zero steady-state error for a constant desired ball position
- Closed-loop poles are selected to ensure second-order dominance and a closed loop damping of ξ=0.7, and natural frequency ω_n=1 rad/s.

[10 marks]

(ii) If we note that there is a decoupling of the beam dynamics from the ball dynamics, it is possible to build a simplified second-order observer to estimate the ball velocity from just the potentiometer output voltages $v_x(t)$ and $v_\theta(t)$.

The potentiometer gains are K_x =5V/m and K_θ= 2V/radian.

Design a second-order Luenberger Observer to provide an estimate of the ball velocity for use in the controller designed in part ii) above.

[9 marks]

6.

(a) Consider the following second-order nonlinear system,

$$\frac{d}{dt}x_1(t) = x_1^2(t) + x_1(t)x_2(t) - x_1(t) - x_2(t)$$
$$\frac{d}{dt}x_2(t) = x_2^2(t) + x_1(t)x_2(t) + x_1(t) - x_2(t)$$

Using the following Lyapunov function,

$$V\left(x_1(t), x_2(t)\right) = x_1^2(t) + x_2^2(t)$$

show that the origin is locally asymptotically stable.

Give a rough sketch that shows the region of asymptotic stability on the state-space.

[12 Marks]

(b) Given the following state-space representation of a linear time-invariant system,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t),$$

show that the origin is globally asymptotically stable, if given a positive-definite, symmetric matrix, P, then another positive-definite, symmetric matrix, Q, can always be found, such that:

$$A^T P + PA = -Q.$$

[6 marks]

(c) A linear system is described by the following coupled differential equations, where $x_1(t)$ and $x_2(t)$ are the states of the system and $u(t)$ is the input,

$$\frac{d}{dt}x_1(t) = -x_1(t) + 2x_2(t)$$
$$\frac{d}{dt}x_2(t) = x_2(t) + u(t).$$

Show, by use of Lyapunov's technique, that the regulatory control-law,

$$u(t) = -k\,x_2(t),$$

can stabilise this system, to make the origin globally asymptotically stable. Within what range should the gain k be selected for stability to be assured?

[7 marks]