OLLSCOIL NA hÉIREANN, CORCAIGH

THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH

UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2011

B.E. DEGREE (ELECTRICAL & ELECTRONIC)

TELECOMMUNICATIONS

EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of log tables and a departmental approved non-programmable calculator is permitted.

Q.1. (a) Illustrate the data and acknowledgement flow between two computers that are communicating over a dedicated link and use the "go back N" ARQ scheme. From this, derive an expression for the utilization of the link in this scheme assuming that the link is error free.

[*8 marks*]

(b) A 50km fibre-optic link with a data rate of 200Mbps is being used to transport data packets with a length of 2000 bits and the system uses an acknowledgement packet with a length of 100 bits. Determine the minimum frame number, N, that should be used to achieve 100% utilization on the link assuming error-free conditions and assuming that the effective velocity of propagation of light in the fibre is 2×10^8 m/s.

[*4 marks*]

(c) Draw two protocol stacks that compare the functions of the OSI and TCP/IP (internet) systems for wide area networks. Clearly label each layer in the diagrams you draw, align them to allow a direct comparison of the layer functions and briefly describe the functions of the layers.

[*8 marks*]

Q.2. (a) Illustrate the logical connection of a set of computers interconnected using a passive bus architecture and describe the functional subdivision of the Data Link layer for this network.

[*4 marks*]

(b) List the sequence of steps followed by a computer that needs to initiate a data transfer if it is connected to a network which uses the CSMA/CD access protocol.

[*6 marks*]

(c) Describe the operation of the truncated binary exponential back-off algorithm for the CSMA/CD network access protocol.

[*6 marks*]

(d) Two computers are at the opposite ends of a passive bus network which uses the CSMA/CD protocol. If the CSMA/CD frames are 200 bytes long, the bus data rate is 1Gbit/s and the propagation velocity along the bus is 2.3×10^8 m/s, determine the maximum length of the bus to ensure proper operation of the CSMA/CD protocol.

[*4 marks*]

Q.3. (a) Illustrate the architecture of a UMTS Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) For mobile telephone networks briefly describe the following, noting any differences between 2nd and 3rd generation systems (2G and 3G) where they exist:

(i) Cell organization and frequency re-use. [4 *marks*]

(ii) The main power control algorithms. [4 *marks*]

(iii) The hand-off algorithms when a user moves between adjacent cells. [4 *marks*]

Q.4. Given that the 2×2 channel matrix $[P(Y|X)]$ for the binary symmetric channel with 2 input symbols, denoted x_i, $1 \le i \le 2$ and 2 output symbols, denoted y_j, $1 \le j \le 2$, is given by: -

$$[P(Y|X)] = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

and the 2×3 channel matrix $[P(Z|Y)]$ for the binary erasure channel with 2 input symbols, denoted y_i, $1 \le i \le 2$ and 3 output symbols, denoted z_j, $1 \le j \le 3$, is given by: -

$$[P(Z|Y)] = \begin{bmatrix} 1-p & p & 0 \\ 0 & p & 1-p \end{bmatrix}$$

where, in both cases, p denotes the probability of error: -

(a) Show that the channel matrix for the composite channel with inputs x_i, $1 \le i \le 2$ and outputs z_j, $1 \le j \le 3$ (i.e. the outputs from the binary symmetric channel become the inputs for the binary erasure channel), denoted $[P(Z|X)]$, is given by: -

$$[P(Z|X)] = \begin{bmatrix} (p-1)^2 & p & p(1-p) \\ p(1-p) & p & (p-1)^2 \end{bmatrix}.$$

[6 *marks*]

(b) Show that if the input symbols x_i, $1 \le i \le 2$ are equiprobable then the entropy, $H[Z]$, of the output from the composite channel is given by: -

$$H[Z] = (p-1)\log_2\left[\frac{1-p}{2}\right] - p\log_2[p].$$

[5 *marks*]

(c) Given that $H[Z]$ in part (b) above corresponds to the maximum possible value of the output entropy, show that the channel capacity C_s (in bits/symbol) of the composite channel is given by: -

$$C_s = (1-p)^2\log_2\left[(1-p)^2\right] + p(1-p)\log_2[p(1-p)] + (p-1)\log_2\left[\frac{1-p}{2}\right].$$

[5 *marks*]

(d) Using a graph, or otherwise, estimate the value of p resulting in a composite channel capacity C_s that is half of the composite channel capacity that would be achieved in an ideal error-free system, denoted C_s^{ideal}, i.e. for which $C_s = \dfrac{C_s^{ideal}}{2}$.

[4 *marks*]

Q.5 A baseband digital communications system uses the following signals to represent its three symbols, respectively denoted x_1, x_2 and x_3: -

$$s_i(t) = \begin{cases} -A & 0 \le t \le T & symbol\ x_1 \\ 0 & 0 \le t \le T & symbol\ x_2 \\ A & 0 \le t \le T & symbol\ x_3, \end{cases}$$

where T denotes the bit signalling interval. The communications are affected by additive white Gaussian noise (AWGN) whose probability density function (pdf) $f_n(v)$ is given by:

$$f_n(v) = \frac{e^{\frac{-v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}.$$

The receiver takes a single sample, z, of the received signal during the bit signalling time and makes the detection decision according to the rule: -

$$decision = \begin{cases} symbol\ x_1 & if\ \ z \le -\tau \\ symbol\ x_2 & if\ \ -\tau < z < \tau \\ symbol\ x_3 & if\ \ z \ge \tau \end{cases}$$

where the threshold τ satisfies $0 < \tau < A$.

(a) Show that, having sent symbol x_2, the probability of error, denoted P_{e,x_2}, is given by

$$P_{e,x_2} = 1 - erf\left[\frac{\tau}{\sigma\sqrt{2}}\right],$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

[8 *marks*]

(b) Given that the overall average probability of error for the communications system, P_e, is given by: -

$$P_e = \left(\frac{1 + P(x_2)}{2}\right) + erf\left[\frac{A - \tau}{\sigma\sqrt{2}}\right]\left(\frac{P(x_2) - 1}{2}\right) - erf\left[\frac{\tau}{\sigma\sqrt{2}}\right] P(x_2)$$

where $P(x_2)$ denotes the probability of sending symbol x_2, prove that the value of τ minimising P_e, denoted τ_{MIN}, is given by: -

$$\tau_{MIN} = \frac{A}{2} - \frac{\sigma^2}{A} \ln\left[\frac{1 - P(x_2)}{2P(x_2)}\right].$$

(Hint : It may be helpful to note that $\frac{d}{dx}\left[\int_0^x G[y]dy\right] = G[x]$).

[9 *marks*]

(c) Prove that, in the equiprobable case, the minimum attainable probability of error,

P_e^{MIN} , is given by: -

$$P_e^{MIN} = \frac{2}{3}\left(1 - erf\left[\frac{A}{2\sqrt{2}\sigma}\right]\right).$$ [3 *marks*]

Q.6. Typical expressions for amplitude shift keying (ASK) and phase shift keying (PSK) modulated waveforms representing binary data, where in each case T is an integer times $1/f_c$, are as follows: -

ASK

$$s_i(t) = \begin{cases} s_1(t) = A_1 \cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = 0 & 0 \le t \le T \end{cases}$$

PSK

$$s_i(t) = \begin{cases} s_1(t) = A_2 \cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = -A_2 \cos[\omega_c t] & 0 \le t \le T. \end{cases}$$

In addition, the probability of error for a binary modulation scheme (denoted MOD) with optimum detection in the presence of AWGN with a power spectral density of $\eta/2\ W/Hz$ is given by: -

$$P_e^{MOD} = Q\left[\sqrt{\frac{E_d}{2\eta}}\right],$$

where E_d denotes the energy difference in the appropriate signal (over a single bit interval).

(a) Derive expressions for P_e^{ASK} and P_e^{PSK}. [6 *marks*]

(b) If the average signal energy per bit, E_b, for the ASK and PSK modulation schemes above is made equal, derive the following expression for the enhancement in reliability, denoted E, achieved by choosing PSK over ASK when both schemes deliver the same bit rate: -

$$E = \frac{P_e^{ASK}}{P_e^{PSK}} = \frac{Q\left[\sqrt{\frac{A_2^2 T}{2\eta}}\right]}{Q\left[\sqrt{\frac{A_2^2 T}{\eta}}\right]}.$$

[6 *marks*]

(c) Using the table of values of $Q[z]$ provided to draw a suitable graph, or otherwise, estimate the amplitude A_2 resulting in $E = 45$ when $\eta/2 = 10^{-12}\ W/Hz$ and the bit rate is 1 Mb/s.

[8 *marks*]

Q.7 Given the following table of field elements of $GF\left(2^5\right)$: -

0	$\alpha^7 = \alpha^4 + \alpha^2$	$\alpha^{15} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$	$\alpha^{23} = \alpha^3 + \alpha^2 + \alpha + 1$
1	$\alpha^8 = \alpha^3 + \alpha^2 + 1$	$\alpha^{16} = \alpha^4 + \alpha^3 + \alpha + 1$	$\alpha^{24} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha$
α	$\alpha^9 = \alpha^4 + \alpha^3 + \alpha$	$\alpha^{17} = \alpha^4 + \alpha + 1$	$\alpha^{25} = \alpha^4 + \alpha^3 + 1$
α^2	$\alpha^{10} = \alpha^4 + 1$	$\alpha^{18} = \alpha + 1$	$\alpha^{26} = \alpha^4 + \alpha^2 + \alpha + 1$
α^3	$\alpha^{11} = \alpha^2 + \alpha + 1$	$\alpha^{19} = \alpha^2 + \alpha$	$\alpha^{27} = \alpha^3 + \alpha + 1$
α^4	$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha$	$\alpha^{20} = \alpha^3 + \alpha^2$	$\alpha^{28} = \alpha^4 + \alpha^2 + \alpha$
$\alpha^5 = \alpha^2 + 1$	$\alpha^{13} = \alpha^4 + \alpha^3 + \alpha^2$	$\alpha^{21} = \alpha^4 + \alpha^3$	$\alpha^{29} = \alpha^3 + 1$
$\alpha^6 = \alpha^3 + \alpha$	$\alpha^{14} = \alpha^4 + \alpha^3 + \alpha^2 + 1$	$\alpha^{22} = \alpha^4 + \alpha^2 + 1$	$\alpha^{30} = \alpha^4 + \alpha$

(a) Show that the generator polynomial for the $(31,21)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^3 + 1.$$

[10 *marks*]

(b) Deduce the codeword $c(x)$ representing the user data polynomial, $i(x)$, given by: -

$$i(x) = x^{15} + x^{11} + x + 1.$$

[2 *marks*]

(c) Due to the presence of errors, represented by the error polynomial, $e(x)$, where: -

$$e(x) = x^{20} + x^{11},$$

affecting the transmission of the codeword $c(x)$ in (b) above, the received polynomial, $v(x) = c(x) + e(x)$, does not equal $c(x)$.

(i) Show that, in general, the error location polynomial is given by: -

$$x^2 + S_1 x + \frac{S_1^3 + S_3}{S_1} = 0.$$

[4 *marks*]

(ii) Show how the syndrome decoding method determines the correct error polynomial, $e(x)$, in this case.

[4 *marks*]