Fourth Year Electrical Engineering

EE4010

Electrical and Electronic Power Supply Systems

Synchronous Machine Worked Examples

Example 1

A three-phase, round-rotor synchronous machine has a synchronous reactance of 1.25 per unit and it supplies full load as a generator at a power factor of 0.9 lagging. The machine is synchronised on a set of infinite busbars at rated voltage and losses can be neglected.

(a) Determine the rated excitation in per unit to sustain this condition.

(b) What excitation would be required to operate the machine at full load current but at a power factor of 0.707 lagging?

(c) With this maximum excitation, what would be the maximum possible power transfer from the machine to the busbars expressed as a percentage of the power delivered in (a) above.

(d) What per unit current would be delivered for Case (c) above.

Solution 1

The per-phase equivalent circuit of a three-phase round-rotor synchronous generator operating on a set of infinite busbars is shown in Figure 1 below. In this circuit, E_f is the per-unit excitation voltage, δ is the load angle, X_s is the per-phase synchronous reactance, I_a is the per-phase armature current and θ is the power factor angle measured with respect to the terminal voltage V_t, which is taken as the reference phase angle.

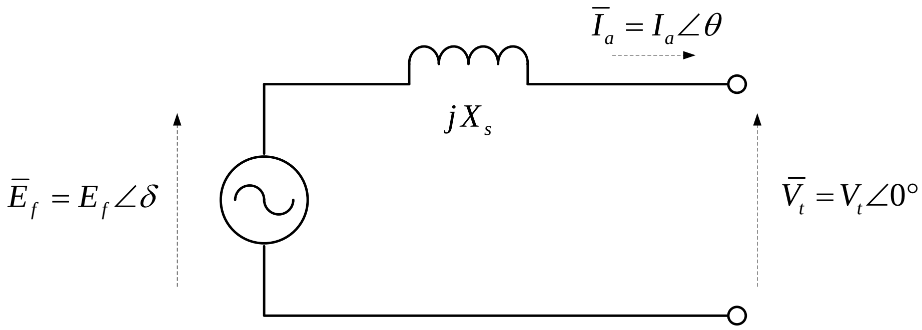

Figure 1

(a)

$$\overline{E}_{f1} = \overline{V}_t + jX_s \overline{I}_{a1}$$

$$\overline{V}_t = 1.0\angle 0° \text{ pu}$$
$$I_{a1} = 1.0 \text{ pu}$$
$$\theta_1 = Cos^{-1}(0.9) \text{ (lagging)}$$
$$\overline{E}_{f1} = 1.0 + j1.25 \times 1.0\left[Cos(\theta_1) - jSin(\theta_1)\right] \text{ pu}$$
$$\overline{E}_{f1} = 1.91 \text{ pu at a phase angle of } 36.1°$$

Note the negative sign in the current phasor due to the lagging power factor operation.

(b)

$$\overline{E}_{f2} = \overline{V}_t + jX_s\overline{I}_{a2}$$

$$I_{a2} = 1.0$$
$$\theta_2 = Cos^{-1}(0.707) \text{ (lagging)}$$
$$\overline{E}_{f2} = 1.0 + j1.25 \times 1.0\left[Cos(\theta_2) - jSin(\theta_2)\right]$$
$$\overline{E}_{f2} = 2.081 \text{ pu at a phase angle of } 25.13°$$

Again note the negative sign in the current phasor due to the lagging power factor operation. Also, note that the excitation voltage magnitude has increased in order to produce the additional reactive power demanded by the drop in power factor.

(c) For maximum power transfer, $\delta \to 90°$ and so

$$P_{\max 2} = \frac{E_{f2}V_t}{X_s}Sin(90°) = \frac{E_{f2}V_t}{X_s}$$

$$E_{f2} = 2.081 \text{ pu}$$
$$V_t = 1.0 \text{ pu}$$
$$X_s = 1.25 \text{ pu}$$
$$P_{\max 2} = 1.665 \text{ pu power}$$

The power delivered to the busbars in Case (a) is given by

$$P_a = \frac{E_{f1}V_t}{X_s}Sin(36.1°) = V_tI_{a1}Cos(\theta_1) = 0.9 \text{ pu}$$

as expected so that the maximum power transfer at this excitation relative to Case (a) is

$$\frac{P_{\max 2}}{P_a} = \frac{1.665}{0.9}100\% = 185\ \%$$

(d) The armature current at this maximum power transfer point is given by

$$\overline{I}_{a2\max} = \frac{\overline{E}_{f2\max} - \overline{V}_t}{jX_s}$$

so that

$$E_{f2\max} = 2.081 \angle 90°$$
$$V_t = 1.0 \text{ pu}$$
$$X_s = 1.25 \text{ pu}$$
$$\overline{I}_{a2\max} = 1.847 \text{ pu at a phase angle of } 25.66° \text{ lagging.}$$

Check:

$$P_{\max} = \frac{E_{f2}V_t}{X_s}Sin(90°) = V_tI_{a2\max}Cos(\theta_{2\max}) = 1.665 \text{ pu power.}$$

Example 2

Two three-phase, 50 Hz, 3.3 kV, round-rotor synchronous generators are connected in parallel to supply a total load of 800 kW at 0.8 power factor lagging. The prime movers to the generators are set so that one machine delivers twice as much power as the other. The more heavily loaded machine has a synchronous reactance of 10 Ω per phase and its excitation is adjusted so that it operates at 0.75 power factor lagging. The synchronous reactance of the second machine is 16 Ω per phase

Calculate the excitation, the load angle, the current and the power factor of each machine. The internal resistances of the machines may be neglected.

Solution 2

The specifications of the system are as follows:

$$
\begin{aligned}
V_t &= \frac{3.3\times 10^3}{\sqrt{3}} \text{ V} \\
X_a &= 10\ \Omega \\
X_b &= 16\ \Omega \\
pf_{total} &= 0.8 \\
\phi_{total} &= Cos^{-1}(pf_{total}) \\
\bar{S}_{total} &= \frac{800\times 10^3}{pf_{total}}\left[Cos\left(\phi_{total}\right) + jSin\left(\phi_{total}\right)\right]
\end{aligned}
$$

Now, since the power delivered by Machine *a* is twice that of Machine *b*, we get

$$
\begin{aligned}
P_{total} &= P_a + P_b \\
P_b &= \frac{P_a}{2} \\
P_a &= \frac{2}{3}P_{total}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
P_a &= \frac{2}{3}\mathrm{Re}[\bar{S}_{total}] \\
&= \frac{2}{3}\mathrm{Re}[\left(800.0 + j600.0\right)] \text{ kVA} \\
&= 533.33 \text{ kW.}
\end{aligned}
$$

The magnitude of the armature current drawn from Machine *a* can now be calculated as follows:

$$
\begin{aligned}
pf_a &= 0.75 \\
\phi_a &= Cos^{-1}\left(pf_a\right) \\
I_a &= \frac{P_a}{3V_t\, pf_a} = 124.41 \text{ A.}
\end{aligned}
$$

Hence, the armature current phasor, the complex power delivered by Machine *a* and its excitation voltage are given by

$$\overline{I}_a = I_a\left[Cos\left(\phi_a\right) - jSin\left(\phi_a\right)\right] = 124.41\angle\text{-}41.41° \text{ A}$$
$$\overline{S}_a = 3V_t\overline{I}_a = 711.11\angle 41.41° \text{ kVA}$$
$$\overline{E}_a = V_t + jX_a\overline{I}_a = 2.88\angle 18.88° \text{ kV}$$

The operating parameters for Machine *b* are then calculated as follows:

$$\overline{S}_b = \overline{S}_{total} - \overline{S}_a = 296.51\angle 25.93° \text{ kVA}$$
$$\overline{E}_b = V_t + jX_b\overline{I}_b = 2.388\angle 18.22° \text{ kV}$$
$$\overline{I}_b = I_b\left[Cos\left(\phi_b\right) - jSin\left(\phi_b\right)\right] = 51.88\angle\text{-}25.93° \text{ A.}$$

Example 3

A generator having a synchronous reactance of .9 pu based on its own rating is connected via a transmission line to a remote bus. The voltage V_{bus} at the remote bus is 1 pu. The line reactance is $X_{line} = 0.15$ pu per phase based on the machine MVA rating. The internal emf E_f of the machine is maintained at 1.35 pu.

(a) Calculate the maximum, or pull-out, power of the machine when operated as a generator.

(b) At the moment of pull-out, calculate the reactive powers at the generator terminals and at the remote bus.

(c) What is the terminal voltage of the generator at the moment of pull-out?

(d) Is the machine current overloaded at pull-out?

Solution 3

$$\begin{aligned} \overline{V}_{bus} &= 1.0\angle 0^\circ \\ jX_{line} &= j0.15 \text{ pu} \\ jX_s &= j0.9 \text{ pu} \\ \overline{E}_f &= 1.35\angle 90^\circ \end{aligned}$$

At this maximum power transfer point,

$$P_{\max} = \frac{E_f V_{bus}}{X_s + X_{line}} Sin(90^\circ) = \frac{1.35 \times 1.0}{0.9 + 0.15} = 1.286 \text{ pu power}$$

$$\begin{aligned} I_{amax} &= \frac{\overline{E}_f - \overline{V}_{bus}}{j(X_s + X_{line})} \\ &= 1.6\ \angle 36.53^\circ \text{ pu.} \end{aligned}$$

Note the leading power factor angle.

The terminal voltage of the generator is given by

$$\begin{aligned} \overline{V}_t &= \overline{V}_{bus} + jX_{line}\overline{I}_{amax} \\ &= 0.879\angle 12.68^\circ \text{ pu.} \end{aligned}$$

The reactive power at the infinite bus is

$$\begin{aligned} Q_{bus} &= \text{Im}[\overline{V}_{bus} \times \overline{I}_{amax}{}^*] \\ &= \text{-}0.952 \text{ pu VAr} \end{aligned}$$

The reactive power at the generator terminals is

$$\begin{aligned} Q_{generator} &= \text{Im}[\overline{V}_t \times \overline{I}_{amax}{}^*] \\ &= \text{-}0.568 \text{ pu VAr} \end{aligned}$$

Note that, as expected,

$$\begin{aligned} Q_{line} &= \left|I_{amax}\right|^2 X_{line} \\ &= 0.384 \text{ pu VAr} \end{aligned}$$

and also that

$$Q_{generator} = Q_{line} + Q_{bus} = \text{-}0.568 \text{ pu VAr} .$$

A phasor diagram, drawn to scale using Mathematica, for this operating point is given in Figure 2 below.

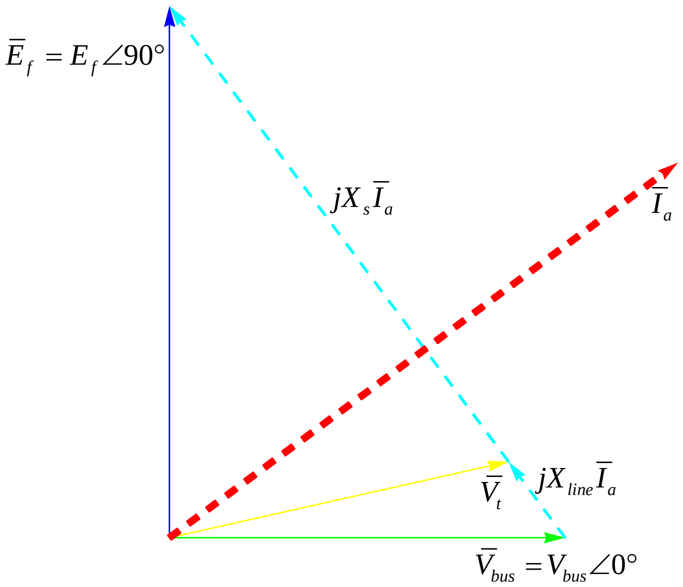

Figure 2

Example 4.

A three-phase, round-rotor synchronous generator is connected to an infinite bus via two identical parallel three-phase transmission lines having a reactance of 0.6 per unit including step-up and step down transformers at each end. The synchronous reactance of the generator is 0.9 per unit. All resistances are negligible and the reactances are expressed taking the generator rating as base. The infinite busbar voltage is 1.0 per unit.

(a) The prime mover input power and the excitation voltage of the generator are adjusted so that it delivers rated current at unity power factor at its terminals in steady state. Calculate the generator terminal voltage, the excitation voltage, the power output and the reactive power delivered to the infinite bus.

(b) The prime mover input power is now set so that there is no real power transfer between the generator and the infinite busbar. The field current of the generator is adjusted until 0.5 per unit lagging reactive power is delivered to the infinite bus. Under these conditions, calculate the terminal voltage and the excitation voltage of the generator.

(c) The system is now returned to the operating point described in Part (a). A fault occurs such that one of the two parallel transmission lines is disconnected by tripping the breakers at its ends. The generator excitation voltage remains constant. Will the generator remain in synchronism with the system? If no, what steps may be taken to ensure stability under these conditions?

Solution 4

This transmission system is illustrated in Figure 3 below.

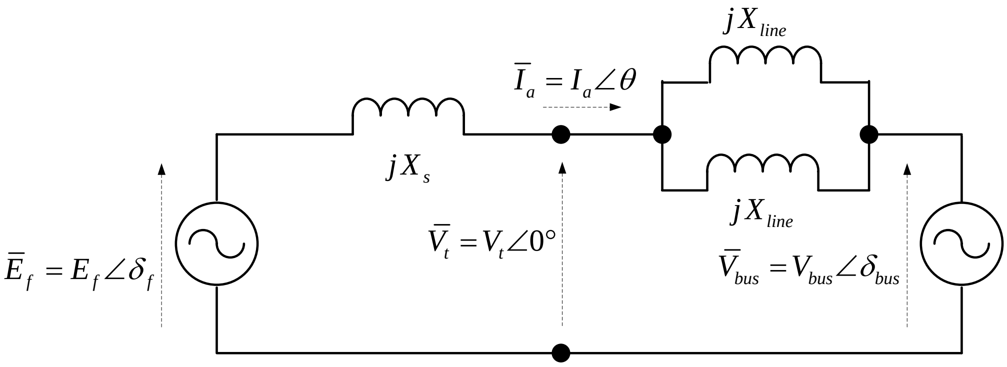

Figure 3

(a) It can be seen that the terminal voltage of the generator is taken as the reference phase angle. In this case, the bus voltage is know in magnitude but not in phase while the generator terminal voltage is known in phase but not in magnitude. The generator current is known in both magnitude and phase since $\bar{I}_a = 1.0\angle 0°$.

Hence,

$$V_t\angle 0° = V_{bus}\angle\delta_{bus} + j\frac{X_{line}}{2}\bar{I}_a$$

$$V_t = V_{bus}Cos\left(\delta_{bus}\right) + jV_{bus}Sin\left(\delta_{bus}\right) + j\frac{X_{line}}{2}\bar{I}_a$$

Thus,

$$V_t = V_{bus} Cos\left(\delta_{bus}\right)$$
$$0 = V_{bus} Sin\left(\delta_{bus}\right) + \frac{X_{line}}{2} I_a$$

and so

$$\delta_{bus} = -Sin^{-1}\left(\frac{X_{line} I_a}{2V_{bus}}\right)$$
$$V_t = V_{bus}\, Cos\left(\delta_{bus}\right).$$

Hence,

$$\delta_{bus} = \text{-17.46}°$$
$$V_t = 0.954.$$

To calculate the generator excitation voltage,

$$E_f \angle \delta = V_t \angle 0° + jX_s \overline{I}_a$$
$$E_f Cos\left(\delta_f\right) + jE_f Sin\left(\delta_f\right) = V_t + jX_s \overline{I}_a$$

Hence,

$$E_f Cos\left(\delta_f\right) = V_t$$
$$E_f Sin\left(\delta_f\right) = X_s I_a$$

so that

$$Tan\left(\delta_f\right) = \frac{X_s I_a}{V_t}$$
$$E_f = \frac{V_t}{Cos\left(\delta_f\right)}$$

Thus

$$\delta_f = Tan^{-1}\left(\frac{X_s I_a}{V_t}\right) = Tan^{-1}\left(\frac{0.9}{0.954}\right) = 43.3°$$
$$E_f = \frac{0.954}{Cos\left(43.3°\right)} = 1.311 \text{ pu}$$

(b) In this case, $\overline{V}_{bus} = 1.0$, and hence the power transfer to the bus $\overline{S}_{bus} = j0.5$. Now, since

$$\overline{I}_a = \left(\frac{\overline{S}_{bus}}{\overline{V}_{bus}}\right)^*$$

then

$$\overline{I}_a = -j0.5.$$

Hence, it is clear that the generator terminal voltage $\overline{V}_t$ is

$$\begin{aligned}\overline{V}_t &= V_{bus}\angle 0° + j\frac{X_{line}}{2}\overline{I}_a \\ &= 1.0 + \left(j\frac{0.6}{2}\right)(-j0.5) \\ &= 1.15 \text{ pu.}\end{aligned}$$

Likewise, the generator excitation voltage $\overline{E}_f$ is

$$\begin{aligned}\overline{E}_f &= V_{bus}\angle 0° + j(\frac{X_{line}}{2} + X_s)\overline{I}_a \\ &= 1.0 + j\left(\frac{0.6}{2} + 0.9\right)(-j0.5) \\ &= 1.6 \text{ pu.}\end{aligned}$$

(c) The prime mover input power in Case (a) is equal to the real power transfer which has been shown to be 0.954 pu. If the circuit breakers at the ends of one of the transmission lines goes open circuit, then the maximum possible power transfer is given by

$$\begin{aligned}P_{\max} &= \frac{E_f V_{bus}}{X_s + X_{line}} \\ &= \frac{1.311 \times 1.0}{0.9 + 0.6} \\ &= 0.874 \text{ pu}\end{aligned}$$

Since this maximum power transfer is now less than the prime mover input power, the system cannot maintain steady state stability and the generator will overspeed and loose synchronism with the grid busbars. A fast acting excitation system and a speed regulator would help to retain stability. A phasor diagram, drawn to scale in Mathematica, for Case (a) is illustrated in Figure 4 below. Note that the phase angle $(\delta_f + \delta_{bus})$ between the two voltage sources is close to the stability limit of $90°$ even with both transmission lines in parallel. Clearly, once the breaker opens, the system is no longer stable.

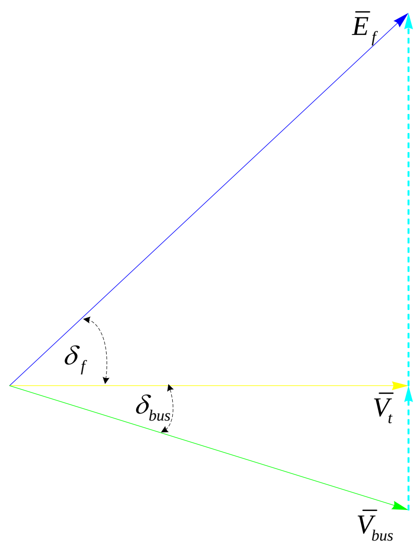

Figure 4

Problems

Q.1 A 1500 kVA, 6.6 kV, three-phase, star-connected, round-rotor synchronous machine has a phase resistance of $0.5\ \Omega/\text{phase}$ and a synchronous reactance of $5.0\ \Omega/\text{phase}$. Calculate the percentage change in the terminal voltage when the rated output of 1500 kVA at a power factor of 0.8 lagging is switched off. Assume that the machine is initially operating at rated voltage and frequency and that the speed and the excitation current remain unchanged.

[+12.43%]

Q.2 Two similar three-phase star-connected round-rotor synchronous generators, designated Set 1 and Set 2, are connected in parallel with each other. Each machine has a synchronous reactance of $j4.5\ \Omega/\text{phase}$ and negligible resistance. The field current excitation to each generator is adjusted until its internal emf has a magnitude of 1910 V/phase. The prime mover inputs are adjusted until it is found that the internal emf of Set 1 has a positive phase displacement of $30°$ relative to that of Set 2. Under these circumstances, calculate (i) the circulating current, (ii) the per-phase terminal voltage, (iii) the active power supplied from Set 1 to Set 2. Sketch the phasor diagram corresponding to this situation.

[109.8 A, 1845 V, 608 kW]

Q.3 The prime mover inputs to the two generator sets in the previous question are altered until their internal emf's are exactly in phase opposition. The field current excitation to Set 1 is adjusted until its internal emf has a magnitude of 2240 V/phase while that of Set 2 is adjusted until its emf is equal to 1600 V/phase. Calculate the circulating current and the per-phase terminal voltage of the machines under these circumstances. Sketch the phasor diagram corresponding to this situation.

[71.1 A, 1920 V]

Q.4 A three-phase, round-rotor synchronous generator is supplying 2.8 MW of power at a power factor of 0.7 lagging to a manufacturing plant. Under these operating conditions, this machine is at its full rated capacity. If the generator power factor is increased to unity by the installation of a three-phase synchronous motor, how much more active power can the generator supply and what must be the power factor of the synchronous motor assuming that this machine absorbs all of the extra real power obtainable from the generator.

[1.2 MW, 0.387 leading]

Q.5 A three-phase, 50 Hz, star-connected, round-rotor synchronous generator is driven by its prime mover at 1000 rpm and at this speed it develops an open circuit line voltage of 460 V when the field current is set to 16 A. The machine has a synchronous reactance of $j2.0\ \Omega/\text{phase}$ and negligible winding resistance. Calculate the driving torque required of the prime mover when the machine is supplying a current of 50 A/phase to a load at a power factor of 0.8 lagging. Assume that the field current and the speed remain constant at 16 A/phase and 1000 rpm, respectively. Neglect magnetic core losses and mechanical friction and windage losses.

[221.4 Nm]

Q.6 A three-phase round-rotor synchronous generator delivers 1.0 pu current at 1.0 pu terminal voltage and a power factor of 0.8 lagging to a set of infinite three-phase busbars. The synchronous reactance of the alternator is $j1.0$ pu and the phase winding resistance may be neglected.

(i) Calculate the internal emf of the machine, the load angle of the machine and the real and reactive power transfers to the set of infinite busbars.

[1.79. 26.56°, 0.8, 0.6]

(ii) If the excitation is now increased by 20% relative to that in (i), calculate the new load angle, the new armature current, the new operating power factor and the new real and reactive power transfers to the set of infinite busbars.

[21.88° 1.27, 0.627, 0.8, 0.992]

(iii) If the excitation were reduced again to that of Case (i), and the mechanical output power from the primer mover is increased by 20%, calculate the new values of the alternator load angle, the armature current and the operating power factor and the real and reactive power transfers to the set of infinite busbars.

[32.45° 1.087, 0.883, 0.96, 0.509]

Q.7 Derive expressions for the real power P and the reactive power Q delivered by a three-phase, round-rotor synchronous generator to an infinite system in terms of the terminal voltage V_t, the generated back-emf, E_f, the synchronous reactance X_s and the load angle δ. Resistive losses, hysteresis and eddy current losses and magnetic saturation may be neglected.

Prove that if the locus of the real and reactive powers are plotted in the complex power plane, $\overline{S} = P + jQ$, then the locus will be a circle of radius

$$r = \frac{E_f V_t}{X_s}$$

with a centre

$$c = \left(0, -\frac{V_t^2}{X_s}\right)$$

on the Q axis. Draw loci corresponding to two values of E_f. For a given operating point, indicate the load angle δ and the power factor angle ϕ on the complex plane. On this plane, draw curves indicating the limits due to armature heating and field current heating.

Q.8 Two star-connected, round-rotor, synchronous generators of identical ratings operate in parallel to deliver 25 MW, 0.9 power factor lagging to a set of 11 kV infinite busbars. The induced line-to-line voltage of Machine A is 15 kV and the prime mover of this machine is set so that it deliver 10 MW of power. The remaining load power is supplied by Machine B.

Calculate the load angle, the current and the power factor of each machine.

[$\delta_A = 16.9°$, $\delta_B = 26.8°$ $I_A = 661.8$ A, $pf_A = 0.793$, $I_B = 820.9$ A, $pf_B = 0.959$]

Calculate also the line-to-line excitation voltage of Machine B. [$14.49\angle 26.8°$ kV]

The synchronous reactances of the machines is $4.8\ \Omega$ and all losses may be neglected.