Power Electronics Homework Problems

Problem 1: ED 4-1
[Ans. 3.33 V, 0.833, -1.67 V, 0.333]

Problems:
H-Bridge: 7, 8, 9, 10, 11
Buck or Boost: 12, 13, 14, 15, 16, 19
Flyback: 17, 20, 21

Problem 2: ED 4-2

Problem 3: ED 4-3
[Ans. 50]

Problem 4: ED 4-4

Problem 5: ED 4-5
[Ans. 0.204 A]

Problem 6: ED 4-6
[Ans. 784 W]

Summer 2002
Problem 7:

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc H-bridge converter are as follows: armature resistance $R_a = 0.35\,\Omega$, armature inductance $L_a = 1.5$ mH, motor moment of inertia 0.02 kg m^2, motor torque constant k = 0.5 Nm/A, converter dc bus voltage V_d = 200 V, switching frequency f = 25 kHz, and amplitude of triangular waveform control voltage V_{tri} = 3 V. The motor is spinning in a forward direction at a speed of 1500 RPM, supplying a load torque of 10 Nm.

i. Sketch the circuit.

ii. Calculate the following: (a) the applied armature voltage vab(t); (b) duty ratios for (i) overall converter, (ii) pole A, and (iii) pole B; (c) control voltage amplitude and (d) peak to peak ripple on the armature current;

iii. Sketch the waveforms for the triangular voltage vtri(t), control voltage vc(t), pole A voltage van(t), the pole b voltage vbn(t), armature voltage va(t), and the armature current iab(t).

iv. Sketch the four different switch configurations of the converter sequenced over one switching cycle. Also note these sequences in your timing diagrams in part (iii) above.

[Ans. (ii) (a) 85.55 V (b) (i) 0.428 (ii) 0.714 (iii) 0.286 (c) 1.284 V (d) 0.653 A, (iv)I-Qau/Qbl, II-Qau/Dbu, III-Qau/Qbl, IV-Dal/Qbl]

Problem 8

Repeat above for the motor running in reverse with a positive torque, and acting as a generator.

[Ans. (ii) (a) -71.55 V (b) (i) -0.358 (ii) 0.321 (iii) 0.678 (c) -1.07 V (d) 0.613 A, (iv)I-Qau/Dbu, II-Dal/Dbu, III-Dal/Qbl, IV-Dal/Dbu]

Summer 2003
Problem 9

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc converter are as follows: armature resistance $R_a = 0.1\Omega$, armature inductance L_a = 1 mH, motor moment of inertia J_m = 0.02 kg m^2, motor constant k = 0.2 V/(rad/s), dc bus voltage V_d = 42 V, switching frequency f_s = 20 kHz, and amplitude of triangular waveform control voltage V_{tri} = 3 V. The motor is spinning in a reverse direction at a speed of 1000 RPM and supplies a load torque of 5 Nm.

(i) Sketch the system.

(ii) Calculate the following: (a) the applied armature voltage V_{AB}; (b) duty ratios for the overall converter, pole A, and pole B; (c) the control voltage; and (d) the peak-to-peak ripple on the armature current.

(iii) Sketch the waveforms for the triangular voltage $v_{tri}(t)$, control voltage $v_c(t)$, pole A voltage $v_A(t)$, pole B voltage $v_B(t)$, armature voltage $v_{AB}(t)$, and armature current $i_{ab}(t)$.

(iv) Sketch the four different switch configurations of the converter sequenced over one switching cycle. Also note these sequences in your timing diagrams in part (iii) above.

[Ans. (ii) (a)-23.44 V (b) (i) –0.558 (ii) 0.222 (iii) 0.778 (c) –1.67 V (d) 0.259 A, (iv)I-Dau/Qbu, II-Qal/Qbu, III-Qal/Dbl, IV-Qal/Qbu]

Summer 2008, Summer 2006
Problem 10

The system parameters of a permanent-magnet dc machine supplied by a switch-mode PWM dc-dc converter are as follows: armature resistance $R_a = 0.1\Omega$, armature inductance L_a = 1 mH, motor moment of inertia J_m = 0.02 kg

m^2, motor constant $k = 0.2$ V/(rad/s), dc bus voltage $V_d = 42$ V, switching frequency $f_s = 20$ kHz, and amplitude of triangular waveform control voltage $V_{tri} = 3$ V. The machine is acting as a **generator** spinning in a forward direction at a speed of 1000 RPM and demanding a torque of 5 Nm.

N+
T-

(v) Sketch the system.

(vi) Calculate the following: (a) the applied armature voltage V_{AB}; (b) duty ratios for the overall converter, pole A, and pole B; (c) the control voltage; and (d) the peak-to-peak ripple on the armature current.

(vii) Sketch the waveforms for the triangular voltage $v_{tri}(t)$, control voltage $v_c(t)$, pole A voltage $v_A(t)$, pole B voltage $v_B(t)$, armature voltage $v_{AB}(t)$, and armature current $i_{ab}(t)$.

(viii) Sketch the four different switch configurations of the converter sequenced over one switching cycle. Also note these sequences in your timing diagrams in part (iii) above.

[Ans. (ii) (a)18.44 V (b) (i) 0.439 (ii) 0.720 (iii) 0.281 (c) 1.317 V (d) 0.259 A, (iv)I-Dau/Dbl, II-Dau/Qbu, III-Dau/Dbl, IV-Qal/Dbl]

Summer 2007, Summer 2005

Problem 11

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc converter are as follows: armature resistance $R_a = 0.1\ \Omega$, armature inductance $L_a = 1$ mH, motor constant $k = 0.07$ V/(rad/s), dc bus voltage $V_d = 12$ V, switching frequency $f_s = 20$ kHz, and amplitude of triangular waveform control voltage $V_{tri} = 5$ V. The motor is spinning forward at a speed of 750 rpm and acts as a *generator* supplied by a full-load torque of -0.7 Nm.

(i) Calculate the following: (a) the applied armature voltage V_{AB}; (b) duty ratios for the overall converter, pole A, and pole B; (c) the control voltage, and (d) the peak-to-peak ripple on the armature current.

(ii) Calculate the rms currents in the upper and lower MOSFET switches of pole A.

[Ans. 4.5 V, 0.375, 0.688, 0.312, 1.875 V, 70.3 mA, upper 8.29 A, lower 5.59 A]

Problem 12

The 2003 Toyota Prius uses a 20 kW bidirectional converter to generate a 500 V dc link voltage from the 200 V NiMH battery. This higher voltage allows the efficiency, range, and emissions of the vehicle to be optimized. The bidirectional converter has an inductance of 435 uH and switches at 10 kHz.

The vehicle is operating in generating mode and the bi-directional converter is required to act as a buck at full power.

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. I_L(rms) = 100.3 A, I_{CLV}(rms) = 8 A, I_{QU}(rms) = 63.4 A, I_{QU}(avg) = 40 A, I_{DL}(rms) = 77.7 A, I_{DL}(avg) = 60 A, I_{CHV}(rms) = 49.2 A; P_Q = 140 W; P_D = 120 W]

Problem 13

The above vehicle is operating in motoring mode and the bi-directional converter is required to act as a boost and provide full power.

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. I_L(rms) = 100.3 A, I_{CLV}(rms) = 8 A, I_{DU}(rms) = 63.4 A, I_{DU}(avg) = 40 A, I_{QL}(rms) = 77.7 A, I_{QL}(avg) = 60 A, I_{CHV}(rms) = 49.2 A; P_Q = 210 W; P_D = 80 W]

Summer 2005

Problem 14

The above vehicle is operating in motoring mode and the bi-directional converter is required to act as a boost and provide a half power level of 10 kW. For this 10 kW condition:

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. 50.6 A, 8 A, 25 A, 39.2, 30 A, 90 W, 32 A, 20 A, 35 W]

Problem 15

Design a Voltage Regulator Module (VRM) for local power regulation of a microprocessor on a mobile phone. The VRM is powered from a 3.3 V source and uses a buck converter. The microprocessor specifications call for a 1.85 V supply with a +/- 1 % allowable fluctuation while consuming 60 mA. The switching frequency is 500 kHz. Neglect the parasitic effects of the controlled MOSFET switch and diode.

(i) Sketch the buck converter.

(ii) Find the duty cycle of the controlled switch, needed to output 1.85 V.
(iii) Choose an inductor that limits the current ripple to +/- 10 %.
(iv) Choose a capacitor to limit the voltage ripple to +/ 1 %.
(v) Determine the rms currents in the inductor, and output and input capacitors.
(vi) Calculate the conduction losses in (a) the MOSFET with $R_{ds(on)}$ of 0.5 ohm, and (b) the diode with a characteristic forward voltage knee of 0.4 V and a slope of 0.1 Ω.
(vii) What much would the diode loss be reduced by using a synchronous buck?

[Ans. (ii) 0.56, (iii) 135 uH, (iv) 81 nF, (v) 60.1 mA, 3.5 mA, 29.9 mA (vi) (a) 1mW, (b) 10.6 mW (vii) diode loss of 10.6 mW reduced to 0.8 mW]

Summer 2006
Problem 16

Design a Voltage Regulator Module (VRM) for local power regulation of a microprocessor on a mobile phone. The VRM is powered from a 3.3 V lithium ion battery and uses a **synchronous** buck converter. The microprocessor specifications call for a 1 V supply with a +/- 1 % allowable fluctuation while consuming 50 mA. The switching frequency is 1 MHz. Neglect the parasitic effects of the controlled MOSFETs.

(i) Sketch the synchronous buck converter.
(ii) Choose an inductor that limits the current ripple to +/- 10 %.
(iii) Choose a capacitor to limit the voltage ripple to +/ 1 %.
(iv) Determine the rms currents in the inductor, and output and input capacitors.
(v) Calculate the conduction losses in the two MOSFETs for $R_{ds(on)}$ of 0.5 ohm.

[Ans. 69.6 uH, 62.5 nF, 50.1 mA, 2.9 mA, 23 mA, 1.3 mW]

Problem 17

In a flyback converter, $V_I = 30$ V, $N_1 = 30$ turns, $N_2 = 15$ turns. The self-inductance of winding 1 is 50 μH and $f_s = 200$ kHz. The output voltage is regulated at 9V. (PED P8-1 to 8-4) Assume ideal components with no parasitics.

8-1. Calculate and draw the waveforms transformer input and output currents along with the ripple current in the output capacitor if the load is 27 W.
8-2 For the same duty ratio in (1) above, calculate the critical power which makes this converter operate at the border of incomplete and complete demagnetization modes.
8-3 Redraw the input and out current waveforms.
8-4 Determine the maximum voltage across the switch.

[Ans. $I_{I,min} = 1.84$ A, $I_{I,min} = 2.96$ A, $I_{O,min} = 3.68$ A, $I_{I,min} = 5.92$ A, $P_{CCM} = 6.3$ W, $V_Q = 48$ V]

Summer 2008, Summer 2007
Problem 19

The 2005 Lexus RX400h hybrid vehicle uses a bidirectional converter to generate a 650 V dc link voltage from the 288 V NiMH battery. This higher voltage allows the efficiency, range, and emissions of the vehicle to be optimized. The bidirectional converter has an inductance of 245 μH and switches at 10 kHz.
The vehicle is operating in motoring mode and the bi-directional converter is required to act as a boost and provide a full power of 30 kW.

(iii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01$ Ω, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005$ Ω.
(iv) Calculate the rms currents in the input and output capacitors.
(v) What voltage rating would be suitable for the IGBTs in this converter?

[Ans. I_L(rms) = 105.9 A, I_{CLV}(rms) = 18.9 A, I_{QL}(rms) = 79 A, I_{QL}(avg) = 58 A, I_{DU}(rms) = 70.5 A, I_{DU}(avg) = 46.2 A, I_{CHV}(rms) = 53.3 A; $P_Q = 179$ W; $P_D = 94.2$ W, 1000 V minimum]

Problem 20

Design a continuous-conduction flyback converter for the following conditions: $V_I = 400$ V, $V_O = 12$ V, $P_O = 20$-60 W and the switching frequency is 40 kHz. Maintain the maximum duty cycle at no greater than 0.45. Calculate the minimum magnetizing inductance required. Assume ideal components.

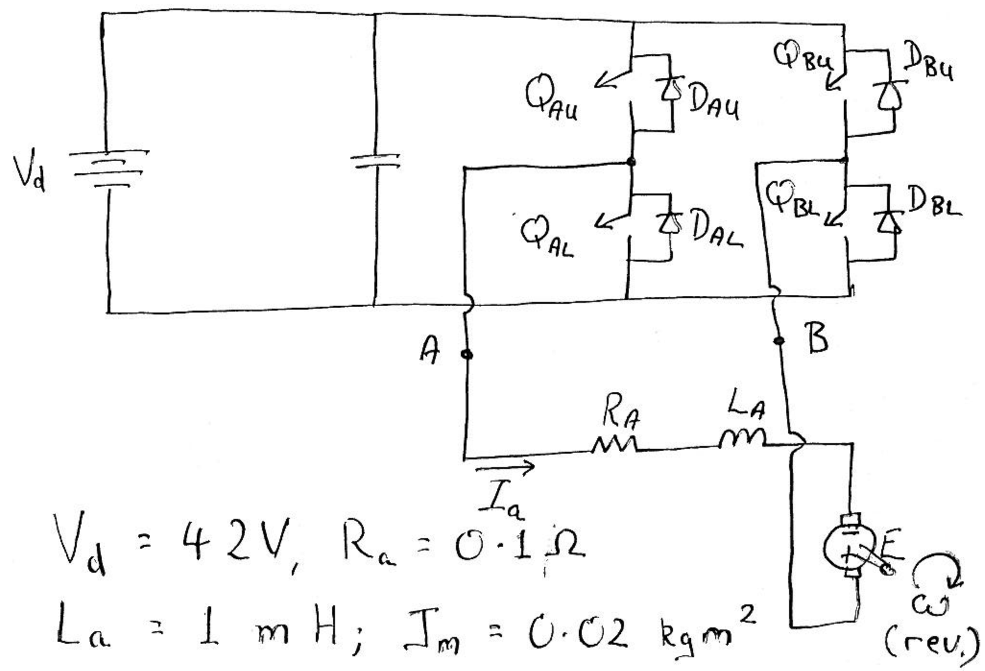
[Ans. 20.3 mH]

Summer 2008
Problem 21

In a regulated flyback converter with a 1:1 turns ratio, $V_O = 12$ V, V_I is 12-24 V, P_{load} is 6-60 W and the switching frequency is 200 kHz. Calculate the maximum value of the magnetizing inductance L_m that can be used if the converter is always required to operate in a complete demagnetization (equivalent to a discontinuous-conduction) mode. Assume ideal components.

[Ans. 1.5 μH]

Q 9, (Summer 2003)



$V_d = 42V$, $R_a = 0.1\,\Omega$

$L_a = 1\ mH$; $J_m = 0.02\ kgm^2$

$k = 0.2\ V/(rad\,s^{-1})$; $f_s = 20\ kHz$

$V_{tri} = \pm 3V$

(ii) $N = -1000\ rpm \Rightarrow \omega = -104.7\ \frac{rad}{s}$

$T = -5\ Nm \Rightarrow I_A = \frac{T}{k} = -25\ A$

$E = k \cdot \omega = -20.94\ V$

$V_{AB} = E + I_A R_A = -23.44V$

$d = \frac{V_{AB}}{V_d} = -0.558$

$V_c = d\,\hat{V}_{tri} = -1.67\ V$

$d_A = \frac{1}{2} + \frac{1}{2}\frac{V_c}{\hat{V}_{tri}} = 0.222$

$d_B = \frac{1}{2} - \frac{1}{2}\frac{V_c}{\hat{V}_{tri}} = 0.778$

$V_L = L_A \frac{\Delta I_{p-p}}{\Delta t}$

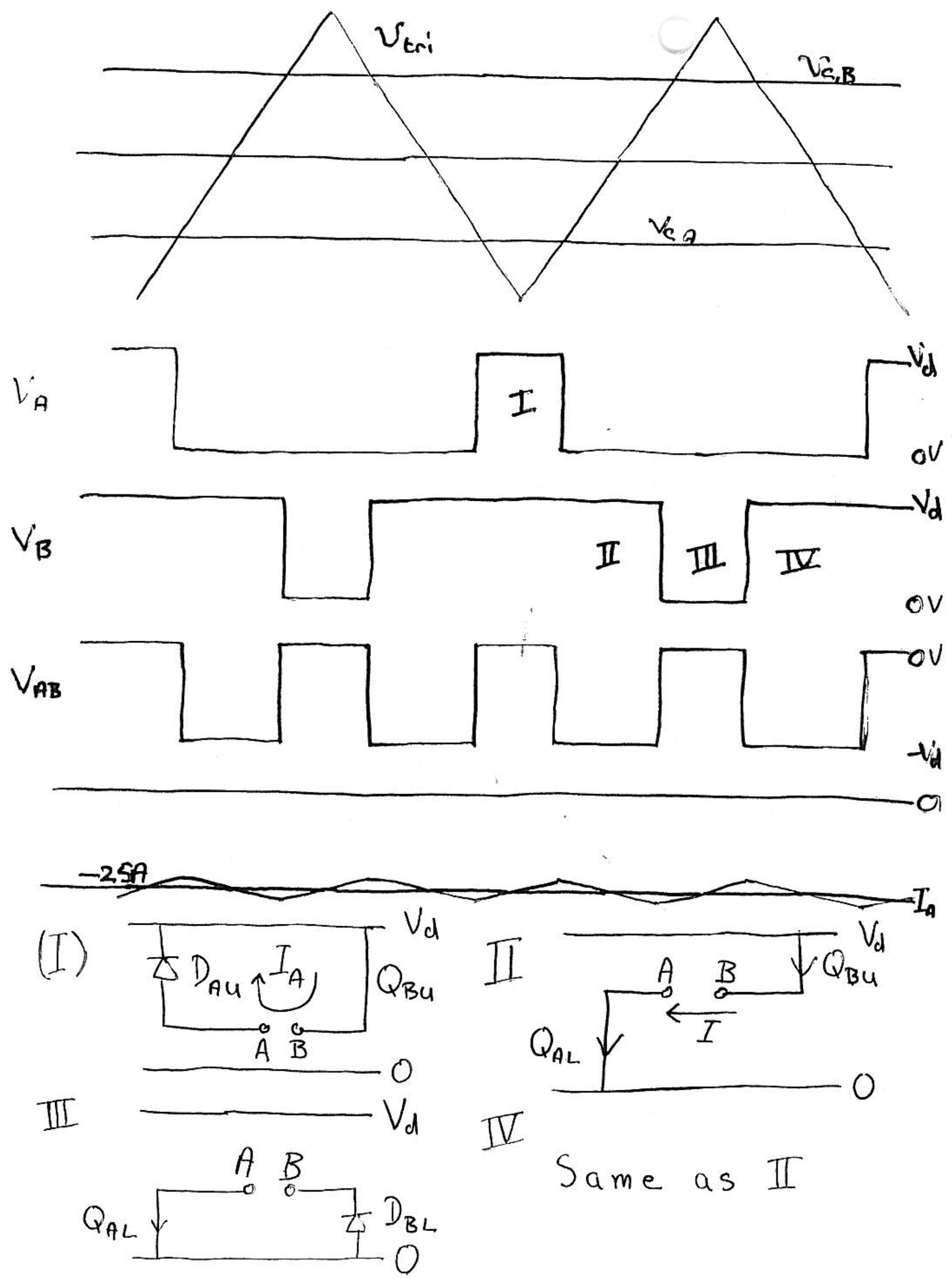
$\Rightarrow \Delta I_{p-p} = \frac{V_L}{L_A} . \Delta t = \cancel{(V_d}$

$= \frac{(V_d - I_a R_a - E)}{L_A} . d . \frac{T_s}{\underline{\underline{2}}}$ N.B.

$= 0.259\ A_{p-p}$

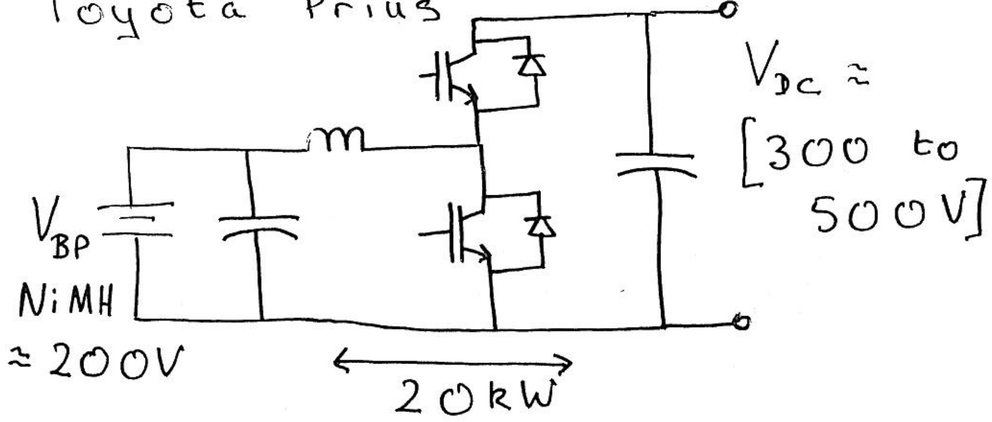
Motoring $\Rightarrow T\omega = +$

Generating $\Rightarrow T\omega = -$



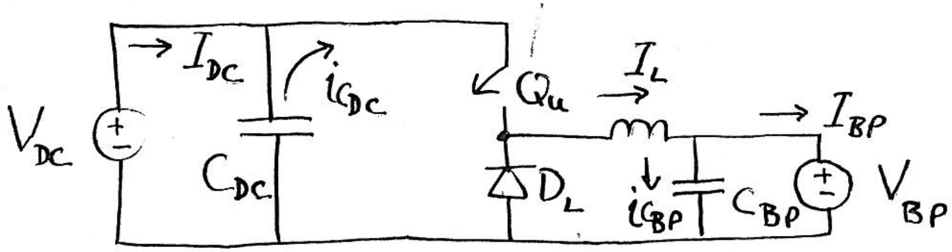
Problem 12

Toyota Prius



Regen => battery recharge

Redraw as buck converter



$P = 20kW$

$V_{BP} = 200V$

$V_{DC} = 500V$

$C_{BP} = 283\mu F$

$C_{DC} = 1130\mu F$

$L = 435\mu H$

$$d_u = \frac{V_{BP}}{V_{DC}} = 0.4$$

$$\Delta I_{p\text{-}p} = \frac{(V_{DC} - V_{BP}) \cdot d_u}{f \cdot L}$$

$$= 27.6 \ A_{p\text{-}p}$$

$$i_L = i_{C_{BP}} = \frac{\Delta I_{p\text{-}p}}{\sqrt{12}}$$

$$= 8 \ A_{rms}$$

$$I_{BP} = \frac{P}{V_{BP}}$$

$$= 100 \ A_{dc}$$

$$I_L = \sqrt{I_{BP}^2 + i_{C_{BP}}^2}$$

$$= 100.3 \ A_{rms}$$

$$I_{Qu(rms)} = \sqrt{d} \ I_L$$

$$= 63.4 \ A_{rms}$$

$$I_{Qu\,(avg)} = d \ I_{BP}$$

$$= 40 \ A_{dc}$$

$$I_{DL(rms)} = \sqrt{1-d} \ I_L$$

$$= 77.7 \ A_{rms}$$

$$I_{DL(avg)} = (1-d) \ I_{BP}$$

$$= 60 \ A$$

$$I_{C_{DC}} = \sqrt{I_{Qu(rms)}^2 - I_{DC}^2}$$

$$= 49.2 \ A_{rms}$$

$$P_{Qu(cond)} = V_{CE} \cdot I_{Qu(avg)} + R_{CE} \, I_{Qu(rms)}^2$$

$$= 2.5 \times 40 + 0.01 \times 63.4^2$$

$$= 140 \ W$$

$$P_{DL(cond)} = V_F \cdot I_{DL(avg)} + R_F \cdot I_{DL(rms)}^2$$

$$= 1.5 \times 60 + 0.005 \times 77.7$$

$$= 120 \ W$$

8-1 PED

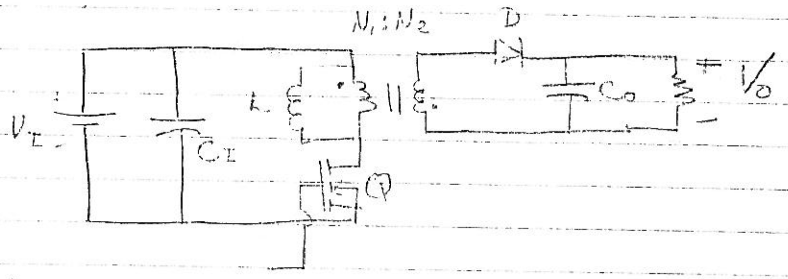
Problem 17

Flyback

$V_I = 30V$ $\quad N_1 = 30$ $\quad L = 50\mu H$

$V_o = 9V$ $\quad N_2 = 15$ $\quad f_s = 200 kHz$

$P_o = 27V$ $\quad \Rightarrow n = \frac{N_2}{N_1} = 0.5$ $\quad \Rightarrow T_s = \frac{1}{f_s} = 5\mu s$



duty ratio

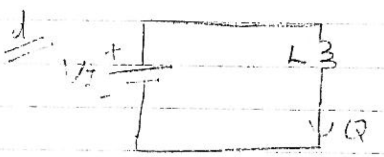
$$\frac{V_o}{V_I} = n \frac{d}{1-d}$$

$\Rightarrow$ rearrange to get

$$d = \frac{V_o}{V_o + n V_I} = \frac{9}{9 + 0.5 \times 30}$$

$$\Rightarrow d = 0.375$$

Input current ripple



$$V_I = L \frac{\Delta I_{I(p-p)}}{d T_s}$$

$$\Rightarrow \Delta I_{I(p-p)} = \frac{V_I \cdot d T_s}{L} = \frac{30 \times 0.375 \times 5\mu}{50\mu}$$

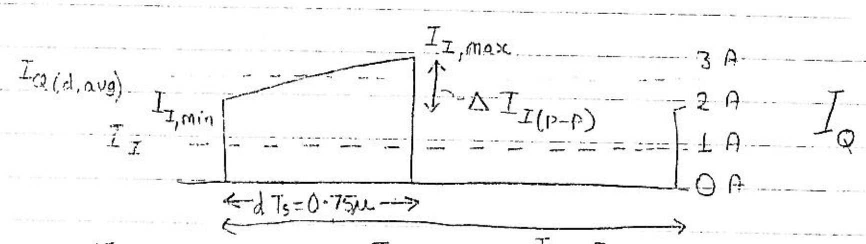
$$\Rightarrow \Delta I_{I(p-p)} = 1.125 A$$

Assume for simplicity $\eta = 100\%$

$$\Rightarrow I_I = \frac{P}{V_I} = \text{Average input current}$$

$$\frac{27}{30} = 0.9 A$$

Let $I_{Q(d,avg)}$ be the average current through switch Q while conducting (d)



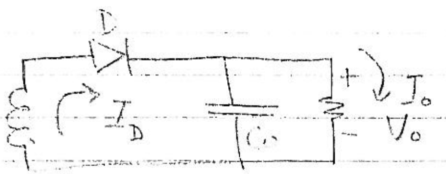
$$I_{Q(d,avg)} = \frac{I_I}{d} = \frac{0.9}{0.375} = 2.4 A$$

Avg. I in switch while conducting
$\Rightarrow$ Over full period T_s must averag 0.9A

$$\Rightarrow I_{I,min} = I_{Q(d,avg)} - \frac{\Delta I_{I(p-p)}}{2} = 2.4 - 1.125/2 = 1.84 A$$

$$I_{I,max} = I_{Q(d,avg)} + \frac{\Delta I_{p-p}}{2} = 2.96 A$$

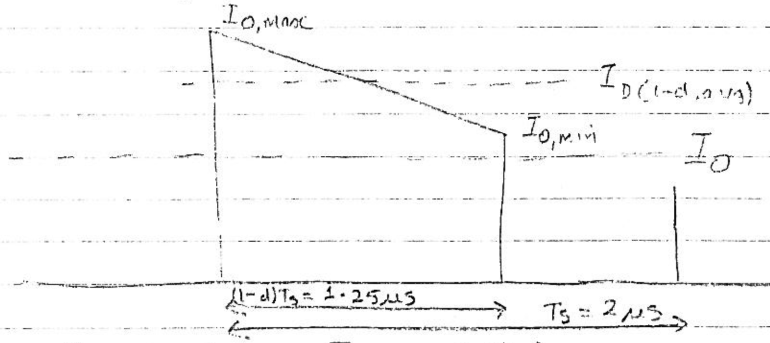
1-d



$$I_o = \frac{P}{V_o}$$

$$= \frac{27}{9} A = 3 A$$

Let $I_{D(1-d,avg)}$ be the average diode current during $(1-d)$



$$\Rightarrow \quad I_{D(1-d,avg)} = \frac{I_o}{1-d} = 4.8 A$$

$$I_{O,max} = I_{I,max} \cdot \frac{N_1}{N_2}$$

$$= 2.96 \times \frac{30}{15} A$$

$$= 5.92 A$$

$$I_{O,min} = I_{I,min} \frac{N_1}{N_2}$$

$$= 3.68 A$$

$$\Delta I_{O(p-p)} = \frac{N_1}{N_2} \Delta I_{I(p-p)} = 2.25 A$$

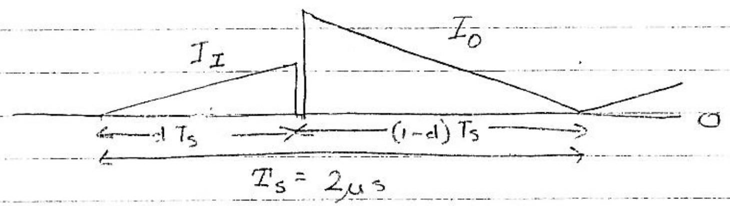
8-2 PED

Flyback converters can operate in continuous-conduction mode, where either switch or diode is conducting, and the transformer is always magnetized or in discontinuous-conduction mode, where neither switch nor diode is conducting for a portion of the cycle, and the transformer is demagnetized for part of the cycle.

The boundary of these two modes is known as the critical-conduction mode. (CCM)

For the problem 8-1, CCM occurs when $I_{I,min} = I_{O,min} = 0$

The waveforms are sketched as follows.



$$I_{Q(d,avg)} = \frac{\Delta I_{I(p-p)}}{2}$$

$$= \frac{1.125 A}{2} = 0.563 A$$

$$I_I = d \; I_{Q(d,avg)}$$

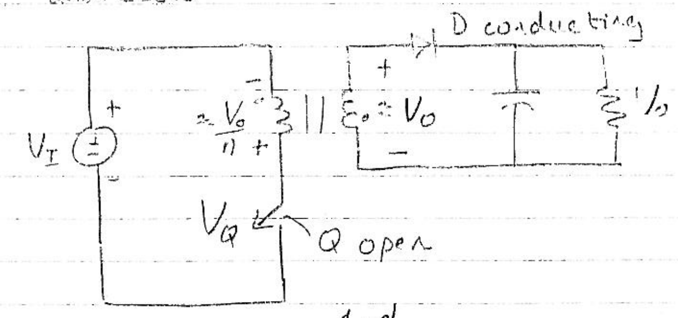
$$= 0.375 \times 0.563 = 0.211 A$$

$$\rightarrow \quad P_I = V_I \cdot I_I$$
$$= 30 \times 0.211$$
$$= 6.33 \text{ W}$$

where $P_I = 6.33\,W$ is the converter power @ CCM, assuming $\eta = 100\%$.

8-4

The maximum voltage across the switch during the off-time $(1-d)$ can be a problem in flybacks and must be addressed



$\underline{\underline{1-d}}$

When Q is open and D conducts, the output voltage V_o is reflected to the primary as V_o/n. Thus the total voltage seen by the switch V_Q is

$$V_Q = V_I + \frac{V_o}{n}$$
$$= 30 + \frac{9}{0.5} \text{ V}$$
$$= 48 \text{ V}$$

The voltage across Q, V_Q, can actually be greater when parasitics are considered

Buck: $\frac{V_O}{V_I} = d$

Boost: $\frac{V_O}{V_I} = \frac{1}{1-d}$

Buck/Boost: $\frac{V_O}{V_I} = -\frac{d}{1-d}$

Flyback: $\frac{V_O}{V_I} = n\frac{d}{1-d}$