EE4011

An RF Model for the BJT

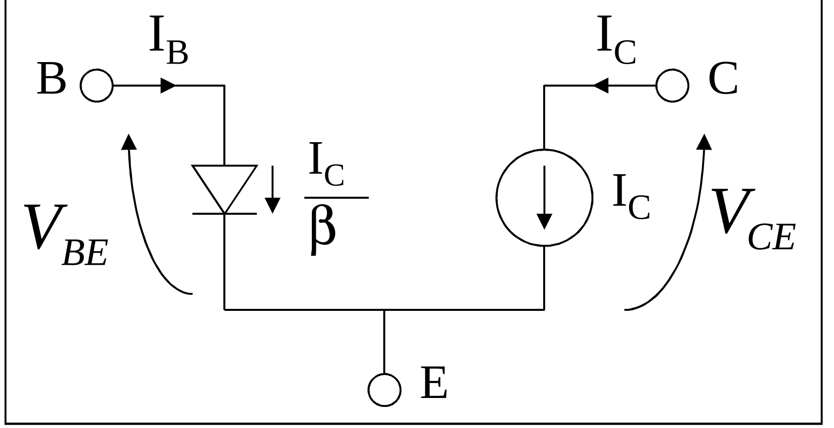
The Cut-Off Frequency, f_T

A Large-Signal Representation of the BJT Common-Emitter Configuration

The core BJT equations

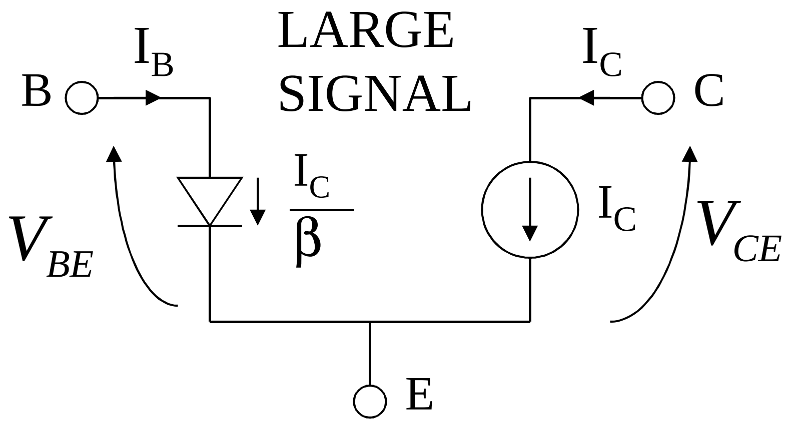
$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1+\frac{V_{CE}}{V_A}\right) \qquad I_B = \frac{I_C}{\beta}$$

Hybrid-Π Large Signal Model



For a BJT in forward-active mode, the device “looks like” a diode connected between the base and emitter, but most of this diode current is “transferred” to the collector and flows as collector current I_C. Thus in forward active mode, the device could be represented by a diode between the base/emitter terminals where the current transfer to the collector is achieved by means of a current source and where only a small fraction of the current (I_C/β) flows in the base terminal. The equivalent circuit is referred to as the large-signal hybrid-π model.

Low-frequency Small-Signal Model (1)



$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{I_C}{\beta} \qquad V_T = \frac{kT}{q} \quad \text{(thermal voltage)}$$

$$V_T = \frac{kT}{q} \approx 25.9mV \text{ at 300K}$$

There are two currents each of which "could be" influenced by each of the two voltages.
Where the current flowing in a terminal changes in response to the voltage on *that* terminal, this can be represented by a resistance or conductance.
Where the current flowing in a terminal changes in response to the voltage on *another* terminal, this can be represented by a transconductance. The combination of resistances and transconductances give rise to the *small-signal* model which is used to determine the response of the device to small changes in the applied bias.

Low-frequency Small-Signal Model (2)

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \approx I_S e^{\frac{V_{BE}}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right) \quad if\ V_{BE} >> V_T$$

The transconductance, g_m, is the derivative of I_C w.r.t. V_{BE}

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{1}{V_T} I_S e^{\frac{V_{BE}}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right) = \frac{I_C}{V_T}$$

so g_m depends linearly on I_C – neat!

The output conductance, g_{out}, is the derivative of I_C w.r.t. V_{CE}

$$g_{out} = \frac{dI_C}{dV_{CE}} = \frac{1}{V_A} I_S e^{\frac{V_{BE}}{V_T}} \approx \frac{I_C}{V_A} \quad \text{if } \frac{V_{CE}}{V_A} << 1$$

so g_{out} depends linearly on I_C also – maybe not so neat, but it's life

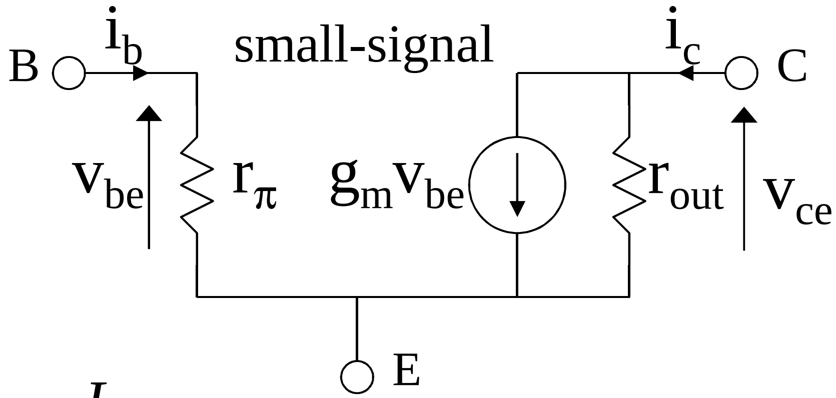
The input conductance, g_π, is the derivative of I_B w.r.t. V_{BE}

$$g_\pi = \frac{dI_B}{dV_{BE}} = \frac{d}{dV_{BE}}\left(\frac{I_C}{\beta}\right) = \frac{1}{\beta}\frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta}$$

so the input conductance is equal to the transconductance divided by the current gain β

Low-frequency Small-Signal Model (3)

It is common to represent the conductances by resistances instead. Thus, the small-signal equivalent circuit for low-frequencies (small-signal hybrid-π) is:

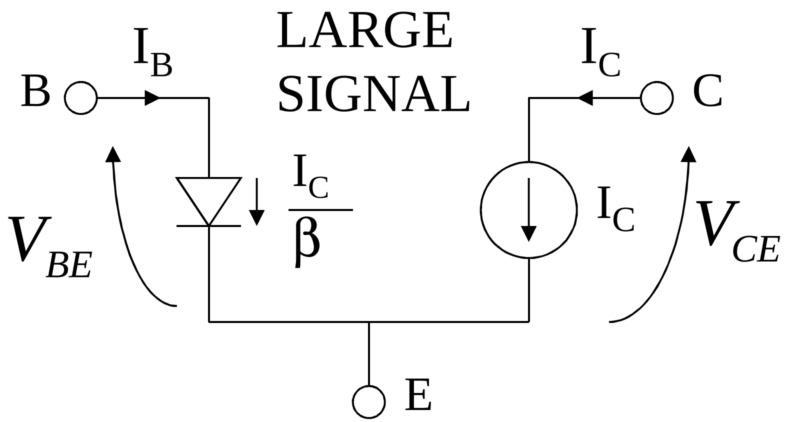


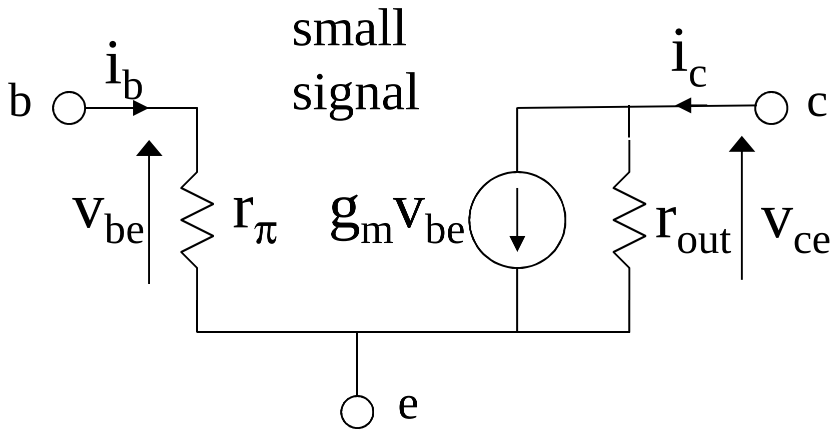
$$g_m = \frac{I_C}{V_T}$$: transconductance proportional to collector current

$$r_{out} = \frac{1}{g_{out}} = \frac{V_A}{I_C}$$: output resistance inversely proportional to collector current

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta}{g_m}$$: input resistance proportional to current gain and inversely proportional to transconductance (and thus collector current)

Summary of BJT Large and Small-Signal Models for Low Frequencies





$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{I_C}{\beta}$$

$$V_T = \frac{kT}{q}$$

I_S, β and V_A are the device parameters

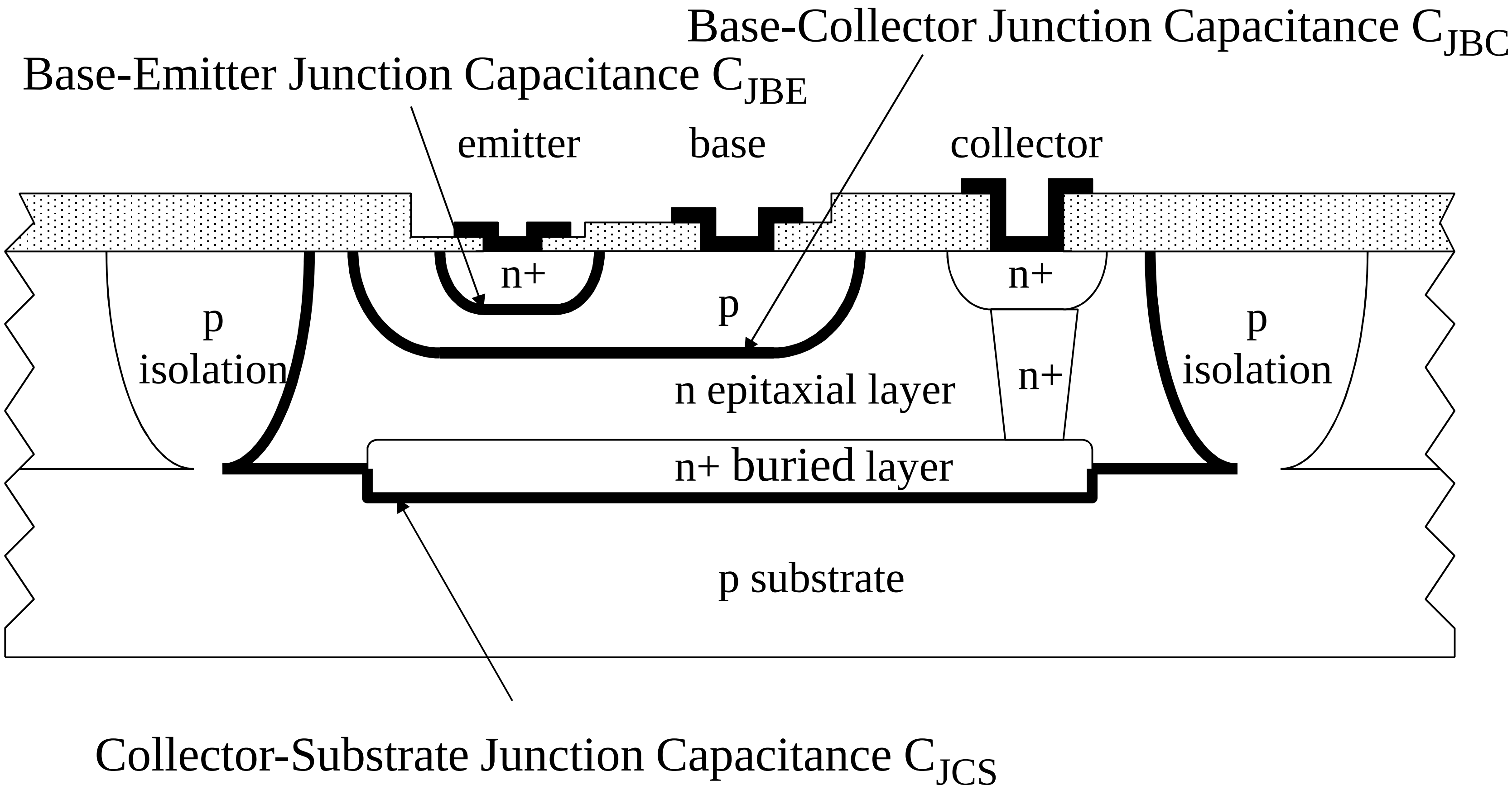
$$g_m = \frac{I_C}{V_T}$$

$$r_{out} = \frac{V_A}{I_C}$$

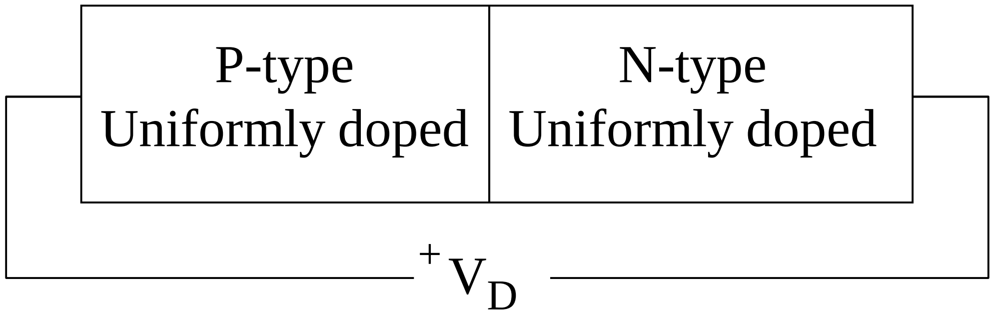
$$r_\pi = \frac{\beta}{g_m}$$

Junction Capacitances

The charge storage elements must be included in the small-signal model to allow the frequency response to be analysed. In a BJT there are several important pn junction capacitances as shown. These are often called depletion capacitances because they are associated with depletion regions.



Capacitance of a pn junction



The capacitance of a depletion region formed at the junction of a uniformly doped p-type material with a uniformly doped n-type material (a step junction) is:

$$C_J = \frac{C_{J0}}{\sqrt{1 - \frac{V_D}{V_J}}}$$

C_{J0} = capacitance at zero bias
V_D = bias across the pn junction
V_J = built-in potential of the junction
In this case the junction grading coefficient is ½ (represented by the square root term). In real devices there isn't usually an ideal "step" change in doping at the junction and a grading coefficient different to ½ is needed.

BJT Junction Capacitance Formulas (in SPICE)

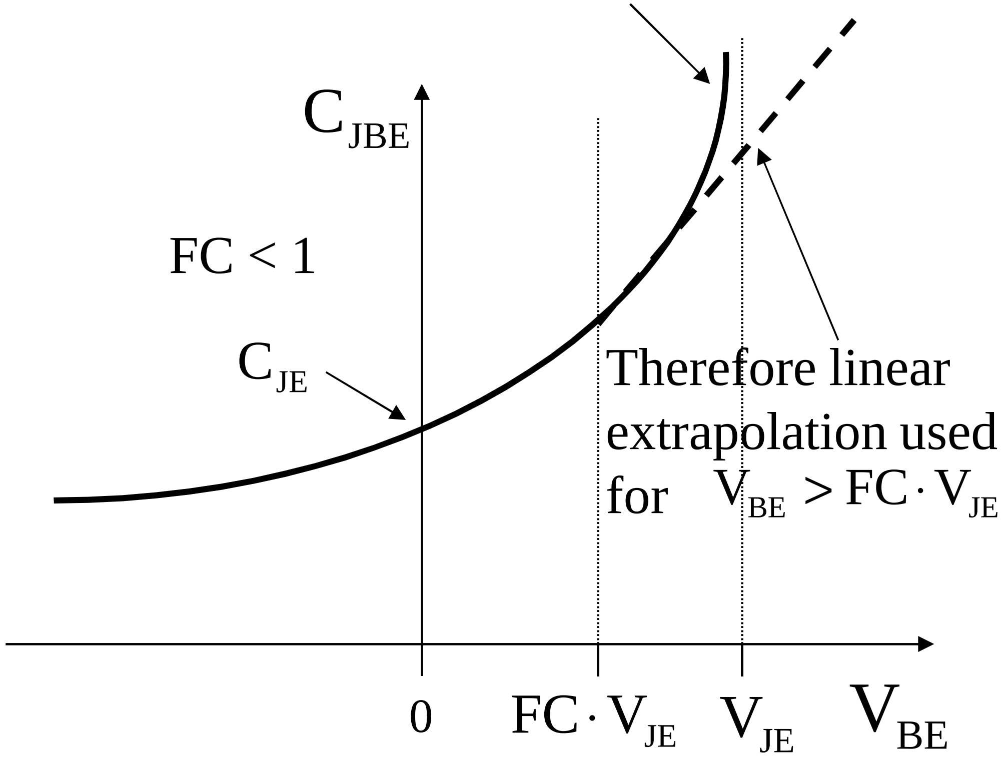
$$C_{JBE} = \frac{C_{JE}}{\left(1 - \frac{V_{BE}}{V_{JE}}\right)^{M_{JE}}} \qquad V_{BE} \le FC \cdot V_{JE}$$

$$C_{JBC} = \frac{C_{JC}}{\left(1 - \frac{V_{BC}}{V_{JC}}\right)^{M_{JC}}} \qquad V_{BC} \le FC \cdot V_{JC}$$

$$C_{JCS} = \frac{C_{JS}}{\left(1 - \frac{V_{CS}}{V_{JS}}\right)^{M_{JS}}} \qquad V_{CS} \le FC \cdot V_{JS}$$

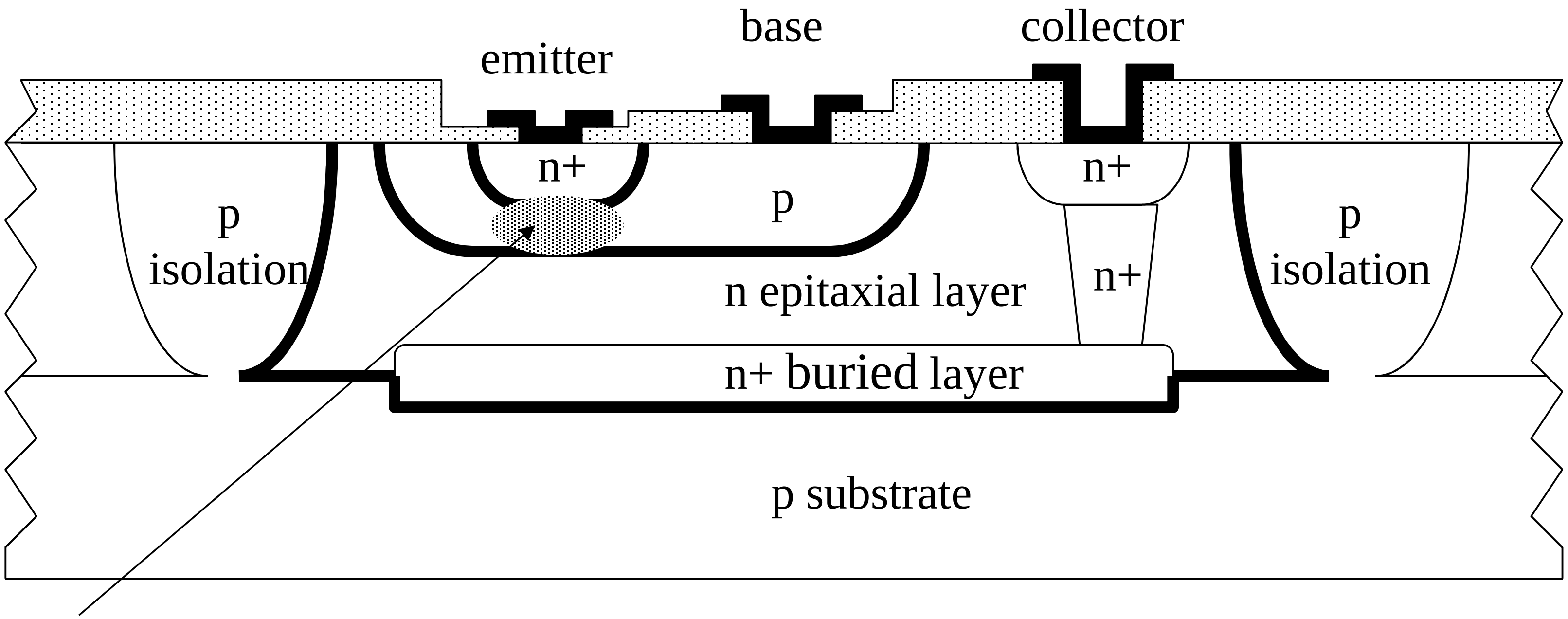
J is for "junction"

Simple capacitance expression would go to infinity at V_{BE}=V_{JE}



Same correction applies to C_{JBC} and C_{JCS}

Minority Carrier Charges in the Base



In the forward active region, there is a build up of minority carriers (electrons in the case of an NPN) in the base region. These minority carriers move mainly by diffusion and their concentration changes in response to changes in V_{BE}. Therefore there is a voltage-dependent minority carrier charge in the base region which is represented by a so-called "diffusion capacitance" between base and emitter.

Diffusion Capacitance

The total base minority carrier charge depends on the collector current and a quantity known as the base transit time, τ, which represents the time it takes a minority carrier to cross from the emitter to the collector.

$$Q = \tau I_C$$

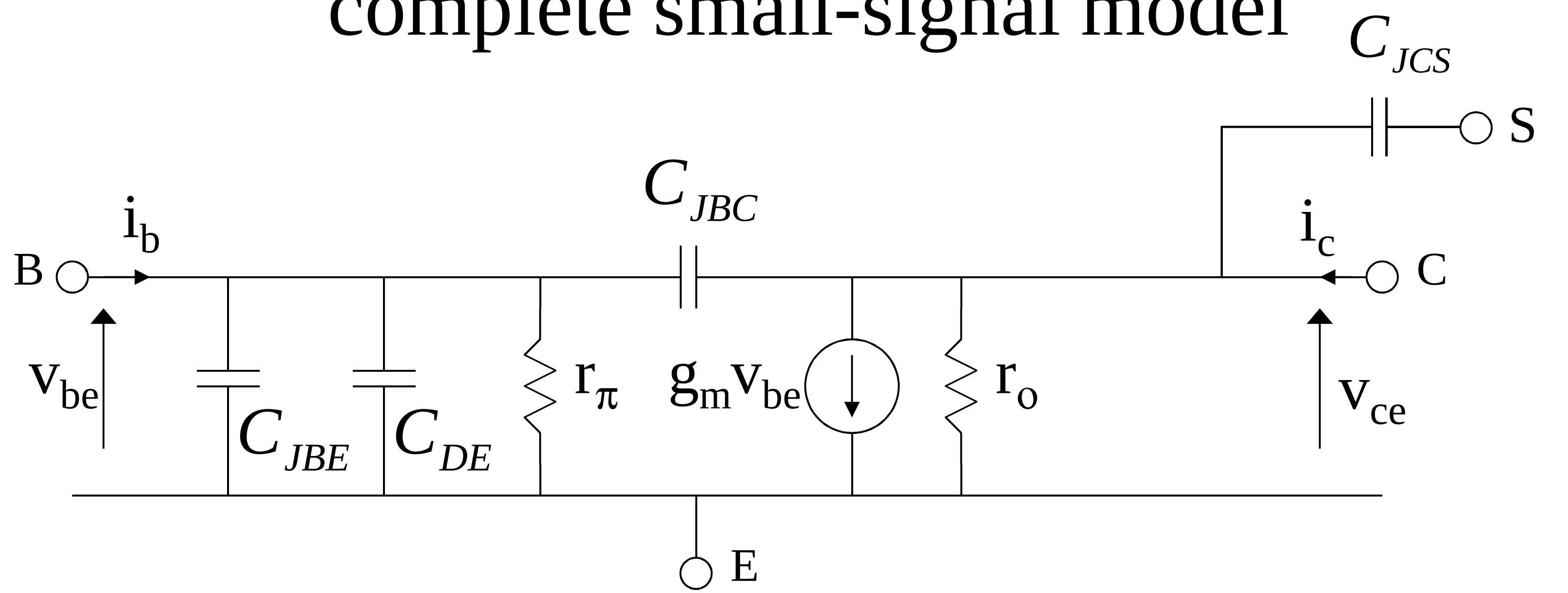
The “diffusion capacitance” is the derivative of Q w.r.t. V_{BE}:

$$C_{DE} = \frac{dQ}{dV_{BE}} = \frac{d(\tau I_C)}{dV_{BE}} = \tau \frac{dI_C}{dV_{BE}} = \tau g_m$$

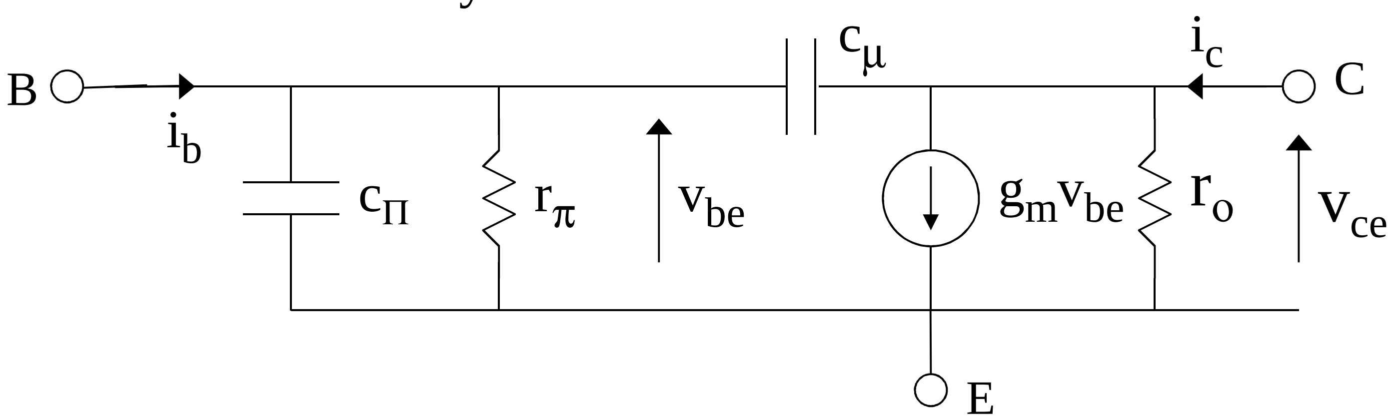
D is for “diffusion”

The diffusion capacitance associated with the minority carrier charges is thus related to the base transit time and the device transconductance.
The base transit time, τ, can be determined from high frequency measurements.

Putting all the capacitances in to make a more complete small-signal model

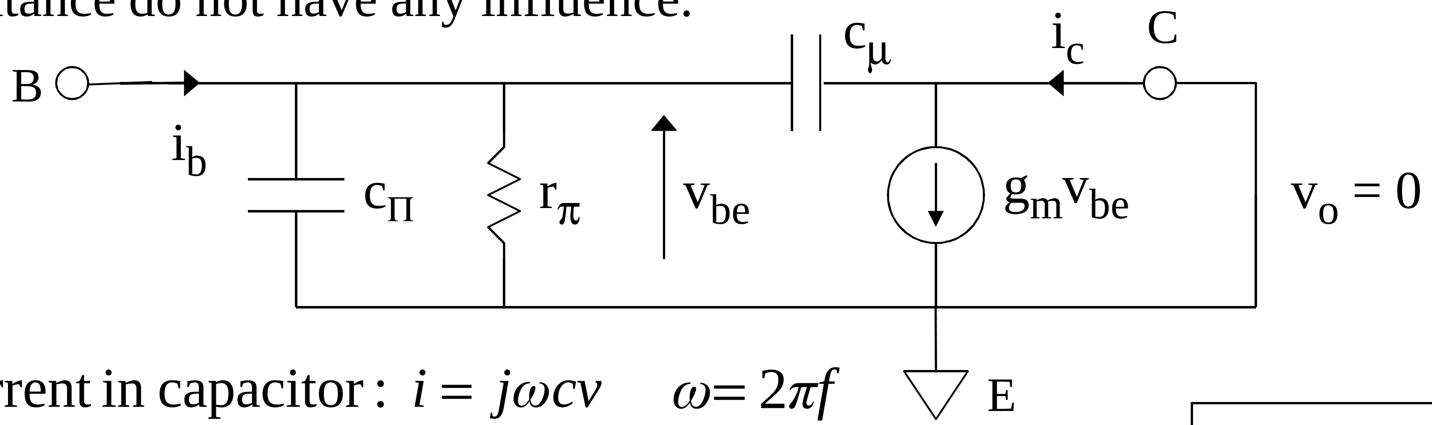


Combining C_{JBE} and C_{DE} and ignoring the substrate gives the widely used small-signal common emitter hybrid-π BJT transistor model:



Short-Circuit Current Gain: h_{fe}

High speed devices are often compared based on a characteristic known as the short-circuit current gain h_{fe} .This is the ratio of output and input currents when the output is short-circuited (the short circuit is for ac signals only – the DC bias is still applied to the device). When the output is short-circuited r_o and collector-substrate capacitance do not have any influence.



ac current in capacitor : $i = j\omega cv \qquad \omega = 2\pi f$

$$i_b = \frac{v_{be}}{r_\pi} + j\omega c_\pi v_{be} + j\omega c_\mu v_{be} = v_{be}\left[g_\pi + j\omega\left(c_\pi + c_\mu\right)\right]$$

$$i_c = g_m v_{be} - j\omega c_\mu v_{be} = v_{be}\left(g_m - j\omega c_\mu\right)$$

$$h_{fe} = \frac{i_c}{i_b} = \frac{g_m - j\omega c_\mu}{g_\pi + j\omega\left(c_\pi + c_\mu\right)}$$

If $\quad g_m >> \omega c_\mu \quad$ then $\quad h_{fe} \approx \dfrac{g_m}{g_\pi + j\omega\left(c_\pi + c_\mu\right)}$

Notation:
$j = \sqrt{-1}$
$i =$ current

Cut-off frequency: f_T

$$h_{fe} = \frac{g_m}{g_\pi + j\omega(c_\pi + c_\mu)} = \frac{g_m}{g_\pi + j2\pi f(c_\pi + c_\mu)}$$

$$\Rightarrow h_{fe} = \frac{g_m / g_\pi}{1 + j2\pi f(c_\pi + c_\mu)/g_\pi} = \frac{\beta}{1 + j2\pi f\, r_\pi(c_\pi + c_\mu)}$$

from earlier definitions:

$$g_\pi = \frac{g_m}{\beta} \Rightarrow \frac{g_m}{g_\pi} = \beta$$

$$r_\pi = \frac{\beta}{g_m} \Rightarrow \frac{\beta}{r_\pi} = g_m$$

For high frequencies:

$$2\pi f\, r_\pi(c_\pi + c_\mu) >> 1 \Rightarrow h_{fe} \approx \frac{\beta}{j2\pi f\, r_\pi(c_\pi + c_\mu)} \Rightarrow |h_{fe}| = \frac{\beta}{2\pi f\, r_\pi(c_\pi + c_\mu)}$$

The **cut-off frequency** is the frequency at which h_{fe} drops to unity

i.e.

$$1 = \frac{\beta}{2\pi f_T\, r_\pi(c_\pi + c_\mu)} \Rightarrow f_T = \frac{\beta}{2\pi r_\pi(c_\pi + c_\mu)} = \frac{g_m}{2\pi(c_\pi + c_\mu)}$$

This is a very common result in transistors – the cut-off frequency is determined by the ratio of the transconductance to the input capacitance. Beyond f_T, the small-signal current flowing in the output circuit is less than the small-signal current flowing in the input circuit so there is no current gain.