OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2012

B.E. (ELECTRICAL AND ELECTRONIC)

DIGITAL SIGNAL PROCESSING
EE4008

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Answer *five* questions.

All questions carry equal marks.

The use of departmental approved non-programmable calculators is permitted

Time Allowed: *3 hours*

Questions Begin on the Next Page

1. (a) Starting with the ideal frequency response $H_d(\omega)$, describe the Windows method of designing a bandpass filter.

[*10 marks*]

(b) Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the Windows method designing a bandpass filter that meets the following specification:

- Passband: $100 - 250$Hz
- Transition Width: 50Hz
- Passband Ripple: < 0.1dB
- Stopband attenuation: > 40dB
- Sampling frequency: 1024Hz

The parameters of common window functions are given in the Appendix.

[*10 marks*]

2. (a) A second order bandstop IIR digital filter has a transfer function

$$H_{BS}(z) = \frac{K(1 - 2\beta z^{-1} + z^{-2})}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$

Where ω_0 is the centre frequency of the Bandstop filter and with $\beta = \cos(\omega_0)$, show that the gain factor K is:

$$K = \frac{1+\alpha}{2}$$

[*6 marks*]

(b) The 3-dB bandwidth of the second order bandstop filter is given by:

$$\Delta\omega_{3db} = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$$

i. Determine the transfer function $H(z)$, of a second order bandstop filter to eliminate 50Hz noise in a system with a sampling frequency of 512Hz. The 3dB bandwidth should be 1Hz.

[*6 marks*]

ii. Determine the constant coefficient difference equation that implements the filter $H(z)$ in the time domain.

[*2 marks*]

iii. Express $H(z)$ in terms of polar co-ordinates and draw the pole/zero plot of $H(z)$.

[*3 marks*]

iv. Sketch the magnitude response $|H(z)|$ of the filter, clearly identifying the bandstop frequency of 50Hz and the 3-dB Bandwidth.

[*3 marks*]

3. (a) The output $y(n)$ of a Causal Linear Time Invariant system is given by

$$y(n) = \sum_{k=0}^{\infty} x(n-k)h(k)$$

where $x(n)$ is the input and $h(n)$ is the impulse response. Show that the Convolution Theorem holds, where the z-transform of the output $Y(z)$ is given by

$$Y(z) = X(z)H(z)$$

where $X(z)$ is the z-transform of $x(n)$ and $H(z)$ is the z-transform of $h(n)$.

[*4 marks*]

(b) Determine the response $y(n)$ of a system with impulse response

$$h(n) = a^n u(n), |a| < 1$$

to the input $x(n) = u(n)$ using z-transforms, the Convolution theorem and partial fractions. For $a = \frac{1}{2}$ determine $y(0), y(1), y(2)$ and $y(3)$.

[*9 marks*]

(c) Using the long division method, determine $y(0), y(1), y(2)$ and $y(3)$, for $a = \frac{1}{2}$ and compare these to the values determined in part (b).

[*7 marks*]

4. (a) Compare the frequency response of the following three Narrow Band Differentiators:

   i. Ideal Differentiator
   ii. First Difference Differentiator
   iii. Central Difference Differentiator

[*10 marks*]

(b) Starting with the ideal frequency response $H_d(\omega)$, determine the ideal impulse response $h_d(n)$ of an ideal broadband differentiator with cutoff frequency ω_c.

[*10 marks*]

5. Describe the steps necessary to design an interpolated lowpass filter using an $L = 2$ comb filter that meets the following specification

   - Passband edge frequency:- $f_{pass} = 0.05$
   - Stopband edge frequency:- $f_{stop} = 0.065$
   - Stopband attenuation:- > 15dB

   Include in your answer the number of multiplications required and a sketch of the magnitude response in dB, over the range $0 \leq f \leq 0.5$ of the original lowpass filter, the prototype filter, the comb filter and the Image reject filter

[*20 marks*]

6. (a) A discrete-time random process $x(n)$ is generated as the output of a linear time-invariant discrete time filter with the following constant coefficient difference equation:

$$x(n) = -0.3x(n-1) + w(n)$$

   where $w(n)$ is the filter input and is white noise with variance σ_w^2.

   i. Find the power spectrum $P_{xx}(\omega)$ of $x(n)$.

[*3 marks*]

   ii. Find the autocorrelation function $\phi_{xx}(k)$

[*7 marks*]

   (b) Describe how the Periodogram method of spectral estimation can be improved by using windowing and averaging to minimise bias and variance at the cost of spectral resolution. Include in your answer a description of the Modified Periodogram using a Hamming window, the Bartlett method and the Welch method with 50% overlap and a Hanning window. Parameters of the window functions are given in the Appendix.

[*10 marks*]

Appendix of Equations

- Window Functions

Window $w(n)$	Sidelobe	$\triangle f$	Stopband Attenuation	Passband Ripple	$\Delta\omega_{3db}$
Rectangular	-13db	$\frac{0.9}{N}$	21db	0.7416db	$0.89\frac{2\pi}{N}$
$w(n) = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hanning	-31db	$\frac{3.1}{N}$	44db	0.0546db	$1.44\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hamming	-41db	$\frac{3.3}{N}$	53db	0.0194db	$1.30\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					

- Table of z-Transforms

Signal $x(n)$	z-Transform $X(z)$	ROC
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > a$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\| < a$

- Integration

$f(x)$	$\int f(x)dx$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\cos x$	$\sin x$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$

- Integration by parts

$$\int u\,dv = uv - \int v\,du$$

- Euler Identity

$$\cos x = \frac{1}{2}\left(e^{-jx} + e^{jx}\right)$$

$$\sin x = \frac{1}{2}j\left(e^{-jx} - e^{jx}\right)$$

- Trigonometric Identities

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$