Summer 07 Q7

Given $GF(2^4)$

0
1
α
α^2
α^3
$\alpha^4 = \alpha + 1$

$\alpha^5 = \alpha^2 + \alpha$
$\alpha^6 = \alpha^3 + \alpha^2$
$\alpha^7 = \alpha^3 + \alpha + 1$
$\alpha^8 = \alpha^2 + 1$
$\alpha^9 = \alpha^3 + \alpha$

$\alpha^{10} = \alpha^2 + \alpha + 1$
$\alpha^{11} = \alpha^3 + \alpha^2 + \alpha$
$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha + 1$
$\alpha^{13} = \alpha^3 + \alpha^2 + 1$
$\alpha^{14} = \alpha^3 + 1$

$\boxed{\alpha^{15} = 1}$

(a) Show

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1$$

for $(15,7)$ code. $t=2$.

$$g(x) = LCM(m_1(x), m_3(x))$$

$$m_1(x) = \underbrace{(x+\alpha)(x+\alpha^2)}_{x^2+\alpha^5 x+\alpha^3}\underbrace{(x+\alpha^4)(x+\alpha^8)}_{x^2+\alpha^5 x+\alpha^{12}}$$

$\Rightarrow m_1(x) = x^4 + x + 1$ (show)

$$m_3(x) = (x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})$$
$$(x+\alpha^9)$$
$$=\alpha^{24} = \alpha^{15}.\alpha^9 = \alpha^9$$
$$= x^4+x^3+x^2+x+1$$

$g(x) = M_1(x).M_3(x)$ etc. [7]

(b) $$V(x) = x^{12}+x^{10}+x^9+x^7+$$
$$x^6+x^4+x^3+x+1$$

Determine $e(x)$

$$S_1 = V(\alpha) = \alpha^3+\alpha^2+1 = \boxed{\alpha^{13} = S_1}$$

$$S_3 = V(\alpha^3) = \alpha^3+\alpha = \alpha^9 = S_3$$

Error locator poly:

$$x^2+S_1x+\frac{S_1^3+S_3}{S_1}=0$$

$=0$

$$x(x+S_1)=0$$

$$\Rightarrow x = S_1 \Rightarrow X_1 = S_1 = \alpha^{13}$$

$\Rightarrow$ $e(x) = x^{13}$, one error occurred (probably)

$\Rightarrow$ $c(x) = v(x) + e(x)$ etc.

(c) $t(x) = \dfrac{c(x)}{g(x)} = x^5 + x^3 + x + 1$

<u>Example</u> $(15,7)$ code, $t=2$, $GF(2^4)$

Assume

$$v(x) = x^{11} + x^8 + x^7 + x^3$$

Decode …

$$v(\alpha) = (\cancel{\alpha^3} + \cancel{\alpha^2} + \cancel{\alpha}) + (\cancel{\alpha^2} + \cancel{1}) + (\cancel{\alpha^3} + \cancel{\alpha} + \cancel{1}) + \alpha^3 = S_1$$

$$v(\alpha^3) = \alpha^{33} + \alpha^{24} + \alpha^{21} + \alpha^{9}$$

$$= \alpha^{15}\cdot\alpha^{15}\cdot\alpha^{3} + \alpha^{15}\cdot\alpha^{9} + \alpha^{15}\cdot\alpha^{6} + \alpha^{9}$$

$$= \alpha^{3} + \cancel{\alpha^{9}} + \alpha^{6} + \cancel{\alpha^{9}}$$

$$= \boxed{\alpha^{2} = S_3}$$

$$\Rightarrow x^2 + \alpha^3 x + \frac{(\alpha^3)^3 + \alpha^2}{\alpha^3} = 0$$

$$\frac{\alpha^9 + \alpha^2}{\alpha^3} = \frac{\alpha^{12}(\alpha^9 + \alpha^2)}{\alpha^{12}\cdot\alpha^3}$$

$$= \alpha^{21} + \alpha^{14}$$

$$= \alpha^{6} + \alpha^{14} = \alpha^{2} + 1$$

$$\Rightarrow x^2 + \alpha^3 x + \alpha^2 + 1 = 0$$

$x = 1 : \alpha^3 + \alpha^2 \neq 0$ NO.

$x = \alpha : \alpha^2 + \alpha^4 + \alpha^2 + 1 = \alpha^4 + 1 \neq 0$

$= \alpha^2 : \alpha^4 + \alpha^5 + \alpha^2 + 1 = 0$ ✓

$\Rightarrow x^2 \in e(x)$

Also,

$X_2 = X_1 + S_1 = \alpha^2 + \alpha^3 = \alpha^6$

$\Rightarrow x^6 \in e(x)$

$\Rightarrow e(x) = x^6 + x^2$

$\Rightarrow c(x) = x^{11} + x^8 + x^7 + x^6 + x^3 + x^2$