S11 Q6. 27/03/13.

(a)

$$\frac{Y}{U} = \frac{k(1+s\tau_2)}{s(1+s\tau_1)} = \frac{k(1+s\tau_2)}{s^2\tau_1+s}$$

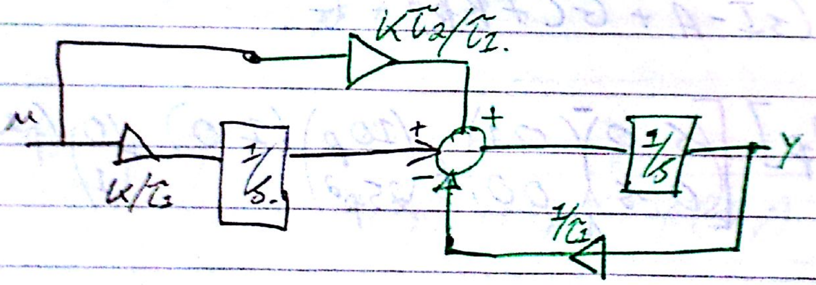
$$\frac{Y}{\mu} = \frac{\frac{K}{\tau_1} + s\frac{k\tau_2}{\tau_1}}{s^2+\frac{1}{\tau_1}s}$$

$$= \frac{\frac{K/\tau_1}{s^2} + \frac{K\tau_2/\tau_1}{s}}{1+\frac{1/\tau_1}{s}}$$

$$\left(1+\frac{1/\tau_1}{s}\right)Y = \left(\frac{k/\tau_1}{s^2} + \frac{K\tau_2/\tau_1}{s}\right)\mu$$

$$Y = \frac{-1/\tau_1}{s}Y + \frac{K/\tau_1}{s^2}\mu + \frac{k\tau_2/\tau_1}{s}\mu.$$

$$= \frac{1}{s}\left[K\tau_2/\tau_1\,\mu - \frac{1}{\tau_1}Y + \frac{K/\tau_1}{s}\mu\right]$$




(c) 1) Design regulator → get $K_1, K_2$.

$\mu = -k_1\theta - k_2\omega$ $\quad k = -[k_1\ k_2]\begin{bmatrix}\theta\\ \omega\end{bmatrix}$

$$|sI - A + Bk| = C_{des} = (s+p)^2.$$
$$= s^2 + 2ps + p^2$$

$$\left|\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(k_1 \quad k_2)\right|$$

$$\begin{vmatrix} s & -1 \\ k_1 & s+k_2 \end{vmatrix} = s^2 + k_2 s + k_1$$

$$= s^2 + 2ps + p^2.$$

$$k_2 = 2p \quad , k_1 = p^2.$$

<u>Estimator</u>

$$|sI - A + GC| = C_{des} = (s+5p)^2 = s^2 + 10ps + 25p^2.$$

$$\left|\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}(1 \quad 0)\right| = s^2 + G_1 s + G_2.$$

$$\therefore G_1 = 10p \,, \quad G_2 = 25p^2.$$

$$C_{eq}(s) = k(sI - A + GC + Bk)^{-1} G.$$

$$[p^2 \quad 2p]\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 10p \\ 25p^2 \end{pmatrix}(1 \quad 0) + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(p^2 \; 2p)\right]^{-1}\begin{bmatrix} 10p \\ 25p^2 \end{bmatrix}$$

$$\Rightarrow [p^2 \quad 2p]\begin{bmatrix} s+10p & -1 \\ 26p^2 & s+2p \end{bmatrix}^{-1}\begin{bmatrix} 10p \\ 25p^2 \end{bmatrix}$$

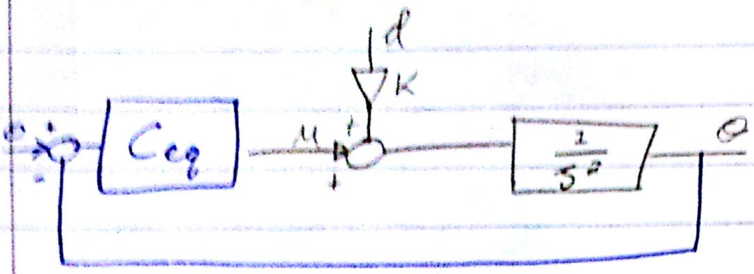
$$\Rightarrow \frac{60p^3 s + 25p^4}{s^2 + 12ps + (20p^2 + 26p^2)}$$

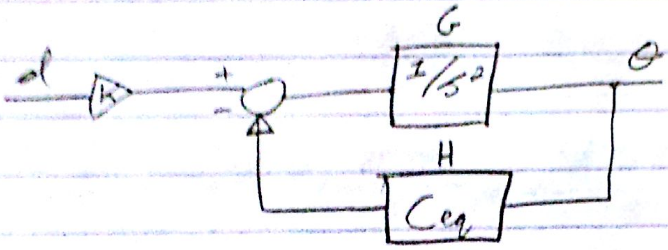
Another possible question:

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ k \end{bmatrix} d.$$

$$G_D(s) = \frac{\theta}{d} = C(sI-A)^{-1}E. = \frac{K}{s^2}$$

$$G(s) = \frac{\theta}{u} = C(sI-A)^{-1}B = \frac{1}{s^2}$$



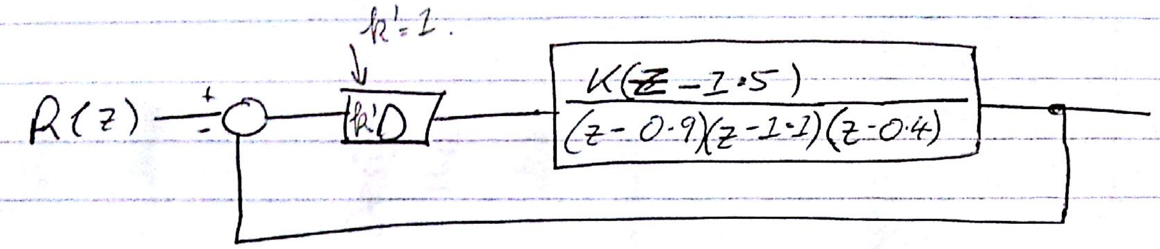

$$\theta/d = \frac{G}{1+GH} \cdot K.$$

$$= \frac{1/s^2 \cdot K}{1 + 1/s^2\, C_{eq}}.$$

$$C_{eq} = \frac{60p^3 s + 25p^4}{s^2 + 12ps + 46p^2},$$

$$G_{cl} = \frac{\theta}{d} = \frac{(\qquad\qquad)}{(s+5p)(s+5p)(s+p)(s+p)}.$$

Generic Q that appears: (Q1 OR Q2 usually). 27/03/13.




Show using a root locus plot how a deadbeat controller cannot satisfactorily control this process!

$$D = \frac{1}{G_m} \frac{C/R}{1 - C/R} \qquad C/R \leftarrow z^{-(N+1)} \qquad \frac{k(z-1.5)}{z^3 \ldots\ldots}$$

$$\frac{Y}{U} = \frac{Kz - 1.5K}{z^3 + \ldots\ldots a}$$

$$(z^3 + \ldots\ldots a)y = (kz - 1.5k)u$$

$$y(k+3) = y(k+2)\, y(k+1)\, y(k) \quad u(k+1)\, u(k).$$

$$y(k+1) = y(k)\, y(k-1)\, y(k-2)\, u(k-1)\, u(k-2).$$

$(N+1) = 2.$ $\qquad C/R = z^{-2}.$

Perfect model except the unstable pole NMP zero

$$G_m = \frac{K(z - (1.5 + \delta_1))}{(z - 0.9)(z - (1.1 + \delta_2))(z - 0.4)}.$$