ME4001 Engineering Management Summer '06

1 a) Reliability: The probability that an item will perform a required function under stated conditions for a stated time.

$$R = x/N$$

$$Q = y/N$$

$$R(t) + Q(t) = 1$$

Failure Rate: The number of failures per unit time expressed as a fraction of the survivors:

$$\lambda(t) = \frac{(dy/dt)}{x}, \quad x = N - y$$

$$\lambda(t) = \frac{(dy/dt)}{N} \cdot \frac{N}{N-y} = \frac{(dy/dt)}{N} \cdot \frac{1}{1-y/N}$$

$$= \frac{f(t)}{R(t)} = \frac{-\frac{dR}{dt}}{R(t)}$$

Failure Density Function: The number of failures per unit time expressed as a fraction of the original population:

$$f(t) = (dy/dt)/N$$

$$y = N \int_0^t f(t)\, dt$$

$$Q = \int_0^t f(t)\, dt$$

$$\Rightarrow R = 1 - \int_0^t f(t)\, dt$$

$$f(t) = \frac{dQ}{dt} = -\frac{dR}{dt}$$

ME4001 Engineering Management Summer '06

1 a) $\lambda = 0.0143$

$R(t) = e^{-\lambda t}$

$f(t) = +\lambda e^{-\lambda t}$

$R(100) = e^{-1.43} = 0.24$

$= 24\%$

b)

@ $t = 250$

$R_A = 0.9$

$R_B = 0.85$

$R_{C1} = 0.95$

$R_{C2} = 0.98$

$R_{C3} = 0.9$

~~$R_C = 1 - (1-R_{C1})(1-R_{C2})(1-R_{C3}) = 0.9999$~~

Prob. all 3 working : 0.8379 $(R_{C1} \times R_{C2} \times R_{C3})$

Prob. C_1, C_2 working : 0.0931 $(R_{C1} \times R_{C2} \times Q_{C3})$

Prob. C_1, C_3 working : 0.0171 $(R_{C1} \times R_{C3} \times Q_{C2})$

Prob. C_2, C_3 working : 0.0441 $(R_{C2} \times R_{C3} \times Q_{C1})$

$R_C = 0.9922$

$R_{SYS} = R_A \times R_B \times R_C$

$= 0.759$

$MTBF = \int_0^t R(t)\, dt$

$= -\frac{1}{\lambda} e^{-\lambda t}$

$= 31,926\, e^{-0.000031322 t}$

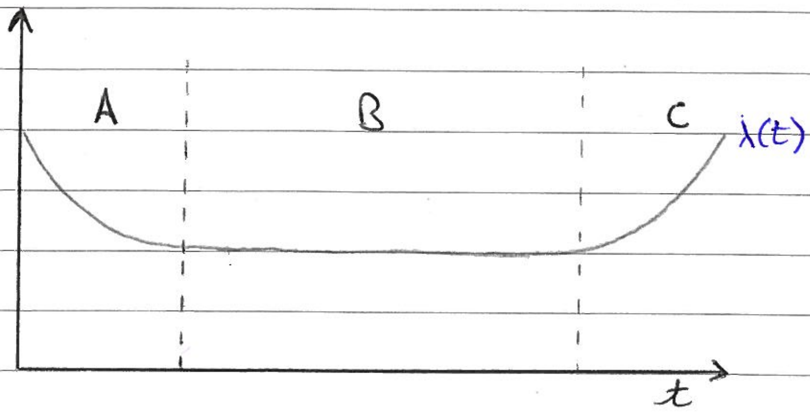
@ $t = 250$ $= 31,677h.$

$R(250) = 0.9922 = e^{-\lambda 250}$

$-\ln(0.9922) = \lambda 250$

$\lambda = 0.000031322$

6a) The typical failure rate curve for a batch of components is known as the bathtub curve. It comprises 3 main sections, marked A, B & C below.



A:

- "Born-in" region, manufacturing defects are found here

B:

- Normal operating region, failure is due to random causes
- Failure analysis centred here

C:

- Wear-out region, maintenance & replacement policies applicable here.

$R_A = R_C = e^{-0.025} = 0.9753$

$R_B = R_D = e^{-0.0375} = 0.9632$

$R_{CD} = 1 - Q_C Q_D = 0.9991$

$R_{SYS} = R_A \times R_B \times R_{CD} = 0.9386$ $\lambda_{sys} = 0.0002535$

$MTBF = \int_0^T R(t) = -\frac{1}{\lambda} e^{-\lambda t} = 3{,}702.5$ hours

ME4001 Engineering Management Summer '05

6 b) $f(t) = \frac{2}{a} - \frac{2t}{a^2}$

$$f(t) = -\frac{dR(t)}{dt} = \frac{dQ(t)}{dt}$$

$$Q(t) = \int_0^t f(t)\,dt$$

$$Q(t) = \frac{2t}{a} - \frac{t^2}{a^2}$$

$$R(t) = 1 - \frac{2t}{a} + \frac{t^2}{a^2}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\frac{2}{a} - \frac{2t}{a^2}}{1 - \frac{2t}{a} - \frac{t^2}{a^2}}$$

Expected no. of operating hours = $R(t) \times$ Total Time

$a = 10$ $\quad R(t) = 1 - \frac{t}{5} + \frac{t^2}{100}$

2 years $\quad R(2) = 0.64$

(or $\int_0^T R(t)\,dt$)

Expected no. of operating hours = 1.28 years

= 11,182.1 hours

ME 4001 Engineering Management Summer '04

5a) Reliability: The probability that an item will perform the required task under stated conditions for a stated period of time.

$$R(t) + Q(t) = 1$$

Failure Density Function: The number of failures per unit time expressed as a fraction of the original population

$$f(t) = (dy/dt)/N$$

$$y = N\int_0^t f(t)\,dt$$

$$Q(t) = \int_0^t f(t)\,dt$$

$$f(t) = \frac{dQ}{dt} = -\frac{dR}{dt}$$

Failure Rate: The number of failures per unit time expressed as a fraction of the ~~original~~ survivors.

$$\lambda(t) = (dy/dt)/x \quad , \; x = N - y$$

$$\lambda(t) = \frac{(dy/dt)}{N} \cdot \frac{1}{1 - y/n}$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

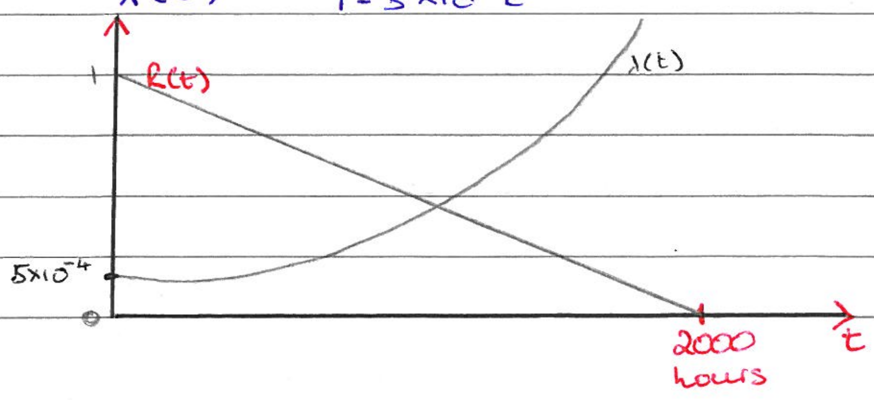
MTBF: The mean number of component hours between breakdowns.

$$MTBF = \int_0^T R(t)\,dt$$

$$f(t) = \text{constant} = 5\times10^{-4}$$

$$R(t) = 1 - (5\times10^{-4}t)$$

$$\lambda(t) = \frac{5\times10^{-4}}{1 - 5\times10^{-4}t}$$



5 b) Mean lifetime = 7,500 hrs

δ = 250 hrs

Max. no. of returns < 1%

$$z = 2.327 = -\frac{(x-\mu)}{\delta} \Rightarrow x = 6918.25 \text{ hours}$$

@ 6,500 hours

$$z = \frac{-(-1000)}{250} = 4$$

R = 0.99997

Q = 0.00003

0.03 failures / 1000 parts

Cost per 1000 sales = €0.90