Solutions UE4010 Summer 2004 Part A

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an nmos in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n = \lambda_p = 0$.
In each question, capacitances other than those mentioned may be ignored.

Question 1

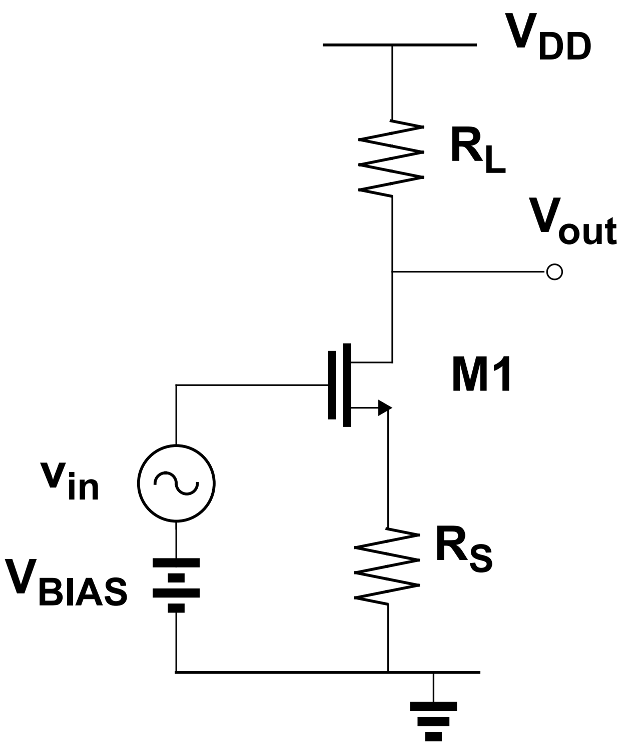


Figure 1

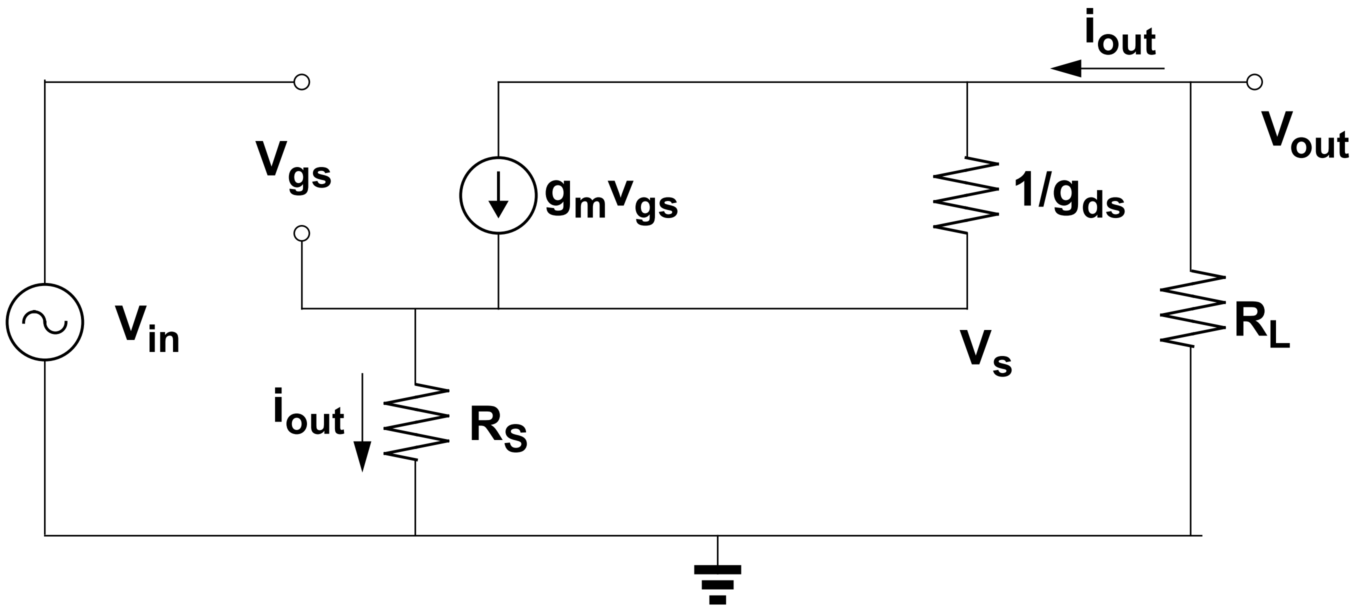
(i) Draw the small-signal equivalent circuit for the gain stage shown in Figure 1.
(ii) Show that (assuming $g_{m1} >> g_{ds1}$, $1/g_{ds1} >> R_L$, $1/g_{ds1} >> R_S$) the small-signal voltage gain is given by

$$\frac{v_{out}}{v_{in}} = -\frac{g_m R_L}{1 + g_m R_S}$$

(iii) Calculate the small-signal voltage gain with the following conditions:
$K_n' = 200\mu A/V^2$, $V_{tn} = 0.7V$, $I_{D1} = 200\mu A$, $R_L = 10k\Omega$, $R_S = 1k\Omega$, $V_{DD} = 5V$, $W/L_{M1} = 12.5/1$
(iv) What is the largest value of gain that can be achieved by increasing R_L?

Solution

(i) Draw the small signal equivalent circuit for the gain stage shown in Figure 1.



(ii) Show that (assuming g_{m1}>>g_{ds1}, $1/g_{ds1}$ >> R_L, $1/g_{ds1}$ >> R_S) the low-frequency small signal voltage gain is given by

$$\frac{v_{out}}{v_{in}} = -\frac{g_m R_L}{1 + g_m R_S}$$

$$v_{out} = -i_{out} R_L$$

$$i_{out} = g_m v_{gs} + (v_{out} - v_s) g_{ds}$$

$$= g_m (v_{in} - v_s) + (v_{out} - v_s) g_{ds}$$

$$= g_m v_{in} - g_m i_{out} R_S + v_{out} g_{ds} - i_{out} R_S g_{ds} \qquad v_s = i_{out} R_S$$

$$= \frac{g_m v_{in} + v_{out} g_{ds}}{1 + g_m R_S + g_{ds} R_S}$$

$$v_{out} = -i_{out} R_L$$

$$v_{out} = -\left(\frac{g_m v_{in} + v_{out} g_{ds}}{1 + g_m R_S + g_{ds} R_S}\right) R_L$$

$$v_{out}\left(1 + \frac{g_{ds} R_L}{1 + g_m R_S + g_{ds} R_S}\right) = -\frac{g_m v_{in} R_L}{1 + g_m R_S + g_{ds} R_S}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_m R_L}{1 + g_m R_S + g_{ds} R_S + g_{ds} R_L}$$

If g_m>>g_{ds} and $1/g_{ds}$>>R_L (i.e. $g_{ds} R_L$ << 1) this reduces to

$$\frac{v_{out}}{v_{in}} = -\frac{g_m R_L}{1 + g_m R_S}$$

(iii) Calculate the low-frequency small signal voltage gain with the following conditions: $K_n{}'=200\mu A/V^2$, $V_{tn}=0.7V$, $I_{D1}=200\mu A$, $R_L=10k\Omega$, $R_S=1k\Omega$, $V_{DD}=5V$, $W/L_{M1}=12.5/1$

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$$V_{GS}-V_t = \sqrt{\frac{2I_{D1}}{K_n{}'\frac{W_1}{L_1}}} = \sqrt{\frac{2\cdot 200\mu A}{200\mu A/V^2\frac{12.5}{1}}} = 400mV$$

$$g_m = \frac{2I_{D1}}{(V_{GS}-V_t)} = \frac{2\times 200\mu A}{0.4} = 1000\mu A/V$$

$$\left(\frac{v_{out}}{v_{in}}\right) = -\frac{g_mR_L}{1+g_mR_S} = -\frac{1000\mu A/V\cdot 10k}{1+1000\mu A/V\cdot 1k} = -\frac{10}{2} = -5 = 14dB$$

(iv) What is the largest value of gain that can be achieved by increasing R_L?

R_L can be increased until the voltage at V_{out} forces M1 out of saturation. For M1 to be still in saturation

$$V_{OUT} - V_S \geq V_{GS} - V_t$$

$$V_{OUTmax} = V_{GS} - V_t + V_S$$

$$V_{OUTmax} = V_{GS} - V_t + I_D R_S$$

$$V_{OUTmax} = 0.4V + 200\mu A \cdot 1k\Omega$$

$$V_{OUTmax} = 0.6V$$

$$V_{DD} - V_{OUTmax} = I_D R_{Lmax}$$

$$R_{Lmax} = \frac{V_{DD} - V_{OUTmax}}{I_D}$$

$$R_{Lmax} = \frac{5 - 0.6}{200\mu A} = 22k\Omega$$

$$\left(\frac{v_{out}}{v_{in}}\right)_{max} = -\frac{g_m R_L}{1 + g_m R_S} = -\frac{1000\mu A/V \cdot 22k}{1 + 1000\mu A/V \cdot 1k} = -\frac{22}{2} = -11$$

$$\left(\frac{v_{out}}{v_{in}}\right)_{max} = -11 = \underline{\underline{20.8dB}}$$

Question 2

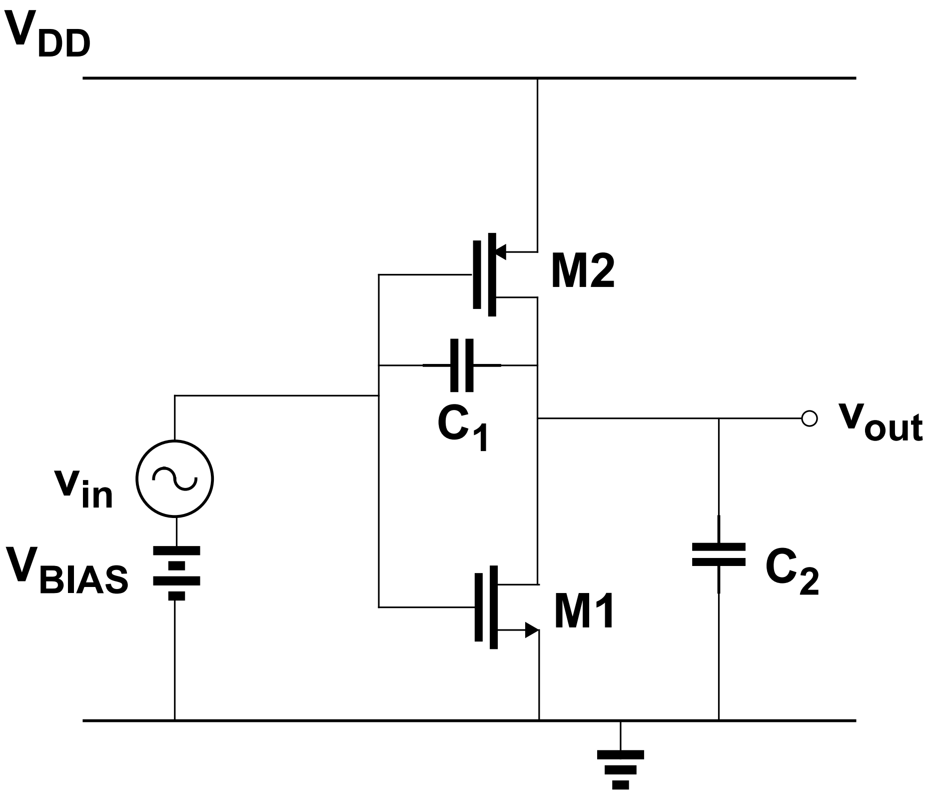

Figure 2

(i) Draw the small signal equivalent circuit for the CMOS Inverter stage shown in Figure 2.
(ii) Ignoring all capacitances except C_1 and C_2 derive an expression for the high frequency transfer function (v_{out}/v_{in})
(iii) Calculate the pole and zero frequencies if
V_{BIAS}=1.5V, V_{tn}=0.7V, V_{tp}=-0.7V, $\lambda_n=\lambda_p$=0.04V^{-1}.C_1=0.1pF, C_2=1.5pF.
Assume both transistors are in saturation with a drain current of 200µA.
(iv) Draw a Bode diagram of the gain. Indicate in dB the dc gain and the gain at frequencies well above the pole and zero frequencies.

(i) Draw the small signal equivalent circuit for the CMOS Inverter stage shown in Figure 2.

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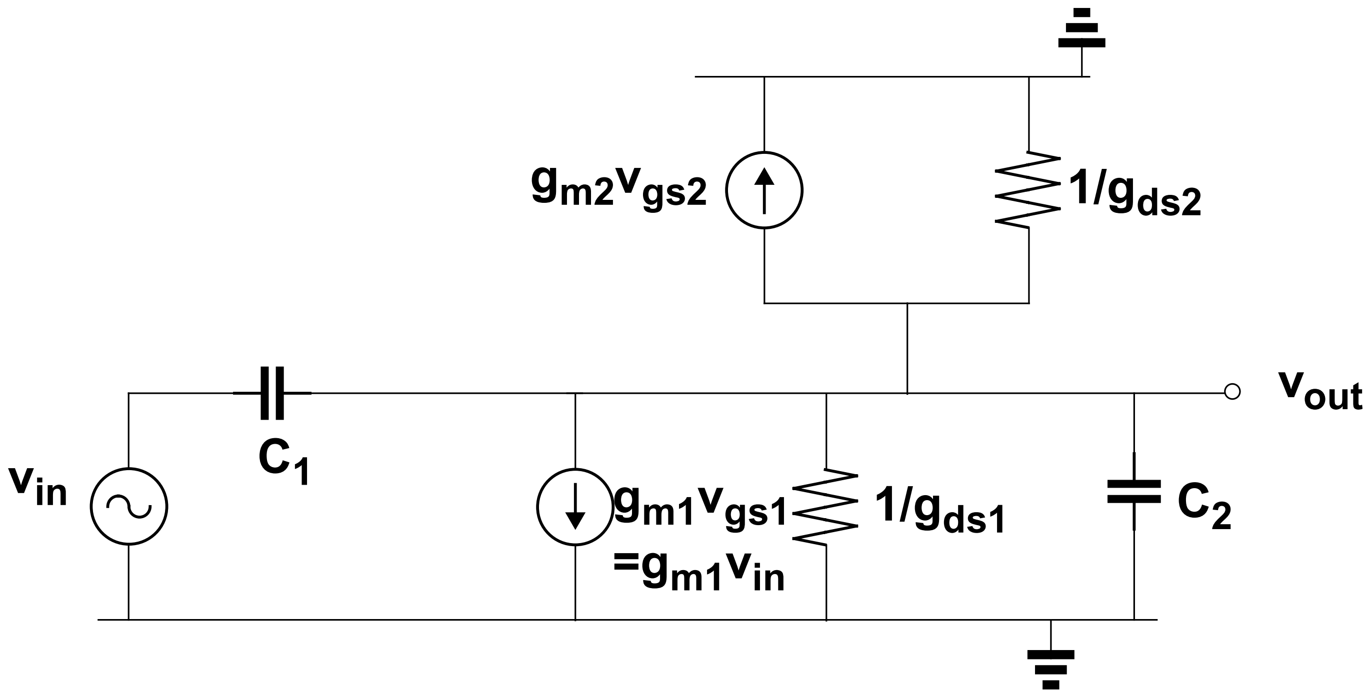


(ii) Ignoring all capacitances except C_1 and C_2 derive an expression for the high frequency transfer function (v_{out}/v_{in})

KCL at output node

$$(v_{out} - v_{in})sC_1 + g_{m1}v_{in} + g_{m2}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} + v_{out}sC_2 = 0$$

$$(g_{m1} + g_{m1} - sC_1)v_{in} = -(g_{ds1} + g_{ds2} + sC_{gs1} + sC_L)v_{out}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2} - sC_1}{g_{ds1} + g_{ds2} + sC_1 + sC_2}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2}} \frac{\left(1 - \frac{sC_1}{g_{m1} + g_{m2}}\right)}{\left(1 + \frac{s(C_1 + C_2)}{g_{ds1} + g_{ds2}}\right)}$$

(iii) Calculate the pole and zero frequencies if
V_{BIAS}=1.5V, V_{tn}=0.7V, V_{tp}=-0.7V, $\lambda_n=\lambda_p$=0.04V^{-1}.C_1=0.1pF, C_2=1.5pF.
Assume both transistors are in saturation with a drain current of 200μA.

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}+g_{m2}}{g_{ds1}+g_{ds2}} \frac{\left(1-\frac{sC_1}{g_{m1}+g_{m2}}\right)}{\left(1+\frac{s(C_1+C_2)}{g_{ds1}+g_{ds2}}\right)}$$

Zero frequency given by

$$|\omega_Z| = \frac{g_{m1}+g_{m2}}{C_1}$$

Pole frequency given by

$$|\omega_p| = \frac{g_{ds1}+g_{ds2}}{(C_1+C_2)}$$

$$g_{m1} = \frac{2I_D}{(V_{GS1}-V_{tn})} = \frac{2\times 200\mu A}{1.5-0.7} = 500\mu A/V$$

$$g_{m2} = \frac{2I_D}{(V_{GS2}-V_{tp})} = \frac{2\times 200\mu A}{1.5-0.7} = 500\mu A/V$$

$$g_{ds1} = \lambda I_D = 0.04V^{-1}\times 200\mu A = 8\mu A/V$$

$$g_{ds2} = \lambda I_D = 0.04V^{-1}\times 200\mu A = 8\mu A/V$$

$$|\omega_p| = \frac{16\mu A/V}{0.1pF+1.5pF} = \underline{\underline{10Mrad/s}}$$

$$|\omega_z| = \frac{1000\mu A/V}{0.1pF} = \underline{\underline{10Grad/s}}$$

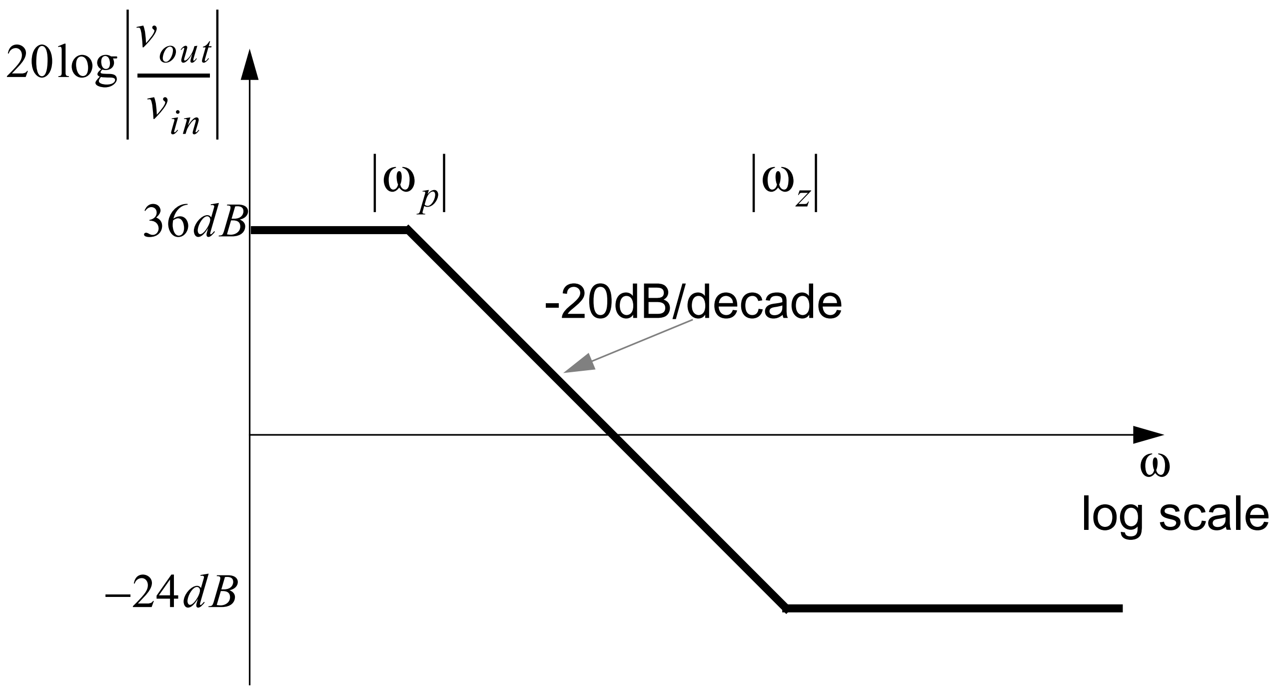
(iv) Draw a Bode diagram of the gain. Indicate in dB the dc gain and the gain at frequencies well above the pole and zero frequencies..

DC gain given by

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}+g_{m2}}{g_{ds1}+g_{ds2}} = -\frac{500\mu A/V + 500\mu A/V}{8\mu A/V + 8\mu A/V} = -62.5$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 36dB$$

Zero is 3 decades up gives HF gain = -24dB.



Question 4

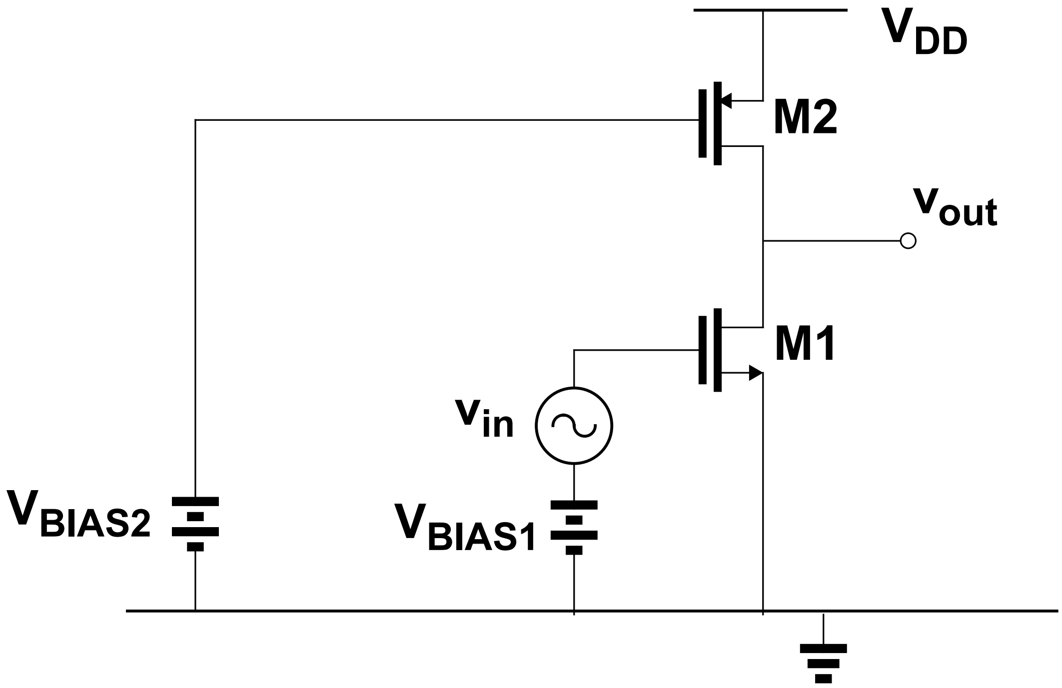


Figure 4

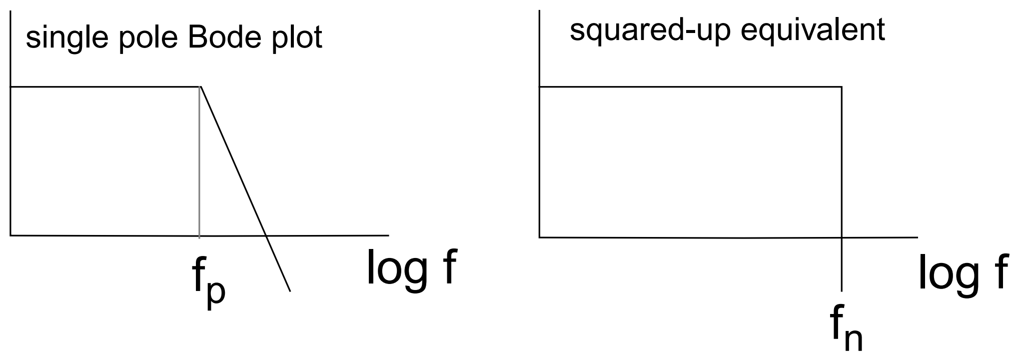
Assume M1 and M2 are operating in saturation. Only thermal noise sources need be considered.

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in})?

(ii) What is the input-referred thermal noise voltage in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T?

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:



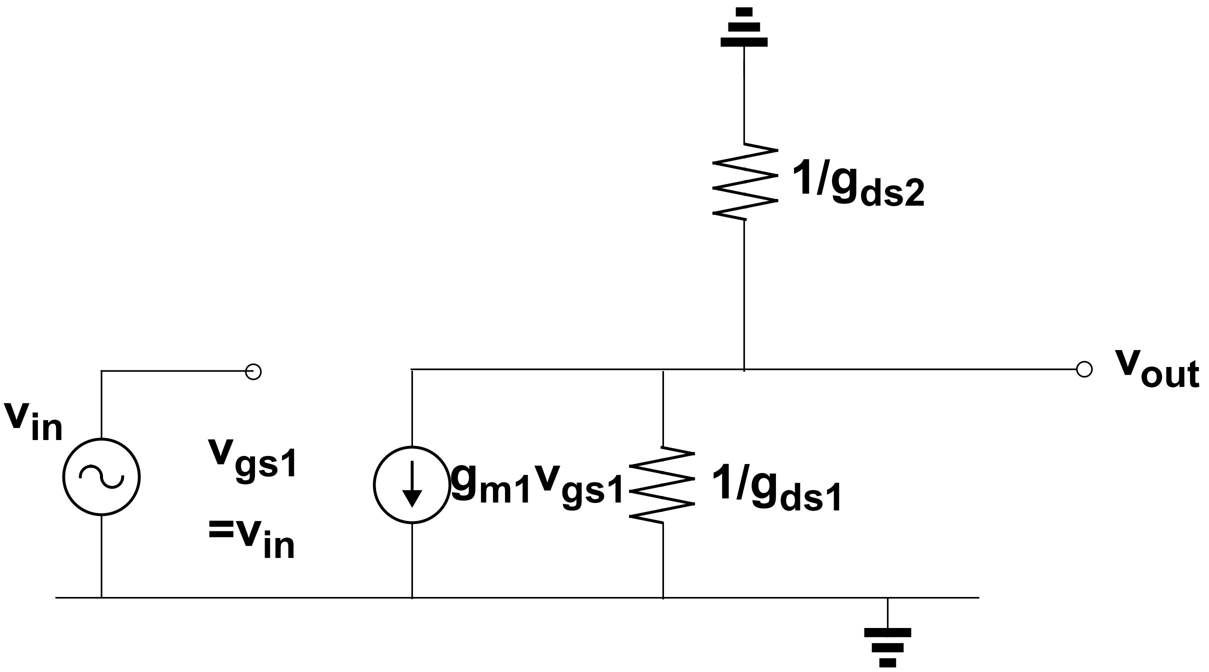
For the area underneath the curves to be the same then $f_n=(\pi/2)*f_p$

(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $1mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.
For this calculation take V_{BIAS1}=1.0V, V_{BIAS2}=2.0V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n=\lambda_p=0.04V^{-1}$, C_L=10pF.
The drain current of M1 is 100μA.
Assume Boltzmann's constant k=$1.38X10^{-23}$J/^{o}K, temperature T=300^{o}K.

Solution

(i) Draw the small signal model for the circuit shown in Figure 3. Ignore all capacitances

What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

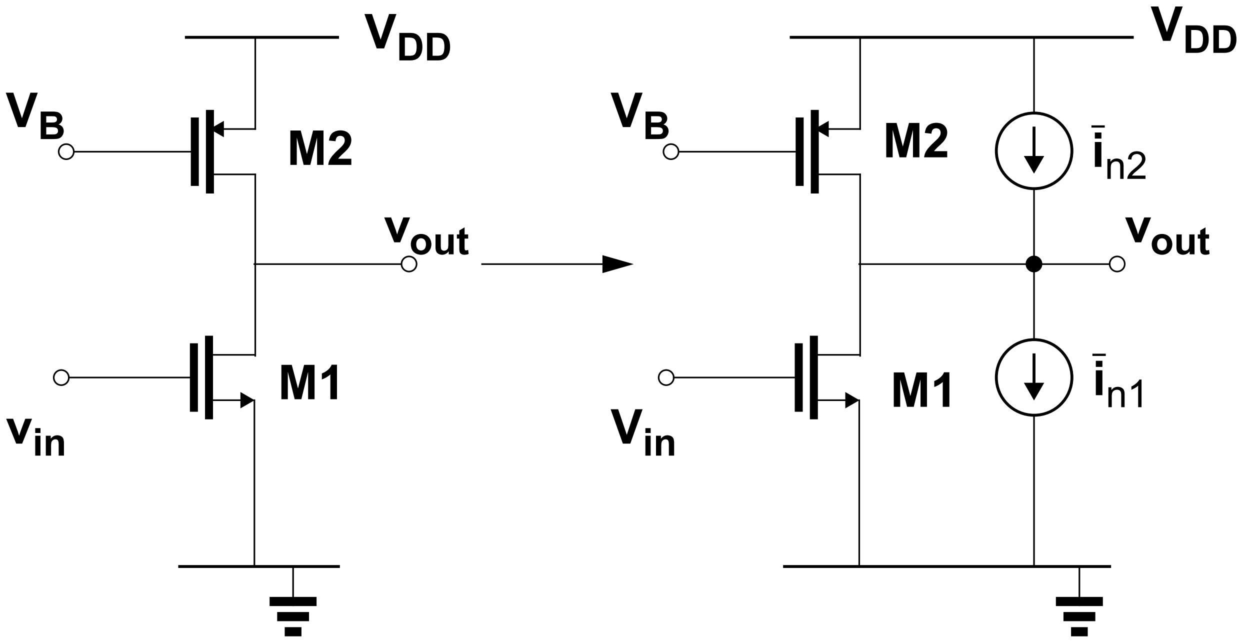


Current at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

(ii) What is the input-referred thermal noise voltage in terms of R_L, the small signal parameters of M1, Boltzmann's constant k and temperature T?



Noise current of MOS: $\overline{i_n^2} = 4kT\left(\frac{2}{3}g_m\right)$

Noise sources uncorrelated => total noise is the sum of squares

$$\overline{i_{nt}^2} = i_{n1}^2 + i_{n2}^2 \quad \text{or} \quad \overline{i_{nt}} = \sqrt{i_{n1} + i_{n2}} \quad \text{rms value}$$

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \quad \text{rms noise} \quad V/\sqrt{Hz}$$

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \sqrt{4kT \cdot \frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)}$$

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

To get voltage noise at output multiply input-referred noise by gain of circuit

$$\overline{v_{no}} = \overline{v_{ni}} \frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

$$\overline{v_{no}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{ds1} + g_{ds2}}$$

To get total noise voltage at output need to integrate this over all frequencies

The circuit is first-order circuit with a pole at

$$\omega_o = -\frac{g_{ds1} + g_{ds2}}{C_L}$$

$$\overline{v_{nototal}^2} = \int_0^{\infty} \left(\frac{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}{(g_{ds1} + g_{ds2})^2} \cdot \frac{1}{1 + \frac{C_L^2}{(g_{ds1} + g_{ds2})^2} \cdot (2\pi f)^2} \right) df$$

This is equal to multiplying by the noise bandwidth

$$v_{nototal}^2 = v_{no}^2 \cdot \frac{\pi}{2} \cdot f_o = \frac{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}{(g_{ds1} + g_{ds2})^2} \cdot \frac{\pi}{2} \cdot \frac{g_{ds1} + g_{ds2}}{2\pi C_L}$$

$$= \left(\frac{\frac{2}{3}(g_{m1} + g_{m2})}{g_{ds1} + g_{ds2}} \cdot \frac{kT}{C_L} \right)$$

(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $1mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.
For this calculation take V_{BIAS1}=1.0V, V_{BIAS2}=2.0V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n=\lambda_p=0.04V^{-1}$, C_L=10pF.
The drain current of M1 is 100μA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

g_m given by

$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V \qquad g_{m2} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V$$

$$g_{ds1} = \lambda_n I_D = 0.04V^{-1} 100\mu A = 4\mu A/V$$

$$g_{ds2} = \lambda_n I_D = 0.04V^{-1} 100\mu A = 4\mu A/V$$

Output signal

$$v_{out} = -\left(\frac{g_{m1}}{g_{ds1}+g_{ds2}}\right)v_{in} = -\frac{800\mu A/V}{8\mu A/V} \cdot 1mV_{rms} = 100mV_{rms}$$

Total output noise:

$$\overline{v_{nototal}} = \sqrt{\frac{\frac{2}{3}(g_{m1}+g_{m2})}{g_{ds1}+g_{ds2}} \cdot \frac{kT}{C_L}}$$

$$\overline{v_{nototal}} = \sqrt{\frac{2}{3}\left(\frac{800\mu A/V + 800\mu A/V}{4\mu A/V + 4\mu A/V}\right) \cdot \frac{1.38\times 10^{-23} 300}{10pF}} = 235\mu V_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{100mV}{235\mu V} = 426$$ or 52.6 dB