EE4001 Study Guide – 2007
(John Hayes, Mar. 2007, Version 1)

Attempt 4 out of 6 questions.

Please note that there may be mistakes in some of the published answers. Check with your classmates as to their results and let me know by email at <u>john.hayes@ucc.ie</u> if there's a problem.

Tutorials as follow: TBD.

Q1 Induction Motor Characterization

Summer 2004
Problem 1
(a)   A symmetrical, four-pole, three-phase, star-connected 70 hp induction motor is characterized as follows. The dc phase-to-phase resistance is measured to be 23.6 mΩ. A no-load test with an applied voltage of 195 V (line-line), 200 Hz, results in a phase current of 64.7 A, and a three-phase power of 1.093 kW. A locked-rotor test with an applied voltage of 35.6 V (line-line), 200 Hz, results in a phase current of 93 A, and a three-phase power of 641 W. Estimate the per-phase equivalent circuit parameters: R_S, L_{LS}, L_M, L_{LR}', and R_R'. Assume that $L_{LR}' = 0.8 \cdot L_{LS}$ for this class of machine.

[Ans. R_S = 11.8 mΩ, L_{LS} = 0.097 mH, L_M = 2.289 mH, L_{LR}'= 0.078 mH́, and R_R'= 12.9 mΩ.]

 (b)   A four-pole, star-connected induction motor interfaces a mechanical load to the 400 V (line-line) 50 Hz power grid (via gearing, contactor and breaker). The machine has the following per-phase equivalent circuit parameters: R_S = 20 mΩ, L_{LS} = 0.2 mH, L_M = 7.2 mH, L_{LR}' = 0.3 mH, and R_R' = 35 mΩ.

At full power, the induction motor requires a per-phase current of 225 A at a power factor of 0.89 and an overall system efficiency of 89.7 %. Calculate approximate values for the following: (i) the electrical input power, (ii) the airgap power, (iii) the per-phase magnetizing current, (iv) the per-phase rotor current, (v) the slip, and (vi) the core, friction and windage losses.

Calculate approximate values for the following when the mechanical load drops to 25 % of its value at full load: (i) the slip, (ii) the per-phase rotor current, (iii) the per-phase current, and (iv) the power factor.

[Ans.138.74 kW, 135.7 kW, 102.6 A, 200.25 A, 3.1 %, 7.04 kW; 0.9 %, 57.4 A, 117.2 A, 0.488]

Autumn 2003, Summer 2001
Problem 2
       A symmetrical, four-pole, three-phase, wye-connected induction motor is characterized as follows. The dc phase-to-phase resistance is measured to be 1.1 Ω. A no-load test with an applied voltage of 208 V (line-line), 60 Hz, results in a phase current of 6.5 A, and a three-phase power of 175 W. A locked-rotor test with an applied voltage of 53 V (line-line), 60 Hz, results in a phase current of 18.2 A, and a three-phase power of 900

       W. Estimate the per-phase equivalent circuit parameters: R_S, L_{LS}, L_M, L_{LR}', and R_R'. Assume that $L_{LR}' = \dfrac{3}{2} L_{LS}$

       for this Class B machine.
[Ans: R_S = 0.55 Ω, L_{LS} = 1.5 mH, L_M = 47.4 mH, L_{LR}'= 2.25 mH́, and R_R'= 0.356 Ω.]

Summer 2003
Problem 3
       (a) A symmetrical, four-pole, three-phase, star-connected induction motor is characterized as follows. The dc phase-to-phase resistance is measured to be 3.54 Ω. A no-load test with an applied voltage of 400 V (line-line), 50 Hz, results in a phase current of 1.8 A, and a three-phase power of 120 W. A locked-rotor test with an applied voltage of 71 V (line-line), 50 Hz, results in a phase current of 4 A, and a three-phase power of 150 W.

       Estimate the per-phase equivalent circuit parameters: R_S, L_{LS}, L_M, L_{LR}', and R_R'. Assume that $L_{LR}' = \dfrac{3}{2} L_{LS}$ for

       this Class B machine.
[Ans. R_S = 1.77 Ω, L_{LS} = 12 mH, L_M = 395 mH, L_{LR}'= 19 mH́, and R_R'= 1.36 Ω]

Problem 4
       The specification table for Westinghouse induction motors is provided as an attachment. Consider the 22 kW, four-pole machine with 400 V (line-line), 50 Hz applied in the delta configuration. What is the slope of the

torque/slip curve in its linear region? Estimate the per-phase equivalent circuit parameters: L_M, R'_R, R_S, P_{CFW}, L_{LS}, and L'_{LR}. Assume that $L'_{LR} = \frac{3}{2}L_{LS}$ for this class of machine.

[Ans. T/Hz = 245 Nm/Hz, L_M = 122 mH, R'_R = 0.44 Ω, R_S = 0.58 Ω, P_{CFW} = 671 W, L_{LS} = 2.9 mH, L_{LR}' = 4.3 mH]

Problem 5
     Consider the 75 kW, four-pole machine with 400 V (line-line), 50 Hz applied in the delta configuration. What is the slope of the torque/slip curve in its linear region? Estimate the per-phase equivalent circuit parameters: L_M, R'_R, R_S, P_{CFW}, L_{LS}, and L'_{LR}. Assume that $L'_{LR} = \frac{3}{2}L_{LS}$ for this class of machine.

[Ans. T/Hz = 1930 Nm/Hz, L_M = 35.6 mH, R'_R = 57 mΩ, R_S = 117 mΩ, P_{CFW} = 1861 W, L_{LS} = 0.9 mH, L_{LR}' = 1.4 mH]

Summer 2005
Problem 6
     The specification table for Westinghouse induction motors is provided as an attachment (see page 6). Consider the 110 kW, four-pole machine with 400 V (line-line), 50 Hz applied in the delta configuration. Assume R_S = 58.3 mΩ.
     (i)   Determine the slope of the torque/speed (Nm/Hz) curve in its linear region?
     (ii)   Estimate the per-phase equivalent circuit parameters: L_M, R'_R, and P_{CFW}.

     (b) The four-pole, 22 kW induction motor in the tables is missing the power factor for the 50 % load point. Calculate an approximate value for the power factor based on the information provided at the 100 % load point.
     (c) Given that the four-pole, 22kW motor has R'_R = 0.44 Ω and R_S = 0.58 Ω, determine the leakage inductances L_{LS} and L'_{LR}. Assume that $L'_{LR} = \frac{3}{2}L_{LS}$ for this class of machine.

[Ans. (a) T/Hz =2828 Nm/Hz, L_M = 24.3 mH, R'_R = 39.3 mΩ, P_{CFW} = 3169 W, (b) PF = 0.7, (c) L_{LS} = 2.9 mH, L_{LR}' = 4.3 mH]

Summer 2006 (a)
Problem 7
     (a) The specification table for Westinghouse induction motors is provided as an attachment (see page 6). Consider the 22 kW, **eight**-pole machine with 400 V (line-line), 50 Hz applied in the delta configuration. Estimate the per-phase equivalent circuit parameters:, R'_R, P_{CFW}, L_{LS}, L_{LR}, and L_M. Assume R_S = 0.432 Ω and L_{LS} equals L'_{LR} for this class of machine.
[Ans. R'_R = 0.49 Ω, P_{CFW} = 785 W, L_{LS} = 2.8 mH, L_{LR}' = 2.8 mH, L_M = 73 mH,]

     (b) A four-pole star-connected induction motor used in an electric vehicle application has the following per-phase equivalent circuit parameters:
R_S = 11.8 mΩ, L_{LS} = 0.0972 mH, L_M = 2.0 mH, L'_{LR} = 0.0772 mH, and R'_R = 12.9 mΩ.
When supplied by a current-controlled inverter outputting 93 A at 200 Hz, the motor generates an output torque of 40 Nm at 5945 rpm. Core, friction and windage losses are estimated at 2.3 kW at this speed. Determine approximate values for the input per-phase voltage, power factor, and efficiency at this operating point.
[Ans.113.3 V, 0.869, 89.7 %]

Q2 Induction Motor Speed Control

Summer 2005
Problem 1
     (a) Sketch the wiring diagram of the star-delta starter for inrush control of the induction machine.

                                         [3 *marks*]
     (b) The specification table for Westinghouse induction motors is provided as an attachment (see page 6). Consider the 22 kW, four-pole machine with 400 V (line-line), 50 Hz applied. What is the direct-on-line starting current when a star-delta starter is used?

                                         [3 *marks*]
     (c)   A four-pole star-connected motor outputs 40 Nm at 1746 RPM when supplied by a 60 Hz line-line voltage of 440 V and a phase current of 10.39 A lagging at a power factor of 0.866. The series resistance is 1.5 Ω.
     (i) By maintaining a constant field flux, what are the electrical line voltage, current, frequency, and power factor sourced from the inverter, when developing 50% of the rated torque at 50% of the rated speed?

(ii) Determine values for the slip frequency, line current and power factor required to ensure constant-power operation of the machine at twice the rated speed.

Use the formula $slope = \dfrac{V_{ph,rated} - R_S \cdot I_{R,rated}'}{f_{rated}}$ for low-voltage boost.

[*14 marks*]

[Ans. (b) 162A, (c) (i) 220.1 V, 30 Hz, 0.654, (ii) 1.8 Hz, 9.37 A, 0.96]

(d) Incorporating low-voltage boost, determine approximate values for the starting frequency, current, and voltage in order to supply 150% of rated torque at startup.

[Ans. 2.7 Hz, 51.2 V, 14.5 A]

Summer 2004
Problem 2

(a) Sketch the wiring diagram of the star-delta starter for inrush control of the induction machine.

[4 *marks*]

(b) A four-pole star-connected induction motor interfaces a mechanical load to the 400 V (line-line) 50 Hz power grid (via gearing, contactor and breaker). The machine has the following per-phase equivalent circuit parameters:

$R_S = 20$ mΩ, $L_{LS} = 0.2$ mH, $L_M = 7.2$ mH, $L_{LR}' = 0.3$ mH, and $R_R' = 35$ mΩ.

(i) Which motor parameters limit the startup current and what value of peak startup current do you expect?

(ii) How much greater would the startup current be if the machine was started in a delta configuration.

A power electronics inverter is now integrated into the system and the motor is connected in star. The motor develops an electromagnetic torque (including friction and windage) of 865 Nm at 1453.5 rpm when supplied by a voltage-source PWM inverter supplying a 50 Hz line-line voltage of 400 V and line current of 225 A lagging at a power factor of 0.89.

(iv) By maintaining a constant airgap flux, what are the electrical line voltage, current, and frequency, and power factor sourced from the inverter, when developing 25 % of the rated torque at 25 % of the rated speed?

(iv) Determine approximate values for the starting electrical line voltage, current, and frequency in order to supply 150% of rated torque at startup.

Use the formula $slope = \dfrac{V_{ph,rated} - R_S \cdot I_{R,rated}'}{f_{rated}}$ for low-voltage boost.

[*16 marks*]

[Ans: 1390 Arms (about 7 times rated!); 2408 Arms; 100 V, 114.2 A, 12.5 Hz, 0.439, 28.7 V, 317 A, 2.326 Hz]

Problem 3

(i) Sketch the wiring diagram, or calculate a start up current from the specs.

(ii) A four-pole star-connected induction motor interfaces a mechanical load to the 400 V (line-line) 50 Hz power grid (via gearing, contactor and breaker). The machine has the following per-phase equivalent circuit parameters:

$R_S = 20$ mΩ, $L_{LS} = 0.2$ mH, $L_M = 7.2$ mH, $L_{LR}' = 0.3$ mH, and $R_R' = 35$ mΩ.

What motor parameters limit the startup current and what value of peak startup current do you expect? How much greater would the startup current be if the machine was started in a delta configuration.

(iii) The specification table for Westinghouse induction motors is provided as an attachment. Consider the 22 kW, four-pole machine with 400 V (line-line), 50 Hz applied. What is the direct-on-line starting current when a star-delta starter is used?

(iii) A four-pole motor outputs 40 Nm at 1140 rpm when supplied by a 50 Hz line-line voltage of 400 V and phase current of 10 A lagging at a power factor of 0.88. The series resistance is 1.5 Ω. Neglecting voltage boost and maintaining a constant field flux, what are the approximate slip and electrical frequencies when outputting 20 Nm at 800 rpm.

Incorporating voltage boost, determine approximate values for the starting frequency, current, and voltage in order to supply 150% of rated torque at startup.

[Ans. 0.5 Hz, 27.67 Hz, 3 Hz, 13.2 A, 26.6 V]

Problem 4

Consider the Westinghouse 22 kW, 8-pole machine with 400 V (line-line), 50 Hz.

(i) What are the initial starting line current and torque for a direct-on-line start?

A volts/hertz controller with voltage boost is integrated into the delta-wired drive. The series resistance is estimated to be 0.43 Ω.

(ii) Determine approximate values for the starting frequency, current, and voltage in order to supply the specified starting torque.

(iii) Maintaining rated airgap flux, what are the electrical line voltage, current, frequency, and power factor sourced from the inverter, when developing 50 % of the rated torque at 75 % of the rated speed?

Use the formula $slope = \dfrac{V_{ph,rated} - R_S \cdot I'_{R,rated}}{f_{rated}}$ for low-voltage boost.

[Ans. 350.5 A, 604. 8 Nm, 2.8 Hz, 99.9 A, 45.6 V, 296.5, 41.8 A, 37.2 Hz, 0.545]

Summer 2006
Problem 5

(a)   Sketch the wiring diagram for the volts/hertz control of the induction machine.

[4 *marks*]

(b)   Consider the Westinghouse 75 kW, 4-pole machine with 400 V (line-line), 50 Hz.

(i)   What are the initial starting line current and torque for a direct-on-line start?

A volts/hertz controller with voltage boost is integrated into the delta-wired drive. The series resistance is estimated to be 117 mΩ.

(ii) Determine approximate values for the starting frequency, current, and voltage in order to supply the specified starting torque.

(iii) Maintaining rated airgap flux, what are the electrical line voltage, current, frequency, and power factor sourced from the inverter, when operating as a **generator** developing 100 % of the rated torque at 50 % of the rated speed?

Use the formula $slope = \dfrac{V_{ph,rated} - R_S \cdot I'_{R,rated}}{f_{rated}}$ for low-voltage boost.

[16 *marks*]

[Ans. 949 A, 1205 Nm, 1.25 Hz, 292.4 A, 29.1 V, 198 V, 130 A, 24.25 Hz, 0.88]

Q3 AC Machines Space Vectors and Vector Control of the Induction Motor

A. AC Machines Space Vectors

Problem 10:

A 2-pole, 3-phase induction motor has the following physical dimensions: radius $r = 7$ cm, length $l = 9$ cm, airgap length $l_g = 0.5$ mm, and number of turns per phase per pole $N_{sp} = 30$. The star-connected motor is supplied by a rated voltage of 208 V (line to line) at a frequency of 60 Hz.

(i)   Develop an expression for the per-phase magnetizing inductance.

(ii)   Calculate the per-phase magnetizing inductance and the per-phase magnetizing current of the machine.

(iii)   Determine the peak magnitudes of the following rotating space vectors: stator current, voltage and flux density.

[Ans. (i) 67.2 mH, 4.74 A (ii) 10.1 A, 254.8 V, 0.76 T]

Problem 11:

A 2-pole, 3-phase induction motor has the following physical dimensions: radius $r = 6$ cm, length $l = 24$ cm, airgap length $l_g = 1.5$ mm, and number of turns per phase per pole $N_{sp} = 50$. The star-connected motor is supplied by a rated voltage of 208 V (line to line) at a frequency of 60 Hz.

(i)   Calculate the per-phase magnetizing inductance and the per-phase magnetizing current of the machine.

(ii)   Determine the peak magnitudes of the following rotating space vectors: stator current, voltage and flux density.

(iii)   Determine the rms per-phase current and output torque when a per-phase reflected current $I'_r = 10$ A flows in the stator.

(iv)   Roughly sketch a space vector diagram showing the approximate phase angles and magnitudes of the space vector voltage, the magnetizing space vector current, the reflected rotor current, and the stator current.

[Ans. (i) 0.142 H, 2.24 A (ii) 4.76 A, 254.8 V, 0.2 T (iii) 10.25 A, 4.53 Nm]

Summer 2005, Summer 2004
Problem 12:

A 4-pole, 3-phase induction motor has the following physical dimensions: radius $r = 6$ cm, length $l = 24$ cm, airgap length $l_g = 0.5$ mm, and number of turns per phase per pole $N_{sp} = 50$. The star-connected motor is supplied by a rated voltage of 400 V (line to line) at a frequency of 50 Hz.

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(i)   Calculate the per-phase magnetizing inductance and the per-phase magnetizing current of the machine.
(ii)   Determine the magnitudes of the following rotating stator space vectors: current, voltage and flux density.
(iii) Determine the rms per-phase current and output torque when a per-phase reflected current $I'_r = 5$ A flows in the stator.

[Ans. (i) 0.426 H, 1.72 A (ii) 3.66 A, 490 V, 0.46 T (iii) 5.29 A, 14.7 Nm]

[6 *marks*]

Summer 2006
Problem 13:
   A 2-pole, 3-phase induction **generator** has the following physical dimensions: radius $r = 6$ cm, length $l = 24$ cm, airgap length $l_g = 1$ mm, and number of turns per phase per pole $N_{sp} = 50$. The star-connected generator is connected to a rated voltage of 400 V (line to line) at a frequency of 50 Hz.
   (i)   Calculate the per-phase rms magnetizing current of the machine.
   (ii)   Determine the peak of the space vector flux density.
   (iii) Determine the input torque when a rms per-phase reflected current $I'_r = 5$ A flows in the stator.
   (iv)   Sketch a space vector diagram showing the approximate phase angles and magnitudes of the space vector voltage, the magnetizing current, the reflected rotor current, and the stator current.

[Ans. 3.45 A, 0.46 T, 7.67 Nm]

B. Vector Control of Induction Motor

Problem 1
   In a 2-pole induction machine, a peak per-phase magnetizing current of 4 A is required to establish the rated airgap flux density. A step change from 0 to rated torque requires a step change in the stator quadrature-axis current $i_{sq} = 10$ A. A slip speed $f_{slip} = 1.8$ Hz is required for steady state operation at rated torque.
   (i)   Calculate the magnitudes of the space-vector current $I_{ms,pk}$, the stator direct-axis current i_{sd}, and the three phase currents, i_a, i_b, and i_c, to establish the rated flux at $t = 0^-$.
   (ii)   Calculate $I_{ms,pk}$ and the three phase currents to establish the rated torque at $t = 0^+$.
   (iii) Assuming that the rotor is blocked, calculate the electrical input frequency and the phase currents at $t = 8$ ms.
   (iv)   Assuming that the rotor speed is constant at 1100 rpm, calculate the electrical input frequency and phase currents at $t = 8$ ms.

[Ans. (i) 6 A, 4.9 A, 4 A, -2 A, -2 A (ii) 13.64 A, 4 A, 5.07 A, -9.07 A (iii) 1.8 Hz, 3.25 A, 5.73 A, -8.98 (iv) 20.13 Hz, -4.80 A, 9.09 A, -4.28 A.]

Summer 2005
Problem 2
   In a 4-pole induction machine, a per-phase current of 106 Arms at an input electrical frequency of 50 Hz is required to establish the rated airgap flux density. A per-phase current of 225 Arms at an input electrical frequency of 51.5 Hz is required to establish rated motoring torque at a mechanical rotor speed of 1500 rpm.
   (i)   Calculate the magnitudes of the space-vector current $I_{ms,pk}$, the stator direct-axis current i_{sd} and quadrature-axis current i_{sq}, and the three phase currents, i_a, i_b, and i_c, to establish the rated flux at $t = 0^-$, the instant just before injection of a step current to develop rated torque.
   (ii)   Recalculate the above currents required to establish the rated flux and motoring torque at $t = 0^+$.
   (iii) Assuming that the rotor speed is constant at 1500 rpm, calculate the per-phase currents at $t = 5$ ms.
   (iv)   Recalculate the above currents required to establish the rated flux and generating torque at $t = 0^+$.
   (v)   Assuming that the generator speed is constant at 1500 rpm, calculate the input electrical frequency and the per-phase currents at $t = 5$ ms.

[Ans. (i) 225 A, 183.7 A, 0 A, 150 A, -75 A, -75 A (ii) 477.3 A, 183.7 A, 343.7 A, 149.9 A, 168.1 A, -318 A (iii) – 287.4 A, 261.9 A, 25.5 A (iv) 477.3 A, 183.7 A, -343.7 A, 149.9 A, -318 A, 168.1 A (v) 48.5 Hz, 287.4 A, -25.5 A, -261.9]

Summer 2004
Problem 3
   In a 4-pole induction machine, a per-phase current of 46 Arms at an input electrical frequency of 200 Hz is required to establish the rated airgap flux density. A per-phase current of 93 Arms at an input electrical frequency of 202 Hz is required to establish rated motoring torque at a mechanical rotor speed of 6000 rpm.
   (i)   Calculate the magnitudes of the space-vector current $I_{ms,pk}$, the stator direct-axis current i_{sd} and quadrature-axis current i_{sq}, and the three phase currents, i_a, i_b, and i_c, to establish the rated flux at $t = 0^-$, the instant just before injection of a step current to develop rated torque.
   (ii)   Recalculate the above currents required to establish the rated flux and a re*generative* torque at $t = 0^+$.

(iii) Assuming that the generator speed is constant at 6000 rpm, calculate the input electrical frequency and the per-phase currents at $t = 1.25$ ms.

[Ans. (i) 97.7 A, 79.8 A, 0 A, 65.1 A, -32.5 A, -32.5 A, (ii) 197.3 A, 79.8 A, -139.9 A, 65.1 A, -131.5 A, 66.4 A; (iii) 198 Hz, 115.2 A, -2.8 A, -112.5 A]

Problem 4

Consider the Westinghouse 22 kW, four-pole machine with 400 V (line-line), 50 Hz applied in the delta configuration.

(i) Calculate the magnitudes of the space-vector current $I_{ms,pk}$, the stator direct-axis current i_{sd} and quadrature-axis current i_{sq}, and the three phase currents, i_a, i_b, and i_c, to establish the rated flux at $t = 0^-$, the instant just before injection of a step current to develop rated torque.

(ii) Recalculate the above currents required to establish the rated flux and motoring torque at $t = 0^+$.

(iii) Assuming rated rotor speed calculate the per-phase currents at $t = 2.5$ ms.

[Ans. 22.06 A, 18.01 A, 0 A, 14.7 A, -7.35 A, -7.35 A, 18.01 A, 35.16 A, 14.76 A, 17.47 A, -32.22 A, -9.85 A, 31.53 A, -21.68 A]

[14 marks]

Summer 2006

Problem 5

The specification table for the Westinghouse 30 kW, four-pole induction motor, with 400 V (line-line), 50 Hz applied in the delta configuration, is provided as an attachment (see page 6). Consider the machine running as an induction generator with a power electronics interface.

(i) Calculate the magnitudes of the space-vector current $I_{ms,pk}$, the stator direct-axis current i_{sd} and quadrature-axis current i_{sq}, and the three phase currents, i_a, i_b, and i_c, to establish the rated flux at $t = 0^-$, the instant just before injection of a step current to develop rated torque.

(ii) Recalculate the above currents required to establish the rated flux and generating torque at $t = 0^+$.

(iv) The generator speed is dropped to 1400 rpm, calculate the input electrical frequency and the per-phase currents at $t = 1$ ms when operating at rated torque and rated flux.

[Ans. (i) 29.66 A, 24.22 A, 0 A, 19.77 A, -9.89 A, -9.89 A (ii) 65.04 A, 24.22 A, 47.25 A, 19.78 A, -43.3 A, 23.53 A (iii) 45.5 Hz, 29.86 A, -42.16 A, -12.3 A]

Q4 Power Electronics Converters

Summer 2002

Problem 7:

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc H-bridge converter are as follows: armature resistance $R_a = 0.35\,\Omega$, armature inductance $L_a = 1.5$ mH, motor moment of inertia 0.02 kg m^2, motor torque constant $k = 0.5$ Nm/A, converter dc bus voltage $V_d = 200$ V, switching frequency $f = 25$ kHz, and amplitude of triangular waveform control voltage $V_{tri} = 3$ V. The motor is spinning in a forward direction at a speed of 1500 RPM, supplying a load torque of 10 Nm.

i. Sketch the circuit.

ii. Calculate the following: (a) the applied armature voltage vab(t); (b) duty ratios for (i) overall converter, (ii) pole A, and (iii) pole B; (c) control voltage amplitude and (d) peak to peak ripple on the armature current;

iii. Sketch the waveforms for the triangular voltage vtri(t), control voltage vc(t), pole A voltage van(t), the pole b voltage vbn(t), armature voltage va(t), and the armature current iab(t).

iv. Sketch the four different switch configurations of the converter sequenced over one switching cycle. Also note these sequences in your timing diagrams in part (iii) above.

[Ans. (ii) (a) 85.55 V (b) (i) 0.428 (ii) 0.714 (iii) 0.286 (c) 1.284 V (d) 0.653 A, (iv)I-Qau/Qbl, II-Qau/Dbu, III-Qau/Qbl, IV-Dal/Qbl]

Problem 8

Repeat above for the motor running in reverse with a positive torque, and acting as a generator.

[Ans. (ii) (a) -71.55 V (b) (i) -0.358 (ii) 0.321 (iii) 0.678 (c) -1.07 V (d) 0.613 A, (iv)I-Qau/Dbu, II-Dal/Dbu, III-Dal/Qbl, IV-Dal/Dbu]

Summer 2003

Problem 9

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc converter are as follows: armature resistance $R_a = 0.1\Omega$, armature inductance $L_a = 1$ mH, motor moment of inertia $J_m = 0.02$ kg m^2, motor constant $k = 0.2$ V/(rad/s), dc bus voltage $V_d = 42$ V, switching frequency $f_s = 20$ kHz, and amplitude of triangular waveform control voltage $V_{tri} = 3$ V. The motor is spinning in a reverse direction at a speed of 1000 RPM and supplies a load torque of 5 Nm.

(i) Sketch the system.

(ii) Calculate the following: (a) the applied armature voltage V_{AB}; (b) duty ratios for the overall converter, pole A, and pole B; (c) the control voltage; and (d) the peak-to-peak ripple on the armature current.

(iii) Sketch the waveforms for the triangular voltage $v_{tri}(t)$, control voltage $v_c(t)$, pole A voltage $v_A(t)$, pole B voltage $v_B(t)$, armature voltage $v_{AB}(t)$, and armature current $i_{ab}(t)$.

(iv) Sketch the four different switch configurations of the converter sequenced over one switching cycle. Also note these sequences in your timing diagrams in part (iii) above.

[Ans. (ii) (a)-23.44 V (b) (i) –0.558 (ii) 0.222 (iii) 0.778 (c) –1.67 V (d) 0.259 A, (iv)I-Dau/Qbu, II-Qal/Qbu, III-Qal/Dbl, IV-Qal/Qbu]

Problem 10

Repeat above for the motor running forward with a negative torque, and acting as a generator.

[Ans. (ii) (a)18.44 V (b) (i) 0.439 (ii) 0.720 (iii) 0.281 (c) 1.317 V (d) 0.259 A, (iv)I-Dau/Dbl, II-Dau/Qbu, III-Dau/Dbl, IV-Qal/Dbl]

Summer 2005
Problem 11

The system parameters of a permanent-magnet dc motor supplied by a switch-mode PWM dc-dc converter are as follows: armature resistance $R_a = 0.1\ \Omega$, armature inductance $L_a = 1$ mH, motor constant $k = 0.07$ V/(rad/s), dc bus voltage $V_d = 12$ V, switching frequency $f_s = 20$ kHz, and amplitude of triangular waveform control voltage $V_{tri} = 5$ V. The motor is spinning forward at a speed of 750 rpm and acts as a *generator* supplied by a full-load torque of -0.7 Nm.

(i) Calculate the following: (a) the applied armature voltage V_{AB}; (b) duty ratios for the overall converter, pole A, and pole B; (c) the control voltage, and (d) the peak-to-peak ripple on the armature current.

(ii) Calculate the rms currents in the upper and lower MOSFET switches of pole A.

[Ans. 4.5 V, 0.375, 0.688, 0.312, 1.875 V, 70.3 mA, upper 8.29 A, lower 5.59 A]

Problem 12

The 2003 Toyota Prius uses a 20 kW bidirectional converter to generate a 500 V dc link voltage from the 200 V NiMH battery. This higher voltage allows the efficiency, range, and emissions of the vehicle to be optimized. The bidirectional converter has an inductance of 435 uH and switches at 10 kHz.

The vehicle is operating in generating mode and the bi-directional converter is required to act as a buck at full power.

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. I_L(rms) = 100.3 A, I_{CLV}(rms) = 8 A, I_{QU}(rms) = 63.4 A, I_{QU}(avg) = 40 A, I_{DL}(rms) = 77.7 A, I_{DL}(avg) = 60 A, I_{CHV}(rms) = 49.2 A; P_Q = 140 W; P_D = 120 W]

Problem 13

The above vehicle is operating in motoring mode and the bi-directional converter is required to act as a boost and provide full power.

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. I_L(rms) = 100.3 A, I_{CLV}(rms) = 8 A, I_{DU}(rms) = 63.4 A, I_{DU}(avg) = 40 A, I_{QL}(rms) = 77.7 A, I_{QL}(avg) = 60 A, I_{CHV}(rms) = 49.2 A; P_Q = 210 W; P_D = 80 W]

Summer 2005
Problem 14

The above vehicle is operating in motoring mode and the bi-directional converter is required to act as a boost and provide a half power level of 10 kW. For this 10 kW condition:

(i) Calculate the rms currents in the inductor, and output and input capacitors.

(ii) Calculate the switch average and rms currents and the resulting conduction losses in (a) the IGBT with $V_{CE(knee)} = 2.5$ V and $R_{CE} = 0.01\ \Omega$, and (b) the diode with $V_{F(knee)} = 1.5$ V and $R_{CE} = 0.005\ \Omega$.

[Ans. 50.6 A, 8 A, 25 A, 39.2, 30 A, 90 W, 32 A, 20 A, 35 W]

Problem 15

Design a Voltage Regulator Module (VRM) for local power regulation of a microprocessor on a mobile phone. The VRM is powered from a 3.3 V source and uses a buck converter. The microprocessor specifications call for a 1.85 V supply with a +/- 1 % allowable fluctuation while consuming 60 mA. The switching frequency is 500 kHz. Neglect the parasitic effects of the controlled MOSFET switch and diode.

(i) Sketch the buck converter.

(ii) Find the duty cycle of the controlled switch, needed to output 1.85 V.
(iii) Choose an inductor that limits the current ripple to +/- 10 %.
(iv) Choose a capacitor to limit the voltage ripple to +/ 1 %.
(v) Determine the rms currents in the inductor, and output and input capacitors.
(vi) Calculate the conduction losses in (a) the MOSFET with $R_{ds(on)}$ of 0.5 ohm, and (b) the diode with a characteristic forward voltage knee of 0.4 V and a slope of 0.1 Ω.
(vii) What much would the diode loss be reduced by using a synchronous buck?

[Ans. (ii) 0.56, (iii) 135 uH, (iv) 81 nF, (v) 60.1 mA, 3.5 mA, 29.9 mA (vi) (a) 1mW, (b) 10.6 mW (vii) diode loss of 10.6 mW reduced to 0.8 mW]

Summer 2006
Problem 16

Design a Voltage Regulator Module (VRM) for local power regulation of a microprocessor on a mobile phone. The VRM is powered from a 3.3 V lithium ion battery and uses a **synchronous** buck converter. The microprocessor specifications call for a 1 V supply with a +/- 1 % allowable fluctuation while consuming 50 mA. The switching frequency is 1 MHz. Neglect the parasitic effects of the controlled MOSFETs.
(i) Sketch the synchronous buck converter.
(ii) Choose an inductor that limits the current ripple to +/- 10 %.
(iii) Choose a capacitor to limit the voltage ripple to +/ 1 %.
(iv) Determine the rms currents in the inductor, and output and input capacitors.
(v) Calculate the conduction losses in the two MOSFETs for $R_{ds(on)}$ of 0.5 ohm.

[Ans. 34.8 uH, 31.3 nF, 50.1 mA, 2.9 mA, 23 mA, 1.3 mW]

Q5 Power Semiconductors

Summer 2005
Problem 1
(a) Sketch the symbol and the vertical structure of the IGBT. Briefly state the advantages of the IGBT over the MOSFET for low frequency operation.

[5 *marks*]

(b) The IRFPS40N60K power MOSFET (see attached specification sheets on pages 7 to 10) from International Rectifier operates in a boost converter with a dc link voltage V_d = 480 V, and load current I_o = 20 A. The MOSFET is driven by a gate drive IC outputting a square-wave voltage v_{GG}, of amplitude 0 V to +10 V, in series with an external gate resistance R_G = 4.3 Ω. Assume the diode has a 1V forward drop and no reverse recovery.

Useful formula: RC discharge time $t = -RC \ln\left[\dfrac{v_c - (-V_{GG})}{V_{ci} - (-V_{GG})}\right]$

(i) Determine the following parameters from the data sheet at a junction temperature of 100°C: maximum threshold voltage, minimum forward transconductance, gate-source capacitance, gate-drain capacitance, maximum on-state resistance, maximum gate voltage at the 20 A load current, and maximum conduction drop across MOSFET at 20 A.
(ii) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, and $i_D(t$) during turn-off of the MOSFET. Note the approximate voltage levels on waveforms.
(iii) Calculate the following (a) turn-off delay time t_{doff}, (b) voltage rise time t_{vr}, and (c) current fall time t_{fv} at a junction temperature of 100°C. Sketch the basic switching circuit under analysis in each case.
(iv) Calculate the turn-off energy loss.

[Ans. 5 V, 21 S, 7895 pF, 75 pF, 0.26 Ω, 6V, 5.2 V, 17.5 ns, 31 ns, 6.2 ns, 0.18 mJ]

[15 *marks*]

Summer 2004
Problem 2
(a) Briefly explain reverse recovery in power diodes and sketch the effects of reverse recovery on the turn-on waveforms of the complementary IGBT or MOSFET.

[6 *marks*]

(b) The IRFP460 power MOSFET (see attached specification sheets on pages 6 and 7) from International Rectifier operates in a boost converter switching at 20 kHz with a dc link voltage V_d = 300 V, and load current I_o = 13 A. The MOSFET is driven by a gate drive IC outputting a square wave voltage v_{GG}, of amplitude –5 V to +15 V, in series with an external gate resistance R_G = 25 Ω. Assume the diode has a 1V forward drop and no reverse recovery.

Useful formulae: RC discharge time $t = -RC \ln\left[\dfrac{v_c - (-V_{GG})}{V_{ci} - (-V_{GG})}\right]$

(v) Determine the following parameters from the data sheet at a junction temperature of 80°C: maximum threshold voltage, minimum forward transconductance, gate-source capacitance, gate-drain capacitance, and on-state resistance.

(vi) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, and $i_D(t)$ during turn-off of the MOSFET.

(vii) Calculate the following (a) turn-off delay time t_{doff}, (b) voltage rise time t_{vr}, and (c) current fall time t_{fv} at a junction temperature of 80°C. Sketch the switching circuit under analysis in each case.

[Ans. 4 V, 13 S, 3850 pF, 350 pF, 0.43 Ω, 72.8 ns, 258 ns, 11 ns]

[14 *marks*]

Summer 2003
Problem 3

An enhancement-mode n-channel vertically diffused power MOSFET operates in a step-up converter switching at 20 kHz, with a dc link voltage $V_d = 400$ V, and load current $I_o = 20$ A. The device characteristics are as follows: threshold voltage $V_{GS(th)} = 4$ V, drain current $I_D = 20$ A at gate voltage $V_{GS} = 6$ V, gate-source capacitance $C_{gs} = 4000$ pF, gate-drain capacitance $C_{gd} = 400$ pF, and on-state resistance $R_{DS(on)} = 0.25$ Ω. The MOSFET is driven by an ideal voltage-source square wave v_{GG}, of amplitude 0 V to 15 V, in series with an external gate resistance $R_G = 25$ Ω. Assume the diode has a 1 V forward drop and no reverse recovery.

(i) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, and $i_D(t)$ during turn-on of the MOSFET.

(ii) Calculate the following times: (a) turn-on delay time $t_{d(on)}$, (b) current rise time t_{ir}, and (c) voltage fall time t_{fv}. For each of the time durations investigated, sketch the equivalent circuit.

[Ans. 34 ns, 22 ns, 440 ns]

Summer 2002
Problem 4

The IRFP460 power MOSFET from International Rectifier operates in a boost converter switching at 20 kHz with a dc link voltage $V_d = 400$ V, and load current $I_o = 20$ A. The MOSFET is driven by a voltage-source square wave v_{GG}, of amplitude 0 V to 15 V, in series with an external gate resistance $R_G = 25$ Ω. Assume the boost diode has a 1V forward drop and no reverse recovery.

(a) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, and $i_D(t)$ during turn-on of the MOSFET.

(b) Determine the following parameters from the data sheet at a junction temperature of 80°C: threshold voltage, forward transconductance, gate-source capacitance, gate-drain capacitance, and on-state resistance.

(c) Calculate the following (i) turn-on delay time t_{don}, (ii) current rise time t_{ir}, (iii) voltage fall time t_{fv} and (iv) turn-on energy loss.

[Ans. 4 V, 13 S, 3850 pF, 350 pF, 0.43 Ω, 33 ns, 15 ns, 363 ns, 1.55 mJ]

Problem 5

An enhancement-mode n-channel vertically diffused power MOSFET operates in a boost converter switching at 20 kHz, with a dc link voltage $V_d = 300$ V, and load current $I_o = 10$ A. The device characteristics are as follows: threshold voltage $V_{GS(th)} = 4$ V, drain current $I_D = 10$ A at gate voltage $V_{GS} = 7$ V, gate-source capacitance $C_{gs} = 1000$ pF, gate-drain capacitance $C_{ds} = 150$ pF, and on-state resistance $R_{DS(on)} = 0.5$ Ω. The MOSFET is driven by an ideal voltage-source square wave v_{GG}, of amplitude 0 V to 15 V, in series with an external gate resistance $R_G = 50$ Ω. Assume the diode has a 1V forward drop and no reverse recovery.

(a) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, $i_D(t)$, $v_{diode}(t)$, and $i_{diode}(t)$ during turn-on of the MOSFET.

(b) Calculate the following (i) turn-on delay time t_{don}, (ii) current rise time t_{ir}, (iii) voltage fall time t_{fv}.

(c) Calculate the following (i) turn-off delay time t_{doff}, (ii) voltage rise time t_{vr}, (iii) current fall time t_{fv}.

(d) Calculate the turn-on and turn-off energy losses.

(e) Calculate the power dissipation in the MOSFET due to switching and conduction losses.

(f) Estimate the MOSFET junction temperature due to turn-on switching loss only, if the MOSFET is bonded to a 70 °C heatsink. The thermal impedance from the heatsink to the junction is $\theta_{J-C} = 0.69$ °C/W.

[Ans. (i) 18 ns (ii) 18 ns (iii) 278 ns, (c)44 ns (ii) 317 ns (iii) 32 ns (d) 0.46 mJ, 0.53 mJ (e) 15.1 W (f) 100.2 °C]

Summer 2006
Problem 6

(a) Sketch the symbol and the vertical structure of the MOSFET. Briefly state the advantages of the MOSFET over the IGBT for high-frequency, low-voltage operation.

[5 *marks*]

(b) An enhancement-mode n-channel vertically diffused power MOSFET operates in a step-up converter switching at 50 kHz, with a dc link voltage $V_d = 300$ V, and load current $I_o = 10$ A. The device characteristics are as

follows: threshold voltage $V_{GS(th)}$ = 4 V, drain current I_D = 10 A at gate voltage V_{GS} = 7 V, gate-source capacitance C_{gs} = 1000 pF, gate-drain capacitance C_{gd} = 150 pF, and on-state resistance $R_{DS(on)}$ = 0.5 Ω. The MOSFET is driven by a voltage-source square wave v_{GG}, of amplitude –15 V to +15 V, in series with an external gate resistance R_G = 50 Ω. Assume the diode has a 1V forward drop and neglect the reverse recovery.

Useful formulae: RC charge time $t = -RC \ln\left[1 - \dfrac{v_c - V_{ci}}{V_{GG} - V_{ci}}\right]$ RC discharge time $t = -RC \ln\left[\dfrac{v_c - (-V_{GG})}{V_{ci} - (-V_{GG})}\right]$

(viii) Sketch $v_{GG}(t)$, $v_{GS}(t)$, $v_{DS}(t)$, and $i_D(t)$ during turn-off of the MOSFET. Note the approximate voltage levels on waveforms.

(ix) Calculate the following (i) turn-on delay time t_{don}, (ii) current rise time t_{ir}, (iii) voltage fall time t_{fv}.

(x) Calculate the turn-on energy losses.

(xi) Estimate the MOSFET junction temperature if the MOSFET is bonded to a 70 °C heatsink. The thermal impedance from the heatsink to the junction is θ_{J-C} = 0.5 °C/W. Assume turn-off energy loss equals the turn-on energy loss, and the conduction loss equals the combined switching loss.

[Ans. 57.7 ns, 18.3 ns, 278 ns, 0.45 mJ, 115 C]

[15 *marks*]

Q6 DC Machines

Summer 2005
Problem 3: ED Problem 7-7

A motor/generator with a pure inertial load is often used as a flywheel to store energy. A motor has a machine constant of 0.5 Nm/A, an armature resistance of 0.35 Ω, and an inertia J = 0.06 kg m^2. Calculate the electrical energy recovered when the machine slows from 1500 rpm to 750 rpm. The braking current is clamped at 10 A during the energy recovery period.

[Ans. 522 J, Hint – calculate mechanical regen. energy of 555 J and subtract RI^2 losses in armature for regen. period, 33 J]

Summer 2004
Problem 9, ED Problem 7-12, Q6(b)

A wound-field dc motor is driving a load whose torque requirement increases linearly with speed (squared-power load) and reaches 5 Nm at a speed of 1400 rpm. The armature terminal voltage is held to its rated value. At the rated flux the no-load speed is 1500 rpm and the full-load speed is 1400 rpm. If the flux is reduced to 80 % of the rated value, calculate the new steady-state speed.

[Ans. 1686 rpm]

Problem 10

Repeat the above question for the case where the torque varies with the square of the speed.

[Ans. 1654 rpm]

Summer 2004
Problem 11

A 100 kW, 250 V dc shunt motor has the attached magnetization curves (including armature-reaction effects) given on page 8. The armature circuit resistance, including brushes is 0.025 Ω. The field rheostat is adjusted for a no-load speed of 1100 rpm.

(i) Determine the field current set point at no load.

(ii) Determine the speed in rpm corresponding to an armature current of 600 A.

Because the speed-load characteristic referred to in (ii) above is considered undesirable, a stabilizing winding of 1.5 cumulative series turns per pole is to be added. The resistance of this winding is negligible. There are 1000 turns per pole in the shunt field. Compute the speed corresponding to an armature current of 600 A.

[Ans. 5.9 A, 1115 rpm, 1041 rpm]

Summer 2005
Problem 12

A 100 kW compound generator, of terminal ratings 250 V and 400 A, has an armature resistance (including brushes) of 0.025 Ω and the attached magnetization curve (see page 11). There are 1000 shunt-field turns per pole and 3 series-field turns per pole. Compute the shunt field current required at full load when the generator speed is 1100 rpm. Include the effects of armature reaction.

[Ans. 6.2 A]

Problem 13

A 25 kW 125 V **separately excited** dc machine is operated at a constant speed of 3000 rpm with a constant field current such that the open-circuit armature voltage is 125 V. The armature resistance is 0.02 Ω. Compute the

armature current, terminal voltage and electromagnetic power and torque when the terminal voltage is (a) 128 V, and (b) 124 V.

[Ans. (a) 150 A, 19.2 kW, 18.75 kW, 59.7 Nm, (b) -50 A, -6.2 kW, -6.25 kW, -19.9 Nm]

Problem 15: ED 7-13

[Ans. $E_{\text{ph-ph}}$ = 75 V, I = 8A]

Problem 16: ED 7-13

Problem 17

The specification sheet for the Maxon 250 W, 48 V, 6500 rpm, EC dc motor is shown on page 161. Compute the armature current, applied voltage, and machine efficiency for the condition shown in line 10 of motor data. What are the amplitude of per-phase back emf and the rms per-phase current?

[Ans. 4.59 A, 41.95 V, 83.2%, 18.6 V, 3.75 A]

Summer 2006 (Q6(a))

Problem 18

A wound-field dc motor is driving a load whose torque requirement increases with the **square** of the speed and reaches 5 Nm at a speed of 1400 rpm. The armature terminal voltage is held to its rated value. At the rated flux the no-load speed is 1500 rpm and the full-load speed is 1400 rpm. If the flux is weakened to 50 % of the rated value, calculate the new steady-state speed.

[Ans. 1686 rpm]

Summer 2006 (Q6(b))

Problem 19

The specification sheet for the Maxon 250 W, 24 V, 5300 rpm, EC dc motor is shown on page 161.

(i)   Compute the armature current, applied voltage, and machine efficiency for the condition shown in line 10 of motor data.

(ii)   What are the amplitude of per-phase back emf and the rms per-phase current?

[Ans. 7.03 A, 25.52 V, 82.6%, 11.23 V, 5.74 A]