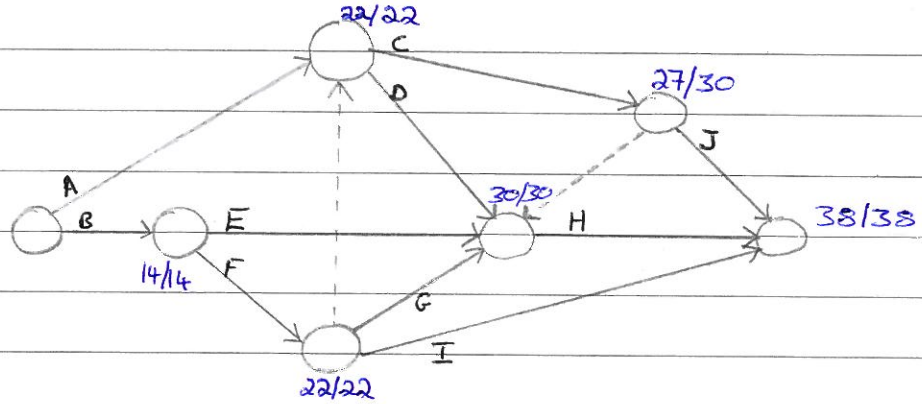
ME4001 Engineering Management Summer '08

4a.

Activity	$\bar{D}$	δ	R.R.	Total Float
A	15	3	75	7
B	14	3	42	0
C	5	0	25	3
D	8	1	40	0
E	4	1	20	12
F	8	2	48	0
G	6	1	36	2
H	8	2	32	0
I	3	0	15	13
J	5	0	25	6



Critical Path = B-F-Dummy-D-H

$\delta_{CP} = \sqrt{18}$

$$z = \frac{2}{\sqrt{18}} = -0.471$$

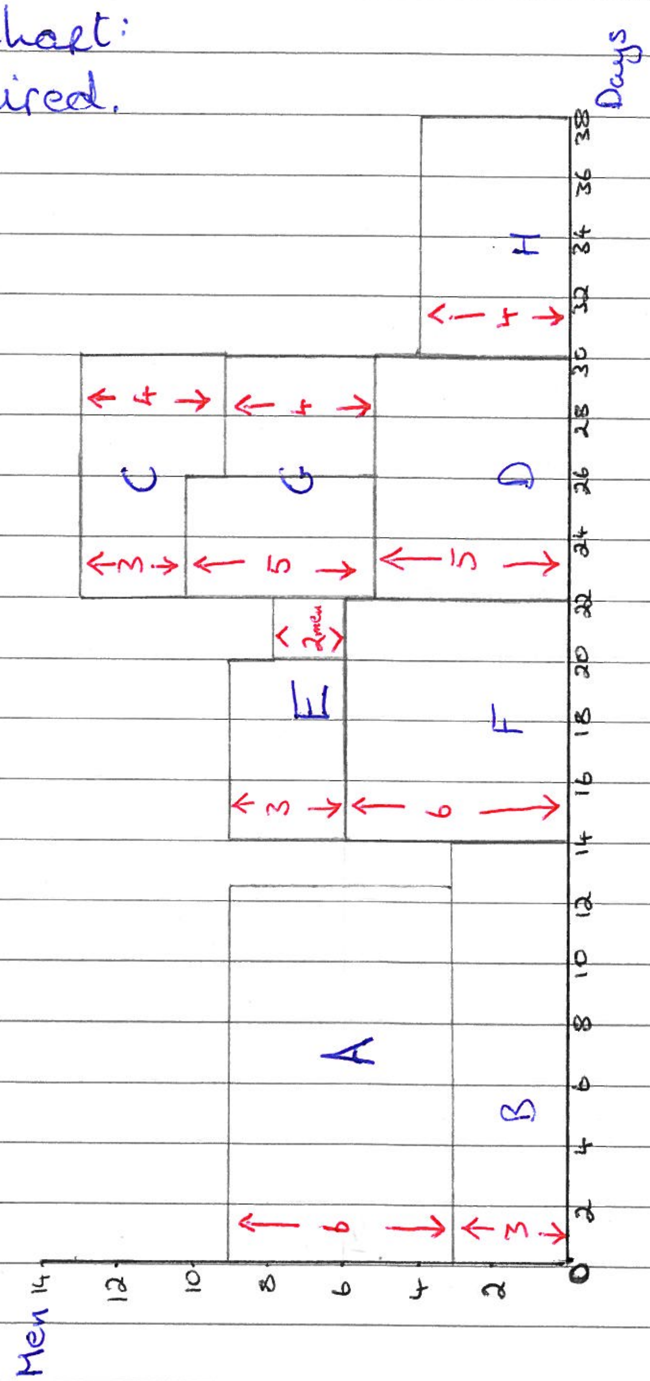
As $z < 0$, $P = 1 - [H(P\{|z|\}) + 0.5]$

$= 1 - 0.319 - 0.5$

$= 18.1\%$

4a) Max. requirement between 22-30 days.
Do critical path first:

From Gantt Chart:
13 men required.



ME4001 Engineering Management Summer '07

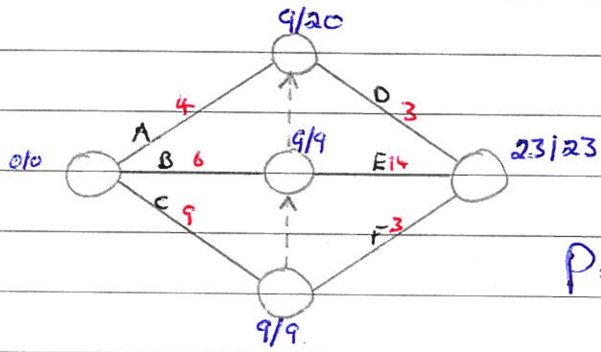
4a)

Activity	$\bar{D}$	δ^2	Total Float
A	4	$^4/_{36}$	16
B	6	$^{16}/_{36}$	3
C	9	$\frac{64}{36}$	0
D	3	$^4/_{36}$	11
E	14	$^{36}/_{36}$	0
F	3	$^4/_{36}$	11

$$\bar{D} = \frac{a+b+4m}{6}$$

$$\delta^2 = \frac{(b-a)^2}{36}$$

Critical Path: C-Dummy-E



$$Z = \frac{-2}{\sqrt{\delta_{cp}^2}}, \quad \delta_{cp}^2 = \frac{100}{36} = 1.\dot{6}$$

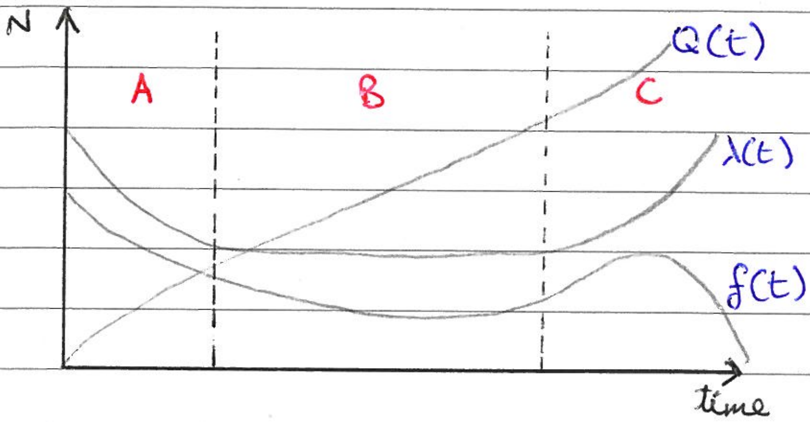
$$Z = -1.2$$

$$P = 1 - \left[(1 - P\{|z|\}) + 0.5 \right]$$

$$P = 1 - 0.1151 - 0.5$$

$$= 38.49\%$$

4 b) The bathtub curve is a graphical representation of failure rates as a function of time. There are 3 regions of interest, marked A, B & C below:



A:

- The infant mortality or "born-in" region
- Defects due to manufacturing are found here
- Reduced by improving manufacturing methods

B:

- Normal operating region
- Failure due to random causes
- Failure analysis centred here, $\lambda(t) = \text{constant} = \lambda$

C:

- Wear-out stage
- Replacement & maintenance policies applicable here
- Correct maintenance in 'B' region extends lifetime & reliability

4b

The failure density function is the number of failures per unit time expressed as a function of the original population:

$$f(t) = (dy/dt)/N$$

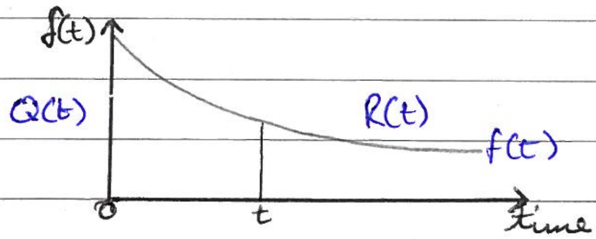
$$y = N\int_0^t f(t)\,dt$$

$$\frac{y}{N} = \int_0^t f(t)\,dt$$

$$\Rightarrow Q(t) = \int_0^t f(t)\,dt \quad \text{but } R(t)+Q(t)=1$$

$$R(t) = 1 - \int_0^t f(t)\,dt$$

$$f(t) = -\frac{dR(t)}{dt} = \frac{dQ(t)}{dt}$$



The failure rate is the number of failures per unit time expressed as a fraction of the number of survivors

$$\lambda(t) = \frac{dy/dt}{x}, \quad x = N - y$$

$$\Rightarrow \lambda(t) = \frac{(dy/dt)}{N} \cdot \frac{N}{N-y} = \frac{(dy/dt)}{N} \cdot \frac{1}{1-y/N} = \frac{f(t)}{R(t)}$$

$$\lambda(t) = \frac{-\left(\frac{dR(t)}{dt}\right)}{R(t)}$$

4 b) $f(0) = 0.4$ $\qquad \dfrac{df(t)}{dt} = -\dfrac{0.4t}{5}$

$f(5) = 0$

$$f(t) = 0.4 - \left(\frac{0.4}{5}\right)t = 0.4 - 0.08t$$

$$Q(t) = \int_0^t f(t)\,dt = 0.4t - \frac{0.08t^2}{2}$$

$$Q(t) = 0.4t - 0.04t^2$$

$$R(t) = 1 - 0.4t + 0.04t^2$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{0.4 - 0.08t}{1 - 0.4t + 0.04t^2}$$

MTBF:

$$\int_0^t R(t)\,dt$$

$$= \int_0^t (1 - 0.4t + 0.04t^2)\,dt$$

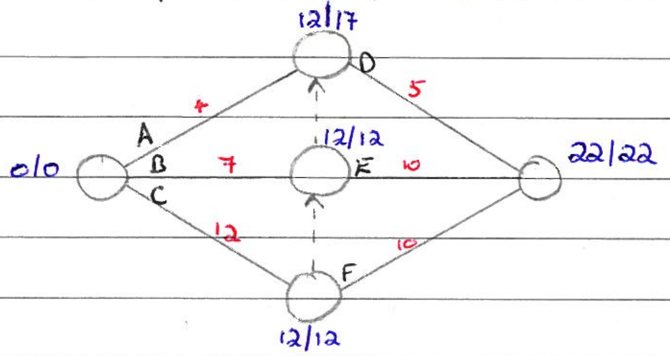
$$= t - 0.2t^2 + \frac{0.04}{3}t^3 \Big|_0^5$$

$$= 1.6 \text{ years}$$

ME4001 Engineering Management Summer '06

4 a)

Activity	$\bar{D}$	S	Total Float
A	4	1	8
B	7	2	5
C	12	2·5	0
D	5	1	5
E	10	2	0
F	10	3	0



2 Potential Critical Paths: C-Dummy-E
C-F

$\delta_{cp1} = \sqrt{10.25}$ $z_1 = \frac{-2}{\delta_{cp1}} = -0.6247$

$\delta_{cp2} = \sqrt{15.25}$ $z_2 = \frac{-2}{\delta_{cp2}} = -0.512$

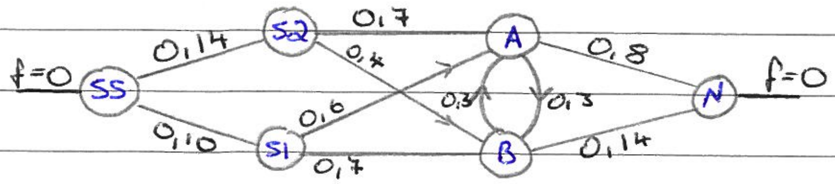
$P_1 = 1 - 0.5 - (R - P\{|z|\}) = 1 - 0.5 - 0.266 = 23.4\%$

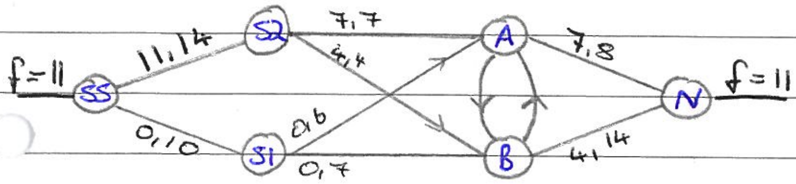
$P_2 = 1 - 0.5 - 0.304 = 19.6\%$

Take lower $P = 19.6\%$

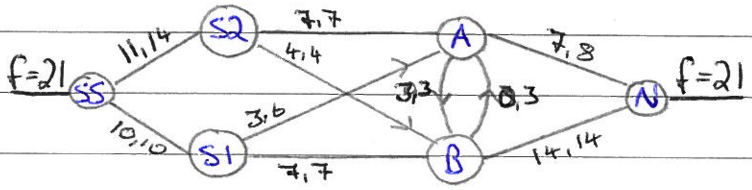
ME4001 Engineering Management Summer '06

4b) A flow augmenting path is a path from source to sink where the flow can take place.





SS - S2 - A - N = 7
SS - S2 - B - N = 4

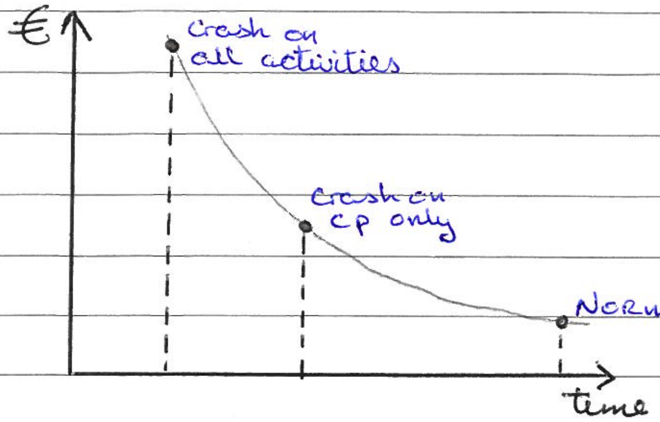


SS - S1 - A - B - N = 3
SS - S1 - B - N = 7

Max. flow = 21

4. Mean Expected Duration = $\frac{a+b+4m}{6}$

Variance $\Rightarrow \delta^2 = \frac{(b-a)^2}{36}$



Normal Cost:

The cost of carrying out the activity in minimum time using normal means (avoiding overtime, use of special staff/resources)

Crash Cost:

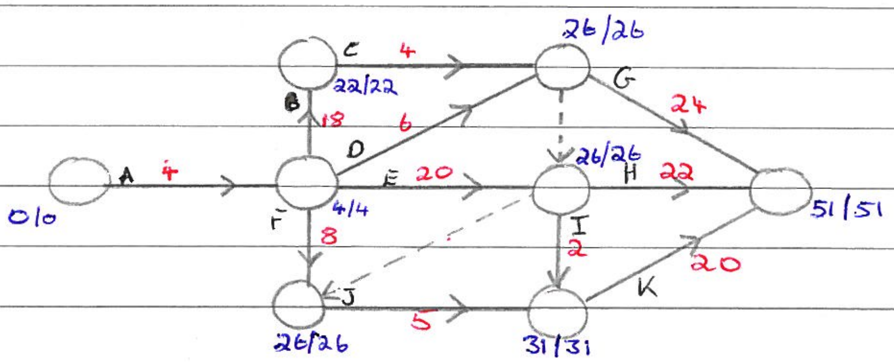
The minimum cost of carrying out the activity using whatever means possible to achieve the absolute minimum time.

As duration decreases, costs increase until crash point is reached. When the critical path duration is reduced, other paths may go critical, increasing costs.

4 b)

i)

Activity	$\bar{D}$	δ	€C.C.	Total Float
A	4	1	90	0
B	18	2	90	0
C	4	0.5	100	0
D	6	0.75	100	16
E	20	1.25	80	2
F	8	1	100	14
G	24	2	80	1
H	22	1.5	80	3
I	2	0.25	100	3
J	5	1	100	0
K	20	1.5	80	0



ii) Critical Path: A - B - C - Dummy - Dummy - J - K

iii) $Z = \frac{-2}{\sqrt{\delta_{cp}^2}}$ $\delta_{cp}^2 = 8.5$ $Z = -0.235$

$$P = 1 - \left(\left[1 - P\{|Z|\}\right] + 0.5\right)$$
$$= 1 - (0.407 + 0.5)$$
$$= 9.3\%$$

4 iv) Lowest crash cost on critical path:
k = €80 /day

When K is reduced by 1 day, total time = 50 days, ABCG becomes another critical path
⇒ Must reduce both paths
⇒ G has lowest cost on CP2

· Reducing total to 48 days
→ Costs 3k + 2G = €400
→ ABC-Dummy-H is now also critical

Reducing 3 CPs for 47 day completion:
→ Costs an additional €240

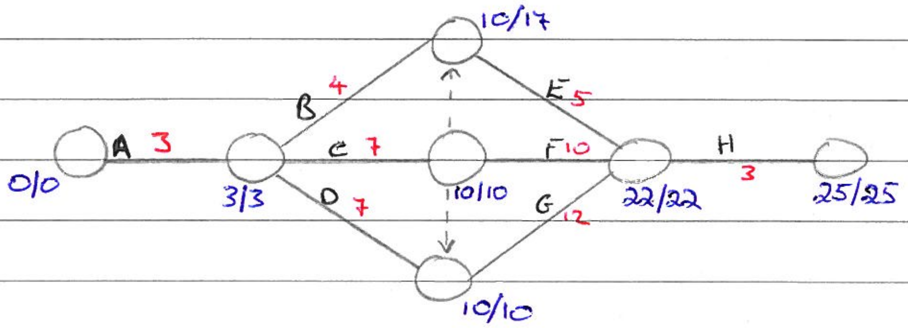
Total Cost of Reducing Period from 51 → 47 days:

€640

ME4001 Engineering Management Summer '04

6a)

Activity	$\bar{D}$	δ	Total Float
A	3	1	0
B	4	1	10
C	7	2	0
D	7	1	0
E	5	1	7
F	10	2	2
G	12	2·5	0
H	3	1	0



CP_1: A - C - Dummy - G-H
CP_2: A - D - G - H

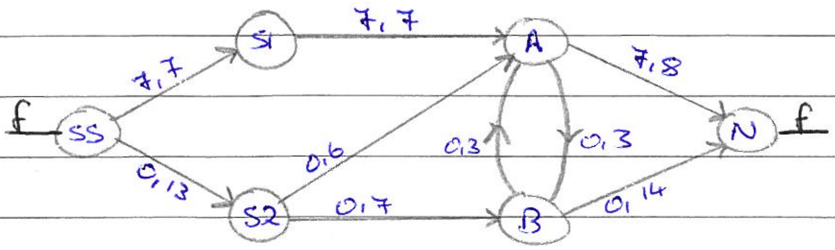
$\delta_1^2 = 12{\cdot}25$ $\qquad Z_1 = -0{\cdot}571$
$\delta_2^2 = 9{\cdot}25$ $\qquad Z_2 = -0{\cdot}658$

$P_1 = 1 - \left(0{\cdot}5 + [1 - P\{|Z_1|\}]\right) = 21{\cdot}57\%$
$P_2 = 1 - \left(0{\cdot}5 + [1 - P\{|Z_2|\}]\right) = 24{\cdot}48\%$

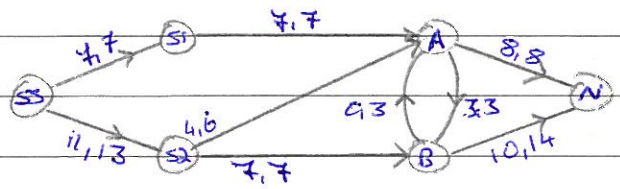
Take $P_1 = 21{\cdot}57\%$

ME4001 Engineering Management Summer '04

6 b)



SS - S1 - A - N ⇒ f = 7



SS - S2 - A - N ⇒ f = 1

SS - S2 - A - B - N ⇒ f = 3

SS - S2 - B - N ⇒ f = 7

Max. flow = 18