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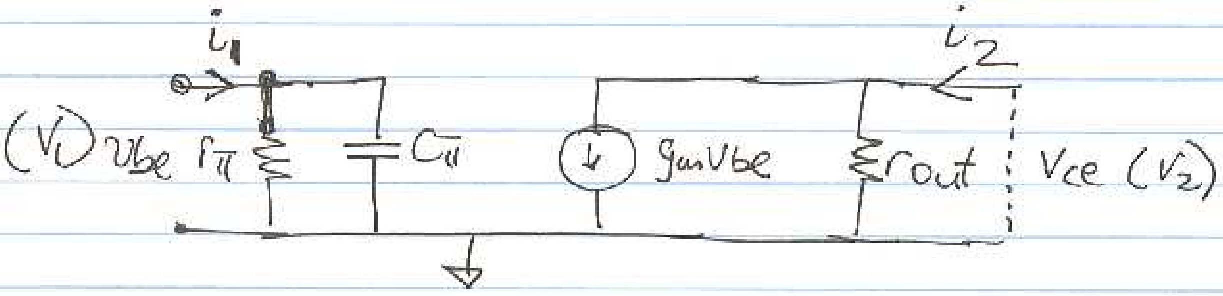
Q1 (a)



$$I_C = I_S e^{\frac{qV_{BE}}{kT}}\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{q}{kT} I_C = \frac{I_C}{V_T} \quad ; \quad V_T = \frac{kT}{q}$$

$$g_{out} = \frac{dI_C}{dV_{CE}} = I_S e^{\frac{qV_{BE}}{kT}} \cdot \frac{1}{V_A} \approx \frac{I_C}{V_A}$$

$$r_{out} = \frac{1}{g_{out}} = \frac{V_A}{I_C}$$

$$r_\pi = \frac{1}{g_\pi} \quad , \quad g_\pi = \frac{dI_B}{dV_{BE}} = \frac{d}{dV_{BE}}\left(\frac{I_C}{\beta}\right) = \frac{g_m}{\beta}$$

$$r_\pi = \beta / g_m$$

f_T is defined as the frequency at which $|h_{21}| = 1$

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0} \Rightarrow h_{21} = \frac{g_m v_{be}}{\left(\frac{1}{r_\pi} + j\omega C_\pi\right) v_{be}} \approx \frac{g_m}{j\omega C_\pi} \text{ at high } \omega$$

$$|h_{21}| = \frac{g_m}{\omega C_\pi} = 1 \ @ \ \omega = 2\pi f_T$$

$\Rightarrow$

$$\Rightarrow f_T = \frac{g_m}{2\pi C_\pi}$$

[8 marks]

Note (when $V_2 = 0$, there is no current in r_{out})

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Q1(b)

$$f_T = \frac{g_m}{2\pi C_\pi}$$

$$C_\pi = C_{je} + C_D = C_{je} + \tau_F g_m$$

$$f_T = \frac{g_m}{2\pi (C_{je} + \tau_F g_m)}$$

$$2\pi f_T = \frac{g_m}{C_{je} + \tau_F g_m}$$

$$= \frac{1}{\tau_F + \frac{C_{je}}{g_m}}$$

$$\Rightarrow \frac{1}{2\pi f_T} = \tau_F + \frac{C_{je}}{g_m} = \tau_F + \frac{V_T . C_{je}}{I_C}$$

From two (f_T, I_C) pairs τ_F and C_{je} can be estimated assuming C_{je} is only weakly bias dependent

$$\frac{1}{2\pi f_{T1}} = \tau_F + \frac{V_T . C_{je}}{I_{C1}}$$

$$\frac{1}{2\pi f_{T2}} = \tau_F + \frac{V_T \, C_{je}}{I_{C2}}$$

$$\Rightarrow \left(\frac{1}{2\pi f_{T1}}\right) - \left(\frac{1}{2\pi f_{T2}}\right) = V_T . C_{je} \left(\frac{1}{I_{C1}} - \frac{1}{I_{C2}}\right)$$

$$\Rightarrow C_{je} = \frac{\left(1/2\pi f_{T1}\right) - \left(1/2\pi f_{T2}\right)}{V_T \left(1/I_{C1} - 1/I_{C2}\right)}$$

at 300K $V_T = \frac{kT}{q} = 25.8\,mV$

Putting in values $C_{je} = \underline{1pF}$

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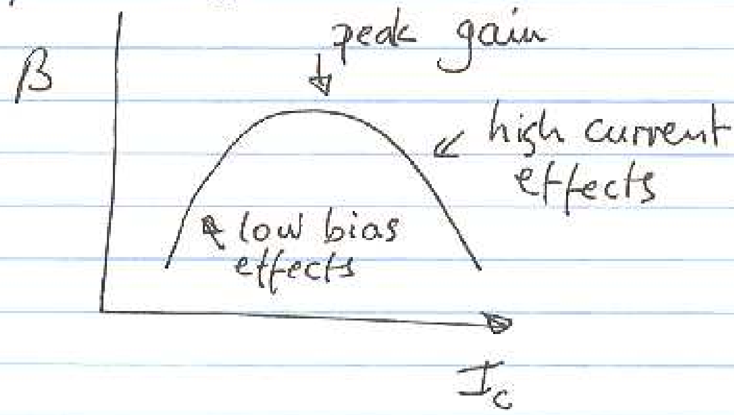
and $\tau_F = \left(\frac{1}{2\pi f_{T1}}\right) - \frac{V_T}{I_{C1}} C_{je}$

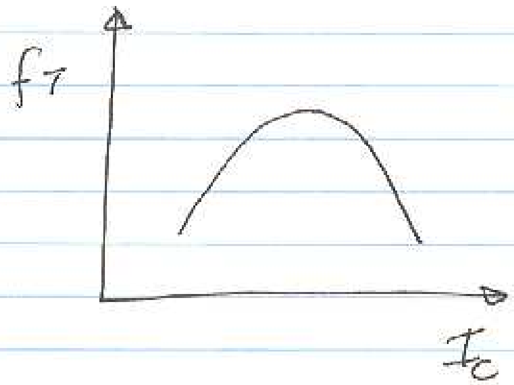
$= 0.1\text{ns}$

[8 marks]

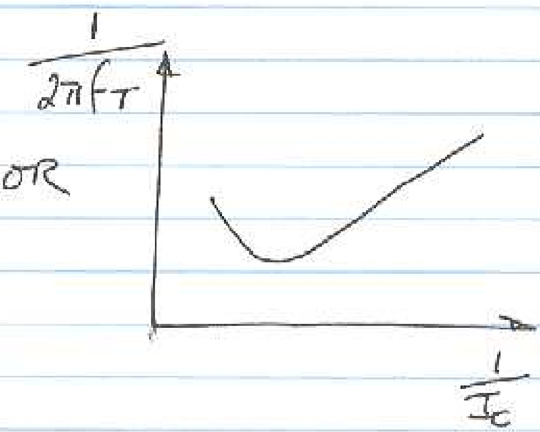
1(c) β vs I_C



f_T vs I_C



OR



[4 marks]

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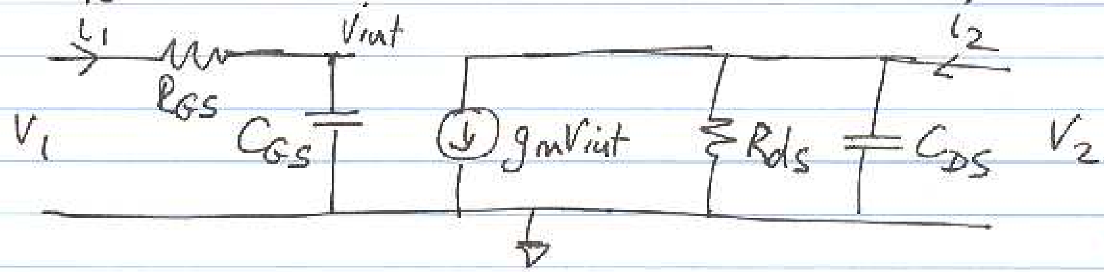
Q2(a)

Ignoring C_{GD}



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$$V_{int} = \frac{1/j\omega C_{GS}}{R_{GS} + 1/j\omega C_{GS}} V_1 = \frac{1}{1 + j\omega R_{GS} C_{GS}} V_1$$

$$\boxed{y_{11}} = \left.\frac{i_1}{V_1}\right|_{V_2=0} = \frac{V_1}{R_{GS} + 1/j\omega C_{GS}} \frac{1}{V_1} = \boxed{\frac{j\omega C_{GS}}{1 + j\omega R_{GS} C_{GS}}}$$ 2

$$\boxed{y_{12}} = \left.\frac{i_1}{V_2}\right|_{V_1=0} = \boxed{0} \quad (\text{no } C_{GD})$$ 1

$$\boxed{y_{21}} = \left.\frac{i_2}{V_1}\right|_{V_2=0} = \frac{g_m V_1}{1 + j\omega R_{GS} C_{GS}} \frac{1}{V_1} = \boxed{\frac{g_m}{1 + j\omega R_{GS} C_{GS}}}$$ 2

$$\boxed{y_{22}} = \left.\frac{i_2}{V_2}\right|_{V_1=0} = \boxed{\frac{1}{R_{DS}} + j\omega C_{DS}}$$ 2

[10 marks]

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Q2(b)

There are 5 equivalent circuit elements to be extracted - gm, rds, Cds, rgs, Cgs.

$$y_{21} = \frac{g_m}{1+j\omega R_{GS} C_{GS}} \Rightarrow \frac{1}{y_{21}} = \frac{1}{g_m} + \frac{j\omega R_{GS} C_{GS}}{g_m}$$

$$\Rightarrow Re\left\{\frac{1}{y_{21}}\right\} = \frac{1}{g_m} \Rightarrow \boxed{g_m = 1 \Big/ Re\left\{\frac{1}{y_{21}}\right\}} \quad (1)$$

$$y_{11} = \frac{j\omega C_{GS}}{1+j\omega R_{GS} C_{GS}} \Rightarrow \frac{1}{y_{11}} = R_{GS} + \frac{1}{j\omega C_{GS}}$$

i.e. $\frac{1}{y_{11}} = R_{GS} - j\frac{1}{\omega C_{GS}}$

$$\Rightarrow \boxed{Re\left\{\frac{1}{y_{11}}\right\} = R_{GS} \quad (2)}$$

$$Im\left\{\frac{1}{y_{11}}\right\} = -\frac{1}{\omega C_{GS}} \Rightarrow \boxed{C_{GS} = -\frac{1}{\omega \cdot Im\left\{\frac{1}{y_{11}}\right\}}}$$

$$= -\frac{1}{2\pi F\, Im\left\{\frac{1}{y_{11}}\right\}} \quad (3)$$

$$y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

$$\Rightarrow \cancel{Im}\ Re\{y_{22}\} = \frac{1}{R_{DS}} \Rightarrow \boxed{R_{DS} = \frac{1}{Re\{y_{22}\}}} \quad (4)$$

4 derivations

$$Im\{y_{22}\} = \omega C_{DS} \Rightarrow \boxed{C_{DS} = \frac{Im\{y_{22}\}}{2\pi F\ \cancel{Im\{y_{22}\}}}} \quad (5)$$

Substituting the given y values into formulas (1) to (5) gives

$g_m = \cancel{\ \ }\ 0.25$, $R_{GS} = 40.17\,\Omega$, $C_{GS} = 0.96\,pF$

$R_{ds} = 50.5\,\Omega$, $C_{DS} = 0.15\,pF$

Also $\boxed{f_T = \frac{g_m}{2\pi C_{GS}}} = 41.5\ GHz$

[10 marks]

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Question 3(a)

Given $y(t) = \left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\omega_1 t)$

$$+ \left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_1^2 A_2\right]\cos(\omega_2 t)$$

$$+ \frac{1}{4}\alpha_3 A_1^3 \cos 3\omega_1 t + \frac{1}{4}\alpha_3 A_2^3 \cos 3\omega_2 t$$

$$+ \frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\omega_1 + \omega_2)t + \frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\omega_1 - \omega_2)t$$

$$+ \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\omega_2 + \omega_1)t + \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\omega_2 - \omega_1)t$$

(i) 1dB gain compression point is defined for a single input frequency

$\therefore$ setting $A_2 = 0$ above the o/p at freq ω_1 is

$$\left[\alpha_1 A_1 + \tfrac{3}{4}\alpha_3 A_1^3\right]\cos(\omega_1 t)$$

$$y'(t) = A_1\left[\alpha_1 + \tfrac{3}{4}\alpha_3 A_1^2\right]\cos(\omega_1 t)$$

for low amplitudes and since $|\alpha_3| << |\alpha_1|$ normally

$$y'(t) \simeq A_1 \alpha_1 \cos(\omega_1 t)$$

and the gain is

$$G = \alpha_1$$

OR in dB

$$G_{dB} = 20\log_{10}(\alpha_1)$$

As the amplitude increases the gain degrades until it drops to 1dB less than this. ie

$$G_{P1dB} = 20\log_{10}(\alpha_1) - 1$$

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At moderate amplitudes the A_1^3 term is important so

$$y'(t) = A_1\left(\alpha_1 + \tfrac{3}{4}\alpha_3 A_1^2\right)\cos\omega_1 t$$

$$= A_1\alpha_1\left[1 + \tfrac{3}{4}\frac{\alpha_3}{\alpha_1}A_1^2\right]\cos\omega_1 t$$

So the gain is

$$G' = \alpha_1\left[1 + \tfrac{3}{4}\frac{\alpha_3}{\alpha_1}A_1^2\right]$$

or in dB $\quad 20\log_{10}G' = 20\log_{10}\left[\alpha_1\left(1+\tfrac{3}{4}\frac{\alpha_3}{\alpha_1}A_1^2\right)\right]$

setting this equal to the previous result

$$20\log_{10}G' = 20\log_{10}G - 1$$

$\Rightarrow$

$$20\log_{10}\left[\alpha_1\left(1+\tfrac{3}{4}\frac{\alpha_3}{\alpha_1}A_1^2\right)\right] = 20\log_{10}(\alpha_1) - 1$$

$$20\log_{10}\alpha_1 + 20\log_{10}\left(1+\frac{3}{4}\frac{\alpha_3}{\alpha_1}A_1^2\right) = 20\log_{10}(\alpha_1) - 1$$

$$\Rightarrow 20\log_{10}\left(1+\frac{3\alpha_3}{4\alpha_1}A_1^2\right) = -1$$

$$\Rightarrow A_1 = \sqrt{\left(10^{-0.05}-1\right)\frac{4}{3}\frac{\alpha_1}{\alpha_3}} = \sqrt{-0.145\frac{\alpha_1}{\alpha_3}}$$

But α_1 and α_3 have opposite signs for compressive behaviour so

$$\frac{\alpha_1}{\alpha_3} = -\left|\frac{\alpha_1}{\alpha_3}\right|$$

$$\Rightarrow A_1 = A_{P1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

[5 marks]

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3(a) (ii) The 3rd order IM intercept is calculated for equal amplitude inputs

$$A_1 = A_2 = A$$

The output at ω_1 is then

$$y'(t) = \left[\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2} A^3\right]\cos(\omega_1 t)$$

$$= \alpha_1 A\left[1 + \frac{9}{4}\alpha_3 A^2\right]\cos(\omega_1 t)$$

Making the approximation that $\frac{9}{4}\alpha_3 A^2 << 1$ give

$$y'(t) = \alpha_1 A \cos(\omega_1 t)$$

Taking any of the 3rd order IM products, eg $(2\omega_1 + \omega_2)$ give

$$y'_{3rd}(t) = \frac{3}{4}\alpha_3 A^3$$

At the 3rd order IM point the amplitudes of the fundamental and the 3rd IM products will be the same so

$$|\alpha_1 A| = \left|\frac{3}{4}\alpha_3 A^3\right|$$

$$\Rightarrow A = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} = A_{IP3}$$

[5 marks]

A_F = o/p at fundamental, A_3 = 3rd Harmonic o/p

3(b) Fundamental Amplitude = $|\alpha_1 A| \Rightarrow \alpha_1 = \frac{A_F}{A} = \frac{100}{1} = 100$

3rd Harmonic Amplitude = $\left|\frac{1}{4}\alpha_3 A^3\right| \Rightarrow \alpha_3 = \frac{A_3}{\frac{1}{4}A^3} = \frac{10^{-9}}{\frac{1}{4}10^{-9}} = 4$

$$\Rightarrow \left|\frac{\alpha_1}{\alpha_3}\right| = \frac{1}{4}A^2 \cdot \frac{A_{Fund}}{A_{3rd\ Harmonic}} = \frac{1}{4}\cdot(10^{-3})^2\ \frac{0.1}{10^{-9}}$$

$$= \cancel{\ldots}\ 25$$

and

$$A_{1dB} = \sqrt{0.145 \times \cancel{\ldots}\ 25} = \cancel{\ldots}\ 1.9V \approx 2V$$

[5 marks]

→ Note: $1mV = 10^{-3}V$

$(1mV)^3 = (10^{-3})^3 = 10^{-9}\ V^3$

~~Very large, but only an example!~~

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3(c) Two other undesired effects are blocking and cross modulation
Looking at the output at frequency ω_1:

$$y(t) = \left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\omega_1 t)$$

Blocking: The gain for this frequency is

$$G = \alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2$$

If $\alpha_1 > 0$ and $\alpha_3 < 0$, then as the amplitude A_2 increases G will decrease - therefore a large signal will de-sensitize the amplifier to weak signals which may be desired.

Cross Modulation
If the frequency ω_2 is a modulated signal such as amplitude modulated then it's amplitude it itself a function of time i.e.

$$y(t) = \left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 \left[A_2'(1 + m\sin\omega_M t)\right]^2\right]\cos\omega_1 t$$

~~The~~ This results in cross modulation products between ω_1 and the modulating frequency ~~of~~ of the second waveform ω_2. Thus the modulation of a stronger signal can transfer to a weaker signal giving unwanted cross-modulation

[5 marks].

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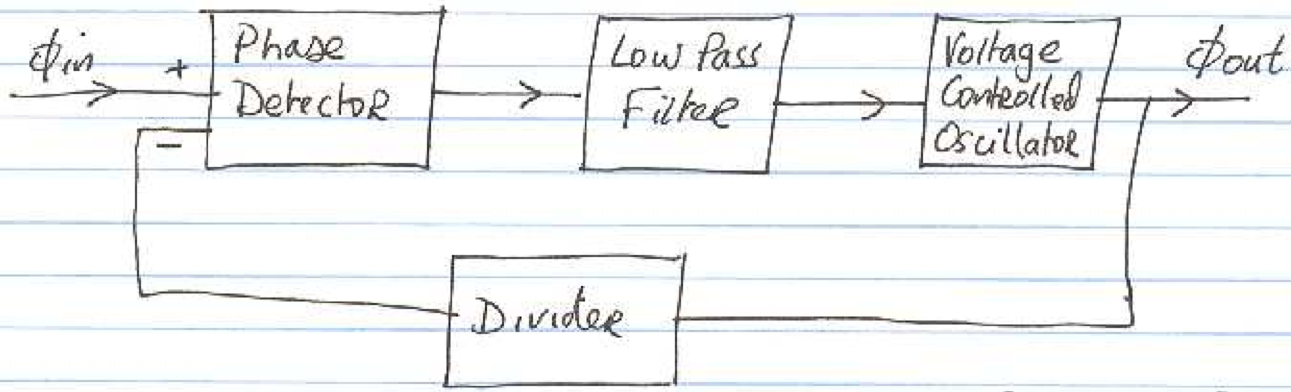
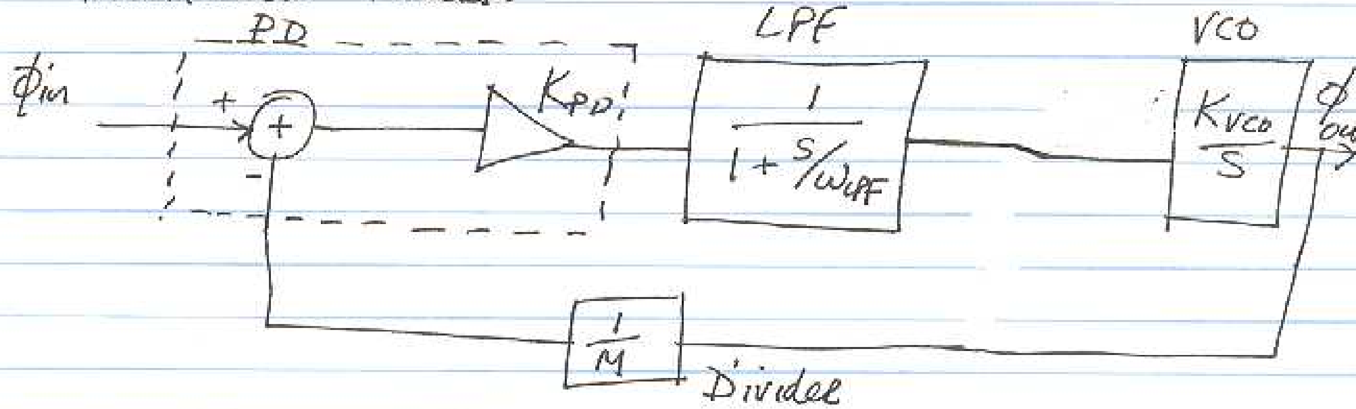
Question 6

Q6(a) Type 1 PLL



[5 marks]

Q6(b) To determine the closed-loop response transfer functions are needed to describe the individual blocks:



Open Loop Response

$$H_{OL}(s) = K_{PD} \cdot \frac{1}{1+s/\omega_{LPF}} \cdot \frac{K_{VCO}}{s}$$

$$= \frac{K_{PD}K_{VCO}}{s^2/\omega_{LPF} + s}$$

Closed Loop Response

$$\phi_{out}(s) = \left[\phi_{in}(s) - \frac{\phi_{out}(s)}{M}\right] H_{OL}(s)$$

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$$\Rightarrow \Phi_{out}(s)\left[1+\frac{H_{OL}(s)}{M}\right] = \Phi_{in}(s)\, H_{OL}(s)$$

$$\Rightarrow H_{CL}(s) = \frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{H_{OL}(s)}{1+\frac{H_{OL}(s)}{M}} = \frac{1}{\frac{1}{H_{OL}(s)}+\frac{1}{M}}$$

$$H_{CL}(s) = \frac{1}{\frac{\frac{s^2}{\omega_{LPF}}+s}{K_{PD}K_{VCO}}+\frac{1}{M}}$$

$$= \frac{K_{PD}K_{VCO}}{\frac{s^2}{\omega_{LPF}}+s+\frac{K_{PD}K_{VCO}}{M}}$$

$$= \frac{K_{PD}K_{VCO}\,\omega_{LPF}}{s^2+s\,\omega_{LPF}+\frac{K_{PD}K_{VCO}\,\omega_{LPF}}{M}}$$

In this case $\omega_n^2 = \frac{K_{PD}K_{VCO}\omega_{LPF}}{M}$, $\omega_n = \sqrt{\frac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$

$$2\zeta\omega_n = \omega_{LPF} \Rightarrow \zeta = \frac{\omega_{LPF}}{2\omega_n} = \frac{\omega_{LPF}}{2}\sqrt{\frac{M}{K_{PD}K_{VCO}\omega_{LPF}}}$$

$$\Rightarrow \zeta = \frac{1}{2}\sqrt{\frac{M\,\omega_{LPF}}{K_{PD}K_{VCO}}}$$

[8 marks]

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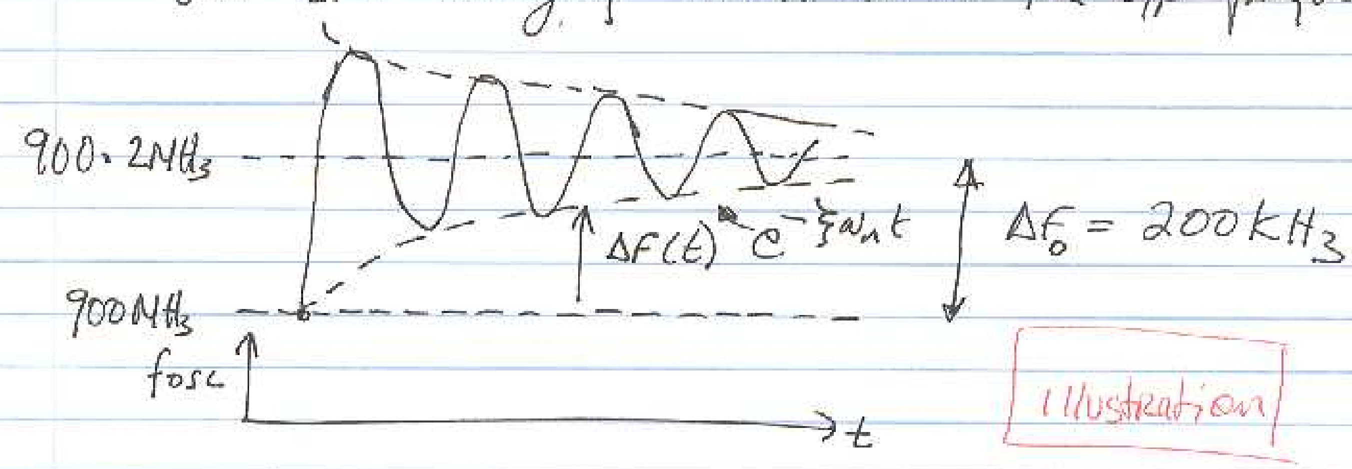
Q6(c) For an underdamped PLL the response to a set frequency change from 900 MHz to 900.2MHz will be a decaying oscillation in the o/p frequency



Illustration

The frequency change in response to a change of set point follows a decaying exponential approximately given by (for small ξ):

$$\Delta F(t) = \Delta F_0\left[1 - e^{-\xi\omega_n t}\right]$$

The error w.r.t. the new set frequency is then

$$e(t) = \Delta f_0 - \Delta F(t)$$

$$= \Delta f_0\left[1 - 1 + e^{-\xi\omega_n t}\right] = \Delta f_0\, e^{-\xi\omega_n t}$$

Want $e(t)$ to be $\le 100\,Hz$

so $$100 = 200\,kHz \cdot e^{-\xi\omega_n t}$$

$$\Rightarrow e^{-\xi\omega_n t} = \frac{100}{200\times10^3}$$

$$-\xi\omega_n t = \ln\left(\frac{1}{2\times10^3}\right)$$

$$\Rightarrow t = -\frac{1}{\xi\omega_n}\ln\left(\frac{1}{2\times10^3}\right)$$

But $\xi\omega_n = \frac{1}{2}\omega_{LPF} = \frac{1}{2}2\pi F_{LPF} = \pi F_{LPF}$

$$\Rightarrow t = -\frac{1}{\pi F_{LPF}}\ln\left(\frac{1}{2\times10^3}\right) = -\frac{1}{\pi(20\times10^3)}\ln\left(\frac{1}{2\times10^3}\right) = 0.12\,ms$$

number

[7 marks]

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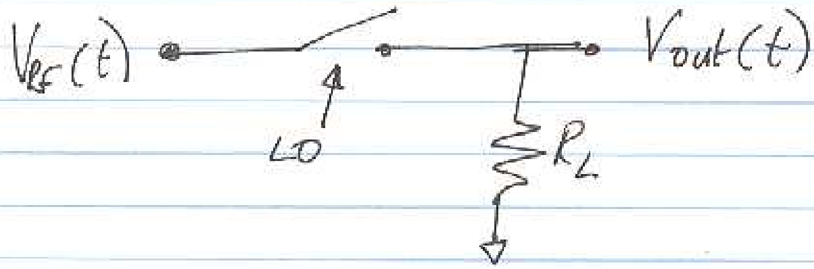
Question 7

7(a) RF Mixer Based on Switch



The LO signal controls a switch. Assuming the LO amplitude is large then it can be approximated that the switch is completely turned on during the positive half cycles of LO and completely turned off during the negative half cycles of LO i.e. $V_{RF}(t)$ is multiplied by a square wave:

$$V_{out}(t) = V_{RF}\cos(\omega_{RF}t)\left[\frac{1}{2} + \frac{2}{\pi}\sin\omega_{LO}t + \frac{2}{3\pi}\sin 3\omega_{LO}t + \cdots\right]$$

Looking at the first two terms of the sq. wave expansion

$$V_{out}(t) = \frac{1}{2}V_{RF}\cos(\omega_{RF}t) + \frac{2}{\pi}V_{RF}\cos(\omega_{RF}t)\sin(\omega_{LO}t) + \cdots$$

$$= \frac{1}{2}V_{RF}\cos\omega_{RF}t + \underbrace{\frac{V_{RF}}{\pi}}_{\text{amplitude of IF signal}}\left[\sin(\omega_{RF}+\omega_{LO})t - \sin(\omega_{RF}-\omega_{LO})t\right] + \cdots$$

Voltage conversion gain

$$A_V = \frac{\text{Amplitude of IF signal}}{\text{Amplitude of RF signal}} = \frac{V_{RF}}{\pi}\frac{1}{V_{RF}} = \frac{1}{\pi}$$

In dB

$$VCG_{dB} = 20\log_{10}\left(\frac{1}{\pi}\right) \simeq -10\,dB.$$

[10 marks]

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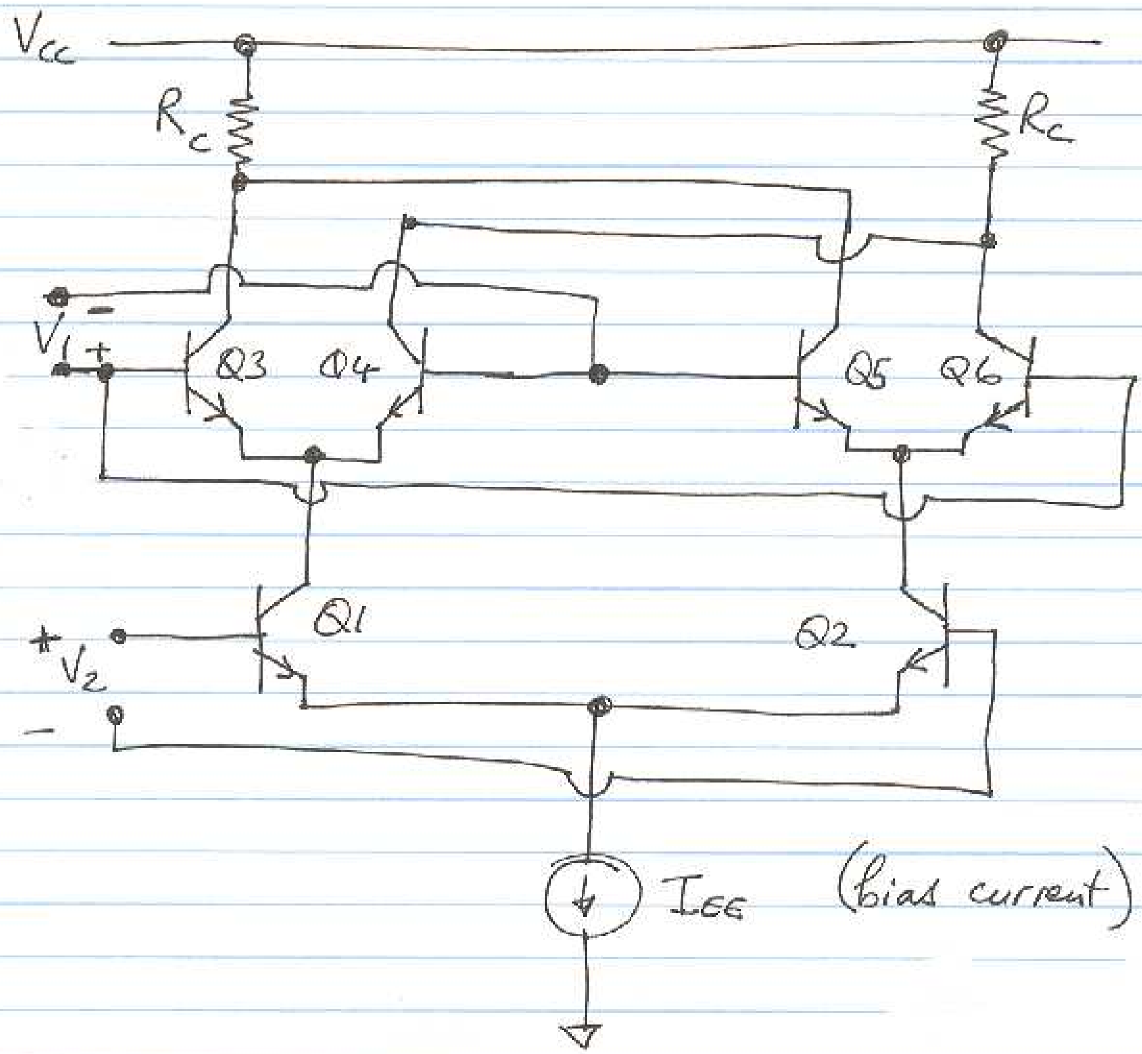
7(b) Gilbert Multiplier Circuit Based on NPNs



The different operating modes and functions are:

1. The magnitudes V_1 and V_2 are small: In this case the hyperbolic transfer function is approximately linear and the output is the product $V_1 V_2$.

2. One signal is large, one signal is small: This is similar to a switch based mixer and does a frequency conversion or modulation function.

3. Both signals are large: The circuit output depends on the phase difference between the two large signals so it performs as a phase detector.

[10 marks]