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THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2007

B.E. DEGREE (ELECTRICAL)

CONTROL ENGINEERING
EE4002

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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of a Casio fx570w or fx570ms calculator is permitted.

1.

(a) Consider the continuous-time PID controller,

$$m(t) = K_P\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\frac{de(t)}{dt}\right)$$

The controller time constants are related according to: $T_D = \dfrac{T_I}{4}$,

Use the matched-pole-zero approach to derive the following difference equation representation of this controller for implementation on a digital computer, with sample time T_s. Show clearly in your derivation how the parameters of the difference equation are related to parameters of the continuous PID controller.

$$m(k) = m(k-1) + \alpha e(k) + \beta e(k-1) + \gamma e(k-2)$$

(Hint: An extra pole at z=0 is required to produce a causal (realisable) control algorithm. Due to the integral action present in the original controller C(s), you

will need to determine the gain of the digital controller D(z) according to:
$\lim_{s \to 0} sC(s) = \lim_{z \to 1} (z-1)D(z)$)

[10 Marks]

(b) Consider the following closed-loop digital control scheme,

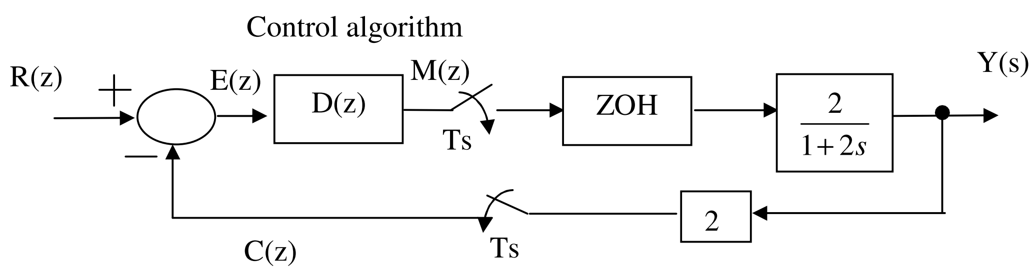

Fig. 1.1 Closed loop digital control system

The following discrete-time control algorithm has been designed, with the sample-time Ts=1 second .

$$m(k) = Ke(k-1) + 0.8m(k-1)$$

Sketch the root locus diagram for this process and use it to explain how the closed-loop dynamics depend on the choice of controller gain K.

Choose K to achieve a closed-loop peak overshoot of 30% for step changes in the setpoint.

[15 Marks]

2.

(a) Consider in Fig. 2.1 the block diagram for a sample and hold.

Derive the transfer function of a Zero-Order Hold and sketch its frequency response.

Briefly explain (without proof) the effect of varying the sampling frequency on the spectrum of the reconstructed signal u(t).

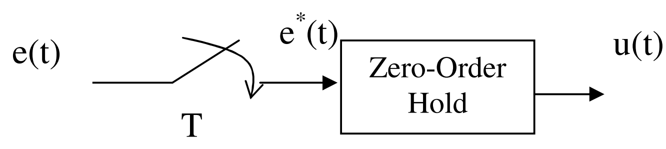

Fig. 2.1 sample and hold

Give Shannon's sampling theorem and comment on the benefits of over-sampling, in particular focussing on control applications. Explain why it is necessary to employ anti-aliasing filters, before sampling. Give some indication how sampling rate and filter bandwidth would be selected.

[12 Marks]

(b) Consider the following general first-order system with time delay (The time delay Td is roughly N samples long), within a closed-loop digital control scheme. The sampling time is T and a zero-order hold is assumed.

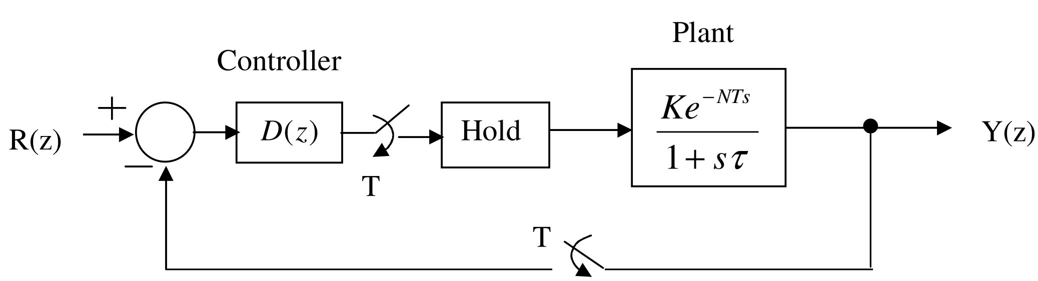

Fig. 2.2 Digital closed-loop control of a first order plant with delay

Derive the following Dahlin's controller for the general first order process, from a basic prescription of the shape of the desired closed-loop step response. Show clearly how the parameters of this controller are determined.

$$D(z) = K_d \frac{1+\gamma z^{-1}}{1+\alpha z^{-1}+\beta z^{-N-1}}.$$

Show that the controller provides integral action.

[13 marks]

3.

(a)

A certain process is known to have an open-loop transfer function of the following structure:

$$G(z) = \frac{\gamma z^{-2}}{1 + \alpha z^{-1} + \beta z^{-2}}.$$

Give the design equations for a Diophantine pole-placement adaptive controller based on estimates of the parameters of this model, provided by a recursive least-squares algorithm. Define the controller polynomials and the desired characteristic equation for this process.

Clearly show the development of the Sylvester matrix used to solve the Diophantine pole-placement design equation.

[10 marks]

(b)

Derive in full, the following least-squares algorithm, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains the sampled process output, up to the current k^{th} sample, *y(k)*.

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1} \Phi(k)^T \underline{Y}(k)$$

If a square matrix *P(k)* is now defined as $P(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1}$, derive the following update equation to obtain *P(k+1)* from process data up to the $(k+1)^{th}$ sample,

$$P(k+1) = \left(P(k)^{-1} + \underline{\psi}(k+1)\underline{\psi}(k+1)^T\right)^{-1},$$

where vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the $(k+1)^{th}$ sample.

Show by application of Householder's Matrix Inversion Lemma,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1},$$

that the following update equation for the model parameter vector can be obtained:

$$\underline{\hat{\theta}}(k+1) = \left[P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^T(k+1)P(k)}{1 + \underline{\psi}^T(k+1)P(k)\underline{\psi}(k+1)}\right]\left[\Phi(k)^T \underline{Y}(k) + \underline{\psi}(k+1)\,y(k+1)\right].$$

[15 marks]

4.

(a) Consider the following state-space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$

i) Develop fully the following solution for the state trajectory $\underline{x}(t)$, for $t \geq 0$, where $\underline{x}(0)$ is the initial state vector at t=0, and $\Phi(t)$ is the transition matrix.

$$\underline{x}(t) = \Phi(t)\underline{x}(0) + \int_0^t \Phi(t-\tau)B\underline{u}(\tau)d\tau$$

ii) If the sample-time is T, and it is assumed that a zero-order hold is applied to the input signal $\underline{u}(t)$, show that this process can be represented by the following discrete-time, state-space equations:

$$\underline{x}(k+1) = e^{AT}\,\underline{x}(k) + A^{-1}\left(e^{AT} - I\right)B\underline{u}(k)$$

[10 marks]

(b) A certain mechatronic system can be modeled by the following differential equation, where $u(t)$ is the input voltage, and $\theta(t)$ is the resulting angle of rotation.

$$\frac{d^2\theta(t)}{dt^2} + 7\frac{d\theta(t)}{dt} + 10\theta(t) = \frac{du(t)}{dt} + u(t)$$

i) Show how this system could be represented as a simulation diagram (eg. Simulink diagram), using only two integrators, a variety of gains and summers.

ii) Use this simulation diagram to derive the control-canonical state-space model of this process.

iii) If the initial conditions are $\theta(0)$=1, $u(0)$=0 and $\frac{d\theta(0)}{dt} = 0$, determine an expression for the zero-input responses of the *states of your model.*

[15 Marks]

5.

(a) Consider the following Nth order open-loop process, with one input *u(t)* and a single output *y(t)*,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

If this process is under the following state space control–law with integral action,

$$u(t) = -K\underline{x}(t) + K_I\int_0^t \left(r(\tau) - y(\tau)\right)d\tau$$

show that the closed-loop characteristic equation is:

$$\det\left[\begin{array}{c|c} sI_N - A + BK & -BK_I \\ \hline C & s \end{array}\right] = 0$$

[8 Marks]

(b) A DC motor driven positioning system can be represented by following block diagram:

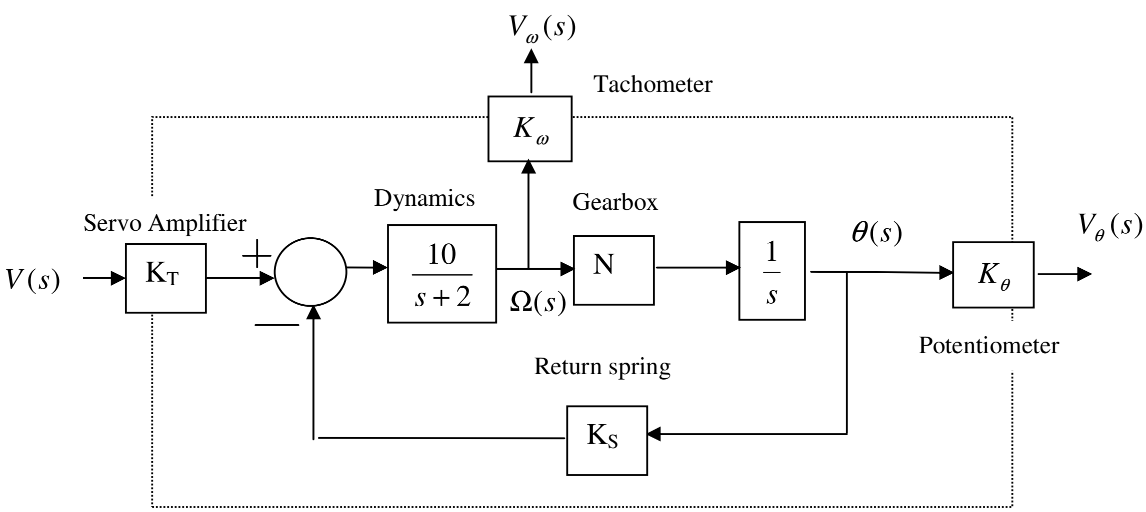

Figure 5.1 Mechatronic servo-system

Where *v(t)* is the applied voltage to the motor, *ω(t)* is the motor speed, (in rad/s), and *θ(t)* is the rotated angle, (in radians). The gearbox tooth ratio is $N > 1$. The motor speed is measured using a tachometer of gain K_ω (V rad^{-1} s) and the rotated angle θ(t) is measured using a potentiometer of gain K_θ (V rad^{-1}).

i) Determine a state-space representation of the process.

ii) Show that for any non-zero choice of gear ratio N, and servo amplifier gain K_T that your state-space representation is controllable.

iii) If N=10, K_T=20 Nm V^{-1} , K_S,=90 Nm rad^{-1}, design the following state space controller to control the angular position $\theta(t)$,

$$v(t) = r(t) - k_1 v_\omega - k_2 v_\theta$$

where the signal $r(t)$ is the setpoint in volts, ie:

$$r(t) = K_\theta \, \theta_d(t)$$

Here $\theta_d(t)$ is the desired rotated angle (in radians), the tacho-meter gain is K_ω =0.2V rad^{-1} s, and the potentiometer gain is K_θ = 1.5V rad^{-1}.

A closed loop damping of ξ=0.5 and a 2% settling time of 0.02 seconds is required.

[17 Marks]

6.

(a) Consider the following N^{th} order open-loop process, with single input *u(t)*, single output *y(t)* , and state-vector $\underline{x}(t)$,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

This process is controlled using a state-space regulator, with gain matrix *K*. The state vector is not measured directly, but is estimated as $\hat{\underline{x}}(t)$ using a full-state Luenberger observer with estimator gain matrix *G*.

Prove that the closed-loop system has 2N poles which are roots of the characteristic equation:

$$|sI - A + BK||sI - A + GC| = 0$$

Explain the "Separation Principle", and how this principle is applied in state-space control design.

[10 marks]

(b) Consider the following simplified model of the attitude dynamics of a satellite:

$$\frac{d^2\theta(t)}{dt^2} = u(t) + d(t)$$

(i) The following state-space regulator has been designed to place both the closed-loop poles at $s = -1 \pm j$:

$$u(t) = -2\frac{d\theta}{dt} - 2\theta(t)$$

Show that the closed-loop system is second-order with no closed-loop zeros in the transfer function $G_D(s)=\theta(s)/D(s)$.

(ii) A full-order Luenberger Observer is now used to estimate the states. The poles of the observer are both placed at s=-10 .

Determine the classical control representation of the state-space controller and show that the presence of the observer has introduced closed-loop zeros in the path from *D(s)* to $\theta(s)$.

[15 marks]