EE4011 RFIC Design

Non-linear Effects in RF Systems: Intermodulation

Two inputs applied to a 3rd order system (1)

An input consisting of two frequency components can be written as:

$$x(t) = A_1 \cos(\varpi_1 t) + A_2 \cos(\varpi_2 t)$$

The output of the 3rd-order system

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

is:

$$\begin{aligned} y(t) = \alpha_1 &\left[A_1 \cos(\varpi_1 t) + A_2 \cos(\varpi_2 t)\right] \\ &+ \alpha_2 \left[A_1 \cos(\varpi_1 t) + A_2 \cos(\varpi_2 t)\right]^2 \\ &+ \alpha_3 \left[A_1 \cos(\varpi_1 t) + A_2 \cos(\varpi_2 t)\right]^3 \end{aligned}$$

The square and cubic terms can be multiplied out and manipulated with trigonometric relationships to give a set of terms at unique frequencies.

Two inputs applied to a 3rd order system (2)

Check this result!

$$y(t) = \frac{1}{2}\alpha_2 A_1^2 + \frac{1}{2}\alpha_2 A_2^2$$
$$+\left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\varpi_1 t)$$
$$+\left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_1^2 A_2\right]\cos(\varpi_2 t)$$
$$+\frac{1}{2}\alpha_2 A_1^2 \cos 2\varpi_1 t + \frac{1}{2}\alpha_2 A_2^2 \cos 2\varpi_2 t + \frac{1}{4}\alpha_3 A_1^3 \cos 3\varpi_1 t + \frac{1}{4}\alpha_3 A_2^3 \cos 3\varpi_2 t$$
$$+\alpha_2 A_1 A_2 \cos(\varpi_1 + \varpi_2)t + \alpha_2 A_1 A_2 \cos(\varpi_1 - \varpi_2)t$$
$$+\frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 + \varpi_2)t + \frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 - \varpi_2)t$$
$$+\frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 + \varpi_1)t + \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 - \varpi_1)t$$

Two inputs applied to a 3rd order system (3)

Looking at the result on the previous slide, the output of the 3rd-order system in response to two frequencies at the input contains the following terms:

DC Offset	
Input Frequencies	ϖ_1, ϖ_2
Second Harmonics	$2\varpi_1, 2\varpi_2$
Third Harmonics	$3\varpi_1, 3\varpi_2$
Second – Order IM Products	$\varpi_1 + \varpi_2, \varpi_1 - \varpi_2$
Third – Order IM Products	$2\varpi_1 + \varpi_2, 2\varpi_1 - \varpi_2, 2\varpi_2 + \varpi_1, 2\varpi_2 - \varpi_1$

IM refers to InterModulation frequencies that occur at the "sums and differences" (beat frequencies) of the input frequencies and their harmonics. The order of an IM term is determined by adding the absolute values of the coefficients of the frequencies making up that term.

In general, a 3rd-order system will produce harmonics up to the 3rd harmonic and IM products up to the 3rd-order.

Third-Order Intermodulation (IM3) Products

If the system is balanced (α_2=0) there will be no 2nd-order intermodulation frequencies (IM2) produced. Therefore, 3rd-order IM products (IM3) are analysed frequently in RF design.

When two signals are applied to a 3rd-order non-linear system the resulting IM3 terms are:

$$\frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 + \varpi_2)t \quad , \quad \frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 - \varpi_2)t$$

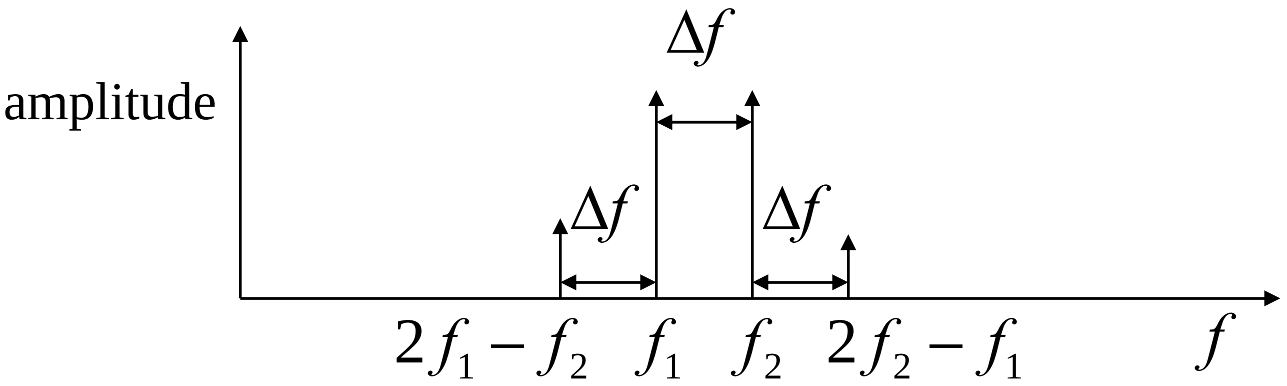
$$\frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 + \varpi_1)t \quad , \quad \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 - \varpi_1)t$$

Usually, it is the difference terms that fall within the bandwidth of the system. Taking a simple case where A_1=A_2=A the amplitudes of IM3 (A_{IM3}) terms in the output are:

$$A_{IM3} = \frac{3}{4}\left|\alpha_3\right| A^3$$

What's so special about the IM3 Terms?

The IM3 products lie at frequencies close to the signal frequencies being used and can cause unwanted interference to adjacent users of the radio spectrum.



$$2f_2 - f_1 = f_2 + f_2 - f_2 = f_2 + (f_2 - f_1) = f_2 + \Delta f$$

$$2f_1 - f_2 = f_1 + f_1 - f_2 = f_1 - (f_2 - f_1) = f_1 - \Delta f$$

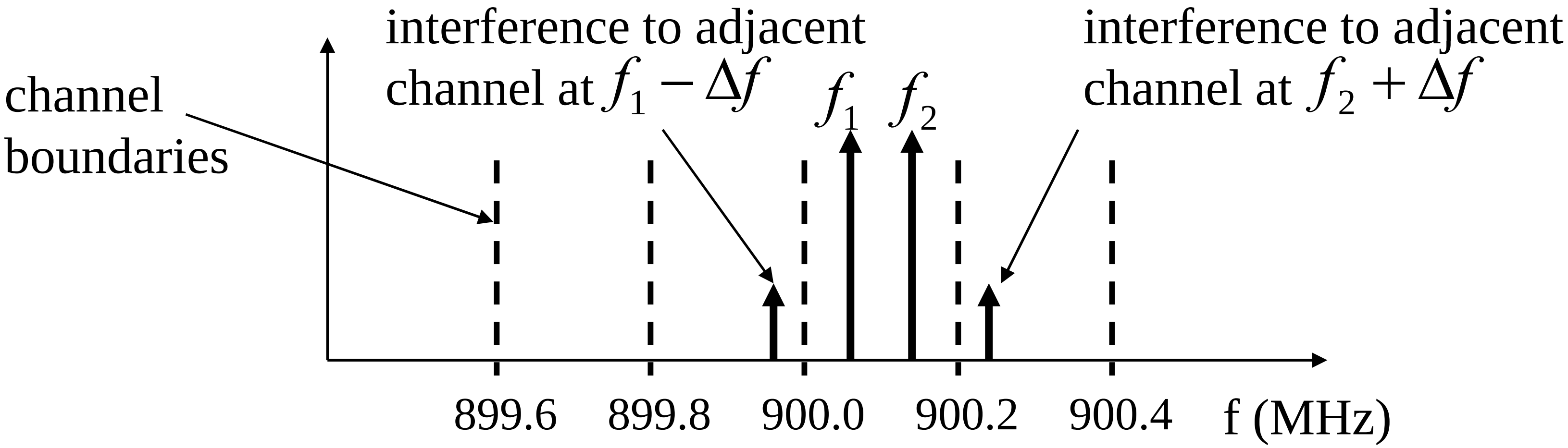
Example of IM3 at GSM Frequencies

In GSM (2G) mobile phone systems, the available spectrum is divided into channels of 200kHz bandwidth which are allocated to different users using a “frequency division multiplexing” system. Consider a user allocated the frequency band 900.0 to 900.2MHz. If there are strong frequencies in this band at 900.05MHz and 900.15MHz, non-linearities in the system can generate IM3 signals at 899.95MHz and 900.25MHz which lie in the adjacent bands allocated to other users and so may cause interference to these users.

$$f_1 = 900.05MHz \quad f_2 = 900.15MHz \quad \Delta f = 0.1MHz$$

$$2f_2 - f_1 = f_2 + \Delta f = 900.15 + 0.1 = 900.25MHz$$

$$2f_1 - f_2 = f_1 - \Delta f = 900.05 - 0.1 = 899.95MHz$$



Outputs at the Fundamentals and IM3

The output terms at the fundamental frequencies are:

$$\left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\varpi_1 t) \quad , \quad \left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_1^2 A_2\right]\cos(\varpi_2 t)$$

Again, taking the simple case of A_1=A_2=A, and considering the outputs at the fundamental frequencies to be the desired outputs, the amplitudes of the desired signals are:

$$A_{SIG} = \left|\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2}\alpha_3 A^3\right| = \left|\alpha_1 A + \frac{9}{4}\alpha_3 A^3\right| \approx |\alpha_1| A \quad \text{if} \quad \alpha_1 >> \frac{9}{4}\alpha_3 A^2$$

So with two signals of equal amplitude applied to a 3rd-order system the desired output signals and the unwanted 3rd-order IM signals are given by:

$$A_{SIG} = |\alpha_1| A \qquad A_{IM3} = \frac{3}{4}|\alpha_3| A^3$$

3rd-Order Intercept Point

$$A_{SIG} = |\alpha_1| A \qquad A_{IM3} = \frac{3}{4}|\alpha_3| A^3$$

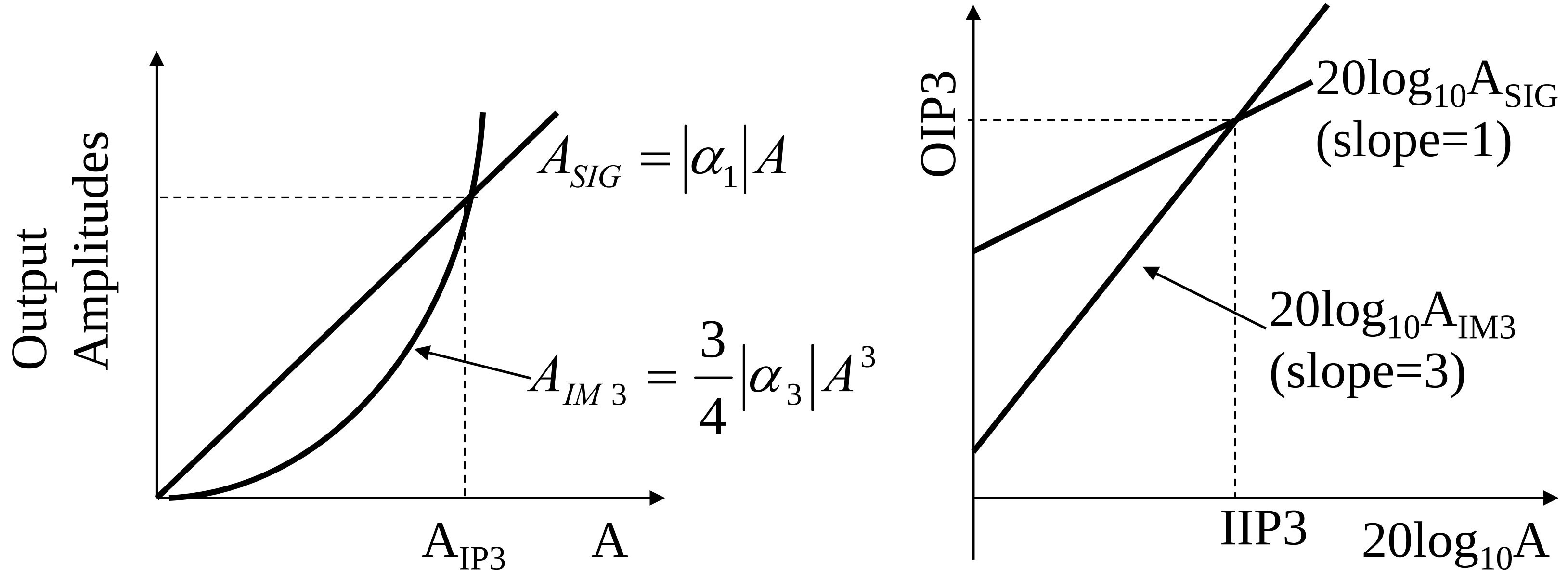
As the amplitude, A, of the input signal increases the desired signal output amplitude increases linearly whereas the unwanted IM3 products increase as the third power of A. Because $|\alpha_1| > |\alpha_3|$, at low input amplitudes the desired signal output is larger than the unwanted IM3 output. However as A increases the IM3 outputs become larger and eventually will reach the same level as the desired signal output. This condition is called the "third-order IM intercept point", IP3. The input power corresponding to this condition is called the input IP3 (IIP3) while the output power at this point is called the output IP3 (OIP3).
The input amplitude corresponding to IP3 is $A=A_{IP3}$ and at this amplitude:

$$A_{SIG} = A_{IM3} \Rightarrow |\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3 \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}}$$

This intercept point is a characteristic of the system and depends only on the linear coefficient α_1 and the third-order coefficient α_3. If A_{IP3} is determined it can be used to evaluate α_3 and the IM3 output levels i.e.:

$$A_{IP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}} \Rightarrow |\alpha_3| = \frac{4}{3}\frac{|\alpha_1|}{A_{IP3}^2} \qquad A_{IM3} = \frac{3}{4}|\alpha_3| A^3 = \frac{3}{4}\frac{4}{3}\frac{|\alpha_1|}{A_{IP3}^2} A^3 = |\alpha_1| \frac{A^3}{A_{IP3}^2}$$

Graphical Interpretation of IP3



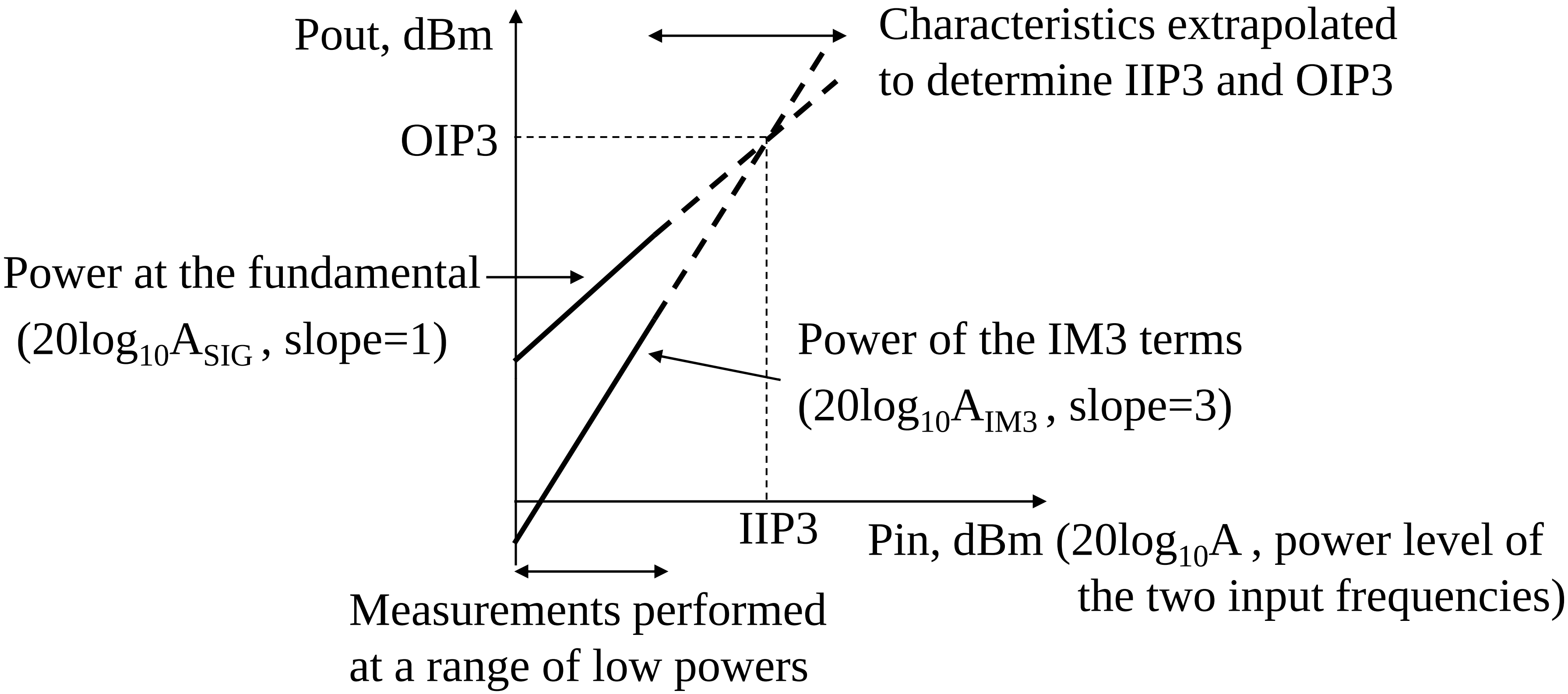
$$20\log_{10}(A_{SIG}) = 20\log_{10}(|\alpha_1| A) = 20\log_{10}|\alpha_1| + 20\log_{10} A$$

$$20\log_{10}(A_{IM3}) = 20\log_{10}\left(\frac{3}{4}|\alpha_3| A^3\right) = 20\log_{10}\left(\frac{3}{4}|\alpha_3|\right) + 60\log_{10} A$$

When the quantities are plotted as dB (power) the IM3 terms in the output grow at 3 times the rate of the desired signal output.

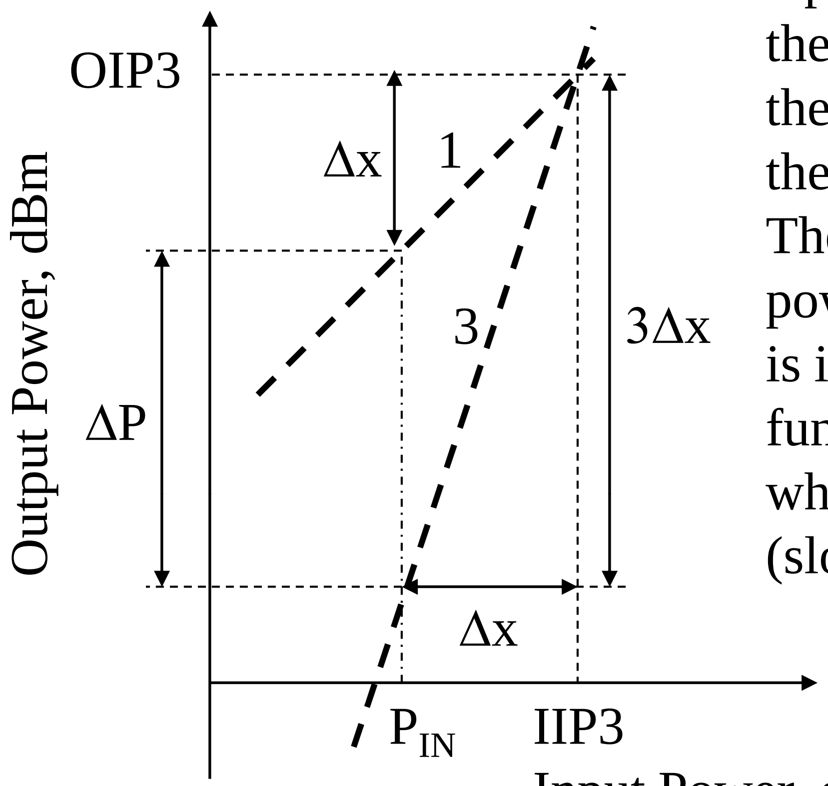
Measuring IP3 by extrapolation

In most cases the input and output IP3 power levels are higher than the normal operating ranges of the system being considered and so they are found by extrapolating measurements at low power:



Measuring IP3 from a single input power level

Because the slope of the output signal power and the IM3 products are known (1 and 3 respectively), IP3 can be determined from measurements at just one input frequency with the help of a graphical construction:



Apply two signals both with power $P_{IN.}$ Measure the output power at the signal frequencies and at the IM3 frequencies - the difference between these is ΔP.

The IM3 intercept will occur at a higher input power denoted by $P_{IN} + \Delta x$. If the input power is increased by Δx the output power at the fundamental will also increase by Δx (slope=1) whereas the IM3 power will increase by 3Δx (slope=3). Hence, graphically:

$$3\Delta x = \Delta P + \Delta x \Rightarrow \Delta x = \frac{\Delta P}{2}$$

$$IIP3 = P_{IN} + \Delta x = P_{IN} + \frac{\Delta P}{2}$$

Slightly more mathematical approach to calculating IIP3

$$\Delta P = 20\log_{10}\left(A_{SIG}\right) - 20\log_{10}\left(A_{IM3}\right)$$

$$= 20\log_{10}\left|\alpha_1\right| + 20\log_{10} A - 20\log_{10}\left(\frac{3}{4}\left|\alpha_3\right|\right) - 60\log_{10} A$$

$$= 20\log_{10}\frac{4}{3}\frac{\left|\alpha_1\right|}{\left|\alpha_3\right|} - 40\log_{10} A = 40\log_{10}\sqrt{\frac{4}{3}\frac{\left|\alpha_1\right|}{\left|\alpha_3\right|}} - 40\log_{10} A$$

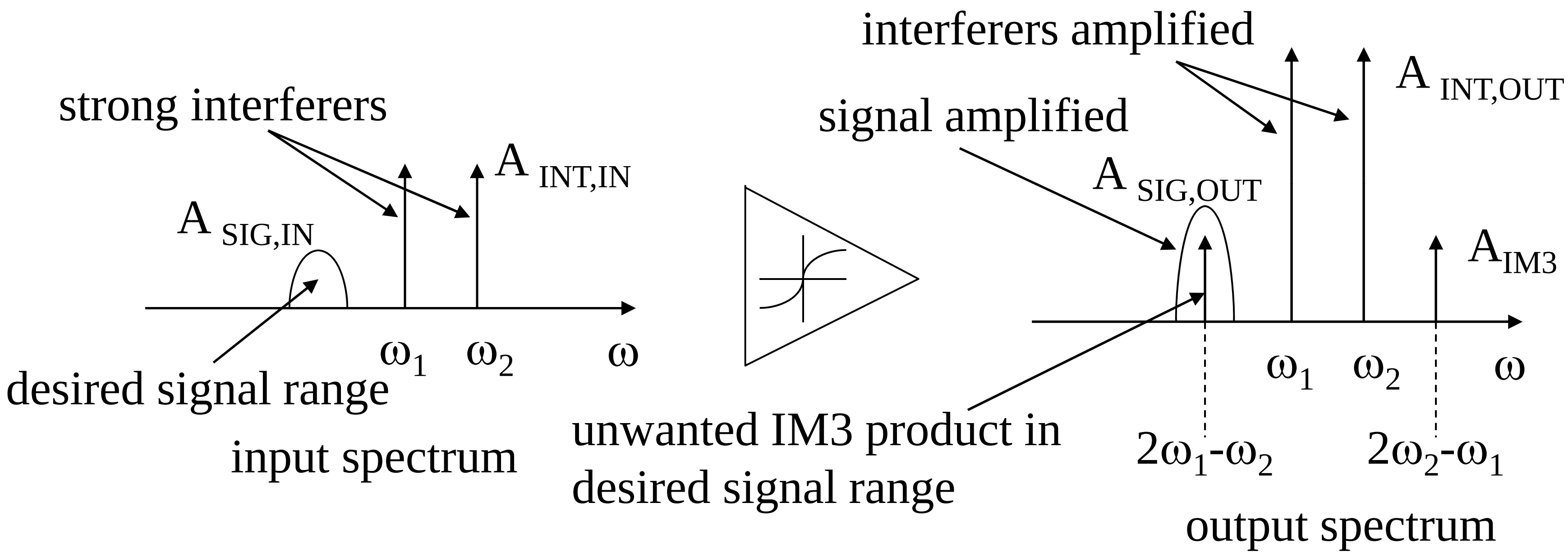
$$\Rightarrow 40\log_{10}\sqrt{\frac{4}{3}\frac{\left|\alpha_1\right|}{\left|\alpha_3\right|}} = 40\log_{10} A + \Delta P$$

$$\Rightarrow 20\log_{10}\sqrt{\frac{4}{3}\frac{\left|\alpha_1\right|}{\left|\alpha_3\right|}} = 20\log_{10} A + \frac{\Delta P}{2}$$

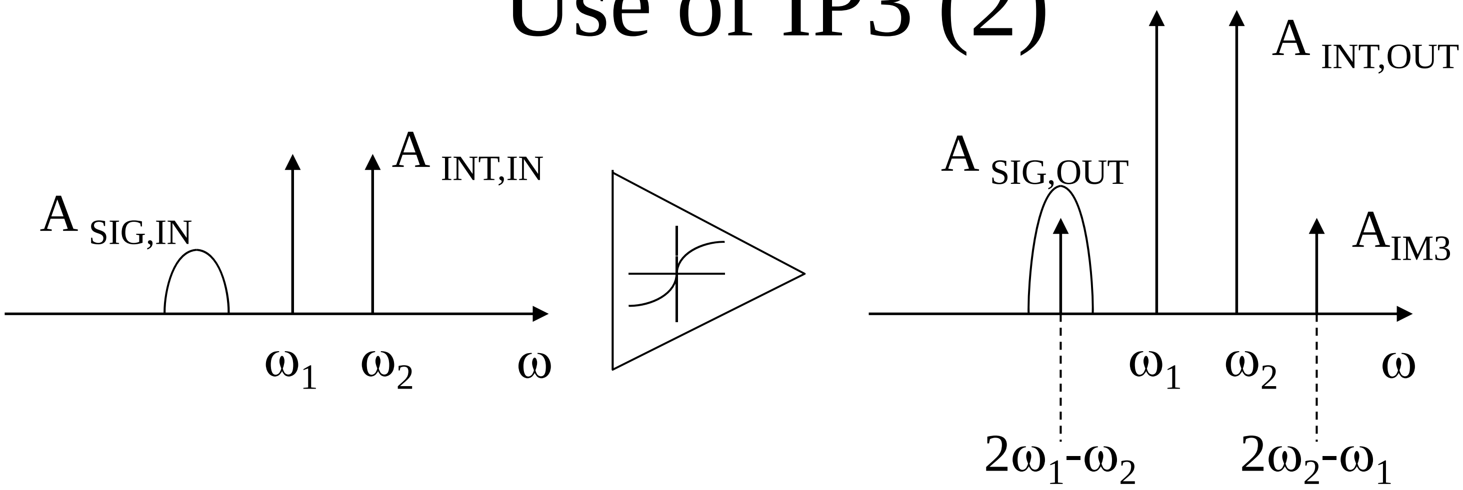
i.e. $IIP3 = P_{IN} + \dfrac{\Delta P}{2}$ and $\Delta P = 2\left(IIP3 - P_{IN}\right)$

Use of IP3 (1)

Consider an amplifier which has a desired range of frequencies at its input and also two relatively large unwanted interference signals. A perfectly linear amplifier will amplify all the signals by the gain factor but will not create any new frequencies. Thus the interferers can easily be separated from the desired signal by a passband filter after the amplifier. However, a real amplifier will generate IM3 products from the strong interferers some of which may lie within the desired signal range. These products cannot subsequently be separated from the desired signal and therefore degrade the signal quality. IP3 is used to quantify these undesired signals.



Use of IP3 (2)



The amplitude of the I_{M3} terms in the output can be found if A_{IP3} is known and if the amplitude of the interferers is known::

$$A_{IM3} = |\alpha_1| \frac{A^3}{A_{IP3}^2} = |\alpha_1| \frac{A_{INT,IN}^3}{A_{IP3}^2}$$

If the gain compression is not severe then: $A_{SIG,OUT} = |\alpha_1| A_{SIG,IN}$

The ratio of the desired signal output to the unwanted IM3 product is:

$$\frac{A_{SIG,OUT}}{A_{IM3}} = \frac{A_{SIG,IN} A_{IP3}^2}{A_{INT,IN}^3}$$

Use of IP3 (3)

$$\frac{A_{SIG,OUT}}{A_{IM3}} = \frac{A_{SIG,IN} A_{IP3}^2}{A_{INT,IN}^3}$$

The IM3 products could be considered as unwanted noise which corrupt the desired signals. Therefore the above ratio is similar to a signal-to-noise (SNR) ratio at the output. As the amplitude of the interfering signals increase this SNR degrades rapidly.

The formulas regarding P1dB, IP3 were derived assuming the "A" referred to amplitude. Where there is a ratio of amplitudes (to the same order) as in the formula above, rms values can also be used as the scaling factors cancel out e.g. taking an example from Razavi:

$$A_{SIG,IN} = 1\mu V\ rms \qquad A_{INT,IN} = 1mV\ rms \qquad A_{IP3} = 70mV\ rms$$

$$\frac{A_{SIG,OUT}}{A_{IM3}} = \frac{10^{-6}\left(70 \times 10^{-3}\right)^2}{\left(10^{-3}\right)^3} = 4.9 \approx 13.8dB$$

Relationship between A_{P1dB} and A_{IP3}

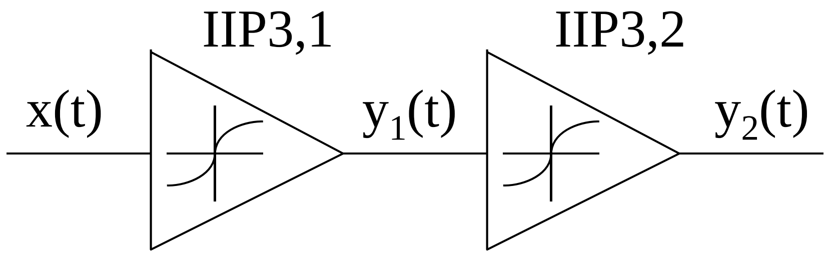
$$A_{P1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad A_{IP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}}$$

$$\frac{A_{IP3}}{A_{P1dB}} = \frac{\sqrt{4/3}}{\sqrt{0.145}} = 3.03 \approx 9.6dB$$

For a balanced system (odd symmetry), gain compression and the third-order IM products are both controlled by the coefficients α_1 and α_3 Therefore it is not surprising that they are closely related. The input signal amplitude at which the output IM3 products become comparable to the desired signal is just over 3 times larger than the input signal amplitude at which 1dB of gain compression occurs i.e. gain compression happens first and it is unlikely that a system will be driven by “in-band” signals large enough to reach the IP3 point. Therefore the IP3 of a system is mainly used to judge how much interference the system will cause to other users or vice versa.

A_{IP3} of 2 Cascaded Stages

A general analysis of the overall IP3 of cascaded stages is tedious. However, if the amplifiers have narrow pass-bands then approximations can be made:



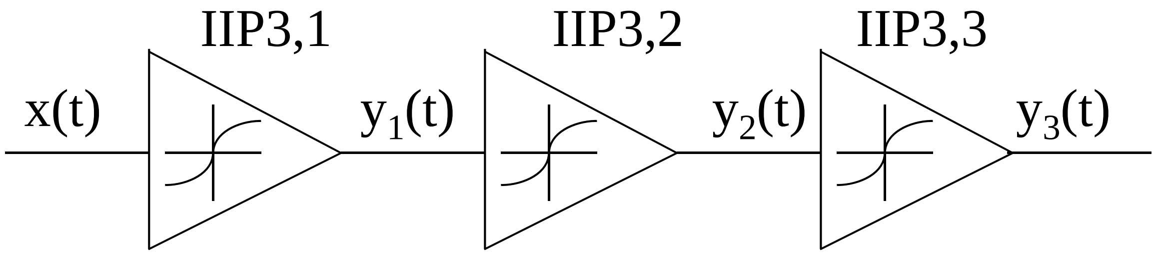
$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

A_{IP3} of 3 or more Cascaded Stages

Assuming that cascaded stages only have a narrow pass-band the effective input IP3 can be determined as follows:



$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{IP3,3}^2} + \cdots$$

If all the stage gains are greater than unity, the non-linearity of each subsequent stage becomes more critical because the IP3 of each stage is “scaled down” by the total gain preceding that stage. On the “transmit path” for instance, the PA (power amplifier) is the last amplifier before the antenna and has a dominant effect on the overall IP3 performance.