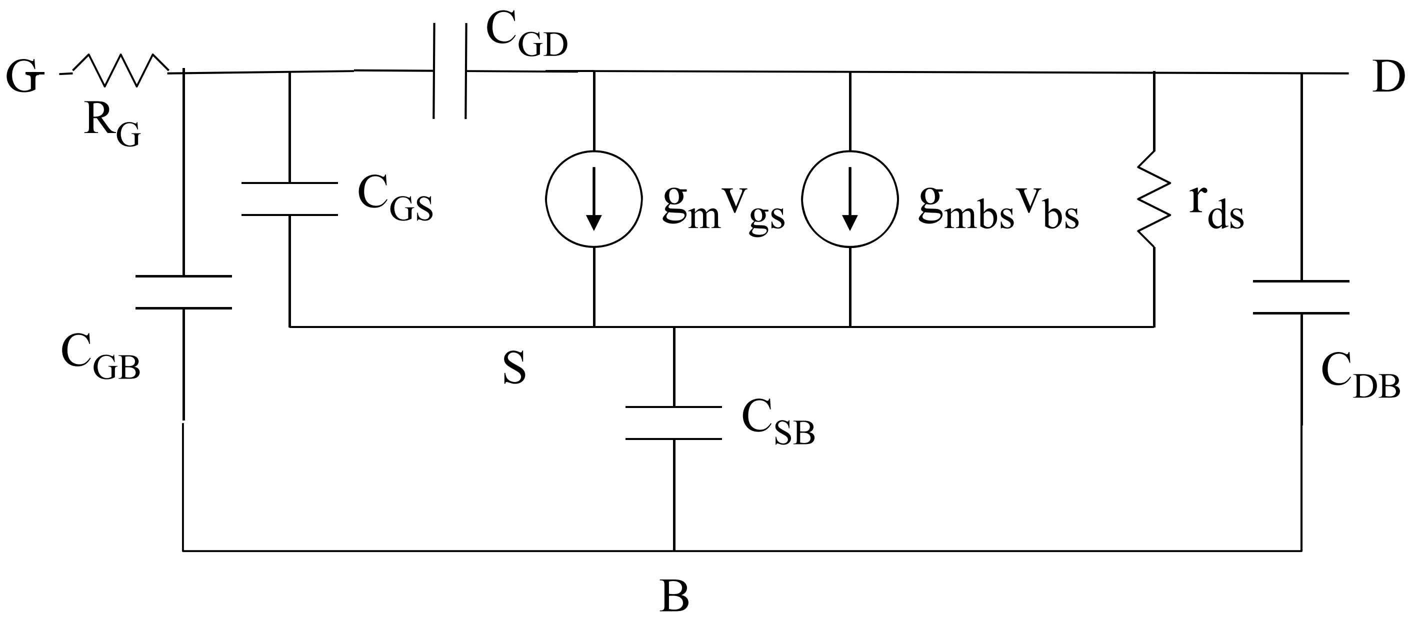
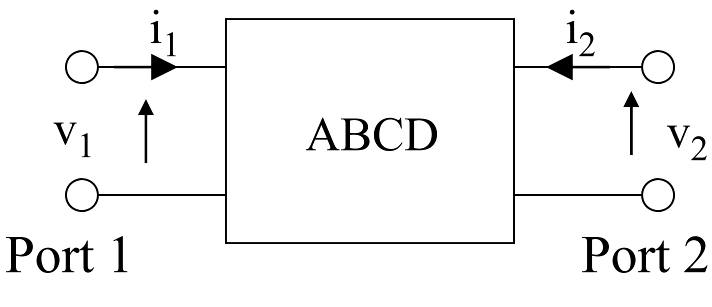
EE4011 Summer 2012 RFIC Design / Kevin McCarthy

Q1 (a) 4 marks Small-signal MOSFET model for RF



C_{GS}: Gate-source capacitance
C_{GB}: Gate-bulk capacitance
C_{SB}: Source-bulk capacitance
g_{mbs}: body-effect transconductance
R_G: gate resistance
C_{GD}: Gate-drain capacitance
C_{DB}: Drain-bulk capacitance
g_m: Transconductance
r_{ds}: output resistance

Q1 (b) 2 marks ABCD Parameters



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$v_1 = Av_2 - Bi_2 \quad i_1 = Cv_2 - Di_2$$

Setting i_2 to 0: $A = \left.\frac{v_1}{v_2}\right|_{i_2=0}$ $\quad C = \left.\frac{i_1}{v_2}\right|_{i_2=0}$

Setting v_2 to 0: $B = -\left.\frac{v_1}{i_2}\right|_{v_2=0}$ $\quad D = -\left.\frac{i_1}{i_2}\right|_{v_2=0}$

Q1 (c) 9 marks Determine the ABCD Parameters

$$\text{Saturation}: \; V_{GS} > V_{TH}, V_{DS} \geq V_{GS} - V_{TH},$$

$$\text{Also} \quad \lambda = 0.01$$

$$\therefore I_{DS} = \frac{1}{2}\frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right) = 136.7mA$$

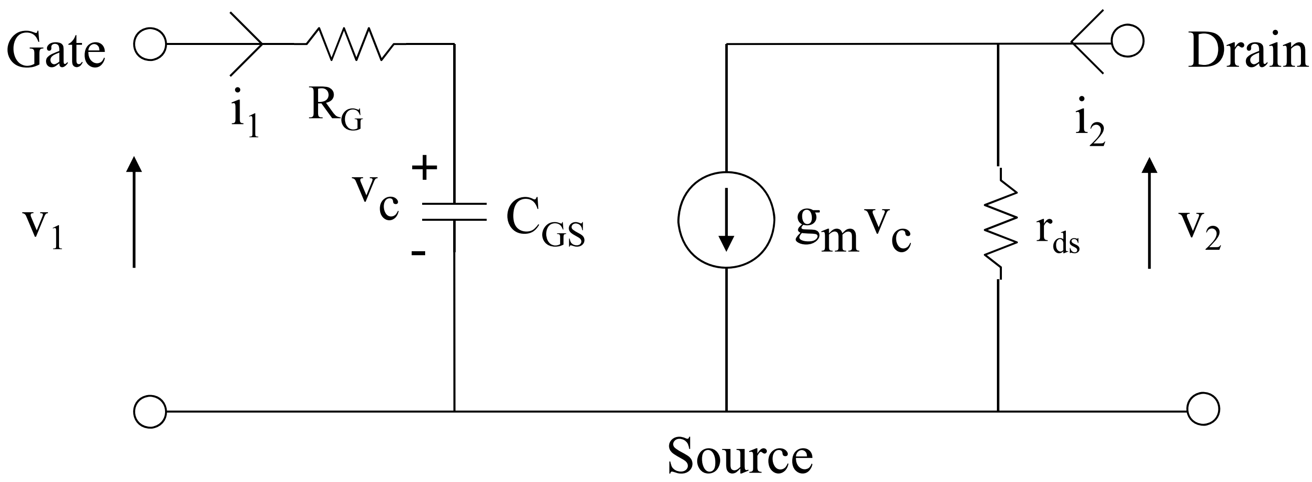
$$g_m = \frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right)\left(1 + \lambda V_{DS}\right)$$

$$g_m = \frac{200}{0.35} \times 400 \times 10^{-4} \times \frac{3.9 \times 8.854 \times 10^{-12}}{5 \times 10^{-9}}\left(2 - 0.7\right)\left(1 + 0.01 \times 2.5\right) = 210.3mS$$

$$g_{ds} = \frac{1}{2}\frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right)^2 \lambda = 1.33mS$$

$$r_{ds} = \frac{1}{g_{ds}} = 749.7\Omega$$

$$C_{GS} = \frac{2}{3}C'_{OX}WL = \frac{2}{3} \times \frac{3.9 \times 8.854 \times 10^{-12}}{5 \times 10^{-9}} \times 200 \times 10^{-6} \times 0.35 \times 10^{-6} = 0.32pF$$



$$v_c = \frac{1/j\omega C_{GS}}{R_G + 1/j\omega C_{GS}}v_1 = \frac{1}{1 + j\omega R_G C_{GS}}v_1$$

$$i_1 = \frac{v_1}{R_G + 1/j\omega C_{GS}} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}}v_1$$

$$\text{with} \quad i_2 = 0$$

$$v_2 = -g_m v_c r_{ds} = -g_m r_{ds}\frac{1}{1 + j\omega R_G C_{GS}}v_1$$

$$A = \left.\frac{v_1}{v_2}\right|_{i_2=0} = -\frac{1 + j\omega R_G C_{GS}}{g_m r_{ds}} \qquad C = \left.\frac{i_1}{v_2}\right|_{i_2=0} = -\frac{j\omega C_{GS}}{g_m r_{ds}}$$

with $\quad v_2 = 0$

$$i_2 = g_m v_c = g_m \frac{1}{1 + j\omega R_G C_{GS}} v_1$$

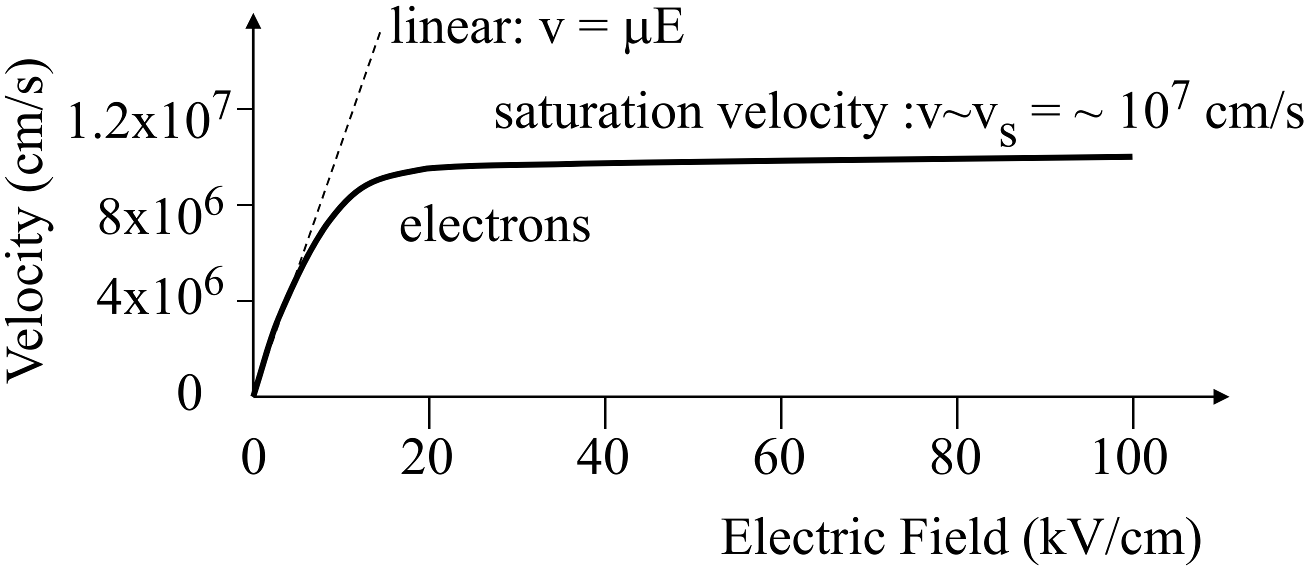
$$B = -\left.\frac{v_1}{i_2}\right|_{v_2=0} = -\frac{1 + j\omega R_G C_{GS}}{g_m} \qquad D = -\left.\frac{i_1}{i_2}\right|_{v_2=0} = -\frac{j\omega C_{GS}}{g_m}$$

Doing the calculations for the ABCD parameters at 10GHz (remember ω=2πF) gives:

$$A = 0.0065\angle -169^\circ \qquad B = 4.85\angle -169^\circ$$

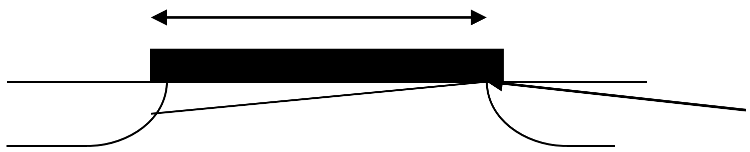
$$C = 1.3\times 10^{-4}\angle -90^\circ \quad D = 0.096\angle -90^\circ$$

Q1 (d) 2 marks Velocity field characteristic of electrons



For electric fields lower that ~ 10kV/cm the electron velocity has a linear relationship with field. For fields higher than about 20kV/cm the velocity saturates at a maximum value (velocity saturation).

Q1 (e) 3 marks Cut-off frequency



For long-channel device

$$E = \frac{V_{DSAT}}{L} \qquad v = \mu E = \mu \frac{V_{DSAT}}{L}$$

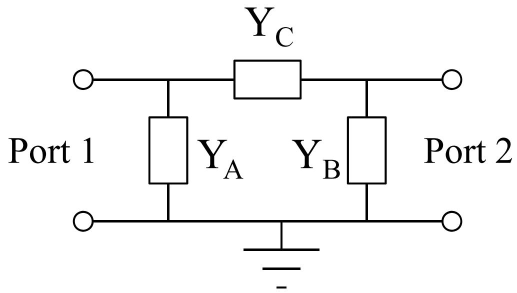
The time for a carrier to cross the channel (the transit time) is approximately:

$$\tau = \frac{L}{v} = \frac{L}{\mu \frac{V_{DSAT}}{L}} = \frac{L^2}{\mu V_{DSAT}} \quad f_T \propto \frac{1}{\tau} \Rightarrow f_T \propto \frac{1}{L^2}$$

For short-channel device – velocity saturation

$$v = v_{sat} \Rightarrow \tau = \frac{L}{v} = \frac{L}{v_{sat}} \Rightarrow \tau \propto L \Rightarrow f_T \propto \frac{1}{L}$$

Q2 (a) 4 marks y-parameters of 3 element circuit



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = Y_A + Y_C \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -Y_C$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = -Y_C \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = Y_B + Y_C$$

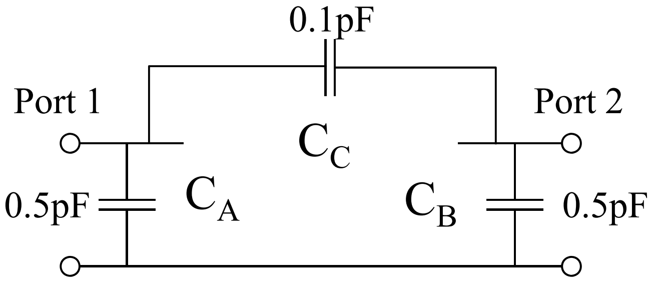
Q2 (b) 2 marks y-parameters of two networks in parallel

For two networks N1 and N2 connected in parallel, the overall y-parameters are the sum of the individual y-parameter matrices:

$$\mathbf{Y}_{\mathbf{Total}} = \mathbf{Y}_{\mathbf{1}} + \mathbf{Y}_{\mathbf{2}}$$

Q2 (c) 4 marks y-parameters of intrinsic MESFET

First calculate the y-parameters of parasitic network at 3GHz



$$y_{11} = Y_A + Y_C = j\varpi C_A + j\varpi C_C \qquad y_{12} = -Y_C = -j\varpi C_C$$

$$y_{21} = -Y_C = -j\varpi C_C \qquad y_{22} = Y_B + Y_C = j\varpi C_A + j\varpi C_B$$

$$\mathbf{Y}_{\mathbf{Par}} = \begin{bmatrix} 0.01131 \angle 90^\circ & 0.001885 \angle -90^\circ \\ 0.001885 \angle -90^\circ & 0.0113 \angle 90^\circ \end{bmatrix}$$

The MESFET y-parameters are then:

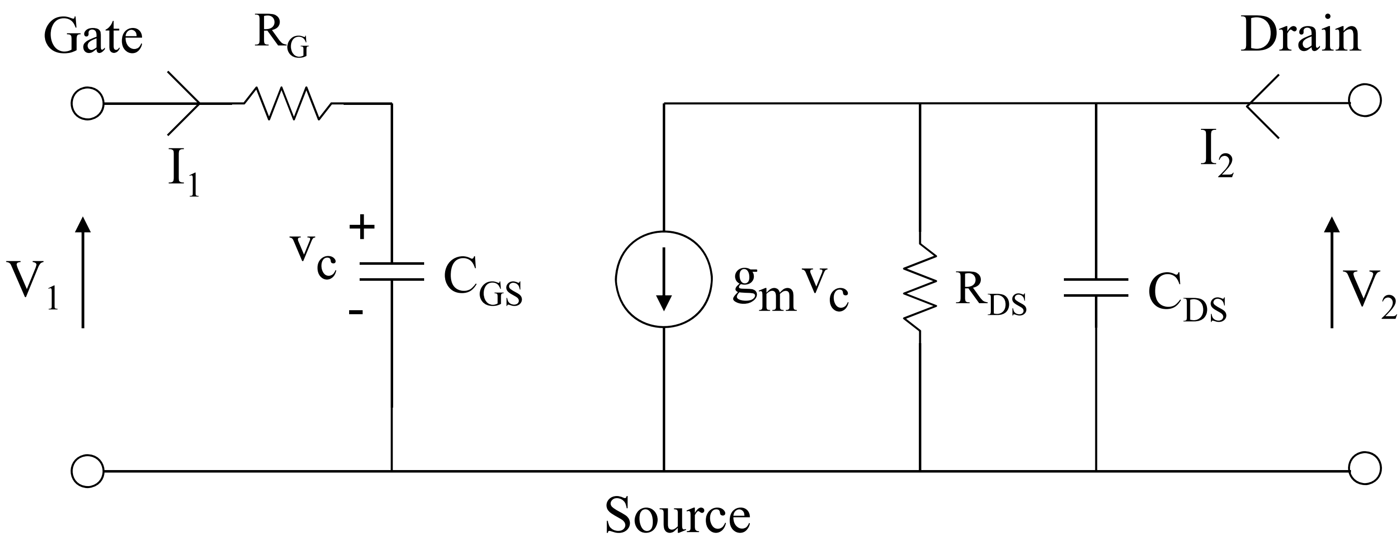
$$\mathbf{Y}_{Meas} = \begin{bmatrix} 0.0264 \angle 88.03° & 0.0019 \angle -90° \\ 0.0999 \angle -4.53° & 0.0247 \angle 57.26° \end{bmatrix}$$

$$\mathbf{Y}_{\mathbf{Par}} = \begin{bmatrix} 0.01131 \angle 90° & 0.001885 \angle -90° \\ 0.001885 \angle -90° & 0.0113 \angle 90° \end{bmatrix}$$

$$\mathbf{Y}_{\mathbf{MESFET}} = \mathbf{Y}_{\mathbf{MEAS}} - \mathbf{Y}_{\mathbf{PAR}} = \begin{bmatrix} 0.01510 \angle 86.55° & 1.504 \times 10^{-5} \angle -90° \\ 0.09977 \angle -3.45° & 0.0164 \angle 35.32° \end{bmatrix}$$

Q2 (d) 10 marks MESFET small-signal components and cur-off frequency

With the specified elements the MESFET small-signal circuit is:



Performing a y-parameter analysis of this gives:

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \quad y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

$$y_{12} = 0 \quad y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

Re-arranging these equations gives the small-signal values:

$$R_G = \text{Re}\left\{\frac{1}{y_{11}}\right\} \approx 4\Omega$$

$$C_{GS} = -\frac{1}{\omega \text{Imag}\left\{\frac{1}{y_{11}}\right\}} \approx 0.8pF$$

$$g_m = \frac{1}{\text{Re}\left\{\frac{1}{y_{21}}\right\}} \approx 0.1S$$

$$R_{DS} = \frac{1}{\text{Re}\{y_{22}\}} \approx 75\Omega$$

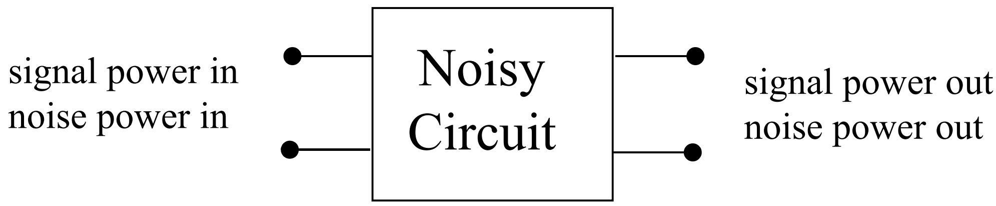
$$C_{DS} = \frac{\text{Imag}\{y_{22}\}}{\omega} \approx 0.5pF$$

The cut-off frequency is:

$$f_T = \frac{\mathrm{g_m}}{2\pi C_{GS}} = 20GHz$$

Q3 (a) 2 marks Noise Factor

The noise factor of a two-port network is defined below:



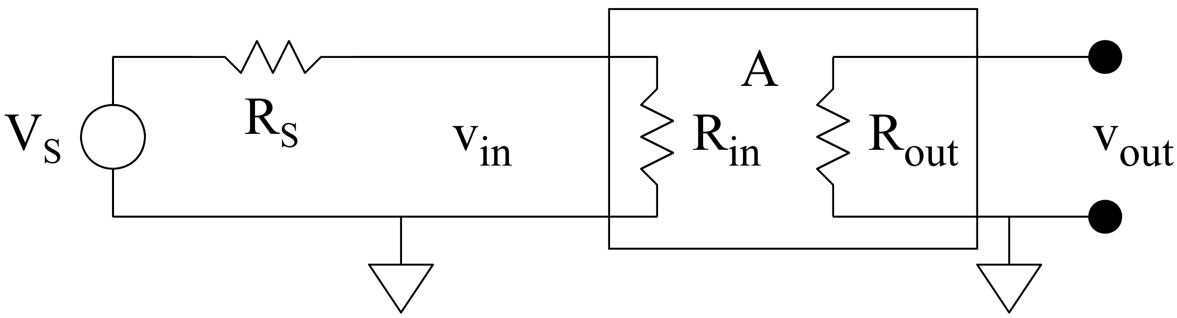
SNR_{in} = (signal power in)/(noise power in)
SNR_{out} = (signal power out)/(noise power out)

$$\text{Noise Factor, F} = \frac{SNR_{in}}{SNR_{out}}$$

Q3 (b) 12 marks Derivation of Noise Factor (12 marks because this is a tedious analysis)

Typical two-port with input and output resistance and voltage gain A:

Determine the input and output signal voltages only (R_{OUT} is optional – it doesn't effect the result):

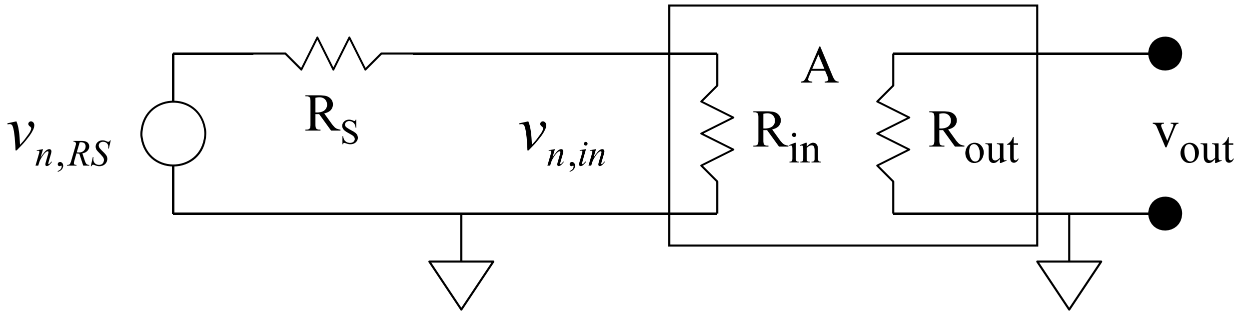


$$v_{in} = \frac{R_{in}}{R_{in}+R_S} V_S = \alpha V_S \qquad \alpha = \frac{R_{in}}{R_{in}+R_S}$$

$$v_{out} = A v_{in} = \alpha A V_S$$

$$v_{out}^2 = A^2 v_{in}^2 = \alpha^2 A^2 V_S^2$$

Analyse the noise taking into account the noise of the source resistance only:



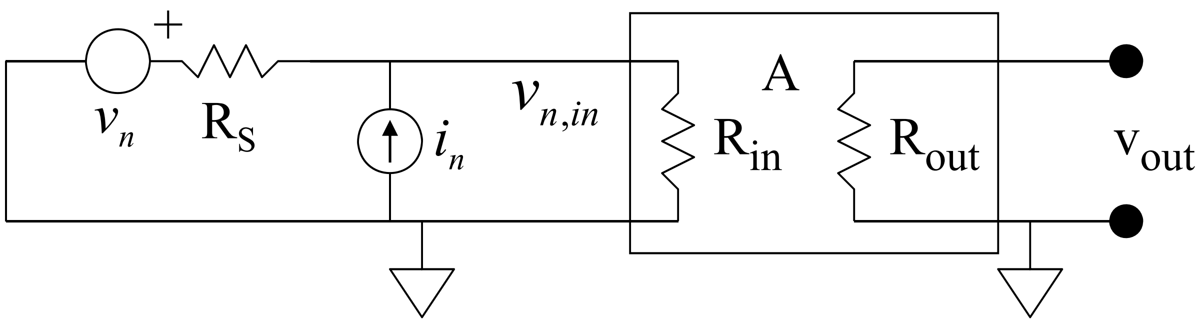
The input noise voltage due to the source resistance alone is:

$$v_{n,in} = \frac{R_{in}}{R_{in} + R_S} v_{n,RS} = \alpha\, v_{n,RS} \qquad \overline{v_{n,in}^2} = \alpha^2 \overline{v_{n,RS}^2}$$

The signal to noise ratio at the input is:

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{n,in}^2}} = \frac{\alpha^2 \overline{V_S^2}}{\alpha^2 \overline{v_{n,RS}^2}} = \frac{\overline{V_S^2}}{\overline{v_{n,RS}^2}} = \frac{\overline{V_S^2}}{4kTR_S\Delta f}$$

Now analyze the effect of the input-referred noise sources alone. The voltage and current sources must be analyzed together because they are correlated. The noise voltage is moved to the "voltage side" of the source resistor to make the circuit analysis easier:



Analysis of the input side of the circuit gives:

$$v_{n,in} = \frac{R_{in}}{R_{in} + R_S}\left(v_n + R_S i_n\right) = \alpha\left(v_n + R_S i_n\right) \Rightarrow \overline{v_{n,in}^2} = \alpha^2 \overline{\left(v_n + R_S i_n\right)^2}$$

Adding the contributions of the source noise and the input-referred noise sources:

$$\left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 \overline{v_{n,RS}^2} + \alpha^2 \overline{\left(v_n + R_S i_n\right)^2}$$

$$\overline{v_{n,out}^2} = A^2 \left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}$$

Determine SNR at the output and F:

$$SNR_{out} = \frac{v_{out}^2}{\overline{v_{n,out}^2}} = \frac{\alpha^2 A^2 V_S^2}{\alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{V_S^2} = \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}}$$

$$F = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S\Delta f} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S} \text{ for } \Delta f = 1Hz$$

Q3 (c) 4 marks Noise Figure Calculation

I_C= 1.5mA , T= 300K, β= 100, r_b= 100Ω., Δf= 1, R_S= 50Ω

$$V_T = \frac{kT}{q} = 25.86mV \qquad g_m = \frac{I_C}{V_T} = 58mS$$

$$\overline{v^2} = 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f \Rightarrow v_n = 1.34nV \quad \overline{i^2} = 2q\frac{I_C}{\beta}\Delta f \Rightarrow i_n = 2.19pA$$

$$F = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S\Delta f} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S} = 3.54 \quad F_{dB} = 10\log_{10}(F) = 5.49dB$$

Q3 (d) 2 marks Noise Temperature

$$T_n = T(F-1) = 300(3.54-1) = 762K$$

Q4 (a) 12 marks Input and Output Matching Network Design

$$s_{11} = 0.55\angle -150^\circ \quad s_{12} = 0.04\angle 20^\circ \quad s_{21} = 2.82\angle 180^\circ \quad s_{22} = 0.45\angle -30^\circ$$

$$F_{\min} = 3.0\ dB \quad \Gamma_{opt} = 0.45\angle 180^\circ \quad R_N = 4\Omega$$

Calculations for the source gain circle G_S=1.2dB

$$G_{S,\max} = \frac{1}{1-|s_{11}|^2} = \frac{1}{1-|0.55|^2} = 1.434$$

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{1.2}{10}} = 1.318$$

$$\Rightarrow g_s = \frac{G_S}{G_{S.\max}} = \frac{1.318}{1.434} = 0.919$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.919 \times |0.55|}{1-|0.55|^2(1-0.919)} = 0.52$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.919}\left(1-|0.55|^2\right)}{1-|0.55|^2(1-0.919)} = 0.20$$

Calculations for the load gain circle G_L=0.7dB

$$G_{L,\max} = \frac{1}{1-|s_{22}|^2} = \frac{1}{1-|0.45|^2} = 1.254$$

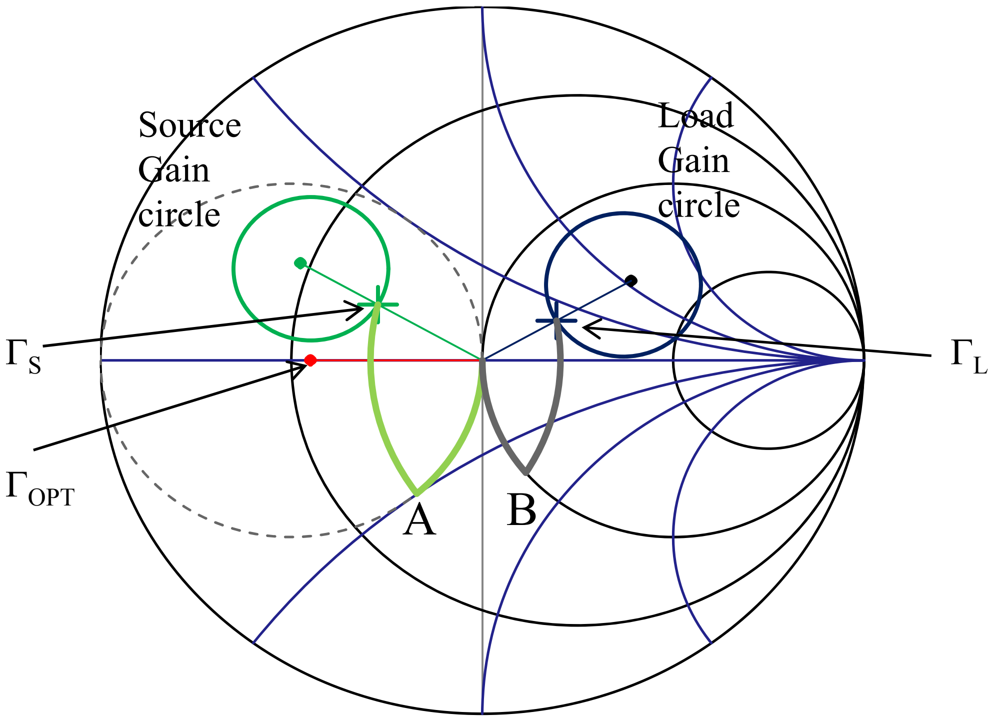
$$G_{L,dB} = 10\log_{10}(G_L) \Rightarrow G_L = 10^{\frac{G_{L,dB}}{10}} = 10^{\frac{0.7}{10}} = 1.175$$

$$\Rightarrow g_l = \frac{G_L}{G_{L,\max}} = \frac{1.175}{1.254} = 0.937$$

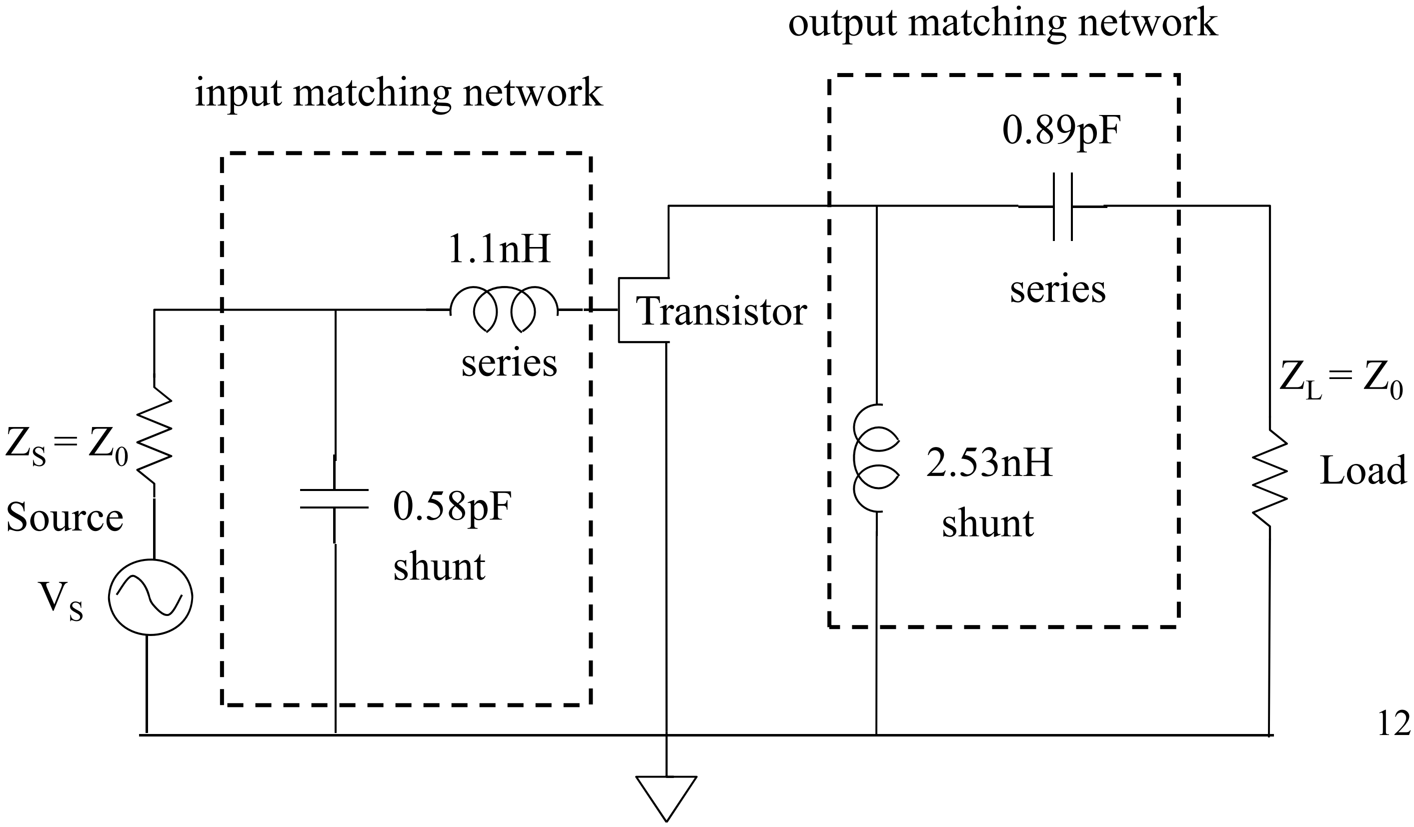
$$|C_L| = \frac{g_l|s_{22}|}{1-|s_{22}|^2(1-g_l)} = \frac{0.937 \times |0.45|}{1-|0.45|^2(1-0.937)} = 0.43$$

$$R_L = \frac{\sqrt{1-g_l}\left(1-|s_{22}|^2\right)}{1-|s_{22}|^2(1-g_l)} = \frac{\sqrt{1-0.937}\left(1-|0.45|^2\right)}{1-|0.45|^2(1-0.937)} = 0.20$$

Draw the gain circles on the Smith Chart – this is a plot generated in Excel but a paper Smith Chart would be used in the exam.



One design option is to place the source reflection coefficient on the nearest point on the source gain circle to the origin and to place the load gain circle on the nearest point on the load gain circle to the origin as illustrated.
The 50Ω source can be transformed to Γ_S using a shunt capacitor and a series inductor.
The 50Ω load can be transformed to GL using a series capacitor and a shunt inductor.



Input Matching Element Values – need to get from origin to source gain circle

Moving from Z_0 (Γ=0) to point A:
Clockwise on conductance circle – shunt capacitor

susceptance at Z_0: b = 0
susceptance at A: b = 0.910

$$C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|0.910|}{2\pi \times 5 \times 10^9 \times 50} = 0.58\,pF$$

Moving from A to Γ_S:
Clockwise on resistance circle – series inductor

reactance at A: x = -0.498
reactance at Γ_S: x = 0.192

$$L = \frac{Z_0 |\Delta x|}{2\pi f} = \frac{50 \times |0.69|}{2\pi \times 5 \times 10^9} = 1.1\,nH$$

Output Matching Element Values – need to get from origin to load gain circle

Moving from Z_0 (Γ=0) to point B:
Anti-clockwise on resistance circle – series capacitor

reactance at Z_0: x = 0   reactance at B: x = -0.718

$$C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 5 \times 10^9 \times |-0.718| \times 50} = 0.89\,pF$$

Moving from B to Γ_L:
Anti-clockwise on conductance circle – shunt inductor

susceptance at B: b = 0.474
susceptance at Γ_L: b = -0.156

$$L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 5 \times 10^9 \times |-0.63|} = 2.53\,nH$$

Note: The values determined from the paper Smith chart will be slightly different because approximate values have to be read from the chart – see next page for the values got using the paper Smith Chart.

$S_{11} = 0.55\angle -150^\circ \Rightarrow S_{11}^* = 0.55\angle 150^\circ$

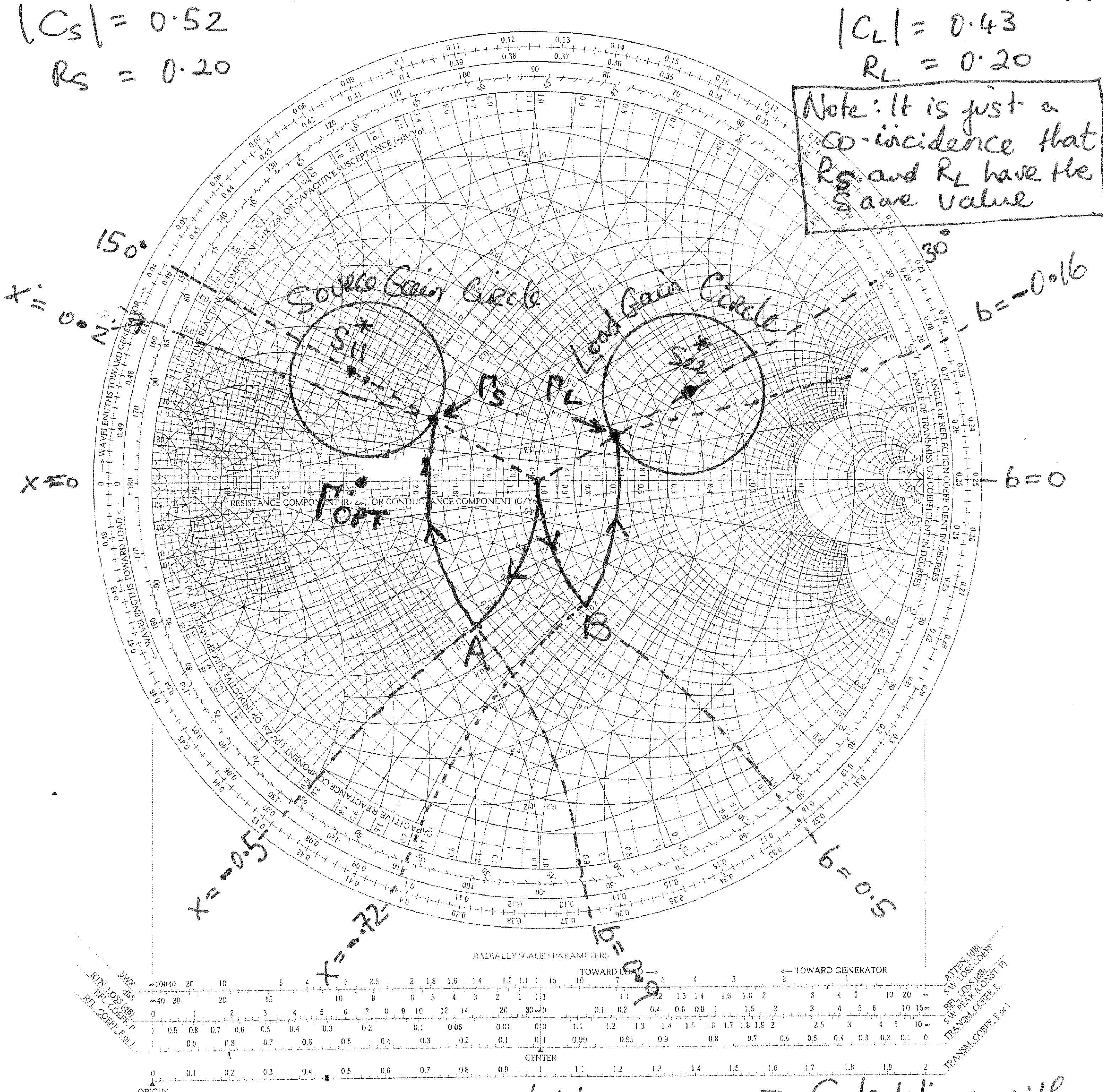
$S_{22} = 0.45\angle -30^\circ \Rightarrow S_{22}^* = 0.45\angle 30^\circ$

EXAMINATION NUMBER	QUESTION NUMBER	DATE
	EE4011 Summer 2012	Q4 (a)
	EE4011 RF IC Design	

Source Gain Circle
$|C_S| = 0.52$
$R_S = 0.20$

Load Gain Circle
$|C_L| = 0.43$
$R_L = 0.20$

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



From Origin to A: $|\Delta b| = 0.9 \Rightarrow C = \frac{|\Delta b|}{2\pi f Z_0} = 0.57\text{pF}$

From A to Γ_S: $|\Delta x| = 0.7 \Rightarrow L = \frac{Z_0|\Delta x|}{2\pi f} = 1.1\text{nH}$

From Origin to B: $|\Delta x| = 0.72 \Rightarrow C = \frac{1}{2\pi f |\Delta x| Z_0} = 0.89\text{pF}$

From B to Γ_L: $|\Delta b| = 0.66 \Rightarrow L = \frac{Z_0}{2\pi f |\Delta b|} = 2.42\text{nH}$

Calculations with values read from paper Smith Chart

Question 4(b)(i) 2 marks Stability Check

Δ and Rollet Stability Factor

$$\Delta = s_{11}s_{22} - s_{12}s_{21} = 0.15 \quad K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} = 2.29$$

This LNA is unconditionally stable because:

$$K > 1 \quad and \quad |\Delta| < 1$$

Question 4(b)(ii) 2 marks G_{TU}

$$G_{TU} = G_S G_0 G_L = G_S |s_{21}|^2 G_L = 1.318(2.82^2)1.175 = 12.32 = 10.91dB$$

Question 4(c)(iii) 2 marks Maximum error with unilateral approximation

This can be approximately estimated with the Unilateral Figure of Merit

$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{\left(1 - |s_{11}|^2\right)\left(1 - |s_{22}|^2\right)} = 0.05$$

$$\frac{1}{(1+M)^2} = 0.907 < \frac{G_T}{G_{TU,\max}} < \frac{1}{(1-M)^2} = 1.108$$

Expressing 0.907 and 1.108 in dB gives a max error of +/- 0.45dB (which is relatively high)

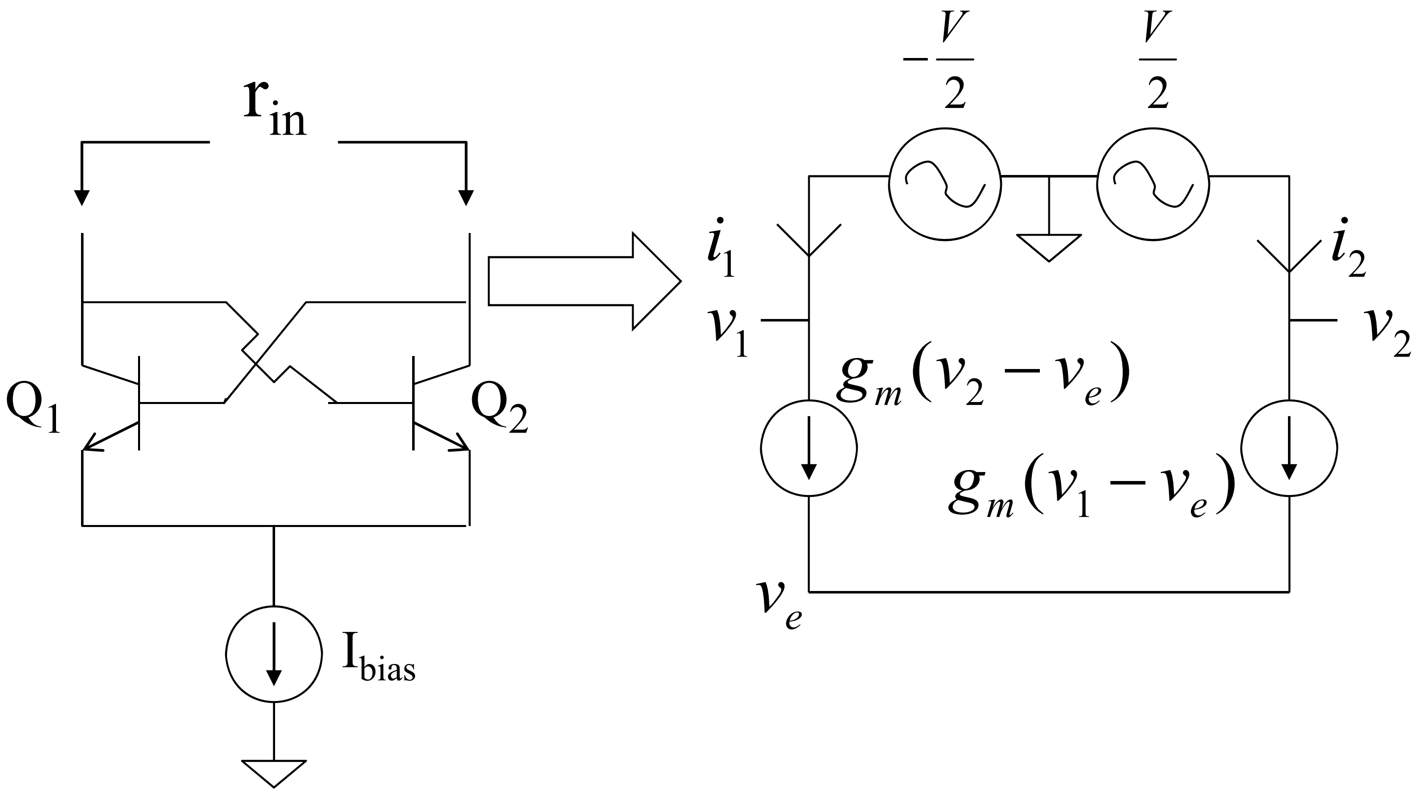
Question 4(c)(iiiv) 2 marks comment on the noise performance

It is seen that to obtain the specified source gain the source reflection coefficient has to be set to a value which is different to the reflection coefficient for optimum noise performance. Thus, the noise figure of the LNA will be higher than the minimum noise figure of 3dB.

Question 5(a) 10 marks negative-gm oscillator

Small-signal analysis to determine input resistance of cross-coupled BJT pair:

Ignore base current and assume: $g_{m1} = g_{m2} = g_m$



$$i_1 = -i_2 \Rightarrow g_m(v_2 - v_e) = -g_m(v_1 - v_e)$$
$$\Rightarrow g_m(v_1 + v_2) = 2g_m v_e$$
$$\Rightarrow v_e = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}\left(\frac{V}{2} - \frac{V}{2}\right) = 0$$
$$\Rightarrow i_2 = g_m v_1 = g_m\left(-\frac{V}{2}\right)$$

$$r_{in} = \frac{v_{tot}}{i_2} = \frac{\frac{V}{2} - \left(-\frac{V}{2}\right)}{g_m\left(-\frac{V}{2}\right)} = -\frac{2}{g_m}$$

In order to sustain oscillation g_m must be large enough so that the negative resistance is large enough to cancel out the parasitic resistances in the circuit, especially the parasitic resistances of the inductors.

Question 5(b) 2 marks Calculate bias current

Calculate the required g_m:

$$r_{in} = -\frac{2}{g_m} \Rightarrow g_m = -\frac{2}{r_{in}} = -\frac{2}{-10} = 0.2S$$

Calculate the required currents:

$$V_T = \frac{kT}{q} = 25.86mV \qquad I_C = g_m V_T = 5.2mA \qquad I_{bias} = 2 \times I_C = 10.4mA$$

Question 5(c) 2 marks Calculate C_{J0}

$$F_{OSC} = \frac{1}{2\pi\sqrt{LC_{TOT}}}$$

$$\Rightarrow C_{TOT} = \frac{1}{(2\pi F_{OSC})^2 L} = \frac{1}{(2\pi \times 1.8 \times 10^9)^2 \times 5 \times 10^{-9}} = 1.56pF$$

$$C_{J0} = C_{TOT} - C_{PAR} = 1.56 - 0.6 = 0.96pF$$

Question 5(d) 4 marks Frequency change

With a reverse bias of 1V on the diodes

$$C_D = \frac{C_{J0}}{(1 - V_D / V_J)^{MJ}} = \frac{0.96 \times 10^{-12}}{(1 - (-1.0)/0.75)^{0.3}} = 0.745pF$$

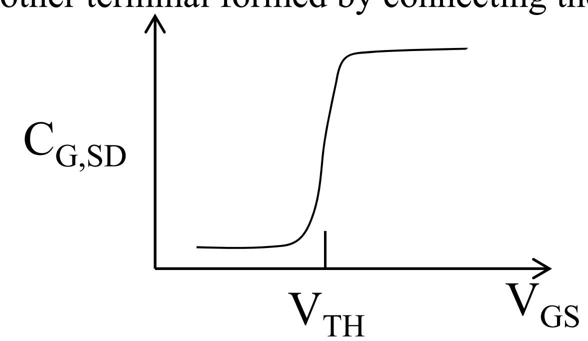
$$C_{TOT} = C_D + C_{PAR} = 0.745 + 0.6 = 1.345pF$$

$$F_{OSC} = \frac{1}{2\pi\sqrt{LC_{TOT}}} = \frac{1}{2\pi\sqrt{5 \times 10^{-9} \times 1.345 \times 10^{-12}}} = 1.94GHz$$

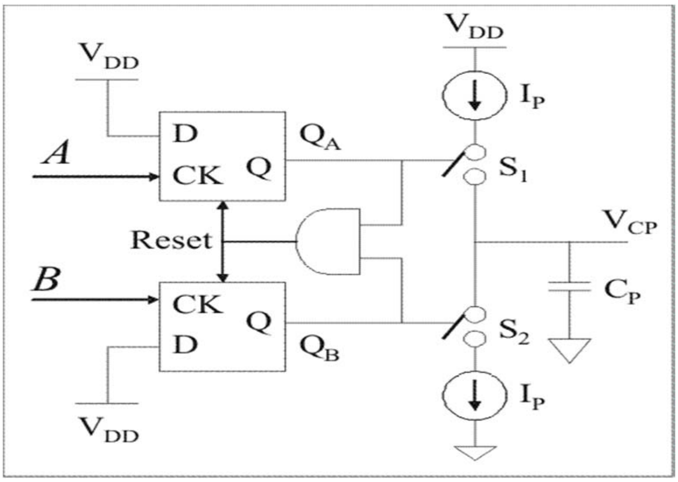
$$\Delta F(\%) = \frac{1.94 - 1.8}{1.8} \times 100 = 7.8\%$$

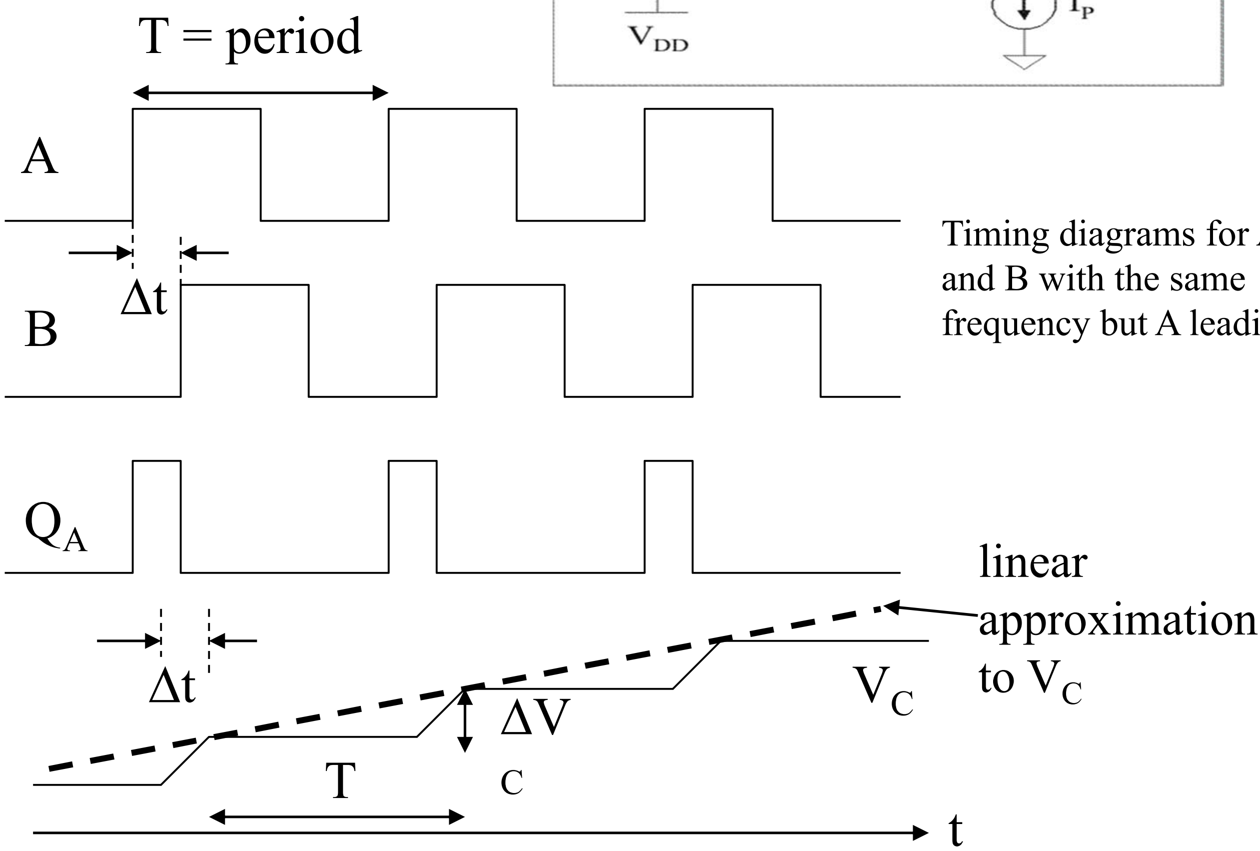
Question 5(e) 2 marks alternative varactor

Many modern CMOS ICs use a MOSFET device as a voltage-controlled capacitance with one terminal of the varactor formed by the gate of the MOSFET and the other terminal formed by connecting the source and drain together.



Question 6(a) 5 marks Transfer function of PFD/CP circuit block





Timing diagrams for A and B with the same frequency but A leading B:

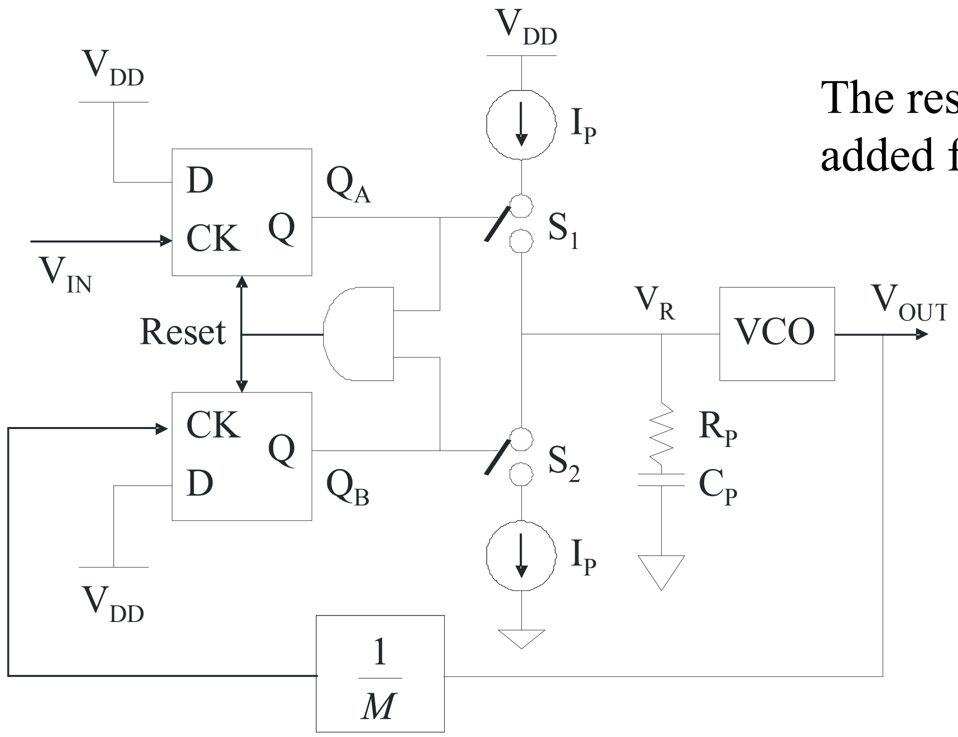
$$\Delta t = \frac{\Delta\varphi}{2\pi}T \qquad \Delta V_C = \frac{I_P}{C_P}\Delta t = \frac{I_P}{C_P}\frac{\Delta\varphi}{2\pi}T$$

$$slope = \frac{dV_C}{dt} \approx \frac{\Delta V_C}{T} = \frac{I_P}{C_P}\frac{\Delta\varphi}{2\pi}T\frac{1}{T} = \frac{I_P}{2\pi C_P}\Delta\varphi$$

$$\Rightarrow \frac{dV_C}{dt} = \frac{I_P}{2\pi C_P}\Delta\varphi \Rightarrow V_C = \frac{I_P}{2\pi C_P}\int\Delta\varphi$$

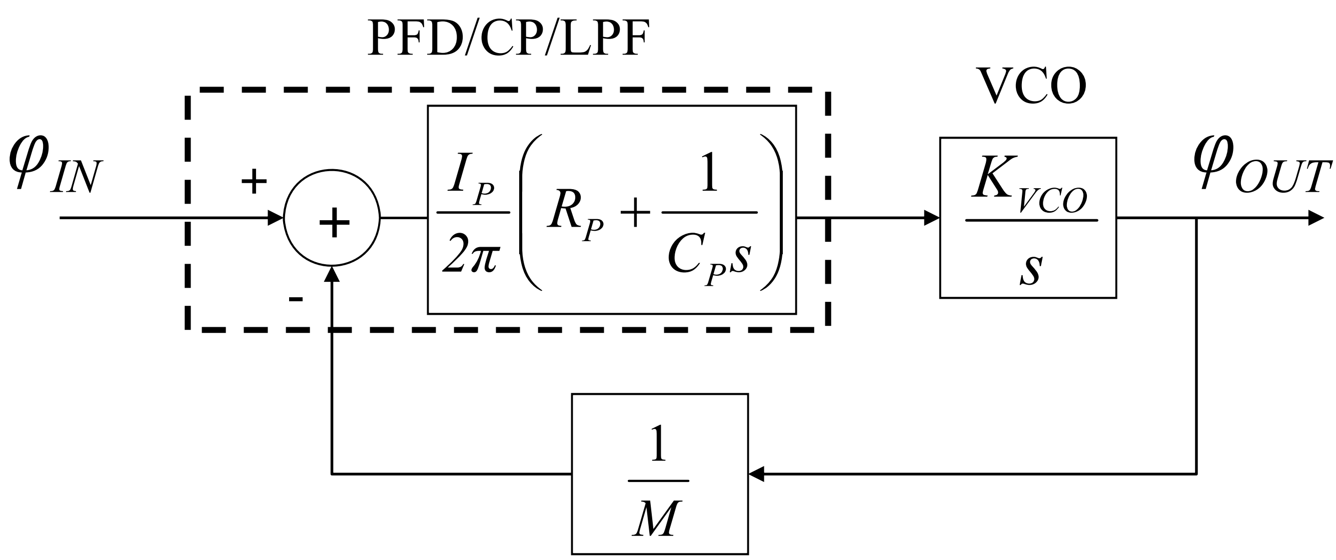
$$\Rightarrow V_C(s) = \frac{I_P}{2\pi C_P}\frac{1}{s}\Delta\varphi(s)$$

Question 6(b) 5 marks Overall Transfer function of Type II PLL



The resistor R_P is added for stability

The individual blocks can be replaced by their transfer functions as follows:



The PD/CP/LPF transfer function is based on the average current method

Open Loop Response

The open loop transfer function is just the product of the individual transfer functions in the forward path

$$H(s) = \frac{I_P K_{VCO}}{2\pi s}\left(R_P + \frac{1}{C_P s} \right)$$

Closed Loop Response

$$\varphi_{OUT}(s) = \left(\varphi_{IN}(s) - \frac{\varphi_{OUT}(s)}{M} \right) H(s)$$

$$\Rightarrow H_{Closed}(s) = \frac{H(s)}{1 + \frac{H(s)}{M}} = \frac{\frac{I_P K_{VCO}}{2\pi s}\left(R_P + \frac{1}{C_P s} \right)}{1 + \frac{I_P K_{VCO}}{2\pi M s}\left(R_P + \frac{1}{C_P s} \right)}$$

$$= \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1 \right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}}$$

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} \equiv \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\varsigma\omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

6(c) Type II Calculations

I_P=0.5mA, C_P=150pF, R_P=20kΩ, K_{VCO}=100MHz/V, M=200

Caution! K_{CVO} is specified as MHz/V but in the formulas below it should be in units of radians/s so don't forget the factor of 2π

6(c)(i) 2 marks natural frequency

$$\omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} = 1.29\times 10^6\ rad/s = 205kHz$$

6(c)(ii) 2 marks damping factor

$$\varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} = 1.94$$

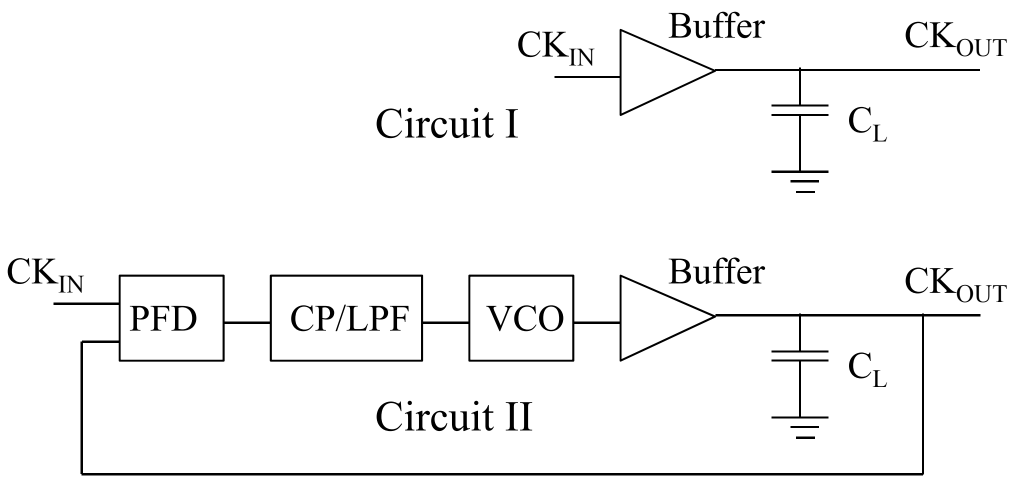
6(c)(ii) 2 marks minimum K_{VCO}

$$\varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \Rightarrow K_{VCO} = \frac{2\pi M}{I_P C_P}\left(\frac{2\varsigma}{R_P}\right)^2$$

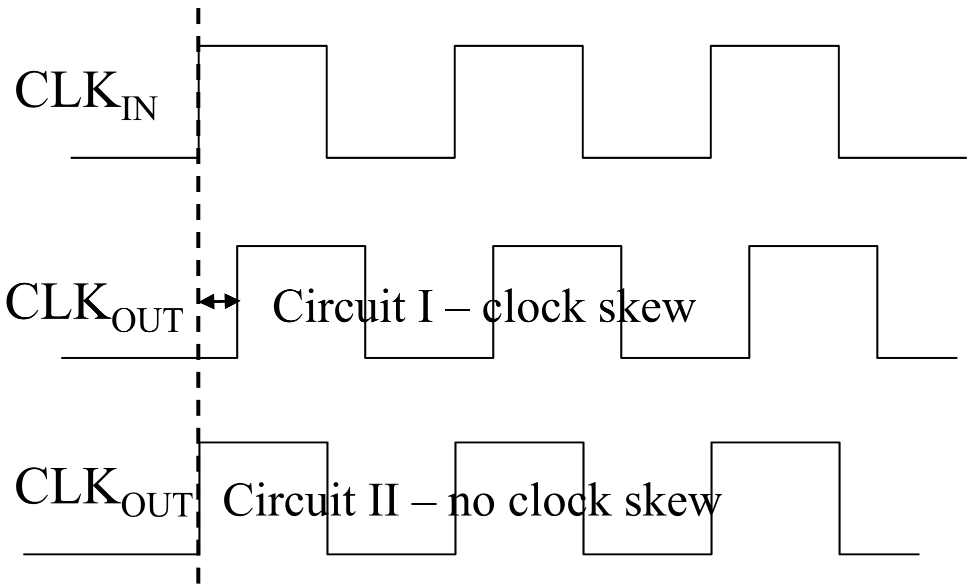
Setting the damping factor to 0.8x1.94=1.552 (i.e. 20% lower than its nominal value) gives

$$K_{VCO} = 4.02\times 108\text{rad/s/V} = 64\text{MHz/V}$$

6(d)(ii) 4 marks Determining what the circuit does



One of the common problems with the clock buffer in Circuit I is that there is a delay caused by the buffer which can create a clock skew in which the edges of the output clock are not aligned to the edges of the input clock and thus lose alignment with the digital data lines giving clock skew.
Circuit II is a PLL based on a CP PFD/LPF thus guaranteeing that the clock edges of CK_{OUT} are aligned with the clock edges of CK_{IN} overcoming the clock skew problem.



Question 7 is an essay type question based on a continuous assessment. It will be graded based on the quantity and quality of relevant information presented.