22/4/09

Autumn 04

Q1(a). $$M(k) = M(k-1) + (X-Y) - \frac{P}{100}\,\frac{(M(k-1)+M(k-2))}{2}$$

$$M(z) = z^{-1}M(z) + \frac{(X-Y)}{1-z^{-1}} - \frac{P}{200}\left(z^{-1}M(z) + z^{-2}M(z)\right)$$

$$M(z) - z^{-1}M(z) + \frac{P}{200}\left(z^{-1}+z^{-2}\right)M(z) = \frac{X-Y}{1-z^{-1}}$$

$$M(z)\left[1 - z^{-1} + \frac{P}{200}z^{-1} + \frac{P}{200}z^{-2}\right] = \frac{(X-Y)z}{z-1}$$

$$M(z) = \frac{(X-Y)z}{(z-1)\left(1-z^{-1}+\frac{P}{200}z^{-1}+\frac{P}{200}z^{-2}\right)}$$

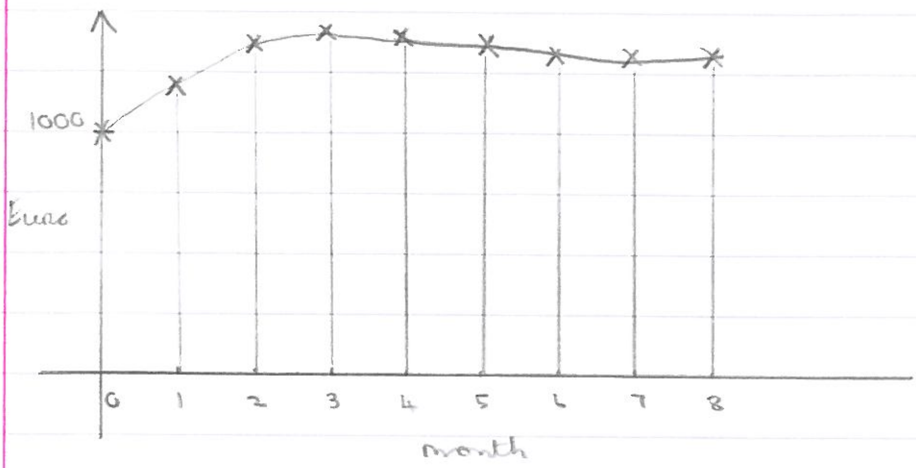
$$M_f = \lim_{z\to1}(z-1)M(z) = \lim_{z\to1}\frac{(X-Y)z}{1-z^{-1}+\frac{P}{200}z^{-1}+\frac{P}{200}z^{-2}}$$

$$= \frac{X-Y}{1-1+\frac{P}{200}+\frac{P}{200}} = \frac{X-Y}{\frac{2P}{200}} = \frac{100(X-Y)}{P}$$

$M_0 = 1000$; $X = 1000$; $Y = 200$; $P = 60$

$$M_f = \frac{100(1000-200)}{60} = 1333\cdot33$$

$$M(1) = 1000 + (1000-200) - \frac{60}{100}\left(\frac{1000+1000}{2}\right) = 1200$$
$$M(2) = 1200 + (1000-200) - \frac{60}{100}\left(\frac{1200+1000}{2}\right) = 1340$$
$$M(3) = 1340 + (1000-200) - \frac{60}{100}\left(\frac{1340+1200}{2}\right) = 1378$$
$$M(4) = 1378 + (1000-200) - \frac{60}{100}\left(\frac{1378+1340}{2}\right) = 1362\cdot6$$
$$M(5) = 1362\cdot6 + (1000-200) - \frac{60}{100}\left(\frac{1362\cdot6+1378}{2}\right) = 1340\cdot42$$
$$M(6) = 1340\cdot42 + (1000-200) - \frac{60}{100}\left(\frac{1340\cdot42+1362\cdot6}{2}\right) = 1329\cdot514$$
$$M(7) = 1329\cdot514 + (1000-200) - \frac{60}{100}\left(\frac{1329\cdot514+1340\cdot42}{2}\right) = 1328\cdot53$$
$$M(8) = 1328\cdot53 + (1000-200) - \frac{60}{100}\left(\frac{1328\cdot53+1329\cdot514}{2}\right) = 1331\cdot41$$

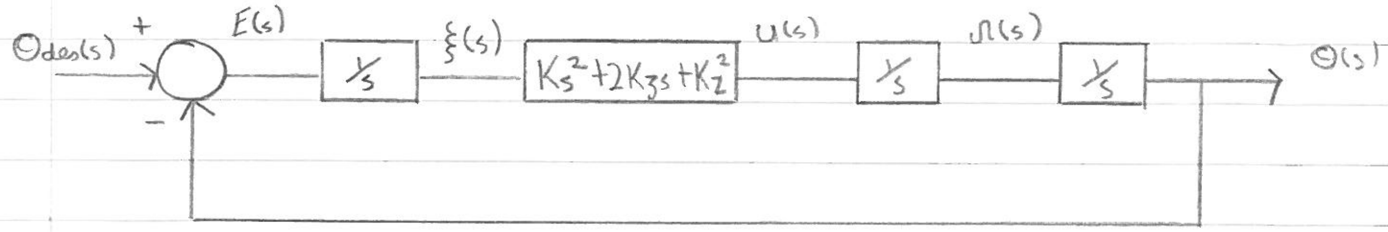


If P > 200%
=> negative balance

Autumn 04

Q 5 (c). $C(s) = \frac{1}{s} K(s^2 + 2zs + z^2)$

$= \frac{1}{s}\left[Ks^2 + 2Kzs + Kz^2\right]$



$\xi(s) = E(s)/s$

$s\,\xi(s) = E(s)$

$\frac{d}{dt}\xi(t) = e(t)$

$= \Theta_{des}(t) - \Theta(t)$

$\Theta(s) = \Omega(s)/s$

$s\,\Theta(s) = \Omega(s)$

$\frac{d}{dt}\Theta(t) = \omega(t)$

$$\Omega(s) = \frac{E(s)(Ks^2 + 2Kzs + Kz^2)}{s^2}$$

$s^2\Omega(s) = E(s)(Ks^2 + 2Kzs + Kz^2)$

$s\,\Omega(s) = KsE(s) + 2KzE(s) + Kz^2\,E(s)/s$

$s\,\Omega(s) = Ks[\Theta_{des}(s) - \Theta(s)] + 2Kz[\Theta_{des}(s) - \Theta(s)] + Kz^2\xi(s)$

$s\,\Omega(s) = Ks\Theta_{des}(s) - K\Omega(s) + 2Kz\Theta_{des}(s) - 2Kz\Theta(s) + Kz^2\xi(s)$

$\Theta_{des}(t) =$ setpoint

$\Rightarrow s\,\Theta_{des}(s) = \frac{d}{dt}\Theta_{des}(t) = 0$

$\Rightarrow \frac{d}{dt}\omega(t) = -K\omega(t) + 2Kz\Theta_{des}(t) - 2Kz\Theta(t) + Kz^2\xi(t)$

$$\frac{d}{dt}\begin{bmatrix}\Theta(t)\\ \omega(t)\\ \xi(t)\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -2Kz & -K & Kz^2\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Theta(t)\\ \omega(t)\\ \xi(t)\end{bmatrix} + \begin{bmatrix}0\\ 2Kz\\ 1\end{bmatrix}\Theta_{des}(t)$$

(ii) $\underline{x}(k+1) = A_d\,\underline{x}(k) + B_d\,\underline{u}(k)$

$$A_d = \phi(T) = e^{AT} = I + AT + \frac{A^2T^2}{2!} + \dots$$

T is very small $\Rightarrow e^{AT} = I + AT$

$$\Rightarrow A_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & T & 0 \\ -2K_\zeta T & -KT & K_\zeta^2 T \\ -T & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T & 0 \\ -2K_\zeta T & 1-KT & K_\zeta^2 T \\ -T & 0 & 1 \end{bmatrix}$$

$$B_d = \int_0^T \phi(\eta) B\, d\eta$$

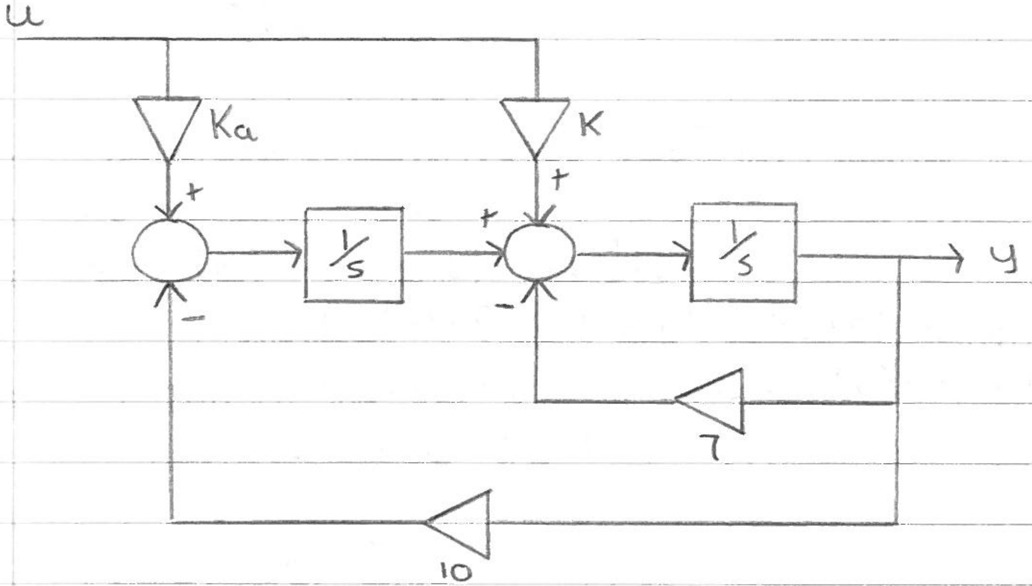
$$= \int_0^T e^{A\eta}\, d\eta\, B$$

$$= \tfrac{1}{A} e^{A\eta}\Big|_0^T B$$
$$= \tfrac{1}{A}(e^{AT} - I) B$$
$$= \tfrac{1}{A}(I + AT - I) B$$
$$= A^{-1}(AT) B$$
$$= IT\, B$$
$$= \begin{bmatrix} T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} 0 \\ 2K_\zeta \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2K_\zeta T \\ T \end{bmatrix}$$

$$\begin{bmatrix} \theta(k+1) \\ \omega(k+1) \\ \xi(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 \\ -2K_\zeta T & 1-KT & K_\zeta^2 T \\ -T & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta(k) \\ \omega(k) \\ \xi(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 2K_\zeta T \\ T \end{bmatrix} \theta_{des}(k)$$

3/4/09 Autumn 2004

Q6 (a). $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{K(s+a)}{s^2+7s+10} = \dfrac{\overset{b_1}{K}s + \overset{b_0}{Ka}}{s^2+\underset{e_1}{7}s+\underset{e_0}{10}}$



The observer canonical state-space equations are

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ -10 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} K \\ Ka \end{bmatrix} u$$

$$AB = \begin{bmatrix} -7 & 1 \\ -10 & 0 \end{bmatrix}\begin{bmatrix} K \\ Ka \end{bmatrix} = \begin{bmatrix} Ka-7K \\ -10K \end{bmatrix}$$

$$C_x = [\, B \mid AB \,] = \begin{bmatrix} K & Ka-7K \\ Ka & -10K \end{bmatrix}$$

$$\det(C_x) = -10K^2 - Ka(Ka-7K) = -10K^2 - K^2a^2 + 7aK^2$$

controllable = $\det(C_x) \neq 0$

not always viable.

regulator

- Design K for regulator to place the N closed loop poles assuming that states are available
- Design G for estimator to provide states with desired error dynamics
- Estimator doesn't affect the position of the regulator poles.

(iii) Consider the estimator

$$\frac{d}{dt}\underline{\hat{x}} = (A-GC)\underline{\hat{x}} + B\underline{u} + G\underline{y}(t)$$

If following regulator is used

$$\underline{u}(t) = -K\underline{\hat{x}}(t)$$

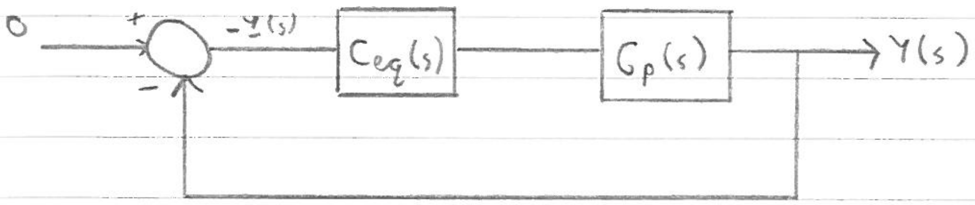
$$\Rightarrow \frac{d}{dt}\underline{\hat{x}} = (A-GC)\underline{\hat{x}} - BK\underline{\hat{x}} + G\underline{y}(t)$$
$$= (A-GC-BK)\underline{\hat{x}} + G\underline{y}(t)$$

Taking Laplace transforms

$$s\underline{\hat{x}}(s) = (A-GC-BK)\underline{\hat{x}}(s) + G\underline{Y}(s)$$
$$(sI-A+GC+BK)\underline{\hat{x}}(s) = G\underline{Y}(s)$$
$$\underline{\hat{x}}(s) = (sI-A+GC+BK)^{-1}G\underline{Y}(s)$$

The controller can then be easily determined as

$$\underline{u} = -K\underline{\hat{x}}(s)$$
$$\Rightarrow \underline{u}(s) = -K(sI-A+GC+BK)^{-1}G\underline{Y}(s)$$



$$C_{eq}(s) = K(sI-A+GC+BK)^{-1}G$$