Fourth Year Electrical Engineering

EE4010

Electrical Power Systems

Transformer Worked Examples

Example 1

The schematic diagram of a three-phase radial transmission system is shown in Figure 1 below. The ratings and the impedances of the various components are as shown along with the nominal transformer line voltages. A load of 50 MW at 0.8 power factor lagging is taken from the 33 kV load busbar which is to be maintained at a line voltage of 30 kV. Calculate the terminal voltage of the synchronous generator.

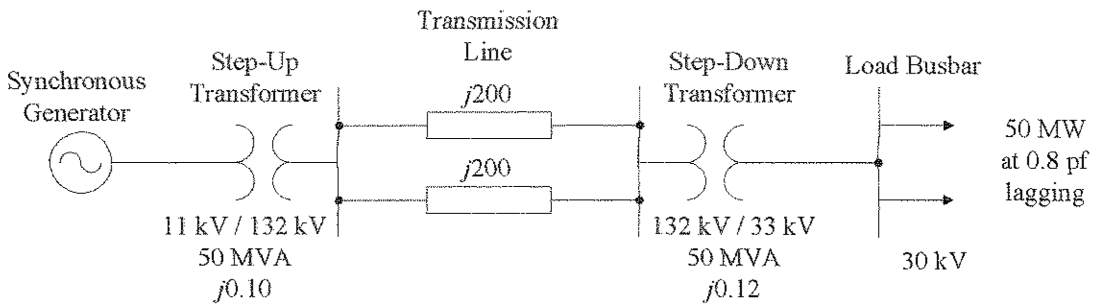


Figure 1

Solution 1

Select 11 kV, 132 kV and 33 kV as the base line-to-line voltages in the generator, transmission line and load zones respectively as determined by the transformer voltages. Select a base of 100 MVA. The reactances of the transformers are expressed on the corresponding rated voltages and volt-amperes. The base impedance for the line is

$$Z_{B\,line} = \frac{(132\times10^3)^2}{100\times10^6} = 174\ \Omega$$

Hence, the per-unit reactance of the line is

$$Z_{pu\,line} = \frac{j200\,//\,j200}{174} = \frac{j100}{174} = \mathrm{j}0.575\ \mathrm{pu}$$

The per-unit reactance of the step-up transformer is

$$Z_{pu\,step-up} = \frac{100}{50}\times j0.1 = \mathrm{j}0.2\ \mathrm{pu}$$

The per-unit reactance of the step-down transformer is

$$Z_{pu\,step-down} = \frac{100}{50}\times j0.12 = \mathrm{j}0.24\ \mathrm{pu}$$

The actual load current is given by

$$I_{load} = \frac{50\times10^6}{\sqrt{3}\times30\times10^3\times0.8} = 1203\ \mathrm{A}.$$

Note that this formula involves the operating power factor since the load power is specified in MW. Note also that it is the *actual* operating voltage which is used in the calculation.

The base current in the load zone is given by

$$I_{base} = \frac{100 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 1750 \text{ A} .$$

Hence, the per-unit load current is given by

$$\bar{I}_{pu\ load} = \frac{1203}{1750} \angle -36.87^\circ = 0.687 \angle -36.87^\circ \text{ pu} .$$

The per unit load voltage is

$$V_{pu\ load} = \frac{30}{33} \angle 0^\circ = 0.91 \angle 0^\circ \text{ pu} .$$

Note that the load voltage is taken as the reference phasor.

Hence, the per-unit value of the voltage at the terminals of the synchronous generator is given by

$$\begin{aligned} \bar{V}_{pu\ generator} &= 0.91 \angle 0^\circ + (0.687 \angle -36.87^\circ)(j0.2 + j0.575 + j0.24) \text{ pu} \\ &= 1.44 \angle 22.78^\circ \text{ pu} \end{aligned}$$

Thus, the magnitude of the actual line voltage at the generator terminals is given by

$$\begin{aligned} V_{generator} &= 11 \times 10^3 \times 1.44 \text{ V} \\ &= 15.84 \text{ kV}. \end{aligned}$$

Example 2

A bank of three single-phase transformers steps up the 13.8 kV line-to-line voltage of a three-phase synchronous generator to a required three-phase transmission line voltage of 138 kV. The generator rating is 41.5 MVA. Specify the voltage, current and MVA rating of each transformer for the following transformer bank connections:

(a) Low voltage – delta	High voltage - star
(b) Low voltage – star	High voltage - delta
(c) Low voltage – star	High voltage - star
(d) Low voltage – delta	High voltage - delta

Solution 2

(a)

$$V_{primary} = 13.8 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{13.8 \times 10^3} = 1002.2 \text{ A}$$

$$V_{secondary} = \frac{138}{\sqrt{3}} = 79.674 \text{ kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{79.67 \times 10^3} = 173.62 \text{ A}$$

(b)

$$V_{primary} = \frac{13.8}{\sqrt{3}} = 7.97 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{7.97 \times 10^3} = 1736.2 \text{ A}$$

$$V_{secondary} = 138 \text{ kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{138 \times 10^3} = 100.24 \text{ A}$$

(c)

$$V_{primary} = \frac{13.8}{\sqrt{3}} = 7.97\ \text{kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83\ \text{MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{7.97 \times 10^3} = 1736.2\ \text{A}$$

$$V_{secondary} = \frac{138}{\sqrt{3}} = 79.67\ \text{kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83\ \text{MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{79.67 \times 10^3} = 173.62\ \text{A}$$

(d)

$$V_{primary} = 13.8\ \text{kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83\ \text{MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{13.8 \times 10^3} = 1002.4\ \text{A}$$

$$V_{secondary} = 138\ \text{kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83\ \text{MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{138 \times 10^3} = 100.24\ \text{A}$$

Example 3

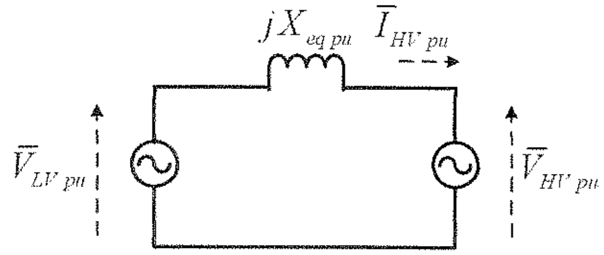
Three single-phase two-winding transformers, each rated at 400 MVA, 13.8 kV, 119.2 kV, with a leakage reactance of 10%, are connected to form a three-phase bank. The resistances of the windings and the excitation currents may be neglected. The high voltage windings are connected in star.

A three-phase load operating under balanced positive-sequence conditions on the high-voltage side draws 1000 MVA at 0.9 power factor lagging at a phase voltage of $\overline{V}_{AN} = 199.2\angle 0° \text{ kV}$.

Determine the voltage on the low voltage $\overline{V}_{an}$ on the low voltage busbar if the low-voltage windings are connected (i) in star and (ii) in delta.

Solution 3

For balanced operation, only the positive sequence equivalent network, as shown below, is required. In this diagram, $\overline{V}_{LV\,pu}$ and $\overline{V}_{HV\,pu}$ are the per-unit voltages at the low and high voltage busbars, respectively. The current $\overline{I}_{HV\,pu}$ is the per-unit current drawn by the HV busbar.



Select $S_{base-1\,phase} = 400$ MVA and $V_{HV\,base-phase} = 199.2$ kV based on the rating of the single-phase transformer and so $V_{HV\,base-line} = \sqrt{3} \times 199.2$ kV=345 kV and $S_{base-3\,phase} = 3 \times 400 = 1200$ MVA. Also, for the star-star connection, $V_{LV\,base-phase} = 13.8$ kV and $V_{HV\,base-line} = \sqrt{3} \times 13.8$ kV=23.9 kV.

Hence, the base current on the HV side is

$$I_{base-HV} = \frac{400 \times 10^6}{199.2 \times 10^3} = 2008 \text{ A}$$

The per-unit load current is then

$$\overline{I}_{HV} = \frac{1000 \times 10^6}{\sqrt{3}\ 345 \times 10^3} \angle -\text{Cos}^{-1}(0.9)° = 1673.5\angle -25.84° \text{ A}$$

so that

$$\overline{I}_{HV\,pu} = \frac{1673.5\angle -25.84°}{2008} = 0.833\ \angle -25.84° \text{ pu}.$$

The per-unit voltage on the HV side is

$$\overline{V}_{HV\,pu} = \frac{\overline{V}_{HV}}{V_{base-phase}} = 1.0\angle 0°.$$

For the star-star connected transformer, the per-unit voltage on the low voltage busbars is given by

$$\begin{aligned}\bar{V}_{LV\,pu} &= \bar{V}_{HV\,pu} + \left(jX_{eq}\right)\left(\bar{I}_{HV\,pu}\right)\\ &= 1.0\angle 0^\circ + j0.1\times\left(0.833\ \angle -25.84^\circ\right)\\ &= 1.039\ \angle 4.139^\circ\ \text{pu}\end{aligned}$$

Hence, the actual phase voltage on the low voltage busbars is given by

$$\begin{aligned}\bar{V}_{LV\,act} &= \bar{V}_{LV\,pu}\ V_{LV\ base-phase}\\ &= 1.039\ \angle 4.139^\circ\ \times\ 13.8\ \text{kV}\\ &= 14.34\ \angle 4.139^\circ\ \text{kV}.\end{aligned}$$

In the star-delta case, there will be a 30° phase shift between the high voltage and the low voltage sides of the transformer. However, the per-unit voltage on the low voltage side is the same irrespective of the transformer connection since the per-unit equivalent circuit is the same in both cases if this phase shift is ignored. Hence, as before,

$$\begin{aligned}\bar{V}_{LV\,pu} &= \bar{V}_{HV\,pu} + \left(jX_{eq}\right)\left(\bar{I}_{HV\,pu}\right)\\ &= 1.0\angle 0^\circ + j0.1\times\left(0.833\ \angle -25.84^\circ\right)\\ &= 1.039\ \angle 4.139^\circ\ \text{pu}\end{aligned}$$

but the actual phase voltage is now

$$\begin{aligned}\bar{V}_{LV\,act} &= \bar{V}_{LV\,pu}\ V_{LV\ base-phase}\\ &= 1.039\ \angle 4.139^\circ\ \times\ \frac{13.8}{\sqrt{3}}\ \text{kV}\\ &= 8.278\ \angle 4.139^\circ\ \text{kV}.\end{aligned}$$

due to the star-delta connection.