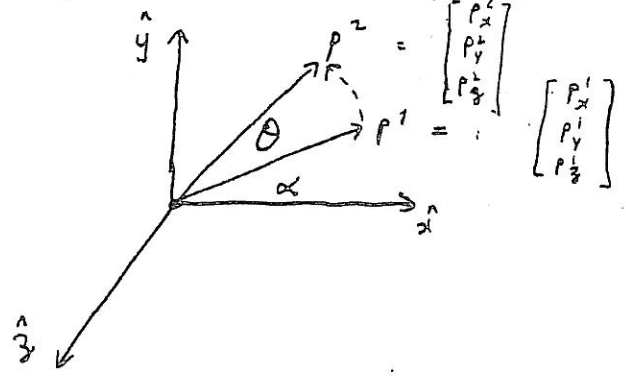
E.G. 2.2 Rotation of θ about $\hat{z}$ (i.e. in $\hat{x}$-$\hat{y}$ plane)



Let $\|p^1\| = \rho = \|p^2\|$

$$p^1_x = \rho \cos\alpha \qquad p^1_y = \rho \sin\alpha$$

$$p^2_x = \rho \cos(\alpha+\theta) \qquad p^2_y = \rho \sin(\alpha+\theta)$$

$$= \rho[\cos\alpha\cos\theta - \sin\alpha\sin\theta] \qquad = \rho[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$$
$$= p^1_x \cos\theta - p^1_y \sin\theta \qquad = p^1_y \cos\theta + p^1_x \sin\theta$$

Now $p^2_z = p^1_z$ $\qquad p^2 = R\,p^1$

$$\Rightarrow \quad R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Rot(\hat{z},\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The gripper of a SCARA robot is at

$$T_{base}^{tool} = \begin{bmatrix} 0.5 & 0.866 & 0 & 0.25 \\ 0.866 & -0.5 & 0 & 1.433 \\ 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

($\theta_1 = 90°$, $\theta_2 = 30°$, $\theta_4 = 0°$, $l_2 = 1.0$, $l_3 = 0.5$, $l_1 - d_3 - l_4 = 0.2\,m$).

If the gripper is rotated by 90° (about the $\hat{z}$ axis defining the gripper), what is the new T_{base}^{tool}

$$Rot(\hat{z}, 90°) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(T_{base}^{tool}\right)_{new} = \left(T_{base}^{tool}\right)_{old} \cdot Rot(\hat{z}, 90°)$$

(Tool (new) relative to Tool (old))

$$= \begin{bmatrix} 0.5 & 0.866 & 0 & 0.25 \\ 0.866 & -0.5 & 0 & 1.433 \\ 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 0.25 \\ -0.5 & 0.866 & 0 & 1.433 \\ 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If, instead, the robot had been rotated by 90° about its base

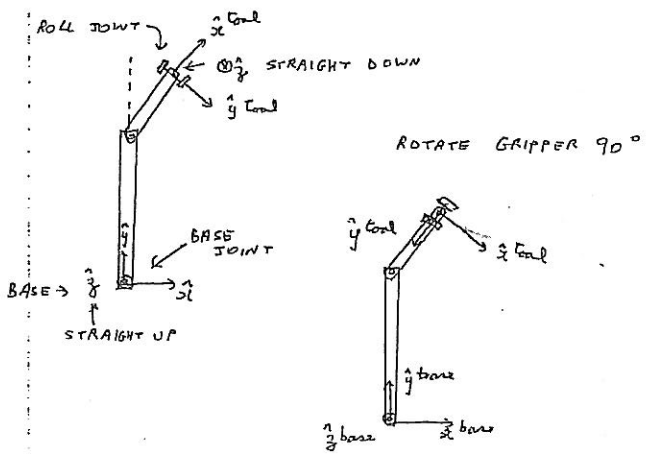
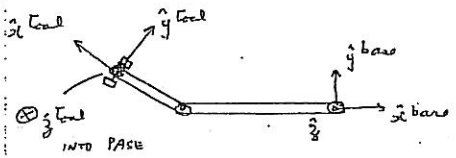
$$\Rightarrow \left(T_{base}^{tool}\right)_{new} = Rot(\hat{z}, 90°) \left(T_{base}^{tool}\right)_{old}$$

(Base (old) relative to base (new))

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.866 & 0 & 0.25 \\ 0.866 & -0.5 & 0 & 1.433 \\ 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.866 & 0.5 & 0 & -1.433 \\ 0.5 & 0.866 & 0 & 0.25 \\ 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TOP VIEW OF SCARA ROBOT



ROTATE BASE 90°