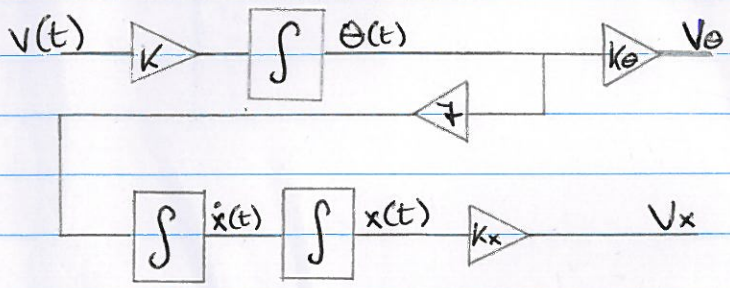
Control Engineering Summer '08

b

$$\frac{d}{dt}\begin{bmatrix}\dot{\theta}(t)\\ \ddot{x}(t)\\ \dot{x}(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 7 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}(t)\\ \ddot{x}(t)\\ \dot{x}(t)\end{bmatrix} + \begin{bmatrix}k\\ 0\\ 0\end{bmatrix} v(t)$$

$$\begin{bmatrix}V_\theta(t)\\ V_x(t)\end{bmatrix} = \begin{bmatrix}K_\theta & 0 & 0\\ 0 & 0 & K_x\end{bmatrix}\begin{bmatrix}\dot{\theta}(t)\\ \ddot{x}(t)\\ \dot{x}(t)\end{bmatrix}$$

i)



$$\frac{V_x(s)}{V(s)} = \frac{7kK_x}{s^3}$$

$$\dot{\theta}(0) = \dot{\theta}_0 \qquad \ddot{x}(0) = 0 \qquad \dot{x}(0) = x_0$$

Zero-input response: $y(t) = C\phi(t)\underline{x}(0)$

$$\phi(t) = e^{At} = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \dots$$

$$\phi(t) = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 7t & 0 & 0\\ 0 & t & 0\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 3.5t^2 & 0 & 0\end{bmatrix}$$

$$\phi(t) = \begin{bmatrix}1 & 0 & 0\\ 7t & 1 & 0\\ 3.5t^2 & t & 1\end{bmatrix}$$

$$y(t) = \begin{bmatrix}K_\theta & 0 & 0\\ 0 & 0 & K_x\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 7t & 1 & 0\\ 3.5t^2 & t & 1\end{bmatrix}\begin{bmatrix}\dot{\theta}_0\\ 0\\ x_0\end{bmatrix}$$

$$= \begin{bmatrix}K_\theta & 0 & 0\\ 0 & 0 & K_x\end{bmatrix}\begin{bmatrix}\dot{\theta}_0\\ 7t\dot{\theta}_0\\ 3.5t^2\dot{\theta}_0 + x_0\end{bmatrix}$$

$$V_x(t) = K_x\left(3.5t^2\dot{\theta}_0 + x_0\right)$$