OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2011

B.E. DEGREE (ELECTRICAL)

CONTROL ENGINEERING
EE4002

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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of departmental approved non-programmable calculators is permitted

1.

(a) Explain why it is necessary to employ anti-aliasing filters, before sampling. Give some indication how sampling rate and the anti-aliasing filter bandwidth would be selected.

[5 Marks]

(b) Derive Tustins's transformation.

A closed-loop speed control scheme for a DC motor is shown below.

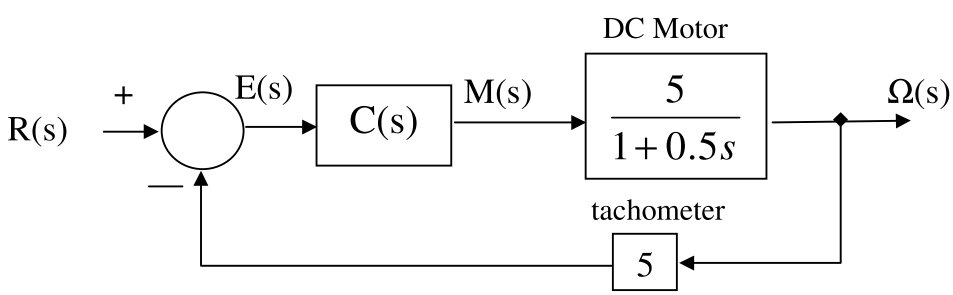

Fig. 1.1: Closed-loop Motor Speed Control

The following PI controller is proposed:

$$m(t) = K_p\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau \right)$$

The controller was tuned to obtain a closed-loop damping factor ζ=0.83. The controller parameters are, K_p=0.19 and T_I=0.2 seconds.

It was decided to implement this PI control-law on a micro-controller, with sample time T=0.2 seconds and assuming a zero-order hold. Tustin's approximation was used to convert the continuous algorithm designed above to a discrete-time PI control algorithm.

Sketch the root locus diagram for the system under digital PI speed control.

Show that the closed-loop performance of the digital speed controller is unsatisfactory.

[20 Marks]

2.

(a) Derive the following design equation for the controller D(z),

$$D(z) = \frac{1}{G(z)} \frac{P(z)}{1-P(z)},$$

where $G(z)=C(z)/U(z)$ is the discrete-time transfer function model of the open-loop process and P(z) is the desired closed loop transfer function. What are the key drawbacks of this design method?

[5 Marks]

(b) A certain process is under digital closed-loop control, with the controller *D(z)* designed using Kalman's method. The following closed-loop time responses have been obtained for the process output *c(k)* and the controller output *m(k)* for the step in the setpoint *r(k)*,

$$r(k) = \begin{cases} 0 & for \quad k < 0 \\ 0.7 & for \quad k \geq 0 \end{cases}$$

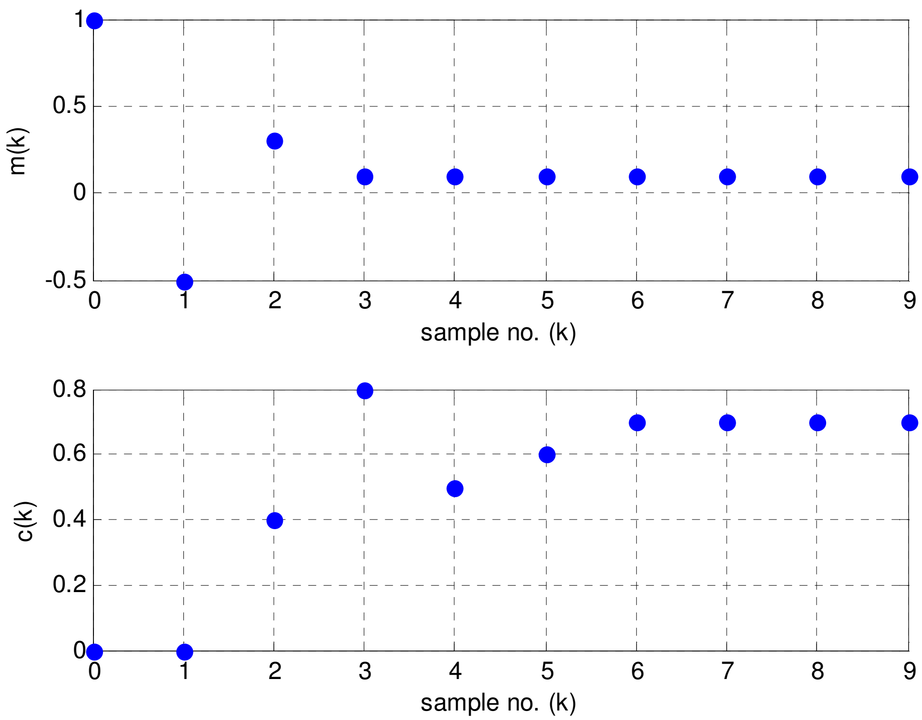

Fig.2.1: Closed-loop Responses for System Under Kalman's Control

Determine the transfer function of the controller *D(z)* that was used to generate these responses.

[8 Marks]

(c) Consider the following general first-order system with time delay, T_d within a closed-loop digital control scheme. The sampling time is T and a zero-order hold is assumed. The time delay T_d is approximately N samples in length.

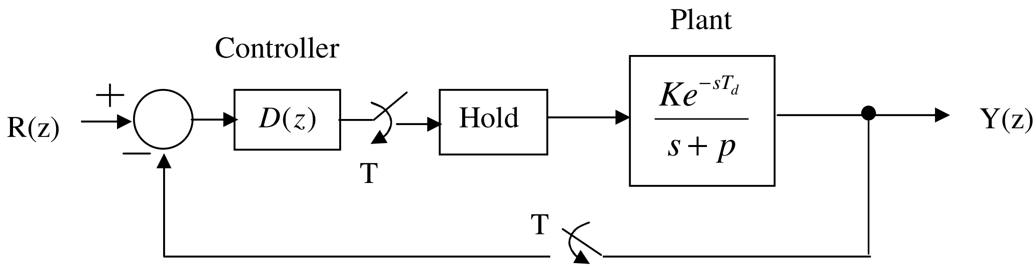

Fig. 2.2: Digital Closed-loop Control of a First-order Plant With Delay

Derive the following Dahlin's controller for the general first order process, from a basic prescription of the shape of the desired closed-loop step response. Show clearly how the parameters of this controller are determined.

$$D(z) = K_d \frac{1+\gamma z^{-1}}{1+\alpha z^{-1}+\beta z^{-N-1}}.$$

Show that the controller provides integral action.

[12 Marks]

3.

(a) Derive in full, the following least-squares algorithm, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function, from a matrix $\Phi(k)$, of input and output data, and a vector $\underline{y}(k)$, of the sampled process output, up to the current kth sample instant.

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1} \Phi(k)^T \underline{y}(k) .$$

If a square matrix P(k) is now defined as $P(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1}$, use Householders Matrix Inversion Lemma,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1},$$

to derive the following update equation for P(k+1) from process data up to the (k+1)th sample,

$$P(k+1) = P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^T(k+1)P(k)}{1 + \underline{\psi}^T(k+1)P(k)\underline{\psi}(k+1)} .$$

Here the vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the (k+1)th sample.

[13 Marks]

(b) The following model structure has been proposed for a certain process that is controlled using an adaptive pole-placement controller:

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + b_1 u(k) + b_2 u(k-1)$$

Here *y(k)* is the plant output, *u(k)* the plant input.

At the kth sampling instant, the estimate of the parameter vector of the process is available from the recursive least squares algorithm, as:

$$\underline{\hat{\theta}}(k) = \left[\hat{a}_1(k) \quad \hat{a}_2(k) \quad \hat{b}_1(k) \quad \hat{b}_2(k)\right]^T = [1.6 \quad -0.64 \quad 0.0 \quad 0.2]^T$$

It is required to place the two dominant poles of the closed-loop process each at z=0.6. It is also desired that the resultant closed-loop system will achieve perfect steady-state tracking of step-like setpoint signals.

Calculate the controller polynomials at the kth sampling instant to achieve the desired closed-loop performance.

[12 Marks]

4.

(a) Consider the following state-space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$

(i) Develop fully the following solution for the state trajectory $\underline{x}(t)$, for $t \geq 0$, where $\underline{x}(0)$ is the initial state vector at t=0, and $\Phi(t)$ is the transition matrix.

$$\underline{x}(t) = \Phi(t)\underline{x}(0) + \int_0^t \Phi(t-\tau)B\underline{u}(\tau)d\tau$$

(ii) Prove the following,

$$\mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\} = e^{At}$$

(iii) If the sample-time is T, and it is assumed that a zero-order hold is applied to the input signal $\underline{u}(t)$, show that this process can be represented by the following discrete-time, state-space equations:

$$\underline{x}(k+1) = e^{AT}\,\underline{x}(k) + A^{-1}\left(e^{AT} - I\right)B\underline{u}(k)$$

[13 Marks]

(b) A certain system can be represented by the block diagram,

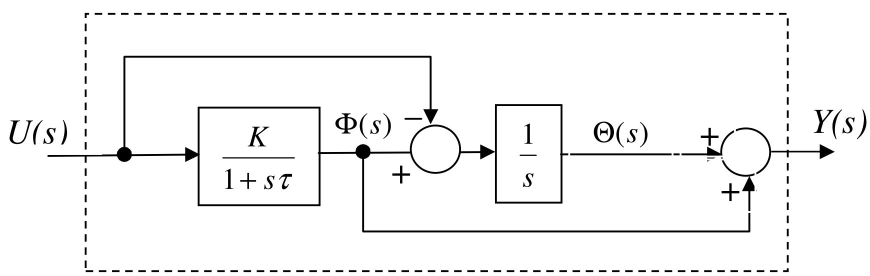

Fig. 4.1: System Block Diagram

(i) Develop a state-space model of this process using the states $\phi(t)$ and $\theta(t)$

(ii) Use this state space model to determine the transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$.

(iii) Determine whether the states are observable for your state space representation.

[12 Marks]

5.

(a) Consider the following Nth order open-loop process, with one input *u(t)* and a single output *y(t)*,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

This process is under the following state space control–law,

$$u(t) = N_u r(t) - K\left(\underline{x}(t) - N_x r(t)\right),$$

where r(t) is the reference. Develop the following design equation, to achieve perfect steady-state tracking of step-like reference signals.

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

[5 Marks]

(b) Consider the following ball-on-beam apparatus consisting of a rigid beam, free to rotate in one plane about its central pivot. A servo-motor is used to rotate the beam. There are two parallel guide rails, on which a steel ball sits.

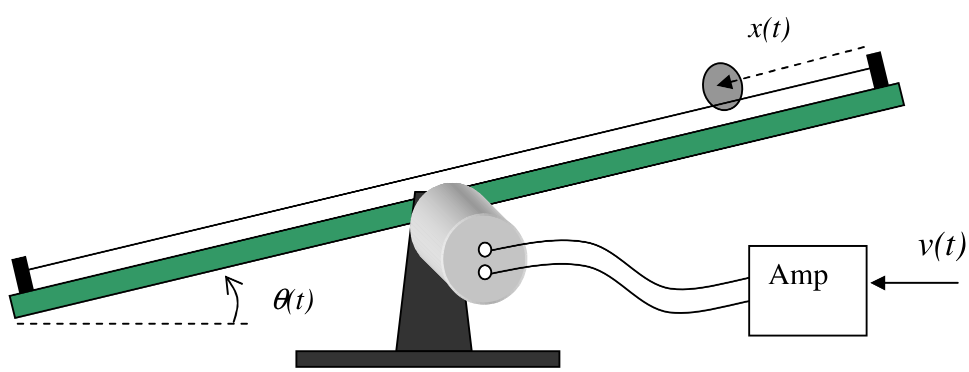

Fig.5.1: Ball-on-Beam Apparatus

Only two sensors are available. The first is a simple rotary potentiometer that is used to provide a measure of the beam angle $\theta(t)$. The second sensor provides a measurement of the ball position *x(t),* using the wire guide rails as a linear potentiometer.

The servo-motor dynamics are so fast that the rotation of the beam can be described by the following first-order differential equation:

$$\frac{d\theta(t)}{dt} = Kv(t).$$

The gains of the linear and rotary potentiometers are K_x and K_θ. respectively

If the moment of inertia, about the axis of rotation, of the ball of mass m and radius r, is $J={}^2/_5\, mr^2$, basic rotational mechanics yields the following expression for the linear acceleration:

$$\frac{d^2 x}{dt^2} = 7\theta(t).$$

The gain K=2Vrad^{-1}s and the potentiometer gains are K_x =5V/m and K_θ = 5V/radian.

Design a state-space ball position controller. It is specified that the peak overshoot in closed-loop ball position should be 10%, with a settling time of $Ts_{2\%} = 4$ seconds, in response to a step change in the desired ball position.

[20 Marks]

6.

(a) A certain process can be modelled by the transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K(1+s\tau_2)}{s(1+s\tau_1)}$$

Develop fully a simulation diagram for the Observer-Canonical representation of this process.

[5 Marks]

(b) Consider the following N^{th} order open-loop process, with single input *u(t)*, single output *y(t)* , and state-vector $\underline{x}(t)$,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

This process is controlled using a state-space regulator, with gain matrix *K*. The state vector is not measured directly, but is estimated as $\hat{\underline{x}}(t)$ using a full-state Luenberger observer with estimator gain matrix *G*.

Develop fully the following characteristic equation for the closed-loop system,

$$|sI - A + BK||sI - A + GC| = 0\,.$$

Use this characteristic equation to explain the "Separation Principle", and how it is applied in state-space control design.

[10 Marks]

(c) Consider the following simplified model of the attitude dynamics of a satellite:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$
$$v_\theta(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix}$$

The following state-space regulator has been designed to place both the closed-loop poles at $s = -p$:

$$u(t) = -k_1\theta(t) - k_2\omega(t)$$

A full-state Luenberger Observer is used to estimate the states, from the input *u(t)* and the sensor output $v_\theta(t)$. The poles of the observer are both placed at $s = -5p$.

Determine the classical control representation of this state-space controller.

[10 Marks]