# Induction Motor

$$\vec{i}_s(t) = \hat{I}_s(t)\angle\theta_{is}(t) = i_a(t)\angle 0^\circ + i_b(t)\angle 120^\circ + i_c(t)\angle 240^\circ$$
$$\vec{i}_r'(t) = \hat{I}_r'(t)\angle\theta_{ir}(t) = i_A'(t)\angle 0^\circ + i_B'(t)\angle 120^\circ + i_C'(t)\angle 240^\circ$$

$$\vec{V}_s(t) = \hat{V}_s(t)\angle\theta_{vs}(t) = V_a(t)\angle 0^\circ + V_b(t)\angle 120^\circ + V_c(t)\angle 240^\circ$$
$$\vec{V}_r'(t) = \hat{V}_r(t)\angle\theta_{vr}(t) = V_A'(t)\angle 0^\circ + V_B'(t)\angle 120^\circ + V_C'(t)\angle 240^\circ$$

$$\vec{\lambda}_s(t) = \hat{\lambda}_s(t)\angle\theta_{\lambda s}(t) = \lambda_a(t)\angle 0^\circ + \lambda_b(t)\angle 120^\circ + \lambda_c(t)\angle 240^\circ$$
$$\vec{\lambda}_r(t) = \hat{\lambda}_r(t)\angle\theta_{\lambda r}(t) = \lambda_A(t)\angle 0^\circ + \lambda_B(t)\angle 120^\circ + \lambda_C(t)\angle 240^\circ$$

$$\begin{bmatrix}\vec{\lambda}_s(t)\\ \vec{\lambda}_r(t)\end{bmatrix} = \begin{bmatrix} L_s & -L_m \\ L_m & -L_r\end{bmatrix}\begin{bmatrix}\vec{i}_s(t)\\ \vec{i}_r'(t)\end{bmatrix}$$

Define an arbitrary, orthogonal, rotating set of d- + q- axis windings producing an equiv. MMF to the 3 phase currents:

To yield appropriate winding inductances

$$\vec{i}_s(t) = \frac{3}{2}\sqrt{2}\, I_s\angle\theta_{is} \quad \text{becomes} \quad \sqrt{\frac{3}{2}}\left(i_{sd}(t) + j\, i_{sq}(t)\right)$$

Broken into d + q axis components:

$\approx$ field $\quad i_{sd} = \sqrt{\frac{2}{3}}$ projection of $\vec{i}_s(t)$ along d-axis

$\approx$ armature $\quad i_{sq} = \sqrt{\frac{2}{3}}$ projection of $\vec{i}_s(t)$ along q-axis

$$\vec{i}_s^{\,a}(t) = i_a(t)e^{j0^\circ} + i_b(t)e^{j120^\circ} + i_c(t)e^{j240^\circ} \quad \text{Sp. Vec. wrt a axis}$$
$$\vec{i}_s^{\,d}(t) = i_a(t)e^{-j(\theta_{da})} + i_b(t)e^{-j(120-\theta_{da})} + i_c(t)e^{j(240-\theta_{da})}$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\begin{bmatrix} i_{sd} \\ i_{sd}\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}\cos\theta_{da} & \cos(\theta_{da}-120^\circ) & \cos(\theta_{da}-240^\circ) \\ -\sin\theta_{da} & \sin(\theta_{da}-120^\circ) & -\sin(\theta_{da}-240^\circ)\end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t)\end{bmatrix}$$

$$\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t)\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}\cos\theta_{da} & -\sin\theta_{da} \\ \cos(\theta_{da}+240^\circ) & -\sin(\theta_{da}+240^\circ) \\ \cos(\theta_{da}+120^\circ) & -\sin(\theta_{da}+120^\circ)\end{bmatrix}\Bigg(\,.$$

# Induction Motor

$$T_{em} = \frac{P}{2}\left(\lambda_{rd} i_{rq} - \lambda_{rq} i_{rd}\right)$$

$$\vec{\lambda}_r^{\,d} = \vec{\lambda}_r^{\,a} e^{-j\theta_{da}} = \sqrt{\tfrac{3}{2}}\left(\lambda_{rd} + j\lambda_{rq}\right)$$

$$\vec{I}_r^{\,d} = \vec{i}_r^{\,a} e^{-j\theta_{da}} = \sqrt{\tfrac{3}{2}}\left(i_{rd} + j i_{rq}\right)$$

$$\omega_{SLIP} = \frac{2}{P}\frac{R_r' i_{rq}}{\lambda_{rd}} \qquad \_\_{SLIP} = \frac{1}{P\pi}\frac{R_r i_{rq}}{\lambda_{rd}}$$

## Eq. Circuit Derivation:

$$\vec{V}_s^{\,a}(t) = R_s \vec{i}_s^{\,a} + \frac{d}{dt}\vec{\lambda}_s^{\,a} = R_s \vec{i}_s^{\,a} + \frac{d}{dt}\left[(L_{ls} + L_m)\vec{i}_s^{\,a} + L_m \vec{i}_r^{\,a}\right]$$

$$\vec{V}_s^{\,d}(t) e^{j\theta_{da}} \quad R_s \vec{i}_s^{\,d} e^{j\theta_{da}} + \frac{d}{dt}\vec{\lambda}_s^{\,d} e^{j\theta_{da}}$$

Note: $\omega_{syn} = \frac{d\theta_{da}}{dt}$

$$R_s \vec{i}_s^{\,d} e^{j\theta_{da}} + e^{j\theta_{da}}\frac{d}{dt}\vec{\lambda}_s^{\,d} + j\omega_{syn} e^{j\theta_{da}} \vec{\lambda}_s^{\,d}$$

$$\Rightarrow \vec{V}_s^{\,d}(t) = R_s \vec{i}_s^{\,d} + \frac{d}{dt}\vec{\lambda}_s^{\,d} + j\omega_e \vec{\lambda}_s^{\,d}$$

also

$$\vec{V}_r^{\,d}(t) = -R_r' \vec{i}_r^{\,d} + \frac{d}{dt}\vec{\lambda}_r^{\,d} + j\omega_{SLIP}\vec{\lambda}_r^{\,d}$$

$$= -R_r' \vec{i}_r^{\,d} + \frac{d}{dt}\vec{\lambda}_r^{\,d} + j\frac{P}{2}\omega_{SLIP}\vec{\lambda}_r^{\,d}$$

Sub in for $\lambda$

$$\vec{V}_s^{\,d}(t) = R_s \vec{i}_s^{\,d} + (L_{ls} + L_m)\frac{d}{dt}\vec{i}_s^{\,d} - L_m \frac{d}{dt}\vec{i}_r^{\,d} + j\omega_e \vec{\lambda}_s^{\,d}$$

$$\vec{V}_r^{\,d}(t) = -R_r' \vec{i}_r^{\,d} - (L_{lr}' + L_m)\frac{d}{dt}\vec{i}_r^{\,d} + L_m \frac{d}{dt}\vec{i}_s^{\,d} + j\left(\omega_e - \frac{P}{2}\omega_m\right)\vec{\lambda}_r^{\,d}$$

## DC Machine

$$T_{em} = P B_{pole} N_a i_a l r$$
$$= \frac{P}{2}\left(N_a \frac{2}{\pi} B_{pole} \pi r l\right) i_a$$
$$= \frac{P}{2}\left(N_a \frac{2}{\pi} B_{pole} A_{pole}\right) i_a = \frac{P}{2}\left(N_a \phi_{pole} \frac{2}{\pi}\right) i_a$$
$$\phi_{avg} = \frac{2}{\pi}\phi_{pole}$$
$$T_{em} = \frac{P}{2}\lambda_{fd} i_a \qquad \lambda_{fd} = \text{field d-axis flux linkage}$$

## PMAC Machine

$$T_{em} = \frac{P}{2}\lambda_{rd} i_{rq}$$

$\vec{I}_s(t)$ leads $\vec{B}_r(t)$ by $90°$

## Induction Motor:

Want to model a similar decoupled torque equation. Stator current space vector is controlled, specifically the components controlling rotor flux & rotor current.

$$\vec{i}_s(t) = \hat{I}_s(t) \angle \theta_{is}(t)$$
$$= i_a(t)\angle 0° + i_b \cancel{(t)} \angle 120° + i_c(t)\angle 240°$$

$$\vec{i}_r'(t) = \hat{I}_r'(t) \angle \theta_{ir}(t)$$
$$= i_A'(t) + i_B'(t)\angle 120° + i_C'(t)\angle 240°$$

$i_A', i_B', i_C'$ are the equiv. reflected rotor currents.

$$\vec{v}_s(t) = \hat{V}_s(t) \angle \theta_{vs}(t)$$
$$= v_a(t)\angle 0° + v_b(t)\angle 120° + v_c(t)\angle 240°$$

$$\vec{v}_r'(t) = \hat{V}_r'(t) \angle \theta_{vr}(t)$$
$$= v_A'(t)\angle 0° + v_B'(t)\angle 120° + v_C' \angle 240°$$

$\Rightarrow$ ~~refle~~ eq. refl. rotor voltages