OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

AUTUMN EXAMINATIONS, 2011

B.E. (ELECTRICAL)

DIGITAL SIGNAL PROCESSING
EE4008

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Time Allowed: *3 hours*

Answer *five* questions.

All questions carry equal marks.

The use of departmental approved non-programmable calculators is permitted

1. (a) Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a band pass filter. [*10 marks*]

   (b) Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the "Windows" method of the band pass filter design that meets the following specification:

   - Passband:- $150 - 250$Hz
   - Transition Width:- 50Hz
   - Passband Ripple:- 0.01dB
   - Stopband attenuation:- > 60dB
   - Sampling frequency:- 1kHz

   The parameters of common window functions are given in the Appendix. [*10 marks*]

2. (a) A First Order Lowpass IIR Digital Filter has a transfer function

$$H_{LP}(z) = G\frac{1 + z^{-1}}{1 - \alpha z^{-1}}$$

Determine the Gain factor G in terms of α. [*5 marks*]

(b) Derive an expression for α in terms of the 3dB cutoff frequency ω_c. [*5 marks*]

(c) Determine the transfer function $H_1(z)$, of a first-order low pass filter with a 3-dB cutoff Frequency of 0.6π. Draw the pole/zero Plot of $H_1(z)$ and sketch the magnitude response of the filter for $0 \leq \omega \leq \pi$, clearly identifying the 3dB cutoff frequency ω_c [*5 marks*]

(d) A comb filter with a transfer function $G_1(z)$ is formed by taking the transfer function $H_1(z)$ and replacing each delay by M delays, such that:

$$G_1(z) = H_1(z^M)$$

Draw the pole/zero plot for $M = 2$ and sketch the magnitude response for the same value of M, for $0 \leq \omega \leq \pi$. [*5 marks*]

3. Consider the z-transform

$$H(z) = \frac{z(z + 0.5)}{z^2 - 0.65z + 0.1}$$

(a) Draw the Pole-Zero plot of $H(z)$ and identify the three possible regions of convergence. [*6 marks*]

(b) Use the partial fractions method to determine the inverse z-transform $h(n)$ where

i. $h(n)$ is a causal sequence

ii. $h(n)$ is an anti-causal sequence

iii. $h(n)$ is a two sided sequence. [*10 marks*]

(c) Determine the first three values of the causal $h(n)$ sequence, using the long division method of inverting the z-transform. [*4 marks*]

4. (a) Let $x(n)$ be a Wide Sense Stationary random process with mean m_X, autocorrelation $\phi_{XX}(k)$ and power spectral density $P_{XX}(\omega)$. $x(n)$ is filtered by a Stable Linear Time Invariant System with impulse response $h(n)$ to produce output $y(n)$. Determine the mean m_Y, autocorrelation $\phi_{YY}(k)$ and power spectral density $P_{YY}(\omega)$ of $y(n)$. [*10 marks*]

(b) Unit variance white noise is filtered by a LTI filter with impulse response:

$$h(n) = \frac{1}{5}\left(-\frac{1}{5}\right)^n u(n) + \frac{1}{5}\left(-\frac{1}{5}\right)^{n-3} u(n-3)$$

Determine the mean and the power spectral density of the filter output in trigonometric form.

[*10 marks*]

5. (a) Show how the N point DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\left(\frac{-j2\pi nk}{N}\right)} \quad k = 0, 1, \ldots, N-1$$

can be reduced to two $\frac{N}{2}$ point DFTs of the odd and even indexed values of $x(n)$.

[*6 marks*]

(b) Hence show that the computational complexity of a radix 2 decimation in time FFT algorithm is

$$O\left(\frac{N}{2}\log_2 N\right)$$

[*4 marks*]

(c) 1024 data values of the Wide Sense Stationary Random Process

$$x(n) = A\sin(n\omega_1 + \phi_1) + A\sin(n\omega_2 + \phi_2) + w(n)$$

are recorded, where A, ω_1 and ω_2 are fixed constants; ϕ_1 and ϕ_2 are random variables uniformly distributed over the interval $-\pi, \pi$ and $w(n)$ is Gaussian White Noise with variance σ_w. For $\Delta\omega = (\omega_2 - \omega_1) = 0.015\pi$ compare the performance of the following 4 methods of spectral estimation in terms of resolution, variance reduction and computation complexity.

i. Periodogram
ii. Modified Periodogram using a Hamming window
iii. Bartlett method
iv. Welch method with 50% overlap and a Hanning window

You may assume that the FFT is used in each of the spectral estimation methods and parameters of the window function are given in the Appendix.

[*10 marks*]

6. (a) Starting with the Yule Walker Equation for an ARMA process of order (p, q)

$$\phi_{XX}(k) = -\sum_{l=1}^{p} a(l)\phi_{XX}(k-l) + \sigma_v^2 \sum_{l=0}^{q-k} b(l+k)h^*(l)$$

derive the Yule Walker Equations for an

i. AR Process of order p

ii. MA Process of order q

[*4 marks*]

(b) Compare the following AR methods of parametric spectral estimation

i. Autocorrelation

ii. Covariance

iii. Modified Covariance

[*9 marks*]

(c) In the covariance method of parametric spectral estimation the value $\hat{c}(j,k)$,where

$$\hat{c}(j,k) = \frac{1}{N-p} \sum_{=p}^{N-1} x^*(n-j)x(n-k)$$

is used as an estimate of the autocorrelation values $\phi_{XX}(j-k)$. Given

$$\begin{aligned}
\hat{c}(0,0) &= 1 \\
\hat{c}(0,1) = \hat{c}(1,0) &= 0.66 \\
\hat{c}(0,2) = \hat{c}(2,0) &= 0.25 \\
\hat{c}(1,1) &= 0.95 \\
\hat{c}(1,2) = \hat{c}(2,1) &= 0.65 \\
\hat{c}(2,2) &= 0.94
\end{aligned}$$

Estimate the spectrum using the covariance method of AR parametric spectral estimation for an AR(2) process.

[*7 marks*]

Appendix of Equations

- Window Functions

Window $w(n)$	Sidelobe	$\triangle f$	Stopband Attenuation	Passband Ripple	$\Delta\omega_{3db}$
Rectangular	-13db	$\frac{0.9}{N}$	21db	0.7416db	$0.89\frac{2\pi}{N}$
$w(n) = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hanning	-31db	$\frac{3.1}{N}$	44db	0.0546db	$1.44\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hamming	-41db	$\frac{3.3}{N}$	53db	0.0194db	$1.30\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					

- Table of Z-Transforms

Signal $x(n)$	Z-Transform $X(z)$	ROC
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > a$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\| < a$