OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2011

B.E. (ELECTRICAL)
M.ENG.SC. (MICROELECTRONICS)
H.DIP. (MICROELECTRONICS)
VISITING EUROPEANS

DIGITAL SIGNAL PROCESSING
EE4008

Dr L. Seed
Dr. M. Creed
Dr. W.P. Marnane

Time Allowed: *3 hours*

Answer *five* questions.

All questions carry equal marks.

The use of departmental approved non-programmable calculators is permitted

The use of Mathematical Tables is permitted.

1. (a) Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a low pass filter. [*10 marks*]

   (b) Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the "Windows" method of the low pass filter design that meets the following specification:

   - Passband edge frequency:- $F_p = 18$kHz
   - Transition Width:- 5kHz
   - Passband Ripple:- 0.01dB
   - Stopband attenuation:- > 40dB
   - Sampling frequency:- 44kHz

   The parameters of common window functions are given in the Appendix. [*10 marks*]

2. (a) A Second order Bandpass IIR Digital Filter has a transfer function

$$H_{BP}(z) = \frac{K(1 - z^{-2})}{1 - \beta(1 + \alpha)z^{-1} + \alpha z^{-2}}$$

Determine the Gain factor K in terms of α and the value β in terms of the centre frequency ω_0. [*5 marks*]

(b) $H_{BP}(z)$ has a pair of complex conjugate poles at $z = re^{\pm j\phi}$ such that

$$H_{BP}(z) = \frac{K(1 - z^{-2})}{(1 - re^{j\phi}z^{-1})(1 - re^{-j\phi}z^{-1})}$$

Show that

$$r = \sqrt{\alpha}$$

and

$$\phi = \cos^{-1}\left(\frac{\beta(1 + \alpha)}{2\sqrt{\alpha}}\right)$$

[*5 marks*]

(c) The 3-dB Bandwidth of the 2nd order Bandpass filter is given by:

$$\Delta\omega_{3db} = \cos^{-1}\left(\frac{2\alpha}{1 + \alpha^2}\right)$$

Determine the transfer function $H(z)$, of a second-order bandpass filter with a centre frequency of 150Hz and a 3-dB bandwidth of 100Hz when the sampling frequency is 1500Hz. Determine the constant coefficient difference equation that implements the filter in the time domain. Draw the pole/zero plot of $H(z)$ and determine the values of the polar co-ordinates r and ϕ. Sketch the magnitude response of the filter, clearly identifying the centre frequency of 150Hz and the 3-dB Bandwidth.

[*10 marks*]

3. Consider the z-transform

$$H(z) = \frac{z(z + 0.5)}{z^2 - 0.7z + 0.12}$$

(a) Draw the Pole-Zero plot of $H(z)$ and identify the three possible regions of convergence. [*6 marks*]

(b) Use the partial fractions method to determine the inverse z-transform $h(n)$ where

i. $h(n)$ is a causal sequence
ii. $h(n)$ is an anti-causal sequence
iii. $h(n)$ is a two sided sequence. [*10 marks*]

(c) Determine the first three values of the causal $h(n)$ sequence, using the long division method of inverting the z-transform. [*4 marks*]

4. (a) Let $x(n)$ be a Wide Sense Stationary random process with mean m_X, autocorrelation $\phi_{XX}(k)$ and power spectral density $P_{XX}(\omega)$. $x(n)$ is filtered by a Stable Linear Time Invariant System with impulse response $h(n)$ to produce output $y(n)$. Determine the mean m_Y, autocorrelation $\phi_{YY}(k)$ and power spectral density $P_{YY}(\omega)$ of $y(n)$. [*10 marks*]

(b) Unit variance white noise is filtered by a LTI filter with impulse response:

$$h(n) = \frac{1}{5}\left(-\frac{1}{5}\right)^{n} u(n) + \frac{1}{5}\left(-\frac{1}{5}\right)^{n-3} u(n-3)$$

Determine the mean and the power spectral density of the filter output in trigonometric form.

[*10 marks*]

5. (a) Show how the N point DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\left(\frac{-j2\pi nk}{N}\right)} \quad k = 0, 1, \ldots, N-1$$

can be reduced to two $\frac{N}{2}$ point DFTs of the odd and even indexed values of $x(n)$. [*6 marks*]

(b) Hence show that the computational complexity of a radix 2 decimation in time FFT algorithm is

$$O\left(\frac{N}{2}\log_2 N\right)$$

[*4 marks*]

(c) 1024 data values of the Wide Sense Stationary Random Process

$$x(n) = A\sin(n\omega_1 + \phi_1) + A\sin(n\omega_2 + \phi_2) + w(n)$$

are recorded, where A, ω_1 and ω_2 are fixed constants; ϕ_1 and ϕ_2 are random variables uniformly distributed over the interval $-\pi, \pi$ and $w(n)$ is Gaussian White Noise with variance σ_w. For $\Delta\omega = (\omega_2 - \omega_1) = 0.025\pi$ compare the performance of the following 4 methods of spectral estimation in terms of resolution, variance reduction and computation complexity.

i. Periodogram
ii. Modified Periodogram using a Hamming window
iii. Bartlett method
iv. Welch method with 50% overlap and a Hanning window

You may assume that the FFT is used in each of the spectral estimation methods and parameters of the window function are given in the Appendix.

[*10 marks*]

6. (a) In Autoregressive Moving Average parametric spectral estimation the signal $x(n)$ and the white noise $v(n)$ are related by the constant coefficient difference equation:

$$x(n) = -\sum_{l=1}^{p} a(l)x(n-l) + \sum_{l=0}^{q} b(l)v(n-l)$$

Starting with this relationship, derive the Yule Walker Equation for an ARMA process of order (p, q)

$$\phi_{XX}(k) = -\sum_{l=1}^{p} a(l)\phi_{XX}(k-l) + \sigma_v^2 \sum_{l=0}^{q-k} b(l+k)h^*(l)$$

[*10 marks*]

(b) From the Yule Walker Equation for an ARMA process in part (a) derive the Yule Walker Equations for an

i. AR Process of order p

ii. MA Process of order q

[*4 marks*]

(c) In the covariance method of parametric spectral estimation the value $\hat{c}(j,k)$,where

$$\hat{c}(j,k) = \frac{1}{N-p} \sum_{=p}^{N-1} x^*(n-j)x(n-k)$$

is used as an estimate of the autocorrelation values $\phi_{XX}(j-k)$. Given

$$\begin{aligned}
\hat{c}(0,0) &= 1 \\
\hat{c}(0,1) = \hat{c}(1,0) &= 0.66 \\
\hat{c}(0,2) = \hat{c}(2,0) &= 0.25 \\
\hat{c}(1,1) &= 0.95 \\
\hat{c}(1,2) = \hat{c}(2,1) &= 0.65 \\
\hat{c}(2,2) &= 0.9
\end{aligned}$$

Estimate the spectrum using the covariance method of AR parametric spectral estimation for an AR(2) process.

[*6 marks*]

Appendix of Equations

- Window Functions

Window $w(n)$	Sidelobe	$\triangle f$	Stopband Attenuation	Passband Ripple	$\Delta\omega_{3db}$
Rectangular	-13db	$\frac{0.9}{N}$	21db	0.7416db	$0.89\frac{2\pi}{N}$
$w(n) = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hanning	-31db	$\frac{3.1}{N}$	44db	0.0546db	$1.44\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hamming	-41db	$\frac{3.3}{N}$	53db	0.0194db	$1.30\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					

- Table of Z-Transforms

Signal $x(n)$	Z-Transform $X(z)$	ROC
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > a$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\| < a$