OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2010

B.E. DEGREE (ELECTRICAL)

CONTROL ENGINEERING
EE4002

Professor C. Delabie
Dr. M. Creed
Dr. G. Lightbody

Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of departmental approved non-programmable calculators is permitted

1.

(a) Consider the following closed-loop discrete-time system.

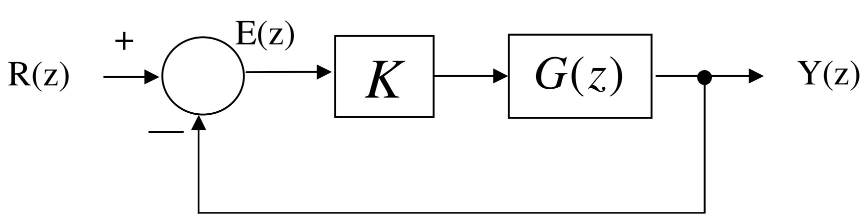

Fig. 1.1 Closed-loop digital control system

The following unit step response has been obtained for the open-loop process G(z).

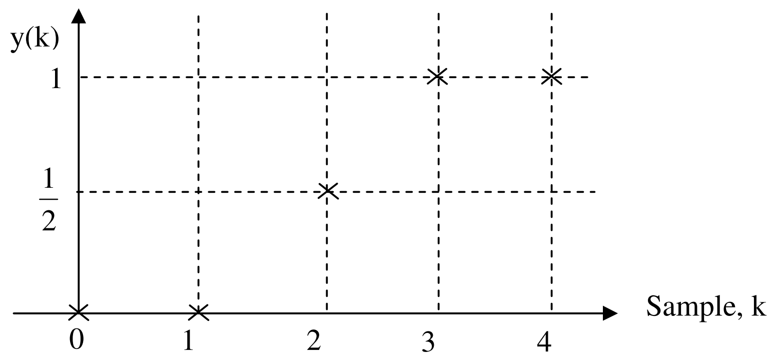

Fig. 1.2 Discrete unit step response for the open-loop process G(z)

The controller gain is now set as K=0.5. Use the difference equation method to sketch the unit step response *y(k)* for the closed loop process.

[5 marks]

(b) Derive Tustins's transformation.

Consider the following PI controller.

$$m(t) = K_p\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau\right).$$

Tustin's approximation was used to convert the continuous algorithm designed above to a discrete-time PI control algorithm, with sample time T.

Show that the transfer function of the equivalent digital controller is,

$$D(z) = K_d\frac{z-\gamma}{z-1}.$$

where the digital controller parameters are related to the continuous controller parameters as follows:

$$K_d = \left(1 + \frac{T}{2T_I}\right)K_p \quad and \quad \gamma = \frac{1 - \dfrac{T}{2T_I}}{1 + \dfrac{T}{2T_I}}.$$

[8 Marks]

(c) Consider in Fig. 1.3 the block diagram for a sample and hold.

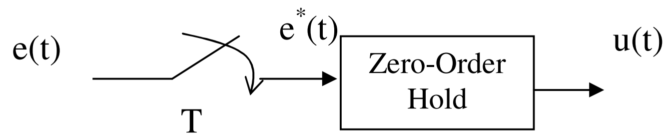

Fig. 1.3 sample and hold

Derive the transfer function of a Zero-Order Hold and sketch its frequency response.

Briefly explain (without proof) the effect of varying the sampling frequency on the spectrum of the reconstructed signal u(t).

Give Shannon's sampling theorem and comment on the benefits of over-sampling, in particular focussing on control applications. Explain why it is necessary to employ anti-aliasing filters, before sampling. Give some indication how sampling rate and filter bandwidth would be selected.

[12 Marks]

2.

(a)

Derive the following design equation for the controller D(z),

$$D(z) = \frac{1}{G(z)} \frac{P(z)}{1 - P(z)}$$

Where *G(z)=C(z)/U(z)* is the discrete-time transfer function model of the open-loop process and P(z) is the desired closed loop transfer function.

Use this design equation to derive the following Dahlin's controller, from a basic prescription of the shape of the desired closed-loop step response.

$$D(z) = \frac{M(z)}{E(z)} = \frac{1}{G(z)} \frac{(1-\alpha)z^{-(N+1)}}{1 - \alpha z^{-1} - (1-\alpha)z^{-(N+1)}}$$

Here, N and α are tuning parameters, used to determine the shape of the closed-loop response.

Consider the following discrete-time closed-loop system.

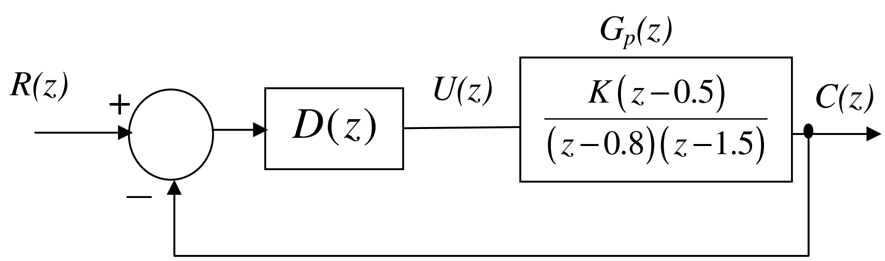

Fig. 2.1 Closed-loop, Discrete-time Process

Show by use of a root-locus plot, why the Dahlin's controller will provide unsatisfactory closed-loop performance.

[13 Marks]

(b) Consider the following closed-loop digital control scheme:

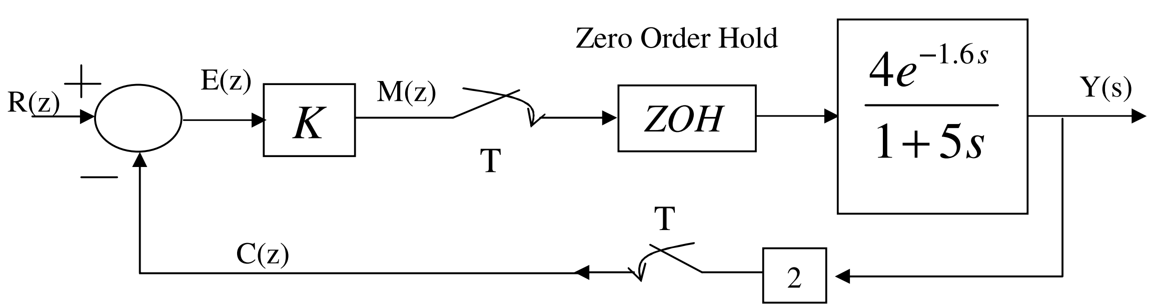

Fig. 2.2 Closed loop digital control system

The sample time T=1 second.

Sketch the root locus diagram for this process, on the Z plane template, and use it to explain how the closed-loop dynamics depend on the choice of the controller gain K.

Use you root locus plot to determine an approximate range of K for stability.

[12 Marks]

3.

In order to emphasise more recent information, "forgetting" can be incorporated within the least squares algorithm. A common choice for the least squares cost function over N valid test points is then:

$$J\left(\underline{\hat{\theta}}(k)\right) = \sum_{i=0}^{N-1} \lambda^{i} e(k-i)^{2}$$

Where, the forgetting factor $\lambda \le 1$, and e(k) is the prediction error.

(i) Derive in full, the following least-squares algorithm with forgetting, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains N valid samples of the process output, up to the current kth sample, *y(k).*

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^{T} \Lambda_{N} \Phi(k)\right)^{-1} \Phi(k)^{T} \Lambda_{N} \underline{Y}(k)$$

Where the weighting matrix for N valid points is the diagonal matrix, defined as:

$$\Lambda_N = \begin{bmatrix} \lambda^{N-1} & 0 & \cdots & 0 & 0 \\ 0 & \ddots & & & 0 \\ 0 & & \lambda^{2} & & \vdots \\ \vdots & & & \lambda & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

[13 Marks]

(ii) If a square matrix *P(k)* is now defined as $P(k) = \left(\Phi(k)^{T} \Lambda_{N} \Phi(k)\right)^{-1}$, derive the following update equation for $P^{-1}(k+1)$ from process data up to the (k+1)th sample, where the vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the (k+1)th sample,

$$P^{-1}(k+1) = \lambda P^{-1}(k) + \underline{\psi}(k+1)\underline{\psi}^{T}(k+1)\,.$$

Use Householders Matrix Inversion Lemma,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1}\text{,}$$

to derive the following update equation:

$$P(k+1) = \frac{1}{\lambda}\left[P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^{T}(k+1)P(k)}{\lambda + \underline{\psi}^{T}(k+1)P(k)\underline{\psi}(k+1)} \right].$$

[12 Marks]

4.

(a) Consider the following state-space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$

i) Develop fully the following solution for the state trajectory $\underline{x}(t)$, for $t \geq 0$, where $\underline{x}(0)$ is the initial state vector at t=0, and $\Phi(t)$ is the transition matrix.

$$\underline{x}(t) = \Phi(t)\underline{x}(0) + \int_0^t \Phi(t-\tau)B\underline{u}(\tau)d\tau$$

ii) If the sample-time is T, and it is assumed that a zero-order hold is applied to the input signal $\underline{u}(t)$, show that this process can be represented by the following discrete-time, state-space equations:

$$\underline{x}(k+1) = e^{AT}\underline{x}(k) + A^{-1}\left(e^{AT} - I\right)B\underline{u}(k)$$

[10 Marks]

(b) A permanent magnet DC motor can be modelled by the following block diagram, where *V(s)* is the applied voltage, *D(s)* the load torque, *I(s)* the armature current and $\Omega(s)$ the rotational speed.

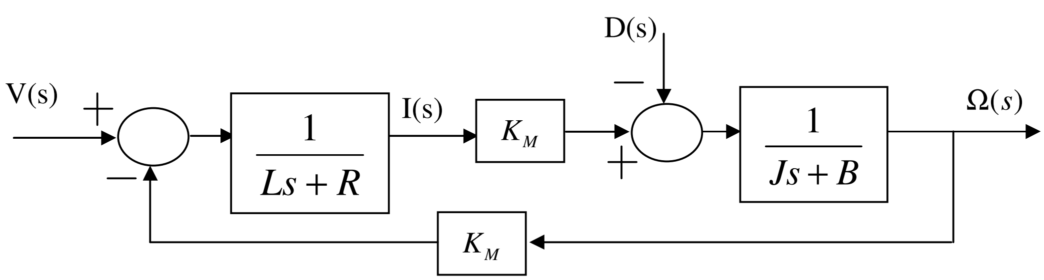

Fig. 4.1 Block Diagram of a DC motor

(i) Determine a state-space model of this process where the states are the current $i(t)$ and the speed $\omega(t)$. Represent this model as a simulation diagram based on only two integrators.

[5 Marks]

(ii) A constant voltage $v(t)=V_0$ is applied to this motor for all time. The load torque $d(t)$ obeys:

$$d(t) = \begin{cases} d_0\ (Nm) & for \quad t \leq 0^+ \\ d_0 + D\ (Nm) & for \quad t > 0^+ \end{cases}$$

What are the initial conditions for the current, $i(0)$ and the speed, $\omega(0)$ if the motor is in equilibrium at time $t=0$?

Determine a first order polynomial approximation to the transition matrix using the matrix exponential method.

Show that the state trajectory for a small time, $t>0$, can be approximated by:

$$\begin{bmatrix} \omega(t) \\ i(t) \end{bmatrix} \approx \begin{bmatrix} \omega(0) \\ i(0) \end{bmatrix} + D \begin{bmatrix} \dfrac{1}{2J^2}\left(Bt^2 - 2Jt\right) \\ \dfrac{K_M t^2}{2JL} \end{bmatrix}$$

[10 Marks]

5.

(a) Consider the following N[th] order open-loop process, with one input *u(t)* and a single output *y(t)*,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

If this process is under the following state space control–law with integral action,

$$u(t) = -K\underline{x}(t) + K_I \int_0^t \big(r(\tau) - y(\tau)\big)\,d\tau$$

show that the closed-loop characteristic equation is:

$$\det\left[\begin{array}{c|c} sI_N - A + BK & -BK_I \\ \hline C & s \end{array}\right] = 0$$

[8 Marks]

(b) Consider the N[th] order MIMO process described by the following state space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$
$$\underline{y}(t) = C\underline{x}(t)$$

(i) Write down the condition which must be satisfied for this process to be controllable.

(ii) The state $\underline{x}(t)$ is now transformed to a new state vector $\underline{z}(t)$, using the linear transformation $\underline{z}(t)=T\underline{x}(t)$, where T is a square, invertible matrix.

Determine the relationship between the controllability matrix of the original system and the controllability matrix of this new transformed set of equations.

What can you conclude about the effect of this transformation on the controllability of the system?

[7 Marks]

(c) A certain open-loop mechatronic system could be represented by the block diagram,

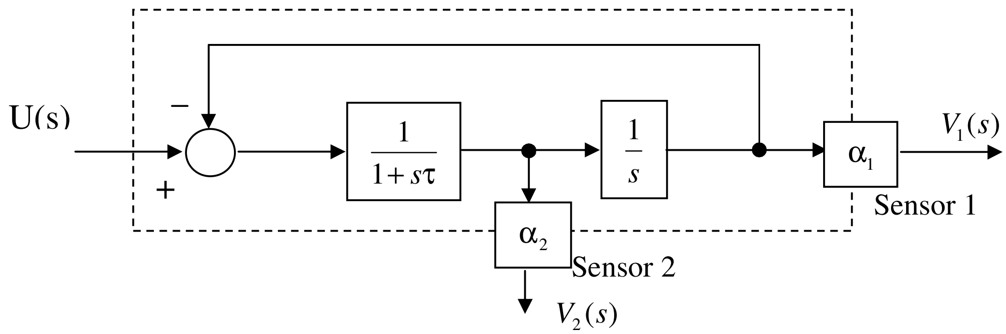

Fig. 5.1 Open-loop Mechatronic System

Design a state-space regulator to place both closed loop poles at $s = -\frac{2}{\tau}$.

[10 Marks]

6.

(a) Consider the following N^{th} order open-loop process, with one input u(t), a single output y(t) and a single *unmeasured* disturbance d(t),

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + Bu(t) + Ed(t)$$
$$y(t) = C\underline{x}(t)$$

Develop the following representation of the full-state, continuous-time, Luenberger observer for this system,

$$\frac{d}{dt}\begin{bmatrix}\underline{\hat{x}}\\ \underline{e}\end{bmatrix} = \begin{bmatrix}A & GC\\ 0 & A-GC\end{bmatrix}\begin{bmatrix}\underline{\hat{x}}\\ \underline{e}\end{bmatrix} + \begin{bmatrix}B & 0\\ 0 & E\end{bmatrix}\begin{bmatrix}u(t)\\ d(t)\end{bmatrix}$$

where G is the estimator gain matrix and the estimation error $\underline{e}(t)$ is defined as,

$$\underline{e}(t) = \underline{x}(t) - \underline{\hat{x}}(t)$$

[10 Marks]

(b) Consider the following open-loop second order process, with two states $\phi(t)$ and $\theta(t)$, a single input u(t) and a single *unmeasured* disturbance d(t).

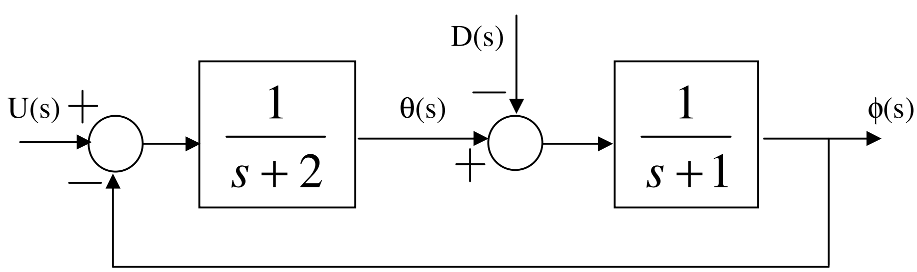

Fig. 6.1 Block Diagram of Second-order Open-loop Process

The angle ϕ is sensed using a potentiometer, which could be modelled as, $V_\phi(t)=0.5\phi(t)$, where $V_\phi(t)$ is the potentiometer output voltage.

(i) Design a full order, continuous-time, Luenberger observer to construct estimates of $\phi(t)$ and $\theta(t)$, from u(t) and the measurement $V_\phi(t)$.

Place the poles of the closed loop estimator both at s =-5.

[10 Marks]

(ii) We have no measurement of the disturbance signal d(t), but we have been informed that this unmeasured disturbance d(t) obeys,

$$\lim_{t \to \infty} d(t) = d_0$$

Determine an expression for the steady state estimation errors in the estimates of $\phi(t)$ and $\theta(t)$ provided by the Luenberger observer designed above.

[5 Marks]