OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

AUTUMN EXAMINATIONS, 2008

B.E. DEGREE (ELECTRICAL)
B.E. DEGREE (MICROELECTRONIC)

TELECOMMUNICATIONS
EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of log tables and departmental approved non-programmable calculators is permitted.

1. (a) Illustrate the OSI model of a communications system with the layers named and stacked in their usual order and describe the function of each of these layers.

[10 *marks*]

(b) (i) Illustrate the timing of the packet and acknowledgement transfers for a data-link which uses a "go-back-N" ARQ scheme and from this determine an expression for the utilization, U, of the data-link.

[6 *marks*]

(ii) For a 150 km data-link with a data rate of 400 Mbps, determine the minimum frame window, N, which is needed to guarantee a utilization of 100% assuming an error-free line. The packet size is 5000 bits, the acknowledgement size is 300 bits and the propagation delay is 5 μs/km.

[*4 marks*]

2. (a) Describe the concept and operation of statistical multiplexing in data communications links. Include a definition of multiplexing gain in your discussion and indicate the types of data sources which are likely to give rise to both low and high multiplexing gain.

[10 *marks*]

(b) For Local Area Networks based on the Ethernet protocol describe the following:

(i) The CSMA/CD algorithm.

[5 *marks*]

(ii) The truncated binary exponential back-off algorithm.

[5 *marks*]

3. (a) Illustrate the architecture of a UMTS Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) For a cellular telephone system (2G and 3G) briefly discuss the following:

(i) Cell organization and frequency re-use.

[4 *marks*]

(ii) The main power control algorithms.

[4 *marks*]

(iii) The hand-off algorithms when a user moves between adjacent cells.

[4 *marks*]

Q.4. Given that the 2×2 channel matrix $\left[P\left(Y_1|X\right)\right]$ for the generalised binary channel with 2 input symbols, denoted x_i, $1 \le i \le 2$ and 2 output symbols, denoted y_j, $1 \le j \le 2$, is given by: -

$$\left[P\left(Y_1|X\right)\right] = \begin{bmatrix} 1-e_1 & e_1 \\ e_2 & 1-e_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & -\frac{e_1}{e_2} \\ 1 & 1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1-e_1-e_2 \end{bmatrix}}_{D} \underbrace{\left(\frac{1}{e_1+e_2}\begin{bmatrix} e_2 & e_1 \\ -e_2 & e_2 \end{bmatrix}\right)}_{F^{-1}}$$

where $e_1 > 0,\ e_2 > 0,\ D$ is a diagonal matrix and the columns of F are eigenvectors of $\left[P\left(Y_1|X\right)\right]$, show that if n such generalised binary channels are connected in series (i.e. the outputs of channel i become the inputs of channel $i+1,\ 1 \le i \le n-1$), then: -

(a) The composite channel matrix $\left[P\left(Y_n|X\right)\right]$ is given by: -

$$\left[P\left(Y_n|X\right)\right]=\frac{1}{e_1+e_2}\begin{bmatrix} e_2+e_1\lambda & e_1(1-\lambda) \\ e_2(1-\lambda) & e_1+e_2\lambda \end{bmatrix}$$

where $\lambda=\left(1-e_1-e_2\right)^n$.

[6 *marks*]

(b) Show that if the output symbols, denoted y_1^c and y_2^c, from the <u>composite</u> channel in part (a) above are to be equiprobable then we require: -

$$P\left(x_1\right)=\frac{e_1-e_2+2e_2\lambda}{2\lambda\left(e_1+e_2\right)}$$

where $P\left(x_1\right)$ denotes the probability of the input symbol being x_1 (noting that, if x_1 is sent and no error occurs, the output symbol will be y_1^c, etc.).

[5 *marks*]

(c) If both the input symbols and the output symbols are equiprobable, show that: -

(i) This condition requires $e_1=e_2$.

[3 *marks*]

(ii) The composite channel capacity, denoted C_s^c, is given by

$$C_s^c=1+\left(\frac{1-\left(1-2e_1\right)^n}{2}\right)\log_2\left(\frac{1-\left(1-2e_1\right)^n}{2}\right)+\left(\frac{1+\left(1-2e_1\right)^n}{2}\right)\log_2\left(\frac{1+\left(1-2e_1\right)^n}{2}\right).$$

[6 *marks*]

Q.5 A baseband digital communications system uses rectangular wave signaling with A_1 volts representing logic 1 and A_2 volts representing logic 0 (where $A_2<A_1$). The receiver takes a single sample of the received signal during the bit signaling time and compares this sample with a decision threshold T. If the communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n\left(v\right)=\frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

and P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1, show that: -

(a) To minimize the resulting overall probability of error P_e, the threshold T is given by: -

$$T=\frac{A_1+A_2}{2}+\frac{\sigma^2}{A_1-A_2}\ln\left[\frac{P_0}{P_1}\right].$$

[10 *marks*]

(b) If $P_0 > P_1$ in (a) above, show that P_e is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{T - A_2}{\sqrt{2\sigma^2}}\right] + (1 - P_0) erf\left[\frac{A_1 - T}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

[10 *marks*]

Q.6. (a) Given that the output signal to noise ratio (SNR) of a matched filter receiver subject to additive white Gaussian noise (AWGN) with power spectral density $\eta/2 \ W/Hz$ is given by $2E_d/\eta$, where E_d denotes the energy in the difference signal, show using the Schwarz inequality (which states: -

$$\left|\int_{-\infty}^{\infty} f_1(\omega) f_2(\omega) d\omega\right|^2 \leq \int_{-\infty}^{\infty} |f_1(\omega)|^2 d\omega \int_{-\infty}^{\infty} |f_2(\omega)|^2 d\omega),$$

or otherwise, that the optimum output SNR is given by: -

$$\left(\frac{S}{N}\right)_{Optimum} = \frac{8E}{\eta}$$

when we stipulate that the signaling waveforms $s_1(t)$ and $s_2(t)$ must have the same signal energy E.

[10 *marks*]

(b) Typical expressions for ASK and PSK modulated waveforms representing binary data, where in each case T is an integer times $1/f_c$, are as follows: -

ASK

$$s_i(t) = \begin{cases} s_1(t) = A_1\cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = 0 & 0 \le t \le T \end{cases}$$

PSK

$$s_i(t) = \begin{cases} s_1(t) = A_2\cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = -A_2\cos[\omega_c t] & 0 \le t \le T \end{cases}$$

In addition, the probability of error for a binary modulation scheme (denoted MOD) with optimum detection in the presence of AWGN with a power spectral density of $\eta/2\ W/Hz$ is given by: -

$$P_e^{MOD} = Q\left[\sqrt{\frac{E_d}{2\eta}}\right]$$

where E_d denotes the energy difference in the appropriate signal (over a single bit interval).

(i) Derive expressions for P_e^{ASK} and P_e^{PSK}.

[5 *marks*]

(ii) If the average signal energy per bit for the ASK and PSK modulation schemes above is made equal, derive the following expression for the enhancement in reliability, denoted E, achieved by choosing PSK over ASK when both schemes deliver the same bit rate: -

$$E = \frac{P_e^{ASK}}{P_e^{PSK}} = \frac{Q\left[\sqrt{\frac{A_2^2 T}{2\eta}}\right]}{Q\left[\sqrt{\frac{A_2^2 T}{\eta}}\right]}.$$

[5 *marks*]

Q.7. (a) Using the primitive polynomial $p(x) = x^5 + x^2 + 1$, generate the field $GF(2^5)$.

[9 *marks*]

(b) Show that for the $(31,21)$ double error correcting primitive BCH code based upon $GF(2^5)$: -

(i) The generator polynomial $g(x)$ is given by: -

$$g(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^3 + 1.$$

[8 *marks*]

(ii) The generator polynomial, $g(x)$, in (i) above satisfies: -

$$g\left(\alpha^2\right) = g\left(\alpha^6\right) = 0 .$$

[3 *marks*]