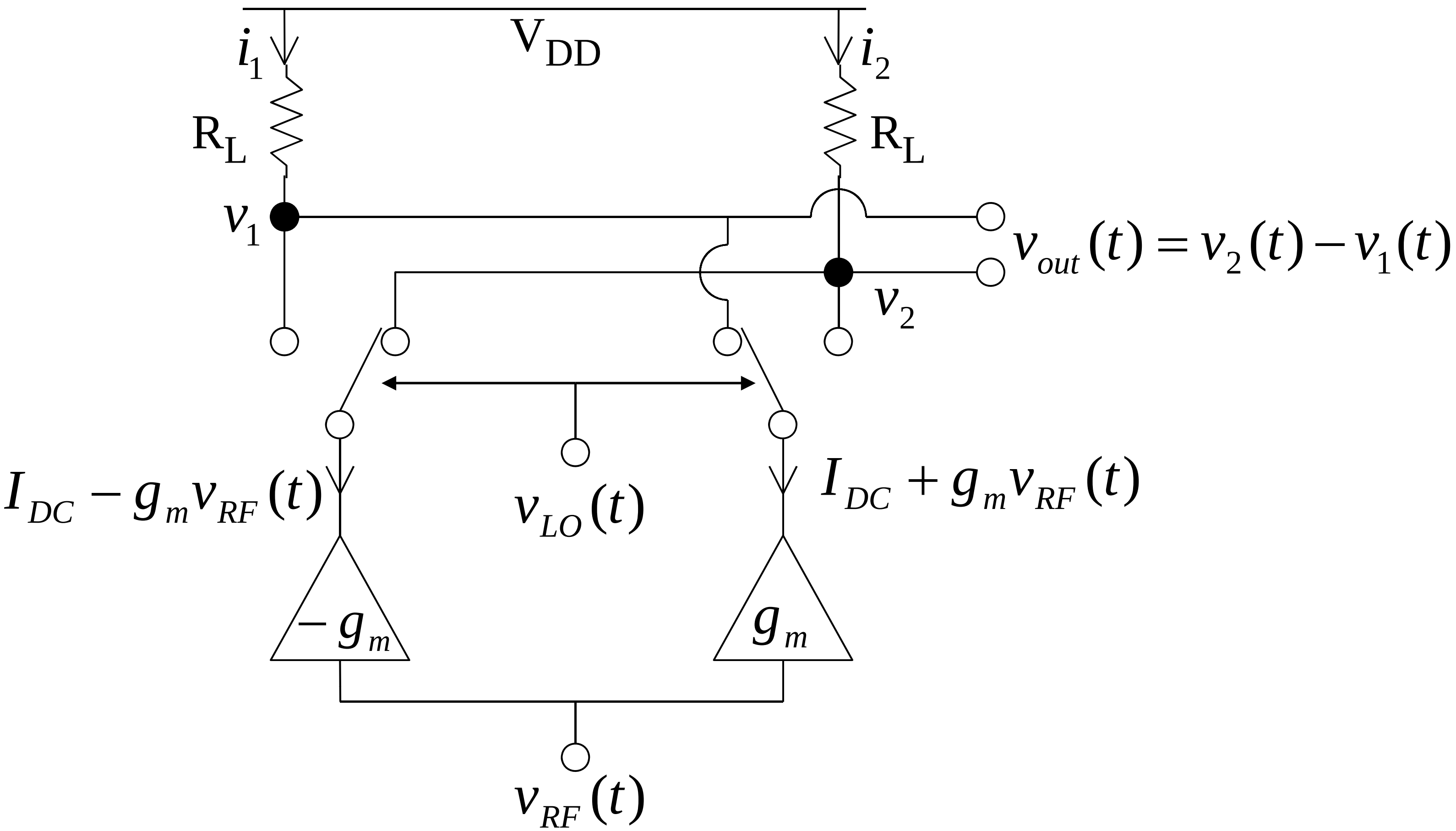
EE4011: RF IC Design

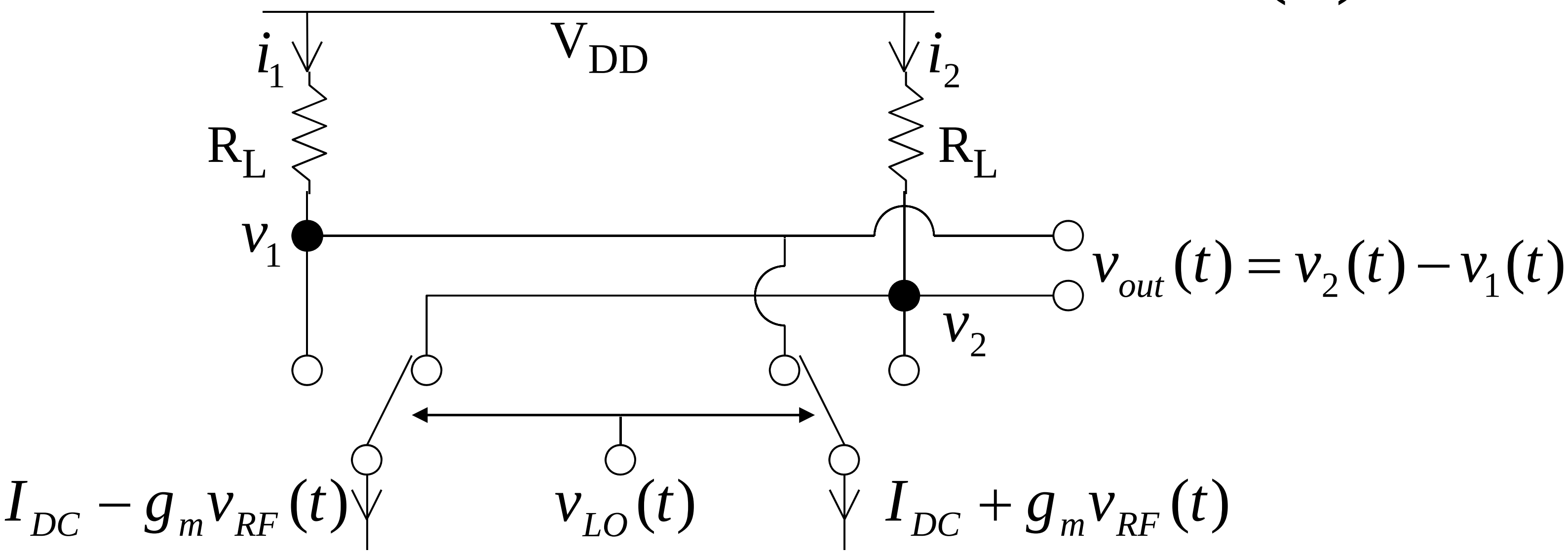
Double-Balanced Mixers and The Gilbert Cell

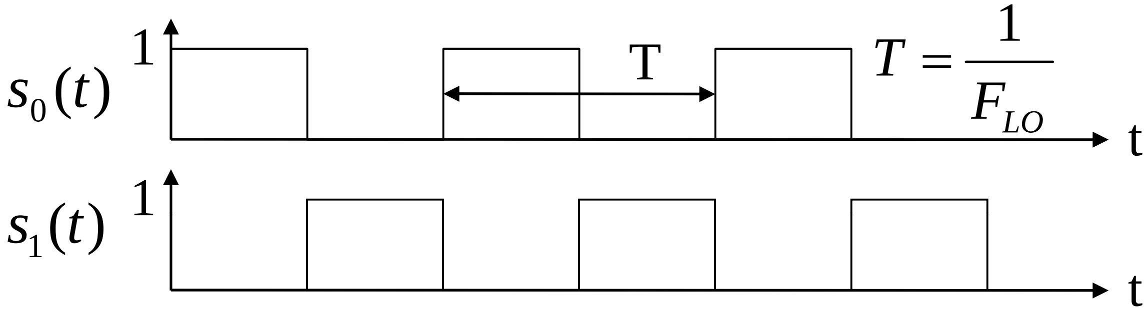
The Double-Balanced Mixer (1)



The LO controls both switches and the interconnects are arranged so that for one half cycle of the LO, the output of the $-g_m$ transconductor is connected to the right hand load resistor and the output of the g_m transconductor is connected to the left hand load resistor and for the next half cycle the transconductor outputs are swapped between the two resistors.

The Double-Balanced Mixer (2)



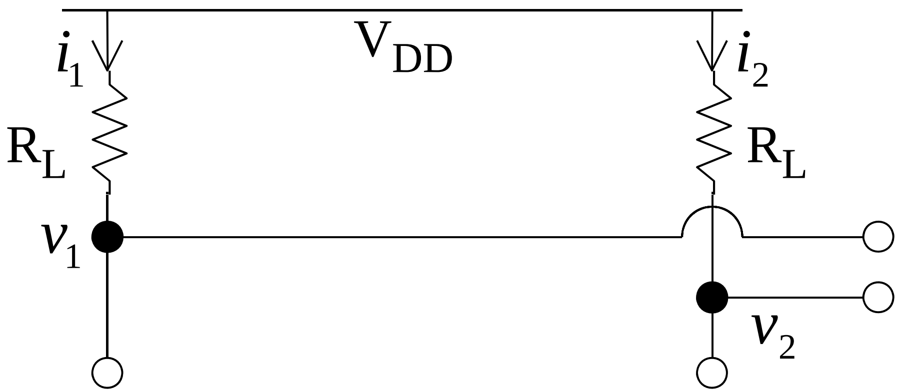


$$i_1(t) = \left(I_{DC} + g_m v_{RF}(t)\right) s_0(t) + \left(I_{DC} - g_m v_{RF}(t)\right) s_1(t)$$

$$i_2(t) = \left(I_{DC} - g_m v_{RF}(t)\right) s_0(t) + \left(I_{DC} + g_m v_{RF}(t)\right) s_1(t)$$

$$i_1(t) - i_2(t) = 2 g_m v_{RF}(t)\left(s_0(t) - s_1(t)\right)$$ All the I_{DC} terms cancel!

The Double-Balanced Mixer (3)



$$v_{out}(t) = v_2(t) - v_1(t)$$

As before and putting in the result of the last slide and the expression for $s_0(t)-s_1(t)$:

$$\begin{aligned}
v_{out}(t) &= v_2(t) - v_1(t) = \left(V_{DD} - i_2(t)\right)R_L - \left(V_{DD} - i_1(t)\right)R_L \\
&= R_L\left(i_1(t) - i_2(t)\right) \\
&= 2g_m R_L v_{RF}(t)\left(s_0(t) - s_1(t)\right) \\
&= 2g_m R_L v_{RF}(t)\frac{4}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right] \\
&= \frac{8g_m R_L}{\pi} v_{RF}(t)\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]
\end{aligned}$$

No I_{DC} term in output!

The Double-Balanced Mixer (4)

Assuming again that the input RF signal is a co-sinusoidal form $V_{RF}\cos(\omega_{RF}t)$ and using the cos(A)sin(B) formula:

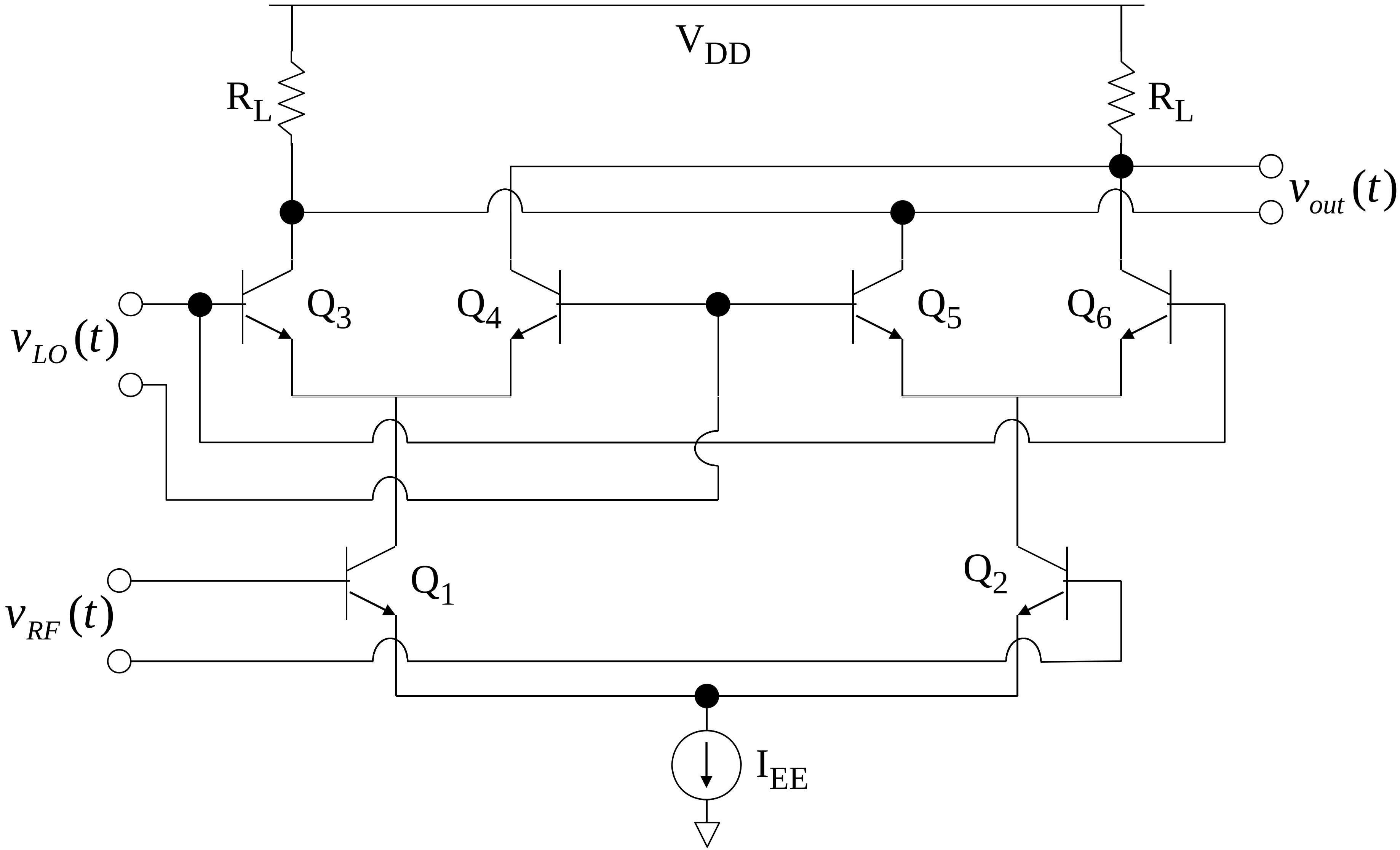
$$v_{out}(t) = \frac{8g_m R_L v_{RF}(t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

$$= \frac{8g_m R_L V_{RF}\cos(\varpi_{RF}t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

$$= \frac{4g_m R_L V_{RF}}{\pi}\left[\begin{array}{l}\sin\big((\varpi_{RF}+\varpi_{LO})t\big) - \sin\big((\varpi_{RF}-\varpi_{LO})t\big) \\ +\frac{1}{3}\sin\big((\varpi_{RF}+3\varpi_{LO})t\big) - \frac{1}{3}\sin\big((\varpi_{RF}-3\varpi_{LO})t\big) + \cdots\end{array}\right]$$

So, the double balanced mixer has eliminated both RF feed-through and LO feed-through and the only outputs are the sum and difference frequencies as desired. Also for the same g_m and R_L the voltage conversion gain is twice that of the single-balanced mixer. Again, the unwanted higher frequency terms can be filtered out.

The Gilbert Cell Mixer



The Gilbert Cell: Operating Modes

1. $v_{LO}(t)$ and $v_{RF}(t)$ both have small amplitudes

   In this situation, all the transistors Q_1 to Q_6 are biased in the forward active region and act as transconductances. The output is approximately the product of $v_{LO}(t)$ and $v_{RF}(t)$ and the circuit is a true analogue multiplier.

2. $v_{LO}(t)$ has a large amplitude and $v_{RF}(t)$ has a small amplitude

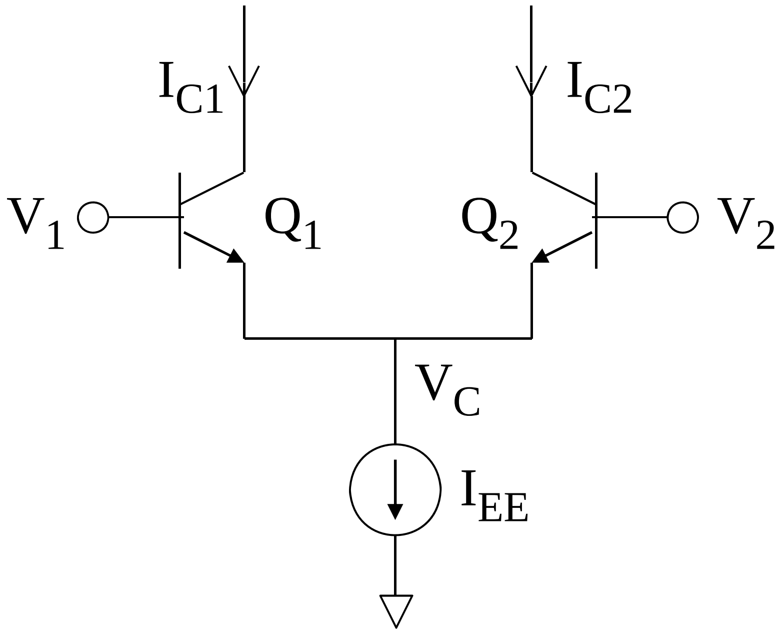
   In this situation, the transistors Q_1 and Q_2 act as transconductances and the transistor pairs Q_3/Q_4 and Q_5/Q_6 act as single-pole double throw switches. This is the double balanced mixer configuration.

3. $v_{LO}(t)$ and $v_{RF}(t)$ both have large amplitudes

   In this situation, all the transistors Q_1 to Q_6 act as switches and the output depends on the phase difference between $v_{LO}(t)$ and $v_{RF}(t)$ i.e. the circuit acts as a phase detector, suitable for use in Type 1 PLLs.

Emitter Coupled Pair (1)

Assuming both transistors are the same i.e. they have the same I_S:



$$I_{C1} = I_S e^{\frac{V_{BE1}}{V_T}} = I_S e^{\frac{V_1 - V_C}{V_T}} \qquad V_1 - V_C >> V_T$$

$$I_{C2} = I_S e^{\frac{V_{BE2}}{V_T}} = I_S e^{\frac{V_2 - V_C}{V_T}} \qquad V_2 - V_C >> V_T$$

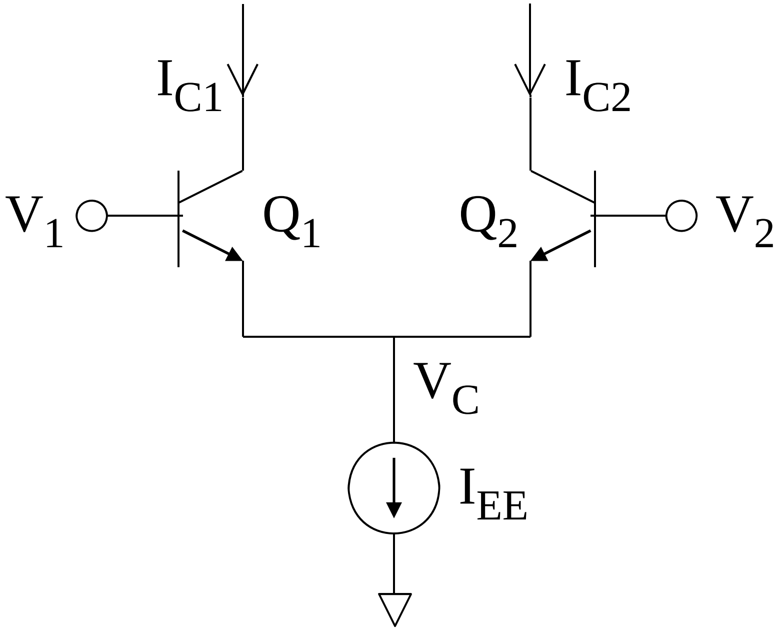
Then:

$$\frac{I_{C1}}{I_{C2}} = \frac{I_S e^{\frac{V_1 - V_C}{V_T}}}{I_S e^{\frac{V_2 - V_C}{V_T}}} = e^{\frac{V_1 - V_2}{V_T}} = e^{\frac{V_d}{V_T}}$$

$$V_T = \frac{kT}{q}$$

$$\Rightarrow \frac{I_{C2}}{I_{C1}} = e^{-\frac{V_d}{V_T}} \qquad \text{where } V_d = V_1 - V_2$$

Emitter Coupled Pair (2)



If the two input voltages are the same (i.e. V_d=0) then I_{EE}/2 flows in both devices. If one input voltage is higher, a larger proportion of I_{EE} flows in the device connected to that input.

Ignoring base current:

$$I_{C1} + I_{C2} = I_{EE}$$

$$\Rightarrow I_{C1} + I_{C1} e^{-\frac{V_d}{V_T}} = I_{EE}$$

$$\Rightarrow I_{C1} = \frac{I_{EE}}{1 + e^{-\frac{V_d}{V_T}}}$$

and

$$\Rightarrow I_{C2} e^{\frac{V_d}{V_T}} + I_{C2} = I_{EE}$$

$$\Rightarrow I_{C2} = \frac{I_{EE}}{1 + e^{\frac{V_d}{V_T}}}$$

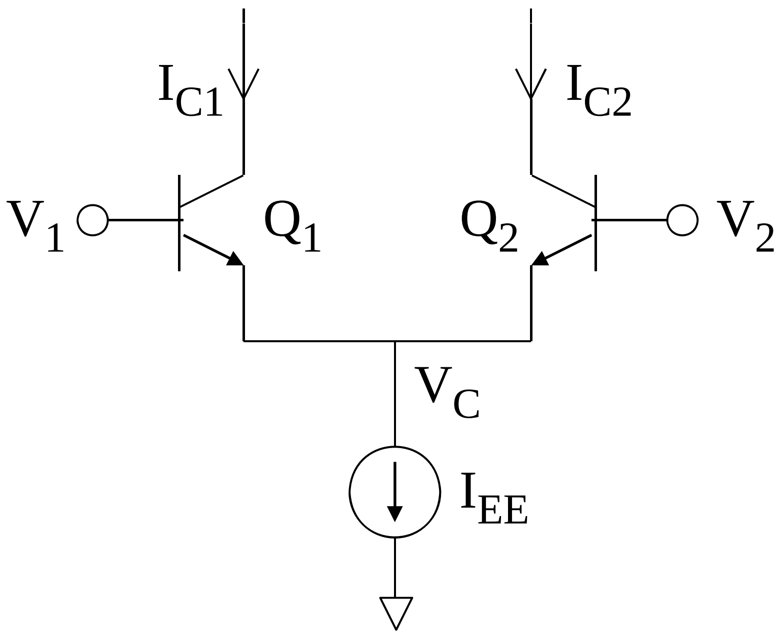
Emitter Coupled Pair (3)

Looking at the difference between the two currents:

$$\Delta I_C = I_{C1} - I_{C2} = \frac{I_{EE}}{1+e^{-\frac{V_d}{V_T}}} - \frac{I_{EE}}{1+e^{\frac{V_d}{V_T}}} = I_{EE}\left(\frac{1}{1+e^{-\frac{V_d}{V_T}}} - \frac{1}{1+e^{\frac{V_d}{V_T}}}\right)$$

$$= I_{EE}\left(\frac{e^{\frac{V_d}{V_T}}}{e^{\frac{V_d}{V_T}}+1} - \frac{1}{1+e^{\frac{V_d}{V_T}}}\right) = I_{EE}\,\frac{e^{\frac{V_d}{V_T}}-1}{e^{\frac{V_d}{V_T}}+1}$$

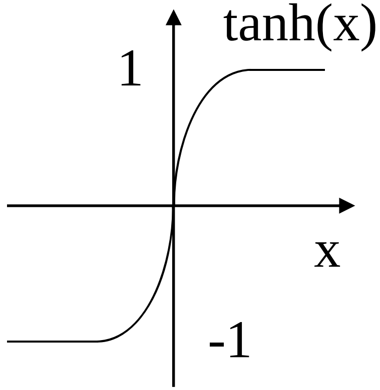
$$= I_{EE}\,\frac{e^{\frac{V_d}{2V_T}} - e^{-\frac{V_d}{2V_T}}}{e^{\frac{V_d}{2V_T}} + e^{-\frac{V_d}{2V_T}}} = I_{EE}\tanh\left(\frac{V_d}{2V_T}\right)$$



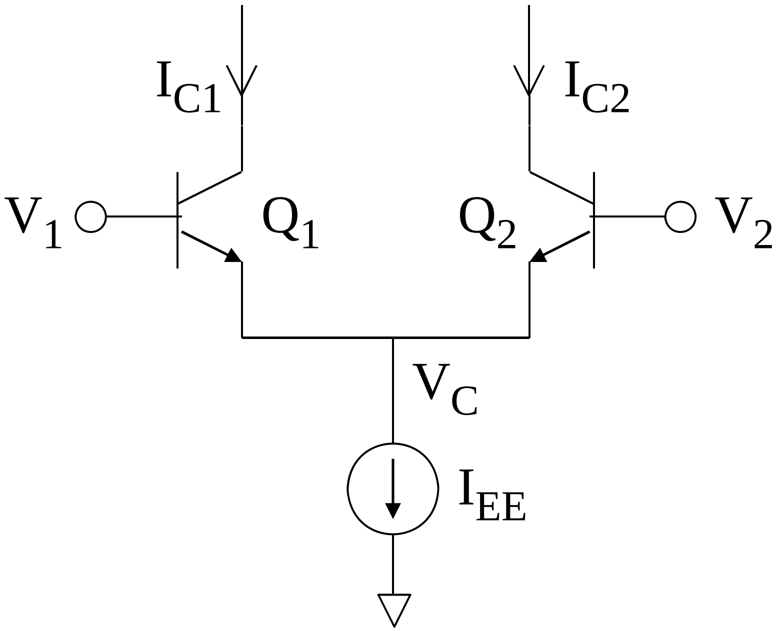
The current difference follows a tanh function.

Emitter Coupled Pair (4)

$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d}{2V_T}\right)$$



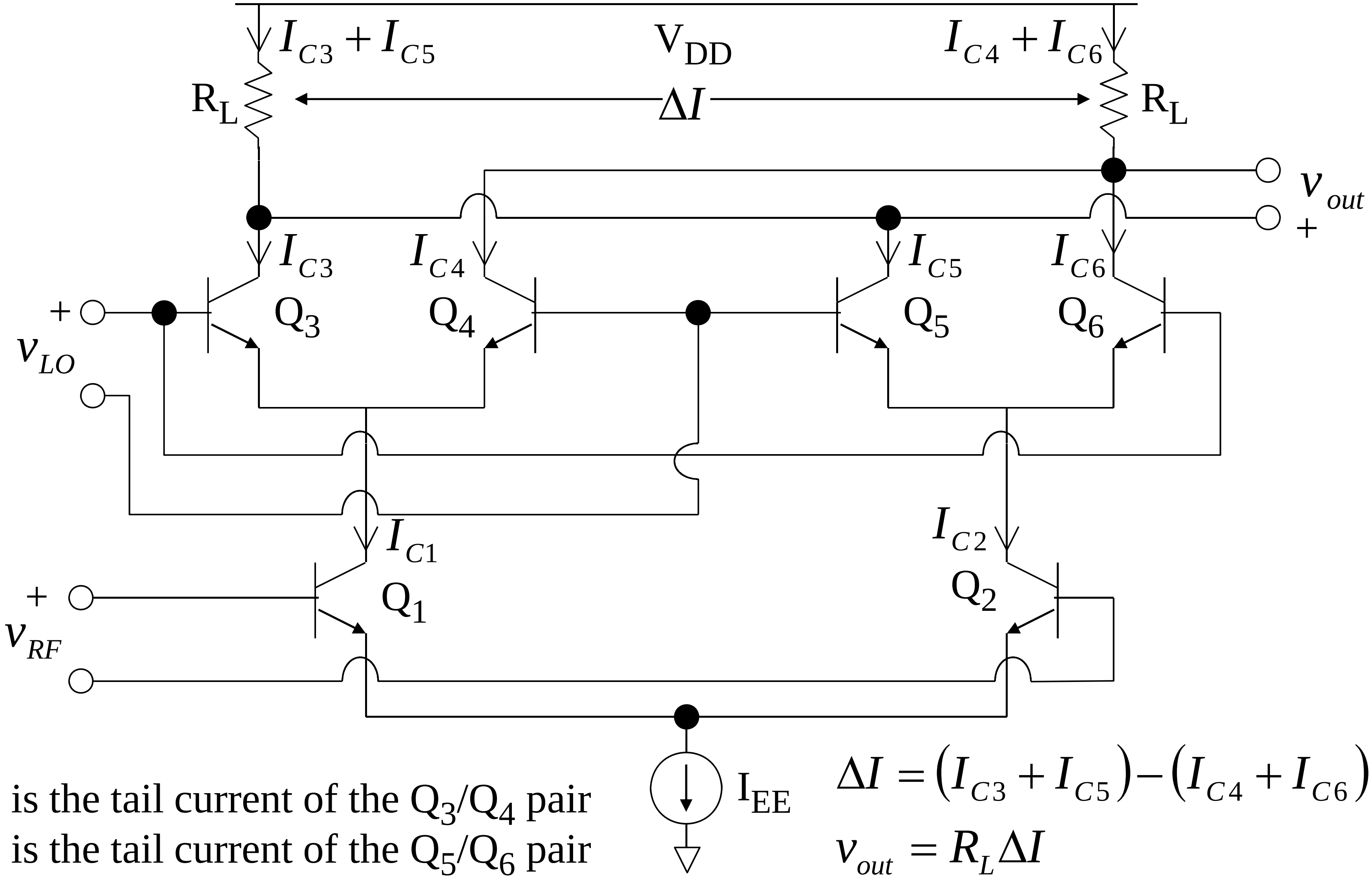
$$\tanh(x) \approx x\,,\ |x| << 1$$
$$\tanh(x) \approx 1\,,\ x >> 1$$
$$\tanh(x) \approx -1\,,\ x << 1$$



For small values of $V_d/2V_T$ the difference in current between the two collectors is linearly proportional to V_d i.e. the circuit acts as a linear transconductance stage.

If $V_d >> 2V_T$ then all of I_{EE} flows in Q_1
If $V_d << 2V_T$ then all of I_{EE} flows in Q_2
In the last two cases the circuit operates as a switch, switching I_{EE} between the two collectors.

Analysis of Gilbert Cell (1)

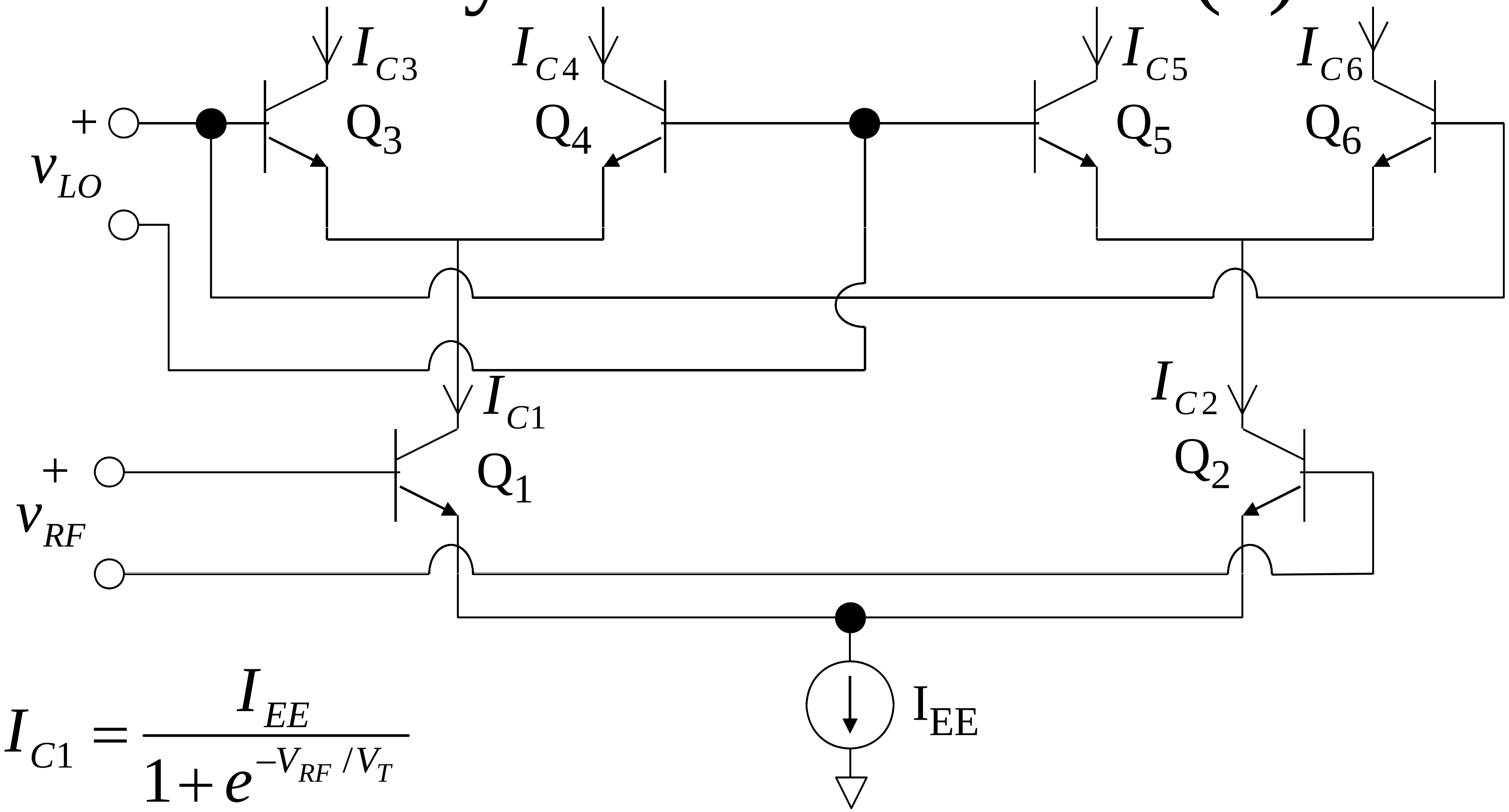


$$\Delta I = \left(I_{C3}+I_{C5}\right)-\left(I_{C4}+I_{C6}\right)$$

$$v_{out} = R_L \Delta I$$

I_{C1} is the tail current of the Q_3/Q_4 pair
I_{C2} is the tail current of the Q_5/Q_6 pair

Analysis of Gilbert Cell (2)

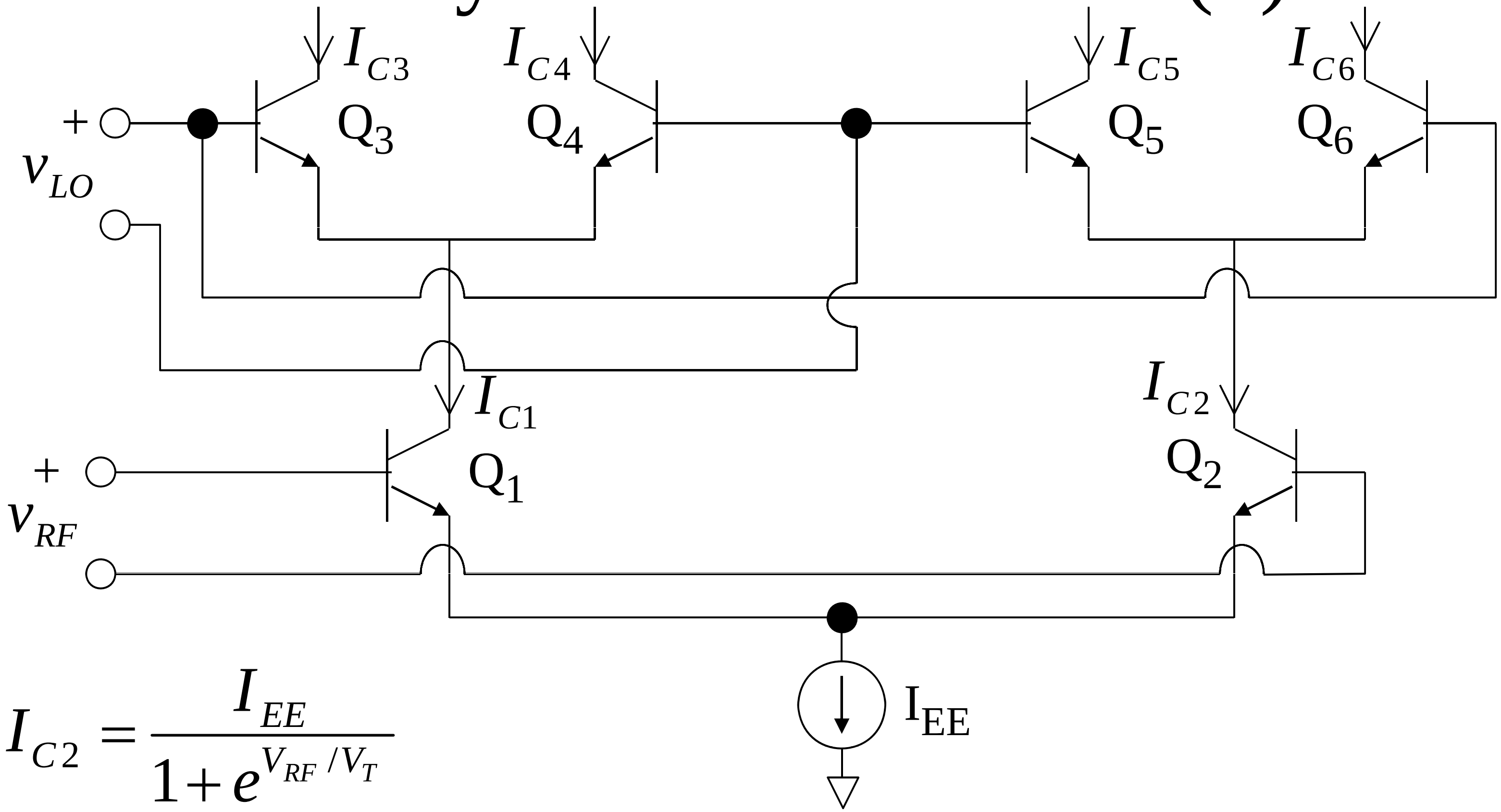


$$I_{C1} = \frac{I_{EE}}{1+e^{-V_{RF}/V_T}}$$

$$I_{C3} = \frac{I_{C1}}{1+e^{-V_{LO}/V_T}} = I_{EE}\,\frac{1}{1+e^{-V_{RF}/V_T}}\,\frac{1}{1+e^{-V_{LO}/V_T}}$$

$$I_{C4} = \frac{I_{C1}}{1+e^{V_{LO}/V_T}} = I_{EE}\,\frac{1}{1+e^{-V_{RF}/V_T}}\,\frac{1}{1+e^{V_{LO}/V_T}}$$

Analysis of Gilbert Cell (3)



$$I_{C2} = \frac{I_{EE}}{1+e^{V_{RF}/V_T}}$$

$$I_{C5} = \frac{I_{C2}}{1+e^{V_{LO}/V_T}} = I_{EE}\frac{1}{1+e^{V_{RF}/V_T}}\frac{1}{1+e^{V_{LO}/V_T}}$$

$$I_{C6} = \frac{I_{C2}}{1+e^{-V_{LO}/V_T}} = I_{EE}\frac{1}{1+e^{V_{RF}/V_T}}\frac{1}{1+e^{-V_{LO}/V_T}}$$

Analysis of Gilbert Cell (4)

$$\Delta I = I_{C3} + I_{C5} - I_{C4} - I_{C6}$$

$$= I_{EE}\left[\begin{array}{l} \dfrac{1}{1+e^{-V_{RF}/V_T}}\dfrac{1}{1+e^{-V_{LO}/V_T}} + \dfrac{1}{1+e^{V_{RF}/V_T}}\dfrac{1}{1+e^{V_{LO}/V_T}} \\ -\dfrac{1}{1+e^{-V_{RF}/V_T}}\dfrac{1}{1+e^{V_{LO}/V_T}} - \dfrac{1}{1+e^{V_{RF}/V_T}}\dfrac{1}{1+e^{-V_{LO}/V_T}} \end{array}\right]$$

$$= I_{EE}\left[\begin{array}{l} \dfrac{1}{1+e^{-V_{RF}/V_T}}\left(\dfrac{1}{1+e^{-V_{LO}/V_T}} - \dfrac{1}{1+e^{V_{LO}/V_T}}\right) \\ -\dfrac{1}{1+e^{V_{RF}/V_T}}\left(\dfrac{1}{1+e^{-V_{LO}/V_T}} - \dfrac{1}{1+e^{V_{LO}/V_T}}\right) \end{array}\right]$$

$$= I_{EE}\left(\frac{1}{1+e^{-V_{RF}/V_T}} - \frac{1}{1+e^{V_{RF}/V_T}}\right)\left(\frac{1}{1+e^{-V_{LO}/V_T}} - \frac{1}{1+e^{V_{LO}/V_T}}\right)$$

Analysis of Gilbert Cell (5)

$$\Delta I = I_{EE}\left(\frac{1}{1+e^{-V_{RF}/V_T}}-\frac{1}{1+e^{V_{RF}/V_T}}\right)\left(\frac{1}{1+e^{-V_{LO}/V_T}}-\frac{1}{1+e^{V_{LO}/V_T}}\right)$$

$$= I_{EE}\left(\frac{e^{V_{RF}/V_T}}{e^{V_{RF}/V_T}+1}-\frac{1}{1+e^{V_{RF}/V_T}}\right)\left(\frac{e^{V_{LO}/V_T}}{e^{V_{LO}/V_T}+1}-\frac{1}{1+e^{V_{LO}/V_T}}\right)$$

$$= I_{EE}\left(\frac{e^{V_{RF}/V_T}-1}{e^{V_{RF}/V_T}+1}\right)\left(\frac{e^{V_{LO}/V_T}-1}{e^{V_{LO}/V_T}+1}\right)$$

$$= I_{EE}\tanh\left(\frac{V_{RF}}{2V_T}\right)\tanh\left(\frac{V_{LO}}{2V_T}\right)$$

$$v_{out} = R_L \Delta I$$

$$= R_L I_{EE}\tanh\left(\frac{V_{RF}}{2V_T}\right)\tanh\left(\frac{V_{LO}}{2V_T}\right)$$

$$\boxed{\tanh\left(\frac{x}{2}\right)=\frac{e^{\frac{x}{2}}-e^{-\frac{x}{2}}}{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}=\frac{e^{x}-1}{e^{x}+1}}$$

Another Look at Gilbert Cell Operating Modes (1)

$$v_{out} = R_L I_{EE} \tanh\left(\frac{V_{RF}}{2V_T}\right)\tanh\left(\frac{V_{LO}}{2V_T}\right)$$

1. $v_{LO}(t)$ and $v_{RF}(t)$ both have small amplitudes: analogue multiplier

$$\tanh\left(\frac{V_{RF}}{2V_T}\right) \approx \frac{V_{RF}}{2V_T}, \tanh\left(\frac{V_{LO}}{2V_T}\right) \approx \frac{V_{LO}}{2V_T} \Rightarrow v_{out} = \frac{R_L I_{EE}}{4V_T^2} V_{RF} V_{LO}$$

2. $v_{LO}(t)$ has a large amplitude and $v_{RF}(t)$ has a small amplitude: mixer

$$\tanh\left(\frac{V_{RF}}{2V_T}\right) \approx \frac{V_{RF}}{2V_T}, \tanh\left(\frac{V_{LO}}{2V_T}\right) \approx \text{LO Square Wave}$$

$$\Rightarrow v_{out} = \frac{R_L I_{EE}}{2V_T} V_{RF} \times \text{LO Square Wave}$$

Another Look at Gilbert Cell Operating Modes (2)

$$v_{out} = R_L I_{EE} \tanh\left(\frac{V_{RF}}{2V_T}\right)\tanh\left(\frac{V_{LO}}{2V_T}\right)$$

3. $v_{LO}(t)$ and $v_{RF}(t)$ both have large amplitudes: phase detector

$$\tanh\left(\frac{V_{RF}}{2V_T}\right) \approx \text{RF Square Wave}$$

$$\tanh\left(\frac{V_{LO}}{2V_T}\right) \approx \text{LO Square Wave}$$

$$\Rightarrow v_{out} = R_L I_{EE} \times \text{RF Square Wave} \times \text{LO Square Wave}$$

The Gilbert Cell Mixer: One Engineer’s Opinion

An interesting comment about the Gilbert Cell:

“The Gilbert-cell mixer is a neat idea, but one that has become a little boring over the years. It has been just a bit too successful, and for a long time we all built them by the dozens and didn’t think much about them.
Then along came the heterojunction bipolar transistor.
Now we can make these critters fly along at frequencies approaching 20GHz.
Without too much trouble, we can make multigigahertz analog multipliers and use these as direct microwave modulators, phase detectors, demodulators, and, of course, mixers. We can even do this cheaply, by using SiGe HBT technology, instead of the more expensive InP and GaAs technologies. In short, they’re fun again.”

Stephen A. Maas
The RF and Microwave Circuit Design Cookbook, p209
Artech House, 1998

Our First-Order Analysis: Words of Caution...

"One of the greatest frustrations in designing Gilbert-cell mixers is that the descriptions of these circuits in most general electronics textbooks focus on mid-frequency performance. By mid-frequency, we mean frequencies where the transistor's parasitic capacitances are negligible; we RF and microwave folks might call this resistive operation. Unfortunately, one of the fundamental dividing lines between us high-frequency people and the folks who deal in "general electronics" is that they operate at mid frequency, while we operate at high frequencies. More often than not, we operate FETs and BJTs at frequencies where they are just barely capable of doing something useful, be it amplifying, oscillating, or mixing. Unfortunately, most of the conventional wisdom about these circuits comes from the mid-frequency assumption. Much of this is invalid at RF and microwave frequencies. For example, one of the ostensible benefits of the HBT is its high dc collector-to-emitter resistance. This should make it ideal as a current-source transistor. However, at high frequencies, the collector-to-emitter impedance of an HBT often is surprisingly low. This is caused by feedback through the collector-to-emitter capacitance combined with the HBT's inherently high transconductance. It is surprising that a feedback capacitance of a few femtofarads could make such a dramatic change in output impedance, but it does. Keeping the base well-grounded helps minimize this effect but does not eliminate it entirely. The moral of the story is simple: be careful of your assumptions."

Stephen A. Maas, The RF and Microwave Circuit Design Cookbook, p222,
Artech House, 1998