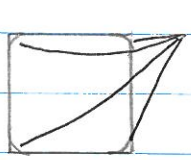
Trajectories

Mechatronics Summer '09

4 (a)

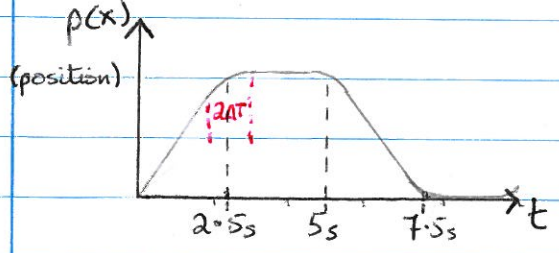


$t_{total} = 10\text{secs}$

$t_{side} = 2{\cdot}5\text{secs}$

1 side = 5 units → 2units/sec

(Ignoring parabolic blends)



Approx. ½ way through parabola at 2·5s, 5s, 7·5s.

We have assumed that initial velocity = max velocity.

Time for entire parabolic blend = $2\Delta T$

$|x,y \text{ acceleration}| = \frac{2}{2\Delta T} = \frac{1}{\Delta T}$ units/s²

$|x,y \text{ deviation}| = \frac{a\Delta T^2}{2} = \frac{\Delta T}{2}$

$|\text{deviation}| = \sqrt{2}\,\frac{\Delta T}{2}$ (Overall taking x_{err} & y_{err} into account, using pythagoras)

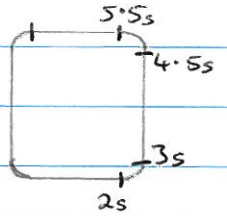
$= \sqrt{2}(0{\cdot}25)$

$\Delta T = 0{\cdot}5\text{s}$

$\Rightarrow |x,y \text{ acceleration}| = 2 \text{ unit/s}^2$

$|x \text{ or } y \text{ acceleration}| = 2\sqrt{2} \text{ unit/s}^2$

b)



Note: Using $s = s_0 + ut + \frac{1}{2}at^2$

x-axis $v_x = 0$, $3\text{s} \le t \le 4{\cdot}5\text{s}$

$P_x = 5$

$v_x = -2\text{units/s}$, $5{\cdot}5 \le t \le 7\text{s}$

$P_x = 5 - 2(t-5) = 15 - 2t$

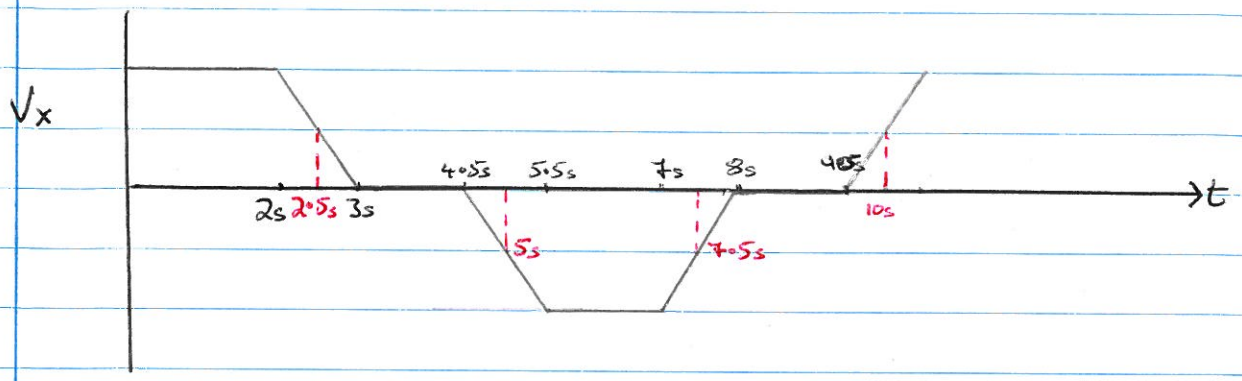
} $a = 0$

Looking for $3{\cdot}5\text{s} \le t \le 6{\cdot}5\text{s}$

$4{\cdot}5\text{s} \le t \le 5{\cdot}5\text{s}$

$P_x = 5 + 0t + \frac{1}{2}(-2)(t-4{\cdot}5)^2$

↑ Initial velocity IN x DIRECTION


v_x
2s
2·5s
3s
4·5s
5·5s
7s
8s
9·5s
10s
t
5s
7·5s

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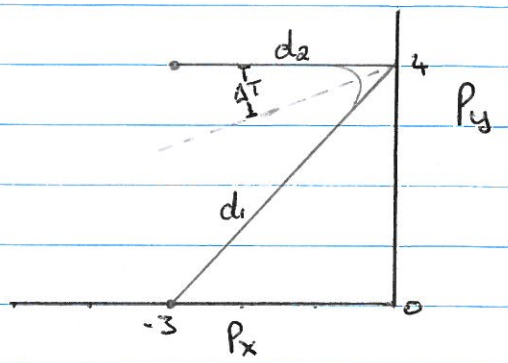
3. $T_{total} = 10s \quad \Delta T = 0.5s$

$w^1 = [-3, 0, 0, 0, 0, -1, 0.5]^T$

$w^2 = [0, 4, 0, 0, 0, -1, 0.8]^T$

$w^3 = [-3, 4, 0, 0, 0, -1, 0.5]^T$



$d_1 = \sqrt{3^2+4^2} = 5 \text{ units}$

$d_2 = 3 \text{ units}$

@ Constant velocity, $v = \frac{8}{10} = 0.8 \text{ units/sec.}$

a) X-Component

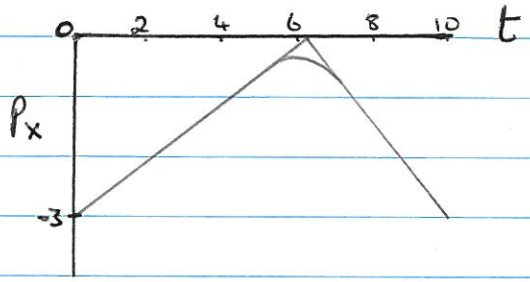
$T_1 = 6.25s$

$T_2 = 3.75s$

$V_{x_1} = \frac{3}{6.25} = 0.48 \text{ units/sec}$

$V_{x_2} = \frac{-3}{3.75} = -0.8 \text{ units/sec}$

$a_x = \frac{V_{x_2} - V_{x_1}}{2\Delta T} = -1.28 \text{ units/sec}^2$



$$W_x(t) = \begin{cases} -3 + 0.48t & 0 \le t \le 5.75 \\ -0.24 + 0.48(t-5.75) - (0.64)(t-5.75)^2 & 5.75 \le t \le 6.75 \\ -0.4 - 0.8(t-6.75) & 6.75 \le t \le 10 \end{cases}$$

b) Y-Component

$V_{y_1} = \frac{4}{6.25} = 0.64 \text{ units/sec}$

$V_{y_2} = 0$

Over first 4s: $p_y = 0 + 0.64t$

$p_x = -3 + 0.48t$

$p_y^2 = 0.4096t^2$

$p_x^2 = 0.2304t^2 - 2.88t + 9$

$$\Theta_2 = \pm \cos^{-1}\left(\frac{0.64t^2 - 2.88t - 41}{20}\right)$$

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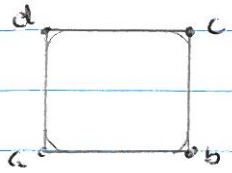
7.



$T_{total} = 10s \Rightarrow v = 2 \text{ units/s}$

$d_{total} = 20 \text{ units} \Rightarrow \Delta T = ?$

a) Half way through parabolae @ 2.5s, 5s, 7.5s, 10s

Length of time for $\frac{1}{2}$ parabola $= \Delta T$

$a_{x,y} = \frac{2}{2\Delta T} = \frac{1}{\Delta T}$

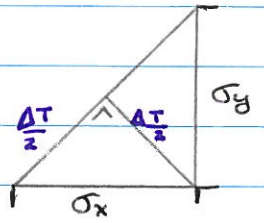
Deviation from corner

$d_{x,y} = \frac{a\Delta T^2}{2} = \frac{\Delta T}{2}$

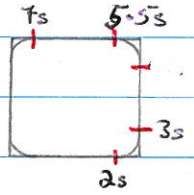
$d_x = d_y = \sqrt{2}\,\frac{\Delta T}{2} = 0.25\sqrt{2}$

$\Rightarrow \Delta T = 0.5$

$a = 2 \text{ units/s}^2$ duration of parabolic blend = 1s



b)



$x_1(t) = 5 \qquad 3.5s \le t \le 4.5s$

$x_2(t) = 5 + 0(t-4.5) - \frac{1}{2}(2)(t-4.5)^2 \qquad 4.5s \le t \le 5.5s$

$x_3(t) = 5 - 2(t-5) \qquad 5.5s \le t \le 6.5s$

$$x(t) = \begin{cases} 5 & 3.5 \le t \le 4.5 \\ 5-(t-4.5)^2 & 4.5 \le t \le 5.5 \\ 5-2(t-5) & 5.5 \le t \le 6.5 \end{cases}$$

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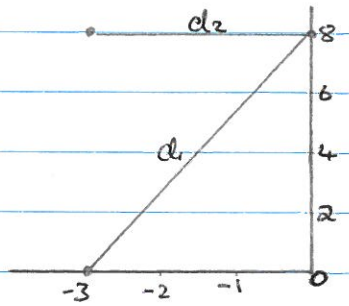
7. $\omega^1 = [-3, 0, 0]^T$ $\quad T_{12} = S_s = T_{23}$

$\omega^2 = [0, 8, 0]^T$

$\omega^3 = [-3, 8, 0]^T$



$d_1 = \sqrt{73}$ $\quad v_1 = \frac{\sqrt{73}}{5}$ units/s

$d_2 = 3$ $\quad v_2 = -0.6$ units/s

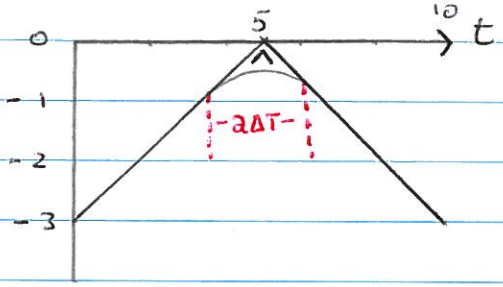
X - Component

$v_{1x} = 0.6$ units/s

$v_{2x} = -0.6$ units/s

$a_x = -\frac{1.2}{2\Delta T}$ units/s^2

$|\sigma_x| = \left|\frac{a\Delta T^2}{2}\right| = \frac{1.2\Delta T}{4} = .3\Delta T$



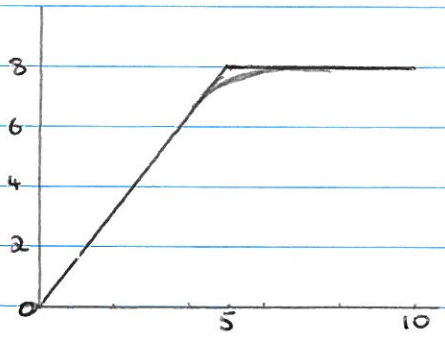
Y - Component

$v_{1y} = 1.6$ units/s

$v_{2y} = 0$ units/s

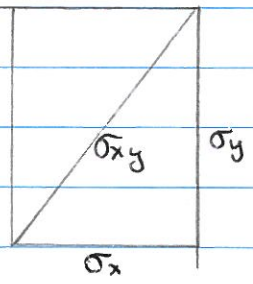
$a_y = -\frac{1.6}{2\Delta T}$

$|\sigma_y| = \left|\frac{a\Delta T^2}{2}\right| = \frac{1.6\Delta T}{4} = .4\Delta T$



$\sigma_{xy} = 0.5\Delta T = 0.125$

$\Delta T = 0.25$



a_{xy}

X - Trajectory

$$x(t) = \begin{cases} -3 + 0.6t & 0 \le t \le 4.75 \\ -0.15 + 0.6(t-4.75) - \frac{1}{2}(2.4)(t-4.75)^2 & 4.75 \le t \le 5.25 \\ -0.6(t-5) & 5.25 \le t \le 10 \end{cases}$$

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Y - Trajectory

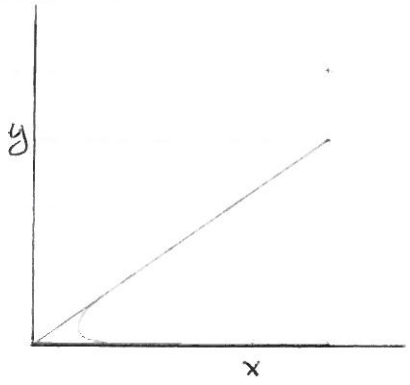
$$y(t) = \begin{cases} 1.6t & 0 \le t \le 4.75 \\ 7.6 + 1.6(t-4.75) - \frac{1}{2}(3.2)(t-4.75)^2 & 4.75 \le t \le 5.25 \\ 8 & 5.25 \le t \le 10 \end{cases}$$

Summer '05

$w^1 = [4, 3, 0, 0, 0, -1, 0.8]^T$ $\quad t_{total} = 18s$

$w^2 = [0, 0, 0, 0, 0, -1, 0.5]^T$ $\quad \Delta T = 1s \quad \bar{v}_1 = \bar{v}_2$

$w^3 = [4, 0, 0, 0, 0, -1, 0.8]^T$



$d_1 = 5 \Rightarrow |\bar{v}| = 0.5$ u/s

$d_2 = 4$

$\bar{v}_{1x} = -0.4$ u/s $\quad \bar{v}_{1y} = -0.3$ u/s

$\bar{v}_{2x} = 0.5$ u/s $\quad \bar{v}_{2y} = 0$ u/s

$a_x = 0.45$ u/s^2 $\quad a_y = 0.15$ u/s^2

$dew_x = 0.225$ units $\quad dew_y = 0.075$ units $\quad dew_{xy} = 0.237171$

X-Trajectory

$0 \le t \le 9s, \quad P_x = 4 - 0.4t$

$9 \le t \le 11s, \quad P_x = 0.4 - 0.4(t-9) + 0.225(t-9)^2$

$11s \le t \le 18s, \quad P_x = 0.5(t-10)$

Y-Trajectory

$0 \le t \le 9s, \quad P_y = 3 - 0.3t$

$9 \le t \le 11s, \quad P_y = 0.3 - 0.3(t-9) + 0.075(t-9)^2$

$11s \le t \le 18s, \quad P_y = 0$

$$\Theta_2 = \pm \text{Arccos}\left(\frac{(4-0.4t)^2 + (3-0.3t)^2 - 50}{50}\right) \quad 0 \le t \le 9s$$

$$\Theta_2 = \pm \text{Arccos}\left(\frac{0.25t^2 - 5t - 25}{50}\right) \quad 0 \le t \le 9s$$

$$\Theta_2 = \pm \text{Arccos}\left(\frac{(0.5t-5)^2 - 50}{50}\right) = \pm \text{Arccos}\left(\frac{0.25t^2 - 5t - 25}{50}\right)$$

Summer '07

$p_x(t) = 5+5t$ $\quad p_y(t) = 25-5t$ $\quad 0 \le t \le 5$

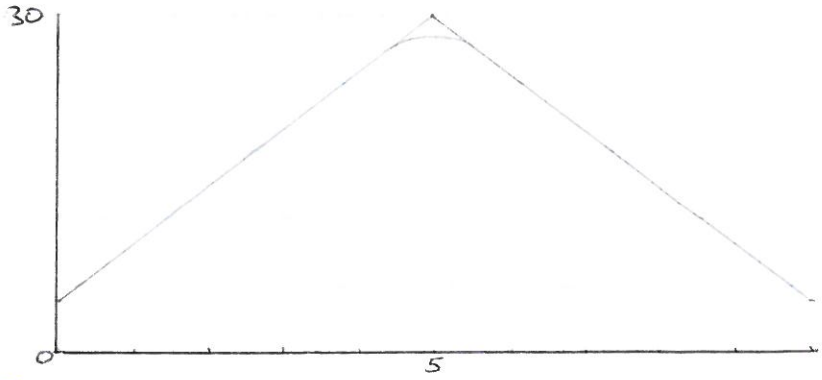
$p_x(t) = 55-5t$ $\quad p_y(t) = 0$ $\quad 5 \le t < 10$

$a = -5\ u/s^2 = \frac{v_{2x} - v_{ix}}{2\Delta T}$

$\bar{v}_{2x} = -5u/s \quad \bar{v}_{ix} = 5u/s$

$\Rightarrow \Delta T = 1s$

$dew_x = 2.5\,cm$



$0 \le t < 4$, $P_x = 5+5t$

$4 \le t < 6$, $P_x = 25 + 5(t-4) - 2.5(t-4)^2$

$6 \le t < 10$, $P_x = 30 - 5(t-5)$

$\bar{v}_{iy} = 5u/s \quad \bar{v}_{2y} = 0$

$a = -5 \Rightarrow \Delta T = 0.5s$

$dew_y = 0.625$

$dew_{xy} = 2.577\,cm$