Table of z-Transforms

Time Function, $f(t)$	Laplace Transform, $F(s)$	z-Transform, $F(z)$	Modified z-Transform, $F(z, m)$
$u(t)$	$\frac{1}{s}$	$\frac{1}{1-z^{-1}}$	$\frac{z^{-1}}{1-z^{-1}}$
t	$\frac{1}{s^2}$	$\frac{Tz^{-1}}{(1-z^{-1})^2}$	$\frac{mTz^{-1}}{1-z^{-1}} + \frac{Tz^{-2}}{(1-z^{-1})^2}$
t^2	$\frac{2!}{s^3}$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$	$T^2\left[\frac{m^2 z^{-1}}{1-z^{-1}} + \frac{(2m+1)z^{-2}}{(1-z^{-1})^2} + \frac{2z^{-3}}{(1-z^{-1})^3}\right]$
t^{n-1}	$\frac{(n-1)!}{s^n}$	$\lim_{a\to 0} (-1)^{n-1} \frac{\partial^{n-1}}{\partial a^{n-1}} \left(\frac{1}{1-e^{-aT}z^{-1}}\right)$	$\lim_{a\to 0} (-1)^{n-1} \frac{\partial^{n-1}}{\partial a^{n-1}} \left(\frac{e^{-amT}z^{-1}}{1-e^{-aT}z^{-1}}\right)$
e^{-at}	$\frac{1}{s+a}$	$\frac{1}{1-e^{-aT}z^{-1}}$	$\frac{e^{-amT}z^{-1}}{1-e^{-aT}z^{-1}}$
$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(\frac{1}{1-e^{-aT}z^{-1}} - \frac{1}{1-e^{-bT}z^{-1}}\right)$	$\frac{z^{-1}}{b-a}\left(\frac{e^{-amT}}{1-e^{-aT}z^{-1}} - \frac{e^{-bmT}}{1-e^{-bT}z^{-1}}\right)$
$\frac{1}{a}(u(t)-e^{-at})$	$\frac{1}{s(s+a)}$	$\frac{1}{a}\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$	$\frac{z^{-1}}{a}\left(\frac{1}{1-z^{-1}} - \frac{e^{-amT}}{1-e^{-aT}z^{-1}}\right)$
$\frac{1}{a}\left(t - \frac{1-e^{-at}}{a}\right)$	$\frac{1}{s^2(s+a)}$	$\frac{1}{a}\left[\frac{Tz^{-1}}{(1-z^{-1})^2} - \frac{(1-e^{-aT})z^{-1}}{a(1-z^{-1})(1-e^{-aT}z^{-1})}\right]$	$\frac{z^{-1}}{a}\left[\frac{Tz^{-1}}{(1-z^{-1})^2} + \frac{amT-1}{a(1-z^{-1})} + \frac{e^{-amT}}{a(1-e^{-aT}z^{-1})}\right]$
$\frac{(a-b)}{a^2}u(t) + \frac{b}{a}t + \frac{1}{a}\left(\frac{b}{a}-1\right)e^{-at}$	$\frac{s+b}{s^2(s+a)}$	$\frac{z^{-1}}{a}\left[\frac{bT}{(1-z^{-1})^2} + \frac{(a-b)(1-e^{-aT})}{a(1-z^{-1})(1-e^{-aT}z^{-1})}\right]$	$\frac{z^{-1}}{a}\left[\frac{bTz^{-1}}{(1-z^{-1})^2} + \left(bmT+1-\frac{b}{a}\right)\frac{1}{1-z^{-1}} + \frac{b-a}{a}\frac{e^{-amT}}{1-e^{-aT}z^{-1}}\right]$
$\frac{1}{ab}\left(u(t) + \frac{b}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt}\right)$	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[\frac{1}{1-z^{-1}} + \frac{b}{(a-b)(1-e^{-aT}z^{-1})} - \frac{a}{(a-b)(1-e^{-bT}z^{-1})}\right]$	$\frac{z^{-1}}{ab}\left[\frac{1}{1-z^{-1}} + \frac{be^{-amT}}{(a-b)(1-e^{-aT}z^{-1})} - \frac{ae^{-bmT}}{(a-b)(1-e^{-bT}z^{-1})}\right]$
te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$	$\frac{Te^{-amT}z^{-1}\left[m+(1-m)e^{-aT}z^{-1}\right]}{(1-e^{-aT}z^{-1})^2}$
$\sin at$	$\frac{a}{s^2+a^2}$	$\frac{z^{-1}\sin aT}{1-2z^{-1}\cos aT + z^{-2}}$	$\frac{z^{-1}\sin amT + z^{-2}\sin(1-m)aT}{1-2z^{-1}\cos aT+z^{-2}}$
$\frac{1}{b}e^{-at}\sin bt$	$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\frac{z^{-1}e^{-aT}\sin bT}{1-2z^{-1}e^{-aT}\cos bT + e^{-2aT}z^{-2}}$	$\frac{z^{-1}e^{-amT}}{b}\frac{\sin bmT + z^{-1}e^{-aT}\sin(1-m)bT}{1-2z^{-1}e^{-aT}\cos bT + e^{-2aT}z^{-2}}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$\frac{1-z^{-1}e^{-aT}\cos bT}{1-2z^{-1}e^{-aT}\cos bT + e^{-2aT}z^{-2}}$	$\frac{e^{-amT}z^{-1}\left[\cos bmT + z^{-1}e^{-aT}\sin(1-m)bT\right]}{1-2z^{-1}e^{-aT}\cos bT + e^{-2aT}z^{-2}}$
$\cos at$	$\frac{s}{s^2+a^2}$	$\frac{1-z^{-1}\cos aT}{1-2z^{-1}\cos aT + z^{-2}}$	$\frac{z^{-1}\cos amT - z^{-2}\cos(1-m)aT}{1-2z^{-1}\cos aT + z^{-2}}$

For a more extensive table see B. C. Kuo, *Analysis and Synthesis of Sampled-Data Control Systems*, Prentice-Hall, Inc. Englewood Cliffs, N.J., 1963.)

10|10

SECOND ORDER DESIGN EQUATIONS

For the second order transfer function:

$$T(s) = \frac{K}{s^2 + 2\xi\omega_N s + \omega_N^2}$$

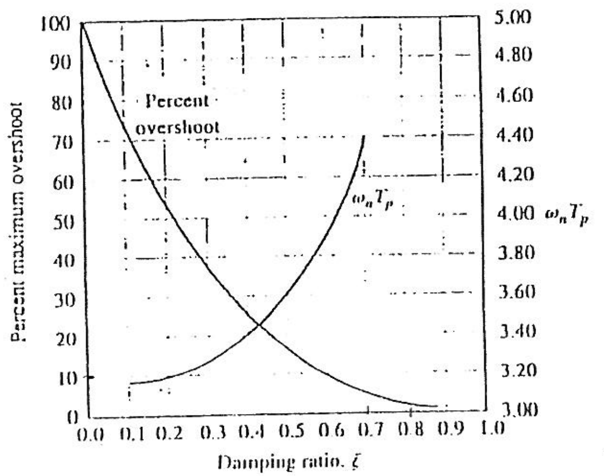
TIME DOMAIN SPECIFICATIONS

a) Peak Overshoot:

$$PO\% = 100EXP\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)$$

b) Time to peak:

$$T_p = \frac{\pi}{\omega_N\sqrt{1-\xi^2}}$$



Graph of PO% and T_p against Damping factor ξ

c) Damped frequency of oscillation:

$$\omega_d = \omega_N\sqrt{1-\xi^2}$$

d) Settling time to within 2% :

$$Ts_{2\%} = \frac{4}{\xi\omega_N}$$

e) Time constant:

$$\tau = \frac{Ts_{2\%}}{4} = \frac{1}{\xi\omega_N}$$

FREQUENCY DOMAIN SPECIFICATIONS

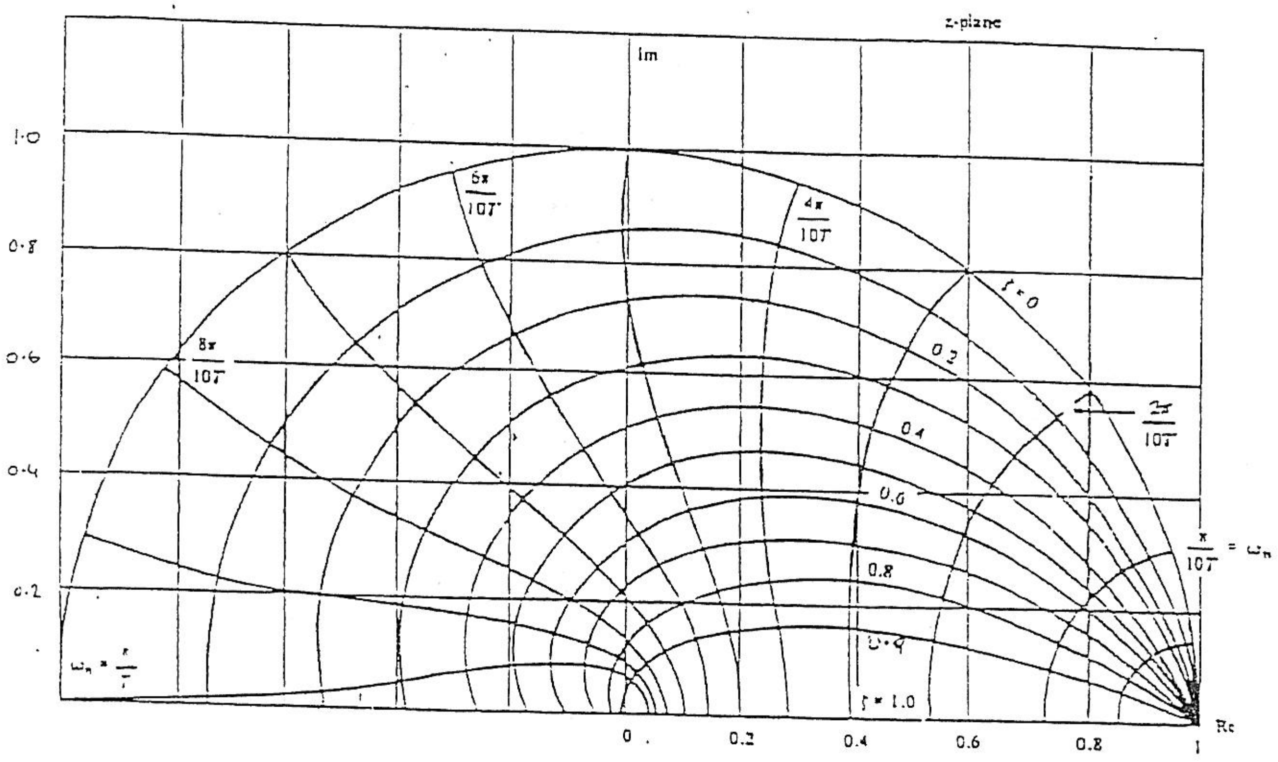
a) Relationship between closed-loop damping factor and phase margin:

$$\phi_m \approx 100\xi$$

b) Settling time (2%) for closed loop step response (in seconds):

$$Ts_{2\%} \approx \frac{8}{\omega_c \tan(\phi_m)}$$

Where ω_c is the gain crossover frequency (rad/s) and ϕ_m is the phase margin (in degrees).



Z Plane Design Template

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