Chapter 4

DESIGNING FEEDBACK CONTROLLERS IN SWITCH-MODE DC POWER SUPPLIES

OBJECTIVES OF FEEDBACK CONTROL

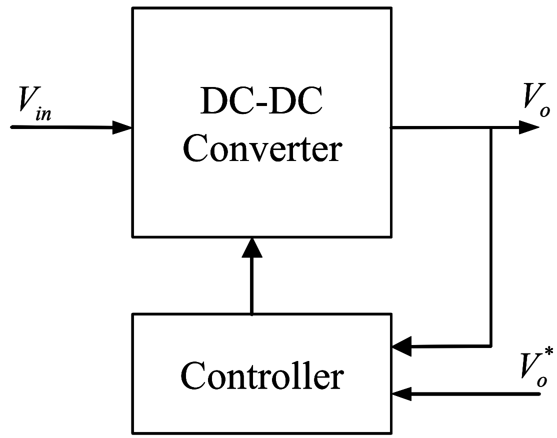


Figure 4-1 Regulated dc power supply.

- zero steady state error
- fast response
- low overshoot
- low noise susceptibility.

The steps in designing the feedback controller:

- Linearize the system for small changes around the dc steady state operating point
- Design the feedback controller using linear control theory
- Confirm and evaluate the system response by simulations for large disturbances

REVIEW OF LINEAR CONTROL THEORY

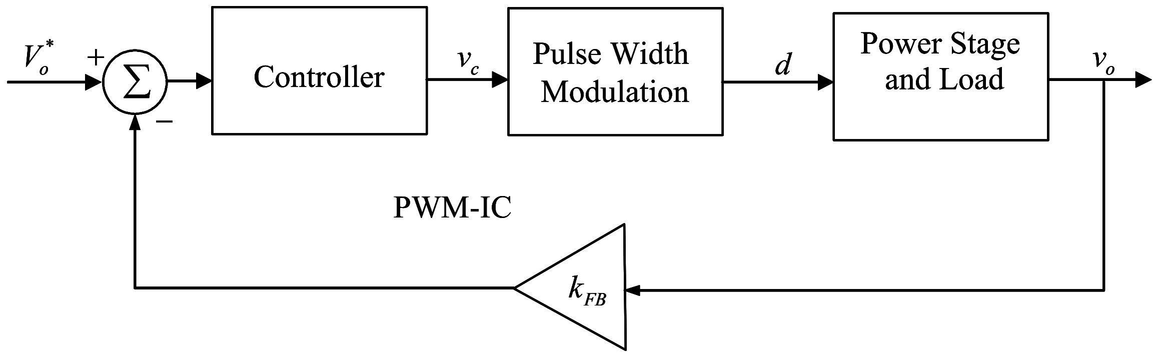


Figure 4-2 Feedback control.

Small signal representation:

$$\bar{v}_o(t) = V_o + \tilde{v}_o(t)$$
$$d(t) = D + \tilde{d}(t)$$
$$v_c(t) = V_c + \tilde{v}_c(t)$$

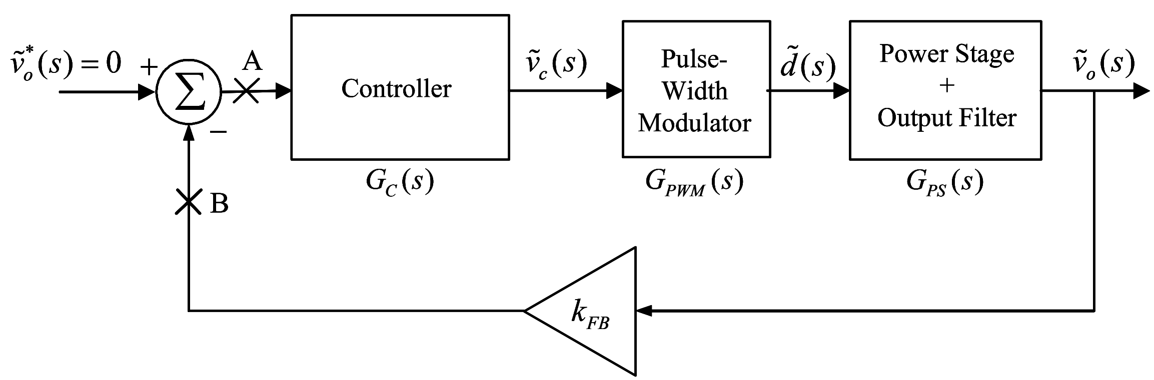


Figure 4-3 Small signal control system representation.

Loop Transfer Function:

$$G_L(s) = G_C(s)\, G_{PWM}(s) G_{PS}(s)\, k_{FB}$$

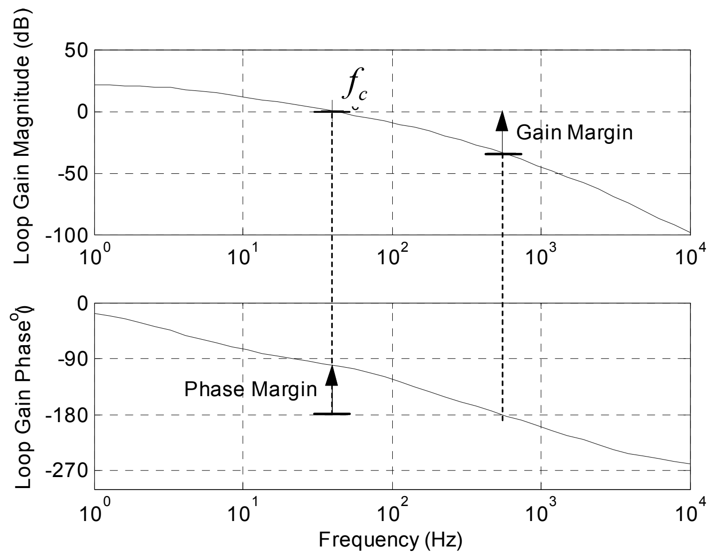


Figure 4-4 Definitions of crossover frequency, gain margin and phase margin.

Phase Margin:

$$\phi_{PM} = \phi_L\big|_{f_c} - (-180^0) = \phi_L\big|_{f_c} + 180^0$$

LINEARIZATION OF VARIOUS TRANSFER FUNCTION BLOCKS

Linearizing the PWM Controller IC

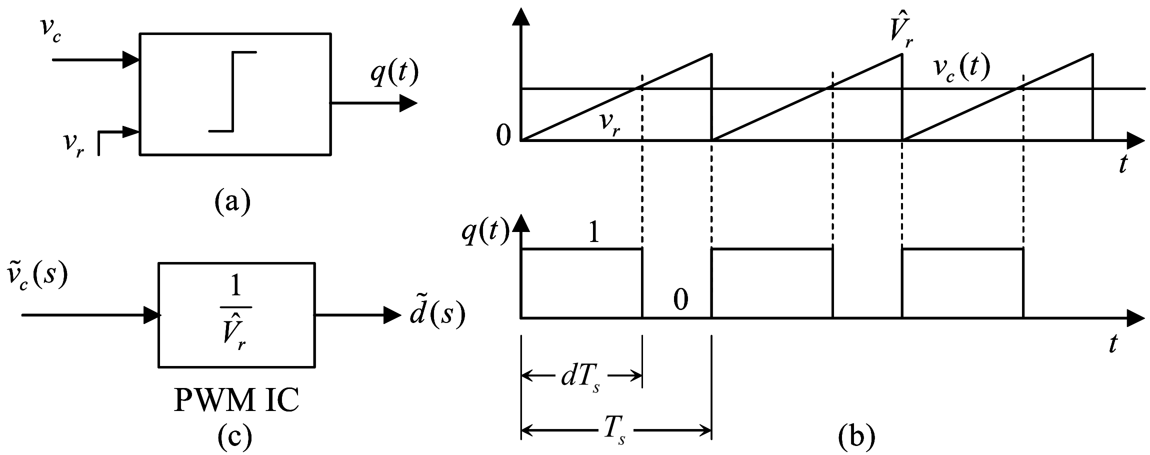


Figure 4-5 PWM waveforms.

$$d(t) = \frac{v_c(t)}{\hat{V}_r}$$

$$v_c(t) = V_c + \tilde{v}_c(t)$$

$$d(t) = \underbrace{\frac{V_c(t)}{\hat{V}_r}}_{D} + \underbrace{\frac{\tilde{v}_c(t)}{\hat{V}_r}}_{\tilde{d}(t)}$$

$$G_{PWM}(s) = \frac{\tilde{d}(s)}{\tilde{v}_c(s)} = \frac{1}{\hat{V}_r}$$

Linearizing the Power Stage of DC-DC Converters in CCM

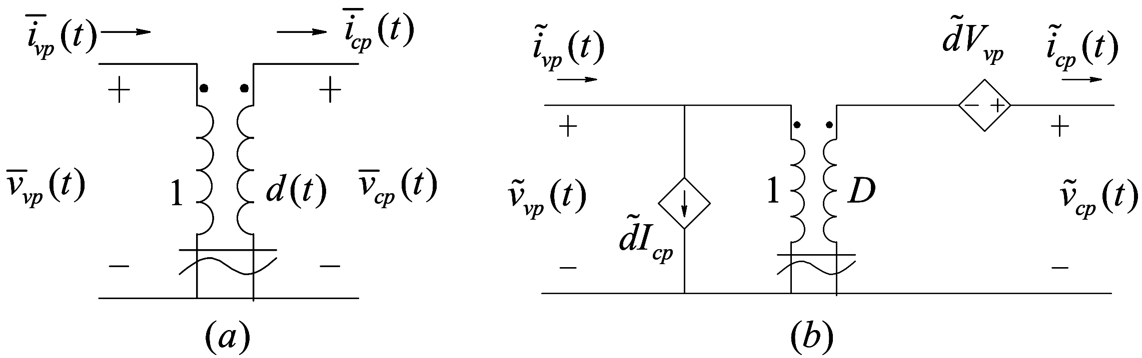


Figure 4-6 Linearizing the switching power-pole.

$$d(t) = D + \tilde{d}(t)$$

$$\bar{v}_{vp}(t) = V_{vp} + \tilde{v}_{vp}(t)$$

$$\bar{v}_{cp}(t) = V_{cp} + \tilde{v}_{cp}(t)$$

$$\bar{i}_{vp}(t) = I_{vp} + \tilde{i}_{vp}(t)$$

$$\bar{i}_{cp}(t) = I_{cp} + \tilde{i}_{cp}(t)$$

Linearizing single-switch converters

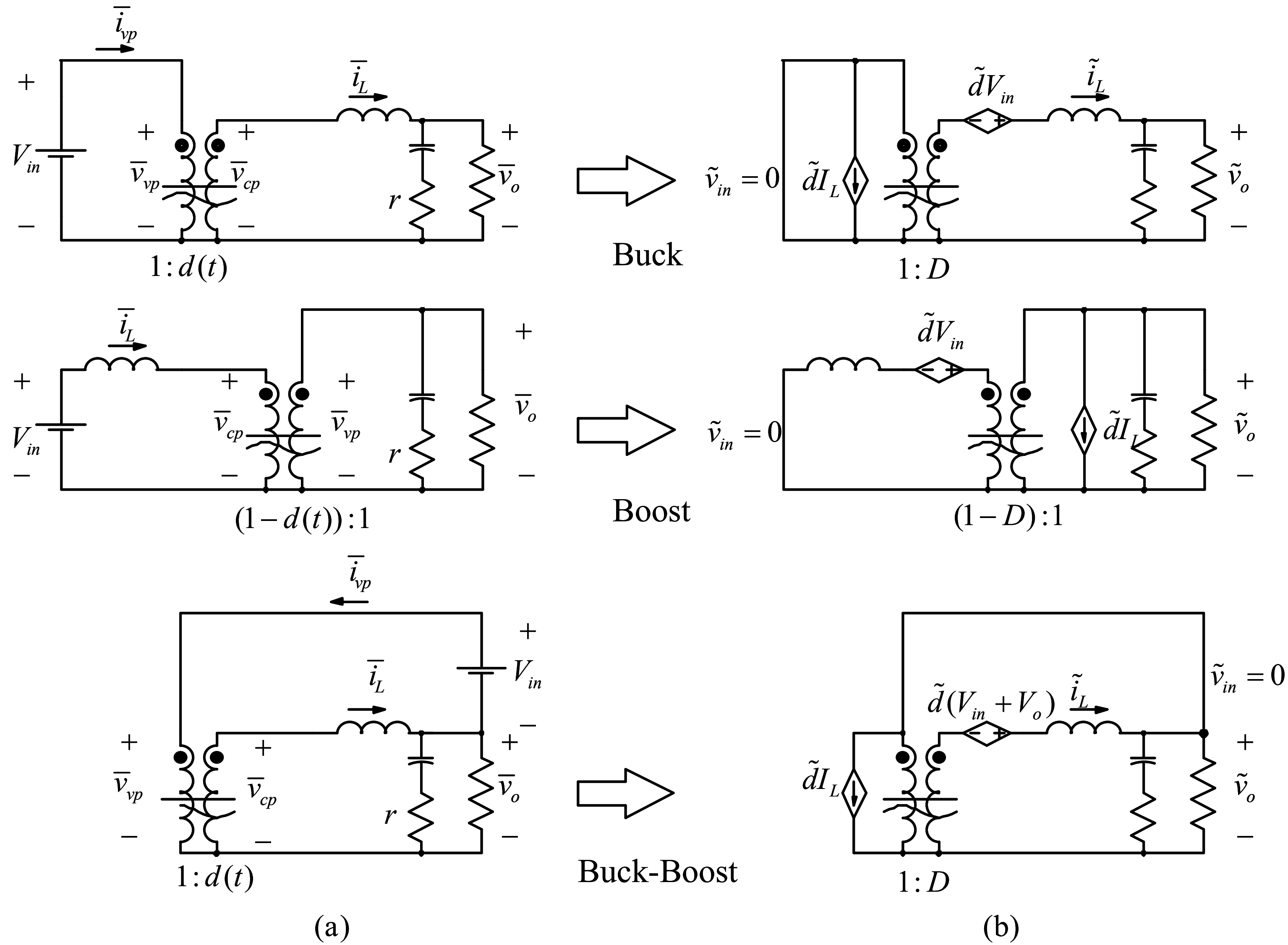
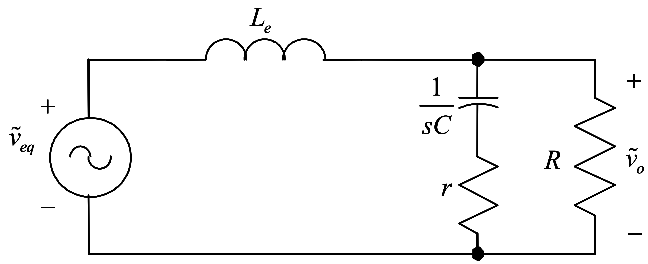


Figure 4-7 Linearizing single-switch converters in CCM.

Small signal equivalent circuit for Buck, Boost and Buck-Boost converters



$$L_e = L \quad \text{(Buck)}$$

$$L_e = \frac{L}{(1-D)^2} \quad \text{(Boost and Buck-Boost)}$$

Figure 4-8 Small signal equivalent circuit for Buck, Boost and Buck-Boost converters.

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{LC} \frac{1 + srC}{s^2 + s\left(\frac{1}{RC} + \frac{r}{L}\right) + \frac{1}{LC}} \qquad \text{(Buck)}$$

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{(1-D)^2} \left(1 - s\frac{L_e}{R}\right) \frac{1 + srC}{L_e C \left(s^2 + s\left(\frac{1}{RC} + \frac{r}{L_e}\right) + \frac{1}{L_e C}\right)} \qquad \text{(Boost)}$$

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{(1-D)^2} \left(1 - s\frac{DL_e}{R}\right) \frac{1 + srC}{L_e C \left(s^2 + s\left(\frac{1}{RC} + \frac{r}{L_e}\right) + \frac{1}{L_e C}\right)} \qquad \text{(Buck-Boost)}$$

Using Computer Simulation to Obtain the transfer function Bode Plots

▲ **Example 4-2** A Buck converter has the following parameters and is operating in CCM: $L = 100\,\mu H$, $C = 697\,\mu F$, $r = 0.1\Omega$, $f_s = 100\,kHz$, $V_{in} = 30V$, and $P_o = 36W$. The duty-ratio D is adjusted to regulate the output voltage $V_o = 12V$. Obtain both the gain and the phase of the power stage $G_{PS}(s)$ for the frequencies ranging from 1 Hz to 100 kHz.

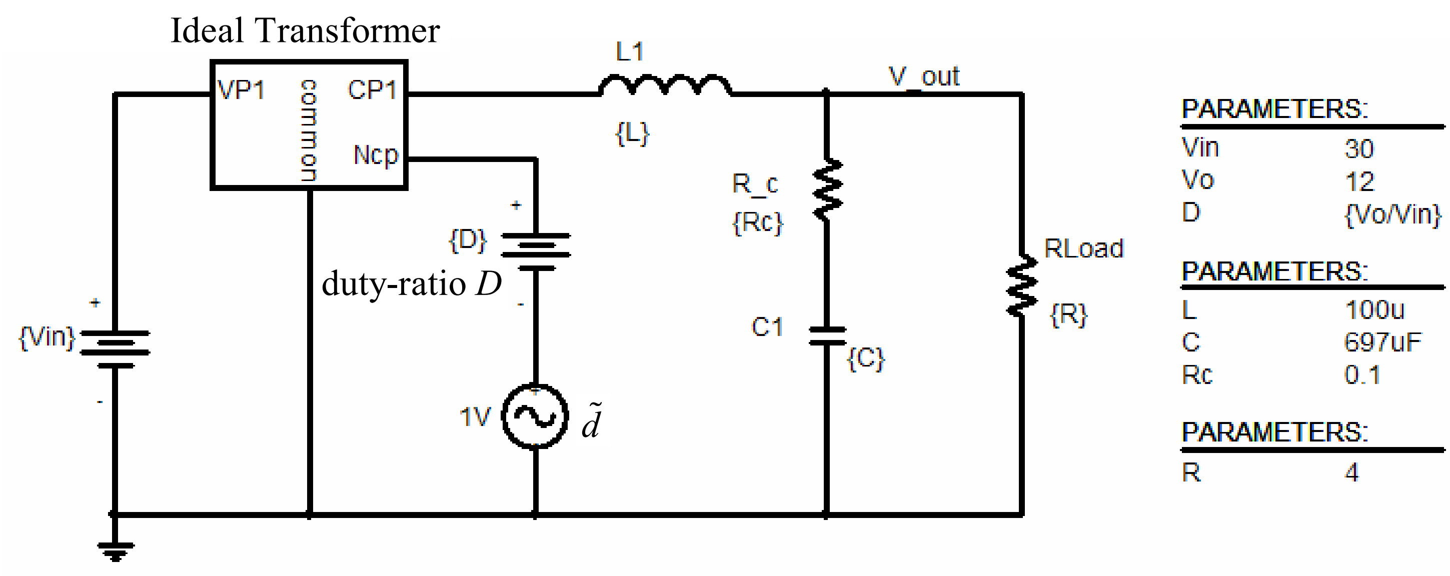
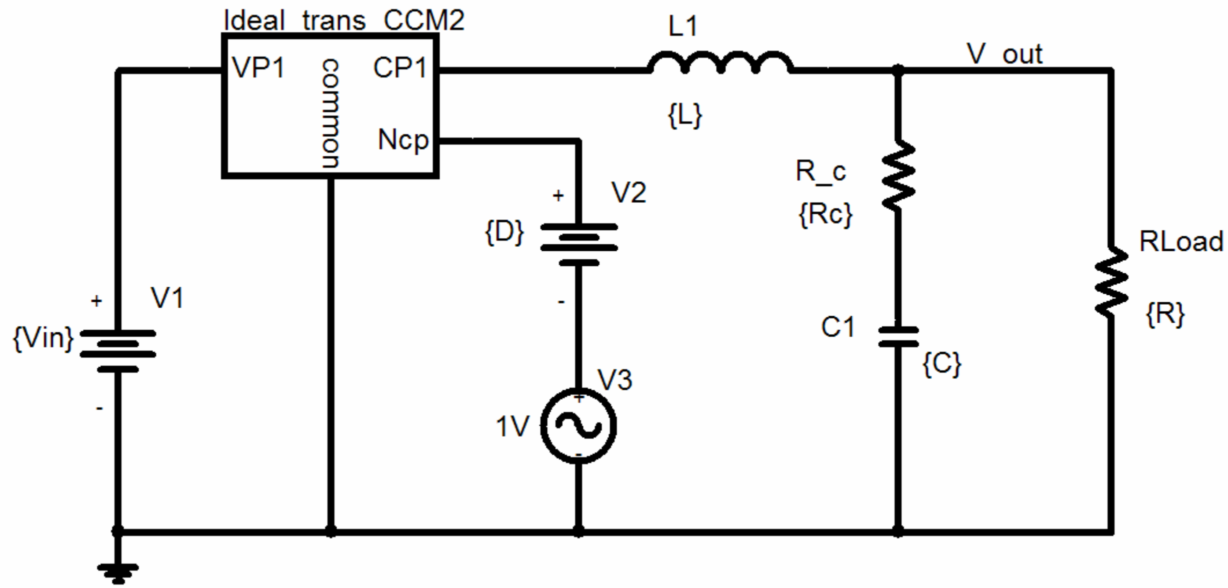


Figure 4-9 PSpice Circuit model for a Buck converter.

PSpice Modeling: C:\FirstCourse_PE_Book03\buck_conv_avg.sch



PARAMETERS:

Vin	30
Vo	12
D	{Vo/Vin}

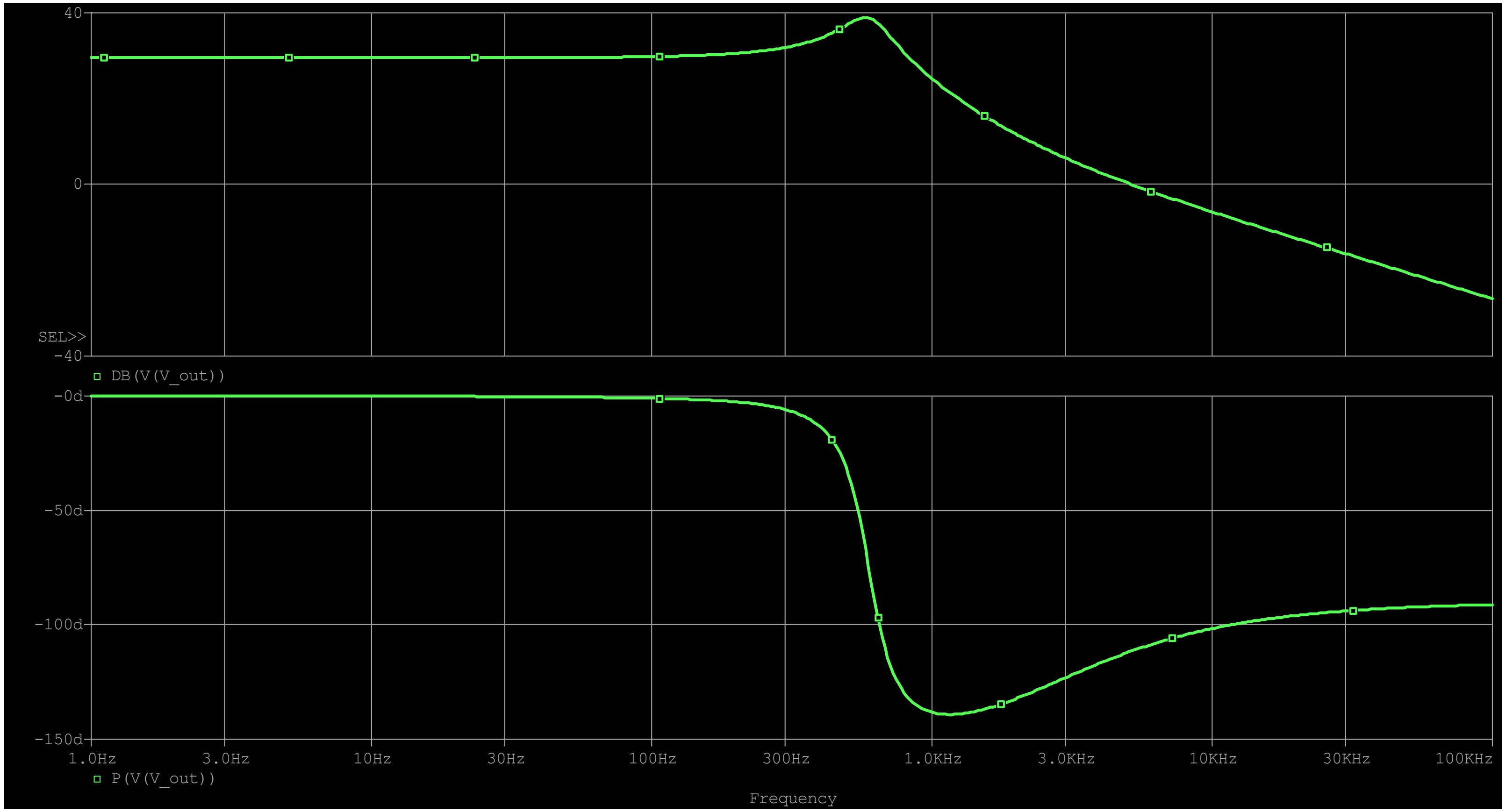
PARAMETERS:

L	100u
C	697uF
Rc	0.1

PARAMETERS:

R	4

Simulation Results



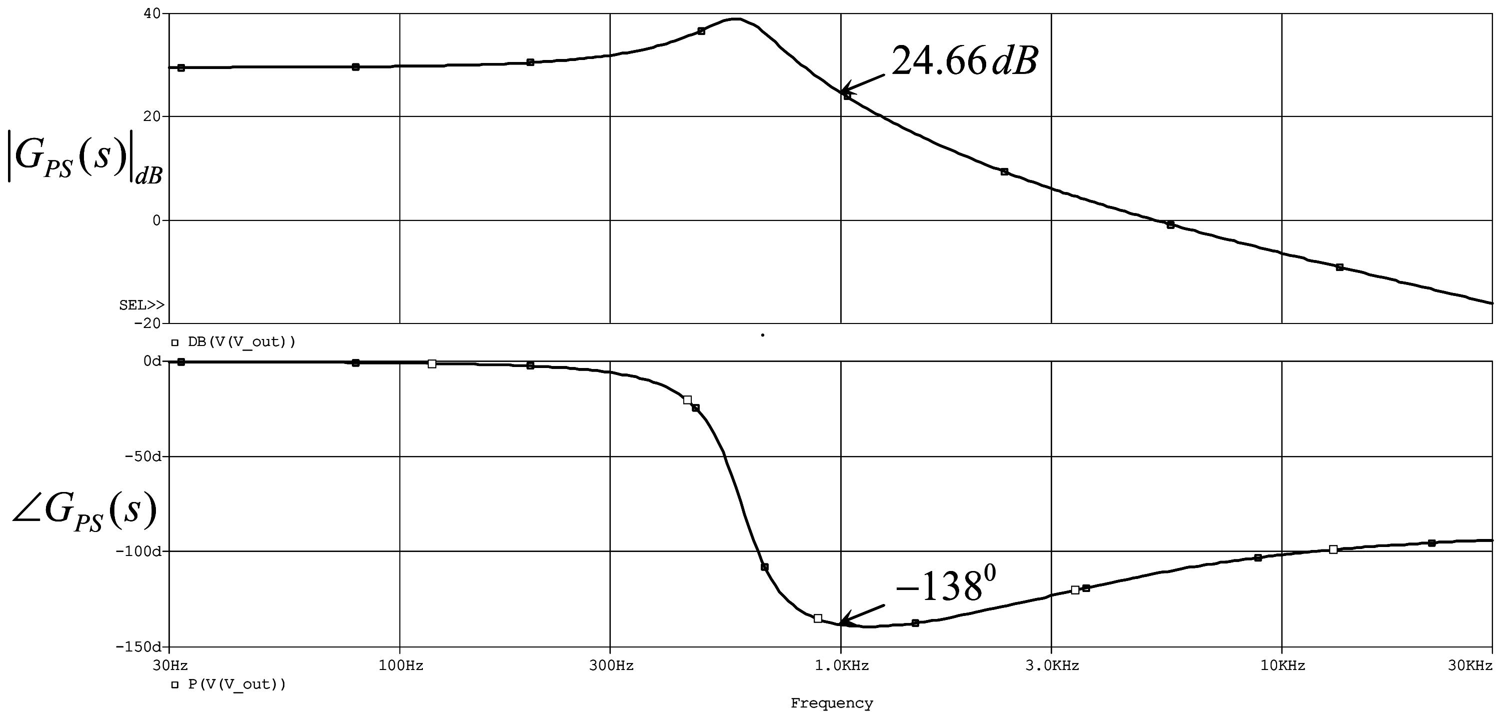


Figure 4-10 The gain and the phase of the power stage

FEEDBACK CONTROLLER DESIGN IN VOLTAGE-MODE CONTROL

▲ **Example 4-3** Design the feedback controller for the Buck converter described in Example 4-2. The PWM-IC is as described in Example 4-1. The output voltage-sensing network in the feedback path has a gain $k_{FB} = 0.2$. The steady state error is required to be zero and the phase margin of the loop transfer function should be 60^0 at as high a crossover frequency as possible.

1. The crossover frequency f_c of the open-loop gain is as high as possible to result in a fast response of the closed-loop system.
2. The phase angle of the open-loop transfer function has the specified phase margin, typically 60^0 at the crossover frequency so that the response in the closed-loop system settles quickly without oscillations.
3. The phase angle of the open-loop transfer function should not drop below -180^0 at frequencies below the crossover frequency.

$$G_c(s) = \frac{k_c}{s} \quad \underbrace{\frac{\left(1 + s/\omega_z\right)^2}{\left(1 + s/\omega_p\right)^2}}_{phase-boost}$$

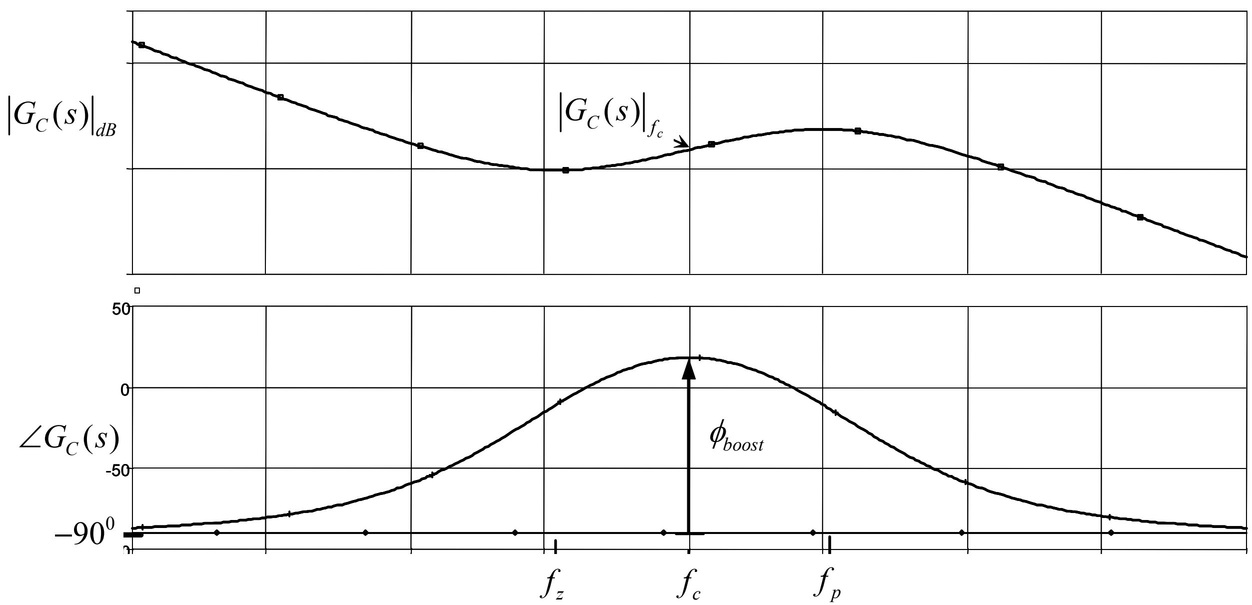


Figure 4-11 Bode plot of $G_C(s)$ in Eq. 4-18.

Step 1: Choose the Crossover Frequency. Choose f_c to be *slightly* beyond the *L-C* resonance frequency $1/(2\pi\sqrt{LC})$, which in this example is approximately 600 Hz. Therefore, we will choose $f_c = 1\text{ kHz}$. This ensures that the phase angle of the loop remains greater than -180^0 at all frequencies.

<u>Step 2: Calculate the needed Phase Boost</u>. The desired phase margin is specified as $\phi_{PM} = 60^0$. The required phase boost ϕ_{boost} at the crossover frequency is calculated as follows, noting that G_{PWM} and k_{FB} produce zero phase shift:

$$\angle G_L(s)\Big|_{f_c} = \angle G_{PS}(s)\Big|_{f_c} + \angle G_C(s)\Big|_{f_c} \quad \text{(from Eq. 4-2)} \qquad (4\text{-}19)$$

$$\angle G_L(s)\Big|_{f_c} = -180^o + \phi_{PM} \quad \text{(from Eq. 4-3)} \qquad (4\text{-}20)$$

$$\angle G_C(s)\Big|_{f_c} = -90^o + \phi_{boost} \quad \text{(from Fig. 4-11)} \qquad (4\text{-}21)$$

Substituting Eqs. 4-20 and 4-21 into Eq. 4-19,

$$\phi_{boost} = -90^o + \phi_{PM} - \angle G_{PS}(s)\Big|_{f_c} \qquad (4\text{-}22)$$

In Fig. 4-10, $\angle G_{PS}(s)\Big|_{f_c} \simeq -138^0$, substituting which in Eq. 4-22 yields the required phase boost $\phi_{boost} = 108^o$.

<u>Step 3: Calculate the Controller Gain at the Crossover Frequency</u>. From Eq. 4-2 at the crossover frequency f_c

$$\left|G_L(s)\right|_{f_c} = \left|G_C(s)\right|_{f_c} \times \left|G_{PWM}(s)\right|_{f_c} \times \left|G_{PS}(s)\right|_{f_c} \times k_{FB} = 1 \tag{4-23}$$

In Fig. 4-10, at $f_c = 1kHz$, $\left|G_{PS}(s)\right|_{f_c=1kHz} = 24.66\,dB = 17.1$. Therefore in Eq. 4-23, using the gain of the PWM block calculated in Example 4-1,

$$\left|G_C(s)\right|_{f_c} \times \underbrace{0.556}_{\left|G_{PWM}(s)\right|_{f_c}} \times \underbrace{17.1}_{\left|G_{PS}(s)\right|_{f_c}} \times \underbrace{0.2}_{k_{FB}} = 1 \tag{4-24}$$

or

$$\left|G_C(s)\right|_{f_c} = 0.5263 \tag{4-25}$$

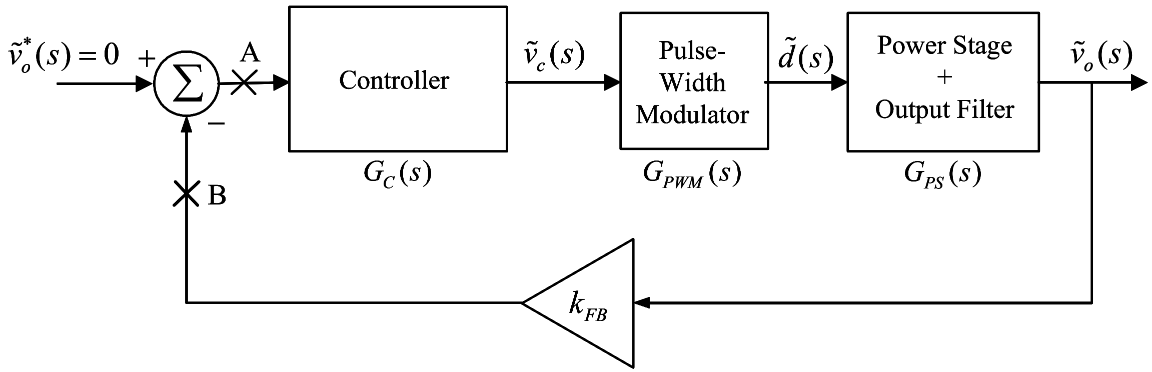


Figure 4-3 Small signal control system representation.

$$G_c(s) = \frac{k_c}{s} \underbrace{\frac{(1 + s/\omega_z)^2}{(1 + s/\omega_p)^2}}_{phase-boost}$$

$$K_{boost} = \sqrt{\frac{\omega_p}{\omega_z}} \qquad K_{boost} = \tan\left(45^o + \frac{\phi_{boost}}{4}\right)$$

$$f_z = \frac{f_c}{K_{boost}} \qquad f_p = K_{boost}\, f_c$$

$$k_c = \left|G_C(s)\right|_{f_c} \frac{\omega_z}{K_{boost}}$$

Implementation of the controller by an op-amp

$$G_c(s) = \frac{k_c}{s} \quad \underbrace{\frac{\left(1 + s/\omega_z\right)^2}{\left(1 + s/\omega_p\right)^2}}_{phase-boost}$$

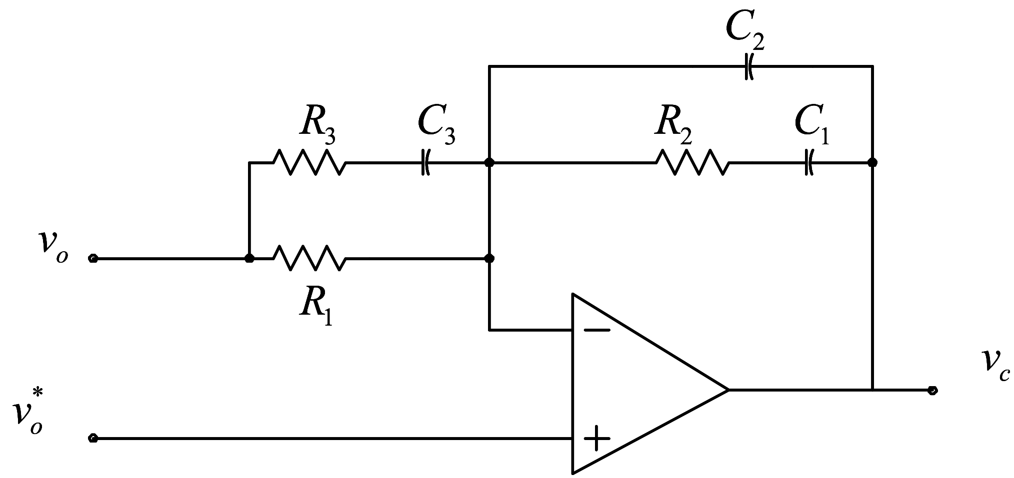


Figure 4-12 Implementation of the controller by an op-amp.

$$C_2 = \omega_z / (k_c \omega_p R_1)$$

$$C_1 = C_2 \left(\omega_p / \omega_z - 1 \right)$$

$$R_2 = 1/(\omega_z C_1)$$

$$R_3 = R_1 / (\omega_p / \omega_z - 1)$$

$$C_3 = 1/(\omega_p R_3)$$

In this numerical example with $f_c = 1$ kHz , $\phi_{boost} = 108^o$, and $\left|G_C(s)\right|_{f_c} = 0.5263$, we can calculate $K_{boost} = 3.078$ in Eq. 4-27. Using Eqs. 4-27 through 4-30, $f_z = 324.9\,Hz$, $f_p = 3078\,Hz$, and $k_c = 349.1$. For the op-amp implementation, we will select $R_1 = 100\,k\Omega$. From Eq. 4-30, $C_2 = 3.0\,nF$, $C_1 = 25.6\,nF$, $R_2 = 19.1\,k\Omega$, $R_3 = 11.8\,k\Omega$, and $C_3 = 4.4\,nF$.

PSpice model of the Buck converter with voltage-mode control

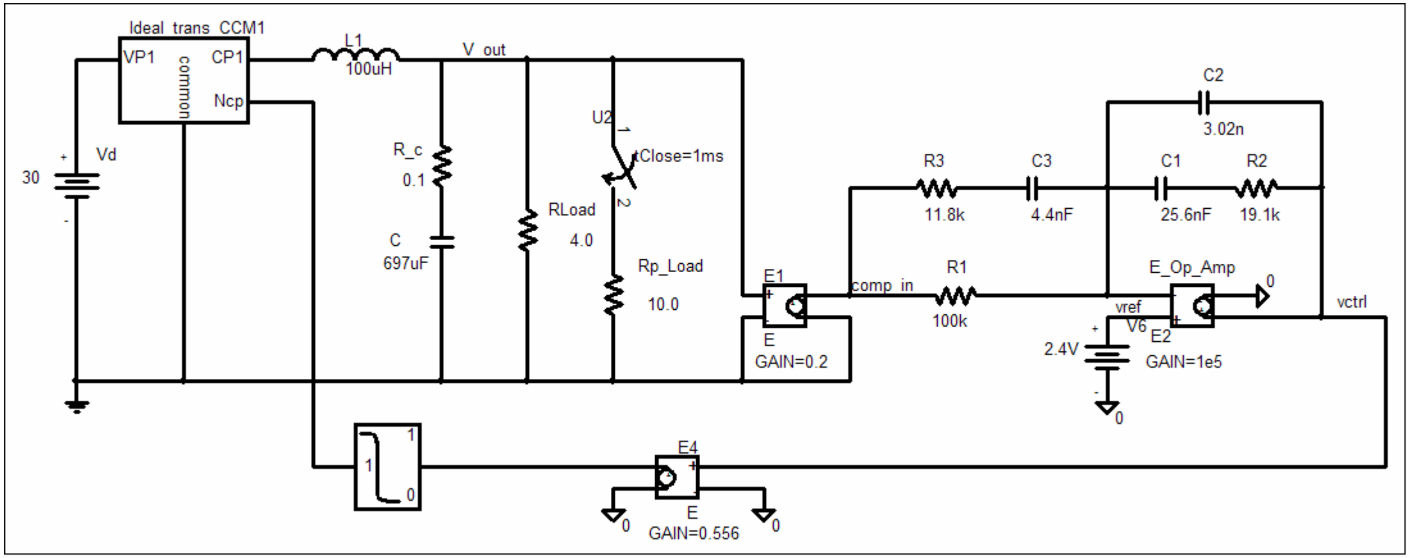


Figure 4-13 PSpice average model of the Buck converter with voltage-mode control.

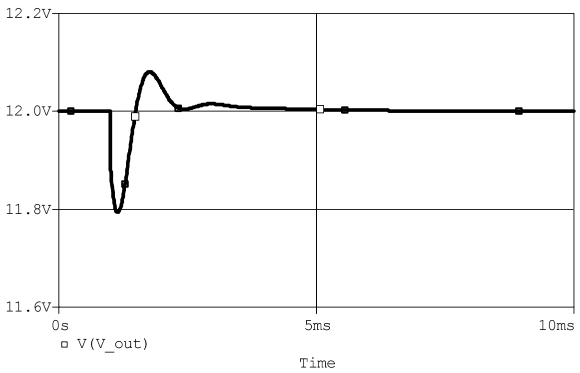


Figure 4-14 Response to a step-change in load.

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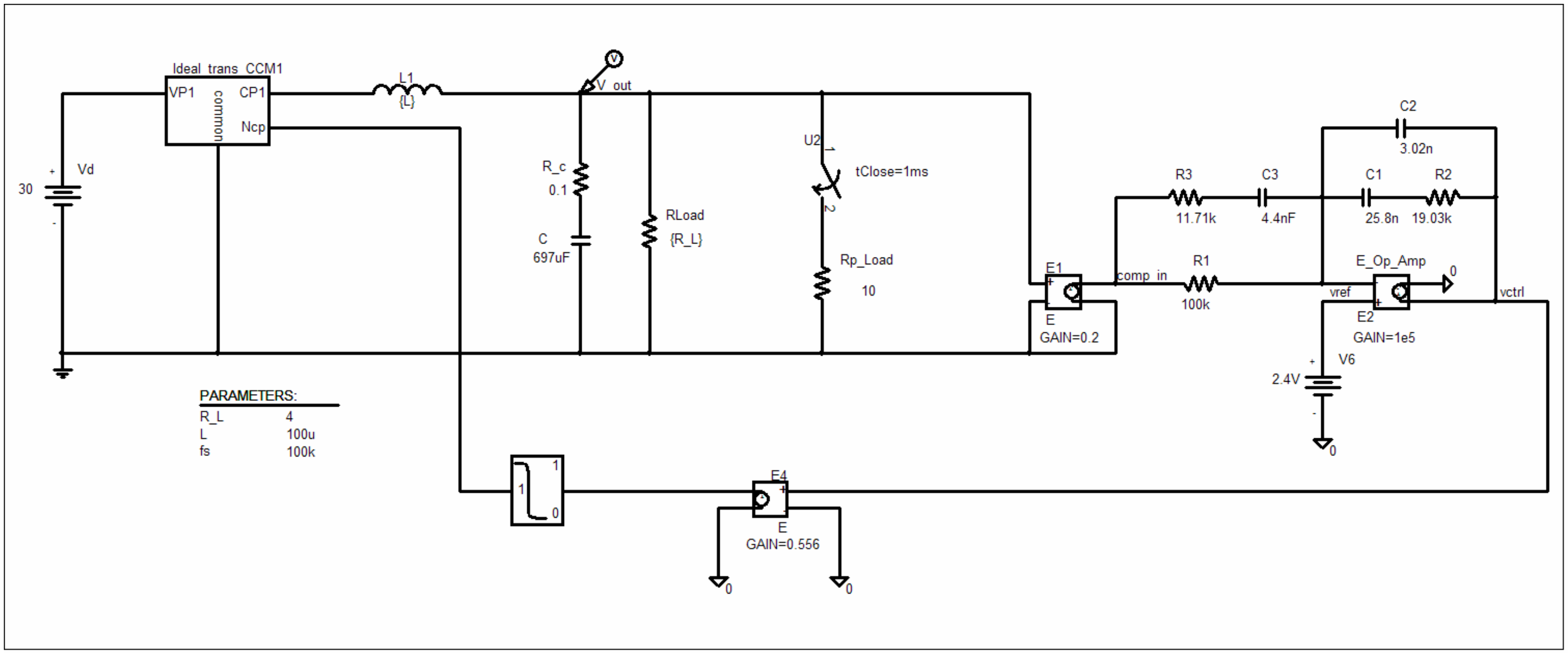
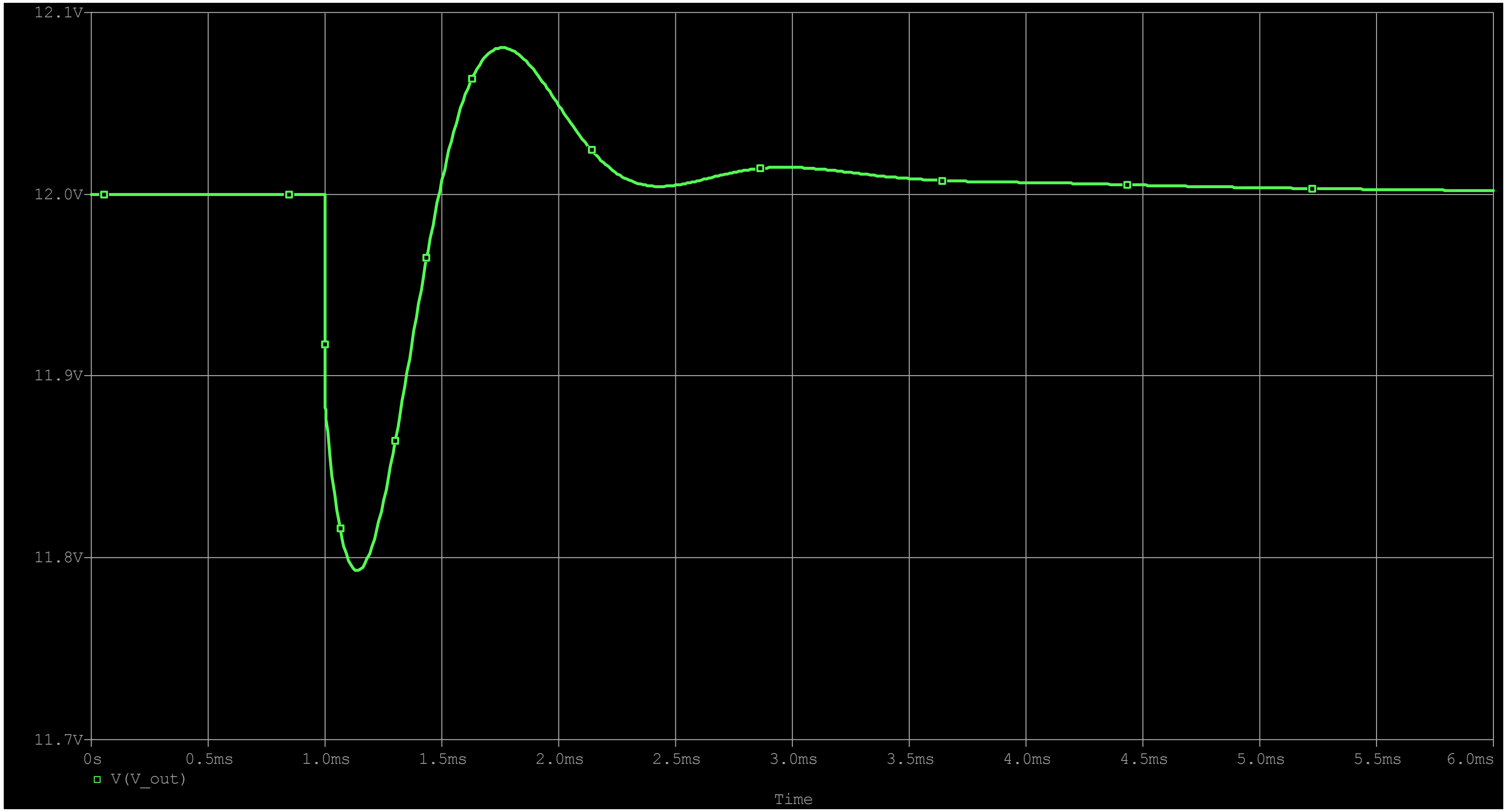


Simulation Results



PEAK-CURRENT MODE CONTROL

- Peak-Current-Mode Control, and
- Average-Current-Mode Control.

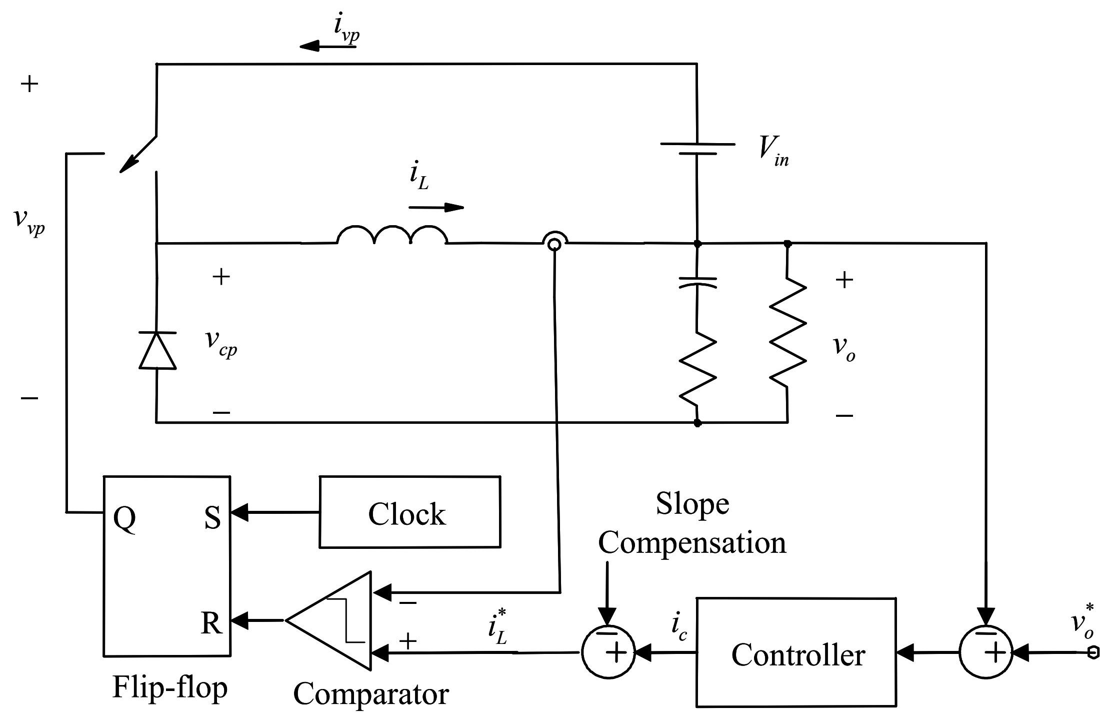
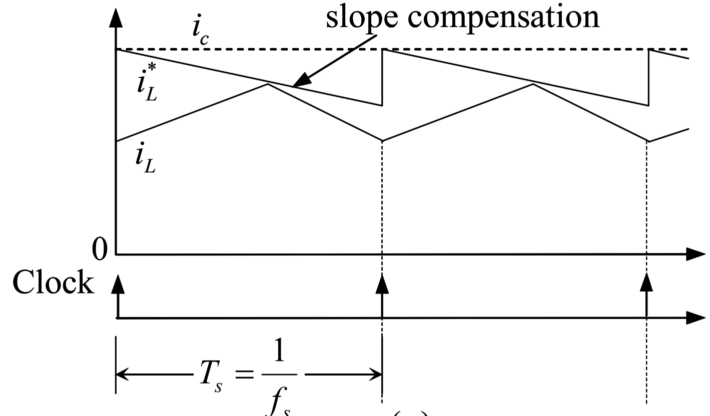
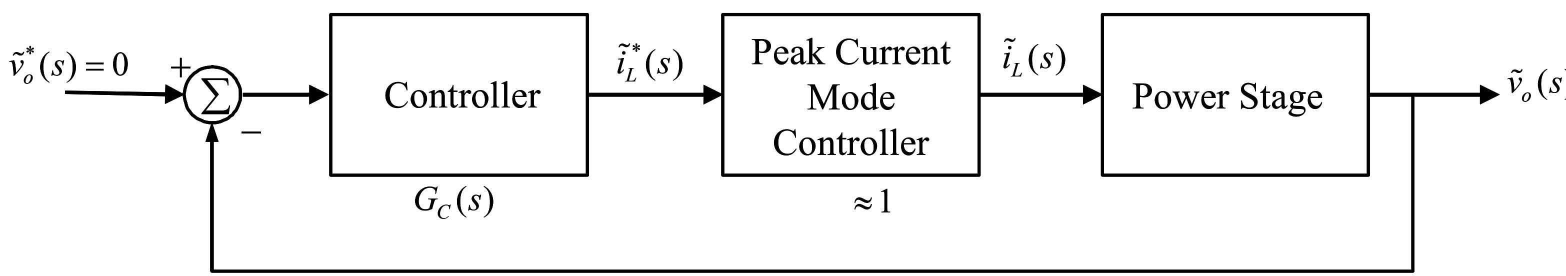


Figure 4-15 Peak current mode control.



(*a*)



(*b*)

Figure 4-16 Peak-current-mode control with slope compensation.

▲ **Example 4-4** In this example, we will design a peak-current-mode controller for a Buck-Boost converter that has the following parameters and operating conditions: $L = 100\,\mu\text{H}$, $C = 697\,\mu\text{F}$, $r = 0.01\Omega$, $f_s = 100\text{ kHz}$, $V_{in} = 30\text{V}$. The output power $P_o = 18\text{W}$ in CCM and the duty-ratio D is adjusted to regulate the output voltage $V_o = 12\text{V}$. The phase margin required for the voltage loop is 60^0. Assume that in the voltage feedback network, $k_{FB} = 1$.

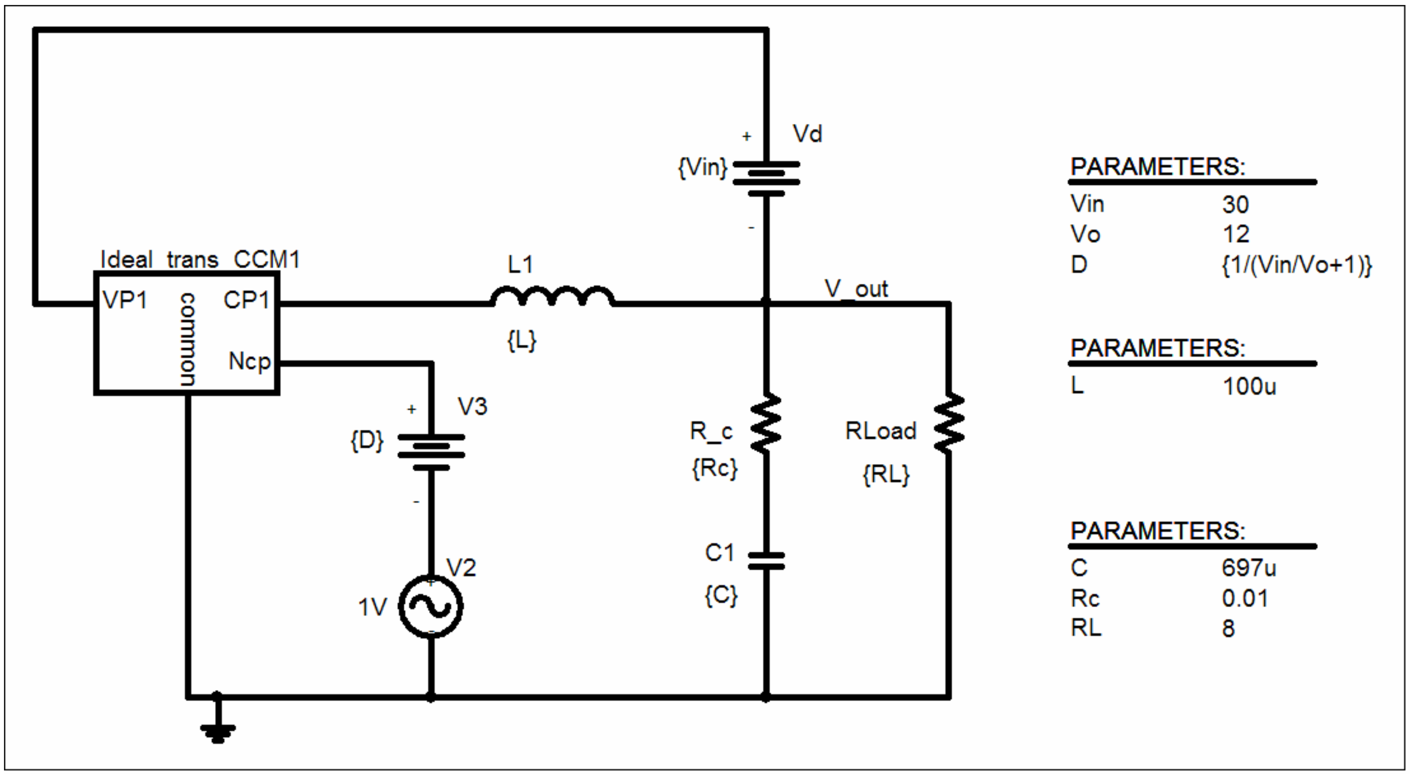


Figure 4-17 PSpice circuit for the Buck-Boost converter.

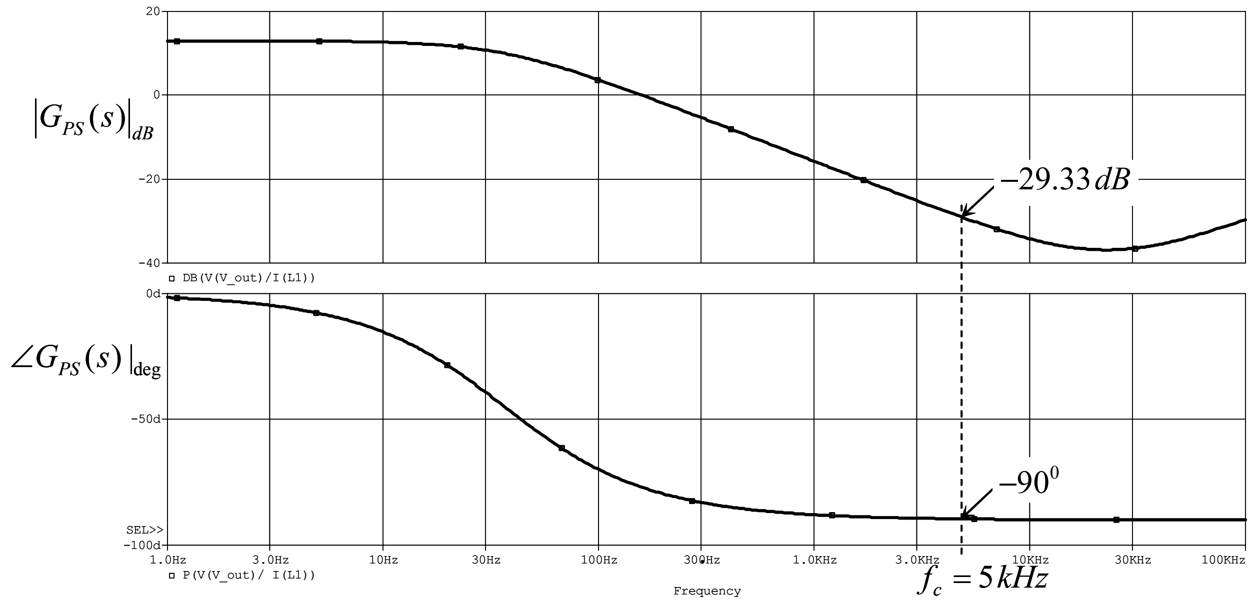


Figure 4-18 Bode plot of $\tilde{v}_o / \tilde{i}_L$.

As shown in Fig. 4-18, the phase angle of the power-stage transfer function levels off at approximately -90^0 at $\sim 1\,kHz$. The crossover frequency is chosen to be $f_c = 5\,kHz$, at which in Fig. 4-18, $\angle G_{PS}(s)\big|_{f_c} \simeq -90^0$. As explained in the Appendix on the accompanying CD, the power-stage transfer function $\tilde{v}_o(s)/\tilde{i}_L(s)$ of Buck-Boost converters contains a right-half-plane zero in CCM. The crossover frequency is chosen well below the frequency of the right-half-plane zero for reasons discussed in the Appendix.

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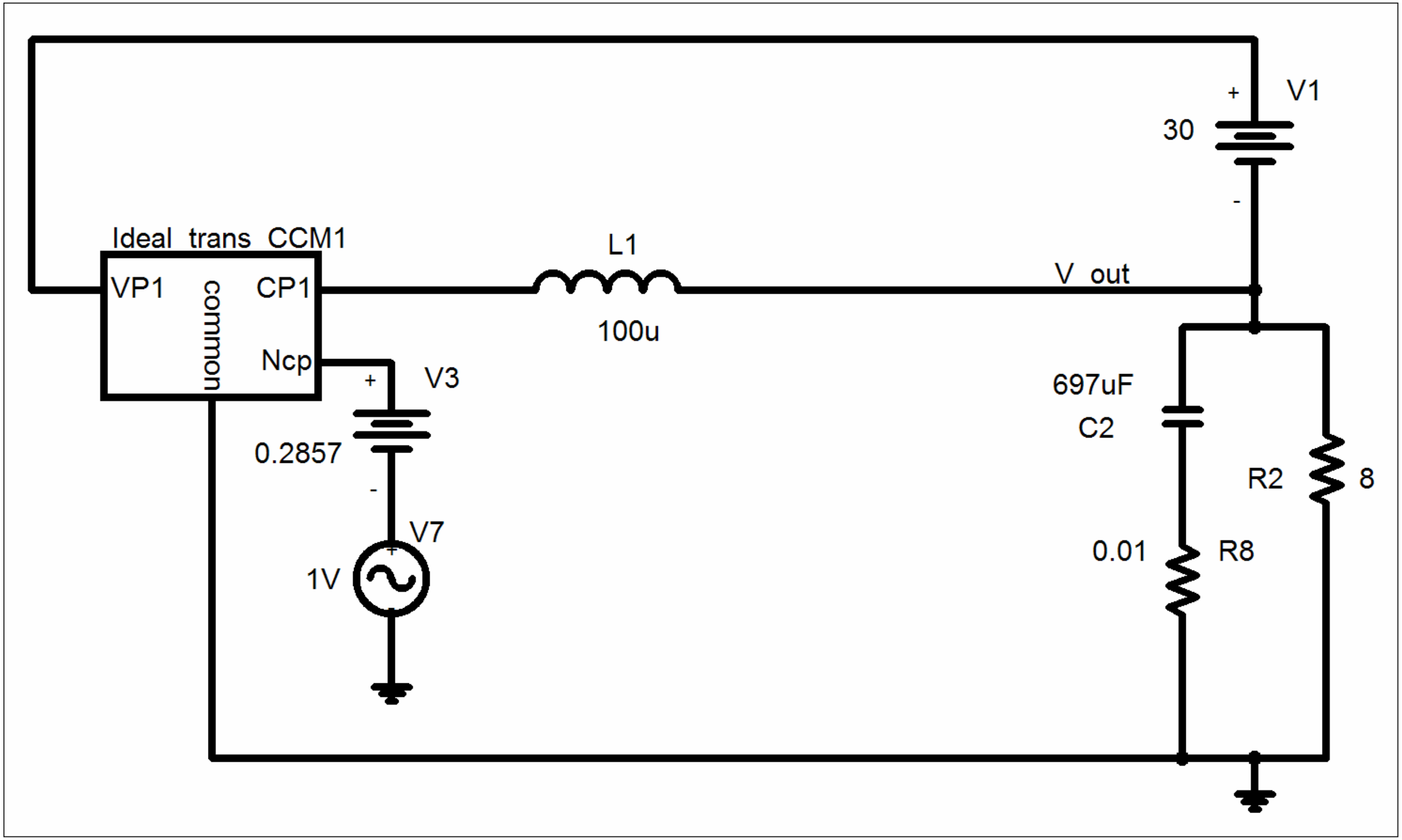
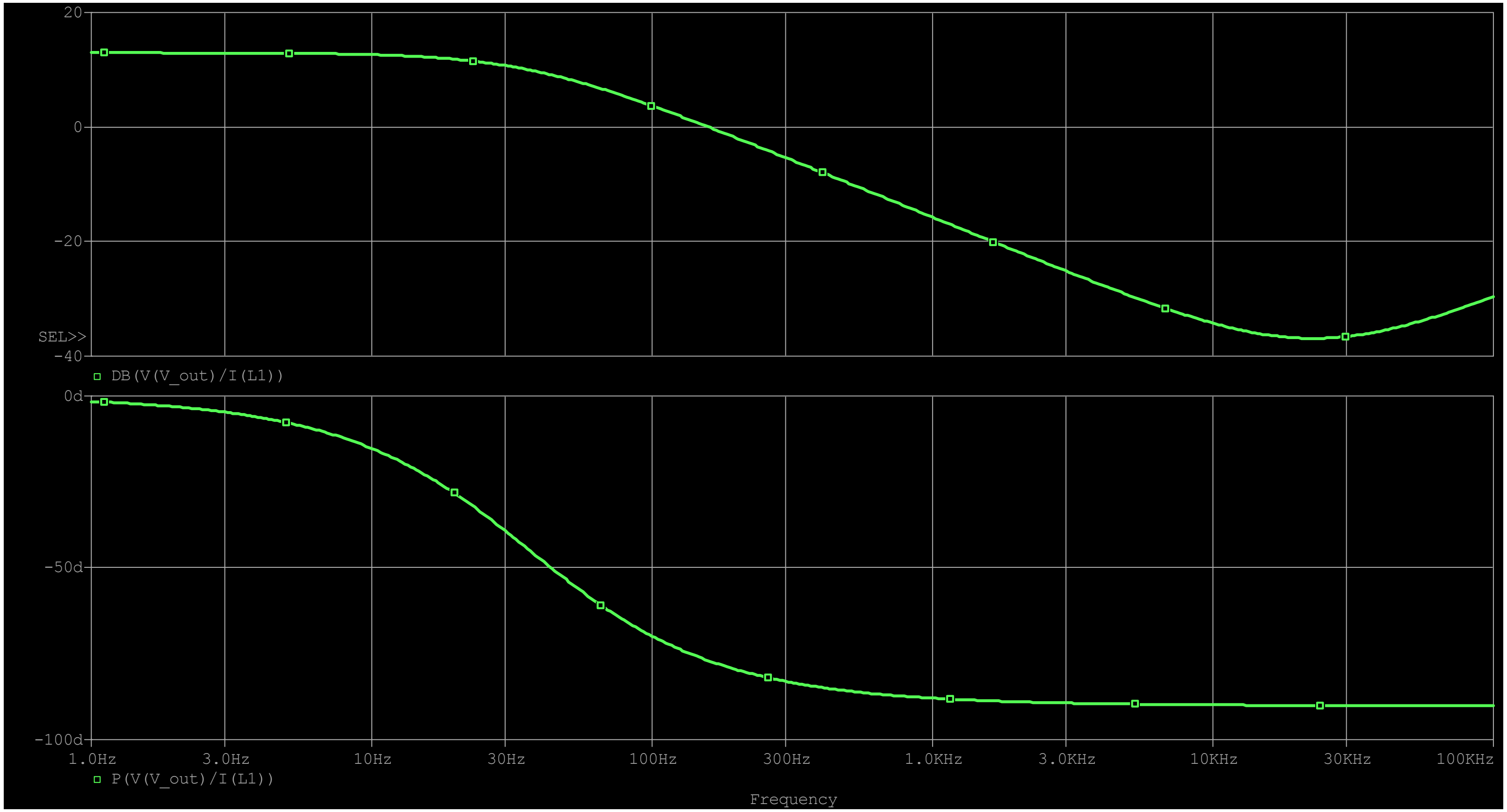


Simulation Results



$$G_c(s) = \frac{k_c}{s} \underbrace{\frac{(1+s/\omega_z)}{(1+s/\omega_p)}}_{phase-boost}$$

$$K_{boost} = \tan\left(45^o + \frac{\phi_{boost}}{2}\right)$$

$$f_z = \frac{f_c}{K_{boost}} \qquad f_p = K_{boost} f_c$$

$$k_c = \omega_z \left|G_C(s)\right|_{f_c}$$

At the crossover frequency, as shown in Fig. 4-18, the power stage transfer function has a gain $\left|G_{PS}(s)\right|_{f_c} = -29.33\,dB$. Therefore, at the crossover frequency, by definition, in Fig. 4-16b

$$\left|G_C(s)\right|_{f_c} \times \left|G_{PS}(s)\right|_{f_c} = 1 \tag{4-37}$$

Hence,

$$\left|G_C(s)\right|_{f_c} = 29.33\,dB = 29.27 \tag{4-38}$$

Using the equations above for $f_c = 5\,kHz$, $\phi_{boost} \simeq 60^0$, and $\left|G_C(s)\right|_{f_c} = 29.27$, $K_{boost} = 3.732$ in Eq. 4-32. Therefore, the parameters in the controller transfer function of Eq. 4-31 are calculated as $f_z = 1340\,Hz$, $f_p = 18660\,Hz$, and $k_c = 246.4 \times 10^3$.

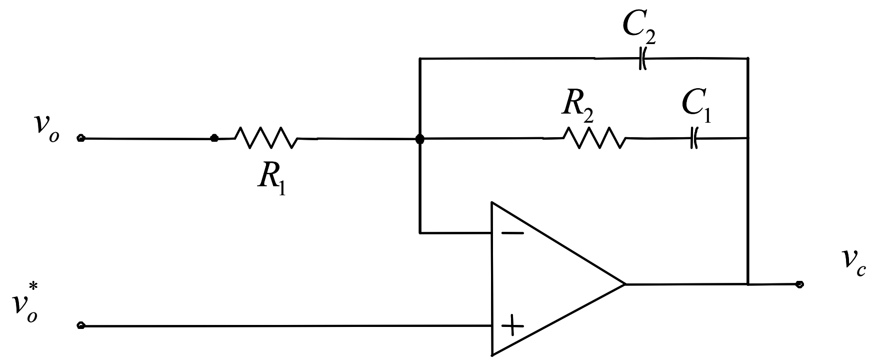


Figure 4-19 Implementation of controller in Eq. 4-32 by an op-amp circuit.

$$R_1 = 10 k\Omega$$

$$C_2 = \frac{\omega_z}{\omega_p R_1 k_c} = 30 \ p\text{F}$$

$$C_1 = C_2 \left(\omega_p / \omega_z - 1\right) = 380 \, p\text{F}$$

$$R_2 = 1/(\omega_z C_1) = 315 k\Omega$$

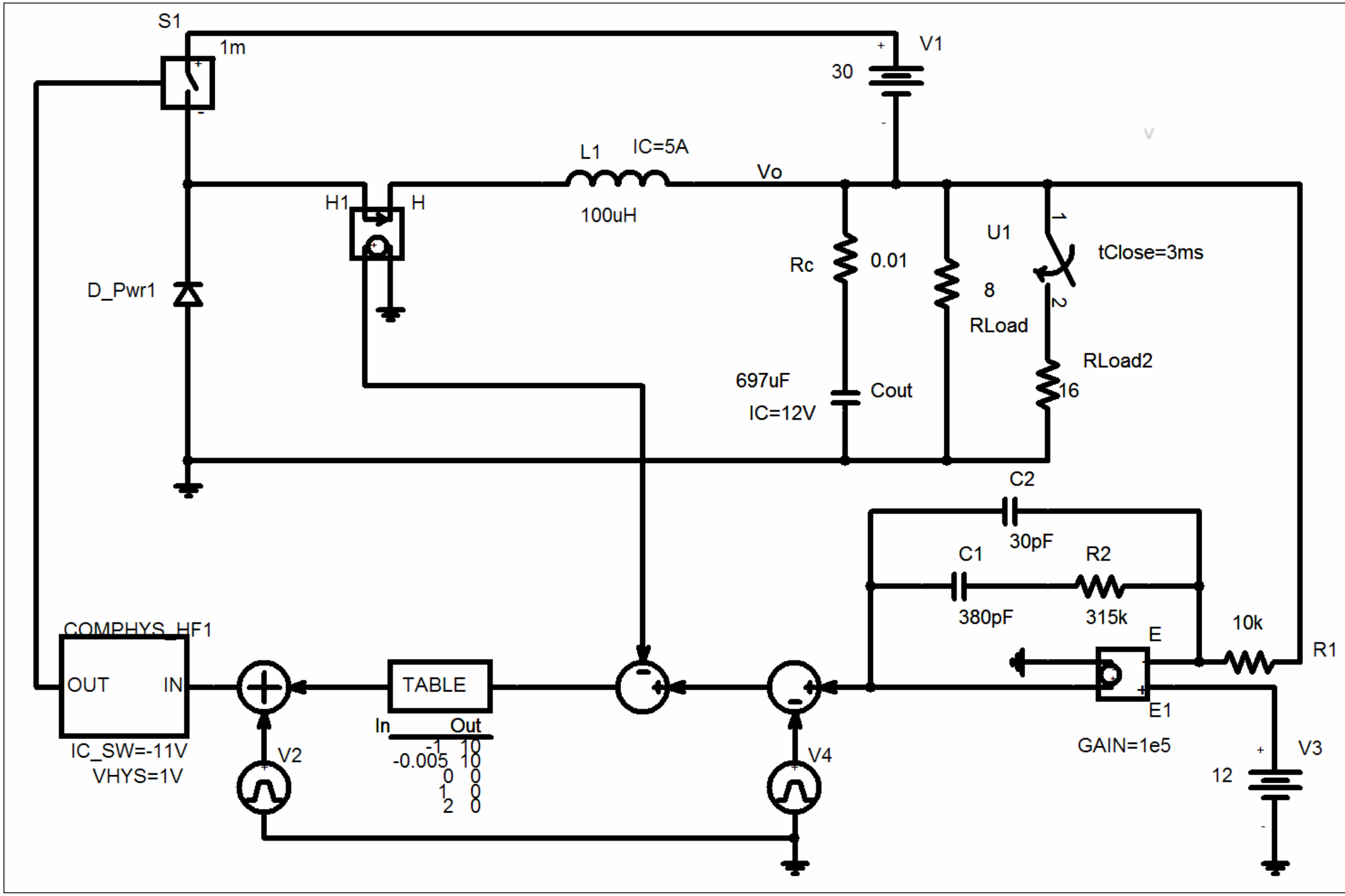


Figure 4-20 PSpice simulation diagram of the peak-current-mode control.

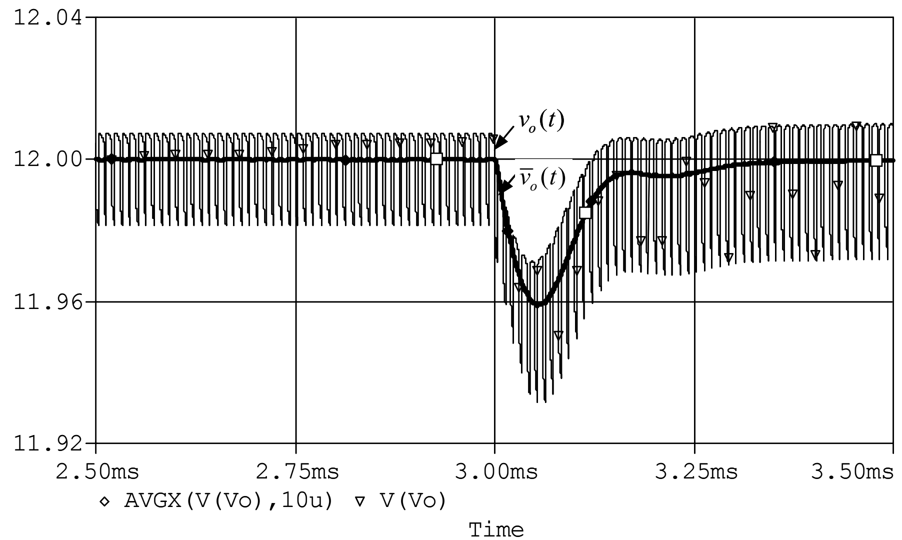


Figure 4-21 Peak current mode control: Output voltage waveform.

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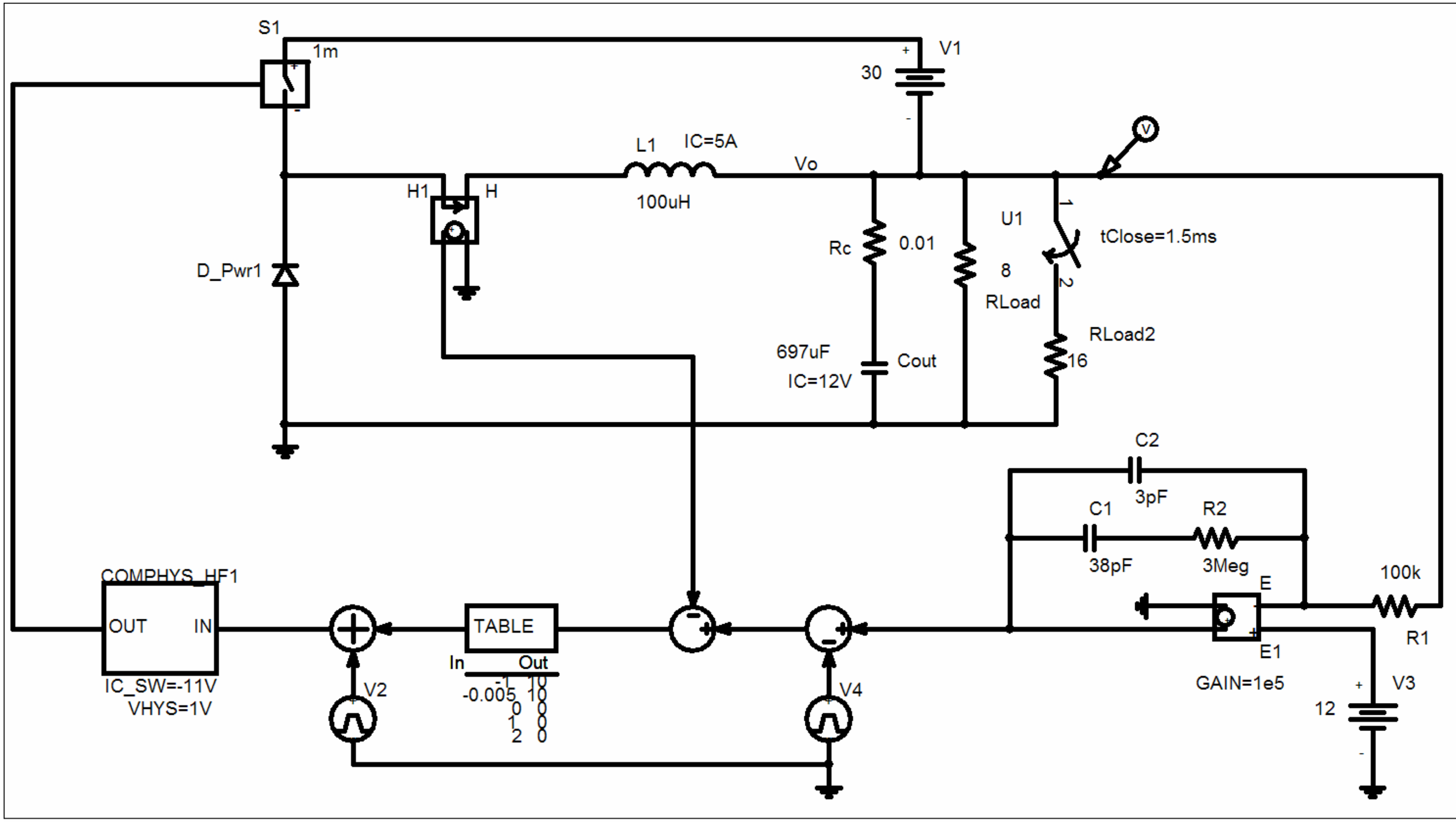
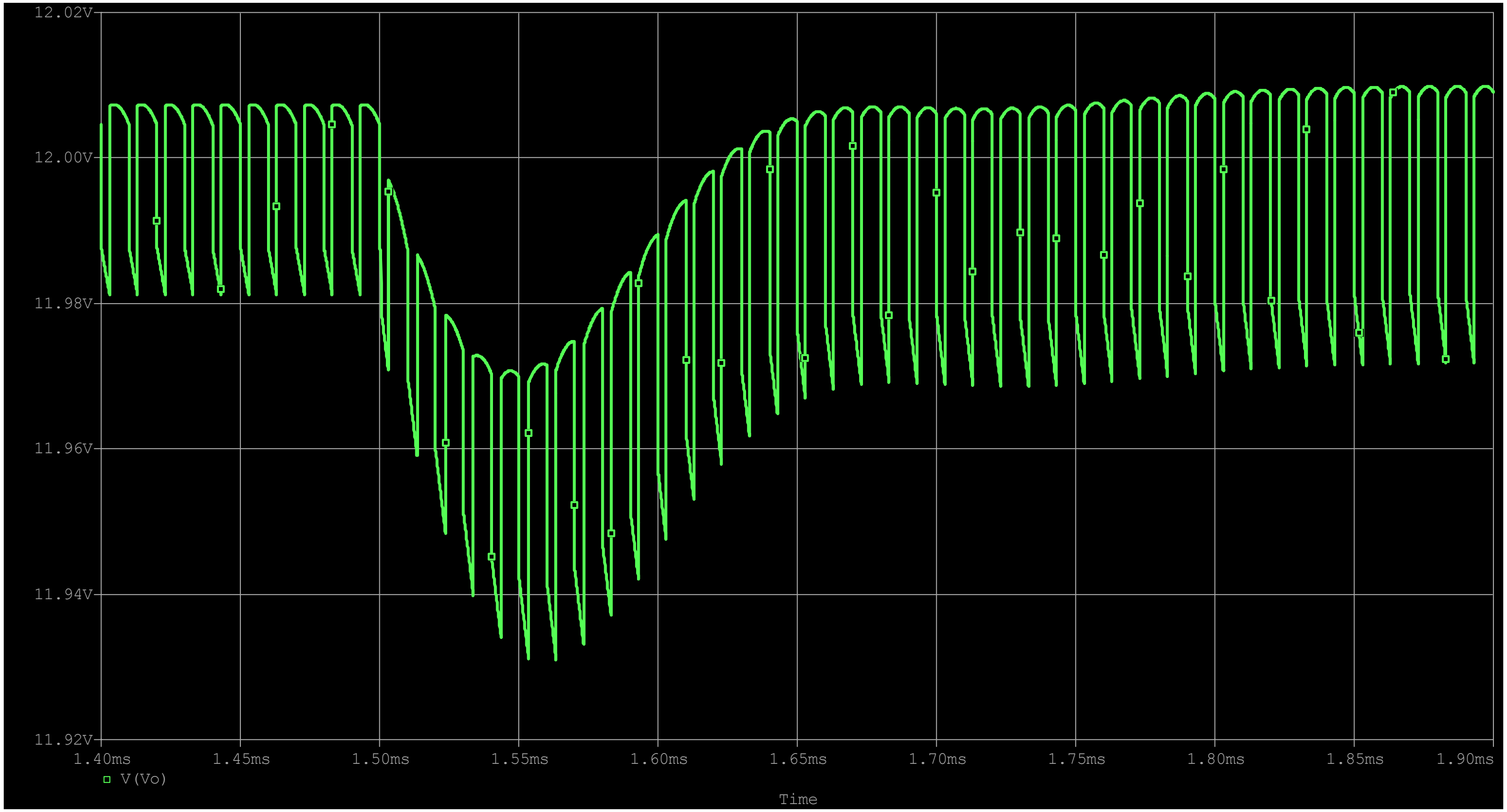
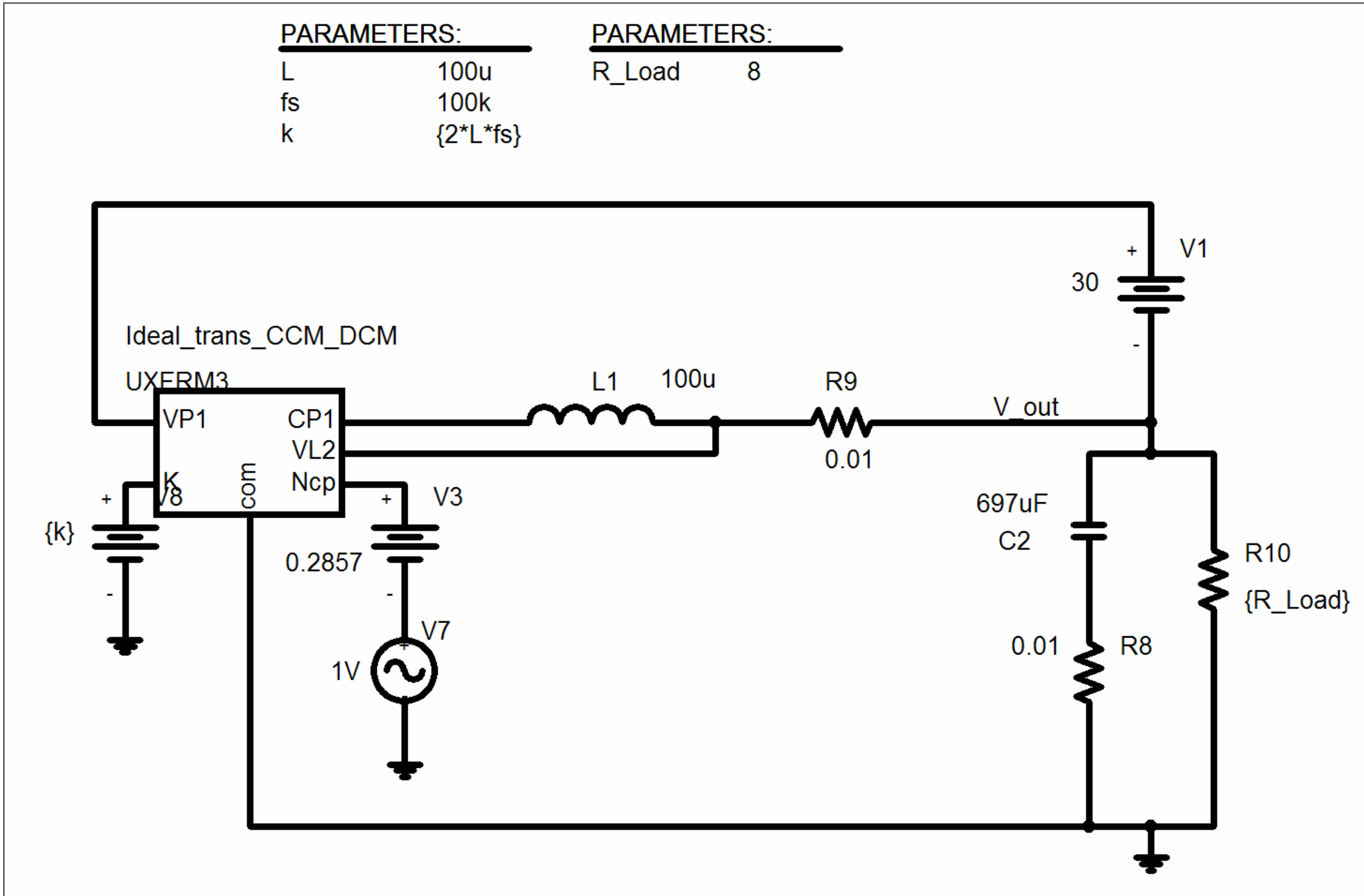


Simulation Results



FEEDBACK CONTROLLER DESIGN IN DCM



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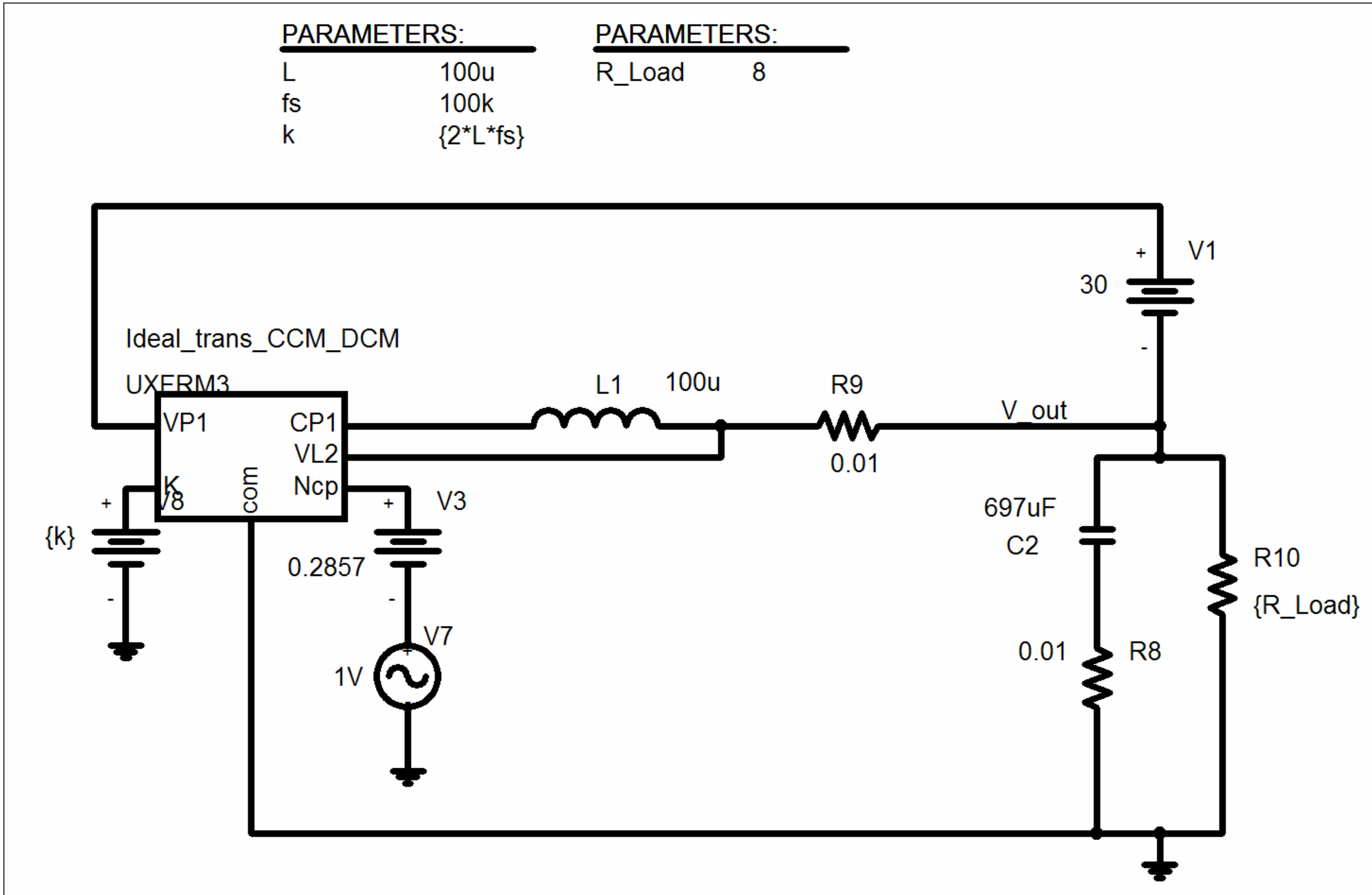

Simulation Results