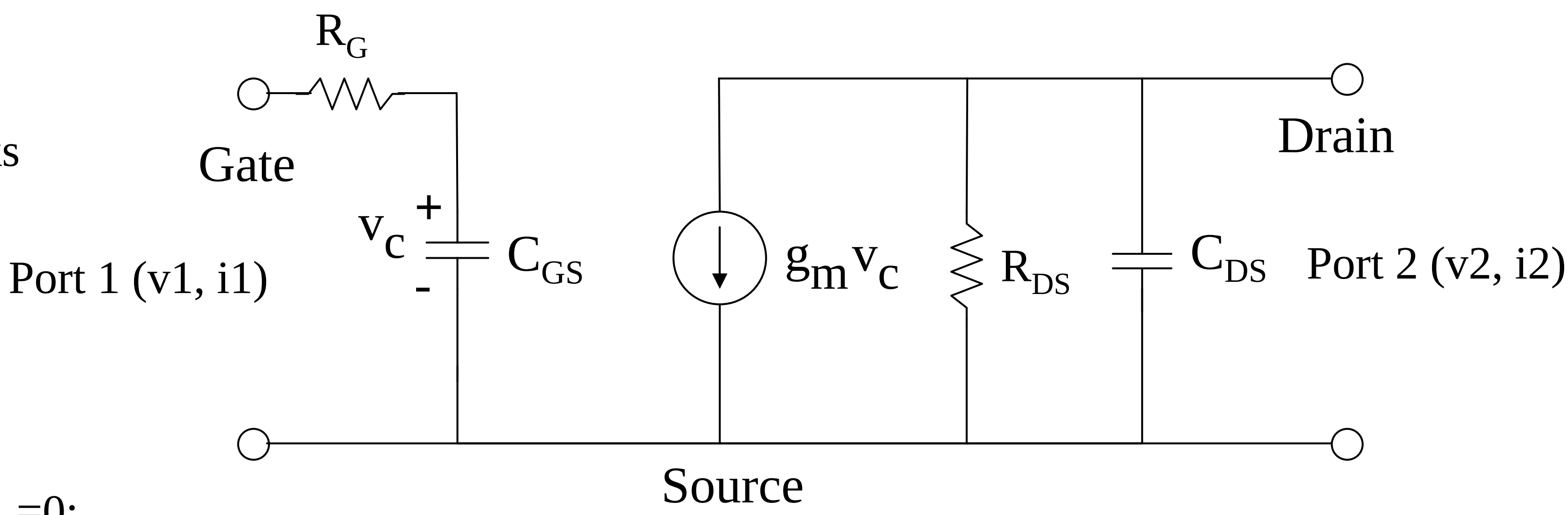
EE4011 RFIC Design Summer 2006

Question 1

(a) 10 marks



Setting v_1=0:

$$v_1 = 0 \Rightarrow i_1 = 0 \Rightarrow v_c = 0 \Rightarrow g_m v_c = 0$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = \frac{0}{v_2} = 0$$ (unilateral property)

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{v_2(1/R_{DS} + j\omega C_{DS})}{v_2} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

Q1(a) continued

Setting v_2=0 gives:

$$i_1 = \frac{v_1}{R_G + \dfrac{1}{j\omega C_{GS}}} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \Rightarrow y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}}$$

$$v_c = v_1 \frac{\dfrac{1}{j\omega C_{GS}}}{R_G + \dfrac{1}{j\omega C_{GS}}} = \frac{v_1}{1 + j\omega R_G C_{GS}} \qquad i_2 = g_m v_c = \frac{g_m v_1}{1 + j\omega R_G C_{GS}} \Rightarrow y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

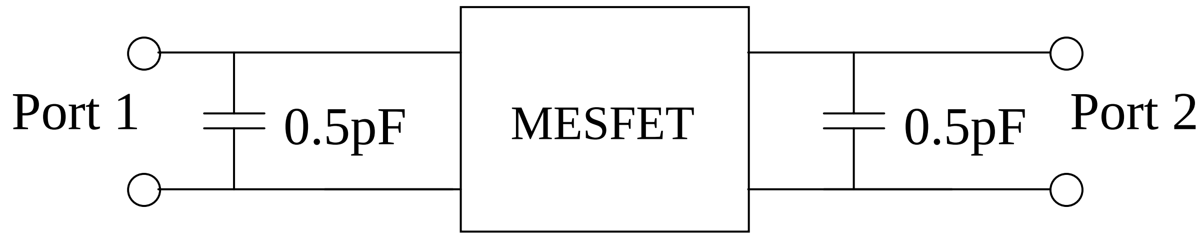
Q1(b) 4 marks

R_G=5Ω, C_{GS}=0.5pF, g_m=0.1S, R_{DS}=100Ω, C_{DS}=0.2pF

Putting the given parameters values into the 4 previous formulas at 1GHz gives:

y_{11} = 0.0001 + j0.0031 = 0.0031 $\angle$ 89°
y_{12} = 0
y_{21} = 0.1 - j0.0016 = 0.1$\angle$ -0.9°
y_{22} = 0.0099 + j0.0012 = 0.01 $\angle$ 7°

Q1(c) 6 marks



The parasitic capacitances will only effect y_{11} and y_{22} by adding an extra capacitive admittance ($j\omega C$ = j0.0031) to each of these. The other two y-parameters are unaffected.

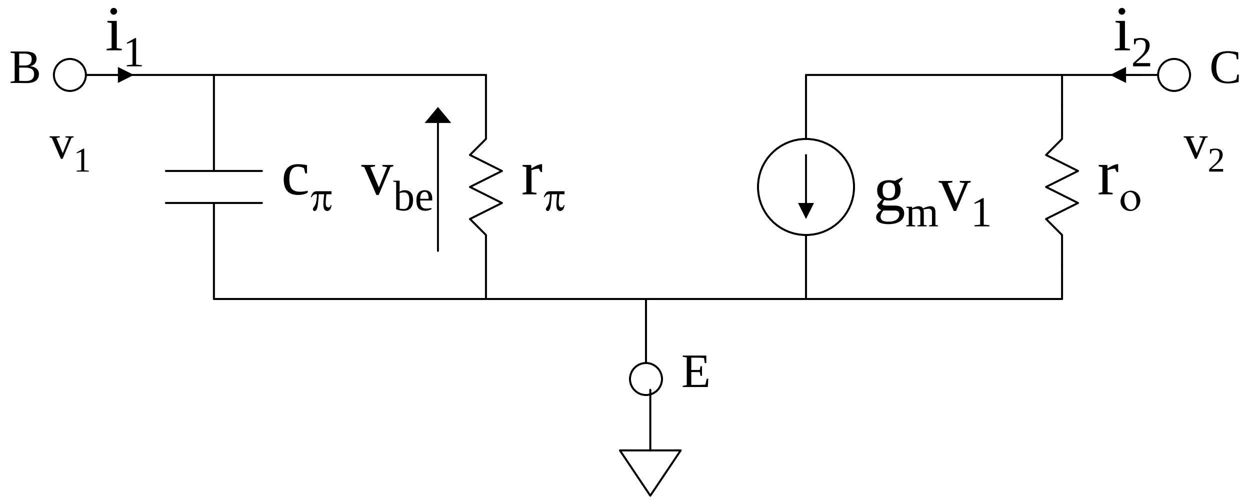
$$y_{11\_NEW} = y_{11\_OLD} + j\omega C_{EXTRA} \quad y_{22\_NEW} = y_{22\_OLD} + j\omega C_{EXTRA}$$

y_{11} = 0.0001 + j0.0062 = 0.0062 $\angle$ 89.1°
y_{12} = 0
y_{21} = 0.1 - j0.0016 = 0.1 $\angle$ -0.9°
y_{22} =0.0099 + j0.0043 = 0.01 $\angle$ 23.7°

Question 2

(a) 12 marks

A BJT small-signal model taking only base-emitter capacitances into account is:



The z-parameters are defined as:

$$v_1 = z_{11} i_1 + z_{12} i_2$$
$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} \qquad z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0}$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} \qquad z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$$

Q2(a) continued

Performing a circuit analysis with $i_2 = 0$ gives:

$$i_1 = v_1\left(j\omega c_\pi + \frac{1}{r_\pi}\right) \Rightarrow z_{11} = \frac{v_1}{i_1} = \frac{1}{j\omega c_\pi + \frac{1}{r_\pi}} = \frac{r_\pi}{1 + j\omega r_\pi c_\pi}$$

$$v_2 = -g_m v_1 r_o \qquad z_{21} = \frac{v_2}{i_1} = \frac{-g_m v_1 r_o}{v_1\left(j\omega c_\pi + \frac{1}{r_\pi}\right)} = \frac{-g_m r_o r_\pi}{1 + j\omega r_\pi c_\pi}$$

Performing a circuit analysis with $i_1 = 0$:
$i_1=0$ can only happen if $v_1=0$. If $v_1=0$ then $g_m v_1=0$ and only r_o contributes to i_2.

$$z_{12} = \frac{v_1}{i_2} = \frac{0}{i_2} = 0$$

$$i_2 = \frac{v_2}{r_o} \Rightarrow z_{22} = \frac{v_2}{i_2} = r_o$$

Question 2(b) 8 marks

The following bias point information is provided:

$k = 1.38 \times 10^{-23}$ J/K $q = 1.602 \times 10^{-19}$ C   f = 2GHz

V_{BE}=0.75 V, V_{CE}=3.0 V, T=300 K $I_S=10^{-15}$ A, β=100, V_A=10V,
$C_{JE}=0.3\times10^{-12}$ F, V_{JE}=1.0 V, M_{JE}=0.5, $\tau_F=0.1\times10^{-9}$ s.

$$V_T = \frac{kT}{q} = 25.8mV \quad I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right) = 5.2mA$$

$$g_m = \frac{I_C}{V_T} = 0.202S \quad \mathrm{r_o} = \frac{V_{AF}}{I_C} = 1914.6\ \Omega$$

$$\mathrm{c}_\pi = \frac{\mathrm{C_{JE}}}{\left(1-\frac{\mathrm{V_{BE}}}{\mathrm{V_{JE}}}\right)^{\mathrm{M_{JE}}}} + g_m\tau_F = 6\times10^{-13} + 2.02\times10^{-11} = 2.08\times10^{-11}F$$

$$\mathrm{r}_\pi = \frac{\beta}{g_m} = 494.8\Omega$$

Question 2(b) continued

Inserting these values into the previous formulas at 2GHz:

$$z_{11} = \frac{r_\pi}{1 + j\omega r_\pi c_\pi}$$

$$z_{21} = \frac{-g_m r_o r_\pi}{1 + j\omega r_\pi c_\pi}$$

$$z_{12} = 0$$

$$z_{22} = r_o$$

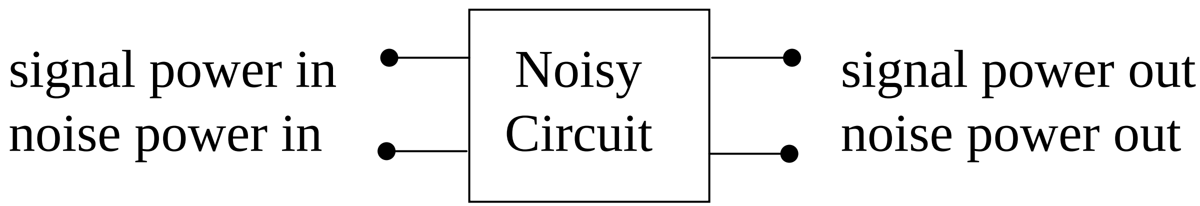
gives:

z_{11} = 0.0295 - j3.823 = 3.823 $\angle$ -89.6°
z_{12} = 0
z_{21} = -11.43 + j1479.6 = 1479.6$\angle$ 90.4°
z_{22} = 1914 = 1914 $\angle$ 0°

Question 3

3(a) 4 marks

The noise factor of a two-port network is defined below:



$$SNR_{in} = (\text{signal power in})/(\text{noise power in})$$
$$SNR_{out} = (\text{signal power out})/(\text{noise power out})$$

$$\text{Noise Factor, F} = \frac{SNR_{in}}{SNR_{out}} \quad (\geq 1)$$

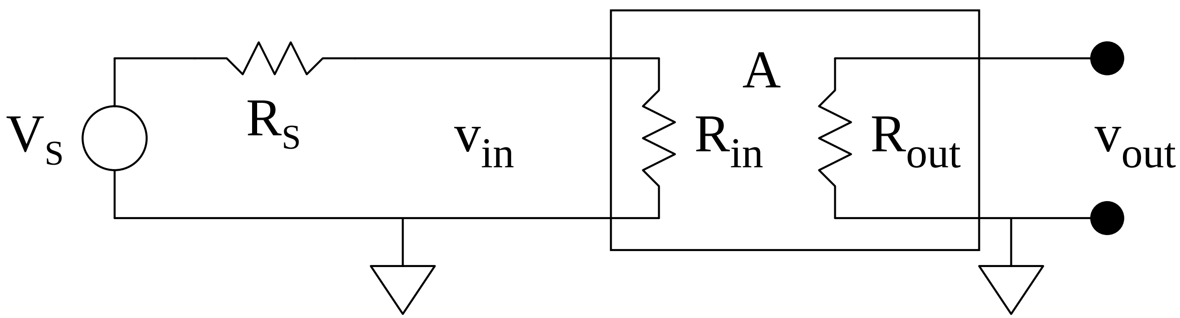
Question 3 continued

3(b) 16 marks

Expression for noise figure of two-port network driven by source impedance R_S.

Typical two-port with input and output resistance and voltage gain A:

Determine the input and output signal voltages only:



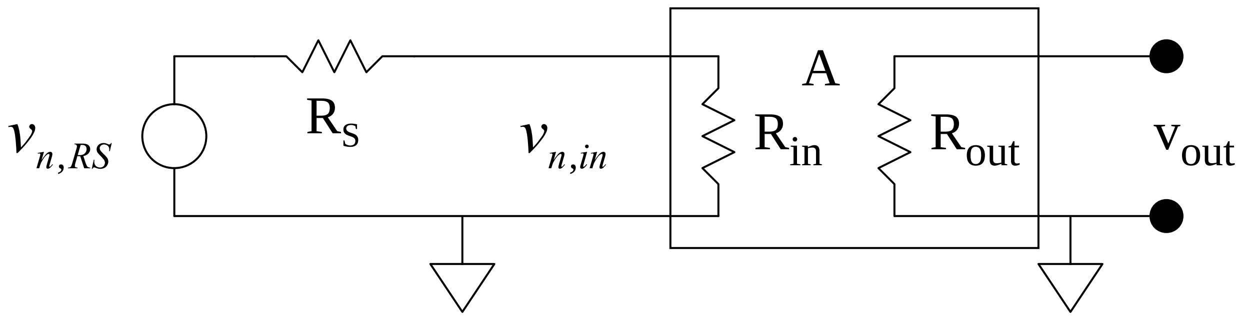
$$v_{in} = \frac{R_{in}}{R_{in}+R_S} V_S = \alpha V_S \qquad \alpha = \frac{R_{in}}{R_{in}+R_S}$$

$$v_{out} = A v_{in} = \alpha A V_S$$

$$v_{out}^2 = A^2 v_{in}^2 = \alpha^2 A^2 V_S^2$$

Question 3(b) continued

Analyse the noise taking into account the noise of the source resistance only:



The input noise voltage due to the source resistance alone is:

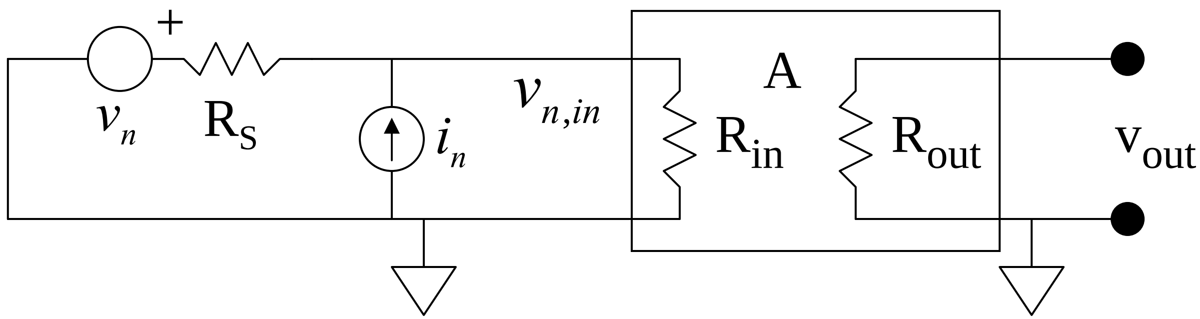
$$v_{n,in} = \frac{R_{in}}{R_{in} + R_S} v_{n,RS} = \alpha\, v_{n,RS} \qquad \overline{v_{n,in}^2} = \alpha^2 \overline{v_{n,RS}^2}$$

The signal to noise ratio at the input is:

$$SNR_{in} = \frac{v_{in}^2}{\overline{v_{n,in}^2}} = \frac{\alpha^2 V_S^2}{\alpha^2 \overline{v_{n,RS}^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} = \frac{V_S^2}{4kTR_S \Delta f}$$

Question 3(b) continued

Now analyze the effect of the input-referred noise sources alone. The voltage and current sources must be analyzed together because they are correlated. The noise voltage is moved to “voltage side” of the source resistor to make the circuit analysis easier:



After some circuit analysis on the input circuit:

$$v_{n,in} = \frac{R_{in}}{R_{in}+R_S}\left(v_n + R_S i_n\right) = \alpha\left(v_n + R_S i_n\right) \Rightarrow \overline{v_{n,in}^2} = \alpha^2\overline{\left(v_n + R_S i_n\right)^2}$$

At this stage we have expressions for the input noise voltage caused by the source resistance and the input noise voltage caused by the input-referred sources of the two-port. Because these are uncorrelated the associated mean square values can be added.

Question 3(b) continued

Adding the contributions of the source noise and the input-referred noise sources:

$$\left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 \overline{v_{n,RS}^2} + \alpha^2 \overline{\left(v_n + R_S i_n\right)^2}$$

$$\overline{v_{n,out}^2} = A^2 \left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}$$

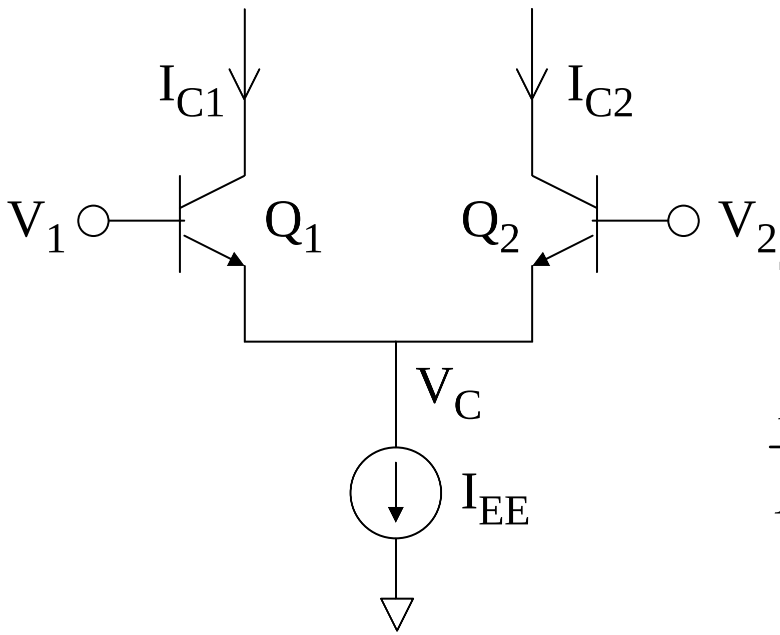
$$SNR_{out} = \frac{v_{out}^2}{\overline{v_{n,out}^2}} = \frac{\alpha^2 A^2 V_S^2}{\alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{V_S^2} = \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}}$$

$$F = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S \Delta f} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S} \text{ for } \Delta f = 1 Hz$$

Question 6

6(a) 12 marks



$$I_{C1} = I_S e^{\frac{V_{BE1}}{V_T}} = I_S e^{\frac{V_1 - V_C}{V_T}} \quad V_1 - V_C >> V_T$$

$$I_{C2} = I_S e^{\frac{V_{BE2}}{V_T}} = I_S e^{\frac{V_2 - V_C}{V_T}} \quad V_2 - V_C >> V_T$$

Then:

$$\frac{I_{C1}}{I_{C2}} = \frac{I_S e^{\frac{V_1 - V_C}{V_T}}}{I_S e^{\frac{V_2 - V_C}{V_T}}} = e^{\frac{V_1 - V_2}{V_T}} = e^{\frac{V_d}{V_T}}$$

$$\Rightarrow \frac{I_{C2}}{I_{C1}} = e^{-\frac{V_d}{V_T}} \quad \text{where } V_d = V_1 - V_2$$

Q6(a) continued

Ignoring base current:

$$I_{C1}+I_{C2}=I_{EE}\Rightarrow I_{C1}+I_{C1}e^{-\frac{V_d}{V_T}}=I_{EE}\Rightarrow I_{C1}=\frac{I_{EE}}{1+e^{-\frac{V_d}{V_T}}}\qquad I_{C2}e^{\frac{V_d}{V_T}}+I_{C2}=I_{EE}\Rightarrow I_{C2}=\frac{I_{EE}}{1+e^{\frac{V_d}{V_T}}}$$

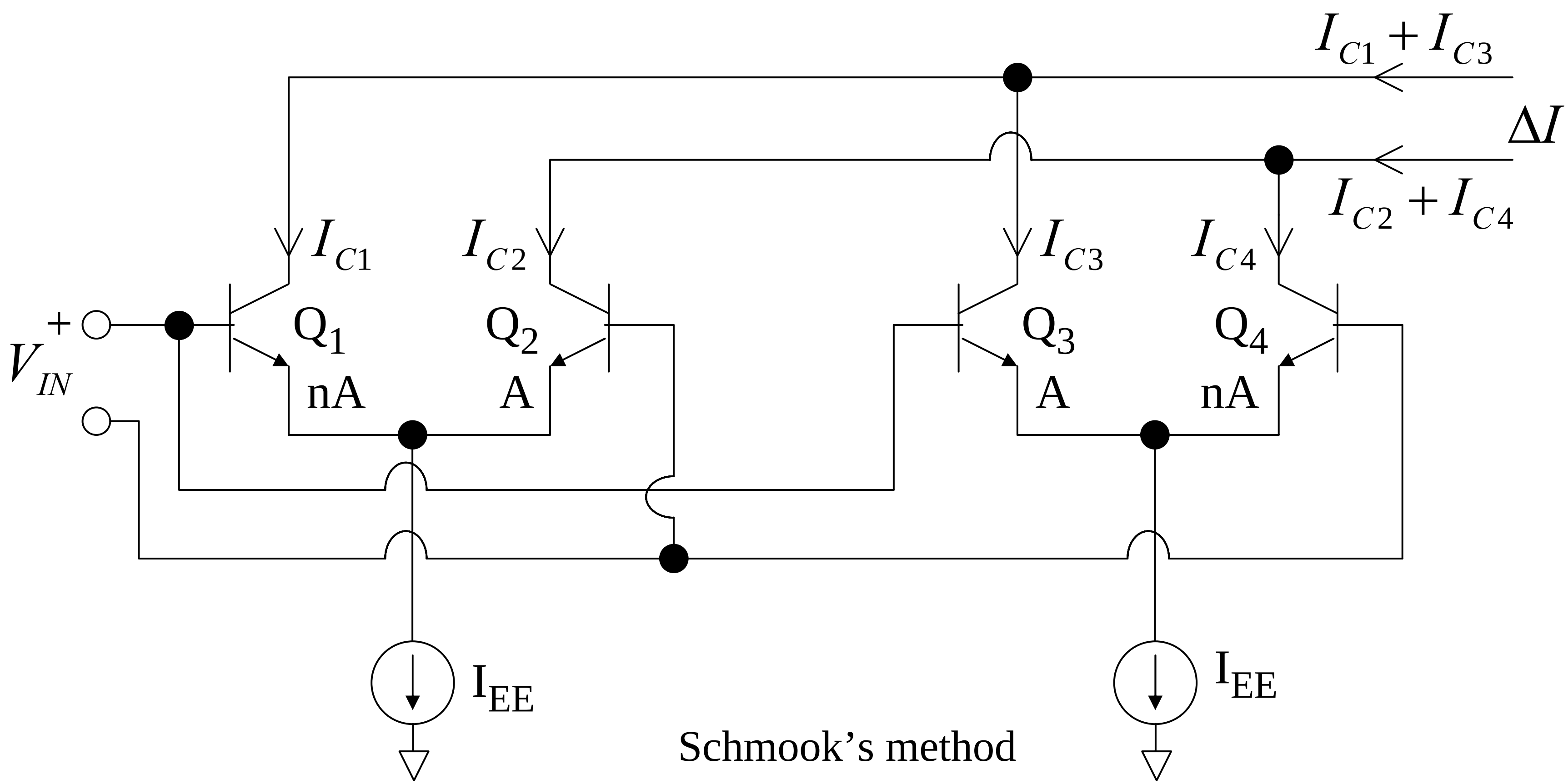
$$\Delta I_C=I_{C1}-I_{C2}=\frac{I_{EE}}{1+e^{-\frac{V_d}{V_T}}}-\frac{I_{EE}}{1+e^{\frac{V_d}{V_T}}}=I_{EE}\left(\frac{1}{1+e^{-\frac{V_d}{V_T}}}-\frac{1}{1+e^{\frac{V_d}{V_T}}}\right)$$

$$=I_{EE}\left(\frac{e^{\frac{V_d}{V_T}}}{e^{\frac{V_d}{V_T}}+1}-\frac{1}{1+e^{\frac{V_d}{V_T}}}\right)=I_{EE}\,\frac{e^{\frac{V_d}{V_T}}-1}{e^{\frac{V_d}{V_T}}+1}$$

$$=I_{EE}\,\frac{e^{\frac{V_d}{2V_T}}-e^{-\frac{V_d}{2V_T}}}{e^{\frac{V_d}{2V_T}}+e^{-\frac{V_d}{2V_T}}}=I_{EE}\tanh\left(\frac{V_d}{2V_T}\right)$$

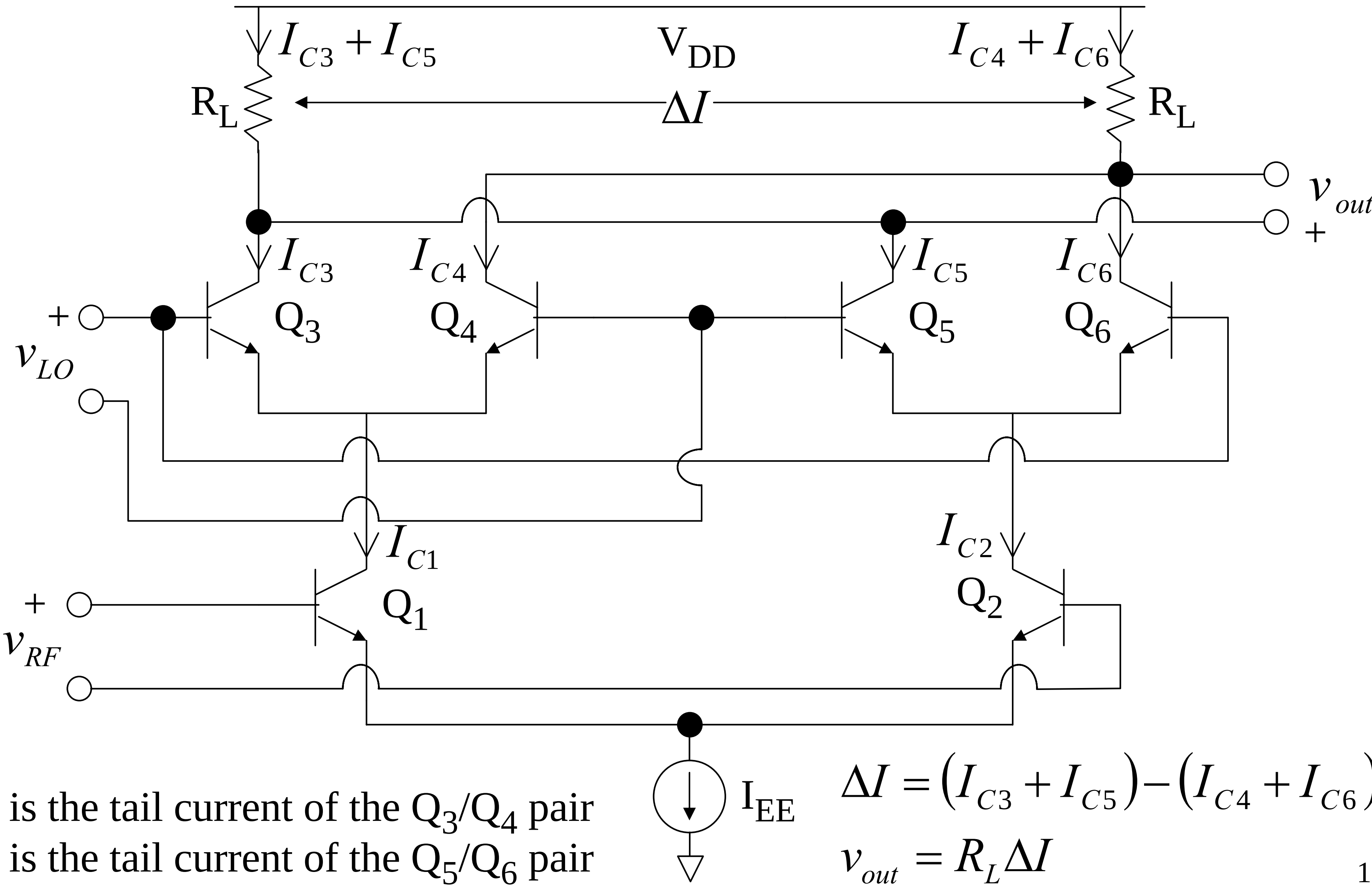
Q6(b) 4 marks

Two methods of increasing the dynamic range of the common emitter pairs are (a) by combining pairs with different emitter ratios (Schmook's method) and (b) by adding resistors in series with the emitter terminals of the devices.



Schmook's method

Q6(c) 4 marks A Gilbert Cell Double Balanced Mixer



I_{C1} is the tail current of the Q_3/Q_4 pair

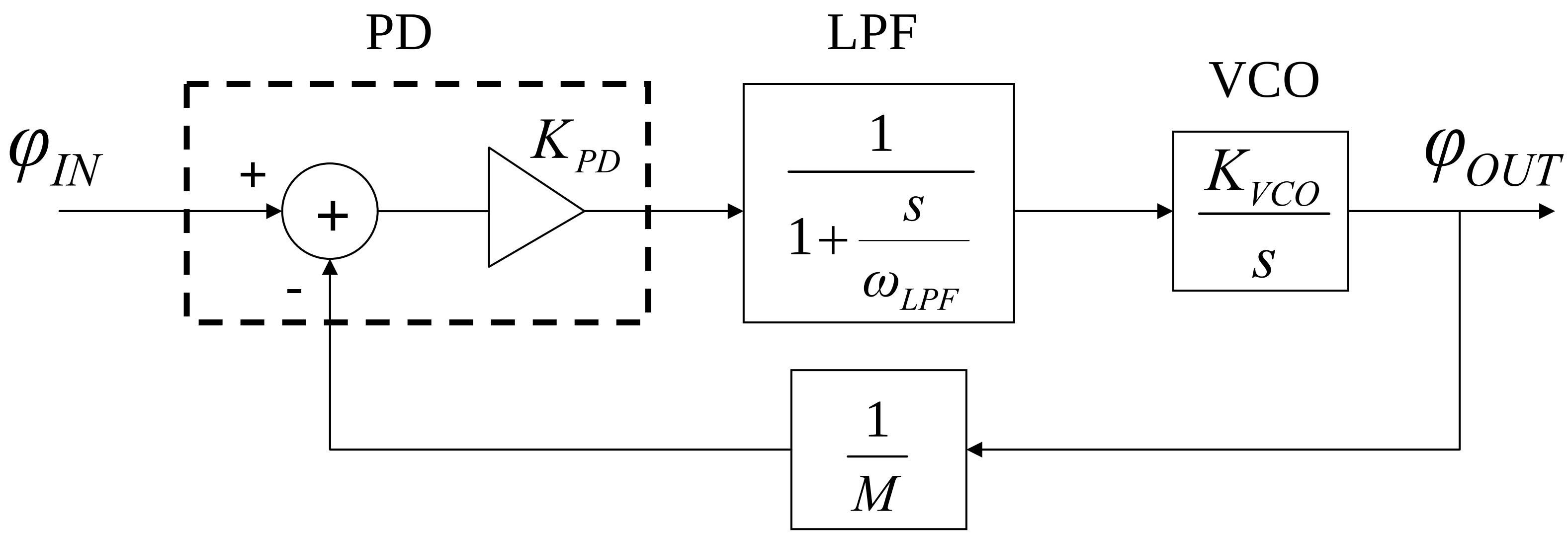
I_{C2} is the tail current of the Q_5/Q_6 pair

$$\Delta I = \left(I_{C3}+I_{C5}\right)-\left(I_{C4}+I_{C6}\right)$$

$$v_{out} = R_L \Delta I$$

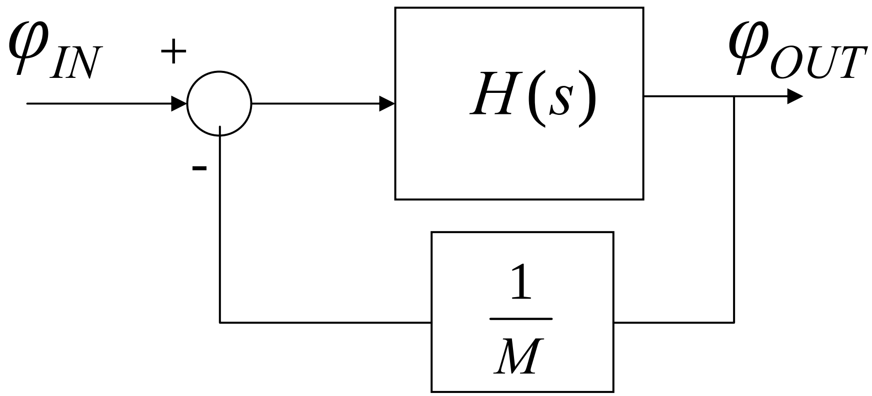
Question 7

Q7(a) 4 marks A Type 1 PLL with integer feedback divider



$$H(s)\Big|_{OPEN} = \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)}\Bigg|_{OPEN} = K_{PD} \cdot \frac{1}{1+\frac{s}{\omega_{LPF}}} \cdot \frac{K_{VCO}}{s} = \frac{K_{PD}K_{VCO}}{s+\frac{s^2}{\omega_{LPF}}}$$

Q7(b) Closed-loop transfer function of Type I PLL:



$$\varphi_{OUT}(s) = H(s)\left(\varphi_{IN}(s) - \frac{\varphi_{OUT}(s)}{M}\right) \Rightarrow \varphi_{OUT}(s)\left(1 + \frac{H(s)}{M}\right) = H(s)\varphi_{IN}(s)$$

$$\Rightarrow \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)} = \frac{H(s)}{1 + \frac{H(s)}{M}} = \frac{1}{\frac{1}{H(s)} + \frac{1}{M}} = \frac{1}{\frac{s + \frac{s^2}{\omega_{LPF}}}{K_{PD}K_{VCO}} + \frac{1}{M}} = \frac{K_{PD}K_{VCO}}{s + \frac{s^2}{\omega_{LPF}} + \frac{K_{PD}K_{VCO}}{M}}$$

$$= \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + \omega_{LPF}s + \frac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$$

Q7(b) continued

$$H(s)\Big|_{CLOSED} = \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + \omega_{LPF}s + \dfrac{K_{PD}K_{VCO}\omega_{LPF}}{M}} \equiv \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + 2\varsigma\,\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$$

$$2\varsigma\,\omega_n = \omega_{LPF} \Rightarrow \varsigma = \frac{1}{2}\frac{\omega_{LPF}}{\omega_n} = \frac{1}{2}\sqrt{\frac{M\omega_{LPF}}{K_{PD}K_{VCO}}}$$

Q7(c)

(i) 2 marks

For an integer feedback the reference frequency must be equal to the desired step size i.e. 200kHz in this case

Q7(c)

(ii) 2 marks

The range of divider values needed are:

$$M = \frac{890}{0.2} = 4450 \qquad to \qquad M = \frac{915}{0.2} = 4575$$

(iii) 2 marks

A rule of thumb to ensure good stability is to set the low-pass filter cut-off frequency to 10% of the reference frequency i.e. 20kHz in this case

(iv) 2 marks

Using an average divide ratio and the values already set:

$$\varsigma = \frac{1}{2}\sqrt{\frac{M\omega_{LPF}}{K_{PD}K_{VCO}}} \Rightarrow K_{PD}K_{VCO} = \frac{M\omega_{LPF}}{4\varsigma^2}$$

$$K_{PD}K_{VCO} = \frac{M\omega_{LPF}}{4\varsigma^2} = \frac{4512.5 \times 2\pi \times 20000}{4 \times 0.707^2} = 2.8 \times 10^8$$