Chapter 9

Introduction to AC Machines

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9-2

Introduction

- Primary AC motor drives
 - x Induction motors (asynchronous)
 - x Squirrel cage brushless
 - x Wound rotor brushed

OFI - Doubly Fed Induction Motories Prince

- x Synchronous Motors
 - x Permanent Magnet brushless
 - x Wound rotor brushed

> Synchronous Generator

x These machines have similar stators but different rotor constructions.

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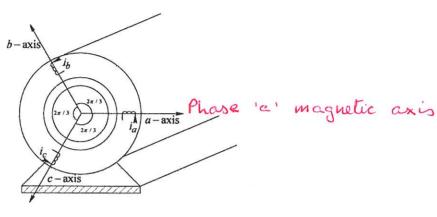




Introduction

Stator windings produce a sinusoidal field distribution in the airgap.

These magnetic field distributions are displaced by 120 ° w.r.t. each other.

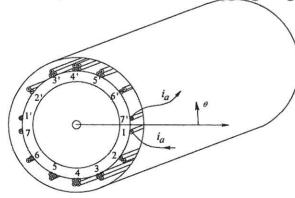


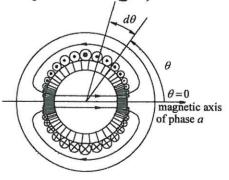
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9-4

Sinusoidally-distributed Stator Windings

(number of windings per phase vary with angle)





Conductor density $n_s(\theta) = \hat{n}_s \sin \theta = number of semiconductors per raclian, 0 LOCA$

Total

$$N_s = \int_0^{\pi} n_s(\theta) d\theta = \int_0^{\pi} \hat{n}_s \sin(\theta) d\theta = 2\hat{n}_s$$

$$\Rightarrow n_s(\theta) = \frac{N_{sp}}{2}\sin(\theta)$$

$$0 < \theta < \pi$$

 $N_{\rm sp}$ is the number of conductors/phase/pole. $N_{\rm sp} = \frac{N_{\rm s}}{a}$ in 2-pote machine I.M. - typically 4 pole; P.M. - multipole

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Introduction

Stator vindings produce a simusoidal field disurbution --:
the airgup.

These magnetic field distributions are displaced by a 2020 war to each other.



Sinusoidally-distributed Stator Windings

(number of windings per phase vary with angle)



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$$(i_{+}^{*} + W_{M}W_{M}) \times \mathbb{R}^{3} + W_{M}W_{M}(i_{+}^{*}) = S$$

s. -li-sti

N_{st} is the number of conductors phase pole I.M. - Typically 4 pole: P.M. - mumpole Air-gap Field Distribution

Apply Ampere's Law.

$$IHdl = Ni = i \int_{0}^{0+\pi} n_s(\xi) d\xi$$

RHR gives direction.

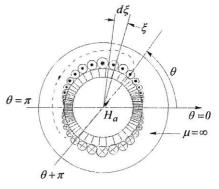
From symmetry

$$H_a(\theta) = -H_a(\theta + \pi)$$

(negative sign because line of integration points inwards at $\theta + \pi$)

$$\sum H.dl = H_m + 2H_a(\theta)l_g = \int_0^{\pi} n_s(\theta + \xi)i_a d\xi$$

$$2H_a(\theta)\ell_g = \frac{N_s}{2}i_a \int_0^{\pi} \sin(\theta + \xi) d\xi = N_s i_a \cos(\theta)$$



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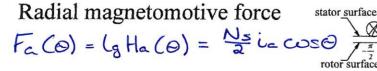
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Air-gap Field Distribution

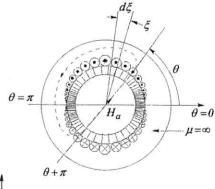
Radial magnetic field strength

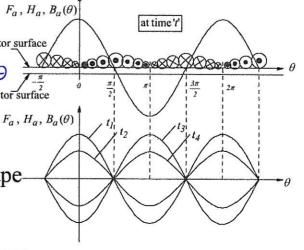
Radial magnetic flux density

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Field quantities have different magnitudes and units but same shape





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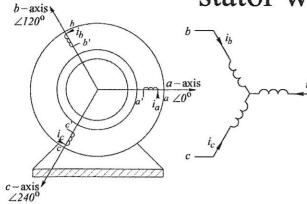
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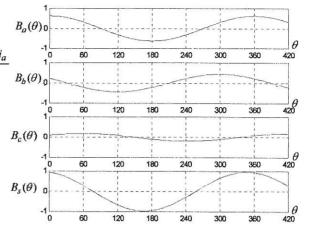
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Three-phase sinusoidally-distributed stator windings





r Example 9-3: 2-pole,
$$l_{\rm g}$$
 = 1mm, $i_{\rm a}$ = 10 A, $i_{\rm b}$ = -7 A, $i_{\rm c}$ = -3 A, $N_{\rm sp}$ = 50.

$$B_a(\theta) = \frac{\mu_o N_s i_a}{2\ell_g} \cos \theta = 0.628 \cos \theta \ Wb/m^2$$

$$B_b(\theta) = -0.440 \times \cos(\theta - 120^0) \ Wb/m^2$$
 $B_c(\theta) = -0.188 \times \cos(\theta - 240^0) \ Wb/m^2$

$$B_c(\theta) = -0.188 \times \cos(\theta - 240^0) Wb/m^2$$

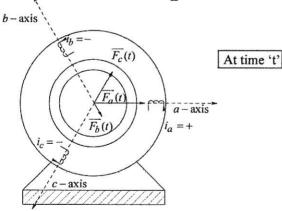
= combined stator-produced flux density

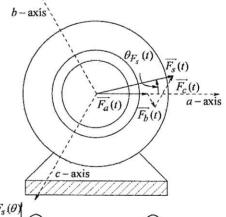
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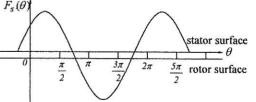
Space Vector to Represent Sinusoidal Distributions





r Complex number representation

$$F_a(\theta, t) = \frac{N_s}{2} i_a(t) \cos(\theta) \Leftrightarrow \overline{F_a}(t) = \frac{N_s}{2} i_a(t) \angle \theta^0$$



Similarly,
$$\overrightarrow{F_b}(t) = \frac{N_s}{2} i_b(t) \angle 120^{\circ}$$
; $\overrightarrow{F_c}(t) = \frac{N_s}{2} i_c(t) \angle 240^{\circ}$

And
$$\overrightarrow{F} = \overrightarrow{F} + \overrightarrow{F} + \overrightarrow{F} = \overrightarrow{F$$

r Similar expressions for B and H





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Example

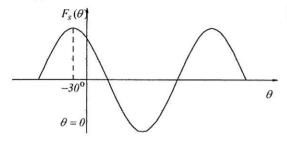
Three-phase, sinusoidally-distributed stator with $\frac{N_s}{2}$ = 50 turns At time t, i_a = 10 A, i_b = -10 A and i_c = 0 A

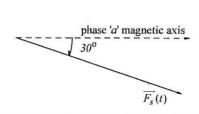
Find \overrightarrow{F}_s

$$\overrightarrow{F_s}(t) = \frac{N_s}{2} \left(i_a \angle 0^0 + i_b \angle 120^0 + i_c \angle 240^0 \right)$$

$$= 50 \left\{ 10 + (-10) \left[\cos 120^0 + j \sin 120^0 \right] + (0) \left[\cos 240^0 + j \sin 240^0 \right] \right\}$$

$$\vec{F}_s(t) = 50 \times 17.32 \angle -30^\circ = 866 \angle -30^\circ A \cdot \text{turns}$$





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9-10

Space Vectors Representation of Combined Phase Currents and Voltages

r Mathematical concept

At time t

$$\vec{l}_s = i_a(t) 20^o + i_b(t) 1120^o + i_c(t) 1240^o$$

$$= \hat{T}_s(t) 10^o$$

= stator current space vector

$$\vec{v}_{s}(t) = v_{a}(t) \angle 0^{0} + v_{b}(t) \angle 120^{0} + v_{c}(t) \angle 240^{0} \angle 240^{0}$$

$$= \hat{V}_{s}(t) \angle \theta_{v_{s}}(t)$$

$$= stator\ voltage\ space\ vector$$

b-axis



a - axis



Physical interpretation of $\overline{i_s}(t)$

$$\frac{N_s}{2}\overrightarrow{i_s}(t) = \underbrace{\frac{N_s}{2}}_{l_a}(t) \angle 0^0 + \underbrace{\frac{N_s}{2}}_{l_b}(t) \angle 120^0 + \underbrace{\frac{N_s}{2}}_{l_c}(t) \angle 240^0 = \overrightarrow{F_s}(t)$$

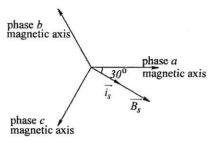
$$\overrightarrow{i_s}(t) = \frac{\overrightarrow{F_s}(t)}{N_s/2} \implies \widehat{I_s}(t) = \frac{\widehat{F_s}(t)}{N_s/2}$$
and $\theta_{i_s}(t) = \theta_{F_s}(t)$

 $\overrightarrow{F_s}(t)$ and $\overrightarrow{i_s}(t)$ are collinear

$$\vec{\beta}_s(t) = \frac{N_s u_o}{2 lg} \vec{l}_s(t)$$

- r Magnetic field is produced by combined effect of i_a , i_b and i_c but could equivalently be produced by hypothetical winding current $\overline{i_s}(t)$ at θ_{i_s}
- r helps in obtaining expression for torque

 $\overline{F_c}(t)$ at time t magnetic axis of hypothetical winding with current \hat{I}_s



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9-12

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Space Vector Components:

Finding Phase Currents from Current Space Vector

$$\operatorname{Re}\left[\vec{i}_{s}\angle 0^{0}\right] = i_{a} + \operatorname{Re}\left[i_{b}\angle 120^{0}\right] + \operatorname{Re}\left[i_{b}\angle 240^{0}\right] = \frac{3}{2}i_{a}$$

$$\Rightarrow i_{a}(t) = \frac{2}{3}\operatorname{Re}(\vec{i}_{s}\angle 0^{\circ}) = \frac{2}{3}\hat{I}_{s}\cos\theta_{i_{s}}$$

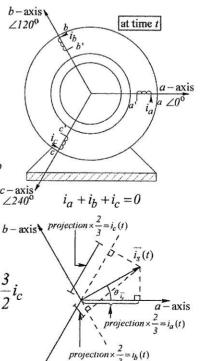
$$\operatorname{Re}\left[\vec{i}_{s}\angle -120^{0}\right] = \operatorname{Re}\left[i_{a}\angle -120^{0}\right] + i_{b} + \operatorname{Re}\left[i_{c}\angle 120^{0}\right] = \frac{3}{2}i_{b}$$

$$\Rightarrow i_{b}(t) = \frac{2}{3}\operatorname{Re}(\vec{i}_{s}\angle -120^{\circ}) = \frac{2}{3}\hat{I}_{s}\cos\left(\theta_{i_{s}} -120^{\circ}\right)$$

$$\operatorname{Re}\left[\vec{i}_{s}\angle -240^{0}\right] = \operatorname{Re}\left[i_{a}\angle -240^{0}\right] + \operatorname{Re}\left[i_{b}\angle -240^{0}\right] + i_{c} = \frac{3}{2}i_{c}$$

$$\Rightarrow i_{c}(t) = \frac{2}{3}\operatorname{Re}(\vec{i}_{s}\angle -240^{\circ}) = \frac{2}{3}\hat{I}_{s}\cos\left(\theta_{i_{s}} -240^{\circ}\right)$$

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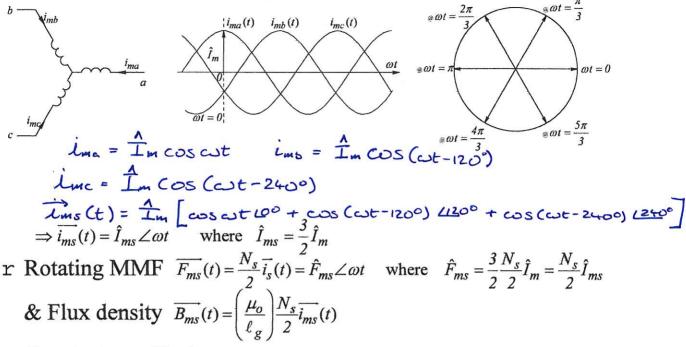






Balanced Sinusoidal Steady-State Excitation

(Rotor Open-Circuited – neglect stator winding resistance and leakage inductance)



r Constant amplitude

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Question:

Derive an expression for the magnetizing current-space vector in an AC machine with a 34 balanced sinusoidal s.s. excitation of sinusoidally distributed windings. Neglect the stator winding resistance of leakage inductance, and assume an open-circuited rotor.

$$\lim_{t \to \infty} (t) = ia(t) L0^{\circ} + ib(t) L120^{\circ}$$

$$+ ic(t) L240^{\circ}$$

$$+ ic(t) L240^{\circ}$$

$$+ ia(cos0+jsin0) + ib(cos120)$$

$$+ ia(cos2+0+jsin2+0)$$

$$= ia(1+j0) + ib(-\frac{1}{2}+j\frac{\sqrt{3}}{2}) + ic(-\frac{1}{2}-j\frac{\sqrt{3}}{2})$$

$$\Rightarrow \lim_{t \to \infty} (t) = [ia - \frac{1}{2}(ib+ia)] + j[\frac{\sqrt{3}}{2}(ib-ia)]$$

$$= laal$$

$$Imag.$$

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leakage reductance)

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r Constant amplitude

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=)
$$i_{a}(t) = I_{m}\cos(\omega t)$$
 $i_{b}(t) = I_{m}\cos(\omega t - 120) = I_{m}\left[\cos(\omega t)\cos(20 t) \sin(\omega t)\cos(120)\right]$
 $= I_{m}\left(-\frac{1}{2}\cos(\omega t) + \frac{\sqrt{3}}{2}\sin(\omega t)\right)$
 $= I_{m}\left(-\frac{1}{2}\cos(\omega t) - \frac{\sqrt{3}}{2}\sin(\omega t)\right)$
 $= I_{m}\left(-\frac{1}{2}\cos(\omega t) - \frac{\sqrt{3}}{2}\sin(\omega t)\right)$
 $= \lim_{n \to \infty} \left[\cos(\omega t) - \frac{1}{2}(i_{b} + i_{c})\right]$
 $= \lim_{n \to \infty} \left[\cos(\omega t) - \frac{1}{2}(i_{b} + i_{c})\right]$
 $= \lim_{n \to \infty} \left[\cos(\omega t) - \frac{1}{2}\cos(\omega t) + \frac{\sqrt{3}}{2}\sin(\omega t) + \frac{1}{2}\sin(\omega t)\right]$
 $= \lim_{n \to \infty} \left[i_{b}(t)\right] = \lim_{n \to \infty} \left(i_{b} - i_{c}\right)$
 $= \lim_{n \to \infty} \left[\sin(\omega t)\right]$
 $= \lim_{n \to \infty} \left[\sin(\omega t)\right]$

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9-16

Simuspidal Distribution

=)
$$\vec{H}(\theta) = \frac{Nsp}{l_8} i \cos \theta$$

Physical $\vec{B}(\theta) = u \cdot H(\theta)$

Vectors

= $u \cdot \frac{Nsp}{l_8} i \cos \theta$
 $\vec{F}(\theta) = l_8 \cdot H(\theta)$

= $Nspi \cos \theta$

Space
$$\begin{aligned}
& \vec{F}_s(t) = \vec{F}_c(t) + \vec{F}_s(t) + \vec{F}_c(t) \\
&= \vec{F}_s \quad Le_{FS} = N_{SP} \quad \vec{L}_s \quad Le_{FS} \\
& \text{Vectors} \\
& (Physical) \\
& \vec{F}_s(t) = \vec{F}_c(t) + \vec{F}_s(t) + \vec{F}_c(t) \\
&= \vec{F}_s \quad Le_{FS} \\
&\vec{F}_s(t) = \vec{F}_c(t) + \vec{F}_s(t) + \vec{F}_c(t) \\
&= \vec{F}_s \quad Le_{FS} \\
&=$$

×

Space
$$J_{s}(t) = J_{a}(t) J_{0}^{o} + J_{b}(t) J_{120}^{o} + J_{c}(t) J_{240}^{o}$$

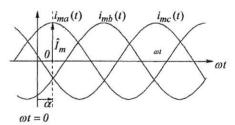
Vectors $J_{s}(t) = J_{a}(t) J_{0}^{o} + J_{b}(t) J_{120}^{o} + J_{c}(t) J_{240}^{o}$
 $J_{s}(t) = J_{a}(t) J_{0}^{o} + J_{b}(t) J_{120}^{o} + J_{c}(t) J_{240}^{o}$
 $= J_{s} J_{0}^{o}$

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Relation Between Space Vectors and **Phasors**

u Time domain $i_{\alpha}(t) = \int_{-\infty}^{\infty} \cos(\omega t - \alpha)$



u Phasor == fm -0

$$\overline{I}_{ma} = \hat{I}_m \angle -\alpha$$

u Space Vector

u Space Vector
$$\overrightarrow{l}_{ms}|_{t=0} = \underbrace{1}_{ms} \stackrel{(a)}{=0} \underbrace{1}_{ms} = \underbrace{1}_{ms} \stackrel{(a)}{=0} \underbrace{1}_{ms} = \widehat{1}_{ms} \stackrel{(a)}{=0} \underbrace{1}_{ms} = \underbrace{1}_{ms} \stackrel{(a)}{=0} \underbrace{1}_{ms} \stackrel{(a)}{$$

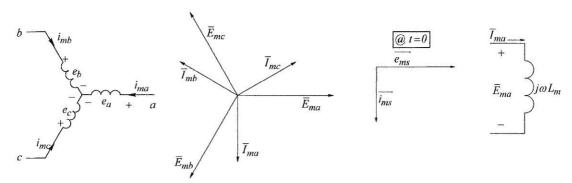
u Space Vector ⇔ phasor

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Voltages in the stator windings



Where the three phase magnetizing inductance (2 pole), $L_m = \frac{3 \pi \mu_0 rl}{2 l} \left(\frac{N_s}{2} \right)^2$

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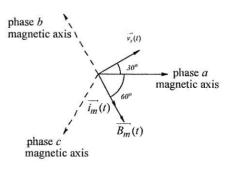
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Example

$$v_a(t) = 120\sqrt{2}\cos\omega t$$
$$v_b(t) = 120\sqrt{2}\cos(\omega t - 120^\circ)$$

$$v_c(t) = 120\sqrt{2}\cos(\omega t - 240^\circ)$$

$$\vec{v}_s = \frac{3}{2} \times 120\sqrt{2} \angle 30^0 = 254.56 \angle 30^0 \ V$$



$$\overrightarrow{i_{ms}} = \frac{\overrightarrow{v}_s}{j\omega L_m} = \frac{254.56 \angle (30^0 - 90^0)}{2\pi \times 60 \times 0.777} = 0.869 \angle -60^0 A$$

$$\overline{B_{ms}} = \frac{\mu_o N_s \vec{i}_{ms}}{2\ell_g} = \frac{4\pi \times 10^{-7} \times 50 \times 0.869 \angle -60^0}{10^{-3}} = 0.055 \angle -60^0 \text{ Wb/m}^2$$

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Derivation of per-phase magnetizing inductance

With only one phase, for example phase a excited by a magnetizing current i_a , the flux distribution in a 2-pole machine is

$$B_a(\theta) = \mu_0 \frac{N_{SP}}{l_g} i_{ma} \cos \theta$$

The energy density at an angle θ is

$$w_{ma}(\theta) = \frac{1 B_a(\theta)^2}{2 \mu_0}$$

The differential energy stored at an angle θ (with respect to the a-axis) in a differential angle $d\theta$ is

$$dW_{ma}(\theta) = w_{ma}(\theta) \cdot d$$
 (volume)

$$=\frac{1}{2}\frac{\left(\mu_0\frac{N_{SP}}{l_g}i_{ma}\cos\theta\right)^2}{\mu_0}.rd\theta.l.l_g$$

$$=\frac{\mu_0 N_{SP} r l}{2 l_{\varrho}} i_{ma}^2 \cdot \cos^2 \theta d\theta$$

Integrating both sides with respect to θ from 0 to 2π gives the total energy stored in the airgap.

$$W_{ma} = \frac{\mu_0 N_{SP} r l}{2l_g} i_{ma}^{2} \cdot \int_{0}^{2\pi} \cos^2 \theta \, d\theta$$

$$=\frac{1}{2}\left(\frac{\pi\mu_0N_{SP}^2rl}{l_g}\right)i_{ma}^2$$

The energy stored in an inductor is given by

$$E = \frac{1}{2} L_{a-a} i_{ma}^2$$

Therefore, the per-phase self inductance is

$$L_{ph}=L_{a-a}=\frac{\pi\mu_{0}N_{SP}^{2}rl}{l_{g}}$$

The magnetizing inductance is easily derived based on an understanding of the relationships between the self and mutual inductances.

(i) Suppose the winding of phase b is aligned with the winding of phase a, i.e. they have aligned magnetic axes.

$$\theta_{mb} = 0^{\circ}$$

$$L_{a-b} = L_{a-a}$$

Thus, any flux generated by phase a is perfectly coupled to phase b. So the mutual inductance between then is equal to the self-inductance of either winding.

(ii) Suppose phase b is 180° out of phase with phase a.

$$\theta_{mb} = 180^{\circ}$$

$$L_{a-b} = -L_{a-a}$$

1

Thus, any flux generated by phase a is perfectly coupled to phase b but has opposite polarity. So the mutual inductance between then is equal and opposite to the self inductance of either winding.

(iii) Suppose phase b is 90° out of phase with phase a,

$$\theta_{mb} = 90^{\circ}$$

$$L_{a-b}=0$$

In this case there is no magnetic flux linkage due to orthogonality of the windings and so the mutual inductance is zero.

The relationship between the mutual and self inductance above is simply a function of $\cos \theta_{mb}$. In the three-phase machine, the phase windings are physically displaced w.r.t. each other by 120° or $\cos \theta_{mb} = -\frac{1}{2}$. Thus the mutual inductances from phases b and c w.r.t. phase a are given by

$$L_{mut} = -\frac{1}{2}L_{ph} = -\frac{1}{2}\frac{\pi\mu_0 N_{Sp}^2 rl}{l_g}$$
 (1)

In a three-phase ac machine the back emf of the three phases are given by

$$\begin{bmatrix} e_{\scriptscriptstyle ma} \\ e_{\scriptscriptstyle mb} \\ e_{\scriptscriptstyle mc} \end{bmatrix} = \begin{bmatrix} L_{\scriptscriptstyle a-a} & L_{\scriptscriptstyle a-b} & L_{\scriptscriptstyle a-c} \\ L_{\scriptscriptstyle a-b} & L_{\scriptscriptstyle b-b} & L_{\scriptscriptstyle b-c} \\ L_{\scriptscriptstyle a-c} & L_{\scriptscriptstyle b-c} & L_{\scriptscriptstyle c-c} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_{\scriptscriptstyle ma} \\ i_{\scriptscriptstyle mb} \\ i_{\scriptscriptstyle mc} \end{bmatrix}$$

Thus, the back emf for phase a is as follows

$$e_{\mathit{ma}} = L_{\mathit{a-a}} \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathit{ma}} + L_{\mathit{a-b}} \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathit{mb}} + L_{\mathit{a-c}} \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathit{mc}}$$

or

$$e_{ma} = L_{ph} \frac{\mathrm{d}}{\mathrm{d}t} i_{ma} + L_{mut} \frac{\mathrm{d}}{\mathrm{d}t} i_{mb} + L_{mut} \frac{\mathrm{d}}{\mathrm{d}t} i_{mc} \tag{2}$$

Substituting (1) into (2) gives

$$e_{ma} = L_{ph} \left(\frac{\mathrm{d}}{\mathrm{d}t} i_{ma} - \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} i_{mb} - \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} i_{mc} \right) \tag{3}$$

We already know that in a balanced three-phase machine

$$i_{ma} + i_{mb} + i_{mc} = 0$$
 or $i_{ma} = -i_{mb} - i_{mc}$ (4)

Substituting (4) into (3) gives

$$e_{ma} = \frac{3}{2} L_{ph} \frac{\mathrm{d}}{\mathrm{d}t} i_{ma} = L_m \frac{\mathrm{d}}{\mathrm{d}t} i_{ma}$$

where

$$L_{m} = \frac{3}{2} L_{ph} = \frac{3}{2} \frac{\pi \mu_{0} N_{SP}^{2} rl}{l_{o}}$$

is the per-phase magnetizing inductance.

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$$\frac{\partial L}{\partial x} = -\frac{1}{2} \left(\frac{\partial L}{\partial x} - \frac{\partial L}{\partial x} - \frac{\partial L}{\partial x} - \frac{\partial L}{\partial x} \right) = 0$$

confert to 10 contents a

$$\left\{\frac{b}{c_1}, \cdots, \frac{a}{c_b}\right\} = \left\{\frac{b}{c_b}\right\} = \sum_{i=0}^{b} \left\{-1, \cdots, \frac{a}{c_b}\right\}$$

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