Note the following equations:

$$\vec{i}_s = \frac{3}{2}\sqrt{2}I_s\angle\theta_{Is} = \sqrt{\frac{3}{2}}\left(i_{sd} + ji_{sq}\right)$$

$$\overline{\lambda_r}(t) = -L_r\vec{i}_r(t) + L_m\vec{i}_s(t)$$

$$\lambda_{rq} = 0 \text{ and steady state} \Rightarrow T_{em} = \frac{P}{2}\lambda_{rd}i_{rq}, \omega_{slip} = \frac{2}{P}\frac{R_r' i_{rq}}{\lambda_{rd}},\ i_{sq} = \frac{L_r}{L_m}i_{rq}, i_{sd} = \frac{\lambda_{rd}}{L_m}$$

$$\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix} = \sqrt{\frac{2}{3}}\begin{pmatrix} \cos\theta_{da} & -\sin\theta_{da} \\ \cos\left(\theta_{da}+240^o\right) & -\sin\left(\theta_{da}+240^o\right) \\ \cos\left(\theta_{da}+120^o\right) & -\sin\left(\theta_{da}+120^o\right) \end{pmatrix}\begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix}$$

$$\theta_{da} = \omega t + \theta_{\lambda r}$$

Problem 1

A four-pole star-connected induction motor used in a servo application has the following per-phase equivalent circuit parameters:

$R_S = 20$ mΩ, $L_{LS} = 0.2$ mH, $L_M = 7.2$ mH, $L_{LR}' = 0.3$ mH, and $R_R' = 35$ mΩ.

At the rated condition of 400 V line-line, 50 Hz, the machine pulls 225 A at a lagging power factor of 0.841. At time $t = 0$, the machine is in steady state.

(i) Taking the per-phase input voltage as the reference, calculate the per-phase rotor current phasor and the per-phase rotor flux-linkage phasor.

(ii) Align the d-axis with the rotating rotor flux linkage state vector and calculate the rotor current and the rotor flux-linkage space vectors.

(iii) Calculate λ_{rd} and i_{rq} and the resulting electromagnetic torque and slip and input currents i_{sd} and i_{sq}.

(iv) Calculate the three phase currents at time $t = 0$.

(v) Calculate the three phase currents at $t = 5$ ms.

$\overline{V_s} = 230.9\angle 0^\circ$ V, $\overline{I_r'} = 195\angle -7.3^\circ$ A; $\overline{\lambda_r} = 0.697\angle -97.3^\circ$ Wb-turns

$\vec{i}_r^{\,d} = 413.6\angle 90^\circ\ A; \vec{\lambda}_r^{\,d} = 1.478\angle 0^\circ\ Wb-turns$

$\lambda_{rd} = 1.207\ Wb-turns; i_{rq} = 337.7\ A; T_{em} = 815.3\,Nm; s = 3.1\%; i_{sd} = 167.7\,A; i_{sq} = 351.8\,A$

$i_a(0) = 267.6A; i_b(0) = -282.9A; i_c(0) = +15.3A$

$i_a(5ms) = 172.2A; i_b(5ms) = 145.7A; i_c(5ms) = -317.8$

Problem 2

A four-pole star-connected induction motor used in a servo application has the following per-phase equivalent circuit parameters:

$R_S = 1.77$ Ω, $L_{LS} = 14$ mH, $L_M = 369$ mH, $L_{LR}' = 12$ mH, and $R_R' = 1.34$ Ω.

At the rated condition of 460 V line-line, 60 Hz, the machine pulls 3.753 A at a lagging power factor of 0.822. At time $t = 0$, the machine is in steady state.

(i) Taking the per-phase input voltage as the reference, calculate the per-phase rotor current phasor and the per-phase rotor flux-linkage phasor.

(ii) Align the d-axis with the rotating rotor flux linkage state vector and calculate the rotor current and the rotor flux-linkage space vectors.

(iii) Calculate λ_{rd} and i_{rq} and the resulting electromagnetic torque and slip.

(iv) Calculate the three phase currents at time $t = 0$.

Note the following equations:

$\overline{V_s} = 265.6\angle 0^\circ$ V, $\overline{I_r'} = 3.19\angle -6.2^\circ$ A; $\overline{\lambda_r} = 0.66\angle -96.2^\circ$ AWb-turns

$\vec{i}_r^{\,d} = 6.77\angle 90^\circ\ A; \vec{\lambda}_r^{\,d} = 1.4\angle -0^\circ\ Wb-turns$

$\lambda_{rd} = 1.144\ Wb-turns; i_{rq} = 5.53\ A; 12.65\,Nm; 1.72\%$

$i_a(0)=4.362A; i_b(0)=-4.79A; i_c(0)=0.44A$

Problem 3

Consider the Westinghouse 22 kW, 8-pole machine at the rated condition of 400 V line-line, 50 Hz with the following parameters.

$R_S = 0.432\ \Omega$, $L_{LS} = 2.8$ mH, $L_M = 73$ mH, $L_{LR}' = 2.8$ mH, and $R_R' = 0.49\ \Omega$. Also include the core loss equivalent resistance $R_c = 417\ \Omega$.

(i) Taking the per-phase input voltage as the reference, calculate the per-phase rotor current phasor and the per-phase rotor flux-linkage phasor.

(ii) Align the d-axis with the rotating rotor flux linkage state vector and calculate the rotor current and the rotor flux-linkage space vectors.

(iii) Calculate λ_{rd} and i_{rq} and the resulting electromagnetic torque and slip.

(iv) Calculate the three phase currents at time $t = 0$.

Note the following equations:

$$\overline{V_s} = 400\angle 0^\circ \text{ V}, \overline{I_r'} = 19.8\angle -1.1^\circ \text{ A}; \overline{\lambda_r} = 1.2\angle -91.1^\circ \text{ AWb-turns}$$

$$\vec{i}_r^{\,d} = 42\angle 90^\circ\ A; \vec{\lambda}_r^{\,d} = 2.54\angle 0^\circ\ Wb-turns$$

$$\lambda_{rd} = 2.07\ Wb-turns; i_{rq} = 34.3\ A; 284\ Nm; 2.6\%$$

$$i_a(0)=28.6A; i_b(0)=-34.8A; i_c(0)=6.2A$$

Problem 4

Consider the Westinghouse 22 kW, 8-pole machine with 400 V (line-line), 50 Hz.

At the rated condition determine the following. For simplicity, approximate the machine model as having parallel magnetizing and rotor resistance branches only, and neglect losses.

(i) Taking the per-phase input voltage as the reference, calculate the per-phase rotor current phasor and the per-phase rotor flux-linkage phasor.

(ii) Align the d-axis with the rotating rotor flux linkage state vector and calculate the rotor current and the rotor flux-linkage space vectors.

(iii) Calculate λ_{rd} and i_{rq} and the resulting electromagnetic torque and slip.

(iv) Calculate the three phase currents at time $t = 0$.

Note the following equations:

$$\overline{V_s} = 400\angle 0^\circ \text{ V}, \overline{I_r'} = 20.23\angle 0^\circ \text{ A}; \overline{\lambda_r} = 1.273\angle -90^\circ \text{ AWb-turns}$$

$$\vec{i}_r^{\,d} = 42.9\angle 90^\circ\ A; \vec{\lambda}_r^{\,d} = 2.70\angle 0^\circ\ Wb-turns$$

$$\lambda_{rd} = 2.21\ Wb-turns; i_{rq} = 35.04\ A; 309.1\ Nm; 2.5\%$$

$$i_a(0)=28.6A; i_b(0)=-34.8A; i_c(0)=6.2A$$

Problem 5

Consider the Westinghouse 22 kW, 8-pole machine with 400 V (line-line), 50 Hz.

At the rated condition, operating in generator mode, determine the following. For simplicity, approximate the machine model as having parallel magnetizing and rotor resistance branches only, and neglect losses.

(i) Taking the per-phase input voltage as the reference, calculate the per-phase rotor current phasor and the per-phase rotor flux-linkage phasor.

(ii) Align the d-axis with the rotating rotor flux linkage state vector and calculate the rotor current and the rotor flux-linkage space vectors.

(iii) Calculate λ_{rd} and i_{rq} and the resulting electromagnetic torque and slip.

(iv) Calculate the three phase currents at time $t = 0$.

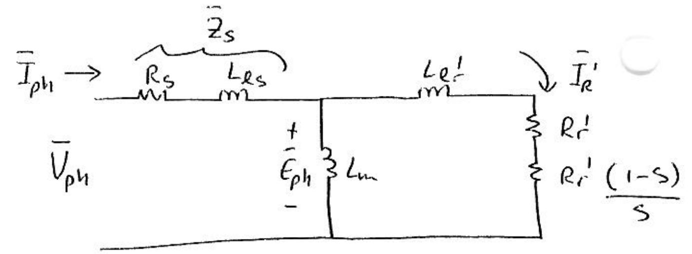
Note the following equations:

$$\overline{V_s} = 400\angle 0^\circ \text{ V}, \overline{I_r'} = 20.23\angle 180^\circ \text{ A}; \overline{\lambda_r} = 1.273\angle -90^\circ \text{ AWb-turns}$$

$$\vec{i}_r^{\,d} = 42.91\angle -90^\circ\ A; \vec{\lambda}_r^{\,d} = 2.701\angle 0^\circ\ Wb-turns$$

$$\lambda_{rd} = 2.21\ Wb-turns; i_{rq} = -35.04\ A; -309.1\ Nm; -2.5\%\ i_a(0)=-28.6A; i_b(0)=-6.2A; i_c(0)=34.8A$$

Q1



$R_s = 20\ m\Omega$ $\quad R_r' = 35\ m\Omega$

$L_{ls} = 0.2\ mH$ $\quad L_{lr}' = 0.3\ mH$ $\quad L_m = 7.2\ mH$

$\Rightarrow L_s = 7.4\ mH$ and $L_r = L_m + L_{lr}' = 7.5\ mH$

@ 400 V, $f_e = 50\ Hz$

$I_{ph} = 225\ A$ @ $PF = 0.841$

$V_{ph} = \frac{V_{LL}}{\sqrt{3}} = 230.9\ V$

$PF = 0.841 \Rightarrow \theta = \cos^{-1} PF = -32.75°$

$\Rightarrow \bar{I}_{ph} = 225 \angle -32.75°\ A$
$= (189.2 - j121.7)\ A$

$\bar{Z}_s$ = Stator Impedance
$= R_s + j\omega_e L_{ls}$
$= (0.02 + j0.0628)\ \Omega$
$= 0.0659 \angle 72.3°\ \Omega$

Let $\bar{V}_{zs} = \bar{Z}_s \cdot \bar{I}_{ph}$
$= (11.4 + j9.4)\ V$

Back emf $\bar{E}_{ph} = \bar{V} - \bar{V}_{zs}$
$= (219.5 - j9.4)\ V$
$= 219.7 \angle -2.5°\ V$

Let $\bar{I}_m$ = magnetizing current

$$= \frac{\bar{E}_{ph}}{j\omega_e L_m} = \frac{(219.5 - j9.4)}{j \cdot 2\cdot\pi\cdot 50 \cdot 7.2\times10^{-3}}\ A$$

$= -4.18 - j97.0\ A$

$= +97.1 \angle -92.5°\ A$

Reflected rotor current

$\bar{I}_R' = \bar{I}_{ph} - \bar{I}_m$
$= (193.4 - j24.7)\ A$
$= 195 \angle -7.3°\ A$

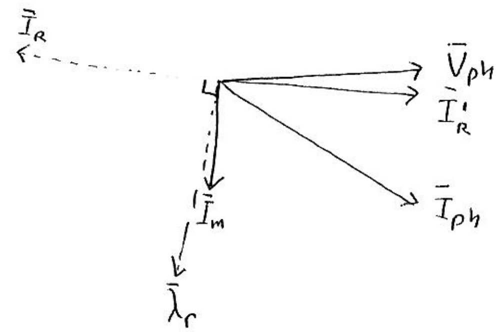
Rotor flux linkage

$\bar{\lambda}_R = -L_r \bar{I}_R' + L_m \bar{I}_{ph}$

$= -7.5\times10^{-3}(193.4 - j24.7)$
$+ 7.2\times10^{-3}(189.2 - j121.7)$

$= (-0.088 - j0.69)$ Wb-turns

$= 0.697 \angle -97.3°$ Wb-turns



Space vectors @ $t=0$

$$\vec{\lambda}_r = \frac{3}{2}\cdot\sqrt{2}\ \hat{\lambda}_r \angle \theta_{\lambda r}$$

$$= 1.478 \angle -97.3^\circ \text{ Wb-turns}$$

Align d-axis with $\vec{\lambda}_r$

$$\Rightarrow \vec{\lambda}_r^{\,d} = 1.478 \angle 0^\circ \text{ Wb-turns}$$

$$\vec{I}_R' = \frac{3}{2}\sqrt{2}\ \hat{I}_R' \angle \theta_{I_r}$$

$$= 413.6 \angle -7.3^\circ \text{ A}$$

$$\Rightarrow \vec{I}_R^{\,\prime d} = 413.6 \angle +90^\circ \text{ A}$$

$$\vec{I}_{ph} = \frac{3}{2}\sqrt{2}\ \bar{I}_{ph} \angle \theta_r$$

$$= 477.3 \angle -32.75^\circ \text{ A}$$

$$\Rightarrow \vec{I}_{ph}^{\,d} = 477.3 \angle 64.5^\circ \text{ A}$$

Given $\vec{\lambda}_r^{\,d} = \sqrt{\frac{3}{2}}\left(\lambda_{rd} + \cancel{j\lambda_{rq}}^{\to 0}\right)$

$$\Rightarrow \lambda_{rd} = 1.207 \text{ Wb-turns}$$

and $\vec{I}_R^{\,\prime d} = \sqrt{\frac{3}{2}}\left(\cancel{i_{rd}}^{\,0} + j\, i_{rq}\right)$

$$\Rightarrow i_{rq} = 337.7\text{A}$$

Given $\vec{I}_{ph}^{\,d} = \sqrt{\frac{3}{2}}\left(i_{sd} + j\, i_{sq}\right)$

$$= (205.4 + j\,430.8)\text{ A}$$

$$\Rightarrow i_{sd} = 167.7 \text{ A}$$

$$i_{sq} = 351.8 \text{ A}$$

$$T_{em} = \frac{P}{2}\lambda_{rd}\, i_{rq}$$

$$= 815.3 \text{ Nm}$$

~~ω_{slip} = 0.0 0.779 rad/s~~

$$\omega_{slip} = \frac{2}{P}\frac{R_r'\, i_{rq}}{\lambda_{rd}}$$

$$= 4.896 \text{ rad/s}$$

$$f_{slip} = 0.779 \text{ Hz}$$

$$s = \frac{f_{slip}}{f_{syn}} = 0.031 \text{ or } 3.1\%$$

Phase currents

@ $t = 0, \quad \theta_{da} = \omega t + \theta_{\lambda r} = \theta_{\lambda r}$

$= -97.3°$

$\Rightarrow \quad i_a(0) = \sqrt{\frac{2}{3}} \left(i_{sd} \cos\theta_{da} - i_{sq} \sin\theta_{da} \right)$

$= \sqrt{\frac{2}{3}} \left(167.7 \cos(-97.3)° - 351.8 \sin(-97.3°) \right)$

$= 267.6 \text{ A}$

$i_b(0) = \sqrt{\frac{2}{3}} \left(i_{sd} \cos(240° - 97.3°) - i_{sq} \sin(240° - 97.3°) \right]$

$= -282.9 \text{ A}$

$i_c(0) = \sqrt{\frac{2}{3}} \left(i_{sd} \cos(120° - 97.3°) - i_{sq} \sin(120° - 97.3°) \right]$

$= -15.3 \text{A}$

@ $t = 5\text{ms} \quad \theta_{da} = 2\pi f_e t + \theta_{\lambda r}$

$= 90° + -97.3°$

$= -7.3°$

$\Rightarrow \quad i_a(\cancel{t}) = 172.2 \text{ A}$

$i_b(5\text{ms}) = 145.7 \text{ A}$

and $i_c(5\text{ms}) = -317.8 \text{ A}$