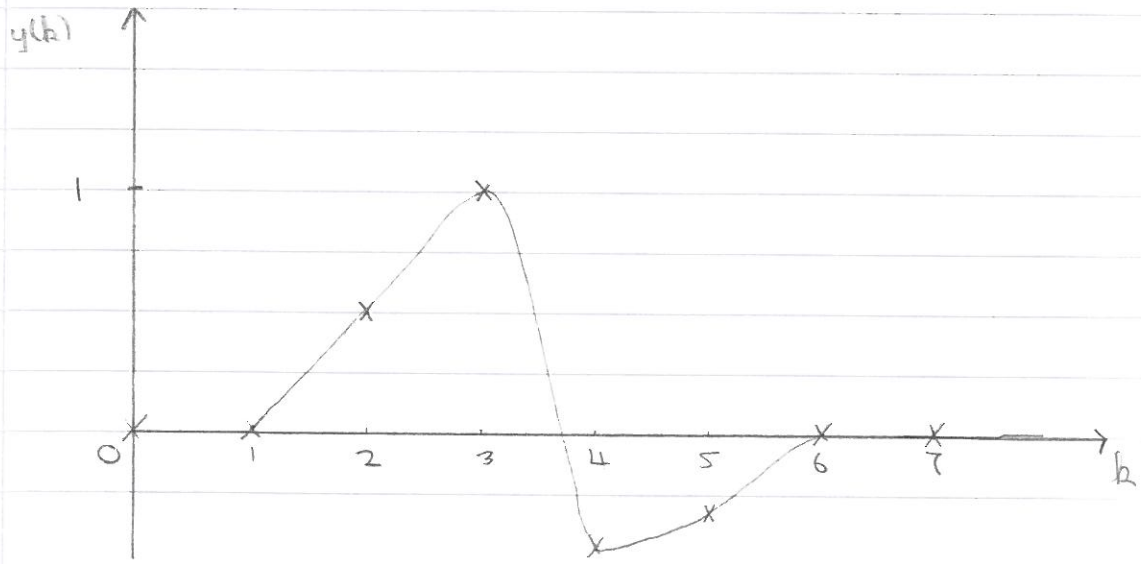
1/5/09 Autumn 08

Q1(a). $h(0) = 0$
$h(1) = 0$
$h(2) = 0{\cdot}5$
$h(3) = 1{\cdot}5$
$h(4) = 1$
$h(5) = 0{\cdot}7$
$h(6) = 0{\cdot}7$
$h(7) = 0{\cdot}7$

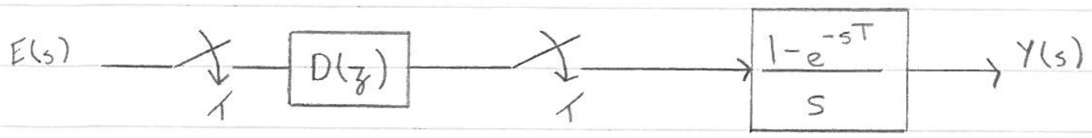
$$Y(z) = G(z)U(z) = \underbrace{(g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots)}_{\text{FROM IMPULSE RESPONSE}} \underbrace{\frac{1}{1-z^{-1}}}_{\text{UNIT STEP}}$$

$$(1-z^{-1})(h_0 + h_1 z^{-1} + h_2 z^{-2} + \dots h_{N-1} z^{-N+1} + h_N z^{-N} + h_N z^{-N-1} \dots) = g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots$$
$$h_0 + (h_1 - h_0) z^{-1} + (h_2 - h_1) z^{-2} + \dots (h_N - h_N) z^{-N-1} = g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots$$

$g_0 = h_0 = 0$
$g_1 = h_1 - h_0 = 0$
$g_2 = h_2 - h_1 = 0{\cdot}5$
$g_3 = h_3 - h_2 = 1$
$g_4 = h_4 - h_3 = -0{\cdot}5$
$g_5 = h_5 - h_4 = -0{\cdot}3$
$g_6 = h_6 - h_5 = 0$
$g_7 = h_7 - h_6 = 0$



(b).



$$\int_0^t e(\tau)\,d\tau \approx I(k) = I(k-1) + \tfrac{T}{2}\left(e(k) + e(k-1)\right)$$

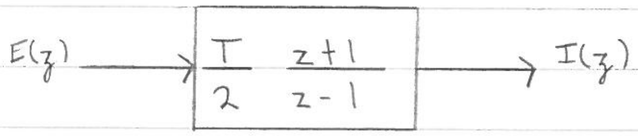
Taking the Z transforms

$$I(z) = z^{-1} I(z) + \tfrac{T}{2}\left(E(z) + z^{-1}E(z)\right)$$

$$(1-z^{-1}) I(z) = \tfrac{T}{2}(1+z^{-1})E(z)$$

$$I(z) = \tfrac{T}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)E(z)$$

This yields the discrete-time transfer function



(ii). $m(t) = K_P\left(e(t) + \tfrac{1}{T_I}\int_0^t e(\tau)\,d\tau\right)$

$$M(s) = K_P\left(E(s) + \tfrac{1}{T_I s}E(s)\right)$$

$$\Rightarrow \frac{M(s)}{E(s)} = K_P\left(1 + \frac{1}{T_I s}\right)$$

Tustins $s = \tfrac{2}{T}\,\frac{z-1}{z+1}$

$$D(z) = K_P\left(1 + \frac{T(z+1)}{2T_I(z-1)}\right)$$

$K_P = 5 \quad T_I = 0.5$

$$\Rightarrow D(z) = \frac{5\left(2(0.5)(z-1) + 1(z+1)\right)}{2(0.5)(z-1)} = \frac{10z}{z-1}$$

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-Ts}}{s} \cdot \frac{2}{s+1} \cdot 0.2\right\} = 0.4(1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s(s+1)}\right\}$$

From tables

$$\mathcal{Z}\left\{\frac{1}{s(s+a)}\right\} = \frac{1}{a} \frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$$

$$\Rightarrow G(z) = 0.4\cancel{(1-z^{-1})} \cdot \frac{1}{1} \frac{(1-e^{-1})z^{-1}}{\cancel{(1-z^{-1})}(1-e^{-1}z^{-1})}$$

$$\Rightarrow G(z) = \frac{0.25z^{-1}}{1-0.37z^{-1}} = \frac{0.25}{z-0.37}$$

$$\Rightarrow D(z)G(z) = \frac{2.5z}{(z-1)(z-0.37)}$$

zero at 0
pole at 1, 0.37

$$\sum_{i=1}^{n} \frac{1}{\sigma - p_i} = \sum_{j=1}^{m} \frac{1}{\sigma - z_j}$$

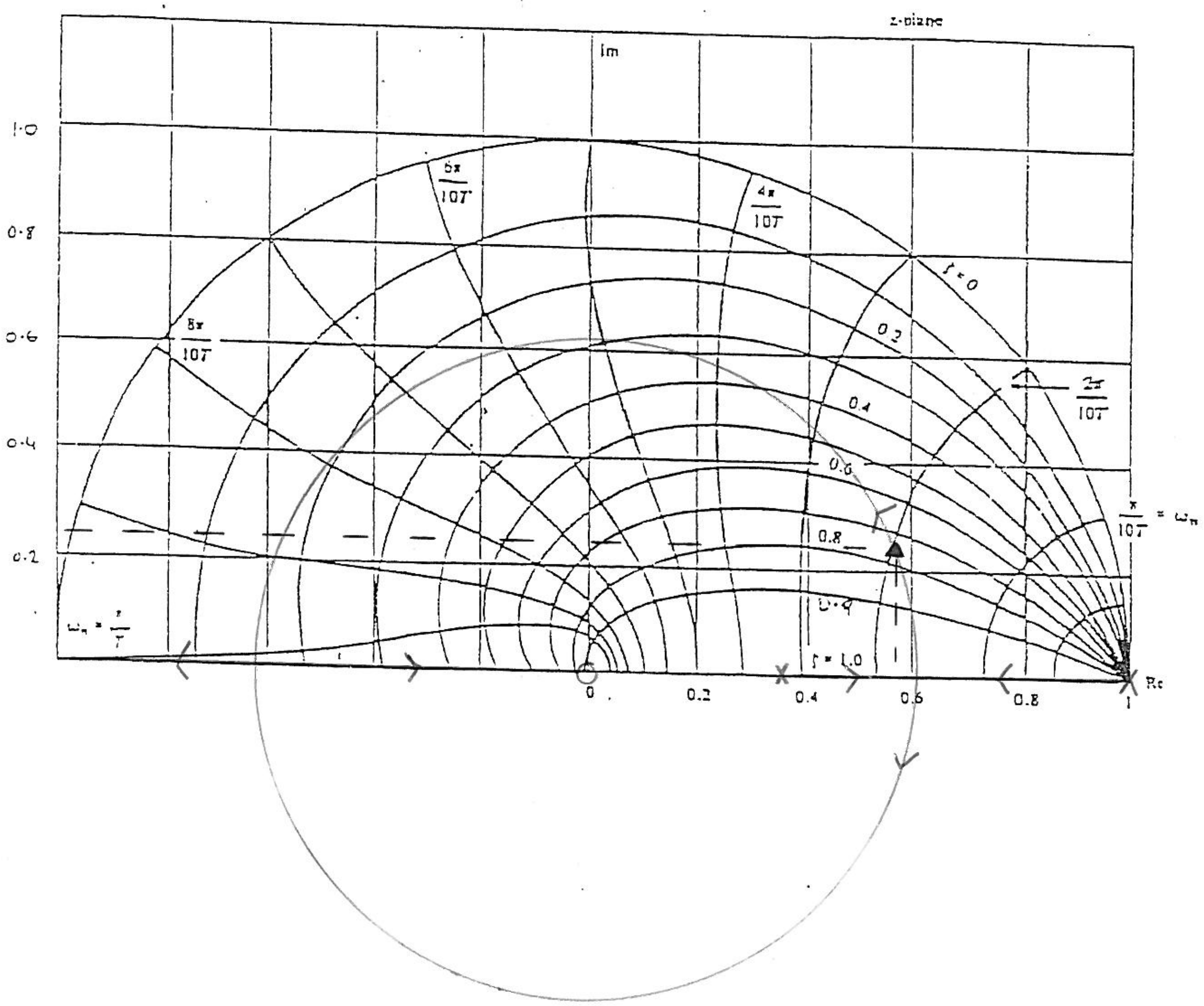
$$\frac{1}{\sigma-1} + \frac{1}{\sigma-0.37} = \frac{1}{\sigma-0}$$

$$(\sigma-0.37)\sigma + \sigma(\sigma-1) = (\sigma-1)(\sigma-0.37)$$

$$\sigma^2 = 0.37$$

$$\sigma = 0.61$$

For $\xi = 0.75$ desired pole location $= 0.56 \pm 0.26j$
$\Rightarrow$ unsatisfactory performance



Z Plane Design Template

Please submit with your script

Autumn 08

Q4 (a). $\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$

Taking Laplace transforms yields:

$\underline{X}(s) = (sI - A)^{-1}(B\underline{U}(s) + \underline{x}(0))$

Define $\phi(s) = (sI-A)^{-1}$

$\underline{X}(s) = \phi(s)\underline{x}(0) + \phi(s)B\underline{U}(s)$

Taking inverse Laplace transforms yields

$\underline{x}(t) = \phi(t)\underline{x}(0) + \phi(t)B \circledast \underline{u}(t)$

$\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(t-\tau)B\underline{u}(\tau)d\tau$

(ii) $\phi(s) = (sI-A)^{-1}$

$\phi(t) = L^{-1}\{(sI-A)^{-1}\}$

Consider the zero-input response

$\underline{x}(t) = \phi(t)\underline{x}(0)$

This is the solution to

$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t)$

Propose the solution

$\underline{x}(t) = e^{At}\underline{x}(0)$

We know the solution is $\underline{x}(t) = \phi(t)\underline{x}(0)$

$\frac{d}{dt}\underline{x}(t) = \frac{d}{dt}(\phi(t)\underline{x}(0)) = \frac{d\phi}{dt}\underline{x}(0)$

$\frac{d^2}{dt^2}\underline{x}(t) = \frac{d^2}{dt^2}(\phi(t)\underline{x}(0)) = \frac{d^2\phi}{dt^2}\underline{x}(0)$

$\frac{d^3}{dt^3}\underline{x}(t) = \frac{d^3}{dt^3}(\phi(t)\underline{x}(0)) = \frac{d^3\phi}{dt^3}\underline{x}(0)$

$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) = A\phi(t)\underline{x}(0)$

$\frac{d^2}{dt^2}\underline{x}(t) = A\frac{d}{dt}\underline{x}(t) = A^2\phi(t)\underline{x}(0)$

$\frac{d^3}{dt^3}\underline{x}(t) = A\frac{d^2}{dt^2}\underline{x}(t) = A^3\phi(t)\underline{x}(0)$

$\Rightarrow \frac{d^i}{dt^i}\phi(t) = A^i\phi(t)$

This will be true if

$\phi(t) = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots$

Define matrix exponential function as

$$e^{At} = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots$$

$$\Rightarrow \phi(t) = e^{At}$$

$$\Rightarrow L^{-1}\{(sI-A)^{-1}\} = e^{At}$$

(iii) The state trajectory is given by

$$\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(t-\tau)B\underline{u}(\tau)d\tau$$

Initial time t_0 with initial state $\underline{x}(t_0)$

$$\underline{x}(t) = \phi(t-t_0)\underline{x}(t_0) + \int_{t_0}^t \phi(t-\tau)B\underline{u}(\tau)d\tau$$

Time step T

$t_0 = kT$

$t = (k+1)T$

$$\underline{x}((k+1)T) = \phi(T)\underline{x}(kT) + \int_{kT}^{(k+1)T} \phi((k+1)T-\tau)B\underline{u}(\tau)d\tau$$

ZOH utilised

$\Rightarrow \underline{u}(t) = \underline{u}(kT)$ for $kT \leq t < (k+1)T$

$$\underline{x}((k+1)T) = \phi(T)\underline{x}(kT) + \int_{kT}^{(k+1)T} \phi((k+1)T-\tau)B\,d\tau\,\underline{u}(kT)$$

$\eta = (k+1)T - \tau$

$d\eta = -d\tau$

$$\underline{x}((k+1)T) = \phi(T)\underline{x}(kT) - \int_T^0 \phi(\eta)B\,d\eta\,\underline{u}(kT)$$

$(k+1)T \rightarrow (k+1)$

$kT \rightarrow k$

$$\underline{x}(k+1) = \phi(t)\underline{x}(k) + \int_0^T \phi(\eta)B\,d\eta\,\underline{u}(k)$$

$$\underline{x}(k+1) = e^{At}\underline{x}(k) + \int_0^T e^{A\eta} B \, d\eta \; \underline{u}(k)$$

$$= e^{At}\underline{x}(k) + \tfrac{1}{A}\left(e^{AT} - e^{A0}\right) B\underline{u}(k)$$

$$= e^{At}\underline{x}(k) + A^{-1}\left(e^{AT} - I\right) B\underline{u}(k)$$

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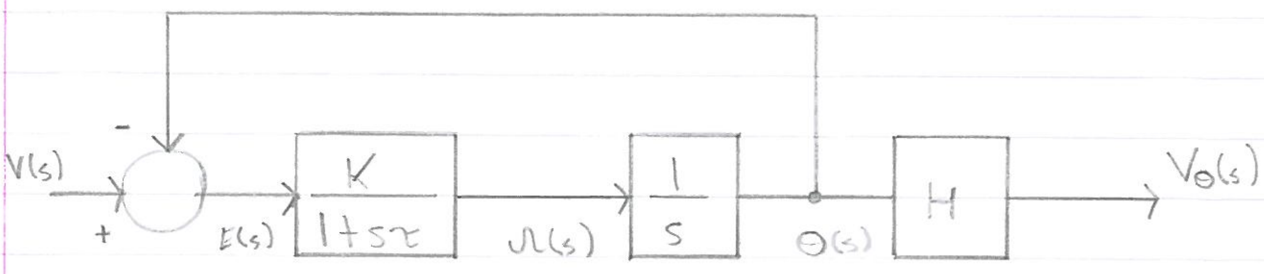
Q4 (b)



(i) $E(s) = V(s) - \Theta(s)$

$\Rightarrow e(t) = v(t) - \theta(t)$

$\Omega(s) = \frac{KE(s)}{1+s\tau}$

$\Omega(s) + s\tau\Omega(s) = KE(s)$

$\omega(t) + \tau \frac{d\omega}{dt} = Ke(t)$

$\frac{d\omega}{dt} = \frac{K}{\tau}e(t) - \frac{1}{\tau}\omega(t)$

$= \frac{K}{\tau}(v(t) - \Theta(t)) - \frac{1}{\tau}\omega(t)$

$= \frac{K}{\tau}v(t) - \frac{K}{\tau}\Theta(t) - \frac{1}{\tau}\omega(t)$

$\Theta(s) = \frac{\Omega(s)}{s}$

$s\Theta(s) = \Omega(s)$

$\frac{d\Theta}{dt} = \omega$

$$\frac{d}{dt}\begin{bmatrix} \omega \\ \Theta \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau} & -\frac{K}{\tau} \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \omega \\ \Theta \end{bmatrix} + \begin{bmatrix} \frac{K}{\tau} \\ 0 \end{bmatrix} v$$

$V_\Theta(s) = H\Theta(s)$

$v_\Theta(t) = H\Theta(t)$

$\Rightarrow V_\Theta(t) = \begin{bmatrix} 0 & H \end{bmatrix}\begin{bmatrix} \omega \\ \Theta \end{bmatrix}$

(iii) Observability matrix

$$O_x = \begin{bmatrix} C \\ \hdashline CA \\ \hdashline \vdots \\ \hdashline CA^{N-1} \end{bmatrix} \qquad N = 2$$

$$O_x = \begin{bmatrix} C \\ CA \end{bmatrix} \qquad CA = \begin{bmatrix} 0 & H \end{bmatrix} \begin{bmatrix} -1/\tau & -K/\tau \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} H & 0 \end{bmatrix}$$

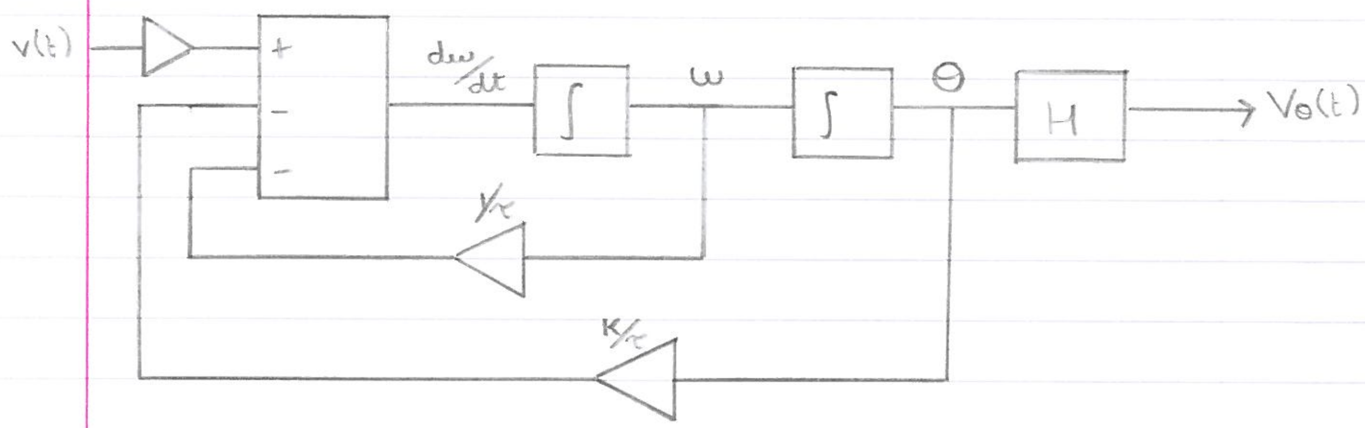
$$O_x = \begin{bmatrix} 0 & H \\ H & 0 \end{bmatrix}$$

States are observable if O_x is full rank

$\Rightarrow \det(O_x) \neq 0$

$\det(O_x) = -H^2 \neq 0$

$\Rightarrow$ observable.



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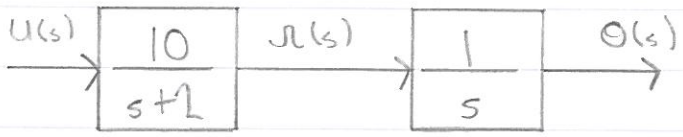
Q 5(b).



$\Omega(s) = \frac{10}{s+2} U(s)$

$s\Omega(s) + 2\Omega(s) = 10 U(s)$

$\frac{d\omega}{dt} + 2\omega(t) = 10 u(t)$

$\frac{d\omega}{dt} = -2\omega(t) + 10 u(t)$

$\Theta(s) = \frac{\Omega(s)}{s}$

$\Omega(s) = s\Theta(s)$

$\Rightarrow \frac{d\Theta}{dt} = \omega$

$$\frac{d}{dt}\begin{bmatrix} \Theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} \Theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

$U(s) = C(s)E(s)$

$E(s) = \Theta_{des}(s) - 0.5\Theta(s)$

$\Rightarrow U(s) = C(s)(\Theta_{des}(s) - 0.5\Theta(s))$

$= (Ks + K_z)(\Theta_{des}(s) - 0.5\Theta(s))$

$= Ks\Theta_{des}(s) - 0.5Ks\Theta(s) + K_z\Theta_{des}(s) - 0.5K_z\Theta(s)$

$\Rightarrow u(t) = K\frac{d\Theta_{des}(s)}{dt} - 0.5K\frac{d\Theta}{dt} + K_z\Theta_{des}(t) - 0.5K_z\Theta(t)$

Setpoint constant

$\Rightarrow \frac{d\Theta_{des}(t)}{dt} = 0$

$\Rightarrow u(t) = K_z\Theta_{des}(t) - 0.5K_z\Theta(t) - 0.5K\omega(t)$

$= K_z\Theta_{des}(t) - K_1\Theta(t) - K_2\omega(t)$

$= K_z\Theta_{des}(t) - [K_1 \;\; K_2]\begin{bmatrix} \Theta \\ \omega \end{bmatrix}$

$C_{des}(s) = \det(sI - A + BK)$

$= \det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \end{pmatrix}(K_1 \;\; K_2)\right]$

$= \det\left[\begin{pmatrix} s & -1 \\ 0 & s+2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 10K_1 & 10K_2 \end{pmatrix}\right]$

$$= \det\begin{pmatrix} s & -1 \\ 10K_1 & s+2+K_2 \end{pmatrix}$$

$$= s(s+2+K_2) + 10K_1$$
$$= s^2 + (2+K_2)s + 10K_1$$

$$C_{des}(s) = (s+(1+j))(s+(1-j))$$
$$= (s+1)^2 - j^2$$
$$= s^2 + 2s + 2$$

$10K_1 = 2$ $\quad\quad$ $2 + K_2 = 2$

$K_1 = 0.2$ $\quad\quad$ $K_2 = 0$? problem with question

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Q 6 (a). The full-state Luenberger observer is:

$\frac{d}{dt}\underline{\hat{x}} = A\underline{\hat{x}} + B\underline{u} + G(\underline{y}(t) - \underline{\hat{y}}(t))$

$\underline{\hat{y}}(t) = C\underline{\hat{x}}(t)$

Define the state estimation vector

$\underline{e}(t) = \underline{x}(t) - \underline{\hat{x}}(t)$

$$\begin{aligned}\frac{d}{dt}\underline{e}(t) &= \frac{d}{dt}\underline{x}(t) - \frac{d}{dt}\underline{\hat{x}}(t)\\ &= A\underline{x}(t) + B\underline{u}(t) - (A\underline{\hat{x}} + B\underline{u} + G(\underline{y} - \underline{\hat{y}}))\\ &= A(\underline{x}(t) - \underline{\hat{x}}(t)) - G(\underline{y}(t) - \underline{\hat{y}}(t))\end{aligned}$$

$\underline{y}(t) = C\underline{x}(t)$ and $\underline{\hat{y}}(t) = C\underline{\hat{x}}(t)$

$$\begin{aligned}\frac{d}{dt}\underline{e}(t) &= A\underline{e}(t) - G(C\underline{x}(t) - C\underline{\hat{x}}(t))\\ &= A\underline{e}(t) - GC(\underline{x}(t) - \underline{\hat{x}}(t))\\ &= A\underline{e}(t) - GC\underline{e}(t)\\ &= (A - GC)\underline{e}(t)\end{aligned}$$

$\underline{u}(t) = -K\underline{\hat{x}}(t)$

The closed loop state equation becomes

$$\begin{aligned}\frac{d}{dt}\underline{x}(t) &= A\underline{x}(t) - BK\underline{\hat{x}}(t)\\ &= A\underline{x}(t) - BK(\underline{x}(t) - \underline{e}(t))\\ &= (A - BK)\underline{x}(t) + BK\underline{e}(t)\end{aligned}$$

$\frac{d}{dt}\underline{x}(t) = (A - BK)\underline{x}(t) + BK\underline{e}(t)$

$\frac{d}{dt}\underline{e}(t) = (A - GC)\underline{e}(t)$

$$\frac{d}{dt}\begin{bmatrix}\underline{x}(t)\\ \underline{e}(t)\end{bmatrix} = \begin{bmatrix}A - BK & BK\\ 0 & A - GC\end{bmatrix}\begin{bmatrix}\underline{x}(t)\\ \underline{e}(t)\end{bmatrix} \updownarrow \text{2N states}$$

The poles of the closed-loop process are then given by the roots of: (2N poles)

$$\det\left[\underset{\substack{2N \times 2N \\ \text{IDENTITY}}}{sI} - \begin{pmatrix} A-BK & BK \\ 0 & A-GC \end{pmatrix}\right] = 0$$

$$\det\left[\begin{pmatrix} sI_N & 0 \\ 0 & sI_N \end{pmatrix} - \begin{pmatrix} A-BK & BK \\ 0 & A-GC \end{pmatrix}\right] = 0$$

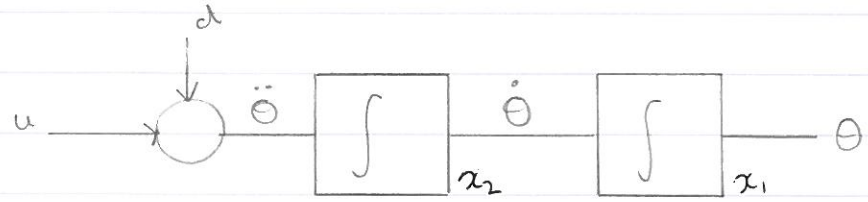
$$\det\begin{pmatrix} sI_N - A + BK & -BK \\ 0 & sI_N - A + GC \end{pmatrix} = 0$$

$$\Rightarrow \overset{A_c(s)}{|sI_N - A + BK|}\;\overset{A_o(s)}{|sI_N - A + GC|} = 0$$

Seperation Principle

Designing the estimator has no effect on the poles of the regulator. So we can design K for the regulator to place the N closed loop poles assuming that states are available. Then we design G for our estimator to provide these states with desired error dynamics. The estimator does not affect the position of the regulator poles.

(b). $\frac{d^2\theta(t)}{dt^2} = u(t) + d(t)$



$\frac{d}{dt} x_1 = x_2$

$\frac{d}{dt} x_2 = u(t) + d(t)$

$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) + Ed(t)$

$y(t) = C\underline{x}(t)$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u + \overset{E}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} d$$

$$\Theta = \overset{C}{[1 \ \ 0]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} u(t) &= -2\, {}^{d\theta}/_{dt} - \theta(t) \\ &= -2x_2 - x_1 \\ &= \underset{K}{[-1 \ \ -2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= -K\underline{x}(t) \end{aligned}$$

$$\Rightarrow \dot{\underline{x}}(t) = (A - BK)\underline{x}(t) + E d(t)$$

$$G(s) = C(sI - A + BK)^{-1} E$$

$$(sI - A + BK) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (+1 \ \ +2)$$

$$= \begin{pmatrix} s & -1 \\ 0 & s \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ +1 & +2 \end{pmatrix} = \begin{pmatrix} s & -1 \\ +1 & s+2 \end{pmatrix}$$

$$\begin{pmatrix} s & -1 \\ -1 & s-2 \end{pmatrix}^{-1} = \frac{1}{s^2 + 2s + 1} \begin{pmatrix} s-2 & 1 \\ 1 & s \end{pmatrix}$$

$$G(s) = (1 \ \ 0) \frac{1}{s^2 + 2s + 1} \begin{pmatrix} s-2 & 1 \\ 1 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{s^2 + 2s + 1} (s-2 \ \ 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{s^2 + 2s + 1}$$ $\Rightarrow$ 2nd order, no closed loop zeros

(i.i) Full order Luenberger observer

$\frac{d}{dt}\hat{\underline{x}} = (A-GC)\hat{\underline{x}} + B\underline{u} + G\underline{y}$

$$G(s) = \frac{1 \leftarrow f_0}{s^2 + 2s + 1}$$

$\uparrow e_1 \quad \nwarrow e_0$

Control canonical format

$$\dot{\underline{x}}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}_{A} \underline{x}(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \overbrace{[1 \ 0]}^{C} \underline{x}(t)$$

$$A - GC = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} (1 \ 0)$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} g_1 & 0 \\ g_2 & 0 \end{pmatrix} = \begin{pmatrix} -g_1 & 1 \\ -1-g_2 & -2 \end{pmatrix}$$

$$\det(sI - F) = 0$$

$$\det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -g_1 & 1 \\ -1-g_2 & -2 \end{pmatrix}\right] = 0$$

$$\det\begin{bmatrix} s+g_1 & -1 \\ g_2+1 & s+2 \end{bmatrix} = 0$$

$$(s+g_1)(s+2) + g_2 + 1 = 0$$

$$s^2 + (g_1+2)s + (g_2 + 2g_1 + 1) = 0$$

2 poles at $s = -5$

$$(s+5)^2 = 0$$

$$s^2 + 10s + 25 = 0$$

$$g_1 + 2 = 10 \qquad g_2 + 2g_1 + 1 = 25$$

$$g_1 = \cancel{12}\ 8 \qquad g_2 = \cancel{50}\ 8$$

$$C_{eq}(s) = K(sI - A + GC + BK)^{-1} G$$

$$(sI - A + GC + BK) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} + \begin{pmatrix} \cancel{12}8 \\ \cancel{50}8 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} +1 & +2 \end{pmatrix}$$

$$= \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} + \begin{pmatrix} 0 & \cancel{12}8 \\ 0 & \cancel{50}8 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ +1 & +2 \end{pmatrix}$$

$$= \begin{pmatrix} s & \cancel{11}7 \\ -2 & s + \cancel{46}8 \end{pmatrix}$$

$$(sI - A + GC + BK)^{-1} = \frac{1}{s^2 + 46s + 22} \begin{pmatrix} s + 46 & -11 \\ 2 & s \end{pmatrix}$$

$$\Rightarrow C_{eq}(s) = \frac{1}{s^2 + 46s + 22} \begin{pmatrix} -1 & -2 \end{pmatrix} \begin{pmatrix} s + 46 & -11 \\ 2 & s \end{pmatrix} \begin{pmatrix} 12 \\ 50 \end{pmatrix}$$

$$= \frac{1}{s^2 + 46s + 22} \begin{pmatrix} -s - 50 & 11 - 2s \end{pmatrix} \begin{pmatrix} 12 \\ 50 \end{pmatrix}$$

$$= \frac{-112s - 50}{s^2 + 46s + 22}$$