Exam Solutions UE4002 Summer 2013

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an NMOS in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{tn})^2(1+\lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n=\lambda_p=0$.
In each question, capacitances other than those shown may be ignored.

Question 1

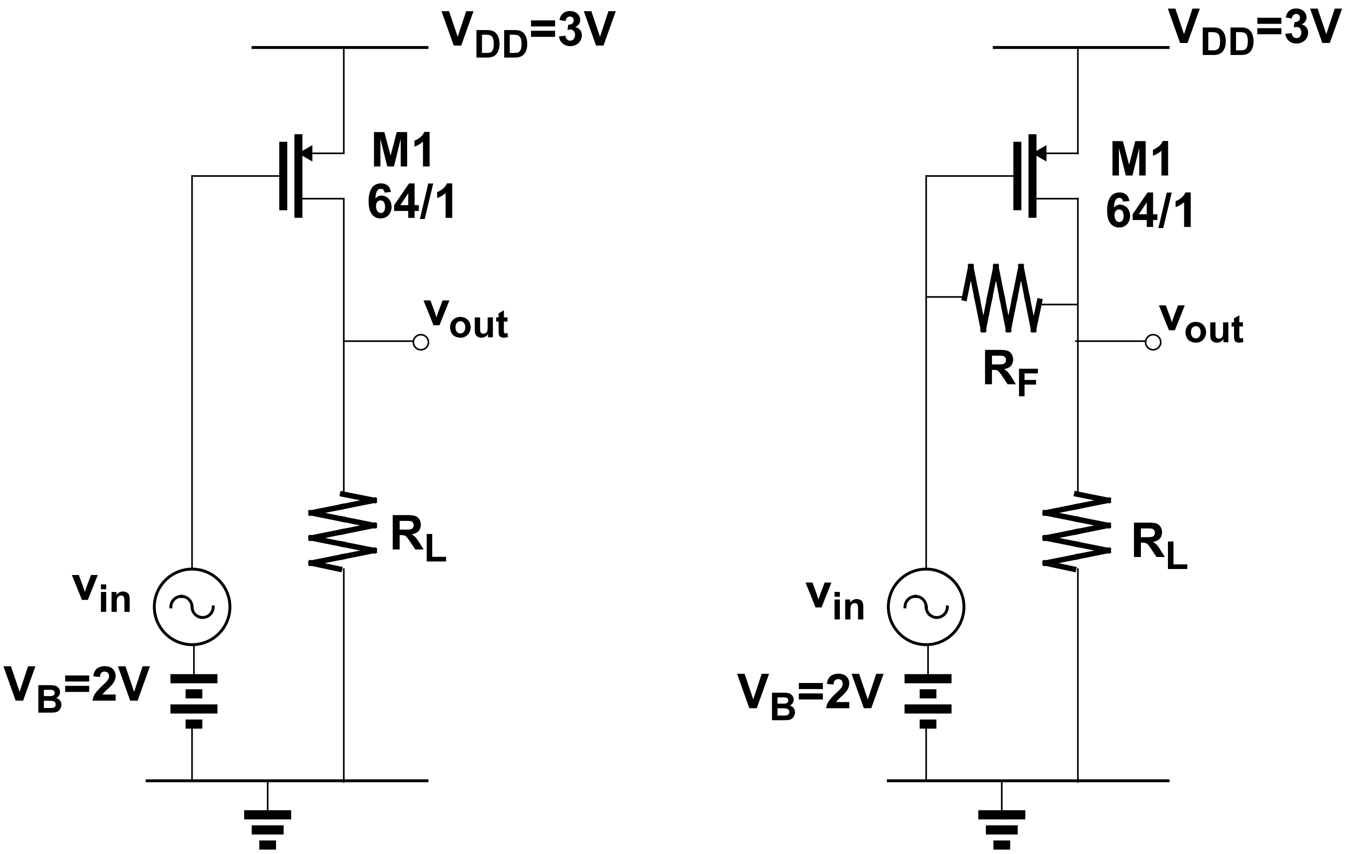

Figure 1a **Figure 1b**

You may assume R_L, R_F << $1/g_{ds1}$. Take $|V_{tp}| = 0.75V$, $K_p' = 50\mu A/V^2$, $R_L = 15k\Omega$, $R_F = 5k\Omega$.
Bias voltages and transistor dimensions (in microns) are as shown in the Figures 1a and 1b.
Assume M1 is in saturation.

(i) Draw the small-signal equivalent circuit for the circuit shown in Figure 1a and derive an expression for the small-signal voltage gain (v_{out}/v_{in}) of the circuit.

(ii) For the circuit in Figure 1a, calculate the value of the small-signal voltage gain in dB.

(iii) Draw the small-signal equivalent circuit for the circuit shown in Figure 1b and derive an expression for the small-signal voltage gain (v_{out}/v_{in}) of the circuit.
(iv) For the circuit in Figure 1b, calculate the value of the small-signal voltage gain in dB.

(i) Draw the small-signal equivalent circuit for the circuit shown in Figure 1a and derive an expression for the small-signal voltage gain (v_{out}/v_{in}) of the circuit.

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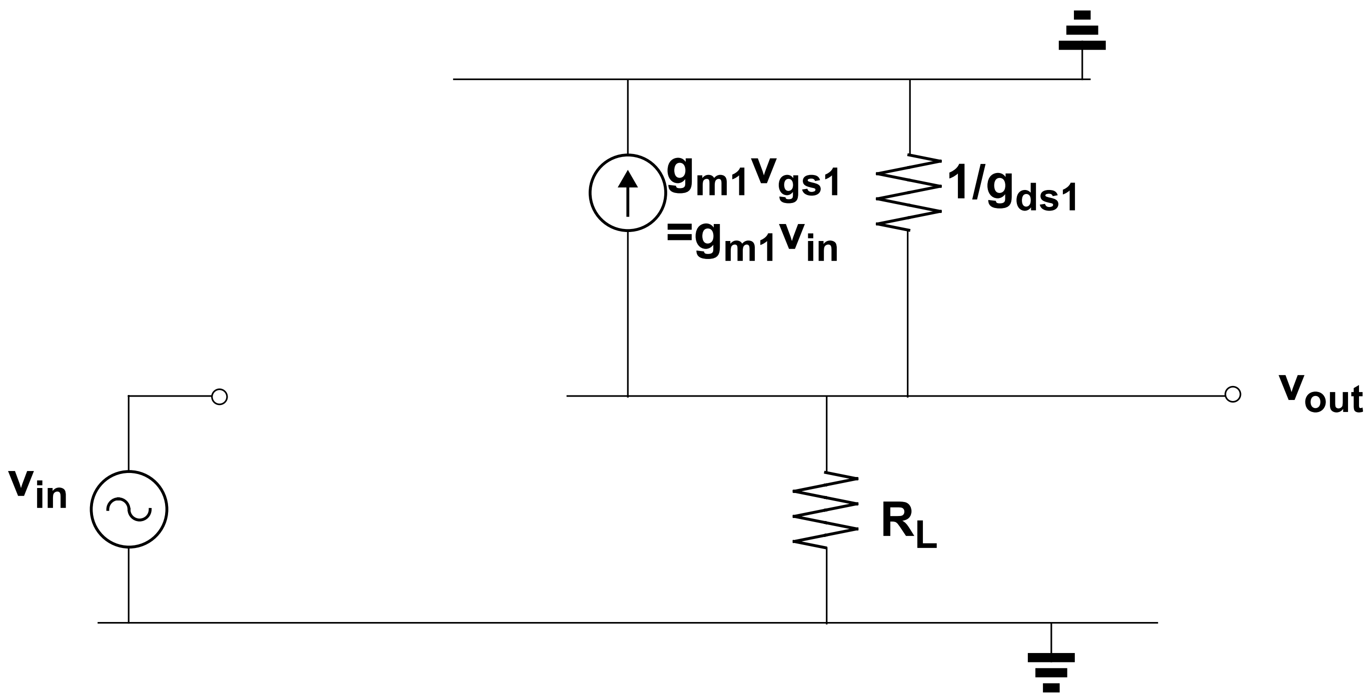


KCL at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + \frac{1}{R_L}} = -g_{m1}R_L$$

(ii) For the circuit in Figure 1a, calculate the value of the small-signal voltage gain in dB.

$$I_{D1} = \frac{K_p'}{2}\frac{W}{L}(|V_{GS1}|-|V_{tp}|)^2 = \frac{50\mu A/V^2}{2} \cdot \frac{64}{1} \cdot (1-0.75)^2 = 100\mu A$$

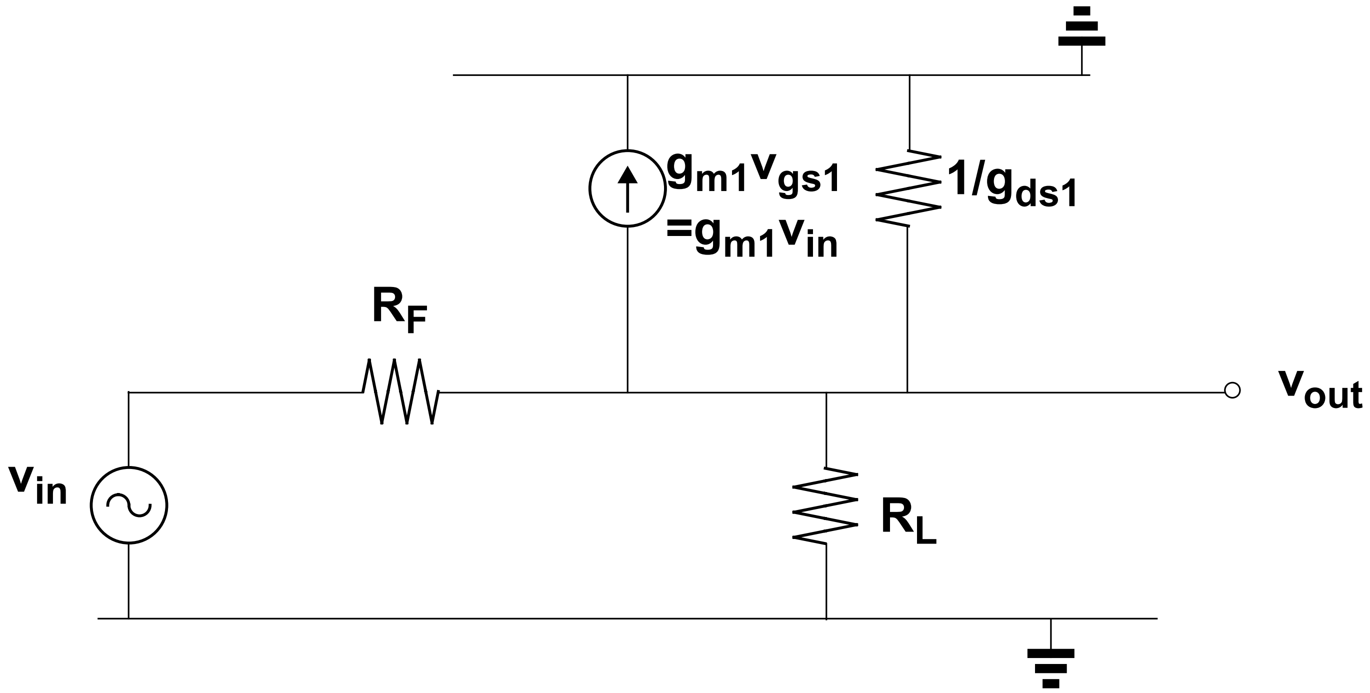
$$|V_{GS1}| - |V_{tp}| = 1V - 0.75V = 0.25V$$

$$g_{m1} = \frac{2I_{D1}}{(|V_{GS1}|-|V_{tp}|)} = \frac{2 \times 100\mu A}{0.25} = 800\mu A/V$$

$$\frac{v_{out}}{v_{in}} \approx -g_{m1}R_{L1} = -800\mu A/V \times 15k\Omega = -12$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 21.6dB$$

(iii) Draw the small-signal equivalent circuit for the circuit shown in Figure 1b and derive an expression for the small-signal voltage gain (v_{out}/v_{in}) of the circuit.



KCL at output node

$$\frac{(v_{out} - v_{in})}{R_F} + g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\left(g_{m1} - \frac{1}{R_F}\right)v_{in} = -\left(g_{ds1} + \frac{1}{R_F} + \frac{1}{R_L}\right)v_{out}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} - \frac{1}{R_F}}{g_{ds1} + \frac{1}{R_F} + \frac{1}{R_L}} \approx -\frac{g_{m1} - \frac{1}{R_F}}{\frac{1}{R_F} + \frac{1}{R_L}}$$

(iv) For the circuit in Figure 1b, calculate the value of the small-signal voltage gain in dB

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} - \dfrac{1}{R_F}}{g_{ds1} + \dfrac{1}{R_F} + \dfrac{1}{R_L}}$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1} - \dfrac{1}{R_F}}{g_{ds1} + \dfrac{1}{R_F} + \dfrac{1}{R_L}} \approx -\frac{g_{m1} - \dfrac{1}{R_F}}{\dfrac{1}{R_F} + \dfrac{1}{R_L}} = -\frac{800\mu A/V - \dfrac{1}{5k\Omega}}{\dfrac{1}{5k\Omega} + \dfrac{1}{15k\Omega}} = -2.3$$

$$20\log\left|\frac{v_{out}}{v_{in}}\right| = 7.0dB$$

Question 2

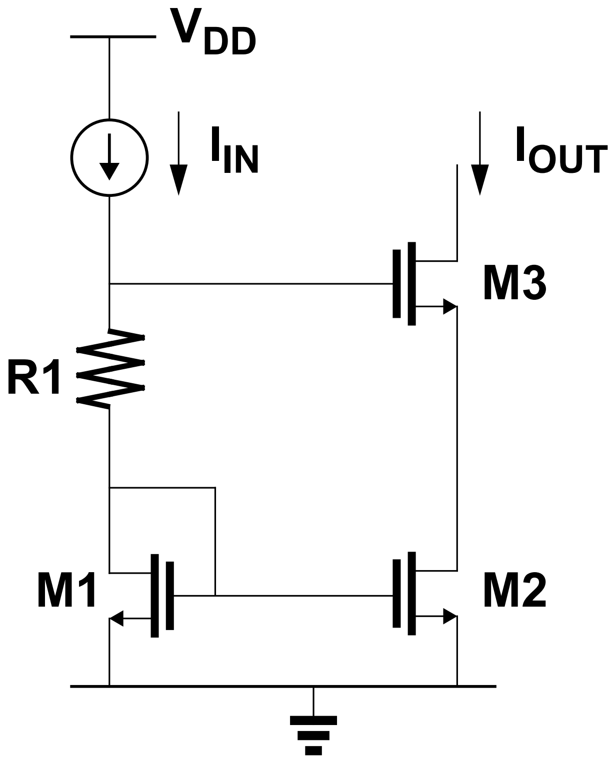


Figure 2

Figure 2 shows a cascoded current mirror.
Assume K_n'=200µA/V^2, V_{tn}=800mV.
All transistors have W/L=25µm/2µm.

(i) If I_{IN}=I_{OUT}=200µA, what is the minimum voltage at the output node, i.e. the drain of M3, such that all transistors are biased in saturation?
What minimum value of R1 is required to ensure M2 is in saturation?

(ii) The current mirror is modified by changing M2 and R1 only.
What W/L of M2 and minimum value of R1 would be required to increase the output current to 800µA and still ensure all transistors are in saturation?

(iii) It is required to measure the small-signal output resistance of the current mirror (i.e. the small-signal resistance looking into the drain of M3). Draw a small-signal equivalent circuit showing how this can be done.
You may assume that the gates of M2 and M3 are at small-signal ground.

(iv) Derive an expression for the small-signal output resistance, and reduce this to its simplest form assuming $g_{m1}, g_{m2}, g_{m3} >> g_{ds1}, g_{ds2}, g_{ds3}$.

(i) If $I_{IN}=I_{OUT}=200\mu A$, what is the minimum voltage at the output node, i.e. the drain of M3, such that all transistors are biased in saturation?
What minimum value of R1 is required to ensure M2 is in saturation?

Bias current of M1, M2, M3 is 200µA. All have same W/L so same V_{GT}

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS1}-V_t)^2 \Rightarrow V_{GS1}-V_t = \sqrt{\frac{2I_{D1}}{K_n'\frac{W}{L}}} = \sqrt{\frac{2\cdot 200\mu A}{200\mu A/V^2\frac{25}{2}}} = 400mV$$

$$V_{GS1}-V_t = 400mV$$

$$V_{D3min} = (V_{GS2}-V_t)+(V_{GS3}-V_t) = 0.4V+0.4V = \underline{\underline{0.8V}}$$

Then

$$V_{G3} = (V_{GS2}-V_t)+(V_{GS3}-V_t)+V_t = 0.4V+0.4V+0.8V = 1.6V$$

$$R_1 = \frac{V_{G3}-V_{G1}}{I_{IN}} = \frac{1.6V-1.2V}{200\mu A} = \underline{\underline{2k\Omega}}$$

(ii) The current mirror is modified by changing M2 and R1 only.
What W/L of M2 and minimum value of R1 would be required to increase the output current to 800µA and still ensure all transistors are in saturation?

$$I_{D2} = \frac{K_n'}{2}\frac{W}{L}(V_{GS2}-V_t)^2$$

For I_{D2} = 800µA, same V_{GS}-V_t, W/L needs to increase by 4 times, e.g. to <u>100/2</u>

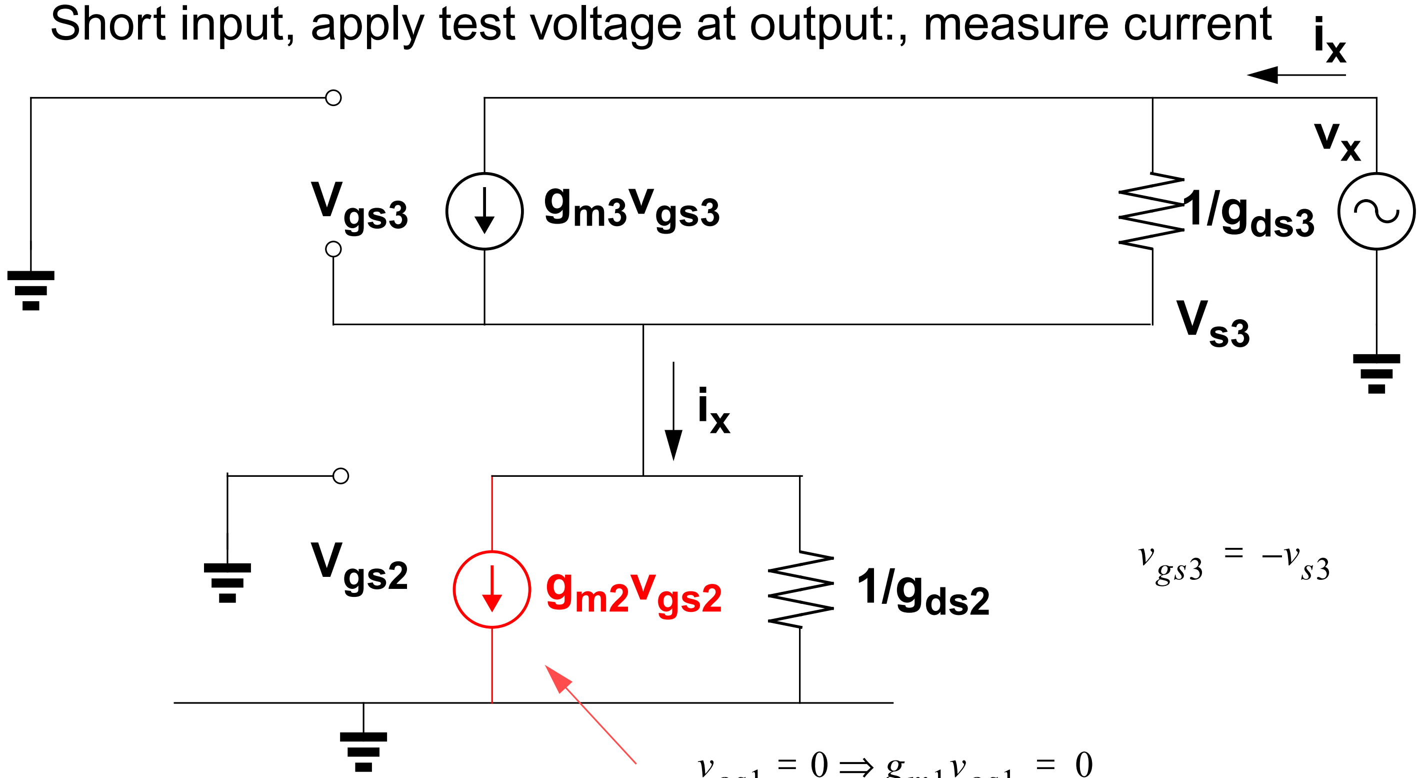
For I_{D3} = 800µA, V_{GS3}-V_t needs to increase by 2 times, i.e. to 0.8V, so V_{G3}=2V

$$R_1 = \frac{V_{G3}-V_{G1}}{I_{IN}} = \frac{2V-1.2V}{200\mu A} = \underline{\underline{4k\Omega}}$$

(iii) It is required to measure the small-signal output resistance of the current mirror (i.e. the small-signal resistance looking into the drain of M3). Draw a small signal model showing how this can be done.
You may assume that the gates of M2 and M3 are at ac ground.

Cascode: Output Resistance R_{out}

Short input, apply test voltage at output:, measure current i_x



$$v_{gs3} = -v_{s3}$$

$$v_{gs1} = 0 \Rightarrow g_{m1}v_{gs1} = 0$$

(iv) Derive an expression for the small-signal output resistance.
Show by assuming g_{m1}, g_{m2}, g_{m3} >> g_{ds1}, g_{ds2}, g_{ds3} that this approximates to

$$r_{out} = \frac{g_{m3}}{g_{ds3}} \cdot \frac{1}{g_{ds2}}$$

$$i_x = g_{m3}v_{gs3} + (v_x - v_s)g_{ds3}$$

$$i_x = -g_{m3}v_{s3} + v_x g_{ds3} - v_s g_{ds3}$$

Since $$v_{s3} = \frac{i_x}{g_{ds2}}$$

$$i_x = -g_{m3}\frac{i_x}{g_{ds2}} + v_x g_{ds3} - \frac{i_x}{g_{ds2}}g_{ds3}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1}{g_{ds3}}\left(1 + \frac{g_{m3}}{g_{ds2}} + \frac{g_{ds3}}{g_{ds2}}\right) \approx \frac{1}{g_{ds2}}\left(\frac{g_{m3}}{g_{ds3}}\right)$$

Question 3

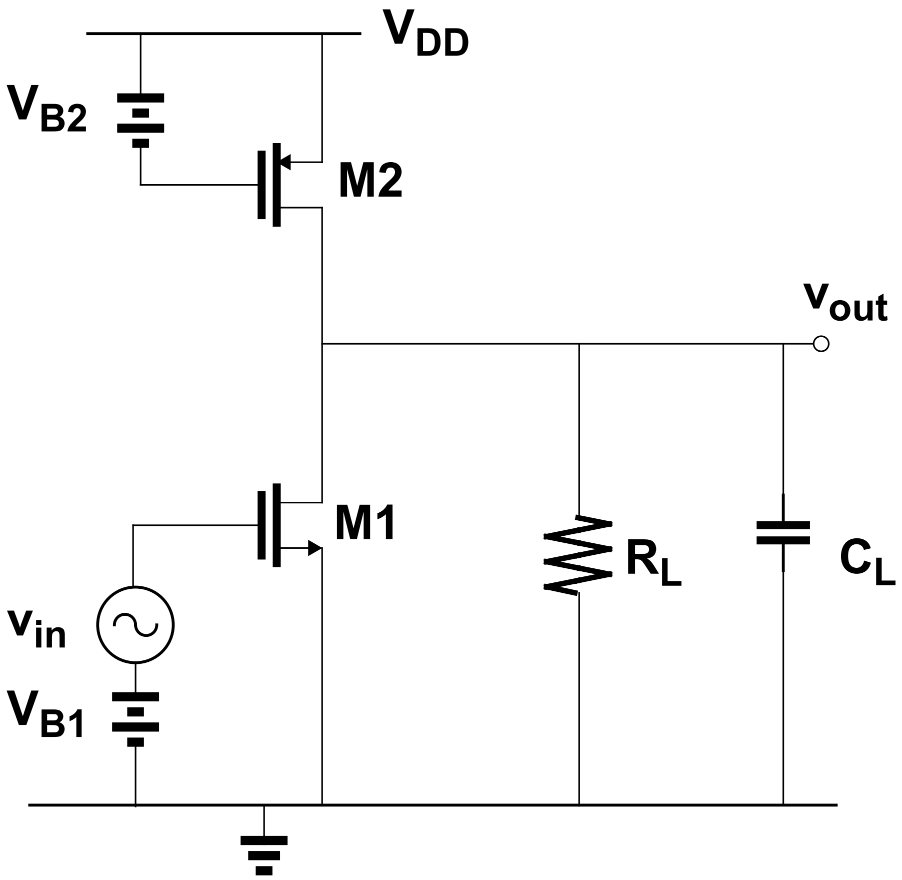


Figure 3

For the questions below you may assume $1/R_L >> g_{ds1}, g_{ds2}$, and that all transistors are biased in saturation.
All transistors have W/L=25µm/1µm.
Take $K_n' = 200\mu A/V^2$, $I_{D1} = 100\mu A$

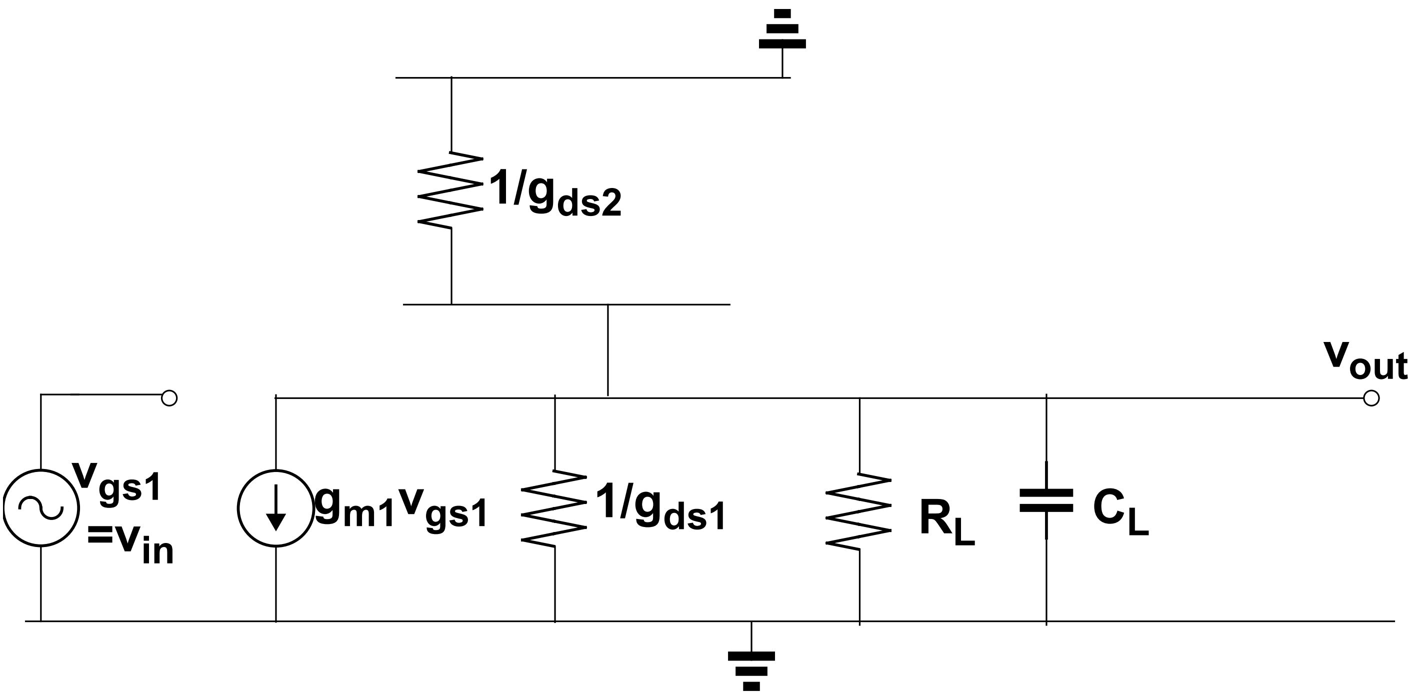
(i) Draw the small-signal equivalent circuit of the gain stage shown in Figure 3.

(ii) Derive an expression for the high-frequency transfer function from v_{in} to v_{out}.

(iii) What value of R_L is required to give a small-signal dc gain (v_{out}/v_{in}) of 20dB?

(iv) With the value of R_L calculated in (iii), what value of C_L is required to give a small-signal gain of -40dB at 10MHz? Draw a Bode diagram of the gain response. Indicate the value of d.c. gain and the pole frequency.

(i) Draw the small-signal equivalent circuit of the gain stage shown in Figure 3.



(ii) Derive an expression for the high-frequency transfer function from v_{in} to v_{out}.

KCL at output node:

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} + \frac{v_{out}}{R_L} + v_{out}sC_L = 0$$

$$g_{m1}v_{in} + v_{out}\left(g_{ds1} + g_{ds2} + \frac{1}{R_L} + sC_L\right) = 0$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_{m1}}{g_{ds1} + g_{ds2} + \frac{1}{R_L} + sC_L}$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_{m1}}{g_{ds1} + g_{ds2} + \frac{1}{R_L}}\left(\frac{1}{1 + \frac{sC_L}{g_{ds1} + g_{ds2} + \frac{1}{R_L}}}\right) \approx g_{m1}R_L\left(\frac{1}{1 + sR_LC_L}\right)$$

(iii) What value of R_L is required to give a small-signal d.c. gain (v_{out}/v_{in}) of 20dB?

$$g_{m1} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{25}{1} \times 100\mu A} = 1000\mu A/V$$

Low-frequency gain given by

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{g_{m1}}{g_{ds1} + g_{ds2} + \frac{1}{R_L}} \approx g_{m1}R_L = 20dB = 10 \Rightarrow R_L = \frac{10}{g_{m1}} = \frac{10}{1000\mu A/V} = 10k\Omega$$

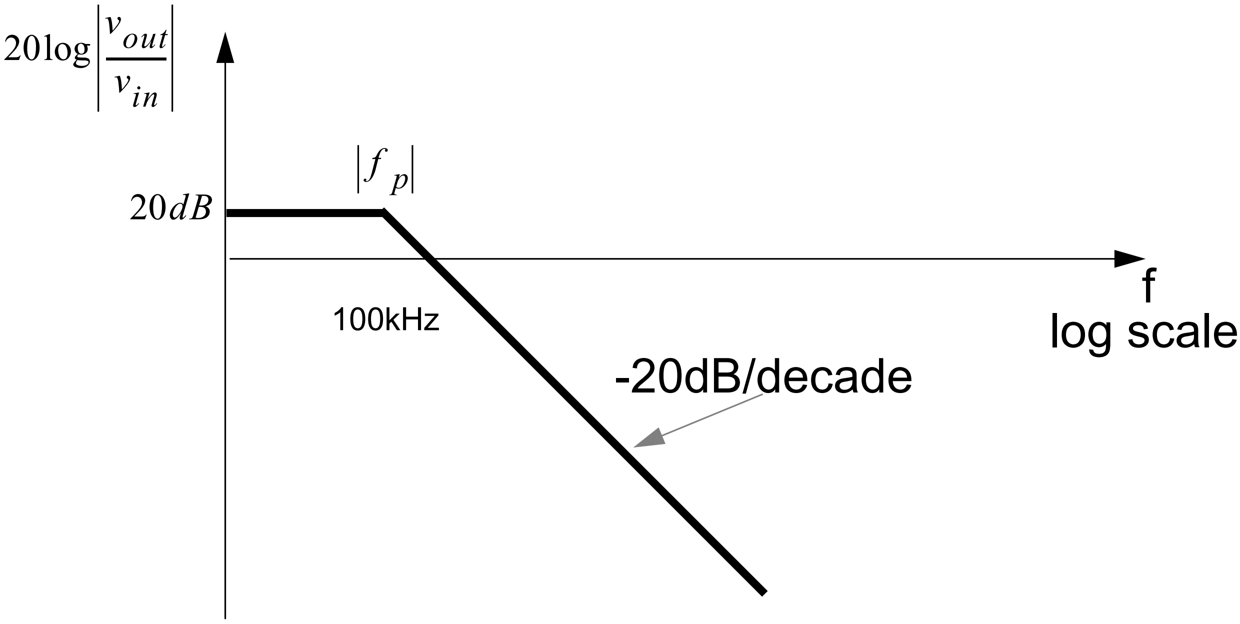
(iv) With the value of R_L calculated in (iii), what value of C_L is required to give a small-signal gain of -40dB at 10MHz? Draw a Bode diagram of the gain response. Indicate the value of d.c. gain and the pole frequency.

Pole frequency given by

$$|\omega_p| = \frac{g_{ds1} + g_{ds2} + \frac{1}{R_L}}{C_L} \approx \frac{1}{R_L C_L} \Rightarrow |f_p| = \frac{1}{2\pi R_L C_L}$$

Gain is required to be -40dB at 100MHz, i.e. 60dB down w.r.t. the dc gain.
For this, the pole needs to be three decades below 10MHz, i.e. at 10kHz

$$C_L = \frac{1}{2\pi f_p R_L} = \frac{1}{2\pi \cdot 10kHz \cdot 10k\Omega} = 1.59nF$$



Question 4

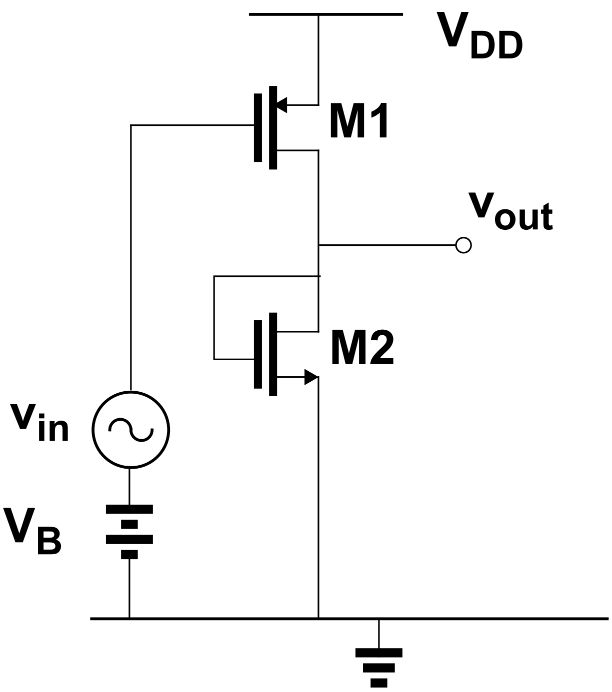


Figure 4

Assume M1 and M2 are operating in saturation.
Only thermal noise sources need be considered.
For calculations take Boltzmann's constant $k=1.38X10^{-23}J/^oK$, temperature $T=300^oK$

For the circuit shown in Figure 4:

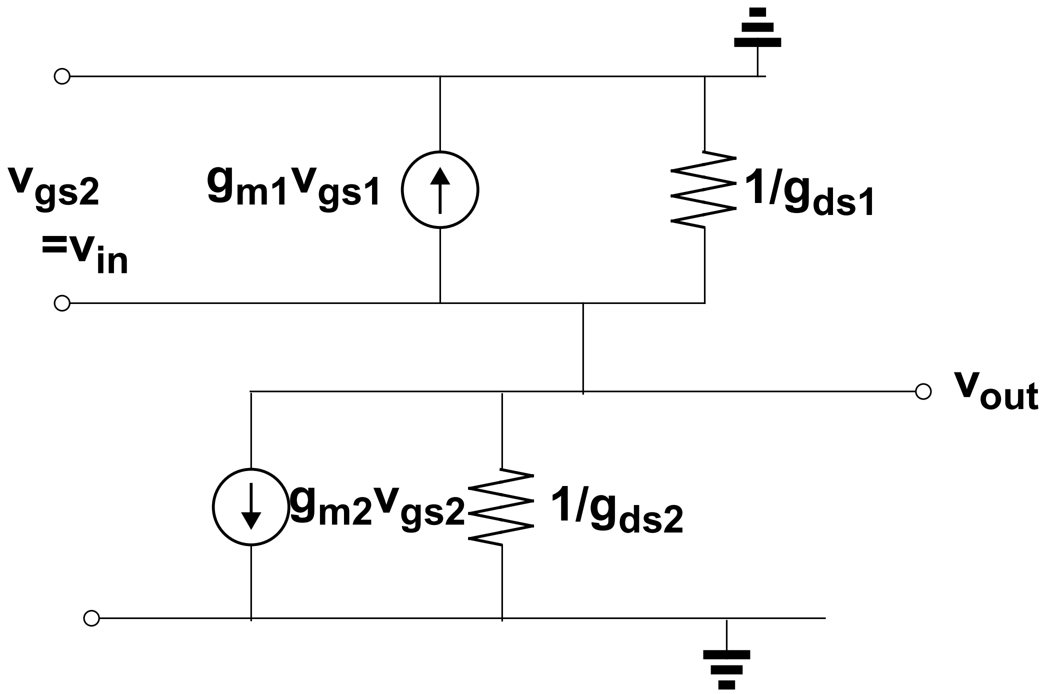
(i) Draw the small-signal equivalent circuit.
Give an expression for the low-frequency small-signal voltage gain (v_{out}/v_{in}) of the circuit?
Assume that $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.

(ii) Give an expression for the input-referred thermal noise voltage density in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T.

(iii) Calculate the input-referred thermal noise voltage density of the circuit.
For this calculation take $|V_{GS1}| = 1V$, $V_{GS2} = 2.8V$, $V_{tn} = 0.8V$, $|V_{tp}| = 0.8V$.
The drain current of M1 is 100μA.

(iv) Calculate the total noise voltage at the output over a bandwidth of 1MHz.
If the input signal v_{in} is a $1mV_{rms}$ sine wave in this bandwidth, calculate the signal-to-noise ratio in dB at the output over the bandwidth of 1MHz.

(i) What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) of the circuit? Assume that $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}$.



Assume that $g_{m1} >> g_{ds1}, g_{ds2}$ and that $g_{m2} >> g_{ds1}, g_{ds2}$

Current at output node

$$g_{m1}v_{gs1} + g_{m2}v_{gs2} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

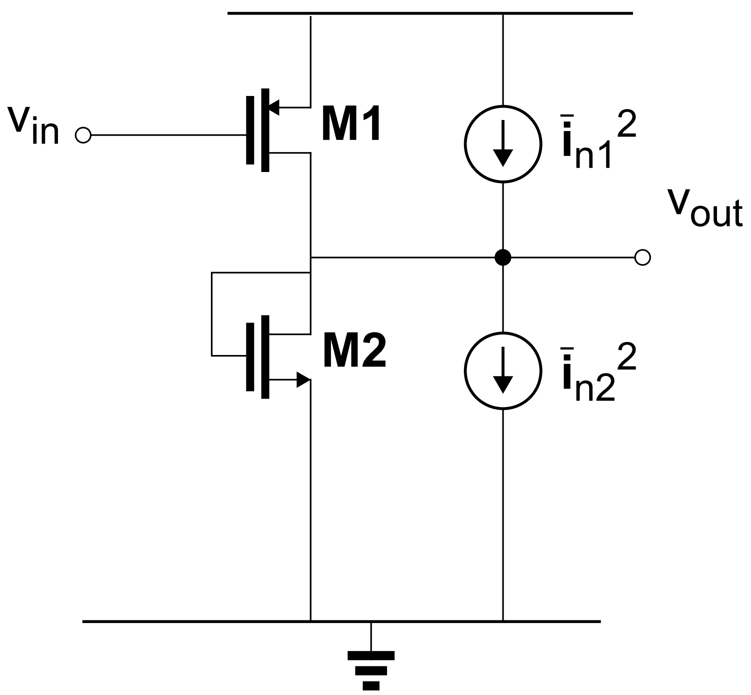
$$g_{m1}v_{in} + g_{m2}v_{out} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + g_{ds1} + g_{ds2}} \cong -\frac{g_{m1}}{g_{m2}}$$

Alternatively recognise that the current of the current-source $g_{m2}v_{gs2}$ is determined by voltage across its terminals i.e. is equivalent to a resistance $1/g_{m2}$.
Since $1/g_{m2} << 1/g_{ds2}$, $1/g_{m2} << 1/g_{ds1}$, can write directly

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

(ii) Derive an expression for the input-referred thermal noise voltage in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T?



Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}$$

Input-referred noise voltage density given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) Calculate the input-referred thermal noise voltage density of the circuit.
For this calculation take $|V_{GS1}| = 1V$, $V_{GS2} = 2.8V$, $V_{tn} = 0.8V$, $|V_{tp}| = 0.8V$.
The drain current of M1 is 100µA.

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

g_m given by

$$g_{m1} = \frac{2I_D}{(V_{GS}-V_T)} = \frac{2 \cdot 100\mu A}{1V - 0.8V} = 1mA/V$$

$$g_{m2} = \frac{2 \cdot 100\mu A}{2.8V - 0.8V} = 100\mu A/V$$

Input-referred noise of M1

$$\overline{v_{ni}} = \frac{\sqrt{(4 \cdot 1.38\times10^{-23} \cdot 300)\left(\frac{2}{3}\right)(1mA/V + 100\mu A/V)}}{1mA/V} = 3.48nV/\sqrt{Hz}$$

(iv) Calculate the total noise voltage at the output over a bandwidth of 1MHz.
If the input signal v_{in} is a $1mV_{rms}$ sine wave in this bandwidth, calculate the signal-to-noise ratio in dB at the output over the bandwidth of 1MHz.

Gain of stage

$$Gain = -\left(\frac{g_{m1}}{g_{m2}}\right) = -\frac{1mA/V}{100\mu A/V} = -10$$

Total noise at output given by

$$\overline{v_{notot}} = \overline{v_{ni}} \cdot \left(\frac{g_{m1}}{g_{m2}}\right) \cdot \sqrt{BW} = 3.48nV/\sqrt{Hz} \cdot 10 \cdot \sqrt{1MHz} = 34.8\mu V_{rms}$$

Output signal

$$v_{out} = -\left(\frac{g_{m1}}{g_{m2}}\right)v_{in} = -10 \cdot 1mV_{rms} = 10mV_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{10mV}{34.8\mu V_{rms}} = 287 \qquad \text{or } 49.2\text{ dB}$$