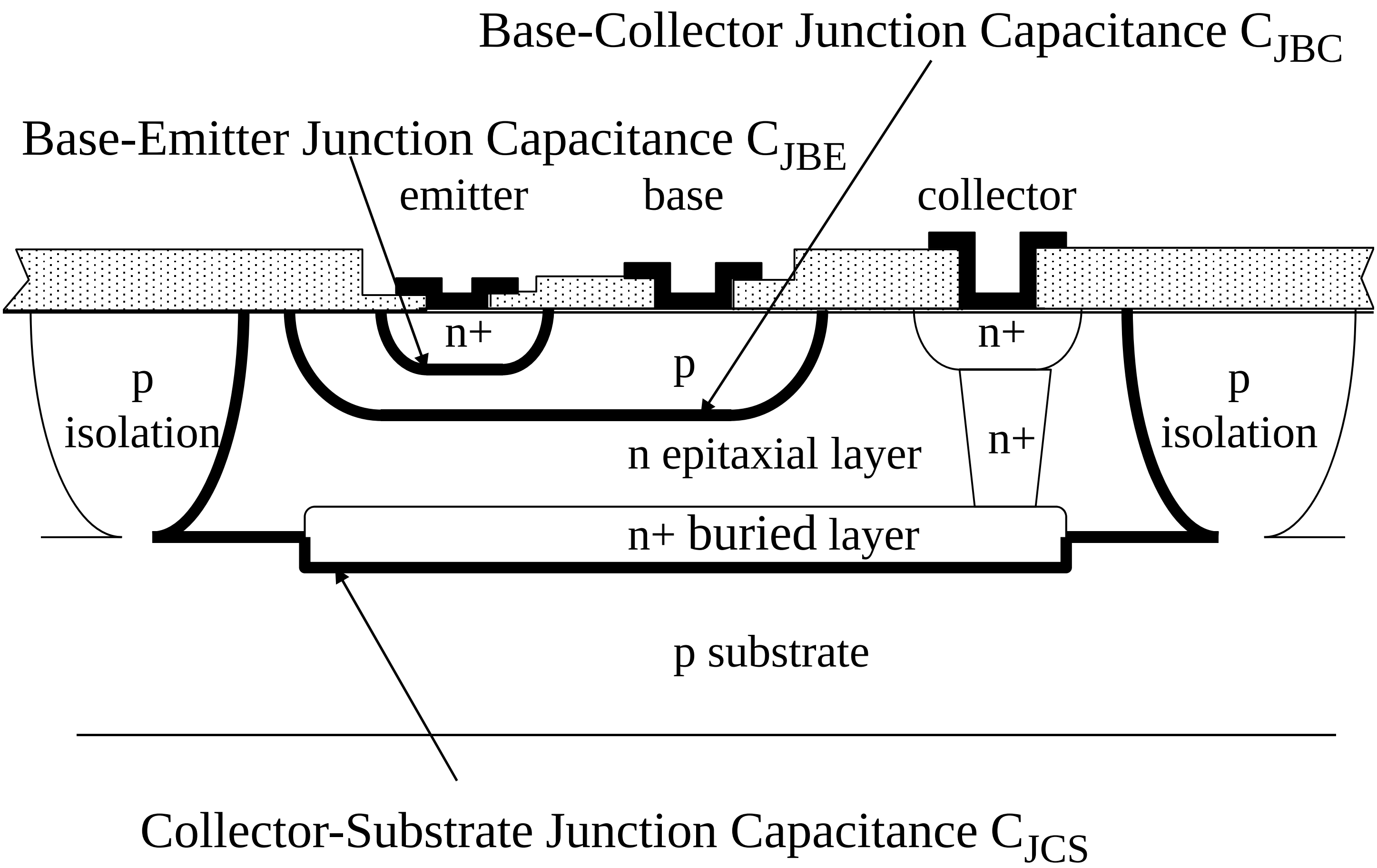
Q1 (b) 4 marks

The main junction capacitances of the BJT are as follows:



Q1 (c) 4 marks   Expression for base-emitter capacitance

$$\mathrm{C_{JBE}} = \frac{\mathrm{C_{JE}}}{\left(1 - \frac{\mathrm{V_{BE}}}{\mathrm{V_{JE}}}\right)^{\mathrm{M_{JE}}}} \qquad \mathrm{V_{BE}} \leq \mathrm{FC} \cdot \mathrm{V_{JE}}$$

$$Q = \tau I_C \qquad C_{DE} = \frac{dQ}{dV_{BE}} = \frac{d(\tau I_C)}{dV_{BE}} = \tau \frac{dI_C}{dV_{BE}} = \tau g_m$$

$$\mathrm{C_{BE}} = \mathrm{C_{JE}} + C_{DE}$$

Q1 (a) 4 marks

The low-frequency small-signal elements of the BJT are as follows:

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \qquad I_B = \frac{I_C}{\beta}$$

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{1}{V_T} I_S e^{\frac{V_{BE}}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right) = \frac{I_C}{V_T}$$

$$g_{out} = \frac{dI_C}{dV_{CE}} = \frac{1}{V_A} I_S e^{\frac{V_{BE}}{V_T}} \approx \frac{I_C}{V_A}$$

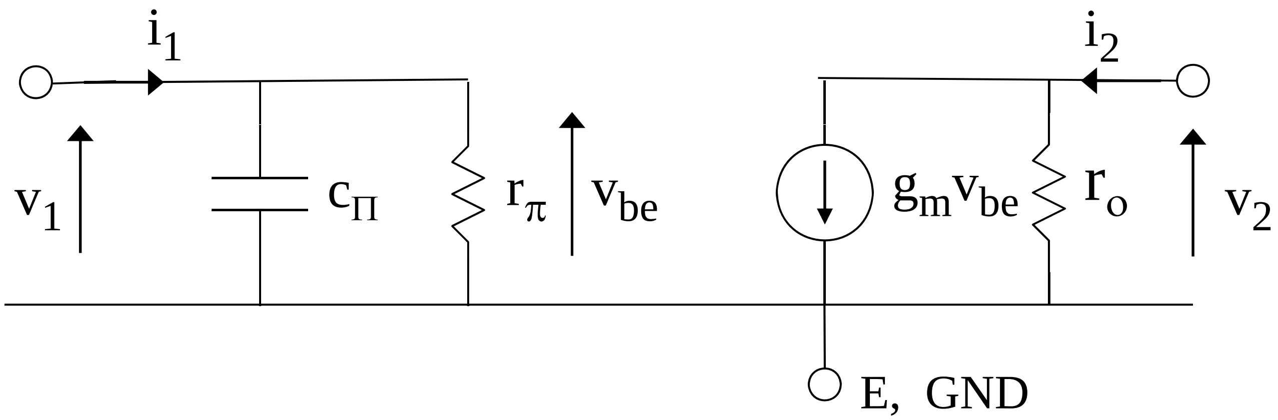
$$g_\pi = \frac{dI_B}{dV_{BE}} = \frac{d}{dV_{BE}}\left(\frac{I_C}{\beta}\right) = \frac{1}{\beta}\frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta}$$

$$r_{out} = \frac{1}{g_{out}} = \frac{V_A}{I_C}$$

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta}{g_m}$$

Q1 (d) 8 marks

y-parameters of BJT small-signal model



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{r_\pi} + j\varpi C_\pi \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = 0$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = g_m \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{r_O} = g_O$$

Putting in the bias voltages and temperature and performing the calculations gives:

i_c = 5.1mA i_b = 51.14uA c_j = 0.6pF, c_d = 19.78pF,
c_{be} = 20.38pF, g_m = 0.198S, g_o = 0.51mS, r_{pi}=505.6Ω,

$y_{11} = 0.002 + j0.192 = 0.192 \angle 89.4$
$y_{12} = 0$
y_{21}=0.198
y_{22}=0.51m

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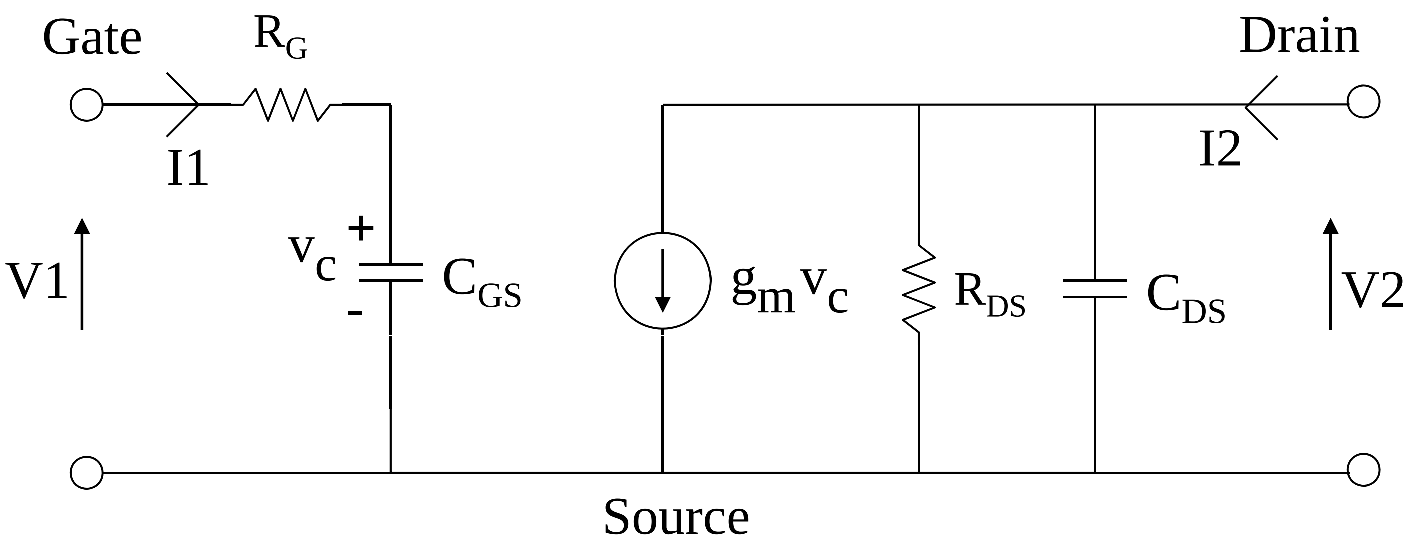
Question 2(a) 8 marks

MESFET small-signal circuit



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the above formulas to the equivalent circuit and simplifying the resulting expressions leads to the final y-parameter formulas:

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \qquad y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

$$y_{12} = 0 \qquad y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

Question 2(b) 8 marks

The previous expressions for the y-parameters can be re-arranged to allow the small-signal element values to be determined from the y-parameters. Using the y-parameters at 2GHz:

$y_{11} = 0.01 \angle 87.7°$
$y_{12} = 0$
$y_{21} = 0.20\angle -2.3°$
$y_{22} = 0.004 \angle 73.6°$

$$R_G = \mathfrak{Re}\left\{\frac{1}{y_{11}}\right\} = 4\Omega$$

$$C_{GS} = -\frac{1}{\omega \mathfrak{Im}\left\{\frac{1}{y_{11}}\right\}} = 0.8pF$$

$$g_m = \frac{1}{\mathfrak{Re}\left\{\frac{1}{y_{21}}\right\}} = 0.2S$$

$$R_{DS} = \frac{1}{\mathfrak{Re}\{y_{22}\}} = 90\Omega$$

$$C_{DS} = \frac{\mathfrak{Im}\{y_{22}\}}{\omega} = 0.3pF$$

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Question 2(c) 6 marks

For a two-port network the ABCD parameters are defined as follows:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

The circuit could be directly analysed to give formulas for the ABCD parameters or they could be determined from the y-parameters:

$$\begin{bmatrix} i_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Using the two sets of relationships, conversion may be achieved as follows:

$$\begin{bmatrix} i_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$A = -\frac{y_{22}}{y_{21}}$$

$$B = -\frac{1}{y_{21}}$$

$$C = \frac{y_{11}y_{22} - y_{12}y_{21}}{y_{21}}$$

$$D = -\frac{y_{11}}{y_{21}}$$

Performing the calculations for this transistor gives:

A= 0.02 ∠ -104°
B = 5 ∠ -177°
C = 1.98e-4∠ 164°
D= 0.05 ∠ -90°

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Q3(a) 5 marks

$$y(t) = \left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\varpi_1 t)$$
$$+\left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_1^2 A_2\right]\cos(\varpi_2 t)$$
$$+\frac{1}{4}\alpha_3 A_1^3 \cos 3\varpi_1 t + \frac{1}{4}\alpha_3 A_2^3 \cos 3\varpi_2 t$$
$$+\frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 + \varpi_2)t + \frac{3}{4}\alpha_3 A_1^2 A_2 \cos(2\varpi_1 - \varpi_2)t$$
$$+\frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 + \varpi_1)t + \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\varpi_2 - \varpi_1)t$$

Taking the case of $A_1=A_2=A$, and considering the outputs at the fundamental frequencies to be the desired outputs, the amplitudes of the desired signals are:

$$A_{SIG} = \left|\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2}\alpha_3 A^3\right| = \left|\alpha_1 A + \frac{9}{4}\alpha_3 A^3\right|$$
$$\approx |\alpha_1| A \quad \text{if} \quad \alpha_1 >> \frac{9}{4}\alpha_3 A^2$$

In this case the unwanted 3rd-order inter-modulation (IM) signals are given by:

$$A_{IM3} = \frac{3}{4}|\alpha_3| A^3$$

As A increases the IM3 outputs will eventually will reach the same level as the desired signal output. This condition is called the “third-order IM intercept point”, IP3. The input amplitude corresponding to this condition is A=AIP3 and at this amplitude:

$$A_{SIG} = A_{IM3} \Rightarrow |\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3 \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}}$$

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Q3(b) 8 marks

The sensitivity of system is defined as the minimum input signal power which is required to give a specified minimum signal-to-noise ratio at the output.

For a given output SNR the input power can be found from the noise figure:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig} / P_{RS}}{SNR_{out}} \Rightarrow P_{sig} = P_{RS} \cdot F \cdot SNR_{out}$$

(per unit bandwidth)

Assuming the system bandwith is B:

$$P_{sig} = P_{RS} \cdot F \cdot SNR_{out} \cdot B$$

Turning the quantities into logs and setting the output SNR to the minimum required value and the input signal power to the minimum value needed to give the required output SNR:

$$P_{\min} = P_{RS}\Big|_{dBm/Hz} + NF + SNR_{\min}\Big|_{dB} + 10\log_{10} B$$

where $NF = 10\log_{10} F$

If the input is conjugate matched to the source the noise power delivered to the input will be:

$$P_{RS} = \frac{\overline{v_n^2}}{4R_S} = \frac{4kTR_S}{4R_S} = kT = -174\, dBm/Hz \quad \text{(at 300K)}$$

$$P_{\text{in,min}} = \underbrace{-174\,\text{dBm/Hz} + NF + 10\log_{10} B}_{\text{noise floor}} + SNR_{\min}\Big|_{dB}$$

$$\text{P}_{\text{Floor}} = -174\,\text{dBm/Hz} + NF + 10\log_{10} B$$

8 marks

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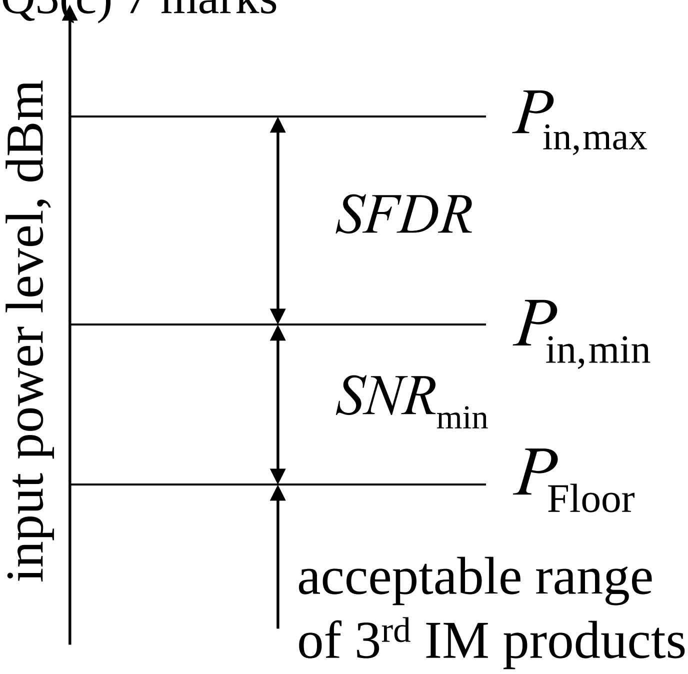
Q3(c) 7 marks



$$P_{in,\min} = P_{Floor} + SNR_{\min}$$

$$P_{in,\max} = \frac{2P_{IIP3} + P_{Floor}}{3}$$

The minimum acceptable power in dB is the noise floor plus the required minimum output SNR. As the input power level increase, two or more signals will give IM3 products. The maximum acceptable input power level is considered to be the input power level at which the IM3 products are as high as the noise floor. The range of power between the minimum level and the maximum level is known as the spurious free dynamic range (SFDR).

$$SFDR = P_{in,\max} - P_{in,\min} = \frac{2P_{IIP3} + P_{Floor}}{3} - \left(P_{Floor} + SNR_{\min}\right)$$

$$= \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min}$$

Using NF = 10 dB, P_{IIP3} = -5dBm, B = 2MHz, T = 300K, SRN_{min} = 15dB gives:

$$P_{\text{Floor}} = -173.83 + 10 + 10\log_{10}(2\times 10^{6}) = -100.82\,\text{dBm}$$

$$SFDR = \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min} = \frac{2(-5+100.82)}{3} - 15 = 48.88\,\text{dB}$$

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Question 4(a)

$$s_{11} = 0.33\angle -150^\circ \quad s_{12} = 0.01\angle 60^\circ \quad s_{21} = 4.0\angle -50^\circ \quad s_{22} = 0.50\angle -45^\circ$$

$$F_{\min} = 3.0\ dB \quad \Gamma_{opt} = 0.75\angle 180^\circ \quad R_N = 10.0\Omega$$

(i) 2 marks

Rollet Stability Factor

$$K = \frac{1-|s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} \qquad \Delta = s_{11}s_{22} - s_{12}s_{21}$$

Test for Unconditional Stability:

$$K > 1 \quad and \quad |\Delta| < 1$$

This device is unconditionally stable because:

$$|\Delta| = 0.2 \quad k = 8.52$$

(ii) 2 marks

Maximum Unilateral Transducer Power Gain

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} = 23.94 = 13.8dB$$

(iii) 2 marks

Unilateral Figure of Merit

$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{\left(1-|s_{11}|^2\right)\left(1-|s_{22}|^2\right)} \qquad \frac{1}{(1+M)^2} < \frac{G_T}{G_{TU,\max}} < \frac{1}{(1-M)^2}$$

Calculating M, gives the error in predicting gain of +/- 0.09dB

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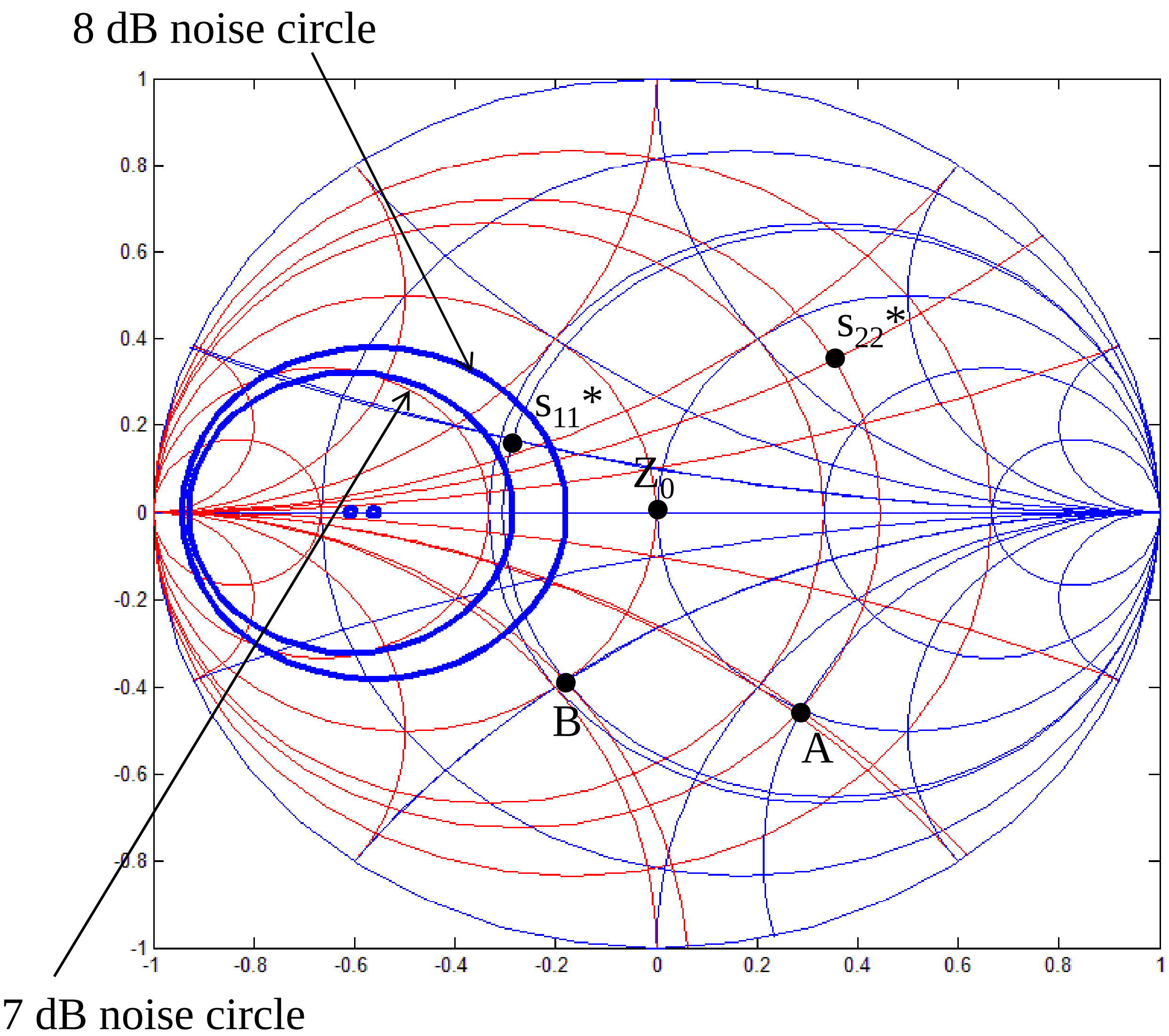
Smith Chart for (b) and (c) This is a Matlab generated plot.
The plot should be drawn on real Smith Chart paper



To transform Z_0 to s_{22}*: Series capacitor, followed by shunt inductor

To transform Z_0 to s_{11}*: Shunt capacitor, followed by series inductor

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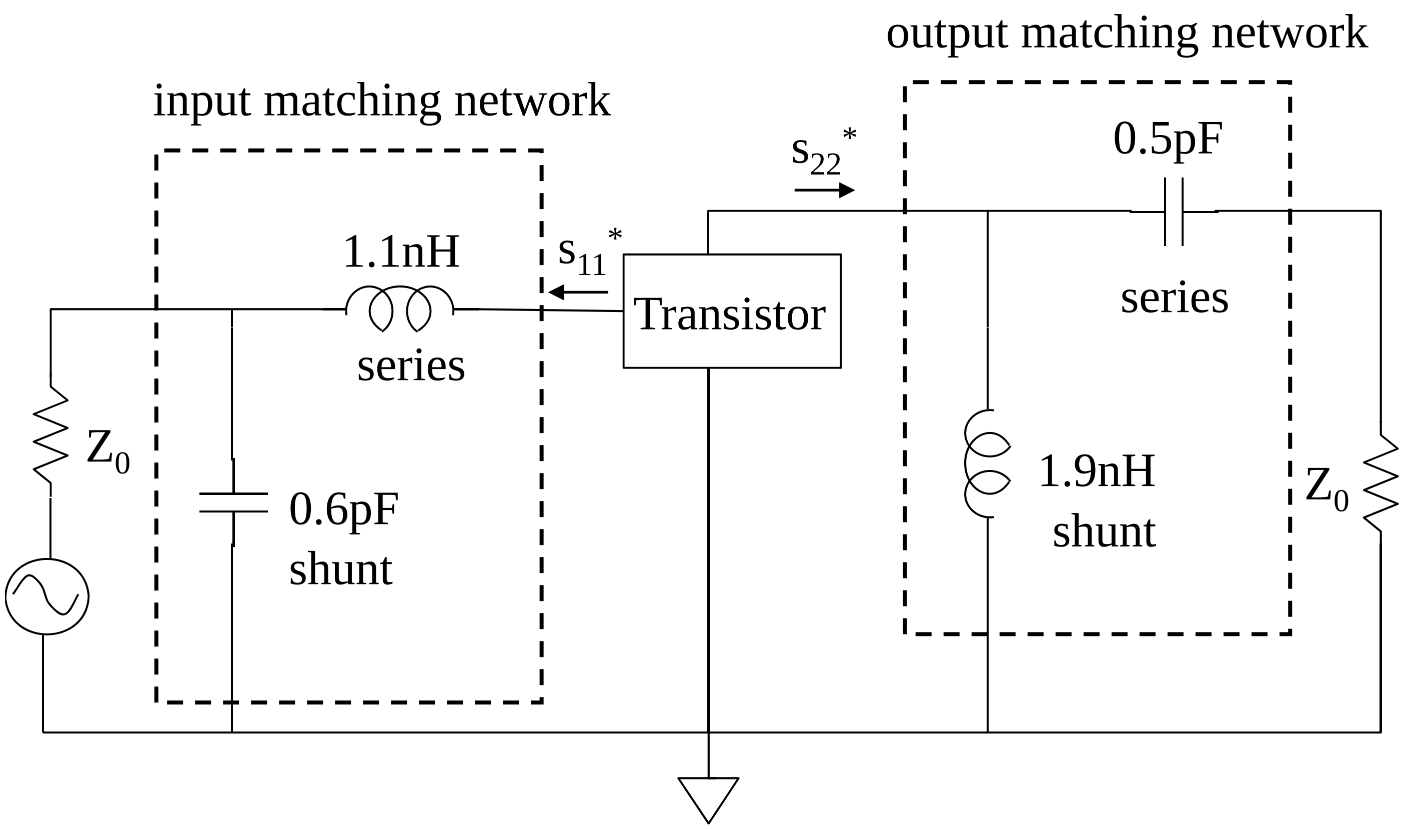
Q4(b) 10 marks

Series inductor: $L = \dfrac{Z_0|\Delta x|}{2\pi f}$ Series capacitor: $C = \dfrac{1}{2\pi f|\Delta x|Z_0}$

Shunt inductor: $L = \dfrac{Z_0}{2\pi f|\Delta b|}$ Shunt capacitor: $C = \dfrac{|\Delta b|}{2\pi f Z_0}$



Q4(c) 4 marks

A series of noise circles need to be drawn to identify those closest to the point s_{11}* where the input reflection coefficient is placed. The nearest circles are 7dB and 8dB so the noise figure that is achieved is between 7 and 8 dB.

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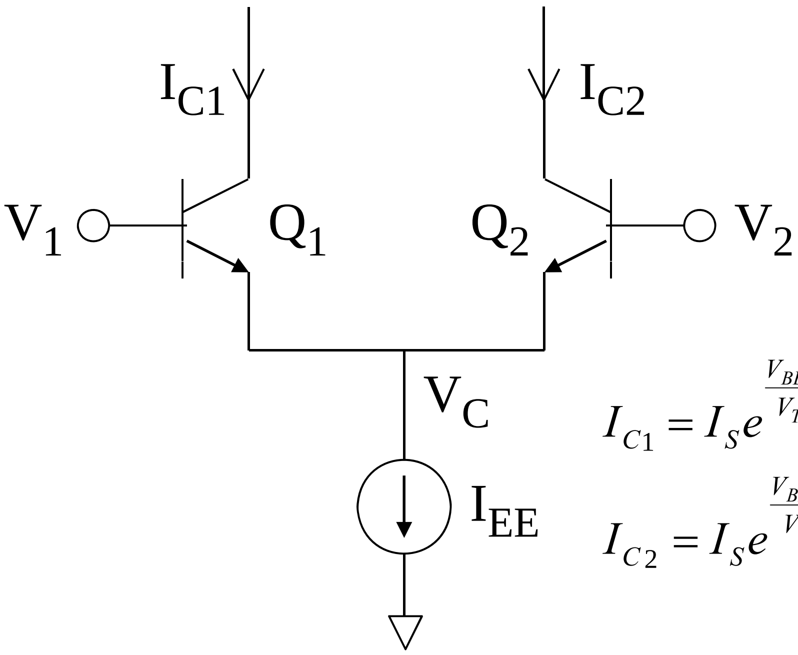
Q5(a) 12 marks



$$I_{C1} = I_S e^{\frac{V_{BE1}}{V_T}} = I_S e^{\frac{V_1 - V_C}{V_T}} \quad V_1 - V_C >> V_T$$

$$I_{C2} = I_S e^{\frac{V_{BE2}}{V_T}} = I_S e^{\frac{V_2 - V_C}{V_T}} \quad V_2 - V_C >> V_T$$

$$\frac{I_{C1}}{I_{C2}} = \frac{I_S e^{\frac{V_1 - V_C}{V_T}}}{I_S e^{\frac{V_2 - V_C}{V_T}}} = e^{\frac{V_1 - V_2}{V_T}} = e^{\frac{V_d}{V_T}} \Rightarrow \frac{I_{C2}}{I_{C1}} = e^{-\frac{V_d}{V_T}} \quad \text{where } V_d = V_1 - V_2$$

$$I_{C1} + I_{C2} = I_{EE} \Rightarrow I_{C1} + I_{C1} e^{-\frac{V_d}{V_T}} = I_{EE} \Rightarrow I_{C1} = \frac{I_{EE}}{1 + e^{-\frac{V_d}{V_T}}} \qquad I_{C2} e^{\frac{V_d}{V_T}} + I_{C2} = I_{EE} \Rightarrow I_{C2} = \frac{I_{EE}}{1 + e^{\frac{V_d}{V_T}}}$$

$$\Delta I_C = I_{C1} - I_{C2} = \frac{I_{EE}}{1 + e^{-\frac{V_d}{V_T}}} - \frac{I_{EE}}{1 + e^{\frac{V_d}{V_T}}} = I_{EE} \left(\frac{1}{1 + e^{-\frac{V_d}{V_T}}} - \frac{1}{1 + e^{\frac{V_d}{V_T}}} \right)$$

$$= I_{EE} \left(\frac{e^{\frac{V_d}{V_T}}}{e^{\frac{V_d}{V_T}} + 1} - \frac{1}{1 + e^{\frac{V_d}{V_T}}} \right) = I_{EE} \frac{e^{\frac{V_d}{V_T}} - 1}{e^{\frac{V_d}{V_T}} + 1}$$

$$= I_{EE} \frac{e^{\frac{V_d}{2V_T}} - e^{-\frac{V_d}{2V_T}}}{e^{\frac{V_d}{2V_T}} + e^{-\frac{V_d}{2V_T}}} = I_{EE} \tanh\left(\frac{V_d}{2V_T} \right)$$

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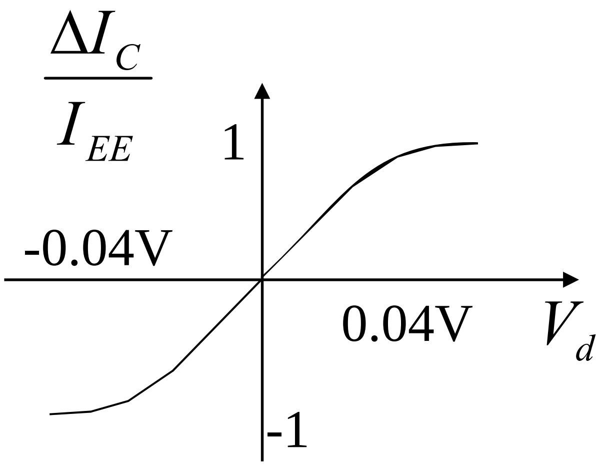
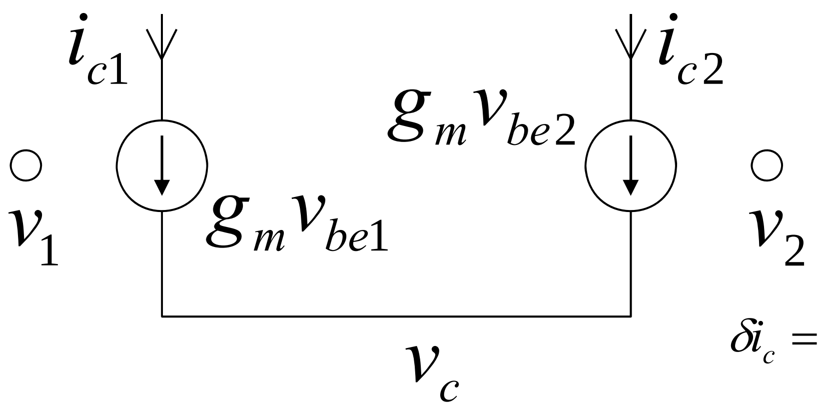
Q5(a) Continued



Q5(b) 4 marks

$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d}{2V_T}\right)$$

For small V_d this becomes: $\Delta I_C = \frac{I_{EE}}{2V_T} V_d$



small-signal analysis

$$\delta i_c = i_{c1} - i_{c2} = g_m \delta v$$

$$g_m = \frac{I_C}{V_T} = \frac{I_{EE}}{2V_T} \Rightarrow \delta i_c = \frac{I_{EE}}{2V_T} \delta v$$

(assuming that I_{EE} is shared equally by Q_1 and Q_2)

The small-signal analysis gives the same result as the large signal

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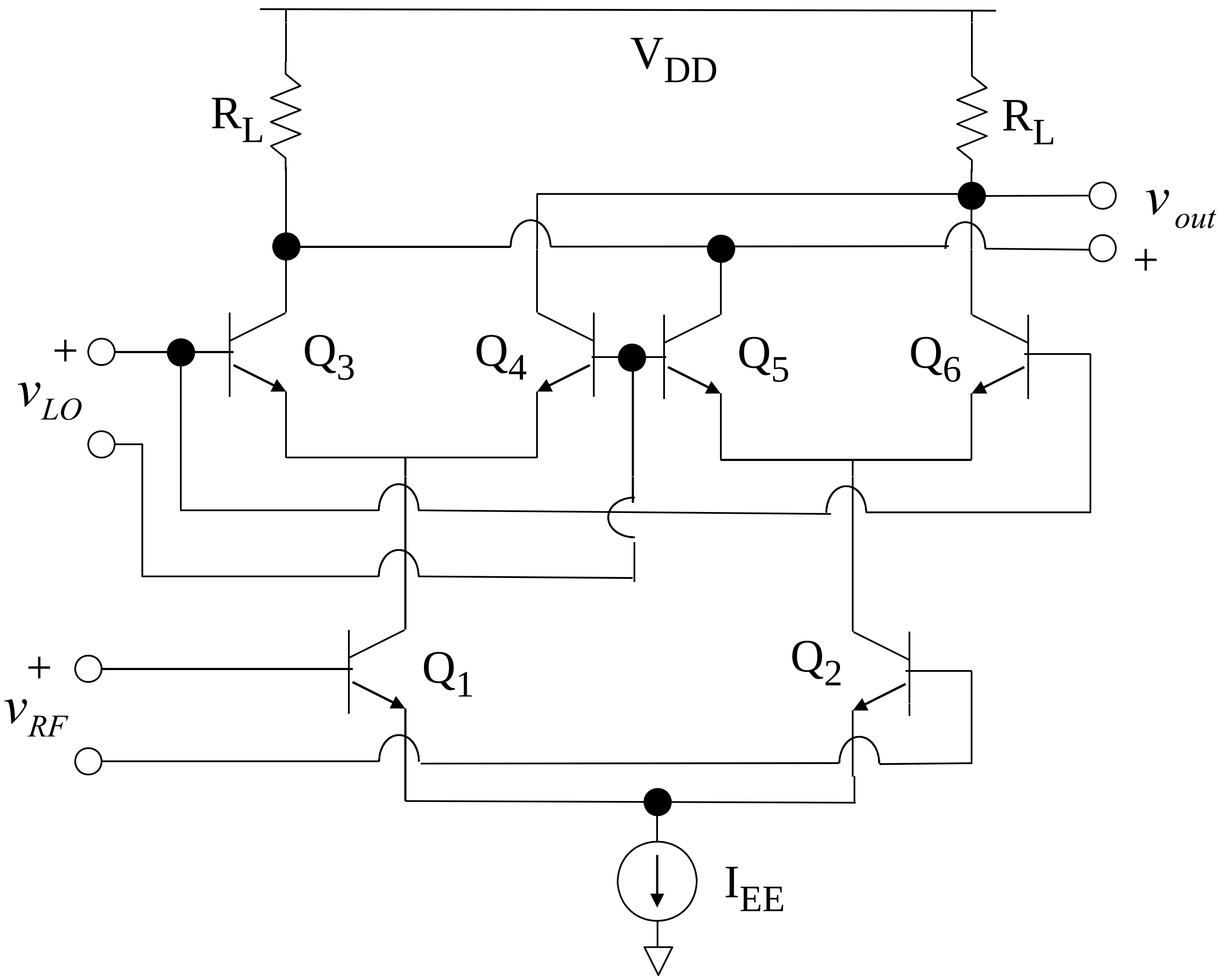
Q5(c) 4 marks Gilbert-cell mixer



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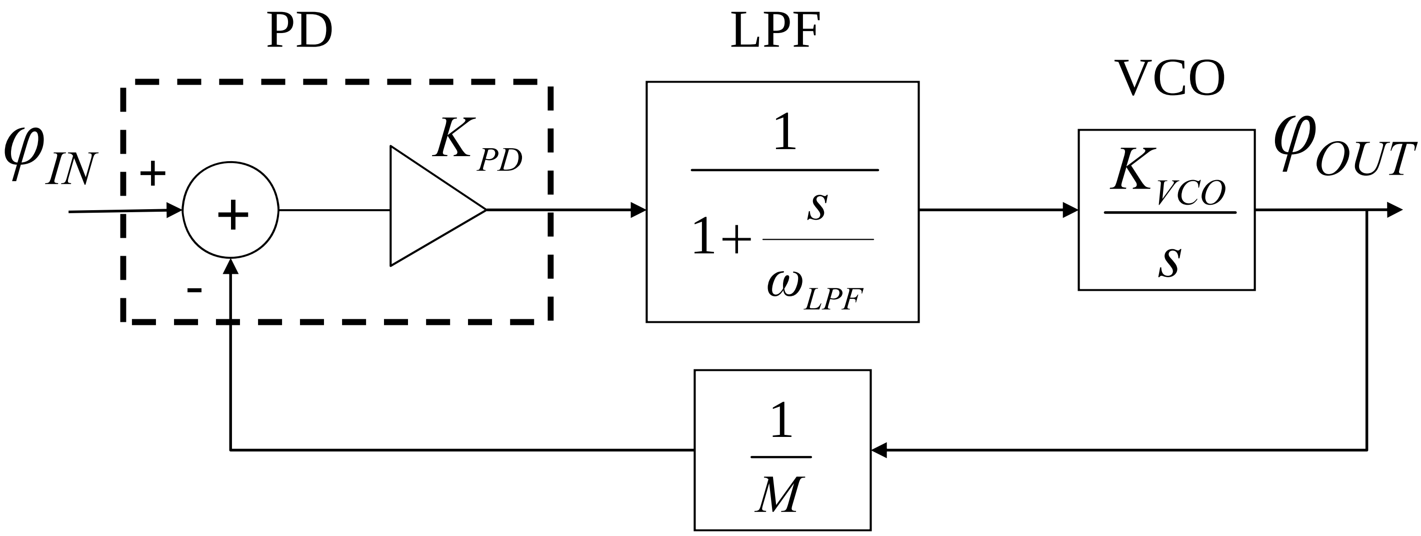
Q6(a) 4 marks

Type 1 PLL with integer feedback divider



Q6(b) 8 marks

$$H(s)\Big|_{OPEN} = \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)}\Bigg|_{OPEN} = K_{PD}\cdot\frac{1}{1+\frac{s}{\omega_{LPF}}}\cdot\frac{K_{VCO}}{s} = \frac{K_{PD}K_{VCO}}{s+\frac{s^2}{\omega_{LPF}}}$$

$$\varphi_{OUT}(s) = H(s)\left(\varphi_{IN}(s)-\frac{\varphi_{OUT}(s)}{M}\right) \Rightarrow \varphi_{OUT}(s)\left(1+\frac{H(s)}{M}\right) = H(s)\varphi_{IN}(s)$$

$$\Rightarrow \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)} = \frac{H(s)}{1+\frac{H(s)}{M}} = \frac{1}{\frac{1}{H(s)}+\frac{1}{M}} = \frac{1}{\frac{s+\frac{s^2}{\omega_{LPF}}}{K_{PD}K_{VCO}}+\frac{1}{M}} = \frac{K_{PD}K_{VCO}}{s+\frac{s^2}{\omega_{LPF}}+\frac{K_{PD}K_{VCO}}{M}}$$

$$= \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2+\omega_{LPF}s+\frac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$$

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Q6(c)

(i) 2 marks

For an integer feedback the reference frequency must be equal to the desired step size i.e. 200kHz in this case

(ii) 2 marks

The range of divider values needed are:

$$M = \frac{925}{0.2} = 4625 \qquad to \qquad M = \frac{960}{0.2} = 4800$$

(iii) 2 marks

A rule of thumb to ensure good stability is to set the low-pass filter cut-off frequency to 10% of the reference frequency i.e. 20kHz in this case.

(iv) 2 marks

With an integer feedback divider, the reference frequency must be the same as the minimum desired frequency step. If this step is small then because the low-pass filter cut-off frequency is usually 10-20% of the reference frequency, the LPF cut-off frequency will be low and the dynamic response of the PLL will be slow. A fractional-N divider allows a small frequency step for a larger reference frequency which facilitates a faster dynamic response.

Question 7 is an essay-type question based on a continuous assessment carried out during the year.