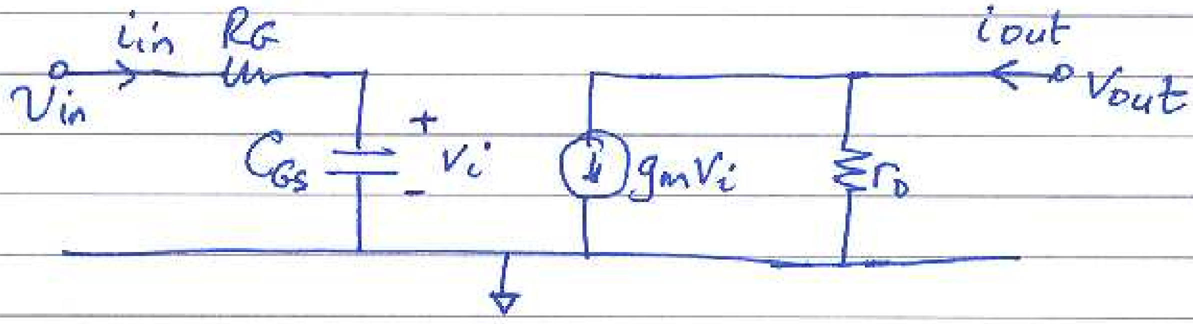
EE4011 RFIC Autumn 2007

Q1(a) Small-signal MOSFET model and f_T



$$h_{21} = \left.\frac{i_{out}}{i_{in}}\right|_{v_{out}=0}$$

$$i_{in} = \frac{v_{in}}{R_G + \frac{1}{j\omega C_{GS}}} = v_{in}\frac{j\omega C_{GS}}{1 + j\omega C_{GS}R_G}$$

when $v_{out} = 0$, there is no current in r_o so

$$i_{out} = g_m v_i = g_m v_{in}\frac{\frac{1}{j\omega C_{GS}}}{R_G + \frac{1}{j\omega C_{GS}}}$$

$$= g_m v_{in}\frac{1}{1 + j\omega C_{GS}R_G}$$

$$\left.\frac{i_{out}}{i_{in}}\right|_{v_{out}=0} = \left(g_m v_{in}\frac{1}{1+j\omega C_{GS}R_G}\right) \Big/ \left(v_{in}\frac{j\omega C_{GS}}{1+j\omega C_{GS}R_G}\right)$$

$$= \frac{g_m}{j\omega C_{GS}}$$

$$|h_{21}| = \frac{g_m}{\omega C_{GS}} = \frac{g_m}{2\pi f C_{GS}}$$

at $f = f_T$, $|h_{21}| = 0$

$$\Rightarrow 1 = \frac{g_m}{2\pi f_T C_{GS}} \Rightarrow f_T = \frac{g_m}{2\pi C_{GS}}$$

$\boxed{10}$

Q1(b) The formula given in part (a) can be used to derive g_m and r_o

$$I_{DS} = \frac{1}{2}\frac{W}{L}\mu C_{ox}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{W}{L}\mu C_{ox}(V_{GS}-V_{TH})(1+\lambda V_{DS})$$

$$g_o = \frac{dI_{DS}}{dV_{DS}} = \frac{1}{2}\frac{W}{L}\mu C_{ox}(V_{GS}-V_{TH})^2\lambda$$

The gain factor $KP = \frac{W}{L}\mu C_{ox} = \frac{W}{L}\mu\frac{\varepsilon_{ox}}{T_{ox}}$

Calculating:

$$KP = \frac{10\times10^{-6}}{0.25\times10^{-6}}\times(400\times10^{-4})\times\frac{3.9\times8.854\times10^{-12}}{4\times10^{-9}}$$

$$= 0.0138 \ A/V^2$$

$$g_m = KP(V_{GS}-V_{TH})(1+\lambda V_{DS}) =$$

$$= (0.0138)(2-0.7)(1+0.1\times3) =$$

$$= 0.0233 \ A/V$$

$$g_o = \frac{1}{2}KP(V_{GS}-V_{TH})^2\lambda = \frac{1}{2}(0.0138)(2-0.7)^2\times0.1$$

$$= 0.0012 \ A/V$$

$$r_o = \frac{1}{g_o} = 857\ \Omega$$

Q1(b)

In saturation

$$C_{GS} = \frac{2}{3} WLC_{ox}$$

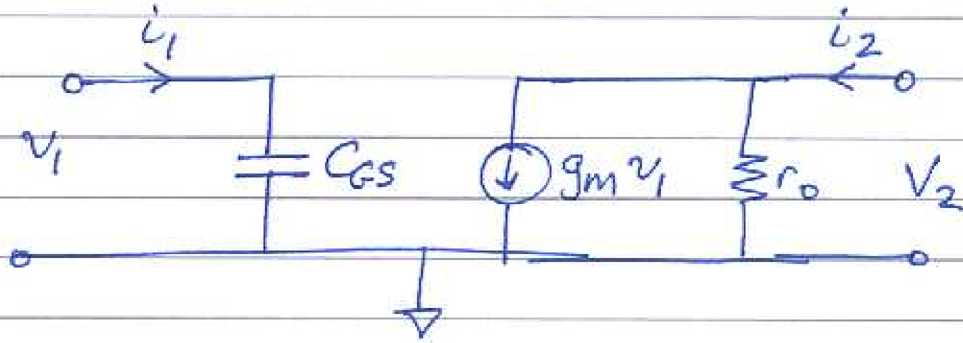
$$= \frac{2}{3}(10\times10^{-6})\times(0.25\times10^{-6})\times\frac{3.9\times8.854\times10^{-12}}{4\times10^{-9}}$$

$$= 1.439\times10^{-14}\ F$$

(i) $f_T = \frac{g_m}{2\pi C_{GS}} = \frac{0.0233}{2\pi\times1.439\times10^{-14}} = 2.58\times10^{11}\ Hz$ $\boxed{2}$

(the value of f_T is physically unrealistic, but it arises from the parameter values given)

(ii) There is no gate resistance specified so this can be assumed to be zero. The simplified equivalent circuit for small-signal analysis is



The z-parameter definitions are

$$z_{11} = \frac{v_1}{i_1}\bigg|_{i_2=0} \qquad z_{12} = \frac{v_1}{i_2}\bigg|_{i_1=0}$$

$$z_{21} = \frac{v_2}{i_1}\bigg|_{i_2=0} \qquad z_{22} = \frac{v_2}{i_2}\bigg|_{i_1=0}$$

First look at $i_2 = 0$

With $i_2 = 0$, the output is open circuited so

$$v_2 = -g_m v_1 r_o$$

Also $i_1 = \dfrac{v_1}{1/j\omega C_{GS}} = v_1 j\omega C_{GS}$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = \frac{v_1}{v_1 j\omega C_{GS}} = \frac{1}{j\omega C_{GS}}$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = \frac{-g_m v_1 r_o}{v_1 j\omega C_{GS}} = -\frac{g_m r_o}{j\omega C_{GS}}$$

With $i_1 = 0$, v_1 must be zero because otherwise there would be current in C_{GS} (and thus i_1 would be non-zero)

If $v_1 = 0$, then $g_m v_1 = 0$ and the only current ~~Z12~~ in the output circuit is flowing through r_o.

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0}, \quad i_2 = g_m v_1 + \frac{v_2}{r_o} = \frac{v_2}{r_o}$$

$$= 0$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = \frac{v_2}{v_2/r_o} = r_o$$

Q1 (b)(ii)

Putting in values for $f = 1.5\ \text{GHz}$

$$Z_{11} = \frac{1}{j\omega C_{GS}} = \frac{1}{j \times 2\pi \times 1.5\times10^{9} \times 1.439\times10^{-14}}$$

$$= 0 - j\,7.373\times10^{3}$$

$$= 7.373\times10^{3} \angle -90^\circ$$

$$Z_{12} = 0 = 0\angle 0^\circ\ \Omega$$

$$Z_{21} = \frac{-g_m r_o}{j\omega C_{GS}} = \frac{-0.0233\times 857}{j\times 2\pi \times 1.439\times10^{-14}}$$

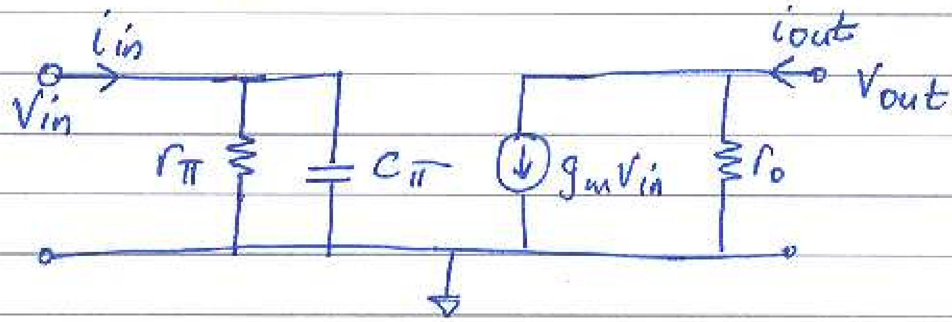
$$= 0 + j\,2.21\times10^{14}$$

$$= 2.2\times10^{14} \angle 90^\circ$$

$$Z_{22} = r_o = 857\,\Omega = 857\angle 0^\circ\ \Omega$$

$\boxed{8}$

Q2(a) Cut-off frequency of BJT



$$h_{21} = \left.\frac{i_{out}}{i_{in}}\right|_{v_{out}=0}$$

$$i_{in} = v_{in}\left(\frac{1}{r_\pi} + j\omega C_\pi\right)$$

when $v_{out}=0$, there is no current in r_o so

$$i_{out} = g_m v_{in}$$

$$\Rightarrow h_{21} = \left.\frac{i_{out}}{i_{in}}\right|_{V_{out}=0} = \frac{g_m v_{in}}{V_{in}\left(\frac{1}{r_\pi} + j\omega C_\pi\right)} = \frac{g_m r_\pi}{1 + j\omega C_\pi r_\pi}$$

when ω is large this is approximately

$$h_{21} \approx \frac{g_m r_\pi}{j\omega C_\pi r_\pi} = \frac{g_m}{j\omega C_\pi}$$

$$|h_{21}| = \frac{g_m}{\omega C_\pi} = \frac{g_m}{2\pi f C_\pi}$$

$$|h_{21}| = 1 \text{ @ } f = f_T \Rightarrow 1 = \frac{g_m}{2\pi f_T C_\pi}$$

$$\Rightarrow f_T = \frac{g_m}{2\pi C_\pi}$$

$\boxed{10}$

Q2(b)

$$f_T = \frac{g_m}{2\pi C_\pi}$$

But $C_\pi = C_{BE} + \tau_F g_m$

$$\therefore f_T = \frac{g_m}{2\pi (C_{BE} + \tau_F g_m)}$$

$$\Rightarrow \frac{1}{f_T} = \frac{2\pi (C_{BE} + \tau_F g_m)}{g_m}$$

$$\Rightarrow \frac{1}{2\pi f_T} = \frac{C_{BE} + \tau_F g_m}{g_m} = \tau_F + \frac{C_{BE}}{g_m}$$

$$I_C = I_S e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{CE}}{V_A}\right)$$

$$\frac{\partial I_C}{\partial V_{BE}} = g_m = I_S \cdot \frac{q}{kT} \cdot e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{CE}}{V_A}\right)$$

$$\approx \frac{I_C}{V_T}$$

$$\therefore \frac{1}{2\pi f_T} = \tau_F + \frac{V_T \cdot C_{BE}}{I_C}$$

C_{BE} is bias dependent and thus varies with I_C but we can assume C_{BE} is constant for this question.

Given two (f_T, I_C) pairs:

$$\frac{1}{2\pi f_{T_1}} = \tau_F + \frac{V_T \cdot C_{BE}}{I_{C_1}} \quad (A)$$

$$\frac{1}{2\pi f_{T_2}} = \tau_F + \frac{V_T C_{BE}}{I_{C_2}} \quad (B)$$

Q2(b) Continued

Subtracting A and B

$$\frac{1}{2\pi}\left(\frac{1}{f_{T_1}} - \frac{1}{f_{T_2}}\right) = V_T C_{BE}\left(\frac{1}{I_{C_1}} - \frac{1}{I_{C_2}}\right)$$

$$\Rightarrow \quad C_{BE} = \frac{1}{2\pi V_T} \; \frac{1/f_{T_1} - 1/f_{T_2}}{1/I_{C_1} - 1/I_{C_2}}$$

$$V_T = \frac{kT}{q} = \frac{1.38\times10^{-23} \times 300}{1.602\times10^{-19}} = 0.0258\,V$$

Using $I_{C_1} = 1mA$, $f_{T_1} = 1.5\,GHz$

$I_{C_2} = 10mA$, $f_{T_2} = 2\,GHz$ gives

$$C_{BE} = \frac{1}{2\pi\times0.0258} \; \frac{\left(\frac{1}{1.5} - \frac{1}{2}\right)\frac{1}{10^9}}{\left(\frac{1}{1} - \frac{1}{10}\right)\frac{1}{10^{-3}}}$$

$$= 1.142\times10^{-12}\,F = 1.142\,pF \qquad \boxed{4}$$

Rearranging A gives

$$\tau_F = \frac{1}{2\pi f_{T_1}} - \frac{V_T C_{BE}}{I_{C_1}}$$

$$= \frac{1}{2\pi\times1.5\times10^9} - \frac{0.0258\times1.142\times10^{-12}}{1\times10^{-3}}$$

$$= 7.658\times10^{-11}\,s = 76.58\,ps \qquad \boxed{4}$$

Q2(c) The cut-off frequency depends on g_m which depends on the bias current I_C. No current is specified for this section so the general formula is repeated

$\boxed{2}$

$$f_T = \frac{g_m}{2\pi C_\pi} = \frac{I_C/V_T}{2\pi\left(C_{BE} + \tau_F \, I_C/V_T\right)}$$

Q3 (a) (i) The 1dB gain compression point is defined for a single input frequency so the formula for y(t) can be simplified by using $A_1 = A$ and $A_2 = 0$ giving

$$y(t) = \left(\alpha_1 A + \frac{3}{4}\alpha_3 A^3\right)\cos(\omega_1 t) + \frac{1}{4}\alpha_3 A^3 \cos(3\omega_1 t)$$

The output at the fundamental frequency ω_1 is

$$y_1(t) = \left(\alpha_1 A + \frac{3}{4}\alpha_3 A^3\right)\cos(\omega_1 t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A^2\right) A \cos(\omega_1 t)$$

The input is $A\cos(\omega_1 t)$ so the gain is

$$G = \alpha_1 + \frac{3}{4}\alpha_3 A^2$$

for small amplitudes A, the A^2 term can be ignored giving

$$G_{small, A} = \alpha_1$$

or in dB:

$$G_{dB, small A} = 20\log_{10}(\alpha_1)$$

for larger amplitudes the A^2 term cannot be ignored so

$$G_{large A} = \alpha_1 + \frac{3}{4}\alpha_3 A^2$$

or in dB

$$G_{dB, large A} = 20\log_{10}\left(\alpha_1 + \frac{3}{4}\alpha_3 A^2\right)$$

because α_3 has the opposite sign to α_1 normally ie $\alpha_1 > 0$ and $\alpha_3 < 0$, $G_{dB, \text{large } A}$ is usually smaller than $G_{dB, \text{small } A}$.

The 1 dB compression point is the value of amplitude A which causes $G_{dB, \text{large } A}$ to be one dB smaller than $G_{dB, \text{small } A}$ ie

$$1 = G_{dB, \text{small } A} - G_{dB, \text{large } A}$$

$$1 = 20\log_{10}(\alpha_1) - 20\log_{10}\left(\alpha_1 + \frac{3}{4}\alpha_3 A^2\right)$$

$$\Rightarrow -1 = 20\log_{10}\left(\alpha_1 + \frac{3}{4}\alpha_3 A^2\right) - 20\log_{10}(\alpha_1)$$

$$\Rightarrow -1 = 20\log_{10}\left(\frac{\alpha_1 + \frac{3}{4}\alpha_3 A^2}{\alpha_1}\right)$$

$$-1 = 20\log_{10}\left(1 + \frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2\right) \Rightarrow -\frac{1}{20} = \log_{10}\left(1 + \frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2\right)$$

$$\Rightarrow 10^{-0.05} = 1 + \frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2$$

$$\Rightarrow 10^{-0.05} - 1 = \frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2$$

usually $\alpha_1 > 0$ and $\alpha_3 < 0$ $\therefore$ $\frac{\alpha_3}{\alpha_1} = -\left|\frac{\alpha_3}{\alpha_1}\right|$

$$\Rightarrow 10^{-0.05} - 1 = -\frac{3}{4}\left|\frac{\alpha_3}{\alpha_1}\right|A^2$$

$$\Rightarrow A = \sqrt{\frac{4}{3}\left(1 - 10^{-0.05}\right)\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$\Rightarrow A = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

this is the amplitude corresponding to the 1dB compression point

$\boxed{5}$

Q3 (a) (ii)

The 3rd order intermodulation intercept point is defined with two inputs having the same amplitudes i.e. $A_1 = A$ and $A_2 = A$

Looking at the output at either of the fundamental frequencies e.g. ω_1 gives

$$y_1(t) = \left(\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2}\alpha_3 A^3\right)\cos(\omega_1 t)$$

for small A the amplitude of this can be approximated as

$$|\alpha_1| A$$

Looking at the outputs of the 3rd order IM terms eg the term for $(2\omega_1 + \omega_2)$ shows that these are of the form

$$y_{IM3}(t) = \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 + \omega_2)t$$

These terms have amplitude

$$\frac{3}{4}|\alpha_3| A^3$$

At the 3rd order intermodulation intercept point the fundamental outputs and the 3rd order IM outputs have the same amplitude i.e.

$$|\alpha_1| A = \frac{3}{4}|\alpha_3| A^3 \Rightarrow |\alpha_1| = \frac{3}{4}|\alpha_3| A^2$$

$$\Rightarrow A = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad \boxed{5}$$

this is the input amplitude corresponding to the 3rd-order intermodulation intercept point (IIP3).

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Q3 (b)

The gain of the amplifier at the fundamental frequency is

$$\alpha_1 = \frac{A_{out}}{A_{in}} = \frac{500\,mV}{10\,mV} = 50$$

The amplitude of the output at the 3rd harmonic frequencies are

$$\left| A_{3rd\ Harmonic} \right| = \frac{1}{4} |\alpha_3| A_{in}^3$$

$$\Rightarrow |\alpha_3| = \frac{|A_{3rd\ harmonic}|}{\frac{1}{4} A_{in}^3} = \frac{8 \times 10^{-9}}{\frac{1}{4}(10 \times 10^{-3})^3}$$

$$= 0.032$$

Then

$$A_{P1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|} = \sqrt{0.145 \times \frac{50}{0.032}} = 15\ V$$

5

Q3(c) Two other undesirable effects are <u>blocking</u> and <u>cross modulation</u>

<u>Blocking</u>: Looking at the full expression for $y(t)$ the gain at the fundamental frequency is

$$G = \alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2$$

because $\alpha_1 > 0$, $\alpha_3 < 0$ usually, as A_2 increases then G will decrease - thus a large signal at one frequency may reduce the effective gain of the amplifier at another frequency.

Q3(c) continued

Cross Modulation

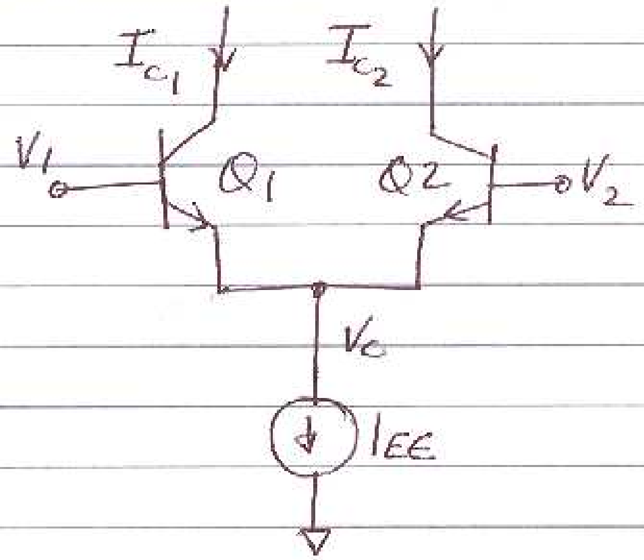
If the frequency ω_2 is a modulated signal e.g. am then the amplitude A_2 depends on this modulation. In that case the output at the frequency ω_1 will be

$$y(t) = \alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 \left[A_2'(1 + m\sin\omega_m t)\right]^2 \cos\omega_1 t$$

where A_2 has been replaced by an amplitude modulated waveform.
This means that the modulating signal ω_m which is effecting the amplitude A_2 is also effecting the amplitude of the signal at ω_1 - that is the modulation is being transferred from one signal to another.

5

Q6(a)



Assume a simple expression for the BJT currents

$$I_{C_1} = I_S e^{(V_1 - V_C)/V_T} \qquad V_T = \frac{kT}{q}$$

$$I_{C_2} = I_S e^{(V_2 - V_C)/V_T}$$

Q1, Q2 identical with same I_S

Dividing the two expressions

$$\frac{I_{C_1}}{I_{C_2}} = e^{(V_1 - V_2)/kT} = e^{\Delta V/V_T}, \quad \Delta V = V_1 - V_2$$

or re-arranging $\quad I_{C_1} = I_{C_2} e^{\Delta V/V_T}$

and $\quad I_{C_2} = I_{C_1} e^{-\Delta V/V_T}$

Assuming the base current can be ignored

$$I_{C_1} + I_{C_2} = I_{EE}$$

Q6(a) continued

$$I_{C_1} + I_{C_2} = I_{EE}$$

expressing I_{C_2} in terms of I_{C_1}:

$$I_{C_1} + I_{C_1} e^{-\Delta V/V_T} = I_{EE}$$

$$\Rightarrow I_{C_1} = \frac{I_{EE}}{1+e^{-\Delta V/V_T}}$$

expressing I_{C_1} in terms of I_{C_2}:

$$I_{C_2} e^{\Delta V/V_T} + I_{C_2} = I_{EE}$$

$$\Rightarrow I_{C_2} = \frac{I_{EE}}{1+e^{\Delta V/V_T}}$$

Then

$$\Delta I = I_{C_1} - I_{C_2} = I_{EE}\left[\frac{1}{1+e^{-\Delta V/V_T}} - \frac{1}{1+e^{\Delta V/V_T}}\right]$$

$$\Delta I = I_{EE}\left[\frac{e^{\Delta V/V_T}}{e^{\Delta V/V_T}+1} - \frac{1}{1+e^{\Delta V/V_T}}\right]$$

$$= I_{EE}\,\frac{e^{\Delta V/V_T}-1}{e^{\Delta V/V_T}+1}$$

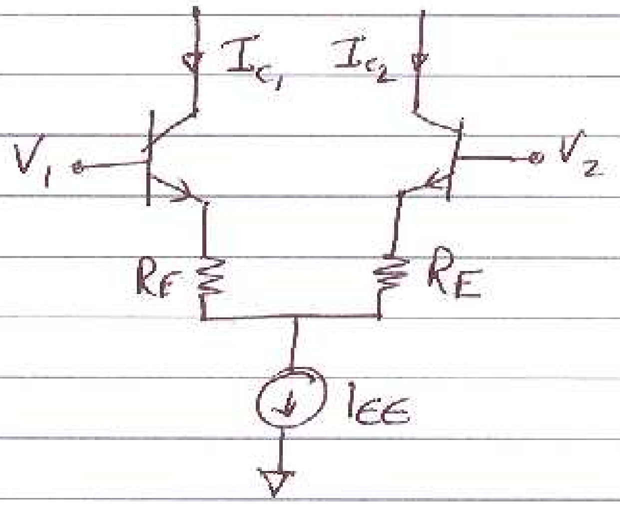
$$\Rightarrow \Delta I = I_{EE}\tanh\left(\frac{\Delta V}{2V_T}\right)$$

$\boxed{12}$

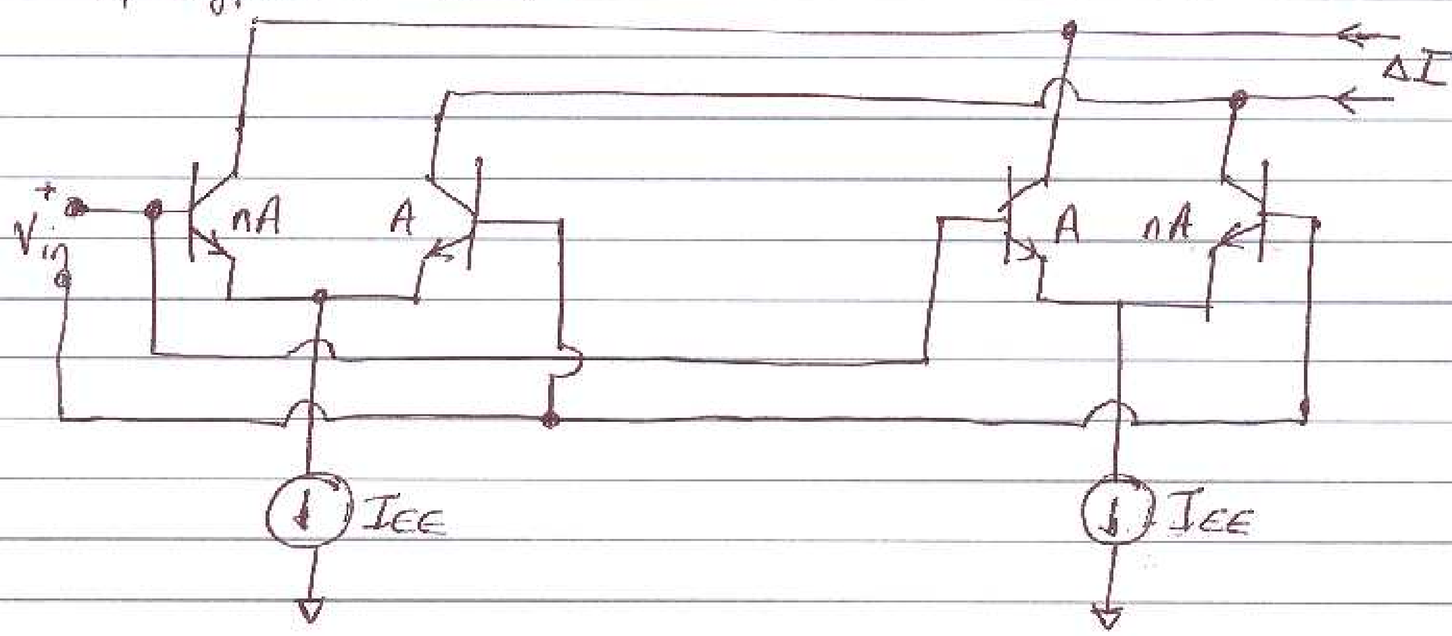
Q6(b)

There are two commonly used techniques to increase the dynamic range of emitter coupled pairs.

Adding emitter resistors:



Using Schmook's method which combines transistors of different areas:



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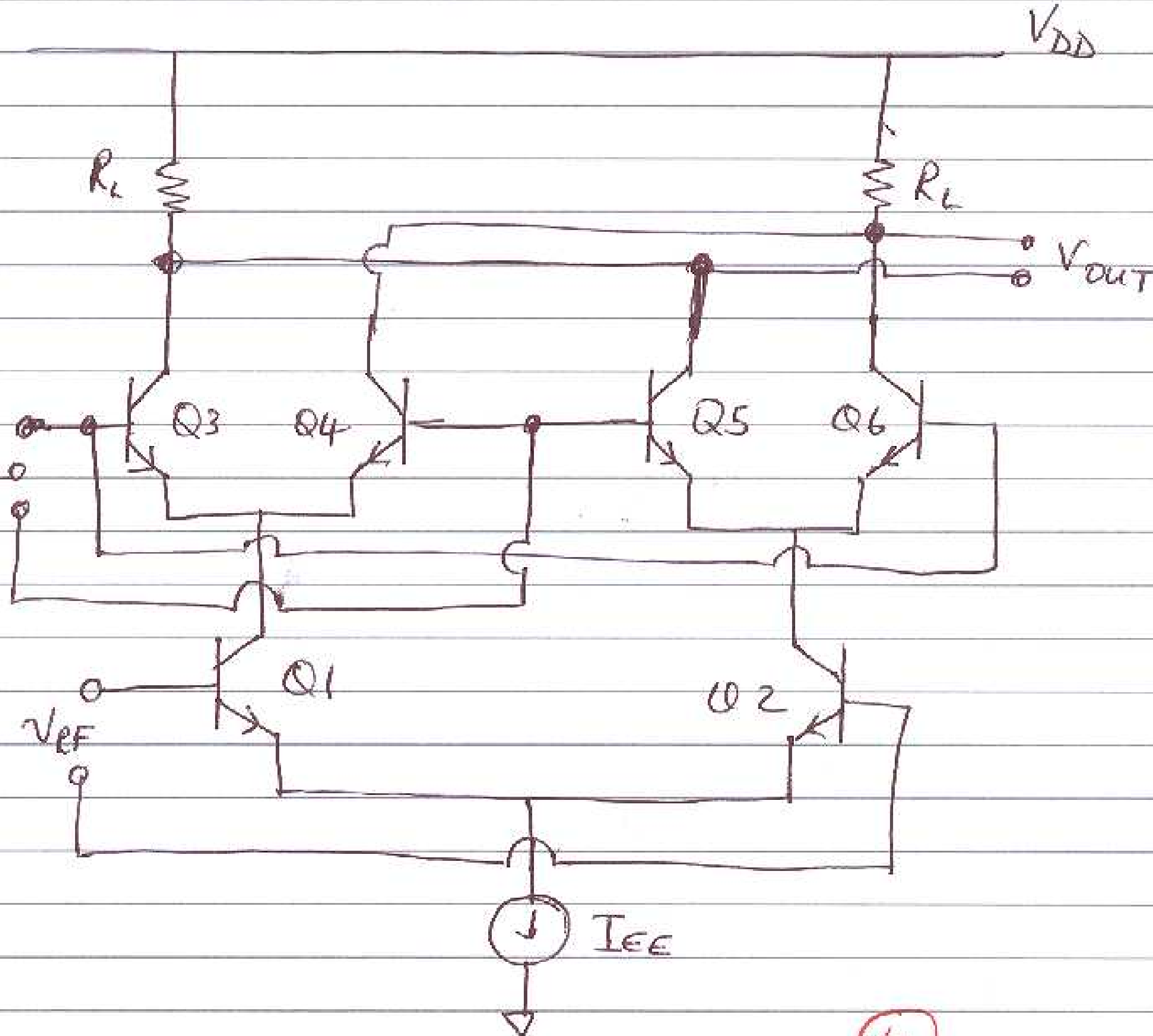
Q6(c) The Gilbert Cell Double Balanced Mixer



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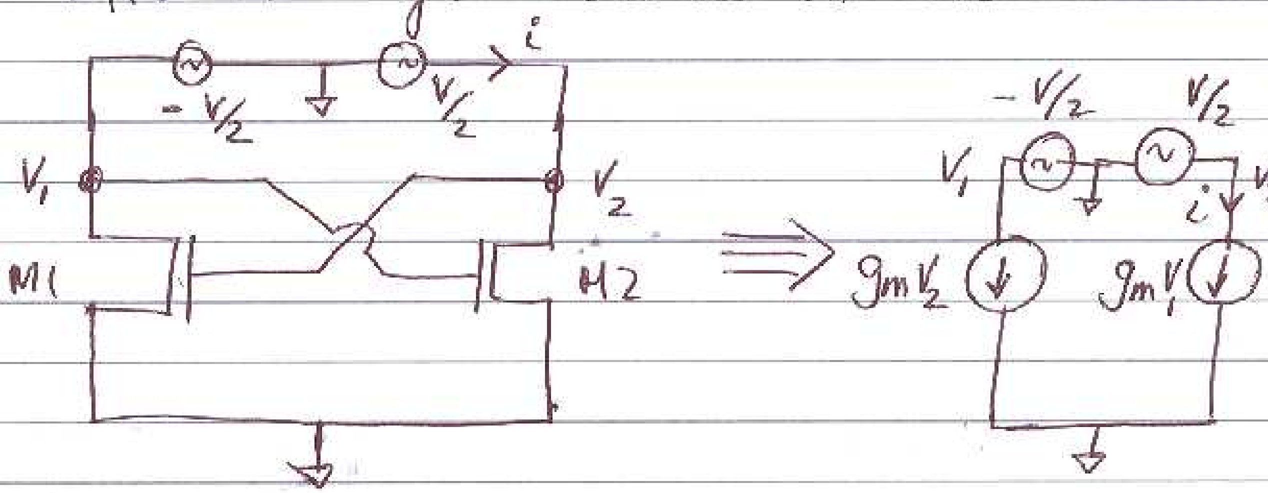
Q7(a) r_{in} is determined by replacing the MOSFETs with the simplest equivalent circuit (just a transconductance) and applying a differential signal: (Also assume the MOSFETs are identical)



$$V_2 = \frac{V}{2}, \quad V_1 = -\frac{V}{2}$$

$$i = g_m V_1 = g_m\left(-\frac{V}{2}\right) = -\frac{V}{2} g_m$$

The effective resistance is calculated by dividing the full differential voltage by the current flowing out of the positive "terminal" i.e.

$$r_{in} = \frac{V}{i} = \frac{V}{-\frac{V}{2} g_m} = -\frac{2}{g_m} \quad \boxed{10}$$

The circuit will oscillate if g_m is chosen so that r_{in} cancels out the series resistance of the inductors in the circuit.

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Q7 (b)

The oscillation frequency of the LC oscillator is given by

$$f = \frac{1}{2\pi\sqrt{LC_{TOT}}} \quad (A)$$

where C_{TOT} is the total capacitance at each of the output nodes.

$$C_{TOT} = C_{DIODE} + C_{PAR}$$

Re-arranging (A) to give capacitance in terms of f

$$(2\pi f)^2 = \frac{1}{LC_{TOT}} \Rightarrow C_{TOT} = \frac{1}{(2\pi f)^2 L}$$

$$\Rightarrow C_{DIODE} + C_{PAR} = \frac{1}{(2\pi f)^2 L}$$

$$C_{DIODE} = \frac{1}{(2\pi f)^2 L} - C_{PAR}$$

When the diode is zero-biased $C_{DIODE} = C_{J0}$ so

$$C_{J0} = \frac{1}{(2\pi \times 1.8\times10^{9})^2 \times 3\times10^{-9}} - 1\times10^{-12}$$

$$= 1.606 \times10^{-12} F = 1.606\,pF \qquad \boxed{5}$$

for a frequency of 2 GHz, the diode capacitance is

$$C_{DIODE} = \frac{1}{(2\pi \times 2\times10^{9})^2 \times 3\times10^{-9}} - 1\times10^{-12}$$

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Q7(b) continued

for 2GHz

$$C_{DIODE} = 1.111 \times 10^{-12} F = 1.111\ pF$$

The diode capacitance is given by

$$C_{DIODE} = \frac{C_{JO}}{\sqrt{1 - V_D/V_J}} \quad (\text{for } MJ = 0.5)$$

$$\Rightarrow \left(\frac{C_{DIODE}}{C_{JO}}\right)^2 = \frac{1}{1 - V_D/V_J}$$

$$\Rightarrow 1 - V_D/V_J = \left(\frac{CJO}{C_{DIODE}}\right)^2$$

$$\Rightarrow V_D/V_J = 1 - \left(\frac{CJO}{C_{DIODE}}\right)^2$$

$$\Rightarrow V_D = V_J\left(1 - \left(\frac{CJO}{C_{DIODE}}\right)^2\right)$$

Therefore the diode voltage is

$$V_D = 0.8\left(1 - \left(\frac{1.606}{1.111}\right)^2\right)$$

$$V_D = -0.872\ V \qquad \boxed{5}$$

i.e. a reverse bias of 0.872V is needed to give oscillation at 2GHz.