OLLSCOIL NA hÉIREANN, CORCAIGH

THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH

UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2009

B.E. DEGREE (ELECTRICAL & ELECTRONIC)

TELECOMMUNICATIONS

EE4004

Professor C. Delabie

Dr. K. G. McCarthy

Dr. C. C. Murphy

Time allowed: *3 hours*

Answer *five* questions.

The use of log tables and a departmental approved non-programmable calculator is permitted.

Q.1. (a) Illustrate the data and acknowledgement flow between two computers which are communicating over a dedicated link and which use the "stop and wait" ARQ scheme. From this derive an expression for the utilization of the link in the case where the link is prone to bit errors. It can be assumed that the only two significant time delays on the link are the propagation delay and the packet (message) holding time.

[*10 marks*]

(b) For a given bit error probability, p, a given line date-rate, R, and a given propagation delay, τ_p, use the expression derived in part (a) of this question to determine the optimum packet size, n, to achieve the maximum line utilization.

[8 marks]

(c) For a data link as described in part (a) of this question, the line data rate is 15kbps, the propagation delay is 15ms and the bit error probability is 0.002. Determine the packet size, n, which will provide the maximum utilization on the link

[2 *marks*]

Q.2. (a) Draw two protocol stacks which compare the functions of the OSI and TCP/IP (internet) systems for wide area networks. Clearly label each layer in the diagrams you draw and align them to allow a direct comparison of the layer functions.

[9 *marks*]

(b) Briefly describe the following topics in TCP/IP (for IPv4):

(i) IP Addressing

[4 *marks*]

(ii) The Domain Name System

[*3 marks*]

(iii) The format of an IP packet.

[*4 marks*]

Q.3. (a) Describe the three main signal degradation mechanisms in Digital Subscriber Line (DSL) technologies for broadband communications.

[5 *marks*]

(b) Describe the four commonly used transmission duplexing methods in DSL technologies.

[6 marks]

(c) Describe the allocation of frequencies in the Asymmetric DSL system and compare the operation of the DMT and CAP approaches to DSL.

[9 *marks*]

Q.4. Given that the 2×2 channel matrix $[P(Y|X)]$ for the binary symmetric channel with 2 input symbols, denoted x_i, $1 \le i \le 2$ and 2 output symbols, denoted y_j, $1 \le j \le 2$, is given by: -

$$[P(Y|X)] = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

and the 2×3 channel matrix $[P(Z|Y)]$ for the binary erasure channel with 2 input symbols, denoted y_i, $1 \le i \le 2$ and 3 output symbols, denoted z_j, $1 \le j \le 3$, is given by: -

$$[P(Z|Y)] = \begin{bmatrix} 1-p & p & 0 \\ 0 & p & 1-p \end{bmatrix}$$

where, in both cases, p denotes the probability of error: -

(a) Show that the channel matrix for the composite channel with inputs x_i, $1 \le i \le 2$ and outputs z_j, $1 \le j \le 3$ (i.e. the outputs from the binary symmetric channel become the inputs for the binary erasure channel), denoted $[P(Z|X)]$, is given by: -

$$[P(Z|X)] = \begin{bmatrix} (p-1)^2 & p & p(1-p) \\ p(1-p) & p & (p-1)^2 \end{bmatrix}.$$

[6 *marks*]

(b) Show that if the input symbols x_i, $1 \le i \le 2$ are equiprobable then the entropy, $H[Z]$, of the output from the composite channel is given by: -

$$H[Z] = (p-1)\log_2\left[\frac{1-p}{2}\right] - p\log_2[p].$$

[5 *marks*]

(c) Given that $H[Z]$ in part (b) above corresponds to the maximum possible value of the output entropy, show that the channel capacity C_s (in bits/symbol) of the composite channel is given by: -

$$C_s = (1-p)^2\log_2\left[(1-p)^2\right] + p(1-p)\log_2[p(1-p)] + (p-1)\log_2\left[\frac{1-p}{2}\right].$$

[5 *marks*]

(d) Using a graph, or otherwise, estimate the value of p resulting in a composite channel capacity C_s that is half of the composite channel capacity that would be achieved in an ideal error-free system, denoted C_s^{ideal}, i.e. for which $C_s = \frac{C_s^{ideal}}{2}$.

[4 *marks*]

Q.5 A baseband digital communications system uses the following signals to represent logic 1 and logic 0: -

$$s_i(t) = \begin{cases} A\sin[\omega t/2] & 0 \le t \le T & \text{logic } 1 \\ -A\sin[\omega t/2] & 0 \le t \le T & \text{logic } 0 \end{cases}$$

where $\omega = 2\pi/T$ and T denotes the bit signalling interval. The receiver takes a single sample of the received signal during the bit signalling time at $t = \frac{T}{2} + \Delta t$ (where Δt represents a clock-induced error in the sampling moment) and compares this sample with a decision threshold D. The communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n(v) = \frac{e^{\frac{-v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}.$$

(a) Show that, to minimize the resulting overall probability of error P_e, the threshold D is given by: -

$$D = \frac{\sigma^2}{2A\sin\left[\omega\frac{T+2\Delta t}{4}\right]}\ln\left[\frac{P_0}{P_1}\right]$$

where P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1.

[10 *marks*]

(b) Show that, if $P_0 > P_1$, then the average probability of error, denoted P_e, is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{D + A\sin\left[\omega\frac{T+2\Delta t}{4}\right]}{\sqrt{2\sigma^2}}\right] + (1-P_0)erf\left[\frac{A\sin\left[\omega\frac{T+2\Delta t}{4}\right] - D}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy.$$

[10 *marks*]

Q.6. (a) An analogue signal having a bandwidth of B_A Hz is sampled at 1.5 times the Nyquist rate and each sample is quantised into one of L equally likely levels. Assuming that successive samples are statistically independent, the signal power at the receiver is S watts and the communication is affected by additive white Gaussian noise with power spectral density $\eta/2\ W/Hz$, show that the minimum channel bandwidth, denoted B_C, required for error-free transmission of the information produced by this source must satisfy the following non-linear equation: -

$$L^{3B_A} = \left(1 + \frac{S}{\eta B_C}\right)^{B_C}.$$

[5 *marks*]

(b) For the system described in (a) above estimate, using a graph or otherwise, the required value of B_C if $B_A = 20\ kHz$, $L = 256$, $S = 0.1\ mW$ and $\eta = 2 \times 10^{-10}\ W/Hz$. Hint: - taking the logarithm of both sides of the non-linear equation in part (a) above helps to avoid very large numbers.

[5 *marks*]

(c) Given that the output signal to noise ratio (SNR) of a matched filter receiver subject to additive white Gaussian noise (AWGN) with power spectral density $\eta/2\ W/Hz$ is given by $2E_d/\eta$ where E_d denotes the energy in the difference signal, show using the Schwarz inequality (which states: -

$$\left|\int_{-\infty}^{\infty} f_1(\omega) f_2(\omega) d\omega\right|^2 \leq \int_{-\infty}^{\infty} |f_1(\omega)|^2 d\omega \int_{-\infty}^{\infty} |f_2(\omega)|^2 d\omega),$$

or otherwise, that the optimum output SNR is given by: -

$$\left(\frac{S}{N}\right)_{Optimum} = \frac{8E}{\eta}$$

where we stipulate that the signaling waveforms $s_1(t)$ and $s_2(t)$ <u>must</u> have the same signal energy E.

[10 *marks*]

Q.7 Given the following table of field elements of $GF(2^5)$: -

0	$\alpha^7 = \alpha^4 + \alpha^2$	$\alpha^{15} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$	$\alpha^{23} = \alpha^3 + \alpha^2 + \alpha + 1$
1	$\alpha^8 = \alpha^3 + \alpha^2 + 1$	$\alpha^{16} = \alpha^4 + \alpha^3 + \alpha + 1$	$\alpha^{24} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha$
α	$\alpha^9 = \alpha^4 + \alpha^3 + \alpha$	$\alpha^{17} = \alpha^4 + \alpha + 1$	$\alpha^{25} = \alpha^4 + \alpha^3 + 1$
α^2	$\alpha^{10} = \alpha^4 + 1$	$\alpha^{18} = \alpha + 1$	$\alpha^{26} = \alpha^4 + \alpha^2 + \alpha + 1$
α^3	$\alpha^{11} = \alpha^2 + \alpha + 1$	$\alpha^{19} = \alpha^2 + \alpha$	$\alpha^{27} = \alpha^3 + \alpha + 1$
α^4	$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha$	$\alpha^{20} = \alpha^3 + \alpha^2$	$\alpha^{28} = \alpha^4 + \alpha^2 + \alpha$
$\alpha^5 = \alpha^2 + 1$	$\alpha^{13} = \alpha^4 + \alpha^3 + \alpha^2$	$\alpha^{21} = \alpha^4 + \alpha^3$	$\alpha^{29} = \alpha^3 + 1$
$\alpha^6 = \alpha^3 + \alpha$	$\alpha^{14} = \alpha^4 + \alpha^3 + \alpha^2 + 1$	$\alpha^{22} = \alpha^4 + \alpha^2 + 1$	$\alpha^{30} = \alpha^4 + \alpha$

(a) Show that the generator polynomial for the $(31,21)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^3 + 1.$$

[10 *marks*]

(b) If the received data, denoted $v(x)$, is given by: -

$$v(x) = x^{28} + x^{14} + x^{13} + x^{11} + x^{10} + x^6 + x^4 + x^3 + 1,$$

use the syndrome decoding method and the error location polynomial: -

$$x^2 + S_1 x + \frac{S_1^3 + S_3}{S_1} = 0$$

to find the error polynomial $e(x)$ and the original codeword $c(x)$, where $v(x) = c(x) + e(x)$.

[10 *marks*]