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7a) $g(x) = LCM(m_1(x), m_2(x))$

$$m_1(x) = (x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8)(x+\alpha^{16})$$
$$= (x^2+\alpha^{19}x+\alpha^3)(x^2+\alpha^{14}x+\alpha^{12})(x+\alpha^{16})$$
$$= (x^2+\alpha^{19}x+\alpha^3)(x^3+\alpha^{16}x^2+\alpha^{14}x^2+\alpha^{30}x+\alpha^{12}x+\alpha^{28})$$
$$= (x^2+\alpha^{19}x+\alpha^3)(x^3+\alpha^{19}x^2+\alpha^{13}x+\alpha^{28})$$
$$= (x^5+\alpha^{19}x^4+\alpha^{13}x^3+\alpha^{28}x^2+\alpha^{19}x^4+\alpha^{38}x^3+\alpha^{32}x^2+\alpha^{47}x+\alpha^3x^3$$
$$+\alpha^{22}x^2+\alpha^{16}x+\alpha^{31}$$
$$= x^5+[\alpha^{19}+\alpha^{19}]x^4+[\alpha^{13}+\alpha^7+\alpha^3]x^3+[\alpha^{28}+\alpha \quad +\alpha^{22}]x^2+[\alpha^{16}+\alpha^{16}]x$$
$$= x^5+x^2+1$$

$$m_3(x) = (x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})(x+\alpha^{24})(x+\alpha^{48})$$
$$= (x^2+\alpha x+\alpha^9)(x^2+\alpha^4x+\alpha^5)(x+\alpha^{48})$$
$$= (x^4+\alpha^{30}x^3+\alpha^9x^2+\alpha^{28}x+\alpha^{14})(x+\alpha^{17})$$
$$= (x^5+[\alpha^{17}+\alpha^{30}]x^4+[\alpha^{47}+\alpha^9]x^3+[\alpha^{26}+\alpha^{28}]x^2+[\alpha^{45}+\alpha^{14}]x+1)$$
$$= x^5+x^4+x^3+x^2+1$$

$$g(x) = (x^5+x^4+x^3+x^2+1)(x^5+x^2+1)$$
$$= x^{10}+\cancel{x^7}+\cancel{x^5}+x^9+x^6+\cancel{x^4}+x^8+\cancel{x^5}+x^3+\cancel{x^7}+\cancel{x^4}+\cancel{x^2}+x^5+\cancel{x^2}+1$$
$$g(x) = x^{10}+x^9+x^8+x^6+x^5+x^3+1$$

b) $v(x) = c(x)+e(x)$

$$v(x) = x^{28}+x^{14}+x^{13}+x^{11}+x^{10}+x^6+x^4+x^3+1$$

$S_1 = v(\alpha^1)$
$S_3 = v(\alpha^3)$

$$S_1 = \alpha^{28}+\alpha^{14}+\alpha^{13}+\alpha^{11}+\alpha^{10}+\alpha^6+\alpha^4+\alpha^3+1$$
$$S_3 = \alpha^{84}+\alpha^{42}+\alpha^{39}+\alpha^{33}+\alpha^{30}+\alpha^{18}+\alpha^{12}+\alpha^9+\alpha^3$$

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7 a) $P(x) = x^5 + x^2 + 1$ $\qquad \alpha^5 = \alpha^2 + 1$

0	0	α^{15}	$\alpha^4+\alpha^3+\alpha^2+\alpha+1$
1	1	α^{16}	$\alpha^4+\alpha^3+\alpha+1$
α	α	α^{17}	$\alpha^4+\alpha+1$
α^2	α^2	α^{18}	$\alpha+1$
α^3	α^3	α^{19}	$\alpha^2+\alpha$
α^4	α^4	α^{20}	$\alpha^3+\alpha^2$
α^5	α^2+1	α^{21}	$\alpha^4+\alpha^3$
α^6	$\alpha^3+\alpha$	α^{22}	$\alpha^4+\alpha^2+1$
α^7	$\alpha^4+\alpha^2$	α^{23}	$\alpha^3+\alpha^2+\alpha+1$
α^8	$\alpha^3+\alpha^2+1$	α^{24}	$\alpha^4+\alpha^3+\alpha^2+\alpha$
α^9	$\alpha^4+\alpha^3+\alpha$	α^{25}	$\alpha^4+\alpha^3+1$
α^{10}	α^4+1	α^{26}	$\alpha^4+\alpha^2+\alpha+1$
α^{11}	$\alpha^2+\alpha+1$	α^{27}	$\alpha^3+\alpha+1$
α^{12}	$\alpha^3+\alpha^2+\alpha$	α^{28}	$\alpha^4+\alpha^2+\alpha$
α^{13}	$\alpha^4+\alpha^3+\alpha^2$	α^{29}	α^3+1
α^{14}	$\alpha^4+\alpha^3+\alpha^2+1$	α^{30}	$\alpha^4+\alpha$

b) i) $m_1(X) = (x-\alpha)(x-\alpha^2)(x-\alpha^4)(x-\alpha^8)(x-\alpha^{16})$

$= (x^2+\alpha^{19}x+\alpha^3)(x^2+\alpha^{14}+\alpha^{12})(x-\alpha^{16})$

$= (x^2+\alpha^{19}x+\alpha^3)(x^3+\alpha^{19}x^2+\alpha^{13}x+\alpha^{28})$

$= x^5 + (\alpha^{19}+\alpha^{19})x^4 + (\alpha^{13}+\alpha^{38}+\alpha^3)x^3 + (\alpha^{28}+\alpha^{32}+\alpha^{22})x^2$
$+ (\alpha^{47}+\alpha^{16})x + \alpha^{31}$

$= x^5 + (\alpha^{13}+\alpha^7+\alpha^3)x^3 + (\alpha^{28}+\alpha+\alpha^{22})x^2 + \alpha^{31}$

$= x^5 + x^2 + 1$

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7 b ii) $m_3(x) = (x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})(x+\alpha^{24})(x+\alpha^{48})$

$= (x^2 + [\alpha^3+\alpha^6]x + \alpha^9)(x^2 + [\alpha^{24}+\alpha^{12}]x + \alpha^{36})(x+\alpha^{48})$

$= (x^2 + \alpha x + \alpha^9)(x^2 + \alpha^4 x + \alpha^5)(x+\alpha^{48})$

$= (x^4 + [\alpha^4+\alpha]x^3 + [\alpha^5+\alpha^5+\alpha^9]x^2 + [\alpha^6+\alpha^{13}]x + \alpha^{14})(x+\alpha^{48})$

$= (x^4 + \alpha^{30}x^3 + \alpha^9 x^2 + \alpha^{28}x + \alpha^{14})(x+\alpha^{17})$

$= x^5 + [\alpha^{30}+\alpha^{17}]x^4 + [\alpha^9+\alpha^{47}]x^3 + [\alpha^{26}+\alpha^{28}]x^2 + [\alpha^{14}+\alpha^{45}]x$
$+ 1$, $\alpha^{47} = \alpha^{16}$ ⁊ $\alpha^{45} = \alpha^{14}$

$= x^5 + x^4 + x^3 + x^2 + 1$

iii) $g(x) = LCM[m_1(x), m_3(x)]$

$= (x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8)(x+\alpha^{16})(x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})$
$(x+\alpha^{24})(x+\alpha^{48})$

$= 0$ if $x = \alpha$, $x = \alpha^3$

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c) $g(x) = \text{LCM}\left[m_1(x), m_3(x)\right]$

$$= (x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8)(x+\alpha^3)(x+\alpha^6)$$
$$(x+\alpha^{12})(x+\alpha^{24})$$

$$= \overbrace{(x^2+\alpha^5 x+\alpha^3)(x^2+\alpha^5 x+\alpha^{12})}^{1}\overbrace{(x^2+\alpha^2 x+\alpha^9)(x^2+\alpha^8 x+\alpha^6)}^{2}$$

$$1 = (x^4+[\alpha^5+\alpha^5]x^3+[\alpha^{12}+\alpha^3+\alpha^{10}]x^2+[\alpha^{17}+\alpha^8]x+\alpha^{15})$$

$$1 = x^4+x+\alpha^{15} = x^4+x+1$$

$$2 = (x^4+[\alpha^8+\alpha^2]x^3+[\alpha^6+\alpha^{10}+\alpha^9]x^2+[\alpha^8+\alpha^{17}]x+\alpha^{15}]$$

$$2 = x^4+x^3+x^2+x+1$$

$$g(x) = (x^4+x^3+x^2+x+1)(x^4+x+1)$$
$$= x^8+x^7+x^6+x^4+1$$

b) $V(x) = c(x)+e(x)$

$$S_1 = V(\alpha^1)$$
$$S_2 = V(\alpha^3)$$

$$S_1 = \alpha^{12}+\alpha^{10}+\alpha^9+\alpha^7+\alpha^6+\alpha^4+\alpha^3+\alpha+1$$

$\Rightarrow$
$$\alpha^3+\alpha^2+\alpha+1$$
$$\alpha^2+\alpha+1$$
$$\alpha^3 \quad +\alpha$$
$$\alpha^3 \quad +\alpha+1$$
$$\alpha^3+\alpha^2$$
$$\alpha+1$$
$$\alpha^3$$
$$\alpha+1$$

$$\alpha^3+\alpha^2+1 = \alpha^{13} = S_1$$

b) $S_2 = \alpha^{36} + \alpha^{30} + \alpha^{27} + \alpha^{21} + \alpha^{18} + \cancel{\alpha^{12}} + \alpha^{9} + \cancel{\alpha^{3}} + \cancel{\alpha}$

$\alpha^{36} = \cancel{\alpha^{6}}$

$\alpha^{30} = \cancel{\alpha}$

$\alpha^{27} = \cancel{\alpha^{12}}$

$\alpha^{21} = \cancel{\alpha^{6}}$

$\alpha^{18} = \cancel{\alpha^{3}}$

$$S_2 = \alpha^9 = \alpha^3 + \alpha$$

Error locator polynomial:

$$X_1^2 + S_1 X_1 + \frac{S_1^3 + S_3}{S_1} = 0$$

$$\frac{S_1^3 + S_3}{S_1} = \frac{\alpha^{39} + \alpha^9}{\alpha^{13}} = 0$$

$$X_1^2 + S_1 X_1 = 0$$

$$X_1 + S_1 = 0$$

$$X_1 = \alpha^{13} = \alpha^3 + \alpha^2 + 1$$

$$c(X) = X^{13} + X^{12} + X^{10} + X^9 + X^7 + X^6 + X^4 + X^3 + X + 1$$

c) $i(X) = \dfrac{c(X)}{g(X)} =$

$$\begin{array}{r|l}
 & X^5 + X^3 + X + 1 \\
X^8 + X^7 + X^6 + X^4 + 1 & \cancel{X^{13}} + \cancel{X^{12}} + X^{10} + \cancel{X^9} + X^7 + X^6 + X^4 + X^3 + X + 1 \\
 & \underline{\cancel{X^{13}} + \cancel{X^{12}} + X^{11} + \cancel{X^9} + X^5} \\
 & X^{11} + X^{10} + X^7 + X^6 + X^5 + X^4 + X^3 + X + 1 \\
 & \underline{X^{11} + X^{10} + X^9 + X^7 + X^3} \\
 & X^9 + X^6 + X^5 + X^4 + X + 1 \\
 & \underline{X^9 + X^8 + X^7 + X^5 + X} \\
 & X^8 + X^7 + X^6 + X^4 + 1
\end{array}$$

$$i(X) = X^5 + X^3 + X + 1$$

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6 a) $p(x) = x^4 + x + 1$ $\qquad \alpha^4 = \alpha + 1$

0	0	α^7	$\alpha^3 + \alpha + 1$
1	1	α^8	$\alpha^2 + 1$
α	α	α^9	$\alpha^3 + \alpha$
α^2	α^2	α^{10}	$\alpha^2 + \alpha + \alpha$
α^3	α^3	α^{11}	$\alpha^3 + \alpha^2 + \alpha$
α^4	$\alpha + 1$	α^{12}	$\alpha^3 + \alpha^2 + \alpha + 1$
α^5	$\alpha^2 + \alpha$	α^{13}	$\alpha^3 + \alpha^2 + 1$
α^6	$\alpha^3 + \alpha^2$	α^{14}	$\alpha^3 + 1$

b) $g(x) = x^8 + x^7 + x^6 + x^4 + 1$ $\qquad i(x) = \dfrac{c(x)}{g(x)}$

$c(x) = x^{11} + x^8 + x^7 + x^6 + x^3 + x^2$

$$i(x) = x^8 + x^7 + x^6 + x^4 + 1 \ \overline{\smash{\big)}\, x^{11} + x^8 + x^7 + x^6 + x^3 + x^2}$$

Quotient: $x^3 + x^2$

$$\cancel{x^{11}} + x^{10} + x^9 + x^7 + x^3$$

$$x^{10} + x^9 + x^8 + x^6 + x^2$$

$i(x) = x^3 + x^2$

c) $e(x) = x^6 + x^2$

$v(x) = c(x) + e(x)$

$= x^{11} + x^8 + x^7 + x^3$

$S_1 = v(\alpha^1)$

$S_2 = v(\alpha^3)$

$S_1 = \alpha^{11} + \alpha^8 + \alpha^7 + \alpha^3 = \alpha^3$

$S_2 = \alpha^{33} + \alpha^{24} + \alpha^{21} + \alpha^9 = \alpha^3 + \alpha^9 + \alpha^6 + \alpha^9$

$S_2 = \alpha^2$

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Syndrome Equations:

$$S_1 = x_1 + x_2$$
$$S_2 = x_1^3 + x_2^3$$

$$(x_1 + x_2)^3 = (x_1 + x_2)^2 (x_1 + x_2)$$
$$= (x_1^2 + x_2^2)(x_1 + x_2)$$
$$= x_1^3 + x_2^3 + x_1 x_2 (x_1 + x_2)$$

$$S_1^3 = S_2 + x_1 x_2 S_1$$
$$S_1^3 + S_2 = x_1 S_1 (S_1 + x_1)$$
$$x_1^2 S_1 + S_1^2 x_1 + S_1^3 + S_2 = 0$$

$$x_1^2 + S_1 x_1 + \frac{S_1^3 + S_2}{S_1} = 0$$
$$x_1^2 + \alpha^3 x_1 + \frac{\alpha^9 + \alpha^2}{\alpha^3} = 0$$

Need to eliminate α^3 term $\Rightarrow \times \alpha^{12}$

$$x_1^2 + \alpha^3 + \frac{\alpha^{21} + \alpha^{14}}{\alpha^{15}} = 0$$

$$x_1^2 + \alpha^3 x_1 + \alpha^8 = 0$$

Evaluate to find errors for different values of x
Since we know $e(x)$, use x^2

$$(\alpha^2)^2 + \alpha^5 + \alpha^8$$
$$\alpha^4 + \alpha^5 + \alpha^8$$
$$\alpha^2 + 1 \quad + \alpha^2 + 1 = 0$$

$\Rightarrow$ Error @ x^2

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6. a) $g(x) = \text{LCM}(m_1(x), m_3(x))$

$$g(x) = (x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8)(x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})(x+\alpha^{24})$$
$$= (x^2+\alpha^5 x+\alpha^3)(x^2+\alpha^5 x+\alpha^{12})(x^2+\alpha^2 x+\alpha^9)(x^2+\alpha^8 x+\alpha^6)$$
$$= (x^4+[\alpha^3+\alpha^{12}+\alpha^{10}]x^2+[\alpha^2+\alpha^8]x+1)(x^4+x^3+x^2+x+1)$$
$$= (x^4+x+1)(x^4+x^3+x^2+x+1)$$
$$= x^8+x^7+x^6+\cancel{x^5}+\cancel{x^4}+\cancel{x^5}+\cancel{x^4}+\cancel{x^3}+\cancel{x^2}+\cancel{x}+x^4+\cancel{x^3}+\cancel{x^2}+\cancel{x}+1$$

$$g(x) = x^8+x^7+x^6+x^4+1$$

b) $c(x) = x^{11}+x^8+x^7+x^6+x^3+x^2$ $\qquad i(x) = \dfrac{c(x)}{g(x)}$

$g(x) = x^8+x^7+x^6+x^4+1$

$$\begin{array}{r|l} & x^3+x^2 \\ \hline x^8+x^7+x^6+x^4+1 & x^{11}+x^8+x^7+x^6+x^3+x^2 \\ & \underline{x^{11}+x^{10}+x^9+x^7+x^3} \\ & x^{10}+x^9+x^8+x^6+x^2 \end{array}$$

$k = 7$

00001100

c) $v(x) = c(x)+e(x)$

$v(x) = x^{11}+x^{10}+x^8+x^7+x^6+x^3$

$S_1 = V(\alpha^1) \qquad S_3 = V(\alpha^3)$

$S_1 = \alpha^{11}+\alpha^{10}+\alpha^8+\alpha^7+\alpha^6+\alpha^3 = \alpha^4$

$S_2 = \alpha^3+1+\alpha^9+\alpha^6+\alpha^3+\alpha^9 = \alpha^{13}$

$$x_1^2 + S_1 x_1 + \frac{S_1^3+S_2}{S_2} = 0$$

$$S_1^3 = \alpha^{12} \Rightarrow \frac{S_1^3+S_2}{S} = \frac{\alpha^{12}+\alpha^{13}}{\alpha^4} = \alpha^8+\alpha^9 = \alpha^{12}$$

$$x_1^2 + \alpha^4{}_1 + \alpha^{12} = 0$$

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Check for values of α that satisfy the eqn.
Since we know $e(x) = x^{10} + x^2$, we will verify:

$(\alpha^{10})^2 + \alpha^4\alpha^{10} + \alpha^{12} = \alpha^5 + \alpha^{14} + \alpha^{12} = 0$ ✓

$(\alpha^2)^2 + \alpha^4\alpha^2 + \alpha^{12} = \alpha^4 + \alpha^6 + \alpha^{12} = 0$ ✓