OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2007

B.E. DEGREE (ELECTRICAL)
B.E. DEGREE (MICROELECTRONIC)

TELECOMMUNICATIONS
EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of a Casio fx570w or fx570ms calculator is permitted.

1. (a) Compare the capacities of BPSK, QPSK, and QAM systems and discuss the factors that influence the choice of a particular modulation scheme?

[*12 marks*]

(b) A PCM-TDM system is to handle four video signals each band limited to 5 MHz. The signals are sampled at the minimum rate and signal to quantisation noise level must be at least 54 dB. If one bit per word is used for synchronisation, and 32QAM is used for transmission, what is the rate of phase change of the carrier?

[8 *marks*]

2. (a) Illustrate the logical connection of computers interconnected using a passive bus architecture and describe the functional subdivision of the Data Link layer for this network.

[*4 marks*]

(b) List the sequence of steps followed by a computer that needs to initiate a data transfer if it is connected to a network which uses the CSMA/CD access protocol.

[*6 marks*]

(c) Describe the operation of the truncated binary exponential back-off algorithm for the CSMA/CD network access protocol.

[*6 marks*]

(d) Two computers are at the opposite ends of a passive bus network which uses the CSMA/CD protocol. If the CSMA/CD frames are 100 bytes long, the bus data rate is 1Gbit/s and the propagation velocity along the bus is 2.3×10^8 m/s, determine the maximum length of the bus to ensure proper operation of the CSMA/CD protocol.

[*4 marks*]

3. (a) Illustrate the architecture of a UMTS Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) For a 3G UMTS cellular telephone system briefly discuss the following:

(i) The channel frequency allocation and downstream data-rates available to the user.

[4 *marks*]

(ii) The power control algorithms in use.

[4 *marks*]

(iii) The use of spreading codes.

[4 *marks*]

Q.4. Given that the 3×3 channel matrix $\left[P\left(Y_1|X\right)\right]$ for the 3-ary uniform channel with 3 input symbols, denoted x_i, $1 \le i \le 3$ and 3 output symbols, denoted y_j, $1 \le j \le 3$, is given by: -

$$\left[P\left(Y_1|X\right)\right] = \begin{bmatrix} 1-p & \alpha & \alpha \\ \alpha & 1-p & \alpha \\ \alpha & \alpha & 1-p \end{bmatrix}$$
$$= \underbrace{\begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} 1-3p/2 & 0 & 0 \\ 0 & 1-3p/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{D} \underbrace{\left(\frac{1}{3}\begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}\right)}_{F^{-1}}$$

where $\alpha = p/2$, D is a diagonal matrix and the columns of F are eigenvectors of $\left[P\left(Y_1|X\right)\right]$, show that if n such 3-ary uniform channels are connected in series (i.e. the outputs of 3-ary uniform channel i become the inputs of 3-ary uniform channel $i+1$, $1 \le i \le n-1$), then: -

(a) The composite channel matrix $\left[P\left(Y_n|X\right)\right]$ is given by: -

$$\left[P\left(Y_n|X\right)\right] = \frac{1}{3}\begin{bmatrix} 1+2q & 1-q & 1-q \\ 1-q & 1+2q & 1-q \\ 1-q & 1-q & 1+2q \end{bmatrix}$$

where $q = \left(1-3p/2\right)^n$.

[8 *marks*]

(b) Given that the input symbols x_i, $1 \le i \le 3$ are equiprobable and the maximum entropy of an m symbol source is given by $\log_2[m]$, the composite channel capacity C_s^c is given by: -

$$C_s^c = \log_2\left[3\left(\frac{1+2q}{3}\right)^{\frac{1+2q}{3}}\left(\frac{1-q}{3}\right)^{2\left(\frac{1-q}{3}\right)}\right]$$

where $q = \left(1-3p/2\right)^n$.

[8 *marks*]

(c) If p is sufficiently small such that its square and higher powers can be neglected, show that the composite channel capacity C_s^c is approximately that of a single 3-ary uniform channel with probability of error free transmission (i.e. probability of receiving y_j having sent x_j) equal to $1 - n \times p$. Note that: -

$$\left(x+y\right)^n = \sum_{i=0}^{n}\binom{n}{i}x^i y^{n-i} \quad \text{and} \quad \binom{n}{i} = \frac{n!}{i!\left(n-i\right)!} .$$

[4 *marks*]

Q.5 A baseband digital communications system uses rectangular wave signaling with A_1 volts representing logic 1 and A_2 volts representing logic 0 (where $A_2 < A_1$). The receiver takes a single sample of the received signal during the bit signaling time and compares this sample with a decision threshold T. If the communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n(v) = \frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

and P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1, show that: -

(a) To minimize the resulting overall probability of error P_e, the threshold T is given by: -

$$T = \frac{A_1 + A_2}{2} + \frac{\sigma^2}{A_1 - A_2}\ln\left[\frac{P_0}{P_1}\right].$$

[10 *marks*]

(b) If $P_0 > P_1$ in (a) above, show that P_e is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{T - A_2}{\sqrt{2\sigma^2}}\right] + (1 - P_0) erf\left[\frac{A_1 - T}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy.$$

[10 *marks*]

6. (a) Given that the output signal to noise ratio (SNR) of a matched filter receiver subject to additive white Gaussian noise (AWGN) with power spectral density $\eta/2\ W/Hz$ is given by $2E_d/\eta$ where E_d denotes the energy in the difference signal, show using the Schwarz inequality (which states: -

$$\left|\int_{-\infty}^{\infty} f_1(\omega)f_2(\omega)d\omega\right|^2 \leq \int_{-\infty}^{\infty}|f_1(\omega)|^2 d\omega \int_{-\infty}^{\infty}|f_2(\omega)|^2 d\omega),$$

or otherwise, that the optimum output SNR is given by: -

$$\left(\frac{S}{N}\right)_{Optimum} = \frac{8E}{\eta}$$

when we stipulate that the signaling waveforms $s_1(t)$ and $s_2(t)$ <u>must</u> have the same signal energy E.

[8 *marks*]

(b) A frequency shift keying modulation scheme is defined by: -

$$s_i(t) = \begin{cases} A\cos(\omega_1 t) & 0 \leq t \leq T_f \\ A\cos(\omega_2 t) & 0 \leq t \leq T_f. \end{cases}$$

Show that if $\omega_1 T_f >> 1$, $\omega_2 T_f >> 1$ and $(\omega_1 - \omega_2)T_f >> 1$ then the probability of error P_e when subject to AWGN with power spectral density $\eta/2\ W/Hz$ and optimum matched filtering detection is used is approximated by: -

$$P_e \approx Q\left[\sqrt{\frac{A^2 T_f}{2\eta}}\right].$$

[7 *marks*]

(c) A phase shift keying modulation scheme is defined by: -

$$s_i(t) = \begin{cases} A\cos(\omega_1 t) & 0 \leq t \leq T_\phi \\ -A\cos(\omega_1 t) & 0 \leq t \leq T_\phi. \end{cases}$$

where T_ϕ is an integer times $1/f_1$ (where $\omega_1 = 2\pi f_1$). If, under the same conditions as (b) above, this scheme must possess the same probability of error P_e as that of (b) above, deduce the value of T_f/T_ϕ and comment upon your result.

[5 *marks*]

Q.7 Given the following table of field elements of $GF\left(2^4\right)$: -

$$
\begin{aligned}
&0\\
&1\\
&\alpha\\
&\alpha^2\\
&\alpha^3\\
&\alpha^4 = \alpha + 1\\
&\alpha^5 = \alpha^2 + \alpha\\
&\alpha^6 = \alpha^3 + \alpha^2\\
&\alpha^7 = \alpha^3 + \alpha + 1\\
&\alpha^8 = \alpha^2 + 1\\
&\alpha^9 = \alpha^3 + \alpha\\
&\alpha^{10} = \alpha^2 + \alpha + 1\\
&\alpha^{11} = \alpha^3 + \alpha^2 + \alpha\\
&\alpha^{12} = \alpha^3 + \alpha^2 + \alpha + 1\\
&\alpha^{13} = \alpha^3 + \alpha^2 + 1\\
&\alpha^{14} = \alpha^3 + 1
\end{aligned}
$$

(a) Show that the generator polynomial for the $(15,7)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1\,.$$

[7 *marks*]

(b) If the received data, denoted $v(x)$, is given by: -

$$v(x) = x^{12} + x^{10} + x^9 + x^7 + x^6 + x^4 + x^3 + x + 1,$$

use the syndrome decoding method to find the error polynomial $e(x)$ and the original codeword $c(x)$, where $v(x) = c(x) + e(x)$.

[10 *marks*]

(c) Deduce the value of the information polynomial, denoted $i(x)$, associated with the value of $c(x)$ calculated in part (b) above.

[3 *marks*]