OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

AUTUMN EXAMINATIONS, 2013

B.E. DEGREE (ELECTRICAL and ELECTRONIC)

TELECOMMUNICATIONS

EE4004

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Time allowed: *3 hours*

Answer *five* questions.
All questions carry equal marks.

The use of departmental approved calculators is permitted.

1. (a) Illustrate a simple wide area network (WAN) consisting of a collection of switches and links and using this diagram describe the typical operation and functions associated with wide area networks.

[*10 marks*]

(b) (i) Illustrate the timing of the packet and acknowledgement transfers for a data-link which uses a "go-back-N" ARQ scheme and from this determine an expression for the utilization, U, of the data-link.

[6 *marks*]

(ii) For a 250*km* data-link with a data rate of 500*Mbps*, determine the minimum frame window, *N*, which is needed to guarantee a utilization of 100% assuming an error-free line. The packet size is 10,000 bits, the acknowledgement size is 300 bits and the propagation delay is $5\mu s/km$.

[*4 marks*]

2. (a) Draw two protocol stacks to compare the functions of the OSI and TCP/IP systems for wide area networks. Clearly label each layer in the diagram and briefly describe the function of each layer.

[9 marks]

(b) Briefly describe the following aspects of TCP/IP:

(i) IP Addressing

[4 *marks*]

(ii) The Domain Name System

[*3 marks*]

(iii) The format of an IP packet.

[*4 marks*]

3. (a) Illustrate the architecture of a UMTS Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) Illustrate a typical FHSS (Frequency Hopping Spread Spectrum) system (both transmitter and receiver sides) and comment briefly on why FHSS systems have advantages with respect to immunity to interference and privacy.

[*7 marks*]

(c) Illustrate and briefly describe the 3 most common architectures for wireless networks and specify which architecture is most suitable if the network spans a range of geographic regions.

[5 *marks*]

4. The channel matrix for the binary symmetric erasure channel (BSEC) is shown below:

$$\left[P\left(Y\left|X\right.\right)\right]=\begin{bmatrix}1-\delta-\varepsilon & \delta & \varepsilon \\ \varepsilon & \delta & 1-\delta-\varepsilon\end{bmatrix}.$$

(a) Show that $H\left(Y|X\right)=-\left(\delta\log_2[\delta]+\varepsilon\log_2[\varepsilon]+(1-\delta-\varepsilon)\log_2[1-\delta-\varepsilon]\right)$.

[8 *marks*]

(b) If the input symbols x_i, $1\le i\le 2$ are equiprobable, show that the channel capacity, C_{BSEC}, in this case is given by: -

$$C_{BSEC}=(\delta-1)\log_2\left[\frac{1-\delta}{2}\right]+\varepsilon\log_2[\varepsilon]+(1-\delta-\varepsilon)\log_2[1-\delta-\varepsilon].$$

[8 *marks*]

(c) By considering the case $\delta\to 0$, or otherwise, deduce the channel capacity of the binary symmetric channel (BSC).

[2 *marks*]

(d) By considering the case $\varepsilon\to 0$, or otherwise, deduce the channel capacity of the binary erasure channel (BEC).

[2 *marks*]

5. A baseband digital communications system uses the following signals to represent its three symbols, respectively denoted x_1, x_2 and x_3: -

$$s_i(t)=\begin{cases}-A & 0\le t\le T \quad symbol\ x_1\\ 0 & 0\le t\le T \quad symbol\ x_2\\ A & 0\le t\le T \quad symbol\ x_3,\end{cases}$$

where T denotes the bit signalling interval. The communications are affected by additive white Gaussian noise (AWGN) whose probability density function (pdf) $f_n(v)$ is given by:

$$f_n(v)=\frac{e^{\frac{-v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}.$$

The receiver takes a single sample, z, of the received signal during the bit signalling time and makes the detection decision according to the rule: -

$$decision=\begin{cases}symbol\ x_1 & if\ \ z\le-\tau\\ symbol\ x_2 & if\ \ -\tau<z<\tau\\ symbol\ x_3 & if\ \ z\ge\tau\end{cases}$$

where the threshold τ satisfies $0<\tau<A$.

(a) Show that, having sent symbol x_2, the probability of error, denoted P_{e,x_2}, is given by

$$P_{e,x_2} = 1 - erf\left[\frac{\tau}{\sigma\sqrt{2}}\right],$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy.$$

[8 *marks*]

(b) Given that the overall average probability of error for the communications system, P_e, is given by: -

$$P_e = \left(\frac{1+P(x_2)}{2}\right) + erf\left[\frac{A-\tau}{\sigma\sqrt{2}}\right]\left(\frac{P(x_2)-1}{2}\right) - erf\left[\frac{\tau}{\sigma\sqrt{2}}\right]P(x_2)$$

where $P(x_2)$ denotes the probability of sending symbol x_2, prove that the value of τ minimising P_e, denoted τ_{MIN}, is given by: -

$$\tau_{MIN} = \frac{A}{2} - \frac{\sigma^2}{A}\ln\left[\frac{1-P(x_2)}{2P(x_2)}\right].$$

(Hint : It may be helpful to note that

$$\frac{d}{dx}\left[\int_0^x G[y]dy\right] = G[x]).$$

[9 *marks*]

(c) Prove that, in the equiprobable case, the minimum attainable probability of error, P_e^{MIN}, is given by: -

$$P_e^{MIN} = \frac{2}{3}\left(1 - erf\left[\frac{A}{2\sqrt{2}\sigma}\right]\right).$$

[3 *marks*]

6. (a) Using the Schwarz inequality, which states: -

$$\left|\int_{-\infty}^{\infty} f_1(\omega) f_2(\omega) d\omega\right|^2 \leq \int_{-\infty}^{\infty} |f_1(\omega)|^2 d\omega \int_{-\infty}^{\infty} |f_2(\omega)|^2 d\omega,$$

or otherwise, show that the signal to noise ratio (SNR) at the output of a linear filter subject to an input signal $s(t)$, with Fourier transform $S(\omega)$, and input coloured noise with power spectral density $S_{nn}(\omega)$ satisfies: -

$$\left(\frac{S}{N}\right)_o \leq \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{S_{nn}(\omega)} d\omega$$

and state the equation for the matched filter for this coloured noise.

[*10 marks*]

(b) Hence, or otherwise, deduce the maximum attainable SNR if the input noise is additive white Gaussian (AWGN) with a power spectral density of $\eta/2$ W/Hz.

[*3 marks*]

(c) The matched filter for an input signal $s(t)$, with Fourier transform $S(\omega)$, and additive white Gaussian noise (AWGN) with a power spectral density $\eta/2$ W/Hz is implemented in a receiver. The actual noise, however, is coloured with a power spectral density of $S_{nn}(\omega)$ (i.e. the system was designed for white noise but, in practice, coloured noise affects the receiver). Show that, in this case, the SNR, $\left(\frac{S}{N}\right)_o$, is given by: -

$$\left(\frac{S}{N}\right)_o = \frac{2\pi \ E^2}{\int_{-\infty}^{\infty} S_{nn}(\omega)|S(\omega)|^2 d\omega}$$

where E denotes the energy content of $s(t)$ (assuming a 1Ω reference resistor).

[*7 marks*]

7. (a) Using the primitive polynomial $p(x) = x^5 + x^2 + 1$, generate the field $GF(2^5)$.

[9 *marks*]

(b) Show that for the $(31,21)$ double error correcting code primitive BCH code based upon $GF(2^5)$: -

(i) The minimal polynomial $m_1(x)$ is given by: -

$$m_1(x) = x^5 + x^2 + 1.$$

[4 *marks*]

(ii) The minimal polynomial $m_3(x)$ is given by: -

$$m_3(x) = x^5 + x^4 + x^3 + x^2 + 1.$$

[4 *marks*]

(iii) The corresponding generator polynomial, $g(x)$, satisfies: -

$$g(\alpha) = g(\alpha^3) = 0.$$

[3 *marks*]