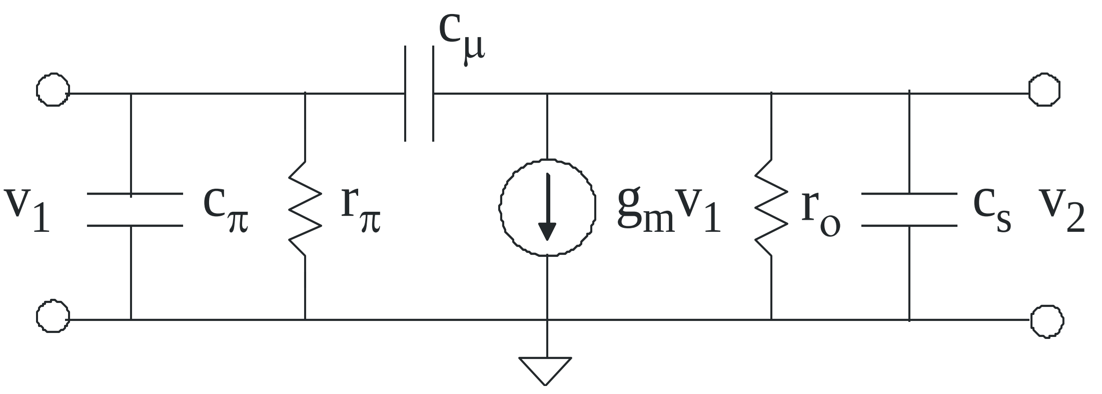
EE4011 RF IC Design Summer 2010
Question 1(a) 14 marks




$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the y-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$y_{11} = \frac{1}{r_\pi} + j\omega\left(c_\pi + c_\mu\right)$$

$$y_{12} = -j\omega c_\mu$$

$$y_{21} = g_m - j\omega c_\mu$$

$$y_{22} = \frac{1}{r_o} + j\omega\left(c_s + c_\mu\right)$$

EE4011 RF IC Design Summer 2010
Question 1(a) continued

The equations on the previous page have to be manipulated to give the small signal-element values as follows:

$$g_m = \mathbf{Re}\left(y_{21}\right) = 0.15S$$

$$r_\pi = \frac{1}{\mathbf{Re}\left(y_{11}\right)} = 250\Omega$$

$$r_o = \frac{1}{\mathbf{Re}\left(y_{22}\right)} = 1.5k\Omega$$

$$c_\mu = \frac{-\mathbf{Im}\left(y_{12}\right)}{2\pi f} = 0.7\,pF$$

$$c_\pi = \frac{\mathbf{Im}\left(y_{11}\right)}{2\pi f} - c_\mu = 4.5\,pF$$

$$c_s = \frac{\mathbf{Im}\left(y_{22}\right)}{2\pi f} - c_\mu = 0.3\,pF$$

Question 1(b) 6 marks

$$V_T = \frac{kT}{q} = 25.9mV$$

(i) $I_C = g_m V_T = 3.9mA$

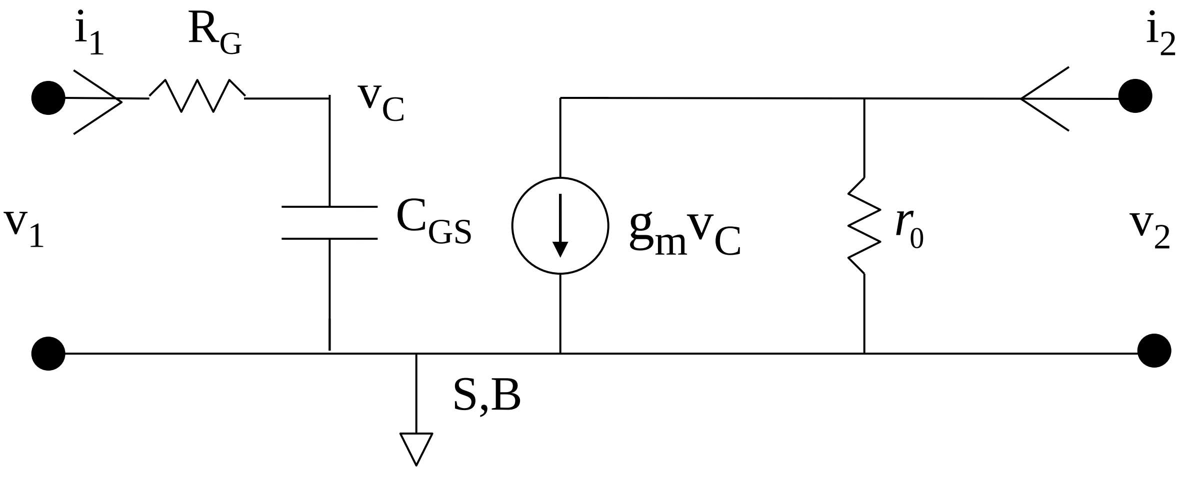
(ii) $V_A = r_o I_C = 5.82V$

(iii) $\beta = g_m r_\pi = 37.5$

(iv) $f_T = \dfrac{g_m}{2\pi\left(c_\pi + c_\mu\right)} = 4.59GHz$

EE4011 RF IC Design Summer 2010

Question 2(a) 8 marks




Applying the z-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = R_G + \frac{1}{j\varpi c_{GS}}$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = -\frac{g_m r_o}{j\varpi c_{GS}}$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = 0$$

$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = r_o$$

Question 2(b)    8 marks

$$C'_{OX} = \frac{\varepsilon_{OX}}{T_{OX}} \qquad V_{DS} > \left(V_{GS} - V_{TH}\right) \text{ so MOSFET in saturation}$$

For a MOSFET in saturation:

$$I_{DS} = \frac{1}{2}\frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$g_m = \frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right) = \sqrt{2\frac{W}{L}\mu C'_{OX}\, I_{DS}}$$

$$g_{ds} = \frac{1}{r_O} = \frac{1}{2}\frac{W}{L}\mu C'_{OX}\left(V_{GS} - V_{TH}\right)^2 \lambda$$

$$C_{GS} = \frac{2}{3}WLC'_{OX}$$

Doing the calculations and inserting these values into the previous formulas for the z-parameters at 1GHz gives:

$z_{11} = 987.7 \angle -89.4°$   $z_{12} = 0$   $z_{21} = 13929 \angle 90°$   $z_{22} = 100 \angle 0°$

(c) 4 marks

Gate resistance with parallel layout and gate contacted at both sides.

$$R_{Geff} = \frac{R_G}{4N^2} = \frac{10}{4 \times 25} = 0.1\Omega$$

EE4011 RF IC Design Summer 2010

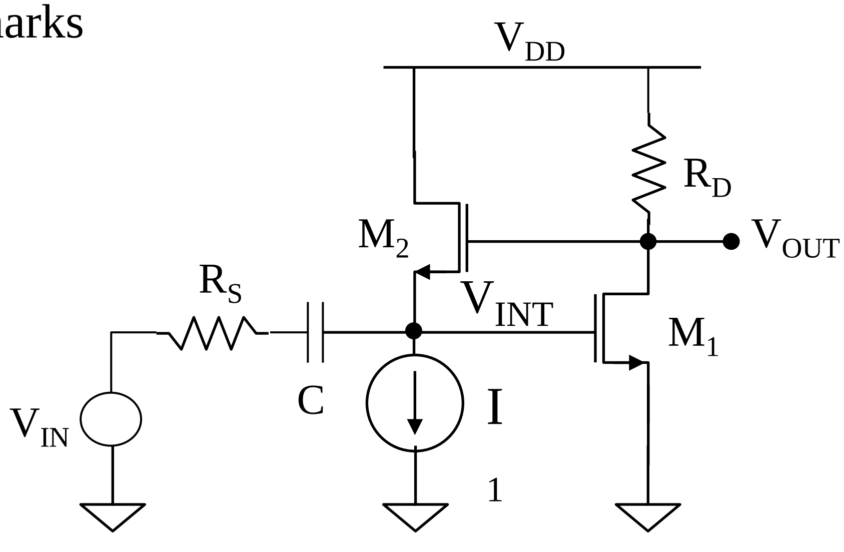
## Question 3

### 3(a) 3 marks

$$\text{Noise Figure, NF} = 10\log_{10}\left(\frac{SNR_{in}}{SNR_{out}}\right) \quad (\text{in dB}, \geq 0)$$

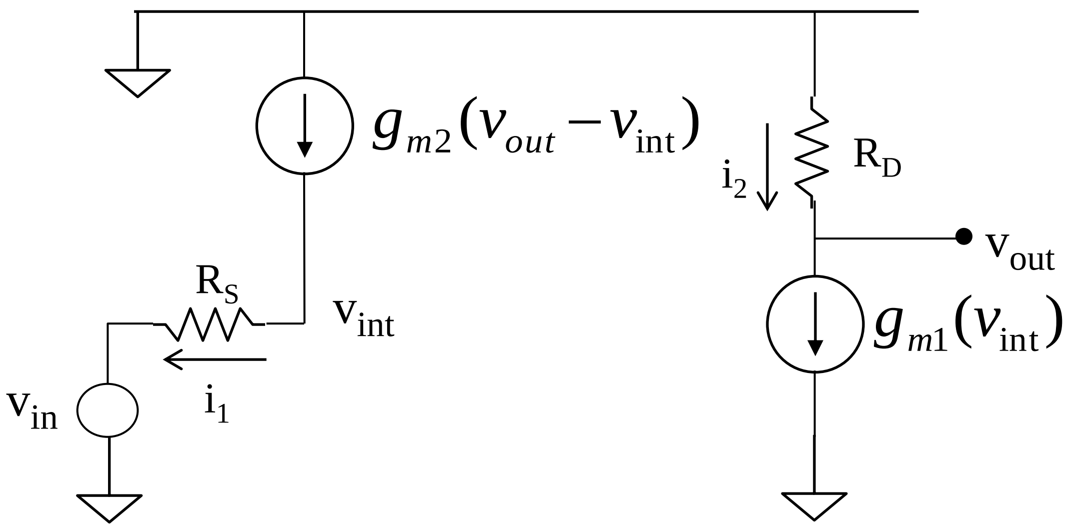
$SNR_{in}$ = (signal power in)/(noise power in)
$SNR_{out}$ = (signal power out)/(noise power out)

### 3(b) 7 marks




A small-signal analysis has to be performed to determine the voltage gain. For the small-signal analysis the DC power supply can be replaced by an ac ground. The current source is ideal so it can be replaced by an open circuit (i.e. just pretend it isn't there) for small-signal analysis. We are told that the impedance of the capacitor is negligible (for ac signals) so it can be "shorted-out" for the ac analysis. We are told that the MOSFETs can be represented by ideal transconductances so we'll only need to represent the MOSFETs by $g_m$ elements in the small-signal analysis. For these $g_m$ elements the current will flow from source to drain (which for M1 is from the $V_{OUT}$ node to ground and for M2 it is from the $V_{DD}$ node to the $V_{INT}$ node). We also need to know what voltage to use to control each of the transconductances. From looking at the circuit we see that for $M_1$ $V_{GS1} = V_{INT}$ and for $M_2$ $V_{GS2} = V_{OUT} - V_{INT}$.
Taking all these considerations together gives the small-signal circuit on the following page.

Question 3(b) continued




$$i_2 = g_{m1} v_{\text{int}}$$

$$v_{out} = -i_2 R_D = -g_{m1} R_D v_{\text{int}} \Rightarrow v_{\text{int}} = -\frac{v_{out}}{g_{m1} R_D} \quad (A)$$

$$i_1 = \frac{v_{\text{int}} - v_{in}}{R_S} \quad also \quad i_1 = g_{m2}(v_{out} - v_{\text{int}}) \Rightarrow \frac{v_{\text{int}} - v_{in}}{R_S} = g_{m2}(v_{out} - v_{\text{int}}) \quad (B)$$

rearrange (B) and substitute $v_{\text{int}}$ from (A):

$$v_{\text{int}} - v_{in} = g_{m2} R_S (v_{out} - v_{\text{int}}) \Rightarrow -\frac{v_{out}}{g_{m1} R_D} - v_{in} = g_{m2} R_S \left( v_{out} - \left( -\frac{v_{out}}{g_{m1} R_D} \right) \right)$$

multiply across by $gm_1 R_D$ and rearrange to find voltage gain, A:

$$-v_{out} - v_{in} g_{m1} R_D = g_{m2} R_S (g_{m1} R_D v_{out} + v_{out})$$

$$\Rightarrow -v_{in} g_{m1} R_D = g_{m2} R_S (g_{m1} R_D v_{out} + v_{out}) + v_{out} = [g_{m2} R_S (g_{m1} R_D + 1) + 1] v_{out}$$

$$\Rightarrow A = \frac{v_{out}}{v_{in}} = -\frac{g_{m1} R_D}{1 + g_{m2} R_S (g_{m1} R_D + 1)}$$

The output voltage and its square (related to power) can then be calculated:

$$v_{out} = A v_{in} \Rightarrow v^2{}_{out} = A^2 v^2{}_{in}$$

3(c) 10 marks

The thermal noise contributed by is the source resistance $R_S$ is

$$v^2{}_{n,in} = 4kTR_S \qquad v^2 / Hz$$

The signal to noise ratio at the input in then:

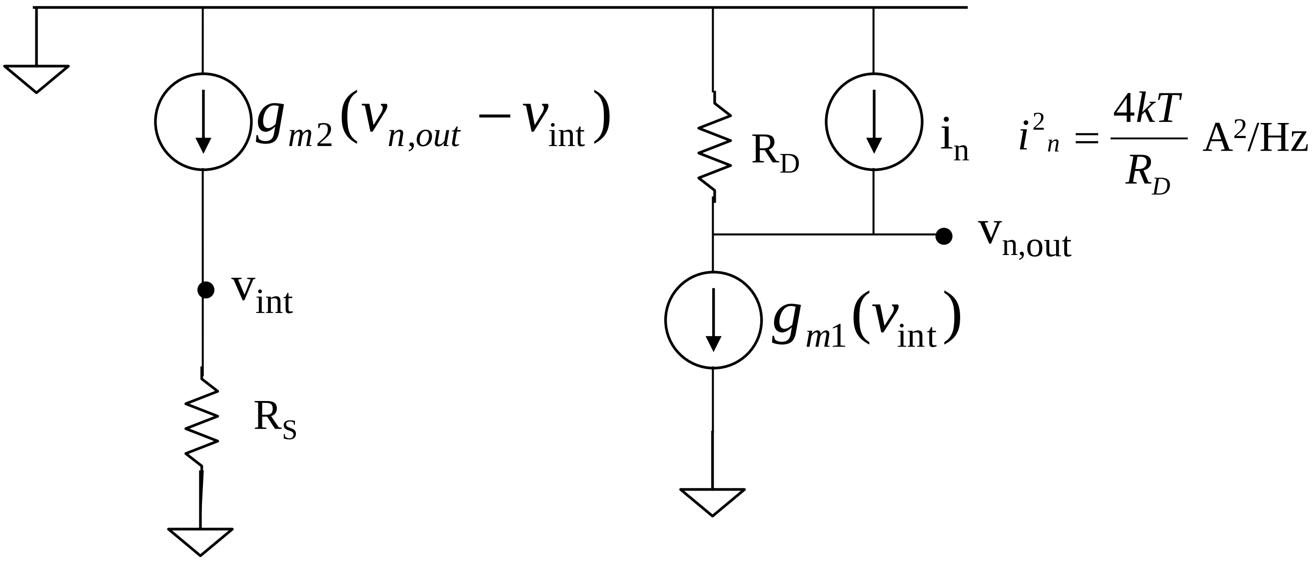
$$SNR_{in} = \frac{v^2{}_{in}}{v^2{}_{n,in}} = \frac{v^2{}_{in}}{4kTR_S}$$

To calculate the SNR at the output, the total noise at the output from both $R_S$ and $R_D$ have to be calculated. The noise from $R_S$ occurs at the input of the amplifier and is gained up by the full gain, A, of the amplifier, just like any other input signal. Therefore, the square noise voltage at the output resulting from the $R_S$ noise can be calculated using the amplifier gain i.e.

$$\left(v^2{}_{n,out}\right)_{RS} = A^2\left(v^2{}_{n,in}\right)_{RS} = A^2 4kTR_S \qquad v^2 / Hz$$

To calculate the noise contributed at the output from $R_D$, a new circuit analysis has to be performed. This is a small-signal analysis where the only signal source will be thermal noise (current or voltage) coming from $R_D$. All other signal sources (including $v_{in}$) will be set to 0 for this analysis. For this analysis it is more convenient to represent the thermal noise coming from $R_D$ as a current source instead of a voltage source. Taking all these considerations into account gives the small-signal circuit on the next page.

Question 3(c) continued




Working through the small-signal analysis of this circuit gives the output voltage noise caused by $i_n$ as follows $R_D$ as:

$$v_{n,out} = i_n \frac{R_D(1+g_{m2}R_S)}{(1+g_{m2}R_S + g_{m1}R_D g_{m2}R_S)}$$

The square noise voltage is then:

$$(v_{n,out})^2{}_{RD} = (i_n)^2{}_{RD} \frac{R_D{}^2(1+g_{m2}R_S)^2}{(1+g_{m2}R_S + g_{m1}R_D g_{m2}R_S)^2} = \frac{4kTR_D(1+g_{m2}R_S)^2}{(1+g_{m2}R_S + g_{m1}R_D g_{m2}R_S)^2}$$

The total o/p noise is then

$$v^2{}_{n,out} = (v^2{}_{n,out})_{RS} + (v^2{}_{n,out})_{RD}$$

The signal to noise ratio at the output can then be calculated and the noise factor:

$$SNR_{out} = \frac{v^2 out}{v^2{}_{n,out}} \qquad F = \frac{SNR_{in}}{SNR_{out}}$$

The final expressions are cumbersome so the formulas determined up to this point are sufficient

EE4011 RF IC Design Summer 2010

Question 4

$$s_{11} = 0.46\angle -143° \quad s_{12} = 0.01\angle 98° \quad s_{21} = 1.70\angle 59° \quad s_{22} = 0.70\angle -30°$$

$$F_{\min} = 2.5\,dB \quad \Gamma_{opt} = 0.55\angle -160° \quad R_N = 5.5\,\Omega$$

$$Z_0 = 50\Omega \quad f = 14GHz$$

(a) (i) 3 marks

For unconditional stability the following conditions are necessary (K is the Rollet Stability Factor):

$$K > 1 \quad and \quad |\Delta| < 1$$

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} \qquad \Delta = s_{11}s_{22} - s_{12}s_{21}$$

Putting in the s-parameters into the formulas gives:

$$K = 11.56 \quad and \quad |\Delta| = 0.31$$

Thus the amplifier will be unconditionally stable.

(a)(ii) 2 marks

$$F_{dB} = 10\log_{10}(F_{ratio}) \Rightarrow F_{ratio} = 10^{\frac{F_{dB}}{10}}$$

The noise circle for $F_i$ = 3dB

$$F_{\min,ratio} = 10^{\frac{2.5}{10}} = 1.778 \quad F_{i,ratio} = 10^{\frac{3}{10}} = 1.995$$

$$N_i = \frac{F_{i,ratio} - F_{\min,ratio}}{4R_N / Z_0}|1 + \Gamma_{opt}|^2 = \frac{1.995 - 1.778}{4 \times 5.5/50}|1 + 0.55\angle{-160°}|^2 = 0.133$$

$$C_{Fi} = \frac{\Gamma_{opt}}{N_i + 1} = \frac{0.55\angle{-160°}}{0.133 + 1} = 0.49\angle{-160°}$$

Note: $\Gamma_{OPT}$ is a complex number in these calculations

$$R_{Fi} = \frac{\sqrt{N_i\left(N_i + 1 - |\Gamma_{opt}|^2\right)}}{(N_i + 1)} = \frac{\sqrt{0.133\left(0.133 + 1 - (0.55)^2\right)}}{(0.133 + 1)} = 0.29$$

The 3dB noise circle can now be drawn on the Smith Chart.

(b) 15 marks

First calculate the maximum unilateral transducer gain:

$$G_{S,\max} \frac{1}{1-|s_{11}|^2} = \frac{1}{1-(0.46)^2} = 1.268(ratio) = 1.03dB$$

$$G_0 = |s_{21}|^2 = (1.70)^2 = 2.89(ratio) = 4.61dB$$

$$G_{L,\max} \frac{1}{1-|s_{22}|^2} = \frac{1}{1-(0.70)^2} = 1.961(ratio) = 2.92dB$$

$$G_{TU,\max,dB} = G_{S,\max,dB} + G_{0,dB} + G_{L,\max,dB} = 1.03dB + 4.61dB + 2.92dB = 8.56dB$$

If there was no constraint on noise, the source reflection coefficient could be set to $s_{11}$* and the load reflection coefficient could be set to $s_{22}$* and in that case the maximum unilateral transducer gain would be 8.56dB. But it is seen that $s_{11}$* is outside the 3dB noise circle and so the noise in that case would be too much. Therefore the input reflection coefficient will have to be moved so that it sits on the 3dB noise circle but this will decrease the source gain term. Some source gain circles will have to be drawn to do the design.
$G_{S,max}$=1.03dB so draw a source gain circle for $G_S$=0.95dB to start

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{0.95}{10}} = 1.245$$

$$g_s = \frac{G_S}{G_{S,\max}} = \frac{1.245}{1.268} = 0.981$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.981\times|0.46|}{1-|0.46|^2(1-0.981)} = 0.45$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.981}\left(1-|0.46|^2\right)}{1-|0.46|^2(1-0.981)} = 0.11$$

The centre of this circle is a distance 0.45 along the line joining the centre of the Smith Chart to $s_{11}$* and the radius is 0.11 and this can be drawn on the Smith Chart.

Question 4(b) continued

It is seen that the source gain circle for 0.95dB doesn't touch the 3dB noise circle so the noise would still be more than 3dB if the source reflection coefficient was placed on the 0.95dB source gain circle. Therefore the gain has to be reduced again – try a source gain of 0.85dB and draw the circle for this.

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{0.85}{10}} = 1.216$$

$$g_s = \frac{G_S}{G_{S,\max}} = \frac{1.216}{1.268} = 0.959$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.959\times|0.46|}{1-|0.46|^2(1-0.959)} = 0.44$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.959}\left(1-|0.46|^2\right)}{1-|0.46|^2(1-0.959)} = 0.16$$

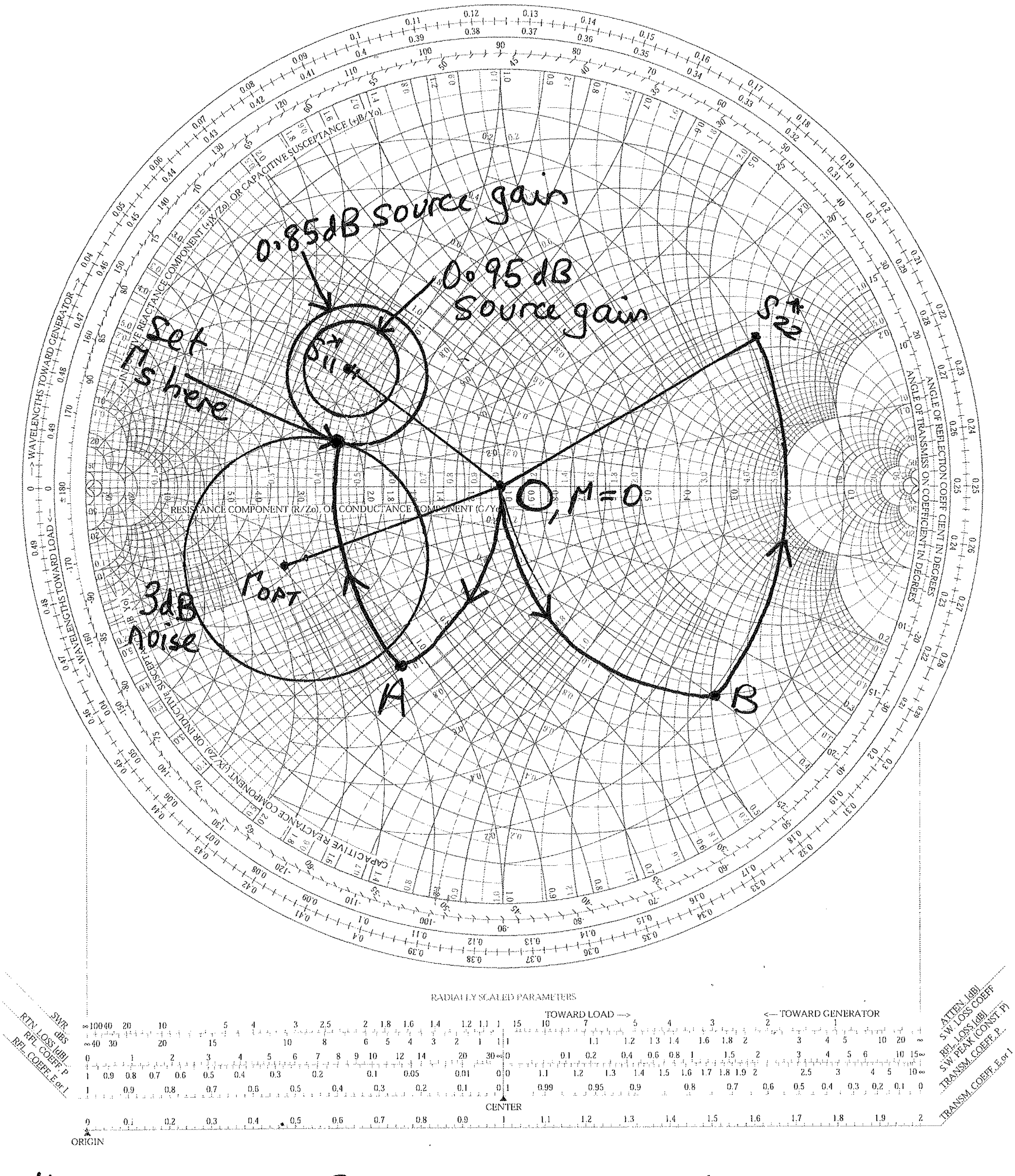
When the source gain circle for 0.85dB is drawn it is seen to touch the 3dB noise circle. Therefore, the source reflection coefficient can be set to the point where the two circles touch – then the source gain will be 0.85dB and the noise figure will be 3dB. Because noise is not influenced by the output reflection coefficient, the load reflection coefficient can be set to $s_{22}$*. Therefore, the maximum unilateral transducer gain that can be achieved is:

$$G_{TU,dB} = G_{S,dB} + G_{0,dB} + G_{L,\max,dB} = 0.85dB + 4.61dB + 2.92dB = 8.38dB \approx 8.4dB$$

Now the matching networks can be designed – see the Smith Chart on the next page.

| EXAMINATION NUMBER | QUESTION NUMBER | DATE |
|---|---|---|
| Q4 Summer 2010 | EE4011 | |
| | EE4011 RF IC Design | |

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES




At the origin of the Smith Chart, O : $x = 0, b = 0$

At point A : $x = -0.5, b = 1.18$ At point B : $x = -2.1, b = 0.4$

At $\Gamma_s$ : $x = 0.11, b = -0.6$ At $S_{22}^*$ : $x = +2.55, b = -0.27$

Question 4(b) continued

Input Matching Element Values

Moving from $Z_0$ ($\Gamma$=0) to point A:
Clockwise on conductance circle – shunt capacitor

susceptance at $Z_0$: b = 0
susceptance at A: b = 1.18

$$C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|1.18|}{2\pi \times 14 \times 10^9 \times 50} = 0.27\,pF$$

Moving from A to $\Gamma_S$:
Clockwise on resistance circle – series inductor

reactance at A: x = -0.5
reactance at $\Gamma_S$: x = 0.11

$$L = \frac{Z_0|\Delta x|}{2\pi f} = \frac{50 \times |0.61|}{2\pi \times 14 \times 10^9} = 0.35nH$$

Output Matching Element Values

Moving from $Z_0$ ($\Gamma$=0) to point B:
Anti-clockwise on resistance circle – series capacitor

reactance at $Z_0$: x = 0
reactance at B: x = -2.1

$$C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 14 \times 10^9 \times |-2.1| \times 50} = 0.11pF$$

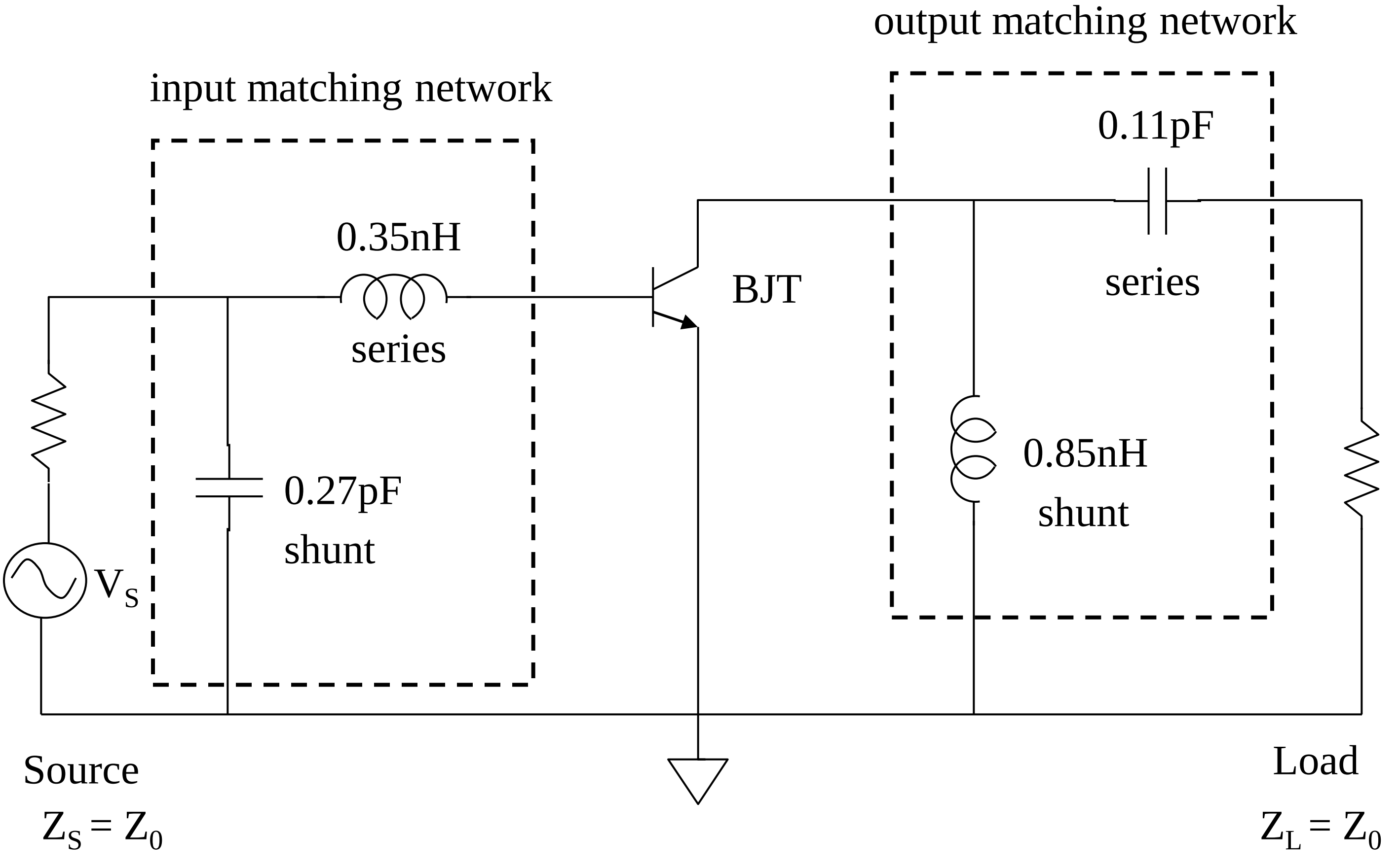
Moving from B to $s_{22}^*$:
Anti-clockwise on conductance circle – shunt inductor

susceptance at B: b = 0.4
susceptance at $s_{22}^*$: b = -0.27

$$L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 14 \times 10^9 \times |-0.67|} = 0.85nH$$

The matching circuit is shown on the next page.

Question 4(b) continued

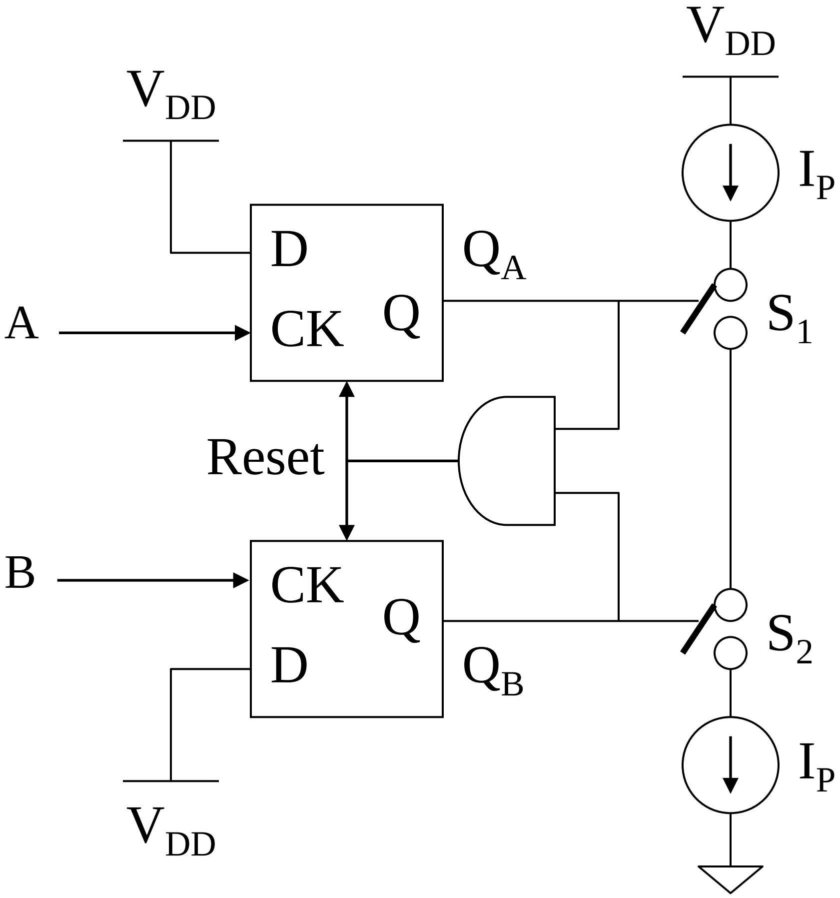



Question 5 concerns derivations from the notes

Question 6

6(a) 5 marks

Suitable PD with transfer function




$$I(s) = \frac{I_P}{2\pi} \Delta\varphi(s)$$

(b) The control voltage is given by:

$$V_C(s) = \frac{I_P}{2\pi}\left(\frac{R_P C_P s + 1}{(R_P C_P C_2 s + C_P + C_2)s}\right)\Delta\varphi(s)$$

The transfer function is found from this through a closed loop analysis.

Q6(c) concerns production of clock phases from the notes

Q7 This is an essay type question based on a continuous assessment assignment.