APPENDIX 3B DISCONTINUOUS-CONDUCTION MODE (DCM) IN DC-DC CONVERTERS

As discussed in section 3-15 and Appendix 3A, all dc-dc converters with a power-pole implemented by one transistor go into a discontinuous conduction mode, DCM, below a certain output load corresponding to the inductor critical current $I_{L,crit}$. In particular, we will select Buck and Buck-Boost converters to analyze in DCM and leave the analysis of Boost converters in DCM as a homework problem.

3B-1 ANALYSIS IN DC STEADY STATE

Fig. 3B-1a shows the power-pole for Buck and Buck-Boost topologies where $V_{vp} = V_{in}$ in the case of a Buck converter and $V_{vp} = (V_{in} + V_o)$ in the case of a Buck-Boost converter. An output load below the critical value results in the inductor current becoming discontinuous as is shown in Fig. 3B-1b, that is, it becomes zero and stays zero for an interval because it cannot reverse through the diode in Fig. 3B-1a. The switch duty-ratio is D, defined in the same manner as in CCM, which equals T_{on} / T_s. The switch-off interval (divided by T_s) now consists of two subintervals: the inductor current flowing through the diode during $D_{off,1}$, and the inductor current remaining at zero during $D_{off,2}$. Note that $D_{off,1} = T_{off,1} / T_s$, and $D_{off,2} = T_{off,2} / T_s$. During $D_{off,2}$, v_A jumps from 0 to V_o as shown in Fig. 3B-1b.

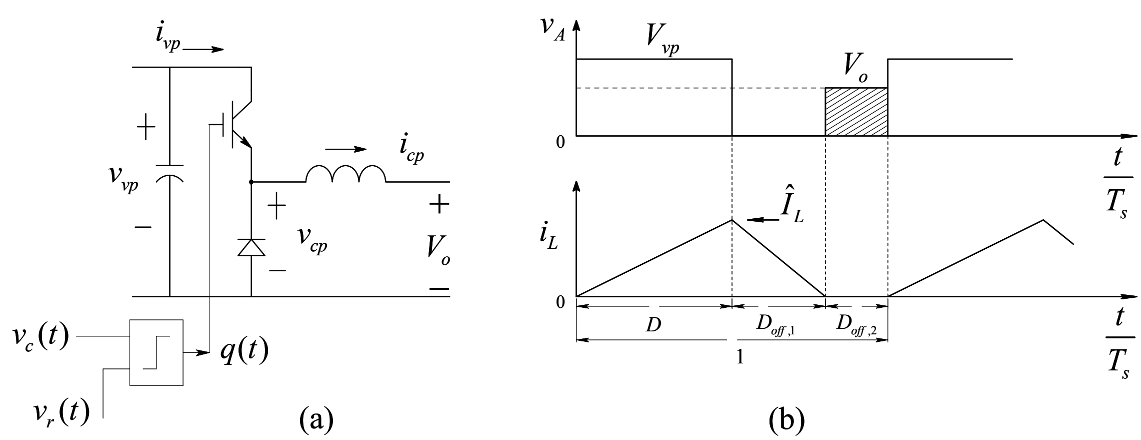

Figure 3B-1 Power-pole in DCM.

In calculating the expressions for the output voltage V_o and the input current I_{in} in DCM, we will assume that V_{in}, D, and f_s are given. Based on the voltages in the circuit of Fig. 3B-1a, and the waveforms in Fig. 3B-1b, the average value can be written in terms of the peak value,

$$I_L = \frac{D + D_{off,1}}{2} \hat{I}_L \tag{3B-1}$$

where,

$$\hat{I}_L = \frac{V_{vp} - V_o}{L} DT_s \tag{3B-2}$$

Substituting Eq. 3B-2 into Eq. 3B-1,

$$D + D_{off,1} = \frac{2LI_L}{(V_{vp} - V_o)DT_s} = \frac{2Lf_s I_L}{(V_{vp} - V_o)D} \tag{3B-3}$$

The average representation is shown in Fig. 3B-2 in which the ideal transformer with the turns-ratio $1:D$ is augmented by two dependent sources, V_k and I_k. The expressions for these dependent sources are calculated below.

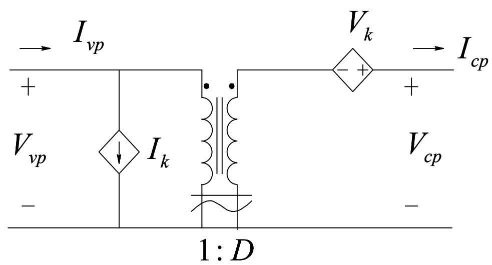

Figure 3B-2 Average representation of the power-pole in DCM in dc steady state.

3B-1-1 V_k and V_o

In Fig. 3B-1b, the average voltage at the current port, noting that $D_{off,2} = 1 - (D + D_{off,1})$, is

$$V_A = V_{vp}D + 0 \cdot D_{off,1} + \underbrace{D_{off,2}V_o}_{V_k} = V_{vp}D + \underbrace{[1 - (D + D_{off,1})]V_o}_{V_k} \tag{3B-4}$$

In Fig. 3B-2,

$$V_k = V_A - DV_{vp} \tag{3B-5}$$

Substituting for V_A from Eq. 3B-4, and for $D + D_{off,1}$ from Eq. 3B-3 into Eq. 3B-5,

$$V_k = [1 - \frac{2Lf_s I_L}{(V_{vp} - V_o)D}]V_o \tag{3B-6}$$

and,

$$V_o = V_A = DV_{vp} + V_k \tag{3B-7}$$

In a Buck converter with $V_{vp} = V_{in}$ and $I_L \le I_{L,crit}$ given by Eq. 3-43

$$V_{k,Buck} = [1 - \frac{2Lf_s I_L}{(V_{in} - V_o)D}]V_o \tag{3B-8}$$

and,

$$V_{o,Buck} = V_{in}\frac{1}{1 + \dfrac{I_L / D^2}{V_{in}/(2Lf_s)}} \tag{3B-9}$$

Similarly, in a Buck-Boost converter with $V_{vp} = (V_{in} + V_o)$ and $I_L \le I_{L,crit}$ given by Eq. 3-44

$$V_{k,Buck-Boost} = [1 - \frac{2Lf_s I_L}{V_{in} D}]V_o \tag{3B-10}$$

and,

$$V_{o,Buck-Boost} = D^2 V_{in}\left(\frac{1}{\dfrac{2Lf_s I_L}{D^2 V_{in}} - 1}\right) \qquad I_L \ge \frac{D^2 V_{in}}{2Lf_s} \tag{3B-11}$$

3B-1-2 I_k and I_{vp}

$$I_k = I_{vp} - DI_L \tag{3B-12}$$

From the waveforms in Fig. 3B-1b,

$$I_{vp} = \frac{D}{D + D_{off,1}} I_L \tag{3B-13}$$

Substituting for I_{vp} from Eq. 3B-13, and for $D + D_{off,1}$ from Eq. 3B-3 into Eq. 3B-13 results in

$$I_k = \frac{D^2}{2Lf_s}(V_{vp} - V_o) - DI_L \tag{3B-14}$$

In a Buck converter with $V_{vp} = V_{in}$ and $I_L \le I_{L,crit}$ given by Eq. 3-43

$$I_{k,Buck} = \frac{D^2}{2Lf_s}(V_{in} - V_o) - DI_L \tag{3B-15}$$

Similarly, in a Buck-Boost converter with $V_{vp} = (V_{in} + V_o)$ and $I_L \le I_{L,crit}$ given by Eq. 3-44 results in

$$I_{k,Buck-Boost} = \frac{D^2}{2Lf_s}V_{in} - DI_L \tag{3B-16}$$

Having calculated I_k, I_{vp} can be calculated in Fig. 3B-2.

3B-2 Dynamic Modeling of Average Representation in CCM and DCM

If the duty ratio varies slowly, with a frequency an order of magnitude smaller than the switching frequency, then the average representation obtained on the basis of dc steady state can be used for dynamic modeling in CCM and DCM by replacing uppercase letters with lowercase letters with a "-" on top to represent average quantities that are shown explicitly to be functions of time:

$$\begin{aligned} D &\Rightarrow d(t) \\ V_o &\Rightarrow \overline{v}_o(t) \\ I_L &\Rightarrow \overline{i}_L(t) \\ I_{in} &\Rightarrow \overline{i}_{in} \\ V_k &\Rightarrow v_k \\ I_k &\Rightarrow i_k \end{aligned} \tag{3B-16}$$

The resulting dynamic average representation id shown in Fig. 3A-4.