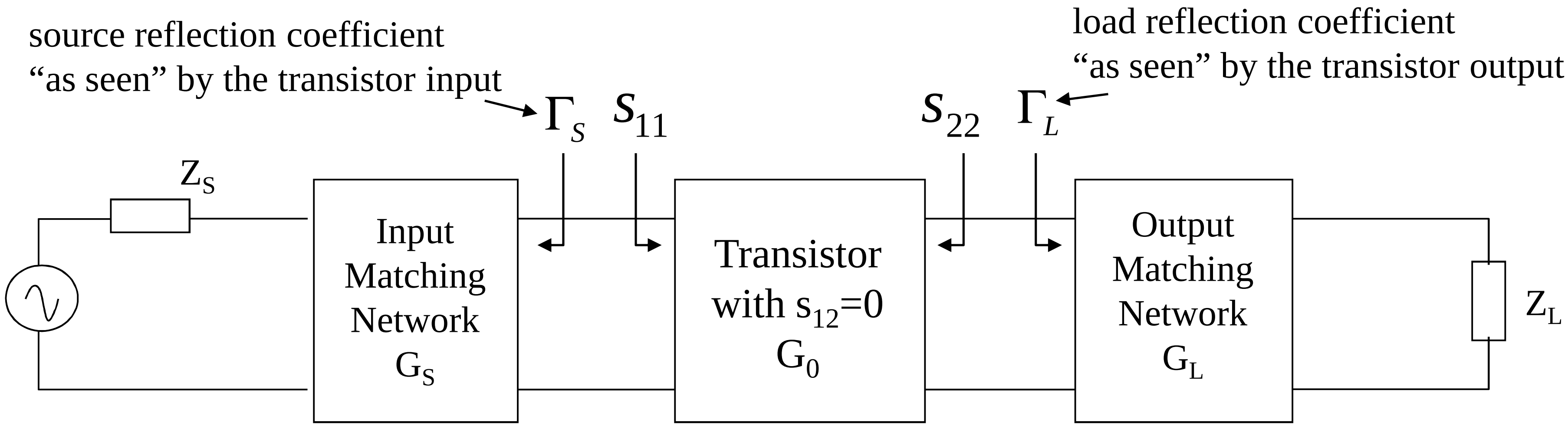
EE4011 RF IC Design

LNA Design Using the Smith Chart

Design for Maximum Gain

Matching For Maximum Gain



Maximum unilateral transducer gain can be achieved by making a conjugate match between the source and the transistor input and between the load and the transistor output i.e.

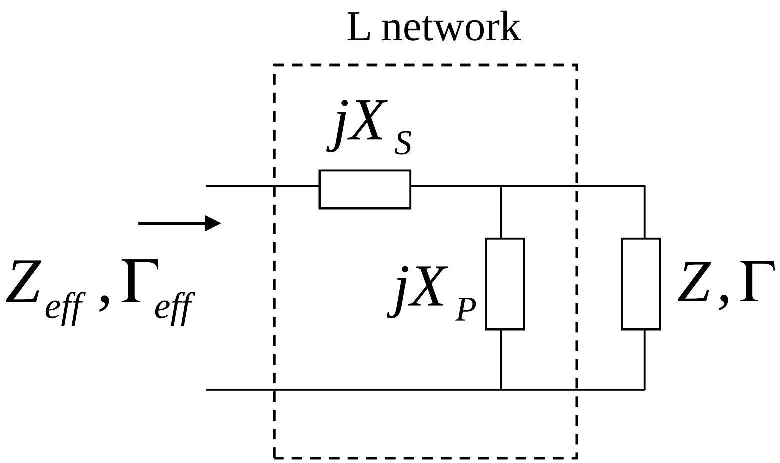
$$\Gamma_S = s_{11}^{*} \quad \Gamma_L = s_{22}^{*}$$

Assuming that these values of source and load reflection coefficients do violate the stability criteria, the problem is then to develop a matching network to transform the source refection coefficient to s_{11}^{*} and the load reflection coefficient to s_{22}^{*}.

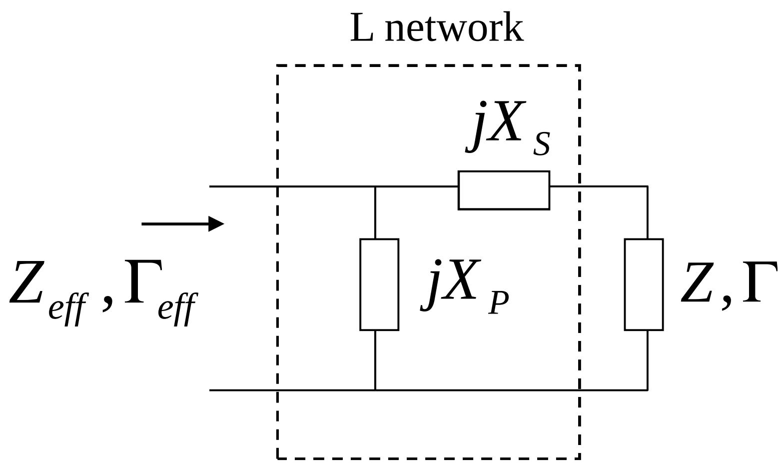
The most straightforward conditions for designing the matching networks are when the source and load impedances are purely resistive and the simplest case of all is where the source and load impedances are equal to the system impedance Z_0.

Two-Element Matching: L Networks

Matching consists of adding L-C networks or lengths of transmissions lines in front of a component to transform its reflection coefficient (or equivalently its impedance) to a new value at a particular frequency. Resistors are not normally used in matching networks because they dissipate power and also add thermal noise. Because reactance is frequency dependent, the matching network will only provide good matching over a small range of frequencies around the frequency used to design the matching components. Therefore, most matching networks have a "band pass" characteristic and designing wideband matching networks is a challenge. The simplest matching networks use two reactances in a "L"-shaped topology:



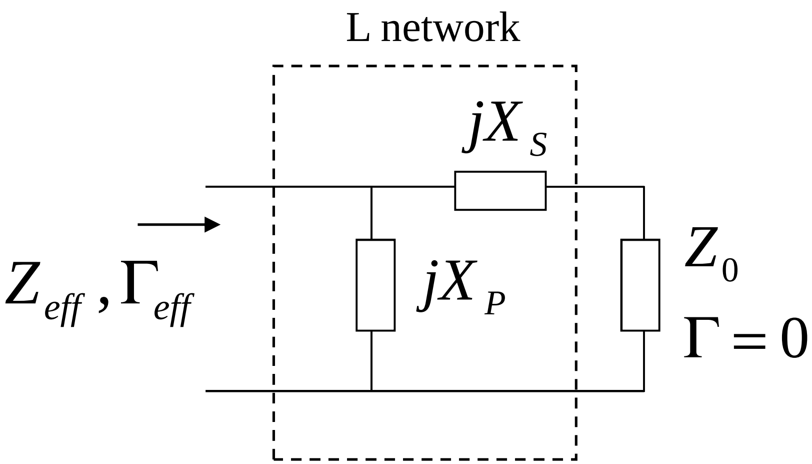
or

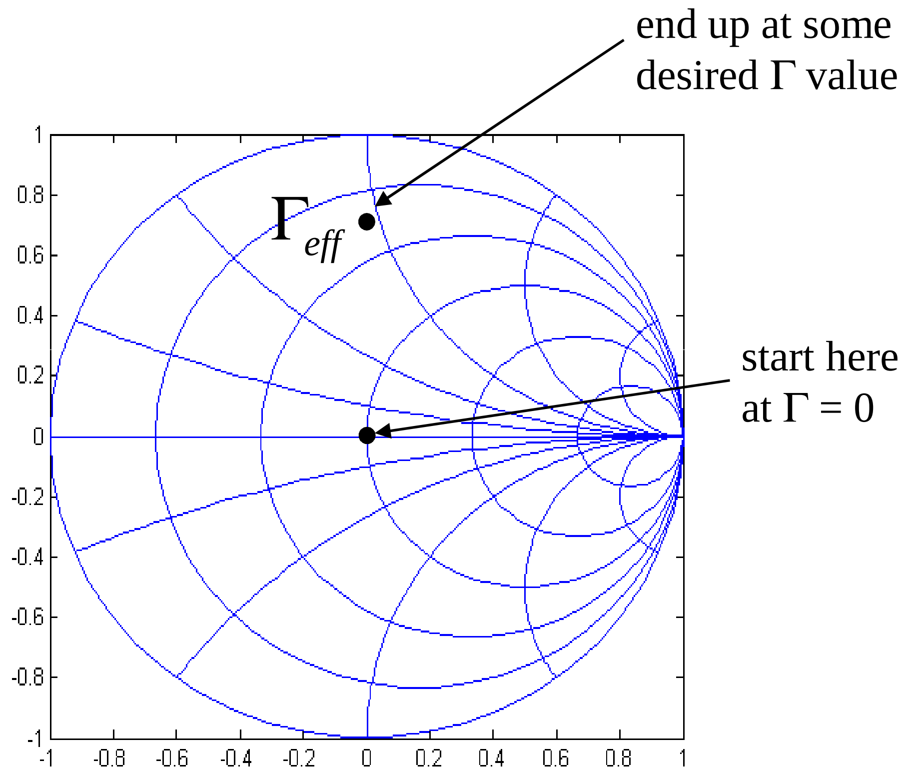


In general, X_P (the parallel reactive element) can be either inductive (positive reactance) or capacitive (negative reactance). If X_P is inductive, X_S (the series reactive element) is capacitive and vice versa.

Using the Smith Chart for Matching

Taking the simplest case of where the source or load impedance is equal to the system impedance (the characteristic impedance used when measuring the transistor s-parameters, Z_0), how can the Smith chart be used to transform this to a new value?

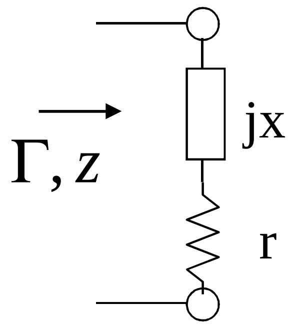




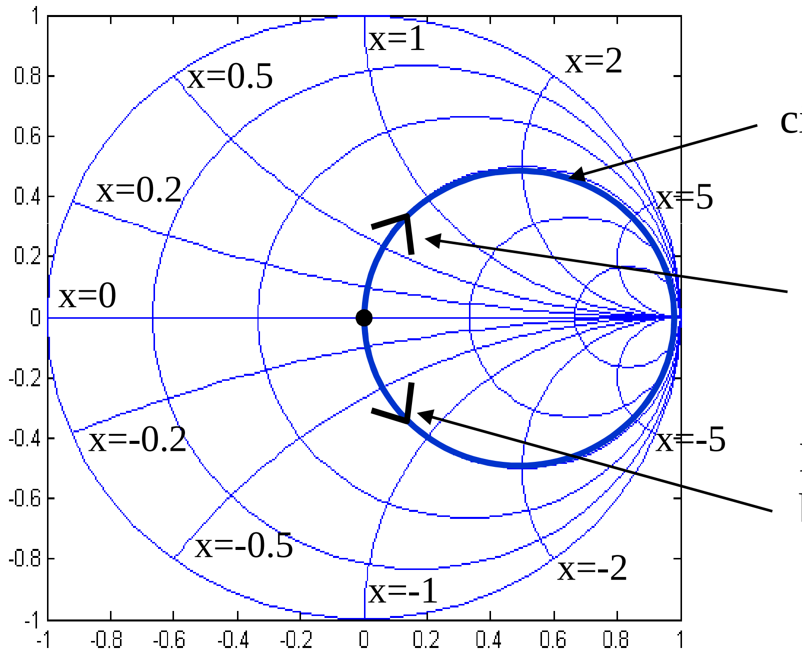
When $Z=Z_0$, $\Gamma=0$, the problem then is to start at the centre of the Smith chart and add one series reactance and one parallel reactance (or vice versa) so that the reflection coefficient moves to the desired value, Γ_{eff}, which could be anywhere inside the unit circle depending on the values of s_{11} and s_{22}.

Starting with Z_0 and adding series reactance

$$Z = R + jX \quad z = r + jx \quad r = \frac{R}{Z_0} \quad x = \frac{X}{Z_0} \quad \text{if } Z = Z_0 \text{ then } r = 1, x = 0$$



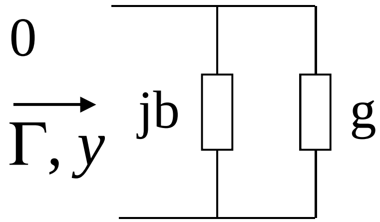
From the construction of the impedance Smith chart, we know that for a fixed resistance as reactance is added in series, the reflection coefficient follows a circle whose radius and centre depends on r.



Here we have started at the origin of the Smith chart but no matter where we start, as series reactance is added to a network, the reflection coefficient will move along a circle determined by the normalised resistance r. It will move clockwise if the reactance is made more positive and anti-clockwise if the reactance is made more negative.

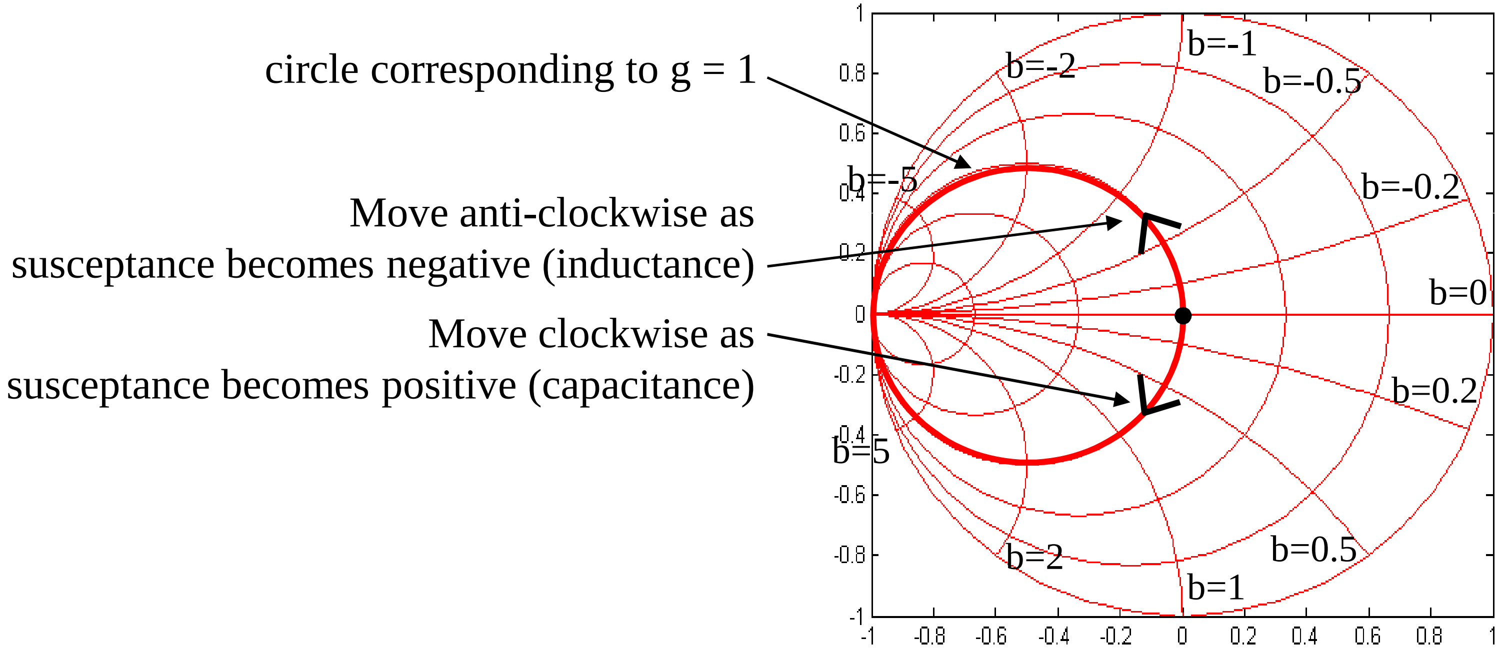
Starting with Z_0 and adding parallel susceptance

$$Y = G + jB \quad y = g + jb \quad g = \frac{G}{Y_0} \quad b = \frac{B}{Y_0} \quad if \;\; Y = Y_0 \;\; then \;\; g = 1, b = 0$$



$$Y_0 = \frac{1}{Z_0}$$

From the construction of the admittance Smith chart, we know that for a fixed conductance as susceptance is added in parallel, the reflection coefficient follows a circle whose radius and centre depends on g.



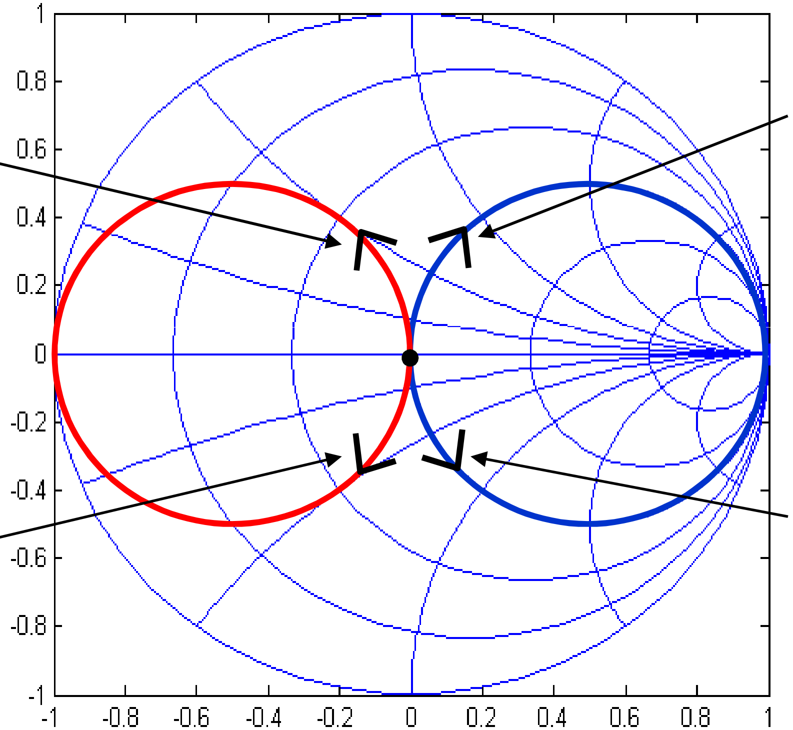
Here we have started at the origin of the Smith chart but no matter where we start, as parallel susceptance is added to a network, the reflection coefficient will move along a circle determined by the normalised condutance g. It will move clockwise if the susceptance is made more positive and anti-clockwise if the susceptance is made more negative.

Series and Shunt Inductance and Capacitance

For any network no matter where the starting point on the Smith chart is:

Adding a shunt (parallel) inductance causes the reflection coefficient to move anti-clockwise on a conductance circle

Adding a series inductance causes the reflection coefficient to move clockwise on a resistance circle

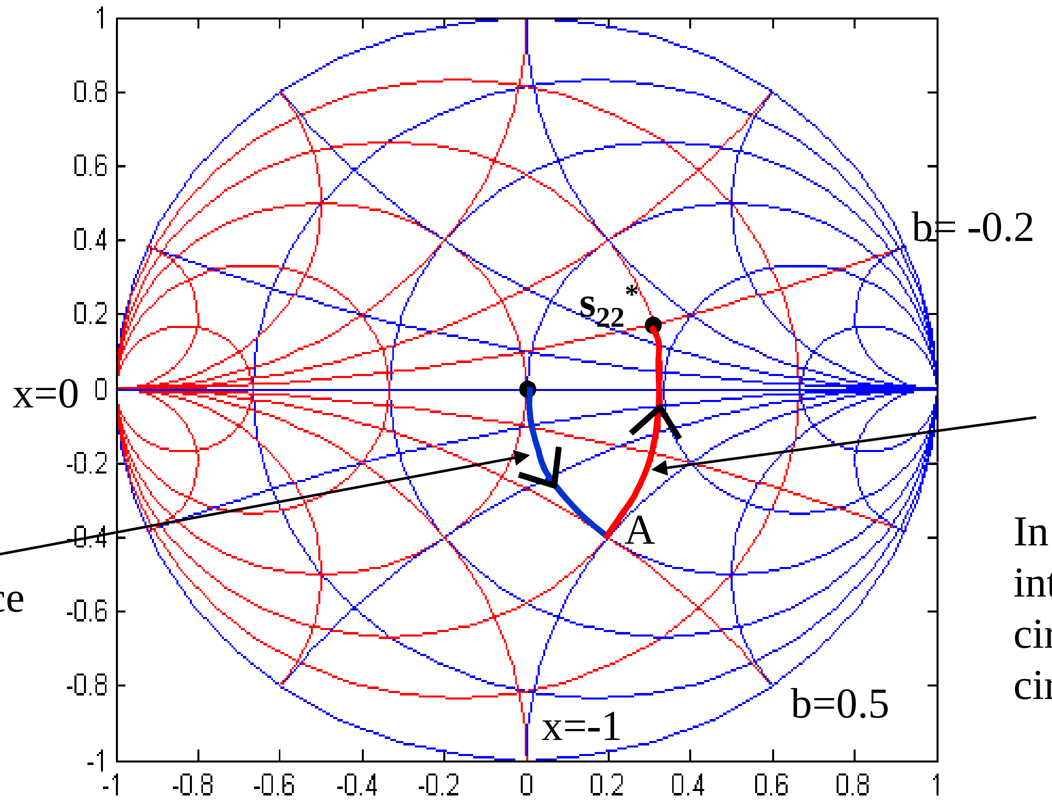


Adding a shunt (parallel) capacitance causes the reflection coefficient to move clockwise on a conductance circle

Adding a series capacitance causes the reflection coefficient to move anti-clockwise on a resistance circle

Obtaining an arbitrary Γ starting with Γ=0

e.g. transform a 50Ω load to give a reflection coefficient s_{22}^* as shown:

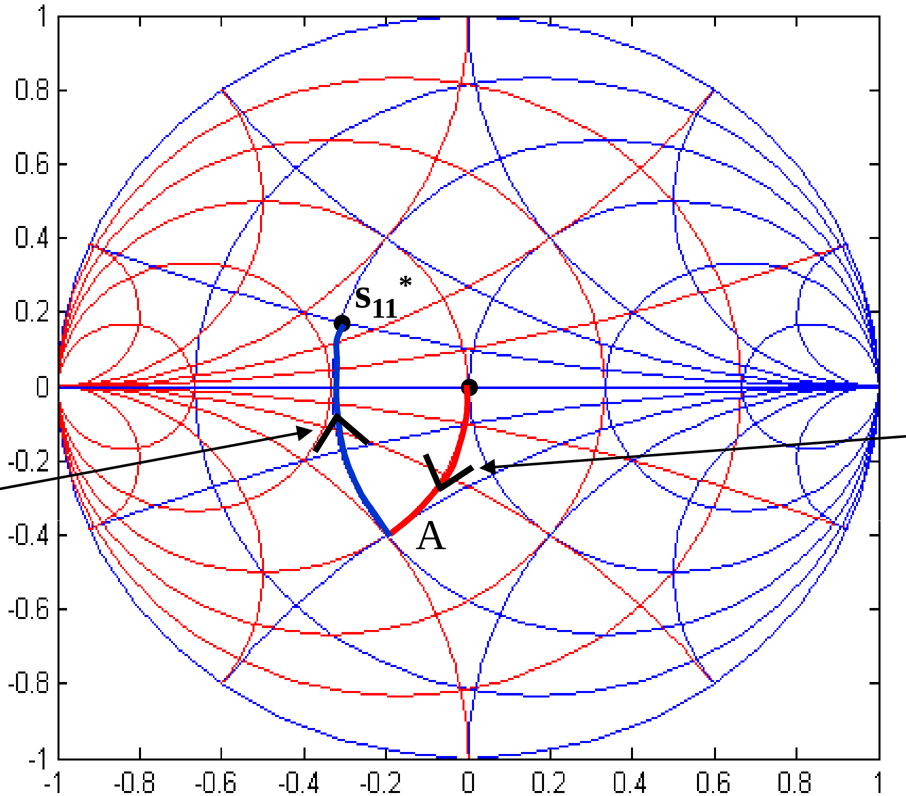


In this case point A is one intersection of the r = 1 circle with the conductance circle going through s_{22}^*

It is possible to transform Γ=0 to Γ=s_{22}^* by first adding a series capacitance and then adding a shunt inductance as shown. At the frequency of operation the capacitance can be calculated from the change of normalised reactance along the resistance circle (x changes from 0 to -1 when Γ moves from 0 to point A) and the inductance can be calculated from the change of normalised susceptance along the conductance circle (b changes from 0.5 to -0.2 when Γ goes from point A to s_{22}^*).

Another example starting with Γ=0

e.g. transform a 50Ω source to give a reflection coefficient s_{11}^* as shown:



In this case point A is one intersection of the g = 1 circle with the resistance circle going through s_{11}^*

It is possible to transform Γ=0 to Γ=s_{11}^* by first adding a shunt capacitance to transform the reflection coefficient to point A and then adding a series inductance to transform it to s_{11}^*. The component values can be determined by the changes in b and x.

To do these operations on the Immittance Smith Chart it is necessary to see the circles corresponding to r=1, g=1 and the resistance and conductance circles going through the desired reflection coefficient. The "circle segments" needed to travel along can then be picked out. If an Immitance Smith chart is not available a rotated version of the Impedance Smith chart can be used to determine the admittance curves.

Example Amplifier Design for High Gain - 1

A MESFET has the following s-parameters at 9GHz:

$$s_{11} = 0.55\angle -150° \quad s_{12} = 0.04\angle 20° \quad s_{21} = 2.82\angle 180° \quad s_{22} = 0.45\angle -30°$$

Design input and output matching networks for maximum stable gain at 9GHz, assuming both source and load impedances are 50Ω.

Check the stability criteria (we'll come back to stability later):

$$\Delta = s_{11}s_{22} - s_{12}s_{21} = 0.15\angle 165° \quad K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} = 2.29$$

$$K > 1 \quad and \quad |\Delta| < 1$$

The device is unconditionally stable so the source and load matching networks can be designed without restriction. Assuming that the device can be approximated as unilateral then:

$$\Gamma_S = s_{11}^* \quad \Gamma_L = s_{22}^*$$

Example Amplifier Design for High Gain - 2

$s_{11} = 0.55\angle -150° \quad s_{12} = 0.04\angle 20° \quad s_{21} = 2.82\angle 180° \quad s_{22} = 0.45\angle -30°$

The maximum unilateral transducer power gain is

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} = 14.3 = 11.55dB$$

The unilateral figure of merit is

$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{\left(1-|s_{11}|^2\right)\left(1-|s_{22}|^2\right)} = 0.05$$

The error in calculating the gain using the unilateral approximation is bounded by:

$$\frac{1}{(1+M)^2} < \frac{G_T'}{G_{TU,\max}} < \frac{1}{(1-M)^2}$$

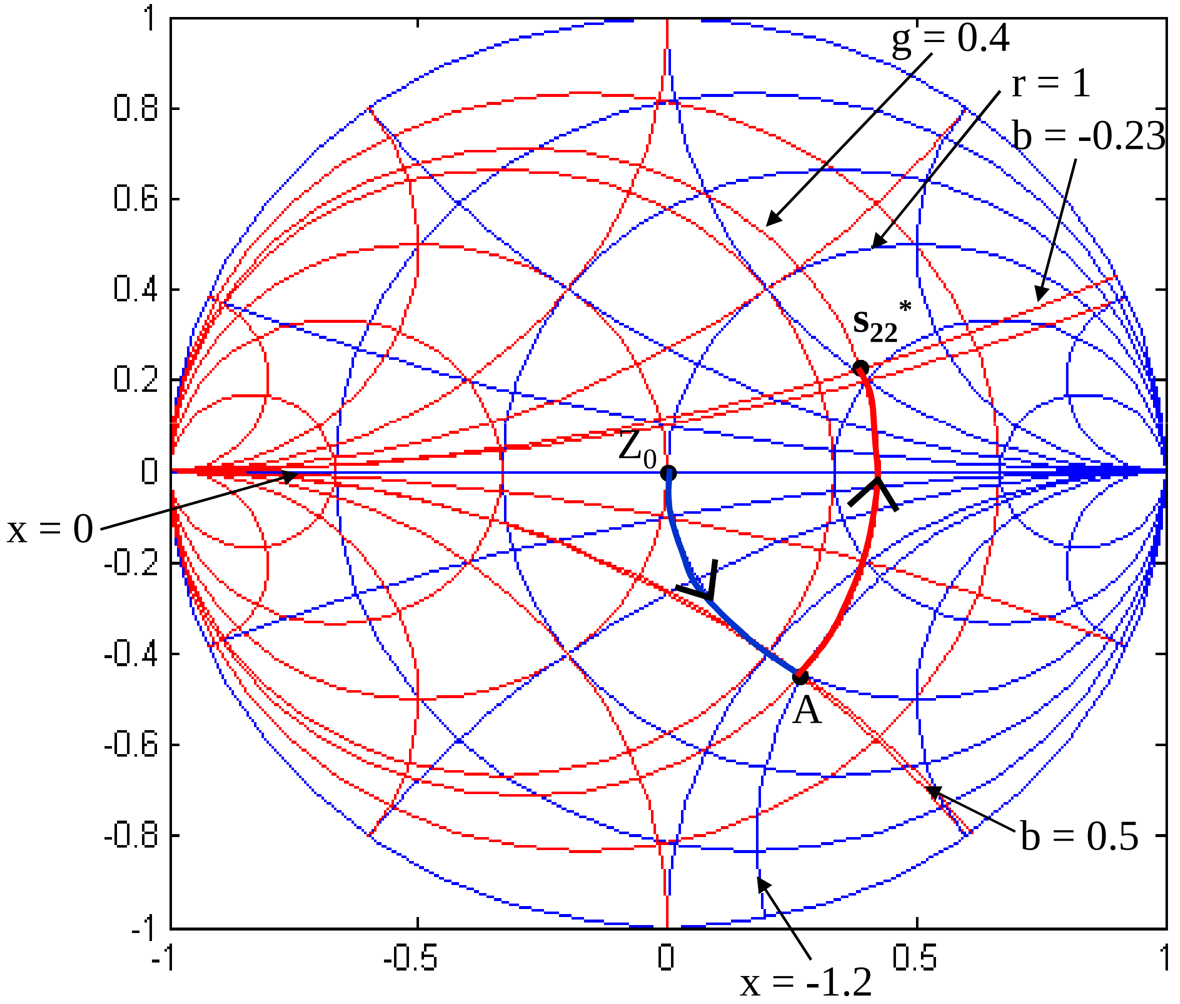
$$0.91 < \frac{G_T'}{G_{TU,\max}} < 1.11$$

$$-0.43dB < \frac{G_T'}{G_{TU,\max}} < 0.45dB$$

So, the error introduced by the unilateral assumption is less than 0.5dB which is acceptable (just!).

Example Amplifier Design for High Gain - 3

Output Matching Network – transforming 50Ω to s_{22}^*



We can get from Z_0 (Γ=0) to point A by moving anticlockwise along the r = 1 circle.

Point A is where the r = 1 circle intersects with the g = 0.4 circle.

s_{22}^* lies on the g = 0.4 circle.

In moving from Γ=0 to point A the reactance changes from x = 0 to x = -1.2. This requires the addition of a series capacitor.

We can get from point A to s_{22}^* by moving anticlockwise along the g = 0.4 circle.

In moving from point A to s_{22}^* the susceptance changes from b = 0.5 to b = -0.23. This requires the addition of a shunt inductor.

Example Amplifier Design for High Gain - 4

Translating the changes in reactance and susceptance to component values

Going from Γ=0 to point A a reactance of -1.2 has to be introduced:

$$Z = R + jX \quad z = r + jx \quad r = \frac{R}{Z_0} \quad x = \frac{X}{Z_0}$$

For a series capacitor:

$$X = -\frac{1}{2\pi f C} \Rightarrow x = -\frac{1}{2\pi f C Z_0} \Rightarrow C = -\frac{1}{2\pi f x Z_0}$$

to obtain x = -1.2 at 9GHz in a 50Ω system we need: $C = -\frac{1}{2\pi \times 9 \times 10^9 \times (-1.2) \times 50} = 0.29\,pF$

Going from point A to s_{22}^* a susceptance of -0.23 - 0.5 = -0.73 has to be introduced:

$$Y = G + jB \quad y = g + jb \quad g = \frac{G}{Y_0} = GZ_0 \quad b = \frac{B}{Y_0} = BZ_0$$

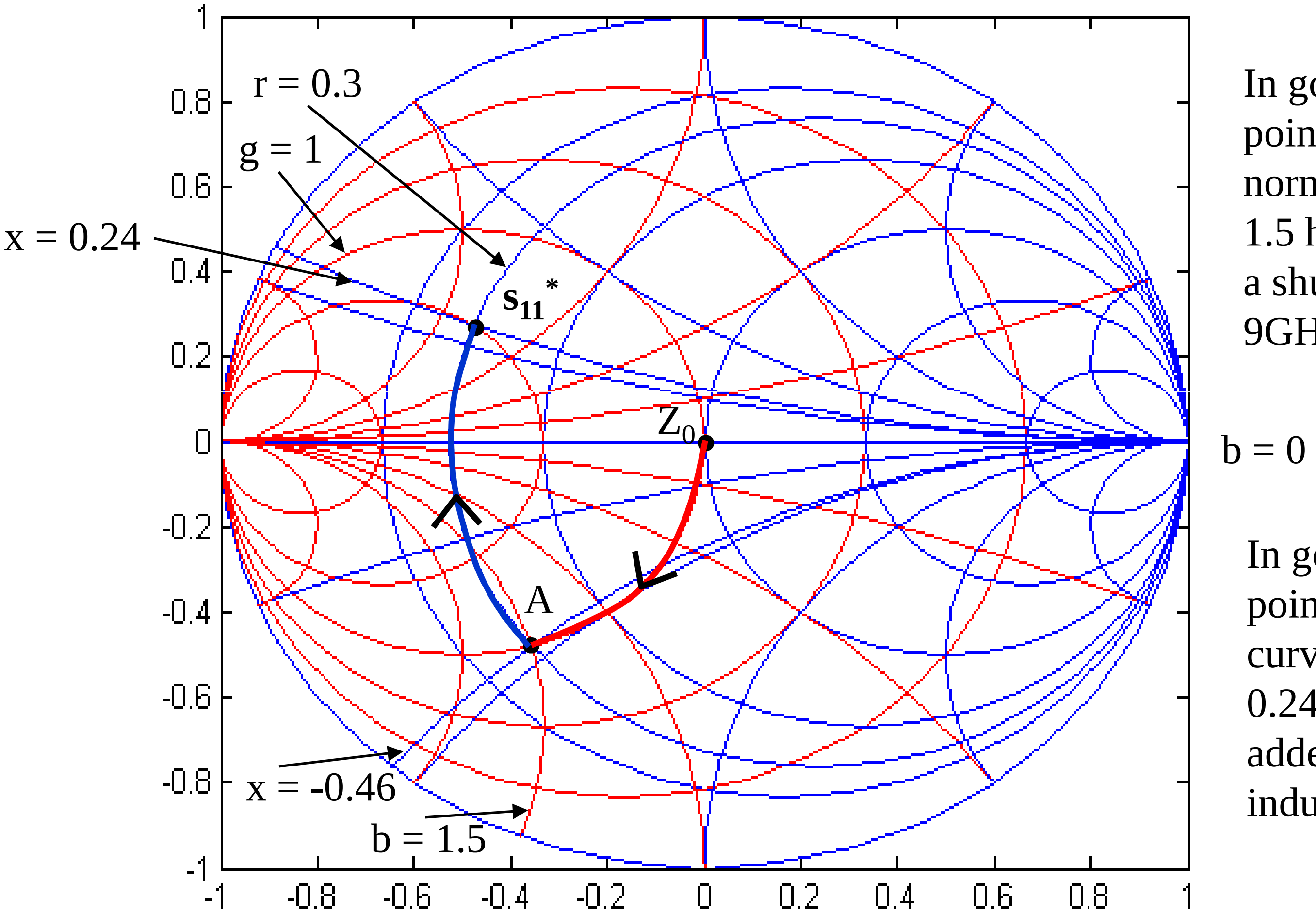
For a shunt inductor:

$$B = -\frac{1}{2\pi f L} \Rightarrow b = -\frac{Z_0}{2\pi f L} \Rightarrow L = -\frac{Z_0}{2\pi f b}$$

to obtain b = -0.73 at 9GHz in a 50Ω system we need: $L = -\frac{50}{2\pi \times 9 \times 10^9 \times (-0.73)} = 1.21nH$

Example Amplifier Design for High Gain - 5

Input Matching Network – transforming 50Ω to s_{11}^*



In going clockwise from Γ = 0 to point A along the g = 1 curve, a normaliszed susceptance of 1.5 has to be added. This needs a shunt capacitor of 0.53pF at 9GHz.

In going clockwise from point A to s_{11}* along the r = 0.3 curve, a normalised reactance of 0.24-(-0.46) = 0.7 has to be added. This needs a series inductor of 0.62nH at 9GHz.

Example Amplifier Design for High Gain - 6

Translating the changes in reactance and susceptance to component values when Z_0 is transformed to s_{11}^*.

In going from Γ=0 to point A, we have moved clockwise along a conductance circle which corresponds to adding a shunt capacitance. The susceptance changes from b=0 to b=1.5 so a susceptance of 1.5 has to be added:

$$Y = G + jB \quad y = g + jb \quad g = \frac{G}{Y_0} = GZ_0 \quad b = \frac{B}{Y_0} = BZ_0$$

For a shunt capacitor

$$B = 2\pi f C \Rightarrow b = 2\pi f CZ_0 \Rightarrow C = \frac{b}{2\pi f Z_0} = \frac{1.5}{2\pi \times 9 \times 10^9 \times 50} \approx 0.53 pF$$

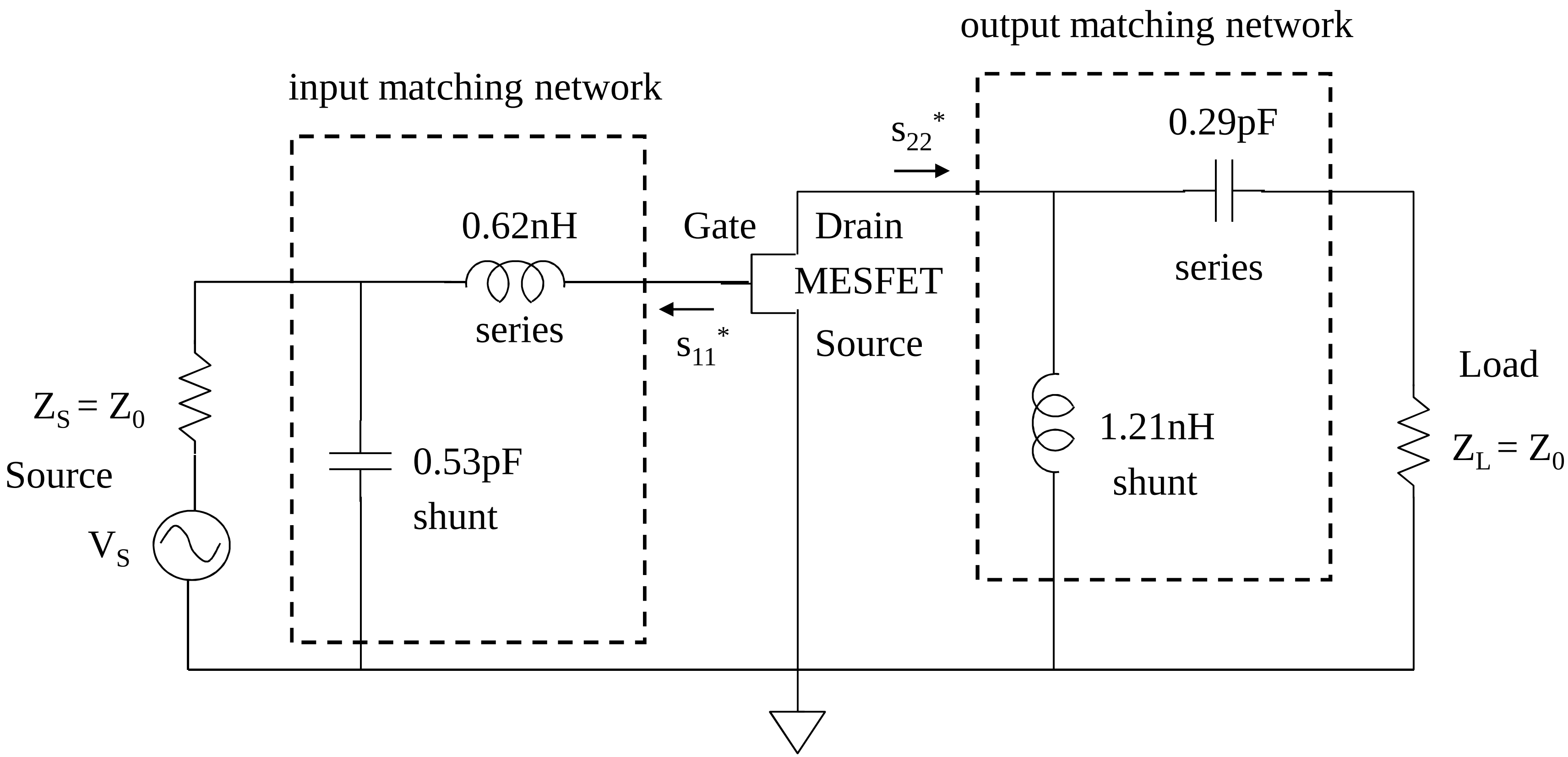
In going from point A to s_{11}^* , we have moved clockwise along a resistance circle which corresponds to adding a series inductance. The reactance changes from x=-0.46 to x=0.24 so a reactance of 0.7 has to be added:

$$Z = R + jX \quad z = r + jx \quad r = \frac{R}{Z_0} \quad x = \frac{X}{Z_0}$$

For a series inductance

$$X = 2\pi f L \Rightarrow x = \frac{2\pi f L}{Z_0} \Rightarrow L = \frac{Z_0 x}{2\pi f} = \frac{50 \times 0.7}{2\pi \times 9 \times 10^9} \approx 0.62 nH$$

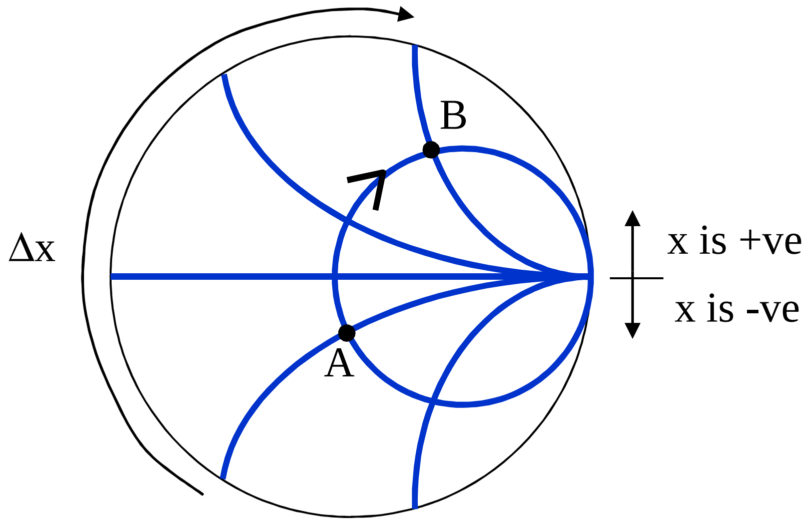
Final Matching Networks

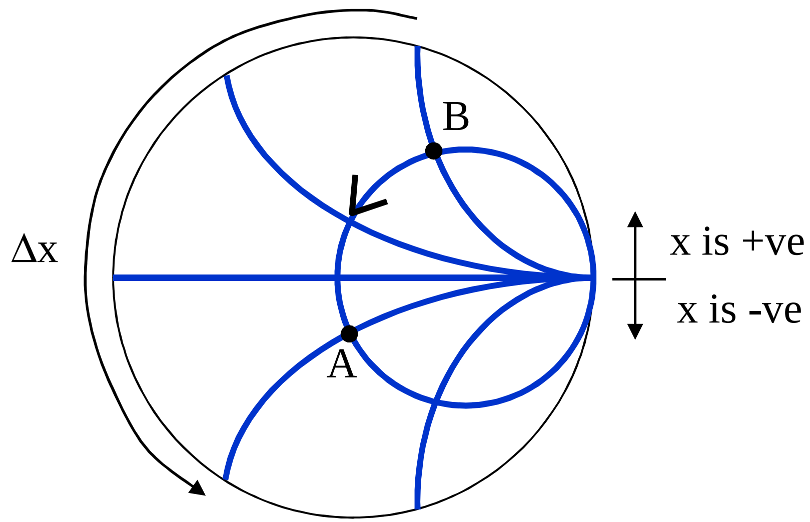


Note: In a practical circuit, DC bias networks are also needed for the active component (the MESFET in this case). The matching networks have to be designed in conjunction with the DC bias networks or else the bias networks may effect the matching or the matching networks may prevent the proper DC bias being applied. DC bias networks have not been shown or incorporated into the design here.

Note that over a broad frequency range the input matching circuit used here will display a "low-pass" characteristic between the source and the gate while the output matching circuit will display a "high-pass" characteristic between the drain and the load.

Note on Calculating Series Component Values





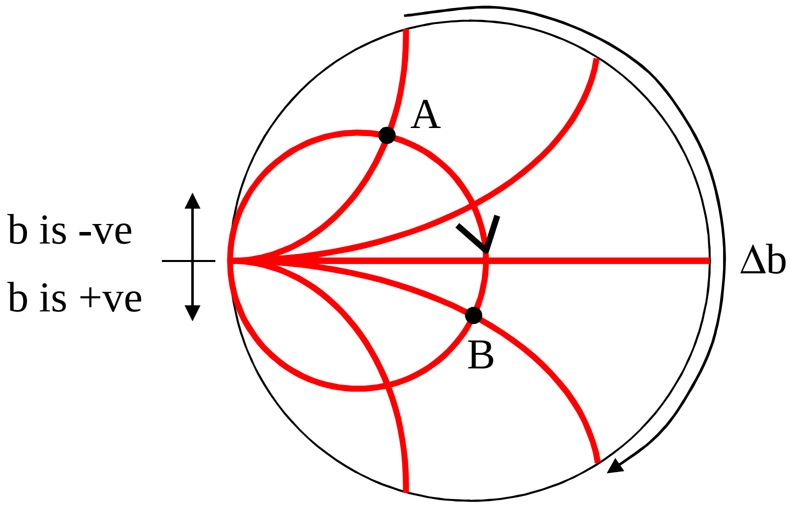
In going from A to B we move clockwise around a resistance circle. This corresponds to adding a series inductor. The value of the inductor can be found from the total change in normalized reactance, Δx, the characteristic impedance, Z_0, and the operating frequency, f.

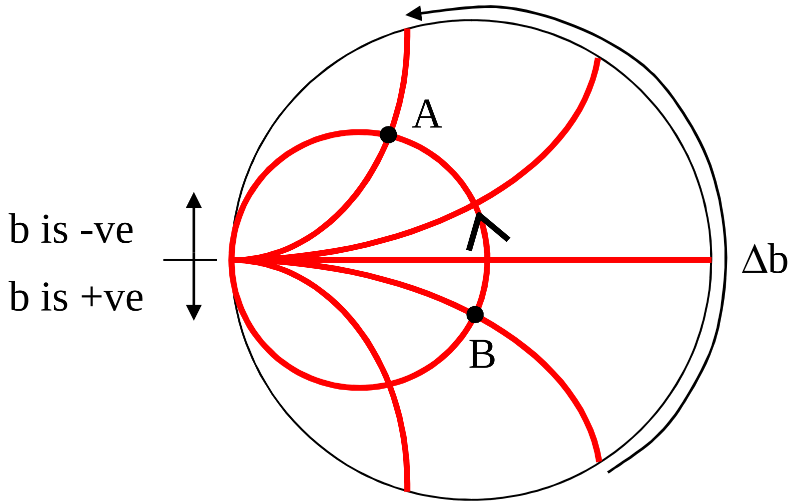
$$L = \frac{Z_0 |\Delta x|}{2\pi f}$$

In going from B to A we move anti-clockwise around a resistance circle. This corresponds to adding a series capacitor. The value of the capacitor can be found from the total change in normalized reactance, Δx, the characteristic impedance, Z_0, and the operating frequency, f.

$$C = \frac{1}{2\pi f |\Delta x| Z_0}$$

Note on Calculating Shunt Component Values





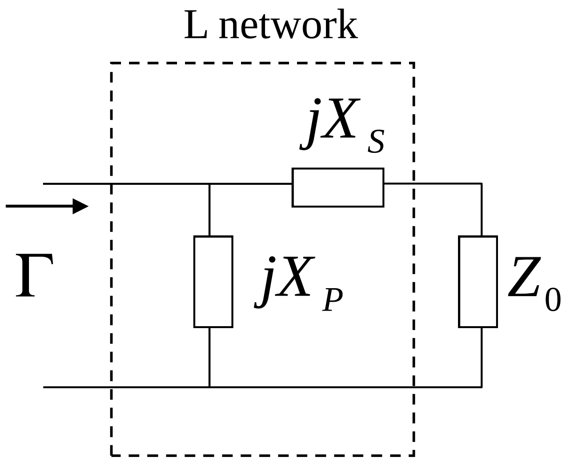
In going from A to B we move clockwise around a conductance circle. This corresponds to adding a shunt capacitor. The value of the capacitor can be found from the total change in normalized susceptance, Δb, the characteristic impedance, Z_0, and the operating frequency, f.

$$C = \frac{|\Delta b|}{2\pi f Z_0}$$

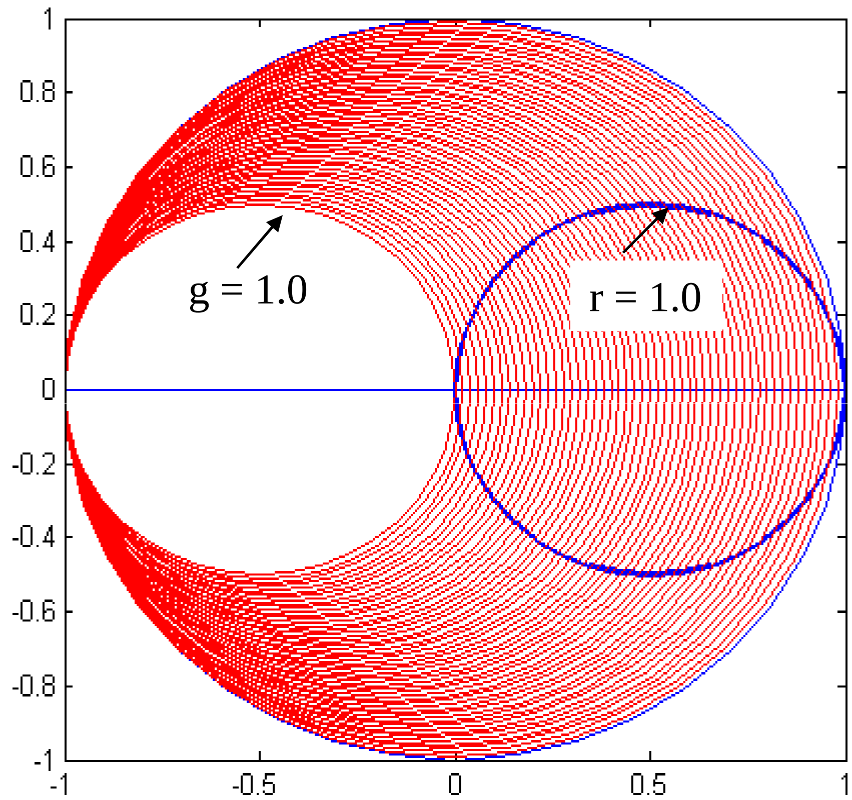
In going from B to A we move anti-clockwise around a conductance circle. This corresponds to adding a shunt inductor. The value of the inductor can be found from the total change in normalized susceptance, Δb, the characteristic impedance, Z_0, and the operating frequency, f.

$$L = \frac{Z_0}{2\pi f |\Delta b|}$$

Limitations to two-element matching when the first element is a series element

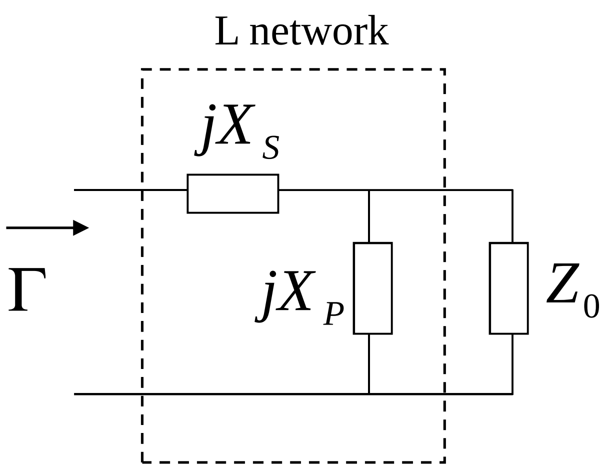


Starting at Γ = 0, if the first element is a series element then this element will move the reflection coefficient along the r = 1 circle. The second shunt element can then only transform the reflection coefficient along any conductance circle which intersects with the r = 1 circle. Therefore, the reflection coefficient cannot be transformed to any value within the g = 1 circle.

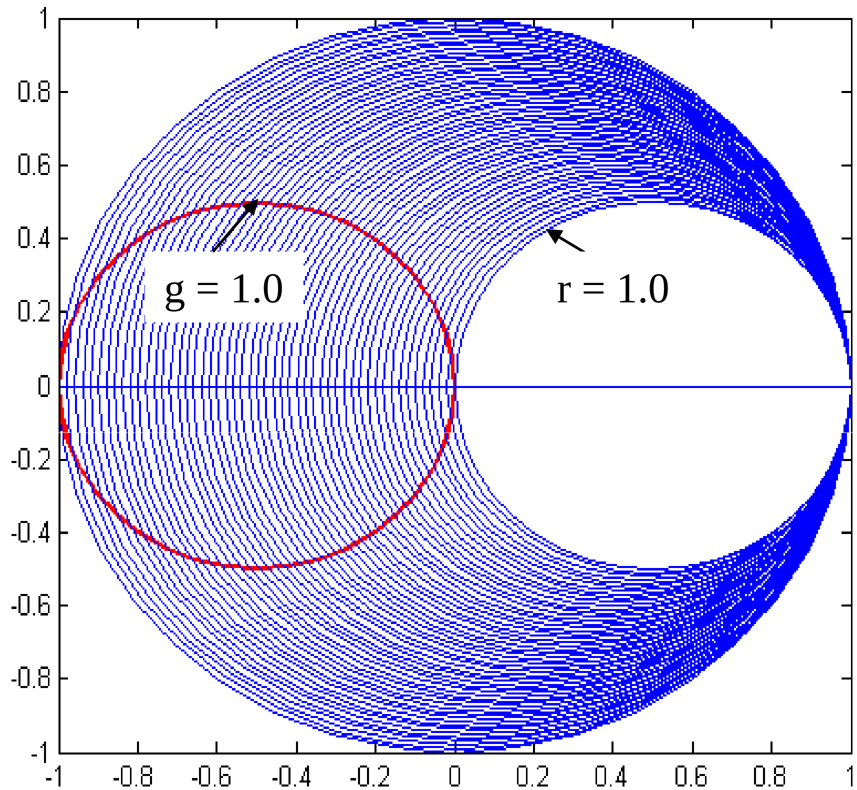


Smith chart showing the range of conductance circles that intersect with the r = 1 circle.

Limitations to two-element matching when the first element is a shunt element

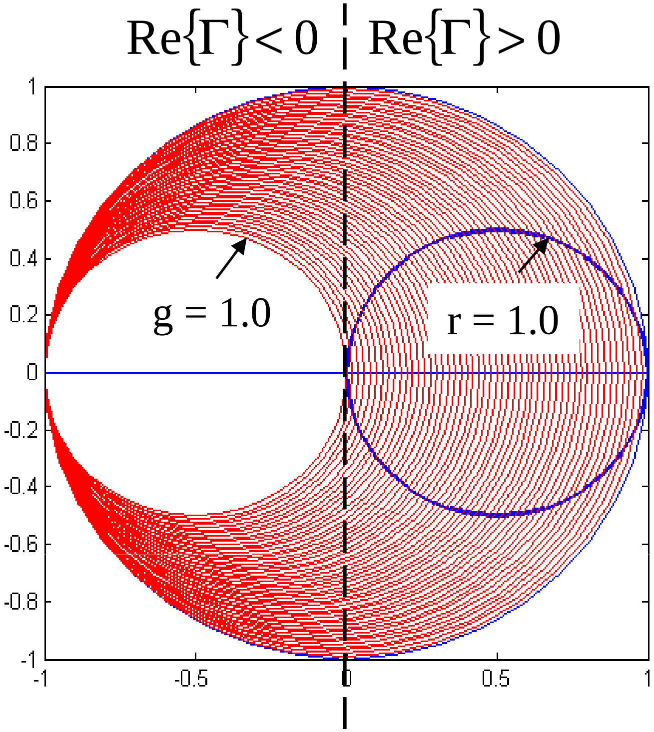


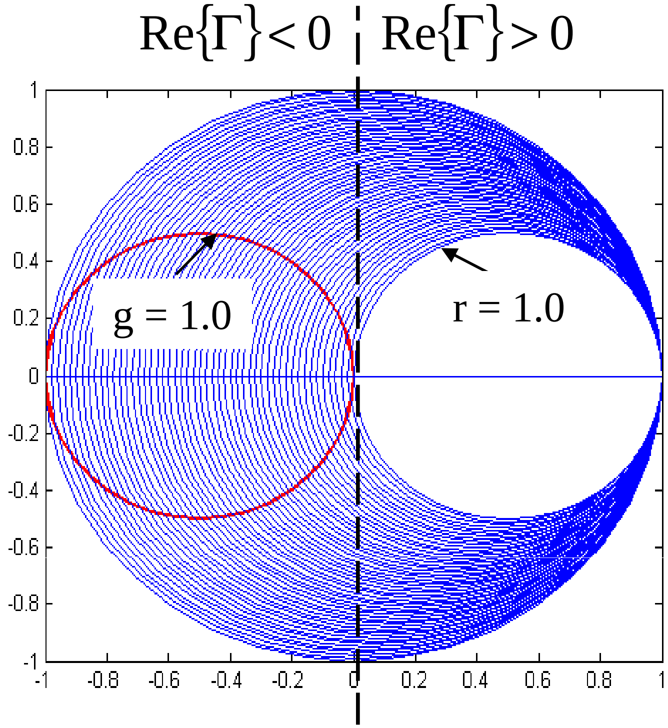
Starting at Γ = 0, if the first element is a shunt element then this element will move the reflection coefficient along the g = 1 circle. The second series element can then only transform the reflection coefficient along any resistance circle which intersects with the g = 1 circle. Therefore, the reflection coefficient cannot be transformed to any value within the r = 1 circle.



Smith chart showing the range of resistance circles that intersect with the g = 1 circle.

Two element matching





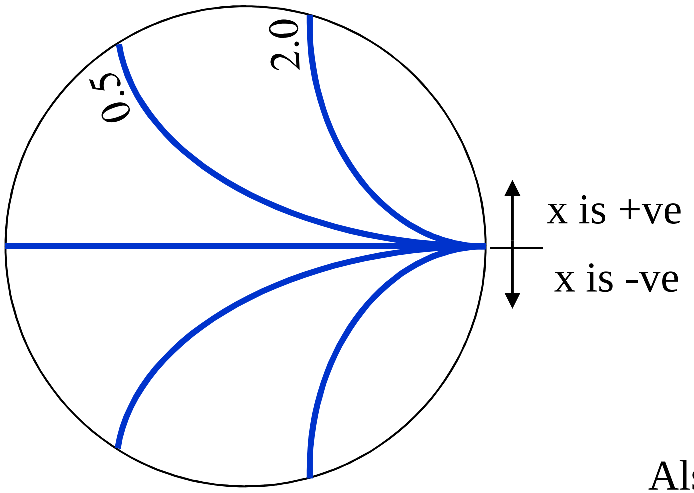
Provided either one of the two-element topologies can be chosen, it is always possible to choose a topology that will transform $\Gamma = 0$ to any point on the Smith chart. Given a final value of Γ that is desired, either one of the topologies can be used in many cases.

If $\mathrm{Re}\{\Gamma\} >= 0$, using a series element first will always allow the final Γ to be reached.

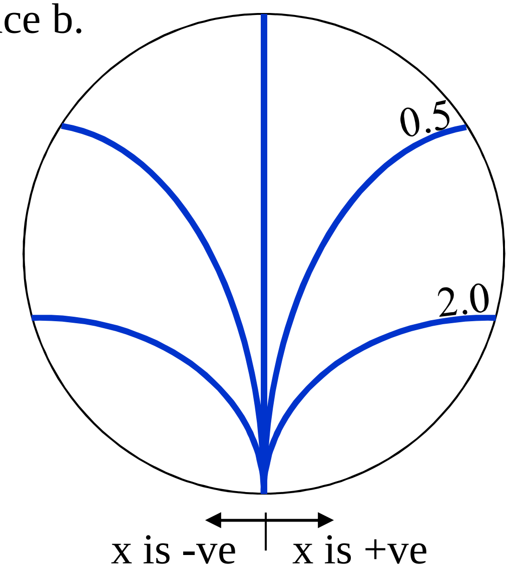
If $\mathrm{Re}\{\Gamma\} <= 0$, using a shunt (parallel) element first will always allow the final Γ to be reached.
(These are just observations, not rules)

Reading the x and b values from the Smith Chart

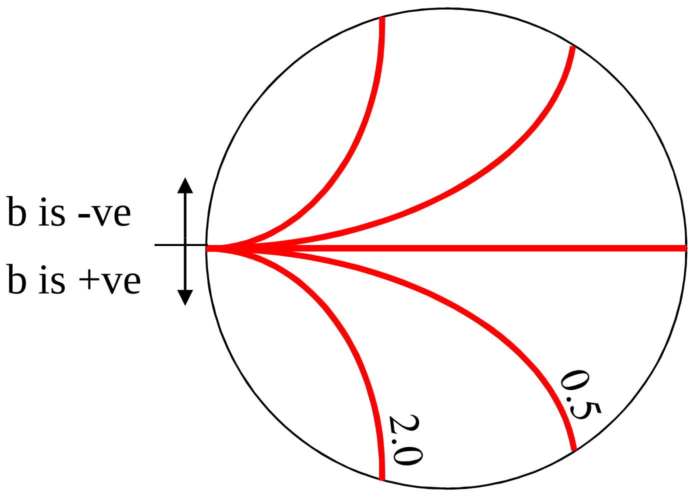
With the Immittance Smith Chart there are two sets of numbers on the chart so it can be confusing as to which number is reactance x and which is susceptance b.



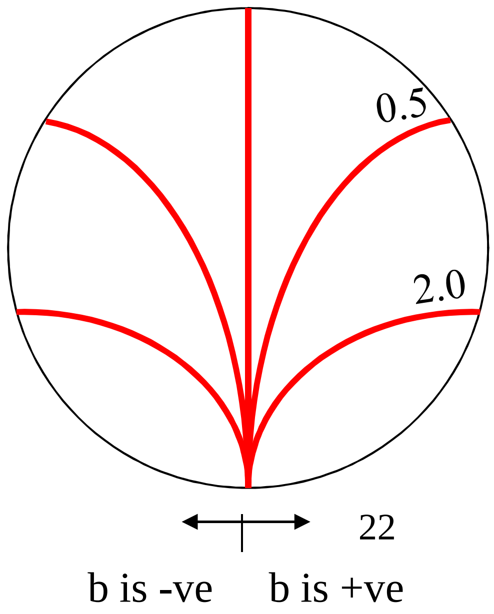
The reactance arcs radiate out from Γ=1 so rotate the Smith Chart 90° clockwise to make the values of x readable the right way around.



Also the signs aren’t marked so you need to know where x and b are positive and negative



The susceptance arcs radiate out from Γ=-1 so rotate the Smith Chart 90° anti-clockwise to make the values of b readable the right way around.



An Example to Try

A bipolar transistor has the following s-parameters at 4GHz in a 50Ω system:

$$s_{11} = 0.552\angle 169^\circ \quad s_{12} = 0.049\angle 23^\circ \quad s_{21} = 1.68\angle 26^\circ \quad s_{22} = 0.839\angle -67^\circ$$

At 4GHz determine the maximum unilateral transducer gain, the unilateral figure of merit and the associated error in predicting the maximum gain with the unilateral assumption.

Design input and output matching networks to obtain maximum gain at 4GHz (you may assume the device is unilateral for this).