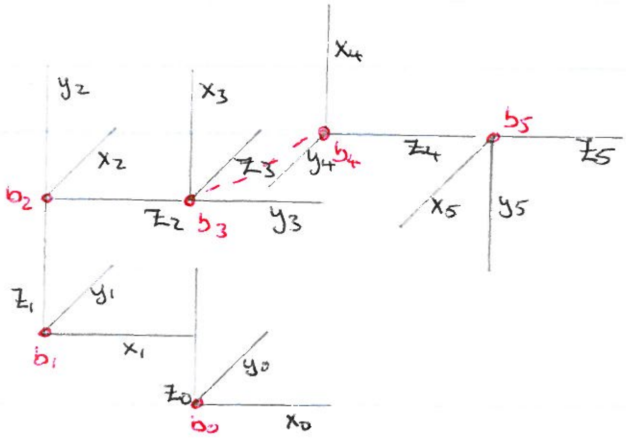
Summer '08

4.



	θ	d	a	α
1	θ_1	0.25	+0.2	0°
2	90°	l_1	0	90°
3	90°	l_2	0	90°
4	θ_4	0	0	-90°
5	θ_5	0.5 m	0	0°

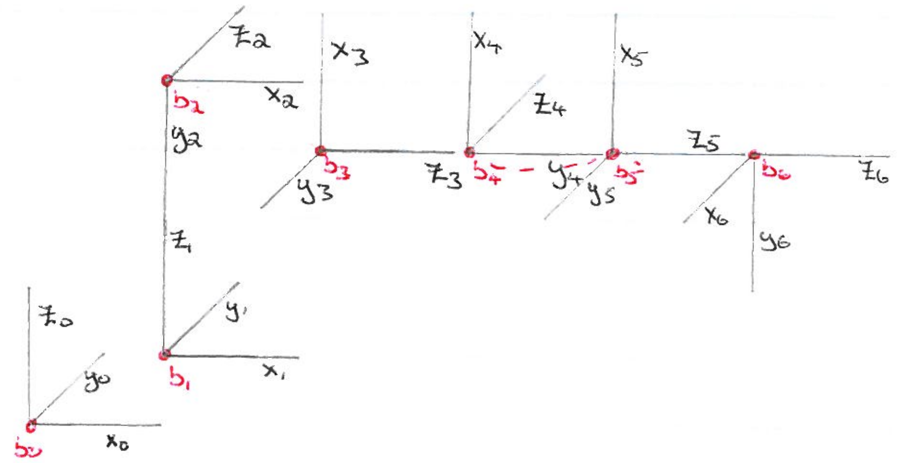
$$T_0^3 = T_0^1 T_1^2 T_2^3$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0.2\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0.2\sin\theta_1 \\ 0 & 0 & 1 & 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 \text{ with } \theta_1 = 0° \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Summer '07

4.



	Θ	d	a	α
1	θ_1	1.3m	0.8m	0°
2	0°	L_1	0	-90°
3	θ_3	-0.5m	0	-90°
4	0°	$1.2+L_2$	0	90°
5	θ_5	0	0	-90°
6	θ_6			

$$T_0^4 = T_0^1 T_1^2 T_2^3 T_3^4$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0.8\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0.8\sin\theta_1 \\ 0 & 0 & 1 & 1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_2+1.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_2^3 @ $\theta_3 = -90^\circ$ as shown

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x_3 in $-y_2$ direction

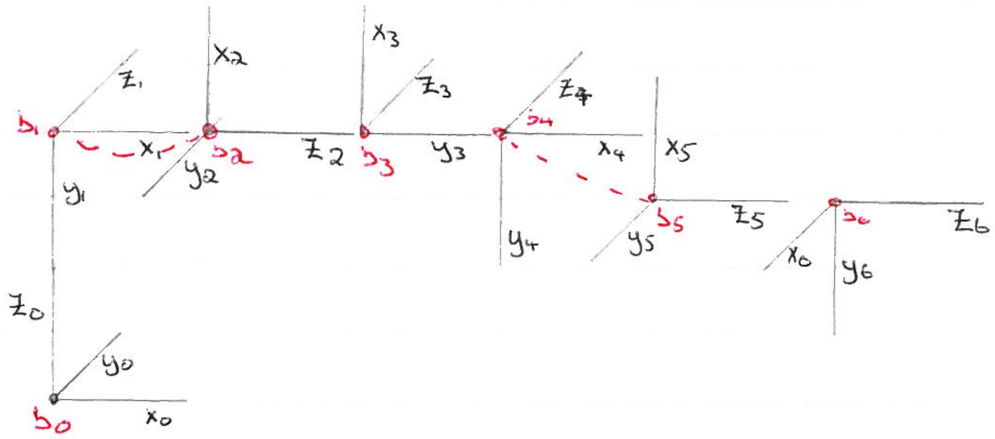
y_3 in $-z_2$ direction

z_3 in x_2 direction

Origin displaced by 0.5m in negative z_2 direction

Summer '06

3.



	θ_i	d	a	α
1	θ_1	2.4m	0	-90°
2	θ_2	0	0	-90°
3	θ_3	1.45m	0	90°
4	θ_4	0	1.2m	0°
5	θ_5	0	0	-90°
6	θ_6	0.7m	0	0°

$$T_0^4 = T_0^1 T_1^2 T_2^3 T_3^4$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 2.4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & +\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & -\cos\theta_3 & 0 \\ 0 & 1 & 0 & 1.45 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 1.2\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 1.2\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

@ $\theta_3 = 0^\circ$

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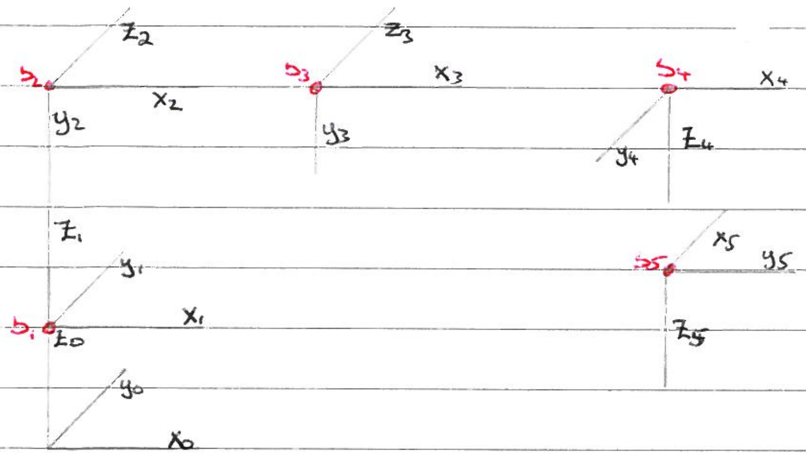
3. a)



b)

	Θ	d	a	α
1	θ_1	0.3m	0	0°
2	0°	L	0	-90°
3	θ_3	0	0.8m	0°
4	θ_4	0	0.65m	-90°
5	θ_5	0.6m	0	0°

c) $T_{BASE}^{TOOL} = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0.8C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & 0.8S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0.65C\theta_4 \\ S\theta_4 & 0 & C\theta_4 & 0.65S\theta_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & 0 \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) T_3^4 @ $\theta_4 = 0°$

From diagram: x_4 dir $= x_3$ dir
y_4 dir $= -z_3$ dir
z_4 dir $= y_3$ dir

b_4 displaced 0.65 m along x^4 axis

$$T_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0.65 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ Verified

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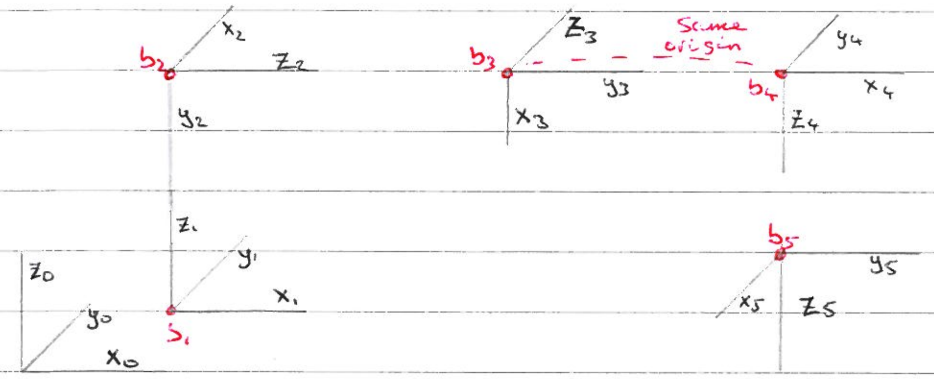
6. a)



b)

	Θ	d	a	α
1	θ_1	0.16m	0.1m	0°
2	90°	L_1	0	90°
3	-90°	L_2	0	-90°
4	θ_4	0	0	-90°
5	θ_5	0.34m	0	0°

c) $T_3^{w} = T_0^1 T_1^2 T_2^3$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0.1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & 0.1S\theta_1 \\ 0 & 0 & 1 & 0.16 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) T_0^1 @ $\theta_1 = 0°$

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.16 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From diagram:

x_1 dir = x_0 dir

dir = y_0 dir

z_1 dir = z_0 dir

b_1 is displaced by 0.16m along z_0 axis & 0.1m along x_1 axis.

$\therefore$ Verified

Forward Kinematics

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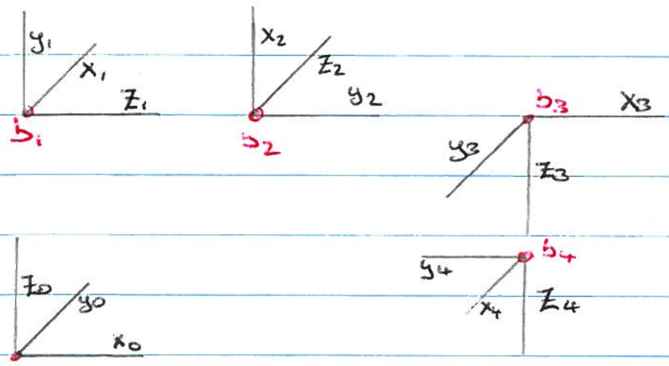
5\. a)



b)

	Θ	d	a	α
1	θ_1	$L_1 + 0.3m$	0	90°
2	90°	$L_2 + 0.25m$	0	90°
3	θ_3	0	$0.7m$	-90°
4	θ_4	$0.75m$	0	0°

c) $T_{base}^{gripper} = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4$

$$= \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & L_1 + 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_2 + 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & 0 & -S\theta_3 & 0.7 \times C\theta_1 \\ S\theta_3 & 0 & C\theta_3 & 0.7 \times S\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $T_2^3 \Rightarrow$ In diagram, $\theta_3 = 90^\circ$

$$T_2^3(90^\circ) = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -0.7 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

x_3 in dir. of y_2
y_3 in -ve dir. of z_2
z_3 in -ve dir of x_2

Origin of L3 is 0.7m along y_2 from L2 origin
∴ Verified

Appendix I
Denavit-Hartenberg Algorithm and Matrix

1. Number the joints from 1 to n starting with the base and ending with the tool yaw, pitch and roll, in that order.
2. Assign a right-handed orthonormal coordinate frame L_0 to the robot base, making sure that z^0 aligns with the axis of joint 1. Set $k = 1$.
3. Align z^k with the axis of joint $k + 1$.
4. Locate the origin of L_k at the intersection of the z^k and z^{k-1} axes. If they do not intersect, use the intersection of z^k with a common normal between z^k and z^{k-1}.
5. Select x^k to be orthogonal to both z^k and z^{k-1}. If z^k and z^{k-1} are parallel, point x^k away from z^{k-1}.
6. Select y^k to form a right-handed orthonormal coordinate frame L_k.
7. Set $k = k + 1$. If $k < n$, go to step 3; else, continue.
8. Set the origin of L_n at the tool tip. Align z^n with the approach vector, y^n with the sliding vector, and x^n with the normal vector of the tool. Set $k = 1$.
9. Locate point b^k at the intersection of the x^k and z^{k-1} axes. If they do not intersect, use the intersection of x^k with a common normal between x^k and z^{k-1}.
10. Compute Θ_k as the angle of rotation from x^{k-1} to x^k measured about z^{k-1}.
11. Compute d_k as the distance from the origin of frame L_{k-1} to point b_k measured along z^{k-1}.
12. Compute a_k as the distance from point b^k to the origin of frame L_k measured along x^k.
13. Compute α_k as the angle of rotation from z^{k-1} to z^k measured about x^k.
14. Set $k = k + 1$. If $k \leq n$, go to step 9; else, stop.

DENAVIT-HARTENBERG MATRIX:

$$T_{i-1}^{i} = \begin{bmatrix} C\Theta_i & -S\Theta_i C\alpha_i & S\Theta_i S\alpha_i & a_i C\Theta_i \\ S\Theta_i & C\Theta_i C\alpha_i & -C\Theta_i S\alpha_i & a_i S\Theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$