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THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2008

B.E. DEGREE (ELECTRICAL)

CONTROL ENGINEERING
EE4002

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Professor P. Murphy
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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of departmental approved non-programmable calculators is permitted

1.

(a) Consider the following unit impulse response from a discrete time process:

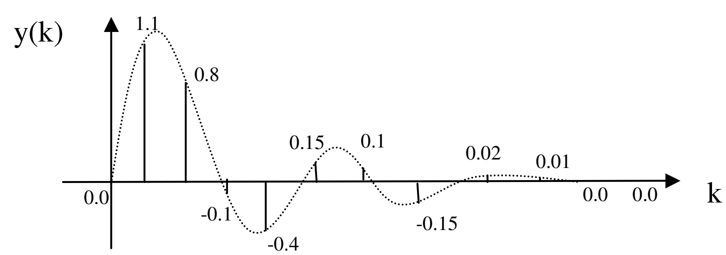

Fig. 1.1 Unit impulse response

Is this process stable?

Sketch the unit step response for this process.

[5 Marks]

(b) Derive Tustin's transform.

Use Tustins method, with sample time T, to develop a discrete-time difference equation representation for the PID compensator,

$$m(t) = K_P\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\frac{de(t)}{dt}\right)$$

What problems do you expect in implementing this discrete PID controller?

[8 Marks]

(c) Consider the following closed-loop digital control scheme:

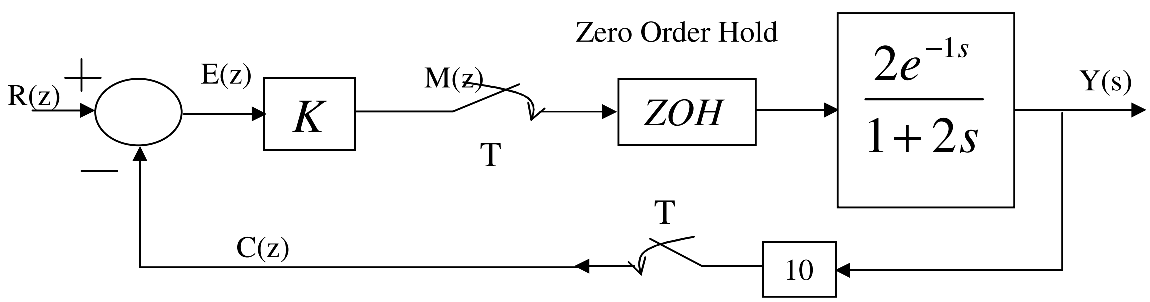

Fig. 1.2 Closed loop digital control system

The sample time T=1 second.

Sketch the root locus diagram for this process, on the Z plane template, and use it to explain how the closed-loop dynamics depend on the choice of the controller gain K.

What is the range of K for stability?

[12 Marks]

2.

(a) Derive the following deadbeat controller, from a basic prescription of the shape of the desired closed-loop step response,

$$D(z) = \frac{1}{G_m(z)} \frac{1}{z^N - 1}$$

Here $G_m(z)=C(z)/U(z)$ is the discrete-time transfer function model of the process, with N representing a tuning parameter, used to determine the desired closed-loop response.

Consider the following discrete-time closed-loop system.

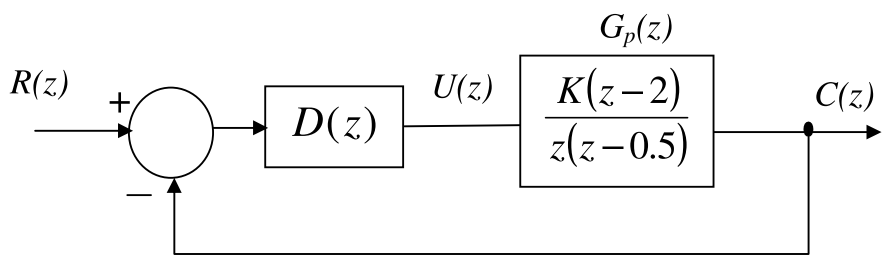

Fig. 2.1 Closed-loop, Discrete-time Process

Show by use of a root-locus plot, why a deadbeat controller will provide unsatisfactory closed-loop performance.

[10 Marks]

(b) Consider in Figure 2.2, the block diagram of a control scheme designed to control a chemical reactor. Here $Q_c(t)$ is the flow-rate of fluid in the heat exchanger jacket – it is manipulated by varying the pump voltage $v(t)$. $Q_{in}(t)$ is the flow-rate of the ingredients.

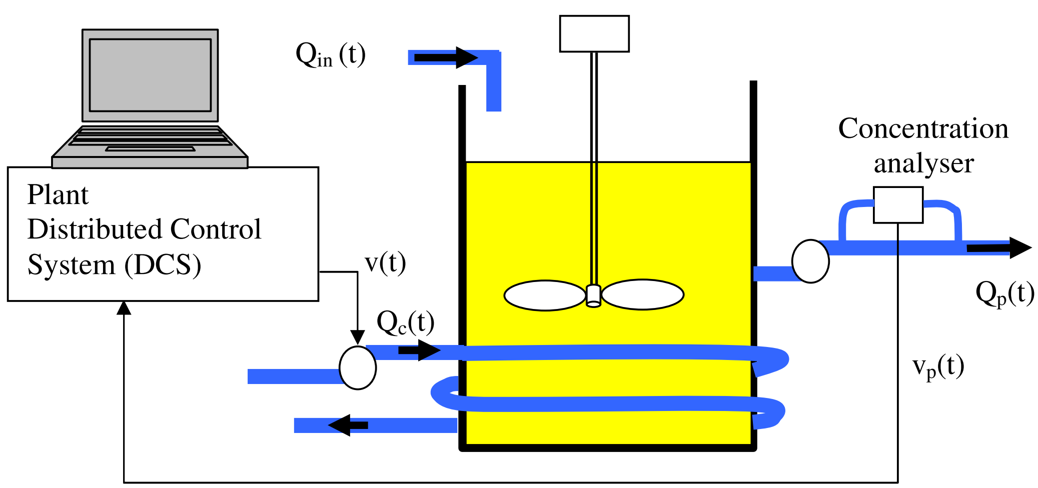

Fig. 2.2 Closed loop control of chemical concentration

The following open-loop transfer function model has been identified between the pump voltage *v(t)* and the analyser voltage $v_p(t)$:

$$\frac{V_p(s)}{V(s)} = \frac{2e^{-5s}}{1+30s}$$

The analyser has been calibrated to output a voltage of 2.0V/(mol l^{-1}).

The sampling time for the DCS is 2.0 seconds, and a zero-order hold is assumed. Design a Dahlin's controller, to achieve a closed-loop time constant of 15 seconds.

Sketch the closed-loop time response for the pump voltage *v(t),* for a unit step in the concentration set-point.

[15 marks]

3.

(a) Derive in full, the following least-squares algorithm, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains the sampled process output, up to the current kth sample, *y(k).*

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1} \Phi(k)^T \underline{Y}(k)$$

If a square matrix *P(k)* is now defined as $P(k) = \left(\Phi(k)^T \Phi(k)\right)^{-1}$, derive the following update equation to obtain *P(k+1)* from process data up to the (k+1)th sample,

$$P(k+1) = \left(P(k)^{-1} + \underline{\psi}(k+1)\underline{\psi}(k+1)^T \right)^{-1},$$

where vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the (k+1)th sample.

Show by application of Householder's Matrix Inversion Lemma,

$$\left(A + BCD\right)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1},$$

that the following update equation for the model parameter vector can be obtained:

$$\underline{\hat{\theta}}(k+1) = \left[P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^T(k+1)P(k)}{1 + \underline{\psi}^T(k+1)P(k)\underline{\psi}(k+1)} \right] \left[\Phi(k)^T \underline{Y}(k) + \underline{\psi}(k+1)\,y(k+1)\right].$$

[13 marks]

(b) Consider the closed-loop scheme for the control of antenna angle *θ(t),*

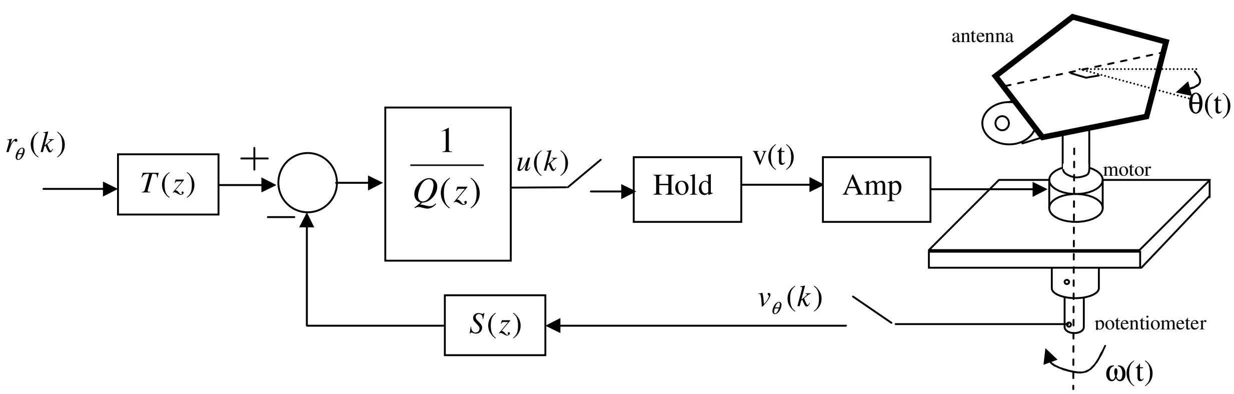

Figure 3.1 Computer control of an antenna positioning system

The following open-loop transfer function has been identified using the least squares technique:

$$\frac{V_\theta(z)}{U(z)} = \frac{0.019z^{-1} + 0.018z^{-2}}{1 - 1.82z^{-1} + 0.82z^{-2}}$$

Use the Diophantine pole-placement technique to design the controller polynomials S,Q and T to position the dominant second order poles at $z = 0.7 \pm 0.3j$. It is also desired that the resultant closed-loop system will achieve perfect steady-state tracking of step-like set-point signals.

[12 Marks]

4.

(a) Consider the following second-order SISO process,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine the transfer function of this process, $G(s)=Y(s)/U(s)$.

Is this system representation controllable and observable?

Determine the transformation $\underline{z}=T\underline{x}$, which would transform this system into the control-canonical form.

[10 Marks]

(b) Consider the following ball-on-beam apparatus consisting of a rigid beam, free to rotate in one plane about its central pivot. A servo-motor is used to rotate the beam. There are two parallel guide rails, on which a steel ball sits.

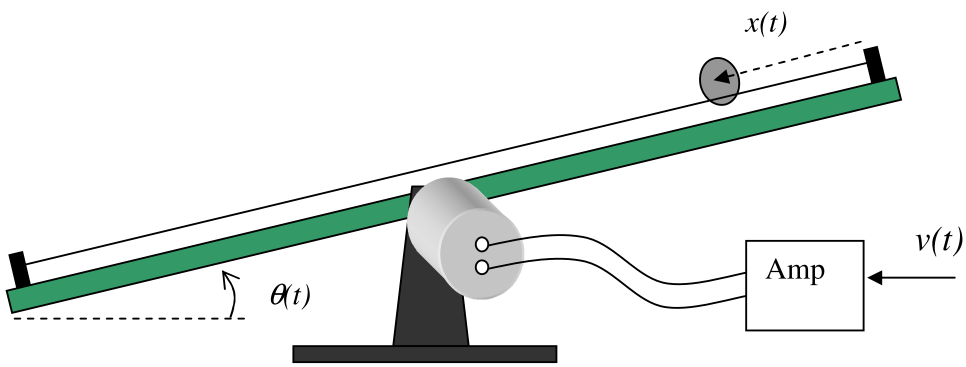

Fig.4.1: Ball-on-Beam Apparatus

Two sensors are available. The first is a simple rotary potentiometer that is used to provide a measure of the beam angle $\theta(t)$. The second sensor provides a measurement of the ball position $x(t)$, using the wire guide rails as a linear potentiometer.

The gains of the linear and rotary potentiometers are K_x and K_θ respectively

The process can be modelled by the following state-space equations.

$$\frac{d}{dt}\begin{bmatrix} \theta(t) \\ \dot{x}(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta(t) \\ \dot{x}(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} v(t)$$

With the output measurement equations:

$$\begin{bmatrix} v_\theta(t) \\ v_x(t) \end{bmatrix} = \begin{bmatrix} K_\theta & 0 & 0 \\ 0 & 0 & K_x \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{x}(t) \\ x(t) \end{bmatrix}$$

(i) Give a Simulink representation of this process based on three integrators.

Use this simulation diagram to determine $G(s) = \dfrac{V_x(s)}{V(s)}$

If the initial states of the process are,

$$\theta(0) = \theta_0 \quad x(0) = x_0 \quad and \quad \dot{x}(0) = 0,$$

show that the zero-input response for the voltage $v_x(t)$ is,

$$v_x(t) = K_x\left(3.5\theta_0 t^2 + x_0\right)$$

[8 marks]

(ii) Show that the discrete-time state-space representation of the process could be approximated by,

$$\begin{bmatrix} \theta(k+1) \\ \dot{x}(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 7T & 1 & 0 \\ 0 & T & 1 \end{bmatrix} \begin{bmatrix} \theta(k+1) \\ \dot{x}(k+1) \\ x(k+1) \end{bmatrix} + \begin{bmatrix} KT \\ 0 \\ 0 \end{bmatrix} v(k)$$

$$\begin{bmatrix} v_\theta(k) \\ v_x(k) \end{bmatrix} = \begin{bmatrix} K_\theta & 0 & 0 \\ 0 & 0 & K_x \end{bmatrix} \begin{bmatrix} \theta(k) \\ \dot{x}(k) \\ x(k) \end{bmatrix}$$

where a zero-order hold is assumed and the sample time T is small.

Confirm that this discrete approximation has three equal poles at z=1.

[7 Marks]

5.

(a) Consider the following N[th] order open-loop process, with one input *u(t)* and a single output *y(t)*,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

If this process is under the following state space control–law with integral action,

$$u(t) = -K\underline{x}(t) + K_I \int_0^t \left(r(\tau) - y(\tau)\right) d\tau$$

show that the closed-loop characteristic equation is:

$$\det\left[\begin{array}{c|c} sI_N - A + BK & -BK_I \\ \hline C & s \end{array}\right] = 0$$

[8 Marks]

(b) Consider the magnetic levitation system:

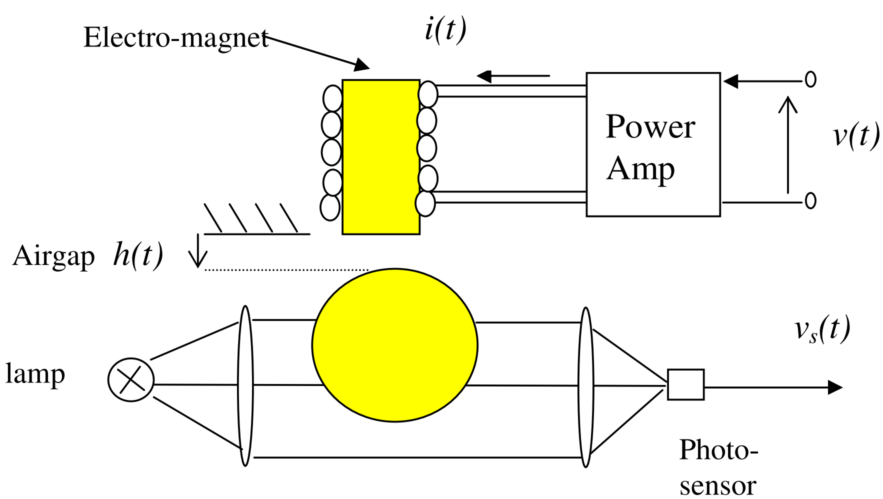

Fig. 5.1 Magnetic Levitation System

This could be modelled by the following differential equation:

$$m\frac{d^2h}{dt^2} = mg - \frac{Kv^2(t)}{h(t)}$$

Here $v(t)$ is the voltage applied to the power amplifier and $h(t)$ the airgap.

The following sensor calibration curve has been obtained that relates the sensor output voltage $v_s(t)$ to the airgap $h(t)$

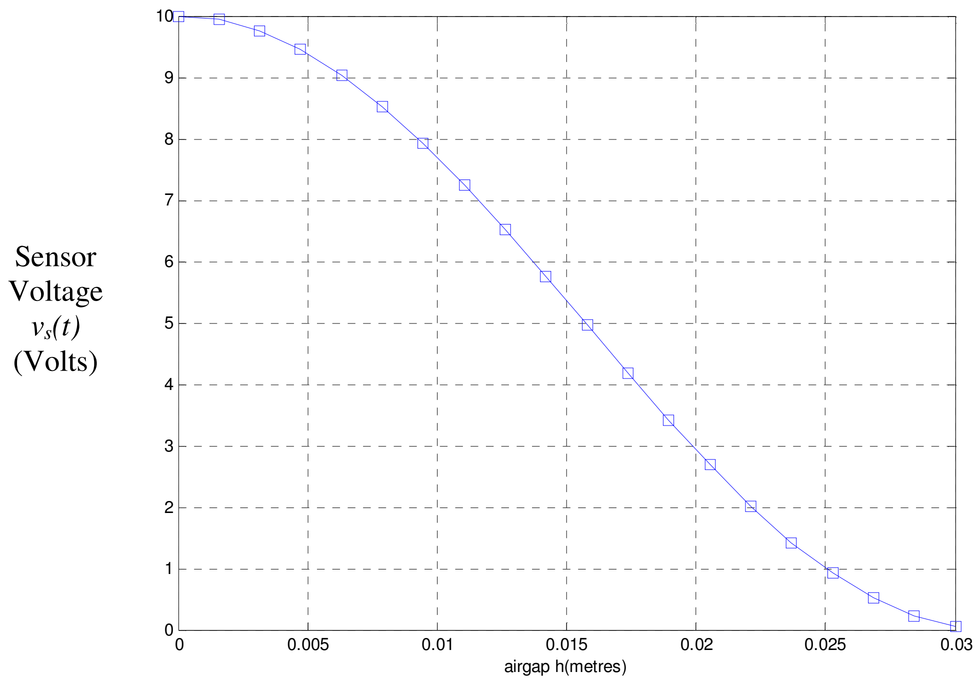

Fig. 5.2 Sensor calibration curve

The process parameters are:

$$m=0.02\text{Kg} \qquad K=2.0\text{x}10^{-5}\ \text{NmV}^{-2} \qquad g=10\ \text{ms}^{-2}$$

Design a state-space controller to maintain a constant airgap of 15mm.

State clearly all the assumptions that you have made in your design.

[17 Marks]

6.

(a) Consider the following Nth order open-loop process, with single input *u(t)*, single output *y(t)* , and state-vector $\underline{x}(t)$,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

This process is controlled using a state-space regulator, with gain matrix *K*. The state vector is not measured directly, but is estimated as $\underline{\hat{x}}(t)$ using a full-state Luenberger observer with estimator gain matrix *G*.

(i) Develop fully the following representation of the closed loop system,

$$\frac{d}{dt}\begin{bmatrix} \underline{x}(t) \\ \underline{e}(t) \end{bmatrix} = \left[\begin{array}{c|c} A - BK & BK \\ \hline 0 & A - GC \end{array}\right] \begin{bmatrix} \underline{x}(t) \\ \underline{e}(t) \end{bmatrix}$$

where the estimation error $\underline{e}(t)$ is defined as, $\underline{e}(t) = \underline{x}(t) - \underline{\hat{x}}(t)$

(ii) Use this representation of the closed-loop process to explain the "Separation Principle", and how this principle is applied in state-space control design.

[10 Marks]

(b) A closed-loop control system for a single joint of a robotic manipulator can be represented by the following block diagram,

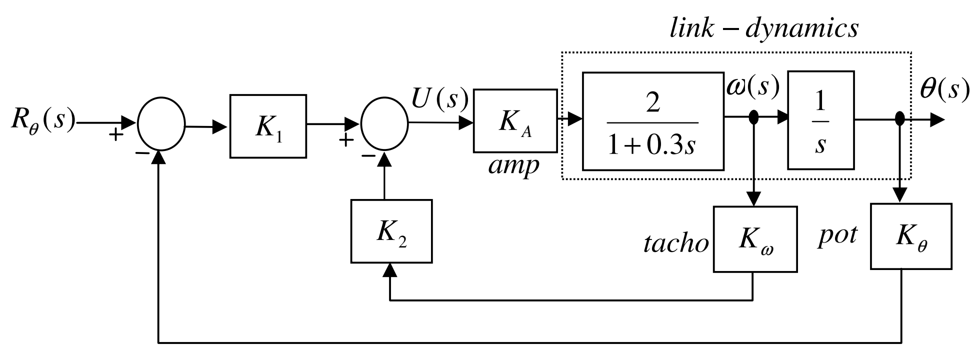

Fig. 6.1 Closed-loop control of a single robotic joint

Where *u(t)* is the input voltage and *θ(t)* the angle of rotation in radians. The sensor gains are $K_\theta = 5 volt/rad$, and $K_\omega = 1 volt\, rad^{-1} s$. The power amplifier gain is $K_A = 0.8$.

(i) Determine the controller gains K_1 and K_2, to place both closed-loop poles at $s = -10$.

(ii) It was decided that it was too expensive to employ a tachometer to measure the rotational speed.

Design a full-order Luenberger observer to provide estimates of the states of this process for use with the controller designed above.

Draw a simulation diagram representing the complete controller, incorporating the control-law and the observer.

[15 Marks]