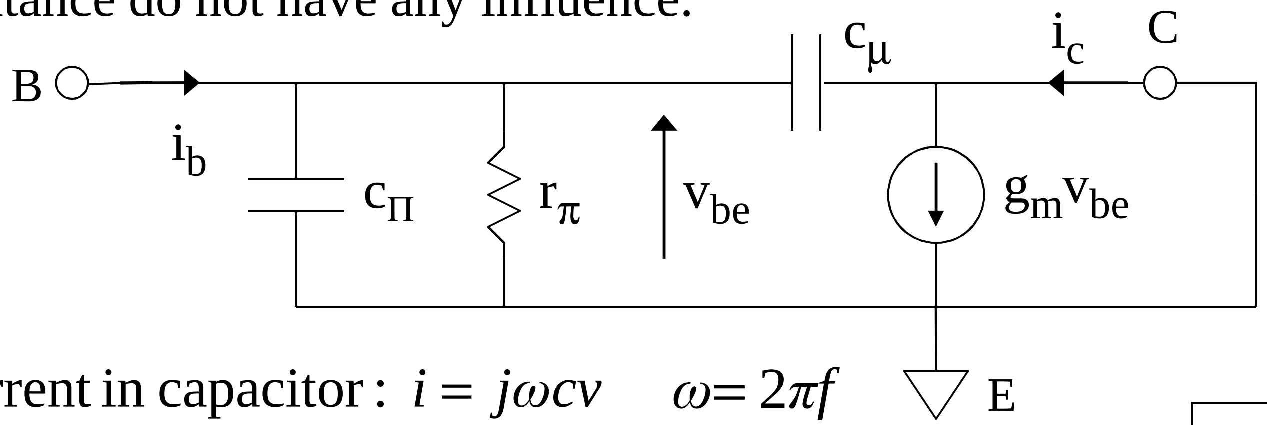
EE4011: RF IC Design

Review of the Expression for BJT Cut-off Frequency

Short-Circuit Current Gain: h_{fe}

High speed devices are often compared based on a characteristic known as the short-circuit current gain h_{fe} .This is the ratio of output and input currents when the output is short-circuited (the short circuit is for ac signals only – the DC bias is still applied to the device). When the output is short-circuited r_o and collector-substrate capacitance do not have any influence.



ac current in capacitor: $i = j\omega c v \qquad \omega = 2\pi f$

$$i_b = \frac{v_{be}}{r_\pi} + j\omega c_\pi v_{be} + j\omega c_\mu v_{be} = v_{be}\left[g_\pi + j\omega\left(c_\pi + c_\mu\right)\right]$$

$$i_c = g_m v_{be} - j\omega c_\mu v_{be} = v_{be}\left(g_m - j\omega c_\mu\right)$$

$$h_{fe} = \frac{i_c}{i_b} = \frac{g_m - j\omega c_\mu}{g_\pi + j\omega\left(c_\pi + c_\mu\right)}$$

If $\quad g_m >> \omega c_\mu \quad$ then $\quad h_{fe} \approx \dfrac{g_m}{g_\pi + j\omega\left(c_\pi + c_\mu\right)}$

Notation:
$j = \sqrt{-1}$
$i = \text{current}$

Cut-off frequency: f_T

$$h_{fe} = \frac{g_m}{g_\pi + j\omega(c_\pi + c_\mu)} = \frac{g_m}{g_\pi + j2\pi f(c_\pi + c_\mu)}$$

$$\Rightarrow h_{fe} = \frac{g_m / g_\pi}{1 + j2\pi f(c_\pi + c_\mu)/g_\pi} = \frac{\beta}{1 + j2\pi f\, r_\pi (c_\pi + c_\mu)}$$

from earlier definitions:

$$g_\pi = \frac{g_m}{\beta} \Rightarrow \frac{g_m}{g_\pi} = \beta$$

$$r_\pi = \frac{\beta}{g_m} \Rightarrow \frac{\beta}{r_\pi} = g_m$$

For high frequencies:

$$2\pi f\, r_\pi (c_\pi + c_\mu) >> 1 \Rightarrow h_{fe} \approx \frac{\beta}{j2\pi f\, r_\pi (c_\pi + c_\mu)} \Rightarrow |h_{fe}| = \frac{\beta}{2\pi f\, r_\pi (c_\pi + c_\mu)}$$

The **cut-off frequency** is the frequency at which h_{fe} drops to unity

i.e.
$$1 = \frac{\beta}{2\pi f_T\, r_\pi (c_\pi + c_\mu)} \Rightarrow f_T = \frac{\beta}{2\pi r_\pi (c_\pi + c_\mu)} = \frac{g_m}{2\pi (c_\pi + c_\mu)}$$

This is a very common result in transistors – the cut-off frequency is determined by the ratio of the transconductance to the input capacitance. Beyond f_T, the small-signal current flowing in the output circuit is less than the small-signal current flowing in the input circuit so there is no current gain.

The pole frequency

$$h_{fe} = \frac{\beta}{1 + j2\pi f\, r_\pi \left(c_\pi + c_\mu\right)}$$

For low frequencies:

$$2\pi f\, r_\pi \left(c_\pi + c_\mu\right) << 1 \Rightarrow h_{fe} \approx \beta$$

The pole frequency, f=f_P is a special case where $2\pi f_P\, r_\pi \left(c_\pi + c_\mu\right) = 1$

$$2\pi f_P\, r_\pi \left(c_\pi + c_\mu\right) = 1 \Rightarrow f_P = \frac{1}{2\pi r_\pi \left(c_\pi + c_\mu\right)}$$

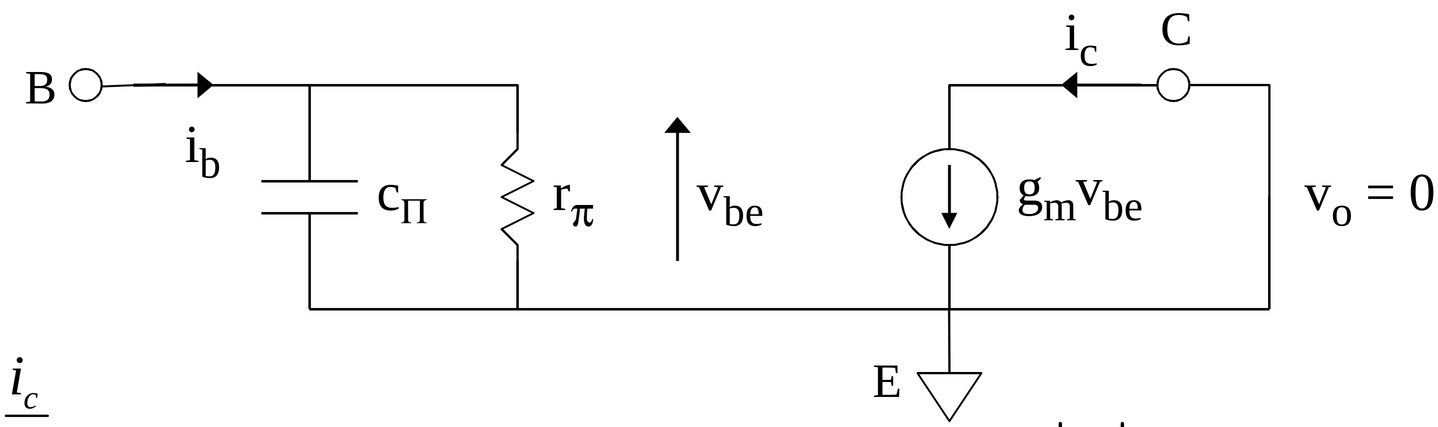
at f=f_P $\quad h_{fe} = \frac{\beta}{1 + j} \Rightarrow \left|h_{fe}\right| = \frac{\beta}{\sqrt{2}}$

The expression for h_{fe} can be rewritten using f_P : $\quad h_{fe} = \frac{\beta}{1 + jf / f_P}$

This is a very common result in transistors – the first pole of the frequency response is often determined by the product of the input resistance and input capacitance

A closer look at the currents

Physical insight (ignoring C_μ):

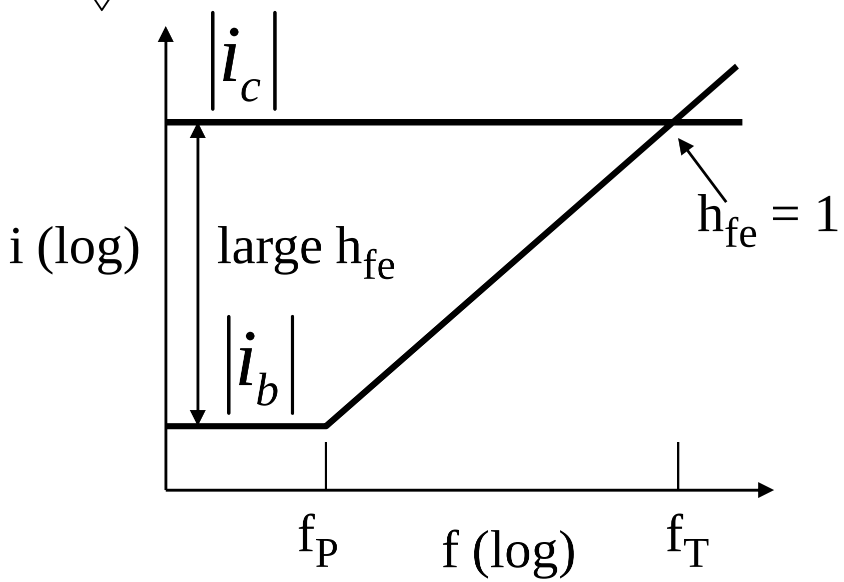


$$h_{fe} = \frac{i_c}{i_b}$$

$i_c = g_m v_{be}$ Stays constant with frequency

$$i_b = v_{be}\left[\frac{1}{r_\pi} + j\omega c_\pi\right]$$

Determined by the resistor at low frequencies and by the capacitor at high frequencies.



Gain-Bandwidth Product

General Expression: $$h_{fe} = \frac{\beta}{1 + j\omega r_\pi \left(c_\pi + c_\mu\right)} = \frac{\beta}{1 + j2\pi f\, r_\pi \left(c_\pi + c_\mu\right)}$$

For low frequencies $h_{fe} \sim \beta$

Cut-off frequency: $$f_T = \frac{g_m}{2\pi\left(c_\pi + c_\mu\right)} = \frac{\beta}{2\pi r_\pi \left(c_\pi + c_\mu\right)}$$

Pole Frequency: $$f_P = \frac{1}{2\pi r_\pi \left(c_\pi + c_\mu\right)}$$ Thus: $$f_T = \beta f_P$$

Using definition of f_P: $$h_{fe} = \frac{\beta}{1 + j2\pi f\, r_\pi \left(c_\pi + c_\mu\right)} = \frac{\beta}{1 + j f / f_P}$$

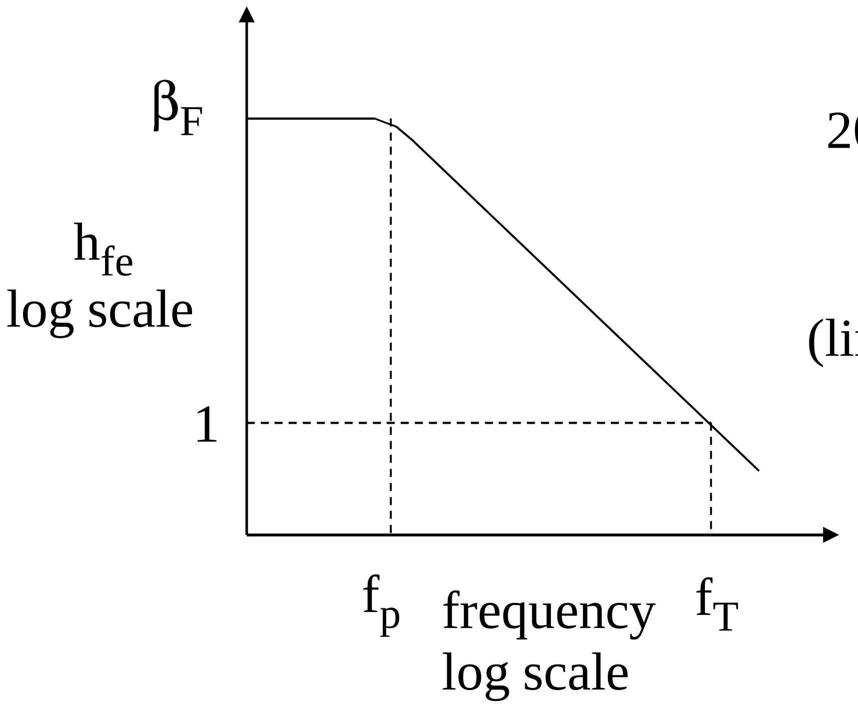
For frequencies $f >> f_P$:

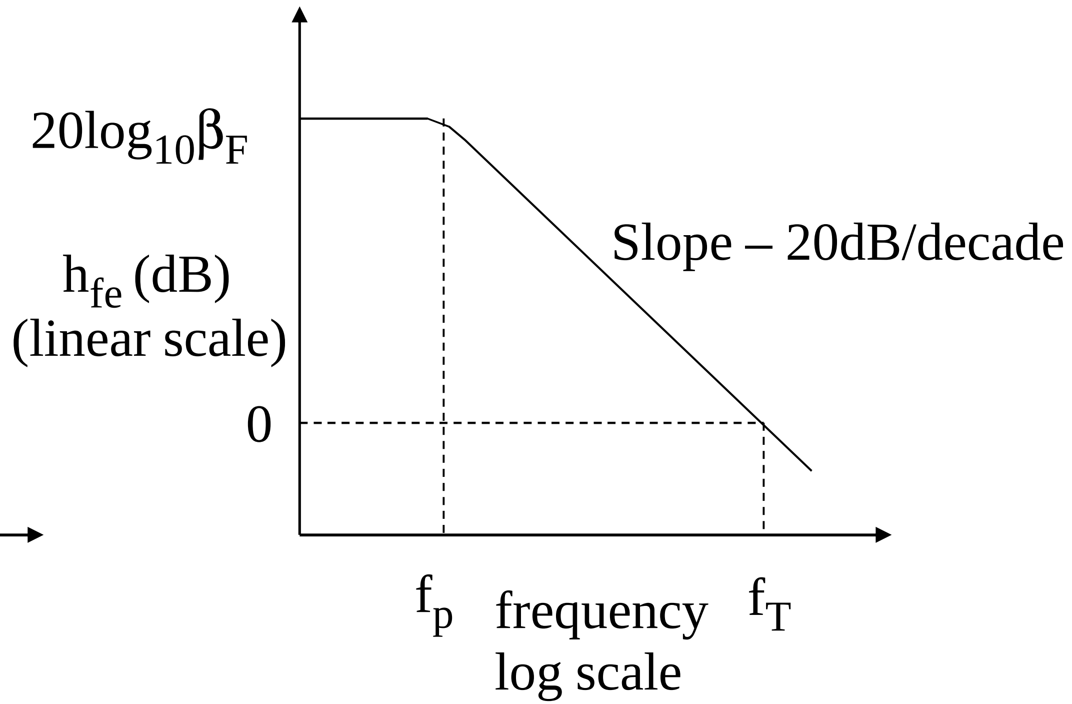
$$h_{fe} \approx \frac{\beta}{j f / f_P} \Rightarrow \left|h_{fe}\right| = \frac{\beta f_P}{f} = \frac{f_T}{f} \Rightarrow \left|h_{fe}\right| f = f_T$$

i.e. for $f >> f_P$, h_{fe} is inversely proportional to frequency and product of current gain and frequency is constant – the so-called gain bandwidth product.

Bode Plots

Gain in decibels: $\left. hfe \right|_{dB} = 20 \log_{10} \left| hfe \right|$





For f >> f_p : $\left| h_{fe} \right| = \dfrac{f_T}{f} \Rightarrow 20 \log_{10} \left| h_{fe} \right| = 20 \log_{10} (f_T) - 20 \log_{10} (f)$

if f increases by a factor of 10, h_{fe} decreases by 20dB giving a slope of –20dB/decade typical of a system with one pole. Note: the analysis of the frequency response was simplified by ignoring the zero associated with c_μ.

f_T variation with I_C (1)

$$f_T = \frac{g_m}{2\pi\left(c_\pi + c_\mu\right)}$$

$$c_\mu = C_{JBC}$$

$$c_\pi = C_{JBE} + C_{DE}$$

$$C_{DE} = g_m \tau_F$$

$$g_m = \frac{I_C}{V_T}$$

Putting all these together:

$$f_T = \frac{g_m}{2\pi(g_m \tau_F + C_{JBE} + C_{JBC})}$$

$$2\pi f_T = \frac{1}{\tau_F + \frac{1}{g_m}(C_{JBE} + C_{JBC})}$$

$$2\pi f_T = \frac{1}{\tau_F + \frac{V_T}{I_C}(C_{JBE} + C_{JBC})}$$

The final formula shows that f_T is directly related to the transit time, the junction capacitances and the collector current.

f_T variation with I_C (2)

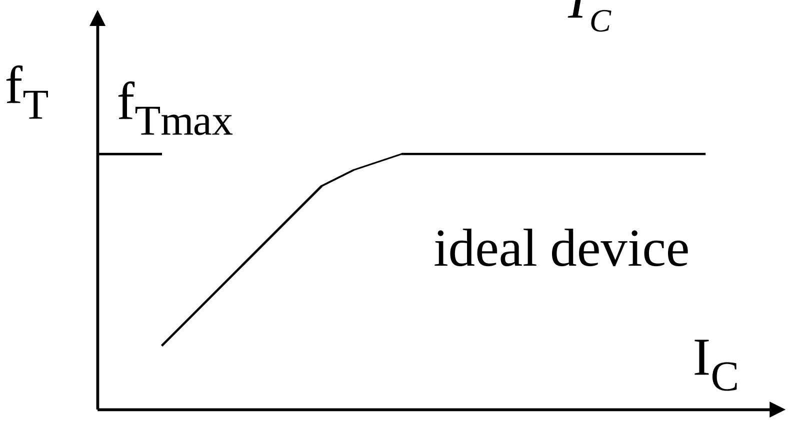
$$2\pi f_T = \frac{1}{\tau_F + \frac{V_T}{I_C}(C_{JBE} + C_{JBC})}$$

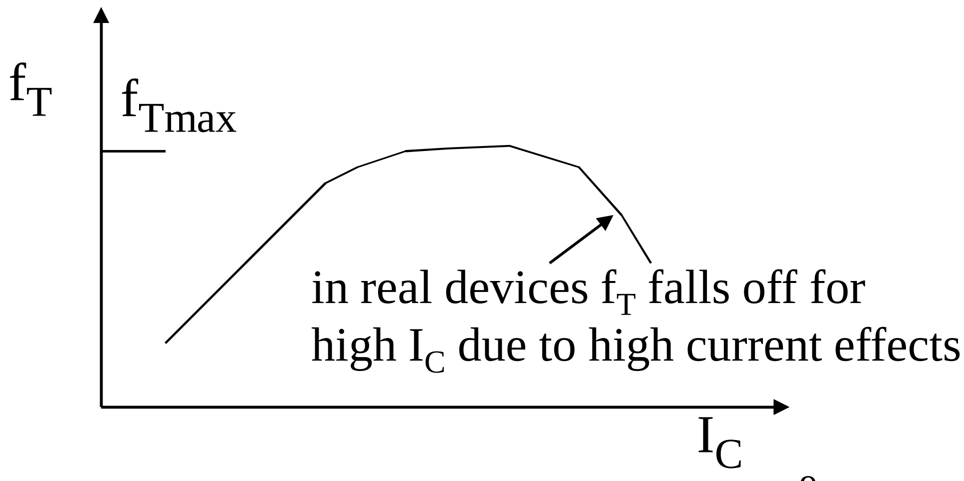
For “moderate” currents the second term in the denominator is small and the cut off frequency is at a maximum value determined by τ_F:

$$2\pi f_T \approx 2\pi f_{T\max} = \frac{1}{\tau_F}$$

For “small” currents the second term in the denominator is large and the cut-off frequency is proportional to I_C:

$$2\pi f_T \approx \frac{1}{\frac{V_T}{I_C}(C_{JBE} + C_{JBC})} = \frac{I_C}{V_T(C_{JBE} + C_{BC})}$$



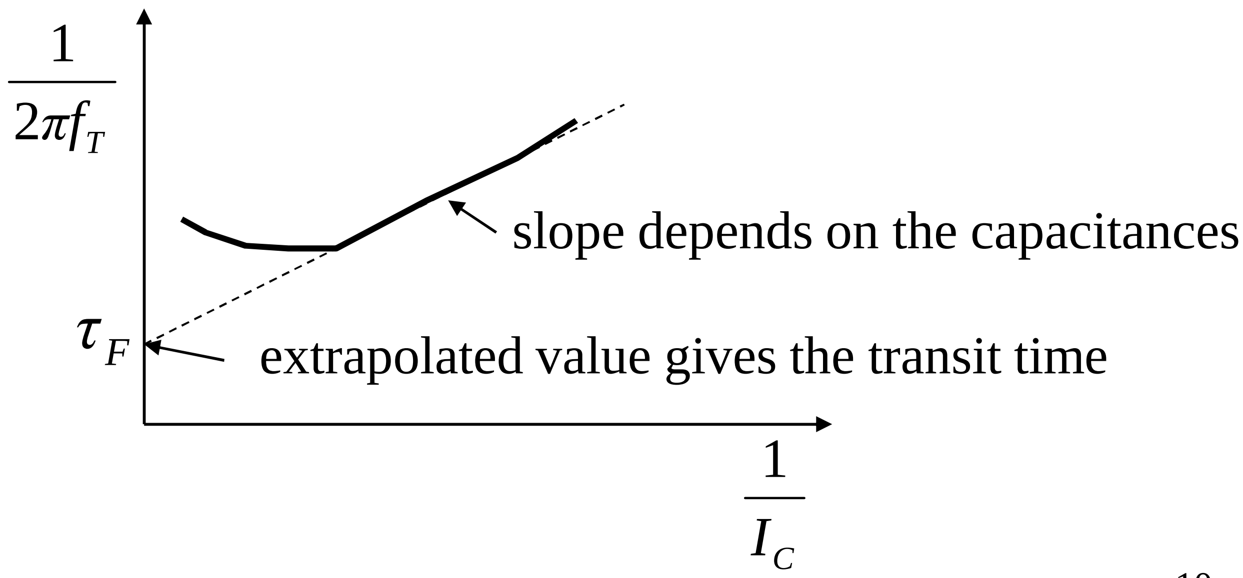


f_T variation with I_C (3)

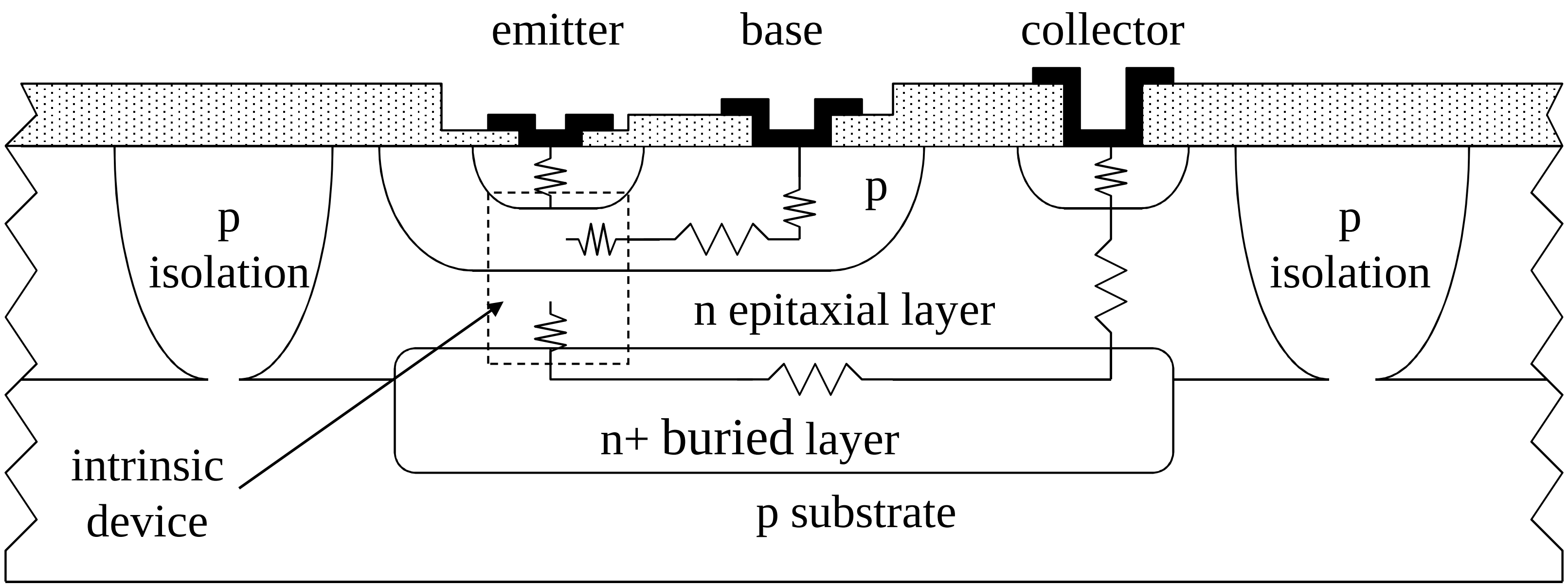
$$2\pi f_T = \frac{1}{\tau_F + \frac{V_T}{I_C}(C_{JBE} + C_{JBC})}$$

$$\Rightarrow \frac{1}{2\pi f_T} = \tau_F + \frac{V_T}{I_C}(C_{JBE} + C_{JBC})$$

The last equation shows how a measurement of f_T vs. I_C can be used to determine the forward transit time and estimate the junction capacitances.



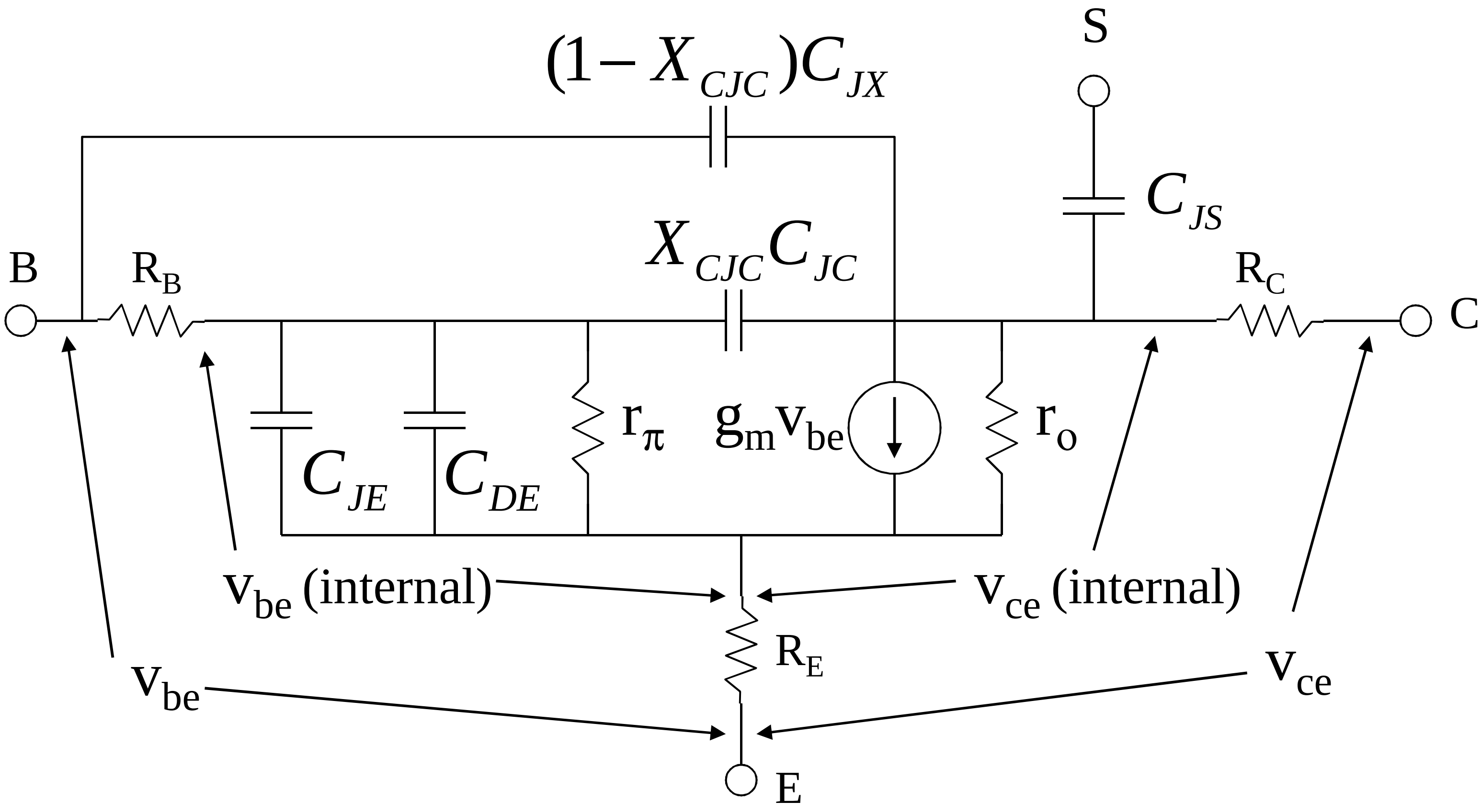
Parasitic Resistances



There are several parasitic resistance components in a real BJT associated with the resistivity of the various diffused/implanted layers and contact resistances as shown above. Because of the voltage drops across these resistances the internal voltages on the intrinsic device are different to the applied bias.

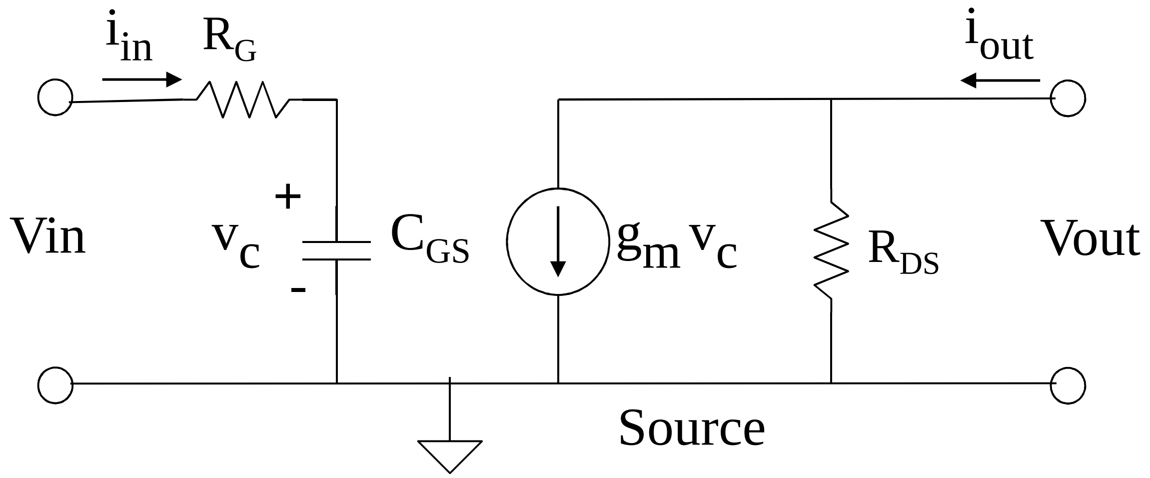
The distributed nature of the base resistance means that the base/collector capacitance needs to be divided between the internal and external base nodes. This is achieved using a parameter X_{CJC} which is the fraction of the total capacitance, C_{JC}, which is connected between the collector and the internal base. The remainder (1-X_{CJC}) is connected between the collector and the external base.

Equivalent Circuit with Parasitic Resistances



Because of the voltage drops across the parasitic resistances, the internal voltages V_{BE} and V_{CE} are less than the external biases, an effect sometimes called “de-biasing”. It reduces the current and effective transconductance compared to the ideal case. The parasitic resistances are also a source of thermal noise.

f_T example to try yourself



Determine the cut-off frequency of a transistor with the small-signal equivalent circuit above. Note that the transconductance element is controlled by the voltage v_c.

From your analysis, sketch how hfe and the input and output currents vary with frequency – do you notice anything different to the BJT from the previous slides?