16/3/09 Control

Summer 2007

Q 1a. $M(s) = K_P(1 + \frac{1}{T_I s} + T_D s)E(s)$

$C(s) = M/E = K_P(1 + \frac{1}{T_I s} + T_D s)$

$= \frac{K_P(T_D s^2 + s + 1/T_I)}{s}$

$= \frac{T_D K_P(s^2 + \frac{1}{T_d}s + \frac{1}{4T_D^2})}{s}$ $\qquad T_D = \frac{T_I}{4}$

$= -\frac{1}{2T_D}$ twice

$C(s) = \frac{\overset{T_d K_P}{K_c}(s + \overset{\frac{1}{2T_D}}{z})^2}{s}$

MPZ: pole @ $s = 0 \xrightarrow{e^{sT}\text{ — sample time}}$ pole @ $z = e^{0T} = 1$

zero @ $s = -z \rightarrow$ zero @ $z = e^{-zT}$

$D(z) = \frac{K_d(z - e^{-zT})^2}{z(z-1)}$ $\qquad C(s) = \frac{K_c(s+z)^2}{s}$

FROM HINT

$\lim_{s \to 0} s\frac{K_c(s+z)^2}{s} = K_c z^2$

$\lim_{z \to 1} (z-1)\frac{K_d(z - e^{-zT})^2}{z-1} = K_d(1 - e^{-zT})^2$

$K_d = \frac{K_c z^2}{(1 - e^{-zT})^2}$

$M/E = D(z) = \frac{K_d(z^2 - 2e^{-zT}z + e^{-2zT})}{z^2 - z}$

$(z^2 - z)M = K_d(z^2 - 2e^{-zT}z + e^{-2zT})E$

$$(1-z^{-1})M(z) = K_d E(z) - 2K_d e^{-zT} z^{-1} E(z) + K_d e^{-2zT} z^{-2} E(z)$$

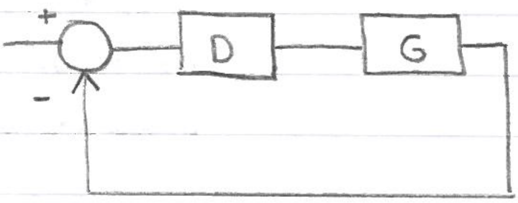
$\underline{\underline{IZT}}$

$$m(k) = m(k-1) + \underbrace{K_d}_{\alpha} e(k) \underbrace{-2K_d e^{-zT}}_{\beta} e(k-1) + \underbrace{K_d e^{-2zT}}_{\gamma} e(k-2)$$

(b). $M(z) = K z^{-1} E(z) + 0.8 z^{-1} M(z)$

$(1-0.8z^{-1})M = K z^{-1} E$

$$\frac{M}{E} = \frac{K z^{-1}}{1-0.8z^{-1}} = \frac{K}{z-0.8} \rightarrow \text{pole @ } z = 0.8$$



$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s} \cdot \frac{2}{1+2s} \cdot 2\right\} \Rightarrow (1-z^{-1})\,\mathcal{Z}\left\{\frac{4}{s(1+2s)}\right\}$$

Tables $\mathcal{Z}\left\{\frac{1}{s(s+a)}\right\} = \frac{1}{a}\,\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$

$$\frac{4}{s(1+2s)} \rightarrow \frac{2}{s(s+\frac{1}{2})} = 2 \times \frac{1}{s(s+\frac{1}{2})}$$

$a = 0.5 \quad T = 1$

$$G(z) = 2 \times \frac{1}{0.5}\,\frac{(1-e^{-0.5})z^{-1}}{(1-z^{-1})(1-e^{-0.5}z^{-1})} \times (1-z^{-1})$$

$$= \frac{1.57z^{-1}}{1-0.606z^{-1}} \Rightarrow \frac{1.57}{z-0.606} \rightarrow \text{pole @ } z = 0.606$$