OLLSCOIL NA hÉIREANN, CORCAIGH

THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH

UNIVERSITY COLLEGE, CORK

AUTUMN EXAMINATIONS, 2007

B.E. DEGREE (ELECTRICAL)

B.E. DEGREE (MICROELECTRONIC)

TELECOMMUNICATIONS

EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of log tables and a Casio fx570w or fx570ms calculator is permitted.

1. (a) Discuss the relationship between quantisation noise and the number of quantisation levels in pulse code modulation (PCM).

[*10 marks*]

(b) A microwave link using 64 QAM modulation is used to carry three signals of 5 MHz, 10 MHz and 15 MHz bandwidth respectively, which have been encoded using standard PCM. If the carrier modulation rate is 6 x 10^7 phase changes per second, what is the maximum possible signal to quantisation noise ratio in dB?

[*10 marks*]

Q.2. (a) Describe the operation of time division multiplexing in a fixed telephone network and illustrate the format and timing of data on a trunk line (E1) in Europe.

[*5 marks*]

(b) Describe the operation of statistical multiplexing in a data communications system. In your discussion, include a definition of multiplexing gain and indicate the expected ranges of multiplexing gain for various data sources.

[*5 marks*]

(c) Illustrate the format of an ATM cell and briefly describe the function of each field of the cell.

[*5 marks*]

(d) Illustrate the error handling procedure for an ATM packet and briefly describe the operation of the procedure.

[*5 marks*]

Q.3. (a) Illustrate the architecture of a UMTS (3G) Radio-Access Network including the core network and the radio network sub-system and briefly describe the function of the main blocks.

[8 *marks*]

(b) For a mobile telephone communications system (2G or 3G) discuss the following:

(i) The use of hexagonal cells with frequency reuse.

[4 *marks*]

(ii) The main types of signal fading.

[4 *marks*]

(iii) The handoff of a mobile station as it moves from one cell to another.

[4 *marks*]

Q.4. Given that the 2×2 channel matrix $\left[P\left(Y_1 | X\right)\right]$ for the generalised binary channel with 2 input symbols, denoted x_i, $1 \le i \le 2$ and 2 output symbols, denoted y_j, $1 \le j \le 2$, is given by: -

$$\left[P\left(Y_1 | X\right)\right] = \begin{bmatrix} 1-e_1 & e_1 \\ e_2 & 1-e_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & -\dfrac{e_1}{e_2} \\ 1 & 1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1-e_1-e_2 \end{bmatrix}}_{D} \underbrace{\left(\frac{1}{e_1+e_2} \begin{bmatrix} e_2 & e_1 \\ -e_2 & e_2 \end{bmatrix} \right)}_{F^{-1}}$$

where $e_1 > 0$, $e_2 > 0$, D is a diagonal matrix and the columns of F are eigenvectors of $\left[P\left(Y_1 | X\right)\right]$, show that if n such generalised binary channels are connected in series (i.e. the outputs of channel i become the inputs of channel $i+1$, $1 \le i \le n-1$), then: -

(a) The composite channel matrix $\left[P\left(Y_n | X\right)\right]$ is given by: -

$$\left[P\left(Y_n | X\right)\right] = \frac{1}{e_1+e_2} \begin{bmatrix} e_2 + e_1\lambda & e_1(1-\lambda) \\ e_2(1-\lambda) & e_1 + e_2\lambda \end{bmatrix}$$

where $\lambda = \left(1-e_1-e_2\right)^n$.

[6 *marks*]

(b) Show that if the output symbols, denoted y_1 and y_2, from the composite channel in part (a) above are to be equiprobable then we require: -

$$P\left(x_1\right) = \frac{e_1 - e_2 + 2e_2\lambda}{2\lambda\left(e_1+e_2\right)}$$

where $P\left(x_1\right)$ denotes the probability of the input symbol being x_1 (noting that, if x_1 is sent and no error occurs, the output symbol will be y_1).

[5 *marks*]

(c) If both the input symbols and the output symbols are equiprobable, show that: -

(i) This condition requires $e_1 = e_2$. [3 *marks*]

(ii) The composite channel capacity, denoted C_s^c, is given by

$$C_s^c = 1 + \left(\frac{1-\left(1-2e_1\right)^n}{2}\right)\log_2\left(\frac{1-\left(1-2e_1\right)^n}{2}\right) + \left(\frac{1+\left(1-2e_1\right)^n}{2}\right)\log_2\left(\frac{1+\left(1-2e_1\right)^n}{2}\right).$$

[6 *marks*]

Q.5 A baseband digital communications system uses rectangular wave signalling with A_1 volts representing logic 1 and A_2 volts representing logic 0 (where $A_2 < A_1$). The receiver takes a single sample of the received signal during the bit signalling time and compares this sample with a decision threshold T. The communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n(v) = \frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}.$$

P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1 and, to minimize the resulting overall probability of error P_e, the threshold T is given by: -

$$T = \frac{A_1 + A_2}{2} + \frac{\sigma^2}{A_1 - A_2}\ln\left[\frac{P_0}{P_1}\right].$$

(a) Show that, if $P_0 > P_1$, then the average probability of error, denoted P_e, is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{T - A_2}{\sqrt{2\sigma^2}}\right] + (1 - P_0) erf\left[\frac{A_1 - T}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy.$$

[10 *marks*]

(b) Consider a system for which $A_1 = 2.5V$, $A_2 = -2.5V$ and $\sigma^2 = 0.45W$. Using the table of values of $erf[x]$ provided: -

(i) Prove that, if the threshold remains fixed at $T = 0V$, then P_e is independent of P_0 and calculate its value in this case.

[4 *marks*]

(ii) When the optimum threshold is employed in each case, calculate the value of P_e when $P_0 = 0.65$ and when $P_0 = 0.75$.

[6 *marks*]

Q.6. Typical expressions for ASK and PSK modulated waveforms representing binary data, where in each case T is an integer times $1/f_c$, are as follows: -

ASK

$$s_i(t) = \begin{cases} s_1(t) = A_1\cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = 0 & 0 \le t \le T \end{cases}$$

PSK

$$s_i(t) = \begin{cases} s_1(t) = A_2\cos[\omega_c t] & 0 \le t \le T \\ s_2(t) = -A_2\cos[\omega_c t] & 0 \le t \le T \end{cases}$$

In addition, the probability of error for a binary modulation scheme (denoted MOD) with optimum detection in the presence of AWGN with a power spectral density of $\eta/2\ W/Hz$ is given by: -

$$P_e^{MOD} = Q\left[\sqrt{\frac{E_d}{2\eta}}\right]$$

where E_d denotes the energy difference in the appropriate signal (over a single bit interval).

(a) Derive expressions for P_e^{ASK} and P_e^{PSK}. [6 *marks*]

(b) If the average signal energy per bit for the ASK and PSK modulation schemes above is made equal, derive the following expression for the enhancement in reliability, denoted E, achieved by choosing PSK over ASK when both schemes deliver the same bit rate: -

$$E = \frac{P_e^{ASK}}{P_e^{PSK}} = \frac{Q\left[\sqrt{\frac{A_2^2 T}{2\eta}}\right]}{Q\left[\sqrt{\frac{A_2^2 T}{\eta}}\right]}.$$

[6 *marks*]

(c) Using the table of values of $Q[z]$ provided to draw a suitable graph, or otherwise, estimate the amplitude A_2 resulting in $E = 45$ when $\eta/2 = 10^{-12}$ W/Hz and the bit rate is 1 Mb/s.

[8 *marks*]

Q.7 Given the following table of field elements of $GF(2^4)$: -

0	$\alpha^7 = \alpha^3 + \alpha + 1$
1	$\alpha^8 = \alpha^2 + 1$
α	$\alpha^9 = \alpha^3 + \alpha$
α^2	$\alpha^{10} = \alpha^2 + \alpha + 1$
α^3	$\alpha^{11} = \alpha^3 + \alpha^2 + \alpha$
$\alpha^4 = \alpha + 1$	$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha + 1$
$\alpha^5 = \alpha^2 + \alpha$	$\alpha^{13} = \alpha^3 + \alpha^2 + 1$
$\alpha^6 = \alpha^3 + \alpha^2$	$\alpha^{14} = \alpha^3 + 1$

(a) Show that the generator polynomial for the $(15,7)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1.$$

[7 *marks*]

(b) If the received data, denoted $v(x)$, is given by: -

$$v(x) = x^{13} + x^{10} + x^9 + x^7 + x^6 + x^4 + x^3 + x,$$

use the syndrome decoding method to find the error polynomial $e(x)$ and the original codeword $c(x)$, where $v(x) = c(x) + e(x)$.

[10 *marks*]

(c) Deduce the value of the information polynomial, denoted $i(x)$, associated with the value of $c(x)$ calculated in part (b) above.

[3 *marks*]

Table of values of $erf(x)$

x	$erf(x)$		x	$erf(x)$
2.5	0.999593		2.63	0.9998
2.505	0.999604		2.635	0.999806
2.51	0.999614		2.64	0.999811
2.515	0.999625		2.645	0.999816
2.52	0.999635		2.65	0.999822
2.525	0.999644		2.655	0.999826
2.53	0.999654		2.66	0.999831
2.535	0.999663		2.665	0.999836
2.54	0.999672		2.67	0.999841
2.545	0.999681		2.675	0.999845
2.55	0.999689		2.68	0.999849
2.555	0.999698		2.685	0.999854
2.56	0.999706		2.69	0.999858
2.565	0.999714		2.695	0.999862
2.57	0.999722		2.7	0.999866
2.575	0.999729		2.705	0.999869
2.58	0.999736		2.71	0.999873
2.585	0.999744		2.715	0.999877
2.59	0.999751		2.72	0.99988
2.595	0.999757		2.725	0.999884
2.6	0.999764		2.73	0.999887
2.605	0.99977		2.735	0.99989
2.61	0.999777		2.74	0.999893
2.615	0.999783		2.745	0.999896
2.62	0.999789		2.75	0.999899
2.625	0.999795			