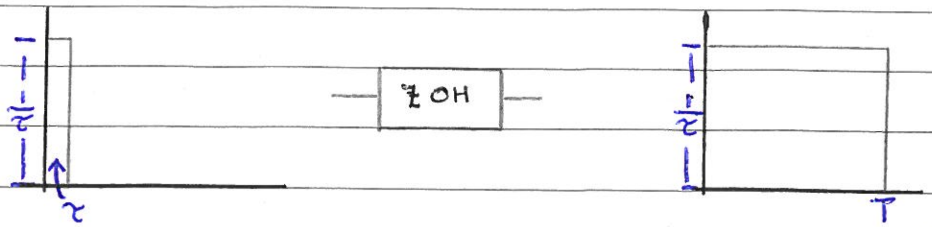
EE4002 Control Engineering Summer '07

2 a)

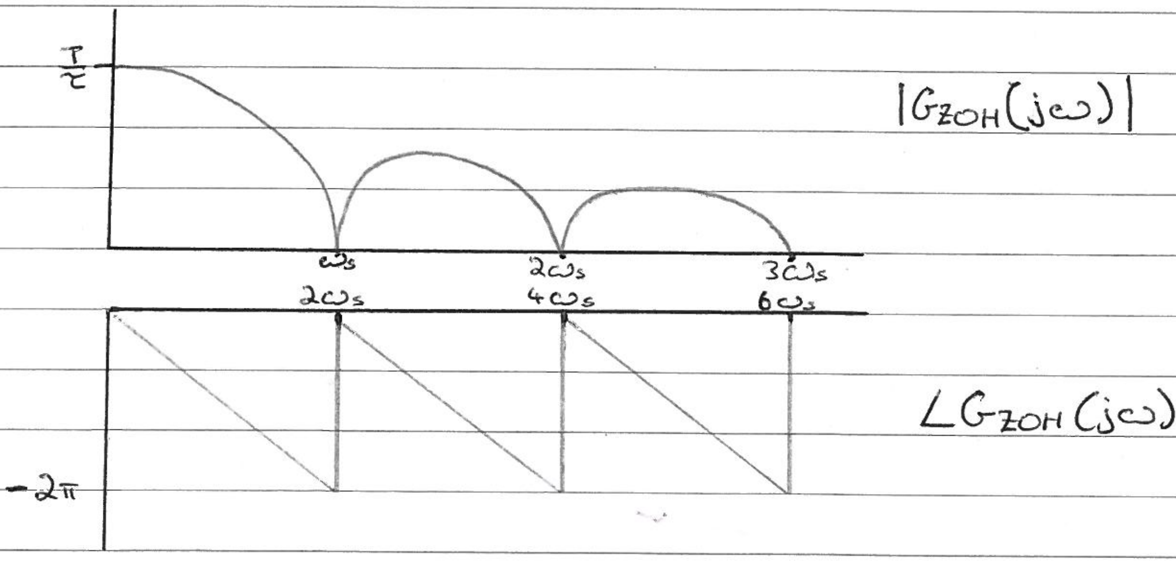


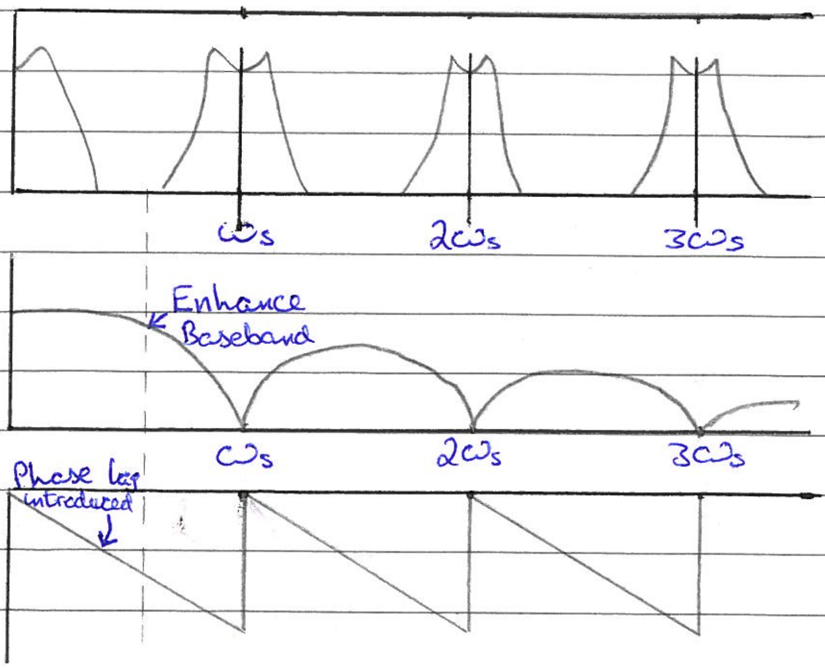
$$G_{ZOH}(Z) = \int_0^\infty g_{ZOH}(t)\,d\tau = \int_0^T \frac{e^{-st}}{\tau}\,dt$$

$$= -\frac{1}{s\tau}\left(e^{-st}\Big|_0^T \right) = \frac{1 - e^{-sT}}{s\tau}$$

$$|G_{ZOH}(j\omega)| = \frac{T}{2}\,\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}$$

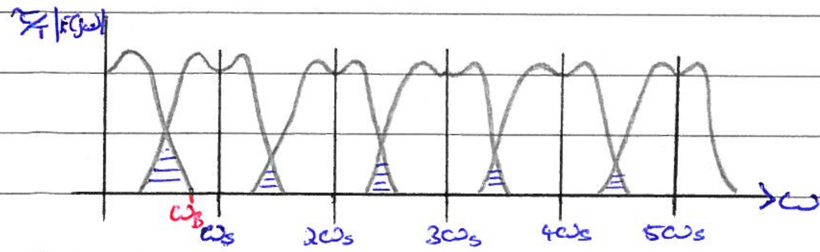
$$\angle G_{ZOH}(j\omega) = -\frac{\omega}{\omega_s}\pi$$





- There are an ifinite number of baseband replications at integer multiples of the sampling frequency

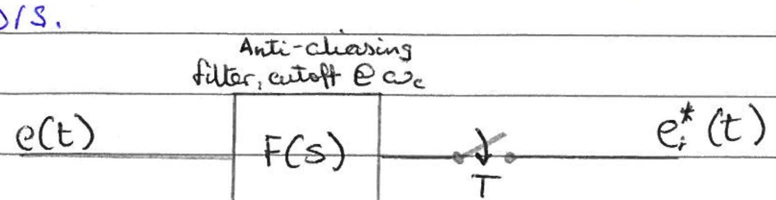
- If $\omega_s > 2\omega_B$, the spectra are distinct: the baseband can easily be extracted using a LPF.
- If $\omega_s < 2\omega_B$, aliasing occurs & we cannot simply reconstruct the baseband from the sampled ~~frequency.~~ signal.



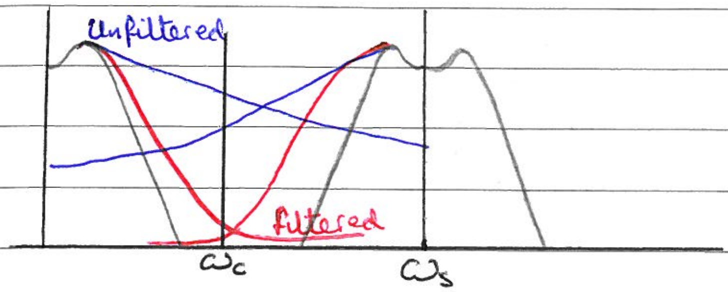
Shannon's Sampling Theorem:

For a continuous time signal ~~with~~ e(t) with $|E(j\omega)| = 0$ for $|\omega| \geq \omega_B$, then the sampling frequency should be chosen as $\omega_s \geq 2\omega_B$ to ensure that aliasing does not occur.

In practice, there is not a finite spectrum due to noise. This can introduce high frequency noise into the frequency range of interest. It is essential to pre-filter e(t) before sampling to avoid large aliasing errors.



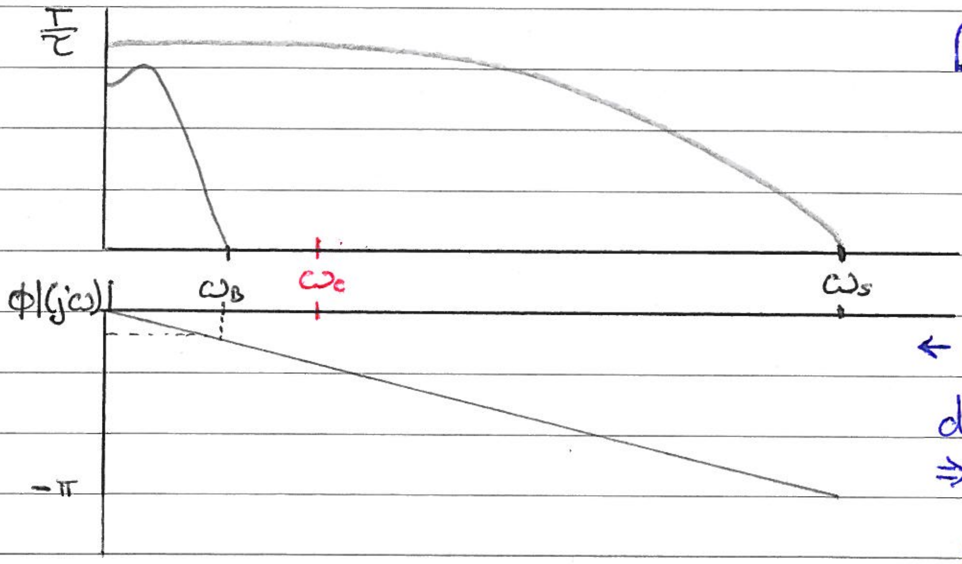
2 c)



• After filtering there is very low noise power in the baseband.

To avoid distortion of the baseband, $\omega_c > \omega_B$

Oversampling:
If we sample at $\omega_s = n\omega_c$, $n \gg 2$



Baseband $\approx \frac{T}{\tau}$

← Reduced phase distortion
⇒ Important for control

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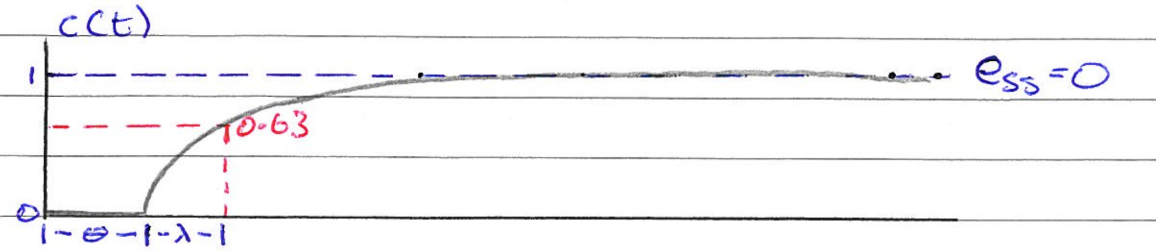
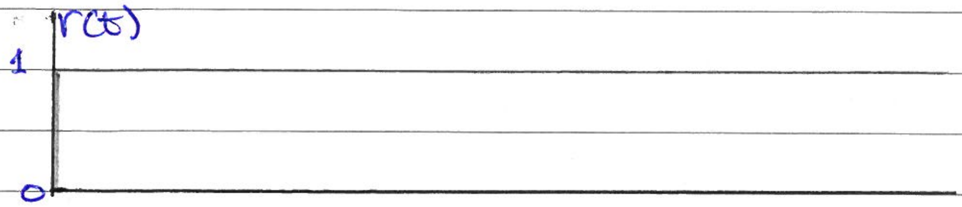
2 b)





$$c(t) = 0 \quad \text{for } t < \theta$$
$$c(t) = 1 - e^{-\frac{t-\theta}{\lambda}} \quad \text{for } t \geq \theta$$

$$C(s) = \mathcal{L}\{c(t)\} = \frac{e^{-\theta s}}{(1+\lambda s)s} \qquad \theta \approx NT$$

$$C(s) = \frac{e^{-NTs}}{(1+\lambda s)s} \xrightarrow{\mathcal{Z}} \frac{(1-e^{-T/\lambda})z^{-(N+1)}}{(1-z^{-1})(1-e^{-T/\lambda}z^{-1})}$$

← 1 extra delay due to sampling

$$R(z) = \frac{1}{1-z^{-1}} \qquad \frac{C(z)}{R(z)} = \frac{(1-e^{-T/\lambda})z^{-(N+1)}}{(1-e^{-T/\lambda}z^{-1})}$$

$$\frac{C/R}{1-C/R} = \frac{(1-e^{-T/\lambda})z^{-(N+1)}}{1-\underbrace{e^{-T/\lambda}}_{\alpha}z^{-1}-\underbrace{(1-e^{-T/\lambda})}_{\beta}z^{-(N+1)}}$$

$$D(z) = \frac{1}{G_m(z)} \cdot \frac{(1-e^{-T/\lambda})z^{-(N+1)}}{1-e^{-T/\lambda}z^{-1}-(1-e^{-T/\lambda})z^{-(N+1)}}$$

$$G_m(z) = (1-z^{-1})\cancel{z^{-N}}\,\mathcal{Z}\left\{\frac{K}{(1+s\tau)s}\right\} = \frac{K\cancel{z^{-N}}}{\tau}\left[\frac{\tau\cdot(1-e^{-T/\lambda})z^{-1}}{(1-e^{-T/\lambda}z^{-1})}\right]$$

$$D(z) = \frac{1-e^{-T/\lambda}z^{-1}}{Kz^{-N}(1-e^{-T/\lambda})z^{-1}} \cdot \frac{(1-e^{-T/\lambda})z^{-N}z^{-1}}{1-e^{-T/\lambda}z^{-1}-(1-e^{-T/\lambda})z^{-(N+1)}}$$

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2b)

$$D(z) = \frac{1 \overbrace{- e^{-T/\lambda}}^{\gamma} z^{-1}}{K\left(1 \underbrace{- e^{-T/\lambda}}_{\alpha} z^{-1} - \underbrace{(1-e^{-T/\lambda})}_{\beta} z^{-(N+1)}\right)} \qquad K_d = \frac{1}{K}$$

$$D(z) = \frac{K_d(1 + \gamma z^{-1})}{1 + \alpha z^{-1} + \beta z^{-(N+1)}}$$

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2 a) $Y(z) = \frac{\alpha}{2} z^{-2} + \alpha z^{-3} + \alpha z^{-4} + \ldots$

$K G(z) = K g_0 + K g_1 z^{-1} + K g_2 z^{-2} + K g_3 z^{-3} + K g_4 z^{-4} + \ldots$

$= Y(z)/R(z) = (1 - z^{-1})\left(\frac{\alpha}{2} z^{-2} + \alpha z^{-3} + \alpha z^{-4} + \ldots\right)$

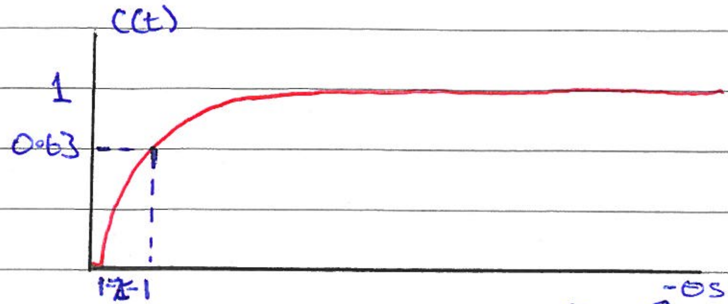
$= \frac{\alpha}{2} z^{-2} + \frac{\alpha}{2} z^{-3}$

$K = \frac{\alpha}{2} \qquad G(z) = \dfrac{z+1}{z^3}$

$|e_{ss}| = |\alpha - 1|$

For $\lim_{k \to \infty} r(k) = r_\infty \qquad |e_{ss}| = r_\infty |\alpha - 1|$

2c)



$$c(t) = \begin{cases} 0, & t < \theta \\ 1 - e^{-\frac{t-\theta}{\tau}}, & t \geq \theta \end{cases}$$

$$C(s) = \mathcal{L}\{c(t)\}$$

$$= \frac{e^{-\theta s}}{s(\lambda s+1)} \xrightarrow{Z} \frac{(1-e^{-T/\tau})z^{-(N+1)}}{(1-z^{-1})(1-e^{-T/\tau}z^{-1})}$$

$$G_m(z) = Z\left\{\frac{K}{1+s\tau} \cdot \frac{1-e^{-sT}}{s}\right\} \Rightarrow N=0$$

$$= \frac{K}{\tau}(1-z^{-1})\, Z\left\{\frac{1}{s(s+\frac{1}{\tau})}\right\} = K\left[\frac{(1-e^{-T/\tau})z^{-1}}{1-e^{-T/\tau}z^{-1}}\right]$$

From $G(z)$, $N=0$

$$C/R = \frac{(1-e^{-T/\tau})z^{-1}}{(1-e^{-T/\tau}z^{-1})} \qquad \frac{C/R}{1-C/R} = \frac{(1-e^{-T/\tau})z^{-1}}{1-e^{-T/\tau}z^{-1}-(1-e^{-T/\tau})z^{-1}}$$

$$D(z) = \frac{1-e^{-T/\tau}z^{-1}}{K\,\cancel{(1-e^{-T/\tau})z^{-1}}} \cdot \frac{\cancel{(1-e^{-T/\tau})z^{-1}}}{1-(e^{-T/\tau}-e^{-T/\tau}+1)z^{-1}}$$

$$D(z) = \frac{1}{K}^{K_d}\,\frac{1-\overbrace{e^{-T/\tau}}^{\gamma}z^{-1}}{1-z^{-1}} = K_d\,\frac{z-\gamma}{z-1}$$

Identical to digital PI controller using Tustin's approximation.

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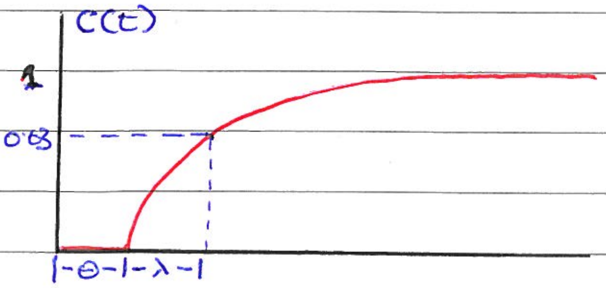
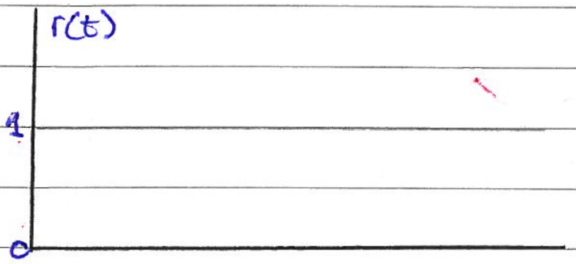
2a)



$$c(t) = \begin{cases} 0, & t < \theta \\ 1 - e^{-\frac{t-\theta}{\lambda}}, & t \geq \theta \end{cases}$$

$$C(s) = \frac{e^{-\theta s}}{s(\lambda s + 1)}$$



$\theta \approx NT$, Delay of N samples

$$C(z) = \frac{(1 - e^{-T/\lambda}) z^{-1}}{(1 - z^{-1})(1 - e^{-T/\lambda} z^{-(N+1)})}$$

$$\frac{C(z)}{R(z)} = \frac{(1 - e^{-T/\lambda}) z^{-1}}{(1 - e^{-T/\lambda} z^{-(N+1)})}$$

$$\frac{\frac{C(z)}{R(z)}}{1 - \frac{C(z)}{R(z)}} = \frac{(1 - e^{-T/\lambda}) z^{-1}}{1 - e^{-T/\lambda} z^{-1} - (1 - e^{-T/\lambda}) z^{-(N+1)}}$$

$e^{-T/\lambda} = \alpha$

$$D(z) = \frac{1}{G_m(z)} \frac{\frac{C(z)}{R(z)}}{1 - \frac{C(z)}{R(z)}}$$

$$= \frac{1}{G_m(z)} \frac{(1 - \alpha) z^{-1}}{1 - \alpha z^{-1} - (1 - \alpha) z^{-(N+1)}}$$

- Possible oscillation in the control signal
- Poles close to the unit circle & close to $z = -1$ ⇒ undamped oscillation with a frequency close to $\frac{\omega_s}{2}$ ⇒ hidden.

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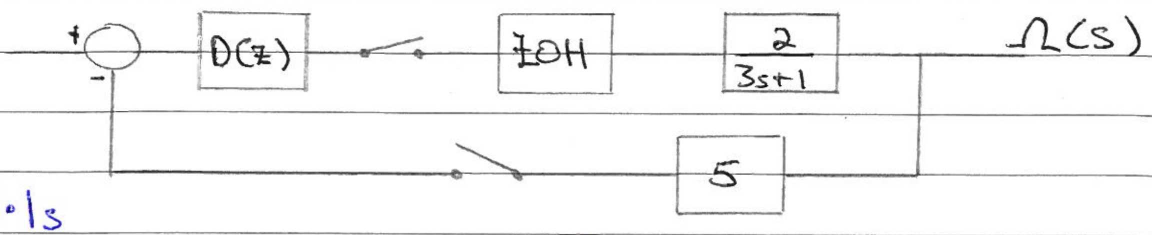
2c)



$T = 0{\cdot}1\,s$

$N = 0$

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{2}{3s+1}\cdot 5\right\}$$

$$= (1-z^{-1})\,\frac{10}{3}\,\mathcal{Z}\left\{\frac{1}{s(s+\frac{1}{3})}\right\} = \frac{10}{3}(3)\left(\frac{(1-e^{-\frac{T}{3}})z^{-1}}{(1-e^{-\frac{T}{3}}z^{-1})}\right)$$

$$= 10\left(\frac{0{\cdot}0328\,z^{-1}}{1-0{\cdot}967\,z^{-1}}\right) = \frac{0{\cdot}328}{z-0{\cdot}967}$$

$$D(z)G(z) = \frac{K\,0{\cdot}328}{z-0{\cdot}967}$$

Pole @ $z = 0{\cdot}967$

Marginal Stability @ $|D(z)G(z)| = 1$

~~$\Rightarrow K = 0{\cdot}1$~~

$\sigma = 0{\cdot}967$ $\quad |z-0{\cdot}967| = 0{\cdot}328K$

$j\omega = \pm j0{\cdot}255$ $\quad$ @ $z = 0{\cdot}967 \pm j0{\cdot}255$

$K = 0{\cdot}777$

Marginally stable @ $K = 0{\cdot}777$

Summer '04 Q2c