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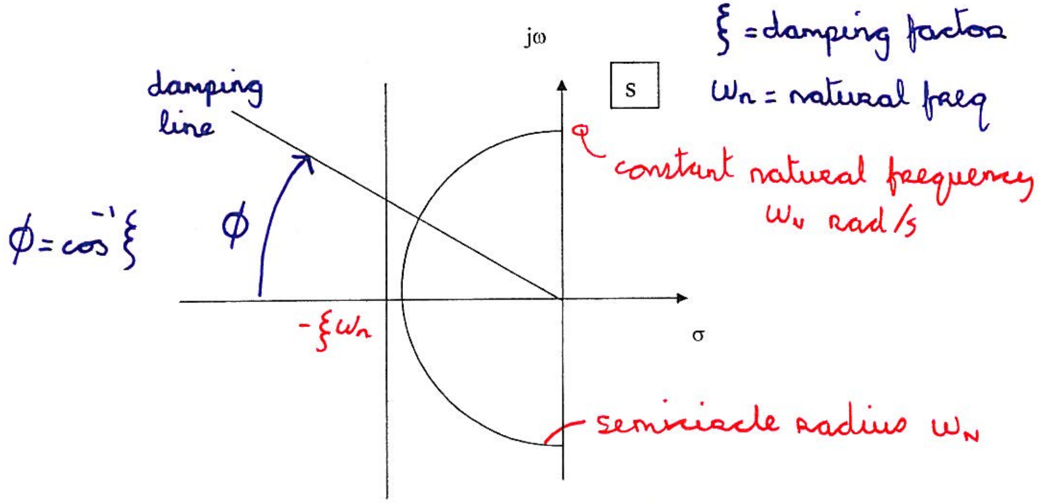
Chapter 7. Pole-Placement Design

7.1 The Z-Grid Template

The following design loci in the s plane are known:

ξ = damping factor

ω_n = natural freq



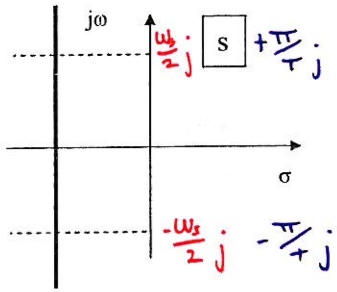
$$\xi\omega_N = \frac{4}{T_{s2\%}}$$

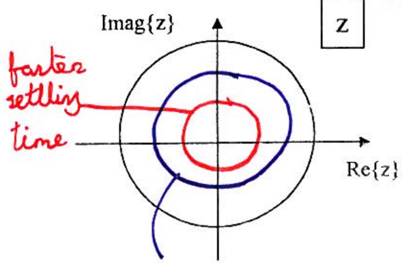
i) Mapping the Settling Time to the Z Plane

$s = -\xi\omega_N + bj$

$$z = e^{(-\xi\omega_N + bj)T} = e^{-\xi\omega_N T} e^{bTj}$$

$M \angle \phi$, ϕ: $-\pi \to \pi$



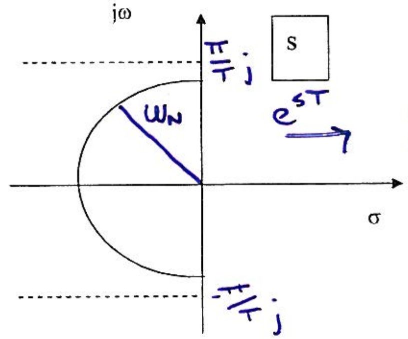


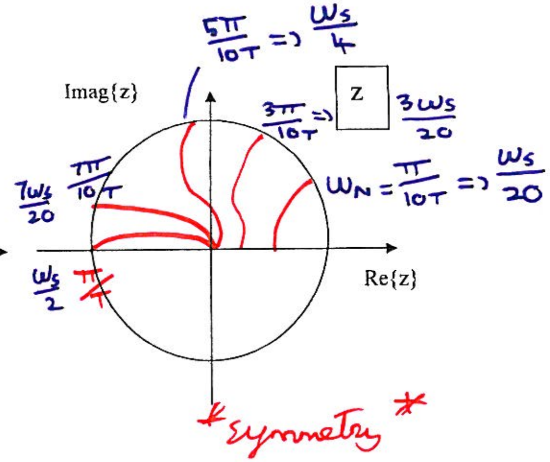
$$T_{s2\%} = \frac{4}{\xi\omega_N}$$

contour of constant settling time is a circle

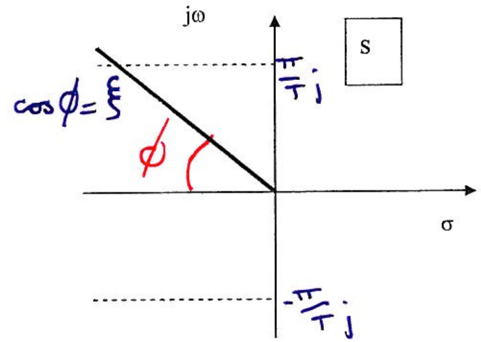
radius is $M = e^{-4T/T_{s2\%}}$

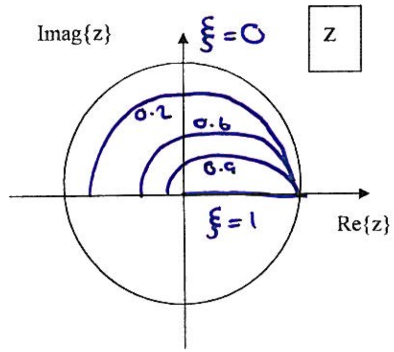
ii) Mapping the Natural frequency loci to the Z Plane





iii) Mapping the Damping Line to the Z Plane

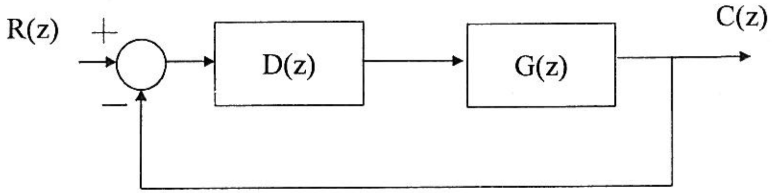




Yields the Z Grid Template:

7.2 Root Locus Design

The closed-loop discrete-time process is:



The characteristic equation is:

$$1 + D(z)G(z) = 0$$

Hence the poles of the closed-loop process obey $D(z)G(z)=-1$

Hence a testpoint $z=\zeta$ on the Z plane will be a pole of the closed-loop process if: $|D(z)G(z)|_{z=\zeta} = 1$

$\angle D(z)G(z)|_{z=\zeta} = 180°$

Consider now the controller is now factorised:

adjustable gain

$$D(z) = KD'(z)$$

Then the poles of the closed-loop process will be a function of the gain controller K. The root locus plot is the locus of the closed-loop poles on the Z plane as K is increased from 0 to∞.

Every point $z=\zeta$ on the root locus must obey:

$$|KD'(z)G(z)|_{z=\zeta} = 1$$

$$\angle KD'(z)G(z)|_{z=\zeta} = 180^\circ$$

7.2.1 Rules for Plotting Root Loci

1) There are as many loci as poles.

2) Loci begin on the poles of the OLTF.

3) Loci end on the zeros of the OLTF or at ∞.

4) Plots are symmetrical about the real axis.

5) For large values of z, the loci are asymptotic to straight lines which intersect the real axis at the point, α, where,

$$\alpha = \frac{sum\ of\ poles - sum\ of\ zeros}{no.of\ poles - no.of\ zeros}$$

6) These lines make angles θ with the real axis of:

$$\theta = \frac{(2k+1)\pi}{no.of\ poles - no.of\ zeros}\quad , k = 0, 1, 2, ...$$

7) On a given section of the real axis, a locus will exist if the sum of the poles and zeros to the right of the section is an odd number.

8) The angles of departure from complex poles and arrival at complex zeros are found by measuring the angle from the pole (or zero) to all other poles and zeros, and obtaining the residue angle:

angle of departure from pole (or arrival at zero) = residue angle - 180°

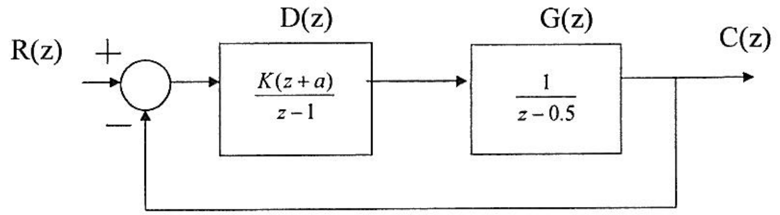
9) The intersection of the locus with the unit circle may be found using Jury's method.

10) If the n OLTF poles are $p_1, p_2, ...p_n$ and the m OLTF zeros are $z_1, z_2, ..z_m$, then the point of departure from the real axis, σ, (known as the breakaway point), must obey:

$$\sum_{i=1}^{n} \frac{1}{\sigma - p_i} = \sum_{j=1}^{m} \frac{1}{\sigma - z_j}$$

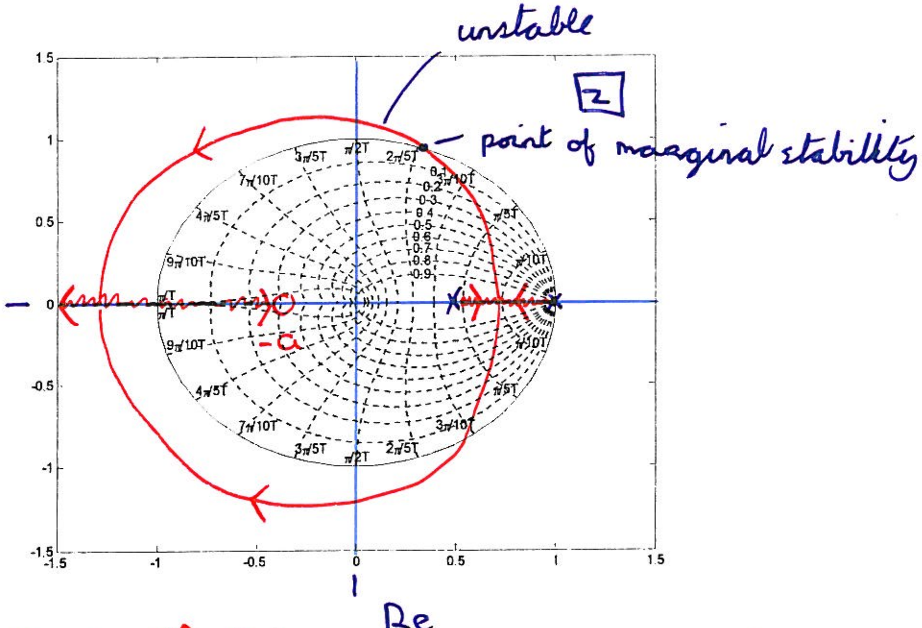
7.2.2 Transient response design via gain adjustment

Consider the example:



Open-loop Poles: $z = 1 \quad z = 0.5$

Open-loop Zeros: $z = -a$

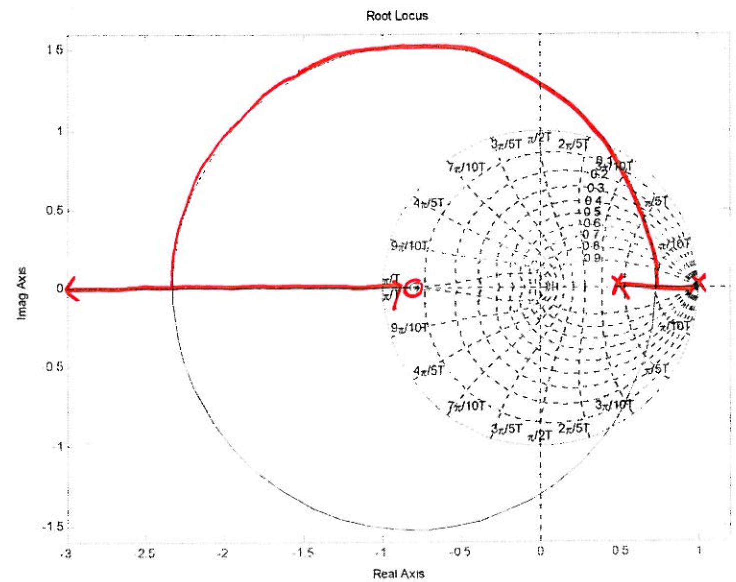


for a fixed a as $K \uparrow$ $\xi \downarrow$ $\omega_n \uparrow$ faster but larger PO%

Consider a=0.8 $\quad D(z) = \frac{K(z+0.8)}{z-1}$

zero @ $z = -0.8$

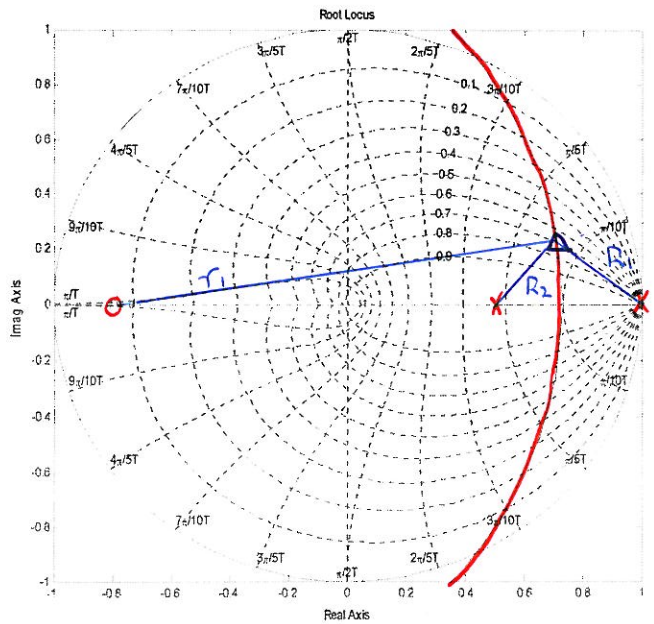
The root-locus diagram for $G(z)D(z) = \frac{K(z+0.8)}{z-1}\frac{1}{z-0.5}$ is:



Design K to achieve a closed-loop damping ξ=0.7

Find K so that locus cuts $\xi = 0.7$ contours

Focus in on the unit circle:



Desired poles are: $z = 0.7 \pm 0.2j$ read off cartesian scale

But we know that:

$$|D(z)G(z)|_{z=0.7+0.2j} = 1$$

That is:

$$|D(z)G(z)|_{z=0.7+j0.2} = 1 = \frac{K\,|0.7+0.2j+0.8|}{|0.7+0.2j-1|\,|0.7+0.2j-0.5|}$$

$$\frac{K(z+0.8)}{(z-1)(z-0.5)}$$

Or using the distances from open-loop poles and zeros:

$$= \frac{K r_1}{R_2 R_1} = \frac{K(1.5133)}{0.2828 \times 0.3606} = 1 \quad \Rightarrow K = 0.067$$

Tutorial: Simulate the closed-loop process in Simulink and verify that you get the desired peak overshoot for a step setpoint.

What is the value of K for stability?
marginal stability when you cut the unit circle
→ closed loop poles are at $0.42 \pm 0.9j$
$|DG|_{z=0.42\pm0.9j} = 1 \Rightarrow$ what is K ?

7.2.3 Designing a Phase-Lead Compensator

Consider the following digital phase lead compensator: "mapped pole zero"

$$D(z) = \frac{K(z-a)}{z-b} \qquad \frac{K(s+z)}{s+p} \xrightarrow{MPZ} \frac{Ka(z-a)}{z-b}$$

$a = e^{-\xi T} \quad b = e^{-pT}$

Place the zero, z=a, directly under the desired pole locations:-
To get the greatest attractive benefit from the zero

Adjust the pole position b:-
so that the desired pole location is now on the root locus

Adjust the gain K:-
$|DG|_{@\text{desired location}} = 1 \qquad \angle DG|_{@\text{desired location}} = 180^\circ$

EXAMPLE:

$$G(z) = \frac{10}{(z-1)(z-0.5)}$$

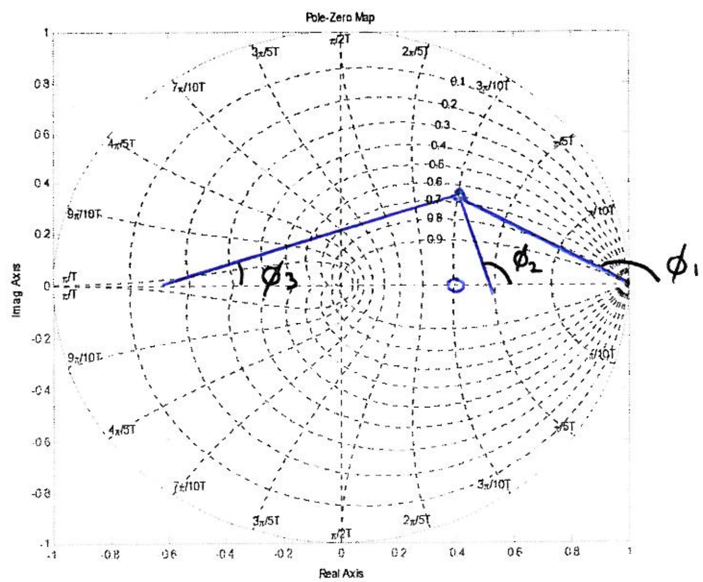
POLES @ $z=1$, $z=0.5$ O/L poles

Design a phase-lead compensator, with sample time T=0.8s to achieve the following closed-loop specifications:

$\omega_n = 1.2$ rad/s $\quad \xi = 0.65$

$1.2 = \frac{\alpha\pi}{10T}$

$1.2 = \frac{\alpha\pi}{16(0.8)} \Rightarrow \alpha \approx 3 \Rightarrow \frac{3\pi}{10T}$



Place the zero of compensator at: $z = 0.4$

The controller is then

$$D(z) = \frac{K(z-0.4)}{z-b}$$

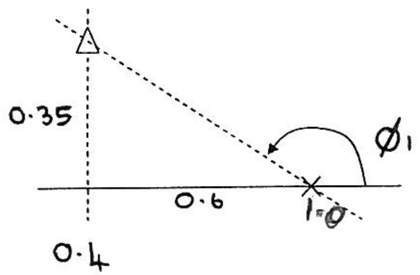
now choose b

Place the controller pole so that:

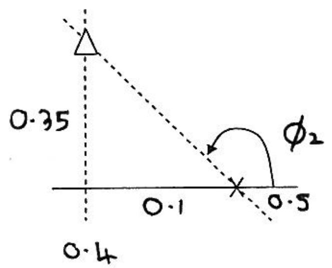
$$ARG(D(z)G(z))\big|_{z=0.4+j0.35} = 180^\circ$$

$$\phi_1 + \phi_2 + \phi_3 - \theta_1 = 180^\circ$$

SUM OF POLES − SUM OF ZEROS = 180°



$$\phi_1 = 180^\circ - \tan^{-1}\left(\frac{0.35}{0.6}\right) = 150^\circ$$



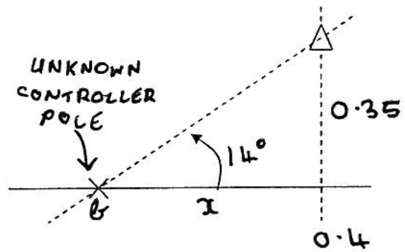
$$\phi_2 = 180^\circ - \tan^{-1}\left(\frac{0.35}{0.1}\right) = 106^\circ$$

Obviously: $\theta = 90^\circ$ because zero @ $z = 0.4$

Hence for the root locus to go through the desired point:

$$\phi_1 + \phi_2 + \phi_3 - \theta = 180^\circ$$

$$\phi_3 = 270^\circ - \phi_1 - \phi_2 = 270 - 150 - 106 = 14^\circ$$



The controller is now:

$$D(z) = \frac{K(z-0.4)}{z+1}$$

CONTROLLER ZERO

CONTROLLER POLE

Now determine the gain K so that at the desired point:

$$\left|\frac{K(z-0.4)}{z+1}\frac{10}{(z-1)(z-0.5)}\right|_{z=0.4+j0.35} = 1$$

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or:

$$\frac{10Kr_1}{R_1R_2R_3} = 1$$

0·68 — R_1, 0·34 — R_2, 1·43 — R_3

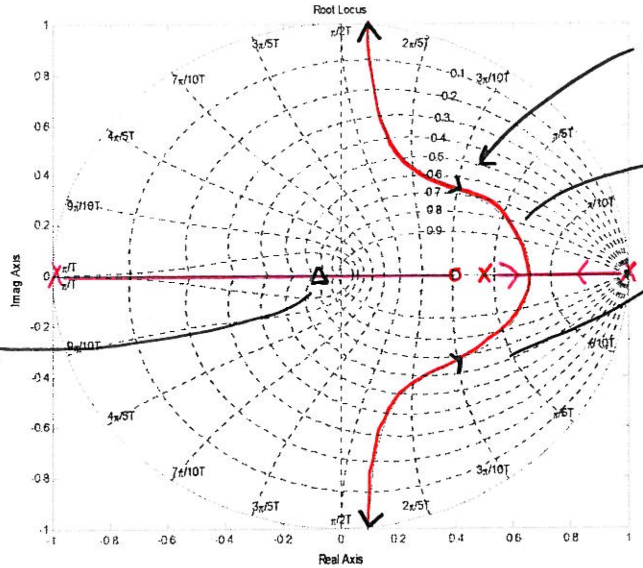
And the controller is then:

$$D(z) = 0.10\,\frac{z-0.4}{z+1}$$

Draw the compensated root locus for D(z)G(z):

$$D(z)G(z) = \frac{0.10(z-0.4)}{z+1}\,\frac{10}{(z-0.5)(z-1)}$$

The compensated root locus is:



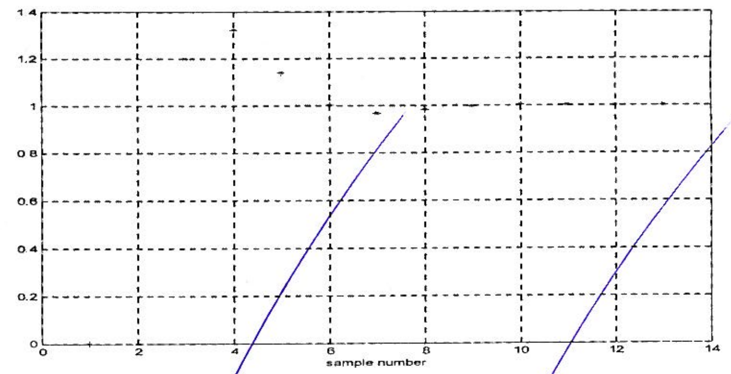
DESIRED LOCATION

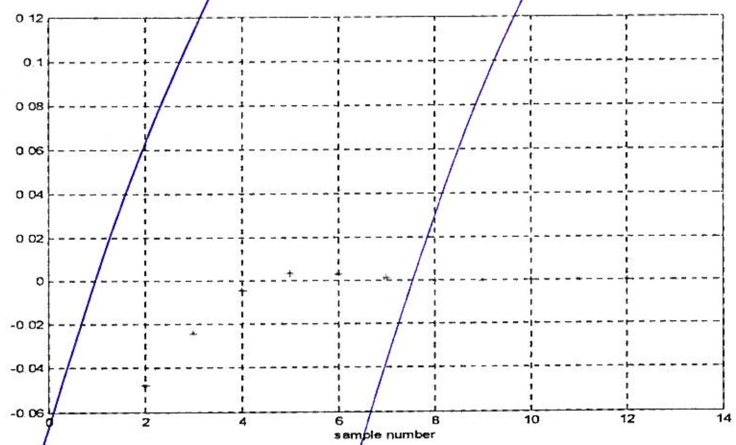
DOMINANT

FAST NEAR CENTRE OF UNIT CIRCLE

3rd order process

Ringing?? Due to pole @ $z=-1$
When $K = 0.104$ 3rd pole has a damping of 0·7
hidden but damped





Notes on Matlab:

rlocus:

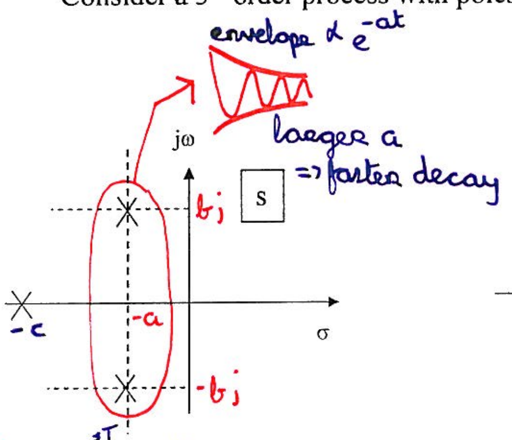
pzmap:

c2dm:

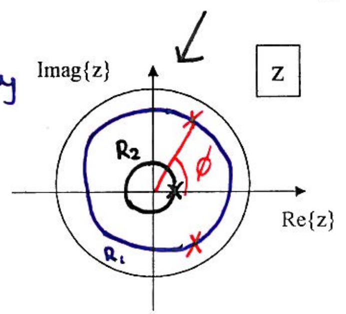
d2cm:

7.3 Note on dominance

Consider a 3rd order process with poles : s=-c and s=-a±bj



dominance is being close to unit circle



$s = -a \pm bj \xmapsto{e^{sT}} e^{-aT} \angle \pm bT = R_1 \angle \pm \phi$

$s = -c \mapsto e^{-cT} = R_2 \angle 0^\circ$

If $c \geq 5a$
five times further out left!

$\xrightarrow{e^{sT}} e^{-cT} \leq e^{-5aT}$

$e^{-cT} \leq (e^{-aT})^5$

$R_2 \leq R_1^5$

$R_1 < 1$

A simple rule of dominance:

- For the s plane a pole s = –a+bj dominates a pole s= –c+dj if: $c \geq 5a$ five times further out left

- For the z plane a pole $r_1\angle\phi$ dominates a pole $r_2\angle\theta$ if:

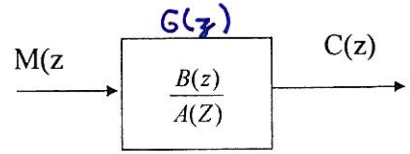
  $R_2 \leq R_1^5$

N.B. fast poles on the z plane are closer to centre of unit circle!

7.4 Pole-Placement Design- A polynomial Approach

7.4.1 The QST Control Scheme

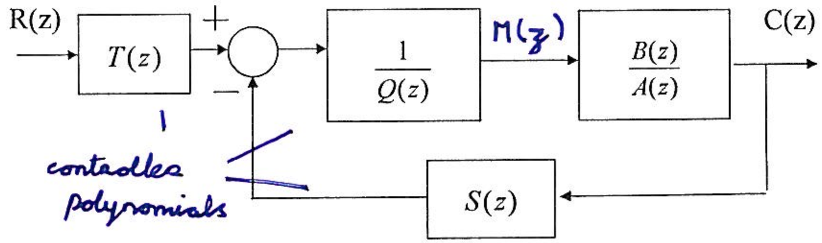
Consider the open-loop process:



Where the process is nth order and:

$$A(z) = z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots a_n$$
$$B(z) = b_1 z^{n-1} + b_2 z^{n-2} + \cdots b_m z^{n-m} \qquad n > m$$

Consider now the closed-loop control scheme: (QST)



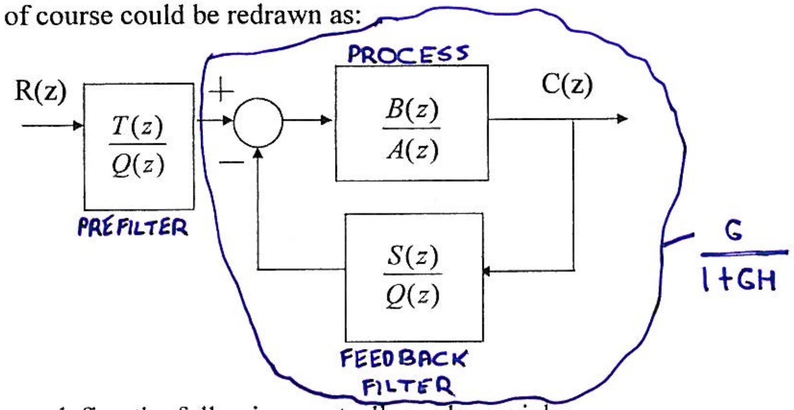
The control-law is then:

$$M(z) = \frac{1}{Q(z)}\left(T(z)R(z) - S(z)C(z)\right)$$

$$= \frac{T(z)}{Q(z)} R(z) - \frac{S(z)}{Q(z)} C(z)$$

$$Q(z)M(z) = T(z)R(z) - S(z)C(z)$$

This of course could be redrawn as:



We now define the following controller polynomials:

$$T(z) = t_0 z^{n_t} + t_1 z^{n_t-1} + t_2 z^{n_t-2} + \cdots + t_{n_t}$$

$$S(z) = s_0 z^{n_s} + s_1 z^{n_s-1} + s_2 z^{n_s-2} + \cdots + s_{n_s}$$

no loss of generality $Q(z) = z^{n_q} + q_1 z^{n_q-1} + q_2 z^{n_q-2} + \cdots + q_{n_q}$

For realisability – ie for causal control

$\frac{T(z)}{Q(z)}$ *and* $\frac{S(z)}{Q(z)}$ must both be causal: $n_q \geq n_t$, $n_q \geq n_s$

The closed-loop transfer function is:

$$\frac{C(z)}{R(z)} = \frac{T(z)}{Q(z)} \underbrace{\frac{\frac{B(z)}{A(z)}}{1+\frac{B(z)}{A(z)}\frac{S(z)}{Q(z)}}}_{\frac{G}{1+GH}} = \frac{T(z)B(z)}{A(z)Q(z)+B(z)S(z)}$$

The characteristic equation for the closed-loop system is:

$$A(z)Q(z) + B(z)S(z) = 0$$

Roots of the characteristic equation give the poles of the closed-loop system.

But how many closed-loop poles?

Remember: $B(z) \rightarrow (n-1)^{th}$ order polynomial

n^{th} order, n_q^{th} order, n_s^{th} order

Then: $\deg\big(\underbrace{A(z)Q(z)}_{n_q+n} + \underbrace{B(z)S(z)}_{(n-1)^{th}\text{ order}}\big) = \overline{}$ $n-1+n_s$

Hence there are $n+n_q$ poles for the closed-loop system.

n from process
n_q from the controller

7.4.2 The Polynomial Pole-Placement Design Route

The pole-placement design problem is then:

i) Select desired poles: $(n+n_q)$ poles to achieve some desired performance

ii) Specify desired closed-loop characteristic equation: $\rightarrow A_{cl}(z) = 0$

iii) Design S(z) and Q(z) $AQ + BS = A_d(z)$

iv) Design T(z) → To achieve desired setpoint tracking
- steady state error etc

The design equation:

$$\boxed{A_{cl}(z) = A(z)Q(z) + B(z)S(z)}$$

Is an example of a Diophantine Equation

Consider now that we require the closed-loop system to remain as n[th] order dominant.

We could factorise the desired closed-loop characteristic equation as follows:

$$A_{cl}(z) = \underbrace{A_C(z)}_{n^{th}\text{ order}}\underbrace{A_O(z)}_{n_q^{th}\text{ order}}$$

where:

$A_c(z) = (z-p_1)(z-p_2)\dots(z-p_n)$ n desired dominant poles

$A_o(z) = (z-p_{n+1})(z-p_{n+2})\dots(z-p_{n+nq})$ nq poles

place these n_q poles much closer to centre of unit circle ∴ FAST!!

We know from the closed loop transfer function that:

$$C(z) = \frac{B(z)T(z)}{\underbrace{A(z)Q(z)+B(z)S(z)}_{A_{cl}(z)}}R(z)$$

when the closed loop poles have been placed:

$$C(z) = \frac{B(z)T(z)}{A_{cl}(z)}R(z) = \frac{B(z)T(z)}{A_o(z)A_c(z)}R(z)$$

It is usual to choose T(z) to cancel out the fast poles: This is ok since $A_o(z)$ is stable

$T(z) = t_o A_o(z)$

This yields:

$$C(z) = \frac{t_o A_o B(z)}{A_o A_c(z)}R(z) = \frac{t_o B(z)}{A_c(z)}R(z)$$

zeros of open loop system are zeros of the closed loop

The gain t_o can now be adjusted to achieve a closed-loop DC gain of unity.

For unity DC gain:

$$\lim_{z\to 1}\frac{t_o B(z)}{A_c(z)} = 1$$

hence: $t_o = \dfrac{A_c(1)}{B(1)}$

<u>EXAMPLE:</u>

$$G(z) = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^2} = \frac{z+1}{(z-1)^2} = \frac{z+1}{z^2-2z+1} = \frac{B(z)}{A(z)}$$

The Diophantine equation is:

$$A_{cl}(z) = A(z)Q(z) + B(z)S(z)$$

First we will specify a simple zero-order controller: $n_q = 0$

l_q^0

$Q(z) = 1$

$T(z) = t_o$

$S(z) = s_o$

$M(z) = t_o R(z) - S_o C(z)$

$$A_{cl}(z) = A(z)Q(z) + B(z)S(z)$$

Which yields:

$$A_{cl}(z) = (z-1)^2\,1 + (z+1)\,s_0$$

$$A_{cl}(z) = z^2 - 2z + 1 + S_o z + S_o$$
$$= z^2 + (S_o-2)z + (S_o+1)$$

can't achieve arbitrary 2nd order response

Now try a first order controller:

$$Q(z) = z + q_1$$
$$S(z) = s_0 z + s_1$$
$$T(z) = t_0 z + t_1$$

The Diophantine equation becomes:

$$A_{cl}(z) = \underbrace{(z^2 - 2z + 1)}_{A}\underbrace{(z + q_1)}_{Q} + \underbrace{(z+1)}_{B}\,\underbrace{(s_0 z + s_1)}_{S}$$

$$A_{cl}(z) = z^3 + (q_1 + s_0 - 2)z^2 + (s_0 + s_1 - 2q_1 + 1)z + (q_1 + s_1)$$

Now consider the desired closed-loop characteristic equation for a 3[rd] order process: 2nd order open loop with a first order control

desired

$$A_{cl}(z) = z^3 + c_1 z^2 + c_2 z + c_3 = A_o(z)A_c(z)$$

choose these to obtain desired closed-loop performance

Comparing similar powers of z:

$q_1 + s_0 - 2 = c_1 \Rightarrow q_1 + s_0 = c_1 + 2$

$s_0 + s_1 - 2q_1 + 1 = c_2 \Rightarrow s_0 + s_1 - 2q_1 = c_2 - 1$

$q_1 + s_1 = c_3 \Rightarrow q_1 + s_1 = c_3$

Which could be written in matrix form as:

unknown controller gains

$$\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} c_1 + 2 \\ c_2 - 1 \\ c_3 \end{bmatrix}}_{y}$$

known for some desired performance

Solveable if $\det A \neq 0$

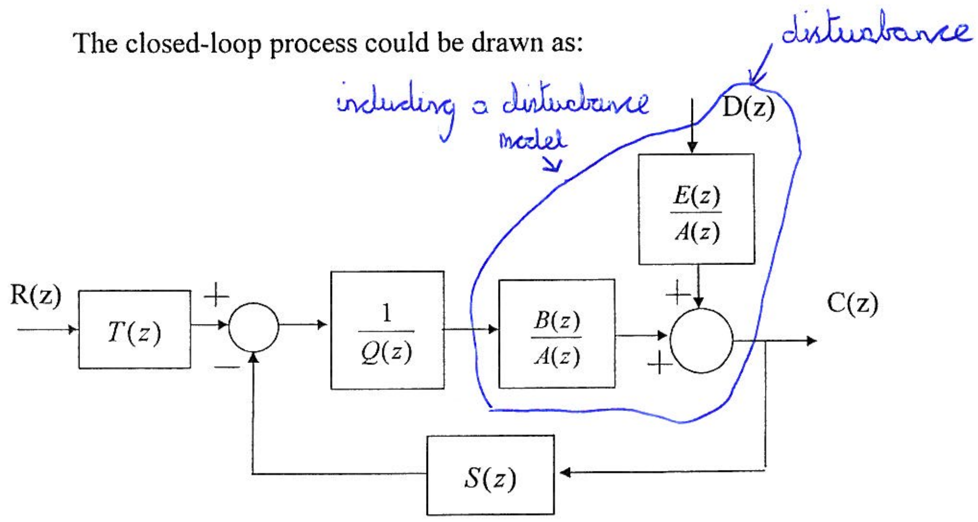
Hence the controller parameters are obtained as:

$$\underbrace{\begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0.25 & -0.25 & 0.25 \\ 0.75 & 0.25 & -0.25 \\ -0.25 & 0.25 & 0.75 \end{bmatrix}}_{A^{-1}}\underbrace{\begin{bmatrix} c_1 + 2 \\ c_2 - 1 \\ c_3 \end{bmatrix}}_{y}$$

$$x = A^{-1}y$$

7.4.3 Steady State Errors

The closed-loop process could be drawn as:



We know that with the choice:

$$T(z) = t_0 A_0(z) \qquad t_0 = \frac{A_c(1)}{B(1)}$$

Yields a unity DC gain: to achieve perfect tracking of steps in $R(k)$

But this technique can be sensitive to errors in the B(z) polynomial:

as process changes $B(1)$ will vary and t_0 will no longer be accurate $\therefore e_{ss} \neq 0$

NOTE: Good tracking of the setpoint does not imply good disturbance rejection.

TUTORIAL: Determine the steady-state error for an asymptotically constant disturbance, if the process B/A is "type 0" and if T(z)=(Ac(1)/B(1))A$_o$ (z).

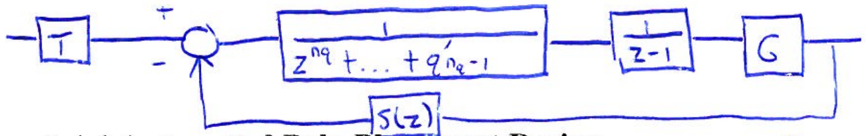
$$\lim_{k\to\infty} d(k) = d_\infty$$

Redo, with B/A as "type 1".

If we need to increase the Type of the process, ie. to introduce integration, we could force a factorisation of Q(z):

$$Q(z) = z^{n_q} + q_1 z^{n_q-1} + q_2 z^{n_q-2} + \cdots + q_{n_q}$$

$\Rightarrow Q(z) = (z-1)(z^{n_{q-1}} + q_1' z^{n_{q-2}} + \ldots + q'_{n_q-1})$ only n_{q-1} "free" parameters



7.4.4 Automated Pole-Placement Design

The Diophantine Equation is:

$$A_{cl}(z) = A(z)Q(z) + B(z)S(z)$$

First assume without loss of generality that:

$\deg(A(z)) = n$ Let $n_q = n_s = n-1$ and $m = n$

or use an incremental model

Hence:

$$\underbrace{\left(z^n + a_1 z^{n-1} + \cdots a_n\right)}_{A}\underbrace{\left(z^{n-1} + q_1 z^{n-2} + \cdots q_{n-1}\right)}_{Q} + \underbrace{\left(b_1 z^{n-1} + b_2 z^{n-2} + \cdots b_n\right)}_{B}\underbrace{\left(s_0 z^{n-1} + s_1 z^{n-2} + \cdots s_{n-1}\right)}_{S}$$

$$= z^{2n-1} + c_1 z^{2n-2} + \ldots c_{2N-1}$$

Compare similar powers of z:

z^{2n-1} : $1 = 1$

z^{2n-2} : $c_1 = a_1 + q_1 + b_1 s_0 \rightarrow c_1 - a_1 = q_1 + b_1 s_0$

z^{2n-3} : $c_2 = a_2 + q_2 + a_1 q_1 + b_2 s_0 + b_1 s_1$

z^{2n-4} : $c_3 = a_3 + q_3 + a_1 q_2 + a_2 q_1 + b_1 s_2 + b_3 s_0 + b_2 s_1$

$\vdots \quad \vdots \quad \vdots$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \ldots & 0 & b_1 & 0 & 0 & \ldots & 0 \\ a_1 & 1 & 0 & \ldots & 0 & b_2 & b_1 & 0 & \ldots & 0 \\ a_2 & a_1 & 1 & 0 \ldots & 0 & b_3 & b_2 & b_1 & 0 \ldots & 0 \\ & & & & & & & & & \end{array}\right] \begin{bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ s_0 \\ \vdots \\ s_{n-1} \end{bmatrix} = \begin{bmatrix} c_1 - a_1 \\ c_2 - a_2 \\ c_3 - a_3 \\ \vdots \\ c_n - a_n \\ c_{n+1} \\ \vdots \\ c_{2n-1} \end{bmatrix}$$

n-1 n n-1 n unknown (2n-1)

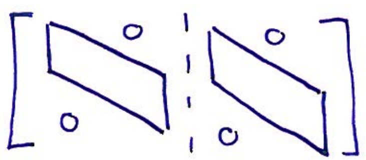
The complete equations are then:

$$\underbrace{\left[\begin{array}{ccccccc|cccccc} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & b_1 & 0 & 0 & 0 & \cdots & 0 \\ a_1 & 1 & 0 & 0 & 0 & \cdots & 0 & b_2 & b_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 1 & 0 & 0 & \cdots & 0 & b_3 & b_2 & b_1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 1 & 0 & \cdots & 0 & b_4 & b_3 & b_2 & b_1 & \cdots & 0 \\ a_4 & a_3 & a_2 & a_1 & 1 & \cdots & 0 & b_5 & b_4 & b_3 & b_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n-2} & a_{n-3} & a_{n-4} & a_{n-5} & a_{n-6} & \cdots & 1 & b_{n-1} & b_{n-2} & b_{n-3} & b_{n-4} & \cdots & 0 \\ a_{n-1} & a_{n-2} & a_{n-3} & a_{n-4} & a_{n-5} & \cdots & a_1 & b_n & b_{n-1} & b_{n-2} & b_{n-3} & \cdots & b_1 \\ a_n & a_{n-1} & a_{n-2} & a_{n-3} & a_{n-4} & \cdots & a_2 & 0 & b_n & b_{n-1} & b_{n-2} & \cdots & b_2 \\ 0 & a_n & a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_3 & 0 & 0 & b_n & b_{n-1} & \cdots & b_3 \\ 0 & 0 & a_n & a_{n-1} & a_{n-2} & \cdots & a_4 & 0 & 0 & 0 & b_n & \cdots & b_4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & a_n & 0 & 0 & 0 & 0 & \cdots & b_n \end{array}\right]}_{n-1 \qquad n} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_{n-1} \\ s_0 \\ s_1 \\ s_2 \\ \vdots \\ s_{n-1} \end{bmatrix}}_{2n-1 \text{ unknowns}} = \begin{bmatrix} c_1 - a_1 \\ c_2 - a_2 \\ c_3 - a_3 \\ \vdots \\ c_n - a_n \\ c_{n+1} \\ c_{n+2} \\ c_{n+3} \\ \vdots \\ c_{2n-1} \end{bmatrix}$$

2n-1

$$A\underline{x} = \underline{Y}$$

Note the structure of the Sylvester Matrix:



Banded

The parameters of the controller polynomials can now be calculated as:

$$\underline{x} = A^{-1}\underline{y}$$

Theory: The Sylvester Matrix is invertible if the polynomials A(z) and B(z) do not have any common factors:

EXAMPLE:

$$G(z) = \frac{z^{-1} + 0.7z^{-2}}{(1 - z^{-1})(1 - 0.8z^{-1})} = \frac{z + 0.7}{(z-1)(z-0.8)} = \frac{\overset{b_1}{1}z + \overset{b_2}{0.7}}{z^2 \underset{a_1}{-1.8}z + \underset{a_2}{0.8}}$$

Choose the following polynomials:

$n_q = n_s = (n-1) = 1$

$$Q(z) = z + q_1$$
$$S(z) = s_0 z + s_1$$

Third order characteristic equation:

$$A_{cl}(z) = z^3 + c_1 z^2 + c_2 z + c_3$$

The following matrix equation could be written:

$$\begin{array}{c} \\ 2n-1 \\ =3 \end{array} \quad \begin{array}{c} \overset{n-1}{} \; \overset{n}{} \\ \begin{bmatrix} 1 & b_1 & 0 \\ a_1 & b_2 & b_1 \\ a_2 & 0 & b_2 \end{bmatrix} \\ 1 \quad 2 \end{array} \begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} c_1 - a_1 \\ c_2 - a_2 \\ c_3 \end{bmatrix} \begin{array}{l} n=2 \\ \\ (n-1)=1 \end{array}$$

The specification for the closed-loop performance is:

SAMPLE TIME

T=0.5seconds $\qquad \omega_n$=2rad/s $\qquad \xi$=0.707

Using the template: place dominant (slow) pole pair

*confirm $\quad z = 0.35 \pm 0.32j \Rightarrow 0.47\angle 42.4^\circ$

Place the fast pole at: first look at

TO ENSURE 2nd ORDER DOMINANCE

$z = (0.47)^3 = 0.023$

choose $z = 0.03$

no problem choosing $z = 0$

The desired closed loop characteristic equation is:

$$A_{cl}(z) = A_O(z)A_C(z) = (z-0.03)(z^2-0.7z+0.22)$$
$$= z^3 - 0.73z^2 + 0.24z - 0.0066$$

TRY THIS BY PLACING THIS FAST POLE AT THE ORIGIN

Then the controller parameters are given by:

$$\begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1.8 & 0.7 & 1 \\ 0.8 & 0 & 0.7 \end{bmatrix}^{-1} \begin{array}{c} c_1 - a_1 \\ \begin{bmatrix} 1.07 \\ -0.56 \\ -0.0066 \end{bmatrix} \end{array} \begin{array}{l} \\ c_2 - a_2 \\ c_3 \end{array} = \begin{bmatrix} 0.3567 \\ 0.7133 \\ \vdots \\ -0.4171 \end{bmatrix}$$

This yields the controller polynomials:

$$Q(z) = z + 0.3567$$
$$S(z) = 0.7133z - 0.4171$$

With the prefilter:

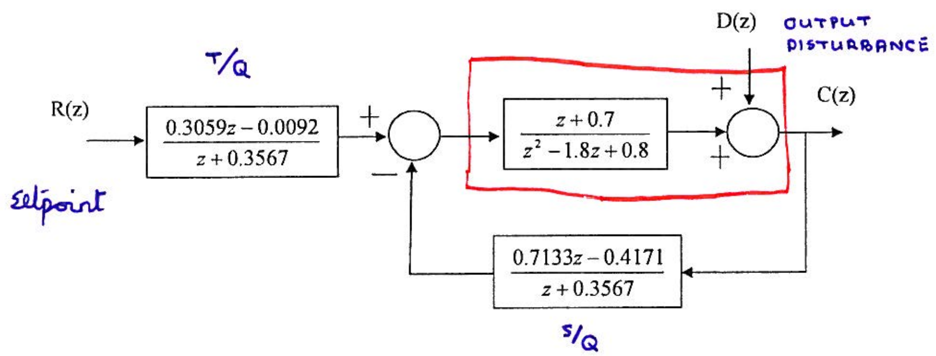
$$T(z) = t_0 A_O = t_0 \overbrace{(z - 0.03)}^{A_o(z)}$$

where:

$$t_0 = \frac{A_C(1)}{B(1)} = \frac{1 - 0.7 + 0.22}{1 + 0.7} = 0.3059$$

$A_c(z) = z^2 - 0.7z + 0.22$

$B(z) = z + 0.7$



T/Q
R(z)
0.3059z − 0.0092 / z + 0.3567
setpoint
+
−
z + 0.7 / z² − 1.8z + 0.8
D(z)
OUTPUT DISTURBANCE
+
+
C(z)
0.7133z − 0.4171 / z + 0.3567
S/Q

Be