EE4004 Telecomms Summer '08

6 a) $B_s = 10\,kHz$ $\quad H(x) = \log_2[256]$ $\quad \frac{\eta}{2} = 10^{-11}\ W/Hz$

$f_s = 30\,kHz$ $\quad H(x) = 8$ $\quad S = 4\times10^{-4}\ W$

Information Rate $= f_s H(x)$

$= 240\,kbps$

For min. channel bandwidth B_c

$$240\times10^3 = C = B_c \log[1 + \tfrac{S}{N}]$$

$$N = \eta B_c = B_c \times 2\times10^{-11}$$

For $B_c = 25\,kHz$
$C = 241.14\,kbps$

For $B_c = 24.9\,kHz$
$C = 240.32\,kbps$

By trial & error, $B_c = 24.861\,kHz$

By linear interpolation:

$B_c = 24.8\,kHz$
$C = 239.5\,kbps$

$$\left(\frac{240-239.5}{240.32-239.5}\right)100Hz + 24.8kHz$$

$B_c = 24.861\,kHz$

b)

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2\,dt$$

$$= \int_0^T [A_1\cos(\omega_1 t) - A_2\cos(\omega_2 t)]^2\,dt$$

$$= \int_0^T A_1^2\cos^2(\omega_1 t) + \int_0^T A_2^2\cos^2(\omega_2 t) - \int_0^T 2A_1A_2\cos(\omega_1 t)\cos(\omega_2 t)$$

$$E_d = A_1^2\int_0^T \cos^2(\omega_1 t) + A_2^2\int_0^T \cos^2(\omega_2 t) - A_1A_2\int_0^T \cos[(\omega_1+\omega_2)t]$$
$$- A_1A_2\int_0^T \cos[(\omega_1-\omega_2)t]\,dt$$

$$E_d = A_1^2\frac{T}{2} + A_2^2\frac{T}{2} - A_1A_2\left[\frac{\sin(\omega_1+\omega_2)T}{\omega_1+\omega_2} + \frac{\sin(\omega_1-\omega_2)T}{\omega_1-\omega_2}\right]$$

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$$E_d = \frac{A_1^2 T}{2} + \frac{A_2^2 T}{2} - \frac{A_1 A_2}{\omega_1+\omega_2}\left[\sin(\omega_1+\omega_2)T - \sin(\omega_1-\omega_2)T\right]$$

$$= \frac{A_1^2 T}{2} + \frac{A_2^2 T}{2} - \frac{A_1 A_2}{\omega_1+\omega_2}\left[2\cos(\omega_1 T)\sin(\omega_2 T)\right]$$

Since $\Rightarrow T = \frac{2\pi}{\omega_{1,2}}$, $\sin(\omega_2 T) = \sin(2\pi) = 0$

$$E_d = (A_1^2 + A_2^2)\frac{T}{2}$$

$$E_b = \frac{1}{2}\int_0^T \left(s_1^2(t) + s_2^2(t)\right)dt$$

$$= \frac{1}{2}\int_0^T A_1^2\cos^2(\omega_1 t)\,dt + \frac{1}{2}\int_0^T A_2^2\cos^2(\omega_2 t)\,dt$$

$$\frac{A_1^2 T}{4} + \frac{A_2^2 T}{4}$$

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6 a) $\left(\frac{S}{N}\right)_{OUTPUT} = \frac{2E_d}{\eta}$ $s_1(t) = k s_2(t)$, $k = \pm 1$

$$\frac{S}{N} = \frac{2}{\eta}\int_0^T [s_1(t) - s_2(t)]^2 dt$$

$$= \frac{2}{\eta}\int_0^T (s_1^2(t) + s_2^2(t))\, dt - \frac{4}{\eta}\int_0^T s_1(t)s_2(t)\, dt$$

$$= \frac{2}{\eta}(2E) - \frac{4}{\eta}\int_0^T s_1(t)s_2(t)\, dt$$

Using Schwartz:

$$\int_0^T s_1(t)s_2(t)\, dt \le \sqrt{\int_0^T s_1^2(t)\,dt \int_0^T s_2^2(t)\,dt}$$

$$= \sqrt{E^2} = E$$

Equality holds for $s_1(t) = k s_2(t)$

SNR is maximised for $k = -1$

$$\left(\frac{S}{N}\right) = \frac{4E}{\eta} + \frac{4E}{\eta} = \frac{8E}{\eta}$$

b) $P_e = Q\sqrt{\frac{E_d}{2\eta}}$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$$

$$E_d = A^2\int_0^T \cos^2(\omega_1 t)\,dt + A^2\int_0^T \cos^2(\omega_2 t)\,dt - A^2\int_0 2\cos(\omega_1 t)\cos(\omega_2 t)\,dt$$

$$= \frac{A^2T}{2} + \frac{A^2T}{2} - \left[A^2\int_0^T \cos(\omega_1+\omega_2)t + A^2\int_0^T \cos(\omega_1-\omega_2)t\, dt\right]$$

$$= A^2T - A^2\left[\cancelto{0}{\frac{\sin[(\omega_1+\omega_2)T]}{(\omega_1+\omega_2)}} + \cancelto{0}{\frac{\sin[(\omega_1-\omega_2)T]}{(\omega_1-\omega_2)}}\right]$$

$$P_e = Q\sqrt{\frac{A^2T}{2\eta}}$$

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6 c) $Ed = \int_0^{T_\phi} [2A\cos(\omega_1 t)]^2 \, dt$

$= 4A^2 \int_0^{T_\phi} \cos^2(\omega_1 t) \, dt$

$= 4A^2 \frac{1}{2\omega_1} \left[\omega_1 T_\phi + \cancel{\frac{\cos(2\omega_1 T_\phi)}{2}} \right]$

$Ed = \frac{4A^2 T_\phi}{2} = 2A^2 T_\phi$

If $Pe_1 = Pe_2$, $\sqrt{\frac{A^2 T_f}{2\eta}} = \sqrt{\frac{2A^2 T_\phi}{2\eta}}$

$T_f = 2T_\phi \Rightarrow \frac{T_f}{T_\phi} = 2$

=> Phase Shift Keying has a higher bitrate

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$$E_d = \int_0^T [s_1(t) - s_2(t)]^2\, dt$$
$$E_b = \tfrac{1}{2}\int_0^T s_1^2(t) + s_2^2(t)\, dt$$
$$s_i(t) = \begin{cases} s_1(t) = A_1\cos(\omega_c t) & 0 \le t \le T \\ s_2(t) = A_2\cos(\omega_c t) & 0 \le t \le T \end{cases}$$

a) $P_e = Q\sqrt{\dfrac{E_d}{2\eta}}$ $\qquad E_d = \int_0^T [(A_1 - A_2)\cos(\omega_c t)]^2\, dt$
$$= (A_1 - A_2)^2 \int_0^T \cos^2(\omega_c t)\, dt$$

$$E_d = (A_1 - A_2)^2 \frac{1}{2\omega_c}\left[\omega_c t + \frac{\sin(2\omega_c t)}{2}\right]_0^T$$
$$= (A_1 - A_2)^2 \left[\frac{2\omega_c T + \cancel{\sin(2\omega_c T)}^{\,0}}{4\omega_c}\right]$$
$$= \frac{(A_1 - A_2)^2\, T}{2}$$
$$P_e = Q\sqrt{\frac{(A_1 - A_2)^2\, T}{4\eta}}$$

b) $E_b = \frac{1}{2}\int_0^T (A_1^2 + A_2^2)\cos^2(\omega_c t)\, dt$
$$= \frac{1}{2}(A_1^2 + A_2^2)\int_0^T \cos^2(\omega_c t)\, dt$$
$$= \frac{(A_1^2 + A_2^2)}{2}\frac{T}{2} = \text{constant} \Rightarrow \therefore\ A_1^2 + A_2^2 = m = \text{constant}$$

P_e is minimised for $y = \frac{(A_1 - A_2)^2 T}{4\eta}$ is maximised

$$\frac{dy}{dA_1} = 2\left(1 - \frac{dA_2}{dA_1}\right)\frac{T}{4\eta}(A_1 - A_2) = 0 \Rightarrow \left(1 - \frac{dA_2}{dA_1}\right)(A_1 - A_2) = 0$$

$\frac{dm}{dA_1} = \frac{d(A_2^2)}{dA_1} + 2A_1 = 0$ Using chain rule: $\frac{d(A_2^2)}{dA_2}\cdot\frac{dA_2}{dA_1} + 2A_1 = 0$

$$\Rightarrow \frac{dA_2}{dA_1} = -\frac{2A_1}{2A_2} \Rightarrow \left(1 + \frac{A_1}{A_2}\right)(A_1 - A_2) = 0$$

But $A_1 \neq A_2$

$$A_1 = -A_2$$

7 a) $B_s = 10\,kHz$ $H(x) = \log_2[128]$

$f_s = 30\,kHz$ $= 7$

$R = f_s H(x)$

$= 210$ kbps

$210 \times 10^3 = C = B_c \log_2\left[1 + \frac{S}{N}\right]$

$N = \eta B_c$

$\eta = 2 \times 10^{-11}$ W/Hz

$S = 4 \times 10^{-4}$ W

For $B_c = 25\,kHz$ $C = 241 \cdot 14$ kbps

For $B_c = 20\,kHz$ $C = 199 \cdot 34$ kbps

By linear interpolation, for $C = 200$ kbps $B_c = 20 \cdot 315\,kHz$

By trial & error, $B_c \simeq 20 \cdot 077\,kHz$

b) $E_A = \int_0^T A_1^2 \, dt$ $E_b = \int_0^T A_2^2 + (-A_2)^2 \, dt$

$= A_1 T$ $= 2A_2^2 T$

$\frac{E_A}{E_b} = \frac{1}{2}$

c)