

# EE4011 RFIC Design

## Non-linear Effects in RF Systems: Other Effects and Dynamic Range

# Another Unwanted Effect: Blocking/Desensitization

When there is a strong interference signal in addition to a weak desired signal the strong interference signal can cause the effective gain of the weak desired signal to be reduced significantly because of gain compression.

e.g. if  $A_1 \cos \omega_1 t$  is the weak desired signal and  $A_2 \cos \omega_2 t$  is the strong interferer ( $A_1 \ll A_2$ ) then from the general formula derived previously the o/p at  $\omega_1$  will be:

$$y_1(t) = \left[ \alpha_1 + \frac{3}{4} \alpha_3 A_1^2 + \frac{3}{2} \alpha_3 A_2^2 \right] A_1 \cos(\omega_1 t)$$
$$\approx \left[ \alpha_1 + \frac{3}{2} \alpha_3 A_2^2 \right] A_1 \cos(\omega_1 t) \quad , \quad A_1 \ll A_2$$

Thus the effective gain for the desired signal component at  $\omega_1$  is:

$$G = \alpha_1 + \frac{3}{2} \alpha_3 A_2^2$$

Assuming  $\alpha_1 > 0$  and  $\alpha_3 < 0$  then as  $A_2$  increases  $G$  decreases causing the system to be desensitized w.r.t. the desired signal.

# Yet Another Unwanted Effect: Cross Modulation

From the previous slide where there is a weak desired signal ( $A_1, \omega_1$ ) in addition to a strong undesired signal ( $A_2, \omega_2$ ), ( $A_1 \ll A_2$ ):

$$y_1(t) = \left[ \alpha_1 + \frac{3}{2} \alpha_3 A_2^2 \right] A_1 \cos(\omega_1 t)$$

If the strong signal is amplitude modulated it can be written as:

$$s(t) = C_2 (1 + m \cos(\omega_m t)) \cos(\omega_2 t) \Rightarrow A_2 = C_2 (1 + m \cos(\omega_m t))$$

Putting this into  $y_1(t)$ :

$$\begin{aligned} y_1(t) &= \left[ \alpha_1 + \frac{3}{2} \alpha_3 C_2^2 (1 + m \cos(\omega_m t))^2 \right] A_1 \cos(\omega_1 t) \\ &= \left[ \alpha_1 + \frac{3}{2} \alpha_3 C_2^2 (1 + 2m \cos(\omega_m t) + m^2 \cos^2(\omega_m t)) \right] A_1 \cos(\omega_1 t) \\ &= \left[ \alpha_1 + \frac{3}{2} \alpha_3 C_2^2 \left( 1 + 2m \cos(\omega_m t) + \frac{m^2}{2} + \frac{m^2}{2} \cos(2\omega_m t) \right) \right] A_1 \cos(\omega_1 t) \end{aligned}$$

$m$  = modulation index  
 $\omega_m$  = modulation frequency

i.e. the desired output signal contains amplitude modulation at  $\omega_m$  and  $2\omega_m$  and the amplitude modulation has transferred from the strong signal to the weak one.

# Signal Range of A System

The ability of a system to cope with weak input signals is determined mainly by the noise performance of the system, whereas the ability to handle strong input signals is determined by gain compression and intermodulation concerns.

Therefore, limits can be set on the lower and higher power levels that should be applied to a system to keep its performance within the desired parameters. These limits give rise to terms such as:

1. Sensitivity
2. Noise Floor
3. Dynamic Range

# Sensitivity - 1

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig} / P_{RS}}{SNR_{out}} \Rightarrow P_{sig} = P_{RS} \cdot F \cdot SNR_{out}$$

$P_{sig}$  is the input signal power.  $P_{RS}$  is the noise power provided by the source resistance. Usually the above quantities are determined at a particular frequency per unit frequency. Assuming the quantities remain flat over the bandwidth  $B$  the total signal power required to obtain a given SNR at the output in a system with noise factor  $F$  and bandwidth  $B$  is:

$$P_{sig,tot} = P_{RS} \cdot F \cdot SNR_{out} \cdot B$$

This can be turned into a criterion for the minimum input signal power that is needed to obtain a minimum SNR at the output – this is the sensitivity of the system. Turning quantities into dBm:

$$\text{Sensitivity: } P_{in,min} = P_{RS}|_{dBm/Hz} + NF + SNR_{min}|_{dB} + 10\log_{10} B$$

$$\text{where } NF = 10\log_{10} F$$

# Sensitivity - 2

If the input is conjugate matched to the source the noise power delivered to the input will be:

$$P_{RS} = \frac{\overline{v_n^2}}{4R_S} = \frac{4kTR_S}{4R_S} = kT = -174 \text{ dBm} / \text{Hz} \quad \text{at room temperature}$$

Thus:

$$\text{Sensitivity: } P_{\text{in,min}} = \underbrace{-174 \text{ dBm/Hz} + NF + 10 \log_{10} B}_{\text{noise floor}} + SNR_{\text{min}} \big|_{\text{dB}}$$

The sum of the first three terms above is the total integrated noise of the system which is sometimes called the “noise floor” of the system. This depends on the temperature, the noise figure of the system and the bandwidth of the system. One way to make the system less noisy is to reduce the bandwidth.

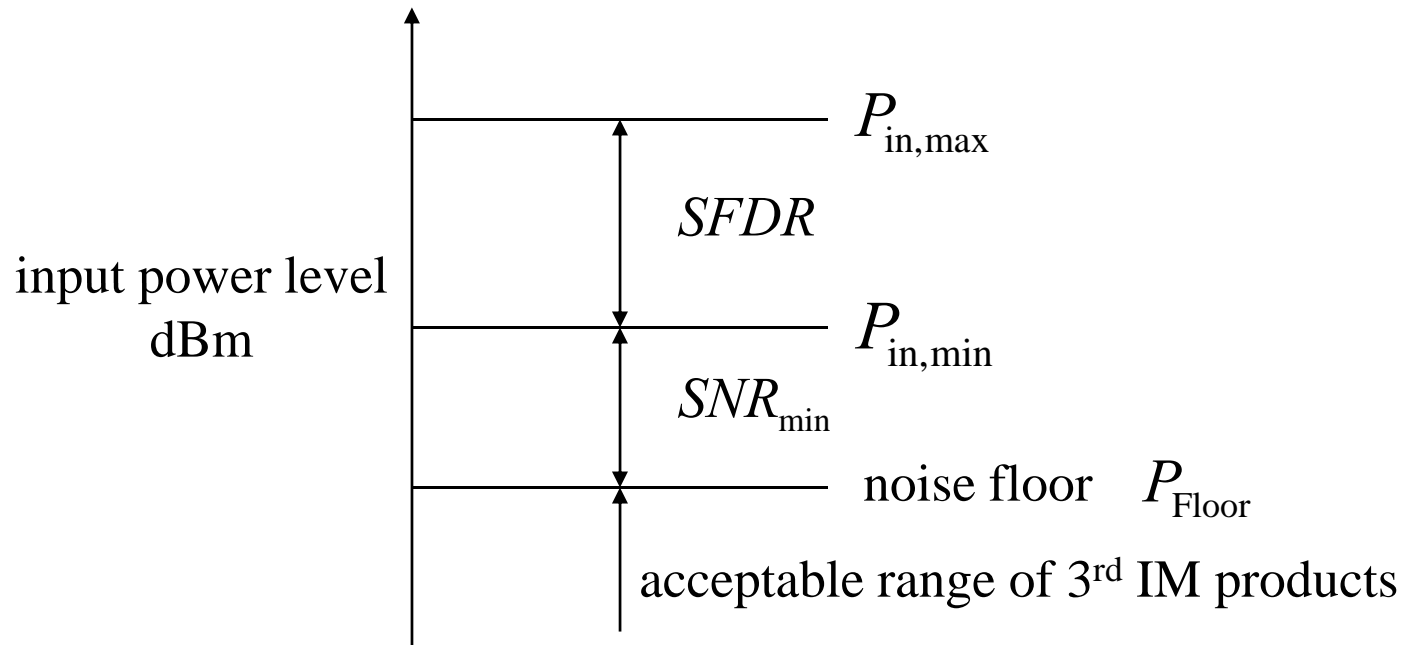
$$P_{\text{Floor}} = -174 \text{ dBm/Hz} + NF + 10 \log_{10} B \quad (\text{room temperature})$$

# Dynamic Range - 1

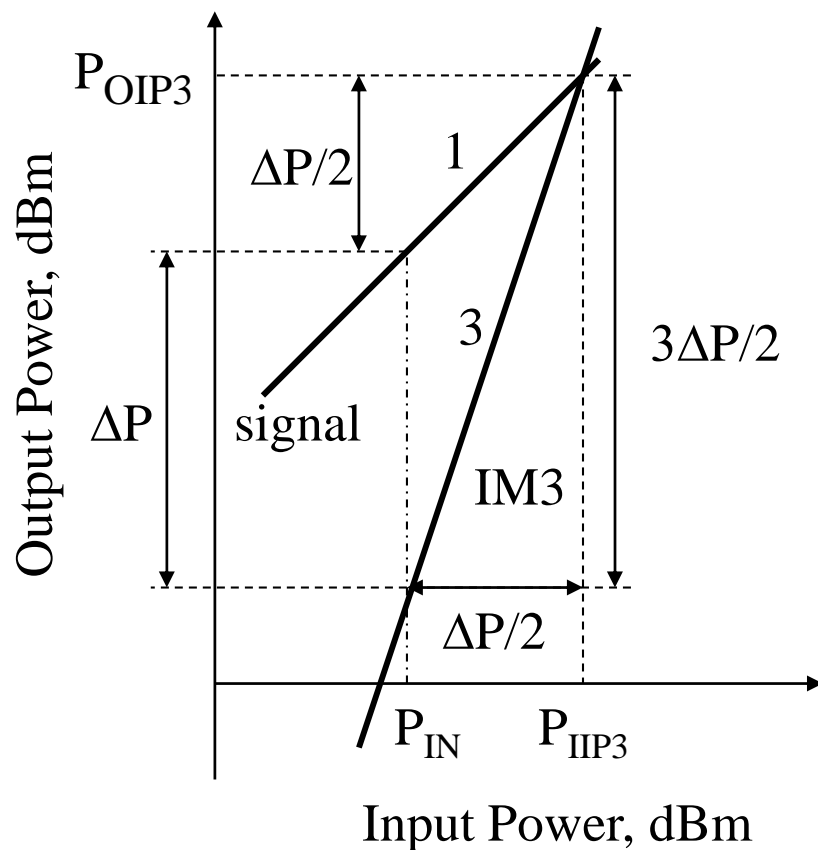
The minimum input signal power is determined by the sensitivity.

As the input power increases, the power of the intermodulation products also increase.

At some point the IM products reach an unacceptable level – the input power at which this occurs could be considered to be the maximum allowable input power. The highest acceptable input power is defined as the maximum input level in a two-tone test for which the input-referred 3<sup>rd</sup> order IM products do not exceed the noise floor. The spurious free dynamic range (SFDR) is the difference (in dB) between the upper and lower acceptable input power levels.



# Dynamic Range - 2



From the previous analysis of IM3:

$$P_{IIP3} = P_{in} + \frac{P_{sig,out} - P_{IM3,out}}{2}$$

If the amplifier has a gain of  $G$  dB then:

$$P_{sig,out} = P_{in} + G$$

$$P_{IM3,out} = P_{IM3,in} + G$$

where  $P_{IM3,in}$  is the input-referred IM3. So:

$$P_{IIP3} = P_{in} + \frac{P_{in} + G - (P_{IM3,in} + G)}{2}$$

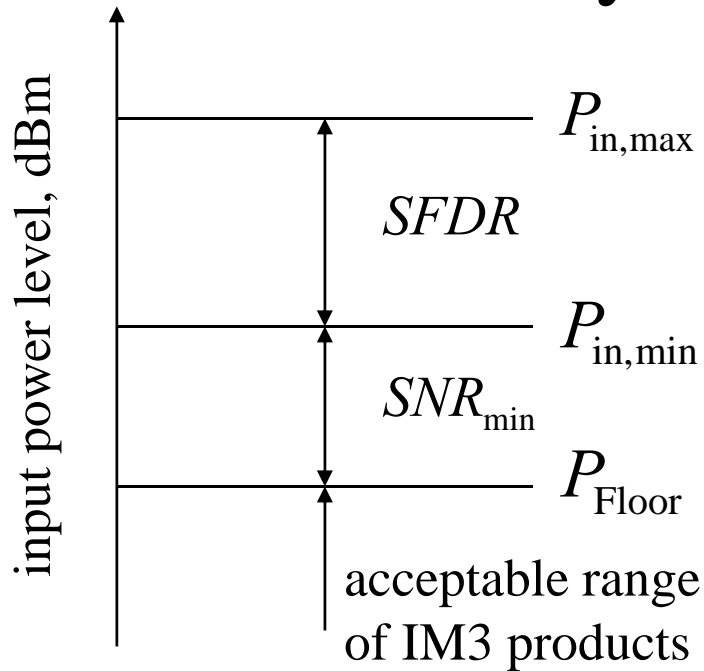
$$\Rightarrow P_{in} = \frac{2P_{IIP3} + P_{IM3,in}}{3}$$

The maximum allowable input level is the power at which the input-referred IM3 becomes equal to the noise floor i.e.

$$P_{in,max} = \frac{2P_{IIP3} + P_{Floor}}{3}$$



# Dynamic Range - 3



$$P_{in,max} = \frac{2P_{IIP3} + P_{Floor}}{3}$$

$$P_{in,min} = P_{Floor} + SNR_{min}$$

$$\begin{aligned} SFDR &= P_{in,max} - P_{in,min} \\ &= \frac{2P_{IIP3} + P_{Floor}}{3} - (P_{Floor} + SNR_{min}) \\ &= \frac{2(P_{IIP3} - P_{Floor})}{3} - SNR_{min} \end{aligned}$$

e.g. calculate the SFDR for a receiver with NF=9 dB,  $P_{IIP3} = -15\text{dBm}$ ,  $B = 200\text{kHz}$  that requires an  $SNR_{min}$  of 12dB.

$$P_{Floor} = -174 + 9 + 10\log_{10}(200 \times 10^3) = -112\text{dBm}$$

$$SFDR = \frac{2(P_{IIP3} - P_{Floor})}{3} - SNR_{min} = \frac{2(-15 - (-112))}{3} - 12 \approx 53\text{dB}$$

# Sample Exam Question

EE4011, Summer 2004, Q2

- (a) Derive an expression for the noise figure of a two-port network driven by a source with impedance  $R_S$ . Assume that the two-port can be represented by a noiseless two-port with equivalent input-referred noise voltage and current sources. (14 marks)
- (b) The equivalent input referred noise voltage and current sources of a bipolar transistor at moderate frequencies are given by:

$$\overline{v^2} = 4kT \left( r_b + \frac{1}{2g_m} \right) \Delta f \quad \overline{i^2} = 2q \frac{I_C}{\beta_f} \Delta f$$

where the symbols have their usual meaning and  $r_b$  is the parasitic base resistance.

A BJT is biased in the forward active region with a collector current,  $I_C=1\text{mA}$  at 300 K.

It has a forward active current gain of 100 and a parasitic base resistance of  $50\Omega$ . Determine the noise figure for a bandwidth of 1Hz if it is driven by a source with the following impedances:

- (i)  $10\Omega$  (3marks)  
(ii)  $100\Omega$  (3marks)

# Sample Exam Question

EE4011, Summer 2004, Q3

(a) Show a graphical means by which the input-referred third-order intercept point (PIIP3) of an amplifier can be determined by measuring the fundamental output power and the third harmonic output power of an amplifier for just one input power level. Clearly identify all important parts of your diagram. (7 marks)

(b) Starting with the definition of the noise factor for an amplifier, develop an expression for the sensitivity of the amplifier which specifies the minimum input power that is required to give an acceptable minimum signal-to-noise ratio, SNR<sub>min</sub>, at the output. Assume the amplifier has bandwidth B, and also assume that the amplifier input forms a conjugate match to the source so that the noise power delivered from the source is given by

$$P_{RS} = kT \quad W / Hz$$

From the expression you derive, identify the noise floor of the system. (7 marks)

(c) Illustrate the concept of spurious free dynamic range (SFDR) using a suitable diagram and calculate the SFDR for a receiver system that requires a minimum SNR of 12dB at the output. The system characteristics are as follows: NF=9 dB, PIIP3= -15 dBm, B=200kHz, T=300K

(6 marks)

# Sample Exam Question

EE4011, Summer 2005, Q3

(a) The input of a balanced (differential) RF amplifier consists of two cosinusoidal waveforms with amplitudes  $A_1$  and  $A_2$  (V) and frequencies  $\omega_1$  and  $\omega_2$  (rad/s) respectively. The output waveform is as follows:

$$\begin{aligned} y(t) = & \left[ \alpha_1 A_1 + \frac{3}{4} \alpha_3 A_1^3 + \frac{3}{2} \alpha_3 A_1 A_2^2 \right] \cos(\omega_1 t) + \left[ \alpha_1 A_2 + \frac{3}{4} \alpha_3 A_2^3 + \frac{3}{2} \alpha_3 A_1^2 A_2 \right] \cos(\omega_2 t) \\ & + \frac{1}{4} \alpha_3 A_1^3 \cos 3\omega_1 t + \frac{1}{4} \alpha_3 A_2^3 \cos 3\omega_2 t \\ & + \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 + \omega_2)t + \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 - \omega_2)t \\ & + \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 + \omega_1)t + \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 - \omega_1)t \end{aligned}$$

Using the formula for  $y(t)$  as a starting point and assuming that  $\alpha_1$  and  $\alpha_3$  have opposite signs, define and derive expressions for the following:

- (i) The 1dB gain compression point (P1dB) (5 marks)
- (ii) The input-referred 3rd-order intermodulation intercept point (IIP3) (5 marks)

# Sample Exam Question

EE4011, Summer 2005, Q3 continued

(b) An amplifier such as the one described in part (a) has a single-frequency input with amplitude  $1\text{mV rms}$ . The output at the fundamental is measured as  $100\text{mV rms}$  and the 3rd-harmonic output is measured as  $1\text{nV rms}$ . Determine the 1dB compression-point of the amplifier. (5 marks)

(c) Based on the expression for  $y(t)$  outlined in part (a) discuss two other undesired effects resulting from amplifier non-linearity in addition to those already considered in part (a). (5 marks)