Solutions UE4002 Summer 2004

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an nmos in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n = \lambda_p = 0$.
In each question, capacitances other than those mentioned may be ignored.
<u>Question 1</u>

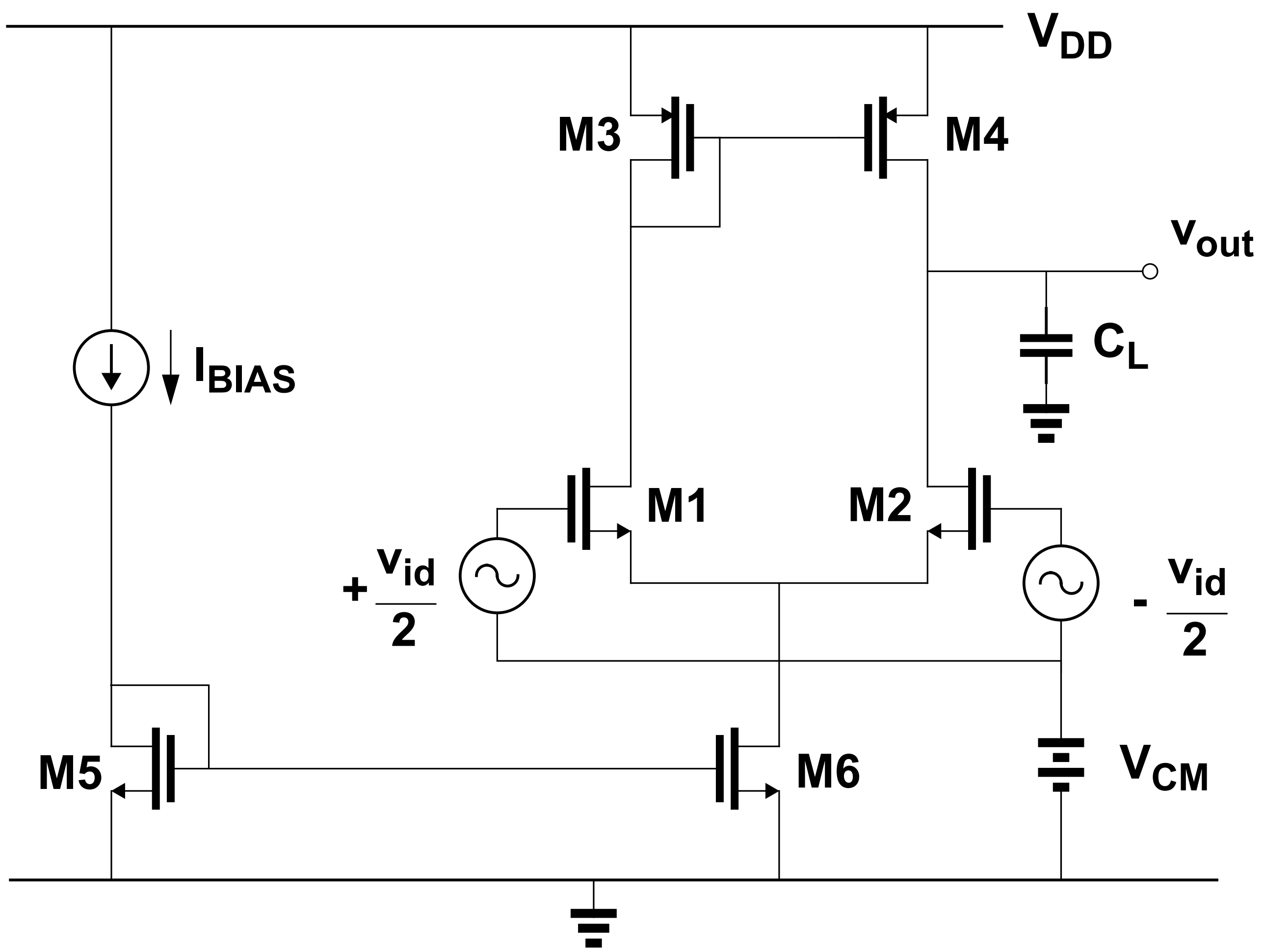


Figure 1

Figure 1 shows a differential to single-ended amplifier.
I_{BIAS} = 400µA, K_n'=200µA/V^2, K_p'=50µA/V^2, V_{tn}=0.8V, V_{tp}=-0.8V, $\lambda_n = \lambda_p$=0.04/L V^{-1}. (L in µm)
Transistor dimensions in µm are:
M1, M2, M3, M4: 32/1
M5, M6: 64/4

(i) What is the low-frequency small-signal gain (v_{out}/v_{id}) of this amplifier ?
You may assume the common source of M1,M2 is at ac ground.
(ii) What is the common-mode input range of this amplifier?
(iii) Calculate the low-frequency small-signal gain.
(iv) Calculate the frequency of the pole at the output node if C_L has a value of 10pF. What is the unity gain frequency?

(i) What is the low-frequency small-signal gain (v_{out}/v_{id}) of this amplifier ?
You may assume the common source of M1,M2 is at ac ground.

Note that the question does not ask for a derivation. It has been shown in the course that the gain of such circuits can be given by inspection. Deriving the gain by the analyses given in the notes is of course also good.

$$Gain = \frac{v_{out}}{v_{id}} = \frac{g_{m1}}{g_{ds2} + g_{ds4}}$$

(ii) What is the common-mode input range of this amplifier?

1. Lower limit on V_{CM}: is given by the requirement that M6 stay in saturation

$$V_{CM} \geq V_{GS1} + (V_{GS6} - V_t)$$

$$V_{GS} - V_t = \sqrt{\frac{2I_D}{K'_n \frac{W}{L}}}$$

M1: $$V_{GS1} - V_t = \sqrt{\frac{2 \cdot 200\mu A}{200\mu A/V \cdot \frac{32}{1}}} = 250mV$$

$$V_{GS1} = 250mV + V_t = 1.05V$$

M7: $$V_{GS7} - V_t = \sqrt{\frac{2 \cdot 400\mu A}{200\mu A/V \cdot \frac{64}{4}}} = 500mV$$

$$V_{CM} \geq 1.05V + 0.5V$$

$$V_{CM} \geq 1.55V$$

2. Upper limit on V_{CM}: is given by the requirement that M1 be in saturation

$$V_{DS1} \geq V_{GS1} - V_{tn}$$

$$V_{DD} - |V_{GS4}| - V_S \geq V_{CM} - V_S - V_{tn}$$

$$V_{CM} \leq V_{DD} - |V_{GS4}| + V_{tn}$$

M4 $$|V_{GS4} - V_{tp}| = \sqrt{\frac{2 \cdot 200\mu A}{50\mu A/V \cdot \frac{32}{1}}} = 500mV$$

$$|V_{GS4}| = 500mV + |V_{tp}| = 1.30V$$

$$V_{CM} \leq V_{DD} - |V_{GS4}| + V_{tn}$$

$$V_{CM} \leq 5 - 1.30V + 0.8V$$

$$V_{CM} \leq 4.5V$$

Common-mode input range given by

$$1.55V \leq V_{CM} \leq 4.5V$$

(iii) Calculate the low-frequency small-signal gain.

$$Gain = \frac{v_{out}}{v_{id}} = \frac{g_{m1}}{g_{ds2} + g_{ds4}}$$

$$g_{m1} = \frac{2I_D}{(V_{GS1} - V_{t1})} = \frac{2 \times 200\mu A}{0.25} = 1600\mu A/V$$

$$g_{ds3} = \lambda I_D = \frac{0.04V^{-1}}{1} \times 200\mu A = 8\mu A/V$$

$$g_{ds4} = \lambda I_D = \frac{0.04V^{-1}}{1} \times 200\mu A = 8\mu A/V$$

$$Gain = \frac{v_{out}}{v_{id}} = \frac{g_{m1}}{g_{ds2} + g_{ds4}} = \frac{1600\mu A/V}{8\mu A/V + 8\mu A/V} = \underline{\underline{100 = 40dB}}$$

(iv) Calculate the frequency of the pole at the output node if C_L has a value of 10pF. What is the unity gain frequency?

Pole frequency given by

$$|\omega_p| = \frac{g_{ds2} + g_{ds4}}{C_L} = \frac{8\mu A/V + 8\mu A/V}{10pF} = \underline{\underline{1.6Mrad/s}}$$

Unity gain frequency given by

$$|\omega_u| = \frac{g_{m1}}{g_{ds2} + g_{ds4}} \frac{g_{ds2} + g_{ds4}}{C_L} = \frac{1600\mu A/V}{10pF} = \underline{\underline{160Mrad/s}}$$

Alternatively observe unity gain frequency is 40dB above pole frequency.

Question 2

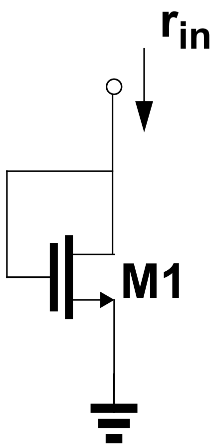


Figure 2

(i) Figure 2 shows a diode connected nmos. Show that, assuming $g_{m1} >> g_{ds1}$, the small-signal resistance looking into the gate-drain node of M1 is given by

$$r_{in} = \frac{1}{g_{m1}}$$

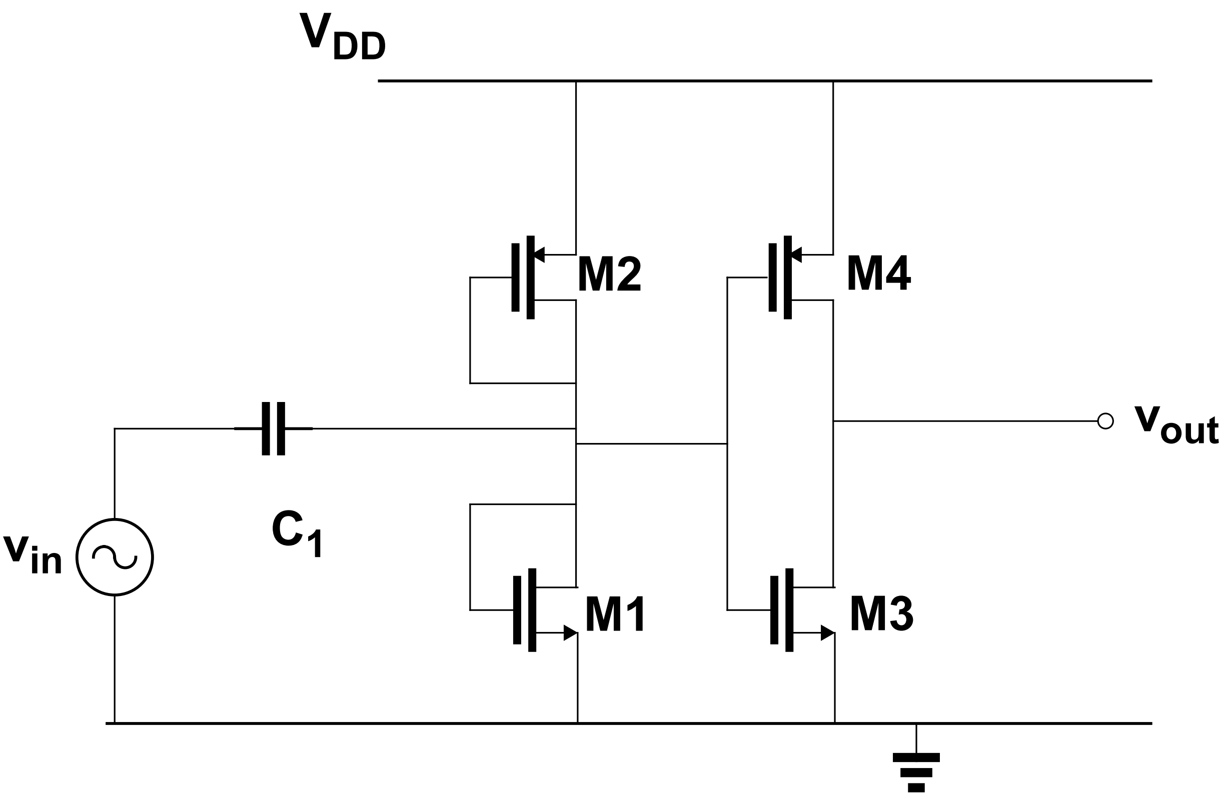


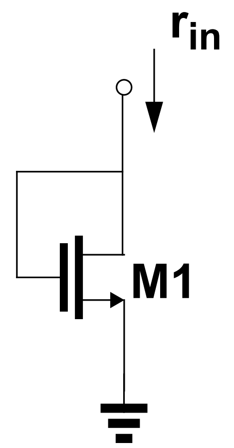
Figure 3

Figure 3 shows a small-signal v_{in} ac-coupled onto an inverter stage.
M1 and M3 have equal dimensions. M2 and M4 have equal dimensions. Assume $g_m >> g_{ds}$ for each transistor.

(ii) Draw the small-signal equivalent circuit for the circuit shown in Figure 3. (The result of (i) may be used).

(iii) Derive an expression for the high frequency transfer function of v_{out}/v_{in}

(iv) Calculate the gain at high frequencies. You may assume that at these frequencies C_1 acts as a short circuit.
V_{tn}=0.7V, V_{tp}=-0.7V, K_n'=200μA/V^2, K_p'=50μA/V^2, $\lambda_n=\lambda_p$=0.04/L V^{-1}. (L in μm)
All nmos transistors have W/L = 10μm/1μm. All pmos transistors have W/L = 40μm/1μm.



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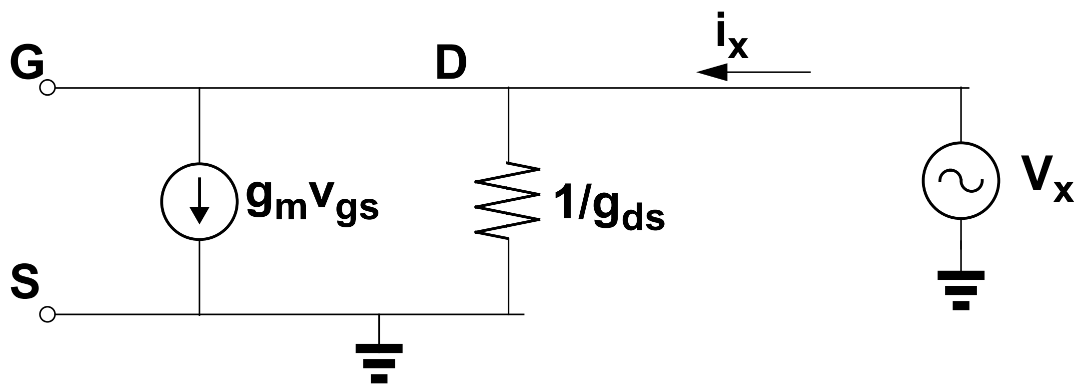


Apply a test voltage at the output

$$i_x = g_m v_{gs} + v_x g_{ds}$$

$$v_{gs} = v_x$$

$$i_x = g_m v_x + v_x g_{ds}$$

$$r_{in} = \frac{v_x}{i_x} = \frac{1}{g_m + g_{ds}} \approx \frac{1}{g_m}$$

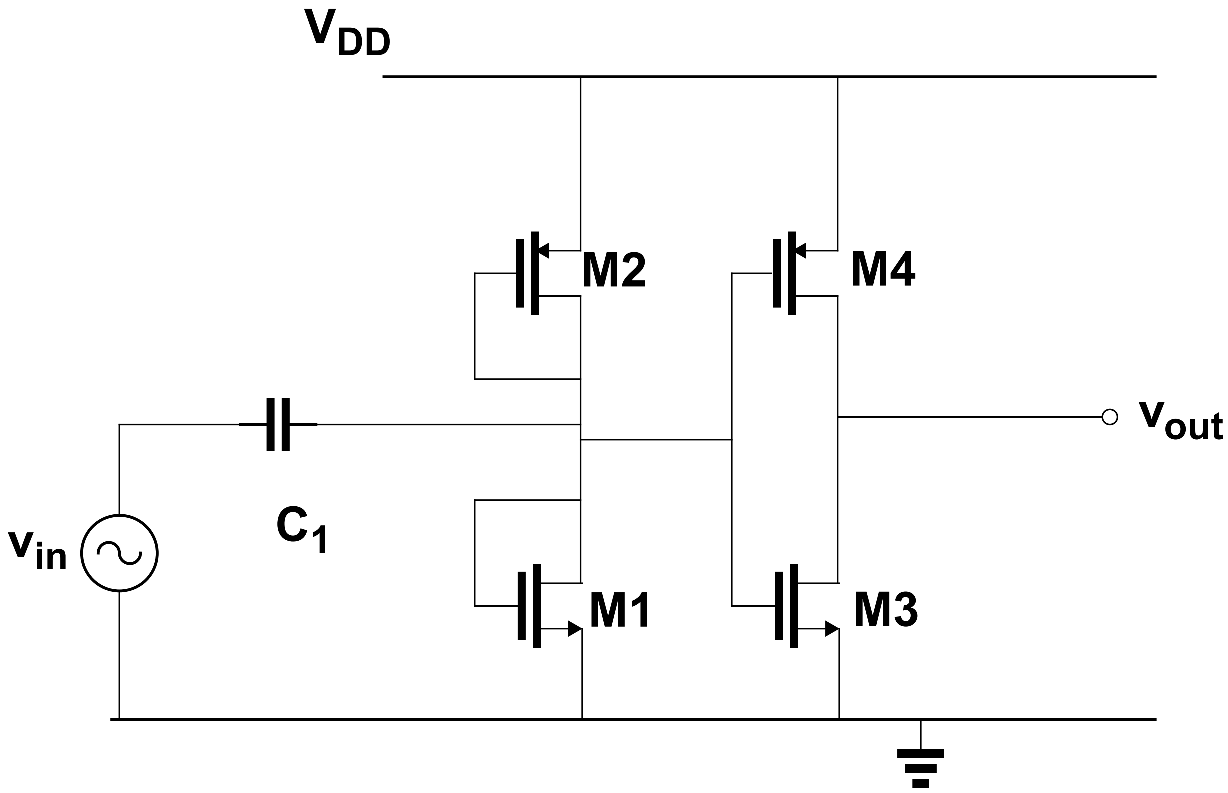


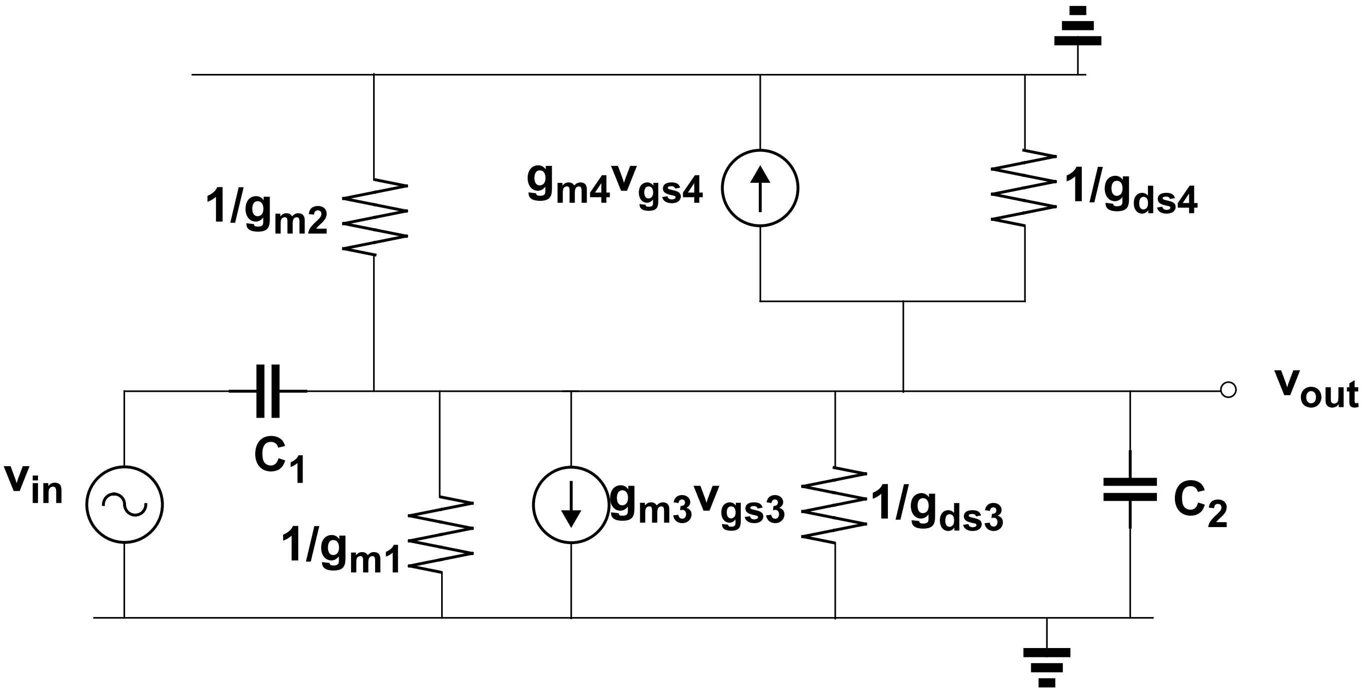
Figure 3

Figure 3 shows a small-signal v_{in} ac-coupled onto an inverter stage.

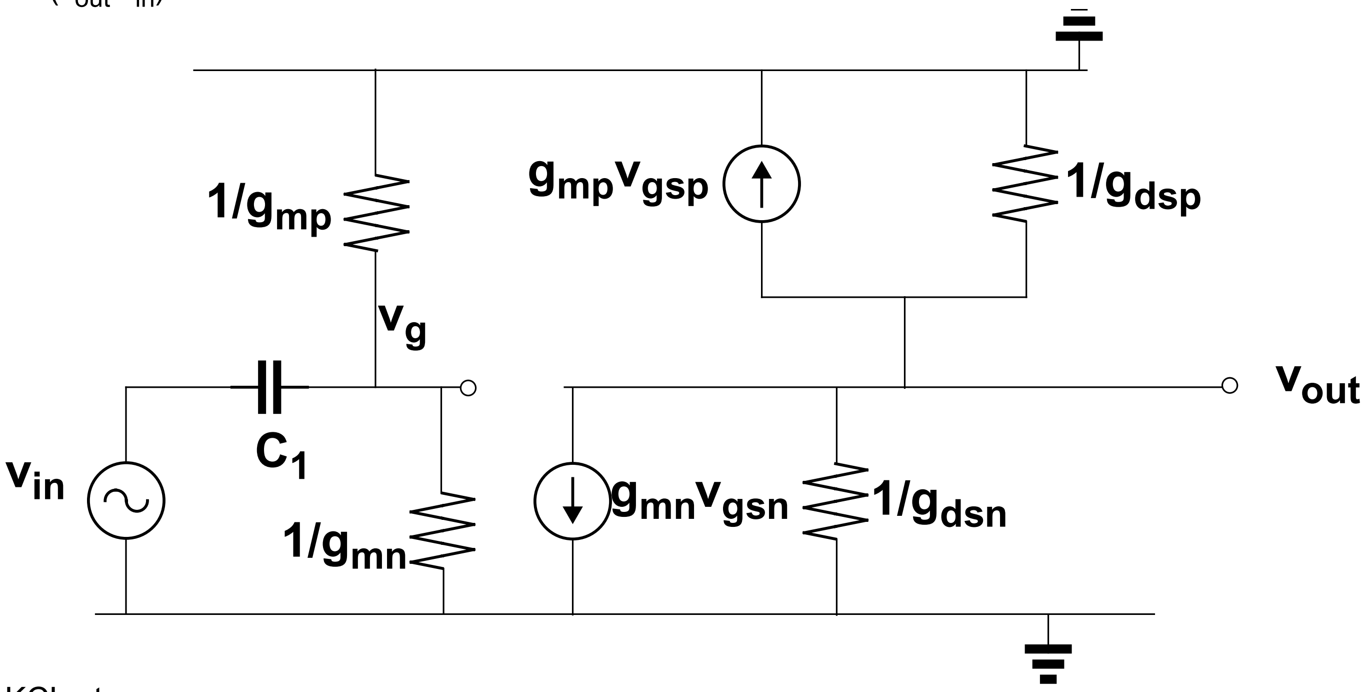
M1 and M3 have equal dimensions. M2 and M4 have equal dimensions. Assume $g_m >> g_{ds}$ for each transistor.

(ii) Draw the small signal equivalent circuit for the circuit shown in Figure 3. (The result of (i) may be used).

Because nmos have same dimensions and gate-source voltage they also have the same current, so $g_{m1}=g_{m3}=g_{mn}$.



(iii) Ignoring all capacitances except C_1 derive an expression for the high frequency transfer function (v_{out}/v_{in})



KCL at v_g

$$(v_g - v_{in})sC_1 + g_{mn}v_g + g_{mp}v_g = 0$$

$$v_g = \frac{v_{in}sC_1}{sC_1 + g_{mn} + g_{mp}}$$

KCL at v_{out}

$$g_{mn}v_{gsn} + g_{mp}v_{gsp} + v_{out}g_{dsn} + v_{out}g_{dsp} = 0$$

$$g_{mn}v_g + g_{mp}v_g + v_{out}g_{dsn} + v_{out}g_{dsp} = 0$$

$$v_{out} = -\frac{(g_{mn} + g_{mp})v_g}{g_{dsn} + g_{dsp}}$$

$$v_{out} = -\frac{(g_{mn} + g_{mp})}{g_{dsn} + g_{dsp}} \frac{v_{in}sC_1}{(sC_1 + g_{mn} + g_{mp})}$$

(iv) Calculate the gain at high frequencies. You may assume that at these frequencies C_1 acts as a short circuit.
Vtn=0.7V, Vtp=-0.7V, K_n'=200µA/V^2, K_p'=50µA/V^2, $\lambda_n=\lambda_p$=0.04/L V^{-1}. (L in µm)
All nmos transistors have W/L = 10µm/1µm. All pmos transistors have W/L = 40µm/1µm.

With C1 shorted the transfer reduces to

$$\frac{v_{out}}{v_{in}} = -\frac{(g_{mn} + g_{mp})}{(g_{dsn} + g_{dsp})}$$

$$= -\frac{\frac{2I}{(V_{GSn} - V_{tn})} + \frac{2I}{(V_{GSp} - V_{tp})}}{(\lambda_n + \lambda_p)I}$$

The biasing of the inverter gain stage is defined by the 2 diode-connected M1 and M2

$$\frac{K_n'}{2}\frac{W}{L}(V_{GSn} - V_{tn})^2 = \frac{K_p'}{2}\frac{W}{L}(V_{GSp} - V_{tp})^2$$

With the values given this reduces to

$$(V_{GSn} - V_{tn}) = (V_{GSp} - V_{tp})$$

With Vtn = Vtp = 0.7V, and with VDD=3V this gives VGSn=VGSp = 1.5V

$$\frac{v_{out}}{v_{in}} = -\frac{4}{(V_{GS} - V_t)(\lambda_n + \lambda_p)}$$

$$= -\frac{4}{(1.5 - 0.7) \cdot 0.08} = -62.5 = 36dB$$

Question 3

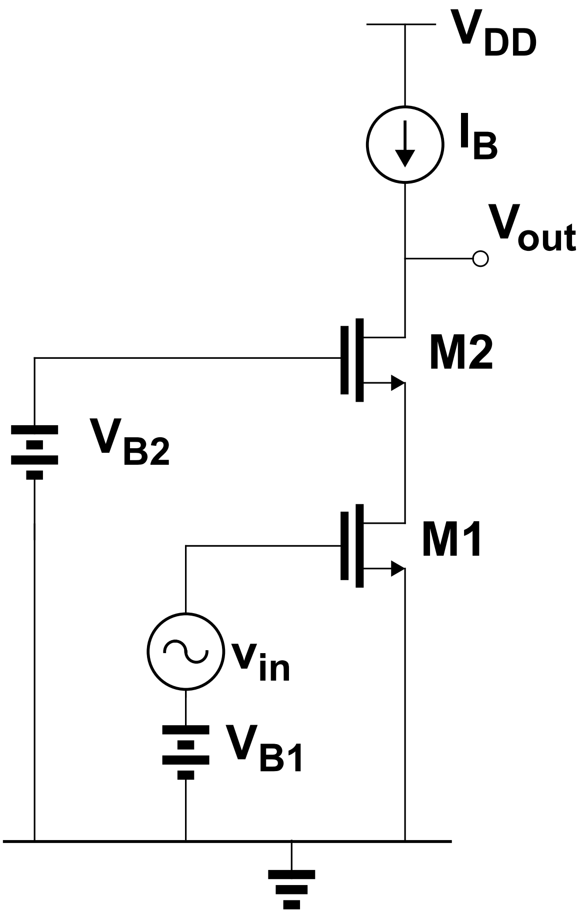


Figure 4(a)

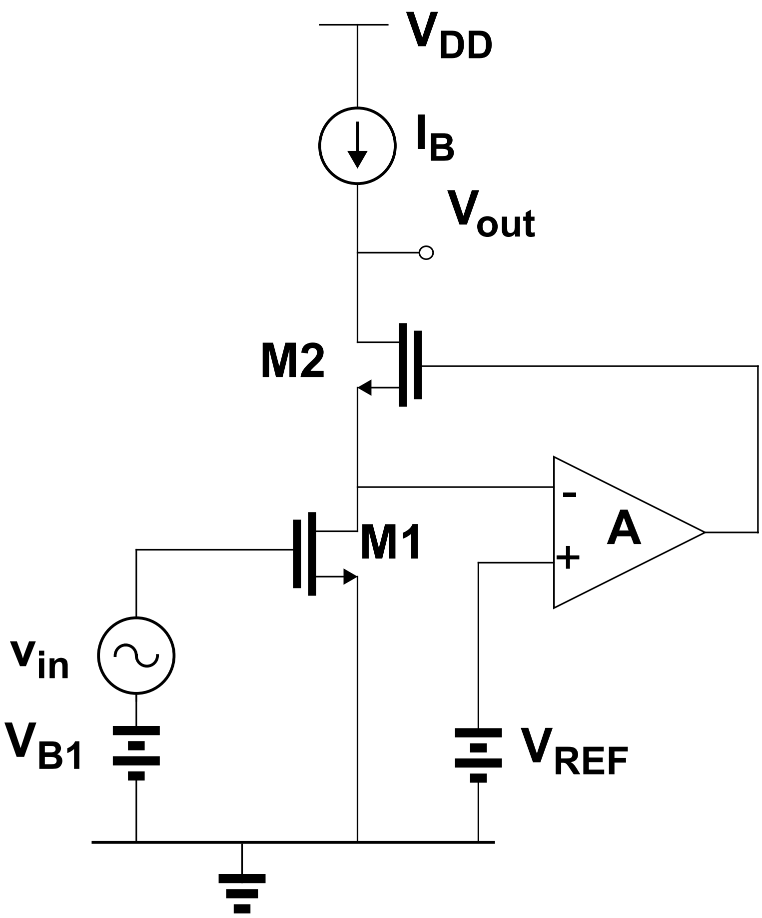


Figure 4(b)

Figures 4(a) and 4(b) show cascoded gain stages.
Assume all devices have equal dimensions and $g_m \gg g_{ds}$
V_{B1}= 1.2V, V_t=0.7V

(i) For the circuit in Figure 4(a) what is the minimum value of V_{B2} such that M1 is in saturation? With this value of V_{B2} what is then the minimum voltage at the output node such that M2 is in saturation?
(ii) Draw the small-signal equivalent circuit for the gain stage shown in Figure 4(a)
(iii) Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) for the circuit shown in Figure 4(a). Simplify the expression assuming $g_m \gg g_{ds}$.
(iv) In Figure 4(b) the gain of the stage has been enhanced by the use of a regulated cascode. Derive an expression for the small-signal voltage gain (v_{out}/v_{in}) of this circuit.

(i) For the circuit in Figure 4(a) what is the minimum value of V_{B2} such that M1 is in saturation? With this value of V_{B2} what is then the minimum voltage at the output node such that M2 is in saturation?

The minimum value of VB2 is the overdrive voltage of M1 plus the gate source voltage of M2

For M1 to be just in saturation

$$V_{DS1} = V_{GS1} - V_t = 1.2 - 0.7 = 0.5$$

$V_{B2} = V_{GS2} + V_{DS1}$

As M2 has the same dimensions and current as M1 then VGS2=VGS1

$$V_{B2} = V_{GS2} + V_{DS1} = 1.2V + 500mV = 1.7V$$

For M2 to be just in saturation

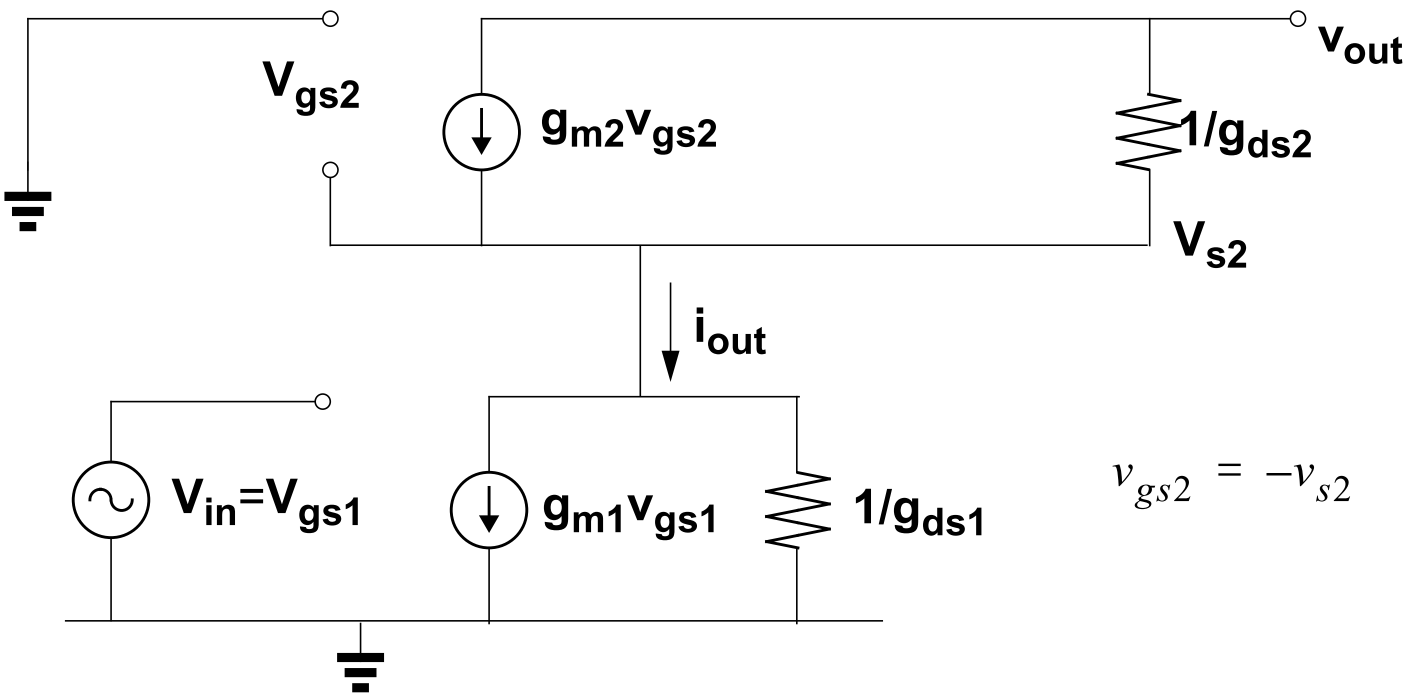
$$V_{DS2} = V_{GS1} - V_t = 1.2 - 0.7 = 0.5V$$

Minimum voltage at the output is voltage required for M1,M2 just in saturation

For M4 to be just in saturation

$$V_{OUT(min)} = V_{DS2} + V_{DS1} = 1V$$

(ii) Draw the small signal equivalent circuit for the gain stage shown in Figure 4(a)



$$v_{gs2} = -v_{s2}$$

(iii) Derive an expression for the low-frequency small signal voltage gain (v_{out}/v_{in}) for the gain stage shown in Figure 4(a). Simplify the expression assuming $g_m >> g_{ds}$.

KCL at source of M2:

$$(v_{out} - v_{s2})g_{ds2} + g_{m2}v_{gs2} = g_{m1}v_{in} + v_{s2}g_{ds1}$$

$$0 = g_{m1}v_{in} + v_{s2}g_{ds1}$$

$$v_{s2} = -\frac{g_{m1}}{g_{ds1}}v_{in}$$

KCL at output node (drain of M2):

$$(v_{out} - v_{s2})g_{ds2} + g_{m2}v_{gs2} = 0$$

$$(v_{out} - v_{s2})g_{ds2} - g_{m2}v_{s2} = 0$$

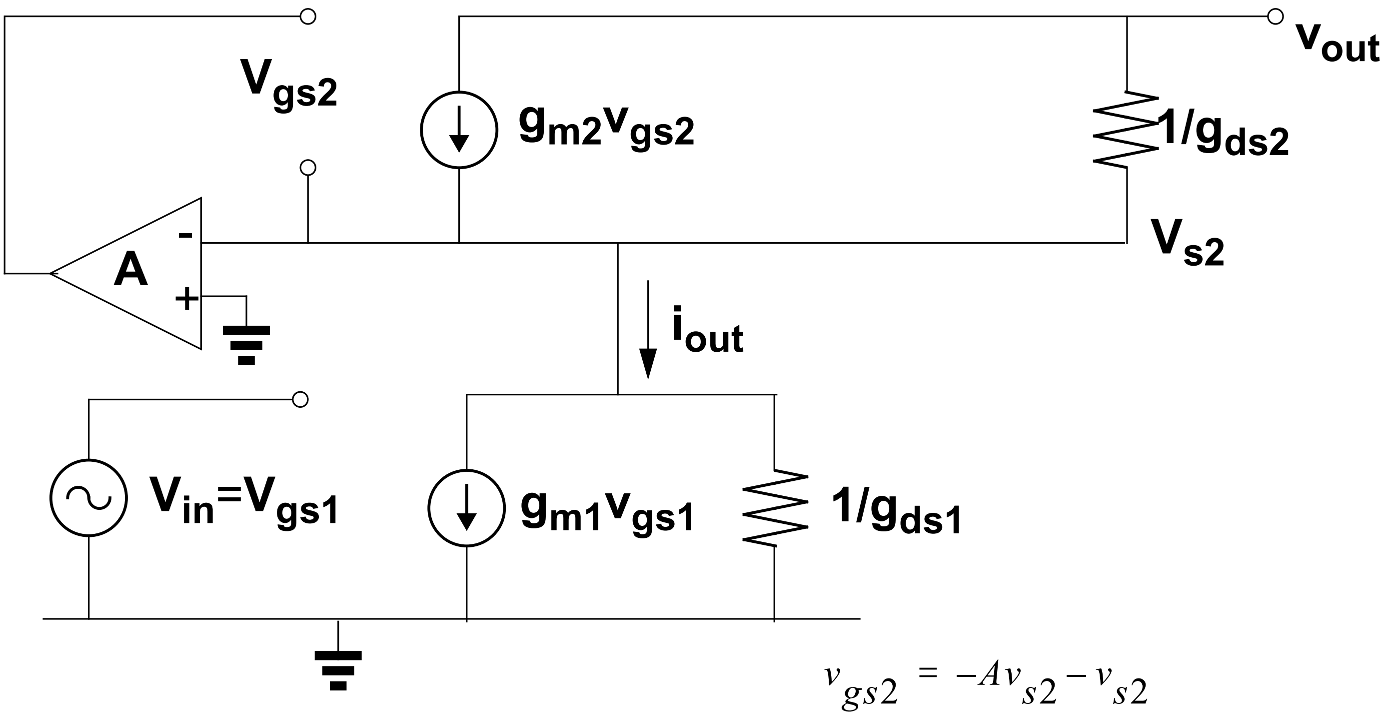
Substitute for V_{s2}

$$v_{out}g_{ds2} + \frac{g_{m1}}{g_{ds1}}g_{ds2}v_{in} + (g_{m2})\left(\frac{g_{m1}}{g_{ds1}}v_{in}\right) = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{1}{g_{ds2}}\left(\frac{g_{m1}}{g_{ds1}}g_{ds2} + g_{m2}\frac{g_{m1}}{g_{ds1}}\right)$$

$$\frac{v_{out}}{v_{in}} = -\left(\frac{g_{m1}}{g_{ds1}} + \frac{(g_{m2})}{g_{ds2}}\frac{g_{m1}}{g_{ds1}}\right) \approx -\frac{g_{m2}}{g_{ds2}}\frac{g_{m1}}{g_{ds1}} = \left(\frac{g_m}{g_{ds}}\right)^2$$

(iv) In Figure 4(b) the gain of the stage has been enhanced by the use of a regulated cascode. Derive an expression for the low-frequency small signal voltage gain (v_{out}/v_{in}) of this gain stage shown in Figure 4(b).



$$v_{gs2} = -Av_{s2} - v_{s2}$$
$$v_{gs2} = -(A+1)v_{s2}$$

KCL at source of M2:

$$(v_{out} - v_{s2})g_{ds2} + g_{m2}v_{gs2} = g_{m1}v_{in} + v_{s2}g_{ds1}$$

$$0 = g_{m1}v_{in} + v_{s2}g_{ds1}$$

$$v_{s2} = -\frac{g_{m1}}{g_{ds1}}v_{in}$$

KCL at output node (drain of M2):

$$(v_{out} - v_{s2})g_{ds2} + g_{m2}v_{gs2} = 0$$

$$(v_{out} - v_{s2})g_{ds2} - (g_{m2})(A+1)v_{s2} = 0$$

Substitute for V_{s2}

$$v_{out}g_{ds2} + \frac{g_{m1}}{g_{ds1}}g_{ds2}v_{in} + (g_{m2})(A+1)\left(\frac{g_{m1}}{g_{ds1}}v_{in}\right) = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{1}{g_{ds2}}\left(\frac{g_{m1}}{g_{ds1}}g_{ds2} + g_{m2}(A+1)\frac{g_{m1}}{g_{ds1}}\right)$$

$$\frac{v_{out}}{v_{in}} = -\left(\frac{g_{m1}}{g_{ds1}} + (A+1)\frac{(g_{m2})}{g_{ds2}}\frac{g_{m1}}{g_{ds1}}\right) \approx -(A+1)\frac{g_{m2}}{g_{ds2}}\frac{g_{m1}}{g_{ds1}} = -(A+1)\left(\frac{g_m}{g_{ds}}\right)^2$$

Question 4

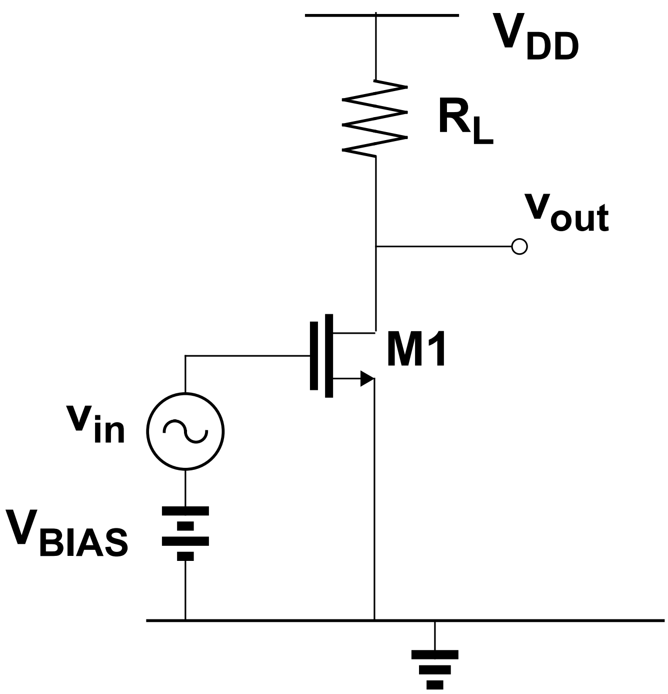


Figure 5

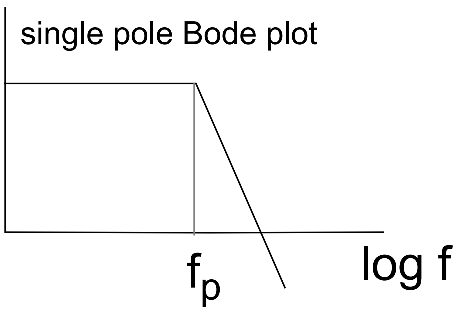
Assume M1 is operating in saturation and that g_{ds1}<<$1/R_L$. Only thermal noise sources need be considered.

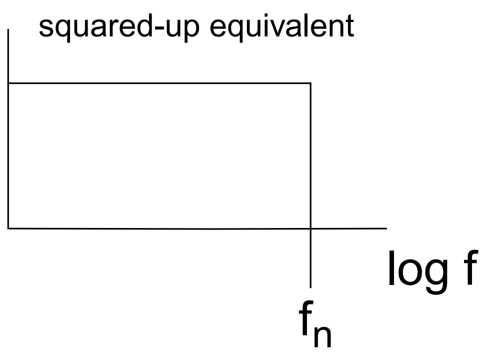
(i) Draw the small-signal model for the circuit shown in Figure 5.
What is the small-signal voltage gain (v_{out}/v_{in})?

(ii) What is the input-referred thermal noise voltage in terms of R_L, the small-signal parameters of M1, Boltzmann's constant k and temperature T?

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:





For the area underneath the curves to be the same then $f_n=(\pi/2)*f_p$

(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $10mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.

For this calculation take V_{GS1}=1V, V_{tn} = 0.75V, λ_n=0.04/L V^{-1}, R_L=5kΩ, C_L=1pF.

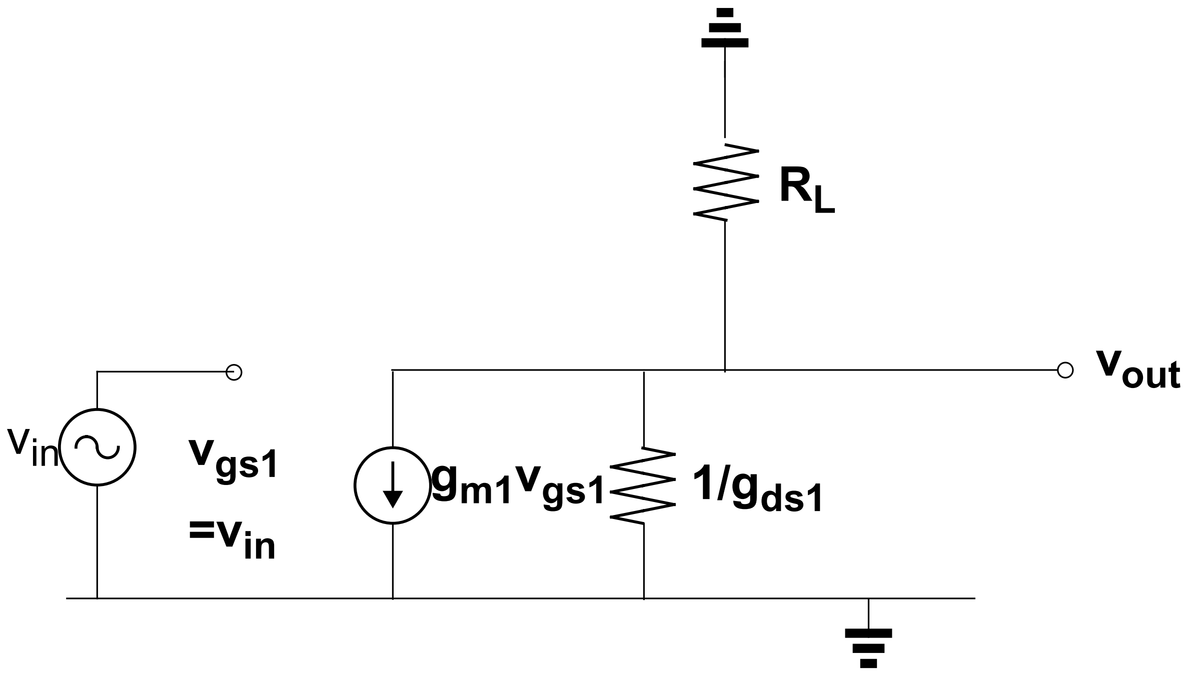
The drain current of M1 is 100μA.

Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

Solution

(i) Draw the small signal model for the circuit shown in Figure 3. Ignore all capacitances

What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

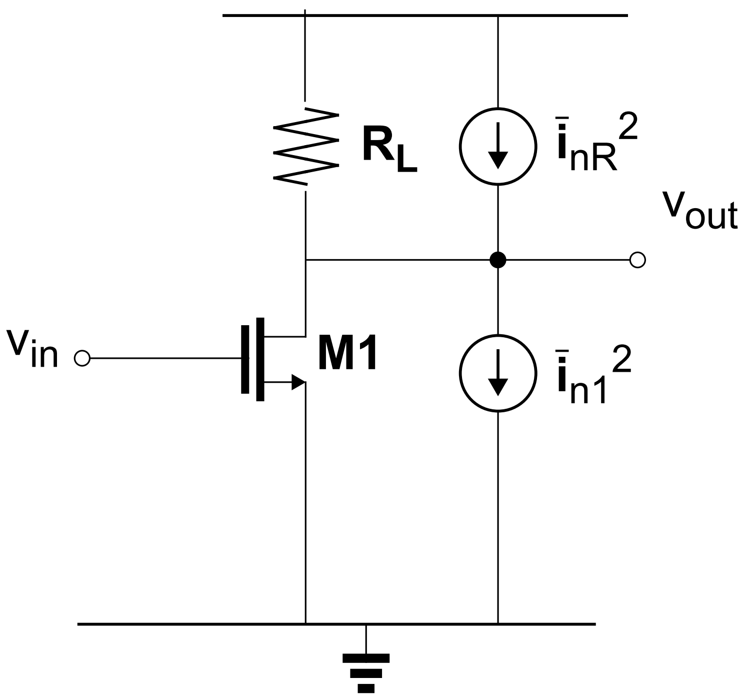


Current at outout node

$$g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + \frac{1}{R_L}} \cong -g_{m1}R_L$$

(ii) What is the input-referred thermal noise voltage in terms of R_L, the small signal parameters of M1, Boltzmann's constant k and temperature T?



Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_{m1}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

Noise voltage at output given by input referred noise multiplied by gain

$$\overline{v_{no}} = \overline{v_{ni}} g_{m1} R_L = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}}{g_{m1}} g_{m1} R_L$$

$$= \left(\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}\right) R_L$$

Capacitor C_L connected between the output node and ground
=> pole at output node given by

$$|f_p| = \frac{1}{2\pi R_L C_L}$$

Total integrated thermal noise power at the output node is given by the product of the thermal noise power and the squared-up equivalent of the first order filter function

$$\overline{v^2_{nototal}} = \overline{v^2_{no}} \frac{\pi}{2} f_p$$

$$\overline{v^2_{nototal}} = \left(4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}\right) R_L^2 \cdot \frac{\pi}{2} \cdot \frac{1}{2\pi R_L C_L}$$

$$\overline{v^2_{nototal}} = \left(\frac{2}{3}g_{m1} + \frac{1}{R_L}\right) R_L \cdot \frac{kT}{C_L}$$

(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $10mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.
For this calculation take V_{GS1}=1V, V_{tn} = 0.75V, λ_n=0.04/L V^{-1}, R_L=5kΩ, C_L=1pF.
The drain current of M1 is 100µA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

g_m given by

$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V$$

Output signal

$$v_{out} = -g_{m1}R_Lv_{in} = -800\mu A/V \cdot 5k \cdot 20mV_{rms} = 80mV_{rms}$$

Total output noise:

$$\overline{v_{nototal}} = \sqrt{\left(\frac{2}{3}g_{m1} + \frac{1}{R_L}\right)R_L \cdot \frac{kT}{C_L}}$$

$$\overline{v_{nototal}} = \sqrt{\left(\frac{2}{3}(800\mu A/V) + \frac{1}{5k}\right) \cdot 5k \cdot \frac{1.38\times10^{-23}300}{1pF}} = 123\mu V_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{80mV}{123\mu V} = 649 \qquad \text{or 56.2 dB}$$