Question 1

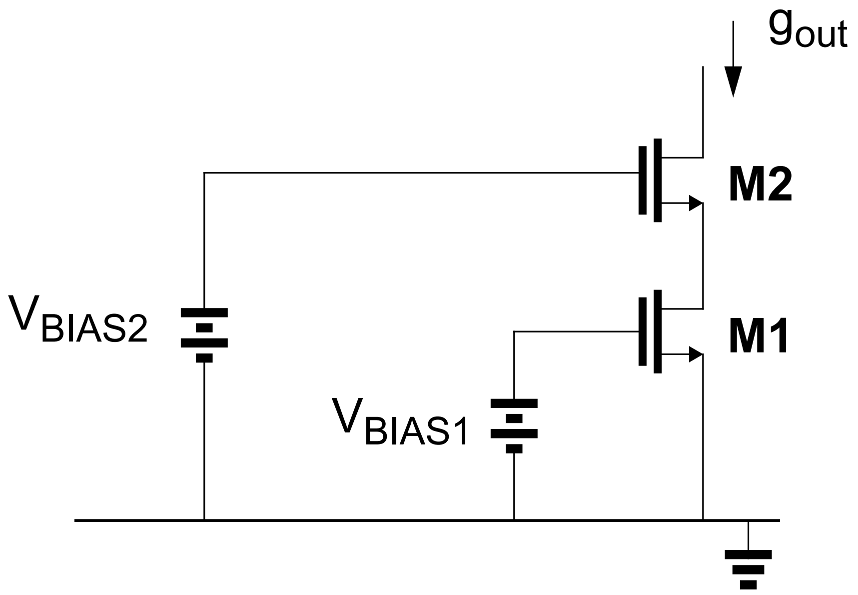


Figure 1

Ignore the body effect.

(i) Draw the small signal model for the circuit shown in Figure 1.
Ignore all capacitances.

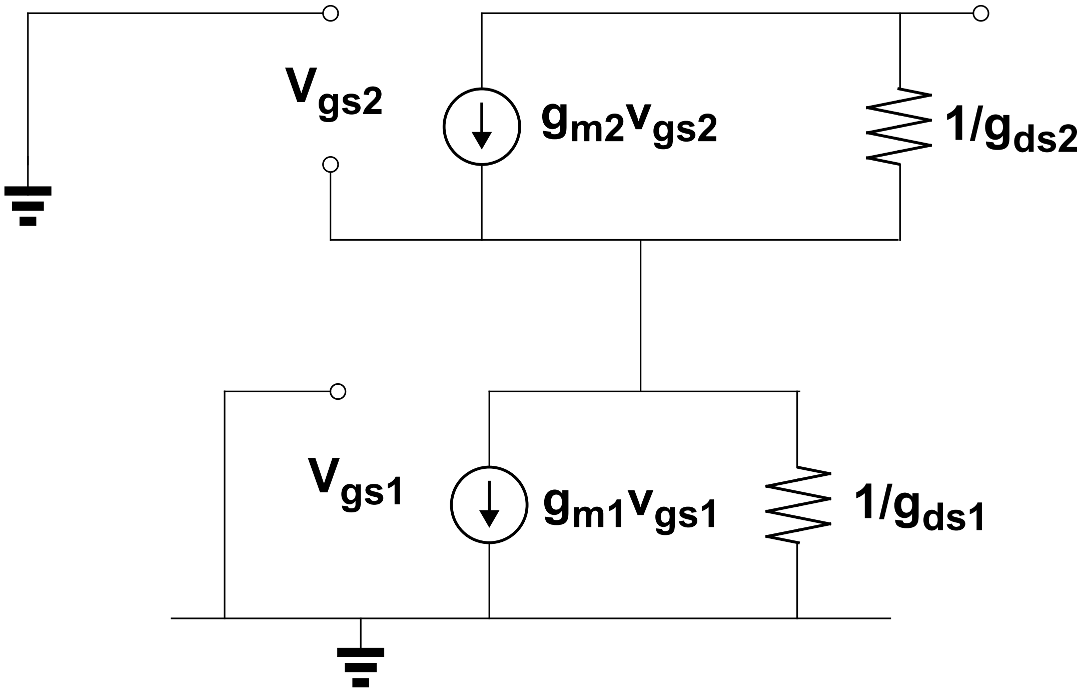
(ii) Derive an expression for the output conductance g_{out} in terms of the small signal parameters of M1 and M2.
Reduce the expression to its simplest form assuming
$g_{m1}=g_{m2}=g_m$, $g_{ds1}=g_{ds2}=g_{ds}$, $g_m>>g_{ds}$

(iii) The circuit is to be biased for optimal low-voltage operation. If
V_{BIAS1}=1.2V
V_T = 0.8V
$(W/L)_{M2}=(W/L)_{M1}$
calculate the minimum value of the voltage at the output node (i.e. at the drain of M2) for both M1 and M2 to be in saturation and the value of V_{BIAS2} necessary to achieve this.
Neglect λ for this calculation.

(iv) Repeat the calculations in (iii) if the aspect ratio of M2 is four times that of M1
i.e $(W/L)_{M2}=4*(W/L)_{M1}$

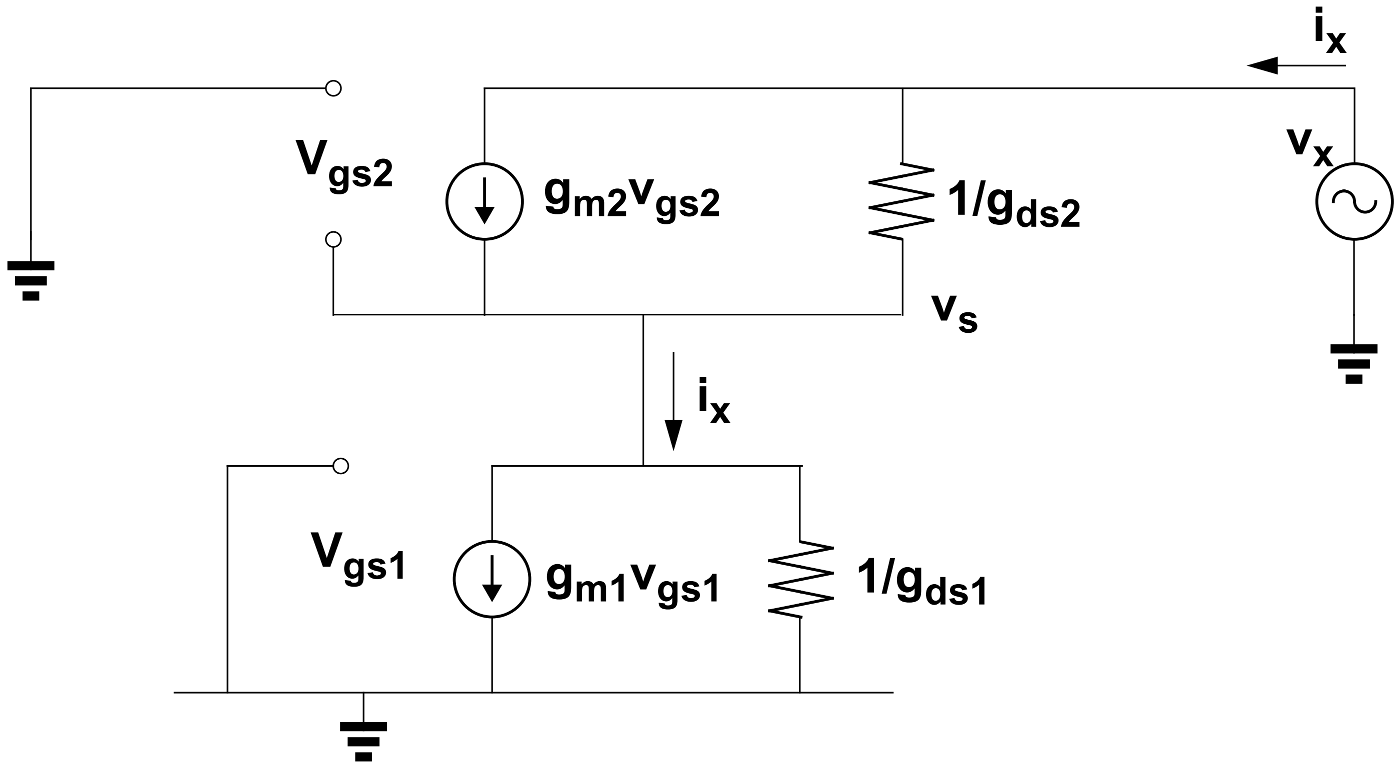
Solution

(i) Draw the small signal model for the circuit shown in Figure 1.
Ignore all capacitances.



(ii) Derive an expression for the output conductance g_{out} in terms of the small signal parameters of M1 and M2.
Reduce the expression to its simplest form assuming
$g_{m1}=g_{m2}=g_m$, $g_{ds1}=g_{ds2}=g_{ds}$, $g_m >> g_{ds}$

To derive the output conductance put a test voltage at the output node and calculate the small-signal current into the circuit.



Note: $v_{gs1} = 0 \Rightarrow g_{m1}v_{gs1} = 0$

$$i_x = g_{m2}v_{gs2} + (v_x - v_s)g_{ds2}$$

Since $v_{gs2} = -v_s$ and $v_s = \frac{i_x}{g_{ds1}}$

$$i_x = -(g_{m2})\frac{i_x}{g_{ds1}} + \left(v_x - \frac{i_x}{g_{ds1}}\right)g_{ds2}$$

$$g_{out} = \frac{i_x}{v_x} = \frac{g_{ds2}}{1 + \frac{g_{m2}}{g_{ds1}} + \frac{g_{ds2}}{g_{ds1}}}$$

Since $g_{m1}=g_{m2}=g_m$, $g_{ds1}=g_{ds2}=g_{ds}$, $g_m>>g_{ds}$ this can be reduced to any of

$$g_{out} \cong \frac{g_{ds2}}{g_{m2}/g_{ds1}} = \frac{g_{ds1}}{g_{m2}/g_{ds2}} = \frac{g_{ds}}{g_m/g_{ds}}$$

(iii)The circuit is to be biased for optimal low-voltage operation. If

V_{BIAS1}=1.2V

V_T = 0.8V

$(W/L)_{M2}=(W/L)_{M1}$

calculate the minimum value of the voltage at the output node (i.e. at the drain of M2) for both M1 and M2 to be in saturation and the value of V_{BIAS2} necessary to achieve this.

Neglect λ for this calculation.

For M1 to be in saturation then

$$V_{DS1} \geq V_{GS1} - V_T$$

$$(V_{DS1})_{min} = V_{GS1} - V_T = 1.2V - 0.8V = 0.4V$$

If M2 is in saturation its drain current is given by

$$I_{D2} = \frac{K'_n}{2}\frac{W}{L}(V_{GS2} - V_T)^2$$

Since M2 has same drain current, W/L and V_T as M1 it will also have the same V_{GS}

$$(V_{DS2})_{min} = V_{GS2} - V_T = 0.4V$$

so minimum voltage at the output for both transistors to be in saturation is given by

$$V_{out} = (V_{DS1})_{min} + (V_{DS2})_{min} = 0.8V$$

The bias voltage V_{BIAS2} necessary to achieve this is given by

$$V_{BIAS2} = V_{GS2} + (V_{DS1})_{min} = 1.2V + 0.4V = 1.6V$$

(iv)Repeat the calculations in (iii) if the aspect ratio of M2 is four times that of M1 i.e $(W/L)_{M2}=4*(W/L)_{M1}$

Since $I_{D1}=I_{D2}$ then

$$\frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS1}-V_T)^2 = \frac{K_n'}{2}4\left(\frac{W}{L}\right)(V_{GS2}-V_T)^2$$

$$V_{GS2}-V_T = \frac{V_{GS1}-V_T}{2} = 0.2V$$

$$V_{GS2} = 1V$$

$$(V_{DS2})_{min} = 0.2V$$

so minimum voltage at the output for both transistors to be in saturation is given by

$$V_{out} = (V_{DS1})_{min} + (V_{DS2})_{min} = 0.6V$$

The bias voltage V_{BIAS2} necessary to achieve this is given by

$$V_{BIAS2} = V_{GS2} + (V_{DS1})_{min} = 1.0V + 0.4V = 1.4V$$

Question 2

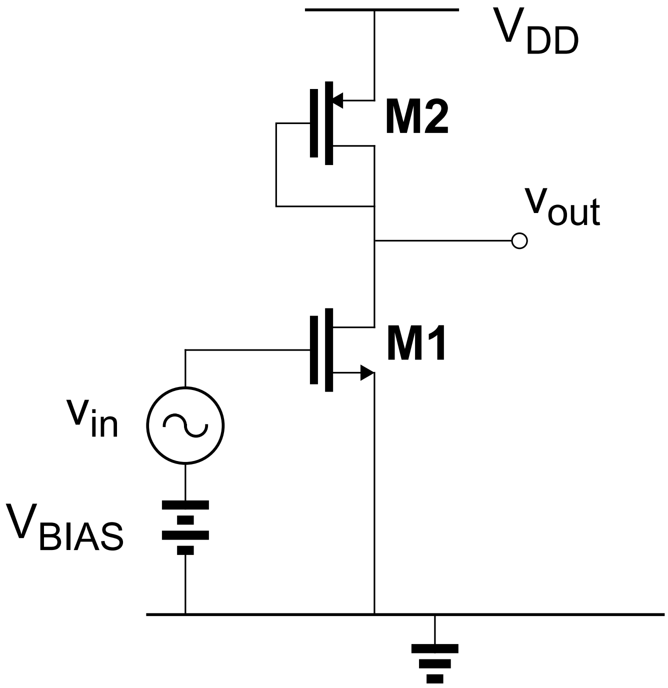


Figure 2

Assume M1 and M2 are operating in saturation and ignore the body effect.

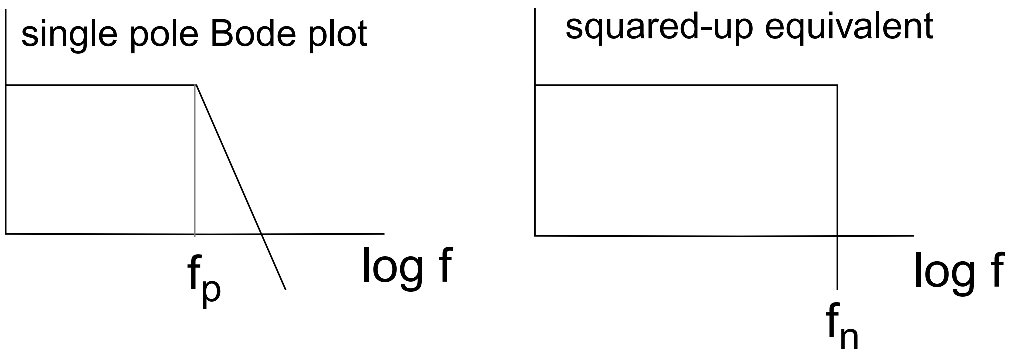
(i) Draw the small signal model for the circuit shown in Figure 2. Ignore all capacitances.

(ii) What is the low-frequency small signal voltage gain (v_{out}/v_{in})? Assume that $g_{m1} >> g_{ds1}, g_{ds2}$ and that $g_{m2} >> g_{ds1}, g_{ds2}$

(iii) What is the input-referred thermal noise voltage in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T?

(iv) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:



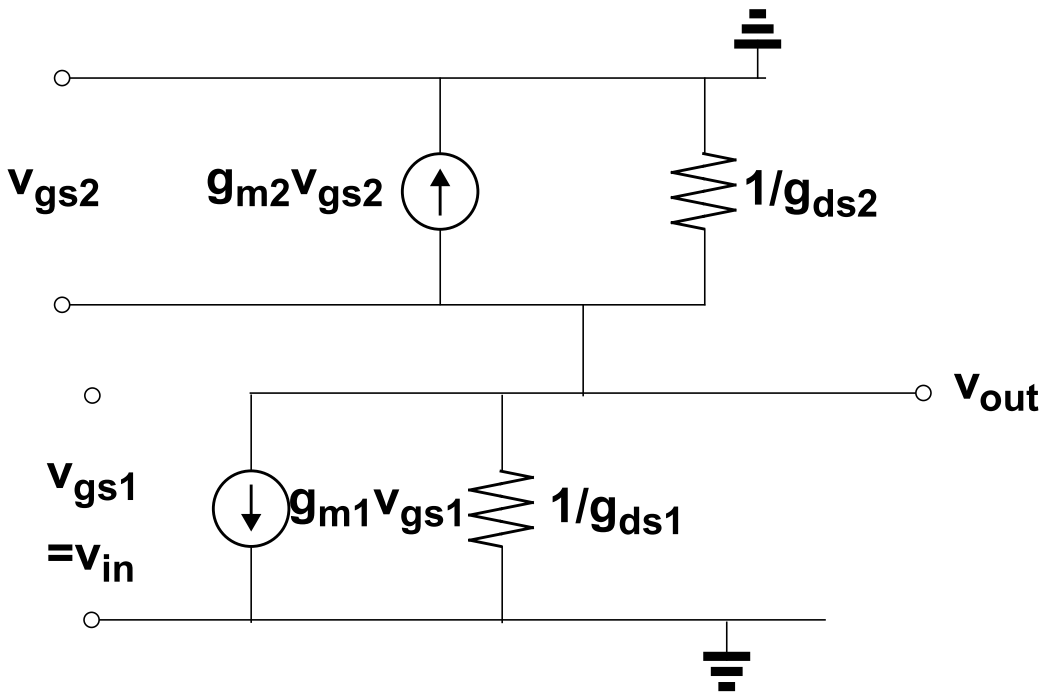
For the area underneath the curves to be the same then $f_n = (\pi/2) * f_p$

(v) Using the result of (iv) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $10mV_{rms}$ sine wave.
For this calculation take $V_{GS1}=1V$, $|V_{GS2}|=2.8V$, $|V_T| = 0.8V$ for M1,M2. $C_L=10pF$.
The drain current of M1 is $100\mu A$.

Assume Boltzmann's constant $k=1.38X10^{-23}J/^oK$, temperature $T=300^oK$.

Solution

(i) Draw the small signal model for the circuit shown in Figure 2.
Ignore all capacitances.



(ii) What is the low-frequency small signal voltage gain (v_{out}/v_{in})?
Assume that $g_{m1} >> g_{ds1}, g_{ds2}$ and that $g_{m2} >> g_{ds1}, g_{ds2}$

Current at outout node

$$g_{m1}v_{gs1} + g_{m2}v_{gs2} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

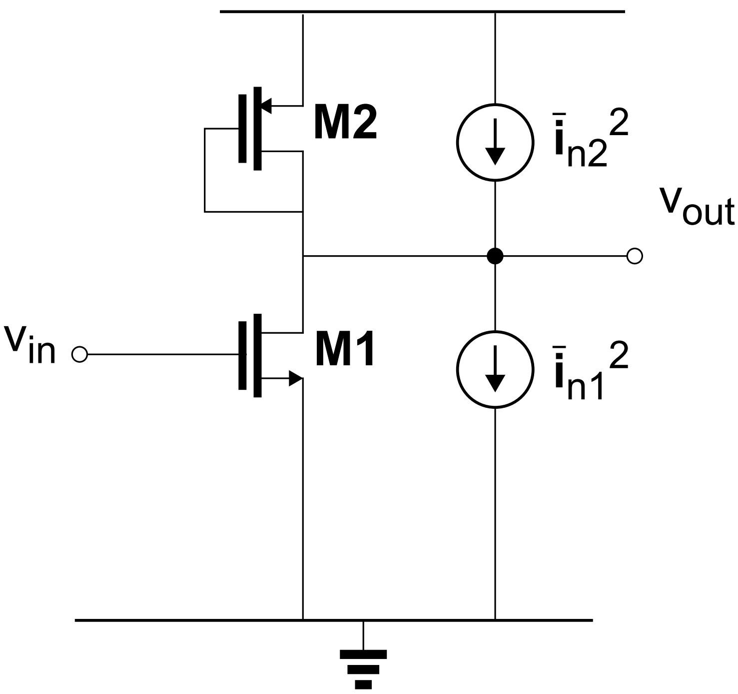
$$g_{m1}v_{in} + g_{m2}v_{out} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + g_{ds1} + g_{ds2}} \cong -\frac{g_{m1}}{g_{m2}}$$

Alternatively recognise that the current of the current-source $g_{m2}v_{gs2}$ is determined by voltage across its terminals i.e. is equivalent to a resistance $1/g_{m2}$.
Since $1/g_{m2} << 1/g_{ds2}$, $1/g_{m2} << 1/g_{ds1}$, can write directly

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

(iii) What is the input-referred thermal noise voltage in terms of the small signal parameters of M1 and M2, Boltzmann's constant k and temperature T?



Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_m} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iv) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

Noise voltage at output given by input referred noise multiplied by gain

$$\overline{v_{no}} = \overline{v_{ni}}\frac{g_{m1}}{g_{m2}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}}\frac{g_{m1}}{g_{m2}}$$

$$= \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m2}}$$

Capacitor C_L connected between the output node and ground
=> pole at output node given by

$$|f_p| = \frac{g_{m2}}{2\pi C_L}$$

Total integrated thermal noise power at the output node is given by the product of the thermal noise power and the squared-up equivalent of the first order filter function

$$\overline{v^2_{nototal}} = \overline{v^2_{no}}\frac{\pi}{2}f_p$$

$$\overline{v^2_{nototal}} = \frac{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}{g^2_{m2}} \cdot \frac{\pi}{2} \cdot \frac{g_{m2}}{2\pi C_L}$$

$$\overline{v^2_{nototal}} = \frac{\frac{2}{3}(g_{m1} + g_{m2})}{g_{m2}} \cdot \frac{kT}{C_L}$$

(v) Using the result of (iv) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $10mV_{rms}$ sine wave.
For this calculation take V_{GS1}=1V, $|V_{GS2}|$=2.8V, $|V_T|$ = 0.8V for M1,M2. C_L=10pF.
The drain current of M1 is 100μA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

g_m given by

$$g_m = \frac{2I_D}{(V_{GS} - V_T)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.8V} = 1mA/V$$

$$g_{m2} = \frac{2 \cdot 100\mu A}{2.8V - 0.8V} = 100\mu A/V$$

Output signal

$$v_{out} = -\frac{g_{m1}}{g_{m2}} v_{in} = -10 \cdot 10mV_{rms} = 100mV_{rms}$$

Total output noise:

$$\overline{v_{nototal}} = \sqrt{\frac{\frac{2}{3}(g_{m1} + g_{m2})}{g_{m2}} \cdot \frac{kT}{C_L}}$$

$$\overline{v_{nototal}} = \sqrt{\frac{\frac{2}{3}(1mA/V + 100\mu A/V)}{100\mu A/V} \cdot \frac{1.38\times 10^{-23} 300}{10pF}} = 55.1\mu V_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{100mV}{55.1\mu V} = 1815 \qquad \text{or 65dB}$$

Question 3

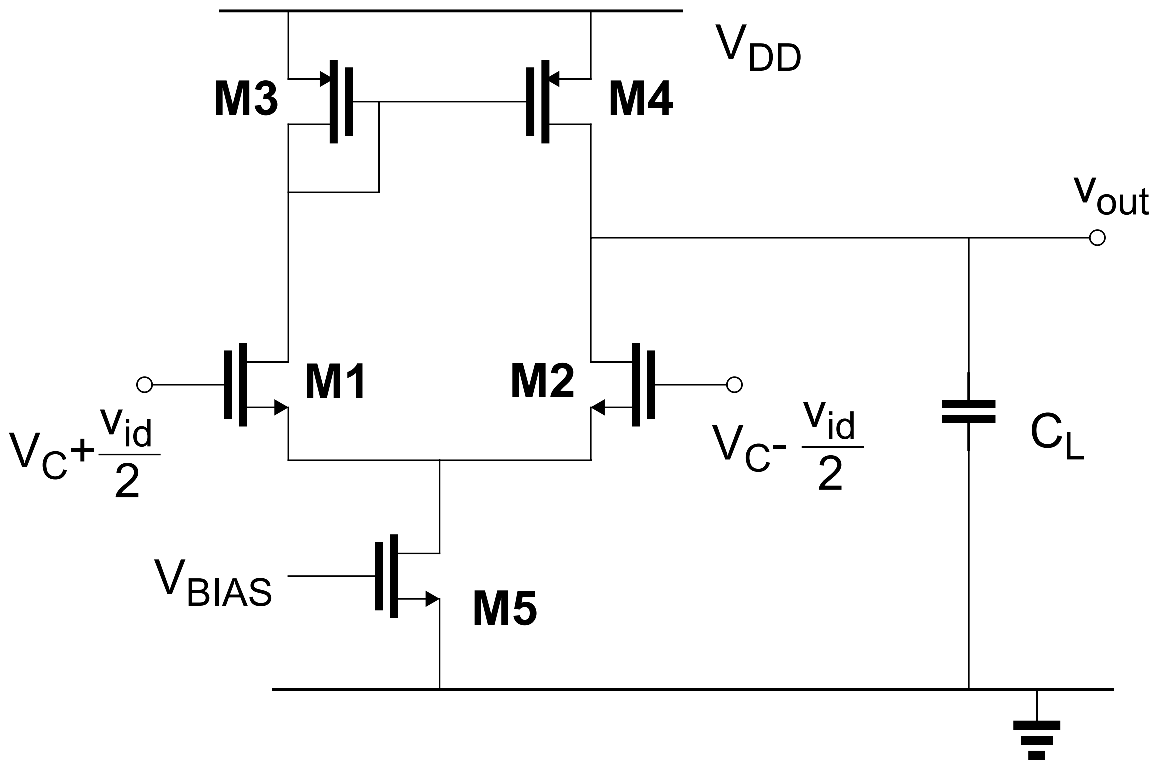


Figure 3.

Assume all devices are operating in saturation. Ignore the body effect.
Use M1=M2, $g_{m1}=g_{m2}=g_{mn}$, $g_{ds1}=g_{ds2}=g_{dsn}$
Use M3=M4, $g_{m3}=g_{m4}=g_{mp}$, $g_{ds3}=g_{ds4}=g_{dsp}$
V_C is the fixed common mode voltage.
A small differential voltage v_{id} is applied to the amplifier.

(i) Derive an expression for the small signal transfer function (V_{out}/v_{id}) of the amplifier in Figure 3 in terms of g_m, g_{ds} and C_L. Consider only capacitance C_L.

(ii) Give expressions for the following: low frequency gain, pole frequency, unity gain frequency.

(iii) Draw a Bode plot identifying the low-frequency gain, pole frequency, and unity gain frequency.

(iv) What is the effect on each of the parameters in (ii) if the bias current is doubled? Assume all devices remain in saturation.

(v) If the signal at the output node is a sine wave given by
$V_{out}=A\sin\omega t$, calculate the maximum frequency such that no slewing occurs.
Take A=0.5V, C_L=10pF. The drain current through M5 is 100μA.

(i) Derive an expression for the small signal transfer function (v_{out}/v_{id}) of the amplifier in Figure 3 in terms of g_m, g_{ds} and C_L. Consider only capacitance C_L.

Source of M1, M2 is at ac ground.

Half signal $v_{id}/2$ and $-v_{id}/2$ are amplified separately to output so that

$$v_{out}(s) = -\frac{g_{m1}}{g_{m3}} \cdot -\frac{g_{m4}}{g_{ds2}+g_{ds4}+sC_L}\left(\frac{v_{id}(s)}{2}\right) - \frac{g_{m2}}{g_{ds2}+g_{ds4}+sC_L}\left(-\frac{v_{id}(s)}{2}\right)$$

Using, $g_{m1}=g_{m2}=g_{mn}$, $g_{ds1}=g_{ds2}=g_{dsn}$ and $g_{m3}=g_{m4}=g_{mp}$, $g_{ds3}=g_{ds4}=g_{dsp}$

$$v_{out}(s) = -\frac{g_{mn}}{g_{mp}} \cdot -\frac{g_{mp}}{g_{dsn}+g_{dsp}+sC_L}\left(\frac{v_{id}}{2}\right) - \frac{g_{mn}}{g_{dsn}+g_{dsp}+sC_L}\left(-\frac{v_{id}}{2}\right)$$

$$\frac{v_{out}}{v_{id}}(s) = \frac{g_{mn}}{g_{dsn}+g_{dsp}+sC_L}$$

(ii) Give expressions for the following: low frequency gain, pole frequency, unity gain frequency

$$\frac{v_{out}}{v_{id}}(s) = \frac{g_{mn}}{g_{dsn} + g_{dsp} + sC_L}$$

Re-write to get

$$\frac{v_{out}}{v_{id}}(s) = \frac{g_{mn}}{g_{dsn} + g_{dsp}} \cdot \frac{g_{mn}}{1 + \dfrac{sC_L}{g_{dsn} + g_{dsp}}}$$

Low-frequency gain A_o given by

$$A_o = \frac{g_{mn}}{g_{dsn} + g_{dsp}}$$

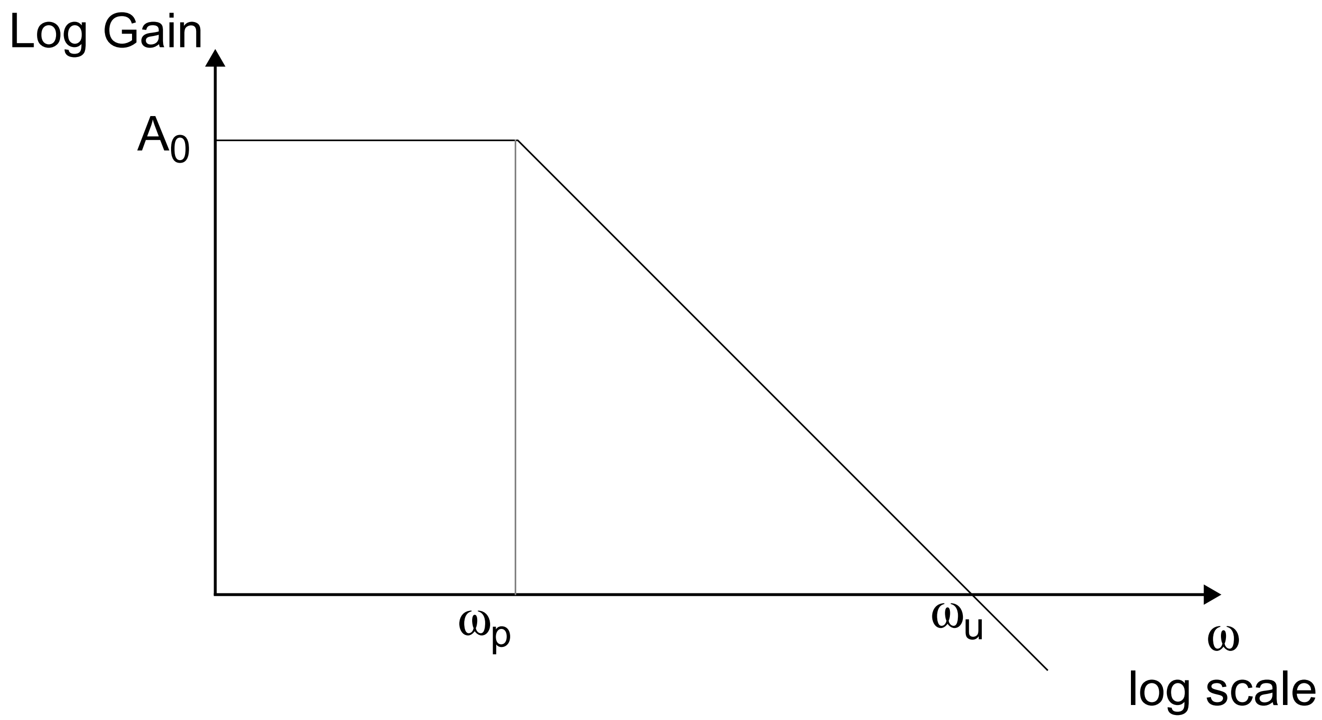
Pole ω_p given by

$$\left|\omega_p\right| = \frac{g_{dsn} + g_{dsp}}{C_L}$$

Unity gain frequency ω_u given by

$$\omega_u = \frac{g_{mn}}{g_{dsn} + g_{dsp}} \frac{g_{dsn} + g_{dsp}}{C_L} = \frac{g_{mn}}{C_L}$$

(iii) Draw a Bode plot identifying the low-frequency gain, pole frequency, and unity gain frequency.



(iv) What is the effect on each of the parameters in (ii) if the bias current is doubled? Assume all devices remain in saturation.

$$g_m = \sqrt{2K_n' \frac{W}{L} I_D}$$

If the bias current is doubled g_m will increase by factor square root 2

$$g_{ds} = \lambda I_D$$

If the bias current is doubled g_{ds} will increase by factor 2

Low-frequency gain Ao

$$A_o = \frac{g_{mn}}{g_{dsn} + g_{dsp}}$$

=> Ao will decrease by factor square root 2

Pole ω_p

$$\omega_p = \frac{g_{dsn} + g_{dsp}}{C_L}$$

=> ω_p will increase by factor 2

Unity gain frequency ω_u

$$\omega_u = \frac{g_{mn}}{C_L}$$

=> ω_u will increase by factor square root. 2

(v) If the signal at the output node is a sine wave given by $V_{out}=A\sin\omega t$, calculate the maximum frequency such that no slewing occurs. Take A=0.5V, C_L=10pF. The drain current through M5 is 100μA.

Slew Rate = $\left(\frac{dv_{out}}{dt}\right)_{max} = \frac{I_B}{C_L}$ where $I_B = I_{DM5}$

$$v_{out} = A\sin\omega t$$

$$\left(\frac{dv_{out}}{dt}\right)_{max} = A\omega$$

For no slewing

$$\left(\frac{dv_{out}}{dt}\right)_{max} = A\omega \le \frac{I_B}{C_L} \Rightarrow \omega \le \frac{I_B}{AC_L}$$

$$\omega_{max} = \frac{I_B}{AC_L} = \frac{100\mu A}{0.5V \cdot 10pF} = 20\times10^6 rad/s$$

or $f_{max} = 3.18MHz$