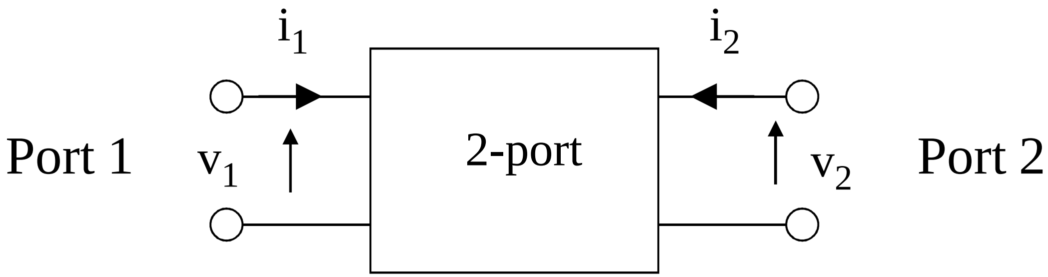
EE4011: RF IC Design

2-port Network Parameters

Two-Port Network Descriptions

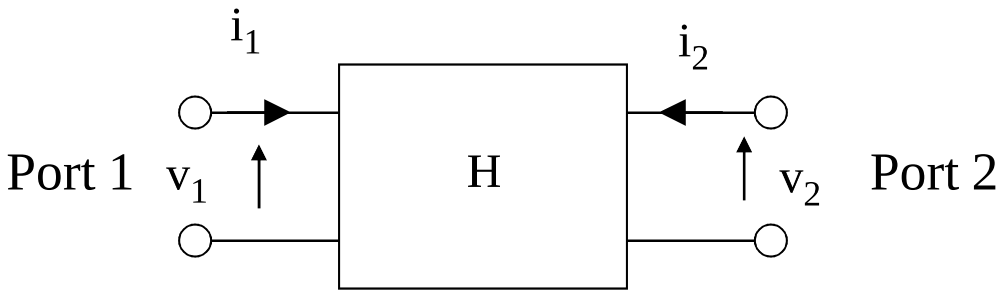
In many cases the transistor or other system that is being measured or simulated can be thought of as a “two-port system” i.e. a system with two connections for RF purposes – usually an input and an output. There are several standard network relationships for relating the quantities in such a system.



Most of the 2-port parameters give relationships between the currents and voltages at the input and output terminals. Every two-port has 4 variables – v_1, i_1, v_2, i_2 – two of which are considered *independent* variables with the other two being considered *dependent* variables. Different parameter sets are defined depending on the choice of independent and dependent variables.

Two-ports can also be defined by making a relationship between the incident and reflected waves at the terminals – this gives rise to scattering-parameters.

h-parameters (hybrid parameters)



$$v_1 = h_{11} i_1 + h_{12} v_2$$
$$i_2 = h_{21} i_1 + h_{22} v_2$$

Setting v_2 to 0: (output short-circuit)

$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0} \qquad h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}$$

Setting i_1 to 0: (input open circuit)

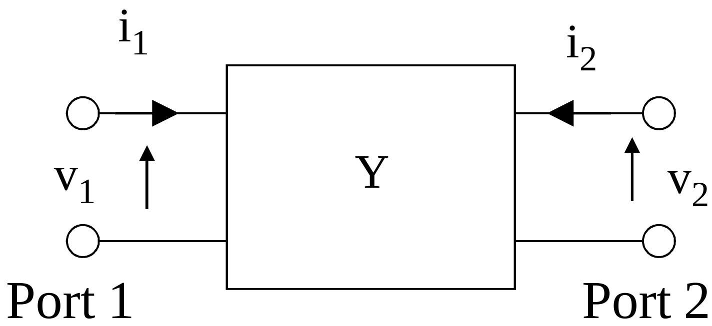
$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0} \qquad h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$$

These are called hybrid parameters because they have different units.
h_{12} and h_{21} are unitless.
h_{11} has units of resistance (Ω).
h_{22} has units of conductance (S).

h-parameters are not widely used nowadays except for h_{21} (which is commonly called h_{fe}) which has become standard to define the cut-off frequency of a transistor.

y-parameters (admittance parameters)

y-parameters treat the terminal voltages as the independent variables and give currents as a function of the voltages. They have units of conductance (S). They are widely used for RF component analysis:



$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$

Setting v_2 to 0:
(output short-circuit)

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

Setting v_1 to 0:
(input short-circuit)

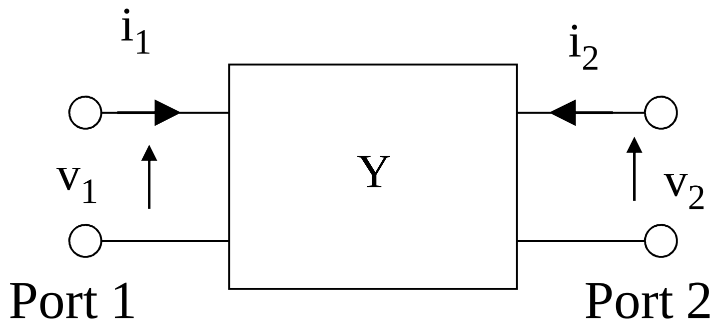
$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Note:

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0} = \frac{y_{21}}{y_{11}}$$

y-parameters – note on subscripts

$$y = \frac{i}{v} = \frac{\text{current}}{\text{voltage}}$$



$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$

The first subscript refers to the current flowing in a particular terminal.

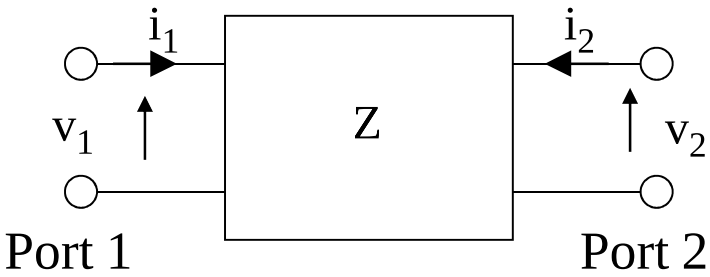
The other voltage is set to 0.

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

The “y” parameter establishes how sensitive this current is w.r.t. a particular voltage. The second subscript is the voltage.

z-parameters (impedance parameters)

z-parameters (impedance parameters) treat the terminal currents as independent and specify the voltages as a function of currents. They have units of impedance (Ω).



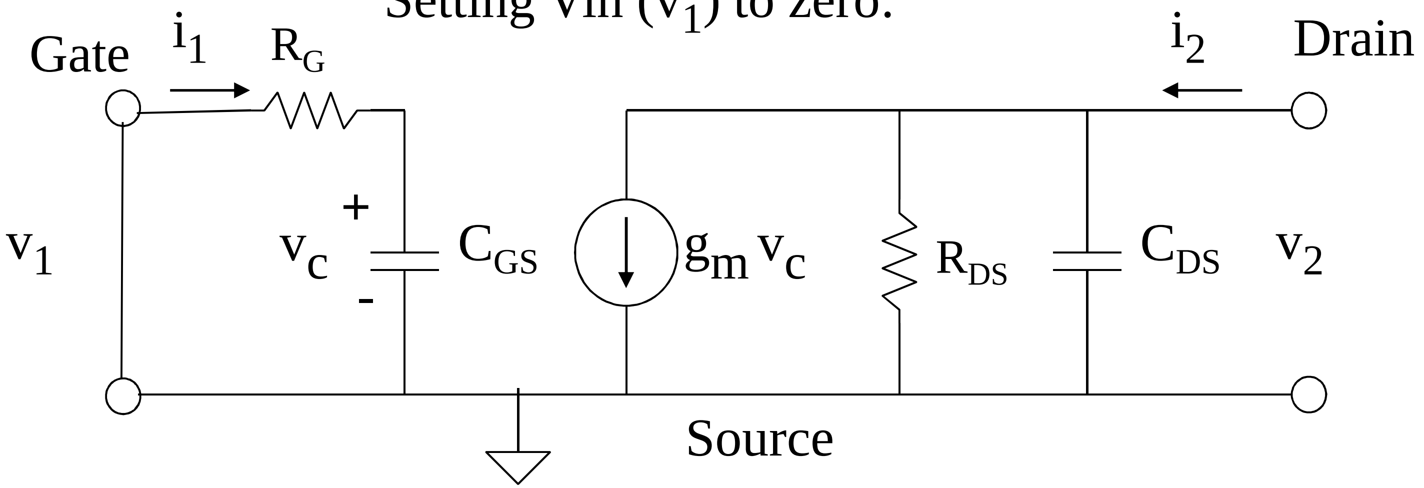
$$v_1 = z_{11} i_1 + z_{12} i_2$$
$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$\text{Setting } i_2 \text{ to } 0: \quad z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} \qquad z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0}$$

$$\text{Setting } i_1 \text{ to } 0: \quad z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} \qquad z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$$

y-parameters for unilateral MESFET (1)

Setting Vin (v_1) to zero:

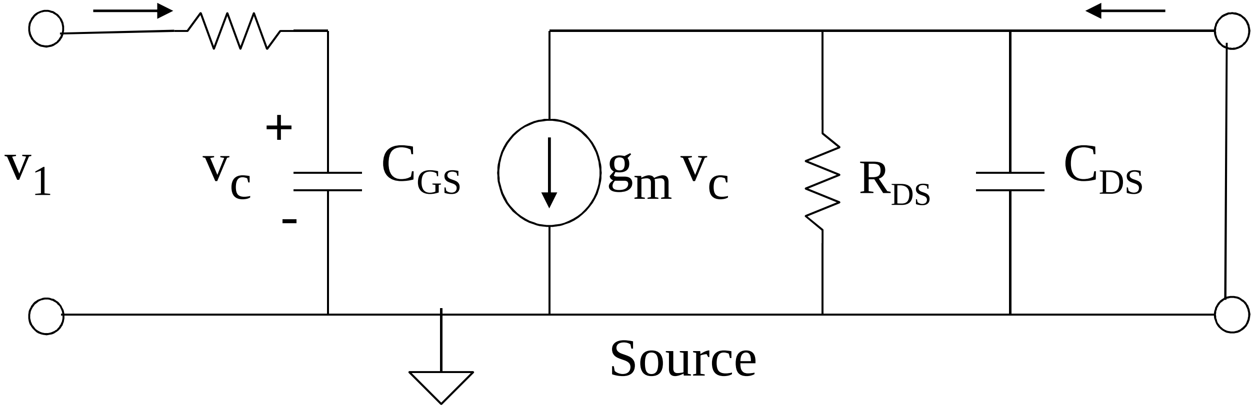


$$v_1 = 0 \Rightarrow i_1 = 0 \Rightarrow v_c = 0 \Rightarrow g_m v_c = 0$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = \frac{0}{v_2} = 0 \qquad \text{(unilateral property)}$$

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{v_2(1/R_{DS} + j\omega C_{DS})}{v_2} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

y-parameters for unilateral MESFET (2)



Because v_2 is zero the current in R_{DS} and C_{DS} is zero.

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = v_1 \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \frac{1}{v_1} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}}$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = v_1 \frac{g_m}{1 + j\omega R_G C_{GS}} \frac{1}{v_1} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

Small-signal elements from y-parameters (1)

Unilateral case

$$y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

$$\mathscr{Re}\{y_{22}\} = \frac{1}{R_{DS}} \Rightarrow R_{DS} = \frac{1}{\mathscr{Re}\{y_{22}\}}$$

$$\mathscr{Im}\{y_{22}\} = \omega C_{DS} \Rightarrow C_{DS} = \frac{\mathscr{Im}\{y_{22}\}}{\omega}$$

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \Rightarrow \frac{1}{y_{11}} = \frac{1 + j\omega R_G C_{GS}}{j\omega C_{GS}} = R_G - \frac{j}{\omega C_{GS}}$$

$$\Rightarrow R_G = \mathscr{Re}\left\{\frac{1}{y_{11}}\right\}$$

$$\mathscr{Im}\left\{\frac{1}{y_{11}}\right\} = -\frac{1}{\omega C_{GS}} \Rightarrow C_{GS} = -\frac{1}{\omega \mathscr{Im}\left\{\frac{1}{y_{11}}\right\}}$$

Small-signal elements from y-parameters (2)

Unilateral case

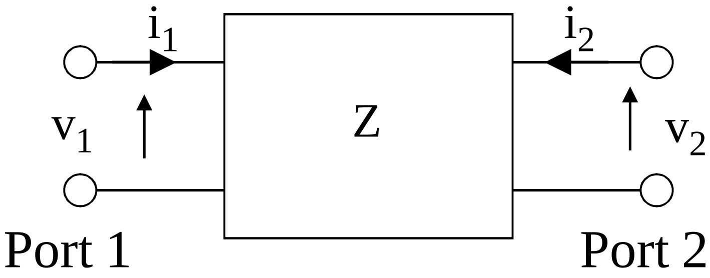
$$y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}} \Rightarrow \frac{1}{y_{21}} = \frac{1 + j\omega R_G C_{GS}}{g_m} = \frac{1}{g_m} + j\frac{\omega R_G C_{GS}}{g_m}$$

$$\mathcal{Re}\left\{\frac{1}{y_{21}}\right\} = \frac{1}{g_m} \Rightarrow g_m = \frac{1}{\mathcal{Re}\left\{\frac{1}{y_{21}}\right\}}$$

This shows how measurements of y-parameters can be used to work backwards and determine the small-signal equivalent circuit values for an RF device. Usually s-parameters are measured and these are converted to y-parameters before using these equations.
The formulas here were simplified by the assumption that the gate-drain coupling capacitance was zero (the unilateral device).

Matrix Relationships

When manipulating the 2-port parameters, it is very convenient to rewrite the equations in matrix format where the terminal voltages and currents are grouped into column matrices and the 2-port parameters are grouped into a 2x2 square matrix.



$$\mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

e.g. for the y-parameters:

$$\left.\begin{aligned} i_1 &= y_{11}v_1 + y_{12}v_2 \\ i_2 &= y_{21}v_1 + y_{22}v_2 \end{aligned}\right\rangle \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \mathbf{I} = \mathbf{YV} \quad \mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

and the z-parameters:

$$\left.\begin{aligned} v_1 &= z_{11}i_1 + z_{12}i_2 \\ v_2 &= z_{21}i_1 + z_{22}i_2 \end{aligned}\right\rangle \Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \Rightarrow \mathbf{V} = \mathbf{ZI} \qquad \mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

Comments on matrix notation

y-parameters: $\mathbf{I} = \mathbf{YV}$

z-parameters: $\mathbf{V} = \mathbf{ZI}$

Note: The “**I**” in these equations refer to the column vector for current. When dealing with matrix notation the symbol “**I**” is usually used to denote the identity matrix – but for these equations with the 2-port parameters we’ll assume that “**I**” refers to the current matrix unless told otherwise.

When manipulating matrices remember that matrix multiplication is not commutative i.e. given two matrices A and B with non-zero elements:

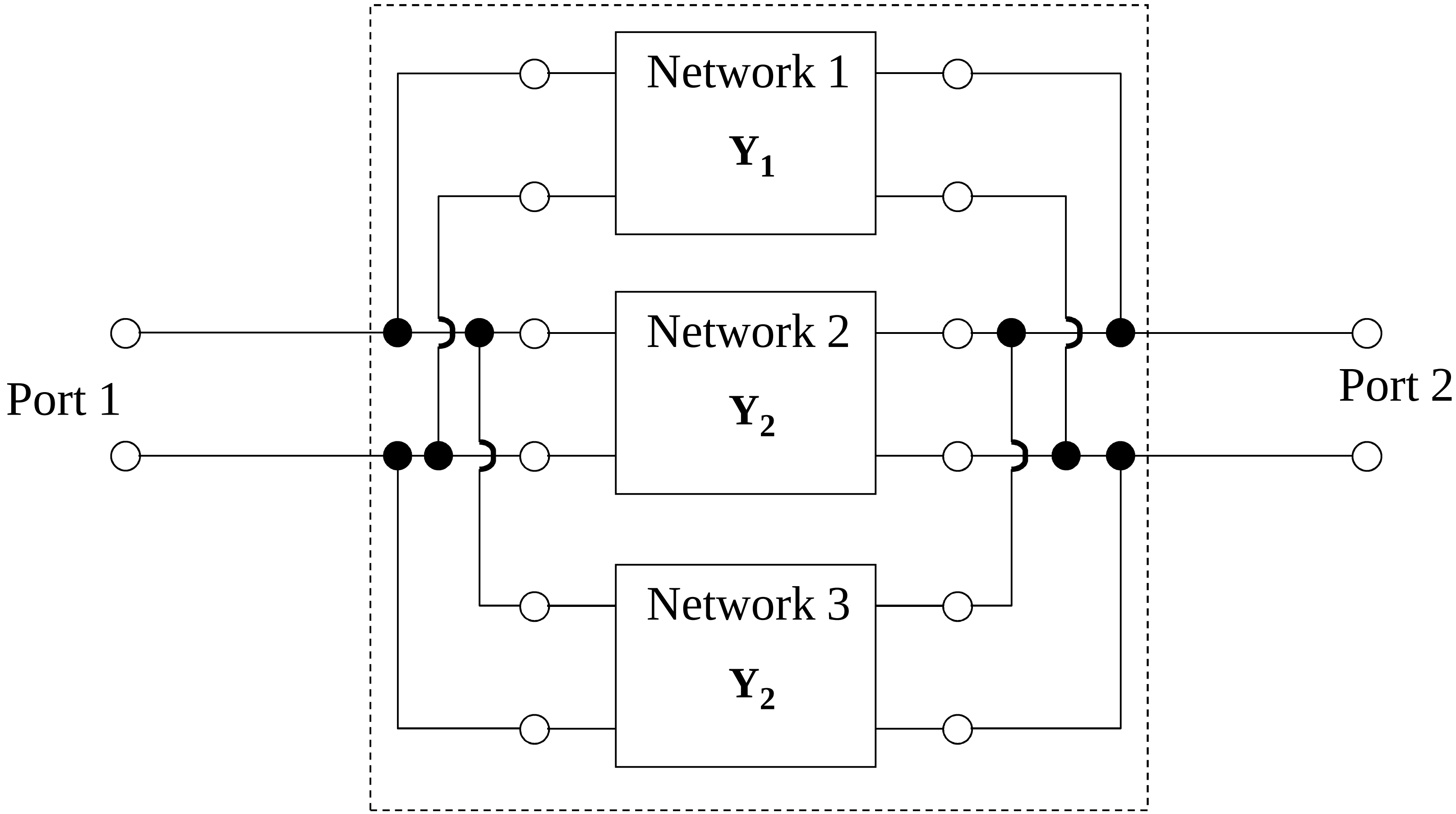
$$\mathbf{AB} \neq \mathbf{BA} \quad , \quad \mathbf{A} \neq \mathbf{B}$$

Relationship between admittance and impedance matrices for a 2-port:

$$\mathbf{I} = \mathbf{YV} \Rightarrow \mathbf{Y}^{-1}\mathbf{I} = \mathbf{Y}^{-1}\mathbf{YV} \Rightarrow \mathbf{Y}^{-1}\mathbf{I} = \mathbf{V}$$

$$\Rightarrow \mathbf{Y}^{-1}\mathbf{I} = \mathbf{ZI} \Rightarrow \mathbf{Y}^{-1} = \mathbf{Z} \quad i.e. \quad \mathbf{Z} = \mathbf{Y}^{-1} \qquad also \quad \mathbf{Y} = \mathbf{Z}^{-1}$$

Parallel combinations of 2-ports/ Y-parameters



The overall y-parameter matrix of a group of two-ports in parallel can be calculated as the sum of the y-parameter matrices of the individual networks i.e.

$$\mathbf{Y_{TOTAL}} = \mathbf{Y_1} + \mathbf{Y_2} + \mathbf{Y_3}$$

Y to Z conversion and vice versa

The inverse of a 2x2 matrix is given by:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \mathbf{A}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \Delta = ad - bc$$

This relationship can be used to convert between y and z parameters:

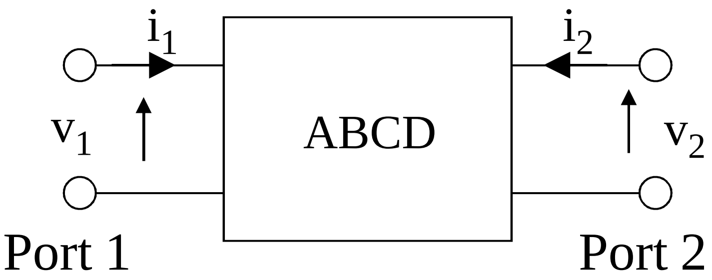
$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \quad \mathbf{Z} = \mathbf{Y}^{-1} = \frac{1}{y_{11}y_{22} - y_{12}y_{21}}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad \mathbf{Y} = \mathbf{Z}^{-1} = \frac{1}{z_{11}z_{22} - z_{12}z_{21}}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

These conversions can be useful when trying to determine the performance of a large network consisting of several interconnected smaller networks.

ABCD-parameters (transmission parameters)

The transmission parameters (the ABCD parameters) specify the relationship between the voltage and current at port 1 and the voltage and current at port 2. By convention, the 2-port parameters treat current as flowing *into* the terminal. The ABCD parameters are written in terms of the current flowing *out of* port 2 so the equations use a minus sign for i_2 i.e. $-i_2$ is the current flowing *out of* port 2.



$$\begin{aligned} v_1 &= Av_2 - Bi_2 \\ i_1 &= Cv_2 - Di_2 \end{aligned} \Rightarrow \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

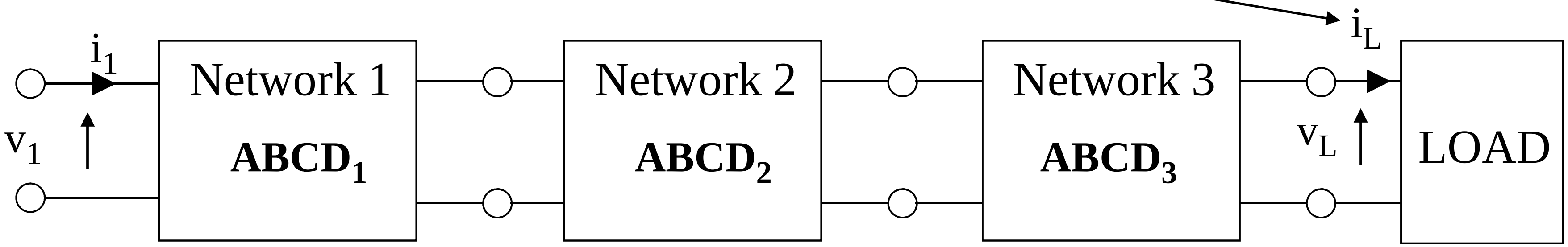
$$\text{Setting } i_2 \text{ to } 0: \quad A = \left.\frac{v_1}{v_2}\right|_{i_2=0} \qquad C = \left.\frac{i_1}{v_2}\right|_{i_2=0}$$

$$\text{Setting } v_2 \text{ to } 0: \quad B = -\left.\frac{v_1}{i_2}\right|_{v_2=0} \qquad D = -\left.\frac{i_1}{i_2}\right|_{v_2=0}$$

Cascading ABCD parameters

For cascaded ABCD networks the overall ABCD parameters can be found by multiplying the individual ABCD matrices in the *correct order*. This is very useful for developing a relationship between the input and the load in multi-stage networks.

Note the load current here is shown as flowing *into the* load so it is the negative of the current flowing into port 2 of network 3.



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Network1} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Network2} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Network3} \begin{bmatrix} v_L \\ i_L \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = (\mathbf{ABCD})_{\mathbf{1}} (\mathbf{ABCD})_{\mathbf{2}} (\mathbf{ABCD})_{\mathbf{3}} \begin{bmatrix} v_L \\ i_L \end{bmatrix} = (\mathbf{ABCD})_{\mathbf{Total}} \begin{bmatrix} v_L \\ i_L \end{bmatrix}$$

$$(\mathbf{ABCD})_{\mathbf{Total}} = (\mathbf{ABCD})_{\mathbf{1}} (\mathbf{ABCD})_{\mathbf{2}} (\mathbf{ABCD})_{\mathbf{3}}$$

A note on complex numbers

Manipulations of 2-port parameters generally involve complex numbers when frequency dependent behaviour is being considered.

If x is a complex number with real part a and imaginary part b, 1/x is given by:

$$x = a + jb$$

$$\frac{1}{x} = \frac{1}{a + jb} = \frac{1}{a + jb}\frac{a - jb}{a - jb} = \frac{a - jb}{a^2 + b^2}$$

The final result of a complex number calculation is best expressed as an amplitude and a phase in degrees

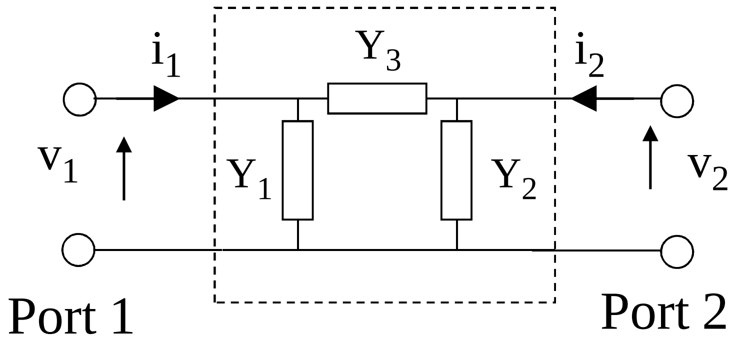
$$x = a + jb \Rightarrow |x| = \sqrt{a^2 + b^2} \quad and \quad \angle x = \tan^{-1}\left(\frac{b}{a}\right)$$

$$e.g. \quad x = 1 + j = \sqrt{2}\angle 45^\circ$$

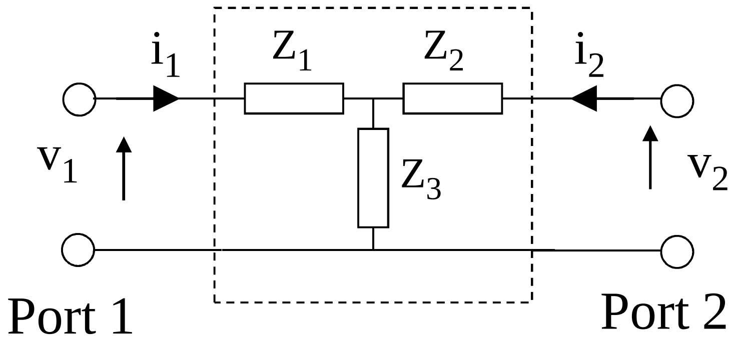
$$\text{angle in degrees} = \frac{180}{\pi} \times \text{angle in radians}$$

Practice: Evaluate the y, z and ABCD parameters of:

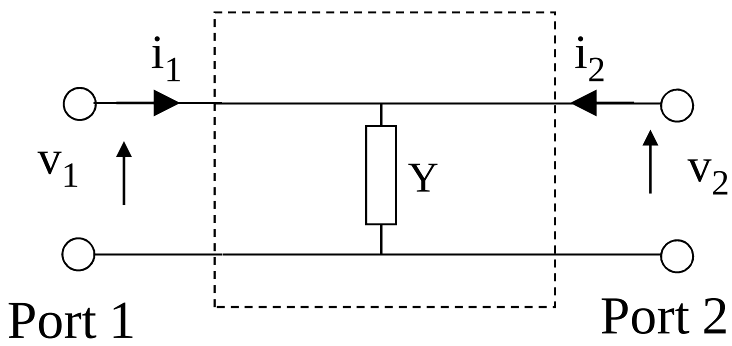
Π-network



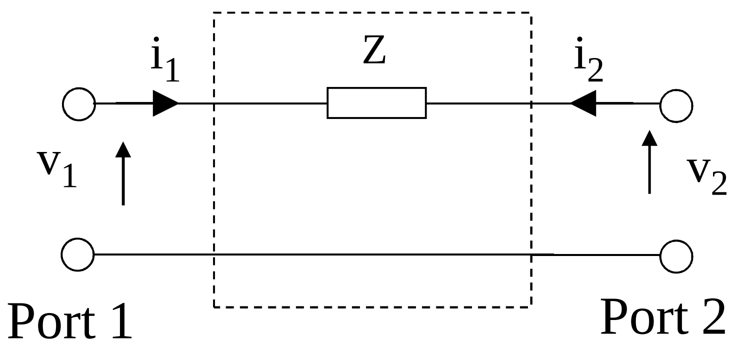
T-network



Shunt element



Series element



Y, Y_1, Y_2 and Y_3 are admittances

Z, Z_1, Z_2 and Z_3 are impedances

Sample Question

Q1, EE4011, Summer 2004

(a) Show a small-signal model of a bipolar junction transistor (BJT) suitable for first-order analysis and from this derive an expression for the cut-off frequency of the transistor in a common-emitter configuration. Assume the transistor is biased in the forward active region with currents given by

$$I_C = I_S e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{CE}}{V_A}\right) \quad , \quad I_B = \frac{I_C}{\beta}$$

where the symbols have their usual meaning. Only consider capacitances associated with the base-emitter circuit.

[10 *marks*]

(b) A BJT is configured as a common-emitter two-port amplifier with the input applied to the base (port 1) and the output taken from the collector (port 2). Determine:

(I) The cut-off frequency [2 *marks*]
(ii) The 4 two-port y-parameters at 1GHz [8 *marks*]

Use the following bias conditions and parameters: V_{BE}=0.75 V, V_{CE}=3.0 V, T=300 K $I_S=10^{-15}$ A,

β=100, V_A=10V, $C_{JE}=10^{-12}$ F, V_{JE}=1 V, M_{JE}=0.5, $\tau_F=10^{-10}$ s

Sample Question

Q1, EE4011, Summer 2005

(a) Show a small-signal model of a bipolar junction transistor (BJT) suitable for first-order analysis and from this derive an expression for the cut-off frequency of the transistor in a common-emitter configuration. Assume the transistor is biased in the forward active region with currents given by

$$I_C = I_S e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{CE}}{V_A}\right) \quad , \quad I_B = \frac{I_C}{\beta}$$

[*8 marks*]

(b) A BJT is configured as a common-emitter two-port amplifier with the input applied to the base (port 1) and the output taken from the collector (port 2). The cut-off frequency has been measured at a temperature of 300K for two values of collector current as follows:

For I_C = 1mA, f_T = 1.26 GHz
For I_C = 5mA, f_T = 1.51 GHz

(c) For this device, estimate (i) the forward base transit time and (ii) the base-emitter junction capacitance (the bias dependence of the base-emitter junction capacitance may be ignored for this calculation)

[*8 marks*]

For a typical BJT illustrate the variation of (i) the current gain and (ii) the cut-off frequency, as a function of collector current.

[*4 marks*]

Sample Question

EE4005, Autumn 2002

Question 5

(a) Define y-parameters which can be used to represent the small-signal behaviour of a two-port network.

(b) Draw a small-signal circuit for a GaAs MESFET which illustrates the most important elements and determine expressions for the four y-parameters of the network considering the input to be at the gate side and the output to be at the drain side.

(c) Determine values for the 4 y-parameters for the equivalent circuit of part (b) at a frequency of 2GHz if the small-signal circuit elements have the following values: R_G=4.5Ω, C_{GS}=0.75pF, R_{DS}=200Ω, C_{DS}=0.07pF and g_m=100mS. Also determine the cut-off frequency of the device for these values.

Sample Question

EE4011, Summer 2005, Q2

(a) Show a small-signal model of a GaAs MESFET suitable for small-signal analysis and derive expressions for the four y-parameters of the device, assuming that port 1 of the network is at the gate/source and port 2 is at the drain/source. The gate-to-drain capacitance may be ignored.

[10 marks]

(b) The y-parameters of a GaAs MESFET in a common-source amplifier configuration have been measured at 3GHz with the following results:

$$y_{11} = 0.018 \angle 85.7°$$
$$y_{12} = 0$$
$$y_{21} = 0.249 \angle -4.31°$$
$$y_{22} = 0.020 \angle 8.05°$$

From these measurements, determine the values of the elements of the small-signal equivalent circuit for the device at 3GHz and also the cut-off frequency of the device.

[10 marks]