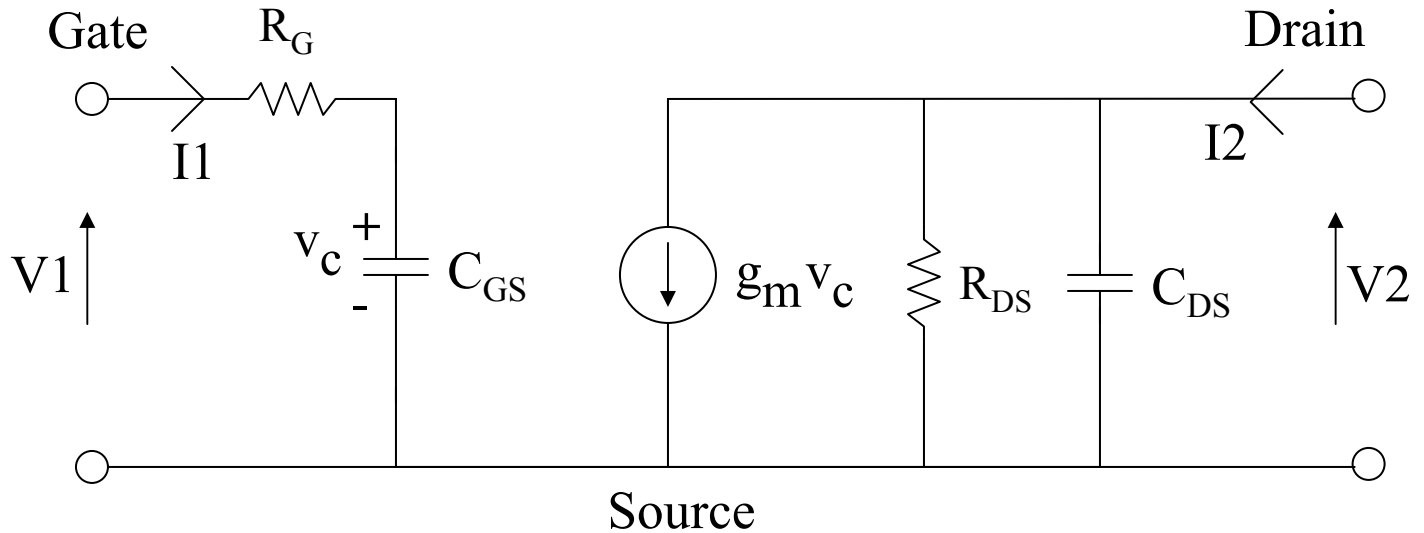


Question 1(a)

Simplified MESFET small-signal circuit, ignoring C_{GD}



$$y_{11} = \left. \frac{i_1}{v_1} \right|_{v_2=0} \quad y_{21} = \left. \frac{i_2}{v_1} \right|_{v_2=0} \quad y_{12} = \left. \frac{i_1}{v_2} \right|_{v_1=0} \quad y_{22} = \left. \frac{i_2}{v_2} \right|_{v_1=0}$$

Applying the above formulas to the equivalent circuit and simplifying the resulting expressions leads to the final y-parameter formulas:

$$y_{11} = \frac{j\omega C_{GS}}{1 + j\omega R_G C_{GS}} \quad y_{21} = \frac{g_m}{1 + j\omega R_G C_{GS}}$$

$$y_{12} = 0 \quad y_{22} = \frac{1}{R_{DS}} + j\omega C_{DS}$$

10 marks

Question 1(b)

The previous expressions for the y-parameters can be re-arranged to allow the small-signal element values to be determined from the y-parameters (assuming $y_{12}=0$). Putting in the values of the y-parameters at 1.5GHz gives:

$$y_{11} = 0.0071 \angle 87.98^\circ$$

$$y_{12} = 0$$

$$y_{21} = 0.1999 \angle -2.02^\circ$$

$$y_{22} = 0.0173 \angle 15.79^\circ$$

$$R_G = \mathcal{Re}\left\{\frac{1}{y_{11}}\right\} = 5\Omega$$

$$C_{GS} = -\frac{1}{\omega \mathcal{Im}\left\{\frac{1}{y_{11}}\right\}} = 0.75 pF$$

$$g_m = \frac{1}{\mathcal{Re}\left\{\frac{1}{y_{21}}\right\}} = 0.2 S$$

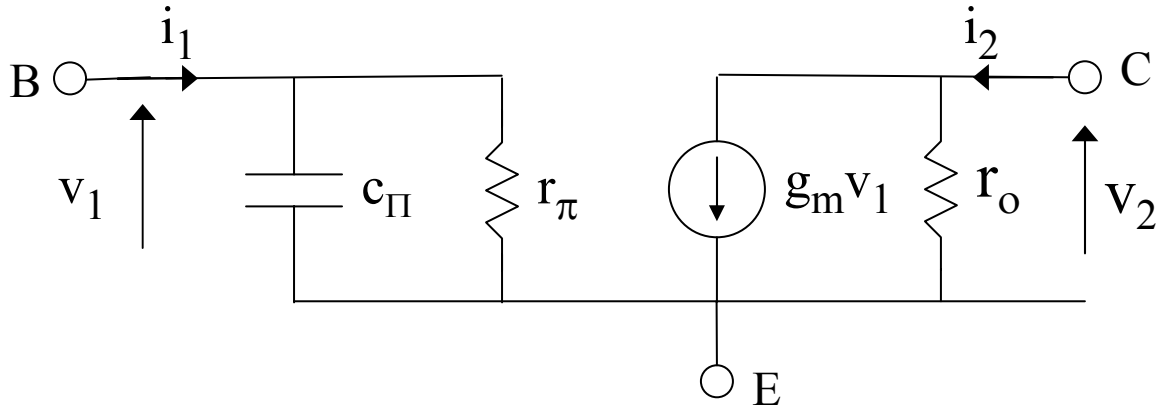
$$R_{DS} = \frac{1}{\mathcal{Re}\{y_{22}\}} = 60\Omega$$

$$C_{DS} = \frac{\mathcal{Im}\{y_{22}\}}{\omega} - 0.5 pF$$

$$f_T = \frac{1}{2\pi C_{GS}} = 42.44 GHz$$

Question 2(a)

A suitable small-signal model for a BJT, accounting only for the base-emitter capacitances is:



Applying the z-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$Z_{11} = \left. \frac{v_1}{i_1} \right|_{i_2=0} = \frac{r_\pi}{1 + j\omega r_\pi c_\pi}$$

$$Z_{21} = \left. \frac{v_2}{i_1} \right|_{i_2=0} = -\frac{g_m r_\pi r_o}{1 + j\omega r_\pi c_\pi}$$

$$Z_{12} = \left. \frac{v_1}{i_2} \right|_{i_1=0} = 0$$

$$Z_{22} = \left. \frac{v_2}{i_2} \right|_{i_1=0} = r_o$$

Question 2(b)

Calculate the z-parameters under the following conditions

$T=300K$, $f=1GHz$, $V_{BE} = 0.8V$, $V_{CE} = 3.0V$, $I_S = 1 \times 10^{-15}A$, $V_A = 10V$,
 $\beta = 100$, $C_{JE} = 0.3pF$, $M_{JE} = 0.5$, $V_{JE} = 1.0V$ and $\tau_F = 0.1ns$.

$$V_T = \frac{kT}{q} = 25.9mV \quad I_C = I_S \exp\left(\frac{qV_{BE}}{kT}\right) \left(1 + \frac{V_{CE}}{V_A}\right) = 35.4mA$$

$$g_m = \frac{I_C}{V_T} = 1.37S \quad r_o \approx \frac{V_A}{I_C} = 283 \Omega$$

$$c_\pi = \frac{C_{JE}}{\left(1 - \frac{V_{BE}}{V_{JE}}\right)^{M_{JE}}} + g_m \tau_F = 0.67 \times 10^{-12} + 1.37 \times 10^{-10} \approx 1.37 \times 10^{-10} F$$

$$r_\pi = \frac{\beta}{g_m} = 73 \Omega$$

Inserting these values into the previous formulas for the z-parameters at 1GHz gives:

$$\begin{aligned} z_{11} &= 1.16 \angle -89.1^\circ \\ z_{12} &= 0 \\ z_{21} &= 448 \angle 90.9^\circ \\ z_{22} &= 283 \angle 0^\circ \end{aligned}$$

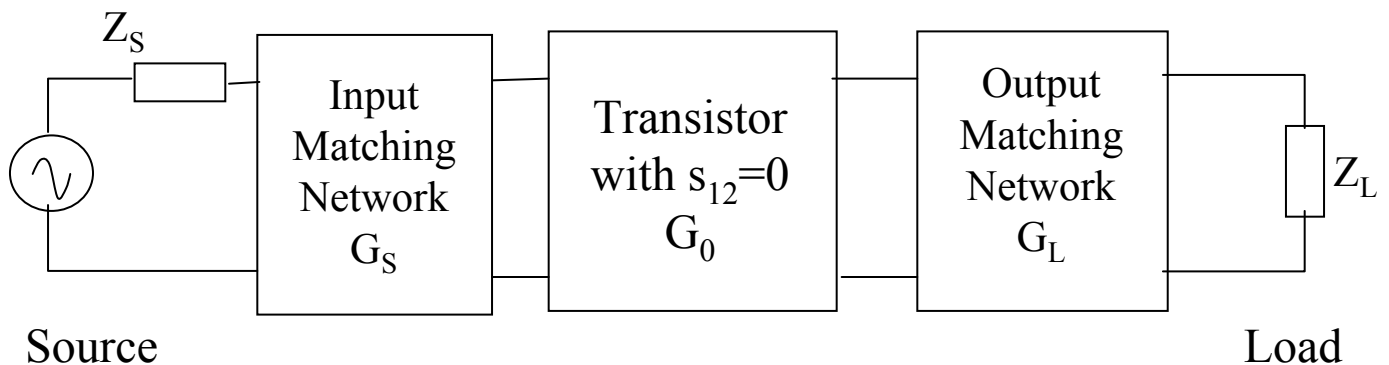
10 marks

Question 3(a)

- (i) The operating power gain (also just called the power gain) is the ratio of the power delivered to the load to the power delivered to the network by the source.
- (ii) The transducer power gain is the ratio of the power delivered to the load to the power *available* from the source.
- (iii) The available power gain is the ratio of the power *available* from the network to the power *available* from the source.

3 marks

(b) Maximum unilateral gain



To obtain the maximum unilateral gain, input and output matching networks are designed to provide conjugate matching to the transistor input and output ports as follows:

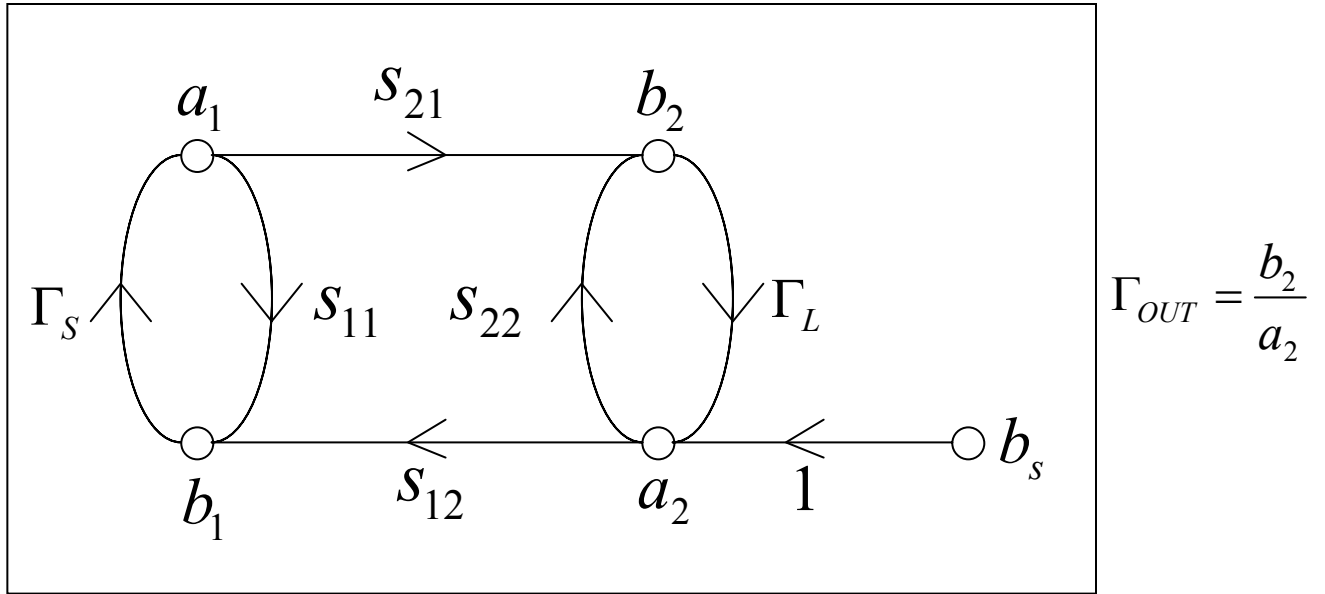
$$\Gamma_S = s_{11}^* \quad \Gamma_L = s_{22}^*$$

If the transistor is not already unilateral then an extra network can be added to force $s_{12}=0$.

3 marks

Question 3(c)

(c) Output reflection coefficient



Following the signal flow diagram from left to right:

$$a_1 = \Gamma_s b_1$$

$$b_1 = s_{11}a_1 + s_{12}a_2 = s_{11}\Gamma_s b_1 + s_{12}a_2 \Rightarrow b_1 = \frac{s_{12}a_2}{1 - s_{11}\Gamma_s} \Rightarrow a_1 = \frac{\Gamma_s s_{12}a_2}{1 - s_{11}\Gamma_s}$$

had s_{11} here and on subsequent lines by mistake

$$b_2 = s_{21}a_1 + s_{22}a_2 = \frac{s_{21}\Gamma_s s_{12}a_2}{1 - s_{11}\Gamma_s} + s_{22}a_2 = \frac{(s_{21}s_{12}\Gamma_s + s_{22} - s_{11}s_{22}\Gamma_s)a_2}{1 - s_{11}\Gamma_s}$$

$$b_2 = \frac{(s_{21}s_{12}\Gamma_s + s_{22}(1 - s_{11}\Gamma_s))a_2}{1 - s_{11}\Gamma_s}$$

$$\Rightarrow \Gamma_{OUT} = \frac{b_2}{a_2} = s_{22} + \frac{s_{21}s_{12}\Gamma_s}{1 - s_{11}\Gamma_s}$$

12 marks

(d)

$$s_{11} = 0.863 \angle -79.1^\circ$$

$$s_{12} = 0.072 \angle 36.5^\circ$$

$$s_{21} = 3.434 \angle 106.2^\circ$$

$$s_{22} = 0.627 \angle -58.3^\circ$$

$$\Gamma_s = 0.1 \angle 0^\circ$$

$$\Gamma_{OUT} = 0.603 \angle -59^\circ$$

Question 4

$$s_{11} = 0.707 \angle -155^\circ \quad s_{12} = 0 \quad s_{21} = 5.00 \angle 180^\circ \quad s_{22} = 0.51 \angle -20^\circ$$

$$Z_0 = 50 \, \Omega \quad F_{\min} = 3 \, \text{dB} \quad \Gamma_{\text{opt}} = 0.45 \angle 180^\circ \quad R_N = 4 \, \Omega$$

(a) (i)

$$G_{S,\max} = \frac{1}{1 - |s_{11}|^2} = \frac{1}{1 - |0.707|^2} = 2 = 3 \, \text{dB}$$

$$G_0 = |5|^2 = 25 = 14 \, \text{dB}$$

$$G_{L,\max} = \frac{1}{1 - |s_{22}|^2} = \frac{1}{1 - |0.51|^2} = 1.35 = 1.3 \, \text{dB}$$

maximum gain:

$$G_{TU,\max,\text{dB}} = G_{S,\max,\text{dB}} + G_{0,\text{dB}} + G_{L,\max,\text{dB}} = 3 \, \text{dB} + 14 \, \text{dB} + 1.3 \, \text{dB} = 18.3 \, \text{dB}$$

(a)(ii)

2 marks

The noise circle for $F_i = 3.1 \, \text{dB}$

$$F_{\text{dB}} = 10 * \log_{10}(F_{\text{ratio}}) \Rightarrow F_{\text{ratio}} = 10^{\frac{F_{\text{dB}}}{10}}$$

$$F_{\min,\text{ratio}} = 10^{\frac{3}{10}} = 1.9953 \quad F_{i,\text{ratio}} = 10^{\frac{3.1}{10}} = 2.0417$$

$$\Gamma_{\text{opt}} = 0.45 \angle 180^\circ = -0.45$$

$$N_i = \frac{F_{i,\text{ratio}} - F_{\min,\text{ratio}}}{4R_N / Z_0} |1 + \Gamma_{\text{opt}}|^2 = \frac{2.0417 - 1.9953}{4 \times 4 / 50} |1 - 0.45|^2 = 0.0439$$

$$C_{Fi} = \frac{\Gamma_{\text{opt}}}{N_i + 1} = \frac{-0.45}{0.0439 + 1} = -0.43 = 0.43 \angle 180^\circ$$

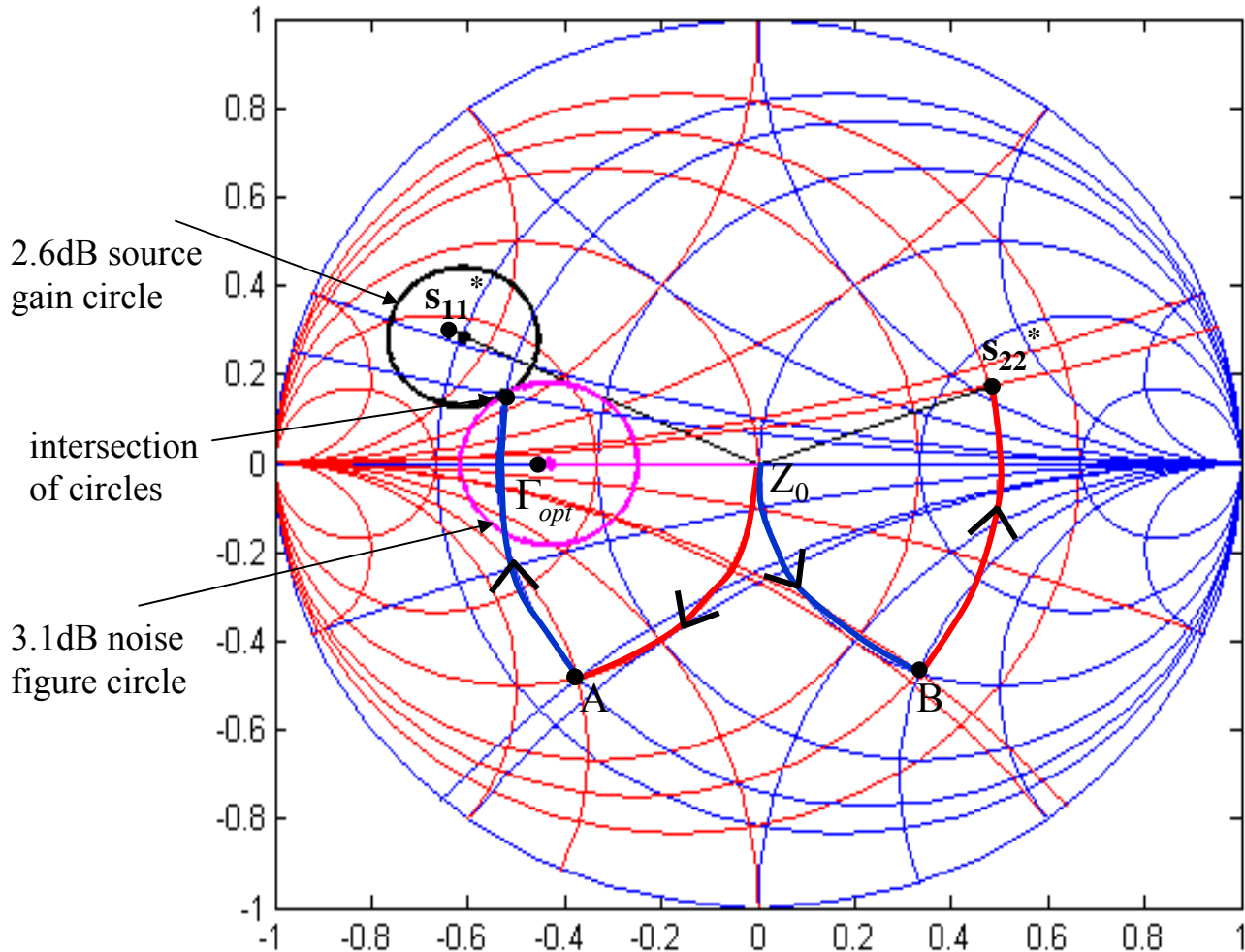
$$R_{Fi} = \frac{\sqrt{N_i(N_i + 1 - |\Gamma_{\text{opt}}|^2)}}{(N_i + 1)} = \frac{\sqrt{0.0439(0.0439 + 1 - |0.45|^2)}}{(0.0439 + 1)} = 0.18$$

The centre of the 3.1dB noise circle is at $\Gamma = -0.43$ and it has a radius of 0.18.

3 marks

Question 4(b)

The maximum source gain is 3dB. But the noise cannot exceed 3.1dB - therefore one or more source gain circles have to be drawn to see which gain circle overlaps slightly with the 3.1dB noise circle. A source gain circle of 2.6dB comes closest – this has to be determined by trial and error.



The source gain circle for $G_{S,dB} = 2.6$

$$G_{S,dB} = 10 \log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{2.6}{10}} = 1.820$$

$$g_s = \frac{G_S}{G_{S,max}} = \frac{1.820}{2} = 0.910$$

$$|C_S| = \frac{g_s |s_{11}|}{1 - |s_{11}|^2 (1 - g_s)} = \frac{0.910 \times |0.707|}{1 - |0.707|^2 (1 - 0.910)} = 0.67$$

$$R_S = \frac{\sqrt{1 - g_s} (1 - |s_{11}|^2)}{1 - |s_{11}|^2 (1 - g_s)} = \frac{\sqrt{1 - 0.910} (1 - |0.707|^2)}{1 - |0.707|^2 (1 - 0.910)} = 0.16$$

The centre of the 2.6dB source gain circle is a distance 0.67 along the line joining the origin and the point s_{11}^* and its radius is 0.16

Question 4(b) continued

Input Matching Element Values

Moving from Z_0 ($\Gamma=0$) to point A:

Clockwise on conductance circle – shunt capacitor

$$\begin{array}{l} \text{susceptance at } Z_0: b = 0 \\ \text{susceptance at A: } b = 1.54 \end{array} \quad C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|1.54|}{2\pi \times 1 \times 10^9 \times 50} = 4.91 pF$$

Moving from A to Γ_{IN} :

Clockwise on resistance circle – series inductor

$$\begin{array}{l} \text{reactance at A: } x = -0.46 \\ \text{reactance at } \Gamma_{IN}: x = 0.13 \end{array} \quad L = \frac{Z_0 |\Delta x|}{2\pi f} = \frac{50 \times |0.59|}{2\pi \times 1 \times 10^9} = 4.69 nH$$

Output Matching Element Values

Moving from Z_0 ($\Gamma=0$) to point B:

Anti-clockwise on resistance circle – series capacitor

$$\begin{array}{l} \text{reactance at } Z_0: x = 0 \\ \text{reactance at B: } x = -1.41 \end{array} \quad C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 1 \times 10^9 \times |-1.41| \times 50} = 2.25 pF$$

Moving from B to s_{22}^* :

Anti-clockwise on conductance circle – shunt inductor

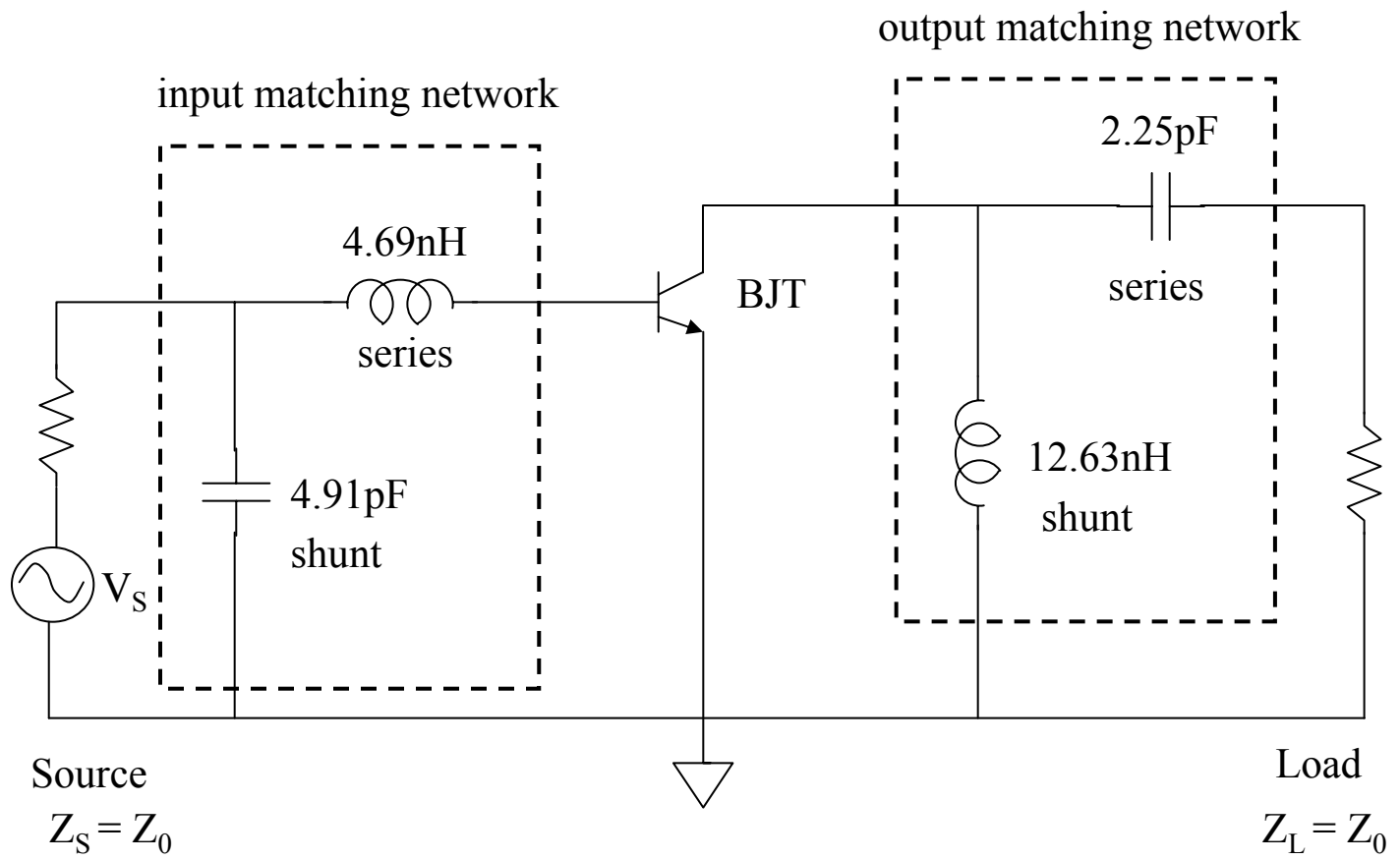
$$\begin{array}{l} \text{susceptance at B: } b = 0.47 \\ \text{susceptance at } s_{22}^*: b = -0.16 \end{array} \quad L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 1 \times 10^9 \times |-0.63|} = 12.63 nH$$

The maximum gain that can be achieved while meeting the noise specification is

$$G_{MAX} = G_S + G_0 + G_{L,\max} = 2.6 + 14 + 1.3 = 17.9 dB$$

The matching elements are shown on the next page.

Question 4(b) continued

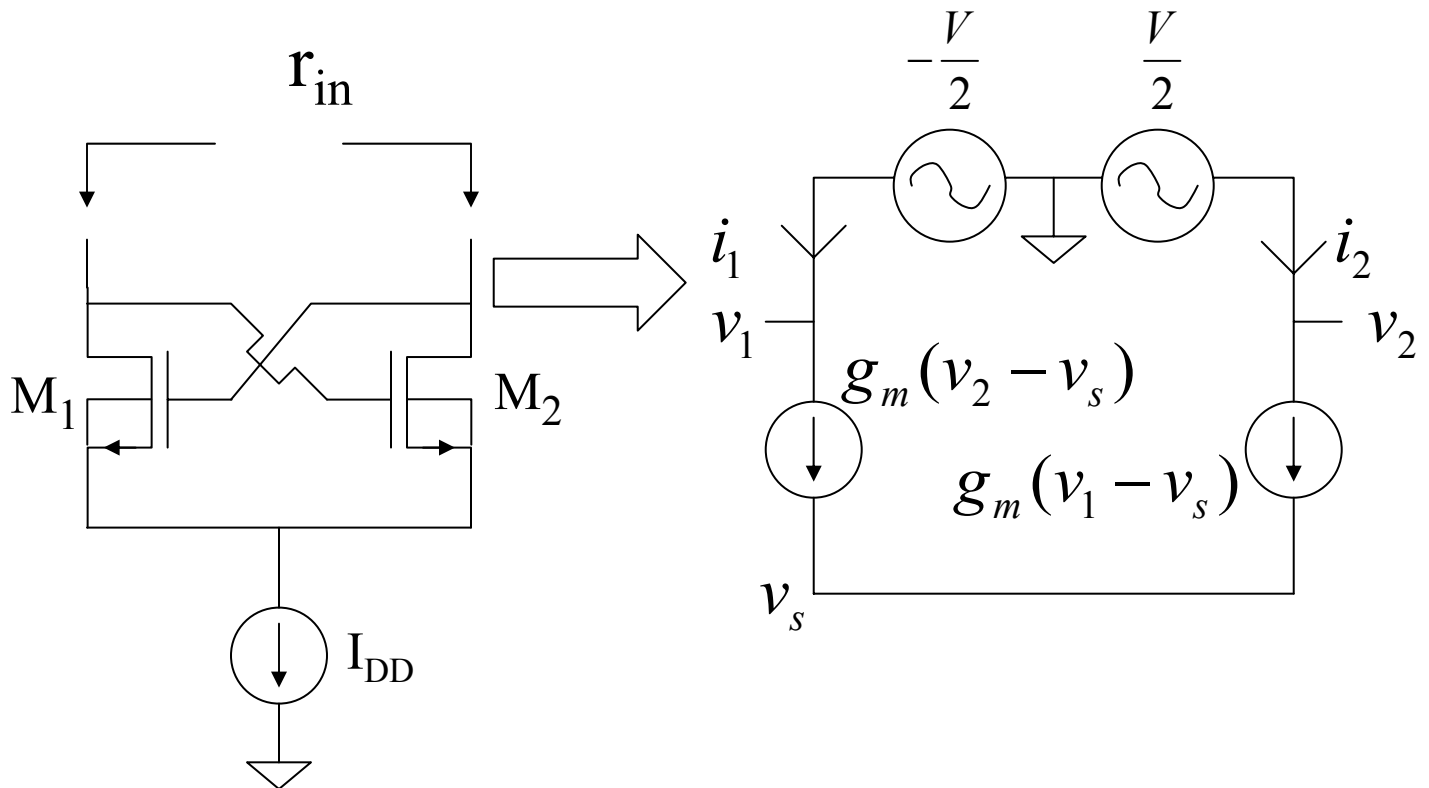


15 marks

Question 5(a)

Small-signal analysis to determine input resistance of cross-coupled MOS pair:

assume: $g_{m1} = g_{m2} = g_m$



$$i_1 = -i_2 \Rightarrow g_m(v_2 - v_s) = -g_m(v_1 - v_s)$$

$$\Rightarrow g_m(v_1 + v_2) = 2g_mv_s$$

$$\Rightarrow v_s = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}\left(\frac{V}{2} - \frac{V}{2}\right) = 0 \quad r_{in} = \frac{v_{tot}}{i_2} = \frac{\frac{V}{2} - \left(-\frac{V}{2}\right)}{g_m\left(-\frac{V}{2}\right)} = -\frac{2}{g_m}$$

$$\Rightarrow i_2 = g_mv_1 = g_m\left(-\frac{V}{2}\right)$$

The MOSFETs sustain oscillation by choosing g_m so that the effect of any parasitic resistances in the circuit (especially in the inductors) are cancelled out.

10 marks

Question 5(b)

$I_{bias} = 5mA$, $L=0.25\mu m$ $T_{OX}=5nm$, $\mu=400cm^2/Vs$,
 $\epsilon_r = 3.9$ (dielectric constant of SiO_2) $R_{IND} = 5\Omega$

Determine W – the total resistance of the inductors in 10Ω and this has to be cancelled out with the MOSFETs by choosing $r_{in}=-10\Omega$

$$g_m = -\frac{2}{r_{in}} = -\frac{2}{-10} = 0.2$$

For MOSFETs in saturation:

$$g_m = \sqrt{2 \frac{W}{L} \mu C'_{OX} I_{DS}} \Rightarrow$$

$$W = \frac{g_m^2 L}{2 \mu C'_{OX} I_{DS}} = \frac{g_m^2 T_{OX} L}{2 \mu \epsilon_0 \epsilon_r I_{DS}}$$

For each MOSFET $I_{DS}=I_{bias}/2$ and putting in the values gives:

$$W = 7.2mm$$

Question 5(c)

5 marks

Diode Parameters: $M_J = 0.5$, $V_J = 0.8V$ and $C_{J0} = 1.5pF$
 $L=4nH$, $V_{out} (DC) = 1.5V$, $V_{bias} = 0V$, $C_{par} = 1pF$

The diode capacitance is:

$$C_D = \frac{C_{J0}}{(1 - V_D / V_J)^{M_J}} = \frac{1.5 \times 10^{-12}}{(1 - (-1.5) / 0.8)^{0.5}} = 0.885 pF$$

The total capacitance at each output node is:

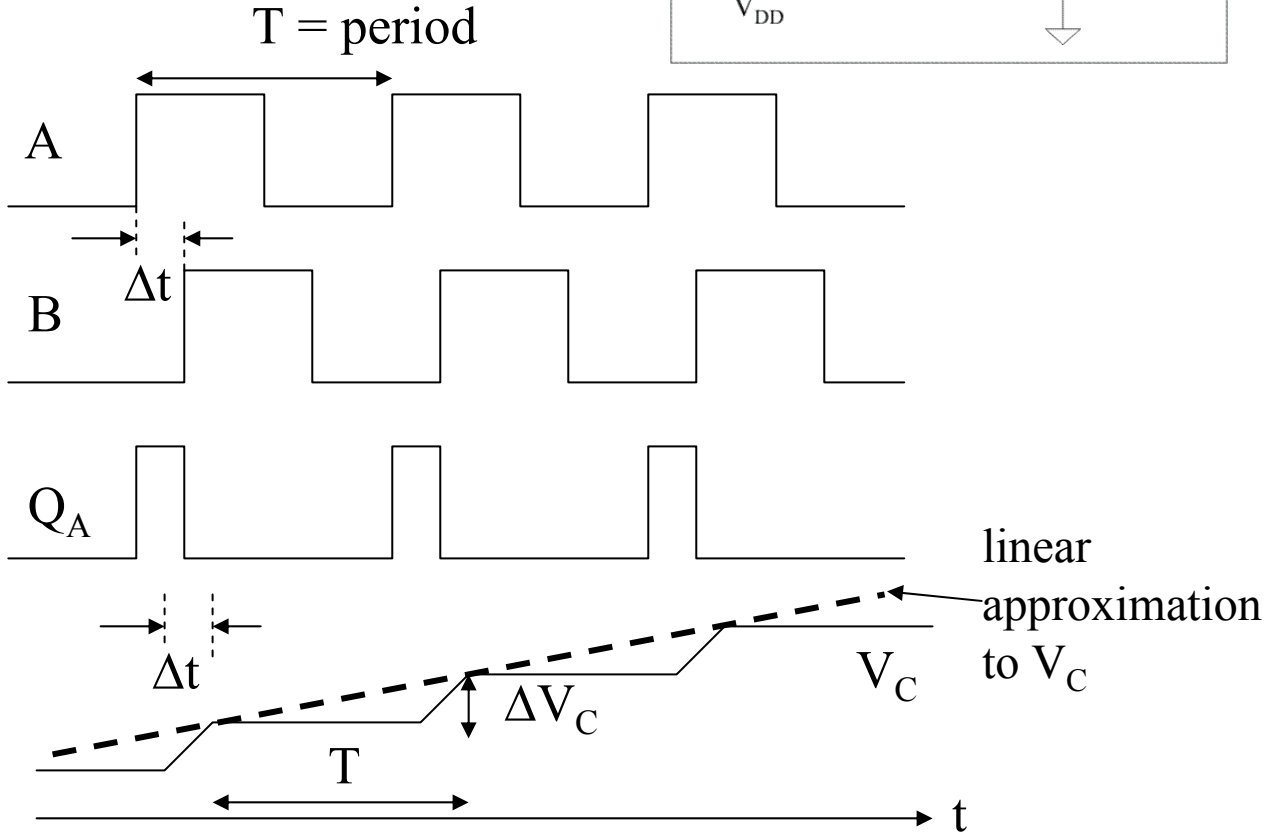
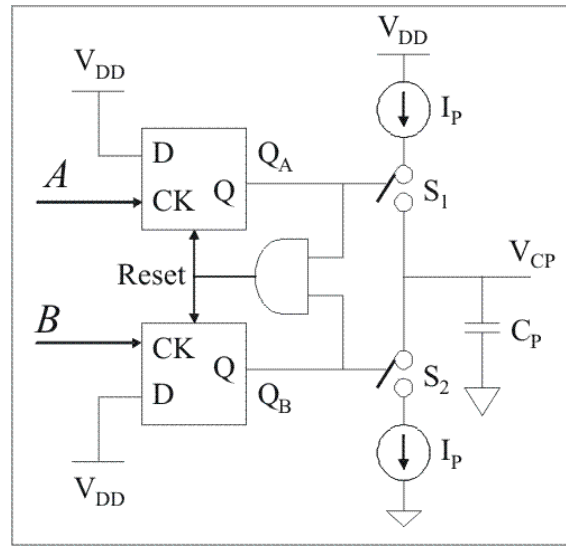
$$C_{TOT} = C_{PAR} + C_D = 1pF + 0.885 pF = 1.885 pF$$

The oscillation frequency is

$$F_{OSC} = \frac{1}{2\pi \sqrt{LC_{TOT}}} = 1.833GHz$$

5 marks ¹²

Question 6(a)



$$\Delta t = \frac{\Delta \phi}{2\pi} T \quad \Delta V_C = \frac{I_P}{C_P} \Delta t = \frac{I_P}{C_P} \frac{\Delta \phi}{2\pi} T$$

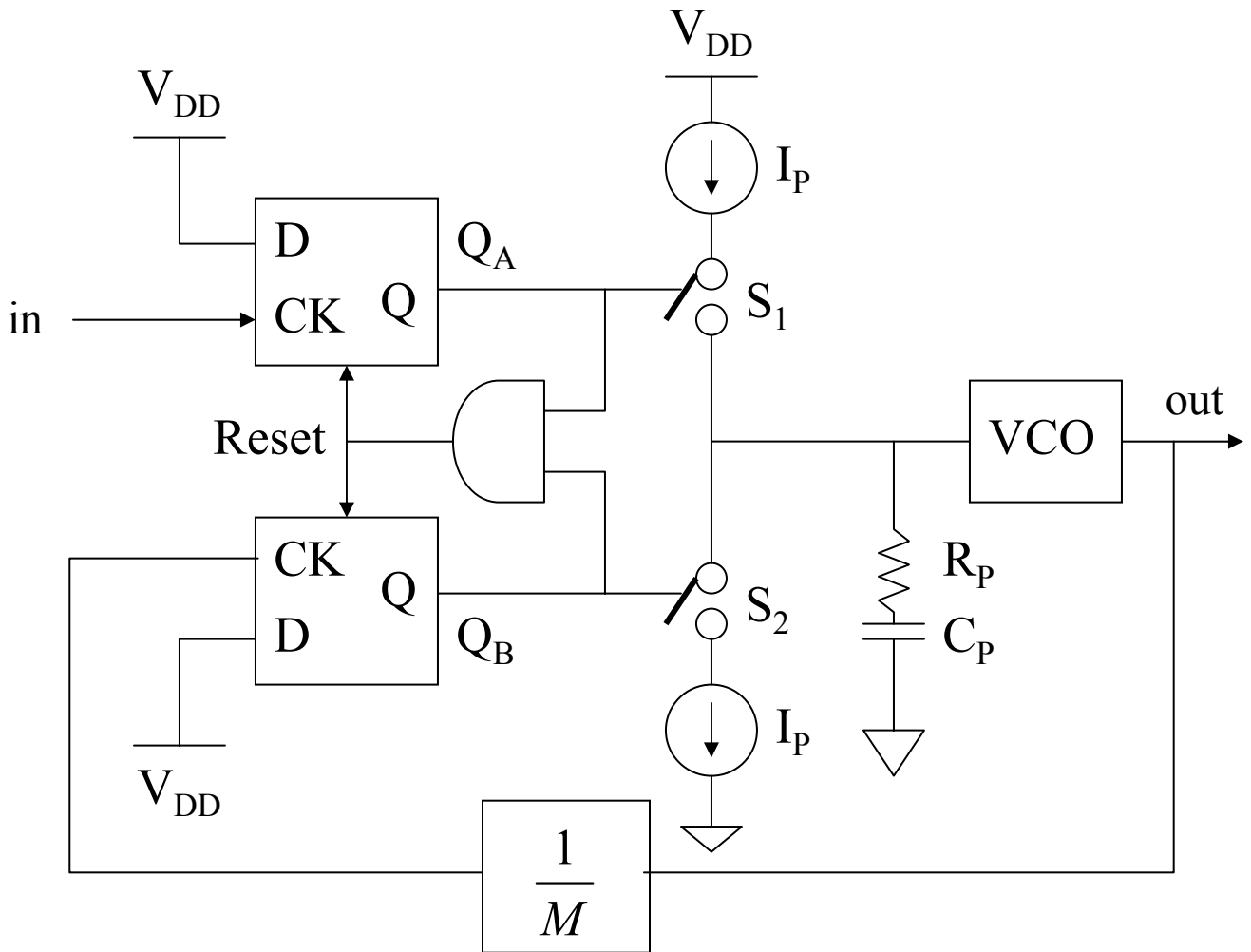
$$\text{slope} = \frac{dV_C}{dt} \approx \frac{\Delta V_C}{T} = \frac{I_P}{C_P} \frac{\Delta \phi}{2\pi} T \frac{1}{T} = \frac{I_P}{2\pi C_P} \Delta \phi$$

$$\Rightarrow \frac{dV_C}{dt} = \frac{I_P}{2\pi C_P} \Delta \phi \Rightarrow V_C = \frac{I_P}{2\pi C_P} \int \Delta \phi$$

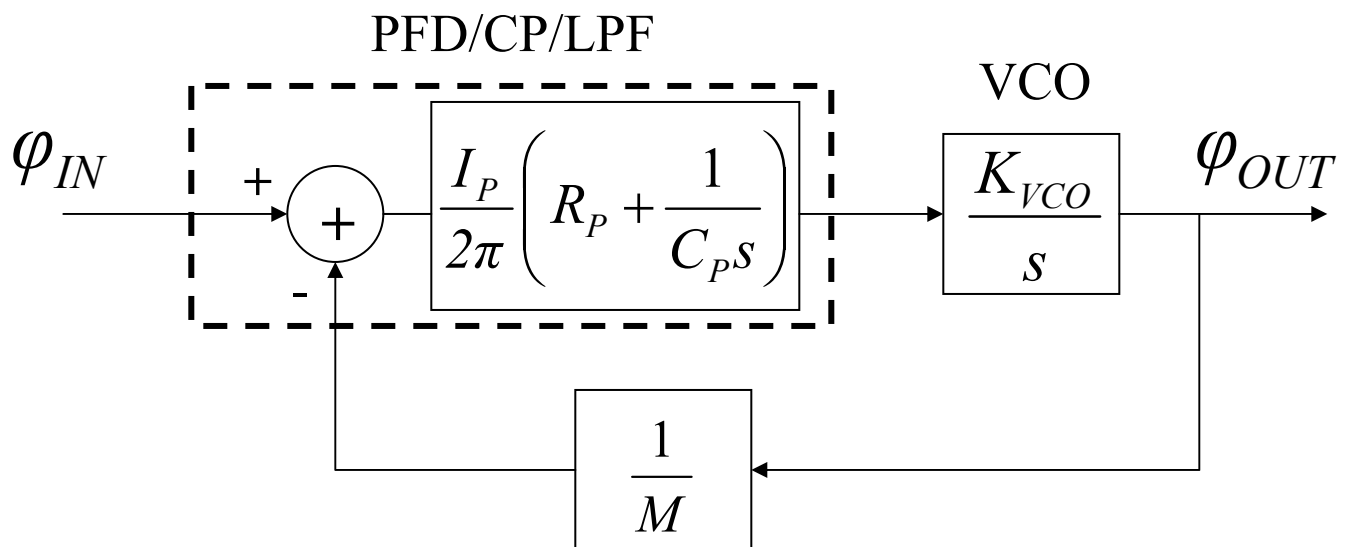
$$\Rightarrow V_C(s) = \frac{I_P}{2\pi C_P} \frac{1}{s} \Delta \phi(s)$$

5 marks

Question 6(b) Type II PLL with integer divider:



In terms of transfer functions:



Question 6(b) continued

The open loop transfer function of the Type II PLL is:

$$H(s) = \frac{I_P K_{VCO}}{2\pi s} \left(R_P + \frac{1}{C_P s} \right)$$

The closed loop transfer function is:

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P} (R_P C_P s + 1)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} = \frac{\frac{I_P K_{VCO}}{2\pi C_P} (R_P C_P s + 1)}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$
$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \zeta = \frac{R_P}{2} \sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\zeta \omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

Question 6(c)

6 marks

Putting the values given into the formulas above:

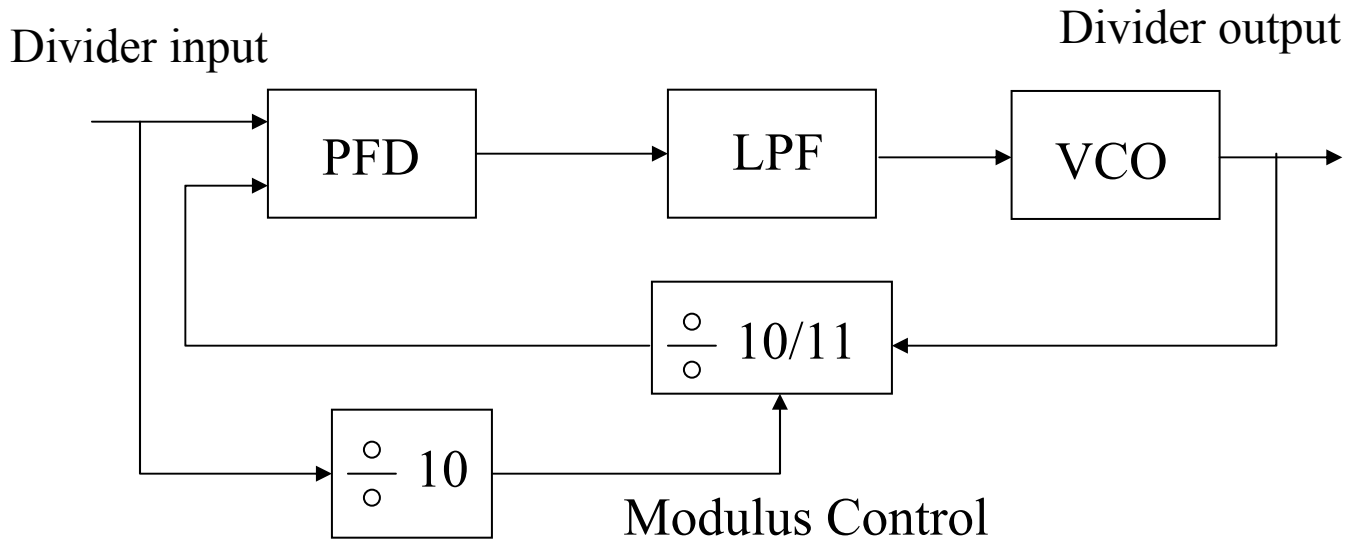
(i) $\omega_n = 1.41 \times 10^6 \text{ rad/s}$

2 marks

(ii) $\zeta = 0.707$

2 marks

Question 6(d)



A Type II PLL with $F_{\text{OUT}} = 10.1F_{\text{IN}}$ can be made with the same overall architecture as shown earlier but with the divider replaced by the above.

The dual-modulus counter normally divides by 10 (with the control line low). When the modulus control line is high it divides by 11. This only happens for every tenth pulse of F_{IN} because of the divide-by-10 counter. The divide ratio is thus 10.1 so $F_{\text{OUT}} = 10.1F_{\text{IN}}$.

Question 7

This is an essay-type question based on a continuous assessment assignment carried out during the year. The resulting short essay is expected to recount as much technical detail as possible regarding the GPS system studied in the assignment.