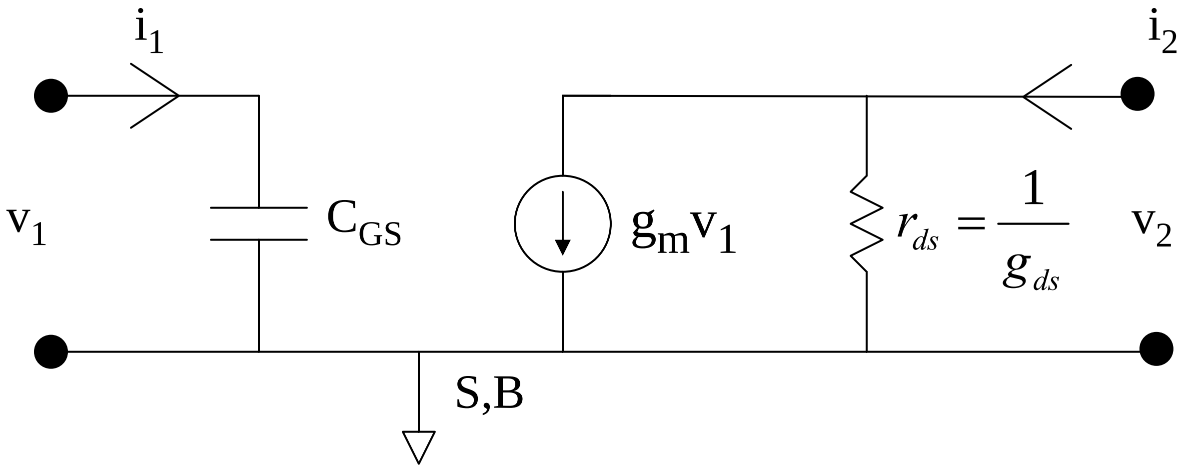
EE4011 RF IC Design Summer 2008
Question 1(a)

Small-signal model of MOSFET ignoring gate resistance and considering C_{GS} as the only important capacitance.



$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0} = \frac{g_m v_1}{j\varpi C_{GS} v_1} = \frac{g_m}{j\varpi C_{GS}} \Rightarrow \left|h_{21}\right| = \frac{g_m}{2\pi f C_{GS}}$$

The cut-off frequency is when $|h_{21}|$ drops to 1:

$$\left|h_{21}\right| = 1 \Rightarrow f = f_T = \frac{g_m}{2\pi C_{GS}}$$

The elements of the model are:

$$I_{DS} = \frac{1}{2}\frac{W}{L}\mu C_{OX}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{W}{L}\mu C_{OX}\left(V_{GS} - V_{TH}\right)\left(1 + \lambda V_{DS}\right)$$

$$g_{ds} = \frac{dI_{DS}}{dV_{DS}} = \frac{1}{2}\frac{W}{L}\mu C_{OX}\left(V_{GS} - V_{TH}\right)^2\left(\lambda\right)$$

$$C_{GS} \approx \frac{2}{3} WLC_{OX}$$

10 marks

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Question 1(b)

$$C_{OX} = \frac{\varepsilon_0 \varepsilon_r}{T_{OX}} = \frac{8.854\times10^{-12}\times3.9}{4\times10^{-9}} = 0.0086\,F/m^2$$

$$k = \frac{W}{L}\mu C_{OX} = \frac{20}{0.25}\times350\times10^{-4}\times0.0086 = 0.0242\,A/V^2$$

$$g_m = k(V_{GS}-V_{TH})(1+\lambda V_{DS}) = 0.0242\times(2.5-0.6)\times(1+0.15\times2) = 0.0597\,A/V$$

$$g_{ds} = \frac{1}{2}k(V_{GS}-V_{TH})^2(\lambda) = \frac{1}{2}\times0.0242\times(2.5-0.6)^2\times0.15 = 0.0065\,A/V$$

$$C_{GS} \approx \frac{2}{3}WLC_{OX} = \frac{2}{3}\times20\times10^{-6}\times0.25\times10^{-6}\times0.0086 = 28.8\times10^{-15}\,F$$

(i)

$$f_T = \frac{g_m}{2\pi C_{GS}} = 330GHz$$

2 marks

(This is unrealistic in practice for this geometry transistor but the model here is very simplistic)

(ii)

Using the definitions of the y-parameters and setting f=1GHz:

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

$$y_{11} = j\omega C_{GS} = 1.8\times10^{-4}\angle 90^\circ$$

$$y_{12} = 0$$

$$y_{21} = g_m = 0.0597\angle 0^\circ$$

$$y_{22} = g_{DS} = 0.0065\angle 0^\circ$$

8 marks

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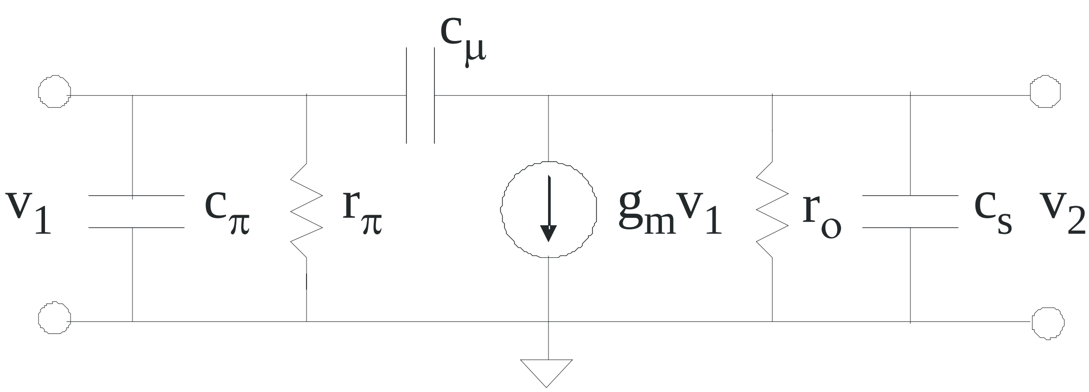
Question 2(a)



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

Applying the y-parameter definitions to the above circuit and performing the circuit analysis under the appropriate conditions gives:

$$y_{11} = \frac{1}{r_\pi} + j\omega\left(c_\pi + c_\mu\right)$$

$$y_{12} = -j\omega c_\mu$$

$$y_{21} = g_m - j\omega c_\mu$$

$$y_{22} = \frac{1}{r_o} + j\omega\left(c_s + c_\mu\right)$$

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Question 2(a) continued

The expressions on the previous page have to be manipulated to express the circuit element values in terms of the y-parameters. The final result of this manipulation is as follows:

$$g_m = \mathbf{Re}(y_{21}) = 0.1S$$

$$r_\pi = \frac{1}{\mathbf{Re}(y_{11})} = 400\Omega$$

$$r_o = \frac{1}{\mathbf{Re}(y_{22})} = 2k\Omega$$

$$c_\mu = \frac{-\mathbf{Im}(y_{12})}{2\pi f} = 1pF$$

$$c_\pi = \frac{\mathbf{Im}(y_{11})}{2\pi f} - c_\mu = 5pF$$

$$c_s = \frac{\mathbf{Im}(y_{22})}{2\pi f} - c_\mu = 2pF$$

16 marks

Question 2(b)

$$f_T = \frac{g_m}{2\pi(c_\pi + c_\mu)} = 2.65GHz$$

4 marks

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Question 3(a)

$$y(t)=\left[\alpha_1 A_1+\frac{3}{4}\alpha_3 A_1^3+\frac{3}{2}\alpha_3 A_1 A_2^2\right]\cos(\varpi_1 t)$$
$$+\left[\alpha_1 A_2+\frac{3}{4}\alpha_3 A_2^3+\frac{3}{2}\alpha_3 A_1^2 A_2\right]\cos(\varpi_2 t)$$
$$+\frac{1}{4}\alpha_3 A_1^3\cos 3\varpi_1 t+\frac{1}{4}\alpha_3 A_2^3\cos 3\varpi_2 t$$
$$+\frac{3}{4}\alpha_3 A_1^2 A_2\cos(2\varpi_1+\varpi_2)t+\frac{3}{4}\alpha_3 A_1^2 A_2\cos(2\varpi_1-\varpi_2)t$$
$$+\frac{3}{4}\alpha_3 A_1 A_2^2\cos(2\varpi_2+\varpi_1)t+\frac{3}{4}\alpha_3 A_1 A_2^2\cos(2\varpi_2-\varpi_1)t$$

(i) P1dB

P1dB is defined for a single frequency input so A_2 can be set to 0 in the above formula and then just looking at the o/p component at the fundamental frequency:

$$y_1(t)=\left(\alpha_1 A+\frac{3\alpha_3 A^3}{4}\right)\cos(\varpi t)=A_{OUT}\cos(\varpi t)$$

The voltage gain of the circuit considering the fundamental is:

$$G_V=\frac{y_1(t)}{x(t)}=\frac{A_{OUT}\cos(\varpi t)}{A\cos(\varpi t)}=\frac{\alpha_1 A+\dfrac{3\alpha_3 A^3}{4}}{A}=\alpha_1+\frac{3\alpha_3 A^2}{4}$$

Converting to dB:

$$G_{dB}=20\log_{10}(G_V)=20\log_{10}\left(\alpha_1+\frac{3\alpha_3 A^2}{4}\right)$$

For small A (amplitude) the A^2 term is very small and the gain is the “ideal gain” for small input amplitudes:

$$G_{dB,small}=20\log_{10}(\alpha_1)$$

Question 3(a)(i) continued

The power gain in dB is:

$$G_{dB} = 20\log_{10}(G_V) = 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right)$$

For a compressive gain stage α_3 has the opposite sign to α_1 i.e. if $\alpha_1 > 0$ then $\alpha_3 < 0$

For small input signals (small A) the power gain is approximately:

$$G_{dB,small} = 20\log_{10}(\alpha_1)$$

At the 1dB point the gain is 1dB smaller than this ideal value i.e.

$$G_{dB,P1dB} = G_{dB,small} - 1 = 20\log_{10}(\alpha_1) - 1$$

The amplitude A corresponding to P1dB can be found by equating the last expression to the first expression:

$$G_{dB} = G_{dB,P1dB} \Rightarrow 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) = 20\log_{10}(\alpha_1) - 1$$

$$20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) = 20\log_{10}(\alpha_1) - 1$$

$$\Rightarrow 20\log_{10}\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) - 20\log_{10}(\alpha_1) = -1$$

$$\Rightarrow \log_{10}\left(\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) / \alpha_1\right) = -0.05$$

$$\Rightarrow 1 + \frac{3\alpha_3 A^2}{4\alpha_1} = 10^{-0.05}$$

$$\Rightarrow A^2 = \left(10^{-0.05} - 1\right)\frac{4}{3}\frac{\alpha_1}{\alpha_3} = -0.145\frac{\alpha_1}{\alpha_3} = 0.145\left|\frac{\alpha_1}{\alpha_3}\right|$$

$$\Rightarrow A = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

Note α_1 and α_2 have opposite signs so:

$$\frac{\alpha_1}{\alpha_3} = -\left|\frac{\alpha_1}{\alpha_3}\right|$$

5 marks

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Question 3(a) continued

(ii) IIP3

Again, taking the case of A_1=A_2=A, and considering the outputs at the fundamental frequencies to be the desired outputs, the amplitudes of the desired signals are:

$$A_{SIG} = \left| \alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2}\alpha_3 A^3 \right| = \left| \alpha_1 A + \frac{9}{4}\alpha_3 A^3 \right|$$

$$\approx |\alpha_1| A \quad \text{if} \quad \alpha_1 >> \frac{9}{4}\alpha_3 A^2$$

In this case the unwanted 3rd-order inter-modulation (IM) signals are given by:

$$A_{IM3} = \frac{3}{4}|\alpha_3| A^3$$

As A increases the IM3 outputs will eventually will reach the same level as the desired signal output. This condition is called the "third-order IM intercept point", IP3. The input amplitude corresponding to this condition is A=AIP3 and at this amplitude:

$$A_{SIG} = A_{IM3} \Rightarrow |\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3 \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}}$$

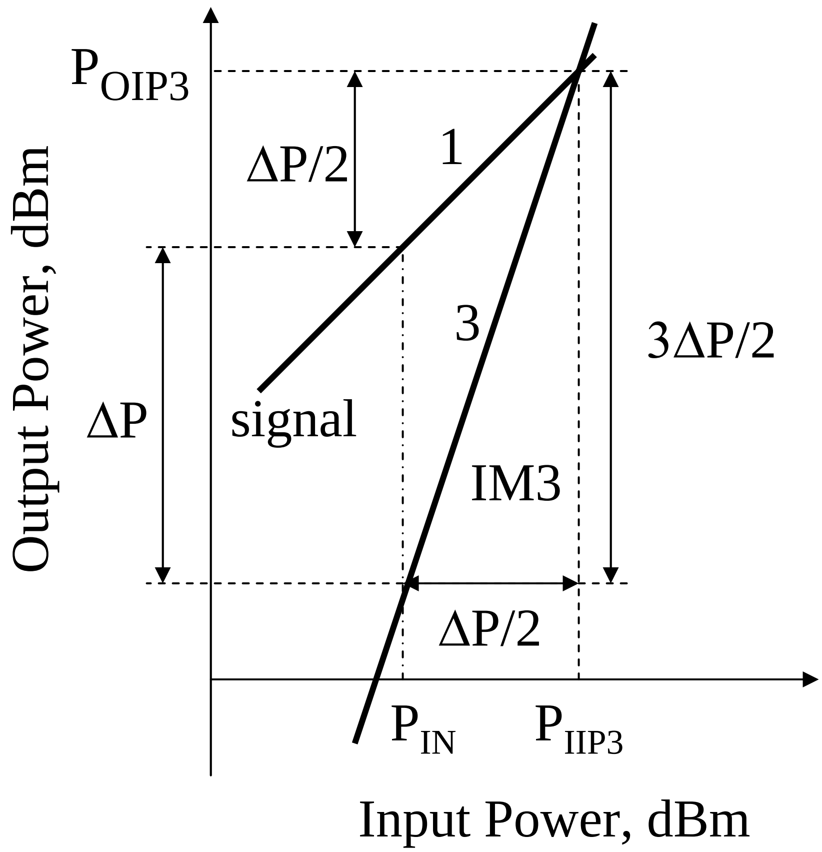
5 marks

(b)

$$|\alpha_1| = \frac{300}{2} = 150 \quad |\alpha_3| = \frac{4\times 2\times 10^{-9}}{\left(2\times 10^{-3}\right)^3} = 1 \Rightarrow A_{IP3} = \sqrt{\frac{3}{4}\frac{|\alpha_1|}{|\alpha_3|}} = \sqrt{\frac{3}{4}\frac{150}{1}} = 10.6V$$

5 marks

EE4011 RF IC Design Summer 2008
Question 3(c)



By applying two signals with input power (P_{in}) and measuring the associated output power at the signal frequency($P_{sig,out}$) and at the IM3 frequencies ($P_{IM3,out}$) it is apparent from the graph that:

$$P_{IIP3} = P_{in} + \frac{P_{sig,out} - P_{IM3,out}}{2}$$

5 marks

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Question 4

(a)

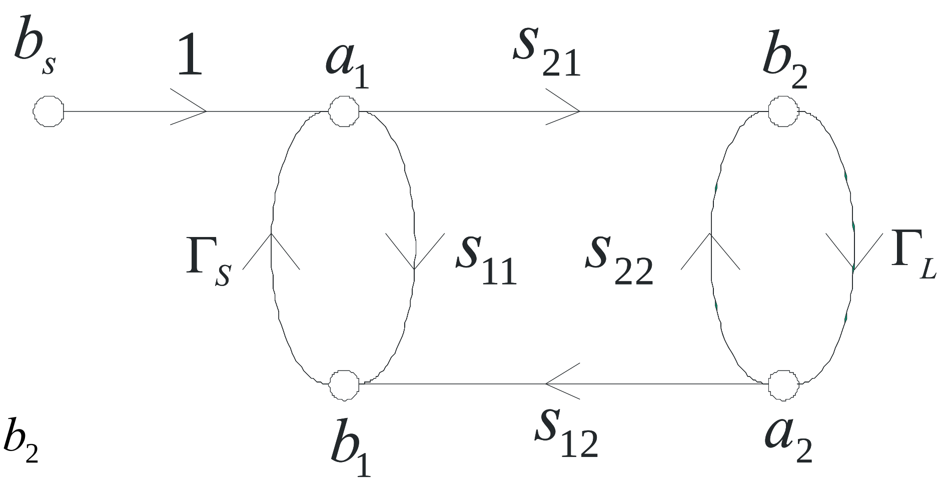
(i) The operating power gain (also just called the power gain) is the ratio of the power delivered to the load to the power delivered to the network by the source.

(ii) The transducer power gain is the ratio of the power delivered to the load to the power *available* from the source.

(iii) The available power gain is the ratio of the power *available* from the network to the power *available* from the source.

3 marks

(b) Operating power gain of the amplifier



$$a_2 = \Gamma_L b_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2 = s_{21}a_1 + s_{22}\Gamma_L b_2 \Rightarrow b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L} \Rightarrow a_2 = \frac{s_{21}\Gamma_L a_1}{1 - s_{22}\Gamma_L}$$

$$b_1 = s_{11}a_1 + s_{12}a_2 = s_{11}a_1 + \frac{s_{12}s_{21}\Gamma_L a_1}{1 - s_{22}\Gamma_L} = \frac{(s_{11} - s_{11}s_{22}\Gamma_L + s_{12}s_{21}\Gamma_L)a_1}{1 - s_{22}\Gamma_L}$$

$$a_1 = b_s + \Gamma_S b_1 = b_s + \frac{(s_{11} - s_{11}s_{22}\Gamma_L + s_{12}s_{21}\Gamma_L)\Gamma_S a_1}{1 - s_{22}\Gamma_L}$$

$$\Rightarrow a_1 = \frac{(1 - s_{22}\Gamma_L)b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

$$= \frac{(1 - s_{22}\Gamma_L)b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

Question 4(b) continued

$$a_1 = \frac{(1 - s_{22}\Gamma_L)b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

$$b_1 = \frac{(s_{11} - s_{11}s_{22}\Gamma_L + s_{12}s_{21}\Gamma_L)a_1}{1 - s_{22}\Gamma_L}$$

$$= \frac{(s_{11} - s_{11}s_{22}\Gamma_L + s_{12}s_{21}\Gamma_L)b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

$$a_2 = \frac{s_{21}\Gamma_L a_1}{1 - s_{22}\Gamma_L} = \frac{s_{21}\Gamma_L b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

$$b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L} = \frac{s_{21}b_s}{1 - s_{11}\Gamma_S - s_{22}\Gamma_L + s_{11}s_{22}\Gamma_S\Gamma_L - s_{12}s_{21}\Gamma_S\Gamma_L}$$

$$G_P = \frac{P_{OUT}}{P_{IN}} = \frac{\frac{1}{2}|b_2|^2 - \frac{1}{2}|a_2|^2}{\frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2}$$

$$= \frac{|s_{21}|^2 - |s_{21}\Gamma_L|^2}{|1 - s_{22}\Gamma_L|^2 - |s_{11} - s_{11}s_{22}\Gamma_L + s_{12}s_{21}\Gamma_L|^2}$$

$$= \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2 - |s_{11}(1 - s_{22}\Gamma_L) + s_{12}s_{21}\Gamma_L|^2}$$

$$= \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2 - |s_{11} - \Delta\Gamma_L|^2} \quad \textit{where} \quad \Delta = s_{11}s_{22} - s_{12}s_{21}$$

14 marks

(c) If $\Gamma_L = 0$

$$G_P = \frac{P_{OUT}}{P_{IN}} = \frac{|s_{21}|^2}{1 - |s_{11}|^2} = \frac{|3.434|^2}{1 - |0.836|^2} = 46.2$$

3 marks

EE4011 RF IC Design Summer 2008
Question 5

(a) Design Procedure for Low Noise Amplifier (LNA)

1. Select transistor based on s-parameters, noise figure, power level, process technology, etc
2. Check the stability – stability factor K, input/output stability circles
3. Check gain – gain circles
4. Check noise – noise circles
5. Design input and output matching networks (and DC biasing)
6. Re-iterate if necessary

5 marks

(b) Transistor with the following s-parameters at 3GHz.

$$s_{11} = 0.38\angle -169^\circ \quad s_{12} = 0 \quad s_{21} = 1.33\angle -39^\circ \quad s_{22} = 0.95\angle -66^\circ$$

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2}$$

$$= \frac{1}{1-|0.38|^2}|1.33|^2\frac{1}{1-|0.95|^2}$$

$$= 1.169 \times 1.769 \times 10.256 = 21.2 \qquad \text{ratio}$$

$$= 0.68dB + 2.48dB + 10.11dB = 13.27dB$$

3 marks

EE4011 RF IC Design Summer 2008
Question 5 continued

(c) Design for source gain = 0.3dB, load gain = 4dB

Identifying the gain circle to give a source gain of 0.3dB

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{0.3}{10}} = 1.0715$$

$$\Rightarrow g_s = \frac{G_S}{G_{S.\max}} = \frac{1.0715}{1.169} = 0.92$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.92\times|0.38|}{1-|0.38|^2(1-0.92)} = 0.35$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.92}\left(1-|0.38|^2\right)}{1-|0.38|^2(1-0.92)} = 0.25$$

The centre of the 0.3dB source gain circle is a distance 0.35 along the line joining the origin and the point s_{11}^* and its radius is 0.25

Identifying the gain circle to give a load gain of 4dB

$$G_{L,dB} = 10\log_{10}(G_L) \Rightarrow G_L = 10^{\frac{G_{L,dB}}{10}} = 10^{\frac{4}{10}} = 2.512$$

$$\Rightarrow g_L = \frac{G_L}{G_{L,\max}} = \frac{2.512}{10.256} = 0.245$$

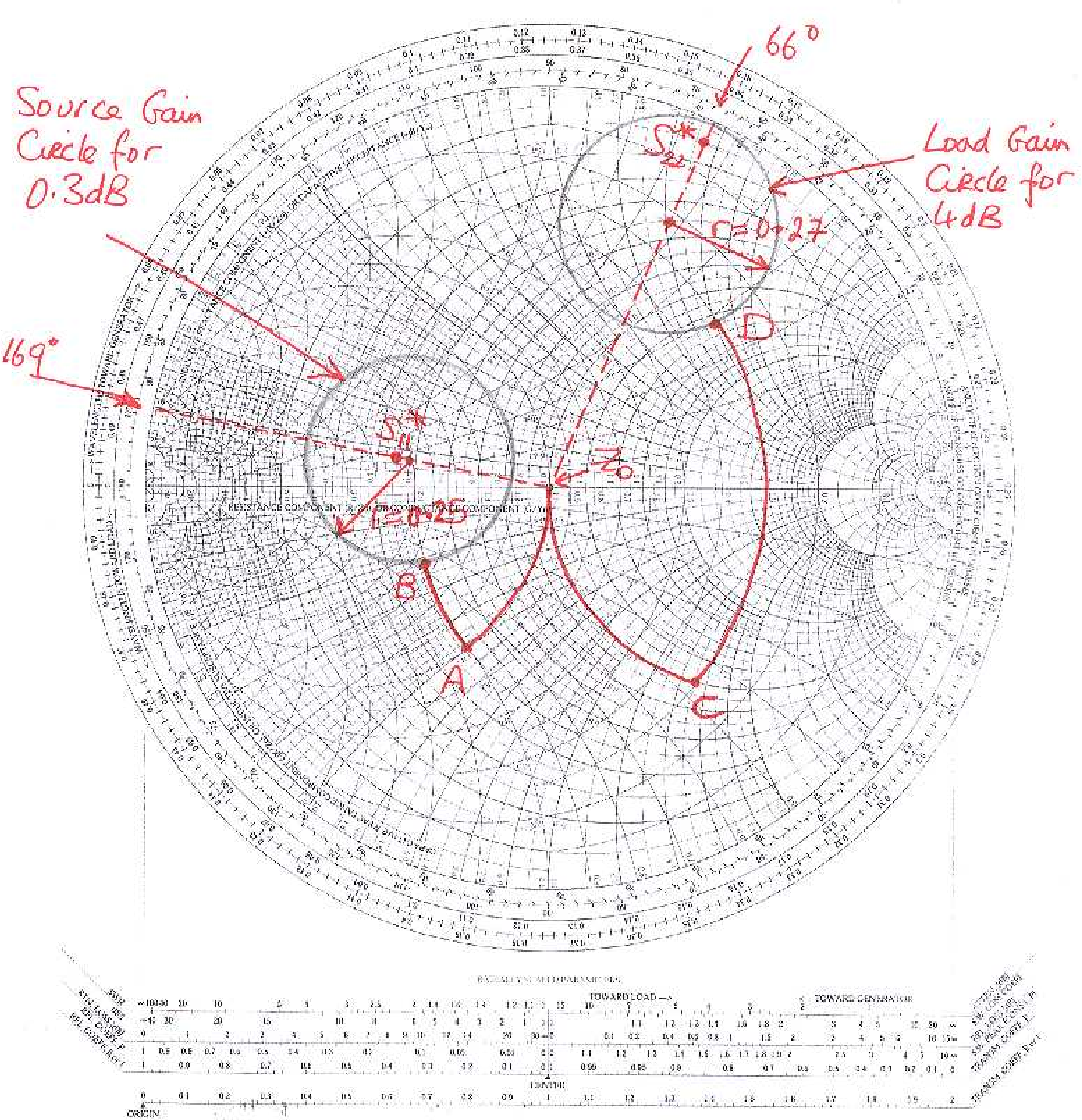
$$|C_L| = \frac{g_L|s_{22}|}{1-|s_{22}|^2(1-g_L)} = \frac{0.245\times|0.95|}{1-|0.95|^2(1-0.245)} = 0.73$$

$$R_L = \frac{\sqrt{1-g_L}\left(1-|s_{22}|^2\right)}{1-|s_{22}|^2(1-g_L)} = \frac{\sqrt{1-0.245}\left(1-|0.95|^2\right)}{1-|0.95|^2(1-0.245)} = 0.27$$

The centre of the 4dB load gain circle is a distance 0.73 along the line joining the origin and the point s_{22}^* and its radius is 0.27

EXAMINATION NUMBER	QUESTION NUMBER	DATE
	EE4011 RF IC Design	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



EE4011 Summer 2008 Q5(c)

Input Matching Element Values – need to get from origin to source gain circle

Moving from Z_0 (Γ=0) to point A:
Clockwise on conductance circle – shunt capacitor

susceptance at Z_0: b = 0
susceptance at A: b = 1.0

$$C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{|1.0|}{2\pi \times 3\times 10^9 \times 50} = 1.06\, pF$$

Moving from A to B:
Clockwise on resistance circle – series inductor

reactance at A: x = -0.5
reactance at B: x = -0.22

$$L = \frac{Z_0|\Delta x|}{2\pi f} = \frac{50\times|0.28|}{2\pi\times 3\times 10^9} = 0.74 nH$$

Output Matching Element Values – need to get from origin to load gain circle

Moving from Z_0 (Γ=0) to point C:
Anti-clockwise on resistance circle – series capacitor

reactance at Z_0: x = 0   reactance at C: x = -1.5

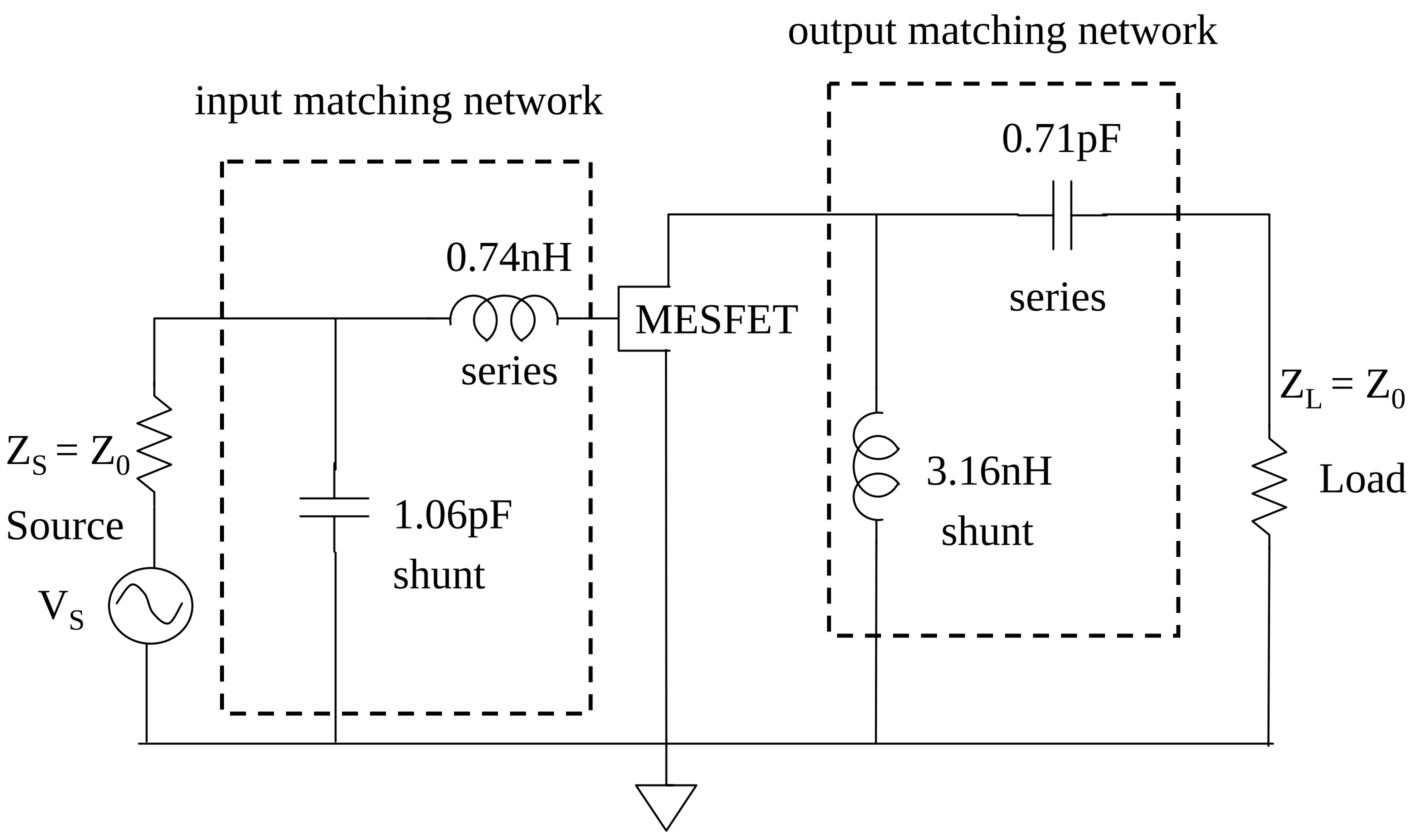
$$C = \frac{1}{2\pi f|\Delta x| Z_0} = \frac{1}{2\pi\times 3\times 10^9 \times|-1.5|\times 50} = 0.71 pF$$

Moving from C to D:
Anti-clockwise on conductance circle – shunt inductor

susceptance at C: b = 0.46
susceptance at D: b = -0.38

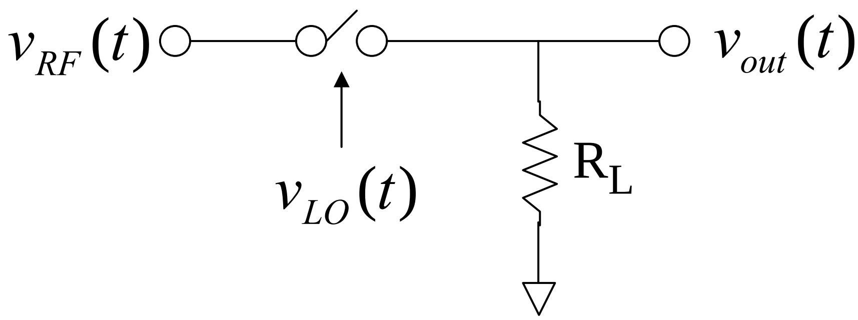
$$L = \frac{Z_0}{2\pi f|\Delta b|} = \frac{50}{2\pi \times 3\times 10^9 \times|-0.84|} = 3.16 nH$$

EE4011 RF IC Design Summer 2008
Question 5(c) continued



12 marks

EE4011 RF IC Design Summer 2008
Question 6(a) Switch-based mixer



The LO signal controls the switch. For alternate half-cycles the switch is on so connecting the RF signal to the o/p or off so grounding the output. This is equivalent to multiplying the RF signal by a square wave at the LO frequency.

$$v_{out}(t) = v_{RF}(t)\left[\frac{1}{2} + \frac{2}{\pi}\sin(\varpi_{LO}t) + \frac{2}{3\pi}\sin(3\varpi_{LO}t) + \frac{2}{5\pi}\sin(5\varpi_{LO}t) + \cdots\right]$$

$$= V_{RF}\cos(\varpi_{RF}t)\left[\frac{1}{2} + \frac{2}{\pi}\sin(\varpi_{LO}t) + \frac{2}{3\pi}\sin(3\varpi_{LO}t) + \frac{2}{5\pi}\sin(5\varpi_{LO}t) + \cdots\right]$$

$$= \frac{1}{2}V_{RF}\cos(\varpi_{RF}t) + \frac{2}{\pi}V_{RF}\cos(\varpi_{RF}t)\sin(\varpi_{LO}t) + \frac{2}{3\pi}V_{RF}\cos(\varpi_{RF}t)\sin(3\varpi_{LO}t)$$

$$+ \frac{2}{5\pi}V_{RF}\cos(\varpi_{RF}t)\sin(5\varpi_{LO}t) + \cdots$$

Using $\cos A \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$ gives

$$v_{out}(t) = \frac{1}{2}V_{RF}\cos(\varpi_{RF}t) + \frac{V_{RF}}{\pi}\left[\sin\left((\varpi_{RF} + \varpi_{LO})t\right) - \sin\left((\varpi_{RF} - \varpi_{LO})t\right)\right]$$

$$+ \frac{V_{RF}}{3\pi}\left[\sin\left((\varpi_{RF} + 3\varpi_{LO})t\right) - \sin\left((\varpi_{RF} - 3\varpi_{LO})t\right)\right]$$

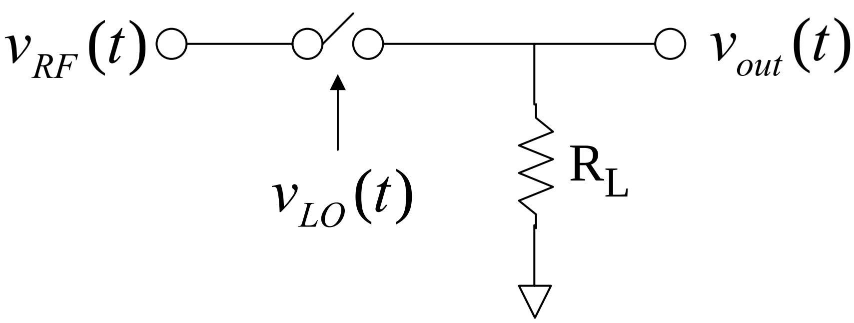
$$+ \frac{V_{RF}}{5\pi}\left[\sin\left((\varpi_{RF} + 5\varpi_{LO})t\right) - \sin\left((\varpi_{RF} - 5\varpi_{LO})t\right)\right] + \cdots$$

EE4011 RF IC Design Summer 2008
Question 6(a) continued

The formula just derived indicates spectral components at:

$$\varpi_{RF}, \varpi_{RF} \pm \varpi_{LO}, \varpi_{RF} \pm 3\varpi_{LO}, \varpi_{RF} \pm 5\varpi_{LO}, etc$$

Voltage conversion gain of mixer:



$$v_{out}(t) = \frac{1}{2}V_{RF}\cos(\varpi_{RF}t) + \frac{V_{RF}}{\pi}\left[\sin\left((\varpi_{RF}+\varpi_{LO})t\right) - \sin\left((\varpi_{RF}-\varpi_{LO})t\right)\right] + \cdots$$

amplitude of desired IF signal

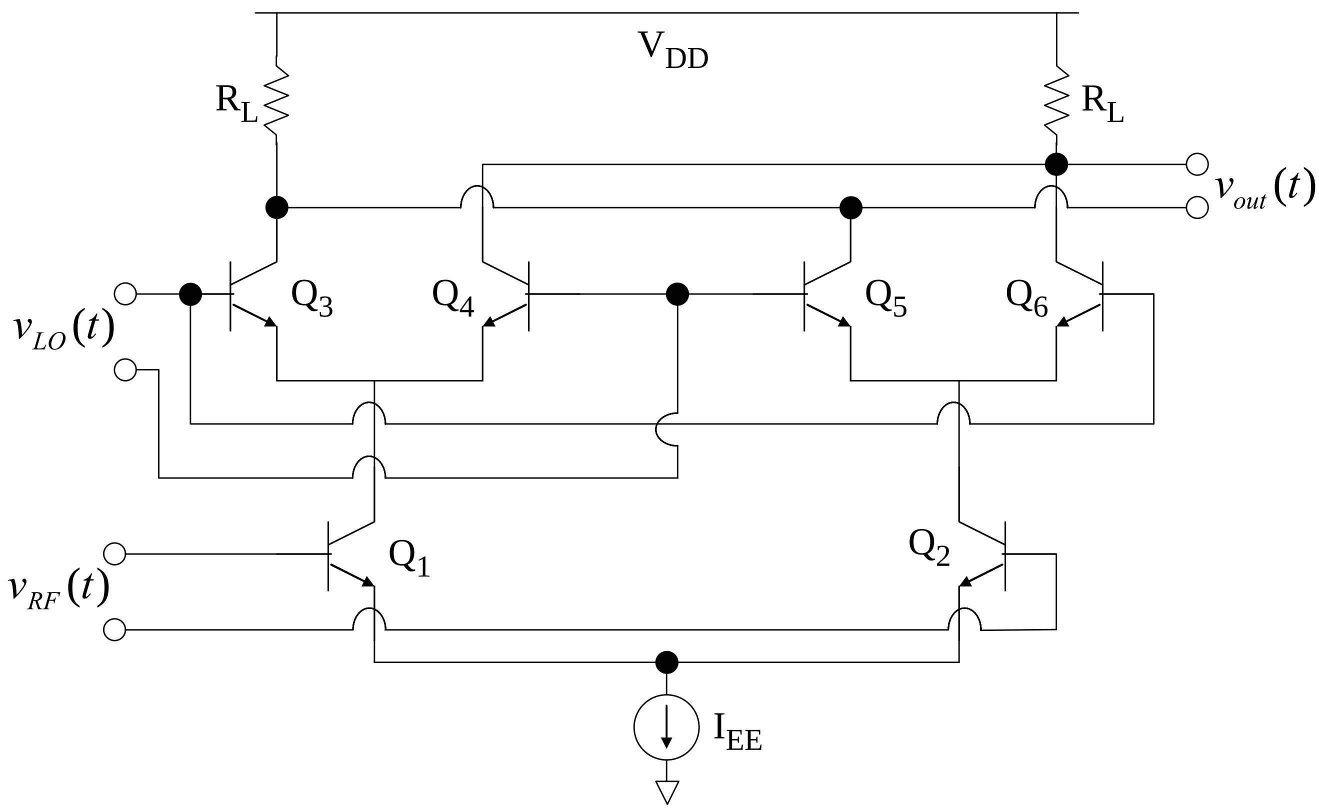
The voltage conversion gain (CG) is the ratio of the amplitudes of output IF signal and the input RF signal i.e.

$$A_{CG} = \frac{V_{IF}}{V_{RF}} = \frac{V_{RF}}{\pi}\frac{1}{V_{RF}} = \frac{1}{\pi} \quad A_{CG,dB} = 20\log_{10}\left(\frac{1}{\pi}\right) \approx -10dB$$

10 marks

EE4011 RF IC Design Summer 2008
Question 6(b)

A Gilbert Cell Double Balanced Mixer



4 marks

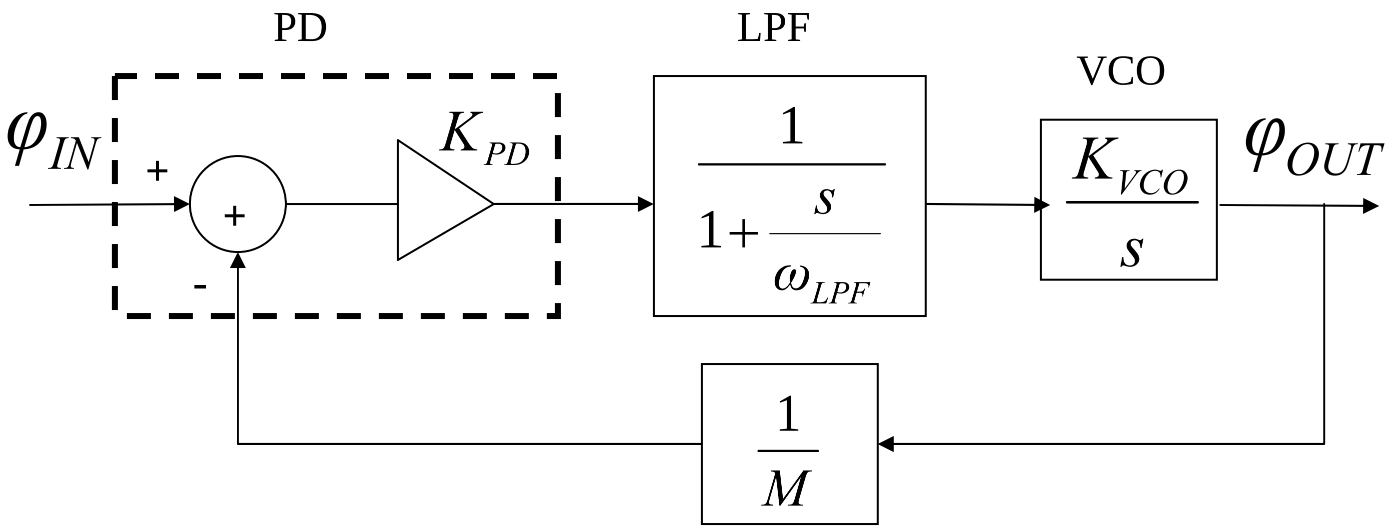
Question 6(c) – 3 possible operating modes

(i) If both V_{RF} and V_{LO} are low-amplitude signals, then the circuit performs as a true analogue multiplier.
(ii) If V_{RF} is a low-amplitude signal and V_{LO} has a large amplitude, then the circuit operated as a frequency mixer.
(iii) If both V_{RF} and V_{LO} are large-amplitude signals, then the circuit acts as a phase detector.

6 marks

EE4011 RF IC Design Summer 2008
Question 7

7(a) Type 1 PLL with integer feedback



4 marks

7(b) Closed loop transfer function

$$H(s)\Big|_{OPEN} = \left.\frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)}\right|_{OPEN} = K_{PD} \cdot \frac{1}{1+\dfrac{s}{\omega_{LPF}}} \cdot \frac{K_{VCO}}{s} = \frac{K_{PD}K_{VCO}}{s+\dfrac{s^2}{\omega_{LPF}}}$$

$$\varphi_{OUT}(s) = H(s)\left(\varphi_{IN}(s) - \frac{\varphi_{OUT}(s)}{M}\right)$$

$$\Rightarrow \varphi_{OUT}(s)\left(1+\frac{H(s)}{M}\right) = H(s)\varphi_{IN}(s)$$

$$\Rightarrow \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)} = \frac{H(s)}{1+\dfrac{H(s)}{M}} = \frac{1}{\dfrac{1}{H(s)}+\dfrac{1}{M}}$$

Question 7(b) continued

$$\frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)} = \frac{1}{\dfrac{1}{H(s)} + \dfrac{1}{M}} = \frac{1}{\dfrac{s + \dfrac{s^2}{\omega_{LPF}}}{K_{PD}K_{VCO}} + \dfrac{1}{M}}$$

$$= \frac{K_{PD}K_{VCO}}{s + \dfrac{s^2}{\omega_{LPF}} + \dfrac{K_{PD}K_{VCO}}{M}}$$

$$= \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + \omega_{LPF}s + \dfrac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$$

$$\frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + \omega_{LPF}s + \dfrac{K_{PD}K_{VCO}\omega_{LPF}}{M}} \equiv \frac{K_{PD}K_{VCO}\omega_{LPF}}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{K_{PD}K_{VCO}\omega_{LPF}}{M}}$$

$$\Rightarrow 2\varsigma\omega_n = \omega_{LPF} \Rightarrow \varsigma = \frac{1}{2}\frac{\omega_{LPF}}{\omega_n} = \frac{1}{2}\sqrt{\frac{M\omega_{LPF}}{K_{PD}K_{VCO}}}$$

8 marks

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Question 7(c)

(i) For an integer feedback the reference frequency must be equal to the desired step size i.e. 200kHz in this case

2 marks

(ii) Range of divider values

$$M = \frac{935}{0.2} = 4675 \qquad to \qquad M = \frac{960}{0.2} = 4800$$

2 marks

(iii) Cut-off frequency of LPF

A rule of thumb to ensure good stability is to set the low-pass filter cut-off frequency to 10% of the reference frequency i.e. 20kHz.

2 marks

(iv) PLL gain constant (use average M value)

$$\varsigma = \frac{1}{2}\sqrt{\frac{M\omega_{LPF}}{K_{PD}K_{VCO}}} \Rightarrow K_{PD}K_{VCO} = \frac{M\omega_{LPF}}{4\varsigma^2}$$

$$K_{PD}K_{VCO} = \frac{M\omega_{LPF}}{4\varsigma^2} = \frac{4737.5 \times 2\pi \times 20000}{4 \times 0.707^2} = 2.98 \times 10^8$$

2 marks