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THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2013

B.E. DEGREE (ELECTRICAL AND ELECTRONIC)
B.E. DEGREE (ENERGY)

CONTROL ENGINEERING
EE4002

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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of departmental approved non-programmable calculators is permitted

1.

(a) Give Shannon's sampling theorem and comment on the benefits of over-sampling, in particular focussing on control applications. Explain why it is necessary to employ anti-aliasing filters, before sampling. Give some indication how sampling rate and filter bandwidth would be selected.

[8 Marks]

(b) Derive Tustins transform.

[5 Marks]

(c) Consider the following closed-loop digital control scheme,

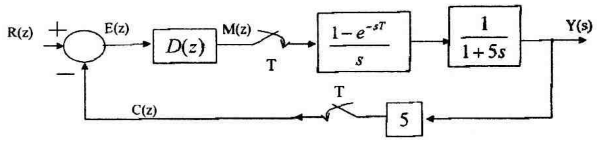

Fig. 1.1 Digital control system

The following discrete-time controller has been designed with the sample time T=1 second,

$$m(k) = Ke(k-1) + 0.7m(k-1).$$

Sketch the root locus diagram for this process, on the Z plane template, and use it to explain how the closed-loop dynamics depend on the choice of the controller gain K.

[12 Marks]

2.

(a) The following unit step response has been obtained for a certain discrete process, $\frac{Y(z)}{U(z)} = G(z)$.

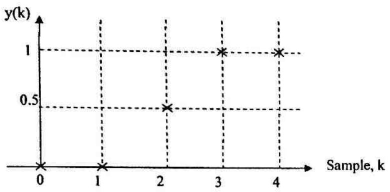

Fig. 2.1 Discrete unit step response

Sketch the response for this process, to the discrete ramp input,

$$u(k) = \begin{cases} 0 & for \quad k < 0 \\ 2k & for \quad k \geq 0 \end{cases}.$$

[5 Marks]

(b) Derive the following deadbeat controller, from a basic prescription of the shape of the desired closed-loop step response,

$$D(z) = \frac{1}{G_m(z)} \frac{1}{z^N - 1}$$

Here $G_m(z)=C(z)/U(z)$ is the discrete-time transfer function model of the process, with N representing a tuning parameter, used to determine the desired closed-loop response.

Consider the discrete-time closed-loop system in Fig. 2.2.

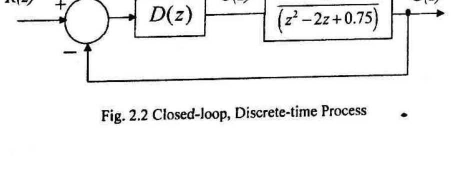

Fig. 2.2 Closed-loop, Discrete-time Process

Show by use of a root-locus plot, why a deadbeat controller will provide unsatisfactory closed-loop performance.

[10 Marks]

(c) Consider the continuous-time PID controller,

$$m(t) = K_p\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\frac{de(t)}{dt}\right)$$

The controller time constants are related according to: $T_D = \frac{T_I}{4}$,

Use the matched-pole-zero approach to derive the following difference equation representation of this controller for implementation on a digital computer, with sample time T_s. Show clearly in your derivation how the parameters of the difference equation are related to parameters of the continuous PID controller.

$$m(k) = m(k-1) + \alpha e(k) + \beta e(k-1) + \gamma e(k-2)$$

(Hint: An extra pole at z=0 is required to produce a causal (realisable) control algorithm. Due to the integral action present in the original controller C(s), you will need to determine the gain of the digital controller D(z) according to: $\lim_{s\to 0} sC(s) = \lim_{z\to 1}(z-1)D(z)$)

[10 Marks]

3.

(a)

A certain process is known to have the following open-loop transfer function,

$$G(z) = \frac{0.5z^{-2}}{(1-0.7z^{-1})(1-0.8z^{-1})}.$$

Design a pole-placement controller for this system which will place the two dominant closed loop poles at $z=-0.5$. It is also desired that this control scheme will provide perfect tracking of step set-point signals.

Clearly show the development of the Sylvester matrix used to solve the Diophantine pole-placement design equation.

[12 Marks]

(b)

Derive in full, the following least-squares algorithm, for the identification of the parameters $\underline{\hat{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains the sampled process output, up to the current k[th] sample, *y(k)*.

$$\underline{\hat{\theta}}(k) = \left(\Phi(k)^T\Phi(k)\right)^{-1}\Phi(k)^T\underline{y}(k)$$

If a square matrix *P(k)* is now defined as $P(k) = \left(\Phi(k)^T\Phi(k)\right)^{-1}$, derive the following update equation to obtain *P(k+1)* from process data up to the (k+1)[th] sample,

$$P(k+1) = \left(P(k)^{-1} + \underline{\psi}(k+1)\underline{\psi}(k+1)^T\right)^{-1}$$

where vector $\underline{\psi}(k+1)$ contains process input and output data sampled up to the (k+1)[th] sample.

Show by application of Householder's Matrix Inversion Lemma,

$$(A+BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1}+DA^{-1}B\right)^{-1}DA^{-1}$$

that the following update equation for the model parameter vector can be obtained:

$$\underline{\hat{\theta}}(k+1) = \left[P(k) - \frac{P(k)\underline{\psi}(k+1)\underline{\psi}^T(k+1)P(k)}{1+\underline{\psi}^T(k+1)P(k)\underline{\psi}(k+1)}\right]\left[\Phi(k)^T\underline{Y}(k) + \underline{\psi}(k+1)y(k+1)\right]$$

[13 Marks]

4.

(a) Consider the following state-space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$

Develop fully the solution for the state trajectory $\underline{x}(t)$, for $t \geq 0$, where $\underline{x}(0)$ is the initial state vector at $t=0$.

[6 Marks]

(b) Consider the following second-order SISO process,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine the discrete-time state-space representation of this system. A zero-order hold is assumed and the sample-time is T=0.1 seconds.

[7 Marks]

(c)

Consider the mechatronic system represented by the block diagram given in Fig. 4.1.

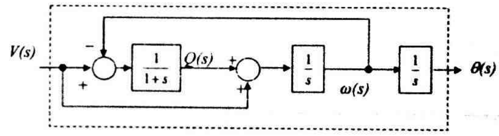

Fig. 4.1: Open-loop servomechanism block diagram

(i) Develop a state-space model of this process using the states q(t), *ω(t)* and *θ(t)*.

(ii) Determine the poles of this system.

(iii) Determine whether the internal states are observable from measurements of the output angle θ and the input voltage *v*.

[12 Marks]

5.

(a) Consider the following N^{th} order open-loop process, with one input $u(t)$ and a single output $y(t)$,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

If this process is under the following state space control–law with integral action,

$$u(t) = -K\underline{x}(t) + K_I \int_0^t \left(r(\tau) - y(\tau)\right) d\tau$$

show that the closed-loop characteristic equation is:

$$\det\left[\begin{array}{c|c} sI_N - A + BK & -BK_I \\ \hline C & s \end{array}\right] = 0$$

[7 Marks]

(b) A classical control scheme for a general DC motor based positioning system is shown in Fig. 5.1.

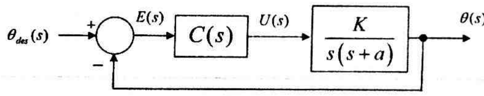

Fig. 5.1: Closed-loop position control system

Here the following PD controller C(s) is used;

$$C(s) = K_C\,(s+z).$$

The set-point is usually constant. Use the state-space method to design the PD controller to place both the desired closed loop poles at $s = -2a$

[8 Marks]

(c) Figure 5.1 shows a variable-speed wind-turbine, connected to the grid via fully rated back-to-back inverters:

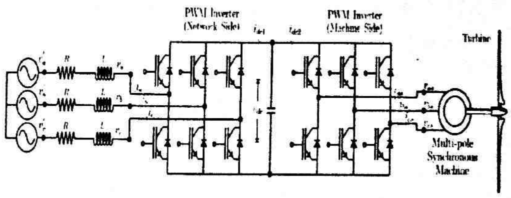

Fig. 5.1 Variable speed wind-turbine

The network-side converter dynamics can be modelled in the DQ domain using the following differential equations,

$$L\frac{di_d}{dt} - \omega L\, i_q(t) + R i_d(t) = v'_d(t) - v_d(t)$$
$$L\frac{di_q}{dt} + \omega L\, i_d(t) + R i_q(t) = v'_q(t) - v_q(t)$$

Here i_d(t)and i_q(t) are the D and Q axis currents. The network voltages in the DQ domain are v'_d and v'_q. The control inputs to this system are the inverter voltages v_d(t) and v_q(t).

At unity power factor, a simplified power balance at the DC link capacitor yields,

$$C\frac{dv_{dc}}{dt} v_{dc}(t) = \tfrac{3}{2}\left(v_d(t)\, i_d(t)\right) + P_w(t).$$

Here v_{dc}(t) is the link capacitor voltage and P_w(t) is the power delivered onto the DC link from the wind-turbine.

The parameters for this system are given in Table I below.

In normal rated operation, this wind turbine will extract 1MW of power from the wind, and deliver this to the grid (neglecting losses) with unity power factor (iq=0A). Determine a linear state-space model of this process at this operating point. (Hint: the equilibrium value for the D axis inverter voltage is positive)

[10 Marks]

TABLE I: System Parameters

	Parameter	Value
Resistance	R	0.003Ω
Inductance	L	80μH
DC-link capacitance	C	0.2F
Nominal D axis grid voltage	v'_{d0}	690V
Nominal Q axis grid voltage	v'_{q0}	0V
Nominal DC-link voltage	V_{dc0}	1050V
Electrical frequency	ω	314.16rad/s

6.

(a) A certain process can be modelled by the transfer function:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K(s+\alpha)}{(s+\beta)(s+\gamma)}$$

Develop fully a simulation diagram for the **Observer Canonical representation** of this process.

[5 Marks]

(b) Consider the following N[th] order open-loop process, with single input $u(t)$, single output $y(t)$, and state-vector $\underline{x}(t)$,

$$\frac{d}{dt}\underline{x}(t)=A\,\underline{x}(t)+B\,u(t)$$
$$y(t)=C\,\underline{x}(t)$$

The state vector is not measured directly, but is estimated as $\hat{\underline{x}}(t)$ using a full-state Luenberger observer with estimator gain matrix G.

The following control-law is utilised, where $r(t)$ is the setpoint signal,

$$u(t)=-K\,\hat{\underline{x}}(t)+Nr(t)$$

(i) Develop fully the following representation of the closed loop system,

$$\frac{d}{dt}\begin{bmatrix}\underline{x}(t)\\ \underline{\varepsilon}(t)\end{bmatrix}=\left[\begin{array}{c|c}A-BK & BK\\ \hline 0 & A-GC\end{array}\right]\begin{bmatrix}\underline{x}(t)\\ \underline{\varepsilon}(t)\end{bmatrix}+\begin{bmatrix}BN\\ \underline{0}\end{bmatrix}r(t)$$

where the estimation error $\underline{\varepsilon}(t)$ is defined as, $\underline{\varepsilon}(t)=\underline{x}(t)-\hat{\underline{x}}(t)$

Use this representation to explain the "Separation Principle", and how this principle is applied in state-space control design.

[10 Marks]

(ii) Show that the closed-loop system could be represented by the following classical realisation, based on a pre-filter and a feedback transfer function:

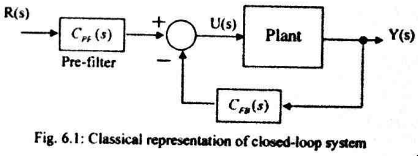

Fig. 6.1: Classical representation of closed-loop system

[10 Marks]