Tutorial Questions for EE4008

1. (a) Starting with the constant coefficient difference quation definition in each case, show that the frequency responses of the first-difference and central-difference differentiators are

   i. First-difference
   $$H_{fd}(\omega) = 2je^{-0.5j\omega} \sin\left(\frac{\omega}{2}\right)$$

   ii. Central-difference
   $$H_{cd}(\omega) = je^{-j\omega} \sin(\omega)$$

   (b) In the continuous time domain a differentiator is defined in Laplace domain as:
   $$H_{diff}(s) = s$$

   Show that a digital differentiator designed using the Bilinear transform has a frequency response of
   $$H_{diff}(\omega) = j \tan\left(\frac{\omega}{2}\right)$$

   You may assume that $T_s = 2$, to ignore the effect of the sampling frequency on the Bilinear transformation.

   (c) Plot the magnitude response of the three differentiators of part (a) and (b).

   (d) Comment on the suitability of the three differentiators for use in an application where there is high frequency noise.

2. (a) Starting with the ideal frequency response $H_d(\omega)$, describe the windows method of designing a broadband differentiator.

   (b) Determine the filter coefficients $h(n), 0 <= n <= M-1$ for $M = 7$ using the Windows method of designing a broadband differentiator using the Hamming window function. Sketch the frequency response of the differentiator.

3. (a) Starting with the ideal frequency response $H_d(\omega)$, describe the windows method of designing a broadband differentiator with cutoff frequency ω_c.

   (b) Determine the filter coefficients $h(n), 0 <= n <= M-1$ for $M = 5$ using the Windows method of designing a broadband differentiator with cutoff frequency 0.75π and a rectangular window function. Sketch the frequency response of the differentiator.

4. (a) Starting with the ideal frequency response $H_d(\omega)$, describe the windows method of designing a broadband differentiator with cutoff frequency ω_c.

   (b) Determine the filter coefficients $h(n), 0 <= n <= M-1$ for $M = 5$ using the Windows method of designing a broadband differentiator with cutoff frequency 0.8π and a rectangular window function. Sketch the frequency response of the differentiator.

5. (a) Describe the steps necessary to transfor a digital Integrator into a stable digital Differentiator

(b) Using the procedure described in part (a) determine the constant coefficient difference equation of the digital differentiator derived from from a Tick's rule integrator, which is defined as:

$$y(n) = 0.3584x(n) + 1.2832x(n-1) + 0.3584x(n-2) + y(n-2)$$