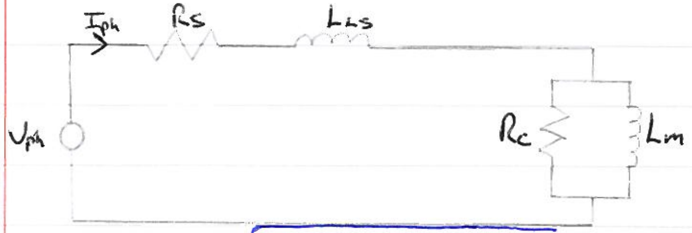
Induction Motor Characterisation Problems

1 a)

	No-Load	Locked-Rotor
4-pole	$V_{L-L} = 195V, 200Hz$	$V_{L-L} = 35.6V, 200Hz$
$P = 51.8kW$	$V_{ph} = 112.6V$	$V_{ph} = 20.55V$
$R_{ph-ph} = 23.6m\Omega$	$I_{NL} = 64.7A$	$I_{BR} = 93A$
$R_s = 11.8m\Omega$	$P_{NL} = 1.093kW$	$P_{BR} = 641W$



$P_{RL} = P_{NL} - 3I^2R_s$
$= 945W$

$Q_{in} = \sqrt{(V_{ph}I_{NL})^2 - \left(\frac{P_{NL}}{3}\right)^2}$
$= 7.276kW = 2\pi f I^2 (L_{LS} + L_m)$

$L_{LS} + L_m = 1.38mH$

$Q_{in} = \sqrt{(V_{ph}I)^2 - \left(\frac{P}{3}\right)^2}$
$= 1.9kW = 2\pi f I^2 (L_{LR}' + L_s)$

$L_{Lr}' + L_s = 174.7\mu H = 1.8L_s$

$\frac{P_{BR}}{3} = I_{BR}^2 (R_s + R_R')$

$L_s = 97\mu H$
$L_{Lr}' = 78\mu H$
$R_R' = 12.9m\Omega$
$L_m = 1.28mH$

b)

4-pole	$L_{LS} = 0.2mH$	$I_{ph} = 225A$
$V_{L-L} = 400V, 50Hz$	$L_M = 7.2mH$	$\cos\phi = 0.89$
$V_{ph} = 230.9V$	$L_{LR}' = 0.3mH$	$\eta = 0.897$
$R_s = 20m\Omega$	$R_R' = 35m\Omega$	

i) $P_i = 3V_{ph}I_{ph}\cos\phi$
$= 138.74kW$

ii) $P_{AG} = P_i - P_{RS}$
$= 135.7kW$

iii) $I_m = I\sin\phi$
$= 102.6A$

iv) $I_R' = I\cos\phi$
$= 200.25A$

v) $P_{AG} = \frac{3I_r^2R_r'}{s}$

$s = 3.1\%$

vi) $P_{cfw} = P_{AG} - P_{Rr} - P_m$
$= 7.04kW$

Induction Motor Characterisation Problems

2. 4-pole

$R_{ph-ph} = 1.1\Omega$

$V_{L-L} = 208V, 60Hz$

$V_{phNL} = 120.1V$

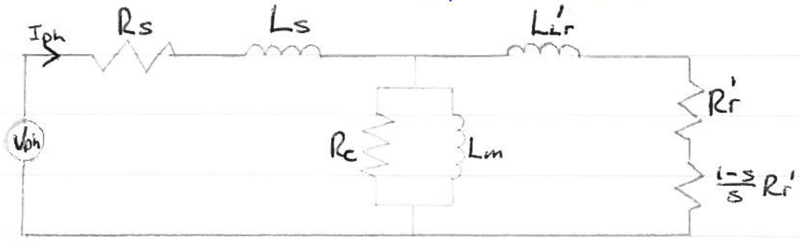
$I_{ph} = 6.5A$

$P_{3\phi} = 175W$

$V_{phBr} = 30.6V$

$I_{ph} = 18.2A$

$P_{BR} = 900W$



$$P_i = 3V_{ph}I_{ph}\cos\phi$$
$$= P_{AG} + 3I^2R_s$$

$$P_{AG} = P_{EM} + P_{Rr}$$
$$= 3I_r^2\left(\tfrac{1-s}{s}\right)R_r' + 3I_r^2R_r'$$
$$= \frac{3I_r^2R_r'}{s}$$

$$P_{EM} = P_m + P_{CFW}$$
$$= 3I_r^2\left(\tfrac{1-s}{s}\right)R_r'$$

$$R_s = \frac{R_{ph-ph}}{2} = 0.55\Omega$$

$$Q_{NL} = \sqrt{(V_{ph}I_{NL})^2 - \left(\tfrac{P_{NL}}{3}\right)^2}$$
$$= 778.5W = 2\pi f I^2(L_s+L_m)$$

$$L_s + L_m = 48.87mH$$

$$R_R' = \frac{\frac{P_{BR}}{3}}{I^2} - R_s = 0.356\Omega$$

$$Q_{BR} = \sqrt{(V_{ph}I_{ph})^2 - \left(\tfrac{P_{BR}}{3}\right)^2}$$
$$= 469W = 2\pi f I^2(L_s + L_{lr})$$

$$L_s + L_{lr} = 3.76mH$$

Class B $\rightarrow$ $L_{lR} = 1.5L_s$

$L_{lR} = 2.25mH$

$L_{ls} = 1.5mH$

$R_R' = 0.356\Omega$

$R_s = 0.55\Omega$

$L_m = 47.37mH$

Induction Motor Characterisation Problems

3\. a) $R_{ph-ph} = 3.54\,\Omega$ $\qquad V_{LR\,ph} = 41\,V$

$V_{ph\,NL} = 230.9\,V$ $\qquad I_{LR\,ph} = 4\,A$

$I_{NL} = 1.8\,A$ $\qquad P_{LR} = 150\,W$

$P_{NL} = 120\,W$

N/L: $\quad Q_{ph} = \sqrt{(V_{ph} I_{ph})^2 - \left(\frac{P_{NL}}{3}\right)^2}$

$= 413.7\ VAr = 2\pi f I^2 (L_{ls} + L_M)$

$L_{ls} + L_M = 406.4\ mH$

L/R: $\quad Q_{ph} = \sqrt{(V_{ph} I_{ph})^2 - \left(\frac{P_{LR}}{3}\right)^2}$

$= 156.2\ VAr = 2\pi f I^2 (L_{ls} + L'_{lr})$

$L_{ls} + L'_{lr} = 31.1\ mH = 2.5\,L_{ls}$

$R_R' = \dfrac{\left(\frac{P_{Lr}}{3}\right)}{I_{LR}^2} - R_s$

$R_s = 1.77\,\Omega$

$R_r' = 1.355\,\Omega$

$L_{ls} = 12.44\ mH$

$L'_{lr} = 18.66\ mH$

$L_M = 394\ mH$

b) $R_s = 11.8\ m\Omega$ $\qquad L'_{lr} = 77.2\ \mu H$ $\qquad T_m = 40\ Nm$

$L_{ls} = 97.2\ \mu H$ $\qquad R_R' = 12.9\ m\Omega$ $\qquad \omega_m = 622.56\ rad\,s^{-1}$

$L_M = 2\ mH$ $\qquad I_{in} = 93\,A,\ 200\,Hz$ $\qquad P_{CFW} = 2.3\ kW$

$P_{EM} = P_m + P_{CFW}$

$= (24.9 + 2.3)\,kW$

$= 27.2\ kW$

$= 3 I_r^2 \left(\frac{1-s}{s}\right) R_r'$

$P_{AG} = P_{EM} + P_{Rr}$

$= \dfrac{3 I_r^2 R_r'}{s}$

$P_{in} = P_{AG} + P_{Rs}$

$= \dfrac{3 I_r^2 R_r'}{s} + 3 I_s^2 R_s$

$= 3 V_{ph} I_{ph} \cos\phi$

$P_{Rs} = 306.2\ W$ $\qquad I_r = 80.78\ A$ $\qquad \cos\phi = 0.869 \left(= \frac{I_r}{I_s}\right)$

$\omega_{SLIP} = 11.52\ rad\,s^{-1}$ $\qquad P_{in} = 27.76\ kW$ $\qquad \eta = \frac{P_m}{P_i} = 0.897$

$s = 0.92\%$ $\qquad V_{in} = 114.5\,V$

Induction Motor Characterisction Problems

4. $P = 22kW$ $\quad N_{SLIP} = 35Rpm$

$V_{ph} = 400V, 50Hz$ $\quad f_{SLIP} = 0.58\dot{3}\,Hz$

$I_{ph} = 22.8\,A$ $\quad T_{rated} = 143\,Nm$

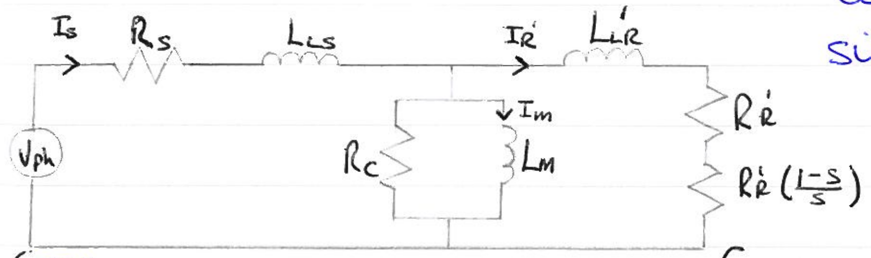
$s = 2.\dot{3}\%$

100% Load
$\eta = 0.913$
$\cos\phi = 0.89$
$\sin\phi = 0.456$

75% Load
$\eta = 0.916$
$\cos\phi = 0.84$
$\sin\phi = 0.543$

i) $\frac{T}{f_{SLIP}} = 245\,Nm/Hz$

ii)



100% $\begin{cases} I_r' = I_{ph}\cos\phi = 20.29A \\ I_m = I_{ph}\sin\phi = 10.4\,A \end{cases}$ $\quad$ 75% $\begin{cases} I_r' = I_{ph_{75\%}}\cos\phi = 15A \\ I_m = I_{ph_{75\%}}\sin\phi = 9.7A \end{cases}$

$V_{ph} \approx I_m 2\pi f_e L_M$ $\quad I_{ph_{75\%}} = \dfrac{P_o}{3V_{ph}\cos\phi\,\eta} = 17.87A$

$L_M \approx 122mH$

$P \approx 3I_{r'}^2\left(\frac{1-s}{s}\right)R_R'$

$R_R' = 426m\Omega$

100%

$P_{LOSS} = \left(\frac{1}{\eta}-1\right)P_o = 2096W$

$= 3R_sI_{ph}^2 + 3R_R'I_{r'}^2 + P_{CFW}$

$P_{CFW} + (1560)R_s = 1570\,W$

75%

$P_{LOSS} = 1{,}513W$

$= 3R_sI_{ph}^2 + 3R_R'I_{R'}^2 + P_{CFW}$

$P_{CFW} + (958)R_s = 1225W$

$602R_s = 345$

$R_s = 573m\Omega$

$P_{CFW} = 676\,W$

$I_{ph,su} = 7.1\,I_{RATED}$

$= 161.9A$

$Z_{ph,su} = \dfrac{V_{ph}}{I_{ph,su}} = 2.47\Omega$

$= \sqrt{(R_s+R_R')^2 + X_L^2}$

$X_L = 2.26\Omega$

$L_{LS} + L_{LR}' = 7.2mH$

$L_{LS} = 2.88mH$

$L_{LR}' = 4.3mH$

Induction Motor Characterisation Problems

5. $P = 75kW$

$V_{ph} = 400V, 50Hz$

$T_{rated} = 482Nm$

$N_{SLIP} = 15rpm$

$s = 1\%$

100% Load

$I_{ph} = 75.1A$

$\eta = 0.942$

$\cos\phi = 0.88$

$\sin\phi = 0.475$

75% Load

$I_{ph} = 57.86A$

$\eta = 0.942$

$\cos\phi = 0.86$

$\sin\phi = 0.51$

i) $\frac{T}{f_{SLIP}} = 1{,}928 \ Nm/Hz$

ii) $I_{R'_{100\%}} = I_{ph}\cos\phi = 66A$

$I_{M_{100\%}} = I_{ph}\sin\phi = 35.67A$

$L_M \simeq \frac{V_{ph}}{2\pi f I_M}$

$L_M \simeq 35.7mH$

$P_M \simeq 3I_{R'}^2\left(\frac{1-s}{s}\right)R_{R'}$

$R_R' \simeq 58m\Omega$

$I_{R'_{75\%}} = 49.76A$

$I_{M_{75\%}} = 29.51A$

100% Load

$P_{LOSS} = \left(\frac{1}{\eta}-1\right)P_O = 4.618kW$

$= 3R_S I_{ph}^2 + 3R_{R'}I_{R'}^2 + P_{cfw}$

$P_{cfw} + (16{,}920)R_S = 3860W$

75% Load

$P_{LOSS} = \left(\frac{1}{\eta}-1\right)P_O = 3.463kW$

$= 3R_S I_{ph}^2 + 3R_{R'}I_{R'}^2 + P_{cfw}$

$P_{cfw} + (10{,}043)R_S = 3{,}032W$

$6{,}877R_S = 828.1$

$R_S = 120m\Omega$

$P_{cfw} = 1.83kW$

$I_{SC} = 7.3I_{rated}$

$= 548.2A$

$Z_{SC} = \frac{V_{ph}}{I_{SC}} = 0.73\Omega$

$= \sqrt{(R_S+R_R')^2 + X_L^2}$

$X_L = 0.708\Omega$

$L_{LS} + L_{LR'} = 2.25mH$

$L_{LS} = 901\mu H$

$L_{LR'} = 1.35mH$

Induction Motor Characterisation Problems

6. $P = 110\,kW$ | $N_{slip} = 15\,rpm$ | 100% | 75%

		100%	75%
$P = 110\,kW$	$N_{slip} = 15\,rpm$	$I_{ph} = 110A$	$I_{ph} = 85.9A$
$V_{ph} = 400V, 50Hz$	$f_{slip} = 0.25Hz$	$\cos\phi = 0.88$	$\cos\phi = 0.85$
$R_s = 58.3\,m\Omega$	$s = 1\%$	$\sin\phi = 0.475$	$\sin\phi = 0.527$
$T_{rated} = 707\,Nm$		$\eta = 0.945$	$\eta = 0.942$

i) $\frac{T}{f_{slip}} = 2828\,Nm/Hz$

ii) $I_M = I_{ph}\sin\phi = 52.25$ $\quad I_{R'} = I_{ph}\cos\phi = 96.8A$

$R_R' = \frac{P_M\, s}{3I_{R'}^2(1-s)} = 39.5\,m\Omega$ $\quad I_{R'75\%} = 73.015A$

@100% Load

$P_{LOSS} = (\frac{1}{\eta} - 1)P_o = 6.4kW$

$= 3R_sI_{ph}^2 + 3R_{R'}I_{R'}^2 + P_{cfw}$

$P_{cfw} + (36{,}300)R_s = 5.29kW$

@75% Load

$P_{LOSS} = (\frac{1}{\eta} - 1)P_o = 5.08kW$

$= 3R_sI_{ph}^2 + 3R_R'I_{R'}^2 + P_{cfw}$

$P_{cfw} + (22{,}136)R_s = 4{,}448\,W$

$14{,}164R_s = 842W$

$R_s = 59.5\,m\Omega$

$P_{cfw} = 3.13kW$

iii) $I_{ph} = 22.8A$ @100% $\quad I_{R'} = 20.3A$ @100%

$V_{ph} = 400V$ $\quad I_{R'} \simeq \frac{I_{R'100\%}}{2} = 10.15A$ @50%

$I_M = 10.4A$

100% $P_{LOSS} = P_{cfw} + 3R_sI_{ph}^2 + 3R_{R'}I_{R'}^2$

50% $P_{LOSS} = P_{cfw} + 3R_s(I_{ph50\%})^2 + 3R_{R'}(I_{R'50\%})^2$

$I_{ph} = \sqrt{I_M^2 + I_{R'}^2}$

$= 14.5A$

$\cos\phi = \frac{10.15}{14.5} = 0.7$

Induction Motor Characterisation Problems

7. a) $P = 22\,kW$ $\quad T_{rated} = 288\,Nm$ $\quad \cos\phi = 0.77$

$N_{SLIP} = 20\,rpm$ $\quad I_{ph} = 26.27A$ $\quad \sin\phi = 0.638$

$s = 2.6\%$ $\quad \eta_{100\%} = 0.907$

$I_M = 16.76A$ $\qquad L_M = \dfrac{V_{ph}}{2\pi f I_M} = 76mH$

$I_{R'} = 20.23A$

$P_{EM} = 3I_{R'}^2\left(\frac{1-s}{s}\right)R_{R'}$ $\qquad P_{LOSS} = \left(\frac{1}{\eta} - 1\right)P_o = 2.256\,kW$

$R_{R'} = 490\,m\Omega$ $\qquad = 3R_S I_{ph}^2 + 3R_{R'} I_{R'}^2 + P_{CFW}$

$P_{CFW} = 760W$

$I_{phsu} = 7.7 I_{rated}$

$= 202.3\,A$

$Z_{phsu} = 1.977\Omega = \sqrt{(0.922)^2 + X_L^2}$

$X_L = 1.075\,\Omega$

$L_{LS} + L_{LR'} = 5.57\,mH$

$L_{LS} = 2.78\,mH$

$L_{LR} = 2.78\,mH$

b) $I_{ph} = 93A,\ 200Hz$ $\qquad P_M = 24.9\,kW$

$T_M = 40\,Nm$ $\qquad P_{EM} = P_M + P_{CFW}$

$N_M = 5945\,rpm$ $\qquad P_{EM} = 27.2\,kW$

$N_{SYN} = 6000\,rpm$ $\qquad = 3I_{R'}^2\left(\frac{1-s}{s}\right)R_{R'}$

$N_{SLIP} = 55\,rpm$ $\qquad I_{R'} = 80.8A$

$s = 0.92\%$ $\qquad \Rightarrow \cos\phi = \frac{I_R}{I_{ph}} = 0.87$

$P_{AG} = P_{EM} + P_{RR}$ $\qquad P_{in} = P_{AG} + P_{RS}$

$= \dfrac{3I_{R'}^2 R_{R'}}{s}$ $\qquad = P_{AG} + 3I_S^2 R_S$

$= 27.77\,kW$

$= 27.46\,kW$ $\qquad = 3V_{ph} I_{ph} \cos\phi$

$V_{ph} = 114.4\,V$

$\eta = \frac{P_o}{P_i} = 0.897$

Squirrel Cage Induction Motor Equations

$$\begin{aligned} P_{EM} &= T_{EM}\omega_m \\ &= P_m + P_{cfw} \\ &= T_m\omega_m + T_{cfw}\omega_m \\ &= 3R_r'\left(\frac{1-s}{s}\right)I_r'^2 \end{aligned}$$

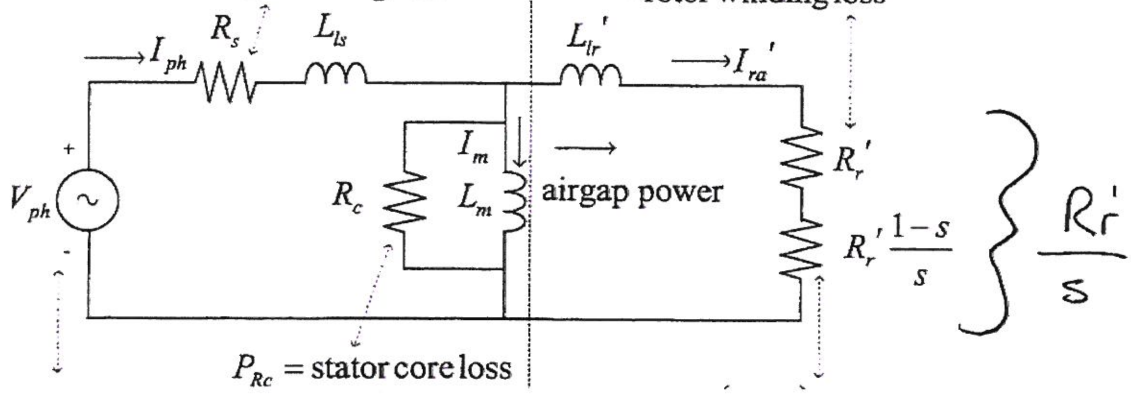
$$\begin{aligned} P_{AG} &= P_{EM} + P_{Rr} \\ &= T_{EM}\omega_{syn} \\ &= T_{EM}\omega_m + T_{EM}\omega_{slip} \\ &= 3R_r'\left(\frac{1-s}{s}\right)I_r'^2 + 3R_r'I_r'^2 \\ &= \frac{3R_r'I_r'^2}{s} \end{aligned}$$

$$\begin{aligned} P_{in} &= P_{AG} + P_{Rs} \\ &= \frac{3R_r'I_r'^2}{s} + 3R_sI_s^2 \\ &= 3V_{ph}I_{ph}\cos\phi \end{aligned}$$

Per-phase IM Eq. Cct Model

$P_{Rs} = 3R_sI_{ph}^2$ = stator winding loss

$P_{Rs} = 3R_r'I_r'^2$ = rotor winding loss



p = # of pole pairs

N_{sp} = # of conductors per phase per pole of the stator

N_{rp} = # of conductors per phase per pole of the rotor

$$R_r' = \left(\frac{N_{sp}}{N_{rp}}\right)^2 R_r$$

$$L_{lr}' = \left(\frac{N_{sp}}{N_{rp}}\right)^2 \frac{1}{p} L_{lr}$$