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THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2012

B.E. DEGREE (ELECTRICAL AND ELECTRONIC)
B.E. DEGREE (ENERGY)

CONTROL ENGINEERING
EE4002

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Time allowed: *3 hours*

Answer *four* questions
All questions carry equal marks

The use of departmental approved non-programmable calculators is permitted

1.

(a) A certain SISO discrete-time process has input *u(k)* and output *y(k)*. The response of this system to a unit step input is given in Fig. 1.1.

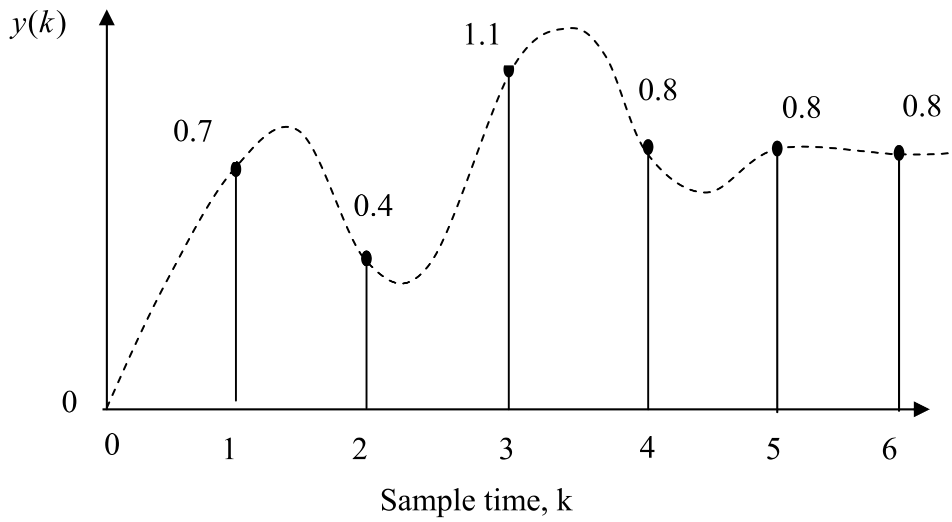

Fig. 1.1: Discrete-time unit step response

Determine an expression for the discrete time transfer function Y(z)/U(z).

Sketch the first 5 samples of the response of this system to the unit ramp input sequence,

$$u(k) = \begin{cases} k & for \quad k \geq 0 \\ 0 & for \quad k < 0 \end{cases}$$

[8 Marks]

(b) Consider the continuous-time PID controller,

$$m(t) = K_P\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\frac{de(t)}{dt} \right)$$

The controller time constants are related according to: $T_D = \frac{T_I}{4}$.

Use the matched-pole-zero approach to derive the following difference equation representation of this controller for implementation on a digital computer, with sample time T_s. Show clearly in your derivation how the parameters of the difference equation are related to parameters of the continuous PID controller.

$$m(k) = m(k-1) + k_1 e(k) + k_2 e(k-1) + k_3 e(k-2)$$

(Hint: An extra pole at z=0 is required to produce a causal (realisable) control algorithm. Due to the integral action present in the original controller C(s), you will need to determine the gain of the digital controller *D(z)* according to: $\lim_{s\to 0} sC(s) = \lim_{z\to 1}(z-1)D(z)$)

[10 Marks]

(c) Consider in Fig. 1.2, the block diagram for a sample and hold.

Derive the transfer function of the Zero-Order Hold and sketch its frequency response. Use the frequency response plots to comment on the benefits of sampling much faster than the Nyquist limit (oversampling).

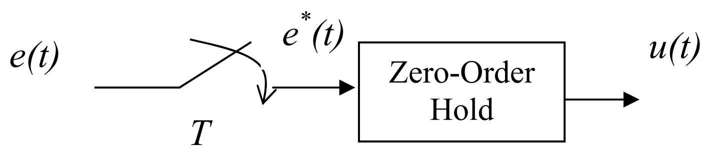

Fig. 1.2 sample and hold

[7 Marks]

2.

(a) The model in Fig. 2.1 has been obtained using system identification techniques, for an air heating system,

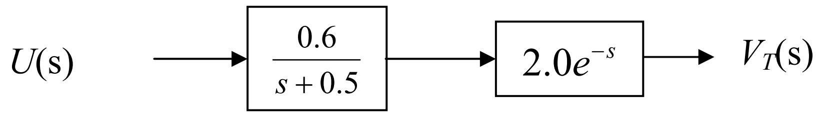

Fig. 2.1 Open-loop air heating system

Here, $u(t)$ is the input voltage and the output is the voltage $v_T(t)$ provided by the temperature sensor.

Design a Dahlin's controller to achieve a settling time (to within 2% of final value) of 4 seconds. The sample-time $Ts = 0.4$ seconds.

Will your design lead to controller ringing?

[15 Marks]

(b) Consider the closed-loop digital speed controller in Fig. 2.2:

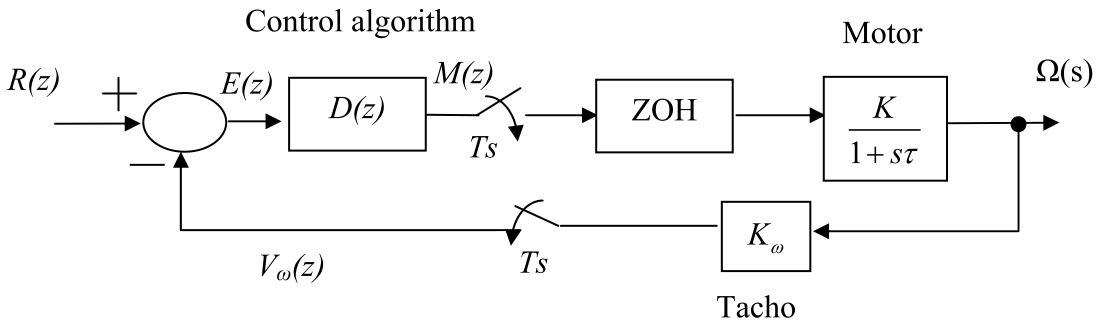

Fig. 2.2 Closed loop digital speed control system

The following discrete-time control algorithm has been designed with a sample-time Ts seconds.

$$m(k) = K_P e(k) + m(k-1)$$

Sketch the root locus diagram for this process and use it to explain how the closed-loop dynamics depend on the choice of controller gain K_p.

[10 Marks]

3.

(a) A certain process is known to have an open-loop transfer function,

$$G(z) = \frac{K}{z\left(z-\alpha\right)} .$$

Design a Diophantine pole-placement controller for this process, to achieve perfect tracking of step-like setpoints and maintain second-order dominance with two dominant poles, both placed at $z=\gamma$. Clearly show the development of all your design equations.

[13 Marks]

(b) Exponential forgetting is usually included in the least squares technique to emphasise more recent information. The least squares cost function over N valid test points is then:

$$J\left(\hat{\underline{\theta}}(k)\right) = \sum_{i=0}^{N-1} \mathbf{e}^{-\alpha i} e(k-i)^2 ,$$

where, the forgetting factor $\lambda = \mathbf{e}^{-\alpha} \leq 1$, and e(k) is the prediction error.

Derive in full, the following least-squares algorithm with forgetting, for the identification of the parameters $\hat{\underline{\theta}}(k)$, of a discrete-time transfer function. Here $\Phi(k)$ is a matrix of input and output data, and the vector $\underline{y}(k)$ contains N valid samples of the process output, up to the current kth sample, $y(k)$.

$$\hat{\underline{\theta}}(k) = \left(\Phi(k)^T \Lambda_N \Phi(k)\right)^{-1} \Phi(k)^T \Lambda_N \underline{Y}(k)$$

Here the weighting matrix for N valid points is the diagonal matrix, defined as:

$$\Lambda_N = \begin{bmatrix} \lambda^{N-1} & 0 & \cdots & 0 & 0 \\ 0 & \ddots & & & 0 \\ 0 & & \lambda^2 & & \vdots \\ \vdots & & & \lambda & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

[12 Marks]

4.

(a) A certain mechatronic system can be modelled by the following differential equation, where *u(t)* is the input voltage, and *θ(t)* is the resulting angle of rotation.

$$\frac{d^2\theta(t)}{dt^2}+5\frac{d\theta(t)}{dt}+4\theta(t)=\frac{du(t)}{dt}+2u(t)$$

i) Show how this system could be represented as a simulation diagram (e.g. Simulink diagram), using only two integrators, a variety of gains and summers.

[4 Marks]

ii) Use this simulation diagram to derive the control-canonical state-space model of this process.

[3 Marks]

iii) If the initial conditions are $\theta(0)=1$ and $\frac{d\theta(0)}{dt}=0$, determine an expression for the zero-input responses of the *states of your model*.

[8 Marks]

(b) Consider the following state-space representation of a second-order SISO process,

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-\lambda & 0\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

i) Determine the transfer function of this process, *G(s)=Y(s)/U(s)*.

[3 Marks]

ii) Is this system controllable?

[3 Marks]

iii) Determine the transformation $\underline{z}=T\underline{x}$, which would transform this system into the control-canonical form

[4 Marks]

5.

(a) Consider the following state-space equations,

$$\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t)$$

If the sample-time is T, and it is assumed that a zero-order hold is applied to the input signal $\underline{u}(t)$, show that this process can be represented by the following discrete-time, state-space equations:

$$\underline{x}(k+1) = e^{AT}\,\underline{x}(k) + A^{-1}\left(e^{AT} - I\right)B\underline{u}(k)$$

[8 Marks]

(b) Consider the roll loop of a certain VTOL aircraft, as depicted in Fig. 5.1:

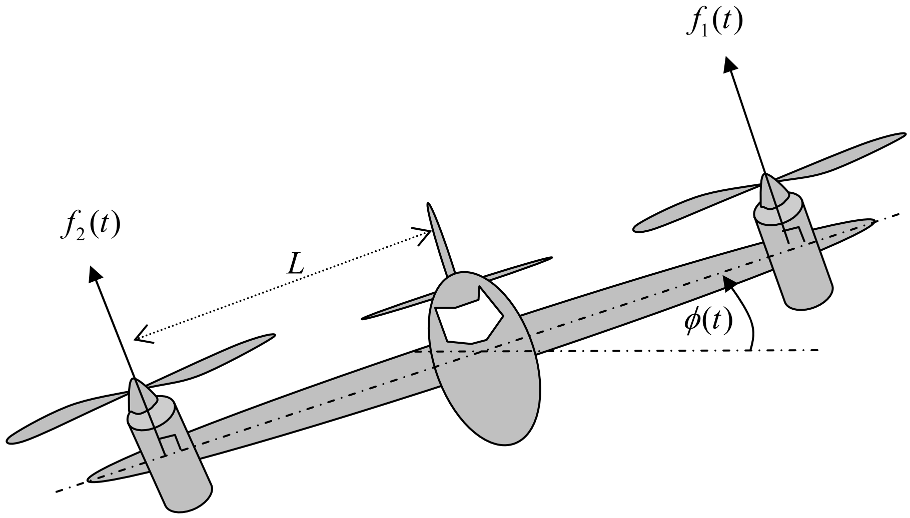

Fig. 5.1 Roll loop of a VTOL aircraft

Here, ϕ(t) is the roll angle in radians, with f_1(t) and f_2(t) the forces acting on the aircraft, generated by each of the turbo-props. In this simple model, the aircraft is hovering, and is free to rotate only about the roll axis. The moment of inertia of the aircraft about the roll axis is J kg m^{-2}. The roll dynamics are modelled as,

$$J\frac{d^2\phi}{dt^2} = LF(t)\,,$$

where $F(t) = f_1(t) - f_2(t)$ is the differential thrust.

The differential thrust can be modelled by the following simplified, linearised differential equation, where β(t) is the pitch command sent to the turbo-prop blades,

$$\tau\frac{dF}{dt} = K\beta(t) - F(t)\,.$$

Show that if the sample time T is relatively small, the following approximate discrete-time state-space model can be obtained:

$$\begin{bmatrix} \phi(k+1) \\ \omega(k+1) \\ F(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & TL/J \\ 0 & 0 & 1-T/\tau \end{bmatrix} \begin{bmatrix} \phi(k) \\ \omega(k) \\ F(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ TK/\tau \end{bmatrix} \beta(k)$$

where ω(k) is the angular velocity about the roll axis. A zero-order hold is utilised.

The following discrete-time state-space regulator is employed:

$$\beta(k) = \beta_0 - K_1\phi(k) - K_2\omega(k) - K_3F(k) .$$

It is desired that all three closed loop poles are placed at $z = \alpha$. It is known that:

$$\frac{T}{\tau} = 0.1 \quad \text{and} \quad \frac{L}{J} = 0.01\left(\text{kg}^{-1}\text{m}^{-1}\right).$$

Show that the gain K_3 should be selected as:

$$K_3 = \frac{29-30\alpha}{K}$$

[17 Marks]

6.

(a) Consider the following N^{th} order open-loop process, with single input *u(t)*, single output *y(t)* , and state-vector $\underline{x}(t)$,

$$\frac{d}{dt}\underline{x}(t) = A\,\underline{x}(t) + B\,u(t)$$
$$y(t) = C\,\underline{x}(t)$$

This process is controlled using a state-space regulator, with gain matrix *K*. The state vector is not measured directly, but is estimated as $\hat{\underline{x}}(t)$ using a full-state Luenberger observer with estimator gain matrix *G*.

Develop fully the following characteristic equation for the closed-loop system,

$$|sI - A + BK||sI - A + GC| = 0\,.$$

Use this characteristic equation to explain the "Separation Principle", and how it is applied in state-space control design.

[10 Marks]

(b) Consider the following schematic diagram representing a section of the water distribution network. Here, the outputs of two reservoirs are combined.

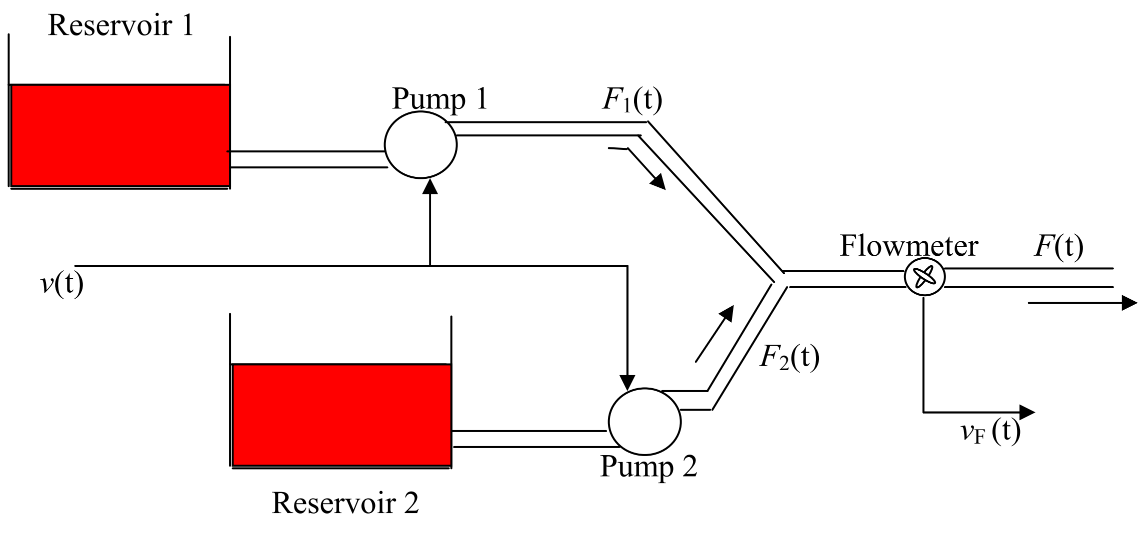

Fig.5.1 Multi-reservoir system

Both pumps are supplied with the same input voltage v(t). The flow-rate generated by pump 1 is F_1(t) (m^3/s) and that generated by pump 2 is F_2(t) (m^3/s). The outlet flow-rate is measured using a rotating vane flow-meter. This flow-meter can be modelled by the following equation, where $K_F = 0.02$ V(m^3/s)$^{-1}$

$$v_F(t) = K_F\,F(t)$$

In a system identification phase, the following transfer functions were obtained:

$$G_1(s) = \frac{F_1(s)}{V(s)} = \frac{100}{1+10s} \qquad\qquad G_2(s) = \frac{F_2(s)}{V(s)} = \frac{200}{1+15s}$$

The process engineer requires that the flowrates F_1(t) and F_2(t) be continuously monitored, but also specifies that cost and maintenance are to be kept to a minimum. It is therefore decided to utilise an observer so that only one flow-meter is required.

i) Show that the flowrates F_1(t) and F_2(t) are observable using the flowmeter output voltage v_F(t) and the pump voltage v(t).

ii) Design an estimator to provide estimates of the flowrates F_1(t) and F_2(t), based only on the measurement voltage v_F(t) and the pump voltage $v(t)$.

iii) Show how this estimator would be constructed, based on only two integrators, and a number of gain and summation blocks.

[15 Marks]