Chapter 9

Introduction to AC Machines

Introduction

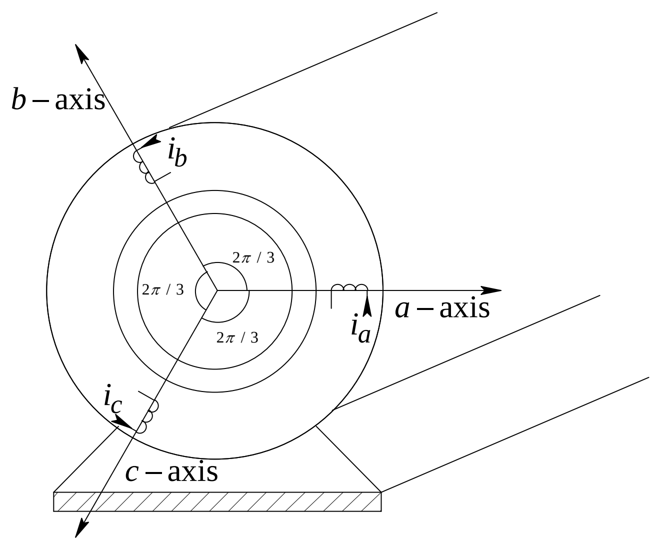
- Primary AC motor drives
  - Induction motors (asynchronous)
    - Squirrel cage – brushless
    - Wound rotor - brushed
  - Synchronous Motors
    - Permanent Magnet – brushless
    - Wound rotor – brushed

- These machines have similar stators but different rotor constructions.

Introduction

Stator windings produce a sinusoidal field distribution in the airgap.

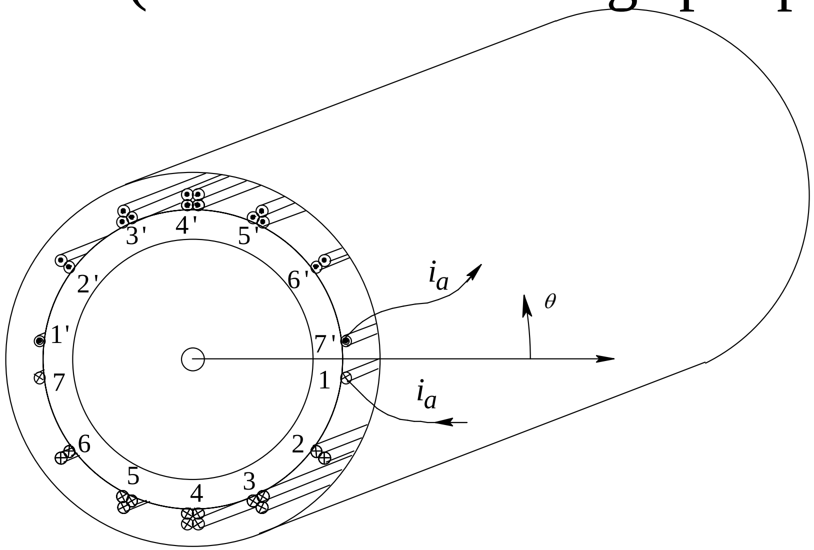
These magnetic field distributions are displaced by 120 º w.r.t. each other.

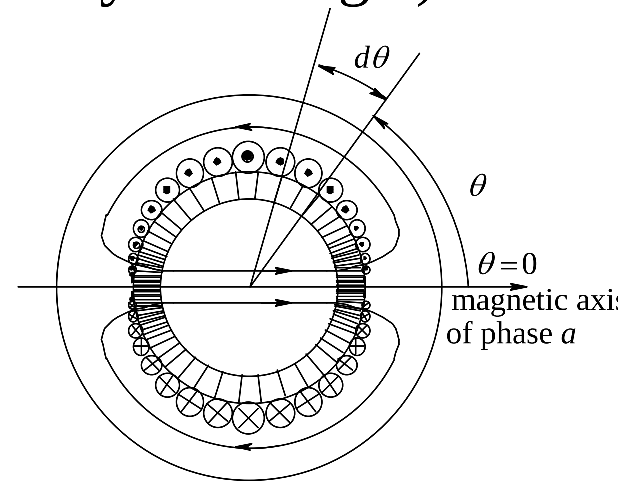


Exit

Sinusoidally-distributed Stator Windings

(number of windings per phase vary with angle)





Conductor density

Total

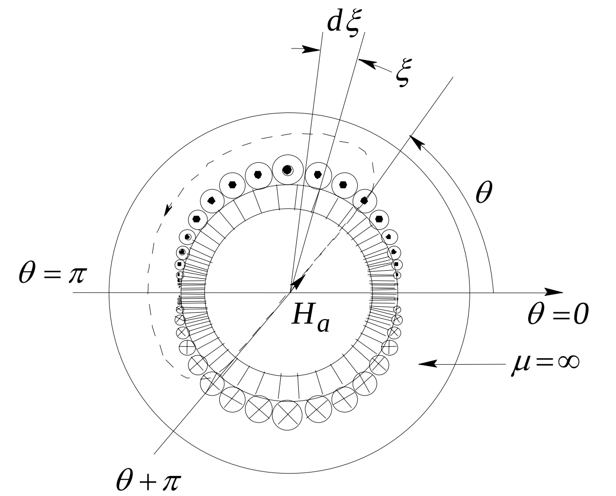
$$N_s = \int_0^{\pi} n_s(\theta)\, d\theta = \int_0^{\pi} \hat{n}_s \sin(\theta)\, d\theta = 2\hat{n}_s$$

$$\Rightarrow\ n_s(\theta) = \frac{N_{sp}}{2} \sin(\theta) \qquad\qquad 0 < \theta < \pi$$

N_{sp} is the number of conductors/phase/pole.

I.M. – typically 4 pole; P.M. - multipole

Air-gap Field Distribution



Apply Ampere's Law.

RHR gives direction.

From symmetry

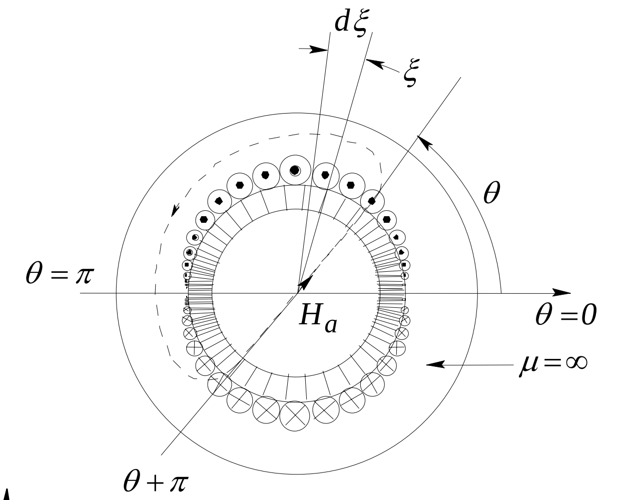
$$H_a(\theta) = -H_a(\theta + \pi)$$

(negative sign because line of integration points inwards at $\theta + \pi$)

$$\sum H.dl = H_m l_m + 2H_a(\theta) l_g = \int_0^{\pi} n_s(\theta + \xi) i_a \, d\xi$$

$$2H_a(\theta)\ell_g = \frac{N_s}{2} i_a \int_0^{\pi} \sin(\theta + \xi)\, d\xi = N_s i_a \cos(\theta)$$

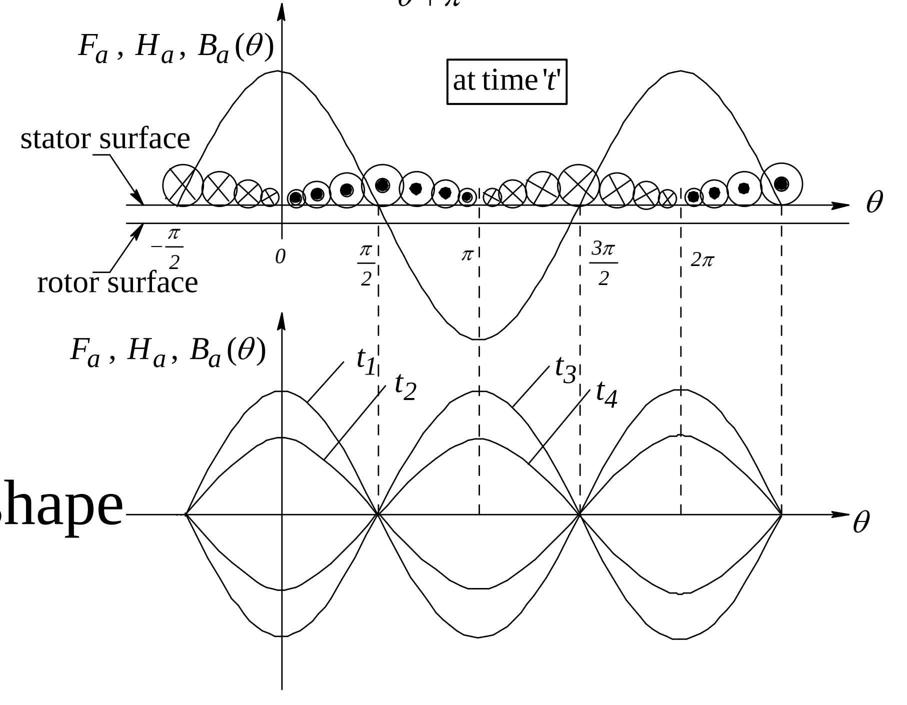
Air-gap Field Distribution



Radial magnetic field strength

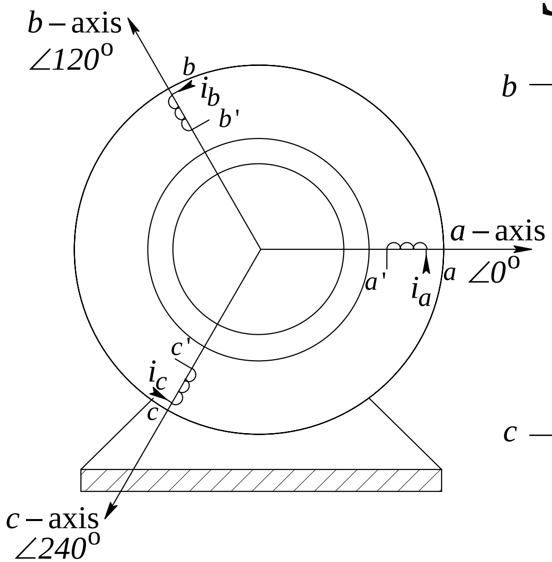
Radial magnetic flux density

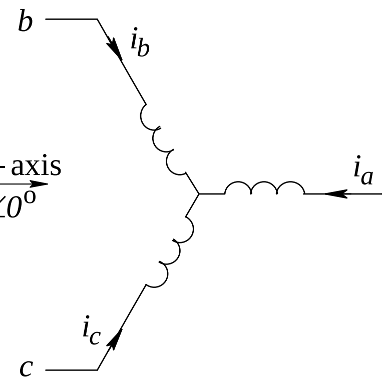
Radial magnetomotive force

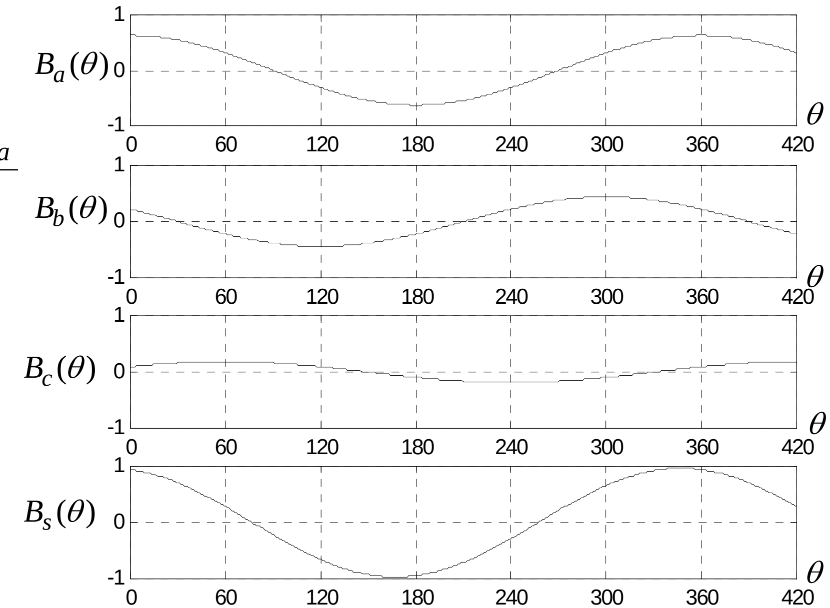


Field quantities have different magnitudes and units but same shape

Three-phase sinusoidally-distributed stator windings







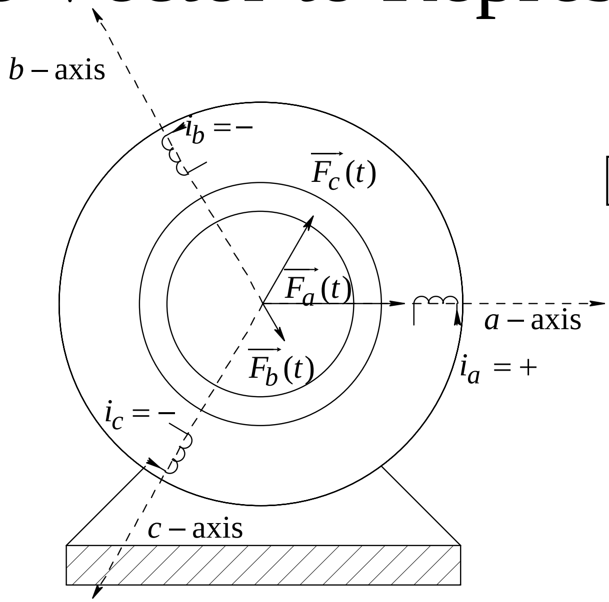
- Example 9-3: 2-pole, l_g = 1mm, i_a = 10 A, i_b = -7 A, i_c = -3 A, N_{sp} = 50.

$$B_a(\theta) = \frac{\mu_o N_s i_a}{2\ell_g} \cos\theta = 0.628 \cos\theta \ Wb/m^2$$

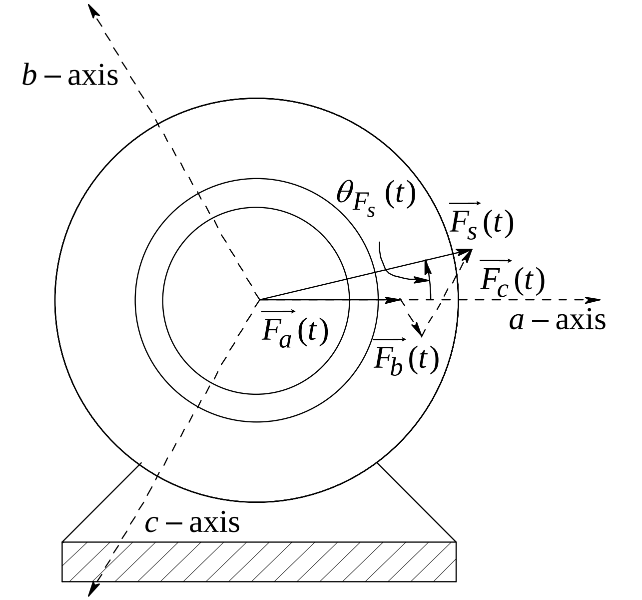
$$B_b(\theta) = -0.440 \times \cos\left(\theta - 120^0\right) \ Wb/m^2 \qquad B_c(\theta) = -0.188 \times \cos\left(\theta - 240^0\right) \ Wb/m^2$$

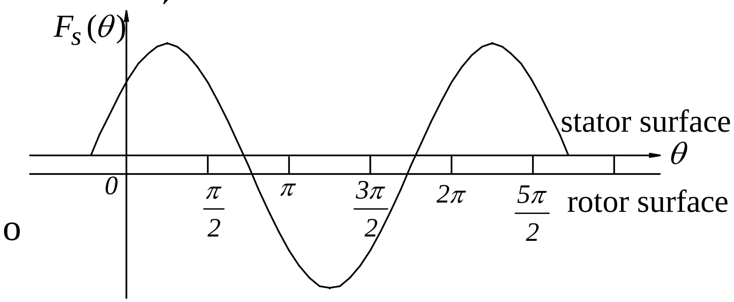
= combined stator-produced flux density

Space Vector to Represent Sinusoidal Distributions



At time ‘t’





- Complex number representation

$$F_a(\theta,t)=\frac{N_s}{2}\,i_a(t)\cos(\theta) \Leftrightarrow \vec{F}_a(t)=\frac{N_s}{2}\,i_a(t)\angle 0^o$$

Similarly, $\vec{F}_b(t)=\frac{N_s}{2}\,i_b(t)\angle 120^o$; $\vec{F}_c(t)=\frac{N_s}{2}\,i_c(t)\angle 240^o$

And

= resultant stator space vector for magnetomotive force

- Similar expressions for *B* and *H*

Example

Three-phase, sinusoidally-distributed stator with $\frac{N_s}{2} = 50$ turns

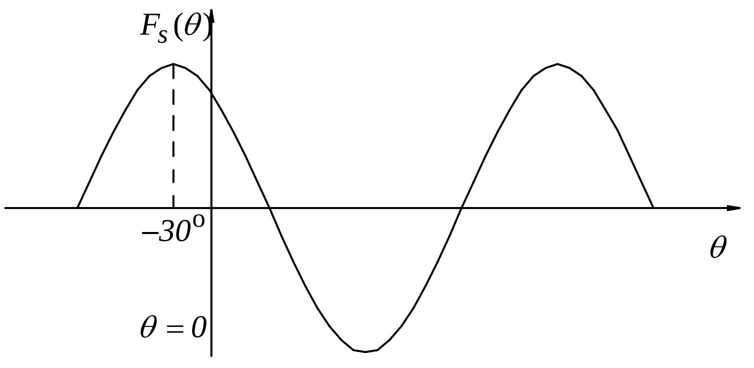
At time t, $i_a = 10A$, $i_b = -10A$ and $i_c = 0A$

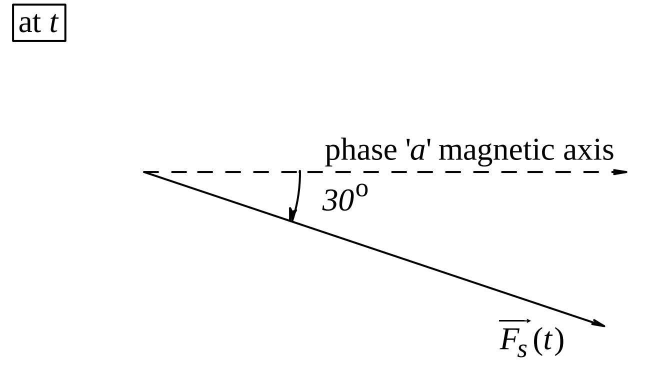
Find $\vec{F}_s$

$$\vec{F}_s(t) = \frac{N_s}{2}\left(i_a\angle 0^0 + i_b\angle 120^0 + i_c\angle 240^0\right)$$

$$= 50\left\{10 + (-10)\,[\cos 120^0 + j\sin 120^0] + (0)\,[\cos 240^0 + j\sin 240^0]\right\}$$

$$\vec{F}_s(t) = 50 \times 17.32\angle -30^{\mathrm{o}} = 866\angle -30^{\mathrm{o}}\ A\cdot\text{turns}$$

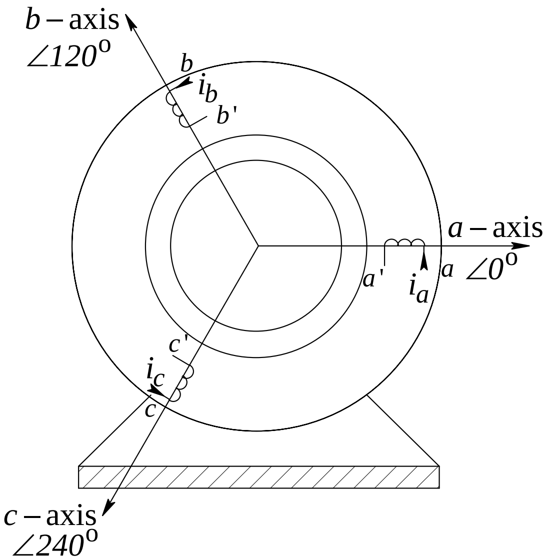




Space Vectors Representation of Combined Phase Currents and Voltages

r Mathematical concept

At time t



$$= stator\ current\ space\ vector$$

$$\vec{v}_s(t) = v_a(t)\angle 0^0 + v_b(t)\angle 120^0 + v_c(t)\angle 240^0$$

$$= \hat{V}_s(t)\angle\theta_{v_s}(t)$$

$$= stator\ voltage\ space\ vector$$

Physical interpretation of $\vec{i}_s(t)$

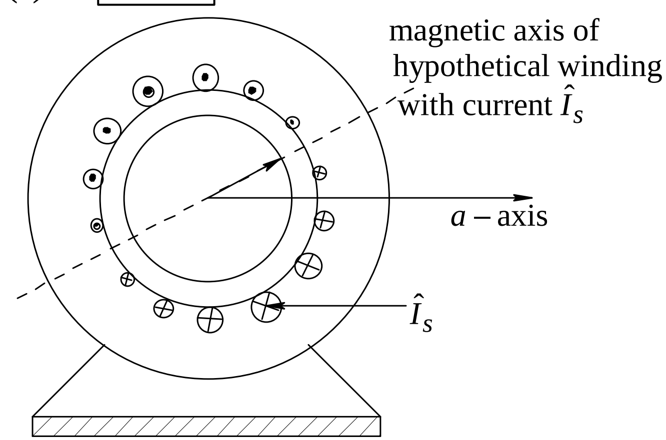
$$\frac{N_s}{2}\vec{i}_s(t) = \underbrace{\frac{N_s}{2} i_a(t)\angle 0^0}_{\vec{F}_a(t)} + \underbrace{\frac{N_s}{2} i_b(t)\angle 120^0}_{\vec{F}_b(t)} + \underbrace{\frac{N_s}{2} i_c(t)\angle 240^0}_{\vec{F}_c(t)} = \vec{F}_s(t)$$

$\boxed{\text{at time } t}$

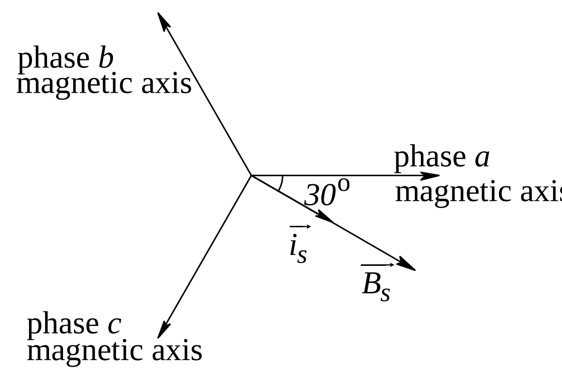
$$\vec{i}_s(t) = \frac{\vec{F}_s(t)}{N_s/2} \quad \Rightarrow \quad \hat{I}_s(t) = \frac{\hat{F}_s(t)}{N_s/2}$$

and $\theta_{i_s}(t) = \theta_{F_s}(t)$

$\vec{F}_s(t)$ and $\vec{i}_s(t)$ are collinear



- Magnetic field is produced by combined effect of i_a, i_b and i_c but could equivalently be produced by hypothetical winding current $\vec{i}_s(t)$ at θ_{i_s}
- helps in obtaining expression for torque



Space Vector Components:

Finding Phase Currents from Current Space Vector

$$\mathrm{Re}\left[\vec{i}_s\angle 0^0\right] = i_a + \underbrace{\mathrm{Re}\left[i_b\angle 120^0\right]}_{-\frac{1}{2}i_b} + \underbrace{\mathrm{Re}\left[i_b\angle 240^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_a$$

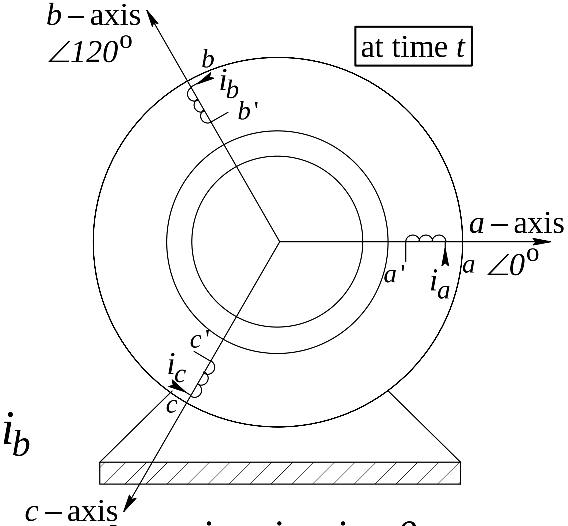
$$\Rightarrow i_a(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle 0^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\theta_{i_s}$$

$$\mathrm{Re}\left[\vec{i}_s\angle -120^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -120^0\right]}_{-\frac{1}{2}i_a} + i_b + \underbrace{\mathrm{Re}\left[i_c\angle 120^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_b$$

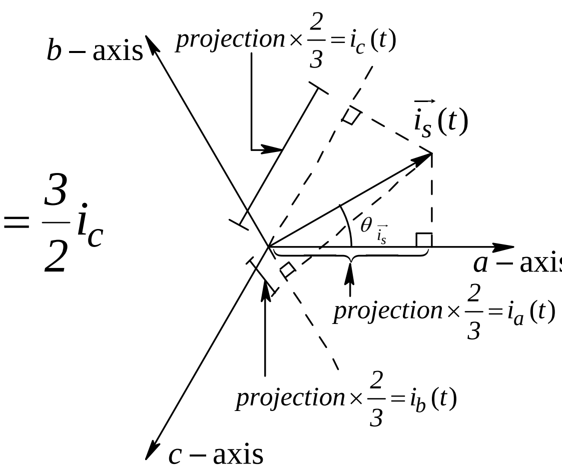
$$\Rightarrow i_b(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle -120^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 120^{\mathrm{o}}\right)$$

$$\mathrm{Re}\left[\vec{i}_s\angle -240^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -240^0\right]}_{-\frac{1}{2}i_a} + \underbrace{\mathrm{Re}\left[i_b\angle -240^0\right]}_{-\frac{1}{2}i_b} + i_c = \frac{3}{2}i_c$$

$$\Rightarrow i_c(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle -240^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 240^{\mathrm{o}}\right)$$

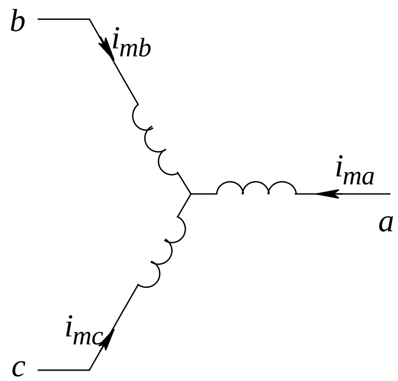


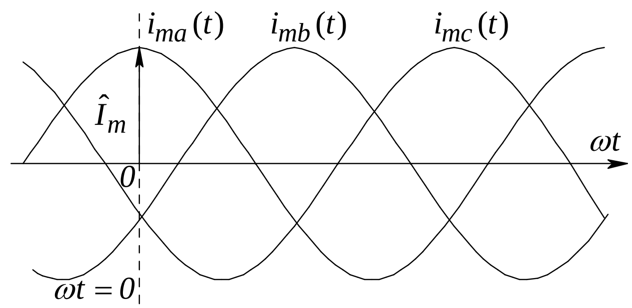
$$i_a + i_b + i_c = 0$$

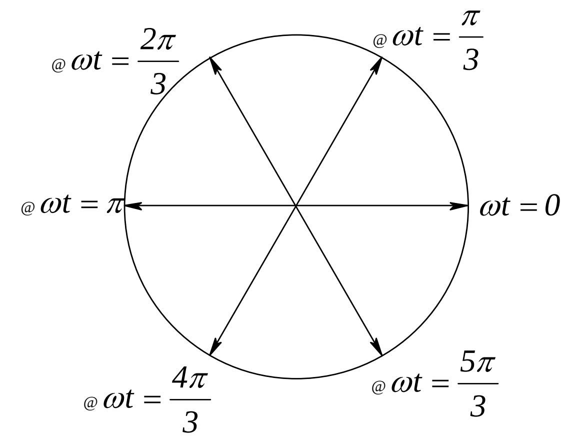


Balanced Sinusoidal Steady-State Excitation

(Rotor Open-Circuited – neglect stator winding resistance and leakage inductance)







$$\Rightarrow \overrightarrow{i_{ms}}(t) = \hat{I}_{ms} \angle \omega t \qquad \text{where} \quad \hat{I}_{ms} = \frac{3}{2}\hat{I}_m$$

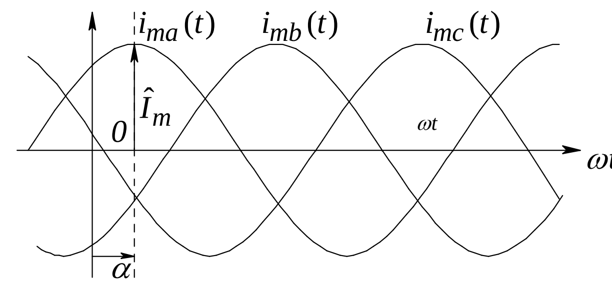
r Rotating MMF $\overrightarrow{F_{ms}}(t) = \frac{N_s}{2}\vec{i}_s(t) = \hat{F}_{ms}\angle\omega t$ where $\hat{F}_{ms} = \frac{3}{2}\frac{N_s}{2}\hat{I}_m = \frac{N_s}{2}\hat{I}_{ms}$

& Flux density $\overrightarrow{B_{ms}}(t) = \left(\frac{\mu_o}{\ell_g}\right)\frac{N_s}{2}\overrightarrow{i_{ms}}(t)$

r Constant amplitude

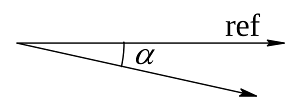
Relation Between Space Vectors and Phasors

u Time domain



$\omega t = 0$

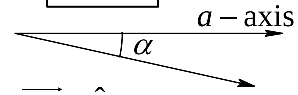
u Phasor



$\bar{I}_{ma} = \hat{I}_m \angle -\alpha$

u Space Vector

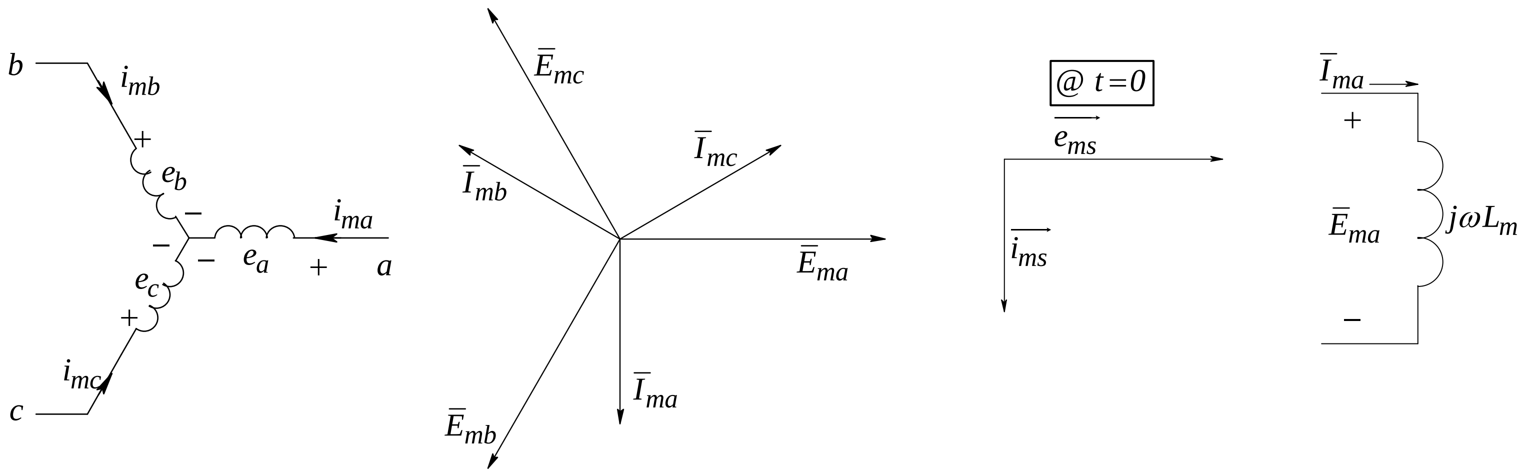
$\boxed{@\ t = 0}$



$\vec{i}_{ms} = \hat{I}_{ms} \angle -\alpha$

u Space Vector $\Leftrightarrow$ phasor

Voltages in the stator windings



Where the three phase magnetizing inductance (2 pole), $L_m = \frac{3}{2}\frac{\pi\mu_o r l}{l_g}\left(\frac{N_s}{2}\right)^2$

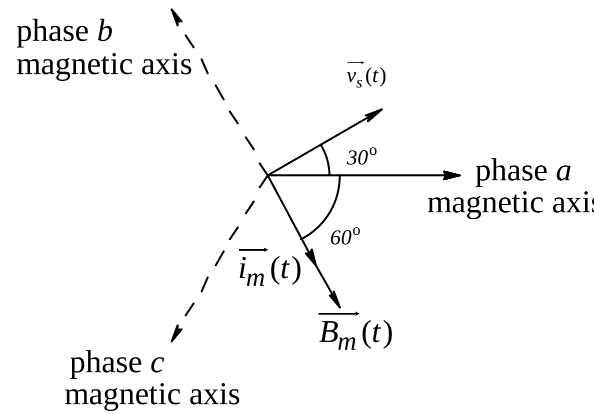
Example

$$v_a(t) = 120\sqrt{2}\cos\omega t$$

$$v_b(t) = 120\sqrt{2}\cos(\omega t - 120°)$$

$$v_c(t) = 120\sqrt{2}\cos(\omega t - 240°)$$

$$\vec{v}_s = \frac{3}{2} \times 120\sqrt{2}\angle 30^0 = 254.56\angle 30^0 \ V$$



$$\vec{i}_{ms} = \frac{\vec{v}_s}{j\omega L_m} = \frac{254.56\angle(30^0 - 90^0)}{2\pi \times 60 \times 0.777} = 0.869\angle -60^0 \ A$$

$$\vec{B}_{ms} = \frac{\mu_o N_s \vec{i}_{ms}}{2\ell_g} = \frac{4\pi \times 10^{-7} \times 50 \times 0.869\angle -60^0}{10^{-3}} = 0.055\angle -60^0 \ Wb/m^2$$