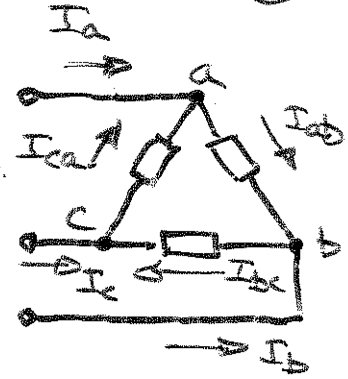
Q.3

PHASE CURRENTS :-

$$\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 20\angle -90^\circ \\ 15\angle 90^\circ \end{bmatrix} A$$



THE SEQUENCE COMPONENTS OF THE PHASE CURRENTS ARE

$$\begin{bmatrix} I_{ph0} \\ I_{ph1} \\ I_{ph2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

$$= \begin{bmatrix} 3.73\angle -26.6^\circ \\ 13.46\angle 3.5^\circ \\ 6.82\angle 172.9^\circ \end{bmatrix} A \ *$$

THE LINE CURRENTS ARE

$$I_a = I_{ab} - I_{ca} = 18.03\angle -56.3^\circ A$$

$$I_b = I_{bc} - I_{ab} = 22.36\angle -116.5^\circ A$$

$$I_c = I_{ca} - I_{bc} = 35.00\angle 90.0^\circ A$$

Q.6 (contd)

THE SEQUENCE COMPONENTS OF THE LINE CURRENTS ARE

$$\begin{bmatrix} I_{L0} \\ I_{L1} \\ I_{L2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 23.32 \angle -26.5^\circ \\ 11.82 \angle -157.0^\circ \end{bmatrix} A \ *$$

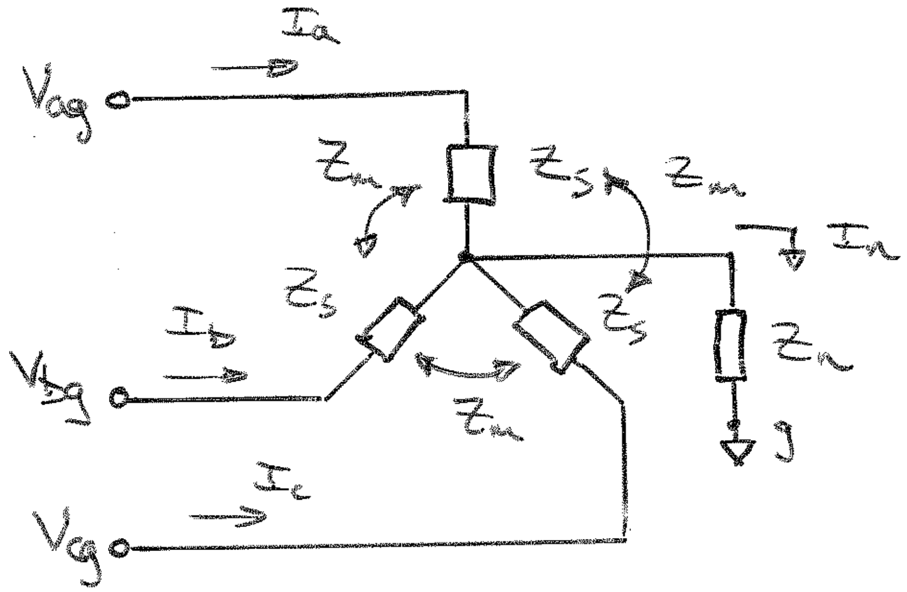
NOTE THAT

$$I_{L0} = 0 \ I_{Ph0}$$

$$I_{L1} = \sqrt{3}\, e^{-j30^\circ} I_{Ph1}$$

$$I_{L2} = \sqrt{3}\, e^{j30^\circ} I_{Ph2}$$

Q. 1.



BY K.V.L. APPLIED TO THE THREE PHASES

$$V_{ag} = I_a Z_s + I_b Z_m + I_c Z_m + I_n Z_n$$

$$V_{bg} = I_a Z_m + I_b Z_s + I_c Z_m + I_n Z_n$$

$$V_{cg} = I_a Z_m + I_b Z_m + I_c Z_s + I_n Z_n$$

BUT SINCE $I_n = I_a + I_b + I_c$ BY K.C.L APPLIED TO THE STAR POINT, WE GET

$$V_{ag} = I_a(Z_s + Z_n) + I_b(Z_m + Z_n) + I_c(Z_m + Z_n)$$

$$V_{bg} = I_a(Z_m + Z_n) + I_b(Z_s + Z_n) + I_c(Z_m + Z_n)$$

$$V_{cg} = I_a(Z_m + Z_n) + I_b(Z_m + Z_n) + I_c(Z_s + Z_n)$$

IN MATRIX FORM

$$\bar{V}_p = \bar{Z}_p \bar{I}_p$$

WHERE

$$\bar{V}_p = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \qquad \bar{I}_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

AND

$$\bar{Z}_p = \begin{bmatrix} (Z_s + Z_n) & (Z_m + Z_n) & (Z_m + Z_n) \\ (Z_m + Z_n) & (Z_s + Z_n) & (Z_m + Z_n) \\ (Z_m + Z_n) & (Z_m + Z_n) & (Z_s + Z_n) \end{bmatrix}$$

TRANSFORMING TO THE SEQUENCE DOMAIN

$$\bar{A}\bar{V}_s = \bar{Z}_p \bar{A} \bar{I}_s$$

$$\Rightarrow \bar{V}_s = (\bar{A}^{-1} \bar{Z}_p \bar{A}) \bar{I}_s$$

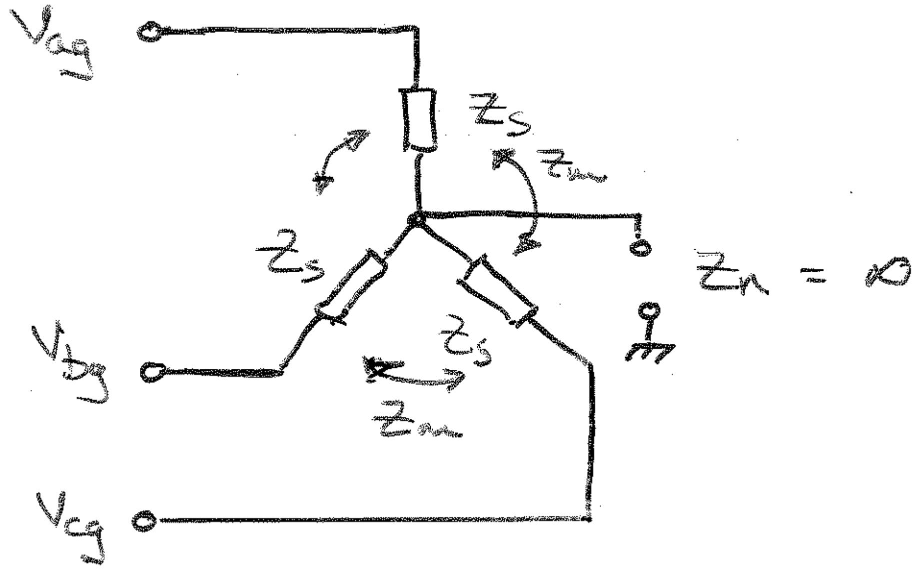
$$= \bar{Z}_s I_s$$

HENCE, SHOW THAT

$$Z_s = \begin{bmatrix} (\bar{Z}_s + 2\bar{Z}_m + 3\bar{Z}_n) & 0 & 0 \\ 0 & (Z_s - Z_m) & 0 \\ 0 & 0 & (Z_s - Z_m) \end{bmatrix}$$

NOTE THAT Z_n APPEARS ONLY IN THE ZERO SEQUENCE NETWORK.

Q.2.



$$\bar{Z}_s = j\,12\,\Omega$$

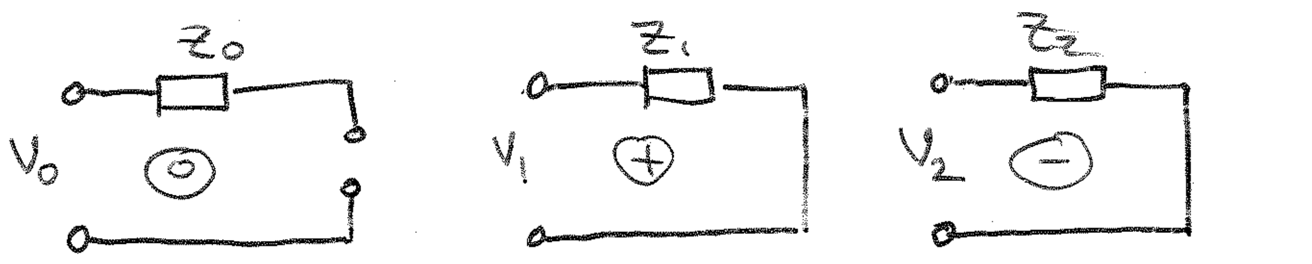
$$\bar{Z}_m = j\,4\,\Omega.$$

FOR BALANCED INPUT VOLTAGES,

$$V_{ag} = \frac{V_{LL}}{\sqrt{3}} = 230\angle 0^\circ$$

TAKING THE PHASE VOLTAGE AS REFERENCE.

FROM PROBLEM 1, THE SEQUENCE IMPEDANCE NETWORKS ARE AS FOLLOWS :-



$$Z_0 = Z_s + 2Z_m + 3Z_n$$

$$Z_1 = Z_s - Z_m$$

$$Z_2 = Z_s - Z_m$$

Q. 2 (contd)

IN THIS CASE $Z_n \to \infty$ GIVING AN OPEN CIRCUIT IN THE ZERO SEQUENCE NETWORK.

SINCE THE INPUT SOURCE VOLTAGE IS BALANCED

$$V_1 = V_{ag} = 230\angle 0^\circ \text{ V}$$

$$V_2 = 0 \text{ V}$$

$$V_0 = 0 \text{ V}$$

NOW,

$$Z_s - Z_m = j12 - j4 = j8\ \Omega$$

HENCE, THE SEQUENCE CURRENTS ARE

$$I_0 = 0$$

$$I_1 = \frac{230}{j8} = -j\,28.75 \text{ A}$$

$$I_2 = 0$$

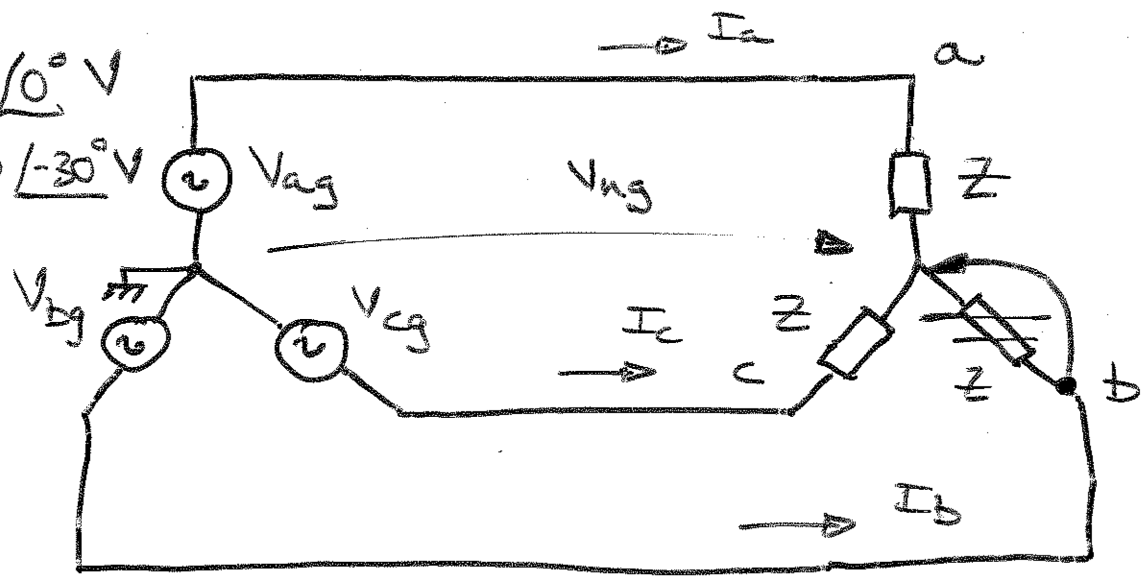
THUS, THE ACTUAL CURRENTS FORM A BALANCED THREE-PHASE SET

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 28.75\angle -90^\circ \\ 28.75\angle -210 \\ 28.75\angle 30^\circ \end{bmatrix} \text{A} \ \cancel{\text{V}}$$

Q. 3 THE THREE-PHASE SYSTEM IS AS SHOWN BELOW. (5)

$V_{ab} = 400\angle 0^\circ$ V

$\Rightarrow V_{ag} = 230\angle -30^\circ$ V



USING K.V.L WE GET IN MATRIX FORM

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} V_{ng} \\ V_{ng} \\ V_{ng} \end{bmatrix}$$

TRANSFORMING TO THE SEQUENCE DOMAIN

$$\begin{bmatrix} 0 \\ 230\angle -30^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_{ng} \\ 0 \\ 0 \end{bmatrix}$$

SINCE $I_n = 3I_0 = 0$ THEN $I_0 = 0$ AND WE CAN SOLVE FOR I_1 AND I_2 FROM THE 2 X 2 MATRIX EQUATION

$$\begin{bmatrix} 230\angle{-30^\circ} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

SO THAT

$$230\angle{-30^\circ} = Z_{11} I_1 + Z_{12} I_2$$
$$0 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow \quad I_2 = -\frac{Z_{21} I_1}{Z_{22}}$$

AND SO

$$230\angle{-30^\circ} = \left[Z_{11} - Z_{12}\left(\frac{Z_{21}}{Z_{22}}\right) \right] I_1$$

$$\Rightarrow \quad I_1 = \frac{230\angle{-30^\circ}}{Z_{11} - \left(\frac{Z_{21}}{Z_{22}}\right) Z_{12}}$$

THE SEQUENCE IMPEDANCE MATRIX IS GIVEN AS USUAL BY

$$\bar{Z}_S = A^{-1} Z_P \bar{A}$$

WHERE

$$Z_P = \begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix}$$

Q.3

HENCE

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z & Z & Z \\ 0 & 0 & 0 \\ Z & aZ & a^2Z \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2Z & Z+aZ & Z+a^2Z \\ Z+a^2Z & 2Z & Z+aZ \\ Z+aZ & Z+a^2Z & 2Z \end{bmatrix}$$

THUS, USING THE USUAL NOTATION

$$Z_S = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix}$$

WE GET

$$Z_{11} = Z_{22} = 2Z = \frac{2}{3}(18 + j6)$$

$$= (12 + j4)\ \Omega$$

$$Z_{12} = \frac{Z}{3}(1+a) = (1.268 + j6.196)\ \Omega$$

$$Z_{21} = \frac{Z}{3}(1+a^2) = (4.732 - j4.196)\ \Omega$$

Q.3.

HENCE, FROM ABOVE

$$I_1 = \frac{230\angle -30^\circ}{Z_{11} - Z_{12}\left(\frac{Z_{21}}{Z_{22}}\right)} = 24.3\angle -48.4^\circ \text{ A}$$

$$I_2 = -\left(\frac{Z_{21}}{Z_{22}}\right) I_1 = 12.2\angle 71.6^\circ \text{ A}$$

AND SO

$$I_a = I_0 + I_1 + I_2$$

$$= 21.1\angle -18.4^\circ \text{ A}.$$

ALTERNATIVE METHOD!

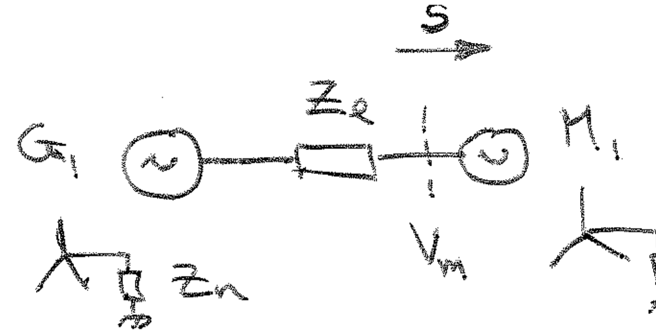
LINE VOLTAGE V_{ab} IS CONNECTED DIRECTLY ACROSS THE IMPEDANCE IN PHASE a SO THAT

$$I_a = \frac{V_{ab}}{Z}$$

$$= \frac{400\angle 0}{18 + j6}$$

$$= 21.1\angle -18.4^\circ \text{ A}. \ !$$

Q.4.



$Z_{0g} = j5\,\Omega$

$Z_{1g} = j15\,\Omega$

$Z_{2g} = j10\,\Omega$

$Z_{ng} = j5\,\Omega$

$Z_{0m} = j5\,\Omega$

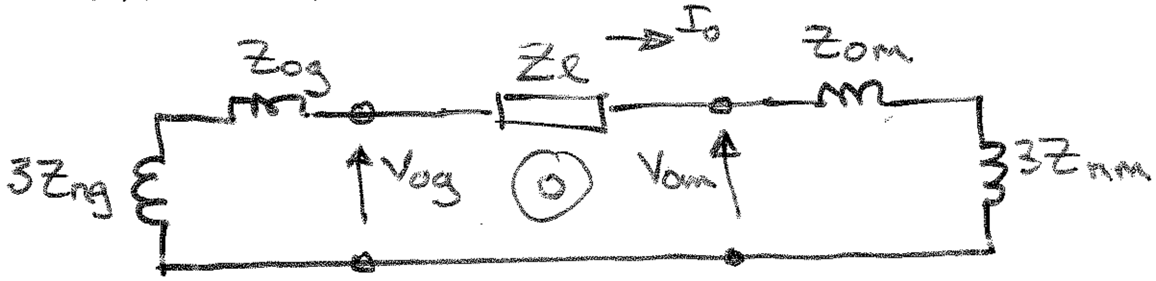
$Z_{1m} = j15\,\Omega$

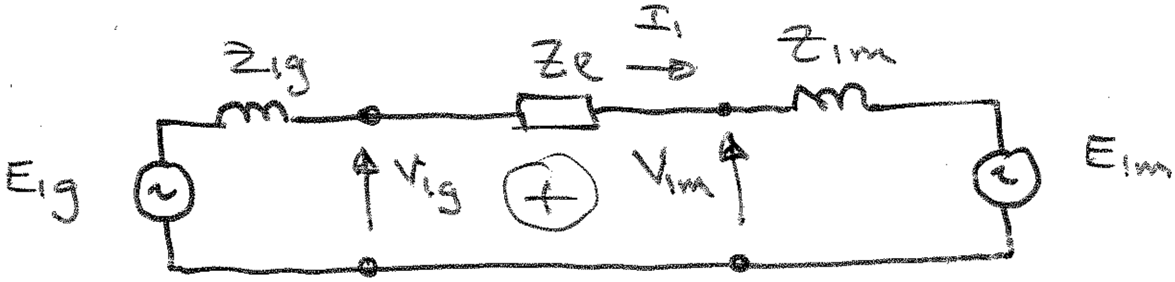
$Z_{2m} = j10\,\Omega$

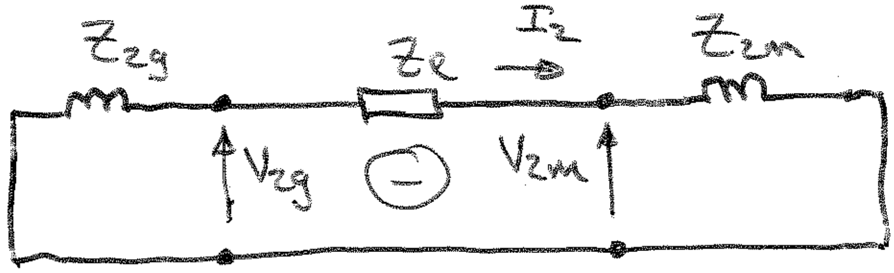
$Z_{nm} = j5\,\Omega$

$$Z_l = 0.5\angle 80^\circ\ \Omega$$

THE SEQUENCE NETWORKS ARE AS FOLLOWS







NOW

$$P = 10\ \text{kW}$$

$$pf = 0.8 \ \text{leading}$$

$$\Rightarrow S = \frac{P}{pf} = 12.5\ \text{kVA}$$

$$\Rightarrow Q = \sqrt{S^2 - P^2} = 7.5\ \text{kVA}$$

Q.4. (contd) $V_{1m} = \frac{400}{\sqrt{3}} \angle 0^\circ$

$$= 230 \angle 0^\circ$$

$$S = P - jQ = 3VI^*$$

$$\Rightarrow I = \left(\frac{P - jQ}{3V}\right)^*$$

$$= 18.1 \angle +36.87^\circ \text{ A}$$

SINCE THE VOLTAGES ARE BALANCED AND THE MACHINES AND THE TRANSMISSION LINE ARE SYMMETRICAL LOADS,

$$I_0 = 0$$

$$I_1 = 18.1 \angle +36.87^\circ \text{ A}$$

$$I_2 = 0$$

SO THAT

$$V_{1g} = V_{1m} + Z_\ell I_1$$

$$\Rightarrow V_{1g} = 230\angle 0^\circ + (0.5 \angle 80^\circ)(18.1 \angle 36.87^\circ)$$

$$= 224.9 \angle 1.89^\circ \text{ V.}$$

$$\Rightarrow V_{1g\,LL} = 389.6 \text{ V.}$$