Fourth Year Electrical Engineering

EE4010

Electrical and Electronic Power Supply Systems

Transformer Worked Examples

Example 1

The schematic diagram of a three-phase radial transmission system is shown in Figure 1 below. The ratings and the impedances of the various components are as shown along with the nominal transformer line voltages. A load of 50 MW at 0.8 power factor lagging is taken from the 33 kV load busbar which is to be maintained at a line voltage of 30 kV. Calculate the terminal voltage of the synchronous generator.

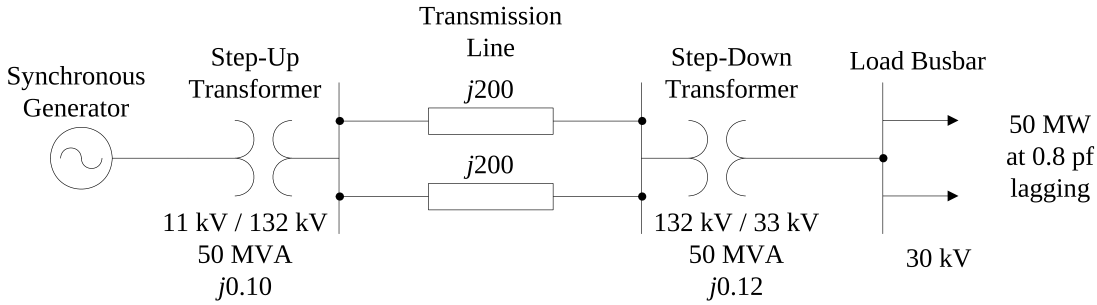


Figure 1

Solution 1

Select 11 kV, 132 kV and 33 kV as the base line-to-line voltages in the generator, transmission line and load zones respectively as determined by the transformer voltages. Select a base of 100 MVA. The reactances of the transformers are expressed on the corresponding rated voltages and volt-amperes. The base impedance for the line is

$$Z_{B\,line} = \frac{\left(132 \times 10^3\right)^2}{100 \times 10^6} = 174\ \Omega$$

Hence, the per-unit reactance of the line is

$$Z_{pu\,line} = \frac{j200 \,//\, j200}{174} = \frac{j100}{174} = \text{ j0.575 pu}$$

The per-unit reactance of the step-up transformer is

$$Z_{pu\,step-up} = \frac{100}{50} \times j0.1 = \text{ j0.2 pu}$$

The per-unit reactance of the step-down transformer is

$$Z_{pu\,step-down} = \frac{100}{50} \times j0.12 = \text{ j0.24 pu}$$

The actual load current is given by

$$I_{load} = \frac{50 \times 10^6}{\sqrt{3} \times 30 \times 10^3 \times 0.8} = \text{ 1203 A}\,.$$

Note that this formula involves the operating power factor since the load power is specified in MW. Note also that it is the *actual* operating voltage which is used in the calculation.

The base current in the load zone is given by

$$I_{base} = \frac{100 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 1750 \text{ A} .$$

Hence, the per-unit load current is given by

$$\bar{I}_{pu\ load} = \frac{1203}{1750} \angle -36.87° = 0.687 \angle -36.87° \text{ pu} .$$

The per unit load voltage is

$$V_{pu\ load} = \frac{30}{33} \angle 0° = 0.91 \angle 0° \text{ pu} .$$

Note that the load voltage is taken as the reference phasor.

Hence, the per-unit value of the voltage at the terminals of the synchronous generator is given by

$$\begin{aligned} \bar{V}_{pu\ generator} &= 0.91 \angle 0° + (0.687 \angle -36.87°)(j0.2 + j0.575 + j0.24) \text{ pu} \\ &= 1.44 \angle 22.78° \text{ pu} \end{aligned}$$

Thus, the magnitude of the actual line voltage at the generator terminals is given by

$$\begin{aligned} V_{generator} &= 11 \times 10^3 \times 1.44 \text{ V} \\ &= 15.84 \text{ kV.} \end{aligned}$$

Example 2

A bank of three single-phase transformers steps up the 13.8 kV line-to-line voltage of a three-phase synchronous generator to a required three-phase transmission line voltage of 138 kV. The generator rating is 41.5 MVA. Specify the voltage, current and MVA rating of each transformer for the following transformer bank connections:

(a) Low voltage – delta	High voltage - star
(b) Low voltage – star	High voltage - delta
(c) Low voltage – star	High voltage - star
(d) Low voltage – delta	High voltage - delta

Solution 2

(a)

$$V_{primary} = 13.8 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83\times10^6}{13.8\times10^3} = 1002.2 \text{ A}$$

$$V_{\mathrm{sec}ondary} = \frac{138}{\sqrt{3}} = 79.674 \text{ kV}$$

$$S_{\mathrm{sec}ondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{\mathrm{sec}ondary} = \frac{13.83\times10^6}{79.67\times10^3} = 173.62 \text{ A}$$

(b)

$$V_{primary} = \frac{13.8}{\sqrt{3}} = 7.97 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83\times10^6}{7.97\times10^3} = 1736.2 \text{ A}$$

$$V_{\mathrm{sec}ondary} = 138 \text{ kV}$$

$$S_{\mathrm{sec}ondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{\mathrm{sec}ondary} = \frac{13.83\times10^6}{138\times10^3} = 100.24 \text{ A}$$

(c)

$$V_{primary} = \frac{13.8}{\sqrt{3}} = 7.97 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{7.97 \times 10^3} = 1736.2 \text{ A}$$

$$V_{secondary} = \frac{138}{\sqrt{3}} = 79.67 \text{ kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{79.67 \times 10^3} = 173.62 \text{ A}$$

(d)

$$V_{primary} = 13.8 \text{ kV}$$

$$S_{primary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{primary} = \frac{13.83 \times 10^6}{13.8 \times 10^3} = 1002.4 \text{ A}$$

$$V_{secondary} = 138 \text{ kV}$$

$$S_{secondary} = \frac{41.5}{3} = 13.83 \text{ MVA}$$

$$I_{secondary} = \frac{13.83 \times 10^6}{138 \times 10^3} = 100.24 \text{ A}$$

Example 3

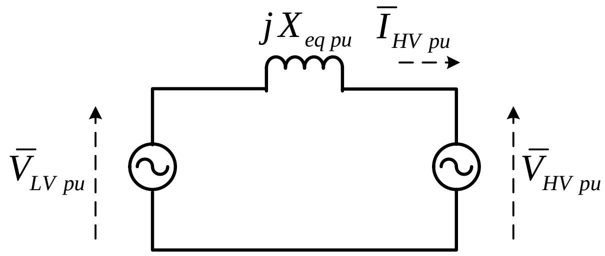
Three single-phase two-winding transformers, each rated at 400 MVA, 13.8 kV, 119.2 kV, with a leakage reactance of 10%, are connected to form a three-phase bank. The resistances of the windings and the excitation currents may be neglected. The high voltage windings are connected in star.

A three-phase load operating under balanced positive-sequence conditions on the high-voltage side draws 1000 MVA at 0.9 power factor lagging at a phase voltage of $\overline{V}_{AN} = 199.2\angle 0°$ kV.

Determine the voltage on the low voltage $\overline{V}_{an}$ on the low voltage busbar if the low-voltage windings are connected (i) in star and (ii) in delta.

Solution 3

For balanced operation, only the positive sequence equivalent network, as shown below, is required. In this diagram, $\overline{V}_{LV\,pu}$ and $\overline{V}_{HV\,pu}$ are the per-unit voltages at the low and high voltage busbars, respectively. The current $\overline{I}_{HV\,pu}$ is the per-unit current drawn by the HV busbar.



Select $S_{base-1phase} = 400$ MVA and $V_{HV\,base-phase} = 199.2$ kV based on the rating of the single-phase transformer and so $V_{HV\,base-line} = \sqrt{3}\times 199.2 \text{ kV} = 345 \text{ kV}$ and $S_{base-3phase} = 3\times 400 = 1200$ MVA. Also, for the star-star connection, $V_{LV\,base-phase} = 13.8$ kV and $V_{HV\,base-line} = \sqrt{3}\times 13.8 \text{ kV} = 23.9 \text{ kV}$.

Hence, the base current on the HV side is

$$I_{base-HV} = \frac{400\times 10^6}{199.2\times 10^3} = 2008 \text{ A}$$

The per-unit load current is then

$$\overline{I}_{HV} = \frac{1000\times 10^6}{\sqrt{3}\ 345\times 10^3}\angle -\text{Cos}^{-1}\left(0.9\right)° = 1673.5\angle -25.84° \text{ A}$$

so that

$$\overline{I}_{HV\text{ pu}} = \frac{1673.5\angle -25.84°}{2008} = 0.833\ \angle -25.84° \text{ pu}.$$

The per-unit voltage on the HV side is

$$\overline{V}_{HV\,pu} = \frac{\overline{V}_{HV}}{V_{base-phase}} = 1.0\angle 0°.$$

For the star-star connected transformer, the per-unit voltage on the low voltage busbars is given by

$$\begin{aligned}\bar{V}_{LV\ pu} &= \bar{V}_{HV\ pu} + \left(jX_{eq}\right)\left(\bar{I}_{HV\ pu}\right) \\ &= \ 1.0\angle 0^\circ + j0.1\times\left(0.833\ \angle -25.84^\circ\right) \\ &= 1.039\ \angle 4.139^\circ \text{ pu}\end{aligned}$$

Hence, the actual phase voltage on the low voltage busbars is given by

$$\begin{aligned}\bar{V}_{LV\,act} &= \bar{V}_{LV\ pu}\ V_{LV\ \ base-phase} \\ &= \ 1.039\ \angle 4.139^\circ\ \times\ 13.8 \text{ kV} \\ &= 14.34\ \angle 4.139^\circ \text{ kV.}\end{aligned}$$

In the star-delta case, there will be a 30° phase shift between the high voltage and the low voltage sides of the transformer. However, the per-unit voltage on the low voltage side is the same irrespective of the transformer connection since the per-unit equivalent circuit is the same in both cases if this phase shift is ignored. Hence, as before,

$$\begin{aligned}\bar{V}_{LV\ pu} &= \bar{V}_{HV\ pu} + \left(jX_{eq}\right)\left(\bar{I}_{HV\ pu}\right) \\ &= \ 1.0\angle 0^\circ + j0.1\times\left(0.833\ \angle -25.84^\circ\right) \\ &= \ 1.039\ \angle 4.139^\circ \text{ pu}\end{aligned}$$

but the actual phase voltage is now

$$\begin{aligned}\bar{V}_{LV\,act} &= \bar{V}_{LV\ pu}\ V_{LV\ \ base-phase} \\ &= \ 1.039\ \angle 4.139^\circ\ \times\ \frac{13.8}{\sqrt{3}} \text{ kV} \\ &= 8.278\ \angle 4.139^\circ \text{ kV.}\end{aligned}$$

due to the star-delta connection.

Problems

1. Two identical 10 MVA, 6.6 kV/33 kV, three-phase, star-star transformers are connected by a 33 kV underground cable. One transformer is located at a power station and the second at a remote substation. The series resistance of the cable per conductor is $7.17\ \Omega$ and the series inductive reactance is $2.0\ \Omega$.

   Each transformer has an equivalent series resistance of $0.04\ \Omega$ per phase and an equivalent leakage reactance of $0.4\ \Omega$ per phase, referred to the low voltage side.

   If 5 MW at 6.0 kV and 0.8 power factor lagging is delivered to a load on the low voltage busbars at the substation transformer, calculate the line voltage on the low voltage busbars at the power station.

   [6.575 kV]

2. A small power distribution system is fed by a generator which supplies power to a feeder line of impedance $(0.15 + j1.0)\ \Omega$ per phase. The line voltage of the generator is set at 4.16 kV. At its remote end, the feeder line supplies three single-phase, 50 Hz, 50 kVA, 2.4 kV/240 V transformers which are connected star/delta in a three-phase, 150 kVA bank to step down the voltage to that which is required by the load.

   On the secondary side, the transformer bank feeds a balanced three-phase load through a low voltage feeder line the impedance of which is $(0.0005 + j0.0020)\ \Omega$ per phase. Find the line voltage at the load when the load draws rated current from the transformers. The power factor of the load is 0.8 lagging, measured at the generator terminals.

   For each single-phase transformer, the short circuit test readings are 48 V, 20.8 A and 617 W when measured on the high voltage side.

   [233 V]

3. A three-phase transformer is connected in star on both the primary and secondary sides. The secondary is connected to a balanced three-phase load via a transmission line. Each transmission line conductor has a series impedance of $(4 + j6)\ \Omega$ per phase. The load has an equivalent per-phase series impedance of $(400 + j600)\ \Omega$. The secondary winding of the transformer has three times as many turns per phase as the primary.

   The parameters of the transformer are as follows:

   Primary impedance: $(0.5 + j2.5)\ \Omega$/phase

   Secondary impedance: $(5.0 + j25.0)\ \Omega$/phase.

   If the primary voltage is 11 kV, calculate the line voltage on the transformer secondary terminals.

   [31.1 kV]