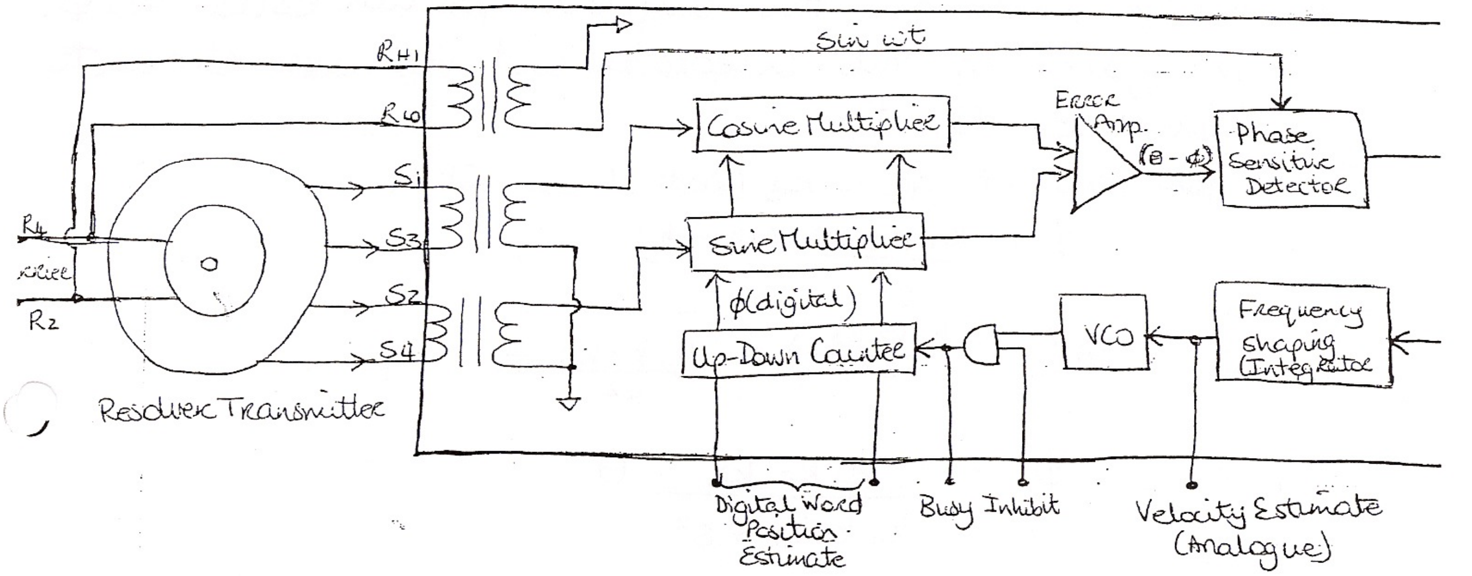
Summer 2003

Q 3 (a) RDC circuit:



$$v_{s1}(t) = V \sin \omega t \sin \theta$$

$$v_{s2}(t) = V \sin \omega t \cos \theta$$

Position Estimate $= \phi$

$$v_{err}(t) = k(v_{s1} \cos\phi - v_{s2} \sin\phi)$$
$$= kV \sin \omega t (\sin\theta \cos\phi - \cos\theta \sin\phi)$$
$$= kV \sin \omega t (\sin(\theta - \phi))$$

For small errors $\theta - \phi$, $\sin(\theta - \phi) \propto (\theta - \phi)$

$$\therefore v_{err}(t) \propto (\theta - \phi)$$

The resolver transmitter measures the position θ of the rotary shaft and $v_{s1}(t)$ and $v_{s2}(t)$ (as above) are used to determine the position error using the position estimate, The error is amplified and then demodulated from the carrier signal, ($\sin \omega t$) in the phase sensitive detector (PSD). The PSD output ~~is~~ undergoes integration in the frequency shaping block (k_1/s). This block supplies an analoge estimate of the velocity of the rotary shaft The velocity signal is the input of the voltage controlled oscillator which provides a pulsed output, with a pulse rate proportional to ~~the~~ its input voltage. The block acts as another integrator (k_2/s). The VCO output is used to clock an up-down counter ~~to~~ which produces the digital position estimate, ϕ. The gains of the integrator and VCO are chosen to minimise $\theta - \phi$.

for constant gain k_3 (rest of circuit):

$$\phi = \frac{k_1 k_2 k_3}{s^2}(\theta - \phi)$$

$$\phi\left(1 + \frac{k_1 k_2 k_3}{s^2}\right) = \frac{k_1 k_2 k_3}{s^2}\theta$$

$$\phi = \frac{k_1 k_2 k_3}{k_1 k_2 k_3 + s^2}\theta$$

(b) $q_1 = C_d \pi d_1 x (P_s - P_1)^{0.5}\left(\frac{2}{\rho}\right)^{0.5}$

$q_2 = C_d \pi d_1 x (P_2 - P_e)^{0.5}\left(\frac{2}{\rho}\right)^{0.5}$

Assuming P_s is constant, P_e is negligible and $q_1 = q_2 = q$ (ideal servo-valve), and letting $P_m = P_1 - P_2$,

$$q = C_d \pi d_1 x (P_s - P_m)^{0.5}\left(\frac{1}{\rho}\right)^{0.5} \approx 6.7\pi d_1 x (P_s - P_m)^{0.5}$$

Linearising $\Rightarrow$ $q \approx K_q x - K_c P_m$

where $K_q \approx 6.7\pi d_1 (P_s)^{0.5}$ $\qquad K_c \approx \dfrac{6.7\pi d_1 x (P_s)^{0.5}}{2 P_s}$

$$\therefore P_m = \frac{K_q x - q}{K_c}$$

Force on piston, $F = P_m A = \frac{A}{K_c}(K_q x - q)$ (1)

Flow Rate, $q = \frac{d(Vol)}{dt} = A\frac{dy}{dt}$ (2)

Load : $F = M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + F_L$ (3)

$$\therefore \frac{A}{K_c}\left(K_q x - A\frac{dy}{dt}\right) = M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + F_L$$

Laplace Transforms:

$$\frac{A K_q}{K_c} X(s) = \left(Ms^2 + \left(B + \frac{A^2}{K_c}\right)s\right)Y(s) + F_L(s)$$

$$\therefore \; Y(s) = \frac{A\, K_q/K_c}{Ms^2 + (B + A^2/K_c)s} X(s) \;\cancel{+} - \frac{F_L(s)}{Ms^2 + (B + A^2/K_c)s}$$

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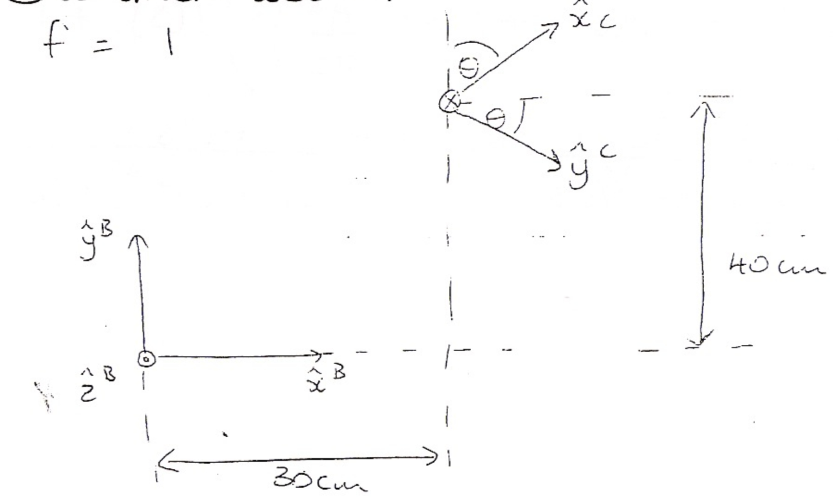
Q4 $f = 1$



$$T_B^{\ c} = \begin{bmatrix} \sin\theta & \cos\theta & 0 & 30 \\ \cos\theta & -\sin\theta & 0 & 40 \\ 0 & 0 & -1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_i^{\ B} = T_i^{\ c}\, T_c^{\ B} = T_i^{\ c}\left[T_B^{\ c}\right]^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/f & 1 \end{bmatrix}\begin{bmatrix} \sin\theta & \cos\theta & 0 & -30\sin\theta-4 \\ \cos\theta & -\sin\theta & 0 & -30\cos\theta+40 \\ 0 & 0 & -1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p^i = T_i^{\ B} p^B$$

$$\therefore\ A^i = T_i^{\ B} A \quad \text{and} \quad D^i = T_i^{\ B} D$$

$$\begin{bmatrix} 1.08743 \\ -0.25977 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 & -30S-40C \\ C & -S & 0 & -30C+40S \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/f & \frac{f-h}{f} \end{bmatrix}\begin{bmatrix} 20 \\ 20 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20S+20C-30S-40C \\ 20C-20S-30C+40S \\ 0 \\ \frac{f-h}{f} \end{bmatrix} \qquad \left(\frac{f-h}{f} = 1-h\right)$$

$$(1.08743)(1-h) = -10S-20C \rightarrow \cancel{-30C = 1.91509(1-h)}$$

$$\begin{bmatrix} 0.70441 \\ 0.06162 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S & C & 0 & -30S-40C \\ C & -S & 0 & -30C+40S \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1-h \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20S + 30C - 30S - 40C \\ 20C - 30S - 30C + 40S \\ 0 \\ 1-h \end{bmatrix}$$

$$(0.70441)(1-h) = -10S - 10C$$
$$(0.06162)(1-h) = -10C + 10S$$

$$\cancel{20C = (1-h)(0.76603)}$$

$$\cancel{1.91509(1-h) = \frac{0.76603(1-h)}{20}}$$

$$\cancel{A^B = T_B^C A^C \quad \therefore A^C = T_C^B A^B}$$

$$\cancel{A_x^i = -\frac{f}{\lambda - f} A_x}$$

$$\cancel{1.08743 = \frac{1}{1-\lambda}(20)}$$
$$\cancel{\therefore \lambda = -\frac{20}{1.08743} + 1 = -17.392}$$

$$\cancel{A^C = \begin{bmatrix} S & C & 0 \\ C & -S & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

$$\frac{-10S - 20C}{1.08743} = \frac{20S - 10C}{-0.25977}$$

$$\frac{S + 2C}{1.08743} = \frac{2S - C}{0.25977}$$

$$0.23888S + 0.47777C = 2S - C$$

$1.76112\,S = 1.47777\,C$

$$\frac{S}{C} = 0.8391$$

(b) $\therefore \theta = 40^\circ$

~~0.70~~

$$1-h = \frac{-10\sin 40 - 10\cos 40}{0.70441}$$

(a) $h = 21$ cm

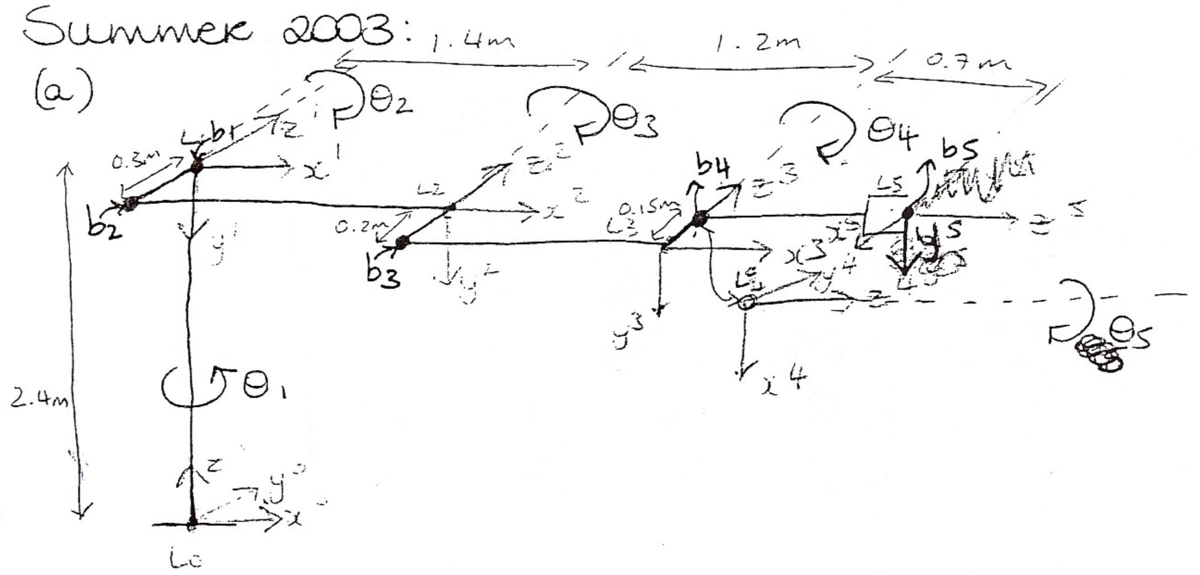
(c)

$$T_B{}^C = \begin{bmatrix} 0.6428 & 0.7660 & 0 & 30 \\ 0.7660 & -0.6428 & 0 & 40 \\ 0 & 0 & -1 & 21 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.26.

Summer 2003:

Q5 (a)



(b)

i	θ_i	d_i	a_i	α_i	Home, q_i
1	θ_1	2.4m	0	$-90°$	$0°$
2	θ_2	-0.3m	1.4m	$0°$	$0°$
3	θ_3	-0.2m	1.2m	$0°$	$0°$
4	θ_4	0.15m	0	$+90°$	$+90°$
5	θ_5	0.7m	0	$0°$	$-90°$

(c) $T_{base}^{tool} = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4 \, T_4^5$

where:

$$T_0^1 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & 2.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 1.4C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 1.4S\theta_2 \\ 0 & 0 & 1 & -0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 1.2C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & 1.2C\theta_3 \\ 0 & 0 & & -0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{\cancel{3}}^{4} = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^{5} = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & 0 \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) $$T_3^{4} = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Home Position : $\theta_4 = 90°$

$C\theta_4 = 0$

$S\theta_4 = 1$

$$T_3^{4} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x^4 \equiv y^3$ ✓

$y^4 \equiv z^3$ ✓

$\cancel{z}^4 \equiv x^3$ ✓

Offsets for L_4 from L_3:

0 in x^3 dirn ✓

0 in y^3 dirn ✓

+0.15 in z^3 dirn ✓

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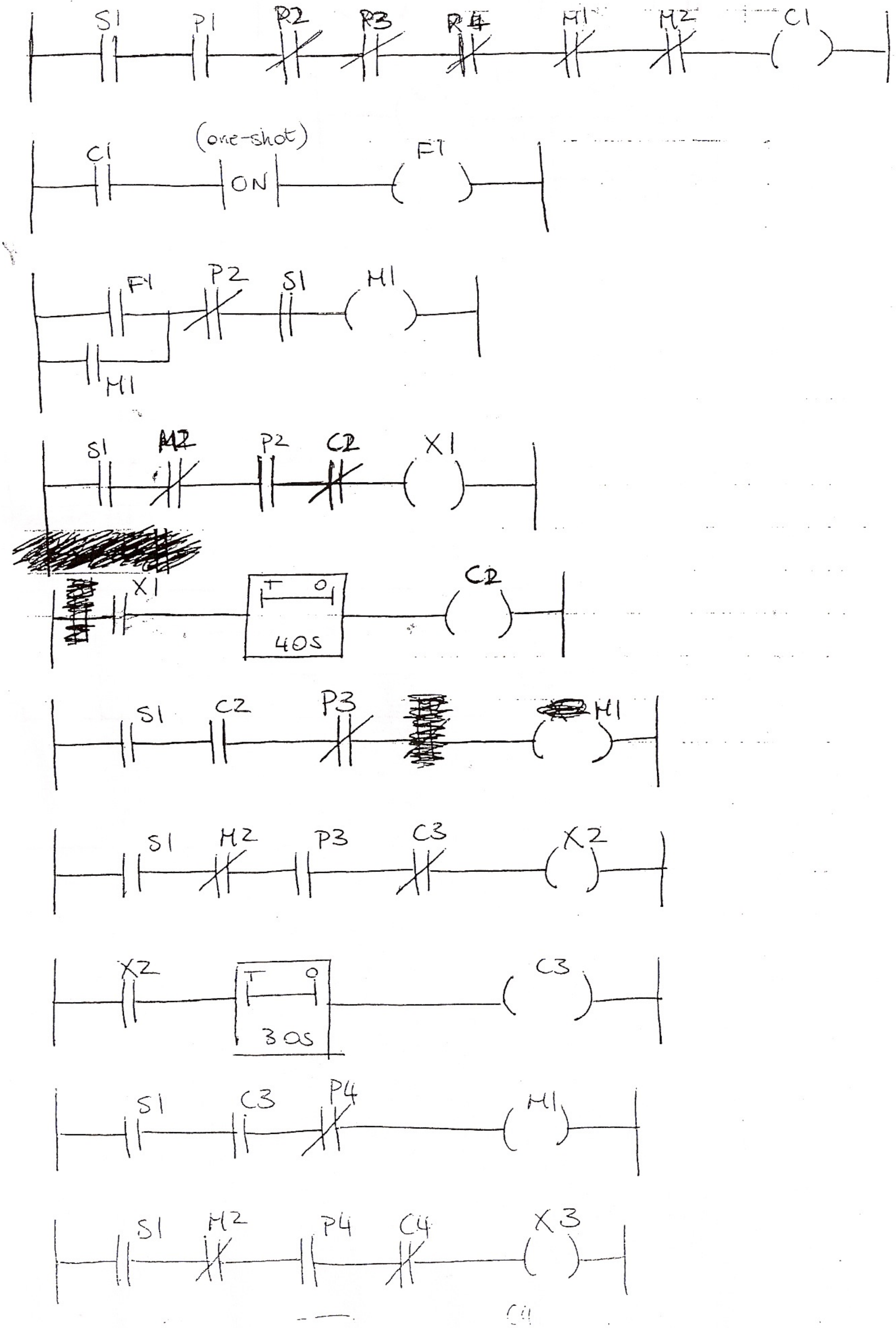
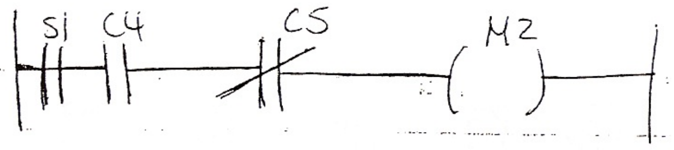
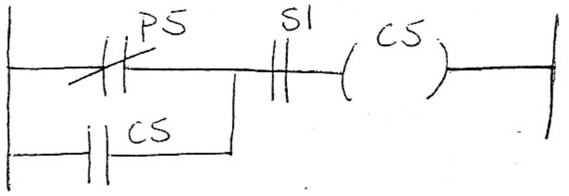
Q6.



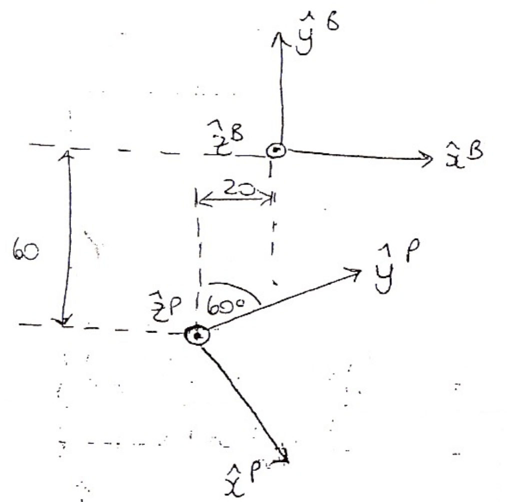

S1
C4
C5
M2


P5
S1
C5
C5

Summer 2003.

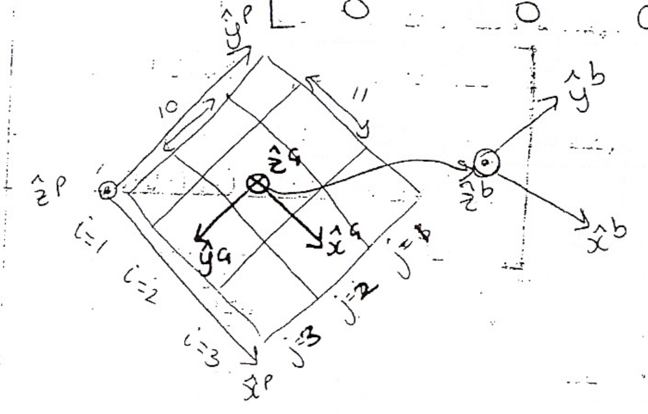
Q7

$$T_B^G = T_B^P \, T_P^b \, T_b^G$$



$$T_B^P = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ & 0 & -20 \\ -\sin 60^\circ & \cos 60^\circ & 0 & -60 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -20 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -60 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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$$\Rightarrow \quad T_P^b = \begin{bmatrix} 1 & 0 & 0 & 5.5 + 11(i-1) \\ 0 & 1 & 0 & 5 + 10(3-j) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_b^G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T_B^G = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 & -20 \\ -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -60 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5.5+11(i-1) \\ 0 & 1 & 0 & 5+10(3-j) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 & -20 \\ -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -60 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \begin{pmatrix} 0 & 5.5+11(i-1) \\ 0 \pm & 5+10(3-j) \\ 1 & 0 \end{pmatrix} \\ 0 & -1 & 0 & \\ 0 & 0 & -1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 0 & \begin{pmatrix} \frac{1}{\sqrt{2}}(5.5+11(i-1))+\frac{3}{\sqrt{2}}(5+10(3-j)) \\ -\frac{3}{\sqrt{2}}(5.5+11(i-1))+\frac{1}{\sqrt{2}}(5+10(3-j)) \\ 1 \end{pmatrix} + \begin{pmatrix} -20 \\ -60 \\ 0 \end{pmatrix} \\ -\frac{3}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \\ 0 & 0 & -1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_B^G = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\left(20.5+11(i-1)+30(3-j)\right)-20 \\ -\frac{3}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\left(-10.5-33(i-1)+10(3-j)\right)-60 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $$(T_B{}^P)_{new} = (T_B{}^P)_{old} \cdot Rot(\hat{y}^P, -45^\circ)$$

$$= (T_B{}^P)_{old} \begin{bmatrix} \cos(-45^\circ) & 0 & -\sin(-45^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-45^\circ) & 0 & \cos(-45^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} s & 0 \\ c & 0 \\ 0 & 1 \end{bmatrix} \Downarrow \begin{bmatrix} 0 & -s \\ 1 & 0 \\ 0 & c \end{bmatrix}$$

$$= (T_B{}^P)_{old} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow (T_B^G)_{new} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 & -20 \\ -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -60 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5.5+11(i-1) \\ 0 & 1 & 0 & 5+10(3-j) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \\ 0 & -1 & \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$