EE4011: RF IC Design

Introduction to Mixers

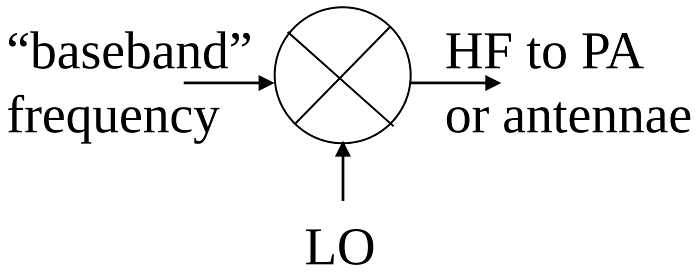
Mixers

Mixers (probably better described as multipliers) are used in RFICs for frequency translation - either as up-converters which take a low frequency signal and translate it to a higher frequency (HF) for transmission by an antenna or as down-converters which take a high frequency signal and translate it to a lower frequencies (often called the intermediate frequency, IF) which can be de-modulated directly or digitised with an ADC and subsequently processed.

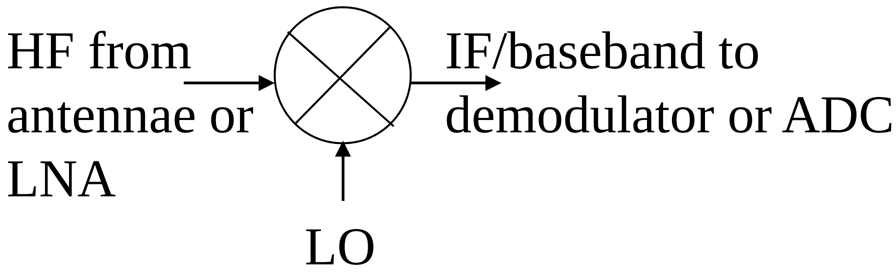
A mixer “multiplies” the incoming frequency with a stable local frequency source known as the Local Oscillator (LO).

Mixers rely on a *non-linear* property of a circuit component or configuration.

Up-conversion:

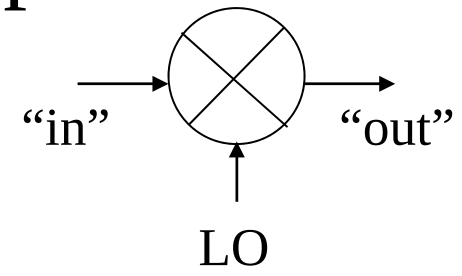


Down-conversion:



Mixer Specifications

Most mixers are 3-port devices:



Mixers have many performance specifications including:

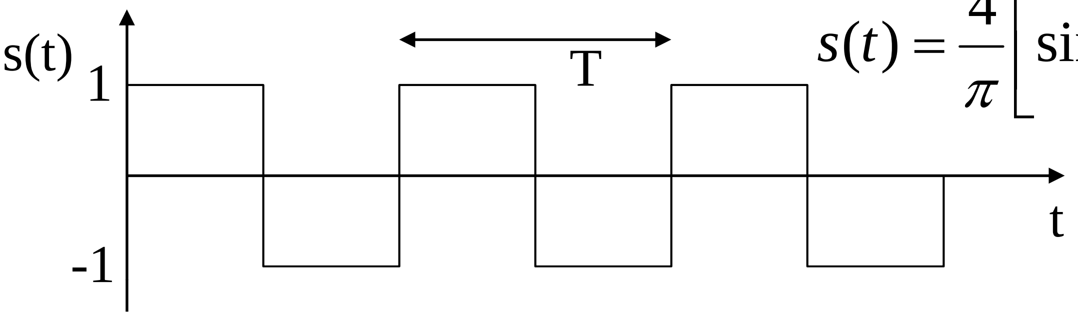
Frequency ranges	the technology has to work at the required frequencies
Gain	active mixers provide gain, passive mixers don’t
Noise Figure	a small NF is desirable
IIP_3	the mixer should cope with a wide range of signals from weak to strong so a high IIP_3 is desirable
Input/output impedance	adequate matching at the ports is important
Port-to-port isolation	unwanted feed-through from port-to-port must be minimized especially LO-to-“in” feed-through which could cause the LO signal to be radiated from a receiving antennae or LO-to-“out” which could cause the LO signal to be radiated from the transmitting antennae

and others…

A Suitable Non-linearity

The multiplication action required for frequency translation can be achieved with any non-linear component such as a diode, transistors etc. An idealised non-linearity switches instantly between two states at some threshold point. The operation of an idealised non-linearity can be represented by square waves in the time domain.

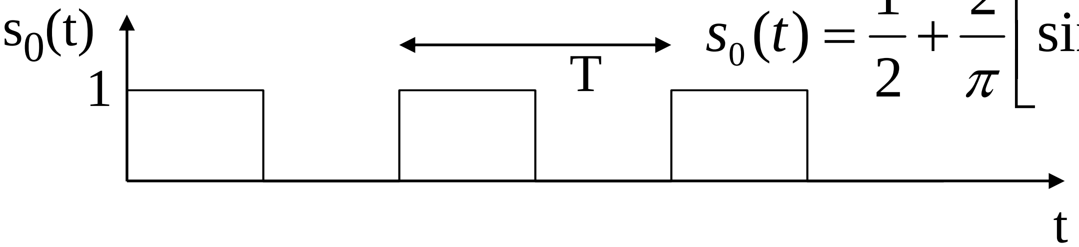
Fourier Series expansion of Square Wave toggling between +1 and -1



$$s(t) = \frac{4}{\pi}\left[\sin(\varpi t) + \frac{1}{3}\sin(3\varpi t) + \frac{1}{5}\sin(5\varpi t) + \cdots\right]$$

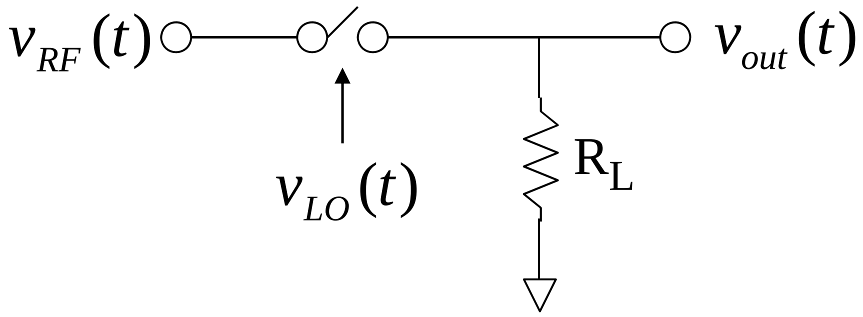
$$\varpi = 2\pi F = 2\pi\frac{1}{T}$$

Fourier Series expansion of Square Wave toggling between 1 and 0



$$s_0(t) = \frac{1}{2} + \frac{2}{\pi}\left[\sin(\varpi t) + \frac{1}{3}\sin(3\varpi t) + \frac{1}{5}\sin(5\varpi t) + \cdots\right]$$

A Mixer Based on a Switch



In this scenario the switch is assumed to be fully turned on for $v_{LO}(t) > 0$ and fully turned off otherwise. When the switch is turned on the output terminal is directly connected to the input terminal. When the switch is off, the output potential is pulled to ground by the load resistor R_L. The operation of the switch is therefore to "multiply" the input voltage by a square wave of frequency f_{LO} which toggles between 0 and 1 i.e.

$$v_{out}(t) = v_{RF}(t)s_0(t) = v_{RF}(t)\left[\frac{1}{2} + \frac{2}{\pi}\sin(\varpi_{LO}t) + \frac{2}{3\pi}\sin(3\varpi_{LO}t) + \frac{2}{5\pi}\sin(5\varpi_{LO}t) + \cdots\right]$$

Note, in this scenario, it is often assumed that the LO is a square wave so that the switch changes from "fully on" to "fully off" every half cyle of the LO waveform. However, a large amplitude sine wave will give almost the same effect in most cases.

Spectral Components in a Switch-Based Mixer

Letting the input have a co-sinusoidal form:

$$v_{out}(t) = v_{RF}(t)\left[\frac{1}{2} + \frac{2}{\pi}\sin(\varpi_{LO}t) + \frac{2}{3\pi}\sin(3\varpi_{LO}t) + \frac{2}{5\pi}\sin(5\varpi_{LO}t) + \cdots\right]$$

$$= V_{RF}\cos(\varpi_{RF}t)\left[\frac{1}{2} + \frac{2}{\pi}\sin(\varpi_{LO}t) + \frac{2}{3\pi}\sin(3\varpi_{LO}t) + \frac{2}{5\pi}\sin(5\varpi_{LO}t) + \cdots\right]$$

$$= \frac{1}{2}V_{RF}\cos(\varpi_{RF}t) + \frac{2}{\pi}V_{RF}\cos(\varpi_{RF}t)\sin(\varpi_{LO}t) + \frac{2}{3\pi}V_{RF}\cos(\varpi_{RF}t)\sin(3\varpi_{LO}t)$$

$$+ \frac{2}{5\pi}V_{RF}\cos(\varpi_{RF}t)\sin(5\varpi_{LO}t) + \cdots$$

$$\boxed{\cos A \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)}$$

$$v_{out}(t) = \frac{1}{2}V_{RF}\cos(\varpi_{RF}t) + \frac{V_{RF}}{\pi}\left[\sin\left((\varpi_{RF} + \varpi_{LO})t\right) - \sin\left((\varpi_{RF} - \varpi_{LO})t\right)\right]$$

$$+ \frac{V_{RF}}{3\pi}\left[\sin\left((\varpi_{RF} + 3\varpi_{LO})t\right) - \sin\left((\varpi_{RF} - 3\varpi_{LO})t\right)\right]$$

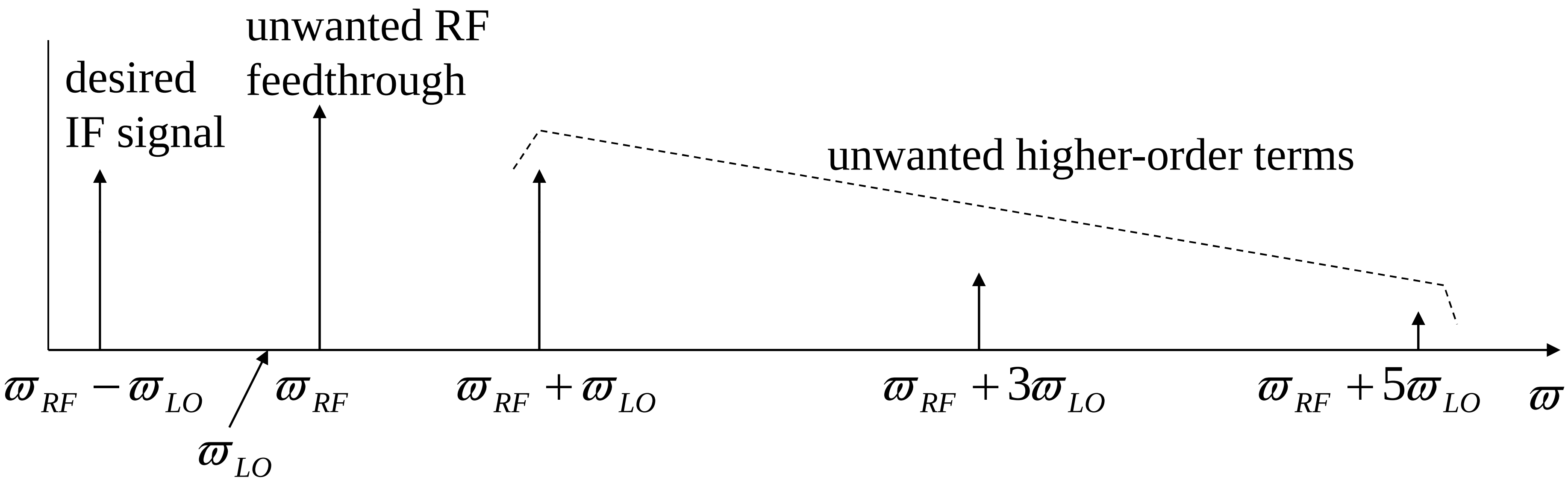
$$+ \frac{V_{RF}}{5\pi}\left[\sin\left((\varpi_{RF} + 5\varpi_{LO})t\right) - \sin\left((\varpi_{RF} - 5\varpi_{LO})t\right)\right] + \cdots$$

Switch-based Mixer: Spectral Plot

The previous formula for o/p voltage indicates spectral components at a range of frequencies:

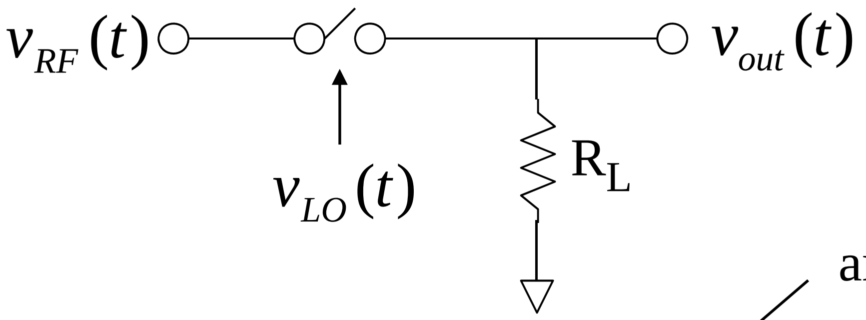
$$\varpi_{RF}, \varpi_{RF} \pm \varpi_{LO}, \varpi_{RF} \pm 3\varpi_{LO}, \varpi_{RF} \pm 5\varpi_{LO}, etc$$

Taking an example where $\omega_{LO} < \omega_{RF}$ and looking only at the positive frequencies the output spectrum looks like:



The unwanted signals can be eliminated by a low pass filter at the output. In this mixer configuration there is no LO feed-through to the output but there is RF feed-through.

Voltage Conversion Gain of Switch-based Mixer



amplitude of desired IF signal

$$v_{out}(t) = \frac{1}{2} V_{RF} \cos(\varpi_{RF} t) + \frac{V_{RF}}{\pi} \left[\sin\left((\varpi_{RF} + \varpi_{LO}) t \right) - \sin\left((\varpi_{RF} - \varpi_{LO}) t \right) \right] + \cdots$$

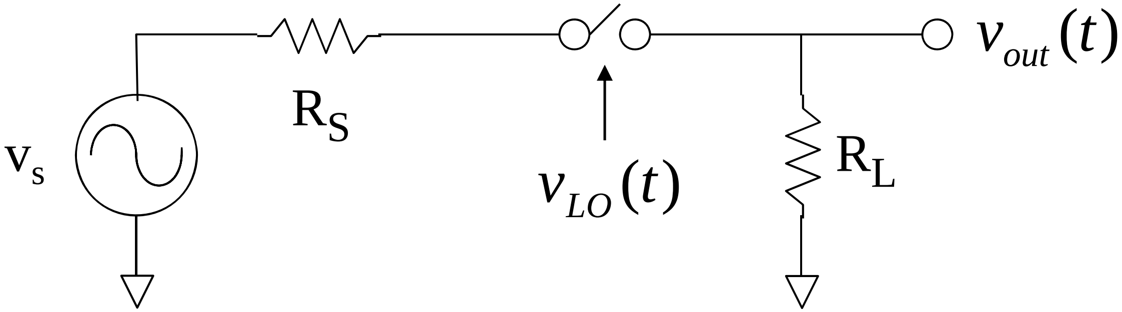
The voltage conversion gain (CG) is the ratio of the amplitudes of output IF signal and the input RF signal i.e.

$$A_{CG} = \frac{V_{IF}}{V_{RF}} = \frac{V_{RF}}{\pi} \frac{1}{V_{RF}} = \frac{1}{\pi} \qquad A_{CG,dB} = 20 \log_{10}\left(\frac{1}{\pi} \right) \approx -10dB$$

Note that conversion gain refers to the ratio of two signals at different frequencies (RF and IF).

Power Conversion Gain of Switch-based Mixer (1)

The power conversion gain of the mixer is the ratio of the output IF power to the *available* RF source power. Because this definition refers to the *available* source power, the source impedance has to be considered.



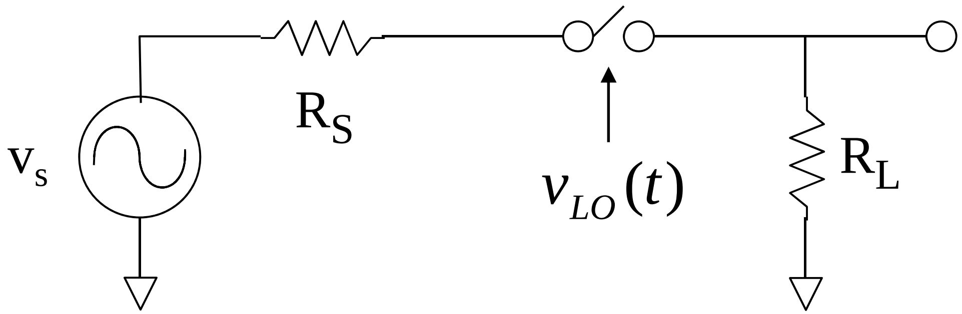
Previously, we have seen that the maximum power which can be delivered from a source to a matched load (the available power) is:

$$P_{AV} = \frac{v_s^2}{4R_S}$$ where v_s is the rms source voltage

If V_{RF} is the amplitude of the sinusoidal source voltage then:

$$v_s = \frac{V_{RF}}{\sqrt{2}} \Rightarrow P_{AV} = \frac{v_s^2}{4R_S} = \frac{V_{RF}^2}{2(4R_S)} = \frac{V_{RF}^2}{8R_S}$$

Power Conversion Gain of Switch-based Mixer (2)



When the source resistance was considered zero the amplitude of the IF signal at the output was:

$$V_{IF} = \frac{V_{RF}}{\pi}$$

With a source resistance, the source and load resistors form a resistor divider so the IF amplitude and rms voltages are then given by:

$$V_{IF} = \frac{R_L}{R_S + R_L}\frac{V_{RF}}{\pi} \qquad v_{IF,rms} = \frac{V_{IF}}{\sqrt{2}}$$

The power delivered to the load is then:

$$P_L = \frac{v_{IF,rms}^2}{R_L} = \frac{1}{2R_L}\left(\frac{R_L}{R_S + R_L}\right)^2 \frac{V_{RF}^2}{\pi^2}$$

Power Conversion Gain of Switch-based Mixer (3)

The power conversion gain is then:

$$P_{CG} = \frac{P_L}{P_{AV}} = \frac{1}{2R_L}\left(\frac{R_L}{R_S + R_L}\right)^2 \frac{V_{RF}^2}{\pi^2}\frac{8R_S}{V_{RF}^2} = \frac{4}{\pi^2}\frac{R_S}{R_L}\left(\frac{R_L}{R_S + R_L}\right)^2$$

Looking at the special case when the source and load are matched i.e. $R_L = R_S$

$$P_{CG} = \frac{4}{\pi^2}\frac{R_S}{R_S}\left(\frac{R_S}{R_S + R_S}\right)^2 = \frac{1}{\pi^2}$$

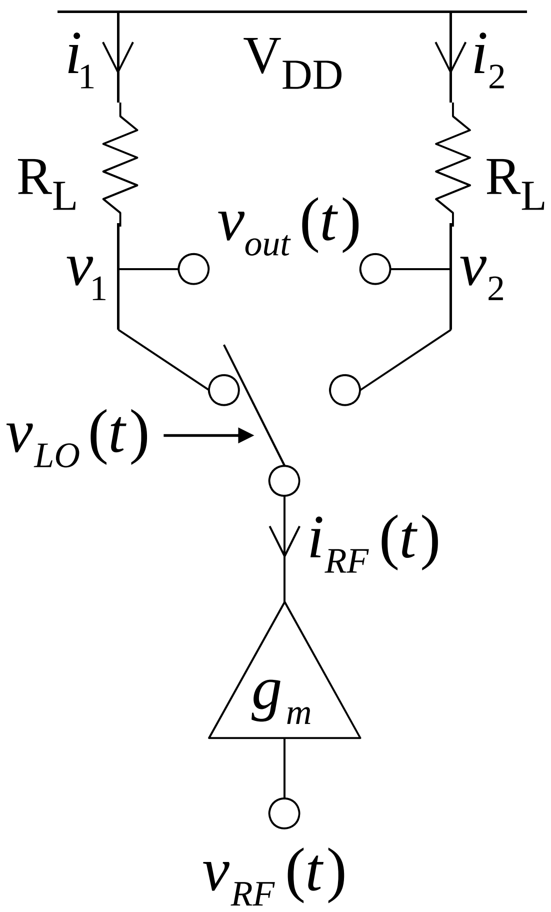
In dB this special case is

$$P_{CG,dB} = 10\log_{10}\left(\frac{1}{\pi^2}\right) \approx -10dB$$

So, when the source and load resistances are the same the power conversion gain of the switch-based mixer is approximately -10dB which is the same as the voltage conversion gain expressed in dB.

The Single-Balanced Mixer (1)

To achieve some gain, the RF signal can be fed to a transconductance element before the switch. A balanced configuration is achieved by switching the output of the transconductance element alternatively between two load resistors and taking the output as the difference voltage across these resistors.



$$i_{RF}(t) = g_m v_{RF}(t)$$

$$\begin{aligned} v_{out}(t) &= v_2(t) - v_1(t) \\ &= \left(V_{DD} - i_2(t)\right)R_L - \left(V_{DD} - i_1(t)\right)R_L \\ &= R_L\left(i_1(t) - i_2(t)\right) \end{aligned}$$

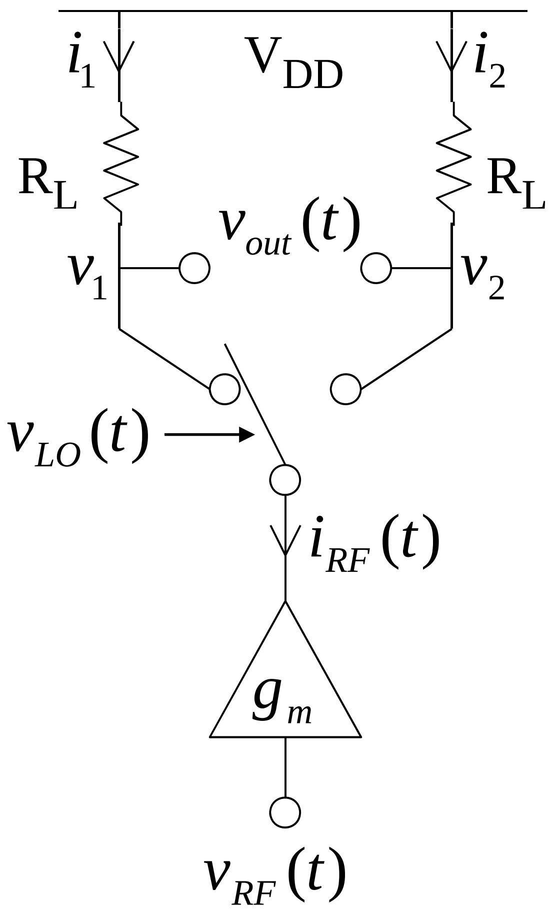
During the positive half cycles of the LO the switch is connected to the left-hand resistor giving:

$$i_1(t) = i_{RF}(t) \quad i_2(t) = 0$$

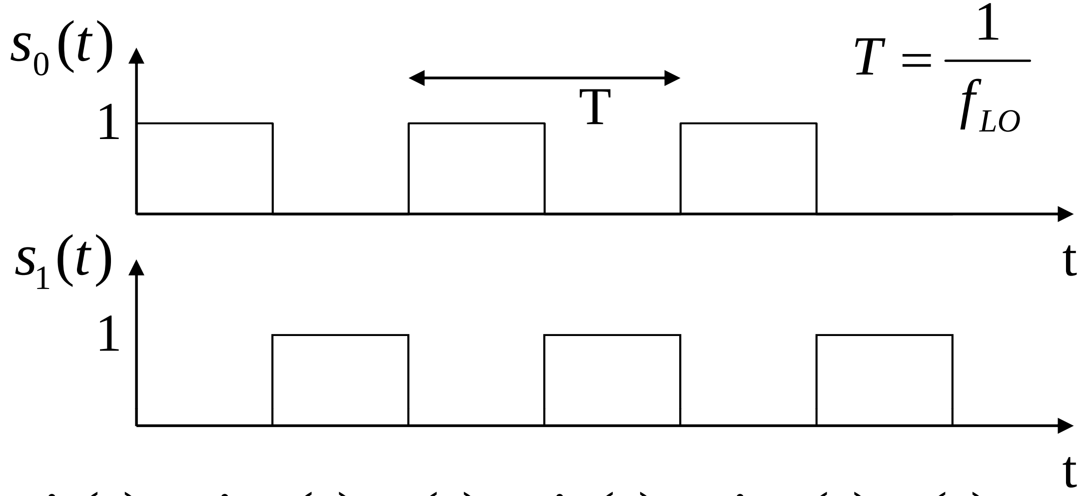
During the negative half cycles of the LO the switch is connected to the right-hand resistor giving:

$$i_1(t) = 0 \quad i_2(t) = i_{RF}(t)$$

The Single-Balanced Mixer (2)



The currents in the left and right load resistors are equivalent to i_{RF} multiplied by two square waves that are 180^0 out of phase i.e.

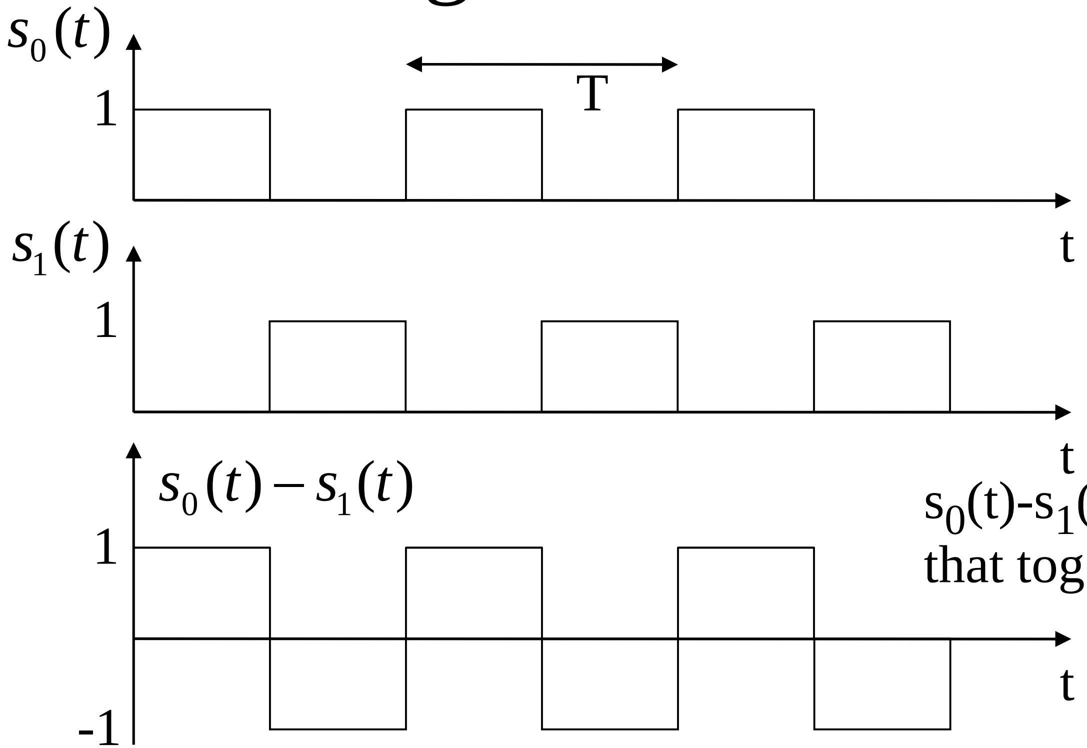


$$T = \frac{1}{f_{LO}}$$

$$i_1(t) = i_{RF}(t)s_0(t) \quad i_2(t) = i_{RF}(t)s_1(t)$$

$$\begin{aligned} v_{out}(t) &= R_L\left(i_{RF}(t)s_0(t) - i_{RF}(t)s_1(t)\right) \\ &= g_m R_L v_{RF}(t)\left(s_0(t) - s_1(t)\right) \end{aligned}$$

The Single-Balanced Mixer (3)



$s_0(t)$-$s_1(t)$ is a square wave that toggles between +1 and -1

$$s_0(t) - s_1(t) = \frac{4}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

$$v_{out}(t) = g_m R_L v_{RF}(t)\big(s_0(t) - s_1(t)\big)$$

$$= \frac{4 g_m R_L v_{RF}(t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

The Single-Balanced Mixer (4)

Using a co-sinusoidal waveform for the RF input and the cos(A)sin(B) formula:

$$v_{out}(t) = \frac{4g_m R_L v_{RF}(t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

$$= \frac{4g_m R_L V_{RF}\cos(\varpi_{RF}t)}{\pi}\left[\sin(\varpi_{LO}t) + \frac{1}{3}\sin(3\varpi_{LO}t) + \cdots\right]$$

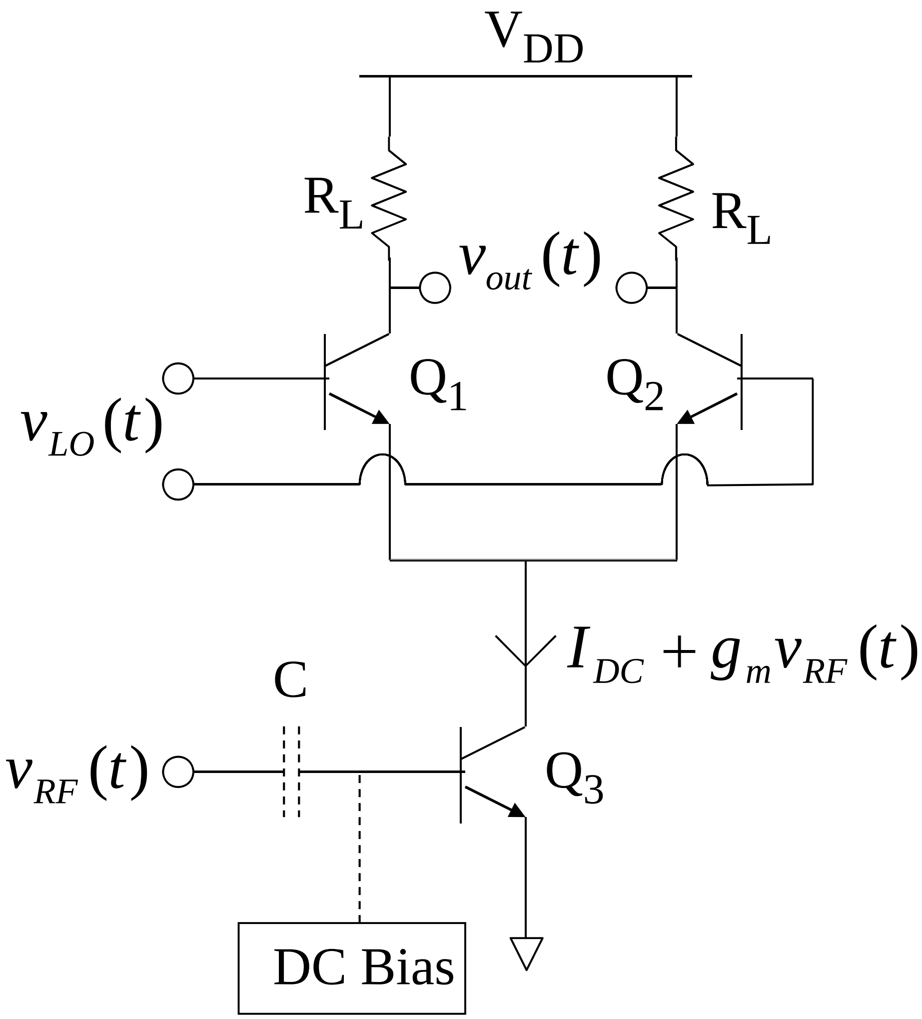
$$= \frac{2g_m R_L V_{RF}}{\pi}\left[\begin{array}{l}\sin\big((\varpi_{RF} + \varpi_{LO})t\big) - \sin\big((\varpi_{RF} - \varpi_{LO})t\big) \\ + \frac{1}{3}\sin\big((\varpi_{RF} + 3\varpi_{LO})t\big) - \frac{1}{3}\sin\big((\varpi_{RF} - 3\varpi_{LO})t\big) + \cdots\end{array}\right]$$

Notice that in the balanced mixer there is no output term at the RF frequency i.e. RF feed-through has been eliminated. The output IF amplitude and voltage conversion gain are given by:

$$V_{IF} = \frac{2g_m R_L V_{RF}}{\pi} \qquad A_{CG} = \frac{V_{IF}}{V_{RF}} = \frac{2g_m R_L}{\pi}$$

By proper choice of g_m and R_L, a conversion gain > 1 can be achieved.

BJT Implementation of Single-Balanced Mixer

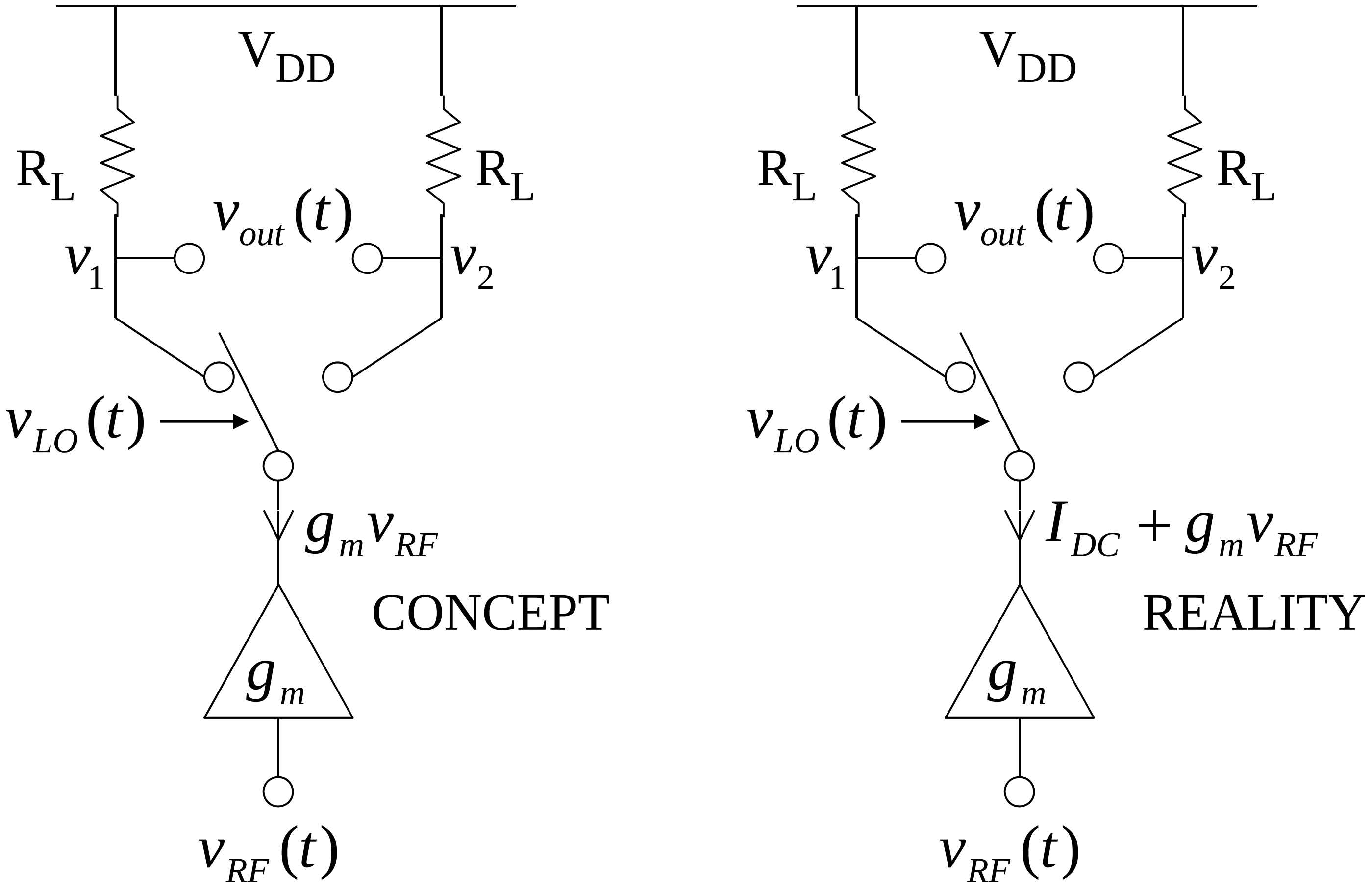


This circuit combines small-signal (Q_3) and large-signal (Q_1, Q_2) operations.

Q_3 is supplied with a DC bias which causes a DC current I_{DC} to flow. This biases Q_3 into it's forward active region where it has a high transconductance g_m. The rf signal applied to the base of Q_3 will therefore give rise to a small signal current at the collector as well as the DC bias current.

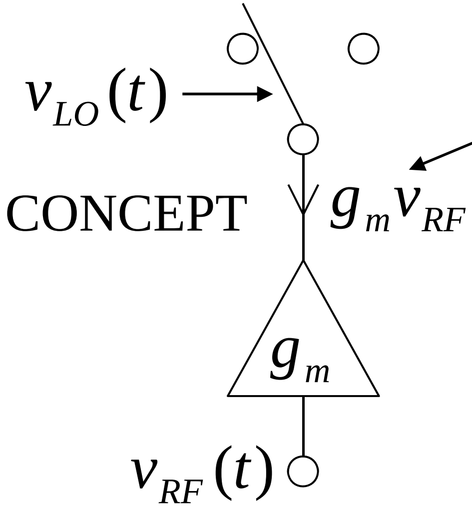
The LO signal is applied differentially across the bases of Q_1 and Q_2. Because the LO signal is relatively large, during alternative half cycles of the LO, Q_1 and Q_2 are either fully on or fully off respectively. Thus, Q_1 and Q_2 act as switches which switch the Q_3 current between the left and right R_Ls.

Single Balanced Mixer: Concept & Reality (1)



In the single balanced mixer concept, the small-signal RF current is switched between two paths. In reality, a DC bias current is needed to set up the transconductance and this also gets switched between the two paths.

Single Balanced Mixer: Concept & Reality (2)

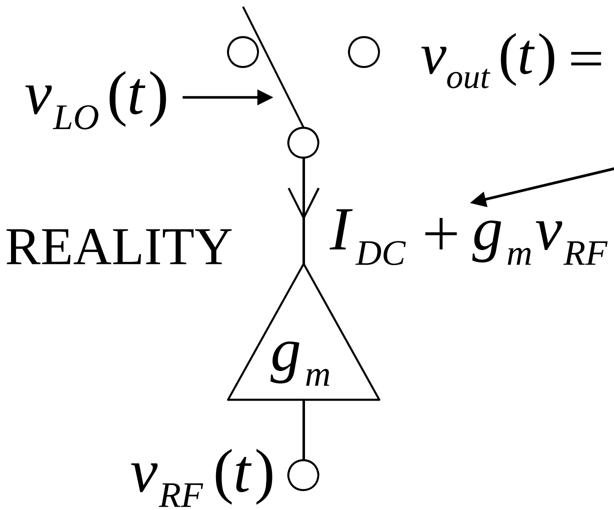


Concept (previous result):

$$v_{out}(t) = \frac{4R_L}{\pi} g_m v_{RF}(t) \left[\sin(\varpi_{LO} t) + \frac{1}{3}\sin(3\varpi_{LO} t) + \cdots \right]$$

Reality:

$$v_{out}(t) = \frac{4R_L}{\pi} \left(g_m v_{RF}(t) + I_{DC} \right) \left[\sin(\varpi_{LO} t) + \frac{1}{3}\sin(3\varpi_{LO} t) + \cdots \right]$$



The I_{DC} term appears in the output multiplied by the odd harmonics of the LO frequency i.e. the practical implementation of the single balanced mixer shown here has LO feed-through which is undesirable.