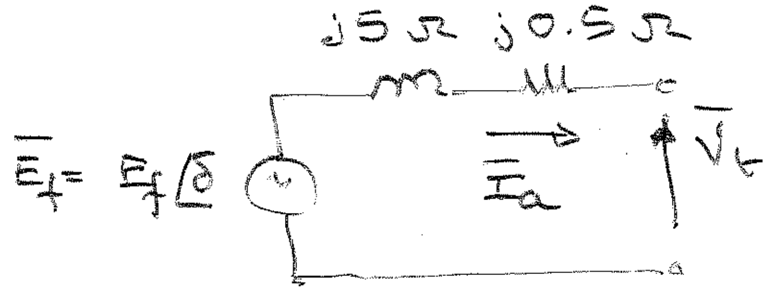
SYNCHRONOUS MACHINE PROBLEMS

Q.1.



$S = 1500$ KVA
$V_t = 6600$ V

$$S = \sqrt{3}\, V_t I_a$$

$$\Rightarrow \quad I_a = \frac{1500 \times 10^3}{\sqrt{3} \times 6600}$$

$$= 131.22 \text{ A}$$

THEN, FOR RATED POWER OPERATION AT 0.8 pf LAGGING,

$$\bar{I}_a = 131.22 \angle -\cos^{-1} 0.8 \ \text{A}$$

$$= 131.22 \angle -36.87^\circ \ \text{A}$$

THE REQUIRED EXCITATION VOLTAGE IS THEN

$$\bar{E}_f = E_f \angle \delta = V_t + \bar{Z}_s \bar{I}_a$$

$$\Rightarrow E_f \angle \delta = \frac{6600}{\sqrt{3}} + (0.5 + j5)\, 131.22 \angle -36.8$$

$$= 3810.5 + 659 \angle 47.42^\circ$$

$$= 4284.3 \angle 6.51^\circ$$

$$\text{Regulation} = \frac{E_f - V_t}{V_t}$$

$$= \left(\frac{4284.3 - 3810.5}{3810.5}\right) 100\%$$

$$= 12.43\%$$

CHECK :-

$$P_o = \sqrt{3}\, V_t I_a \cos\phi$$

$$= \sqrt{3}\ 6600 \times 131.22 \times 0.8$$

$$= 1200 \text{ kW}$$

AS EXPECTED.

$$P_o = S_{RATED}\ Pf$$

$$= 1500 \times 0.8 = 1200 \text{ kW}$$

ALSO

$$P_o \doteq \frac{3 E_f V_t \sin\delta}{X_s} \quad \text{(NEGLECT } R_a\text{)}$$

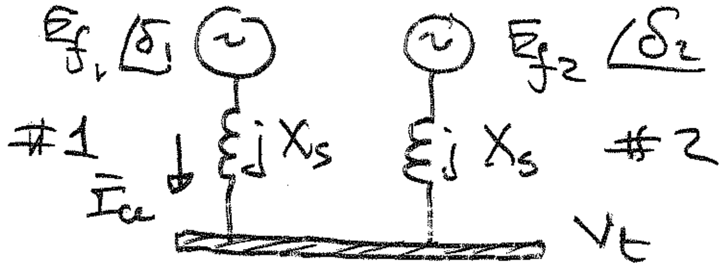
$$\doteq \frac{3 \times 4284.3 \times 3810.5}{5} \sin(6.51^\circ)$$

$$\doteq 1110.5 \text{ kW}$$

EXERCISE :- CALCULATE $3I_a^2R_a$ AND $\bar{S} = 3\bar{E}_f \bar{I}_a^*$

EE4010
SYNCHRONOUS MACHINES

Q.2.



$X_s = 4.5\,\Omega$

$$E_{f_1} = E_{f_2} = 1910 \text{ V/phase.}$$

$$\delta_1 - \delta_2 = 30^\circ$$

HENCE,

$$\bar{I}_a = \frac{E_{f_1}\angle\delta_1 - E_{f_2}\angle\delta_2}{2jX_s}$$

TAKING δ_2 AS THE REFERENCE PHASOR,

$$\bar{I}_a = \frac{1910\angle 30^\circ - 1910\angle 0^\circ}{2j\,4.5}$$

$$= 109.8\angle 15^\circ \text{ A}$$

THE PER-PHASE TERMINAL VOLTAGE IS

$$\bar{V}_t = E_{f2}\angle\delta_2 + jX_s\bar{I}_a$$

$$= 1910 + (j4.5)(109.8\angle 15^\circ)$$

$$= 1845\angle 15^\circ \text{ V/phase}$$

CHECK :-

$$\bar{V}_t = E_{f1} \angle \delta_1 - j X_s \bar{I}_a$$

$$= 1910 \angle 30^\circ - j4.5 (109.8 \angle 30^\circ)$$

$$= 1845 \angle 15^\circ \text{ V/phase}$$

THE POWER TRANSFER IS

$$P = \frac{3 E_{f_1} E_{f_2} \sin(\delta_1 - \delta_2)}{2 X_s}$$

$$= \frac{3 \; 1910^2}{9} \sin(30^\circ)$$

$$= 608 \text{ kW.}$$

CHECK :-

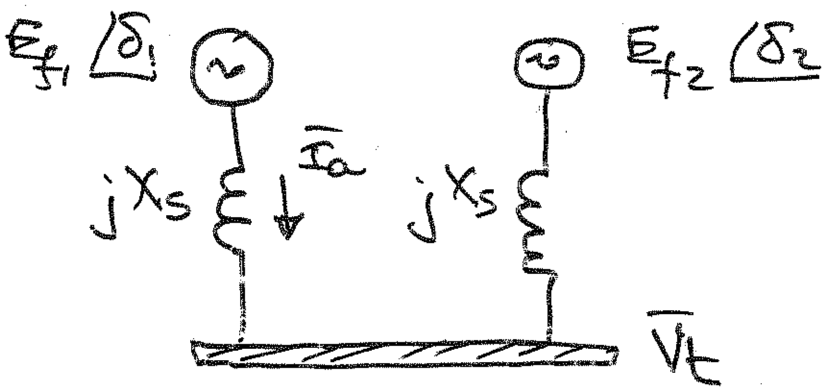
$$P = \mathrm{Re}\left[3 \; \bar{E}_{f_2} \bar{I}_a^*\right]$$

$$= \mathrm{Re}\left[3 \times 1910 \times 109.8 \angle 15^{\circ *}\right]$$

$$= 608 \text{ kW.}$$

EXERCISE :- CONSTRUCT THE PHASOR DIAGRAM.

Q. 3



$X_s = 4.5\,\Omega$

$E_{f1} = 2240$ V / phase

$E_{f2} = 1600$ V / phase

THE EMF'S ARE IN PHASE OPPOSITION.

HENCE

$$\bar{I}_a = \frac{\bar{E}_{f1} - \bar{E}_{f2}}{2jX_s}$$

$$= \frac{2240 - 1600}{2j4.5}$$

$$= 71.1 \angle -90^\circ \text{ A}$$

THE TERMINAL VOLTAGE IS

$$\bar{V}_t = \bar{E}_{f2} + jX_s I_a$$

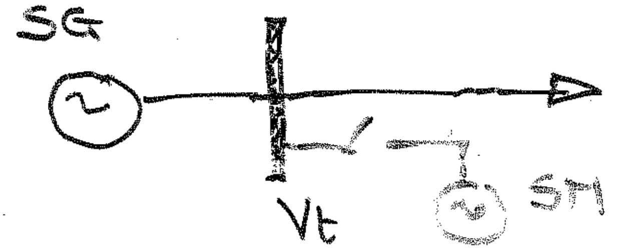
$$\Rightarrow \quad V_t = 1600 + (j4.5)(71.1\angle{-90^\circ})$$

$$= 1920\angle{0^\circ} \ \text{V/phase}$$

EXERCISE :-

CONSTRUCT THE CORRESPONDING PHASOR DIAGRAM.

Q.4. $\bar{S}_G = P + jQ$



S_G = 2.8 MW AT 0.7 pf LAGGIN

$= \frac{2.8}{0.7} \angle \cos^{-1} 0.7$

$= 4.0 \angle 45.6^\circ$ MVA.

HENCE, THE GENERATOR RATING IS 4.0 MW

INCREASE THE GENERATOR POWER FACTOR TO UNITY AND STILL OPERATE AT RATED CONDITIONS TO GIVE A REAL POWER INCREASE OF

$\Delta P = 4 MW - 2.8 MW$

$= 1.2 MW.$

THE SYNCHRONOUS MOTOR MUST PRODUCE $4.0 \sin(45.6^\circ) = 2.85 MVA_r$ OF REACTIVE POWER AND SO ITS OPERATI COMPLEX POWER IS

$\bar{S}_M = P - jQ$

$= 1.2 - j2.85$ MVA

SYNC M/C Q. 4.

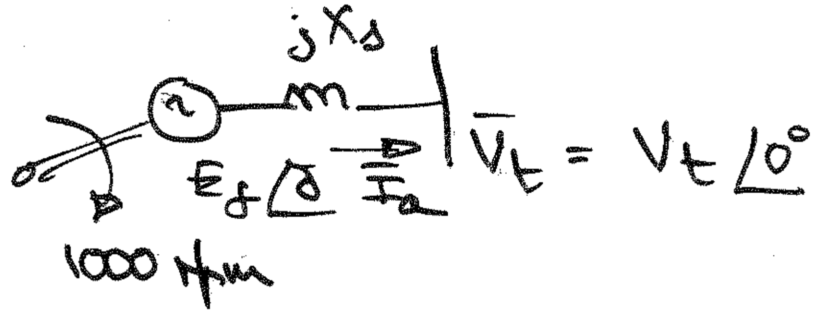
$$\Rightarrow \quad \bar{S}_M = 3.09 \angle -67.2^\circ \text{ MVA}$$

THUS, THE RATED CAPACITY OF THE GENERATOR MUST BE 3.1 MVA AND ITS OPERATING POWER FACTOR IS

$$pf_M = \cos(-67.2^\circ)$$

$$= 0.387 \text{ LEADING.}$$

Q.5.



$X_s = 2\,\Omega$

$I_f = 16\,A \qquad E_f = 460\,V_{LINE}$

$$\Rightarrow \quad E_f = \frac{460}{\sqrt{3}} = 265.6\,V.$$

$\bar{I}_a = 50\,A$ @ 0.8 pf LAGGING

BY KVL,

$$\bar{E}_f = \bar{V}_t + jX_s\bar{I}_a$$

$$\Rightarrow \quad E_f(\cos\delta + j\sin\delta) = V_t\angle 0 + jX_s I_a(\cos\phi - j\sin\phi) \quad (1)$$

WHERE $\phi = \cos^{-1}(0.8) = 36.87°$.

HENCE, EXPANDING EQ (1) INTO REAL AND IMAGINARY PARTS,

$$E_f\cos\delta = V_t + X_s I_a \sin\phi \quad (2)$$

$$E_f\sin\delta = X_s I_a \cos\phi \quad (3)$$

FROM (3)

$$\delta = \sin^{-1}\left[\frac{X_s I_a \cos\phi}{E_f}\right]$$

Q.5.

$$\Rightarrow \quad \delta = \sin^{-1}\left[\frac{2 \times 50 \times 0.8}{265.6}\right]$$

$$= 17.53^\circ$$

HENCE

$$V_t = E_f \cos\delta - X_s I_a \sin\phi$$

$$= 265.6 \cos(17.53^\circ) - 2 \times 50 \times 0.6$$

$$= 193.3\,V.$$

NOW, SINCE

$$P = 3\frac{E_f V_t \sin\delta}{X_s}$$

THEN

$$P = \frac{3 \times 265.6 \times 193.3}{2} \sin(17.53^\circ)$$

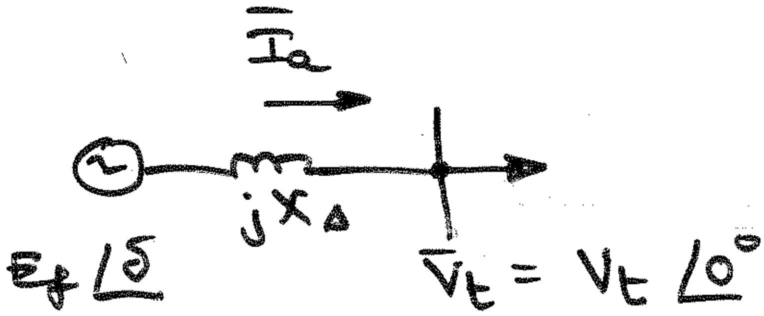
$$= 23.19\ kW$$

BUT THE SHAFT SPEED IS 1000 r/m
SO THAT

$$T_e = \frac{P}{\omega_m} = \frac{23.19 \times 10^3}{2\pi\,1000/60}$$

$$= 221.5\ Nm.$$

Q 6.



$jX_s = j1.0\text{pu}$

$V_t = 1.0\text{pu}$

$$\bar{I}_a = 1.0\angle -\cos^{-1}0.8$$

$$\bar{I}_a = 0.8 - j0.6$$

(i) $$\bar{E}_f = \bar{V}_t + jX_s\bar{I}_a$$

$$\Rightarrow \bar{E}_f = 1.0\angle 0^\circ + j1.0(0.8 - j0.6)$$

$$= 1.0 + j0.8 + 0.6$$

$$= 1.6 + j0.8$$

$$= 1.789\angle 26.57^\circ \text{ pu}$$

ALSO $$\bar{S}_1 = \bar{V}_t\bar{I}_a^*$$

$$= 1.0(0.8 + j0.6)^*$$

$$= 0.8 - j0.6$$

$$\Rightarrow P_1 = 0.8$$

$$Q_1 = 0.6$$

(ii) SINCE THE REAL POWER REMAINS THE SAME

$$P_1 = \frac{E_f' V_t}{X_s} \sin\delta'$$

$$\Rightarrow \quad \delta' = \sin^{-1}\left[\frac{P_1 X_s}{E_f'}\right]$$

$$= \sin^{-1}\left[\frac{0.8 \times 1}{1.2 \times 1.789}\right]$$

$$= 21.88^\circ$$

THUS

$$\bar{I}_a' = \frac{\bar{E}_f - \bar{V}_t}{jX_s}$$

$$= \frac{1.2 \times 1.789 \angle 21.88^\circ - 1\angle 0^\circ}{j1}$$

$$= 1.275 \angle -51.12^\circ \text{ pu}$$

$$\cos\phi' = \cos(51.12^\circ) = 0.627$$

$$\bar{S}' = V_t \bar{I}_a^*$$

$$= 1.0 \times (1.275 \angle -51.12^\circ)^*$$

$$= 0.8 + j0.992$$

(iii) P IS INCREASED BY 20%

HENCE

$$P'' = \frac{E_f V_t}{X_s} \sin \delta''$$

$$\Rightarrow \delta'' = \sin^{-1}\left[\frac{P'' X_s}{E_f V_t}\right]$$

$$= \sin^{-1}\left[\frac{1.2 \times 0.8 \times 1.0}{1.789 \times 1.0}\right]$$

$$\delta'' = 32.45^\circ$$

THUS

$$\bar{I}_a'' = \frac{E_f \angle \delta'' - \bar{V}_t}{j X_s}$$

$$= \frac{1.789 \angle 32.45^\circ - 1.0}{j1}$$

$$= 1.087 \angle -27.97^\circ \text{ pu}$$

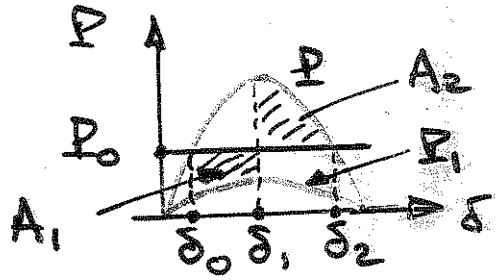
$$\Rightarrow \cos\phi'' = \cos(27.97) = 0.883$$

$$\bar{S}'' = \bar{V}_t \bar{I}_a''^{*}$$

$$= 1.0 (1.087 \angle -27.97^\circ)$$

$$= 0.96 + j0.509 \text{ pu}$$

Q.8



$P_0 = 0.6$ pu.

$P = 1.3$ pu

$P_1 = 0.1$ pu

$$\delta_0 = \sin^{-1}\left(\frac{0.6}{1.3}\right) = 27.5^\circ = 0.48 \text{ rad.}$$

$$\delta_2 = 180^\circ - \sin^{-1}\left(\frac{0.6}{1.3}\right) = 152.5^\circ$$

$$= 2.66 \text{ rad.}$$

THE EQUAL AREA CRITERION REQUIRES THAT

$$A_1 = A_2$$

$$\Rightarrow \int_{\delta_0}^{\delta_1} (P_0 - P_1 \sin\delta)\, d\delta + \int_{\delta_1}^{\delta_2} (P_0 - P\sin\delta)\, d\delta = 0$$

$$\Rightarrow \left[P_0 + P_1 \cos\delta\right]_{\delta_0}^{\delta_1} + \left[P_0 + P\cos\delta\right]_{\delta_1}^{\delta_2} = 0$$

$$\Rightarrow \{P_0\delta_1 + P_1\cos\delta_1 - P_0\delta_0 - P_1\cos\delta_0\} + \{P_0\delta_2 + P\cos\delta_2 - P_0\delta_1 - P\cos\delta_1\} = 0$$

$$\Rightarrow \cos\delta_1 = \frac{P_0(\delta_0 - \delta_2) + P_1\cos\delta_0 - P\cos\delta_2}{(P_1 - P)}$$

$$\Rightarrow \cos\delta_1 = \frac{0.6(0.48 - 2.66) + 0.1\cos(27.5^\circ) - 1.3\cos(152.5^\circ)}{0.1 - 1.3}$$

Q.8.

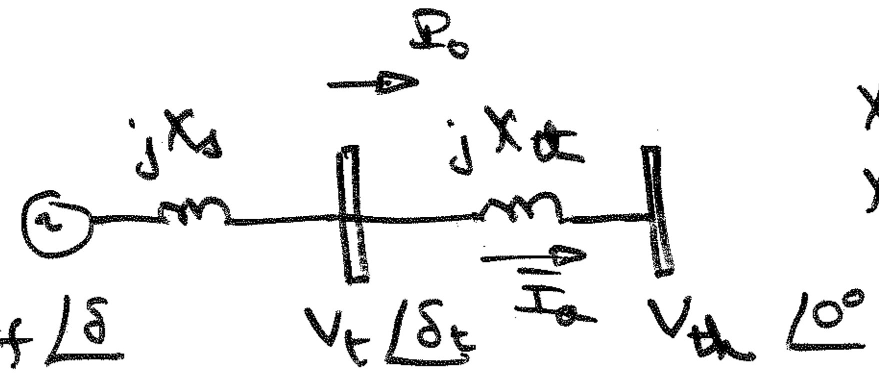
$$\Rightarrow \cos\delta_1 = \frac{-1.308 + 0.0887 + 1.153}{-1.2}$$

$$= 0.055$$

$$\Rightarrow \delta_1 = \cos^{-1}(0.055)$$

$$= 86.8^\circ.$$

Q.9.



$X_s = 0.2$
$X_{th} = 0.3$

$P_o = 450$ MW $\quad S_{base} = 500$ MVA

$$\Rightarrow \quad P_o = \frac{450}{500} = 0.9 \text{ pu.}$$

INITIALLY

$$V_t = V_{th} = 1.0 \text{ pu.}$$

ALSO,

$$P_o = \frac{V_t V_{th}}{X_{th}} \sin \delta_t$$

$$\Rightarrow \quad \delta_t = \sin^{-1}\left[\frac{P_o X_{th}}{V_t V_{th}}\right]$$

$$\delta_t = 15.66°$$

HENCE

$$\bar{I}_a = \frac{V_t \angle \delta_t - V_{th} \angle 0°}{j X_{th}}$$

$$= \frac{1 \angle 15.66° - 1.0 \angle 0°}{j\,0.3}$$

$$= 0.908 \angle 7.83° \text{ pu}$$

$$\Rightarrow \quad E_f \angle \delta = \bar{V}_{th} + j(X_d + X_{th})\bar{I}_a$$

$$\Rightarrow \quad E_f \angle \delta = 1.0 + j0.5\,(0.908 \angle 7.83^\circ)$$

$$= 1.04 \angle 25.62^\circ \text{ pu.}$$

THE INITIAL POWER / LOAD ANGLE CURVE IS

$$P = \frac{E_f V_{th}}{X_d + X_{th}} \sin\delta$$

$$= \frac{1.04 \times 1.0}{0.2 + 0.3} \sin\delta$$

$$= 2.08 \sin\delta$$

SINCE THE TERMINALS OF THE GENERATOR ARE DIRECTLY SHORTED, THE FAULTED POWER CURVE IS

$$P_1 = 0.0 \sin\delta$$

USING THE NOTATION OF PROBLEM 8,

$$\cos\delta_1 = \frac{P_0(\delta_0 - \delta_2) + P_1\cos\delta_0 - P\cos\delta_2}{P_1 - P}$$

WHERE $\delta_0 = \sin^{-1}\left[\frac{0.9}{2.08}\right] = 25.63^\circ$

$$= 0.447 \text{ rad}$$

ALSO $\delta_2 = \pi - \delta_0 = 154.3^\circ$

$= 2.694$ rad.

$P_0 = 0.9\ ,\ P_1 = 0.0\ ,\ P = 2.08$

HENCE,

$$\cos\delta_1 = \frac{0.9(0.447 - 2.694) + 0 - 2.08\cos(154.3^\circ)}{0 - 2.08}$$

$$= \frac{-2.022 + 1.874}{-2.08}$$

$$= 0.071$$

$$\Rightarrow \quad \delta_1 = \cos^{-1}(0.071)$$

$$\delta_1 = 85.9^\circ.$$