

# Control Engineering Summer '08

4a)

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} u(t)$$

$$y(t) = \overset{C}{\begin{bmatrix} 2 & 3 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B$$

$$= \begin{bmatrix} 2 & 3 \end{bmatrix} \left( \begin{bmatrix} s+2 & 0 \\ 0 & s+4 \end{bmatrix}^{-1} \right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{(s+2)(s+4)} \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} s+4 & 0 \\ 0 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+4)} \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} s+4 \\ s+2 \end{bmatrix} = \frac{5s+14}{(s+2)(s+4)} = \frac{5s+14}{s^2+6s+8}$$

Controllability:

$$C_x = [B \mid AB] \quad AB = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$C_x = \begin{bmatrix} 1 & -2 \\ 1 & -4 \end{bmatrix}$$

$$\det(C_x) = -2 \neq 0 \Rightarrow \text{Controllable}$$

Observability:

$$O_x = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -4 & -12 \end{bmatrix} \quad CA = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(O_x) = -12 \neq 0 \Rightarrow \text{Observable}$$

CCF

$$A = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [14 \ 5]$$

$$C_x = [B \mid AB] = \begin{bmatrix} 0 & 1 \\ 1 & -6 \end{bmatrix} = T C_x$$

$$T = C_x C_x^{-1}, \quad C_x^{-1} = -\frac{1}{2} \begin{bmatrix} -4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} +2 & -1 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ -1 & 2 \end{bmatrix}$$