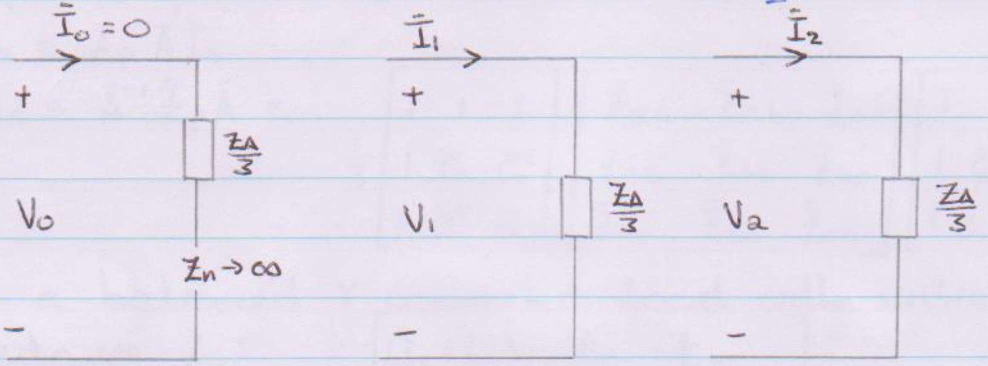
Electrical Power Systems Summer '09

3 a) ii)

$$\bar{Z}_{ph} = \begin{bmatrix} Z_Y+Z_n & Z_n & Z_n \\ Z_n & Z_Y+Z_n & Z_n \\ Z_n & Z_n & Z_Y+Z_n \end{bmatrix} \quad \bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} Z_Y+Z_n & Z_n & Z_n \\ Z_n & Z_Y+Z_n & Z_n \\ Z_n & Z_n & Z_Y+Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

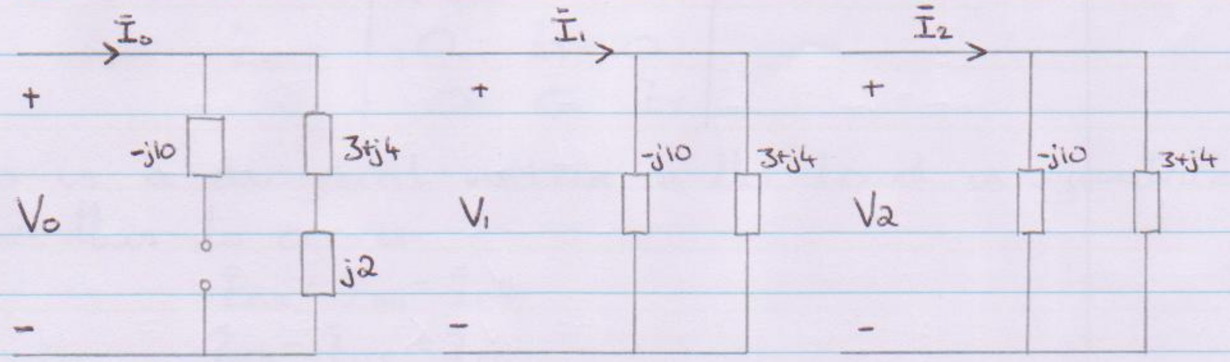
$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} Z_Y+3Z_n & Z_Y & Z_Y \\ Z_Y+3Z_n & \bar{a}^2 Z_Y & \bar{a} Z_Y \\ Z_Y+3Z_n & \bar{a} Z_Y & \bar{a}^2 Z_Y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3(Z_Y+3Z_n) & 0 & 0 \\ 0 & 3Z_Y & 0 \\ 0 & 0 & 3Z_Y \end{bmatrix}$$

$$\bar{Z}_s = \begin{bmatrix} Z_Y+3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$



b) $Z_Y = 3 + j4\ \Omega \qquad Z_{Y_2} = -j10\ \Omega$

$Z_n = +j2\ \Omega$



$$\begin{bmatrix} 32.7\angle 30^\circ \\ 188.3\angle 7.6^\circ \\ 44\angle -109.8^\circ \end{bmatrix} = \begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} \qquad \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 4.875\angle -33.43^\circ \\ 25.263\angle -18.97^\circ \\ 5.9\angle -136.40^\circ \end{bmatrix}$$

$$\bar{I}_a = \bar{I}_0 + \bar{I}_1 + \bar{I}_2 = 28.02\angle -32.3^\circ\ A$$

Electrical Power Systems Summer '08

3a) i) $\bar{V}_s = \bar{A}^{-1}\bar{V}_p \Rightarrow \begin{bmatrix}\bar{V}_0\\ \bar{V}_1\\ \bar{V}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{V}_{an}\\ \bar{V}_{bn}\\ \bar{V}_{cn}\end{bmatrix}$

$\bar{V}_0$ = zero sequence voltage
$\bar{V}_1$ = +ive sequence voltage
$\bar{V}_2$ = -ive sequence voltage

$\bar{a} = 1\angle 120^\circ$

ii) $\bar{I}_s = \bar{A}^{-1}\bar{I}_p \Rightarrow \begin{bmatrix}\bar{I}_0\\ \bar{I}_1\\ \bar{I}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{I}_a\\ \bar{I}_b\\ \bar{I}_c\end{bmatrix}$

$\bar{I}_a + \bar{I}_b + \bar{I}_c = 3\bar{I}_0 = \bar{I}_n$

iii) $\bar{V}_p = \bar{Z}_p\bar{I}_p$

$\bar{A}\bar{V}_s = \bar{Z}_p\bar{A}\bar{I}_s$

$\bar{Z}_s = \bar{A}^{-1}\bar{Z}_p\bar{A} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac}\\ \bar{Z}_{ab} & \bar{Z}_{bb} & \bar{Z}_{bc}\\ \bar{Z}_{ac} & \bar{Z}_{bc} & \bar{Z}_{cc}\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a}^2 & \bar{a}\\ 1 & \bar{a} & \bar{a}^2\end{bmatrix}$

For a balanced Y connected load with neutral-to-ground impedance $\bar{Z}_n$:

$$\bar{Z}_p = \begin{bmatrix}(Z_Y+Z_n) & Z_n & Z_n\\ Z_n & (Z_Y+Z_n) & Z_n\\ Z_n & Z_n & (Z_Y+Z_n)\end{bmatrix}$$

$$\bar{Z}_s = \begin{bmatrix}Z_Y+3Z_n & 0 & 0\\ 0 & Z_Y & 0\\ 0 & 0 & Z_Y\end{bmatrix}$$

Z_s is a diagonal matrix in the load is symmetrical.
For this to occur:

$$\bar{Z}_{aa} = \bar{Z}_{bb} = \bar{Z}_{cc}$$
$$\bar{Z}_{ab} = \bar{Z}_{ac} = \bar{Z}_{bc}$$
$$\Rightarrow \bar{Z}_{01} = \bar{Z}_{10} = \bar{Z}_{02} = \bar{Z}_{20} = \bar{Z}_{12} = \bar{Z}_{21} = 0$$
$$\bar{Z}_0 = \bar{Z}_{aa} + 2\bar{Z}_{ab}$$
$$\bar{Z}_1 = \bar{Z}_2 = \bar{Z}_{aa} - \bar{Z}_{ab}$$

Mutual coupling is eliminated.

Electrical Power Systems Summer '08

3 a)

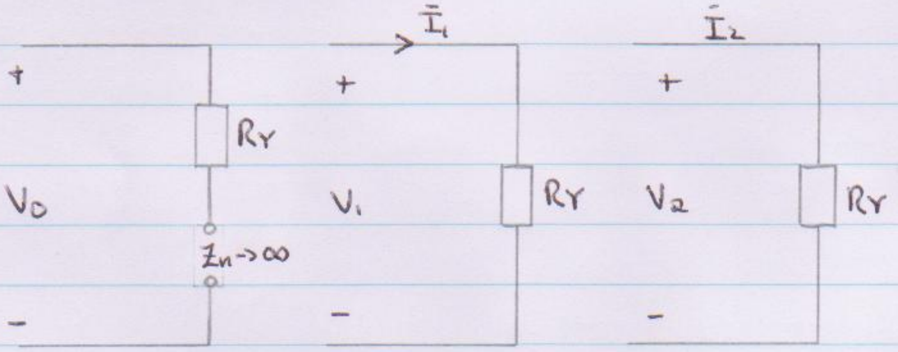
$$\bar{Z}_p = \begin{bmatrix} R_Y & 0 & 0 \\ 0 & R_Y & 0 \\ 0 & 0 & R_Y \end{bmatrix} \quad \bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} R_Y & 0 & 0 \\ 0 & R_Y & 0 \\ 0 & 0 & R_Y \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} R_Y & R_Y & R_Y \\ R_Y & \bar{a}^2 R_Y & \bar{a} R_Y \\ R_Y & \bar{a} R_Y & \bar{a}^2 R_Y \end{bmatrix} = \begin{bmatrix} R_Y & 0 & 0 \\ 0 & R_Y & 0 \\ 0 & 0 & R_Y \end{bmatrix}$$

$$\bar{V}_{phg} = \bar{V}_{phn} + \bar{V}_{ng}$$
$$\bar{A}\bar{V}_s = \bar{Z}_{ph}\bar{I}_{ph} + \bar{V}_{ng}$$
$$\bar{V}_s = \bar{Z}_s\bar{I}_s + \bar{A}^{-1}\bar{V}_{ng}$$

$$\bar{A}^{-1}\bar{V}_{ng} = \begin{bmatrix} \bar{V}_{ng} \\ 0 \\ 0 \end{bmatrix} \Rightarrow \bar{V}_{ng} = \bar{V}_0$$

3-Wire $\Rightarrow \bar{I}_0 = 0$



$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} \qquad \bar{I}_a = \bar{I}_1 + \bar{I}_2$$

b)

$$\begin{bmatrix} \bar{V}_{ag} \\ \bar{V}_{bg} \\ \bar{V}_{cg} \end{bmatrix} = \begin{bmatrix} 100\angle 0^\circ \\ 200\angle 270^\circ \\ 100\angle 120^\circ \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} 100\angle 0^\circ \\ 200\angle 270^\circ \\ 100\angle 120^\circ \end{bmatrix} = \begin{bmatrix} 41.31\angle -66.2^\circ \\ 128.8\angle 115^\circ \\ 41.31\angle 173.8^\circ \end{bmatrix}$$

$$\bar{V}_{ng} = 41.31\angle -66.2^\circ \qquad \bar{V}_{bg} = \bar{V}_{bn} + \bar{V}_{ng}$$
$$\bar{V}_{bn} = 163.06\angle -96^\circ\ V$$

$\Rightarrow$ Voltmeter reads 163.06 V

Electrical Power Systems Summer '07

3 a) i $\bar{V}_s = \bar{A}^{-1}\bar{V}_p \Rightarrow \begin{bmatrix}\bar{V}_0\\ \bar{V}_1\\ \bar{V}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{V}_{an}\\ \bar{V}_{bn}\\ \bar{V}_{cn}\end{bmatrix}$ $\bar{a} = 1\,\underline{/120^\circ}$

$\bar{V}_0$ = zero sequence voltage
$\bar{V}_1$ = positive sequence voltage
$\bar{V}_2$ = negative sequence voltage

ii $\bar{I}_s = \bar{A}^{-1}\bar{I}_p \Rightarrow \begin{bmatrix}\bar{I}_0\\ \bar{I}_1\\ \bar{I}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{I}_a\\ \bar{I}_b\\ \bar{I}_c\end{bmatrix}$ $\bar{I}_a + \bar{I}_b + \bar{I}_c = \bar{I}_n = 3\bar{I}_0$

iii $\bar{V}_p = \bar{Z}_p\bar{I}_p$

$\bar{A}\bar{V}_s = \bar{Z}_p\bar{A}I_s$

$\bar{V}_s = \underbrace{\left[\bar{A}^{-1}\bar{Z}_p\bar{A}\right]}_{\bar{Z}_s} I_s$

$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a} & \bar{a}^2\\ 1 & \bar{a}^2 & \bar{a}\end{bmatrix}\begin{bmatrix}\bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac}\\ \bar{Z}_{ab} & \bar{Z}_{bb} & \bar{Z}_{bc}\\ \bar{Z}_{ac} & \bar{Z}_{bc} & \bar{Z}_{cc}\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \bar{a}^2 & \bar{a}\\ 1 & \bar{a} & \bar{a}^2\end{bmatrix}$$

For a balanced load with neutral-to-ground impedance $\bar{Z}_N$:

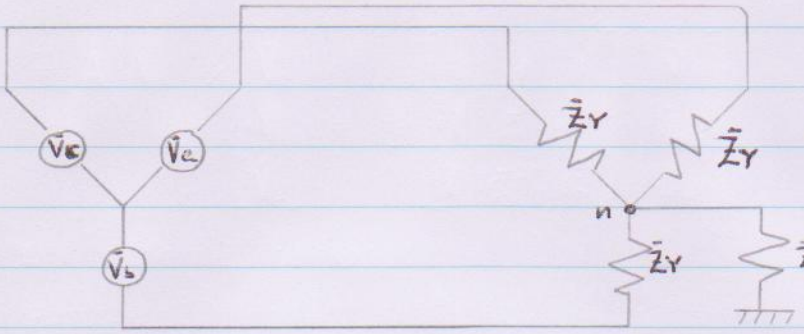
$$\bar{Z}_s = \begin{bmatrix}\bar{Z}_Y + 3\bar{Z}_n & 0 & 0\\ 0 & \bar{Z}_Y & 0\\ 0 & 0 & \bar{Z}_Y\end{bmatrix}$$

For this to hold:

$$\bar{Z}_{aa} = \bar{Z}_{bb} = \bar{Z}_{cc}$$
$$\bar{Z}_{bc} = \bar{Z}_{ac} = \bar{Z}_{ab}$$
$$\Rightarrow \bar{Z}_{01} = \bar{Z}_{10} = \bar{Z}_{02} = \bar{Z}_{20} = \bar{Z}_{12} = \bar{Z}_{21} = 0$$

Electrical Power Systems Summer '07

3 a)



$\bar{V}_{ag} = \bar{I}_a \bar{Z}_Y + [\bar{I}_a + \bar{I}_b + \bar{I}_c]\bar{Z}_n$

$\bar{V}_{bg} = \bar{I}_b \bar{Z}_Y + [\bar{I}_a + \bar{I}_b + \bar{I}_c]\bar{Z}_n$

$\bar{V}_{cg} = \bar{I}_c \bar{Z}_Y + [\bar{I}_a + \bar{I}_b + \bar{I}_c]\bar{Z}_n$

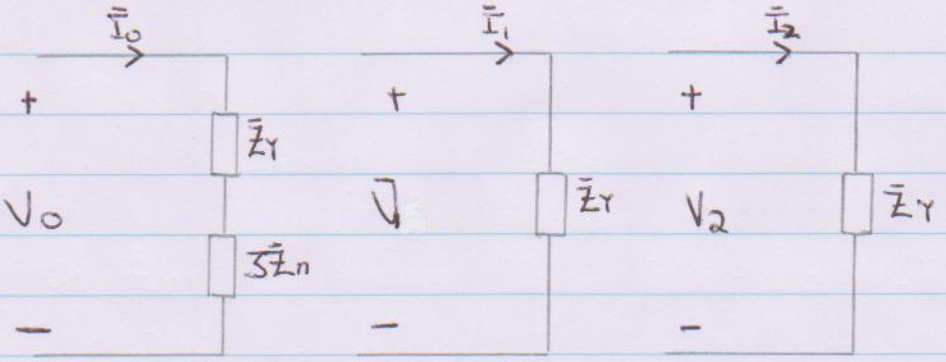
$$\begin{bmatrix} \bar{V}_{ag} \\ \bar{V}_{bg} \\ \bar{V}_{cg} \end{bmatrix} = \begin{bmatrix} Z_Y + Z_N & Z_N & Z_N \\ Z_N & Z_Y + Z_N & Z_N \\ Z_N & Z_N & Z_Y + Z_N \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} Z_Y + Z_N & Z_N & Z_N \\ Z_N & Z_Y + Z_N & Z_N \\ Z_N & Z_N & Z_Y + Z_N \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} Z_Y + 3Z_N & Z_Y & Z_Y \\ Z_Y + 3Z_N & \bar{a}^2 Z_Y & \bar{a} Z_Y \\ Z_Y + 3Z_N & \bar{a} Z_Y & \bar{a}^2 Z_Y \end{bmatrix}$$

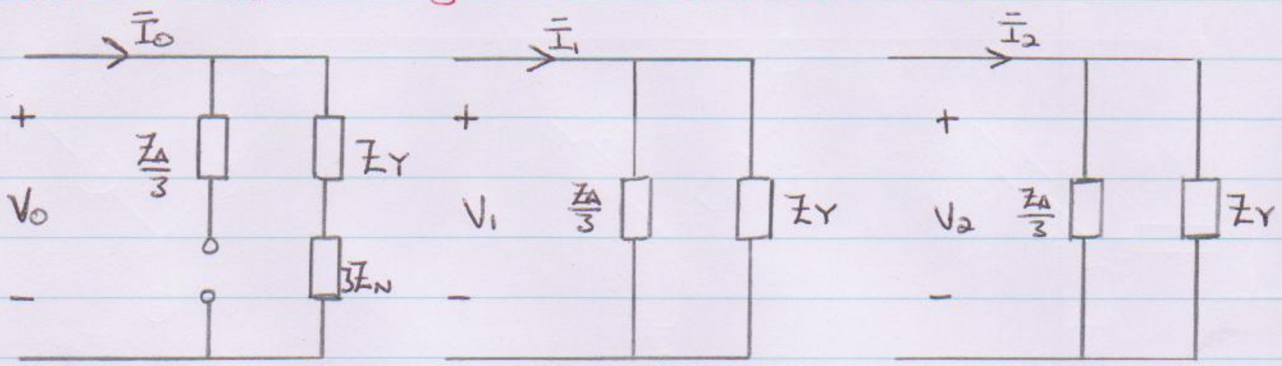
$$= \frac{1}{3}\begin{bmatrix} 3(\bar{Z}_Y + 3\bar{Z}_N) & 0 & 0 \\ 0 & 3\bar{Z}_Y & 0 \\ 0 & 0 & 3\bar{Z}_Y \end{bmatrix}$$

$$\bar{Z}_S = \begin{bmatrix} \bar{Z}_Y + 3\bar{Z}_N & 0 & 0 \\ 0 & \bar{Z}_Y & 0 \\ 0 & 0 & \bar{Z}_Y \end{bmatrix}$$



Electrical Power Systems Summer '07

3 b)



$$I_0 = 1.857 \angle -8.2^\circ \text{ A}$$
$$I_1 = 70.71 \angle -45^\circ \text{ A}$$
$$I_2 = 10.61 \angle 155^\circ \text{ A}$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} 1.857 \angle -8.2^\circ \\ 70.71 \angle -45^\circ \\ 10.61 \angle 155^\circ \end{bmatrix}$$

$$\bar{I}_a = 62.28 \angle -47.3^\circ \text{ A}$$

Electrical Power Systems Summer '06

3 b) $\bar{Z}_{phase} = \begin{bmatrix} \bar{Z}_s & \bar{Z}_m & \bar{Z}_m \\ \bar{Z}_m & \bar{Z}_s & \bar{Z}_m \\ \bar{Z}_m & \bar{Z}_m & \bar{Z}_s \end{bmatrix}$ $\qquad \bar{Z}_s = \bar{A}^{-1} \bar{Z}_p \bar{A}$

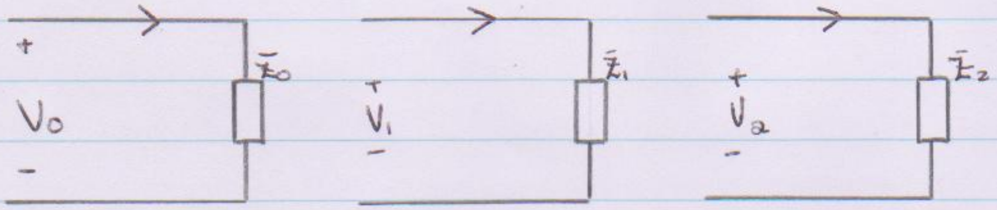
$$\bar{Z}_s = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} \bar{Z}_s & \bar{Z}_m & \bar{Z}_m \\ \bar{Z}_m & \bar{Z}_s & \bar{Z}_m \\ \bar{Z}_m & \bar{Z}_m & \bar{Z}_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} (\bar{Z}_s + 2\bar{Z}_m) & (\bar{Z}_s + \bar{Z}_m(\bar{a}^2 + \bar{a})) & (\bar{Z}_s + \bar{Z}_m(\bar{a} + \bar{a}^2)) \\ (\bar{Z}_s + 2\bar{Z}_m) & (\bar{a}^2 \bar{Z}_s + \bar{Z}_m(1 + \bar{a})) & (\bar{a}\bar{Z}_s + \bar{Z}_m(1 + \bar{a}^2)) \\ (\bar{Z}_s + 2\bar{Z}_m) & (\bar{a}\bar{Z}_s + \bar{Z}_m(1 + \bar{a}^2)) & (\bar{a}^2 \bar{Z}_s + \bar{Z}_m(1 + \bar{a})) \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 3(\bar{Z}_s + 2\bar{Z}_m) & 0 & 0 \\ 0 & 3(\bar{Z}_s - \bar{Z}_m) & 0 \\ 0 & 0 & 3(\bar{Z}_s - \bar{Z}_m) \end{bmatrix} \Rightarrow \text{As } (\bar{a}^2 + \bar{a}) = -1$$

$$\bar{Z}_s = \begin{bmatrix} \bar{Z}_s + 2\bar{Z}_m & 0 & 0 \\ 0 & \bar{Z}_s - \bar{Z}_m & 0 \\ 0 & 0 & \bar{Z}_s - \bar{Z}_m \end{bmatrix}$$

$$\begin{bmatrix} \bar{Z}_0 \\ \bar{Z}_1 \\ \bar{Z}_2 \end{bmatrix} = \begin{bmatrix} \bar{Z}_s + 2\bar{Z}_m \\ \bar{Z}_s - \bar{Z}_m \\ \bar{Z}_s - \bar{Z}_m \end{bmatrix} = \begin{bmatrix} 20 + j70 \\ 5 + j10 \\ 5 + j10 \end{bmatrix}$$



$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix} \begin{bmatrix} 277\angle 0^\circ \\ 260\angle -120^\circ \\ 295\angle 115^\circ \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 47.74\angle 62.1^\circ \\ 831.3\angle -1.77^\circ \\ 27.65\angle -143.4^\circ \end{bmatrix} = \begin{bmatrix} 15.91\angle 62.1^\circ \\ 277.1\angle -1.77^\circ \\ 9.22\angle -143.4^\circ \end{bmatrix}$$

$$\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 0.22\angle -11.95^\circ \\ 24.78\angle -65.2^\circ \\ 0.825\angle 153.2^\circ \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} \Rightarrow \bar{I}_a = 24.27\angle -66^\circ \text{ A}$$

Electrical Power Systems Summer '05

3. c)

$$\bar{Z}_{ph} = \begin{bmatrix} \bar{Z}_s+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n \\ \bar{Z}_m+\bar{Z}_n & \bar{Z}_s+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n \\ \bar{Z}_m+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n & \bar{Z}_s+\bar{Z}_n \end{bmatrix}$$

$$\bar{Z}_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} \bar{Z}_s+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n \\ \bar{Z}_m+\bar{Z}_n & \bar{Z}_s+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n \\ \bar{Z}_m+\bar{Z}_n & \bar{Z}_m+\bar{Z}_n & \bar{Z}_s+\bar{Z}_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} 3\bar{Z}_n+2\bar{Z}_m+\bar{Z}_s & \bar{Z}_s-\bar{Z}_m & \bar{Z}_s-\bar{Z}_m \\ 3\bar{Z}_n+2\bar{Z}_m+\bar{Z}_s & \bar{a}^2\bar{Z}_s+(1+\bar{a})\bar{Z}_m & \bar{a}\bar{Z}_s+(1+\bar{a}^2)\bar{Z}_m \\ 3\bar{Z}_n+2\bar{Z}_m+\bar{Z}_s & \bar{a}\bar{Z}_s+(1+\bar{a}^2)\bar{Z}_m & \bar{a}^2\bar{Z}_s+(1+\bar{a})\bar{Z}_m \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3(3\bar{Z}_n+2\bar{Z}_m+\bar{Z}_s) & 0 & 0 \\ 0 & 3(\bar{Z}_s-\bar{Z}_m) & 0 \\ 0 & 0 & 3(\bar{Z}_s-\bar{Z}_m) \end{bmatrix}$$

$$\bar{Z}_S = \begin{bmatrix} \bar{Z}_0 \\ \bar{Z}_1 \\ \bar{Z}_2 \end{bmatrix} = \begin{bmatrix} 3\bar{Z}_n+2\bar{Z}_m+\bar{Z}_s \\ \bar{Z}_s - \bar{Z}_m \\ \bar{Z}_s - \bar{Z}_m \end{bmatrix}$$

b) $V_{L-L} = 400V$ Balanced $\Rightarrow$ Positive sequence only

$V_{ph} = 230.94V$

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} 230.94\angle 0^\circ \\ 230.94\angle -120^\circ \\ 230.94\angle +120^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 230.94\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Rightarrow \bar{I}_0 = \bar{I}_2 = 0 \qquad \bar{I}_1 = \frac{230.94}{8+j20} = 10.72\angle -68.2^\circ \text{ A}$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}\begin{bmatrix} 0 \\ 10.72\angle -68.2 \\ 0 \end{bmatrix} \Rightarrow \bar{I}_a = 10.72\angle -68.2^\circ \text{ A}$$

Electrical Power Systems Summer '05

3. b)

$$\bar{V}_s = \begin{bmatrix} 32.7 \angle 30^\circ \\ 188.3 \angle 7.6^\circ \\ 44 \angle -109.8^\circ \end{bmatrix} = \begin{bmatrix} 8+j50 \\ 8+j20 \\ 8+j20 \end{bmatrix} \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 0.646 \angle -50.91^\circ \\ 8.742 \angle -60.6^\circ \\ 2.04 \angle -178^\circ \end{bmatrix}$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} 0.646 \angle -50.91 \\ 8.742 \angle -60.6 \\ 2.04 \angle -178^\circ \end{bmatrix}$$

$$\bar{I}_a = 8.61 \angle -72^\circ \text{ A}$$

Electrical Power Systems Summer '04

3 a)

$$\bar{Z}_{ph} = \begin{bmatrix} \bar{Z}_{aa} & 0 & 0 \\ 0 & \bar{Z}_{bb} & 0 \\ 0 & 0 & \bar{Z}_{cc} \end{bmatrix} \qquad \bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} \bar{Z}_{aa} & 0 & 0 \\ 0 & \bar{Z}_{bb} & 0 \\ 0 & 0 & \bar{Z}_{cc} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}$$

$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{aa} & \bar{Z}_{aa} \\ \bar{Z}_{bb} & \bar{a}^2\bar{Z}_{bb} & \bar{a}\bar{Z}_{bb} \\ \bar{Z}_{cc} & \bar{a}\bar{Z}_{cc} & \bar{a}^2\bar{Z}_{cc} \end{bmatrix}$$

$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix} (\bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc}) & \bar{Z}_{aa}+\bar{a}^2\bar{Z}_{bb}+\bar{a}\bar{Z}_{cc} & \bar{Z}_{aa}+\bar{a}\bar{Z}_{bb}+\bar{a}^2\bar{Z}_{cc} \\ \bar{Z}_{aa}+\bar{a}\bar{Z}_{bb}+\bar{a}^2\bar{Z}_{cc} & (\bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc}) & \bar{Z}_{aa}+\bar{a}^2\bar{Z}_{bb}+\bar{a}\bar{Z}_{cc} \\ \bar{Z}_{aa}+\bar{a}^2\bar{Z}_{bb}+\bar{a}\bar{Z}_{cc} & \bar{Z}_{aa}+\bar{a}\bar{Z}_{bb}+\bar{a}^2\bar{Z}_{cc} & (\bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc}) \end{bmatrix}$$

b)

$$\bar{Z}_{aa} = 10\angle 0^\circ\,\Omega \qquad \bar{Z}_{bb} = 10\angle 0^\circ\,\Omega \qquad \bar{Z}_{cc} = 20\angle 0^\circ\,\Omega$$

$$\bar{Z}_s = \frac{1}{3}\begin{bmatrix} 40\angle 0^\circ\,\Omega & 10\angle 120^\circ & 10\angle -120^\circ \\ 10\angle -120^\circ & 40\angle 0^\circ\,\Omega & 10\angle 120^\circ \\ 10\angle 120^\circ & 10\angle -120^\circ & 40\angle 0^\circ\,\Omega \end{bmatrix}$$

$$V_s = \begin{bmatrix} 0 \\ 230.94\angle 0^\circ \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 40\angle 0^\circ & 10\angle 120^\circ & 10\angle -120^\circ \\ 10\angle -120^\circ & 40\angle 0^\circ & 10\angle 120^\circ \\ 10\angle 120^\circ & 10\angle -120^\circ & 40\angle 0^\circ \end{bmatrix}\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a} & \bar{a}^2 \\ 1 & \bar{a}^2 & \bar{a} \end{bmatrix}\begin{bmatrix} V_{ng} \\ V_{ng} \\ V_{ng} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 230.94\angle 0^\circ \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 40\angle 0 & 10\angle 120 & 10\angle -120 \\ 10\angle -120 & 40\angle 0 & 10\angle 120 \\ 10\angle 120 & 10\angle -120 & 40\angle 0 \end{bmatrix}\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} + \begin{bmatrix} V_{ng} \\ 0 \\ 0 \end{bmatrix}$$

3-Wire $\Rightarrow \bar{I}_0 = 0$

$$692.82 = 40\angle 0^\circ\,\bar{I}_1 + 10\angle 120^\circ\,\bar{I}_2 \qquad (\times 4\angle -120)$$

$$0 = 10\angle -120\,\bar{I}_1 + \cancel{40\angle 0\,\bar{I}_2}$$

$$2771.28\angle -120 = 160\angle -120\,\bar{I}_1 + \cancel{40\angle 0\,\bar{I}_2}$$

$$2771.28\angle -120 = 150\angle -120\,\bar{I}_1$$

$$\bar{I}_1 = 18.48\angle 0^\circ\,A$$

$$\bar{I}_2 = 4.62\angle 60^\circ\,A$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix}\begin{bmatrix} 0 \\ 18.48\angle 0^\circ \\ 4.62\angle 60^\circ \end{bmatrix} \Rightarrow \bar{I}_a = 21.17\angle 10.9^\circ\,A$$

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3b) $\frac{1}{3}(10\angle 120^\circ \bar{I}_1 + 10\angle -120^\circ \bar{I}_2) = -\bar{V}_{ng}$

$$\bar{V}_{ng} = 46.2\angle -60^\circ \text{ V}$$