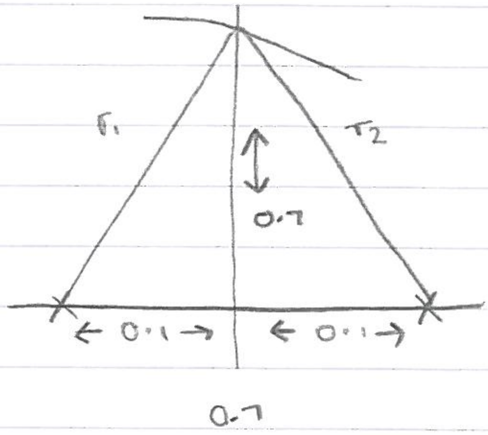
18/3/09

Control

$z = 0.7 + 0.7j$ (marginal stability)



Q 2 (a). Anti-aliasing
Faster than Nyquist
Flat gain response for zero order hold
Introduces phase lag which can be reduced by oversampling
Distorting the baseband

(b). Dahlins

$$D(z) = \frac{1}{G} \frac{C/R}{1 - C/R}$$

Continuous

$$\frac{C}{R} = \frac{1}{1 + \lambda s} e^{-sT_d}$$

Discrete

$$\frac{C}{R} = \frac{1 - e^{-T/\lambda}}{1 - e^{-T/\lambda} z^{-1}} z^{-(N+1)}$$

$$D(z) = \frac{1}{G} \frac{\dfrac{1 - e^{-T/\lambda}}{1 - e^{-T/\lambda} z^{-1}} z^{-(N+1)}}{1 - \dfrac{1 - e^{-T/\lambda}}{1 - e^{-T/\lambda} z^{-1}} z^{-(N+1)}}$$

$$= \frac{1}{G} \quad \frac{(1-e^{-T/\tau})z^{-(N+1)}}{1-e^{-T/\tau}z^{-1} - (1-e^{-T/\tau})z^{-(N+1)}}$$

$$G(z) = \mathcal{Z}\left\{ \frac{1-e^{-sT}}{s} \cdot \frac{Ke^{-NTs}}{1+s\tau} \right\}$$

N delays

$\Rightarrow e^{-NTs} \rightarrow z^{-N}$

$\Rightarrow (1-e^{-sT}) \rightarrow 1-z^{-1}$

$$G(z) = (1-z^{-1})z^{-N} \mathcal{Z}\left\{\frac{K}{s(1+s\tau)}\right\} = \frac{K(1-e^{-T/\tau})z^{-(N+1)}}{1-e^{-T/\tau}z^{-1}}$$

$\vdots$

$$D(z) = \underbrace{\frac{1}{K}\left(\frac{1-e^{-T/\tau}}{1-e^{-T/\tau}}\right)}_{K_d} \frac{1 \underbrace{-e^{-T/\tau}}_{\gamma} z^{-1}}{1 \underbrace{-e^{-T/\tau}}_{\alpha} z^{-1} \underbrace{-(1-e^{-T/\tau})}_{\beta} z^{-(N+1)}}$$

Integral action

$\Rightarrow$ pole @ $s=0$ or $z=1$

$1-e^{-T/\tau}z^{-1} - (1-e^{-T/\tau})z^{-(N+1)}$

$z=1$

$1-e^{-T/\tau} - (1-e^{-T/\tau})(1) = 0$

$\Rightarrow z=1$ is a root of the denominator of $D(z)$

$\Rightarrow$ pole of $D(z)$

$\Rightarrow$ integral action

Q 3 (a). $G(z) = \dfrac{\gamma z^{-2}}{1+\alpha z^{-1}+\beta z^{-2}} = \dfrac{\gamma}{z^2+\alpha z+\beta} = \dfrac{B}{A}$

second order process

$Q = z + q_1$

$S = s_0 z + s_1$

$T = t_0(z - p_1)$

3rd order desired char. equation

$A_{cl} = (\underbrace{z-p_1}_{\text{FAST}})\underbrace{(z-p_2)(z-p_3)}_{\text{2nd order dominance}}$

$A_{cl}(z) = z^3 + c_1 z^2 + c_2 z + c_3$

Diophantine pole placement eqn:

$AQ + BS = A_{cl}$

$(z^2+\alpha z+\beta)(z+q_1) + \gamma(s_0 z + s_1) = z^3 + c_1 z^2 + c_2 z + c_3$

compare coeffs of similar powers of z:

$z^2: q_1 + \alpha = c_1$

$z^1: \alpha q_1 + \beta + \gamma s_0 = c_2$

$z^0: \beta q_1 + \gamma s_1 = c_3$

In matrix form

$$\begin{bmatrix} 1 & 0 & 0 \\ \alpha & \gamma & 0 \\ \beta & 0 & \gamma \end{bmatrix} \begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} c_1 - \alpha \\ c_2 - \beta \\ c_3 \end{bmatrix}$$

The design eqn is:

$$\begin{bmatrix} q_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \alpha & \gamma & 0 \\ \beta & 0 & \gamma \end{bmatrix}^{-1} \begin{bmatrix} c_1 - \alpha \\ c_2 - \beta \\ c_3 \end{bmatrix}$$

Finally $T(z) = t_0(\underbrace{z-p_1}_{\text{FAST POLE}})$

where $t_0 = \lim_{z \to 1} \frac{(z-P_2)(z-P_3)}{\gamma}$

$$T(z) = \frac{(1-P_2)(1-P_3)}{\gamma}(z-P_1)$$

(b). $G(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z^{-1} + \ldots + b_m z^{-m}}{1 - a_1 z^{-1} - \ldots - a_n z^{-n}}$

This yields the diff eqn:

$$y(k) = a_1 y(k-1) \ldots + a_n y(k-n) + b_1 u(k-1) + \ldots + b_m u(k-m)$$

At k^{th} sample
$\hat{y}(n) = \hat{a}_1 y(k-1) \ldots \hat{b}_m u(k-m)$

$\hat{\Theta} = [\hat{a}_1 \ldots \hat{b}_m]^T$
Can be repeated for all valid data points

$$\underline{\hat{Y}}(k) = \begin{bmatrix} \vdots \\ \hat{y}(k-2) \\ \hat{y}(k-1) \\ \hat{y}(k) \end{bmatrix} = \left[\begin{array}{c|c} & \\ y(k-3) & u(k-3) \\ y(k-2) \ldots y(k-n-1) & u(k-2) \ldots u(k-m-1) \\ y(k-1) \ldots y(k-n) & u(k-1) \ldots u(k-m) \end{array}\right] \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{b}_m \end{bmatrix}$$

$\hat{Y} = \Phi \hat{\Theta}$ $\quad$ $\hat{Y}(k) = \Phi(u)\hat{\Theta}(u)$ $\quad$ $J = \sum_{i=0} e^2(k-i)$