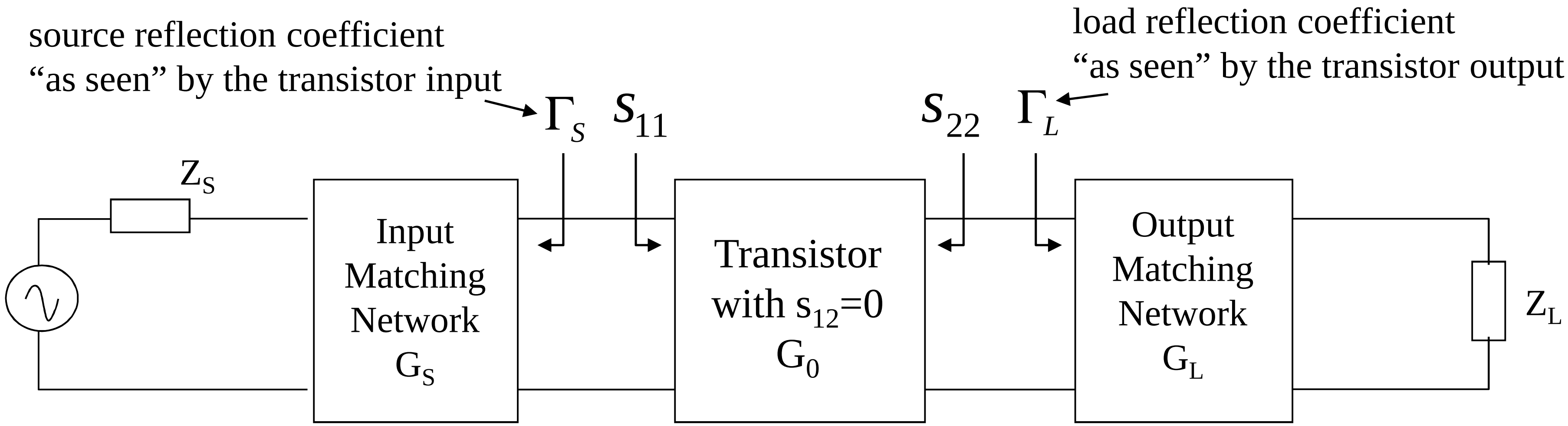
EE4011: RFIC Design

Gain Circles and Designing for a Specified Gain

Designing for Maximum Gain



Maximum unilateral transducer gain can be achieved by making a conjugate match between the source and the transistor input and between the load and the transistor output i.e.

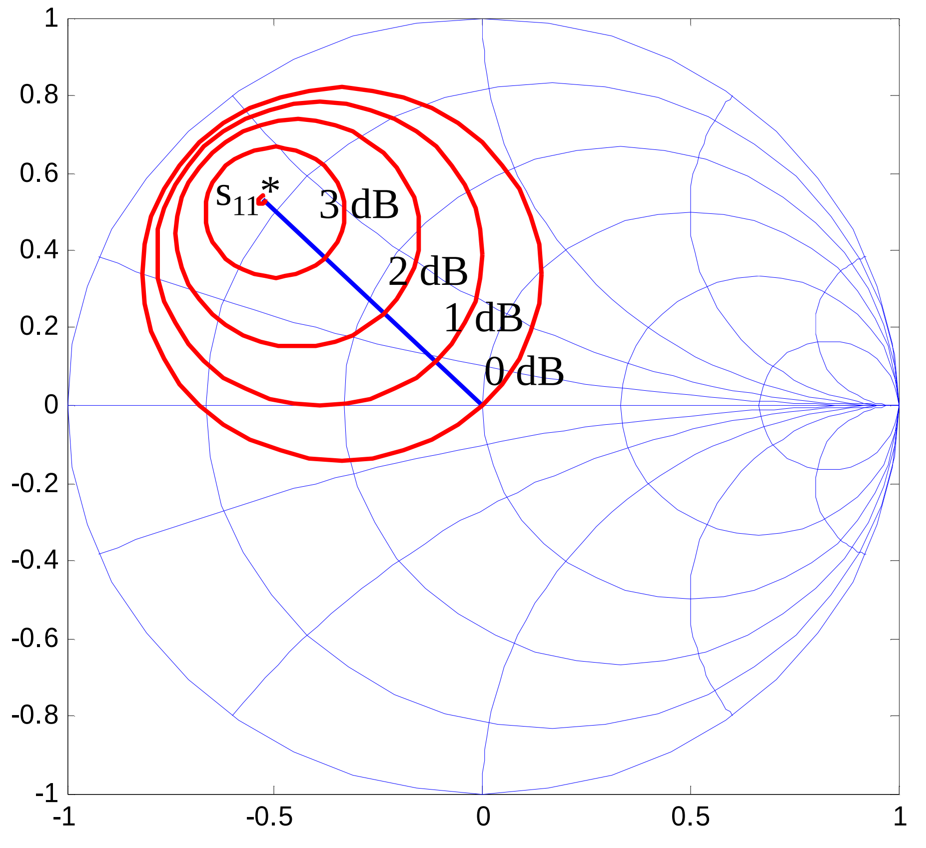
$$\Gamma_S = s_{11}^* \quad \Gamma_L = s_{22}^*$$

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} = G_{S,\max}G_O G_{L,\max} \quad G_{S,\max} = \frac{1}{1-|s_{11}|^2} \quad G_O = |s_{21}|^2 \quad G_{L,\max} = \frac{1}{1-|s_{22}|^2}$$

Sometimes we don't want the maximum possible gain – we want to design for a specific gain and sometimes other considerations such as noise or stability mean that we cannot set Γ_S to s_{11}* and Γ_L to s_{22}* - what happens then?

Dependence of Gain on Γ

The variation of gain with Γ_S and Γ_L can be seen by means of circles on the Smith Chart and these are a very useful design aid when we want to design for a specific gain or see how gain would degrade when the input and output reflection coefficients cannot be set to the maximum gain values due to other considerations such as noise or stability. The following are examples of source gain circles:



Normalised Source and Load Gains

The unilateral transducer power gain was defined as:

$$G_{TU}=\frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2}|s_{21}|^2\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}=G_S G_O G_L \quad G_S=\frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2} \quad G_O=|s_{21}|^2 \quad G_L=\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}$$

The maximum unilateral transducer power gain was:

$$G_{TU,\max}=\frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2}=G_{S,\max}G_O G_{L,\max} \quad G_{S,\max}=\frac{1}{1-|s_{11}|^2} \quad G_O=|s_{21}|^2 \quad G_{L,\max}=\frac{1}{1-|s_{22}|^2}$$

For this maximum gain condition, the source and load gain terms G_S and G_L are at their maximum values.

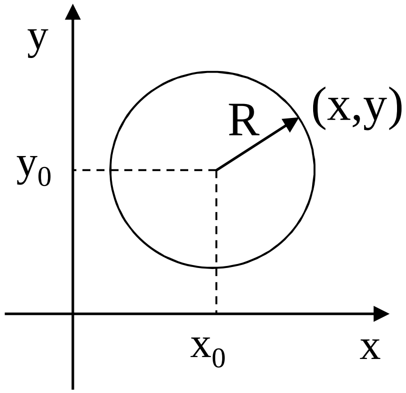
If Γ_S cannot be set to exactly s_{11}* and Γ_L cannot be set to exactly s_{22}* the gain will be less than its maximum value. The roll-off in gain as Γ_S moves away from s_{11}* and as Γ_L moves away from s_{22}* can be seen by defining normalised source and load gains and plotting circles on the Smith chart to represent the loci of reflection coefficients that give the same gain values.

$$g_s=\frac{G_S}{G_{S,\max}}=\frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2}\left(1-|s_{11}|^2\right) \qquad g_l=\frac{G_L}{G_{L,\max}}=\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}\left(1-|s_{22}|^2\right)$$

Note: by definition

$$g_s\le 1 \quad g_l\le 1$$

Equations for Circles on the Complex Plane

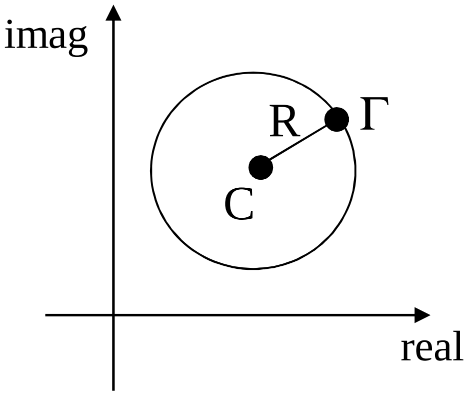


The equation of a circle in cartesian co-ordinates is:

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

where (x_0,y_0) is the centre of the circle and R is the radius

On the complex plane, if C is the centre of a circle and Γ is any point on the circle the equation of the circle can be written:



$$\left|C - \Gamma\right|^2 = R^2$$

Doing a little rearranging:

$$\left|C - \Gamma\right|^2 = (C - \Gamma)(C - \Gamma)^* = (C - \Gamma)(C^* - \Gamma^*)$$

$$= CC^* - C^*\Gamma - C\Gamma^* + \Gamma\Gamma^* = \left|C\right|^2 - C^*\Gamma - C\Gamma^* + \left|\Gamma\right|^2$$

So, the equation of a circle on the complex plane can also be written as:

$$\left|\Gamma\right|^2 - C^*\Gamma - C\Gamma^* + \left|C\right|^2 = R^2$$

If we can put any equation involving Γ into this format, it shows that the locus of the Γ points is a circle with centre C and radius R.

Gain Circles for the Input Matching Circuit (1)

The normalised gain of the source term was

$$g_s = \frac{G_S}{G_{S,\max}} = \frac{1-\left|\Gamma_S\right|^2}{\left|1-s_{11}\Gamma_S\right|^2}\left(1-\left|s_{11}\right|^2\right)$$

What values of Γ_S would give a constant g_s? Well…

$$g_s = \frac{1-\left|\Gamma_S\right|^2}{\left|1-s_{11}\Gamma_S\right|^2}\left(1-\left|s_{11}\right|^2\right)$$

$$\Rightarrow g_s\left|1-s_{11}\Gamma_S\right|^2 = \left(1-\left|\Gamma_S\right|^2\right)\left(1-\left|s_{11}\right|^2\right)$$

$$\Rightarrow g_s\left(1-s_{11}\Gamma_S\right)\left(1-s_{11}{}^*\Gamma_S{}^*\right) = \left(1-\left|\Gamma_S\right|^2\right)\left(1-\left|s_{11}\right|^2\right)$$

$$\Rightarrow g_s\left(1-s_{11}\Gamma_S - s_{11}{}^*\Gamma_S{}^* + \left|s_{11}\right|^2\left|\Gamma_S\right|^2\right) = 1-\left|\Gamma_S\right|^2 - \left|s_{11}\right|^2 + \left|\Gamma_S\right|^2\left|s_{11}\right|^2$$

$$\Rightarrow \left|\Gamma_S\right|^2\left(1-\left|s_{11}\right|^2\left(1-g_s\right)\right) - g_s s_{11}\Gamma_S - g_s s_{11}{}^*\Gamma_S{}^* = 1-\left|s_{11}\right|^2 - g_s$$

$$\Rightarrow \left|\Gamma_S\right|^2 - \frac{g_s s_{11}}{1-\left|s_{11}\right|^2\left(1-g_s\right)}\Gamma_S - \frac{g_s s_{11}{}^*}{1-\left|s_{11}\right|^2\left(1-g_s\right)}\Gamma_S{}^* = \frac{1-\left|s_{11}\right|^2 - g_s}{1-\left|s_{11}\right|^2\left(1-g_s\right)}$$

Gain Circles for the Input Matching Circuit (2)

$$\left|\Gamma_S\right|^2 - \frac{g_s s_{11}}{1-\left|s_{11}\right|^2(1-g_s)}\Gamma_S - \frac{g_s s_{11}^*}{1-\left|s_{11}\right|^2(1-g_s)}\Gamma_S^* = \frac{1-\left|s_{11}\right|^2 - g_s}{1-\left|s_{11}\right|^2(1-g_s)}$$

This looks similar to the circle formula $\left|\Gamma\right|^2 - C^*\Gamma - C\Gamma^* + \left|C\right|^2 = R^2$ with $C = \frac{g_s s_{11}^*}{1-\left|s_{11}\right|^2(1-g_s)}$

But to follow the circle formula a term $|C|^2$ has to be added to the L.H.S. (and thus to the R.H.S.):

$$\left|\Gamma_S\right|^2 - \frac{g_s s_{11}}{1-\left|s_{11}\right|^2(1-g_s)}\Gamma_S - \frac{g_s s_{11}^*}{1-\left|s_{11}\right|^2(1-g_s)}\Gamma_S^* + \frac{g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2} = \frac{1-\left|s_{11}\right|^2 - g_s}{1-\left|s_{11}\right|^2(1-g_s)} + \frac{g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2}$$

Looking at the RHS:

$$\frac{1-\left|s_{11}\right|^2 - g_s}{1-\left|s_{11}\right|^2(1-g_s)} + \frac{g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2} = \frac{\left(1-\left|s_{11}\right|^2 - g_s\right)\left(1-\left|s_{11}\right|^2(1-g_s)\right) + g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2}$$

$$= \frac{1-2\left|s_{11}\right|^2 + \left|s_{11}\right|^4 - g_s\left(1-2\left|s_{11}\right|^2 + \left|s_{11}\right|^4\right) - g_s^2\left|s_{11}\right|^2 + g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2} = \frac{(1-g_s)\left(1-\left|s_{11}\right|^2\right)^2}{\left(1-\left|s_{11}\right|^2(1-g_s)\right)^2} = \left(\frac{\sqrt{1-g_s}\left(1-\left|s_{11}\right|^2\right)}{1-\left|s_{11}\right|^2(1-g_s)}\right)^2$$

Gain Circles for the Input Matching Circuit (3)

Using the new expression for the RHS gives:

$$\left|\Gamma_S\right|^2 - \frac{g_s s_{11}}{1-\left|s_{11}\right|^2\left(1-g_s\right)}\Gamma_S - \frac{g_s s_{11}^*}{1-\left|s_{11}\right|^2\left(1-g_s\right)}\Gamma_S^* + \frac{g_s^2\left|s_{11}\right|^2}{\left(1-\left|s_{11}\right|^2\left(1-g_s\right)\right)^2} = \left(\frac{\sqrt{1-g_s}\left(1-\left|s_{11}\right|^2\right)}{1-\left|s_{11}\right|^2\left(1-g_s\right)}\right)^2$$

which directly corresponds to the circle formula $\left|\Gamma\right|^2 - C^*\Gamma - C\Gamma^* + \left|C\right|^2 = R^2$

with $C = \dfrac{g_s s_{11}^*}{1-\left|s_{11}\right|^2\left(1-g_s\right)}$ $\quad R = \dfrac{\sqrt{1-g_s}\left(1-\left|s_{11}\right|^2\right)}{1-\left|s_{11}\right|^2\left(1-g_s\right)}$ $\quad g_s = \dfrac{G_S}{G_{S,\max}}$

As g_s varies from 0 to 1, the centre of the resulting circles will lie along a line connecting the origin and the point s_{11}^* and the distance from the origin to the centre of the circle will be:

$$\left|C\right| = \frac{g_s\left|s_{11}\right|}{1-\left|s_{11}\right|^2\left(1-g_s\right)}$$

When g_s=1 the source gain term has its maximum value i.e. G_S=$G_{s,max}$ and $C = s_{11}^*$ $\quad R = 0$
i.e. the circle centre is at s_{11}^* and the radius is 0 which means that the maximum gain is only achieved for a single value of source reflection coefficient, Γ_S=S_{11}^*.

Usually, circles are plotted to show the gain dropping off in regular steps such as 0.1dB, 1dB, etc.

Examples of Constant Gain Circles

Draw constant gain circles for the source matching network for a transistor with $s_{11} = 0.75\angle -135°$

$s_{11} = 0.75\angle -135° \Rightarrow s_{11}^{\ *} = 0.75\angle 135°$ The maximum source gain occurs for $\Gamma_S = s_{11}^{\ *} = 0.75\angle 135°$

$$G_{S,\max} = \frac{1}{1-|s_{11}|^2} = \frac{1}{1-|0.75|^2} = 2.29 = 3.59dB$$

The maximum achievable gain from the source matching network in 3.59dB so we will plot gain circles for gains lower than the maximum value e.g. 3dB,2dB,1dB and 0dB. For each of these values, G_S has to be determined first, then g_s is determined and finally C and R are evaluated using the formulas.

$$G_{S,dB} = 10\log10(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} \quad g_s = \frac{G_S}{G_{S,\max}} \quad |C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} \quad R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)}$$

Doing the calculations:

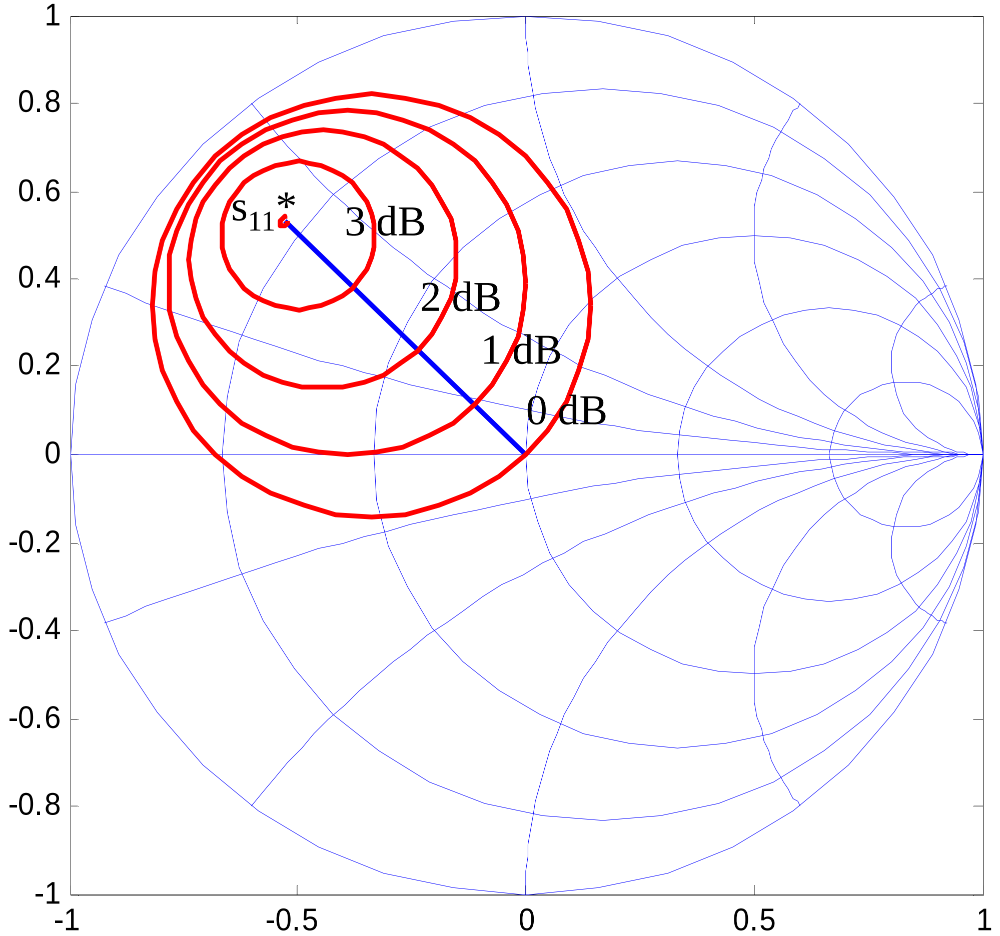
$$s_{11}^{\ *} = 0.75\angle 135° \quad G_{S,\max} = 3.59dB$$

$G_{S,dB}$	G_S	g_s	$\|C_S\|$	R_S
3	1.995	0.873	0.705	0.168
2	1.585	0.693	0.628	0.293
1	1.259	0.551	0.553	0.392
0	1	0.438	0.480	0.480

The subscript “S” in the various quantities indicate that the values apply to the source matching network.

Example Gain Circles

The source gain circles for the previous example are:



Note the 0dB circle goes through the origin of the Smith Chart. This can be shown to be generally the case. This 0 dB circle is the limit for gain. The gain of the source matching term is less than 1 (i.e. a loss) for reflection coefficients outside the 0dB circle.

Note as the radius of the gain circle increases, the gain decreases.

Gain Circles for the Load Matching Network

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2}|s_{21}|^2\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2} = G_S G_O G_L \quad G_S = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2} \quad G_O = |s_{21}|^2 \quad G_L = \frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}$$

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} = G_{S,\max}G_O G_{L,\max} \quad G_{S,\max} = \frac{1}{1-|s_{11}|^2} \quad G_O = |s_{21}|^2 \quad G_{L,\max} = \frac{1}{1-|s_{22}|^2}$$

Just like the source matching network, gain circles can be drawn to show the roll-off in gain when the output of the device is not terminated in a reflection coefficient equal to exactly s_{22}^*. The formulas for the centre and radius of the load matching gain circles are similar to those for the source matching network as follows:

$$g_l = \frac{G_L}{G_{L,\max}} = \frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}\left(1-|s_{22}|^2\right)$$

The subscript "L" in the various quantities indicate that the values apply to the load matching network.

$$C_L = \frac{g_l s_{22}^*}{1-|s_{22}|^2(1-g_l)} \qquad |C_L| = \frac{g_l|s_{22}|}{1-|s_{22}|^2(1-g_l)} \qquad R_L = \frac{\sqrt{1-g_l}\left(1-|s_{22}|^2\right)}{1-|s_{22}|^2(1-g_l)}$$

When $Z_S = Z_L = Z_0$

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2}|s_{21}|^2\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2} = G_S G_O G_L \quad G_S = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2} \quad G_O = |s_{21}|^2 \quad G_L = \frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}$$

If $Z_S = Z_0$ then $\Gamma_S = 0$ and if $Z_L = Z_0$ then $\Gamma_L = 0$ and in that case:

$$G_S = 1 \quad G_L = 1 \Rightarrow G_{TU} = G_S G_O G_L = |s_{21}|^2$$

This corresponds to the situation when the s-parameters of the device are being measured i.e. the device is connected using source and load impedances set to the system impedance Z_0 so that the source and load reflection coefficients will be zero and in that case $|s_{21}|^2$ corresponds directly to the measured transducer power gain.

Designing for a Specified Gain - 1

Previously we designed an amplifier to give the maximum possible gain at a given frequency by designed the input matching network to give a source reflection coefficient of s_{11}^* and designing the output matching network to give a load reflection coefficient of s_{22}^*.
Gain circles can be used to design for other gain values, apart from the maximum gain.

A BJT has the following s-parameters at 3GHz (at an appropriate bias condition):

$$s_{11} = 0.707\angle -155^\circ \quad s_{12} = 0 \quad s_{21} = 4\angle 180^\circ \quad s_{22} = 0.51\angle -20^\circ$$

If the source and load impedances are 50Ω design input and output matching networks to give a power gain of 15dB at 3GHz (note we are to design for a specified gain, not the maximum possible gain).

Before doing anything, check the stability:

$$\Delta = s_{11}s_{22} - s_{12}s_{21} = 0.36\angle -175^\circ \quad K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} = \infty$$

$$K > 1 \quad and \quad |\Delta| < 1$$

The device is unconditionally stable so the source and load matching networks can be designed without any restrictions due to stability.

Designing for a Specified Gain - 2

$$s_{11} = 0.707\angle -155^\circ \quad s_{12} = 0 \quad s_{21} = 4\angle 180^\circ \quad s_{22} = 0.51\angle -20^\circ$$

Determine the maximum unilateral transducer gain:

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} = \frac{1}{1-|0.707|^2}|4|^2\frac{1}{1-|0.51|^2} = 2\times 16\times 1.35 = 43.2$$

In dB:

$$G_{TU,\max} = 10\log_{10}(2) + 10\log_{10}(16) + 10\log_{10}(1.35) = 3 + 12 + 1.3 = 16.3dB$$

Note: the transistor is unilateral in this case so there is no error in assuming it is unilateral! i.e. M=0.

Here $G_{S,max}$=3dB and $G_{L,max}$=1.4dB. The maximum possible gain is 16.4dB but only a gain of 15dB is required.

The forward power gain of the transistor $|s_{21}|^2$ cannot be changed but the input and output matching networks can be moved away from their optimum gain points to give a slightly lower gain. Many options are available but we'll choose:

$$G_S = 2dB, \quad G_L = 1dB \qquad \text{giving} \qquad G = G_S + 12 + G_L = 2 + 12 + 1 = 15dB$$

To determine appropriate values for the source and load matching networks, the 2dB gain circle for the source matching network and the 1dB gain circle for the load matching network have to be drawn.

Designing for a Specified Gain - 3

Identifying the gain circle for $G_{S,dB} = 2$

$$G_{S,dB} = 10\log_{10}(G_S) \Rightarrow G_S = 10^{\frac{G_{S,dB}}{10}} = 10^{\frac{2}{10}} = 1.585 \quad g_s = \frac{G_S}{G_{S.\max}} = \frac{1.585}{2} = 0.793$$

$$|C_S| = \frac{g_s|s_{11}|}{1-|s_{11}|^2(1-g_s)} = \frac{0.793\times|0.707|}{1-|0.707|^2(1-0.793)} = 0.63$$

$$R_S = \frac{\sqrt{1-g_s}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.793}\left(1-|0.707|^2\right)}{1-|0.707|^2(1-0.793)} = 0.25$$

The centre of the 2dB source gain circle is a distance 0.63 along the line joining the origin and the point s_{11}^* and its radius is 0.25

Identifying the gain circle for $G_{L,dB} = 1$

$$G_{L,dB} = 10\log_{10}(G_L) \Rightarrow G_L = 10^{\frac{G_{L,dB}}{10}} = 10^{\frac{1}{10}} = 1.259 \quad g_L = \frac{G_L}{G_{L,\max}} = \frac{1.259}{1.35} = 0.93$$

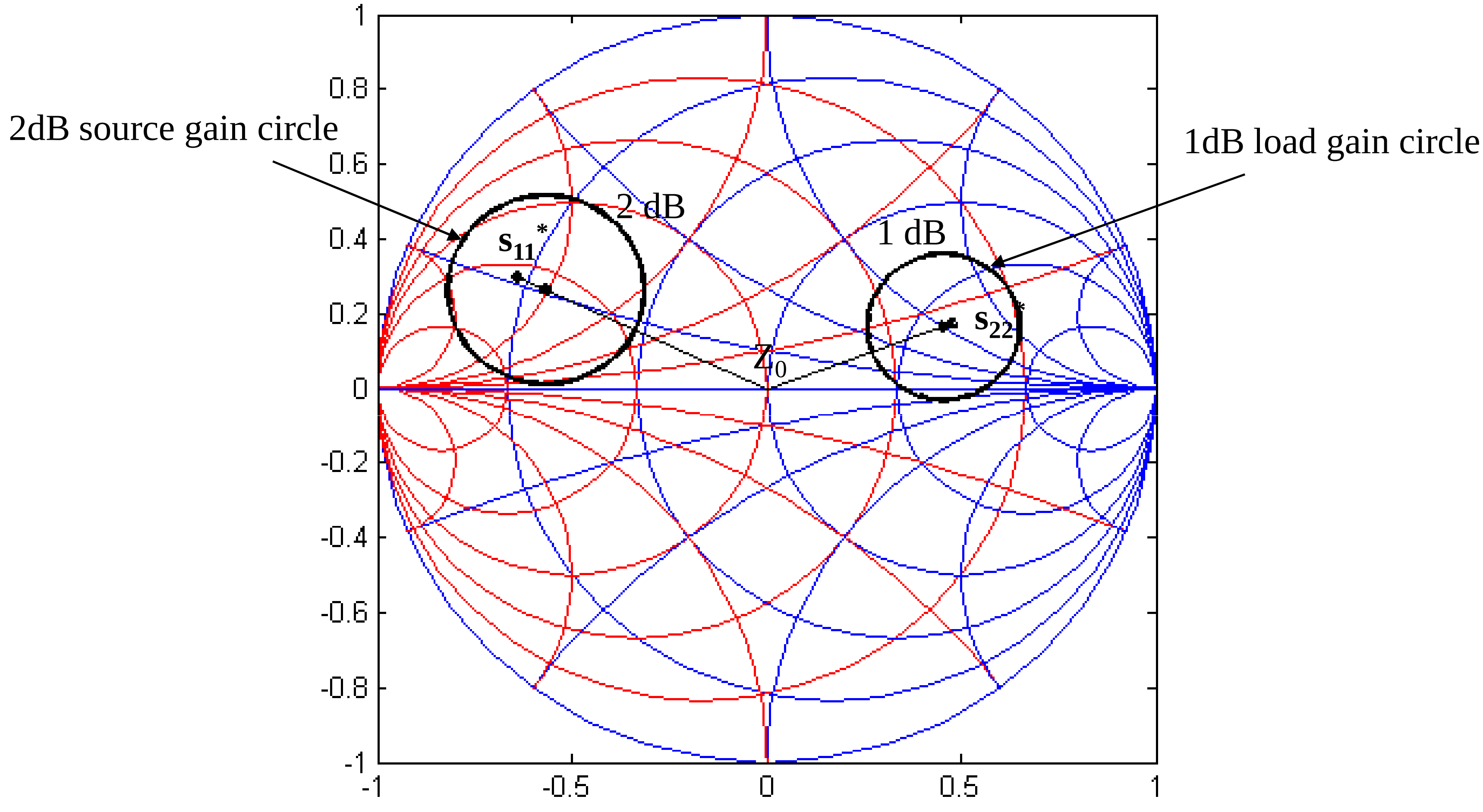
$$|C_L| = \frac{g_L|s_{22}|}{1-|s_{22}|^2(1-g_L)} = \frac{0.93\times|0.51|}{1-|0.51|^2(1-0.93)} = 0.48$$

$$R_L = \frac{\sqrt{1-g_L}\left(1-|s_{22}|^2\right)}{1-|s_{22}|^2(1-g_L)} = \frac{\sqrt{1-0.93}\left(1-|0.51|^2\right)}{1-|0.51|^2(1-0.93)} = 0.2$$

The centre of the 1dB load gain circle is a distance 0.48 along the line joining the origin and the point s_{22}^* and its radius is 0.2

Designing for a Specified Gain - 4

Smith Chart showing the 2dB source gain circle and the 1dB load gain circle

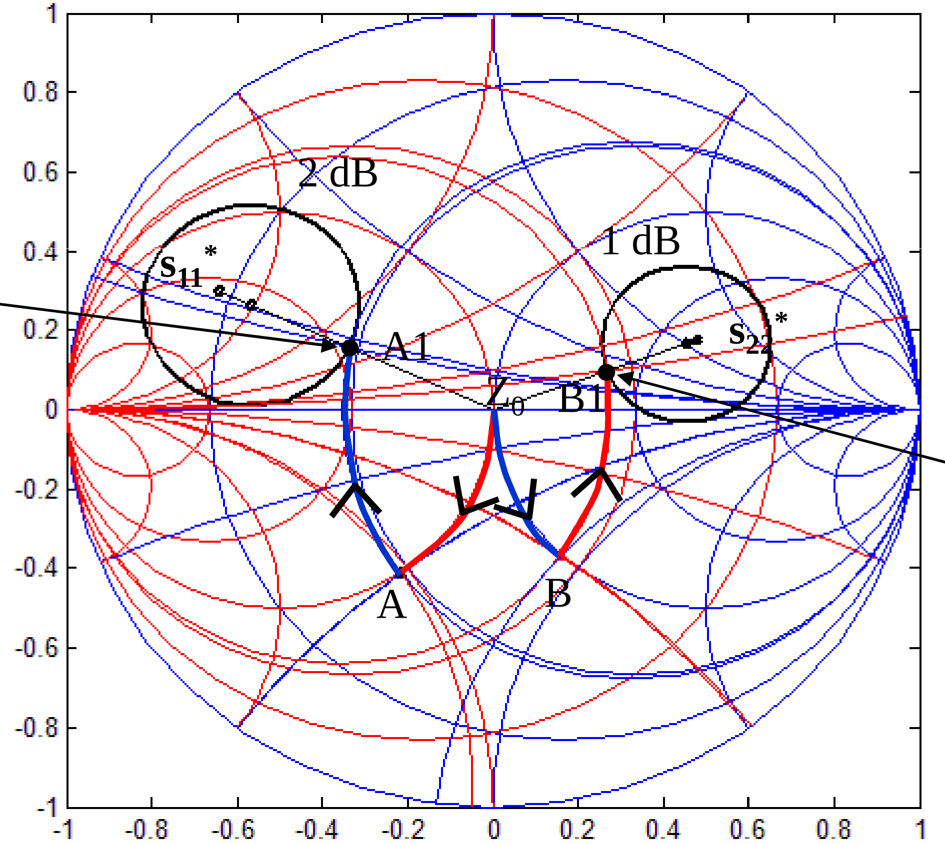


The input reflection coefficient can be any point on the source 2dB circle. The output reflection coefficient can be any point on the load 1dB circle. So there's a big choice of reflection coefficients and thus matching networks.

Designing for a Specified Gain - 5

The reflection coefficients can be moved to any point on the desired gain circles. However, the most common approach is to move the reflection coefficients by the minimum amount necessary. In this example the source and load reflection coefficients are both equal to Z_0 so the reflection coefficients can be moved to the intersection between the source and load gain circles and the lines from the origin to s_{11}* and s_{22}* respectively. This approach minimizes the size of the components needed for the matching elements.

Nearest point from the origin to the edge of the 2 dB source gain circle.



Nearest point from the origin to the edge of the 1 dB load gain circle.

Designing for a Specified Gain - 6

Input (Source) Matching Element Values

Moving from Z_0 (Γ=0) to point A: Clockwise on conductance circle – shunt capacitor

susceptance at Z_0: b = 0
susceptance at A: b = 1.049

$$C = \frac{|\Delta b|}{2\pi f Z_0} = \frac{1.049}{2\pi \times 3 \times 10^9 \times 50} = 1.12\, pF$$

Moving from A to A1: Clockwise on resistance circle – series inductor

reactance at A: x = -0.5
reactance at A1: x = 0.173

$$L = \frac{Z_0 |\Delta x|}{2\pi f} = \frac{50 \times |0.673|}{2\pi \times 3 \times 10^9} = 1.78\, nH$$

Output (Load) Matching Element Values

Moving from Z_0 (Γ=0) to point B: Anti-clockwise on resistance circle – series capacitor

reactance at Z_0: x = 0
reactance at B: x = -0.875

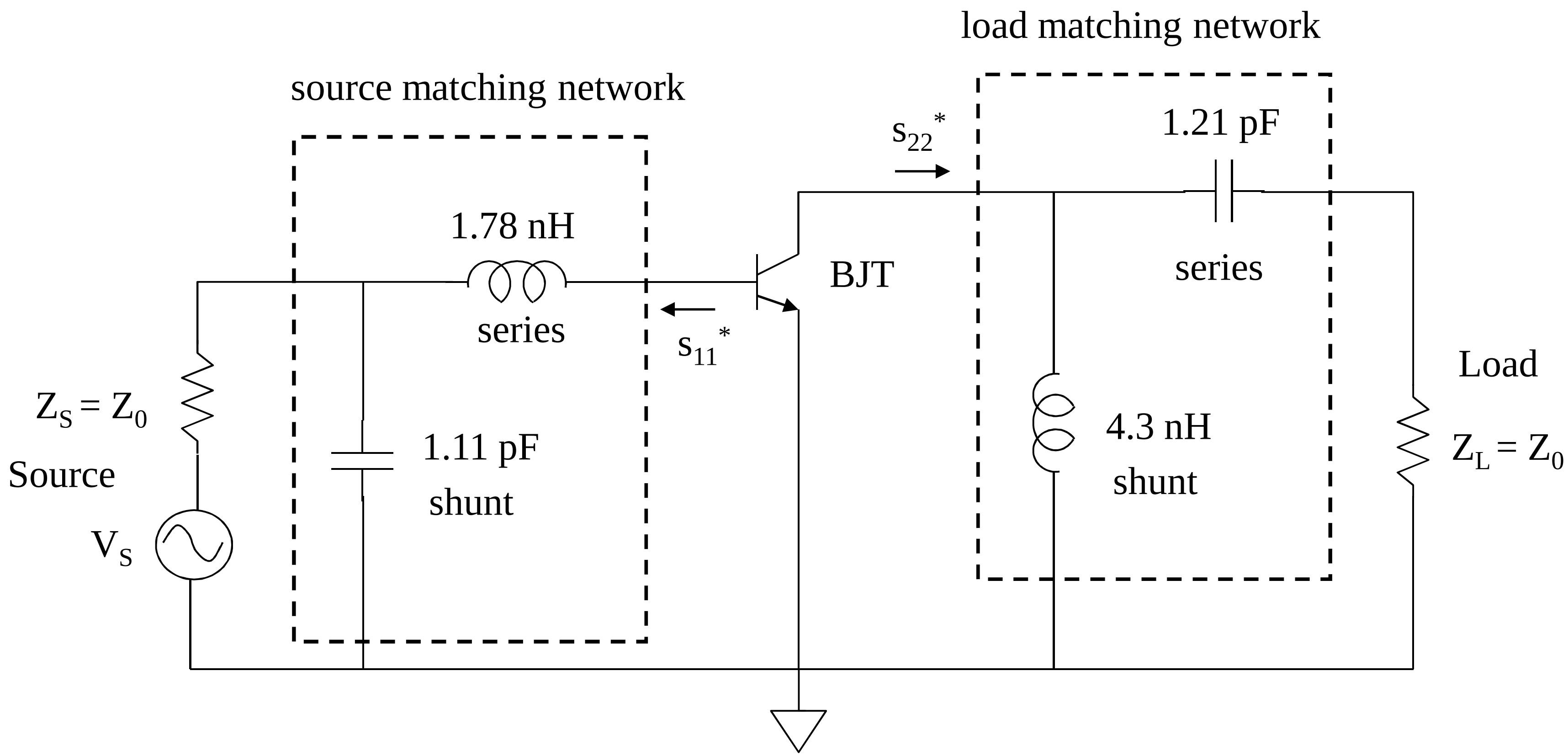
$$C = \frac{1}{2\pi f |\Delta x| Z_0} = \frac{1}{2\pi \times 3 \times 10^9 \times |0.875| \times 50} = 1.21\, pF$$

Moving from B to B1: Anti-clockwise on conductance circle – shunt inductor

susceptance at B: b = 0.496
susceptance at B1: b = -0.121

$$L = \frac{Z_0}{2\pi f |\Delta b|} = \frac{50}{2\pi \times 3 \times 10^9 \times |0.617|} = 4.3\, nH$$

Designing for a Specified Gain - 7



Example for you to try

A GaAs MESFET has the following s-parameters at 3GHz (at V_{DS}=5V, I_{DS}=10mA):

$$s_{11} = 0.38\angle -169^\circ \quad s_{12} = 0 \quad s_{21} = 1.33\angle -39^\circ \quad s_{22} = 0.95\angle -66^\circ$$

If the source and load impedances are 50Ω design input and output matching networks to give a power gain of 6dB at 3GHz.