EE4011: RF IC Design

Stability Considerations for LNA Design

Checklist for LNAs and Smith Charts

Stability

Certain combinations of source and load impedances may result in unwanted oscillations of amplifier circuits depending on the parameters of the active device, making the circuits useless as amplifiers.
In particular, if the real part of the input resistance or the real part of the output resistance of the two-port circuit is negative, the circuit will have a tendency to oscillate. This is equivalent to the condition where the magnitude of the input or output reflection coefficient is greater than 1.
The two-port is unconditionally stable if Re{Z_{IN}} > 0 and Re{Z_{OUT}} > 0, for all positive real source and load impedances. This is equivalent to the condition:

$$\left|\Gamma_{IN}\right| \leq 1 \quad \left|\Gamma_{OUT}\right| \leq 1$$

A common criterion for stability is defined using the Rollet stability factor, K, and the factor Δ as follows

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} \qquad \Delta = s_{11}s_{22} - s_{12}s_{21}$$

The device is *unconditionally stable* if:

$$K > 1 \quad and \quad |\Delta| < 1$$

The device is *conditionally stable* (i.e. *potentially unstable*) if:

$$K < 1 \quad and \quad |\Delta| < 1 \quad or \quad K > 1 \quad and \quad |\Delta| > 1$$

If the device is only conditionally stable, stability circles can be used to indicate the range of source and load reflection coefficients for which stability is achievable.

Load Stability Circles - 1

The source and load reflection coefficients which give rise to stable (and unstable) conditions can be identified by drawing circles on the Smith chart. The input reflection coefficient of the two-port is:

$$\Gamma_{IN} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L} = \frac{s_{11}\left(1-s_{22}\Gamma_L\right)+s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L} = \frac{s_{11}-\Delta\Gamma_L}{1-s_{22}\Gamma_L}$$

One of the criteria for the loaded network to be unconditionally stable is that $\left|\Gamma_{IN}\right| < 1$

The limit of the stable region can be determined by setting $\left|\Gamma_{IN}\right| = 1$ i.e.

$$\left|\frac{s_{11}-\Delta\Gamma_L}{1-s_{22}\Gamma_L}\right| = 1 \Rightarrow \left|\frac{s_{11}-\Delta\Gamma_L}{1-s_{22}\Gamma_L}\right|^2 = 1 \Rightarrow \left|s_{11}-\Delta\Gamma_L\right|^2 = \left|1-s_{22}\Gamma_L\right|^2$$

$$\Rightarrow \left(s_{11}-\Delta\Gamma_L\right)\left(s_{11}-\Delta\Gamma_L\right)^* = \left(1-s_{22}\Gamma_L\right)\left(1-s_{22}\Gamma_L\right)^*$$

$$\Rightarrow \left(s_{11}-\Delta\Gamma_L\right)\left(s_{11}^*-\Delta^*\Gamma_L^*\right) = \left(1-s_{22}\Gamma_L\right)\left(1-s_{22}^*\Gamma_L^*\right)$$

$$\Rightarrow \left|s_{11}\right|^2 - \Delta\Gamma_L s_{11}^* - \Delta^*\Gamma_L^* s_{11} + \left|\Delta\right|^2\left|\Gamma_L\right|^2 = 1 - s_{22}\Gamma_L - s_{22}^*\Gamma_L^* + \left|s_{22}\right|^2\left|\Gamma_L\right|^2$$

$$\Rightarrow \left|\Gamma_L\right|^2\left(\left|\Delta\right|^2 - \left|s_{22}\right|^2\right) + \left(s_{22}-\Delta s_{11}^*\right)\Gamma_L + \left(s_{22}^*-\Delta^* s_{11}\right)\Gamma_L^* = 1 - \left|s_{11}\right|^2$$

$$\Rightarrow \left|\Gamma_L\right|^2\left(\left|s_{22}\right|^2 - \left|\Delta\right|^2\right) - \left(s_{22}-\Delta s_{11}^*\right)\Gamma_L - \left(s_{22}^*-\Delta^* s_{11}\right)\Gamma_L^* = \left|s_{11}\right|^2 - 1$$

$$\Rightarrow \left|\Gamma_L\right|^2 - \frac{s_{22}-\Delta s_{11}^*}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L - \frac{s_{22}^*-\Delta^* s_{11}}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L^* = \frac{\left|s_{11}\right|^2-1}{\left|s_{22}\right|^2-\left|\Delta\right|^2}$$

Load Stability Circles - 2

Recall the expression for a circle on the complex plane: $\left|\Gamma\right|^2 - C^*\Gamma - C\Gamma^* + \left|C\right|^2 = R^2$

The expression on the previous slide looks similar to this except that a term $|C|^2=CC^*$ has to be added to each side i.e.:

$$\left|\Gamma_L\right|^2 - \frac{s_{22}-\Delta s_{11}^{\ *}}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L - \frac{s_{22}^{\ *}-\Delta^* s_{11}}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L^{\ *} + \frac{\left(s_{22}-\Delta s_{11}^{\ *}\right)\left(s_{22}^{\ *}-\Delta^* s_{11}\right)}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2} = \frac{\left|s_{11}\right|^2-1}{\left|s_{22}\right|^2-\left|\Delta\right|^2} + \frac{\left(s_{22}-\Delta s_{11}^{\ *}\right)\left(s_{22}^{\ *}-\Delta^* s_{11}\right)}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2}$$

Looking at the R.H.S. terms:

$$\frac{\left|s_{11}\right|^2-1}{\left|s_{22}\right|^2-\left|\Delta\right|^2} + \frac{\left(s_{22}-\Delta s_{11}^{\ *}\right)\left(s_{22}^{\ *}-\Delta^* s_{11}\right)}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2} = \frac{\left(\left|s_{11}\right|^2-1\right)\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)+\left(s_{22}-\Delta s_{11}^{\ *}\right)\left(s_{22}^{\ *}-\Delta^* s_{11}\right)}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2}$$

Looking at the numerator of the new R.H.S. term:

$$\begin{aligned}
&\left(\left|s_{11}\right|^2-1\right)\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)+\left(s_{22}-\Delta s_{11}^{\ *}\right)\left(s_{22}^{\ *}-\Delta^* s_{11}\right)\\
&=\left|s_{11}\right|^2\left|s_{22}\right|^2-\left|s_{22}\right|^2-\left|s_{11}\right|^2\left|\Delta\right|^2+\left|\Delta\right|^2+\left|s_{22}\right|^2-\Delta s_{11}^{\ *}s_{22}^{\ *}-\Delta^* s_{11}s_{22}+\left|\Delta\right|^2\left|s_{11}\right|^2\\
&=\left|s_{11}\right|^2\left|s_{22}\right|^2-\Delta s_{11}^{\ *}s_{22}^{\ *}-\Delta^* s_{11}s_{22}+\left|\Delta\right|^2\\
&=\left(s_{11}s_{22}-\Delta\right)\left(s_{11}^{\ *}s_{22}^{\ *}-\Delta^*\right)=\left(s_{11}s_{22}-\Delta\right)\left(s_{11}s_{22}-\Delta\right)^*=\left|s_{11}s_{22}-\Delta\right|^2\\
&=\left|s_{11}s_{22}-\left(s_{11}s_{22}-s_{12}s_{21}\right)\right|^2=\left|s_{12}s_{21}\right|^2
\end{aligned}$$

Source and Load Stability Circles

The full equation is then:

$$\left|\Gamma_L\right|^2 - \frac{s_{22}-\Delta s_{11}^{\,*}}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L - \frac{s_{22}^{\,*}-\Delta^* s_{11}}{\left|s_{22}\right|^2-\left|\Delta\right|^2}\Gamma_L^{\,*} + \frac{\left(s_{22}-\Delta s_{11}^{\,*}\right)\left(s_{22}^{\,*}-\Delta^* s_{11}\right)}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2} = \frac{\left|s_{12}s_{21}\right|^2}{\left(\left|s_{22}\right|^2-\left|\Delta\right|^2\right)^2}$$

This is the equation of a circle with centre and radius as follows:

$$CS_L = \frac{s_{22}^{\,*}-\Delta^* s_{11}}{\left|s_{22}\right|^2-\left|\Delta\right|^2} \qquad RS_L = \frac{\left|s_{12}s_{21}\right|}{\left|\left|s_{22}\right|^2-\left|\Delta\right|^2\right|}$$

A variety of notation is used to denote the centre and radius of the stability circles associated with the load and source networks. Here I'm using "CS" and "RS" to mean the centre and radius, respectively, of the stability circle and the subscript "L" to denote the load network.
A similar analysis of the output reflection coefficient leads to formulas for the circle which gives the boundary of stability for the source network:

$$\Gamma_{OUT} = s_{22} + \frac{s_{12}s_{21}\Gamma_S}{1-s_{11}\Gamma_S} = \frac{s_{22}\left(1-s_{11}\Gamma_S\right)+s_{12}s_{21}\Gamma_S}{1-s_{11}\Gamma_S} = \frac{s_{22}-\Delta\Gamma_S}{1-s_{11}\Gamma_S}$$

Using $\left|\Gamma_{OUT}\right| = 1$ gives:

$$CS_S = \frac{s_{11}^{\,*}-\Delta^* s_{22}}{\left|s_{11}\right|^2-\left|\Delta\right|^2} \qquad RS_S = \frac{\left|s_{12}s_{21}\right|}{\left|\left|s_{11}\right|^2-\left|\Delta\right|^2\right|}$$

Here the subscript "S" refers to the source network.

Example Stability Circles

$$s_{11} = 0.64\angle -170°$$

$$s_{12} = 0.05\angle 15°$$

$$s_{21} = 2.10\angle 30°$$

$$s_{22} = 0.57\angle -95°$$

$$\Delta = s_{11}s_{22} - s_{12}s_{21} = 0.30\angle 110°$$

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} = 1.72$$

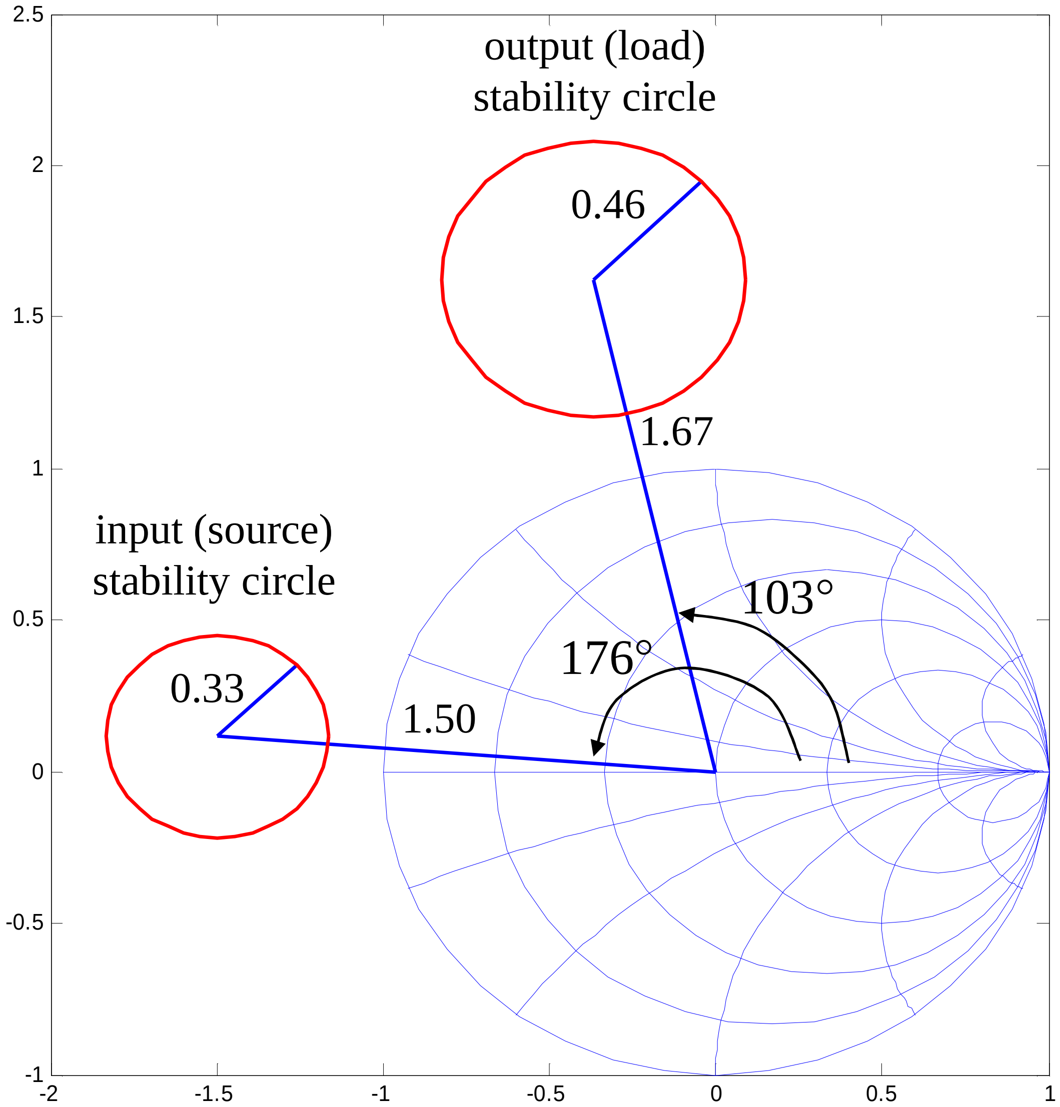
K > 1 and |Δ| < 1 so the device is unconditionally stable.

$$CS_S = \frac{s_{11}^{\ *} - \Delta^* s_{22}}{|s_{11}|^2 - |\Delta|^2} = 1.50\angle 176°$$

$$RS_S = \frac{|s_{12}s_{21}|}{\left||s_{11}|^2 - |\Delta|^2\right|} = 0.33$$

$$CS_L = \frac{s_{22}^{\ *} - \Delta^* s_{11}}{|s_{22}|^2 - |\Delta|^2} = 1.67\angle 103°$$

$$RS_L = \frac{|s_{12}s_{21}|}{\left||s_{22}|^2 - |\Delta|^2\right|} = 0.46$$



More Example Stability Circles

$$s_{11} = 0.92\angle -69°$$

$$s_{12} = 0.047\angle 43°$$

$$s_{21} = 2.34\angle 112°$$

$$s_{22} = 0.63\angle -52°$$

$$\Delta = s_{11}s_{22} - s_{12}s_{21} = 0.58\angle -110°$$

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} = 0.42$$

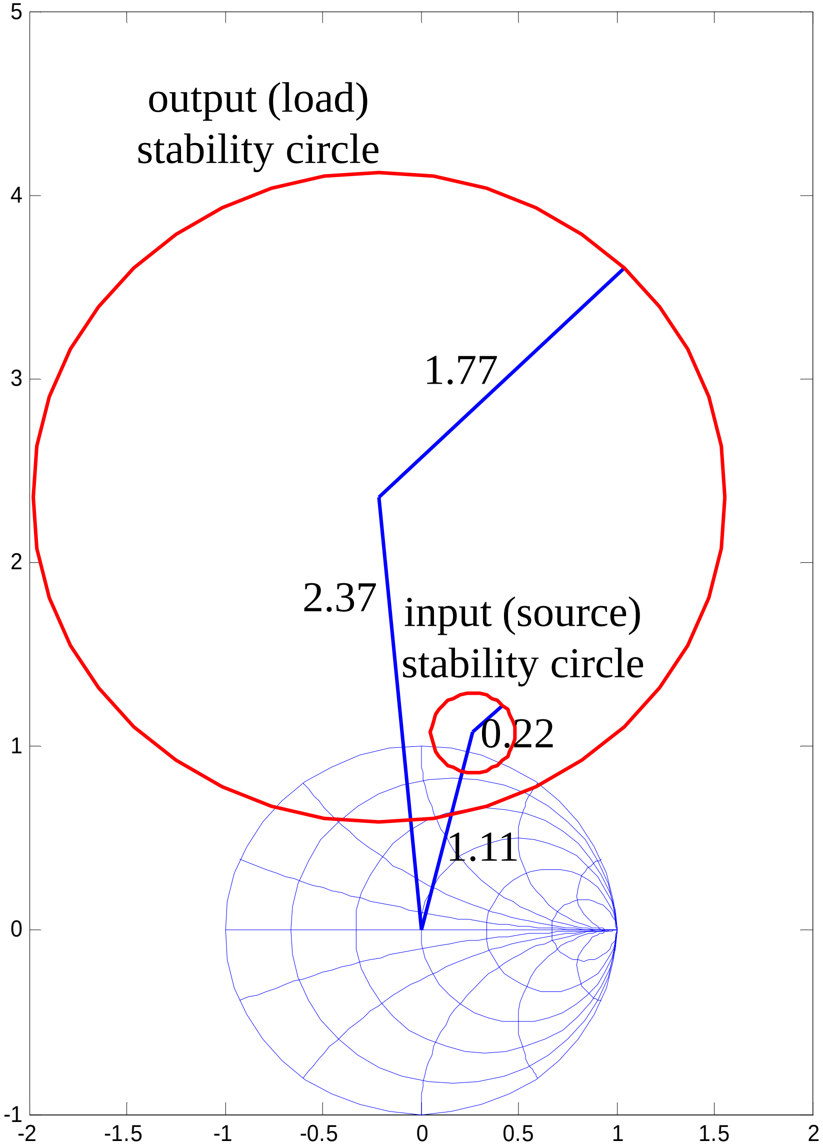
K <1 and |Δ| < 1 so the device is ***conditionally stable***.

$$CS_S = \frac{s_{11}^* - \Delta^* s_{22}}{|s_{11}|^2 - |\Delta|^2} = 1.11\angle 76°$$

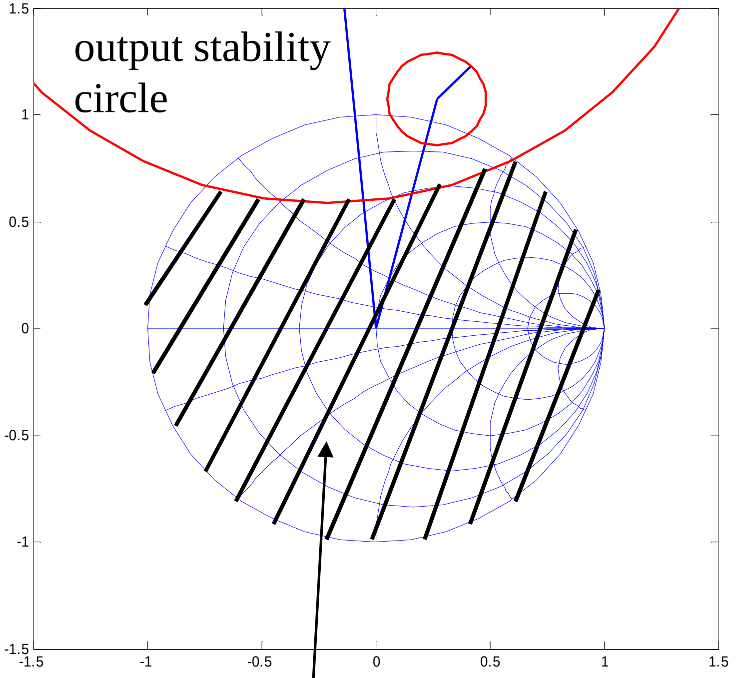
$$RS_S = \frac{|s_{12}s_{21}|}{\left||s_{11}|^2 - |\Delta|^2\right|} = 0.22$$

$$CS_L = \frac{s_{22}^* - \Delta^* s_{11}}{|s_{22}|^2 - |\Delta|^2} = 2.37\angle 95°$$

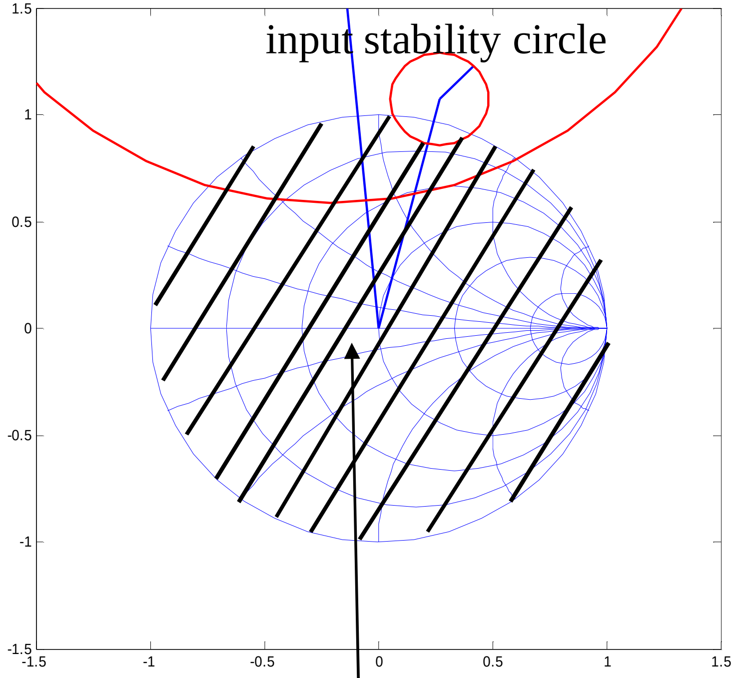
$$RS_L = \frac{|s_{12}s_{21}|}{\left||s_{22}|^2 - |\Delta|^2\right|} = 1.77$$



In practice (at least for $|s_{11}| < 0$ and $|s_{22}| < 0$) only the portion of the stability circles that overlap with the unit circle on the Smith chart are important and need to be drawn.



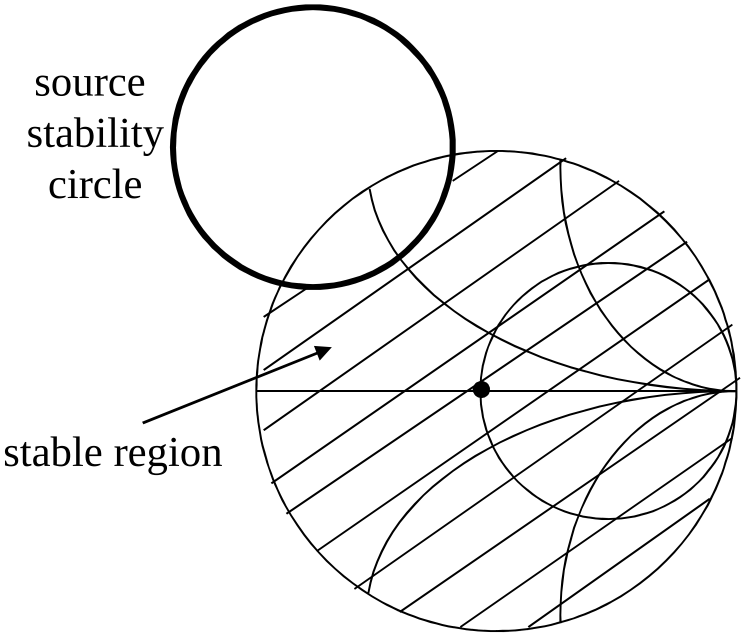
stable operating region
for the output matching network
(inside the Smith chart but outside
the output stability circle)



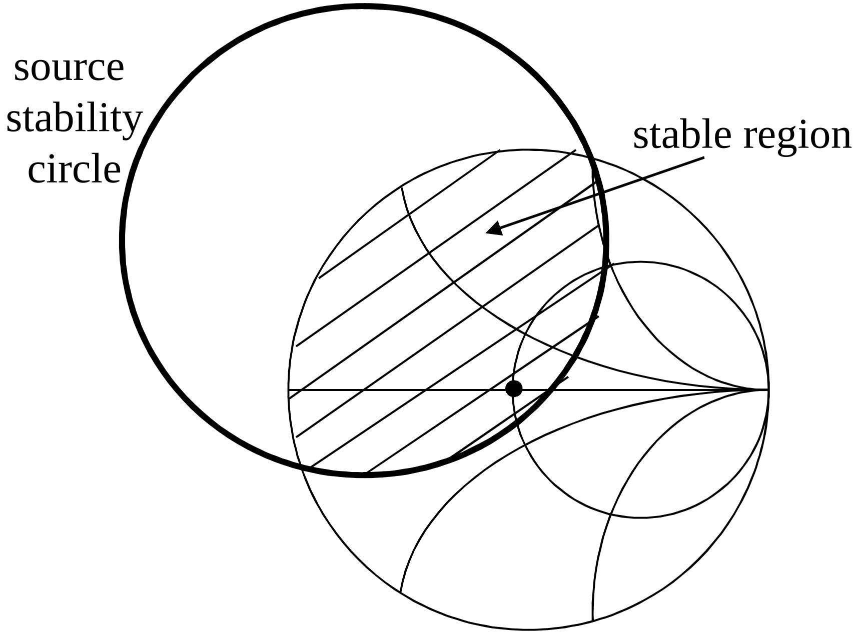
stable operating region
for the input matching network
(inside the Smith chart but outside
the input stability circle)

Which Side of the Circles are Stable?

The regions of stability depend on whether the centre of the Smith Chart falls inside the stability circles and also on s_{11} and s_{22}. The centre of the Smith chart corresponds to the measurement conditions for the s-parameters (i.e. the source and load impedances are set to Z_0 giving $\Gamma_S=\Gamma_L=0$).
The device is stable if $|s_{11}| < 1$ and $|s_{22}| < 1$ and in this case the centre of the Smith Chart will correspond to stable operation. If the centre of the Smith chart falls within the stability circles then it is the area inside the stability circles that is stable. If the centre of the Smith chart is outside the stability circles then it is the area outside the stability circle that is stable.
In the diagrams below, those areas which are stable and inside the Smith chart are crosshatched.

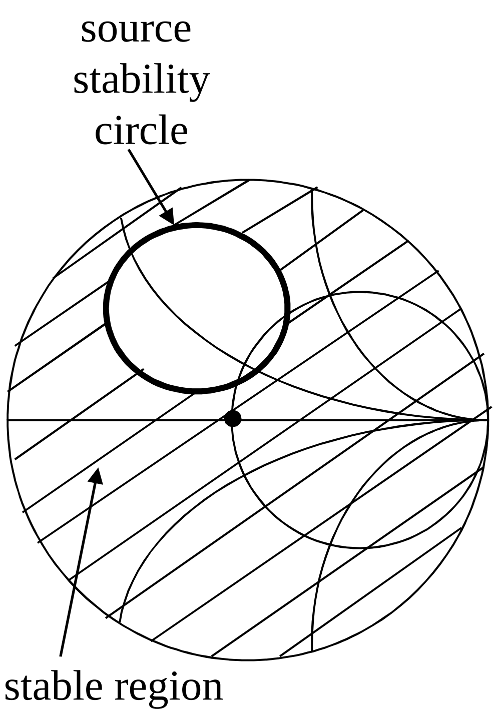


$$\left|s_{11}\right| < 1$$

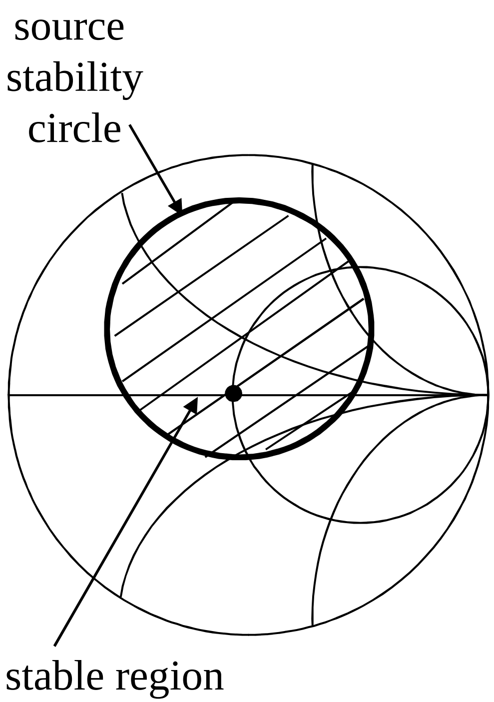


$$\left|s_{11}\right| < 1$$

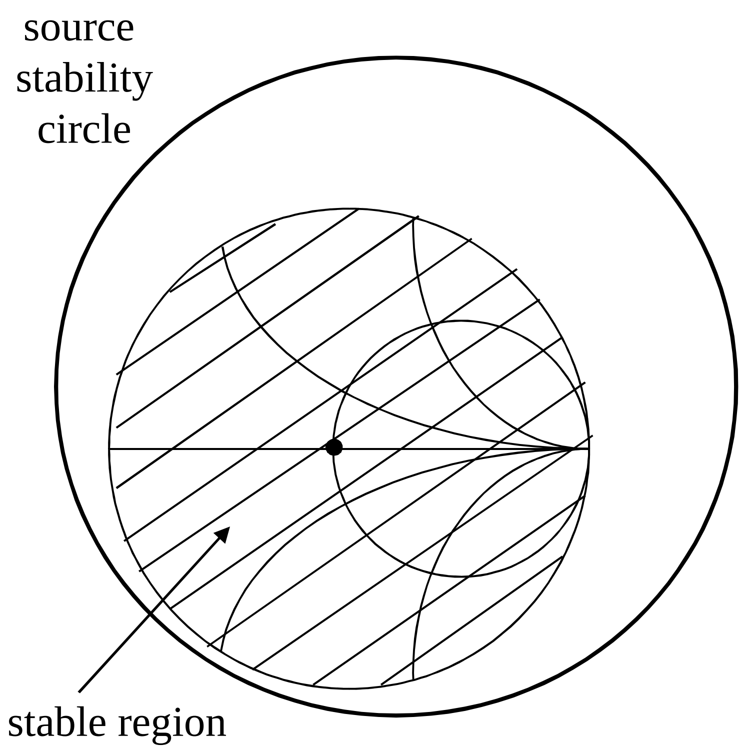
More Stability Circle Scenarios for $|s_{11}| < 1$



$$|s_{11}| < 1$$



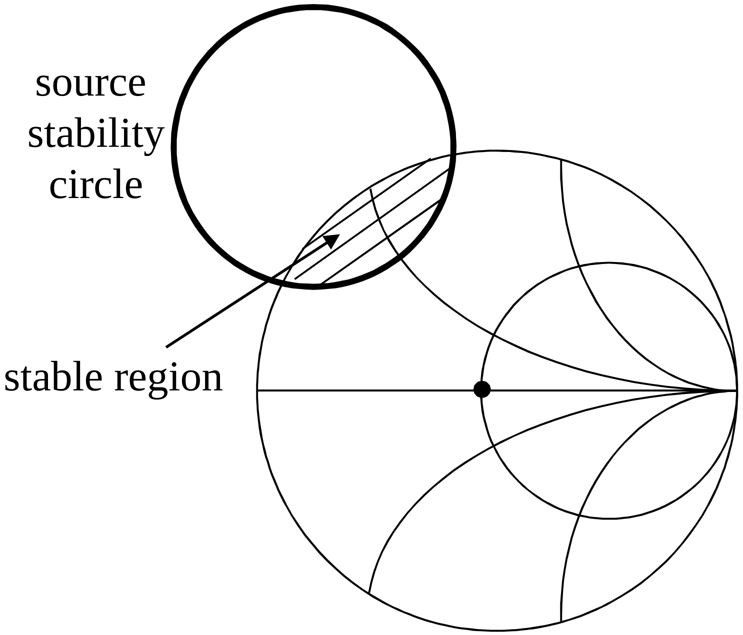
$$|s_{11}| < 1$$



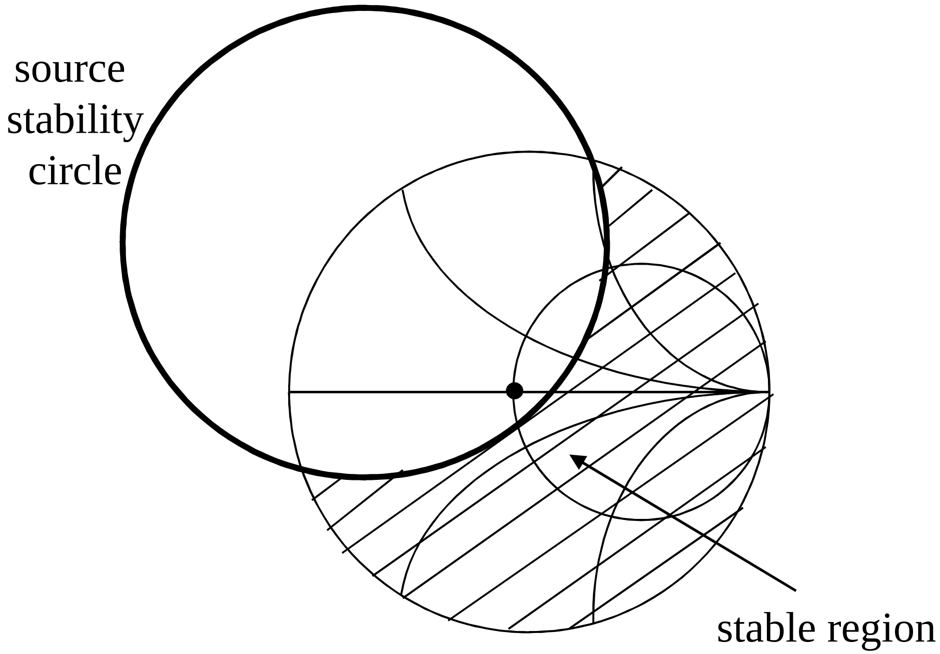
$$|s_{11}| < 1$$

Other Stability Scenarios

Load stability circles can also be drawn and follow the same behaviour for $|s_{22}| < 1$ as seen previously with the source stability circles with $|s_{11}| < 1$.
What about if $|s_{11}| > 1$ or $|s_{22}| > 1$? In this case the device is unstable during the s-parameter measurement and the centre of the Smith chart corresponds to unstable behaviour. The stability regions are then the opposite way around compared to the patterns already seen e.g.



$$|s_{11}| > 1$$



$$|s_{11}| > 1$$

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Checklist for Smith Charts/LNAs

Checklist for Studying LNAs and the Smith Chart (1)

1. Definition of reflection coefficient. Formula for Γ in terms of Z and Z_0.
2. Incident and reflected quantities on transmission line – relationship of total voltage and current to incident and reflected values
3. Characteristic impedance Z_0 and VSWR ("viswar").
4. Wave quantities a and b and power flow in terms of a and b
5. Familiarity with Smith Chart – showing reflection coefficients on the complex plane.
6. Familiarity with impedance, admittance and immittance versions of Smith Chart
7. Familiarity with characteristic reflection curves for different types of loads vs. frequency – series and parallel combinations of resistance and inductor or resistor and capacitor.
8. Familiarity with reflection coefficient of standard loads: Z_0, short circuit, open circuit.
9. For a given point on Smith Chart, know how to read off normalized resistance, conductance, reactance and susceptance for this point (including which parts of the Smith Chart correspond to positive and negative reactance and susceptance) and convert these to resistance, conductance, inductance or capacitance values at a given frequency.
10. Know the definitions of the s-parameters for a two-port.
11. Know the signal flow diagram for a one-stage amplifier.

Checklist for Studying LNAs and the Smith Chart (2)

12. Know the definitions in words of the various types of power gain that can be defined for a two-port:
    G_P - operational gain (also just called gain), G_T - transducer gain and G_A - available gain.
13. Familiarity with concept of power available from the source and power available from the amplifier.
14. Be able to derive the formulas for the different gain starting with an appropriate signal flow diagram.
15. Be able to derive formulas for the amplifier input and output reflection coefficients Γ_{IN} and Γ_{OUT} from appropriate signal flow diagrams.
16. Remember definitions and formulas for unilateral transducer gain G_{TU} and maximum unilateral transducer gain $G_{TU,max}$ and Unilateral Figure of Merit M.
17. Remember that G_{TU} can be partititoned into 3 gain terms G_S, G_0 and G_L
18. Remember the principle of conjugate matching for matching gain.
19. Know the procedure for designing two-element matching networks with the Smith Chart.
20. Remember the conditions for stability of an amplifier and the formulas for Δ and the Rollet stability factor K in terms of the device's s-parameters.
21. Given formulas for gain circles, noise circles or stability circles, be able to do the appropriate calculations and draw these circles on the Smith Chart.
22. Be familiar with LNA design procedures for maximum gain and for a set target gain, with or without a constraint on noise figure, always remembering to check the stability criteria and choose the matching accordingly.

Common Formulas for LNA Design - 1

Note: All the gains terms are power ratios in these formulas

Maximum Unilateral Transducer Power Gain

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} \qquad G_{S,\max} = \frac{1}{1-|s_{11}|^2} \qquad G_0 = |s_{21}|^2 \qquad G_{L,\max} = \frac{1}{1-|s_{22}|^2}$$

Unilateral Figure of Merit

$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{\left(1-|s_{11}|^2\right)\left(1-|s_{22}|^2\right)} \qquad \frac{1}{(1+M)^2} < \frac{G_T}{G_{TU,\max}} < \frac{1}{(1-M)^2}$$

Source Gain Circles

$$g_S = \frac{G_S}{G_{S,\max}} \qquad |C_S| = \frac{g_S|s_{11}|}{1-|s_{11}|^2(1-g_S)} \qquad R_S = \frac{\sqrt{1-g_S}\left(1-|s_{11}|^2\right)}{1-|s_{11}|^2(1-g_S)}$$

Load Gain Circles

$$g_L = \frac{G_L}{G_{L,\max}} \qquad |C_L| = \frac{g_L|s_{22}|}{1-|s_{22}|^2(1-g_L)} \qquad R_L = \frac{\sqrt{1-g_L}\left(1-|s_{22}|^2\right)}{1-|s_{22}|^2(1-g_L)}$$

Common Formulas for LNA Design - 2

Converting between gain as a power ratio (G) and gain in dB (G_{dB})

$$G_{dB} = 10\log 10(G) \qquad G = 10^{\left(G_{dB}/10\right)}$$

Series element calculations – moving on a constant resistance circle

$$L = \frac{Z_0 |\Delta x|}{2\pi f}$$

Series inductor calculation
Moving clockwise on resistance circle with reactance change of Δx

$$C = \frac{1}{2\pi f |\Delta x| Z_0}$$

Series capacitor calculation
Moving anti-clockwise on resistance circle with reactance change of Δx

Shunt (parallel) element calculations – moving on a constant conductance circle

$$C = \frac{|\Delta b|}{2\pi f Z_0}$$

Shunt capacitor calculation
Moving clockwise on conductance circle with susceptance change of Δb

$$L = \frac{Z_0}{2\pi f |\Delta b|}$$

Shunt inductor calculation
Moving anti-clockwise on conductance circle with susceptance change of Δb

Common Formulas for LNA Design - 3

Rollet Stability Factor

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|} \qquad \Delta = s_{11}s_{22} - s_{12}s_{21}$$

Test for Unconditional Stability

$$K > 1 \quad and \quad |\Delta| < 1$$

Source Stability Circles

$$CS_S = \frac{s_{11}^{\ *} - \Delta^* s_{22}}{|s_{11}|^2 - |\Delta|^2} \quad RS_S = \frac{|s_{12}s_{21}|}{\left||s_{11}|^2 - |\Delta|^2\right|}$$

Load Stability Circles

$$CS_L = \frac{s_{22}^{\ *} - \Delta^* s_{11}}{|s_{22}|^2 - |\Delta|^2} \quad RS_L = \frac{|s_{12}s_{21}|}{\left||s_{22}|^2 - |\Delta|^2\right|}$$

Common Formulas for LNA Design - 4

Constant Noise Circles

$$N_i = \frac{F_i - F_{\min}}{4R_N / Z_0}\left|1+\Gamma_{opt}\right|^2 \qquad C_{Fi} = \frac{\Gamma_{opt}}{N_i + 1} \qquad R_{Fi} = \frac{\sqrt{N_i\left(N_i + 1 - \left|\Gamma_{opt}\right|^2\right)}}{\left(N_i + 1\right)}$$

Note: F_{min} and F_i are noise power ratios (i.e. noise factors) in these formulas

Converting between noise factor (F) and noise figure (F_{dB})

$$F_{dB} = 10\log 10(F) \qquad F = 10^{\left(F_{dB}/10\right)}$$