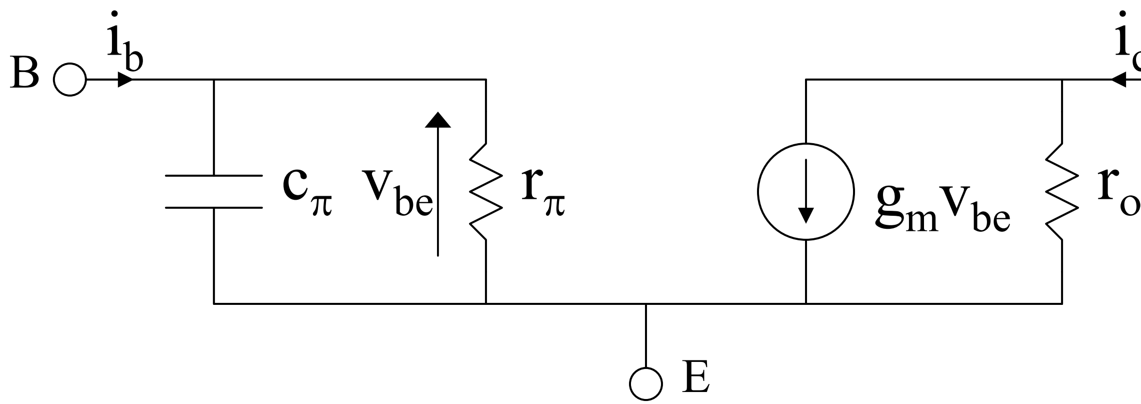
EE4011: RF IC Design
Summer 2004 Answers

Question 1

(a) 10 marks

A BJT small-signal model taking only base-emitter capacitances into account is:



Defining hfe with the collector short-circuited:

$$i_b = j\omega c_\pi v_{be} + \frac{v_{be}}{r_\pi} = v_{be}\left[g_\pi + j\omega c_\pi\right] \qquad i_c = g_m v_{be} \qquad h_{fe} = \frac{i_c}{i_b} = \frac{g_m}{g_\pi + j\omega c_\pi}$$

For high frequencies:

$$\Rightarrow h_{fe} \approx \frac{g_m}{j2\pi f\, c_\pi} \Rightarrow \left|h_{fe}\right| = \frac{g_m}{2\pi f\, c_\pi}$$

The cut-off frequency:

$$\left|h_{fe}\right| = 1\Big|_{f=f_T} \Rightarrow f_T = \frac{g_m}{2\pi c_\pi}$$

Question 1

(b) The following bias point information is needed

$k = 1.38 \times 10^{-23}$ J/K $q = 1.602 \times 10^{-19}$ C

V_{BE}=0.75 V, V_{CE}=3.0 V, T=300 K $I_S=10^{-15}$ A, β=100, V_A=10V,
$C_{JE}=10^{-12}$ F, V_{JE}=5 V, M_{JE}=0.5, $\tau_F=10^{-10}$ s.

$$I_C \approx I_S \exp\left(\frac{qV_{BE}}{kT}\right) = 0.004A \quad g_m = \frac{q}{kT} I_C = 0.1555A/V \quad r_o = \frac{V_{AF}}{I_C} = 2489\,\Omega$$

$$c_\pi = \frac{C_{JE}}{\left(1 - \frac{V_{BE}}{V_{JE}}\right)^{M_{JE}}} + g_m \tau_F = 1.08 \times 10^{-12} + 1.55 \times 10^{-1}1 = 1.66 \times 10^{-11} F$$

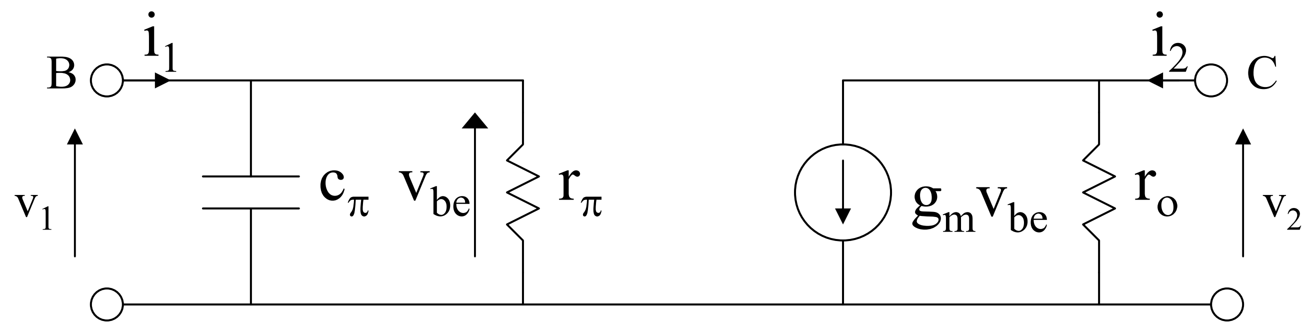
$$r_\pi = \frac{\beta}{g_m} = 643\,\Omega$$

(i) 2 marks

$$f_T = \frac{g_m}{2\pi c_\pi} = 1.48 GHz$$

Question 1

(b)(ii) 8 marks



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{r_\pi} + j\varpi C_\pi \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = 0$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = g_m \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{r_o}$$

If these are evaluated at 1 GHz the values are:

$y_{11} = 0.0016 + j0.1045$, $|y_{11}|=0.1$, angle(y_{11})=89 degrees, $y_{12}=0$, $y_{21}=0.1555$, $y_{22}=4\text{x}10^{-4}$

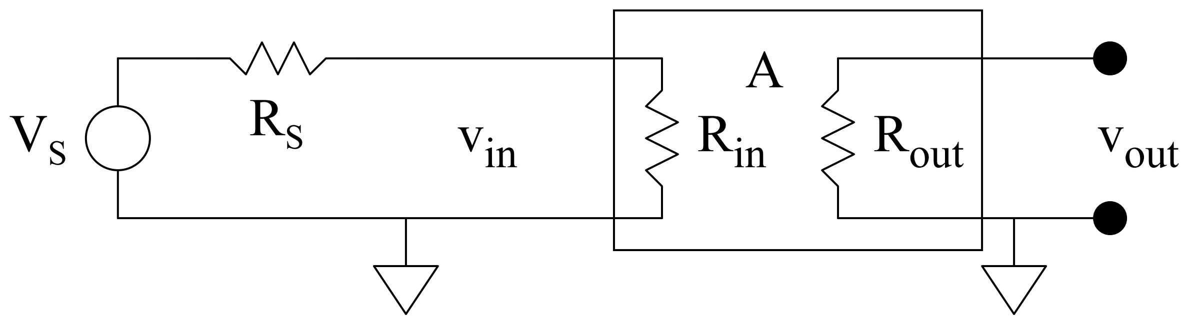
Question 2

2(a) 14 marks

Expression for noise figure of two-port network driven by source impedance R_S.

Typical two-port with input and output resistance and voltage gain A:

Determine the input and output signal voltages only:

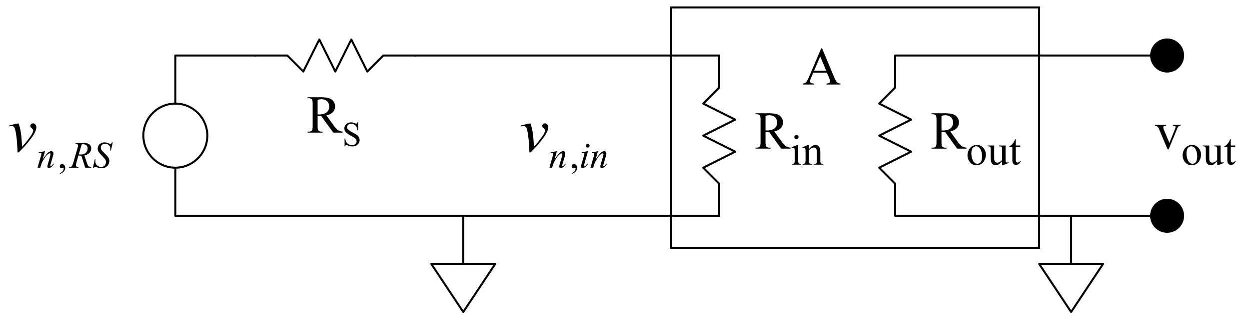


$$v_{in} = \frac{R_{in}}{R_{in} + R_S} V_S = \alpha V_S \qquad \alpha = \frac{R_{in}}{R_{in} + R_S}$$

$$v_{out} = A v_{in} = \alpha A V_S$$

$$v_{out}^2 = A^2 v_{in}^2 = \alpha^2 A^2 V_S^2$$

Question 2(a)
Analyse the noise taking into account the noise of the source resistance only:



The input noise voltage due to the source resistance alone is:

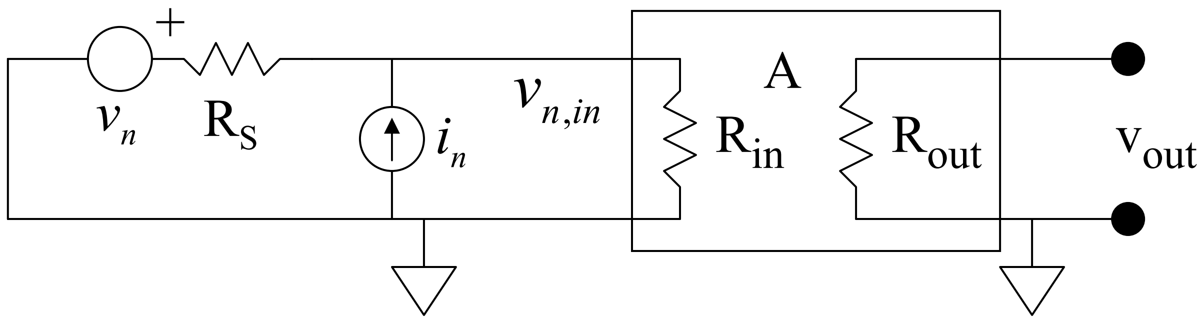
$$v_{n,in} = \frac{R_{in}}{R_{in}+R_S} v_{n,RS} = \alpha\, v_{n,RS} \qquad \overline{v_{n,in}^2} = \alpha^2 \overline{v_{n,RS}^2}$$

The signal to noise ratio at the input is:

$$SNR_{in} = \frac{v_{in}^2}{\overline{v_{n,in}^2}} = \frac{\alpha^2 V_S^2}{\alpha^2 \overline{v_{n,RS}^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} = \frac{V_S^2}{4kTR_S\Delta f}$$

Question 2(a)

Now analyze the effect of the input-referred noise sources alone. The voltage and current sources must be analyzed together because they are correlated. The noise voltage is moved to "voltage side" of the source resistor to make the circuit analysis easier:



After some circuit analysis on the input circuit:

$$v_{n,in} = \frac{R_{in}}{R_{in} + R_S}\left(v_n + R_S i_n\right) = \alpha\left(v_n + R_S i_n\right) \Rightarrow \overline{v_{n,in}^2} = \alpha^2 \overline{\left(v_n + R_S i_n\right)^2}$$

At this stage we have expressions for the input noise voltage caused by the source resistance and the input noise voltage caused by the input-referred sources of the two-port. Because these are uncorrelated the associated mean square values can be added.

Question 2(a)

Adding the contributions of the source noise and the input-referred noise sources:

$$\left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 \overline{v_{n,RS}^2} + \alpha^2 \overline{\left(v_n + R_S i_n\right)^2}$$

$$\overline{v_{n,out}^2} = A^2 \left(\overline{v_{n,in}^2}\right)_{TOT} = \alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}$$

$$SNR_{out} = \frac{v_{out}^2}{\overline{\overline{v_{n,out}^2}}} = \frac{\alpha^2 A^2 V_S^2}{\alpha^2 A^2 \overline{v_{n,RS}^2} + \alpha^2 A^2 \overline{\left(v_n + R_S i_n\right)^2}} = \frac{V_S^2}{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{V_S^2}{\overline{v_{n,RS}^2}} \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{V_S^2} = \frac{\overline{v_{n,RS}^2} + \overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}}$$

$$F = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{\overline{v_{n,RS}^2}} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S \Delta f} = 1 + \frac{\overline{\left(v_n + R_S i_n\right)^2}}{4kTR_S} \text{ for } \Delta f = 1Hz$$

Question 2(b)

For simplicity the parasitic base resistance resistance is only used for the calculation of the equivalent input noise sources and is not assumed to effect the noise figure otherwise.

Using I_C=1mA, T=300K, r_b=50Ω, B_f=100, Δf=1Hz in the following formula:

$$\overline{v^2} = 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f \quad \overline{i^2} = 2q\frac{I_C}{\beta_f}\Delta f$$

gives:

v=1nV (per root Hz)
i=1.79pA (per root Hz)

$$F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S} \text{ for } \Delta f = 1Hz$$

2(b)(i) 3 marks

Using R_S=10Ω gives F = 7.51

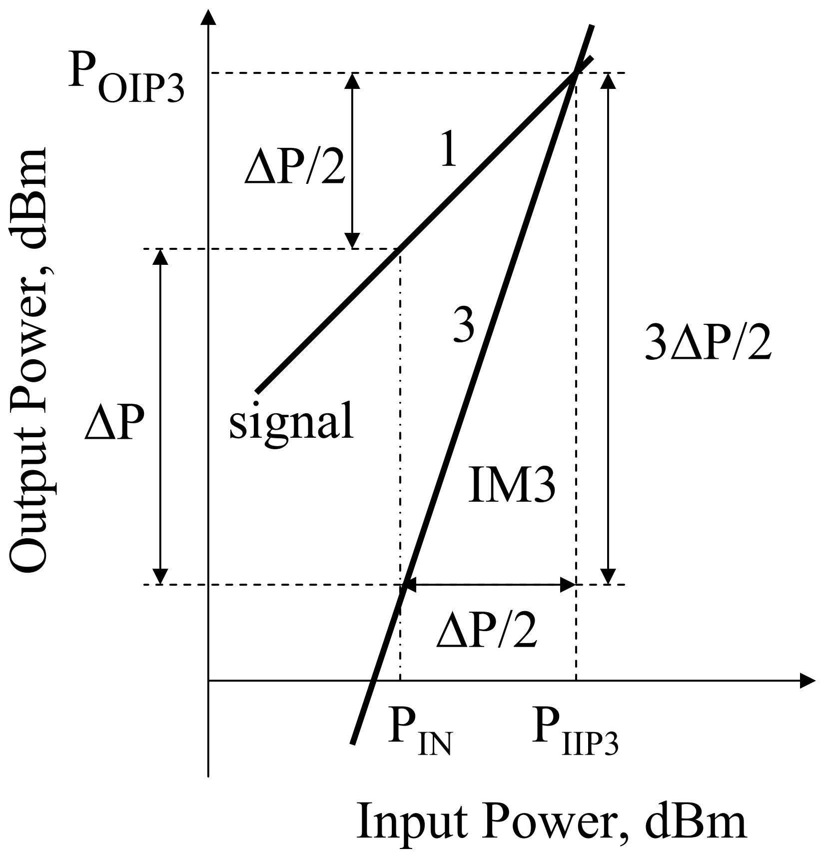
2(b)(ii) 3 marks

Using R_S=100Ω gives F = 1.87

Question 3

3(a) 7 marks

A graphical means of estimating the input-referred third-order intercept point.



By applying two signals with input power (P_{in}) and measuring the associated output signal power ($P_{sig,out}$) and at the IM3 frequencies ($P_{IM3,out}$) it is apparent from the graph that:

$$P_{IIP3} = P_{in} + \frac{P_{sig,out} - P_{IM3,out}}{2}$$

Question 3

3(b) 7 marks

The sensitivity of system is defined as the minimum input signal power which is required to give a specified minimum signal-to-noise ratio at the output.

For a given output SNR the input power can be found from the noise figure:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig} / P_{RS}}{SNR_{out}} \Rightarrow P_{sig} = P_{RS} \cdot F \cdot SNR_{out} \qquad \text{(per unit bandwidth)}$$

Assuming the system bandwith is B:

$$P_{sig} = P_{RS} \cdot F \cdot SNR_{out} \cdot B$$

Turning the quantities into logs and setting the output SNR to the minimum required value and the Input signal power to the minimum value needed to give the required output SNR:

$$P_{\min} = P_{RS}\Big|_{dBm/Hz} + NF + SNR_{\min}\Big|_{dB} + 10\log_{10} B \quad \text{where} \quad NF = 10\log_{10} F$$

3(b) continued

If the input is conjugate matched to the source the noise power delivered to the input will be:

$$P_{RS} = \frac{\overline{v_n^2}}{4R_S} = \frac{4kTR_S}{4R_S} = kT = -174\, dBm / Hz$$

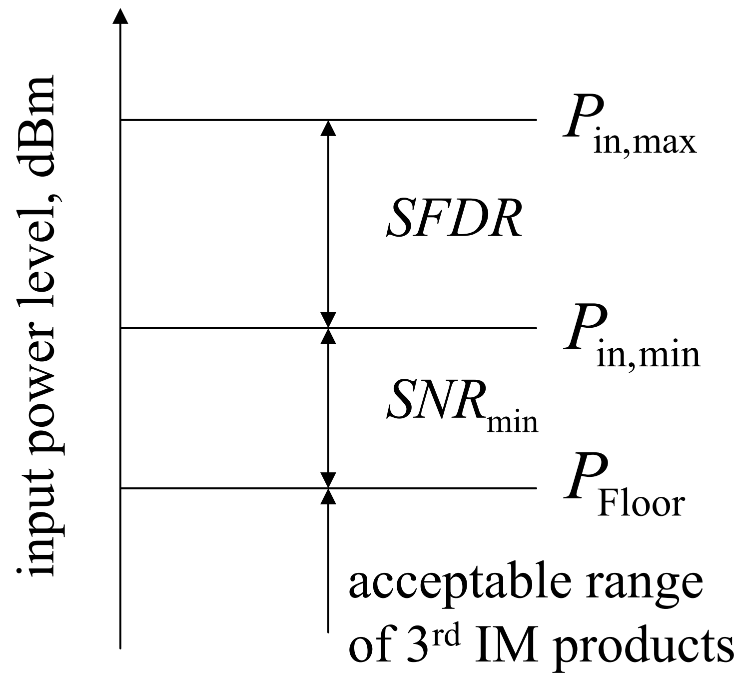
$$P_{\text{in,min}} = \underbrace{-174\,\text{dBm/Hz} + NF + 10\log_{10} B}_{\text{noise floor}} + SNR_{\text{min}}\Big|_{dB}$$

The noise floor of the system is the set of constants shown and corresponds to the input power level which gives and output SNR of 0dB (i.e. the output noise power is the same as the output signal power).

$$\text{P}_{\text{Floor}} = -174\,\text{dBm/Hz} + NF + 10\log_{10} B$$

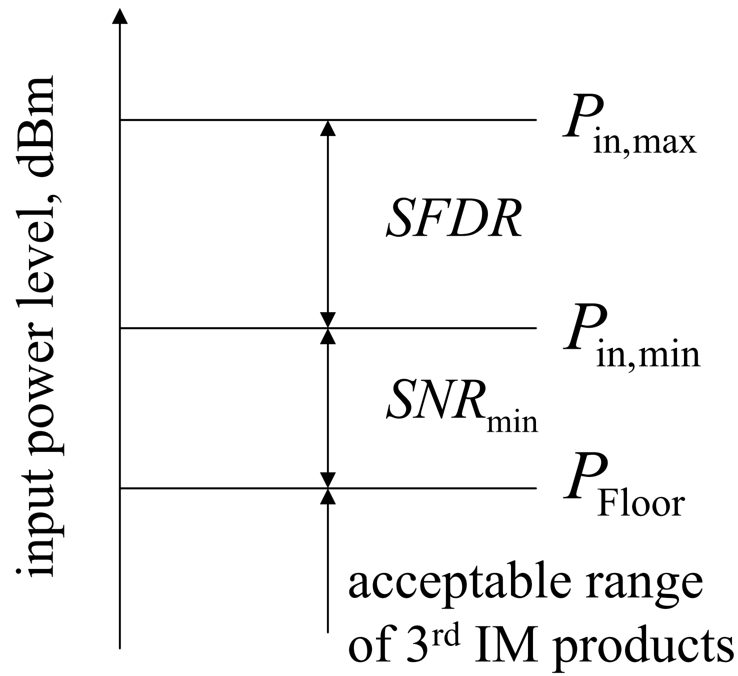
Question (3)

3(c) 6 marks



The minimum acceptable power in dB is the noise floor plus the required minimum output SNR. As the input power level increase, two or more signals will give IM3 products. The maximum acceptable input power level is considered to be the input power level at which the IM3 products are as high as the noise floor.
The range of power between the minimum level and the maximum level is known as the spurious free dynamic range (SFDR).

Q3(c) continued



$$P_{in,\max} = \frac{2P_{IIP3} + P_{Floor}}{3}$$

At this power the IM3 products reach the noise floor.

$$P_{in,\min} = P_{Floor} + SNR_{\min}$$

$$SFDR = P_{in,\max} - P_{in,\min}$$

$$= \frac{2P_{IIP3} + P_{Floor}}{3} - \left(P_{Floor} + SNR_{\min}\right)$$

$$= \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min}$$

Using NF = 9 dB, P_{IIP3} = -15dBm, B = 200kHz, T = 300K, SRN_{min} = 12dB gives:

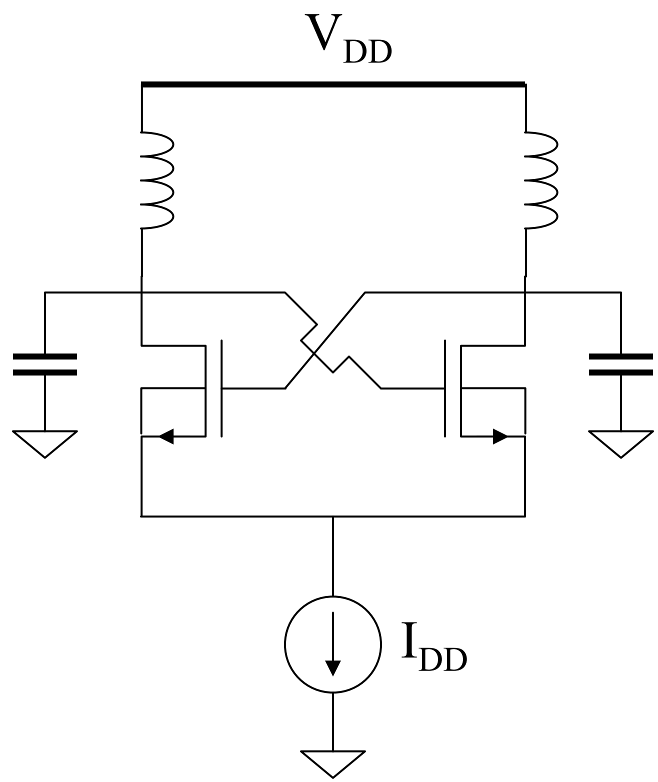
$$\mathrm{P}_{\text{Floor}} = -174 + 9 + 10\log_{10}(200\times10^{3}) = -112\,\text{dBm}$$

$$SFDR = \frac{2\left(P_{IIP3} - P_{Floor}\right)}{3} - SNR_{\min} = \frac{2(15+112)}{3} - 12 \approx 53\,\text{dB}$$

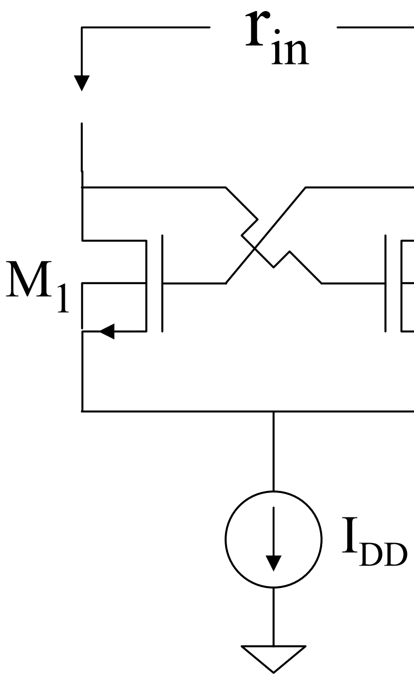
Question 6

6(a) 5 marks

An negative-g_m VCO based on cross-coupled MOSFETs

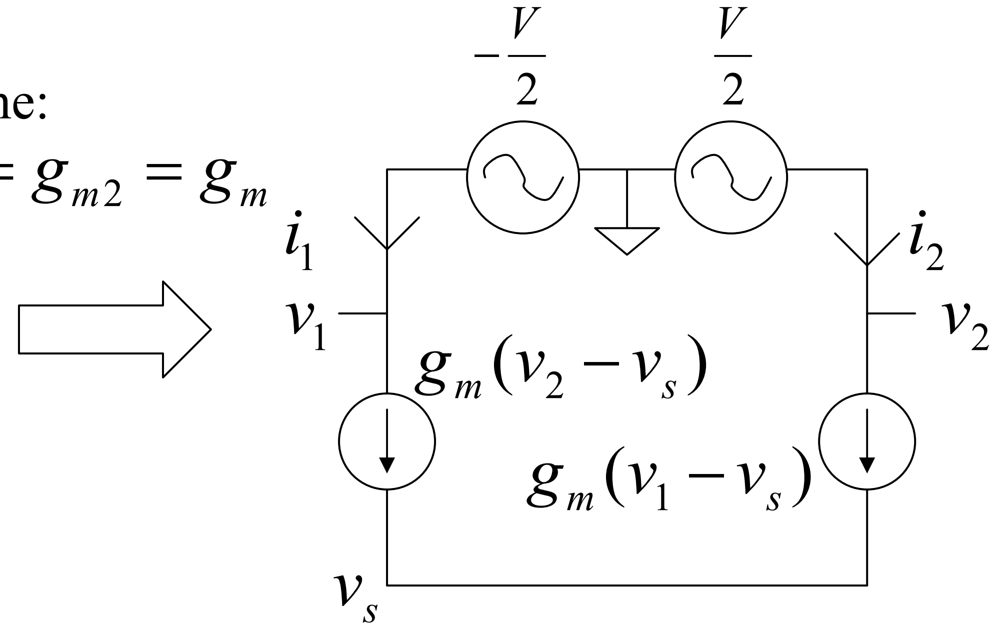


6(b) 10 marks



assume:

$$g_{m1} = g_{m2} = g_m$$



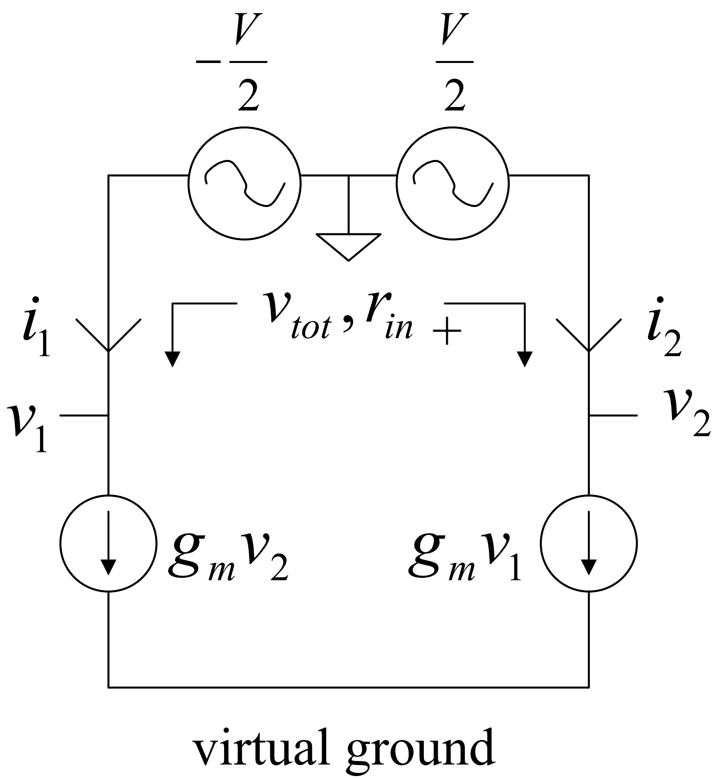
$$i_1 = -i_2 \Rightarrow g_m(v_2 - v_s) = -g_m(v_1 - v_s)$$

$$\Rightarrow g_m(v_1 + v_2) = 2g_m v_s$$

$$\Rightarrow v_s = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}\left(\frac{V}{2} - \frac{V}{2}\right) = 0$$

$$\Rightarrow i_2 = g_m v_1 = g_m\left(-\frac{V}{2}\right)$$

6(b) continued

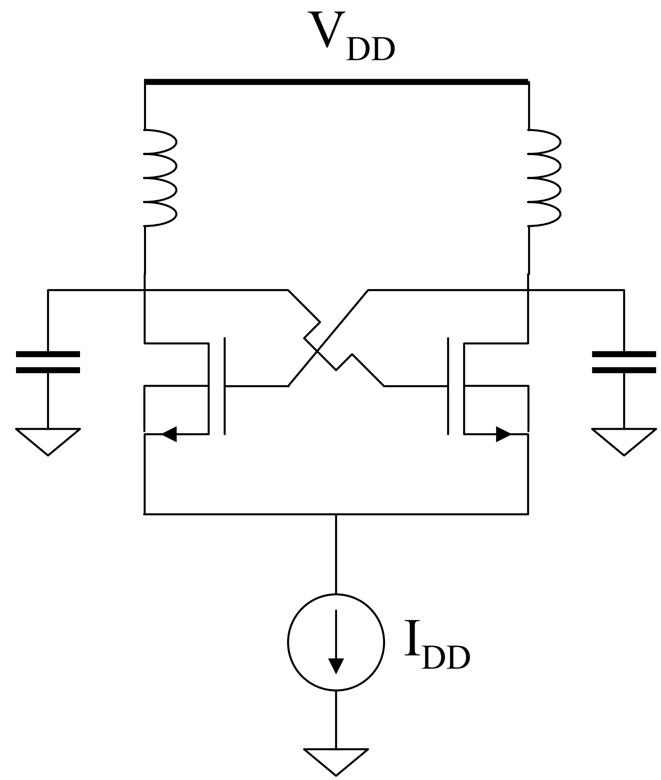


The small-signal input resistance is the total voltage across the input divided by the input current:

$$r_{in} = \frac{v_{tot}}{i_2} = \frac{\frac{V}{2} - \left(-\frac{V}{2}\right)}{g_m\left(-\frac{V}{2}\right)} = -\frac{2}{g_m}$$

The small-signal input resistance of the cross-coupled pair configuration is negative and determined by the device transconductances. These can be set by the DC current and the W/L ratio of the devices. These can be chosen to give an r_{in} which cancels the resistance of the resonator and thus allows oscillations to continue.

6(c) 5 marks



$$r_{in} = -\frac{2}{g_m} \Rightarrow g_m = -\frac{2}{r_{in}}$$

The input resistance must cancel the parasitic resistance of the inductors which are 20Ω in total. Therefore

$$g_m = -\frac{2}{-20} = 0.1$$

i.e. the transistors must be biased to give this g_m.

For MOSFETs in saturation:

$$g_m = \sqrt{2\frac{W}{L}\mu C_{OX} I_D} \Rightarrow \frac{W}{L} = \frac{g_m^2}{2\mu C_{OX} I_D} \qquad C_{OX} = \frac{\varepsilon_{OX}}{T_{OX}}$$

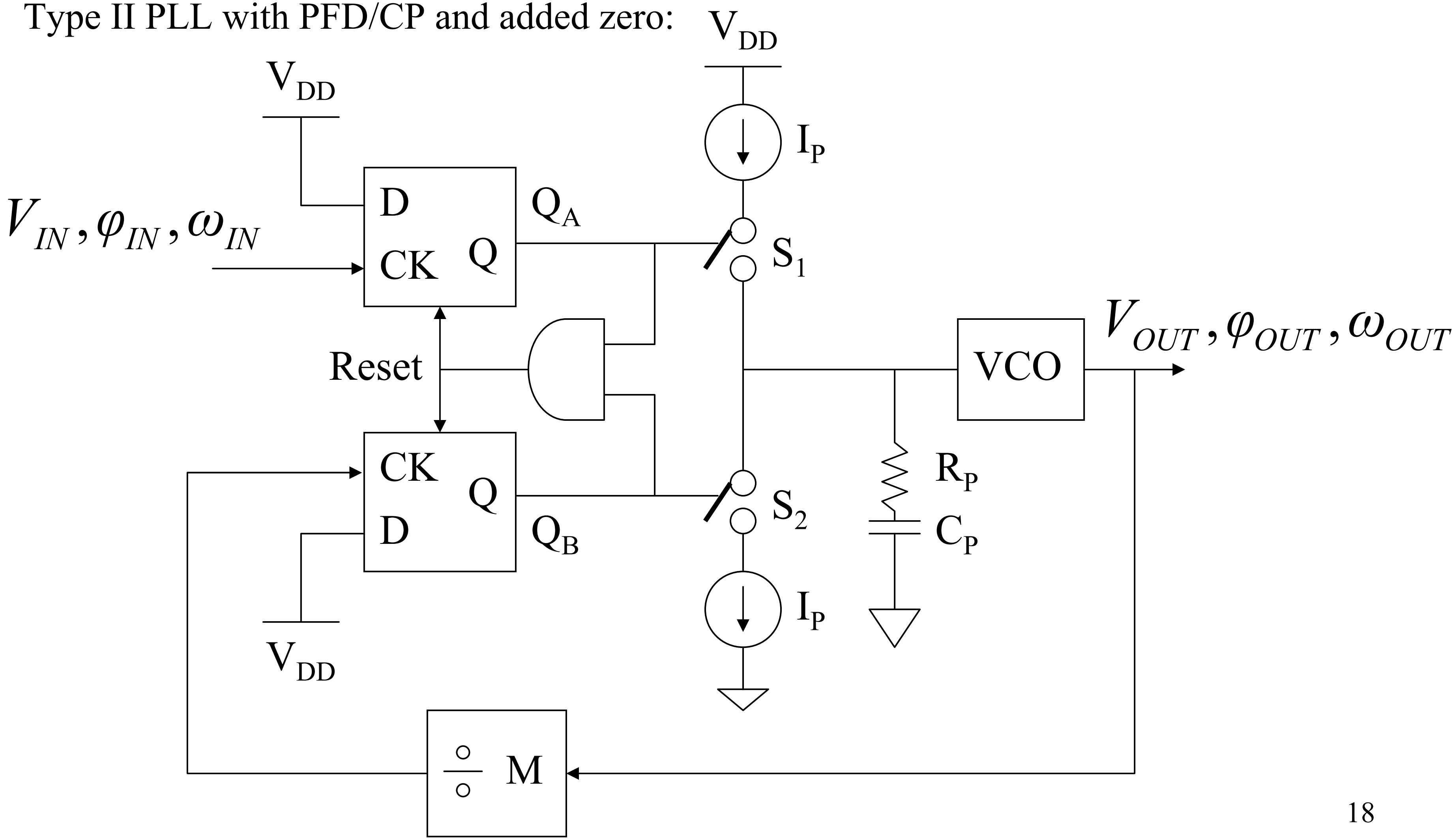
Using I_D=5mA (half of bias current of 10mA), L=0.25μm, T_{OX}=5nm, μ=400cm^2/Vs gives:

$$\frac{W}{L} = 3.62\times10^3 \Rightarrow W = 905\mu m$$

Question 7

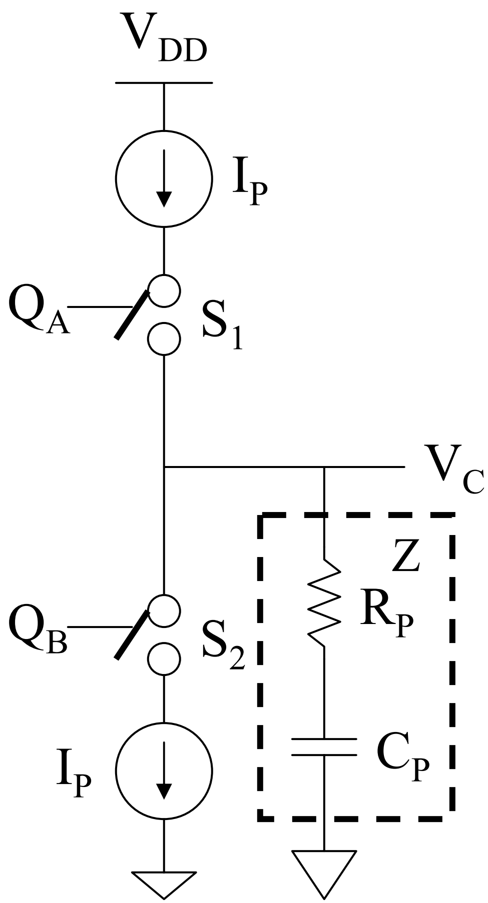
7(a) 5 marks

Type II PLL with PFD/CP and added zero:

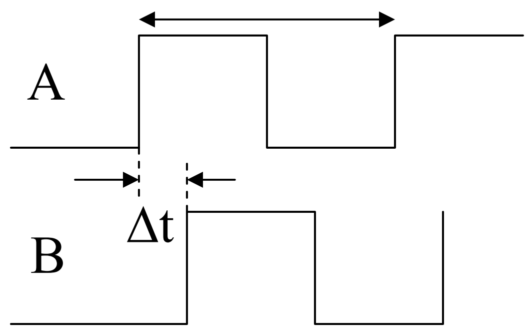


7(b)

The transfer function of the PFD/CP using the "average current" method:



T = period



$\Delta\varphi$ is the phase difference

$$\Delta t = \frac{\Delta\varphi}{2\pi}T$$

$$\Delta Q = I_P \Delta t = I_P \frac{\Delta\varphi}{2\pi}T$$

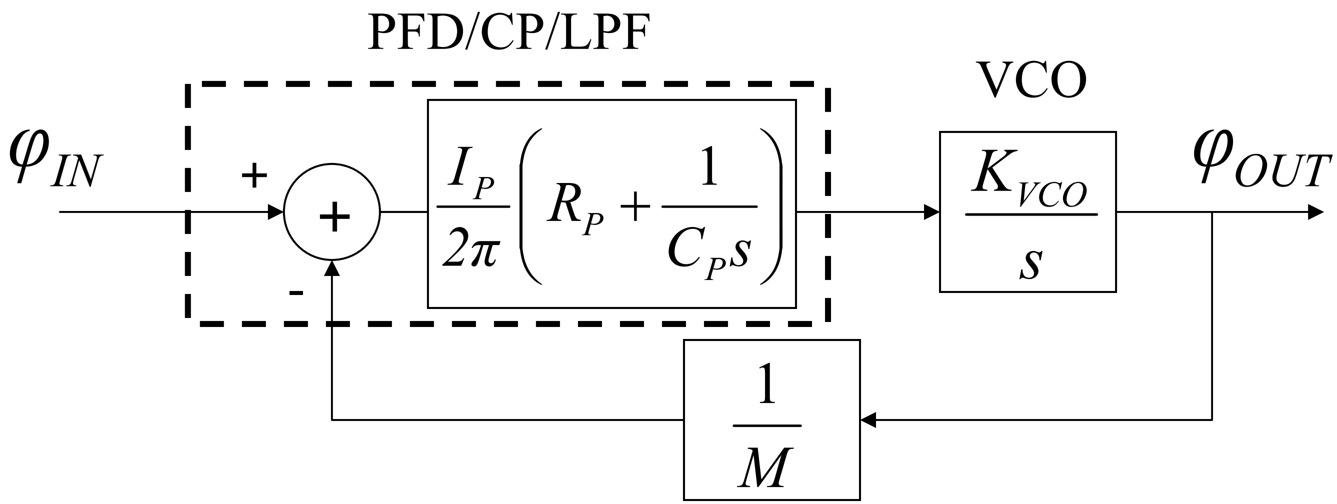
$$I_{AVG} = \frac{\Delta Q}{T} = I_P \frac{\Delta\varphi}{2\pi}\frac{T}{T} = \frac{I_P}{2\pi}\Delta\varphi \Rightarrow I(s) = \frac{I_P}{2\pi}\Delta\varphi(s)$$

$$Z(s) = R_P + \frac{1}{sC_P}$$

$$\Rightarrow V_C(s) = \left(R_P + \frac{1}{sC_P}\right)I(s) = \frac{I_P}{2\pi}\left(R_P + \frac{1}{sC_P}\right)\Delta\varphi(s)$$

7(b)(i) 7 marks

Replacing the blocks by their transfer functions including the VCO:

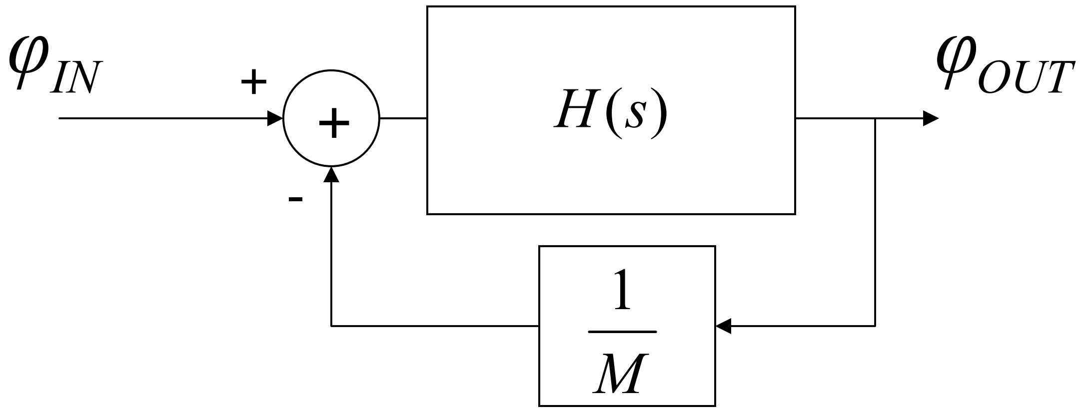


Open loop transfer function:

$$H(s) = \frac{\varphi_{OUT}(s)}{\varphi_{IN}(s)} = \frac{I_P K_{VCO}}{2\pi s}\left(R_P + \frac{1}{C_P s}\right)$$

7(b)(ii) 3 marks

Closed loop transfer function:



$$\varphi_{OUT}(s) = \left(\varphi_{IN}(s) - \frac{\varphi_{OUT}(s)}{M}\right) H(s)$$

$$\Rightarrow H_{Closed}(s) = \frac{H(s)}{1 + \frac{H(s)}{M}} = \frac{\frac{I_P K_{VCO}}{2\pi s}\left(R_P + \frac{1}{C_P s}\right)}{1 + \frac{I_P K_{VCO}}{2\pi M s}\left(R_P + \frac{1}{C_P s}\right)} = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}}$$

7(c) 5 marks

Re-arranging the closed loop transfer function so that the denominator looks like a standard second-order system gives the natural frequency, the damping factor and the loop time constant:

$$H_{Closed}(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + \frac{I_P K_{VCO}}{2\pi M} R_P s + \frac{I_P K_{VCO}}{2\pi C_P M}} = \frac{\frac{I_P K_{VCO}}{2\pi C_P}\left(R_P C_P s + 1\right)}{s^2 + 2\varsigma \omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P M}} \quad \varsigma = \frac{R_P}{2}\sqrt{\frac{I_P C_P K_{VCO}}{2\pi M}} \quad \tau = \frac{1}{\varsigma \omega_n} = \frac{4\pi M}{I_P R_P K_{VCO}}$$

Using I_P=1mA, C_P=100pF, R_P=10kΩ, K_{VCO}=100MHz/V, M=1000 gives:

$$\omega_n = 10^6\, rads/s\ (159kHz) \quad \varsigma = 0.5 \quad \tau = 2\mu s$$