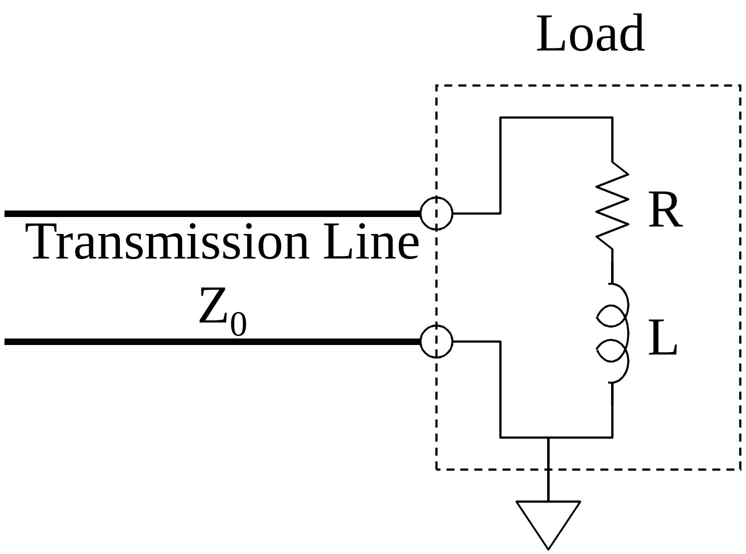
EE4011 RFIC Design

The Smith Chart

Γ vs. frequency for resistive inductive load



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} \quad \text{where} \quad \bar{Z}_L = \frac{Z_L}{Z_0}$$

$$Z_L = R + j\varpi L \Rightarrow \bar{Z}_L = \frac{R}{Z_0} + j\frac{\varpi L}{Z_0} = r + jx$$

$$\text{where} \quad r = \frac{R}{Z_0} \quad \textit{and} \quad x = \frac{\varpi L}{Z_0}$$

$$\Gamma = \frac{(r-1) + jx}{(r+1) + jx}$$

As the frequency varies from zero to infinity x also varies from 0 to infinity.

At very low frequencies: $x << r \Rightarrow \Gamma = \dfrac{(r-1)}{(r+1)}$

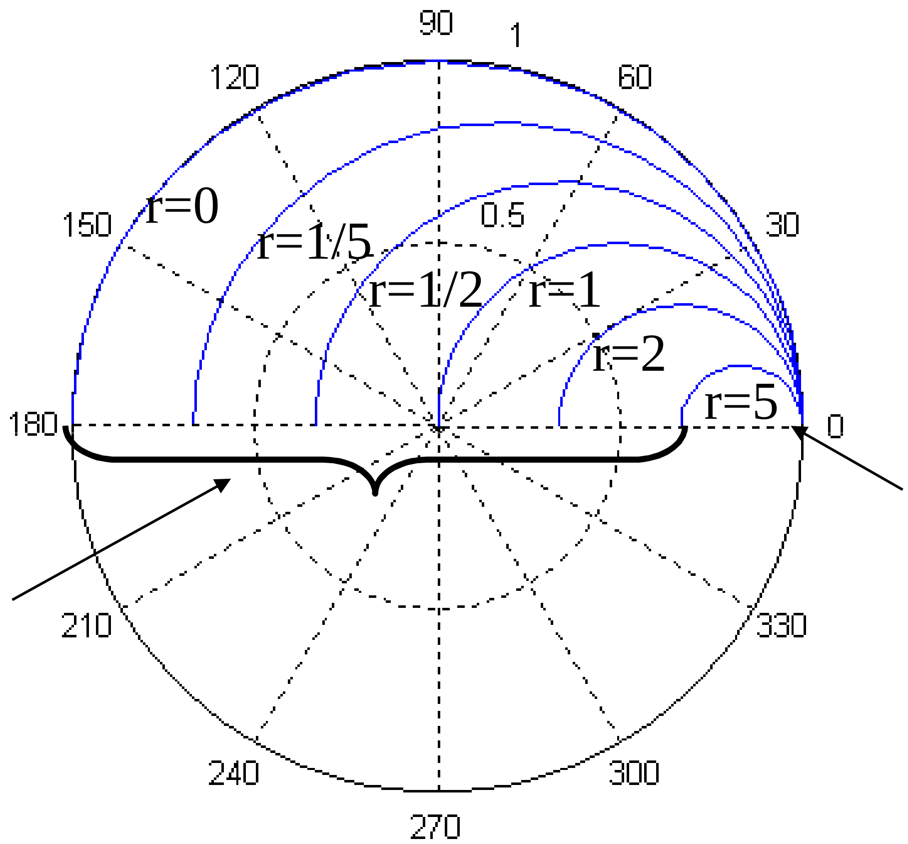
At very high frequencies: $x >> r \Rightarrow \Gamma = \dfrac{jx}{jx} = 1$

This condition is similar to an “open circuit” giving complete reflection.

Polar plots of Γ vs. frequency for different resistive inductive loads

$$\Gamma = \frac{(r-1)+jx}{(r+1)+jx}$$

$$r = \frac{R}{Z_0} \qquad x = \frac{\varpi L}{Z_0}$$

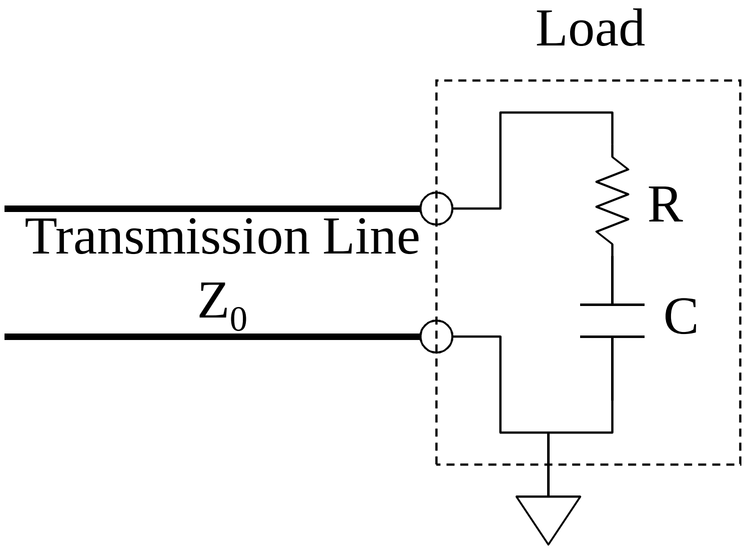


the curves begin on the real axis between -1 and 1 for ω = 0

the curves end at the co-ordinate (1,0) for ω = ∞

These curves are made by picking sample values of normalised resistance, r, and any inductance, L. Then Γ is calculated as a function of frequency, ω, as ω varies from 0 to infinity and graphed on a polar plot.

Γ vs. frequency for resistive capacitive load



$$Z_L = R + \frac{1}{j\varpi C} \Rightarrow \bar{Z}_L = \frac{R}{Z_0} + j\left(-\frac{1}{Z_0 \varpi C}\right) = r + jx$$

where $\quad r = \dfrac{R}{Z_0} \quad and \quad x = -\dfrac{1}{Z_0 \varpi C}$

$$\Gamma = \frac{(r-1) + jx}{(r+1) + jx}$$

As the frequency varies from zero to infinity x varies from minus infinity to 0.

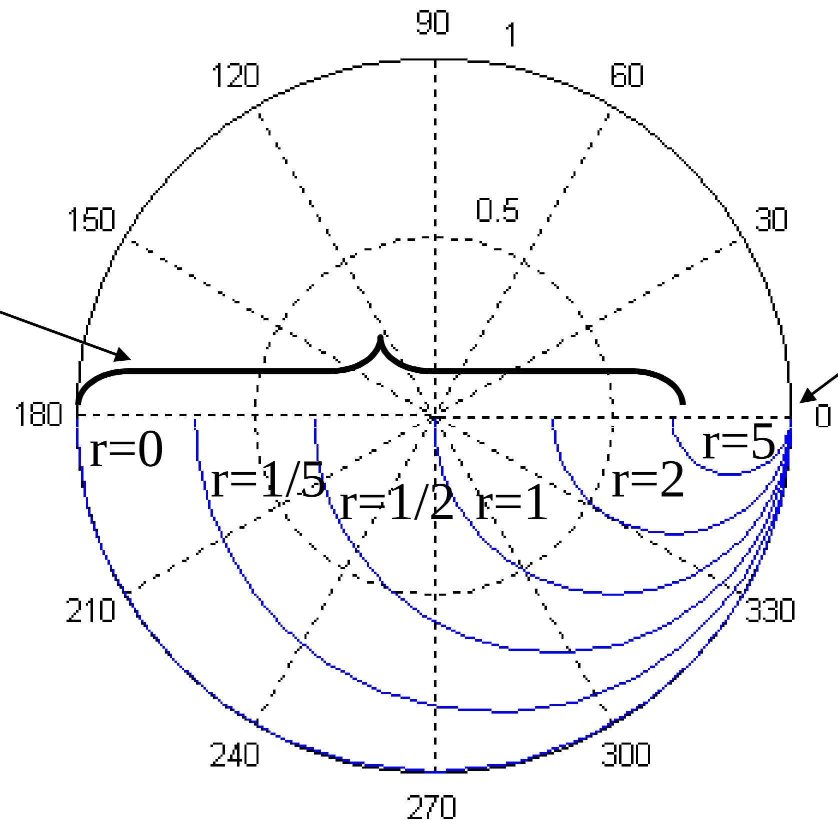
At very low frequencies: $\quad |x| >> r \Rightarrow \Gamma = \dfrac{jx}{jx} = 1$

This condition is similar to an "open circuit" giving complete reflection.

At very high frequencies: $\quad |x| << r \Rightarrow \Gamma = \dfrac{(r-1)}{(r+1)}$

Polar plots of Γ vs. frequency for different resistive capacitive loads

the curves end on
the real axis between
-1 and 1 for $\omega = \infty$



the curves all start at
the co-ordinate (1,0)
for $\omega = 0$

$$\Gamma = \frac{(r-1)+jx}{(r+1)+jx}$$

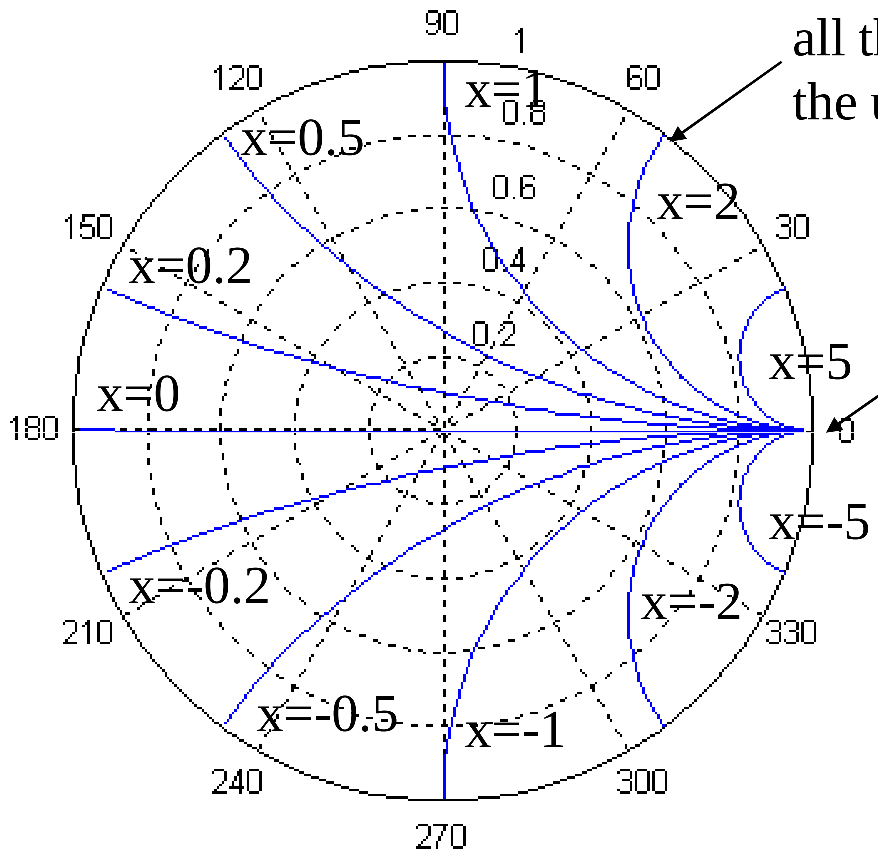
$$r = \frac{R}{Z_0} \qquad x = -\frac{1}{Z_0 \varpi C}$$

These curves are made by picking sample values of normalised resistance, r, and any capacitance, C. Then Γ is calculated as a function of frequency, ω, as ω varies from 0 to infinity and graphed on a polar plot.

Polar plots of Γ vs. frequency for constant reactance but varying resistance

$$\overline{Z}_L = r + jx$$

$$\Gamma = \frac{(r-1)+jx}{(r+1)+jx}$$



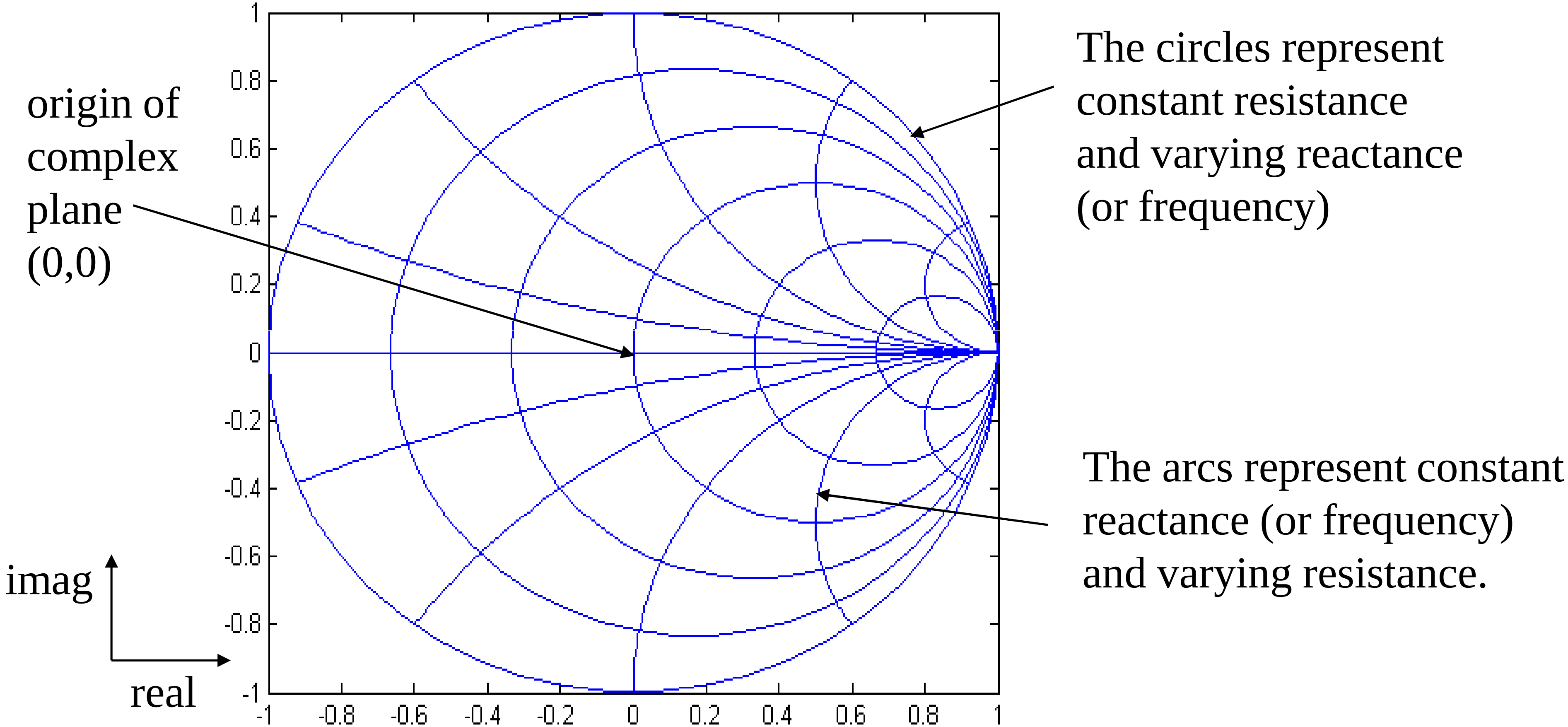
all the curves begin on the unit circle for r = 0.

all the curves end at the point (1,0) for r = ∞

These curves are made by picking sample values of normalised reactance, x. Then Γ is calculated as a function of resistance, r, as r varies from 0 to infinity and graphed on a polar plot.

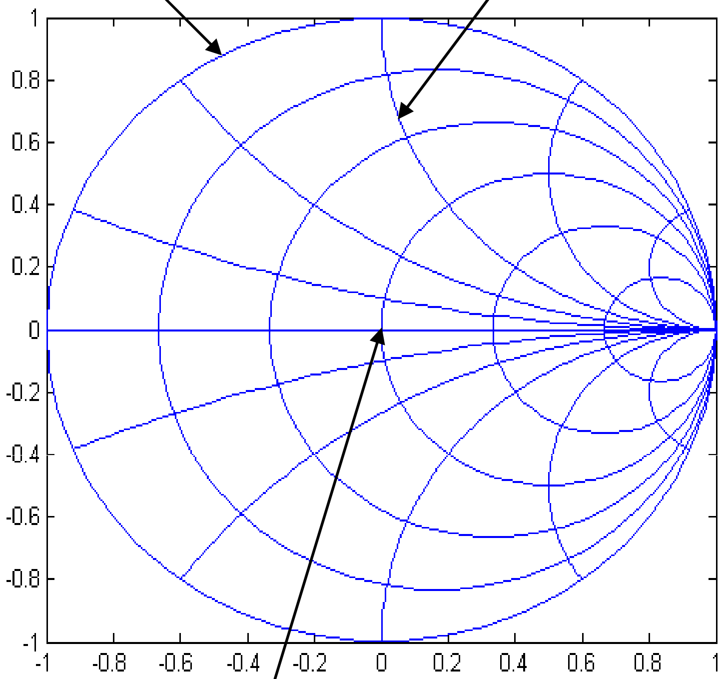
The Smith Chart

The Smith Chart (inventor Philip H. Smith) is a superposition of the graphs we have seen up to now i.e. a plot of the reflection coefficients of a range of standard loads with constant resistance and varying reactance or varying resistance and constant reactance.



These are plots of reflection coefficient, Γ, on the complex plane but in the standard Smith chart the real and imaginary axes are not labelled.

The Impedance Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\overline{Z}_L - 1}{\overline{Z}_L + 1} \quad \text{where} \quad \overline{Z}_L = \frac{Z_L}{Z_0}$$

$$Z_L = R + jX \Rightarrow \overline{Z}_L = \frac{R}{Z_0} + j\frac{X}{Z_0} = r + jx$$

$$\Gamma = \frac{\overline{Z}_L - 1}{\overline{Z}_L + 1} = \frac{(r-1) + jx}{(r+1) + jx}$$

$$= \frac{[(r-1) + jx][(r+1) - jx]}{[(r+1) + jx][(r+1) - jx]}$$

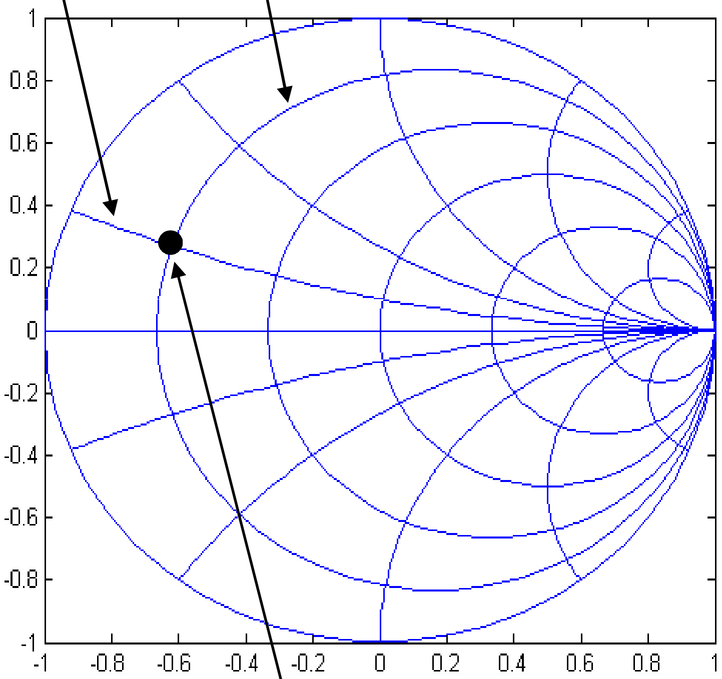
$$= \frac{r^2 + x^2 - 1}{(r+1)^2 + x^2} + j\frac{2x}{(r+1)^2 + x^2}$$

Impedance consists of a real part (resistance) and an imaginary part (reactance). Smith charts which show the reflection coefficient for standard normalized resistances and reactances are known as Impedance Smith Charts. Looking at the final formula, Γ will be in the top semicircle for positive reactance (x>0).

Locating the Reflection Coefficient

arc corresponding to x

circle corresponding to r



Γ *corresponding to* $\overline{Z}_L = r + jx$

Given a load Z_L hooked up to a transmission line of characteristic impedance Z_0

$$Z_L = R + jX \Rightarrow \overline{Z}_L = \frac{R}{Z_0} + j\frac{X}{Z_0} = r + jx$$

instead of having to calculate the reflection coefficient using the formula

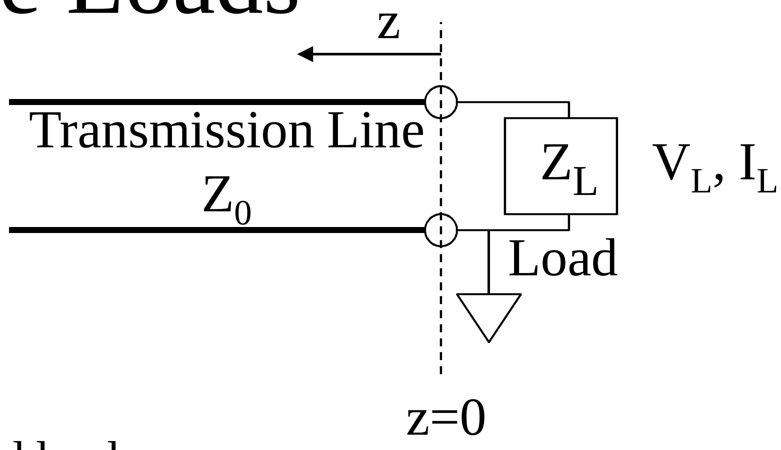
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\overline{Z}_L - 1}{\overline{Z}_L + 1}$$

we can instead look for the circle corresponding to r and the arc corresponding to x and the intersection of these is the reflection coefficient Γ that we are interested in.

We can easily see then how Γ would vary if resistance or reactance (or frequency) changes.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Some Example Loads



$$\Gamma = 0\Big|_{Z_L = Z_0} \qquad \Gamma = -1\Big|_{Z_L = 0} \qquad \Gamma = 1\Big|_{Z_L = \infty}$$

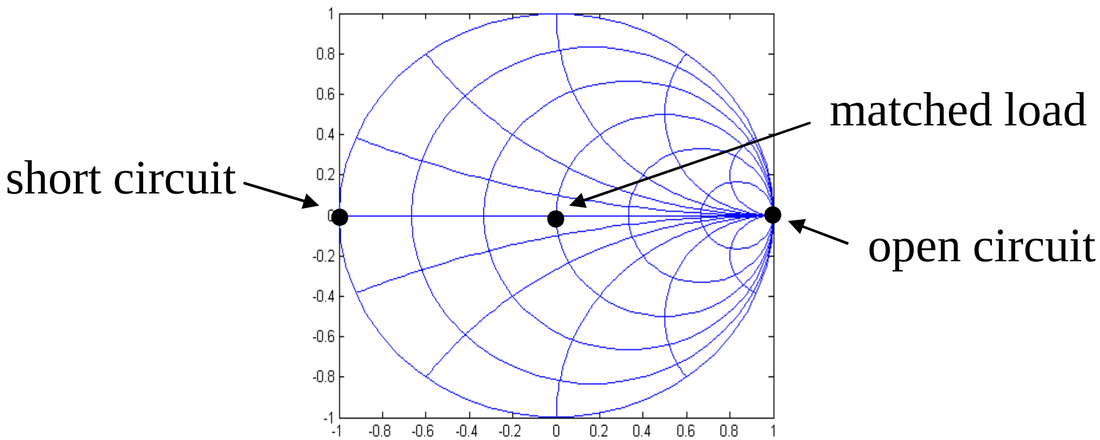
matched load — short circuit — open circuit

At z=0:

$$V_r(0) = \Gamma V_i(0) \qquad \Gamma = 0 \Rightarrow V_r(0) = 0 \qquad \text{matched load}$$

$$V_L = Z_L I_L = V_i(0) + V_r(0) \qquad V_L = 0 \Rightarrow V_r(0) = -V_i(0) \quad \text{short circuit}$$

$$I_L = \frac{1}{Z_0}\left[V_i(0) - V_r(0)\right] \qquad I_L = 0 \Rightarrow V_r(0) = V_i(0) \qquad \text{open circuit}$$

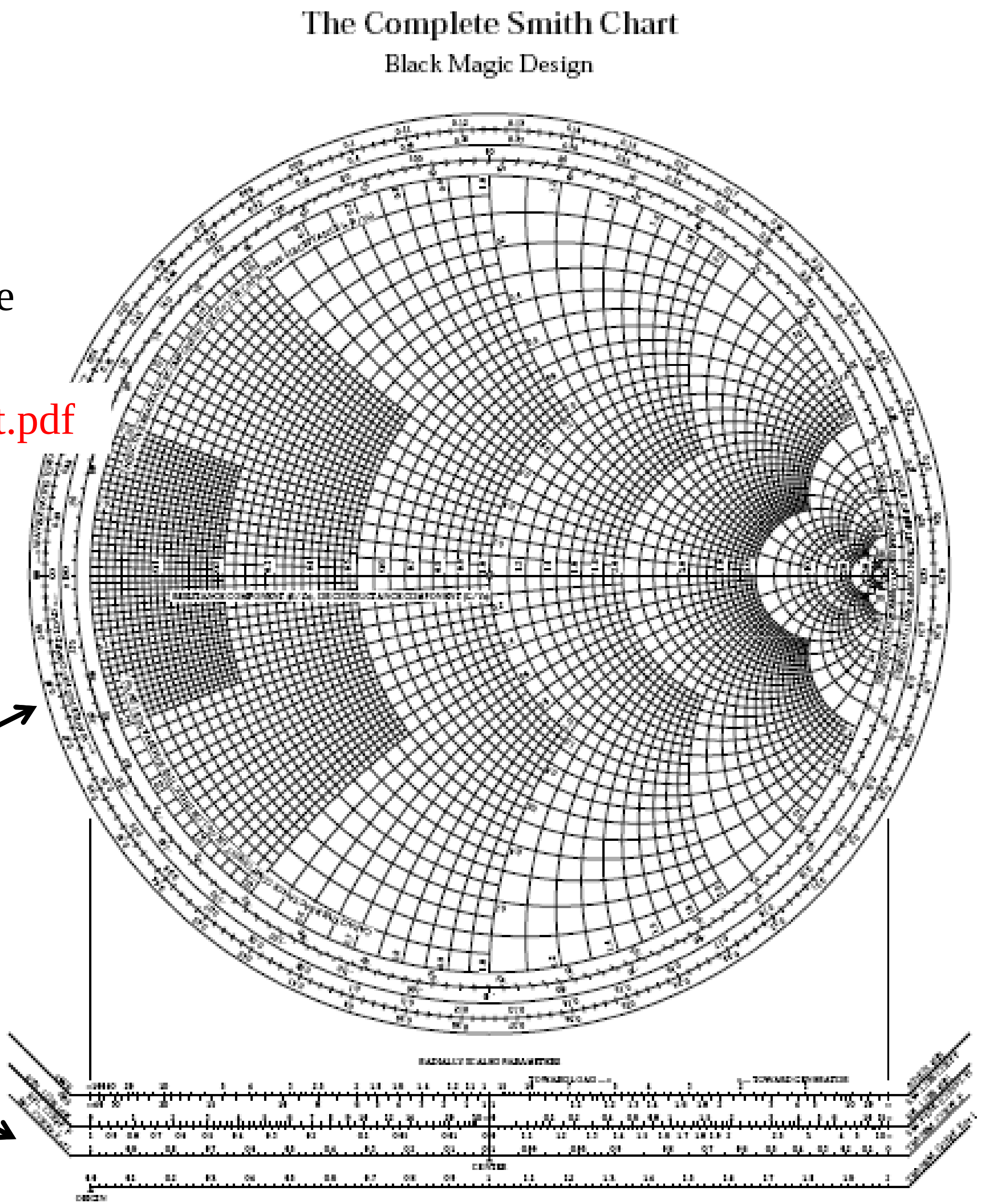


Example Smith Chart

Many sample Smith Charts are available on the internet including this one from

http://www.sss-mag.com/pdf/smithchart.pdf

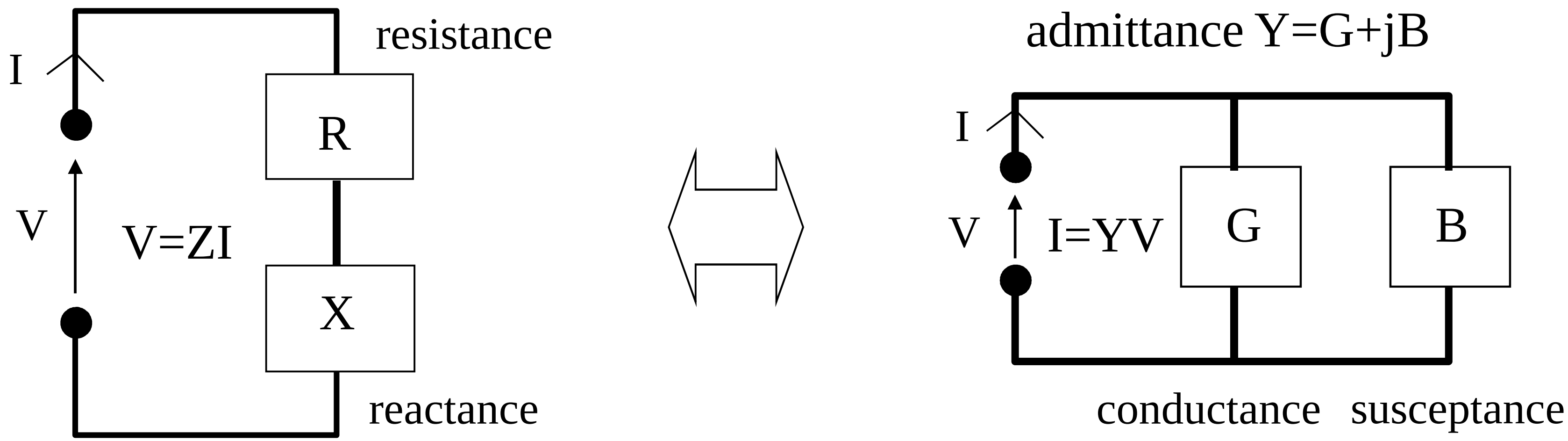
To use the Smith Chart it is important to be familiar with the radial and linear scales on the chart.



Looking at the load as an admittance

So far we have considered the load to be an impedance consisting of a series combination of a resistance and a reactance (inductive or capacitive). A load can also be represented by an admittance consisting of a parallel combination of a conductance and a susceptance (inductive or capacitive).

impedance Z=R+jX



For some RF design techniques it is convenient to swap between the two representations and versions of the Smith chart exist to facilitate this.

Calculating Γ Using Admittance

The reflection coefficient can also be calculated using the admittance of the load (Y_L) and the characteristic admittance of the transmission line (Y_0):

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \text{Using} \quad Z_L = \frac{1}{Y_L} \quad \text{and} \quad Z_0 = \frac{1}{Y_0} \quad \text{gives:}$$

$$\Gamma = \frac{\frac{1}{Y_L} - \frac{1}{Y_0}}{\frac{1}{Y_L} + \frac{1}{Y_0}} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - \overline{Y}_L}{1 + \overline{Y}_L} = -\frac{\overline{Y}_L - 1}{\overline{Y}_L + 1} \quad \text{where} \quad \overline{Y}_L = \frac{Y_L}{Y_0}$$

Admittance consists of a real part (conductance) and an imaginary part (susceptance):

$$Y = G + jB \Rightarrow \overline{Y}_L = \frac{G}{Y_0} + j\frac{B}{Y_0} = g + jb$$

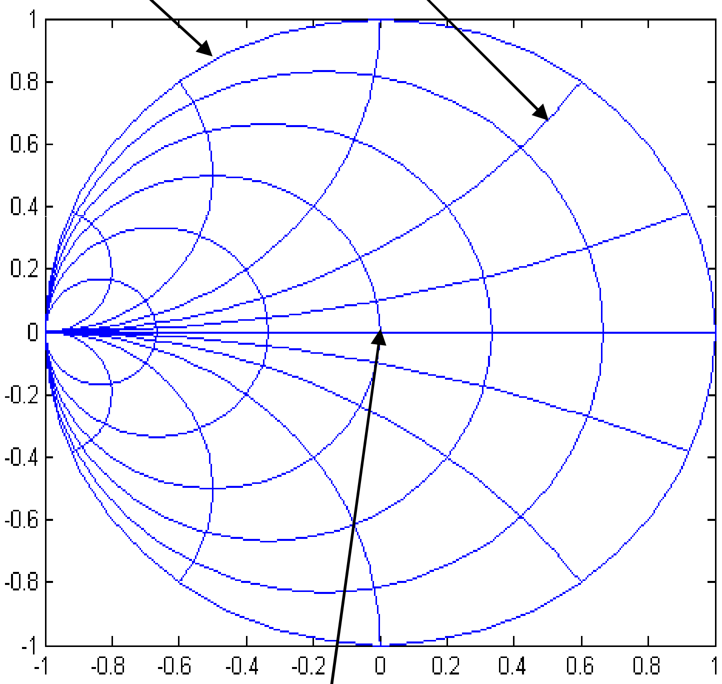
$$\Gamma = -\frac{\overline{Y}_L - 1}{\overline{Y}_L + 1} = -\frac{(g-1) + jb}{(g+1) + jb} = -\frac{g^2 + b^2 - 1}{(g+1)^2 + b^2} - j\frac{2b}{(g+1)^2 + b^2}$$

Admittance Smith Charts

Smith charts which show the reflection coefficients for standard normalized conductances and susceptances are known as Admittance Smith Charts.

circles denoting constant g

arcs denoting constant b



origin of complex plane (0,0)

$$\Gamma = -\frac{\overline{Y}_L - 1}{\overline{Y}_L + 1} =$$

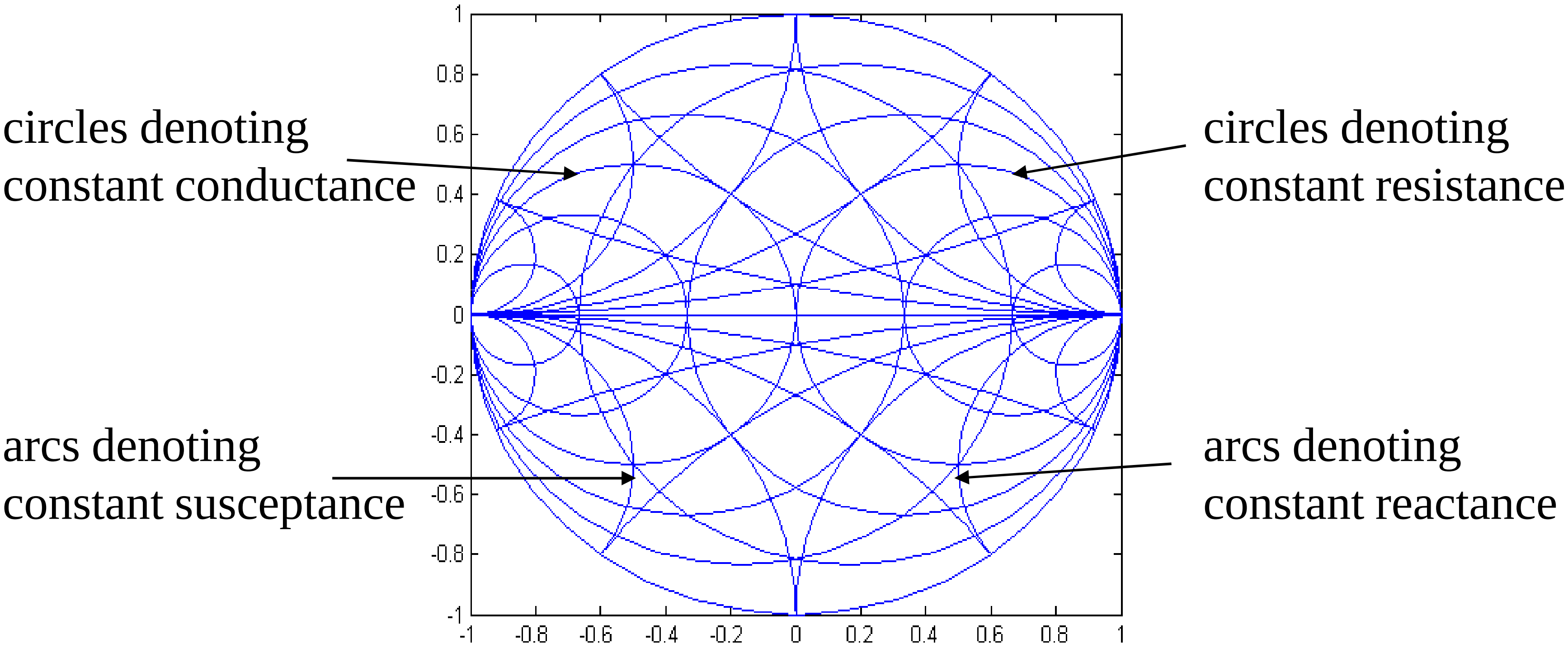
$$-\frac{g^2 + b^2 - 1}{(g+1)^2 + b^2} - j\frac{2b}{(g+1)^2 + b^2}$$

The Admittance Smith Chart looks like the Impedance Smith Chart rotated around the origin by 180°

Looking at the formula, Γ will be in the bottom semicircle for positive susceptance (b>0).

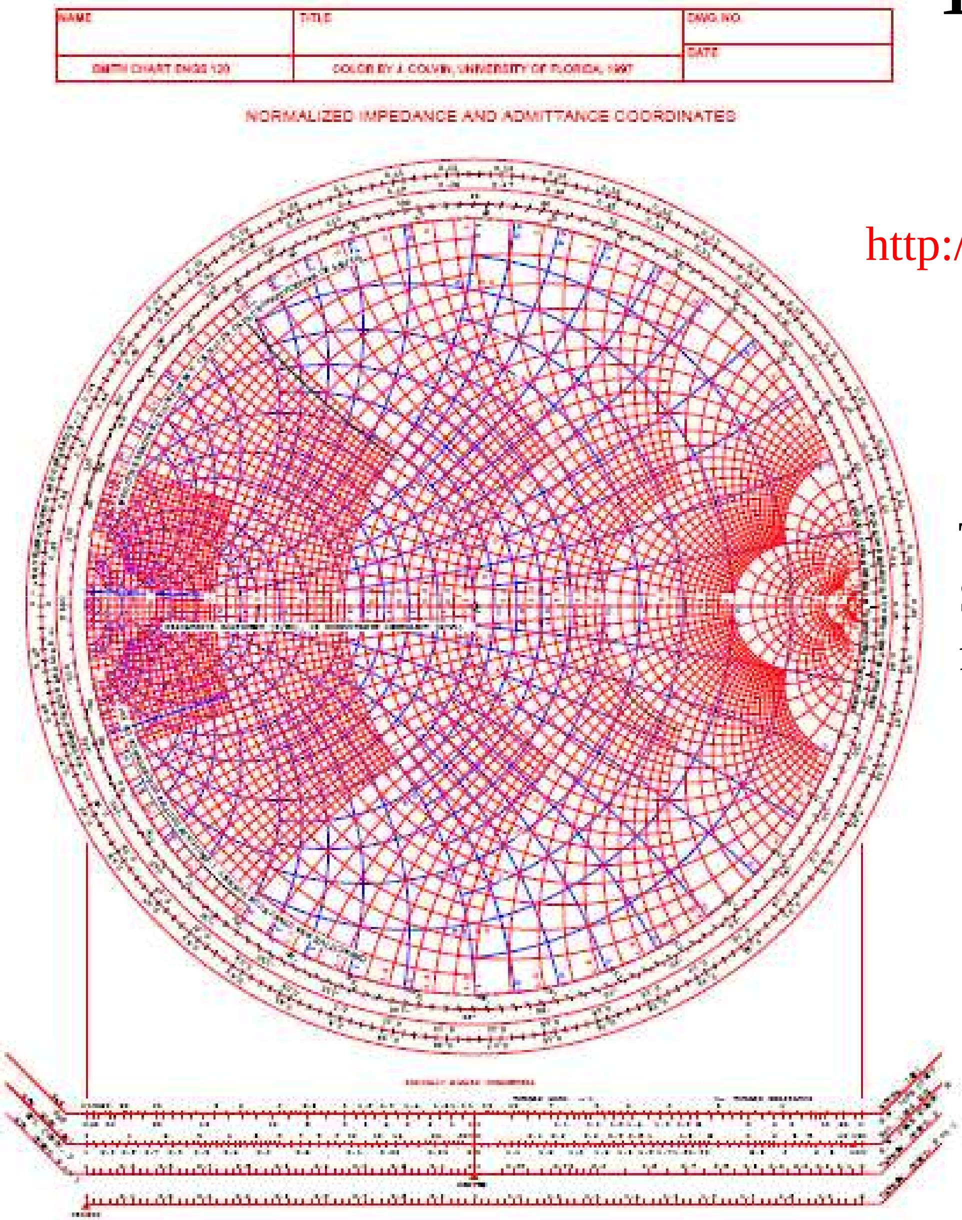
Immittance Smith Charts

Smith charts which show the reflection coefficient for standard normalized ***Im***pedances and Ad***mittances*** are sometimes known as Immittance Smith Charts and allow “easy” movement between the impedance and admittance representations.



To convert from impedance to admittance, a point is located on the Smith chart based on the impedance curves, and the corresponding normalized conductance and susceptance are read from the admittance curves which pass through the point.

Immittance Smith Chart

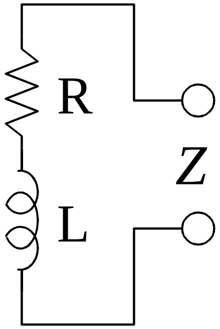


An example of this type of Smith Chart is available at

http://www.dartmouth.edu/~sullivan/colorsmith.pdf

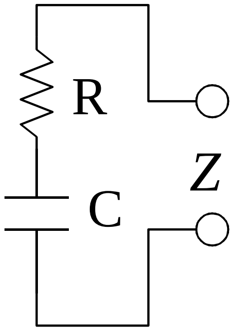
The red circles and arcs form the Impedance Smith Chart and the blue circles and arcs form the Admittance Smith Chart

Reactances and Susceptances



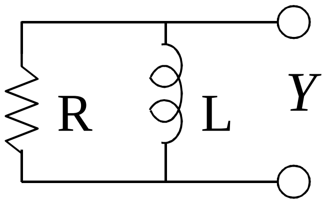
$$Z = R + j\varpi L$$

inductive reactance is positive



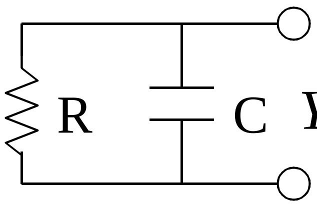
$$Z = R + \frac{1}{j\varpi C} = R - j\frac{1}{\varpi C}$$

capacitive reactance is negative



$$Y = G + \frac{1}{j\varpi L} = \frac{1}{R} - j\frac{1}{\varpi L}$$

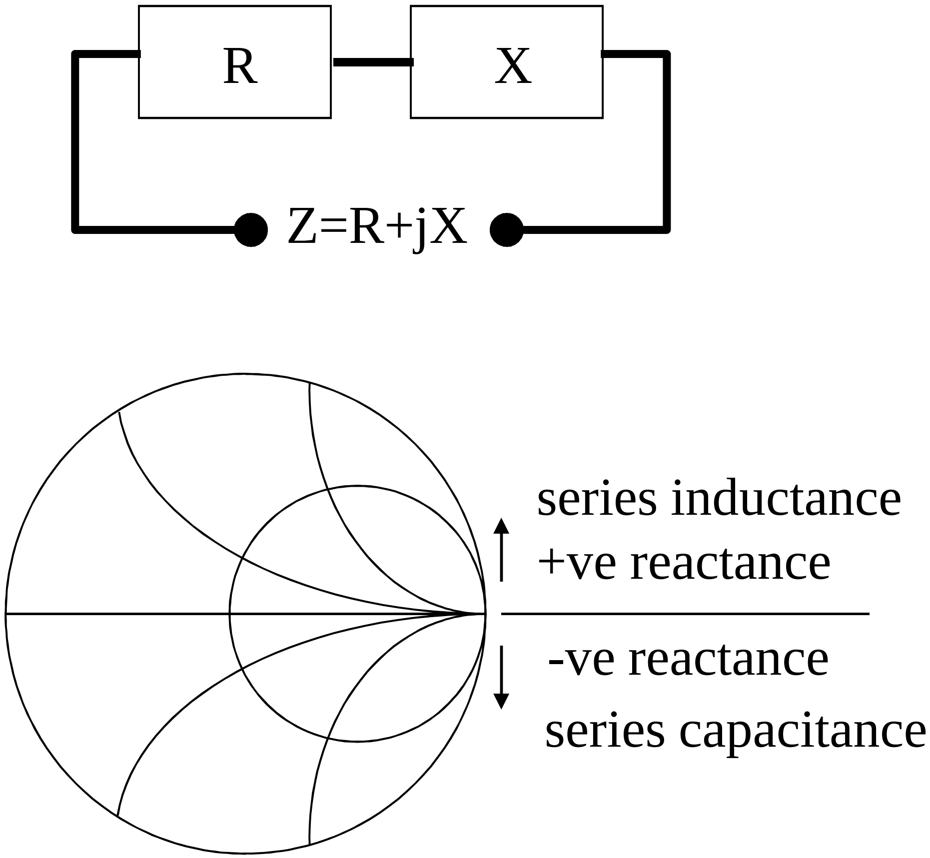
inductive susceptance is negative



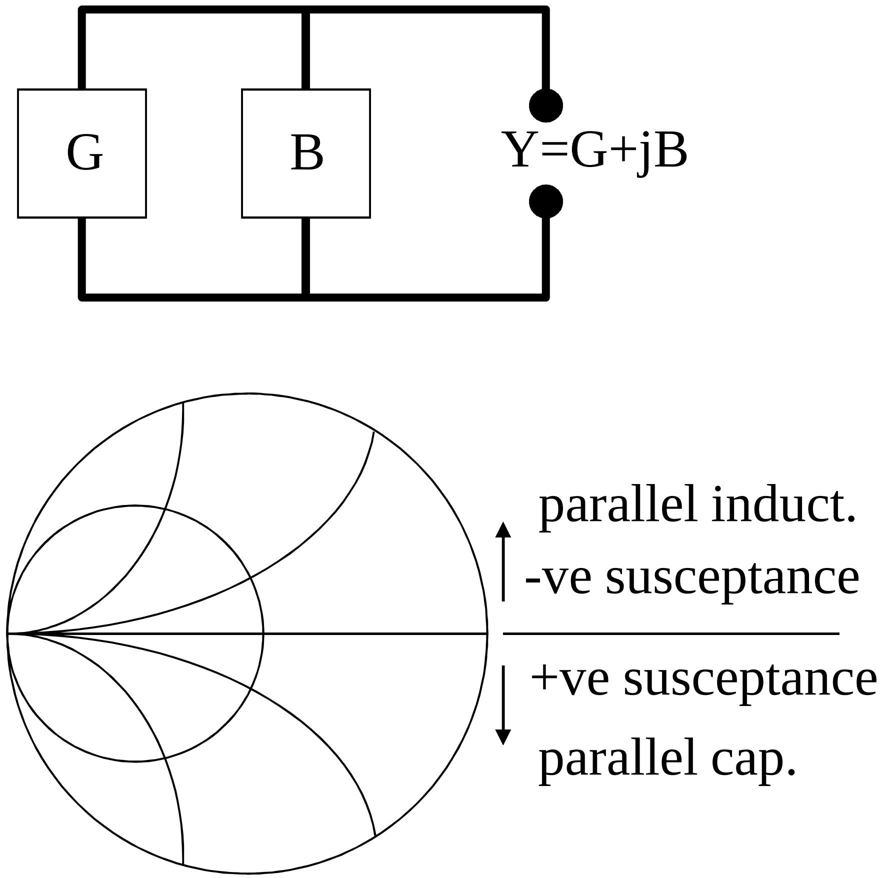
$$Y = G + j\varpi C = \frac{1}{R} + j\varpi C$$

capacitive susceptance is positive

Location of Γ for Inductances and Capacitances

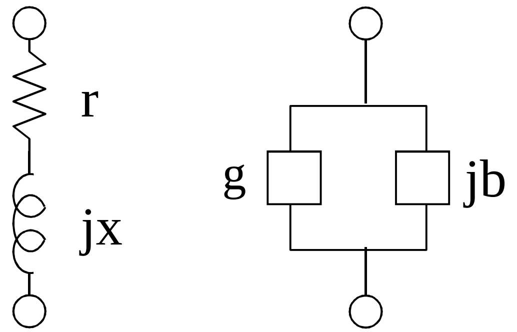


Impedance Smith Chart



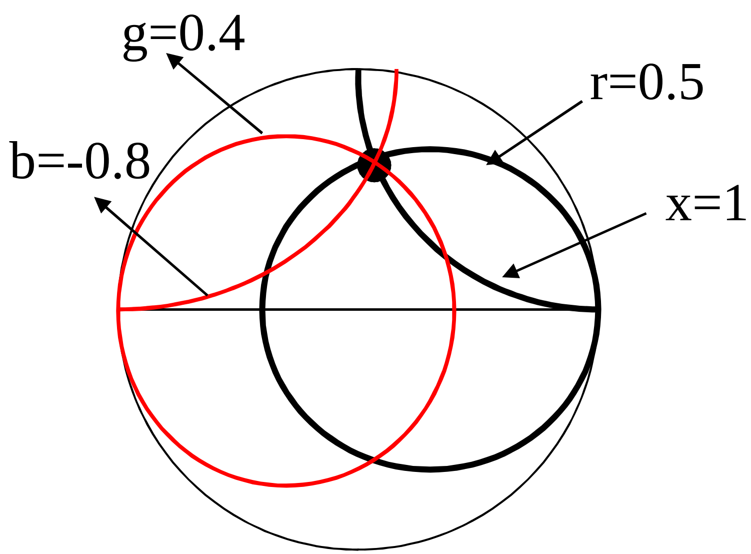
Admittance Smith Chart

Using the Immittance Smith Chart to translate from a Series Circuit to a Parallel Circuit



Resistances and reactances are normalised to Z_0.
Conductances and susceptances are normalised to Y_0.

e.g. for $z = r+jx = 0.5 + j1.0$



1. Locate the circle corresponding to r = 0.5 and the arc corresponding to x=1.0 on the Impedance Smith Chart. This is the reflection coefficient for the impedance z=0.5 + j1.0
2. Identify the conductance circle and the susceptance arc of the Admittance Smith Chart which pass through the reflection coefficient. These give the admittance value:

$$y = g+jb = 0.4\text{-}j0.8$$