Solutions UE4002 Summer 2010

Each part of each question carries equal marks.
The body effect may be ignored in each question.
The following equation is given for the drain current of an NMOS in saturation:

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{tn})^2(1+\lambda_n V_{DS})$$

For dc biasing calculations take $\lambda_n=\lambda_p=0$.
In each question, capacitances other than those mentioned may be ignored.

Question 1

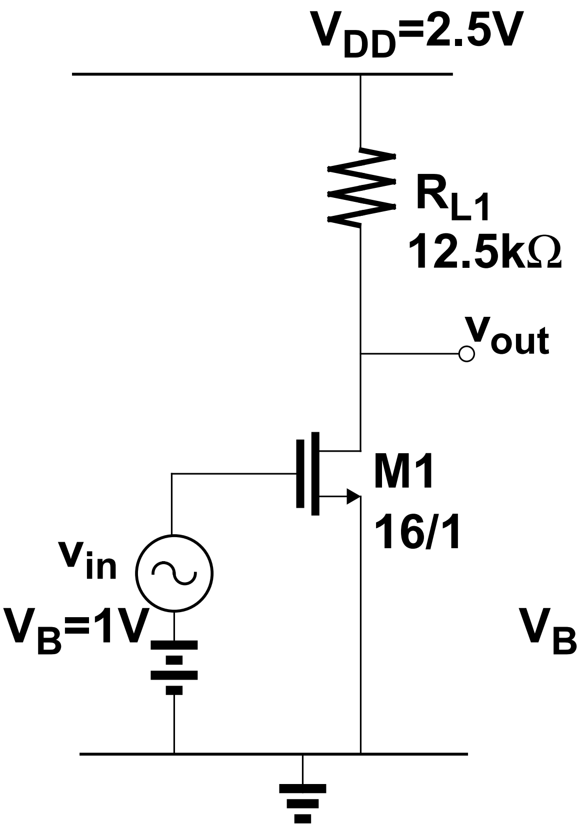


Figure 1a

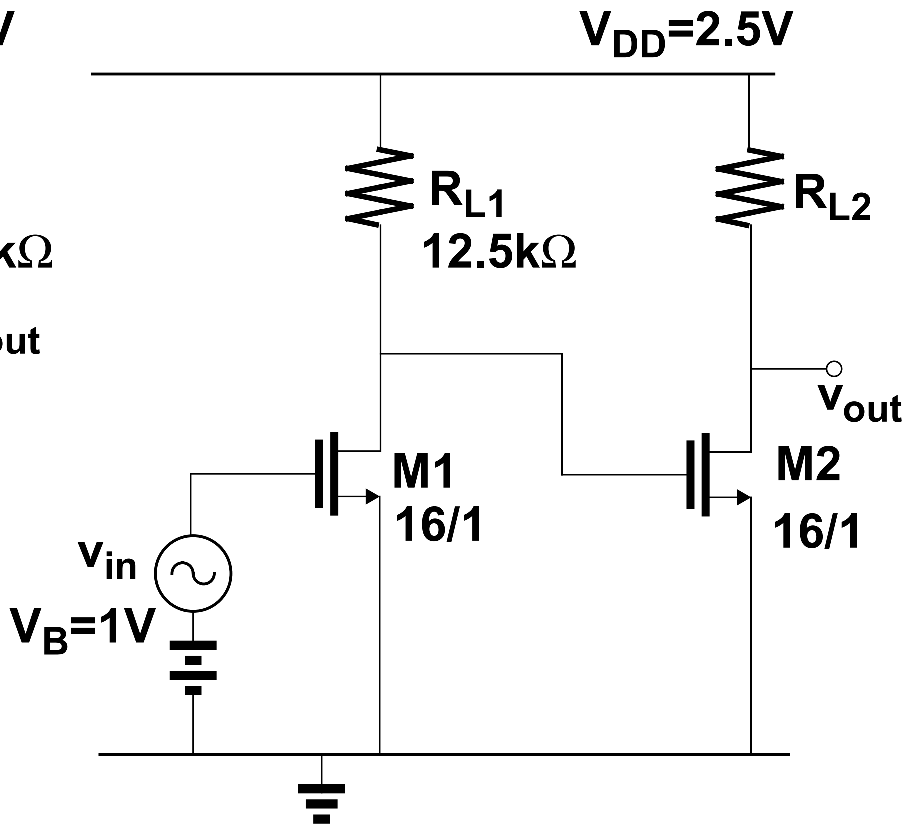


Figure 1b

For the questions below you may assume $g_{ds1}, g_{ds2} << 1/R_{L1}, 1/R_{L2}$ and that all devices are biased in saturation.
DC bias voltages, transistor dimensions and resistor values are as shown in the figures.
Take $K_n'=200\mu A/V^2$, $V_{tn}= 0.75V$.

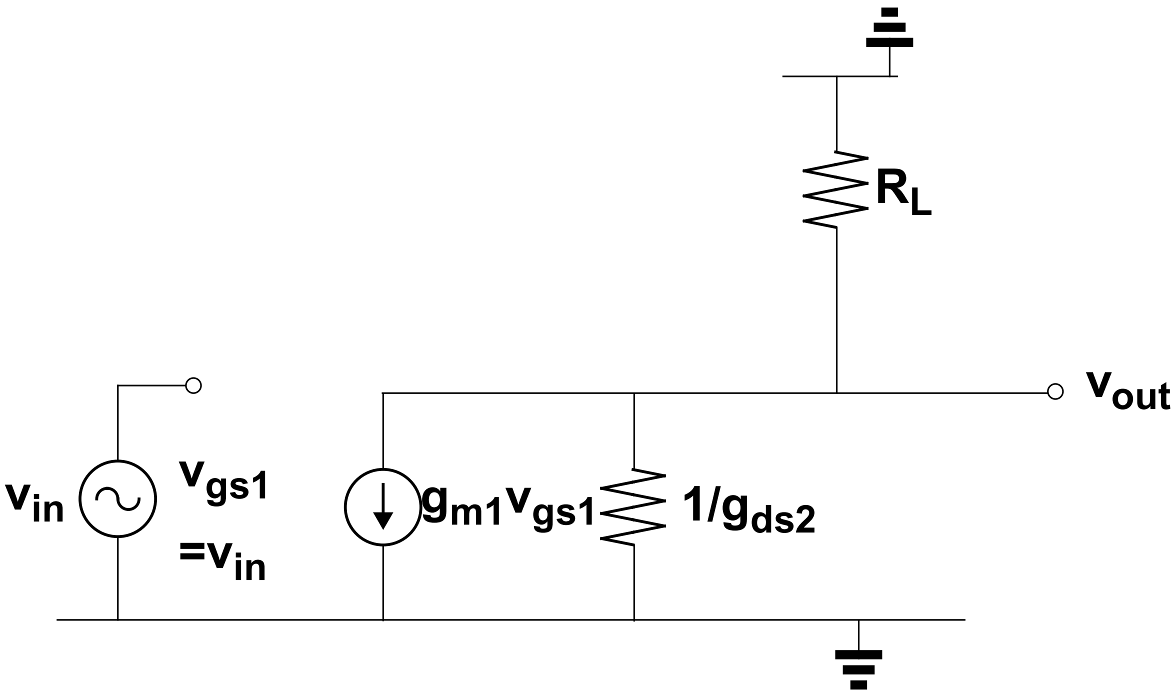
(i) Figure 1a shows a gain stage with a resistive load.
Draw the small-signal model for this circuit.
Derive an expression for the small signal voltage gain (v_{out}/v_{in}).

(ii) For the circuit in Figure 1a calculate the drain current of M1 and the small-signal voltage gain (v_{out}/v_{in}) in dB.

(iii) Calculate the drain current of M2 in the circuit shown in Figure 1b.

(iv) Calculate the maximum small-signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 1b.

(i) Figure 1a shows a gain stage with a resistive load.
Draw the small-signal model for this circuit.
Derive an expression for the low-frequency small signal voltage gain (v_{out}/v_{in}).



Current at output node

$$g_{m1}v_{gs1} + v_{out}g_{ds1} + v_{out}R_L = 0$$

$$g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + \frac{1}{R_L}} \cong \underline{\underline{-g_{m1}R_L}}$$

(ii) Calculate the drain current of M1 and the small-signal voltage gain (v_{out}/v_{in}) in dB.

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS1}-V_{tn})^2 = \frac{200\mu A/V^2}{2} \cdot \frac{16}{1} \cdot (1-0.75)^2 = \underline{\underline{100\mu A}}$$

$$g_{m1} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2\times 200\mu A/V \times \frac{16}{1} \times 100\mu A} = 800\mu A/V$$

Low-frequency gain given by

$$\frac{v_{out}}{v_{in}} \cong -g_{m1}R_L = 800\mu A/V \times 12.5k\Omega = -10 = \underline{\underline{20dB}}$$

(iii) Calculate the drain current of M2 in the circuit shown in Figure 1b.

The gate source voltage of M2 is set by the voltage at the drain of M1

$$V_{GS2} \cong V_{DD} - I_{D1}R_{L1} = 2.5V - 100\mu A \times 12.5k\Omega = 1.25V$$

The drain current of M2 is then given by

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS2}-V_{tn})^2 = \frac{200\mu A/V^2}{2} \cdot \frac{16}{1} \cdot (1.25-0.75)^2 = \underline{\underline{400\mu A}}$$

(iv) Calculate the maximum small-signal voltage gain (v_{out}/v_{in}) of the circuit shown in Figure 1b.

The circuit in Figure 1b has max. gain with the largest value of R_{L2} such that M2 is still just in saturation

The minimum voltage at the output for M2 to be in saturation is then given by

$$V_{OUTmin} \cong V_{GS2} - V_t = 1.25V - 0.75V = 0.5V$$

The max value of R_{L2} is then given by

$$R_{L2max} \cong \frac{V_{DD} - V_{OUTmin}}{I_{D2}} = \frac{2.5V - 0.5}{400\mu A} = 5k\Omega$$

The transconductance of M2 is given by

$$g_{m2} = \frac{2I_D}{V_{GS2} - V_t} = \frac{2\times 400\mu A}{0.5V} = 1600\mu A/V$$

The overall gain is then given by

$$\frac{v_{out}}{v_{in}} \cong -g_{m1}R_{L1} \times -g_{m2}R_{L2} = -10 \times -1600\mu A/V \times 5k\Omega = 80 = \underline{\underline{38dB}}$$

Question 2

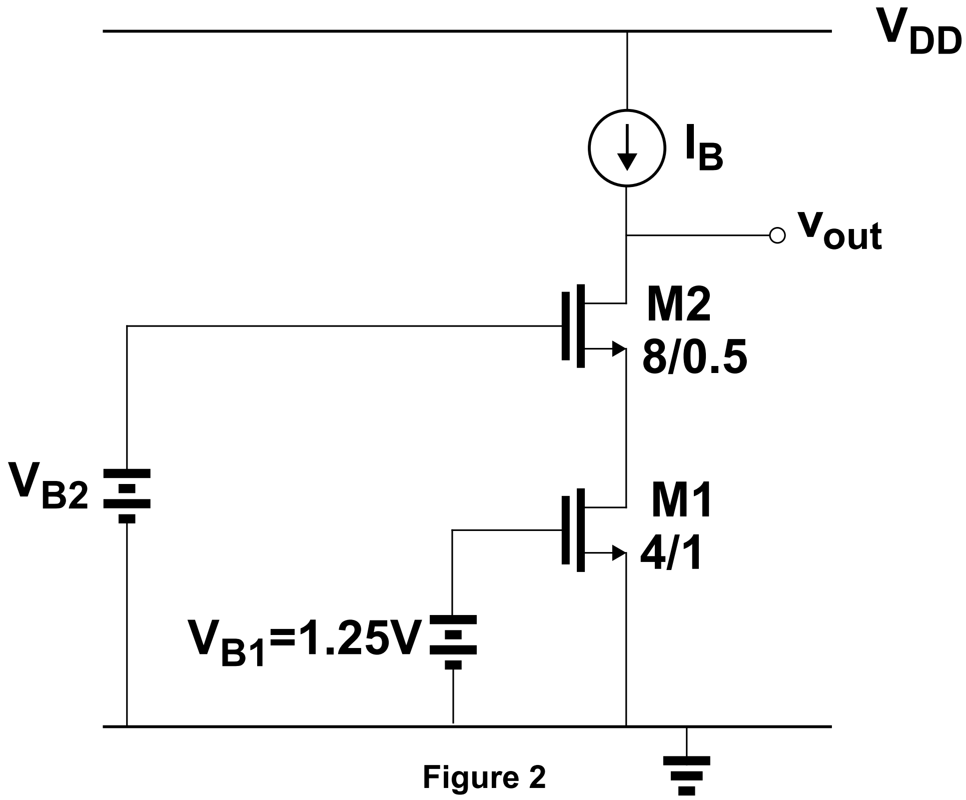


Figure 2

Figure 2 shows a cascode stage with an ideal current source load.
Biasing and transistor dimensions are as shown in Figure 2.
Take $K_n' = 200\mu A/V^2$, $V_{tn} = 0.75V$.
Assume all transistors are biased in saturation.

(i) What is the minimum value of V_{B2} such that M1 is in saturation?
What is then the minimum voltage at the output node (v_{out}) such that M2 is in saturation?

(ii) Draw a small-signal equivalent circuit of the cascode stage, showing how to measure the small-signal output resistance i.e. the resistance looking into the node v_{out}.

(iii) Derive an expression for the small-signal output resistance in terms of the small-signal parameters of M1 and M2.
Simplify the expression assuming $g_{m1}, g_{m2}, >> g_{ds1}, g_{ds2}$.

(iv) Calculate the small-signal output resistance.
Take $\lambda_n = 0.04/L\ V^{-1}$ with L in microns.

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(i) What is the minimum value of V_{B2} such that M1 is in saturation?
What is then the minimum voltage at the output node (v_{out}) such that M2 is in saturation?

For M1 to be in saturation then

$$V_{DS1} \geq V_{GS1} - V_{tn}$$

$$(V_{DS1})_{min} = V_{GS1} - V_{tn} = 1.25V - 0.75V = 500mV$$

$$I_{D1} = \frac{K_n'}{2}\frac{W}{L}(V_{GS1} - V_{tn})^2 = \frac{200\mu A/V^2}{2} \cdot \frac{4}{1} \cdot (1.25 - 0.75)^2 = 100\mu A$$

As

$$V_{GS2} - V_t = \sqrt{\frac{2I_{D2}}{K_n' \frac{W_2}{L_2}}} = \sqrt{\frac{2 \cdot 100\mu A}{200\mu A/V^2 \frac{8}{0.5}}} = 250mV$$

$$V_{B2} = V_{GS2} + (V_{DS1})_{min}$$

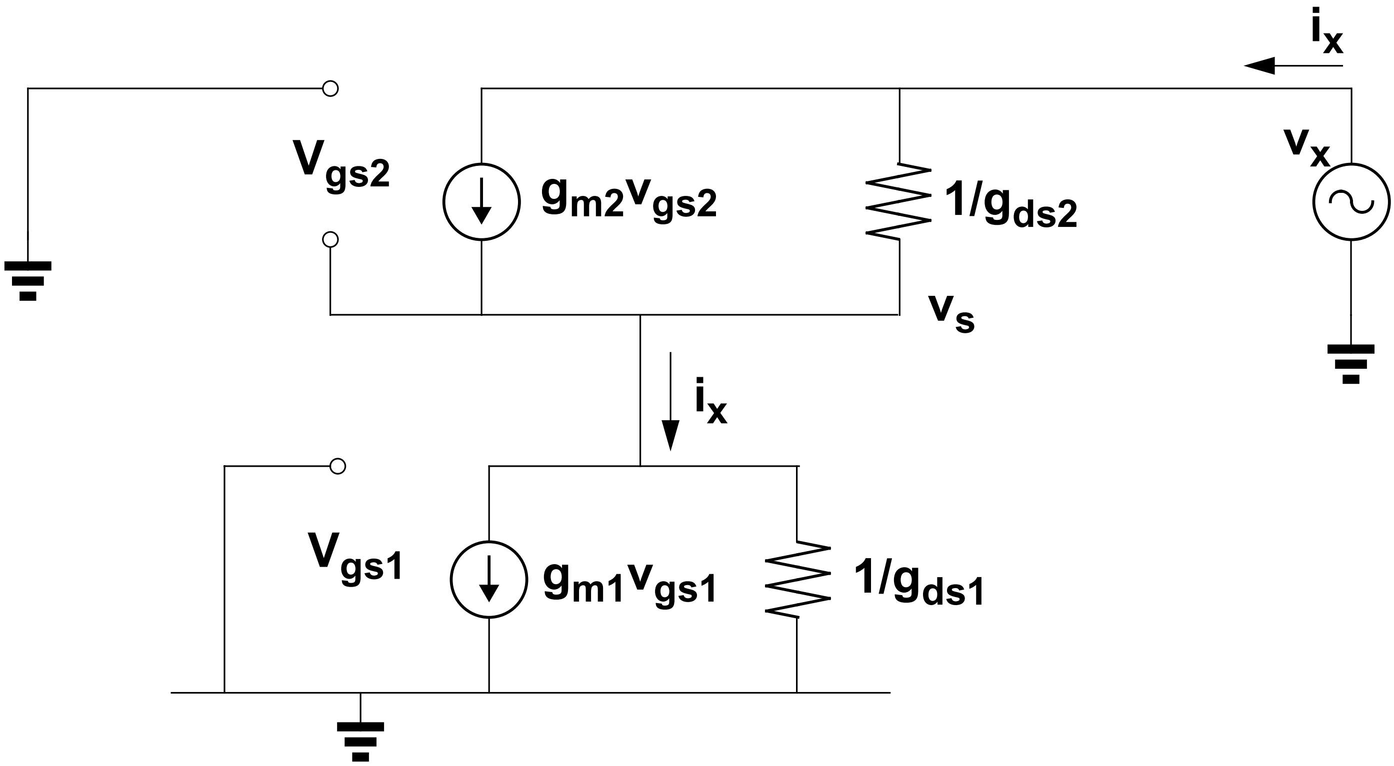
$$V_{BIAS2} = (V_{GS2} - V_{tn}) + V_{tn} + (V_{DS1})_{min} = 0.25V + 0.75V + 0.5V = \underline{\underline{1.5V}}$$

$$V_{DS2} \geq V_{GS2} - V_{tn} = 0.25V$$

so minimum voltage at the output for both transistors to be in saturation is given by

$$V_{outmin} = (V_{DS1})_{min} + (V_{DS2})_{min} = 0.25V + 0.5V = \underline{\underline{0.75V}}$$

(ii) Draw a small-signal equivalent circuit of of the cascode stage, showing how to measure the small-signal output resistance i.e. the resistance looking into the node v_{out}.



(iii) Derive an expression for the small-signal output resistance in terms of the small-signal parameters of M1 and M2. Simplify the expression assuming g_{m1},g_{m2},>>g_{ds1},g_{ds2},.

Note: $v_{gs1} = 0 \Rightarrow g_{m1}v_{gs1} = 0$

$$i_x = g_{m2}v_{gs2} + (v_x - v_s)g_{ds2}$$

Since $v_{gs2} = -v_s$ and $v_s = \frac{i_x}{g_{ds1}}$

$$i_x = -(g_{m2})\frac{i_x}{g_{ds1}} + \left(v_x - \frac{i_x}{g_{ds1}}\right)g_{ds2}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1 + \frac{g_{m2}}{g_{ds1}} + \frac{g_{ds2}}{g_{ds1}}}{g_{ds2}}$$

Since g_{m1},g_{m2} >> g_{ds1},g_{ds2} this can be reduced to

$$r_{out} = \frac{g_{m2}}{g_{ds2}} \cdot \frac{1}{g_{ds1}}$$

(iv) Calculate the small-signal output resistance assuming all transistors are in saturation. Take λ_n=0.04/L V^{-1} with L in microns.

$$r_{out} = \frac{g_{m2}}{g_{ds2}} \cdot \frac{1}{g_{ds1}}$$

$$g_{m2} = \frac{2I_{D2}}{(V_{GS2} - V_{tn})} = \frac{2 \times 100\mu A}{0.25V} = 800\mu A/V$$

$$g_{ds1} = \lambda I_{D1} = \frac{0.04}{L_1} I_{D1} = \frac{0.04}{1} 100\mu A = 4\mu A/V$$

$$g_{ds2} = \lambda I_{D2} = \frac{0.04}{L_2} I_{D2} = \frac{0.04}{0.5} 100\mu A = 8\mu A/V$$

$$r_{out1} = \frac{g_{m2}}{g_{ds2}} \cdot \frac{1}{g_{ds1}} = \frac{800\mu A/V}{8\mu A/V} \cdot \frac{1}{4\mu A/V} = 25M\Omega$$

Question 3

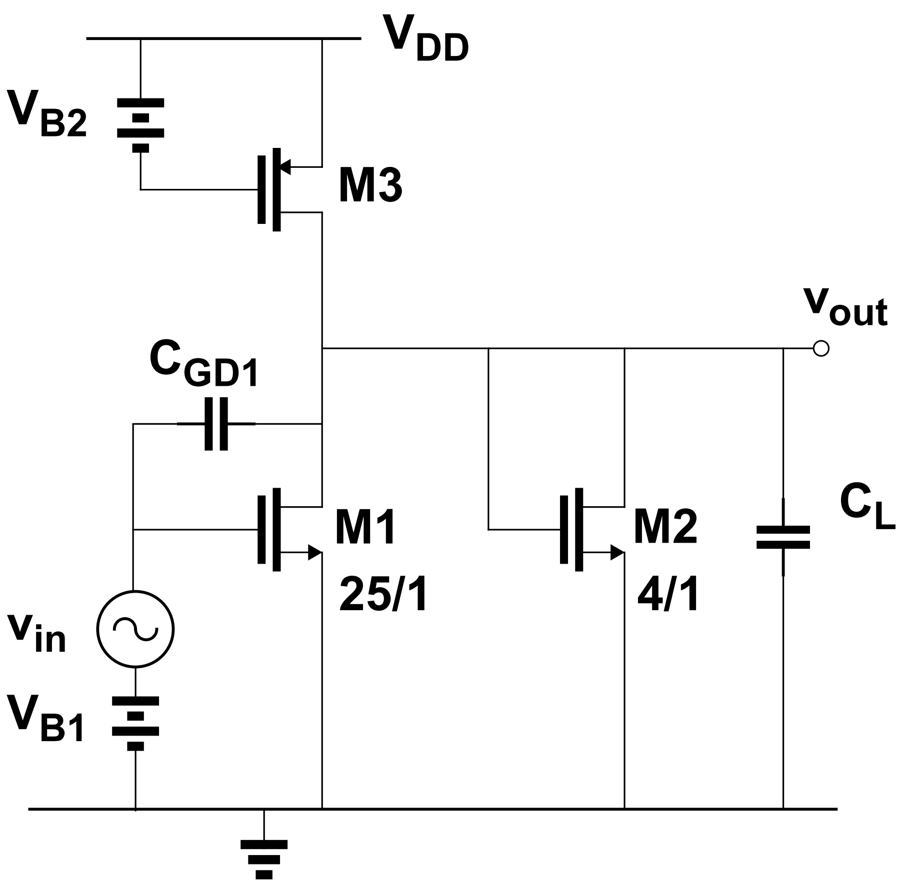
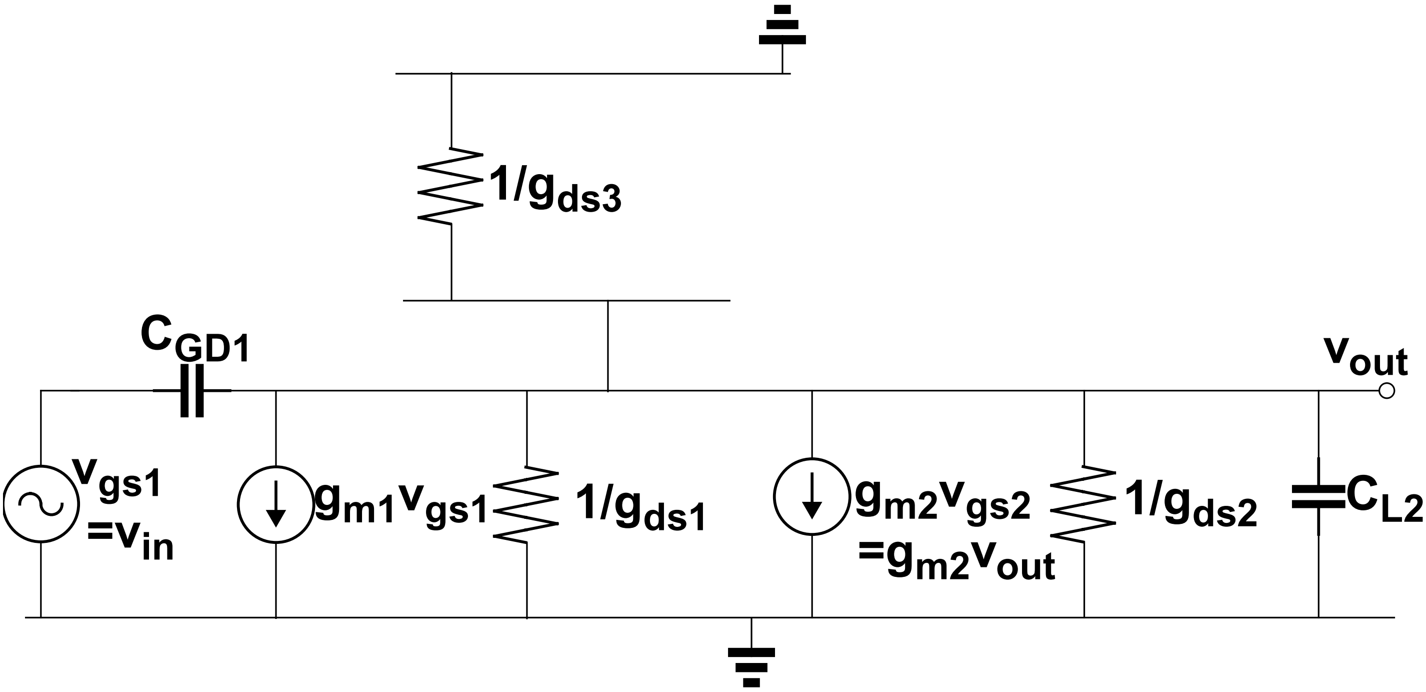


Figure 3

For the questions below you may assume $g_{m1}, g_{m2} >> g_{ds1}, g_{ds2}, g_{ds3}$, and that all transistors are biased in saturation. Transistor dimensions are as shown in Figure 3.

(i) Figure 3 shows a capacitively loaded gain stage. Draw the small-signal model for this circuit.
(ii) Ignoring all capacitances except C_{GD1}, and C_L, derive an expression for the high-frequency transfer function from v_{in} to v_{out}.
(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the break frequencies (i.e. pole and/or zero frequencies) if $K_n'=200\mu A/V^2$, $I_{D1}=100\mu A$, $|I_{D3}|=200\mu A$, $C_{GD1}=0.1pF$, $C_L=7.9pF$.
(iv) Draw a Bode diagram of the gain response.
Indicate the values of gain at d.c. and at frequencies well above the pole and/or zero frequencies.

(i) Figure 3 shows a capacitively loaded gain stage. Draw the small-signal model for this circuit.



(ii) Ignoring all capacitances except C_{GD1}, C_L, derive an expression for the high-frequency transfer function from v_{in} to v_{out}.

KCL at output node:

$$(v_{out} - v_{in})sC_{GD1} + g_{m1}v_{in} + v_{out}g_{m2} + v_{out}g_{ds2} + v_{out}g_{ds3} + v_{out}sC_L = 0$$

$$v_{in}(g_{m1} - sC_{GD1}) + v_{out}(g_{ds1} + g_{m2} + g_{ds2} + g_{ds3} + s(C_{GD1} + C_L)) = 0$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_{m1} - sC_{GD1}}{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3} + s(C_{GD1} + C_L)}$$

$$a(s) = \frac{v_{out}}{v_{in}}(s) = -\frac{g_{m1}}{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3}}\left(\frac{1 - s\frac{C_{GD1}}{g_{m1}}}{1 + \frac{s(C_{GD1} + C_L)}{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3}}}\right)$$

(iii) Calculate the low-frequency gain (v_{out}/v_{in}) and the break frequencies (i.e. pole and/or zero frequencies) if $K_n' = 200\mu A/V^2$, $I_{D1} = 100\mu A$, $|I_{D3}| = 200\mu A$, $C_{GD1} = 0.1pF$, $C_L = 7.9pF$.

Current from M3 splits between M1 and M2 => 100µA in M2

$$g_{m1} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{25}{1} \times 100\mu A} = 1000\mu A/V$$

$$g_{m2} = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2 \times 200\mu A/V \times \frac{4}{1} \times 100\mu A} = 400\mu A/V$$

Low-frequency gain given by

$$\frac{v_{out}}{v_{in}} \cong -\frac{g_{m1}}{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3}} \approx -\frac{g_{m1}}{g_{m2}} = -\frac{1000\mu A/V}{400\mu A/V} = -2.5 \Rightarrow \underline{\underline{8dB}}$$

Zero frequency given by

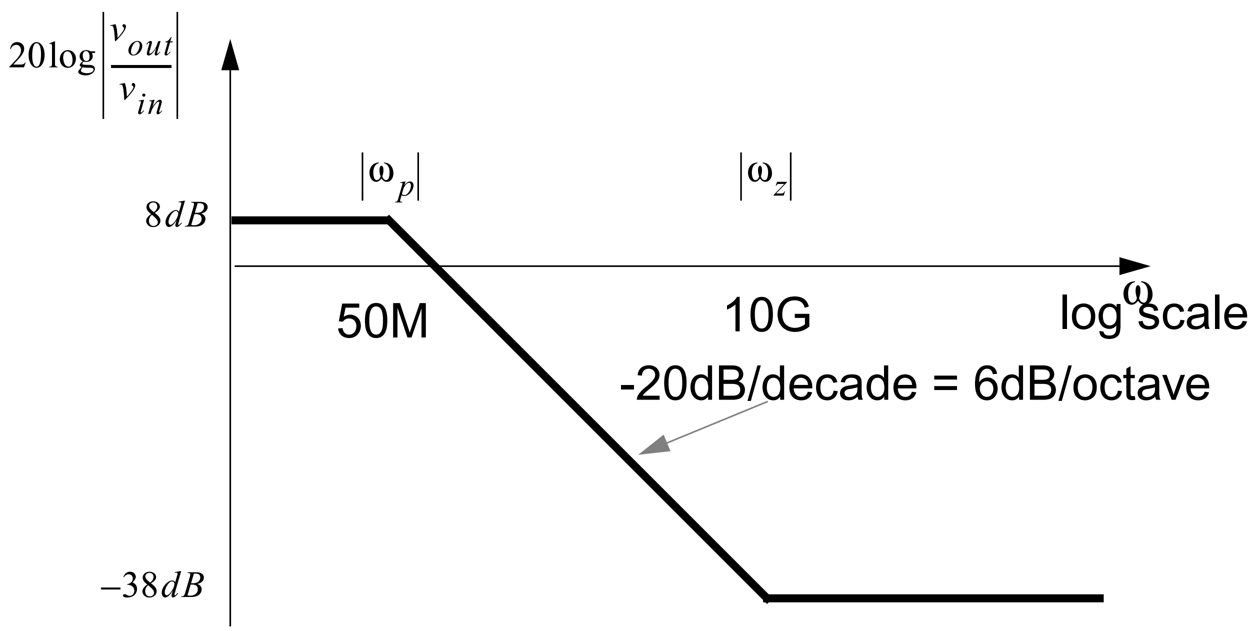
$$|\omega_z| = \frac{g_{m1}}{C_{GD1}} = \frac{1000\mu A/V}{0.1pF} = \underline{\underline{10Grad/s}}$$

Pole frequency given by

$$|\omega_p| = \frac{g_{ds1} + g_{m2} + g_{ds2} + g_{ds3}}{C_{GD1} + C_L} \approx \frac{g_{m2}}{C_{GD1} + C_L} = \frac{400\mu A/V}{7.9pF + 0.1pF} = \underline{\underline{50Mrad/s}}$$

(iv) Draw a Bode diagram of the gain response. Indicate the values of gain at d.c. and at frequencies well above the pole and zero frequencies.

Zero is 2 decades and 1 octave down, so gain at high frequecies is 46dB down = -38dB



Question 4

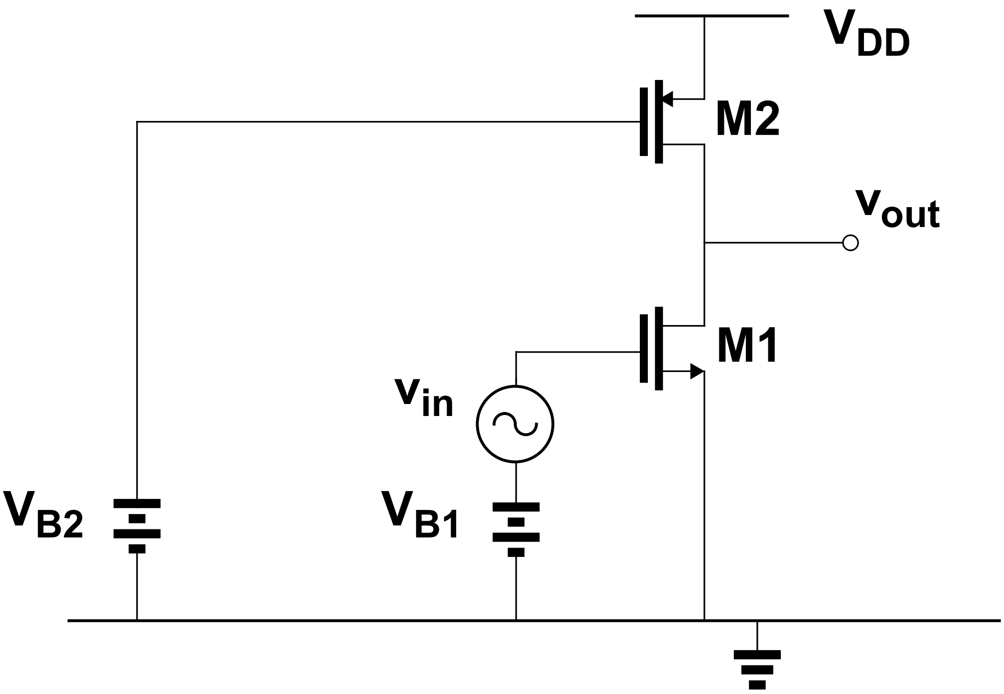


Figure 4

Assume M1 and M2 are operating in saturation. Only thermal noise sources need be considered.

For calculations take Boltzmann's constant $k=1.38X10^{-23}J/^{o}K$, temperature $T=300^{o}K$.

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters of M1 and M2?

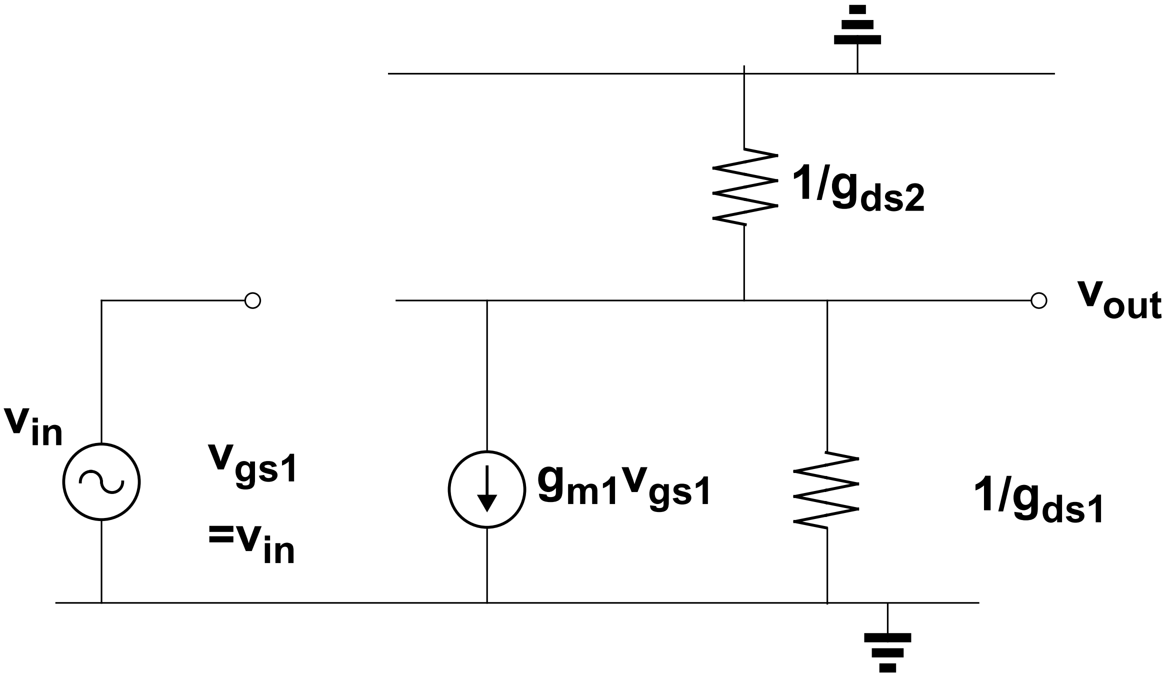
(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4?
The answer should be in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T.

(iii) Calculate the input-referred thermal noise voltage density of the circuit if
V_{B1}=1.0V, V_{B2}=1.75V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n=\lambda_p=0.04V^{-1}$.
The drain current of M2 is 100µA.
Calculate the thermal noise voltage density at the output of the circuit.

(iv) What minimum value of input signal is required for an output signal-to-noise ratio of 60dB over a bandwidth of 1MHz?

Solution

(i) Draw the small-signal model for the circuit shown in Figure 4.
What is the low-frequency small-signal voltage gain (v_{out}/v_{in}) in terms of the small-signal parameters of M1 and M2?

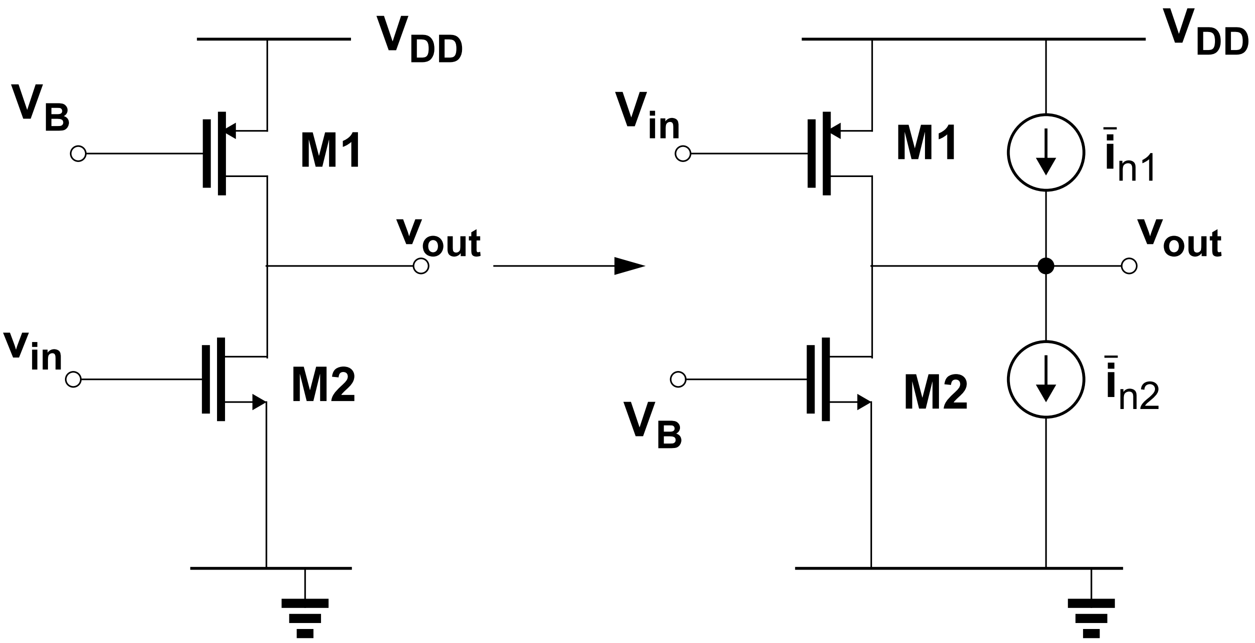


Current at output node

$$g_{m1}v_{in} + v_{out}g_{ds1} + v_{out}g_{ds2} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds2}}$$

(ii) What is the input-referred thermal noise voltage density of the circuit shown in Figure 4? The answer should be in terms of the small-signal parameters of M1 and M2, Boltzmann's constant k and temperature T.?



Noise current density of MOS:

$$\overline{i_n^2} = 4kT\left(\frac{2}{3}g_m\right) \qquad A^2/Hz$$

$$\overline{i_n} = \sqrt{4kT\left(\frac{2}{3}g_m\right)} \qquad A/\sqrt{Hz}$$

Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_{m1}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + 4kT\left(\frac{2}{3}g_{m2}\right)}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) Calculate the input-referred thermal noise voltage density of the circuit if V_{B1}=1.0V, V_{B2}=1.75V, V_{DD}=3V, V_{tn} = 0.75V, V_{tp} = -0.75V, $\lambda_n=\lambda_p=0.04V^{-1}$.
The drain current of M2 is 100µA.
Calculate the thermal noise voltage density at the output of the circuit.

g_m given by

$$g_m = \frac{2I_D}{(|V_{GS}|-|V_{tp}|)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V \qquad g_{m2} = \frac{2 \cdot 100\mu A}{(3V - 1.75V) - 0.75V} = 400\mu A/V$$

$$\overline{v_{ni}} = \frac{\sqrt{4kT\left(\frac{2}{3}(g_{m1} + g_{m2})\right)}}{g_{m1}}$$

$$\overline{v_{nitot}} = \frac{\sqrt{(4 \cdot 1.38\times10^{-23} \cdot 300)\left(\frac{2}{3}\right)(800\mu A/V + 400\mu A/V)}}{800\mu A/V} = 4.55nV/\sqrt{Hz}$$

$$g_{ds1} = \lambda_n I_D = 0.04V^{-1}100\mu A = 4\mu A/V$$
$$g_{ds2} = \lambda_n I_D = 0.04V^{-1}100\mu A = 4\mu A/V$$

Gain of stage

$$Gain = -\left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) = -\frac{800\mu A/V}{8\mu A/V} = -100$$

$$\overline{v_{notot}} = \overline{v_{nitot}} \cdot \left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) = 455nV/\sqrt{Hz}$$

(iv) What minimum value of input signal is required for an output signal-to-noise ratio of 60dB over a bandwidth of 1MHz?

Signal-to-Noise ratio at the output is given by

$$\frac{S}{N} = \frac{v_{out}}{\overline{v_{nitot}} \cdot \left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) \cdot \sqrt{BW}} = 1000$$ i.e. 60dB

So required signal at the output is

$$v_{out} = \overline{v_{nitot}} \cdot \left(\frac{g_{m1}}{g_{ds1} + g_{ds2}}\right) \cdot \sqrt{BW} \cdot \frac{S}{N}$$

So required signal at the input is

$$v_{in} = \overline{v_{nitot}} \cdot \sqrt{BW} \cdot \frac{S}{N} = 4.55nV/\sqrt{Hz} \cdot \sqrt{1MHz} \cdot 1000 = \underline{\underline{4.55mV}}$$