2/5/09 Autumn 03

Q2(a) $M(z) = \sum_{k=1}^{\infty} m(k) z^{-k}$

$M(z) = m_1 z^{-1} + m_2 z^{-2} + \dots + m_n z^{-n} + m_n z^{-n-1} + \dots$

Since $R(z) = 1 - z^{-1}$

$M(z)/R(z) = (1-z^{-1})(m_1 z^{-1} + m_2 z^{-2} + \dots + m_n z^{-n} + m_n z^{-n-1} + \dots)$

$= m_1 z^{-1} + (m_2 - m_1) z^{-2} + (m_3 - m_2) z^{-3} + (m_4 - m_3) z^{-4}$

$\Rightarrow Q(z) = M(z)/R(z) = z^{-1} + (-0.5-1) z^{-2} + (0.3+0.5) z^{-3} + (0.1-0.3) z^{-4}$

$= z^{-1} - 1.5 z^{-2} + 0.8 z^{-3} - 0.2 z^{-4}$

$C(z) = z^{-d}(c_1 z^{-1} + c_2 z^{-2} + \dots c_n z^{-n} + z^{-n-1} + \dots)$

$C(z)/R(z) = (1-z^{-1}) z^{-d}(c_1 z^{-1} + c_2 z^{-2} + \dots c_n z^{-n} + z^{-n-1} + \dots)$

$= z^{-d}(c_1 z^{-1} + (c_2 - c_1) z^{-2} + (1 - c_n) z^{-n-1})$

$\Rightarrow P(z) = C(z)/R(z) = z^{-2}(0.4 z^{-1} + 0.4 z^{-2} - 0.3 z^{-3} + 0.1 z^{-4} + 0.1 z^{-5}$

$$D(z) = \frac{Q(z)}{1-P(z)} = \frac{z^{-1} - 1.5 z^{-2} + 0.8 z^{-3} - 0.2 z^{-4}}{1 - z^{-2}(0.4 z^{-1} + 0.4 z^{-2} - 0.3 z^{-3} + 0.1 z^{-4} + 0.1 z^{-5}}$$

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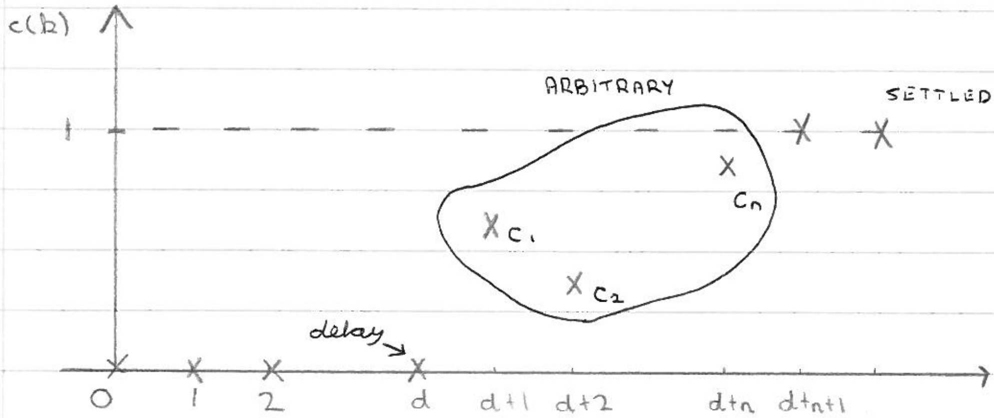
Q2(a). Specify $D(z)$ to achieve the following for a unit step in $r(k)$

(i) Output $c(k)$ will settle to a steady-state of 1 within $(n+d+1)$ samples



Taking Z transform of the sequence $c(k)$

$$C(z) = \sum_{k=0}^{\infty} c(k) z^{-k}$$

$$\Rightarrow C(z) = z^{-d}(c_1 z^{-1} + c_2 z^{-2} + \dots c_n z^{-n} + z^{-n-1} + z^{-n-2} \dots)$$

Since $R(z) = \frac{1}{1-z^{-1}}$ (unit step)

Then we can specify the desired step response $C(z)/R(z)$ as:

$$C(z)/R(z) = (1-z^{-1}) z^{-d} (c_1 z^{-1} + c_2 z^{-2} + \dots c_n z^{-n} + z^{-n-1} + z^{-n-2} \dots)$$

$$= z^{-d}(c_1 z^{-1} + (c_2 - c_1) z^{-1} + \dots (c_n - c_{n-1}) z^{-n} + (1-c_n) z^{-n-1} + (1-1) z^{-n-2} \dots)$$

$$= z^{-d}(c_1 z^{-1} + (c_2 - c_1) z^{-2} + \dots + (1-c_n) z^{-n-1})$$

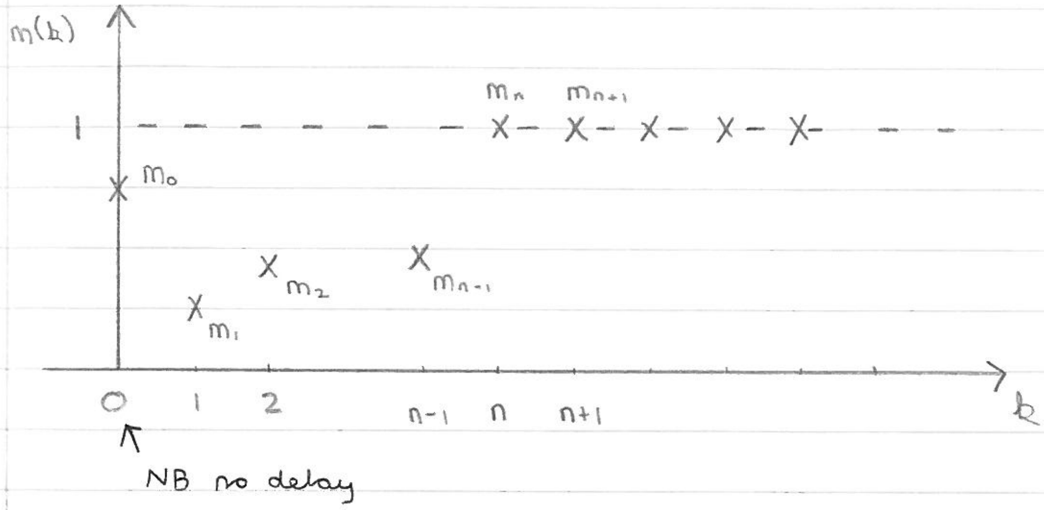
We then define

$$C(z)/R(z) = P(z) = z^{-d}(p_1 z^{-1} + p_2 z^{-2} + \dots p_n z^{-n} + p_{n+1} z^{-(n+1)})$$

Note the constraint DC gain = 1

$$\Rightarrow \sum_{i=1}^{n+1} p_i = 1$$

(ii) The controller output $m(k)$ will settle to a steady state value within n samples



Taking Z transform of the sequence $m(k)$

$$M(z) = \sum_{k=0}^{\infty} m(k) z^{-k}$$

hence

$$M(z) = m_0 + m_1 z^{-1} + m_2 z^{-2} + \ldots + m_{n-1} z^{-n+1} + m_n z^{-n} + m_n z^{-n-1} \ldots$$

Since $R(z) = \frac{1}{1-z^{-1}}$

We can specify the desired controller response $M(z)/R(z)$ as

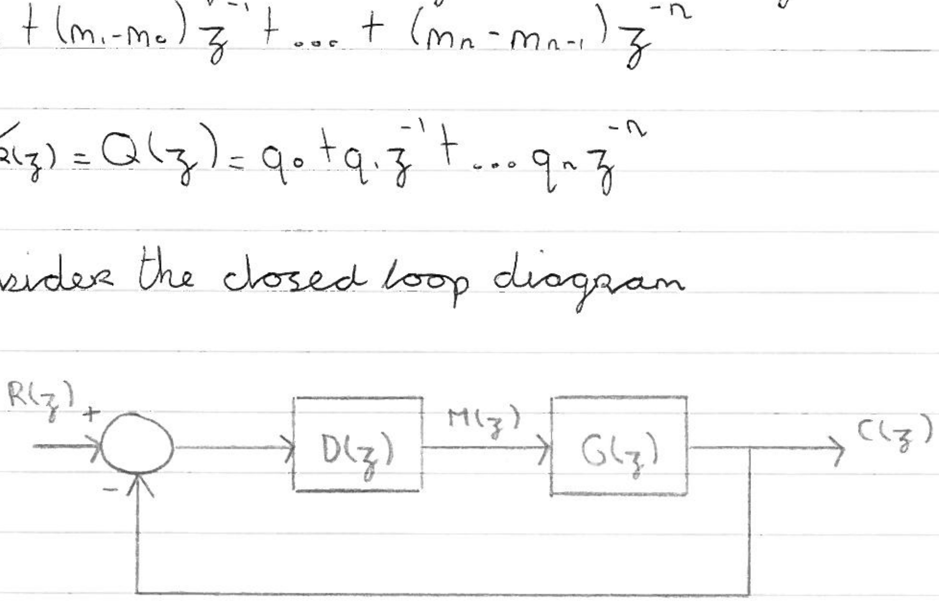
$$M(z)/R(z) = (1-z^{-1})(m_0 + m_1 z^{-1} + m_2 z^{-2} + \ldots + m_{n-1} z^{-n+1} + m_n z^{-n} + m_n z^{-n-1} \ldots)$$

$$= m_0 + (m_1 - m_0) z^{-1} + (m_2 - m_1) z^{-2} + \ldots (m_n - m_{n-1}) z^{-n} + (m_n - m_n) z^{-n+1} + \ldots$$

$$= m_0 + (m_1 - m_0) z^{-1} + \ldots + (m_n - m_{n-1}) z^{-n}$$

$$M(z)/R(z) = Q(z) = q_0 + q_1 z^{-1} + \ldots q_n z^{-n}$$

Consider the closed loop diagram



Then we can say

$$G(z) = \frac{z^{-d}(b_1 z^{-1} + b_2 z^{-2} + \ldots b_m z^{-m})}{1 - a_1 z^{-1} - a_2 z^{-2} \ldots - a_n z^{-n}} = \frac{C(z)}{M(z)} = \frac{C(z)/R(z)}{M(z)/R(z)} = \frac{P(z)}{Q(z)}$$

Since $P(z)$ is arbitrary, then why not specify it as

$$P(z) = \frac{z^{-d}(b_1 z^{-1} + b_2 z^{-2} + \dots b_m z^{-m})}{b_1 + b_2 + \dots b_m}$$

Then this choice of $P(z)$ forces the following choice of $Q(z)$

$$Q(z) = \frac{1 - a_1 z^{-1} - a_2 z^{-2} \dots - a_n z^{-n}}{b_1 + b_2 + \dots b_m}$$

$$D(z) = \frac{1}{G(z)} \frac{C(z)/R(z)}{1 - C(z)/R(z)} = \frac{Q(z)}{1 - P(z)}$$

$$\Rightarrow D(z) = \frac{1 - a_1 z^{-1} - a_2 z^{-2} \dots - a_n z^{-n}}{1 - z^{-d}(b_1 z^{-1} + b_2 z^{-2} + \dots b_m z^{-m})}$$

$$\sum P_i = 1$$

$$\Rightarrow D(z) = \frac{1 - a_1 z^{-1} - a_2 z^{-2} \dots - a_n z^{-n}}{(b_1 + b_2 + b_3 + \dots + b_m) - z^{-d}(b_1 z^{-1} + b_2 z^{-2} \dots b_m z^{-m})}$$

$$= \frac{1 - a_1 z^{-1} - a_2 z^{-2} \dots - a_n z^{-n}}{\sum_{j=1}^{m} b_j - \sum_{j=1}^{m} b_j \sum_{j=1}^{m} z^{-d-j}}$$

$$= \frac{1 - \sum_{i=1}^{n} a_i z^{-i}}{\sum_{j=1}^{m} b_j (1 - z^{-d-j})}$$

Benefit

- eliminates ringing

Drawback

- if poles of $G(z)$ are outside unit circle, small errors in representing $G(z)$ poles will exponentially increase

$\Rightarrow$ closed loop process is unstable

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Q6a. $\frac{d}{dt}\underline{x}(t) = A\underline{x}(t) + Bu(t) + Ed(t)$

$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) + BU(s) + ED(s)$

$(sI - A)\underline{X}(s) = BU(s) + ED(s) + \underline{x}(0)$

$\underline{X}(s) = (sI-A)^{-1}(BU(s) + ED(s) + \underline{x}(0))$

Let $(sI-A)^{-1} = \phi(s)$

$\underline{X}(s) = \phi(s)\underline{x}(0) + \phi(s)(BU(s) + ED(s))$

$\underline{x}(t) = \phi(t)\underline{x}(0) + \phi(t) * (Bu(t) + Ed(t))$

$\underline{x}(t) = \phi(t)\underline{x}(0) + \int_0^t \phi(t-\tau)(Bu(\tau) + Ed(\tau))\,d\tau$

(b) $I(s) = \frac{V(s) - K_m\Omega(s)}{Ls + R}$

$LsI(s) + RI(s) = V(s) - K_m\Omega(s)$

$L\frac{di}{dt} + Ri(t) = v(t) - K_m\omega(t)$

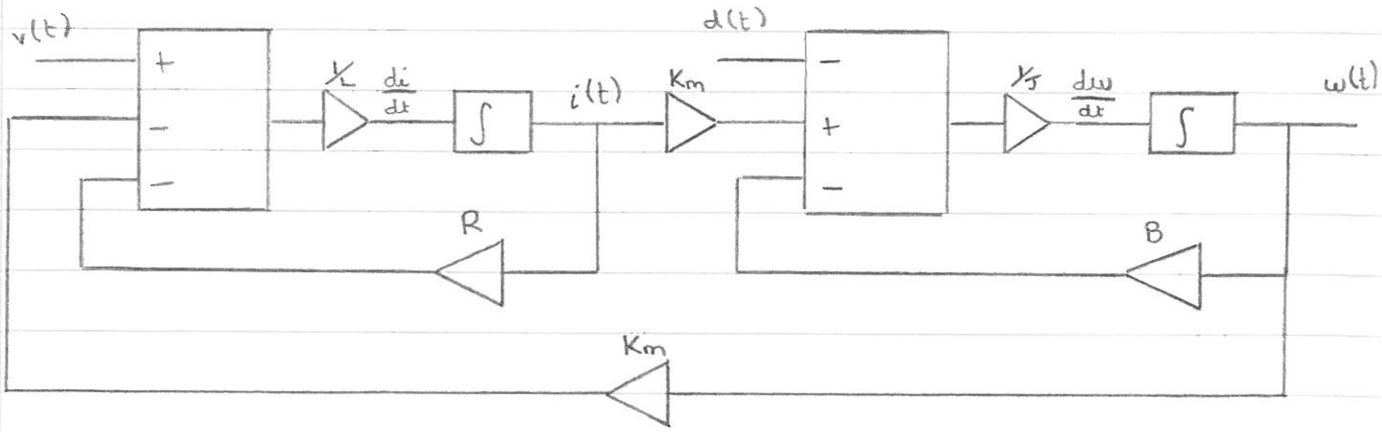
$\frac{di}{dt} = \frac{1}{L}(v(t) - K_m\omega(t) - Ri(t))$

$\Omega(s) = \frac{K_mI(s) - D(s)}{Js + B}$

$Js\Omega(s) + B\Omega(s) = K_mI(s) - D(s)$

$J\frac{d\omega}{dt} + B\omega(t) = K_mi(t) - d(t)$

$\frac{d\omega}{dt} = \frac{1}{J}(K_mi(t) - B\omega(t) - d(t))$



$$\frac{d}{dt}\begin{bmatrix} \omega \\ i \end{bmatrix} = \begin{bmatrix} -B/J & K_m/J \\ -K_m/L & -R/L \end{bmatrix}\begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} v + \begin{bmatrix} -1/J \\ 0 \end{bmatrix} d$$

(ii)