Question 1

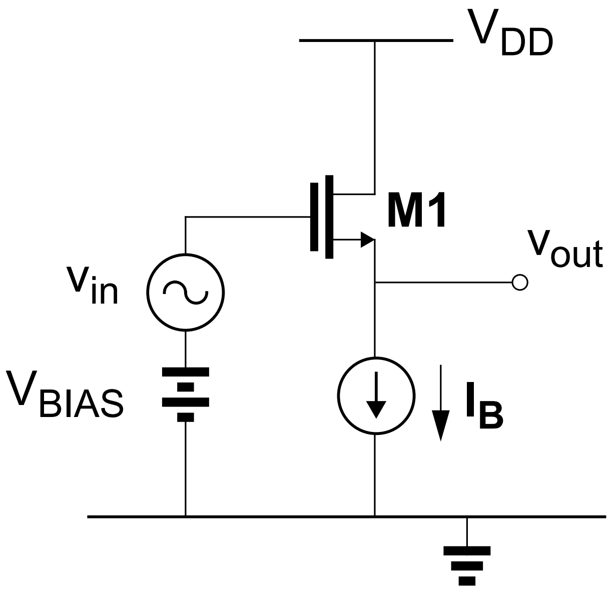


Figure 1

Assume M1 is operating in saturation and that $g_{m1} >> g_{ds1}$.
I_B is an ideal current source
The body effect may be ignored in the analysis.

(i) Draw the small signal model for the circuit shown in Figure 1. Ignore all capacitances.

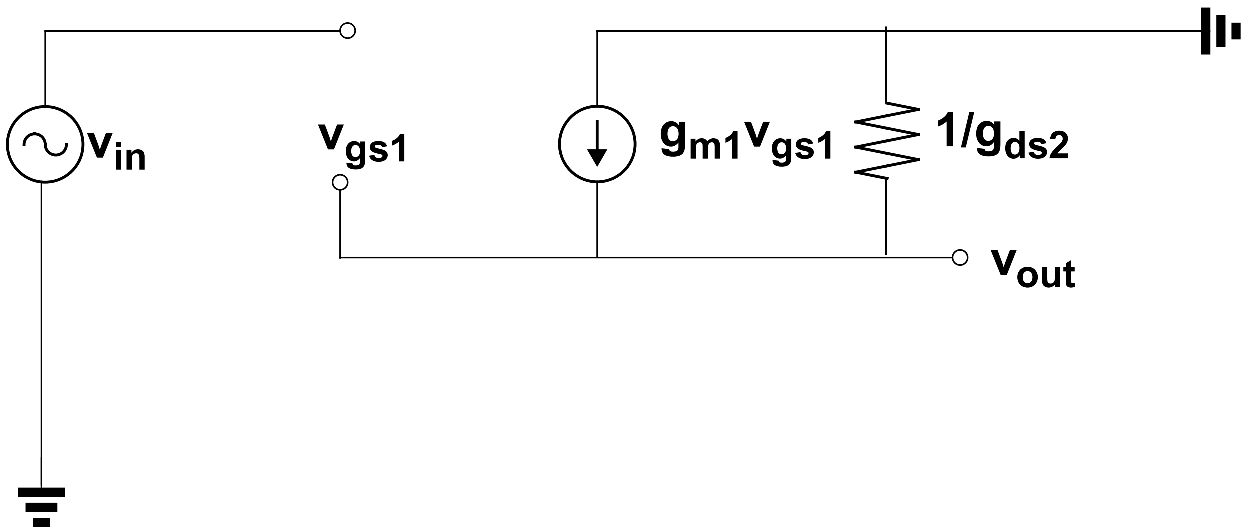
(ii) Derive an expression for the impedance at the output node

(iii) What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

(iv) The circuit shown in Figure 1 is required to drive a resistive load of 1kΩ. What is the requirement on g_{m1} if the small-signal attenuation of the stage is not to be greater than 6dB?

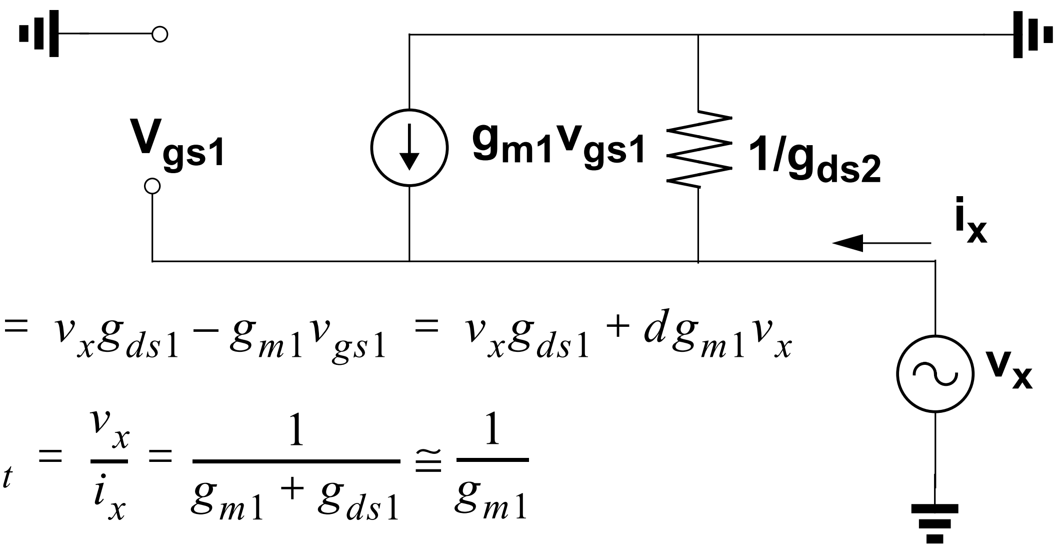
Solution

(i) Draw the small signal model for the circuit shown in Figure 1. Ignore all capacitances.



(ii) Derive an expression for the impedance at the output node

To calculate output impedance put a test voltage at the output node and calculate the small-signal current into the circuit. Ground the input node.

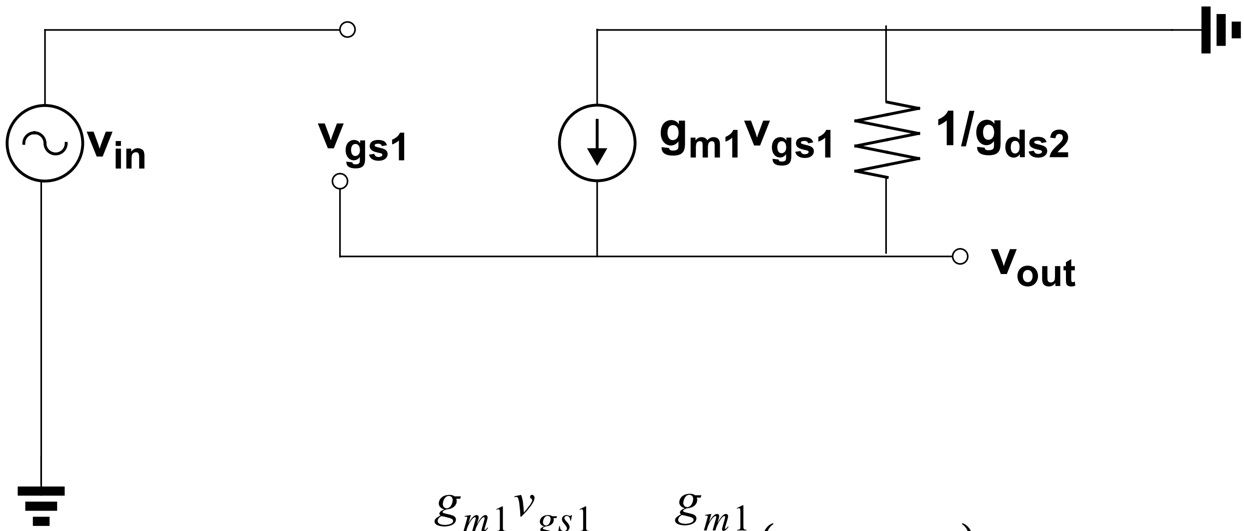


$$i_x = v_x g_{ds1} - g_{m1} v_{gs1} = v_x g_{ds1} + d g_{m1} v_x$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m1} + g_{ds1}} \cong \frac{1}{g_{m1}}$$

Alternatively recognise that the current of the current-source $g_{m1}v_{gs1}$ is determined by voltage across its terminals i.e. is equivalent to a resistance $1/g_{m1}$ and write result directly

(iii) What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

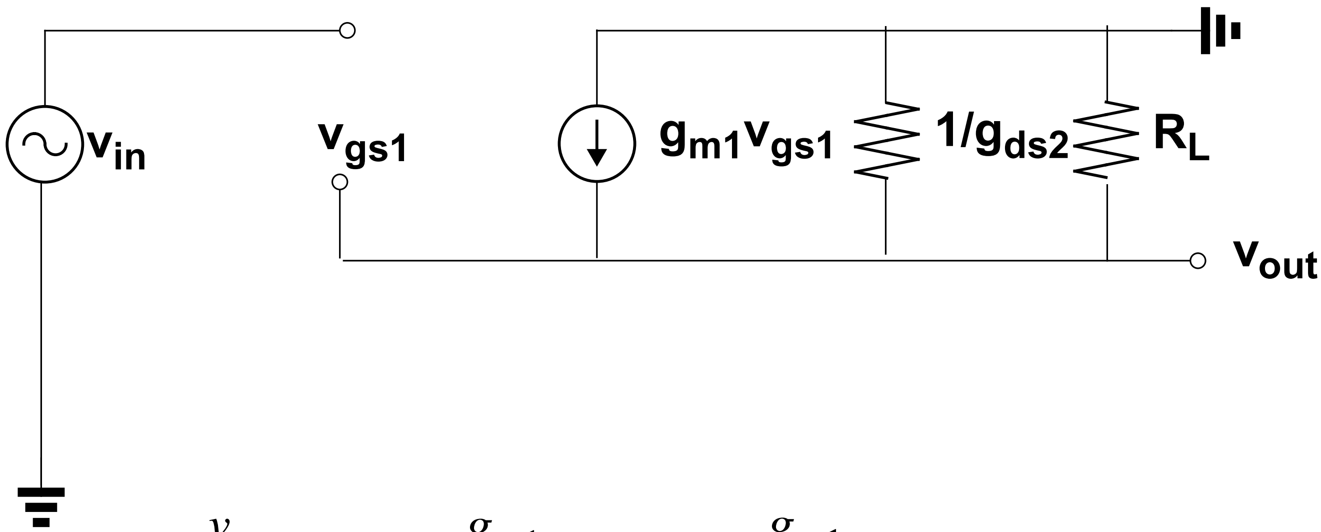


$$v_{out} = \frac{g_{m1}v_{gs1}}{g_{ds1}} = \frac{g_{m1}}{g_{ds1}}(v_{in} - v_{out})$$

$$\frac{v_{out}}{v_{in}} = \frac{\dfrac{g_{m1}}{g_{ds1}}}{1 + \dfrac{g_{m1}}{g_{ds1}}} = \frac{g_{m1}}{g_{m1} + g_{ds1}} \approx 1$$

Alternatively recognise that the current of the current-source $g_{m1}v_{gs1}$ is determined by voltage across its terminals i.e. is equivalent to a resistance $1/g_{m1}$ and write result directly

(iv) The circuit shown in Figure 1 is required to drive a resistive load of 1kΩ. What is the requirement on g_{m1} if the small-signal attenuation of the stage is not to be greater than 6dB?



$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{ds1} + R_L} \approx \frac{g_{m1}}{g_{m1} + \frac{1}{R_L}} > 0.5$$

$$g_{m1} > \frac{1}{R_L}$$

$$g_{m1} > 1mA/V$$

Question 2

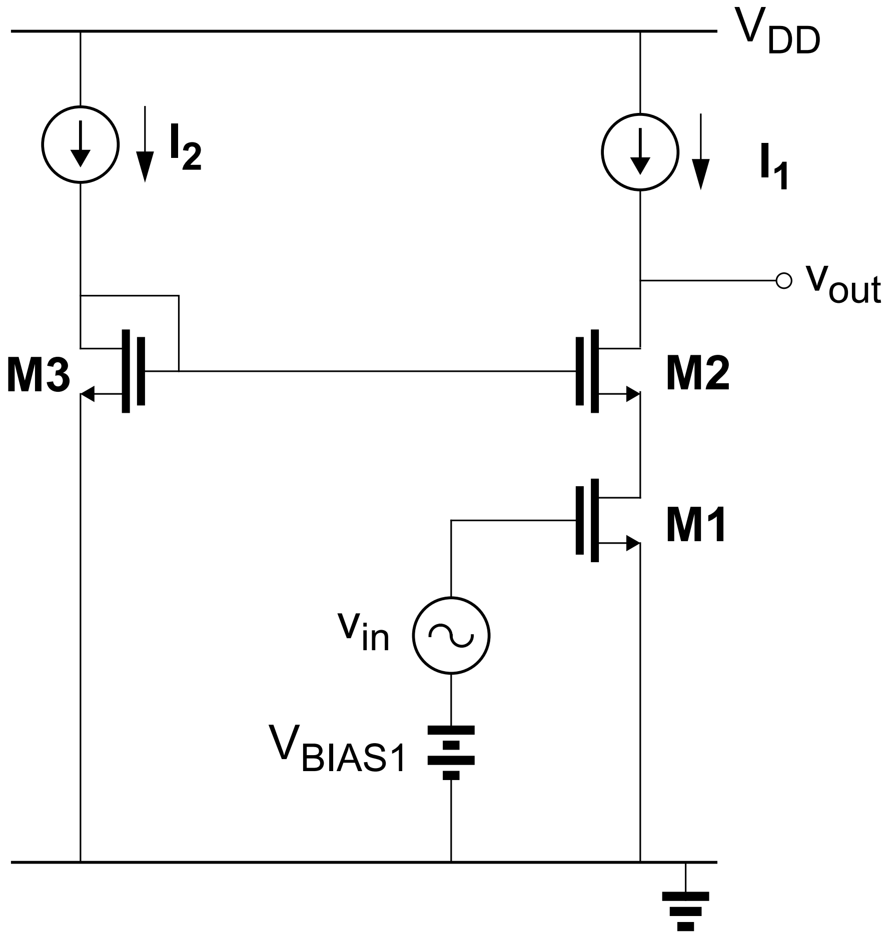


Figure 2

Figure 2 shows a cascode gain stage. I_1 and I_2 are ideal current sources.
For the questions below ignore the body effect.

(i) Draw the small signal model for the circuit shown in Figure 2. Ignore all capacitances.

(ii) The voltage gain of this stage can be approximated by
$v_{out}/v_{in}=g_m r_{out}$
where r_{out} is the impedance at the output node.
Derive an expression for v_{out}/v_{in} in terms of the small signal transistor parameters.
Reduce the expression to its simplest form assuming
$g_{m1}=g_{m2}=g_{m3}=g_m$, $g_{ds1}=g_{ds2}=g_{ds3}=g_{ds}$, $g_m>>g_{ds}$

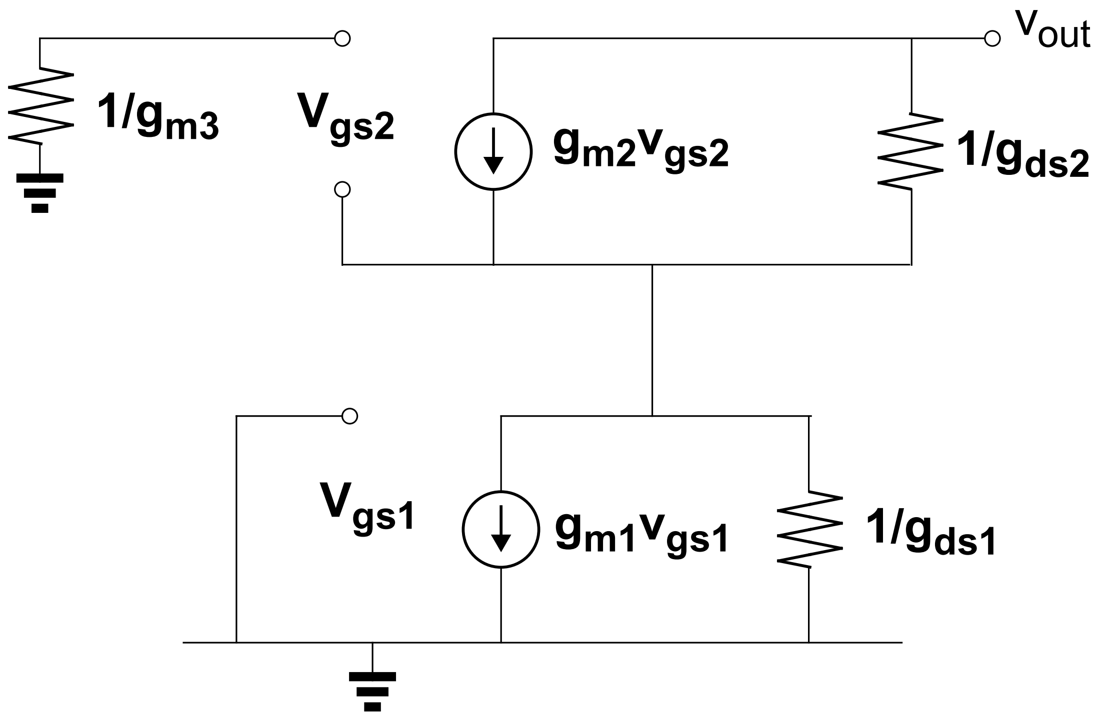
(iii) The circuit is to be biased for optimal low-voltage operation. If
V_{BIAS1}=1.25V, V_T = 1V, I_1=100μA
$(W/L)_{M1}=(W/L)_{M2}=(W/L)_{M3}=16\mu m/1\mu m$
calculate the minimum value of the voltage at the output node (i.e. at the drain of M2) for both M1 and M2 to be in saturation and the value of I_2 necessary to achieve this.
Neglect λ for this calculation.

(iv) For low power I_2 is changed to 40μA.

What value of $(W/L)_{M3}$ is required to preserve the bias conditions of M1 and M2.

Solution

(i) Draw the small signal model for the circuit shown in Figure 2.
Ignore all capacitances.



(ii) The voltage gain of this stage can be approximated by

$v_{out}/v_{in}=g_{m1}r_{out}$

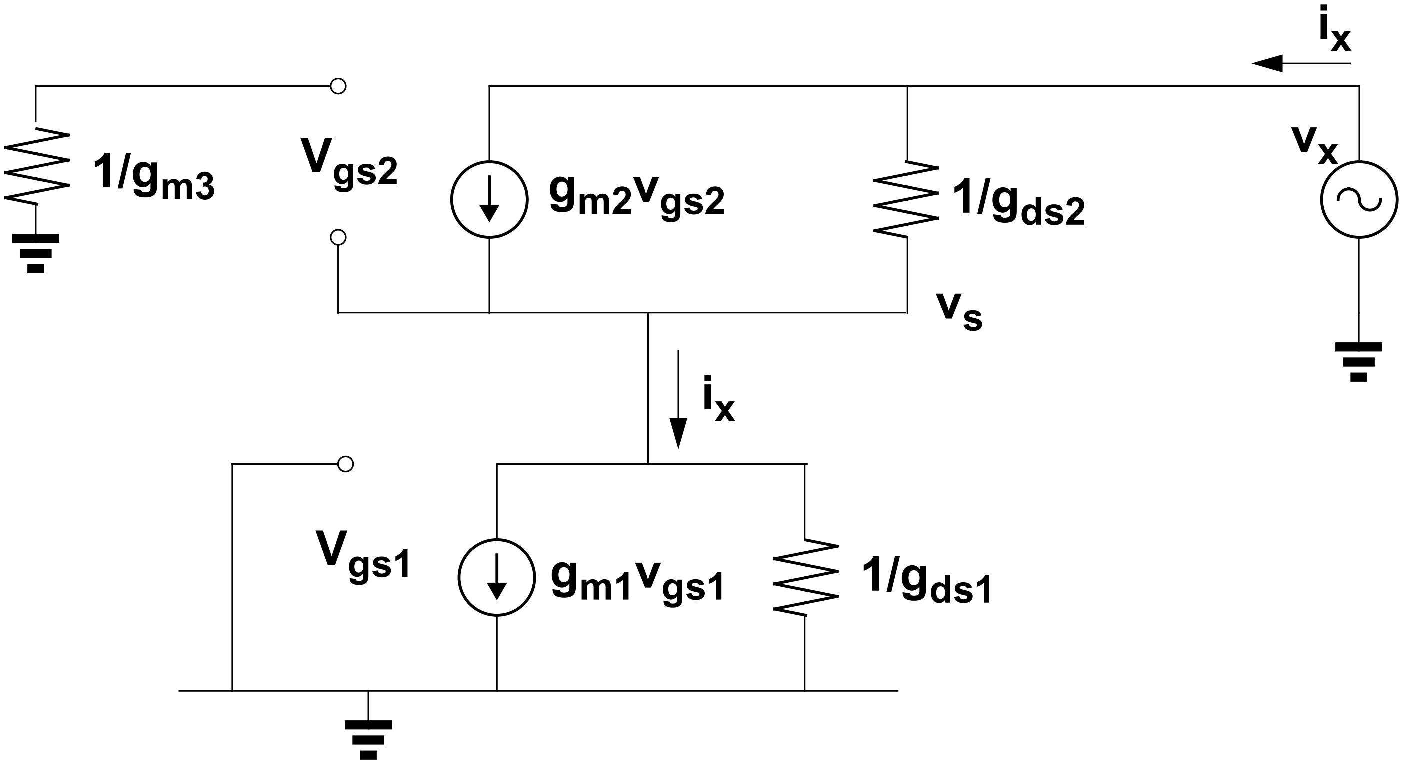
where r_{out} is the impedance at the output node.

Derive an expression for v_{out}/v_{in} in terms of the small signal transistor parameters.

Reduce the expression to its simplest form assuming

$g_{m1}=g_{m2}=g_{m3}=g_m$, $g_{ds1}=g_{ds2}=g_{ds3}=g_{ds}$, $g_m >> g_{ds}$

To derive the output impedance/conductance put a test voltage at the output node and calculate the small-signal current into the circuit.



Note: $v_{gs1} = 0 \Rightarrow g_{m1} v_{gs1} = 0$

$$i_x = g_{m2} v_{gs2} + (v_x - v_s) g_{ds2}$$

Since $v_{gs2} = -v_s$ and $v_s = \dfrac{i_x}{g_{ds1}}$

$$i_x = -(g_{m2}) \frac{i_x}{g_{ds1}} + \left(v_x - \frac{i_x}{g_{ds1}} \right) g_{ds2}$$

$$r_{out} = \frac{v_x}{i_x} = \frac{1 + \dfrac{g_{m2}}{g_{ds1}} + \dfrac{g_{ds2}}{g_{ds1}}}{g_{ds2}}$$

$$\frac{v_{out}}{v_{in}} = g_{m1} r_{out} = g_{m1} \frac{1 + \dfrac{g_{m2}}{g_{ds1}} + \dfrac{g_{ds2}}{g_{ds1}}}{g_{ds2}}$$

$$\frac{v_{out}}{v_{in}} = g_{m1} r_{out} = g_{m1} \frac{1 + \frac{g_{m2}}{g_{ds1}} + \frac{g_{ds2}}{g_{ds1}}}{g_{ds2}}$$

Since $g_{m1}=g_{m2}=g_m$, $g_{ds1}=g_{ds2}=g_{ds}$, $g_m>>g_{ds}$ this can be reduced to any of

$$\frac{v_{out}}{v_{in}} \cong g_{m1} \frac{g_{m2}/g_{ds1}}{g_{ds2}} = \frac{g_{m1}}{g_{ds1}} \cdot \frac{g_{m2}}{g_{ds2}} = \left(\frac{g_m}{g_{ds}}\right)^2$$

(iii) The circuit is to be biased for optimal low-voltage operation. If
V_{BIAS1}=1.25V, V_T = 1V, I_1=100μA
$(W/L)_{M1}=(W/L)_{M2}=(W/L)_{M3}$=16μm/1μm$_{M1}$
calculate the minimum value of the voltage at the output node (i.e. at the drain of M2) for both M1 and M2 to be in saturation and the value of I_2 necessary to achieve this.
Neglect λ for this calculation.

For M1 to be in saturation then

$$V_{DS1} \geq V_{GS1} - V_T$$

$$(V_{DS1})_{min} = V_{GS1} - V_T = 1.25V - 1V = 0.25V$$

If M2 is in saturation its drain current is given by

$$I_{D2} = \frac{K_n'}{2}\frac{W}{L}(V_{GS2} - V_T)^2$$

Since M2 has same drain current, W/L and V_T as M1 it will also have the same V_{GS}

$$(V_{DS2})_{min} = V_{GS2} - V_T = 0.25V$$

so minimum voltage at the output for both transistors to be in saturation is given by

$$V_{out} = (V_{DS1})_{min} + (V_{DS2})_{min} = 0.5V$$

The bias voltage V_{GS3} necessary to achieve this is given by

$$V_{GS3} = V_{GS2} + (V_{DS1})_{min} = 1.25V + 0.25V = 1.5V$$

$$I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS1} - V_T)^2$$

$$I_{D3} = \frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS3} - V_T)^2$$

$$\frac{I_{D1}}{I_{D3}} = \frac{(V_{GS1}-V_T)^2}{(V_{GS3}-V_T)^2}$$

$$I_{D3} = I_{D1}\frac{(V_{GS3}-V_T)^2}{(V_{GS1}-V_T)^2} = 100\mu A\frac{(0.5)^2}{(0.25)^2} = 400\mu A$$

(iv) For low power I_2 is changed to 40µA.
What value of $(W/L)_{M3}$ is required to preserve the bias conditions of M1 and M2.

Need same V_{GS3}-V_T

$$I_{D3} = \frac{K'_n}{2}\left(\frac{W}{L}\right)_3(V_{GS3}-V_T)^2$$

If ID3 is reduced by a factor 10, then W/L also needs to be reduced by ten

$$\left(\frac{W}{L}\right)_{M3} = 1.6$$

Question 3

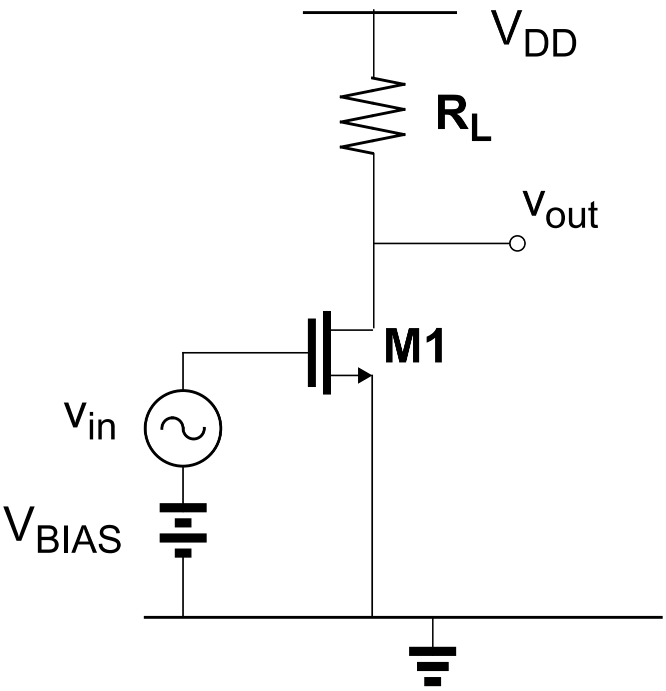


Figure 3

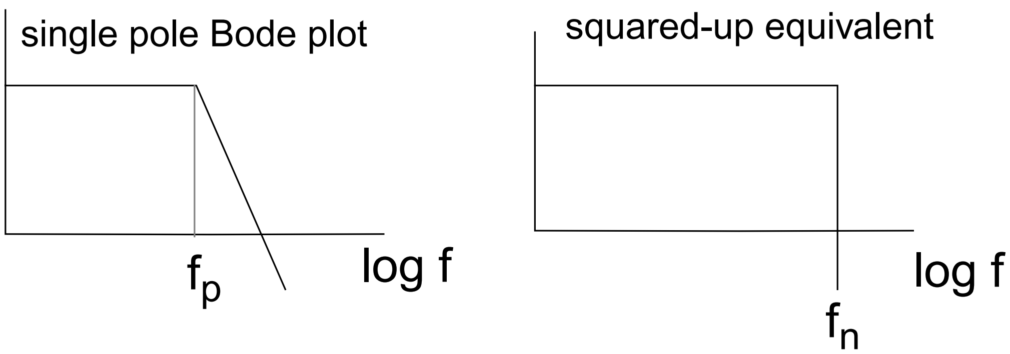
Assume M1 is operating in saturation and ignore the body effect.
Assume that $g_{m1} >> g_{ds1}$ and that $g_{ds1} << 1/R_L$

(i) Draw the small signal model for the circuit shown in Figure 3.
Ignore all capacitances.
What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

(ii) What is the input-referred thermal noise voltage in terms of R_L, the small signal parameters of M1, Boltzmann's constant k and temperature T?

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

You may assume the following:



For the area underneath the curves to be the same then $f_n=(\pi/2)*f_p$

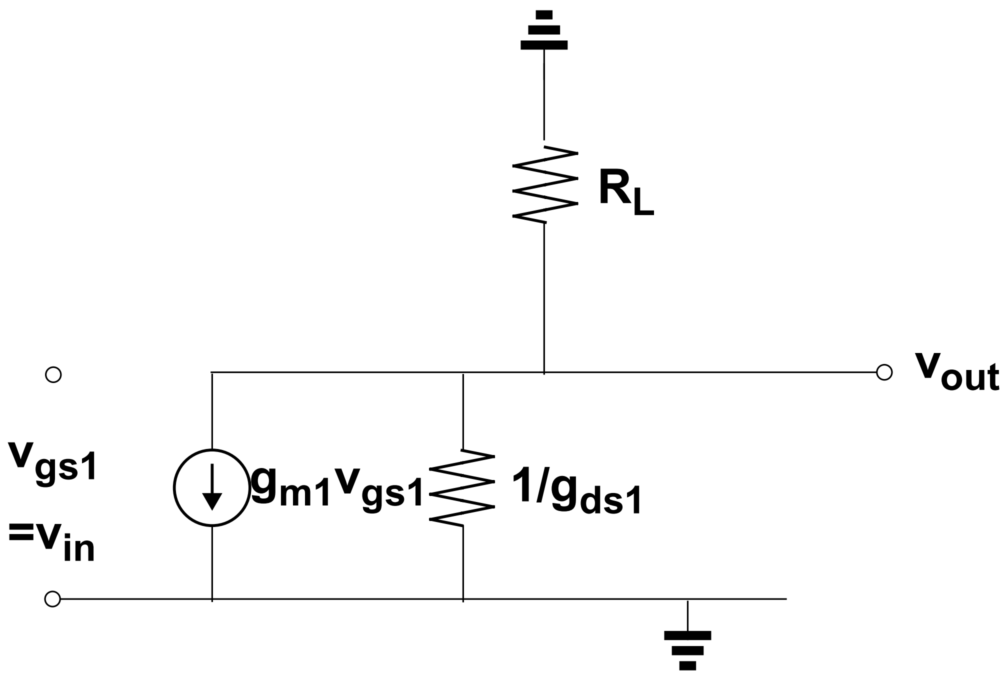
(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $10mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.
For this calculation take V_{GS1}=1V, $|V_T|$ = 0.75V , R_L=5k C_L=1pF.
The drain current of M1 is 100μA.

Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

Solution

(i) Draw the small signal model for the circuit shown in Figure 3.
Ignore all capacitances

What is the low-frequency small signal voltage gain (v_{out}/v_{in})?

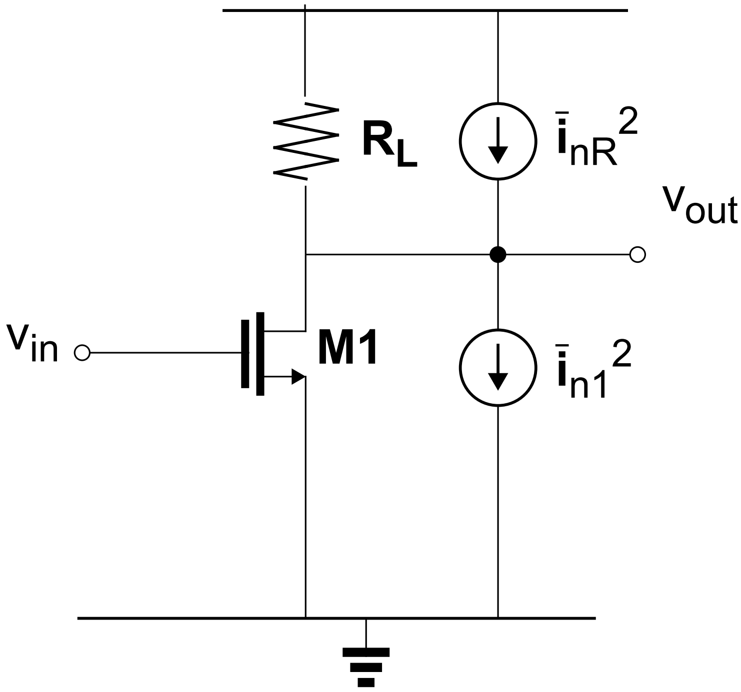


Current at outout node

$$g_{m1}v_{in} + v_{out}g_{ds1} + \frac{v_{out}}{R_L} = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{ds1} + \frac{1}{R_L}} \cong -g_{m1}R_L$$

(ii) What is the input-referred thermal noise voltage in terms of R_L, the small signal parameters of M1, Boltzmann's constant k and temperature T?



Total noise current at output is square root of the individual noise currents

$$\overline{i_{nt}} = \sqrt{i_{n1}^2 + i_{n2}^2} = \sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}$$

Input-referred noise voltage given by

$$\overline{v_{ni}} = \frac{\overline{i_{nt}}}{g_{m1}} = \frac{\sqrt{4kT\left(\frac{2}{3}g_{m1}\right) + \frac{4kT}{R_L}}}{g_{m1}} \qquad V/\sqrt{Hz}$$

(iii) If a capacitor C_L is connected between the output node and ground what is the total integrated thermal noise at the output node?

Noise voltage at output given by input referred noise multiplied by gain

$$\overline{v_{no}} = \overline{v_{ni}} g_{m1} R_L = \frac{\sqrt{4kT\left(\frac{2}{3} g_{m1}\right) + \frac{4kT}{R_L}}}{g_{m1}} g_{m1} R_L$$

$$= \left(\sqrt{4kT\left(\frac{2}{3} g_{m1}\right) + \frac{4kT}{R_L}}\right) R_L$$

Capacitor C_L connected between the output node and ground
=> pole at output node given by

$$|f_p| = \frac{1}{2\pi R_L C_L}$$

Total integrated thermal noise power at the output node is given by the product of the thermal noise power and the squared-up equivalent of the first order filter function

$$\overline{v^2_{nototal}} = \overline{v^2_{no}} \frac{\pi}{2} f_p$$

$$\overline{v^2_{nototal}} = \left(4kT\left(\frac{2}{3} g_{m1}\right) + \frac{4kT}{R_L}\right) R_L^2 \cdot \frac{\pi}{2} \cdot \frac{1}{2\pi R_L C_L}$$

$$\overline{v^2_{nototal}} = \left(\frac{2}{3} g_{m1} + \frac{1}{R_L}\right) R_L \cdot \frac{kT}{C_L}$$

(iv) Using the result of (iii) calculate the signal-to noise ratio at the output if the input signal v_{in} is a $20mV_{rms}$ sine wave with a frequency much lower than the frequency of the pole at the output node.
For this calculation take V_{GS1}=1V, $|V_T|$ = 0.75V , R_L=5kΩ, C_L=1pF.
The drain current of M1 is 100μA.
Assume Boltzmann's constant k=1.38X10^{-23}J/^{o}K, temperature T=300^{o}K.

g_m given by

$$g_m = \frac{2I_D}{(V_{GS}-V_T)}$$

$$g_{m1} = \frac{2 \cdot 100\mu A}{1V - 0.75V} = 800\mu A/V$$

Output signal

$$v_{out} = -g_{m1}R_L v_{in} = -800\mu A/V \cdot 5k \cdot 20mV_{rms} = 80mV_{rms}$$

Total output noise:

$$\overline{v_{nototal}} = \sqrt{\left(\frac{2}{3}g_{m1} + \frac{1}{R_L}\right)R_L \cdot \frac{kT}{C_L}}$$

$$\overline{v_{nototal}} = \sqrt{\left(\frac{2}{3}(800\mu A/V) + \frac{1}{5k}\right) \cdot 5k \cdot \frac{1.38\times10^{-23}300}{1pF}} = 123\mu V_{rms}$$

Signal-to-Noise ratio given by

$$\frac{S}{N} = \frac{80mV}{123\mu V} = 649 \qquad \text{or 56.2 dB}$$