Department
of
Electrical and Electronic Engineering

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Fourth Year Electrical Engineering

EE4010

Electrical and Electronic Power Supply Systems

Power System Faults Worked Examples

Example 1

Three identical star-connected, 10 kV, 50 Hz, round-rotor synchronous generators in a generating station are connected each in series with a similar current limiting reactor of reactance X to a common busbar *A*. The generators have a rating of 10 MVA and a per-phase sub-transient reactance of 0.06 pu.

Two 50 Hz 10 kV/38 kV three-phase transformers, one of 15 MVA rating and 0.03 pu reactance and the second of 10 MVA rating and 0.02 pu reactance, are connected in parallel to this busbar. The two transformers in parallel supply at busbar *B* a 38 kV overhead transmission line of impedance $(0.2 + j0.7)\ \Omega/\text{km}$. At a sub-station 10 km from the generating station is a 25 MVA, 38 kV/10 kV transformer of reactance 0.06 pu connecting busbar *C* to busbar *D*.

Calculate the reactance X of the current limiting reactors if each generator is not to carry more than $2\,{}^1\!/_3$ times its rated current. Pre-fault current may be neglected and the system is operating at rated voltage prior to the fault.

Solution 1

A single-line diagram of the power distribution system is illustrated in Figure 1 below.

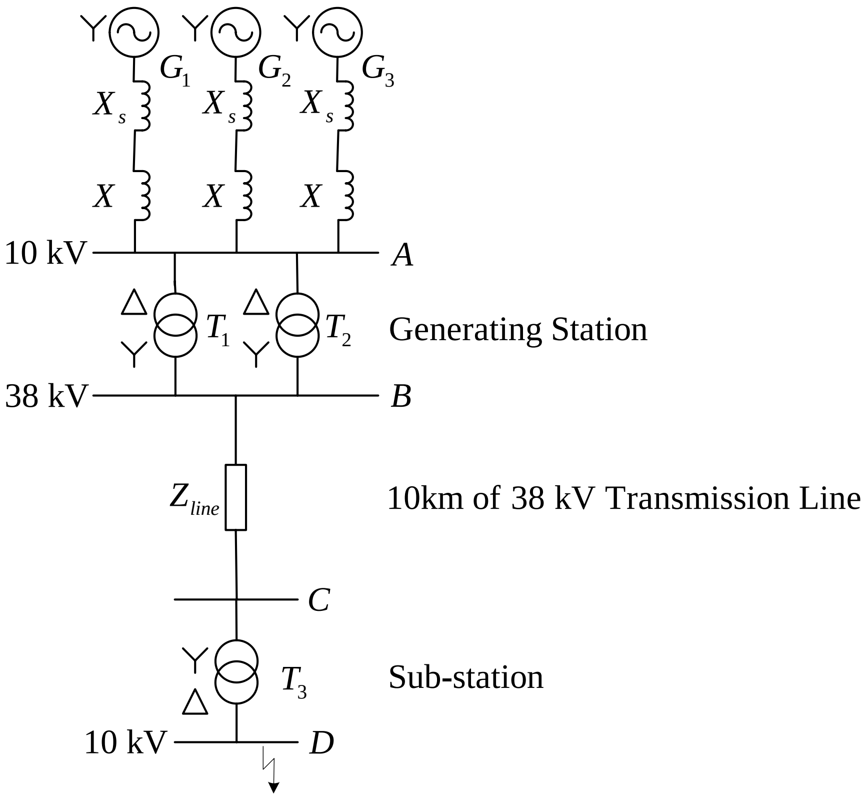

Figure 1 Single line diagram for Example 1.

Take 10 MVA as base volt-amperes. Take 10 kV as base voltage in the generating station and the sub-station. Take 38 kV as base voltage in the transmission line zone.

Hence,

$$Z_{base1} = \frac{V_{base1}{}^2}{S_{base}} = \frac{\left(10 \times 10^3\right)^2}{10 \times 10^6} = 10\ \Omega$$

$$Z_{base2} = \frac{V_{base2}{}^2}{S_{base}} = \frac{\left(38\times10^3\right)^2}{10\times10^6} = 144.4\ \Omega$$

The specifications of the generators are as follows:

G1: 10 kV, 10 MVA, $Z_s = 0.06$ pu.
G2: 10 kV, 10 MVA, $Z_s = 0.06$ pu.
G3: 10 kV, 10 MVA, $Z_s = 0.06$ pu.

The specifications of the transformers are as follows:

T1: 10 kV/38 kV, 10 MVA, $Z_{1pu} = 0.02$.
T2: 10 kV/38 kV, 15 MVA, $Z_{2pu\ \mathrm{old}} = 0.03$

$$\begin{aligned} Z_{2pu} &= 0.03\times\frac{S_{base\,new}}{S_{base\,old}} \\ &= 0.03\times\frac{10\times10^6}{15\times10^6} \\ &= 0.02\ \text{pu} \end{aligned}$$

T3: 38 kV/10 kV, 25 MVA, $Z_{3pu\ \mathrm{old}} = 0.06$

$$\begin{aligned} Z_{3pu} &= 0.06\times\frac{S_{base\,new}}{S_{base\,old}} \\ &= 0.06\times\frac{10\times10^6}{25\times10^6} \\ &= 0.024\ \text{pu} \end{aligned}$$

The per unit impedance of the transmission line is

$$Z_{line\,pu} = \frac{\left(0.2 + j0.7\right)\times10}{Z_{base2}} = \left(0.01385 + j0.0484\right)\ \text{pu..}$$

When a bolted three-phase fault occurs at the low voltage terminals of Transformer 3, the fault current is given by

$$I_{fault} = \frac{V_f}{\left(\frac{Z_s + X}{3}\right) + \left(\frac{Z_{T1}\times Z_{T2}}{Z_{T1} + Z_{T2}}\right) + Z_{line} + Z_{T3}}.$$

Assuming that the pre-fault voltage is the rated per unit value, $V_f = 1.0$, then the maximum fault current to be fed into a symmetrical, three-phase short circuit by the three generators is given by

$$I_{fault\,\max} = 3\times 2\,{}^1\!/_3 = 7\ \text{pu.}$$

Thus,

$$I_{fault\max} \le \left| \frac{1.0}{j\left(\frac{0.06+X}{3}\right)+\left(\frac{0.02\times 0.02}{0.02+0.02}\right)+\left(0.01385+j0.0484\right)+0.024} \right|$$

This gives one unknown in a single inequality and hence it can be solved for the value of the required current limiting reactor *X*.

Solving this inequality gives

$$X_{pu} \ge 0.1191 \text{ pu}$$

Since $Z_{base1} = 10\ \Omega$, we get

$$X_{act} \ge 1.191\ \Omega$$

as the required value.

Example 2

A power station consists of three identical 50 Hz round-rotor synchronous generators each of 200 MVA and 0.9 per unit synchronous reactance. These machines are running equally loaded and to each is connected a 10 kV/110 kV transformer of 200 MVA rating and 0.09 per unit reactance. The 110 kV busbar from the transformers feeds a short 110 kV transmission line having a resistance of $0.5\ \Omega$ per phase and an inductance of 8 mH per phase. The resistances of the generators and the transformers are negligible.

A load of 400 MW at 0.8 power factor lagging is supplied at 110 kV at the load end of the transmission line. Calculate the excitation voltage of the synchronous generators.

If a bolted three-phase short circuit occurs at the load end of the transmission, calculate the steady state short circuit current.

Solution 2

A single-line diagram of the power transmission system is illustrated in Figure 2 below.

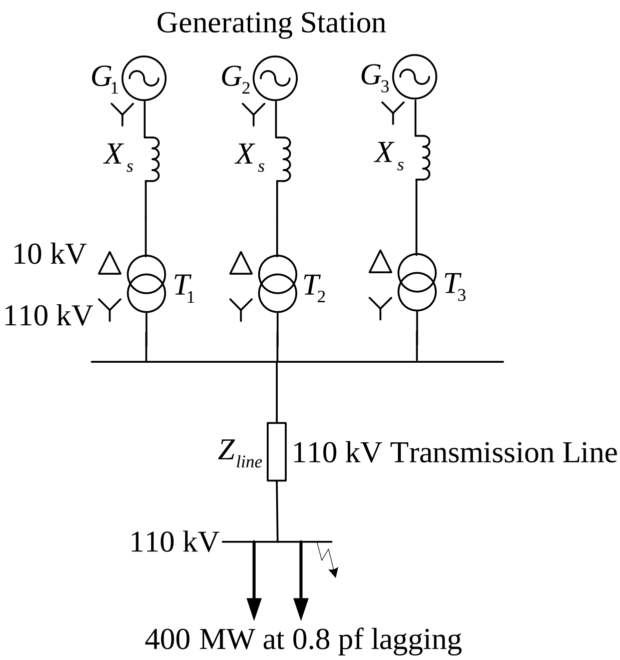

Figure 2 Single line diagram for Example 2

Take $S_{base} = 200\ \text{MVA}$, $V_{base1} = 10\ \text{kV}$ and $V_{base2} = 110\ \text{kV}$ so that the current and impedance quantities

$$I_{base1} = \frac{S_{base}}{\sqrt{3}\ V_{base1}} = 11.547\ \text{kA}$$

$$I_{base2} = \frac{S_{base}}{\sqrt{3}\ V_{base2}} = 1.049\ \text{kA}$$

$$Z_{base1} = \frac{V_{base1}^{\ 2}}{\text{S}_{base}} = 0.5\ \Omega$$

$$Z_{base2} = \frac{V_{base2}^{\ 2}}{\text{S}_{base}} = 60.5\ \Omega$$

give a consistent set of base parameters.

The synchronous reactance of the generators and the transformers are

$$\bar{Z}_S = j0.9 \text{ pu}$$
$$\bar{Z}_T = j0.09 \text{ pu.}$$

The per unit impedance of the transmission line is

$$\bar{Z}_{Line} = \frac{0.5 + j(2\pi \times 50 \times 8 \times 10^{-3})}{Z_{base2}} = (0.008264 + j0.04154) \text{ pu}$$

Assuming identical generators and transformers, the total equivalent series impedance of the transmission circuit is then

$$\begin{aligned}\bar{Z}_{eq} &= \frac{\bar{Z}_G}{3} + \frac{\bar{Z}_T}{3} + \bar{Z}_{Line} \\ &= \frac{0.9}{3} + \frac{0.09}{3} + \frac{0.5 + j(2\pi \times 50 \times 8 \times 10^{-3})}{Z_{base1}} \\ &= (0.008264 + j0.3714) \text{ pu.}\end{aligned}$$

The load volt-amperes is given by

$$\phi = Cos^{-1}(0.8) = 36.87°$$
$$\bar{S}_{load} = \frac{400}{0.8}\left[Cos(\phi) + j\,Sin(\phi)\right] = 500\left[Cos(\phi) + j\,Sin(\phi)\right].$$

Hence, the per unit load volt-amperes and voltage are

$$\bar{S}_{load\ pu} = (2.0 + j1.5) \text{ pu}$$
$$\bar{V}_{t\ pu} = \frac{110 \times 10^3}{110 \times 10^3} \angle 0° = 1.0\angle 0°$$

Thus, the per unit load current is

$$\bar{I}_{load\ pu} = \left(\frac{\bar{S}_{load}}{\bar{V}_t}\right)^* = (2.0\ \text{-}\ j1.5) \text{ pu}$$

Hence, the common excitation voltage of the generators is

$$\begin{aligned}\bar{E}_{f\ pu} &= \bar{V}_{t\ pu} + \bar{Z}_{eq}\bar{I}_{load\ pu} \\ &= \bar{V}_{t\ pu} + \left(\frac{\bar{Z}_G}{3} + \frac{\bar{Z}_T}{3} + \bar{Z}_{Line}\right)\bar{I}_{load\ pu}\end{aligned}$$

giving

$$\begin{aligned}\bar{E}_{f\ pu} &= (1.5738 + j0.7306) \text{ pu} \\ &= 1.7352\angle 24.90° \text{ pu.}\end{aligned}$$

Finally, the per unit short circuit current for a bolted short circuit fault directly on the load 110 kV busbars is given by

$$\begin{aligned}
\overline{I}_{f\,pu} &= \frac{\overline{E}_{f\,pu}}{\overline{Z}_{eq}} = \frac{\overline{E}_{f\,pu}}{\dfrac{\overline{Z}_G}{3} + \dfrac{\overline{Z}_T}{3} + \overline{Z}_{Line}} \\
&= \frac{1.7352\angle 24.90^\circ}{0.3716\angle 88.72^\circ} \\
&= 2.0598 \ - \ j4.1902 \\
&= 4.6691\angle\text{-}63.82^\circ \ \ \text{pu.}
\end{aligned}$$

The actual load current is calculated as

$$\begin{aligned}
\overline{I}_f &= \overline{I}_{f\,pu} I_{base2} \\
&= 4901.27\angle - 63.82^\circ \ \text{A.}
\end{aligned}$$

Example 3

A 6.6 kV, three-phase, synchronous generator has positive, negative and zero sequence impedances of $j4.5\,\Omega$, $j3.0\,\Omega$ and $j1.5\,\Omega$ respectively. The machine is star-connected and the star point is solidly earthed. Calculate the current when a single line-to-earth fault occurs at the terminals of the machine.

Solution 3

The following is a *Mathematica*-based solution. It is based on the formulae presented in the section on single-line-to-earth fault. The total impedance to the zero sequence fault current is equal to the sum of the sequence impedances. All sequence currents are equal. Note that the phase-to-ground voltage on Phase *a* is of course zero as expected due to the nature of the fault.

```
(* Define the a operator *)
Clear[a]
a = Cos[120 Degree] + i Sin[120 Degree];

(* Define the sequence impedance of the machine *)
Z0 = i 1.5;
Z1 = i 4.5;
Z2 = i 3.0;

(* Calculate the phase voltage *)
Vline = 6.6 10^3;
Vphase = Vline/Sqrt[3];

(* Calculate the sequence currents *)
I0 = Vphase/(Z0 + Z1 + Z2);
I1 = I0;
I2 = I0;

(* Calculate the Phase a fault current *)
Ia = 3 I0;
Print["Fault current = ", Abs[Ia], " A at a phase angle of ", N[Arg[Ia]/Degree], " Degrees"]

(* Calculate the sequence components of the terminal voltage *)
V0 = - I0 Z0;
V1 = Vphase - I1 Z1;
V2 = -I2 Z2;

(* Calculate the a phase voltage *)
Va = V0 + V1 + V2;
Vb = V0 + a^2 V1 + a V2;
Vc = V0 + a V1 + a^2 V2;
```

```
Fault current = 1270.17 A at a phase angle of -90. Degrees

Phase a voltage = 0 V at a phase angle of 0. Degrees
Phase b voltage = 2910.33 V at a phase angle of -109.107 Degrees
Phase c voltage = 2910.33 V at a phase angle of 109.107 Degrees
```

Example 4

A 38 kV electrical source has a three-phase fault level of 2000 MVA and it may be assumed to have equal internal reactances to all three sequences. It supplies a 45 MVA, 110 kV/38 kV solidly-earthed star/delta connected transformer bank T_1 having a leakage reactance of 10%. This transformer is connected to a 110 kV transmission line which is 100 km long and has per phase positive sequence and per-phase zero sequence reactances of $0.7\ \Omega/\text{km}$ and $1.5\ \Omega/\text{km}$ respectively. At the remote end of the transmission line is connected a transformer T_2 identical to that at the sending end but the circuit breaker on the 38 kV side of this transformer is open circuited.

If a zero-impedance single-phase-to-earth fault occurs at the mid-point of the transmission line, calculate the fault current to earth and the line current flowing in the two healthy phases from the generator side.

Solution 4

The single line diagram of the system is as shown in Figure 3 below.

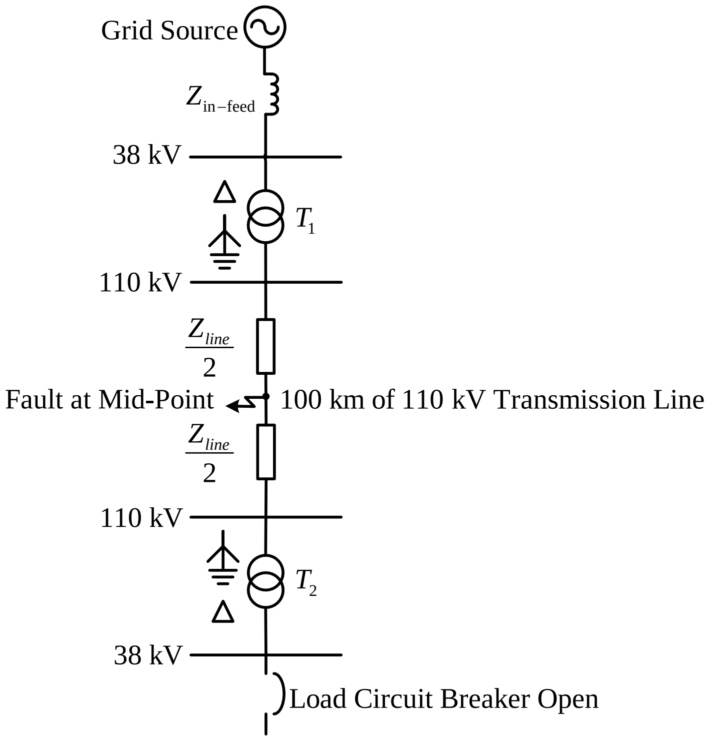

Figure 3 Single line diagram for Example 4.

Take $S_{base} = 45\ \text{MVA}$ as base volt-amperes. Also, specify $V_{base\ \text{LV}} = 38\ \text{kV}$ and $V_{base\ \text{HV}} = 110\ \text{kV}$ as the base voltages on either side of the transformers. In this case, the base parameters are as follows

$$I_{baseLV} = \frac{S_{base}}{\sqrt{3}\ V_{baseLV}} = 683.70\ \text{A}$$

$$I_{baseHV} = \frac{S_{base}}{\sqrt{3}\ V_{baseHV}} = 236.19\ \text{A}$$

$$Z_{baseLV} = \frac{{V_{baseLV}}^2}{\text{S}_{base}} = 32.09\ \Omega$$

$$Z_{baseHV} = \frac{{V_{baseHV}}^2}{\text{S}_{base}} = 268.89\ \Omega.$$

The positive, negative and zero sequence networks of the transmission system are as shown below

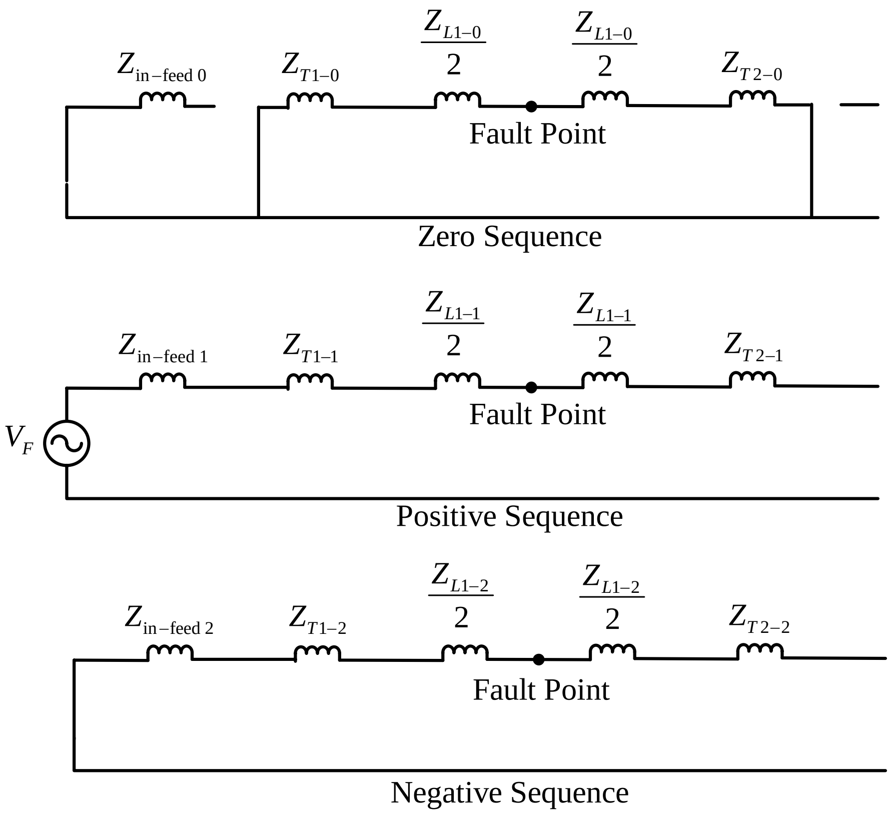

Figure 4 Sequence networks for the system of Example 4.

Hence, the Thevenin equivalent sequence networks for this system are as in Figure 5 below.

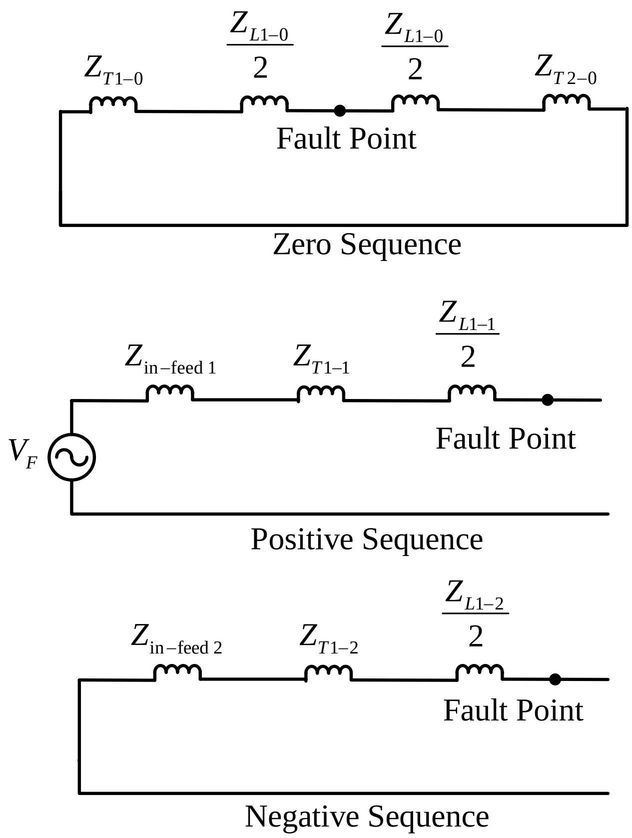

Figure 5 Thevenin equivalent sequence networks for Example 4.

Since the fault is a single-line-to-earth fault at the mid-point of the transmission line, the Thevenin equivalent sequence networks are connected in series at the fault point as illustrated in Figure 6 below.

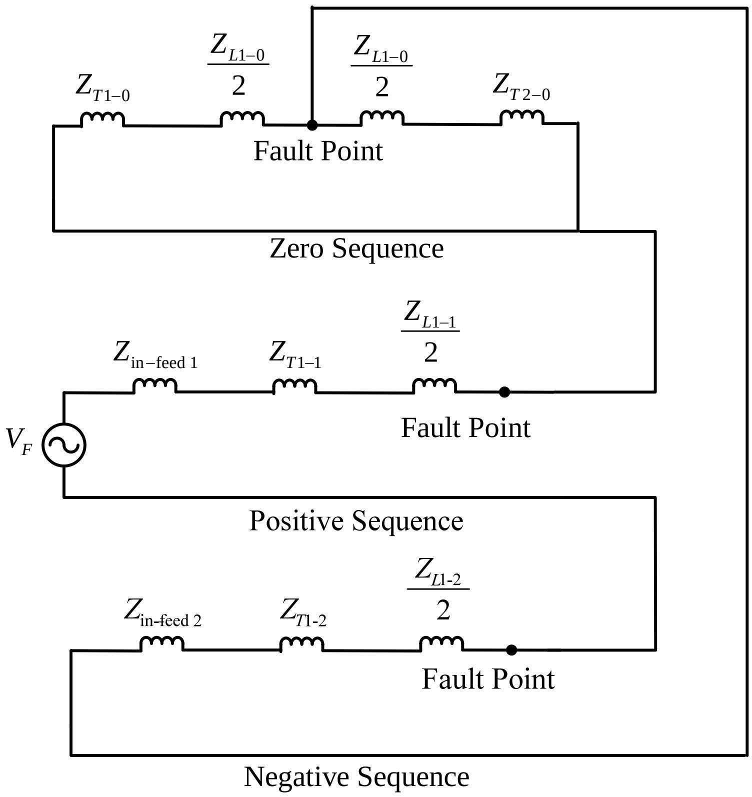

Figure 6 Sequence network connection for the single-line-to-earth fault.

The sequence impedances of the two transformers are as follows.

$$\bar{Z}_{T1-0} = \bar{Z}_{T2-0} = j0.1 \text{ pu}$$
$$\bar{Z}_{T1-1} = \bar{Z}_{T2-1} = j0.1 \text{ pu}$$
$$\bar{Z}_{T1-2} = \bar{Z}_{T2-2} = j0.1 \text{ pu.}$$

The sequence impedances of the transmission line are given by

$$\bar{Z}_{L1-0} = j\frac{1.5\times100}{268.89} = j0.5578 \text{ pu}$$
$$\bar{Z}_{L1-1} = j\frac{0.7\times100}{268.89} = j0.2603 \text{ pu}$$
$$\bar{Z}_{L1-2} = j\frac{0.7\times100}{268.89} = j0.2603 \text{ pu.}$$

The source, or grid, in-feed circuit has a fault impedance of $MVA_f = 2000$ MVA. The base MVA is $S_{base} = 45$ MVA and hence the equivalent per unit impedance of the source is calculated as

$$MVA_{f\ \text{pu}} = \frac{MVA_f}{S_{base}}$$
$$\bar{Z}_{in-feed\ 0} = \bar{Z}_{in-feed\ 1} = \bar{Z}_{in-feed\ 2} = j\frac{1}{MVA_{f\ \text{pu}}} = j\frac{45}{2000} = j0.0225 \text{ pu.}$$

It is assumed that the system is initially operating at rated voltage and that the pre-fault current is negligible. Hence, the pre-fault voltage is $V_f = 1.0$ pu.

The equivalent sequence impedances are then

$$\overline{Z}_0 = \frac{\overline{Z}_{T1\text{-}0} + \dfrac{\overline{Z}_{L1\text{-}0}}{2}}{2} = j0.1894 \text{ pu}$$

$$\overline{Z}_1 = \overline{Z}_{\text{infeed 1}} + \overline{Z}_{T1\text{-}1} + \frac{\overline{Z}_{L1\text{-}1}}{2} = j0.2527 \text{ pu}$$

$$\overline{Z}_2 = \overline{Z}_{\text{infeed 2}} + \overline{Z}_{T1\text{-}2} + \frac{\overline{Z}_{L1\text{-}2}}{2} = j0.2527 \text{ pu.}$$

The zero sequence fault current can now be calculated as

$$I_0 = \frac{V_f}{\overline{Z}_0 + \overline{Z}_1 + \overline{Z}_1} = \text{ - } j1.4393 \text{ pu.}$$

The per unit and actual fault current in Phase *a* is given by

$$\overline{I}_{af\ \text{pu}} = 3\overline{I}_0 = \ -j4.3178 \text{ pu.}$$

$$\overline{I}_{af\ \text{actual}} = \overline{I}_{af\ \text{pu}} I_{base\ \text{hv}} = \ -j4.3178 \times 236.189 = 1019.8 \text{ A.}$$

Exercise: Prove that the currents from Phases *b* and *c* flowing into the fault are both zero.

The sequence currents flowing in the transmission line to the left of the fault point are as follows

$$\underline{\overline{I}}_{lineL\ \text{sequence}} = \begin{bmatrix} \overline{I}_{lineL\text{-}0} \\ \overline{I}_{lineL\text{-}1} \\ \overline{I}_{lineL\text{-}2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\overline{I}_0 \\ \overline{I}_0 \\ \overline{I}_0 \end{bmatrix}$$

The sequence component to phase current transformation is by definition

$$\underline{\overline{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \overline{a}^2 & \overline{a} \\ 1 & \overline{a} & \overline{a}^2 \end{bmatrix}$$

where $\overline{a} = 1.0\angle 120°$. Hence the actual currents flowing in the three phases of the transmission line to the left of the fault are given by

$$\underline{\overline{I}}_{lineL\ \text{pu}} = \underline{\overline{A}}.\underline{\overline{I}}_{lineL\ \text{sequence}}$$

so the

$$\underline{\overline{I}}_{lineL\ \text{actual}} = \begin{bmatrix} -j849.852 \\ \text{-}j169.97 \\ \text{-}j169.97 \end{bmatrix}$$

Exercise: Calculate the currents flowing in the transmission line on the right hand side of the fault and prove that the fault currents in the three phases are $\overline{I}_a = -j1019.8$ A and $\overline{I}_b = \overline{I}_c = 0$ A as previously calculated.

Problems

1. A large electrical power generating station consists of N busbar sections. A generator of rated volt-amperes S_b and per-unit reactance X_g is connected to every busbar section. The busbar sections are each connected to a common tie-bar by a reactor of per-unit reactance X_t on the same base parameters as the generators. Draw a single-line diagram illustrating this system. Show that, in the limit as $N \to \infty$, the bolted, three-phase symmetrical per-unit fault volt-amperes on any busbar section is given by

$$MVA_f = \left[\frac{1}{X_g} + \frac{1}{X_t} \right] S_b \, .$$

2. The 11 kV three-phase busbars of a small electrical power station are in three sections, designated *A*, *B* and *C*. These three sections are interconnected by three ring reactors as follows.

   Between *A* and *B* :- *X* = 10% on a base of 20 MVA.
   Between *B* and *C* :- *X* = 8% on a base of 20 MVA.
   Between *C* and *A* :- *X* = 12% on a base of 20 MVA.

   On each section, there is one alternator, rated at 30 MVA with a reactance of 20%. In addition, on section *B*, there is a transformer, rated at 45 MVA, 11 kV/132 kV, with an equivalent series reactance of 12.5%.

   Construct a single-line diagram of this system. Draw also a per-phase equivalent circuit for the system if a three-phase symmetrical short circuit occurs on the high voltage busbars of the transformer and estimate the three-phase fault volt-amperes corresponding to the bolted three-phase fault at this point.

   [172 MVA]

3. A synchronous generator G_1, generating 1.0 per-unit voltage, is connected via a star-star transformer of reactance 0.12 per-unit to two transmissions lines in parallel. The other two ends of the lines are connected through a star-star transformer of reactance 0.1 per-unit to a second generator G_2, also generating 1.0 per-unit voltage. The system is initially unloaded. Calculate the fault current fed into a double-line-to-ground fault on the line side terminals of the transformer fed from generator G_1. The per-unit specifications of the equipment used are as follows.

   Transformers: $X_0 = X_1 = X_2$
   Transmission lines: $X_0 = 0.70,\ X_1 = 0.30,\ X_2 = 0.30$
   Generator G_1: $X_0 = 0.05,\ X_1 = 0.30,\ X_2 = 0.20$
   Generator G_2: $X_0 = 0.03,\ X_1 = 0.25,\ X_2 = 0.15.$

   The star-points of G_1 and both of the transformers are solidly grounded while that of G_2 is floating.

   [$\overline{I}_f = 4.86\angle 90°$ pu]

4. An 11 kV, three-phase, round rotor, synchronous generator is connected to an 11 kV/66 kV three-phase two-winding transformer which, in turn, feeds a three-phase, three-winding, 66 kV/11 kV/3.3 kV transformer bank through a short feeder line of negligible impedance.

Draw the single-line diagram for this transmission system. Construct the positive, negative and zero sequence equivalent networks and hence calculate the fault current which flows when a single-line-to-ground fault occurs on a terminal of the 11 kV winding of the three-winding transformer bank.

The base volt-amperes is 10 MVA and the specifications of the equipment employed in the power systems are as follows.

Generator *G*: Connected in star, grounded via $Z_n = (3 + j0)\ \Omega$,
$X_0 = 0.03,\ X_1 = 0.15,\ X_2 = 0.10.$

Transformer T_1: 11 kV winding in delta.
66 kV winding in grounded star,
$X_0 = X_1 = X_2 = 0.10.$

Transformer T_2: 66 kV winding in grounded star,
$X_0 = X_1 = X_2 = 0.04.$
11 kV winding in star grounded via $Z_n = (3 + j0)\ \Omega$,
$X_0 = X_1 = X_2 = 0.03.$
3.3 kV winding in delta,
$X_0 = X_1 = X_2 = 0.05.$

[$\bar{I}_f = 1579\angle -41^\circ$ A]

5. A 38 kV busbar has a three-phase fault infeed level of 1000 MVA. The negative and zero sequence source reactances of this infeed are $2/3$ and $1/3$ that of the positive sequence reactance. In addition, the zero sequence resistance is $60\ \Omega$.

A three-phase, 30 MVA, 110 kV /38 kV, solidly earthed star/delta connected transformer having a per unit reactance of 0.1 is fed from the 38 kV busbars.

Calculate the fault current in kA at the 110 kV terminals of the transformer for the following faults

(a) a three-phase fault

(b) a single-line-to-earth fault

(c) a line-to-line fault

(d) a line-to-line-to-earth fault.

$$\left[\begin{array}{l} I_{3\Phi} = 1.211 \text{ kA} \\ I_{1\Phi} = 1.349 \text{ kA} \\ I_{2\Phi} = 1.091 \text{ kA} \\ I_{2\Phi-\text{earth}} = 1.396 \text{ kA} \end{array}\right]$$