Tutorial Questions for EE4008

1. Determine the inverse z-transform of:

$$X(z) = \frac{-\frac{1}{24}z^{-1}}{1 - \frac{7}{24}z^{-1} + \frac{1}{48}z^{-2}}$$

where the region of convergence is the exterior of a circle, using

   (a) The partial Fraction Method

   (b) The long division method (determine $x(n)$ for $n = 0, 1, \dots 4$)

2. An LTI System has the system function:

$$H(z) = \frac{-18 - 25z^{-1}}{6 - 5z^{-1} - 6z^{-2}}$$

Draw the Pole-zero Plot of the system. Specify the ROC of $H(z)$ and determine $h(n)$ using the partial fractions method for the following cases:

   (a) The system is stable.

   (b) The system is causal.

   (c) The system is anti-causal.

3. Consider the z-transform

$$H(z) = \frac{z(z - 0.3)}{z^2 - 0.1z - 0.12}$$

   (a) Draw the Pole-Zero plot of $H(z)$ and identify the three possible regions of convergence.

   (b) Use the partial fractions method to determine the inverse z-transform $h(n)$ where

      i. $h(n)$ is a causal sequence

      ii. $h(n)$ is an anti-causal sequence

      iii. $h(n)$ is a two sided sequence.

   (c) Determine the first three values of the causal $h(n)$ sequence, using the long division method of inverting the z-transform.

4. (a) The output $y(n)$ of a Causal Linear Time Invariant system is given by

$$y(n) = \sum_{k=0}^{\infty} x(n-k)h(k)$$

where $x(n)$ is the input and $h(n)$ is the impulse response. Show that the Convolution Theorem holds, where the z-transform of the output $Y(z)$ is given by

$$Y(z) = X(z)H(z)$$

where $X(z)$ is the z-transform of $x(n)$ and $H(z)$ is the z-transform of $h(n)$.

(b) Determine the response $y(n)$ of a system with impulse response

$$h(n) = a^n u(n), |a| < 1$$

to the input $x(n) = u(n)$ using z-transforms, the Convolution theorem and partial fractions. For $a = \frac{1}{2}$ determine $y(0), y(1), y(2)$ and $y(3)$.

(c) Using the long division method, determine $y(0), y(1), y(2)$ and $y(3)$, for $a = \frac{1}{2}$ and compare these to the values determined in part (b).

5. (a) The output $y(n)$ of a Causal Linear Time Invariant system is given by

$$y(n) = \sum_{k=0}^{\infty} x(n-k)h(k)$$

where $x(n)$ is the input and $h(n)$ is the impulse response. Show that the Convolution Theorem holds, where the Z-transform of the output $Y(z)$ is given by

$$Y(z) = X(z)H(z)$$

with $X(z)$ the Z-transform of $x(n)$ and $H(z)$ the Z-transform of $h(n)$.

(b) For input $x(n) = a^n u(n)$ and impulse response $h(n) = b^n u(n)$, determine the output $y(n)$ using Z-transforms, the convolution Theorem and partial fractions.

(c) For $a = \frac{1}{4}$ and $b = -\frac{1}{3}$ determine the Region of Convergence of $Y(z)$.

(d) Using the long division method, determine $y(0), y(1), y(2), y(3)$, for $a = \frac{1}{4}$ and $b = -\frac{1}{3}$.

6. A causal IIR filter has a rational transfer function:

$$H(z) = \frac{1 - z^{-2}}{1 - 1.131z^{-1} + 0.64z^{-2}}$$

(a) What are the filter coefficients that implement this transfer function.

(b) Determine the first four values of the impulse response of this filter using the long division method.

(c) Draw the pole/zero plot of this filter.

(d) Explain how an approximation of the frequency response can be determined from the pole-zero plot.

(e) Using the pole/zero plot, sketch the magnitude response of the filter $H(z)$.

7. (a) The crosscorrelation of two real deterministic signal $x(n)$ and $y(n)$ is defined as:

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)y(n) \quad l = 0, \pm 1, \pm 2, \ldots$$

Show that the Z-transform is given by:

$$R_{xy}(z) = X(z)Y(\frac{1}{z})$$

(b) Using Z-transforms determine the autocorrelation sequence $r_{xx}(k)$ of the signal:

$$x(n) = a^n u(n) \quad -1 < a < 1$$