OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2013

B.E. (ELECTRICAL)
M.ENG.SC. (MICROELECTRONICS)
H.DIP. (MICROELECTRONICS)
VISITING EUROPEANS

DIGITAL SIGNAL PROCESSING
EE4008

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Answer *five* questions.

All questions carry equal marks.

The use of departmental approved non-programmable calculators is permitted

Time Allowed: *3 hours*

Questions Begin on the Next Page

1. (a) Starting with the ideal frequency response $H_d(\omega)$, describe the Windows method of designing a bandstop filter. [*10 marks*]

   (b) Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the Windows method designing a bandstop filter that meets the following specification:

   - Stopband: $45-55$Hz
   - Transition width: 5Hz
   - Passband ripple: <0.1dB
   - Stopband attenuation: >40dB
   - Sampling frequency: 256Hz

   The parameters of common window functions are given in the Appendix. [*10 marks*]

2. (a) A first order lowpass IIR digital filter has a transfer function

$$H_{LP}(z) = G\frac{1+z^{-1}}{1-\alpha z^{-1}}$$

   Show that the gain factor G is

$$G = \frac{1-\alpha}{2}$$

   [*2 marks*]

   (b) For the first order lowpass filter of part (a) show that

$$\alpha = \frac{1-\sin\omega_c}{\cos\omega_c}$$

   where ω_c is the 3dB cutoff frequency. [*3 marks*]

   (c) Determine the system function $H_{LP}(z)$, of a first order lowpass filter with a 3-dB cutoff frequency of $F_c = 128$Hz where the sampling frequency is $F_s = 1024$Hz

   [*3 marks*]

   (d) Determine the system function $H_{LP-B}(z)$ of a digital lowpass filter using the bilinear transformation of a Butterworth filter. Sketch the magnitude response of the filter. The digital filter specifications are:

   - Passband edge frequency: 128Hz.
   - Stopband edge frequency: 176Hz.
   - Passband ripple: 3dB.
   - Minimum stopband attenuation: 10dB
   - Sampling frequency: 1024Hz.

   [*8 marks*]

   (e) Sketch the magnitude response of the filters in part (c) and (d). [*4 marks*]

3. A causal IIR filter has a rational transfer function:

$$H(z) = \frac{1 - z^{-2}}{1 - 1.131z^{-1} + 0.64z^{-2}}$$

(a) What are the filter coefficients that implement this transfer function. [*2 marks*]

(b) Determine the first four values of the impulse response of this filter using the long division method. [*6 marks*]

(c) Draw the pole/zero plot of this filter. [*4 marks*]

(d) Explain how an approximation of the frequency response can be determined from the pole-zero plot. [*5 marks*]

(e) Using the pole/zero plot, sketch the magnitude response of the filter $H(z)$. [*3 marks*]

4. (a) Starting with the constant coefficient difference quation definition in each case, show that the frequency responses of the first-difference and central-difference differentiators are

i. First-difference

$$H_{fd}(\omega) = 2je^{-0.5j\omega} \sin\left(\frac{\omega}{2}\right)$$

ii. Central-difference

$$H_{cd}(\omega) = je^{-j\omega} \sin(\omega)$$

[*6 marks*]

(b) In the continuous time domain a differentiator is defined in Laplace domain as:

$$H_{diff}(s) = s$$

Show that a digital differentiator designed using the Bilinear transform has a frequency response of

$$H_{diff}(\omega) = j \tan\left(\frac{\omega}{2}\right)$$

You may assume that $T_s = 2$, to ignore the effect of the sampling frequency on the Bilinear transformation. [*6 marks*]

(c) Plot the magnitude response of the three differentiators of part (a) and (b). [*6 marks*]

(d) Comment on the suitability of the three differentiators for use in an application where there is high frequency noise. [*2 marks*]

5. (a) Describe the steps necessary to transform a digital integrator into a stable digital differentiator. [*8 marks*]

(b) Using the procedure described in part (a) determine the constant coefficient difference equation of the digital differentiator derived from a Tick's rule integrator, which is defined as:

$$y(n) = 0.3584x(n) + 1.2832x(n-1) + 0.3584x(n-2) + y(n-2)$$

[*12 marks*]

6. (a) Let $x(n)$ be a Wide Sense Stationary random process with mean m_X, autocorrelation $\phi_{XX}(k)$ and power spectral density $P_{XX}(\omega)$. $x(n)$ is filtered by a Stable Linear Time Invariant System with impulse response $h(n)$ to produce output $y(n)$. Determine the mean m_Y, autocorrelation $\phi_{YY}(k)$ and power spectral density $P_{YY}(\omega)$ of $y(n)$. [*10 marks*]

(b) Unit variance white noise is filtered by a LTI filter with impulse response:

$$h(n) = \frac{1}{4}\left(-\frac{1}{4}\right)^{n} u(n) + \frac{1}{4}\left(-\frac{1}{4}\right)^{n-2} u(n-2)$$

Determine the mean and the power spectral density of the filter output in trigonometric form.

[*10 marks*]

Appendix of Equations

- Window Functions

Window $w(n)$	Sidelobe	$\triangle f$	Stopband Attenuation	Passband Ripple	$\Delta\omega_{3db}$
Rectangular	-13db	$\frac{0.9}{N}$	21db	0.7416db	$0.89\frac{2\pi}{N}$
$w(n) = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hanning	-31db	$\frac{3.1}{N}$	44db	0.0546db	$1.44\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hamming	-41db	$\frac{3.3}{N}$	53db	0.0194db	$1.30\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$					

- Butterworth Filter Order given by:

$$n = \left\lceil \frac{\log_{10}\left[\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}\right]}{2\log_{10}\left[\frac{\Omega_s}{\Omega_p}\right]} \right\rceil$$

- Butterworth 3-dB cutoff frequency given by:

$$\Omega_c = \Omega_p 10^{-\left[\frac{\log_{10}\left[10^{0.1A_{max}}-1\right]}{2n}\right]}$$

- Table of Butterworth Polynomials:

n	
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.76536s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

- Table of z-Transforms

Signal $x(n)$	z-Transform $X(z)$	ROC
$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < a$

- Integration

$f(x)$	$\int f(x)dx$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\cos x$	$\sin x$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$

- Integration by parts

$$\int udv = uv - \int vdu$$

- Euler Identity

$$\cos x = \frac{1}{2}\left(e^{-jx} + e^{jx}\right)$$

$$\sin x = \frac{1}{2}j\left(e^{-jx} - e^{jx}\right)$$

- Trigonometric Identities

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$