Tutorial Questions on FIR Filter Design

1. Starting with the ideal frequency response for a Lowpass filter

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

   derive the ideal impulse response

$$h_d(n) = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\sin \omega_c n}{\pi n} & n \ne 0 \end{cases}$$

2. Starting with the ideal frequency response for a Highpass filter

$$H_d(\omega) = \begin{cases} 0 & |\omega| \le \omega_c \\ 1 & \omega_c < |\omega| \le \pi \end{cases}$$

   derive the ideal impulse response

$$h_d(n) = \begin{cases} 1 - \frac{\omega_c}{\pi} & n = 0 \\ -\frac{\sin(\omega_c n)}{\pi n} & n \ne 0 \end{cases}$$

3. Starting with the ideal frequency response for a Bandpass filter

$$H_d(\omega) = \begin{cases} 0 & -\pi \le \omega < -\omega_{c2} \\ 1 & -\omega_{c2} \le \omega \le -\omega_{c1} \\ 0 & -\omega_{c1} < \omega < \omega_{c1} \\ 1 & \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & \omega_{c2} < \omega \le \pi \end{cases}$$

   derive the ideal impulse response

$$h_d(n) = \begin{cases} \frac{\omega_{c2} - \omega_{c1}}{\pi} & n = 0 \\ \frac{\sin(\omega_{c2} n)}{\pi n} - \frac{\sin(\omega_{c1} n)}{\pi n} & n \ne 0 \end{cases}$$

4. Starting with the ideal frequency response for a Bandstop filter

$$H_d(\omega) = \begin{cases} 1 & -\pi \le \omega < -\omega_{c2} \\ 0 & -\omega_{c2} \le \omega \le -\omega_{c1} \\ 1 & -\omega_{c1} < \omega < \omega_{c1} \\ 0 & \omega_{c1} \le \omega \le \omega_{c2} \\ 1 & \omega_{c2} < \omega \le \pi \end{cases}$$

   derive the ideal impulse response

$$h_d(n) = \begin{cases} 1 + \frac{\omega_{c1} - \omega_{c2}}{\pi} & n = 0 \\ \frac{\sin(\omega_{c1} n)}{\pi n} - \frac{\sin(\omega_{c2} n)}{\pi n} & n \ne 0 \end{cases}$$

5. Starting with the expression for the Frequency response for an FIR Filter:

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

Show that a FIR filter with positive symmetric coefficients ($h(n) = h(M-1-n)$) and M odd has a frequency response

$$H(\omega) = e^{-j\omega(M-1)/2}\left[h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{(M-3)/2} h(n)\cos\left(\omega\left(\frac{M-1}{2} - n\right)\right)\right]$$

6. Starting with the expression for the Frequency response for an FIR Filter:

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

Show that a FIR filter with positive symmetric coefficients ($h(n) = h(M-1-n)$) and M even has a frequency response

$$H(\omega) = e^{-j\omega(M-1)/2}\left[2\sum_{n=0}^{M/2-1} h(n)\cos\left(\omega\left(\frac{M-1}{2} - n\right)\right)\right]$$

7. Starting with the expression for the Frequency response for an FIR Filter:

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

Show that a FIR filter with negative symmetric coefficients ($h(n) = -h(M-1-n)$), M odd and $h\left(\frac{M-1}{2}\right) = 0$ has a frequency response

$$H(\omega) = e^{j[-\omega(M-1)/2+\pi/2]}\left[2\sum_{n=0}^{(M-3)/2} h(n)\sin\omega\left(\frac{M-1}{2} - n\right)\right]$$

8. Starting with the expression for the Frequency response for an FIR Filter:

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

Show that a FIR filter with negative symmetric coefficients ($h(n) = -h(M-1-n)$) and M even has a frequency response

$$H(\omega) = e^{j[-\omega(M-1)/2+\pi/2]}\left[2\sum_{n=0}^{(M/2)-1} h(n)\sin\omega\left(\frac{M-1}{2} - n\right)\right]$$

9. A FIR low pass filter designed using the rectangular window has the impulse response

$$h(n) = \begin{cases} \frac{\sin\left(\omega_c\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} & 0 \le n \le M-1 \;\; n \ne \frac{M-1}{2} \\ \frac{\omega_c}{\pi} & n = \frac{M-1}{2} \;\text{ for } M \text{ odd} \\ 0 & 0 > n, M-1 < n \end{cases}$$

With M odd, show that the filter coefficients have positive symmetry

$$h(n) = h(M-1-n)$$

10. A FIR high pass filter designed using the rectangular window has the impulse response

$$h(n) = \begin{cases} -\frac{\sin\left(\omega_c\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} & 0 \le n \le M-1 \;\; n \ne \frac{M-1}{2} \\ 1-\frac{\omega_c}{\pi} & n = \frac{M-1}{2} \;\text{ for } M \text{ odd} \\ 0 & 0 > n, M-1 < n \end{cases}$$

With M odd, show that the filter coefficients have positive symmetry

$$h(n) = h(M-1-n)$$

11. A FIR band pass filter designed using the rectangular window has the impulse response

$$h(n) = \begin{cases} \frac{\sin\left(\omega_{c2}\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} - \frac{\sin\left(\omega_{c1}\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} & 0 \le n \le M-1 \;\; n \ne \frac{M-1}{2} \\ \frac{\omega_{c2}-\omega_{c1}}{\pi} & n = \frac{M-1}{2} \;\text{ for } M \text{ odd} \\ 0 & 0 > n, M-1 < n \end{cases}$$

With M odd, show that the filter coefficients have positive symmetry

$$h(n) = h(M-1-n)$$

12. A FIR band pass filter designed using the rectangular window has the impulse response

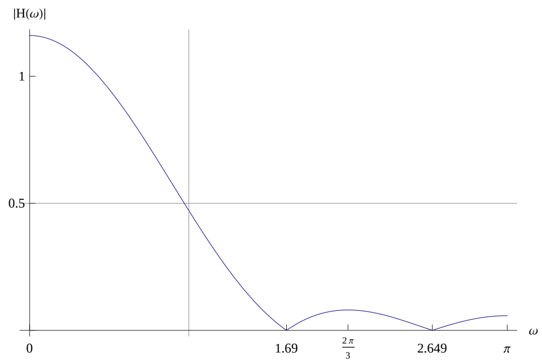
$$h(n) = \begin{cases} \frac{\sin\left(\omega_{c1}\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} - \frac{\sin\left(\omega_{c2}\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)} & 0 \le n \le M-1 \;\; n \ne \frac{M-1}{2} \\ 1+\frac{\omega_{c1}-\omega_{c2}}{\pi} & n = \frac{M-1}{2} \;\text{ for } M \text{ odd} \\ 0 & 0 > n, M-1 < n \end{cases}$$

With M odd, show that the filter coefficients have positive symmetry

$$h(n) = h(M-1-n)$$

13. For a low pass filter designed using a rectangular window, with $M = 5$ and $\omega_c = \frac{\pi}{3}$ determine the frequency response

The plot of the magnitude response of this filter for $0 \le \omega \le \pi$ is:



Plot the phase response $\theta(\omega)$ for $-\pi \le \omega \le \pi$

14. Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a low pass filter.

15. Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a high pass filter.

16. Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a bandpass filter.

17. Starting with the ideal frequncy response $H_d(\omega)$, describe the windows method of designing a bandstop filter.

18. Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the "Windows" method of the low pass filter design that meets the following specification:

    - Passband edge frequency:- $F_p = 15$kHz
    - Transition Width:- 4kHz
    - Passband Ripple:- 0.02dB
    - Stopband attenuation:- > 40dB
    - Sampling frequency:- 40kHz

    The parameters of common window functions are given in the Appendix.

19. Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the "Windows" method of the High pass filter design that meets the following specification:

    - Passband edge frequency:- $F_p = 300$Hz
    - Stopband edge frequency:- $F_{stop} = 250$Hz
    - Passband Ripple:- 0.1dB
    - Stopband attenuation:- > 42dB

- Sampling frequency:- 1kHz

The parameters of common window functions are given in the Appendix.

20. Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the Windows method designing a bandpass filter that meets the following specification:

    - Passband: $12.5 - 20$kHz
    - Transition Width: 5kHz
    - Passband Ripple: < 1dB
    - Stopband attenuation: > 15dB
    - Sampling frequency: 52kHz

    The parameters of common window functions are given in the Appendix.

21. Determine the filter length M and the coefficients $h(1)$ and $h(\frac{M-1}{2})$ using the Windows method designing a bandstop filter that meets the following specification:

    - Stopband: $40 - 60$Hz
    - Transition Width: 5Hz
    - Passband Ripple: < 0.8dB
    - Stopband attenuation: > 20dB
    - Sampling frequency: 150Hz

    The parameters of common window functions are given in the Appendix.

Appendix of Equations

- Window Functions

Window $w(n)$	Sidelobe	$\triangle f$	Stopband Attenuation	Passband Ripple	$\Delta\omega_{3db}$
Rectangular	-13db	$\frac{0.9}{N}$	21db	0.7416db	$0.89\frac{2\pi}{N}$
$w(n) = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hanning	-31db	$\frac{3.1}{N}$	44db	0.0546db	$1.44\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					
Hamming	-41db	$\frac{3.3}{N}$	53db	0.0194db	$1.30\frac{2\pi}{N}$
$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$					

- Integration

$f(x)$	$\int f(x)dx$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\cos x$	$\sin x$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$

- Integration by parts

$$\int udv = uv - \int vdu$$

- Euler Identity

$$\cos x = \frac{1}{2}\left(e^{-jx} + e^{jx}\right)$$

$$\sin x = \frac{1}{2}j\left(e^{-jx} - e^{jx}\right)$$