

**OLLSCOIL NA hÉIREANN, CORCAIGH**  
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH  
UNIVERSITY COLLEGE, CORK

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**SUMMER EXAMINATIONS, 2012**

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**B.E. DEGREE (ELECTRICAL & ELECTRONIC)**

TELECOMMUNICATIONS

EE4004

Dr. L. Seed

Professor N. Riza

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Dr. C. Murphy

Time allowed: *3 hours*

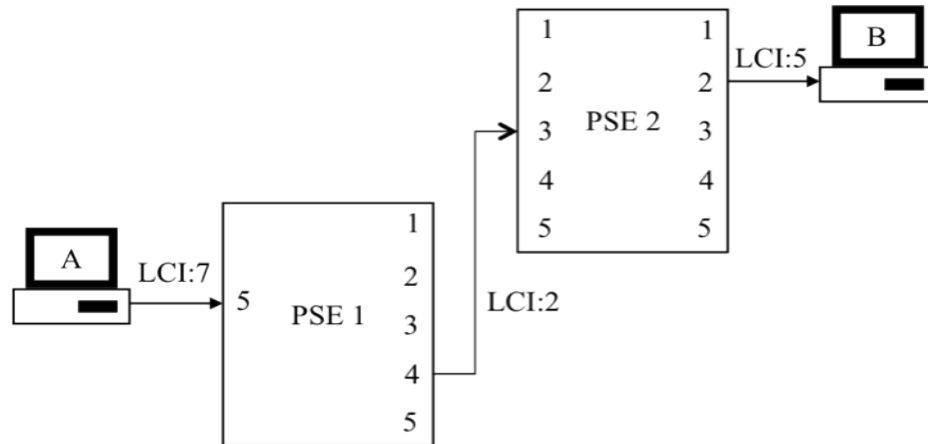
Answer *five* questions.

All questions carry equal marks.

The use of log tables and a departmental approved non-programmable calculator is permitted.

1. (a) Draw a simplified representation of a wide area data network illustrating its main elements and briefly describe the tasks that the network must do to carry data successfully from one user to another.  
[8 marks]
- (b) Illustrate the operation of a store and forward packet data network considering source and destination nodes and two intermediate nodes (nodes 1 and 2). From this, determine formulas for the network delay (ND) and total message transmission time (TT). You may ignore the propagation delay.  
[8 marks]
- (c) A message of 1000 bits is to be sent over a data network from source to destination via two intermediate nodes. Each link along the route has a data-rate of 1Mbps and is totally dedicated to this transmission. A header size of 50 bits is used by the protocol. Calculate the network delay and the total transmission time if the message is sent as 10 packets.  
[4 marks]
2. (a) Using diagrams, as appropriate, describe the following aspects of a public telephone system in Europe:
- (i) The interface used at the telephone exchange for a voice connection giving rise to the commonly specified data-rate associated with one voice call.  
[6 marks]
- (ii) The first level of multiplexing used in European trunk lines giving rise to the trunk data rate commonly referred to as the E1 rate.  
[4 marks]
- (b) Briefly describe the operation of an X.25 data network.  
[6 marks]

- (c) For the X.25 data network below, show suitable routing tables for PSE1 and PSE2 to allow data transfer from user A to user B.



[4 marks]

3. (a) (i) Illustrate the format of an ATM cell and briefly describe the function of each field in the cell, noting the main differences between an ATM cell and other commonly used wide-area network packet formats.

[4 marks]

- (ii) Describe how the cell boundaries are identified in an ATM system and specify why this is such an important task.

[4 marks]

- (b) (i) Describe the three main signal degradation mechanisms in Digital Subscriber Line (DSL) technologies for broadband communications and state briefly how these are overcome.

[6 marks]

- (ii) Describe the four commonly used transmission duplexing methods in DSL technologies and specify which ones are used for ADSL.

[6 marks]

4. Given that the  $2 \times 2$  channel matrix  $[P(Y_1|X)]$  for the generalised binary channel with 2 input symbols, denoted  $x_i, 1 \leq i \leq 2$  and 2 output symbols, denoted  $y_j, 1 \leq j \leq 2$ , is given by: -

$$[P(Y_1|X)] = \begin{bmatrix} 1-e_1 & e_1 \\ e_2 & 1-e_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & -e_1 \\ 1 & 1 \end{bmatrix}}_F \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1-e_1-e_2 \end{bmatrix}}_D \underbrace{\left( \frac{1}{e_1+e_2} \begin{bmatrix} e_2 & e_1 \\ -e_2 & e_2 \end{bmatrix} \right)}_{F^{-1}}$$

where  $e_1 > 0, e_2 > 0$ ,  $D$  is a diagonal matrix and the columns of  $F$  are eigenvectors of  $[P(Y_1|X)]$ , show that if  $n$  such generalised binary channels are connected in series (i.e. the outputs of channel  $i$  become the inputs of channel  $i+1, 1 \leq i \leq n-1$ ), then: -

- (a) The composite channel matrix  $[P(Y_n|X)]$  is given by: -

$$[P(Y_n|X)] = \frac{1}{e_1+e_2} \begin{bmatrix} e_2+e_1\lambda & e_1(1-\lambda) \\ e_2(1-\lambda) & e_1+e_2\lambda \end{bmatrix}$$

where  $\lambda = (1-e_1-e_2)^n$ .

[6 marks]

- (b) Show that if the output symbols, denoted  $y_1$  and  $y_2$ , from the composite channel in part (a) above are to be equiprobable then we require: -

$$P(x_1) = \frac{e_1-e_2+2e_2\lambda}{2\lambda(e_1+e_2)}$$

where  $P(x_1)$  denotes the probability of the input symbol being  $x_1$  (noting that, if  $x_1$  is sent and no error occurs, the output symbol will be  $y_1$ ).

[5 marks]

(c) If both the input symbols and the output symbols are equiprobable, show that: -

(i) This condition requires  $e_1 = e_2$ . [3 marks]

(ii) The composite channel capacity, denoted  $C_s^c$ , is given by

$$C_s^c = 1 + \left( \frac{1 - (1 - 2e_1)^n}{2} \right) \log_2 \left( \frac{1 - (1 - 2e_1)^n}{2} \right) + \left( \frac{1 + (1 - 2e_1)^n}{2} \right) \log_2 \left( \frac{1 + (1 - 2e_1)^n}{2} \right).$$

[6 marks]

5. A baseband digital communications system uses rectangular wave signalling with  $A_1$  volts representing logic 1 and  $A_2$  volts representing logic 0 (where  $A_2 < A_1$ ). The receiver takes a single sample of the received signal during the bit signalling time and compares this sample with a decision threshold  $T$ . The communications are affected by zero-mean additive Gaussian noise whose probability density function  $f_n$  is given by: -

$$f_n(v) = \frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}},$$

$P_0$  and  $P_1$  respectively denote the probability of sending logic 0 and logic 1 and, to minimize the resulting overall probability of error  $P_e$ , the threshold  $T$  is given by: -

$$T = \frac{A_1 + A_2}{2} + \frac{\sigma^2}{A_1 - A_2} \ln \left[ \frac{P_0}{P_1} \right].$$

- (a) Show that, if  $P_0 > P_1$ , then the average probability of error, denoted  $P_e$ , is given by: -

$$P_e = \frac{1}{2} \left( 1 - \left( P_0 \operatorname{erf} \left[ \frac{T - A_2}{\sqrt{2\sigma^2}} \right] + (1 - P_0) \operatorname{erf} \left[ \frac{A_1 - T}{\sqrt{2\sigma^2}} \right] \right) \right)$$

where: -

$$\operatorname{erf}[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

[10 marks]

- (b) Consider a system for which  $A_1 = 2.5V$ ,  $A_2 = -2.5V$  and  $\sigma^2 = 0.45W$ . Using the table of values of  $\operatorname{erf}(x)$  provided on page 8: -

- (i) Prove that, if the threshold remains fixed at  $T = 0V$ , then  $P_e$  is independent of  $P_0$  and calculate its value in this case.

[4 marks]

- (ii) When the optimum threshold is employed in each case, calculate the value of  $P_e$  when  $P_0 = 0.65$  and when  $P_0 = 0.75$ .

[6 marks]

6. (a) An analogue signal having a 20kHz bandwidth is sampled at 1.5 times the Nyquist rate and each sample is quantised into one of 128 equally likely levels. Assuming that successive samples are statistically independent, the signal power at the receiver is 0.2mW and the communication is affected by additive white Gaussian noise with power spectral density  $\eta/2 = 10^{-11} W/Hz$ , estimate via the use of a suitable graph, or otherwise, the minimum channel bandwidth required for error-free transmission of the information produced by this source.

[10 marks]

- (b) A binary modulation scheme is described by: -

$$s_i(t) = \begin{cases} s_1(t) = A_1 \cos(\omega_1 t) & 0 \leq t \leq T \\ s_2(t) = A_2 \cos(\omega_2 t) & 0 \leq t \leq T \end{cases}$$

where  $T$  is an integer multiple of  $2\pi/\omega_i, i=1,2$ . For this modulation scheme, given

that (under the usual assumptions)  $P_e = Q\left[\sqrt{\frac{E_d}{2\eta}}\right]$ , show that: -

$$P_e = Q\left[\sqrt{\frac{E_b}{\eta}}\right]$$

where  $E_b$  denotes the average signal energy per bit.

[10 marks]

7. (a) Using the primitive polynomial  $p(x) = x^4 + x + 1$ , generate the field  $GF(2^4)$ .

[6 marks]

- (b) Prove that the generator polynomial for the (15,7) double error correcting primitive BCH code based upon  $GF(2^4)$ , denoted  $g(x)$ , is given by: -

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1.$$

[6 marks]

- (c) Use the syndrome decoding method to correct the received sequence: -

$$r(x) = x^{11} + x^8 + x^7 + x^3.$$

[8 marks]

**Table of values of  $\operatorname{erf}(x)$**

$x$	$\operatorname{erf}(x)$		$x$	$\operatorname{erf}(x)$
2.5	0.999593		2.63	0.9998
2.505	0.999604		2.635	0.999806
2.51	0.999614		2.64	0.999811
2.515	0.999625		2.645	0.999816
2.52	0.999635		2.65	0.999822
2.525	0.999644		2.655	0.999826
2.53	0.999654		2.66	0.999831
2.535	0.999663		2.665	0.999836
2.54	0.999672		2.67	0.999841
2.545	0.999681		2.675	0.999845
2.55	0.999689		2.68	0.999849
2.555	0.999698		2.685	0.999854
2.56	0.999706		2.69	0.999858
2.565	0.999714		2.695	0.999862
2.57	0.999722		2.7	0.999866
2.575	0.999729		2.705	0.999869
2.58	0.999736		2.71	0.999873
2.585	0.999744		2.715	0.999877
2.59	0.999751		2.72	0.99988
2.595	0.999757		2.725	0.999884
2.6	0.999764		2.73	0.999887
2.605	0.99977		2.735	0.99989
2.61	0.999777		2.74	0.999893
2.615	0.999783		2.745	0.999896
2.62	0.999789		2.75	0.999899
2.625	0.999795			