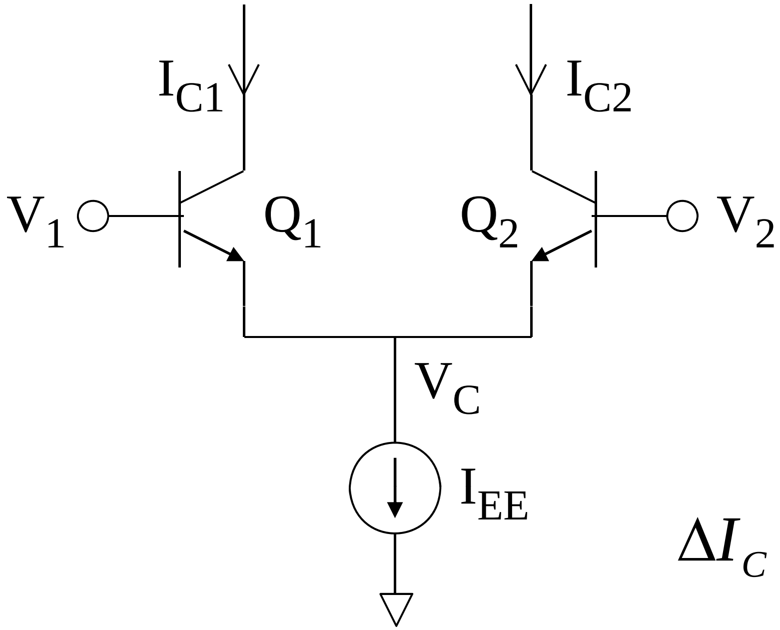
EE4011 RFIC

Common-Emitter Pairs and Dynamic Range

Another Look at the Emitter Coupled Pair



$$V_d = V_1 - V_2$$

$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d}{2V_T}\right)$$

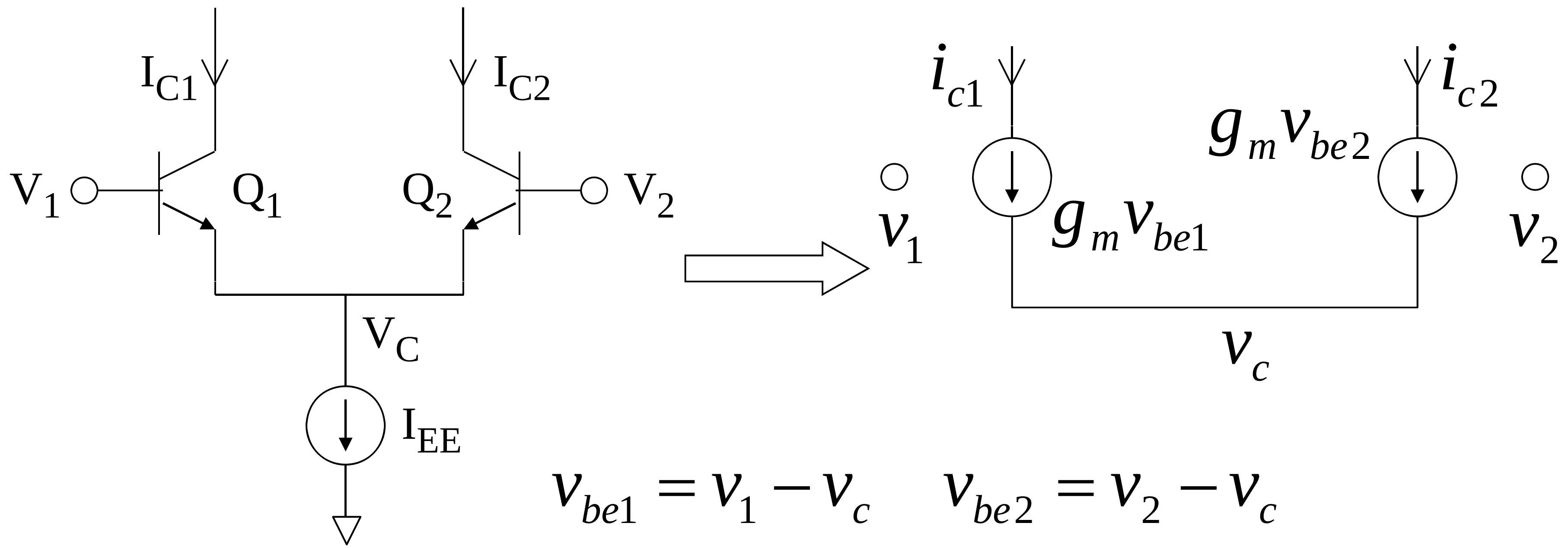
If V_d is small then because of the behaviour of tanh(x) for small x:

$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d}{2V_T}\right) \approx I_{EE} \frac{V_d}{2V_T} = \frac{I_{EE}}{2V_T} V_d$$

i.e. the differential output current is linearly proportional to the differential input voltage

Common Emitter Pair – Small Signal (1)

Small-signal: Q1/Q2 replaced by transconductances. Ignoring i/p resistance and output conductance for simplicity. Current source is a small-signal “open”:

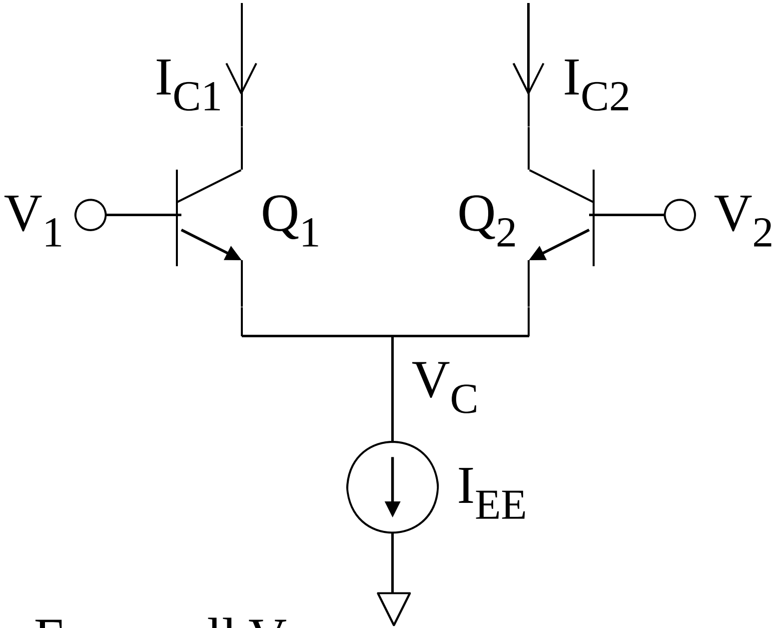


$$v_{be1} = v_1 - v_c \qquad v_{be2} = v_2 - v_c$$

$$v_{be1} - v_{be2} = v_1 - v_2 = \delta v$$

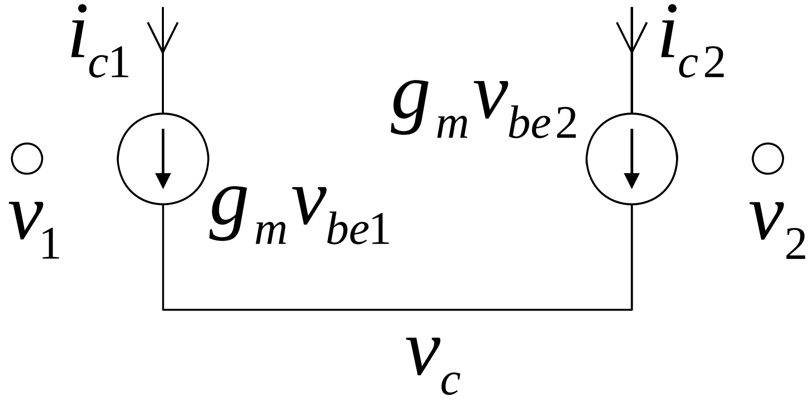
$$i_{c1} - i_{c2} = g_m v_{be1} - g_m v_{be2} = g_m \delta v$$

Common Emitter Pair – Small Signal (2)



For small V_d:

$$\Delta I_C = \frac{I_{EE}}{2V_T} V_d$$



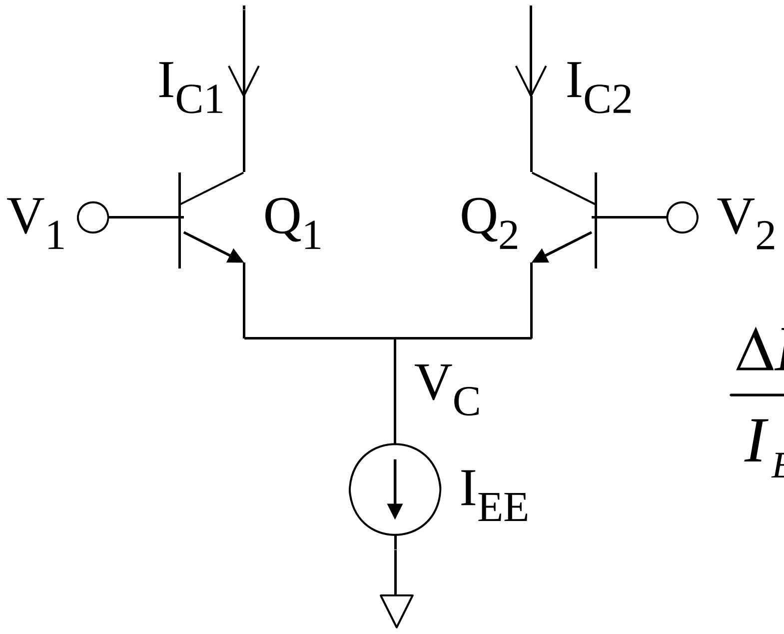
$$\delta i_c = i_{c1} - i_{c2} = g_m \delta v$$

$$g_m = \frac{I_C}{V_T} = \frac{I_{EE}}{2V_T} \Rightarrow \delta i_c = \frac{I_{EE}}{2V_T} \delta v$$

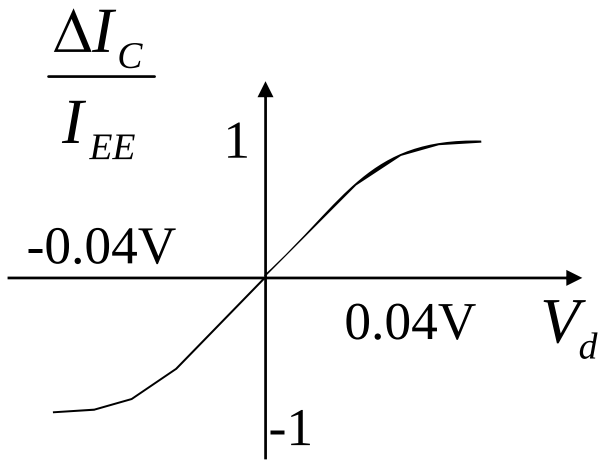
(assuming the DC biases V_1 and V_2 are the same, I_{EE} is shared equally by Q_1 and Q_2)

The large-signal analysis gives the same results as the small-signal analysis if the input differential voltage is made small in the large-signal analysis – this is a reassuring consistency check.

Common Emitter Pair – Symmetry Property



$$\frac{\Delta I_C}{I_{EE}} = \tanh\left(\frac{V_d}{2V_T}\right)$$



$$\tanh(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n} (2^{2n} - 1) B_n x^{2n-1}}{(2n)!}, |x| < \frac{\pi}{2}$$

B_n is the Bernoulli number n.

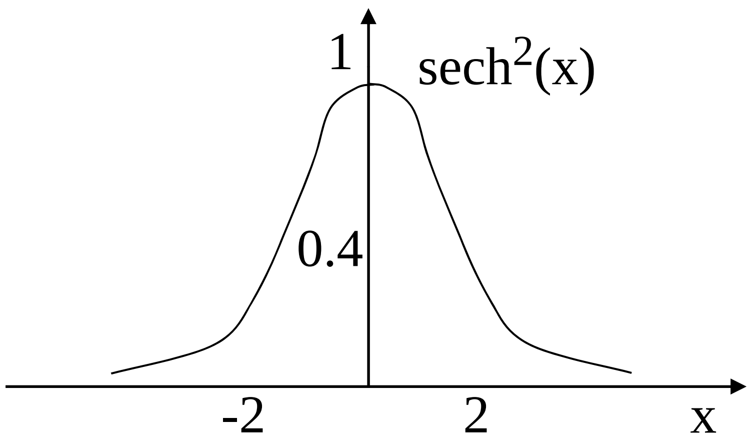
This is a typical balanced or differential system encountered in RF systems. All the even order terms in the transfer function are zero. Also, the system is compressive i.e. the coefficient of x^3 has the opposite sign to the coefficient of x. These could be used to find the 1dB compression point of the system.

Common Emitter Pair – Transconductance

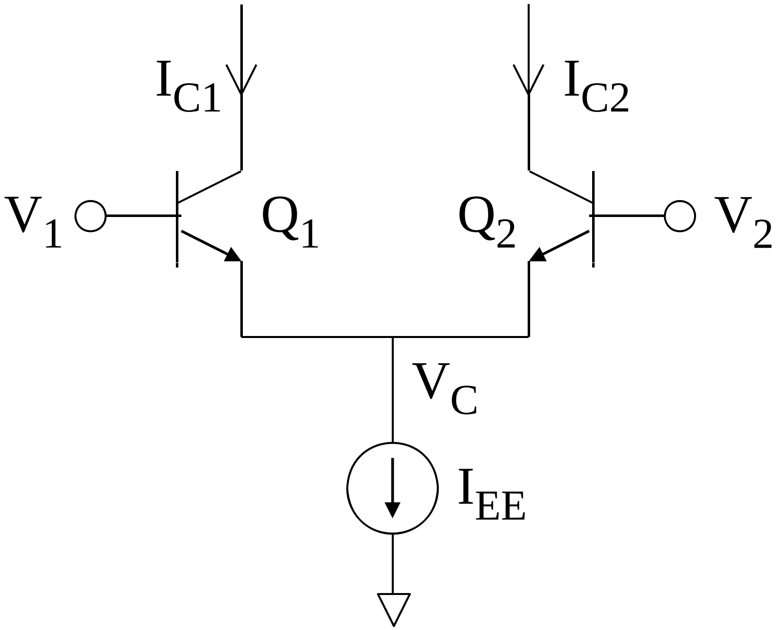
$$\frac{d}{dx}\tanh(x) = \mathrm{sech}^2(x)$$

$$\Delta I_C = I_{EE}\tanh\left(\frac{V_d}{2V_T}\right) \Rightarrow g_m = \frac{d\Delta I_C}{dV_d} = \frac{I_{EE}}{2V_T}\mathrm{sech}^2\left(\frac{V_d}{2V_T}\right)$$

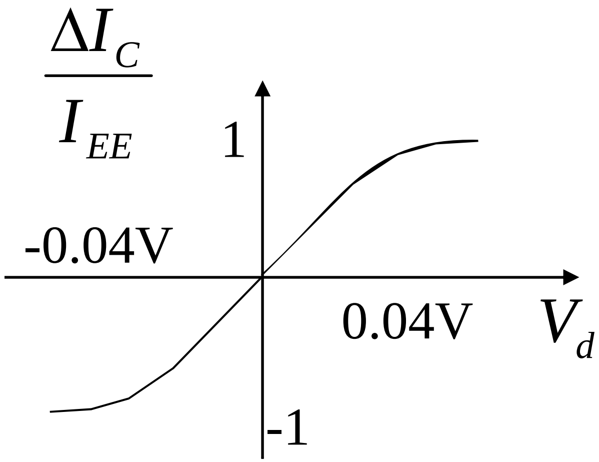
When V_d=0: $g_m = \frac{I_{EE}}{2V_T}\mathrm{sech}^2(0) = \frac{I_{EE}}{2V_T}$ as expected



Common Emitter Pair – Dynamic Range



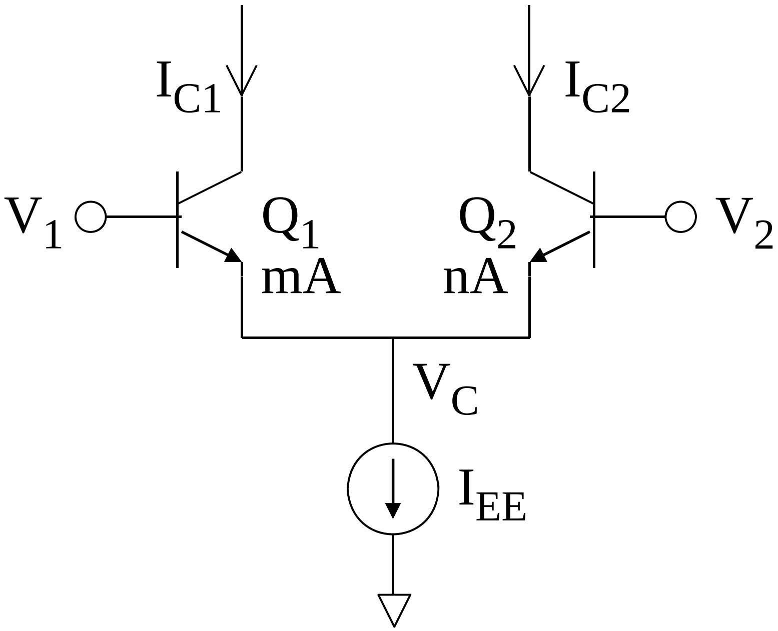
$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d}{2V_T}\right)$$



The linear range of the standard common emitter pair is not very large – when the differential input voltage exceeds about 40mV (at 300K) the output characteristic deviates strongly from a linear response. Various techniques are used to increase the dynamic range of common emitter pairs for both amplifiers and mixers – these are mainly based on the use of emitter degeneration resistors or cross coupled pairs (Schmoock's method) or both.

Emitter-Coupled Pair: Different Emitter Areas (1)

The transistors emitter areas are multiples of a unit size A.



$$I_{C1} = mI_S e^{\frac{V_1 - V_C}{V_T}} \quad , I_{C2} = nI_S e^{\frac{V_2 - V_C}{V_T}}$$

$$\frac{I_{C1}}{I_{C2}} = \frac{mI_S e^{\frac{V_1 - V_C}{V_T}}}{nI_S e^{\frac{V_2 - V_C}{V_T}}} = \frac{m}{n} e^{\frac{V_1 - V_2}{V_T}} = re^{\frac{V_d}{V_T}}$$

$$V_d = V_1 - V_2 \quad , r = \frac{m}{n}$$

$$\Rightarrow \frac{I_{C2}}{I_{C1}} = \frac{1}{r} e^{-\frac{V_d}{V_T}}$$

Emitter-Coupled Pair: Different Emitter Areas (2)

$$I_{C1} + I_{C2} = I_{EE} \Rightarrow I_{C1}\left(1 + \frac{I_{C2}}{I_{C1}}\right) = I_{EE} \Rightarrow I_{C1} = \frac{I_{EE}}{1 + \frac{I_{C2}}{I_{C1}}} = \frac{I_{EE}}{1 + \frac{1}{r} e^{-\frac{V_d}{V_T}}}$$

$$I_{C1} + I_{C2} = I_{EE} \Rightarrow I_{C2}\left(1 + \frac{I_{C1}}{I_{C2}}\right) = I_{EE} \Rightarrow I_{C2} = \frac{I_{EE}}{1 + \frac{I_{C1}}{I_{C2}}} = \frac{I_{EE}}{1 + re^{\frac{V_d}{V_T}}}$$

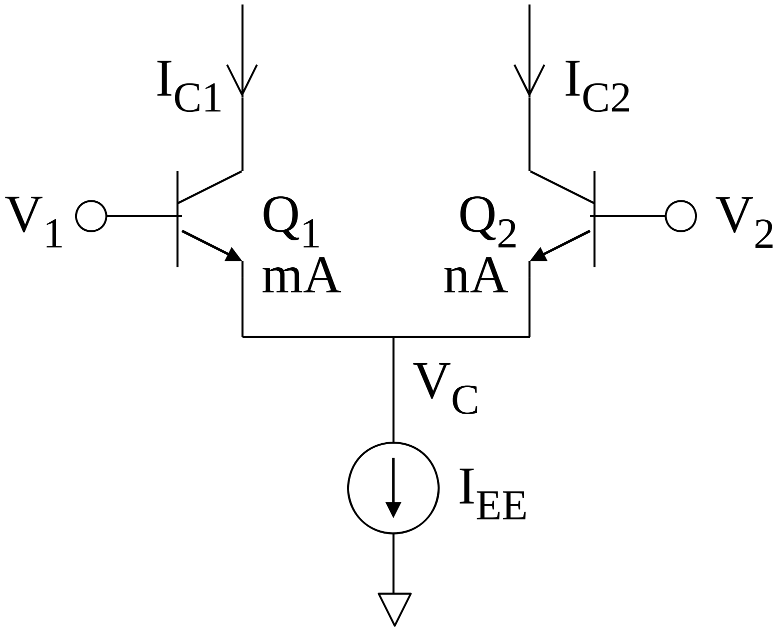
$$\Delta I = I_{C1} - I_{C2} = I_{EE}\left(\frac{1}{1 + \frac{1}{r} e^{-\frac{V_d}{V_T}}} - \frac{1}{1 + re^{\frac{V_d}{V_T}}}\right) = I_{EE}\left(\frac{re^{\frac{V_d}{V_T}}}{re^{\frac{V_d}{V_T}} + 1} - \frac{1}{1 + re^{\frac{V_d}{V_T}}}\right)$$

$$\Delta I = I_{EE}\left(\frac{re^{\frac{V_d}{V_T}} - 1}{re^{\frac{V_d}{V_T}} + 1}\right)$$

(ignoring base current as before)

Emitter-Coupled Pair: Different Emitter Areas (3)

$$\Delta I = I_{EE}\left(\frac{re^{\frac{V_d}{V_T}}-1}{re^{\frac{V_d}{V_T}}+1}\right)$$



The current is the two transistors is the same when ΔI=0 i.e.

$$\Delta I = 0 \Rightarrow re^{\frac{V_d}{V_T}} - 1 = 0 \Rightarrow e^{\frac{V_d}{V_T}} = \frac{1}{r} \Rightarrow V_d = -V_T \ln(r)$$

So the transfer characteristic can be moved left or right by choosing r. If r > 1 (i.e. m>n) V_d is negative, if r<1 (i.e. n>m) V_d is positive and if r=1 (m=n) V_d is zero.

Emitter-Coupled Pair: Different Emitter Areas (4)

$$\Delta I = I_{EE}\left(\frac{re^{\frac{V_d}{V_T}}-1}{re^{\frac{V_d}{V_T}}+1}\right) \qquad \tanh\left(\frac{x}{2}\right)=\frac{e^x-1}{e^x+1}$$

define $\quad s=\ln(r)=\ln\left(\frac{m}{n}\right)\Rightarrow r=e^s$

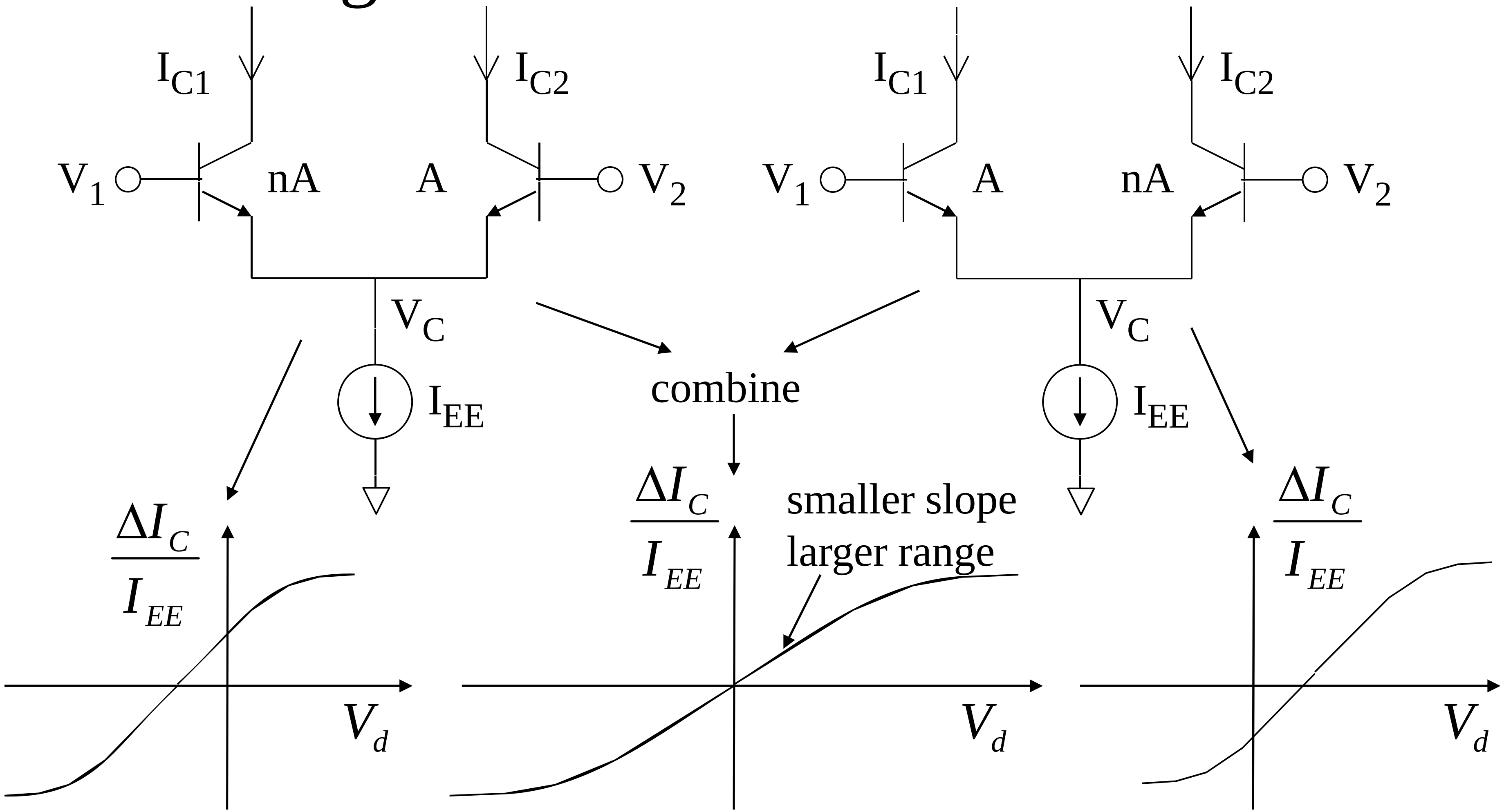
$$\Delta I = I_{EE}\left(\frac{re^{\frac{V_d}{V_T}}-1}{re^{\frac{V_d}{V_T}}+1}\right)=I_{EE}\left(\frac{e^s e^{\frac{V_d}{V_T}}-1}{e^s e^{\frac{V_d}{V_T}}+1}\right)=I_{EE}\left(\frac{e^{\left(\frac{V_d}{V_T}+s\right)}-1}{e^{\left(\frac{V_d}{V_T}+s\right)}+1}\right)$$

$$\Rightarrow \Delta I = I_{EE}\tanh\left(\frac{1}{2}\left(\frac{V_d}{V_T}+s\right)\right)$$

So the characteristic is a tanh function offset on the x-axis by an amount s

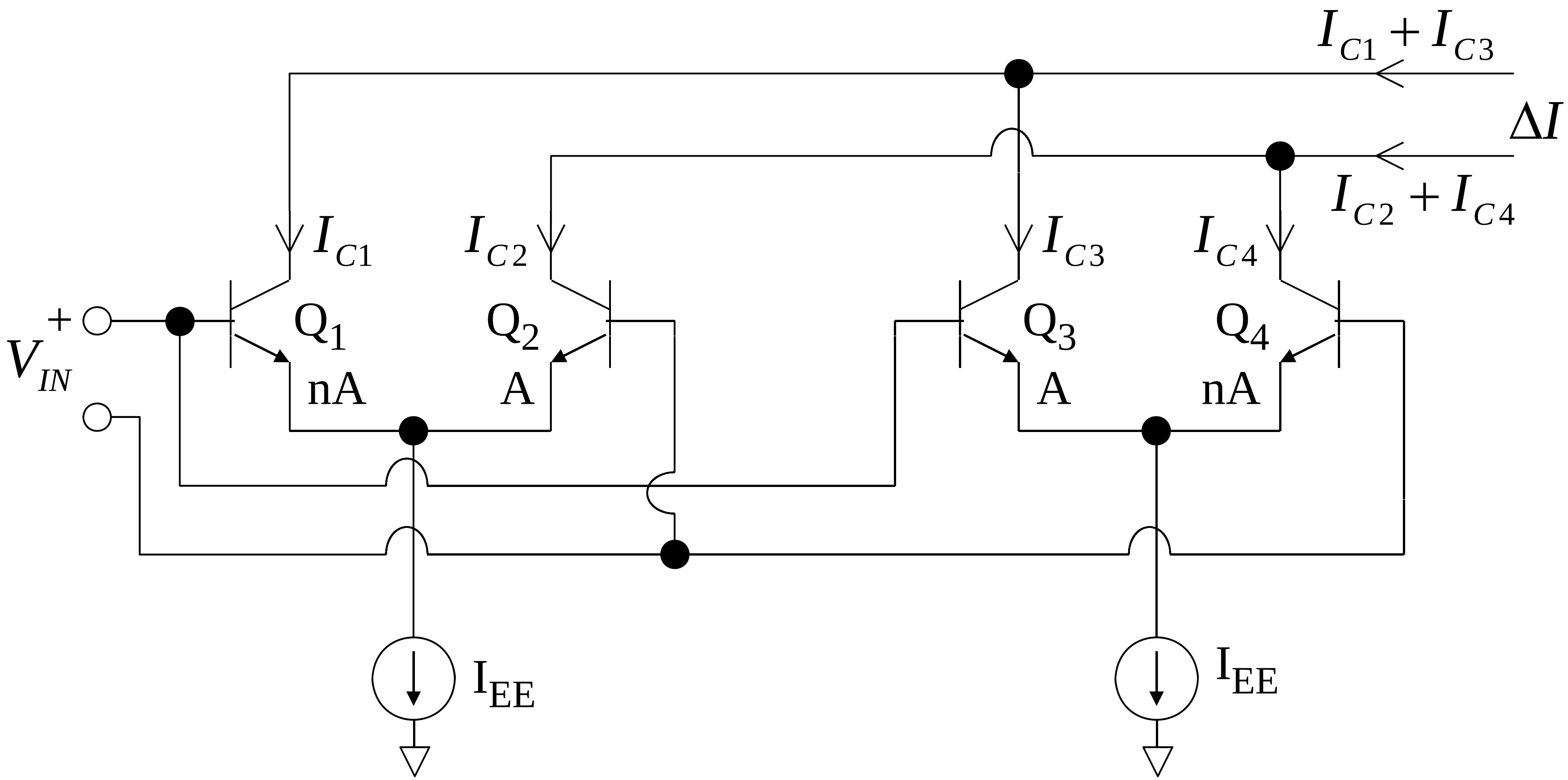
$$g_m=\frac{d\Delta I}{dV_d}=\frac{I_{EE}}{2V_T}\operatorname{sech}^2\left(\frac{1}{2}\left(\frac{V_d}{V_T}+s\right)\right)$$

Background to Schmoock's Method



Common emitter pairs with different emitter areas shift the balance point ($I_{C1}=I_{C2}$) left or right along the V_d axis. Combining the currents of multiple such pairs gives an overall smaller transconductance but it remains flatter over a wider range of V_d.

A Version of Schmook's Circuit

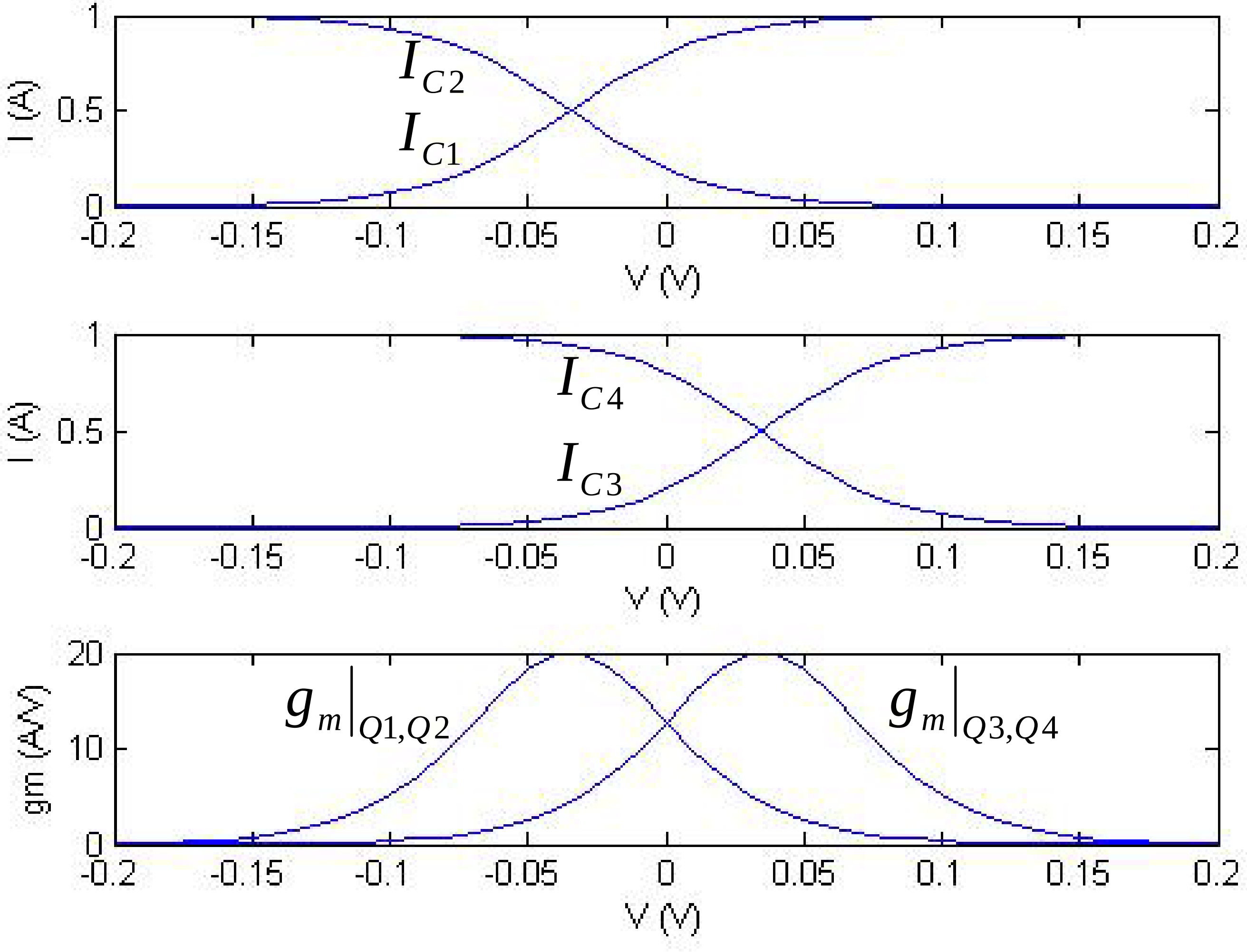


$$\Delta I = I_{C1} + I_{C3} - \left(I_{C2} + I_{C4}\right) = \left(I_{C1} - I_{C2}\right) + \left(I_{C3} - I_{C4}\right)$$

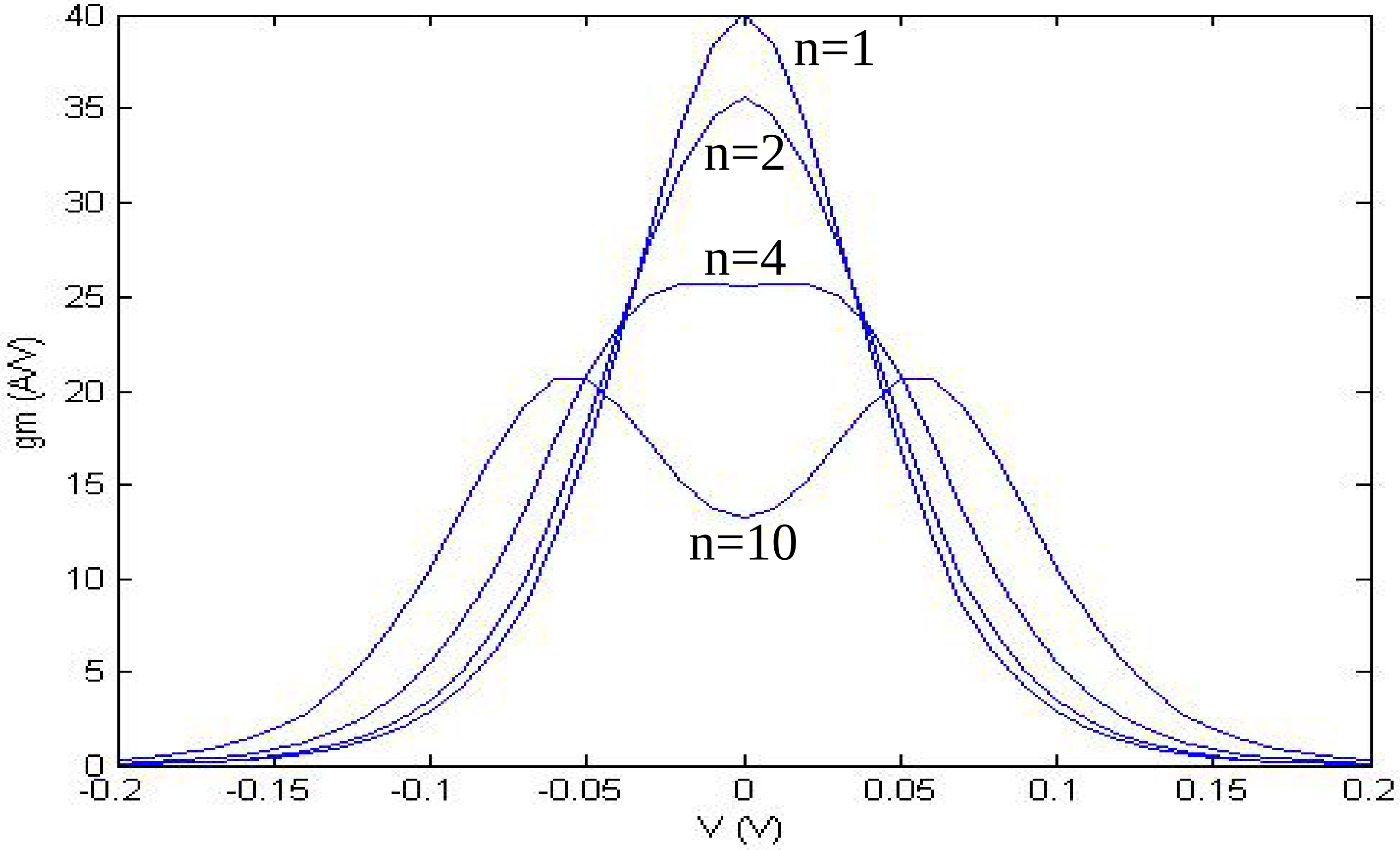
$$g_m = \frac{d\Delta I}{dV_{IN}} = \frac{d\left(I_{C1} - I_{C2}\right)}{dV_{IN}} + \frac{d\left(I_{C3} - I_{C4}\right)}{dV_{IN}} = g_m\Big|_{Q1,Q2} + g_m\Big|_{Q3,Q4}$$

Schmook Circuit: Currents and g_m

I_{EE}=1A, n=4



Total g_m from Schmook Circuit

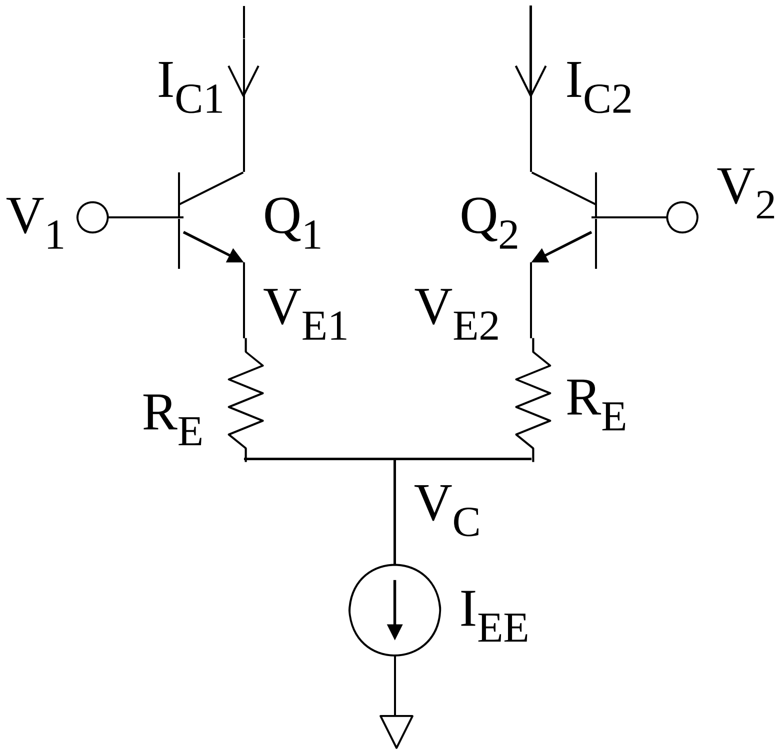


I_{EE}=1A

For n=4, the transconductance is flat over a “wide” range of V_{IN} (+/- 30mV)

CE Pair with Emitter Degeneration Resistors

same transistor areas
same emitter resistors



$$I_{C1} = I_S e^{\frac{V_1 - V_{E1}}{V_T}}, \; I_{C2} = I_S e^{\frac{V_2 - V_{E2}}{V_T}}$$

$$\Rightarrow \frac{I_{C1}}{I_{C2}} = e^{\left[(V_1 - V_2) - (V_{E1} - V_{E2})\right]/V_T}$$

$$V_{E1} = V_C + R_E I_{C1}, \; V_{E2} = V_C + R_E I_{C2}$$

$$\Rightarrow V_{E1} - V_{E2} = R_E \Delta I_C$$

$$\Rightarrow \frac{I_{C1}}{I_{C2}} = e^{(V_d - R_E \Delta I_C)/V_T}$$

This leads eventually to:

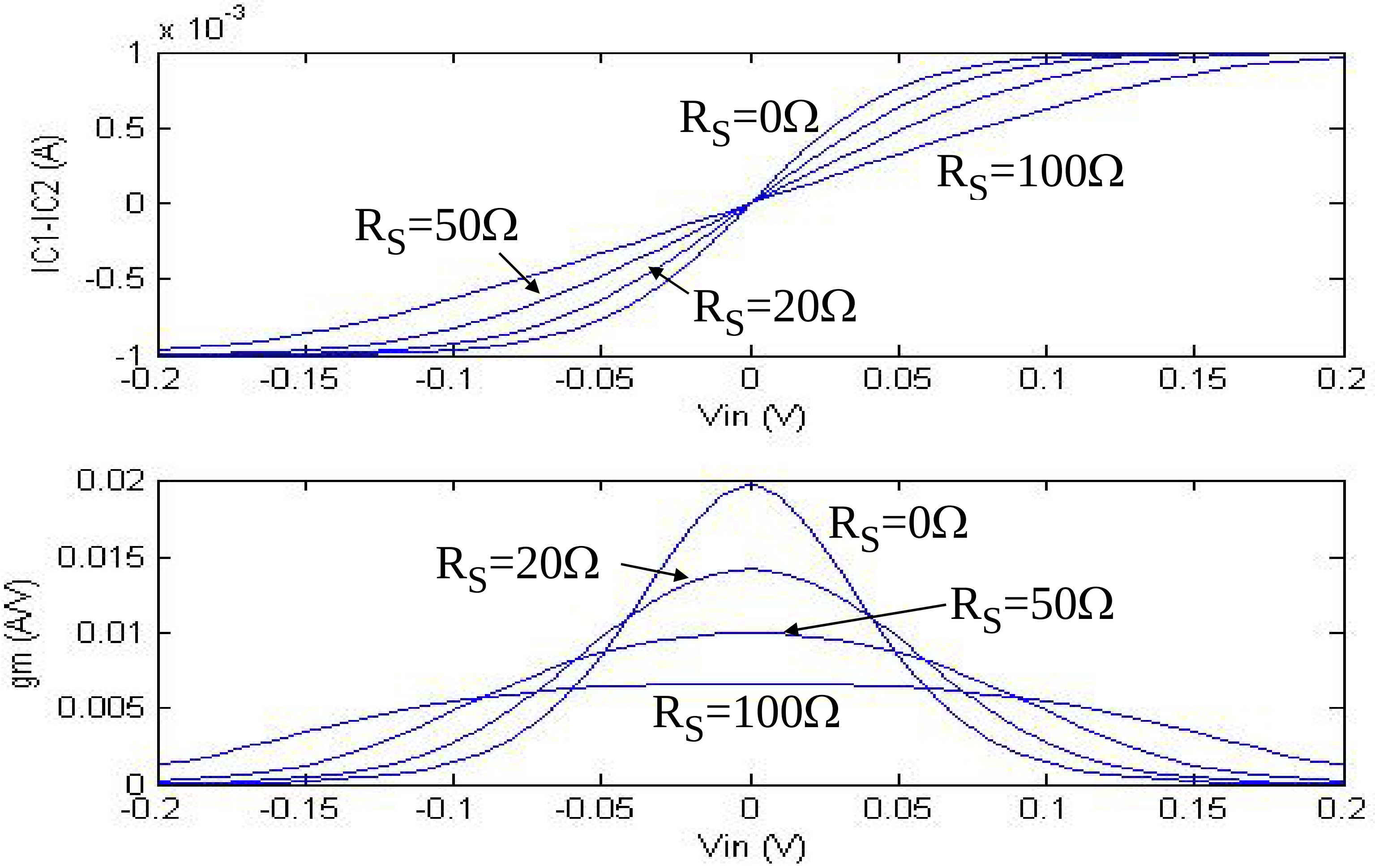
$$\Delta I_C = I_{EE} \tanh\left(\frac{V_d - R_E \Delta I_C}{2V_T}\right)$$

This is a non-linear equation in ΔI_C requiring a non-linear solution or circuit simulator. The general trend is to flatten out the transfer characteristic.

Effect of Degeneration Resistors

I_{EE}=1mA

Degeneration resistors give a wider and flatter dynamic range at the expense of overall transconductance (and gain).



If you’re fascinated by common emitter pairs, see…

“An Input Stage Transconductance Reduction Technique for High-Slew Rate Operational Amplifiers”,
James. C. Schmoock,
IEEE Journal of Solid-State Circuits, Vol. SC-10, No. 6, Dec. 1975.

“The Multi-tanh Principle: A Tutorial Overview”,
Barrie Gilbert,
IEEE Journal of Solid-State Circuits, Vol. 33, No. 1, Jan. 1998.

Both are available from www.ieeexplore.ieee.org