Q.1. (a) $$\frac{1+a}{1+a-a^2}$$

RECALL: $$1 + a + a^2 = 0$$

$$\Rightarrow \quad 1 + a = -a^2$$

THUS

$$\frac{1+a}{1+a-a^2} = \frac{-a^2}{-a^2-a^2} = \frac{a^2}{2a^2}$$

$$= \tfrac{1}{2}\angle 0^\circ$$

(b) $$\frac{a^2+a+j}{ja-a^2} = \frac{-1+j}{a(j-a)}$$

$$= \frac{\sqrt{2}\angle 135^\circ}{1\angle 120^\circ\left[j-\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]}$$

$$= \frac{\sqrt{2}\angle 135^\circ}{1\angle 120^\circ\left[\frac{1}{2}+j\left(1-\frac{\sqrt{3}}{2}\right)\right]}$$

$$= \frac{\sqrt{2}\angle 15^\circ}{0.5177\angle 15^\circ}$$

$$= \frac{\sqrt{2}}{0.5177}\angle 0^\circ$$

$$= 2.732\angle 0^\circ$$

(c) $(1-a)(1+a^2) = 1 + a^2 - a - a^3$

BUT $a^3 = 1$ so

$$(1-a)(1+a^2) = a^2 - a$$
$$= \left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) - \left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right)$$
$$= -j\sqrt{3}$$
$$= \sqrt{3}\,\underline{/-90^\circ}$$

(d) $(a + a^2)(1 + a^2) = (-1)(-a)$

$$= a$$
$$= 1\,\underline{/120^\circ}$$

(e) $a^{10} = \dfrac{a^{10}}{1}$

$$= \frac{a^{10}}{(a^3)^3}$$
$$= \frac{a^{10}}{a^9}$$
$$= a.$$

(f) $(ja)^{10} = (j)^{10} a^{10}$

$$= (j^2)^5 a (a^9)$$
$$= (-1)^5 a$$
$$= -a$$

(g) $(1-a)^3 = \left[1 - \left(-\frac{1}{2} + j\,\sqrt{3}/2\right)\right]^3$

$$= \left(\frac{3}{2} - j\,\frac{\sqrt{3}}{2}\right)^3$$

$$= \left[\sqrt{3}\left(\frac{\sqrt{3}}{2} - j\,\frac{1}{2}\right)\right]^3$$

$$= \left[\sqrt{3}\;1\,\angle -30^\circ\right]^3$$

$$= 3\sqrt{3}\,\underline{\angle -90^\circ}$$

$$= -j\,5.196$$

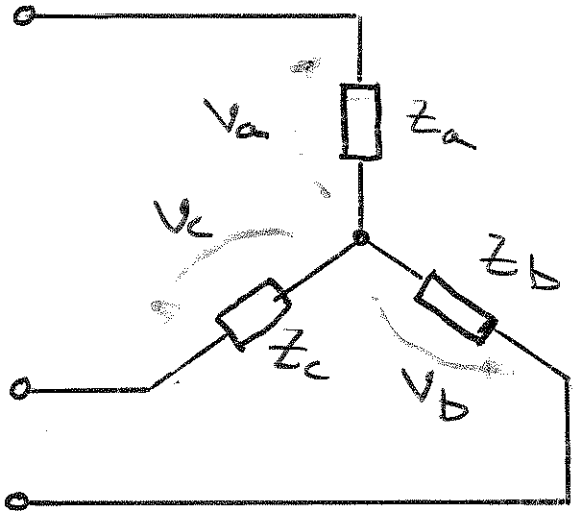
(h) $e^{\bar{a}} = e^{(-1/2 + j\,\sqrt{3}/2)}$

$$= e^{-1/2}\,e^{j\,\sqrt{3}/2}$$

$$= e^{-1/2}\,\underline{\angle \sqrt{3}/2 \text{ rad}}$$

$$= 0.606\,\underline{\angle 49.6^\circ}$$

Q.2.



BY DEFINITION,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

THE LINE VOLTAGES ARE DEFINED AS

$$V_{ab} = V_a - V_b$$

$$V_{bc} = V_b - V_c$$

$$V_{ca} = V_c - V_a .$$

HENCE, THE SYMMETRICAL COMPONENTS OF THE LINE VOLTAGES ARE GIVEN BY

$$\begin{bmatrix} V_{ab0} \\ V_{ab1} \\ V_{ab2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a - V_b \\ V_b - V_c \\ V_c - V_a \end{bmatrix}$$

$$\Rightarrow \quad V_{ab0} = \frac{1}{3}\left[(V_a - V_b) + (V_b - V_c) + (V_c - V_a)\right]$$

$$V_{ab0} = 0 \qquad (*)$$

ALSO, FOR THE POSITIVE SEQUENCE

$$V_{ab1} = 1/3\left[(V_a - V_b) + a(V_b - V_c) + a^2(V_c - V_a)\right]$$

$$= 1/3\left[V_a + aV_b + a^2V_c - (a^2V_a + V_b + aV_c)\right]$$

$$= 1/3\left[(V_a + aV_b + a^2V_c) - a^2(V_a + aV_b + a^2V_c)\right]$$

$$= 1/3\left[(1-a^2)(V_a + aV_b + a^2V_c)\right]$$

$$= \frac{1}{3}(1-a^2)(3V_{a1})$$

$$= (1-a^2)\,V_{a1}$$

$$= \sqrt{3}\,\angle 30^\circ\, V_{a1}$$

$$V_{ab1} = \sqrt{3}\, e^{j30^\circ}\, V_{a1} \qquad (*)$$

FOR THE NEGATIVE SEQUENCE,

$$V_{ab2} = 1/3\left[(V_a - V_b) + a^2(V_b - V_c) + a(V_c - V_a)\right]$$

$$= 1/3\left[(V_a + a^2V_b + aV_c) - (aV_a + V_b + a^2V_c\right.$$

$$\Rightarrow \quad V_{ab2} = \frac{1}{3}\left[\left(V_a + a^2 V_b + aV_c\right) - a\left(V_a + a^2 V_b + aV_c\right)\right]$$

$$= \frac{1}{3}\left[(1-a)\left(V_a + a^2 V_b + aV_c\right)\right]$$

$$= \frac{1}{3}\left[(1-a)\left(3 V_{a2}\right)\right]$$

$$= (1-a) V_{a2}$$

$$= \sqrt{3}\,\underline{\angle -30^\circ}\; V_{a2}$$

$$= \sqrt{3}\, e^{-j30^\circ} V_{a2}. \qquad (*)$$

Q.3.

$$\bar{V}_S = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 80\angle 30^\circ \\ 40\angle -30^\circ \end{bmatrix}$$

THE CORRESPONDING PHASE VOLTAGES ARE

$$\bar{V}_P = \bar{A}\, V_S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

HENCE,

$$V_a = V_0 + V_1 + V_2$$

$$= 10\angle 0^\circ + 80\angle 30^\circ + 40\angle -30^\circ$$

$$= 115.6\angle 9.96^\circ \text{ V}$$

$$V_b = V_0 + a^2 V_1 + a V_2$$

$$= 10\angle 0 + (1\angle -120^\circ)\, 80\angle 30^\circ + (1\angle 120)\, 40\angle -30^\circ$$

$$= 10\angle 0^\circ + 80\angle -90^\circ + 40\angle 90^\circ$$

$$= 41.2\angle -75.96^\circ \text{ V}$$

$$V_c = V_0 + a V_1 + a^2 V_2$$

$$\Rightarrow V_c = 10\angle 0^\circ + (1\angle 120^\circ)\, 80\angle 30^\circ + (1\angle -120^\circ)\, 40\angle -30^\circ$$

$$= 10\angle 0^\circ + 80\angle 150^\circ + 40\angle -150^\circ$$

$$= 96.0\angle 167.98^\circ \text{ V.}$$

HENCE, THE LINE VOLTAGES ARE

$$V_{ab} = V_a - V_b = 120\angle 30^\circ \text{ V}$$

$$V_{bc} = V_b - V_c = 120\angle -30^\circ \text{ V}$$

$$V_{ca} = V_c - V_a = 207.8\angle 180^\circ \text{ V.}$$

NEXT CALCULATE THE SYMMETRICAL COMPONENTS OF THE LINE VOLTAGES

$$\begin{bmatrix} V_{LL0} \\ V_{LL1} \\ V_{LL2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 120\angle 30^\circ \\ 120\angle -30^\circ \\ 207.8\angle 180 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 138.6\angle 60^\circ \\ 69.3\angle -60^\circ \end{bmatrix} \text{ V}$$

Q.3 (contd.)

NOW SINCE

$$V_{ab0} = 0$$

$$V_{ab1} = \sqrt{3}\, e^{j30^\circ}\, V_{a1}$$

$$V_{ab2} = \sqrt{3}\, e^{-j30}\, V_{a2}$$

WE GET THAT THE SEQUENCE COMPONENTS OF THE CORRESPONDING PHASE VOLTAGES ARE

$$V_{a0} = 0 \quad V$$

$$V_{a1} = \frac{V_{ab1}}{\sqrt{3}\, e^{j30}}$$

$$= \frac{138.6 \angle 60^\circ}{\sqrt{3} \angle 30^\circ}$$

$$= 80 \angle 30^\circ \quad V$$

$$V_{a2} = \frac{V_{ab2}}{\sqrt{3}\, e^{-j30^\circ}}$$

$$= \frac{69.3 \angle -60^\circ}{\sqrt{3}\, e^{-j30}}$$

$$= 40 \angle -30^\circ \quad V$$

RE-CONSTRUCTING THE PHASE VOLTAGES WE GET

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\Rightarrow \quad V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= 0 + 80\angle 30^\circ + 40\angle -30^\circ$$

$$= 105.8\angle 10.9^\circ \text{ V}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$= 0 + (1\angle -120^\circ)\,80\angle 30^\circ + (1\angle 120^\circ)\,40\angle -30^\circ$$

$$= 80\angle -90^\circ + 40\angle 90^\circ$$

$$= 40\angle -90^\circ \text{ V}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

$$= 0 + (1\angle 120^\circ)(80\angle 30^\circ) + (1\angle -120^\circ)(40\angle -30^\circ)$$

$$= 80\angle 150^\circ + 40\angle -150^\circ$$

$$= 105.8\angle 169.1^\circ \text{ V}$$

NOTE THAT THE RE-CONSTRUCTED PHASE VOLTAGES ARE NOT THE SAME AS THE ORIGINAL PHASE VOLTAGES.

Q.3. (contd)

HOWEVER, EITHER SET WILL RESULT IN THE SAME LINE VOLTAGES.

NOTE THAT THE ZERO SEQUENCE LINE VOLTAGE IS ALWAYS ZERO, EVEN THOUGH THE ZERO SEQUENCE PHASE VOLTAGE MAY EXIST.

THUS, IT IS NOT POSSIBLE TO CONSTRUCT THE COMPLETE SET OF SYMMETRICAL COMPONENTS OF PHASE VOLTAGES EVEN WHEN THE UNBALANCED SET OF LINE VOLTAGES IS KNOWN.

HOWEVER, WE CAN OBTAIN A SET OF PHASE VOLTAGES WITH NO ZERO SEQUENCE COMPONENT TO REPRESENT THE UNBALANCED SYSTEM.

Q. 4

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 280 \angle 0^\circ \\ 290 \angle -130^\circ \\ 260 \angle 110^\circ \end{bmatrix}$$

HENCE,

$$\begin{bmatrix} V_{Lg0} \\ V_{Lg1} \\ V_{Lg2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}$$

$$= \begin{bmatrix} 7.55 \angle 78.1^\circ \\ 275.73 \angle -6.6^\circ \\ 24.87 \angle 79.4^\circ \end{bmatrix} \quad *$$

THE LINE-TO-LINE VOLTAGES ARE GIVEN BY

$$V_{ab} = V_{ag} - V_{bg} = 516.6 \angle 25.5^\circ \text{ V}$$

$$V_{bc} = V_{bg} - V_{cg} = 476.6 \angle -101.8^\circ \text{ V}$$

$$V_{ca} = V_{cg} - V_{ag} = 442.5 \angle 146.5^\circ \text{ V}$$

THE SYMMETRICAL COMPONENTS OF THE LINE VOLTAGES CAN BE CALCULATED DIRECTLY

$$\begin{bmatrix} V_{LL0} \\ V_{LL1} \\ V_{LL2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 \\ 477.6 \angle 23.4^\circ \\ 43.1 \angle 49.4^\circ \end{bmatrix} V}_{} \quad (*)$$

THESE COMPONENTS CAN ALSO BE CALCULATED AS

$$V_{LL0} = 0 \times V_{Lg0} = 0$$

$$V_{LL1} = \sqrt{3}\, e^{j30} V_{Lg1}$$

$$= \sqrt{3}\, e^{j30} (275.73 \angle -6.6^\circ)\, V$$

$$= 477.6 \angle 23.4^\circ \ V$$

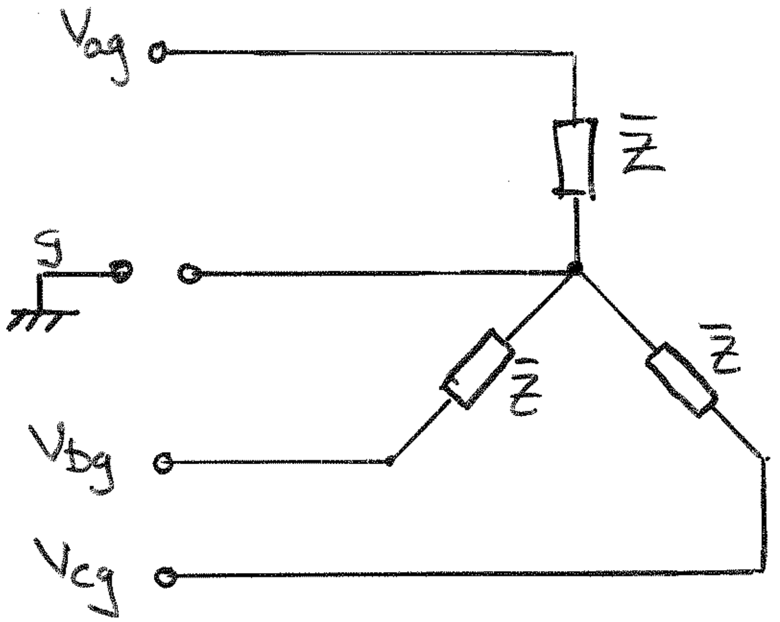
$$V_{LL2} = \sqrt{3}\, e^{-j30} V_{Lg2}$$

$$= \sqrt{3}\, e^{-j30} (24.87 \angle 79.4^\circ)\, V$$

$$= 43.1 \angle 49.4^\circ \ V$$

AS COMPUTED DIRECTLY ABOVE IN EQ. (*)

Q.5.



$S_{rated} = 500\,kVA$
$V_{rated} = 10\,kV$
$\bar{Z} = (200 + j0)\,\Omega$

THE INPUT VOLTAGES ARE UNBALANCED AND DEFINED BY THE LINE VOLTAGES

$$\bar{V}_{LL} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 8000\,\angle 82.8^\circ \\ 12000\,\angle -41.4^\circ \\ 10000\,\angle 180^\circ \end{bmatrix} V$$

SOLVE THE PROBLEM USING THE PER-UNIT SYSTEM. SELECT

$$S_{base} = S_{rated}$$
$$= 500\ kVA$$

$$V_{base} = V_{rated}$$
$$= 10\ kV_{LL}$$

HENCE

$$Z_{base} = \frac{V_{base}^2}{S_{base}}$$

Q.5 (contd)

$$Z_{base} = 200\ \Omega.$$

THUS

$$\bar{Z}_{pu} = \frac{\bar{Z}}{Z_{base}} = \frac{200 + j0}{200} = 1.0\ pu$$

ALSO

$$\bar{V}_{LL\,pu} = \frac{\bar{V}_{LL}}{V_{base}} = \begin{bmatrix} 0.8 \angle 82.8^\circ \\ 1.2 \angle -41.4^\circ \\ 1.0 \angle 180^\circ \end{bmatrix} pu$$

HENCE, TRANSFORMING TO SEQUENCE COMPONENTS, WE GET

$$\bar{V}_{S\,LL\,pu} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0.8 \angle 82.8^\circ \\ 1.2 \angle -41.4^\circ \\ 1.0 \angle 180^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.986 \angle 73.5^\circ \\ 0.235 \angle -139.7^\circ \end{bmatrix} pu$$

USING THE RELATIONSHIP BETWEEN THE LINE AND PHASE SEQUENCE COMPONENTS

$$V_{LL0} = 0, \quad V_{LL1} = \sqrt{3}\, e^{j30^\circ} V_{PH1}$$

$$V_{LL2} = \sqrt{3}\, e^{-j30^\circ} V_{PH2}$$

Q.5 (contd)

HOWEVER, IN THE PER-UNIT SYSTEM

$$V_{LL\,base} = \sqrt{3}\, V_{PH\,base}$$

SO THAT, IN PER UNIT

$$V_{LL0\,pu} = 0$$

$$V_{LL1\,pu} = e^{j30}\, V_{PH1\,pu}$$

$$V_{LL2\,pu} = e^{-j30}\, V_{PH2\,pu}$$

THUS,

$$V_{PH0\,pu} = 0$$

$$V_{PH1\,pu} = \frac{V_{LL1\,pu}}{e^{j30}}$$

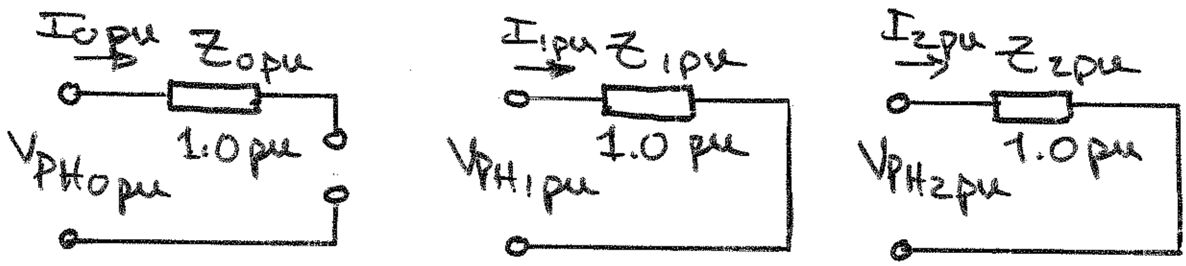
$$= 0.986\,\angle 43.5^\circ \text{ pu}$$

$$V_{PH2\,pu} = \frac{V_{LL2\,pu}}{e^{-j30}}$$

$$= \frac{0.235\,\angle -139.7^\circ}{e^{-j30}}$$

$$= 0.235\,\angle -109.7^\circ \text{ pu}$$

THE SEQUENCE IMPEDANCE NETWORKS OF THE BALANCED THREE-PHASE, THREE-WIRE LOAD ARE AS FOLLOWS



HENCE,

$$I_{0pu} = 0$$

$$I_{1pu} = \frac{V_{PH1pu}}{Z_{1pu}} = \frac{0.986\angle 43.5^\circ}{1.0} = 0.986\angle 43.5^\circ \ pu$$

$$I_{2pu} = \frac{V_{PH2pu}}{Z_{2pu}} = \frac{0.235\angle -109.7^\circ}{1.0} = 0.235\angle -109.7^\circ \ pu$$

Q. 5 (contd.)

THE ACTUAL CURRENTS CAN BE CALCULATED USING THE BASE CURRENT

$$I_{base} = \frac{S_{base}}{\sqrt{3}\, V_{LL\,base}}$$

$$= \frac{500 \times 10^3}{\sqrt{3} \times 10 \times 10^3}$$

$$= 28.87 \text{ A}.$$

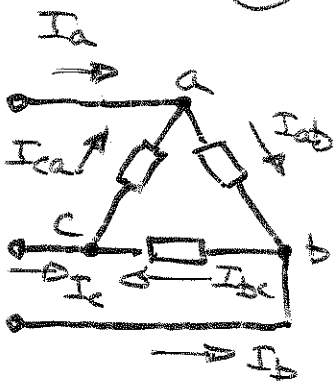
THE ACTUAL LINE CURRENTS CAN BE CALCULATED IN PHASOR FORM AS

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{0pu} \\ I_{1pu} \\ I_{2pu} \end{bmatrix} I_{base}$$

Q.3

PHASE CURRENTS :-

$$\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 20\angle -90^\circ \\ 15\angle 90^\circ \end{bmatrix} A$$



THE SEQUENCE COMPONENTS OF THE PHASE CURRENTS ARE

$$\begin{bmatrix} I_{ph.0} \\ I_{ph.1} \\ I_{ph.2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

$$= \begin{bmatrix} 3.73\angle -26.6^\circ \\ 13.46\angle 3.5^\circ \\ 6.82\angle 172.9^\circ \end{bmatrix} A \ *$$

THE LINE CURRENTS ARE

$$I_a = I_{ab} - I_{ca} = 18.03\angle -56.3^\circ A$$

$$I_b = I_{bc} - I_{ab} = 22.36\angle -116.6^\circ A$$

$$I_c = I_{ca} - I_{bc} = 35.00\angle 90.0^\circ A$$

Q.6 (contd.)

THE SEQUENCE COMPONENTS OF THE LINE CURRENTS ARE

$$\begin{bmatrix} I_{L0} \\ I_{L1} \\ I_{L2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 23.32 \angle -26.5^\circ \\ 11.82 \angle -157.0^\circ \end{bmatrix} \text{A} \; *$$

NOTE THAT

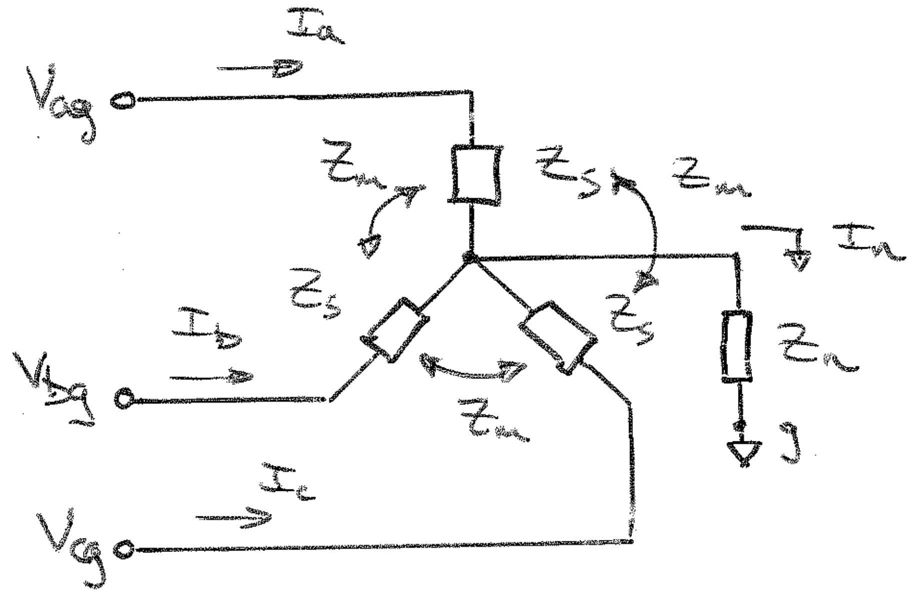
$$I_{L0} = 0 \; I_{Ph0}$$

$$I_{L1} = \sqrt{3}\, e^{-j30^\circ} I_{Ph1}$$

$$I_{L2} = \sqrt{3}\, e^{j30^\circ} I_{Ph2}$$

SYMMETRICAL COMPONENTS II

Q.1.



BY K.V.L. APPLIED TO THE THREE PHASES

$$V_{ag} = I_a Z_s + I_b Z_m + I_c Z_m + I_n Z_n$$

$$V_{bg} = I_a Z_m + I_b Z_s + I_c Z_m + I_n Z_n$$

$$V_{cg} = I_a Z_m + I_b Z_m + I_c Z_s + I_n Z_n$$

BUT SINCE $I_n = I_a + I_b + I_c$ BY K.C.L APPLIED TO THE STAR POINT, WE GET

$$V_{ag} = I_a(Z_s + Z_n) + I_b(Z_m + Z_n) + I_c(Z_m + Z_n)$$

$$V_{bg} = I_a(Z_m + Z_n) + I_b(Z_s + Z_n) + I_c(Z_m + Z_n)$$

$$V_{cg} = I_a(Z_m + Z_n) + I_b(Z_m + Z_n) + I_c(Z_s + Z_n)$$

IN MATRIX FORM

$$\overline{V_p} = \overline{Z_p}\,\overline{I_p}$$

Q. 1. (contd.)

WHERE

$$\bar{V}_p = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \qquad \bar{I}_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

AND

$$\bar{Z}_p = \begin{bmatrix} (Z_s + Z_n) & (Z_m + Z_n) & (Z_m + Z_n) \\ (Z_m + Z_n) & (Z_s + Z_n) & (Z_m + Z_n) \\ (Z_m + Z_n) & (Z_m + Z_n) & (Z_s + Z_n) \end{bmatrix}$$

TRANSFORMING TO THE SEQUENCE DOMAIN

$$\bar{A}\bar{V}_s = \bar{Z}_p \bar{A} \bar{I}_s$$

$$\Rightarrow \bar{V}_s = (\bar{A}^{-1} \bar{Z}_p \bar{A}) \bar{I}_s$$

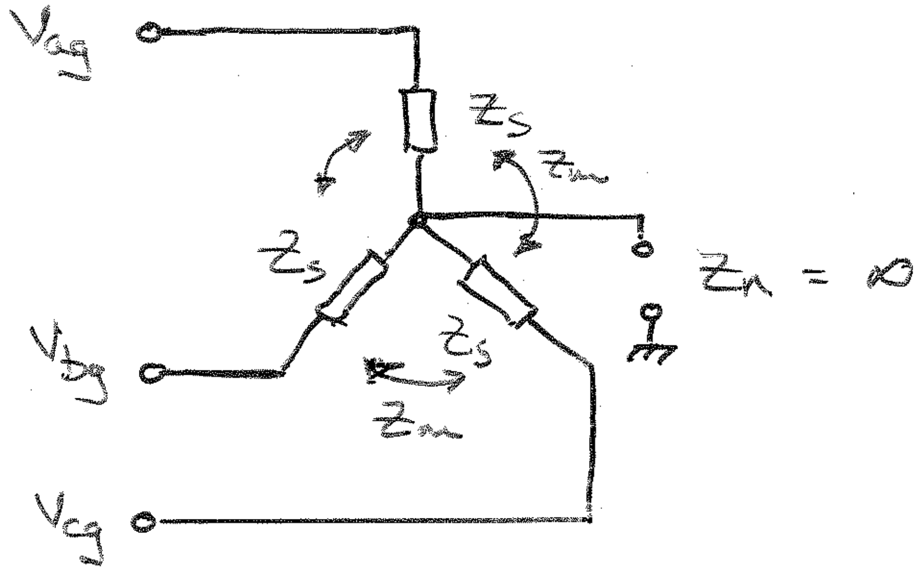
$$= \bar{Z}_s I_s$$

HENCE, SHOW THAT

$$Z_s = \begin{bmatrix} (\bar{Z}_s + 2\bar{Z}_m + 3Z_n) & 0 & 0 \\ 0 & (Z_s - Z_m) & 0 \\ 0 & 0 & (Z_s - Z_m) \end{bmatrix}$$

NOTE THAT Z_n APPEARS ONLY IN THE ZERO SEQUENCE NETWORK.

Q.2.



$$\bar{Z}_s = j12\,\Omega$$

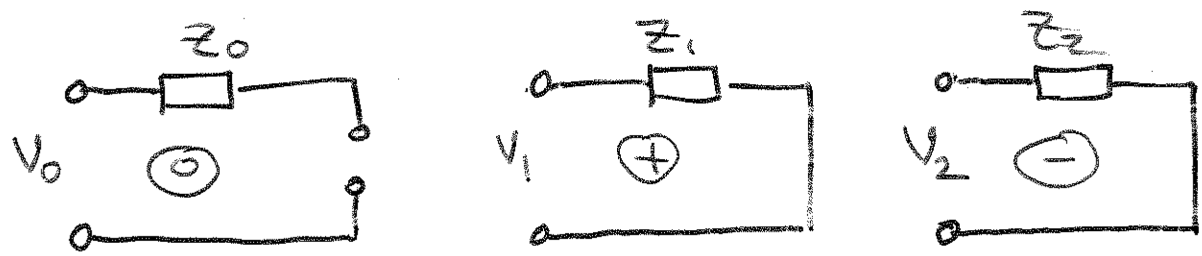
$$\bar{Z}_m = j4\,\Omega.$$

FOR BALANCED INPUT VOLTAGES,

$$V_{ag} = \frac{V_{LL}}{\sqrt{3}} = 230\angle 0^\circ$$

TAKING THE PHASE VOLTAGE AS REFERENCE.

FROM PROBLEM 1, THE SEQUENCE IMPEDANCE NETWORKS ARE AS FOLLOWS :-



$$Z_0 = Z_s + 2Z_m + 3Z_n$$

$$Z_1 = Z_s - Z_m$$

$$Z_2 = Z_s - Z_m$$

Q.2 (contd)

IN THIS CASE $Z_n \rightarrow \infty$ GIVING AN OPEN CIRCUIT IN THE ZERO SEQUENCE NETWORK.

SINCE THE INPUT SOURCE VOLTAGE IS BALANCED

$$V_1 = V_{ag} = 230\angle 0^\circ \text{ V}$$

$$V_2 = 0 \text{ V}$$

$$V_0 = 0 \text{ V}$$

NOW, $\quad Z_s - Z_m = j12 - j4 = j8\ \Omega$

HENCE, THE SEQUENCE CURRENTS ARE

$$I_0 = 0$$

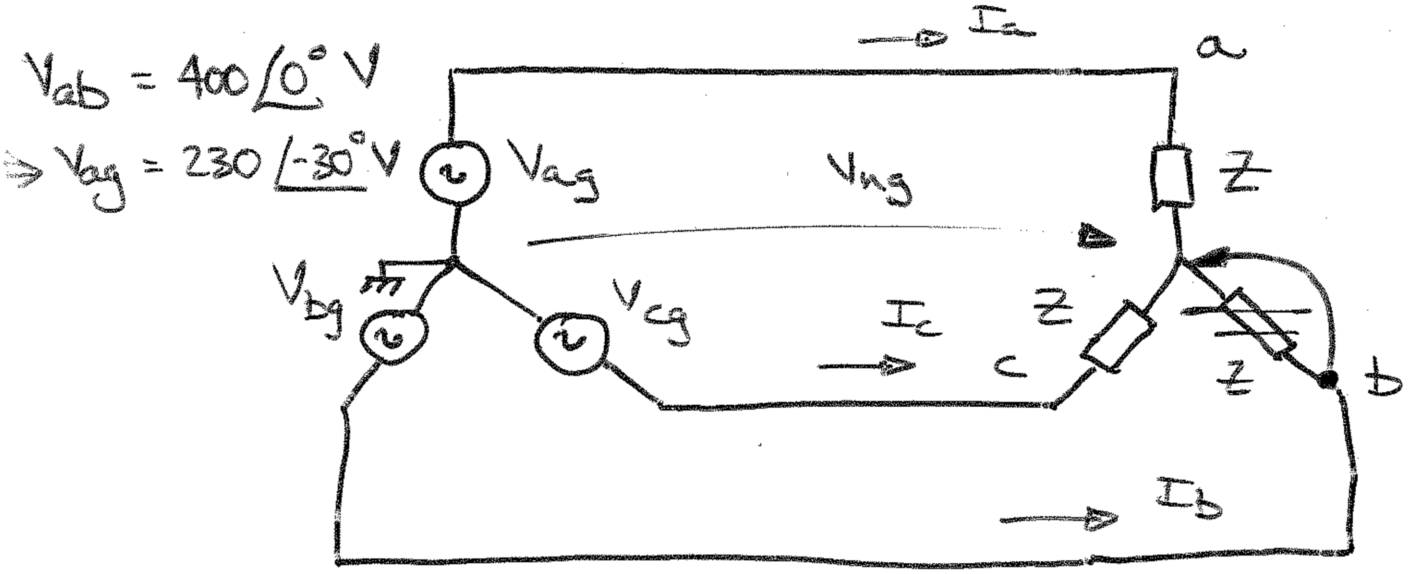
$$I_1 = \frac{230}{j8} = -j\,28.75 \text{ A}$$

$$I_2 = 0$$

THUS, THE ACTUAL CURRENTS FORM A BALANCED THREE-PHASE SET

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 28.75\angle -90^\circ \\ 28.75\angle -210 \\ 28.75\angle 30^\circ \end{bmatrix} \text{ A } \cancel{\text{A}}$$

Q.3 THE THREE-PHASE SYSTEM IS AS SHOWN BELOW. (5)



USING KVL WE GET IN MATRIX FORM

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} V_{ng} \\ V_{ng} \\ V_{ng} \end{bmatrix}$$

TRANSFORMING TO THE SEQUENCE DOMAIN

$$\begin{bmatrix} 0 \\ 230\angle -30^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_{ng} \\ 0 \\ 0 \end{bmatrix}$$

SINCE $I_n = 3I_0 = 0$ THEN $I_0 = 0$ AND WE CAN SOLVE FOR I_1 AND I_2 FROM THE 2 X 2 MATRIX EQUATION

$$\begin{bmatrix} 230\angle{-30^\circ} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

SO THAT

$$230\angle{-30^\circ} = Z_{11} I_1 + Z_{12} I_2$$
$$0 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow \quad I_2 = -\frac{Z_{21}}{Z_{22}} I_1$$

AND SO

$$230\angle{-30^\circ} = \left[Z_{11} - Z_{12}\left(\frac{Z_{21}}{Z_{22}}\right)\right] I_1$$

$$\Rightarrow \quad I_1 = \frac{230\angle{-30^\circ}}{Z_{11} - \left(\frac{Z_{21}}{Z_{22}}\right) Z_{12}}$$

THE SEQUENCE IMPEDANCE MATRIX IS GIVEN AS USUAL BY

$$\bar{Z}_S = A^{-1} Z_p \bar{A}$$

WHERE

$$Z_p = \begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix}$$

Q.3

HENCE

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z & Z & Z \\ 0 & 0 & 0 \\ Z & aZ & a^2Z \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2Z & Z+aZ & Z+a^2Z \\ Z+a^2Z & 2Z & Z+aZ \\ Z+aZ & Z+a^2Z & 2Z \end{bmatrix}$$

THUS, USING THE USUAL NOTATION

$$Z_S = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix}$$

WE GET

$$Z_{11} = Z_{22} = 2Z = \frac{2}{3}(18 + j6)$$
$$= (12 + j4)\ \Omega$$

$$Z_{12} = \frac{Z}{3}(1+a) = (1.268 + j6.196)\ \Omega$$
$$Z_{21} = \frac{Z}{3}(1+a^2) = (4.732 - j4.196)\ \Omega$$

Q.3.

HENCE, FROM ABOVE

$$I_1 = \frac{230\angle -30^\circ}{Z_{11} - Z_{12}\left(\frac{Z_{21}}{Z_{22}}\right)} = 24.3\angle -48.4^\circ \text{ A}$$

$$I_2 = -\left(\frac{Z_{21}}{Z_{22}}\right) I_1 = 12.2\angle 71.6^\circ \text{ A}$$

AND SO

$$I_a = I_0 + I_1 + I_2$$
$$= 21.1\angle -18.4^\circ \text{ A}.$$

ALTERNATIVE METHOD!

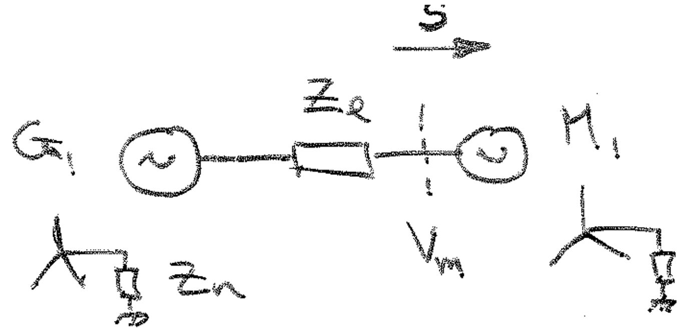
LINE VOLTAGE V_{ab} IS CONNECTED DIRECTLY ACROSS THE IMPEDANCE IN PHASE a SO THAT

$$I_a = \frac{V_{ab}}{Z}$$

$$= \frac{400\angle 0}{18 + j6}$$

$$= 21.1\angle -18.4^\circ \text{ A}. !$$

Q.4.



$Z_{0g} = j5\,\Omega$

$Z_{1g} = j15\,\Omega$

$Z_{2g} = j10\,\Omega$

$Z_{ng} = j5\,\Omega$

$Z_{0m} = j5\,\Omega$

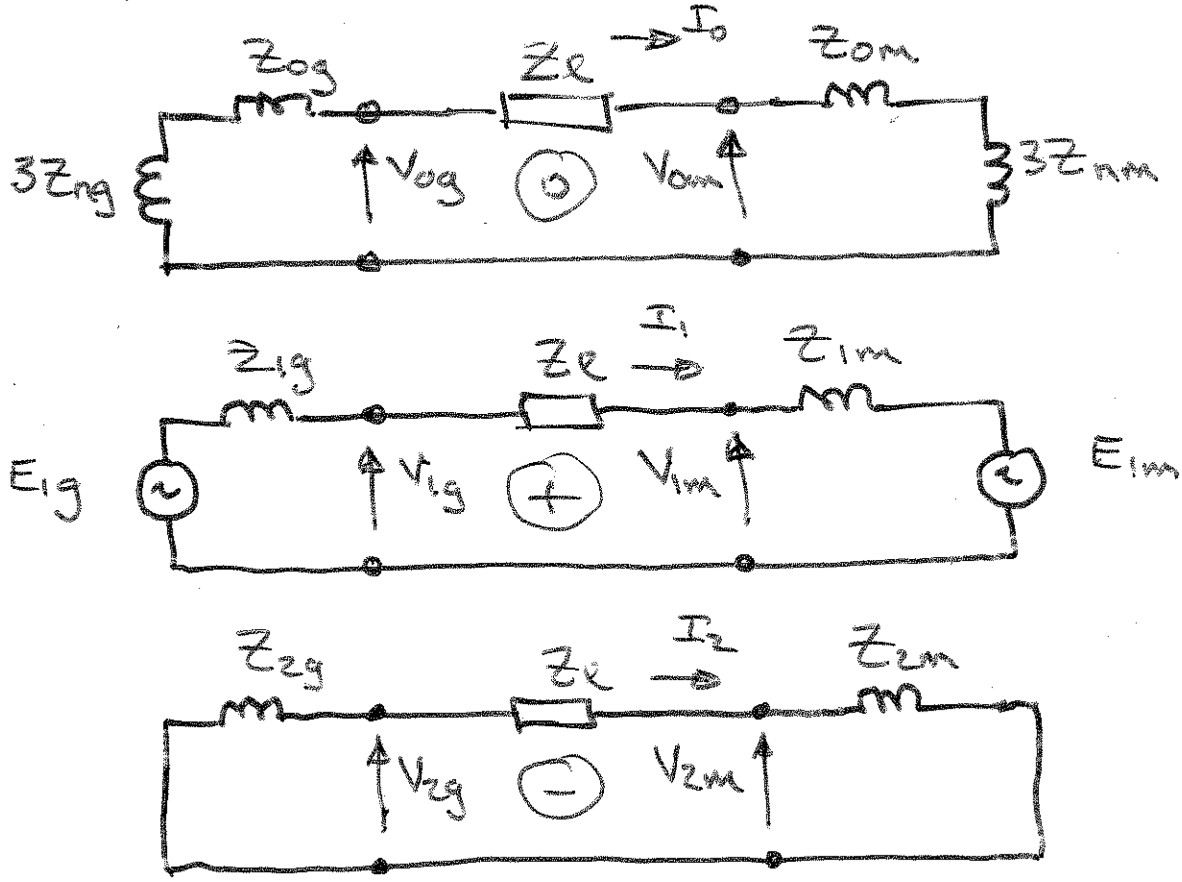
$Z_{1m} = j15\,\Omega$

$Z_{2m} = j10\,\Omega$

$Z_{nm} = j5\,\Omega$.

$$Z_e = 0.5\angle 80^\circ\ \Omega$$

THE SEQUENCE NETWORKS ARE AS FOLLOWS



NOW

$$P = 10\text{ kW}$$

$$pf = 0.8 \text{ leading}$$

$$\Rightarrow S = \frac{P}{pf} = 12.5\text{ kVA}$$

$$\Rightarrow Q = \sqrt{S^2 - P^2} = 7.5\text{ kVA}$$

Q.4. (contd)
$$V_{1m} = \frac{400}{\sqrt{3}} \angle 0^\circ$$

$$= 230 \angle 0^\circ$$

$$S = P - jQ = 3VI^*$$

$$\Rightarrow I = \left(\frac{P - jQ}{3V}\right)^*$$

$$= 18.1 \angle +36.87^\circ \text{ A}$$

SINCE THE VOLTAGES ARE BALANCED AND THE MACHINES AND THE TRANSMISSION LINE ARE SYMMETRICAL LOADS,

$$I_0 = 0$$

$$I_1 = 18.1 \angle +36.87^\circ \text{ A}$$

$$I_2 = 0$$

SO THAT

$$V_{1g} = V_{1m} + Z_\ell I_1$$

$$\Rightarrow V_{1g} = 230\angle 0^\circ + (0.5\angle 80^\circ)(18.1\angle +36.87^\circ)$$

$$= 224.9 \angle 1.89^\circ \text{ V.}$$

$$\Rightarrow V_{1g\,LL} = 389.6\text{ V.}$$