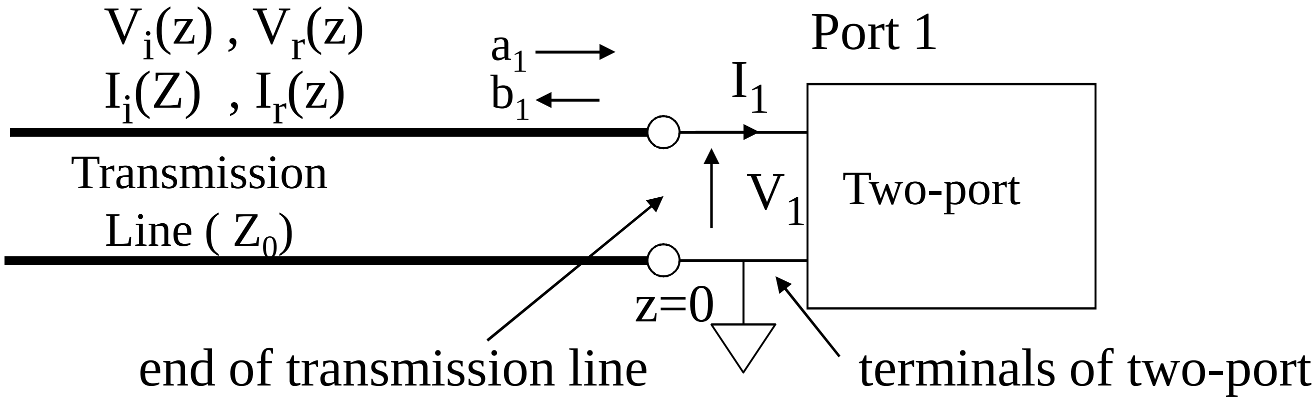
EE4011 RFIC Design

Relationship of s-parameters to y-parameters

Example MESFET s-parameters on a Smith Chart

Scattering Transfer (t) Parameters

Wave and "total" quantities at edge of a two-port - 1



Relating the transmission line expressions at the edge of port 1 to the total voltage and current at port 1 gives:

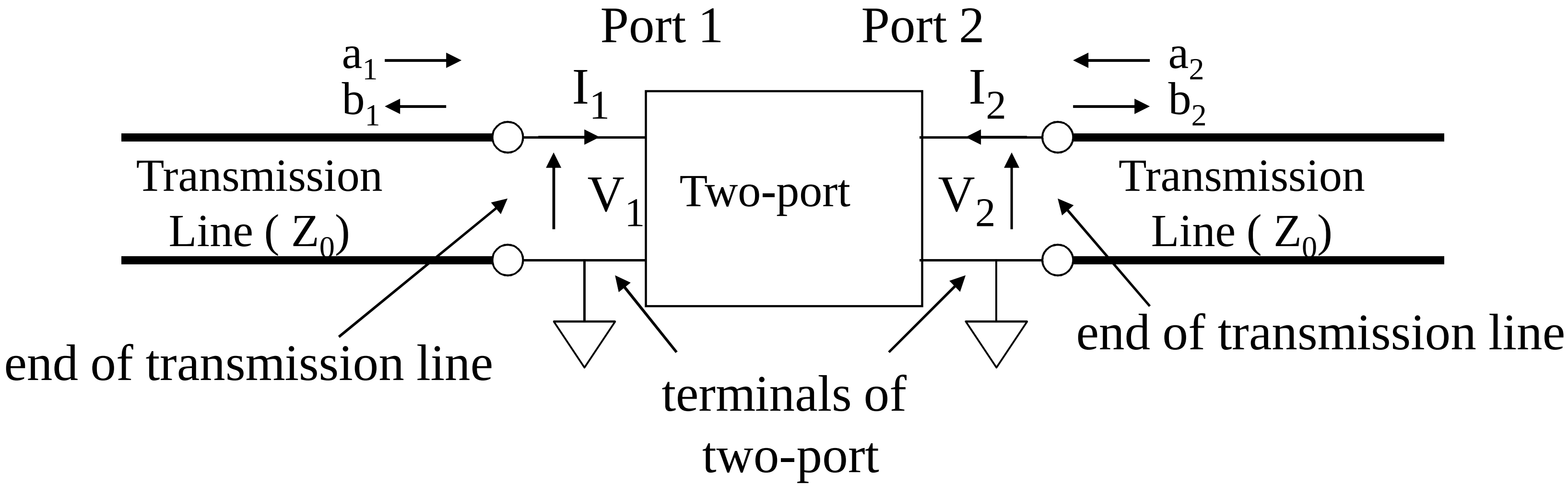
$$V(z) = V_i(z) + V_r(z) \Rightarrow V(0) = V_i(0) + V_r(0) = V_1$$

$$I(z) = \frac{V_i(z) - V_r(z)}{Z_0} \Rightarrow I(0) = \frac{V_i(0) - V_r(0)}{Z_0} = I_1 \Rightarrow V_i(0) - V_r(0) = Z_0 I_1$$

Adding and subtracting the final equations on the two lines above gives:

$$V_i(0) = \frac{V_1 + I_1 Z_0}{2} \text{ and } V_r(0) = \frac{V_1 - I_1 Z_0}{2}$$

Wave and “total” quantities at edge of a two-port - 2



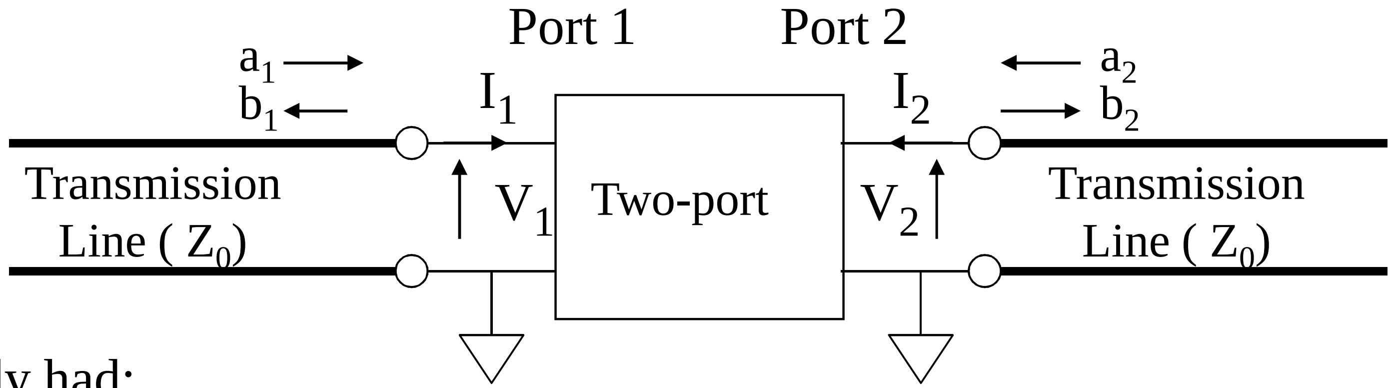
Generalizing the last result to both sides of the two-port:

$$V_{i1} = \frac{V_1 + I_1 Z_0}{2} \quad V_{r1} = \frac{V_1 - I_1 Z_0}{2} \quad V_{i2} = \frac{V_2 + I_2 Z_0}{2} \quad V_{r2} = \frac{V_2 - I_2 Z_0}{2}$$

where V_{i1},V_{r1},V_{i2} and V_{r2} are the incident and reflected voltages at the end of the transmission lines connecting to ports 1 and 2.

These equations allow the wave quantities to be determined if the total voltages and currents are known at the terminals of the two-port. These relationships are useful in circuit simulators such as SPICE which solve for total voltages and currents.

Matrix Relationship: Wave and Total Variables



We already had:

$$V_{i1} = \frac{V_1 + I_1 Z_0}{2} \quad V_{i2} = \frac{V_2 + I_2 Z_0}{2} \quad \mathrm{V}_{\mathrm{r}1} = \frac{V_1 - I_1 Z_0}{2} \quad \mathrm{V}_{\mathrm{r}2} = \frac{V_2 - I_2 Z_0}{2}$$

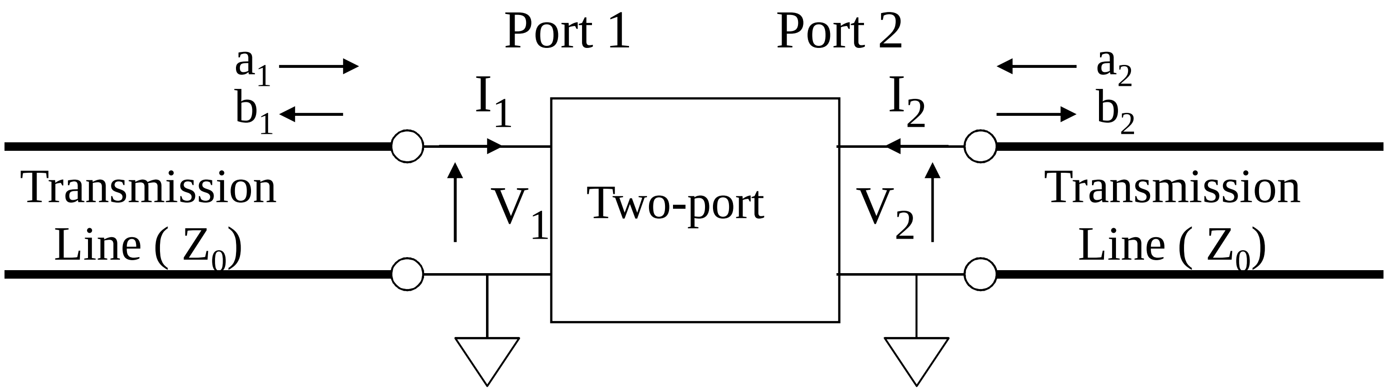
This can be written in matrix form as:

$$\mathbf{V}_{\mathbf{i}} = \frac{1}{2}\mathbf{V} + \frac{Z_0}{2}\mathbf{I} \quad \text{and} \quad \mathbf{V}_r = \frac{1}{2}\mathbf{V} - \frac{Z_0}{2}\mathbf{I}$$

where

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \mathbf{V}_i = \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix} \quad \mathbf{V}_r = \begin{bmatrix} V_{r1} \\ V_{r2} \end{bmatrix}$$

Matrix Relationship: Incident and Reflected Voltages



From the definition of s-parameters and the incident and reflected waves:

$$b_1 = s_{11}a_1 + s_{12}a_2 \Rightarrow \frac{V_{r1}}{\sqrt{Z_0}} = s_{11}\frac{V_{i1}}{\sqrt{Z_0}} + s_{12}\frac{V_{i2}}{\sqrt{Z_0}}$$

$$b_2 = s_{21}a_1 + s_{22}a_2 \Rightarrow \frac{V_{r2}}{\sqrt{Z_0}} = s_{21}\frac{V_{i1}}{\sqrt{Z_0}} + s_{22}\frac{V_{i2}}{\sqrt{Z_0}}$$

$$\Rightarrow \begin{bmatrix} V_{r1} \\ V_{r2} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}$$

This can be written in matrix form as:

$$\mathbf{V_r} = \mathbf{sV_i} \quad \text{where} \quad \mathbf{V_r} = \begin{bmatrix} V_{r1} \\ V_{r2} \end{bmatrix} \quad \mathbf{V}_i = \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix} \quad \mathbf{s} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

S-parameter/y-parameter conversion - 1

$$\mathbf{V_r} = \mathbf{s}\mathbf{V_i}$$

Using previous relationships for V_i and V_r in terms of V and I::

$$\mathbf{V_r} = \mathbf{s}\mathbf{V_i} \Rightarrow \frac{1}{2}\mathbf{V} - \frac{Z_0}{2}\mathbf{I} = \mathbf{s}\left[\frac{1}{2}\mathbf{V} + \frac{Z_0}{2}\mathbf{I}\right] \Rightarrow \mathbf{V} - Z_0\mathbf{I} = \mathbf{s}[\mathbf{V} + Z_0\mathbf{I}]$$

Recall the definition of y-parameters:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{i.e.} \quad \mathbf{I} = \mathbf{y}\mathbf{V}$$

Substituting for **I** in the previous expression:

$$\mathbf{V} - Z_0\mathbf{y}\mathbf{V} = \mathbf{s}[\mathbf{V} + Z_0\mathbf{y}\mathbf{V}] \Rightarrow [\mathbf{II} - Z_0\mathbf{y}]\mathbf{V} = \mathbf{s}[\mathbf{II} + Z_0\mathbf{y}]\mathbf{V} \Rightarrow [\mathbf{II} - Z_0\mathbf{y}] = \mathbf{s}[\mathbf{II} + Z_0\mathbf{y}]$$

where **II** is the 2x2 identity matrix: $\mathbf{II} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

S-parameter/y-parameter conversion - 2

$$[\mathbf{II} - Z_0\mathbf{y}] = \mathbf{s}[\mathbf{II} + Z_0\mathbf{y}]$$

Define normalized y-parameters as follows using: $y_0 = \dfrac{1}{Z_0}$

$$\mathbf{y'} = Z_0\mathbf{y} = Z_0\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} Z_0 y_{11} & Z_0 y_{12} \\ Z_0 y_{21} & Z_0 y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}/y_0 & y_{12}/y_0 \\ y_{21}/y_0 & y_{22}/y_0 \end{bmatrix} = \begin{bmatrix} y'_{11} & y'_{12} \\ y'_{21} & y'_{22} \end{bmatrix}$$

This gives

$$[\mathbf{II} - \mathbf{y'}] = \mathbf{s}[\mathbf{II} + \mathbf{y'}] \Rightarrow [\mathbf{II} - \mathbf{y'}][\mathbf{II} + \mathbf{y'}]^{-1} = \mathbf{s}[\mathbf{II} + \mathbf{y'}][\mathbf{II} + \mathbf{y'}]^{-1} \Rightarrow \underline{\mathbf{s} = [\mathbf{II} - \mathbf{y'}][\mathbf{II} + \mathbf{y'}]^{-1}}$$

and

$$[\mathbf{II} - \mathbf{y'}] = \mathbf{s}[\mathbf{II} + \mathbf{y'}] \Rightarrow \mathbf{II} - \mathbf{y'} = \mathbf{s} + \mathbf{sy'} \Rightarrow \mathbf{II} - \mathbf{s} = \mathbf{y'} + \mathbf{sy'} = [\mathbf{II} + \mathbf{s}]\mathbf{y'}$$

i.e. $\mathbf{II} - \mathbf{s} = [\mathbf{II} + \mathbf{s}]\mathbf{y'} \Rightarrow \underline{\mathbf{y'} = [\mathbf{II} + \mathbf{s}]^{-1}[\mathbf{II} - \mathbf{s}]}$

Thus if **y** is known, **s** can be determined and vice versa – note that the characteristic impedance Z_0 has to be specified to perform this conversion. The y-parameters are independent of Z_0 but the s-parameters depend on Z_0.

S-parameter/y-parameter conversion - 3

$$\mathbf{s} = [\mathbf{II} - \mathbf{y}'][\mathbf{II} + \mathbf{y}']^{-1} = \begin{bmatrix} 1 - y'_{11} & -y'_{12} \\ -y'_{21} & 1 - y'_{22} \end{bmatrix} \begin{bmatrix} 1 + y'_{11} & y'_{12} \\ y'_{21} & 1 + y'_{22} \end{bmatrix}^{-1}$$

$$\mathbf{s} = \frac{1}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}} \begin{bmatrix} 1 - y'_{11} & -y'_{12} \\ -y'_{21} & 1 - y'_{22} \end{bmatrix} \begin{bmatrix} 1 + y'_{22} & -y'_{12} \\ -y'_{21} & 1 + y'_{11} \end{bmatrix}$$

$$y'_{11} = Z_0 y_{11}$$

$$y'_{12} = Z_0 y_{12}$$

$$y'_{21} = Z_0 y_{21}$$

$$y'_{22} = Z_0 y_{22}$$

$$s_{11} = \frac{(1 - y'_{11})(1 + y'_{22}) + (-y'_{12})(-y'_{21})}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}} = \frac{(1 + y'_{22})(1 - y'_{11}) + y'_{12}\, y'_{21}}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}}$$

$$s_{12} = \frac{(1 - y'_{11})(-y'_{12}) + (-y'_{12})(1 + y'_{11})}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}} = \frac{-2 y'_{12}}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}}$$

$$s_{21} = \frac{(-y'_{21})(1 + y'_{22}) + (1 - y'_{22})(-y'_{21})}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}} = \frac{-2 y'_{21}}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}}$$

$$s_{22} = \frac{(-y'_{21})(-y'_{12}) + (1 - y'_{22})(1 + y'_{11})}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}} = \frac{(1 + y'_{11})(1 - y'_{22}) + y'_{12}\, y'_{21}}{(1 + y'_{11})(1 + y'_{22}) - y'_{12}\, y'_{21}}$$

S-parameter/y-parameter conversion - 4

$$\mathbf{y'} = [\mathbf{II}+\mathbf{s}]^{-1}[\mathbf{II}-\mathbf{s}] = \begin{bmatrix} 1+s_{11} & s_{12} \\ s_{21} & 1+s_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1-s_{11} & -s_{12} \\ -s_{21} & 1-s_{22} \end{bmatrix}$$

$$\mathbf{y'} = \frac{1}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} \begin{bmatrix} 1+s_{22} & -s_{12} \\ -s_{21} & 1+s_{11} \end{bmatrix} \begin{bmatrix} 1-s_{11} & -s_{12} \\ -s_{21} & 1-s_{22} \end{bmatrix}$$

$$y'_{11} = \frac{(1+s_{22})(1-s_{11})+(-s_{12})(-s_{21})}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} = \frac{(1+s_{22})(1-s_{11})+s_{12}s_{21}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} \qquad y_{11} = \frac{y'_{11}}{Z_0}$$

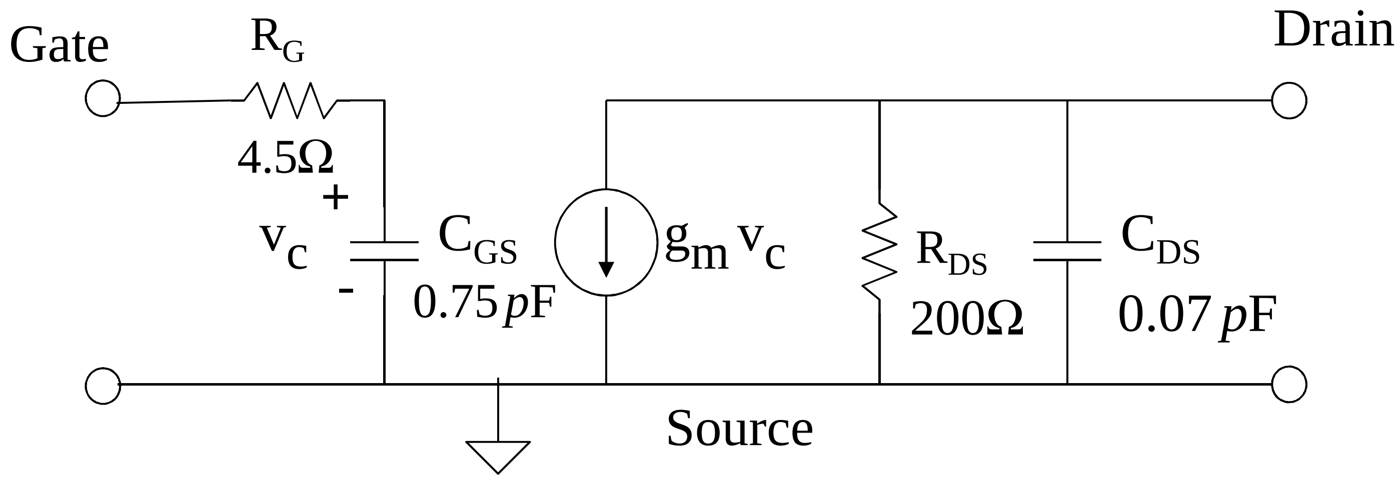
$$y'_{12} = \frac{(1+s_{22})(-s_{12})+(-s_{12})(1-s_{22})}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} = \frac{-2s_{12}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} \qquad y_{12} = \frac{y'_{12}}{Z_0}$$

$$y'_{21} = \frac{(-s_{21})(1-s_{11})+(1+s_{11})(-s_{21})}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} = \frac{-2s_{21}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} \qquad y_{21} = \frac{y'_{21}}{Z_0}$$

$$y'_{22} = \frac{(-s_{21})(-s_{12})+(1+s_{11})(1-s_{22})}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} = \frac{s_{12}s_{21}+(1+s_{11})(1-s_{22})}{(1+s_{11})(1+s_{22})-s_{12}s_{21}} \qquad y_{22} = \frac{y'_{22}}{Z_0}$$

MESFET s-parameters

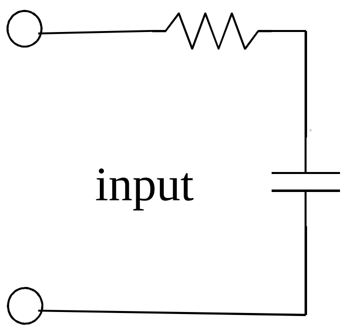
The simplified equivalent circuit for a sample MESFET used before:

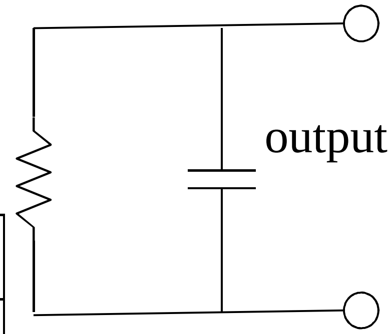


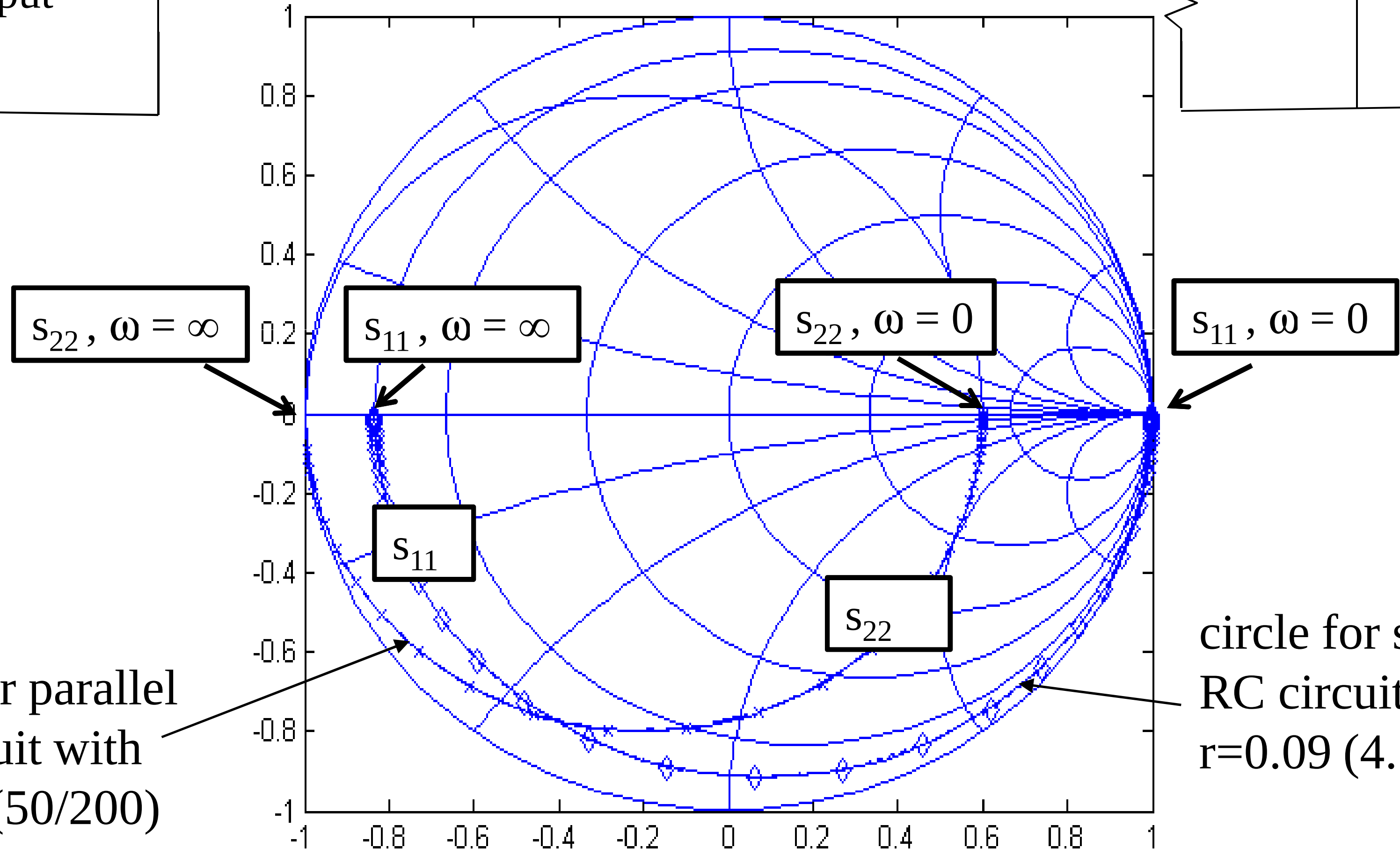
If this MESFET is connected for RF measurements with port 1 on the gate side and port 2 on the drain side, then the series connected circuit formed by R_G and C_{GS} will determine s_{11} and the parallel connected circuit formed by R_{DS} and C_{DS} will determine s_{22}. s_{11} and s_{22} are reflection coefficients and are usually plotted on a Smith Chart. s_{21} and s_{12} are usually plotted on a polar plot.

Note in this simplified case there is no capacitive coupling between the gate and drain i.e. it is a unilateral configuration with $y_{12} = s_{12} = 0$.

Example s_{11},s_{22} for MESFET



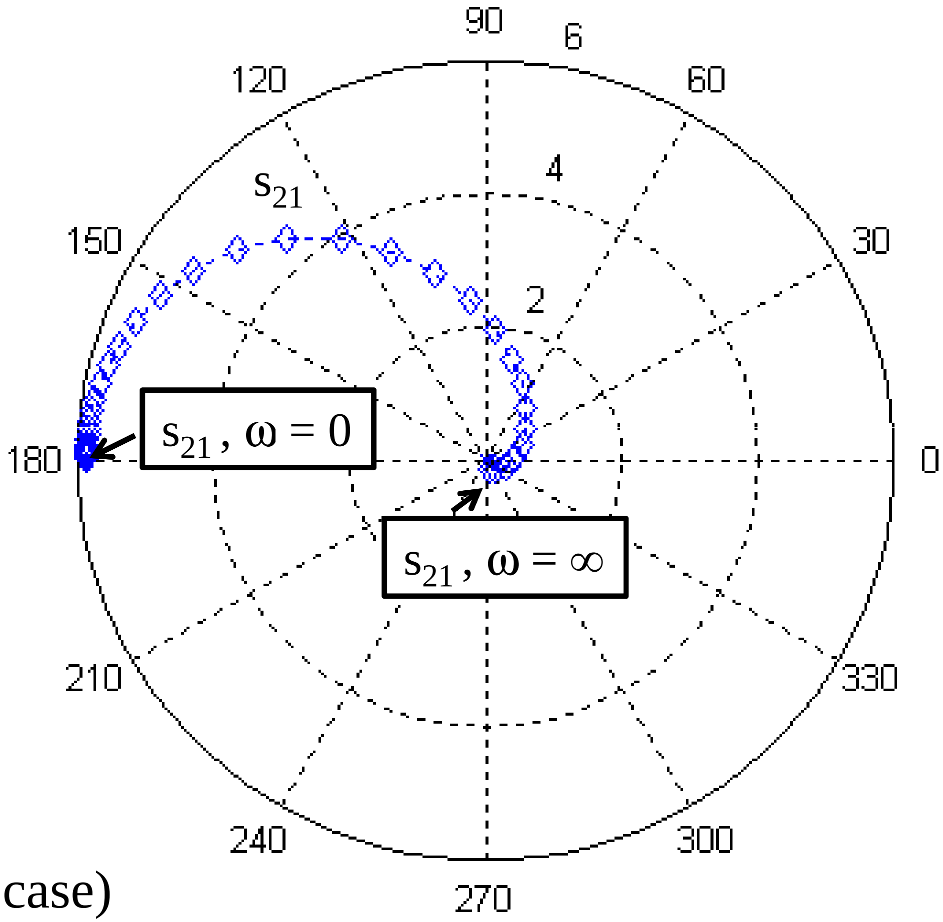




circle for parallel RC circuit with g=0.25 (50/200)

circle for series RC circuit with r=0.09 (4.5/50)

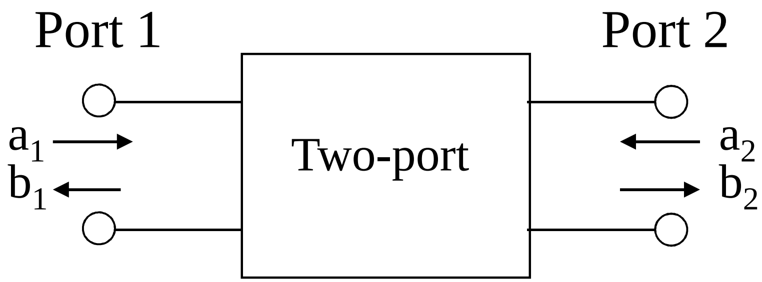
Example s_{12},s_{21} for MESFET



(s_{12} is zero in this case)

Scattering Transfer (t) Parameters

The scattering parameters of a 2-port network treat the incident waves as the independent quantities and give expressions for the reflected waves as a function of the incident waves.



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

In a similar fashion to the ABCD parameters, the ***scattering transfer parameters*** (also called the ***chain scattering parameters*** or ***t-parameters***), are defined to express the input wave variables as a function of the output wave variables i.e.:

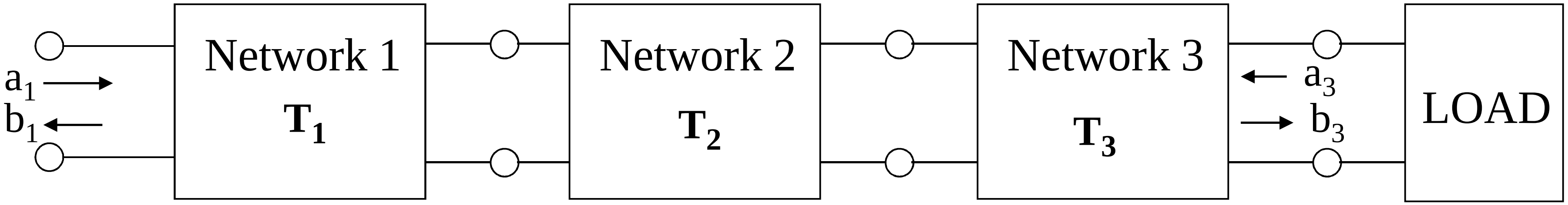
wave reflected by port 1
wave incident on port 1

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

wave reflected back towards port 2
wave transmitted through port 2

notice the different order of a and b on the left and right sides !

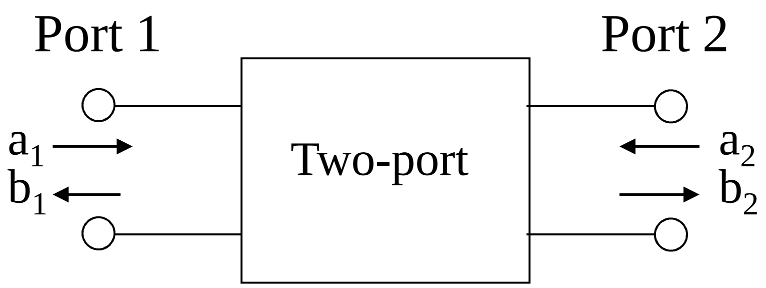
t-parameters of cascaded networks



Similar to ABCD parameters, the overall t-parameter matrix of a cascaded network can be found by multiplying the individual t-parameter matrices of the individual 2-ports in the correct order - so for the network above:

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \mathbf{T_1 T_2 T_3} \begin{bmatrix} a_3 \\ b_3 \end{bmatrix}$$

Conversion between s- and t-parameters



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$ s-parameters

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$ t-parameters

If one set of parameters are known, the other can set can be determined:

$$t_{11} = s_{12} - \frac{s_{11}s_{22}}{s_{21}} \qquad s_{11} = \frac{t_{12}}{t_{22}}$$

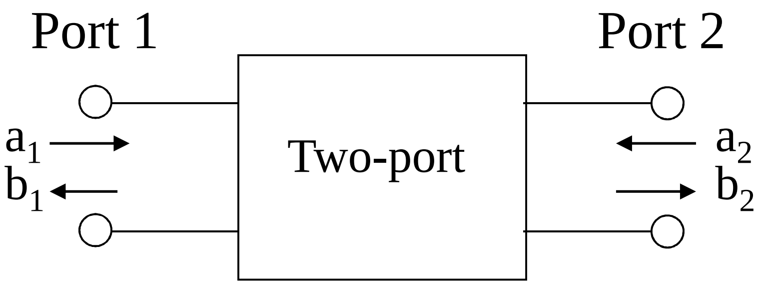
$$t_{12} = \frac{s_{11}}{s_{21}} \qquad s_{12} = t_{11} - \frac{t_{12}t_{21}}{t_{22}}$$

$$t_{21} = -\frac{s_{22}}{s_{21}} \qquad s_{21} = \frac{1}{t_{22}}$$

$$t_{22} = \frac{1}{s_{21}} \qquad s_{22} = -\frac{t_{21}}{t_{22}}$$

An Alternative Definition of t-Parameters

Sometimes a different definition is used for the t-parameters :



$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$ t-parameters

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$ "alternative" t-parameters

The conversion between s-parameters and the "alternative" t-parameters are:

$$t_{11} = \frac{1}{s_{21}} \qquad s_{11} = \frac{t_{21}}{t_{11}}$$

$$t_{12} = -\frac{s_{22}}{s_{21}} \qquad s_{12} = t_{22} - \frac{t_{12}t_{21}}{t_{11}}$$

$$t_{21} = \frac{s_{11}}{s_{21}} \qquad s_{21} = \frac{1}{t_{11}}$$

$$t_{22} = s_{12} - \frac{s_{11}s_{22}}{s_{21}} \qquad s_{22} = -\frac{t_{12}}{t_{11}}$$

So if you are given a set of t-parameters, make sure you know which version of the t-parameters are being used.
The "alternative" t-parameters can also be cascaded.