```
A4 = 3×3 Matrix{Rational{Int64}}:
      1//1    7//4   3//2
      0//1    1//1  10//27
      0//1  -19//2  -4//1
```

```
A4 = [1 0 0;
      0 -2//27 0;
      0 0 1] * A3
```

Finally, I replace the third row with $19/2$ times the second row plus the third row. This gets the matrix in upper triangular form.

```
A5 = 3×3 Matrix{Rational{Int64}}:
      1//1  7//4    3//2
      0//1  1//1   10//27
      0//1  0//1  -13//27
```

```
A5 = [1 0 0;
      0 1 0;
      0 19//2 1] * A4
```

Lastly, I express the answer in a more readable form using latexify

$$\begin{bmatrix} 1 & \frac{7}{4} & \frac{3}{2} \\ 0 & 1 & \frac{10}{27} \\ 0 & 0 & \frac{-13}{27} \end{bmatrix}$$

```
latexify(A5)
```

Your turn

Reduce a (3×3) matrix

```
B0 = 3×3 Matrix{Rational}:
       7//1   1//1  3//1
      -2//1  -4//1  1//1
      -9//1  -6//1  5//1
```

```
B0 = convert(Matrix{Rational}, rand(-9:9, (3,3)))
```

```
B1 = 3×3 Matrix{Rational{Int64}}:
       7//1   1//1  3//1
      -2//1  -4//1  1//1
      -9//1  -6//1  5//1
```

```
B1=[7//1   1//1  3//1;
 -2//1  -4//1  1//1;
 -9//1  -6//1  5//1]
```

```
E31 = 3×3 Matrix{Rational{Int64}}:
       1//1   0//1  0//1
       0//1   1//1  0//1
       1//1  -1//1  1//1
```

```
E31 = [1//1 0//1 0//1;
0//1 1//1 0//1;
1//1 -1//1 1//1]
```

```
B2 = 3×3 Matrix{Rational{Int64}}:
      7//1   1//1  3//1
     -2//1  -4//1  1//1
      0//1  -1//1  7//1
```

```
B2 = E31*B1
```

```
E21 = 3×3 Matrix{Rational{Int64}}:
      1//1  0//1  0//1
      2//7  1//1  0//1
      0//1  0//1  1//1
```

```
E21 = [1//1 0//1 0//1;
2//7 1//1 0//1;
0//1 0//1 1//1]
```

```
B3 = 3×3 Matrix{Rational{Int64}}:
     7//1    1//1   3//1
     0//1  -26//7  13//7
     0//1   -1//1   7//1
```

```
B3 = E21*B2
```

```
E32 = 3×3 Matrix{Rational{Int64}}:
      1//1   0//1   0//1
      0//1   1//1   0//1
      0//1  -7//26  1//1
```

```
E32 = [1//1 0//1 0//1;
0//1 1//1 0//1;
0//1 -7//26 1//1]
```

```
U1 = 3×3 Matrix{Rational{Int64}}:
     7//1    1//1   3//1
     0//1  -26//7  13//7
     0//1    0//1  13//2
```

```
U1 = E32*B3
```

$$\begin{bmatrix} 7 & 1 & 3 \\ 0 & \frac{-26}{7} & \frac{13}{7} \\ 0 & 0 & \frac{13}{2} \end{bmatrix}$$

```
latexify(U1)
```

Reduce a (4×4) matrix

After you finish reducing the (3×3) matrix to upper triangular form, reduce this (4×4) matrix using the same method as described above.

```
C0 = 4×4 Matrix{Rational}:
      1//1  -2//1   2//1  -4//1
      6//1   4//1   5//1   6//1
     -1//1  -3//1   2//1   4//1
     -3//1  -4//1  -3//1  -9//1
```

```
C0 = convert(Matrix{Rational}, rand(-9:9, (4,4)))
```

```
C1 = 4×4 Matrix{Rational{Int64}}:
       1//1  -2//1   2//1  -4//1
       6//1   4//1   5//1   6//1
      -1//1  -3//1   2//1   4//1
      -3//1  -4//1  -3//1  -9//1
```

```
C1=[1//1  -2//1   2//1  -4//1;
  6//1   4//1   5//1   6//1;
 -1//1  -3//1   2//1   4//1;
 -3//1  -4//1  -3//1  -9//1]
```

```
E31_2 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  0//1
        1//1  0//1  1//1  0//1
        0//1  0//1  0//1  1//1
```

```
E31_2 =[
1//1  0//1   0//1  0//1;
0//1   1//1   0//1 0//1;
1//1  0//1   1//1 0//1;
0//1  0//1  0//1  1//1
]
```

```
C2 = 4×4 Matrix{Rational{Int64}}:
       1//1  -2//1   2//1  -4//1
       6//1   4//1   5//1   6//1
       0//1  -5//1   4//1   0//1
      -3//1  -4//1  -3//1  -9//1
```

```
C2 = E31_2*C1
```

```
E21_2 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  2//1
        0//1  0//1  1//1  0//1
        0//1  0//1  0//1  1//1
```

```
E21_2 =[
1//1  0//1   0//1  0//1;
0//1   1//1   0//1 2//1;
0//1  0//1   1//1 0//1;
0//1  0//1  0//1  1//1
]
```

```
C3 = 4×4 Matrix{Rational{Int64}}:
       1//1  -2//1   2//1   -4//1
       0//1  -4//1  -1//1  -12//1
       0//1  -5//1   4//1    0//1
      -3//1  -4//1  -3//1   -9//1
```

```
C3=E21_2*C2
```

```
E41_2 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  0//1
        0//1  0//1  1//1  0//1
        3//1  0//1  0//1  1//1
```

```
E41_2 =[
1//1  0//1   0//1  0//1;
0//1   1//1   0//1 0//1;
0//1  0//1   1//1 0//1;
3//1  0//1  0//1  1//1
]
```

```
C4 = 4×4 Matrix{Rational{Int64}}:
      1//1   -2//1   2//1   -4//1
      0//1   -4//1  -1//1  -12//1
      0//1   -5//1   4//1    0//1
      0//1  -10//1   3//1  -21//1
```

```
C4 = E41_2*C3
```

```
E42_2 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1   0//1  0//1
        0//1  1//1   0//1  0//1
        0//1  0//1   1//1  0//1
        0//1  0//1  -2//1  1//1
```

```
E42_2 = [
1//1  0//1   0//1  0//1;
0//1  1//1   0//1  0//1;
0//1  0//1   1//1  0//1;
0//1  0//1   -2//1  1//1
]
```

```
C5 = 4×4 Matrix{Rational{Int64}}:
      1//1  -2//1   2//1   -4//1
      0//1  -4//1  -1//1  -12//1
      0//1  -5//1   4//1    0//1
      0//1   0//1  -5//1  -21//1
```

```
C5 = E42_2*C4
```

```
E32_2 = 4×4 Matrix{Rational{Int64}}:
        1//1   0//1  0//1  0//1
        0//1   1//1  0//1  0//1
        0//1  -5//4  1//1  0//1
        0//1   0//1  0//1  1//1
```

```
E32_2 = [
1//1  0//1   0//1  0//1;
0//1  1//1   0//1  0//1;
0//1  -5//4   1//1  0//1;
0//1  0//1   0//1  1//1
]
```

```
C6 = 4×4 Matrix{Rational{Int64}}:
      1//1  -2//1   2//1   -4//1
      0//1  -4//1  -1//1  -12//1
      0//1   0//1  21//4   15//1
      0//1   0//1  -5//1  -21//1
```

```
C6 = E32_2*C5
```

```
E43_2 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1   0//1   0//1
        0//1  1//1   0//1   0//1
        0//1  0//1   1//1   0//1
        0//1  0//1  20//21  1//1
```

```
E43_2  = [
1//1  0//1   0//1  0//1;
0//1  1//1   0//1  0//1;
0//1  0//1   1//1  0//1;
0//1  0//1   20//21  1//1
]
```

```
U2 = 4×4 Matrix{Rational{Int64}}:
      1//1  -2//1   2//1   -4//1
      0//1  -4//1  -1//1  -12//1
      0//1   0//1  21//4   15//1
      0//1   0//1   0//1  -47//7
```

```
U2 = E43_2*C6
```

$$\begin{bmatrix} 1 & -2 & 2 & -4 \\ 0 & -4 & -1 & -12 \\ 0 & 0 & \frac{21}{4} & 15 \\ 0 & 0 & 0 & \frac{-47}{7} \end{bmatrix}$$

```
latexify(U2)
```