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LU Factorization of a Matrix

In this assignment you will need to find the LU factorization of 3×3 and 4×4 matrices. Some with no row exchanges and others which require row exchanges (permutation matrices).

There is also an extra-credit problem that you should do.

To get credit, you must verify that your factorizations are correct by multiplying them out and showing that their product is the original matrix.

Be sure to **disable the cells** containing the matrices, so their values don't get accidentally changed.

1. Find the LU factorization of this 3×3 matrix:

```
A = 3×3 Matrix{Rational}:
 -3//1  4//1   5//1
 -2//1  6//1  -6//1
 -4//1  0//1  -4//1
```

```
A = no_zero_pivots(3)
```

```
A = 3×3 Matrix{Rational{Int64}}:
 -3//1  4//1   5//1
 -2//1  6//1  -6//1
 -4//1  0//1  -4//1
```

```
A = [
-3//1  4//1   5//1;
 -2//1  6//1  -6//1;
 -4//1  0//1  -4//1
]
```

```
E31 = 3×3 Matrix{Rational{Int64}}:
       1//1   0//1  0//1
       0//1   1//1  0//1
       0//1  -2//1  1//1
```

```
E31 = [
    1//1 0//1 0//1;
    0//1 1//1 0//1;
    0//1 -2//1 1//1;
]
```

```
E21 = 3×3 Matrix{Rational{Int64}}:
        1//1  0//1  0//1
       -2//3  1//1  0//1
        0//1  0//1  1//1
```

```
E21 = [
    1//1 0//1 0//1;
    -2//3 1//1 0//1;
    0//1 0//1 1//1;
]
```

```
E32 = 3×3 Matrix{Rational{Int64}}:
       1//1   0//1  0//1
       0//1   1//1  0//1
       0//1  18//5  1//1
```

```
E32 = [
    1//1 0//1 0//1;
    0//1 1//1 0//1;
    0//1 36//10 1//1;
]
```

```
U = 3×3 Matrix{Rational{Int64}}:
     -3//1   4//1     5//1
      0//1  10//3   -28//3
      0//1   0//1  -128//5
```

```
U = E32*E21*E31*A
```

```
L = 3×3 Matrix{Rational{Int64}}:
     1//1   0//1  0//1
     2//3   1//1  0//1
     4//3  -8//5  1//1
```

```
L = [
1//1 0//1 0//1;
2//3 1//1 0//1;
4//3 -8//5 1//1
]
```

```
true
```

```
L*U==A
```

2. Find the LU factorization of this 3×3 matrix:

```
B = 3×3 Matrix{Rational}:
     0//1   5//1  -2//1
    -2//1  -8//1   9//1
    -7//1  -1//1   6//1
```

```
B = zero_pivots(3)
```

```
B = 3×3 Matrix{Rational{Int64}}:
     0//1   5//1  -2//1
    -2//1  -8//1   9//1
    -7//1  -1//1   6//1
```

```
B = [
    0//1   5//1  -2//1;
 -2//1  -8//1   9//1;
 -7//1  -1//1   6//1
]
```

```
P213 = 3×3 Matrix{Rational{Int64}}:
       0//1  1//1  0//1
       1//1  0//1  0//1
       0//1  0//1  1//1
```

```
P213 =[
0//1 1//1 0//1;
1//1 0//1 0//1;
0//1 0//1 1//1
]
```

```
E31_2 = 3×3 Matrix{Rational{Int64}}:
        1//1  0//1  0//1
        0//1  1//1  0//1
       -7//2  0//1  1//1
```

```
E31_2 =
[
1//1 0//1 0//1;
0//1 1//1 0//1;
-7//2 0//1 1//1
]
```

```
E32_2 = 3×3 Matrix{Rational{Int64}}:
        1//1    0//1  0//1
        0//1    1//1  0//1
        0//1  -27//5  1//1
```

```
E32_2 =
[
1//1 0//1 0//1;
0//1 1//1 0//1;
0//1 -27//5 1//1
]
```

```
U_2 = 3×3 Matrix{Rational{Int64}}:
      -2//1  -8//1     9//1
       0//1   5//1    -2//1
       0//1   0//1  -147//10
```

```
U_2 = E32_2*E31_2*P213*B
```

```
L_2 = 3×3 Matrix{Rational{Int64}}:
       0//1   1//1  0//1
       1//1   0//1  0//1
       7//2  27//5  1//1
```

```
L_2 = [
0//1 1//1 0//1;
1//1 0//1 0//1;
7//2 27//5 1//1
]
```

```
true
```

```
L_2*U_2==B
```

3. Find the LU factorization of this 4×4 matrix:

```
C = 4×4 Matrix{Rational}:
 -6//1   0//1  -6//1  -5//1
  0//1  -4//1   3//1  -9//1
 -9//1  -2//1  -9//1   1//1
  5//1  -2//1   1//1   5//1
```

```
C = no_zero_pivots(4)
```

```
C = 4×4 Matrix{Rational{Int64}}:
 -6//1   0//1  -6//1  -5//1
  0//1  -4//1   3//1  -9//1
 -9//1  -2//1  -9//1   1//1
  5//1  -2//1   1//1   5//1
```

```
C =
[
 -6//1   0//1  -6//1  -5//1;
  0//1  -4//1   3//1  -9//1;
 -9//1  -2//1  -9//1   1//1;
  5//1  -2//1   1//1   5//1
]
```

```
E31_3 = 4×4 Matrix{Rational{Int64}}:
  1//1  0//1  0//1  0//1
  0//1  1//1  0//1  0//1
 -3//2  0//1  1//1  0//1
  0//1  0//1  0//1  1//1
```

```
E31_3 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
-9//6 0//1 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E41_3 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  0//1
        0//1  0//1  1//1  0//1
        5//6  0//1  0//1  1//1
```

```
E41_3 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
5//6 0//1 0//1 1//1
]
```

```
E32_3 = 4×4 Matrix{Rational{Int64}}:
        1//1   0//1  0//1  0//1
        0//1   1//1  0//1  0//1
        0//1  -1//2  1//1  0//1
        0//1   0//1  0//1  1//1
```

```
E32_3 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 -1//2 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E42_3 = 4×4 Matrix{Rational{Int64}}:
        1//1   0//1  0//1  0//1
        0//1   1//1  0//1  0//1
        0//1   0//1  1//1  0//1
        0//1  -1//2  0//1  1//1
```

```
E42_3 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 -1//2 0//1 1//1
]
```

```
E43_3 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1    0//1  0//1
        0//1  1//1    0//1  0//1
        0//1  0//1    1//1  0//1
        0//1  0//1  -11//3  1//1
```

```
E43_3 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 0//1 -22//6 1//1
]
```

```
U_3 = 4×4 Matrix{Rational{Int64}}:
      -6//1   0//1  -6//1    -5//1
       0//1  -4//1   3//1    -9//1
       0//1   0//1  -3//2    13//1
       0//1   0//1   0//1  -127//3
```

```
U_3 = E43_3*E42_3*E32_3*E41_3*E31_3*C
```

```
L_3 = 4×4 Matrix{Rational{Int64}}:
       1//1  0//1   0//1  0//1
       0//1  1//1   0//1  0//1
       3//2  1//2   1//1  0//1
      -5//6  1//2  11//3  1//1
```

```
L_3 = [
  1//1  0//1   0//1  0//1;
  0//1  1//1   0//1  0//1;
  3//2  1//2   1//1  0//1;
 -5//6  1//2  11//3  1//1
]
```

```
true
```

```
L_3*U_3==C
```

4. Find the LU factorization of this 4×4 matrix:

```
D = 4×4 Matrix{Rational}:
     0//1   0//1  -9//1   2//1
     5//1  -6//1  -4//1   5//1
     6//1   8//1   9//1   7//1
     3//1  -4//1   6//1  -7//1
```

```
D = zero_pivots(4)
```

```
D = 4×4 Matrix{Rational{Int64}}:
     0//1   0//1  -9//1   2//1
     5//1  -6//1  -4//1   5//1
     6//1   8//1   9//1   7//1
     3//1  -4//1   6//1  -7//1
```

```
D =
[
 0//1   0//1  -9//1   2//1;
 5//1  -6//1  -4//1   5//1;
 6//1   8//1   9//1   7//1;
 3//1  -4//1   6//1  -7//1
]
```

```
P4231_4 = 4×4 Matrix{Rational{Int64}}:
           0//1  0//1  0//1  1//1
           0//1  1//1  0//1  0//1
           0//1  0//1  1//1  0//1
           1//1  0//1  0//1  0//1
```

```
P4231_4 =
[
0//1 0//1 0//1 1//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
1//1 0//1 0//1 0//1
]
```

```
E31_4 = 4×4 Matrix{Rational{Int64}}:
           1//1  0//1  0//1  0//1
           0//1  1//1  0//1  0//1
          -2//1  0//1  1//1  0//1
           0//1  0//1  0//1  1//1
```

```
E31_4 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
-2//1 0//1 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E21_4 = 4×4 Matrix{Rational{Int64}}:
           1//1  0//1  0//1  0//1
          -5//3  1//1  0//1  0//1
           0//1  0//1  1//1  0//1
           0//1  0//1  0//1  1//1
```

```
E21_4 =
[
1//1 0//1 0//1 0//1;
-5//3 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E32_4 = 4×4 Matrix{Rational{Int64}}:
           1//1    0//1  0//1  0//1
           0//1    1//1  0//1  0//1
           0//1  -24//1  1//1  0//1
           0//1    0//1  0//1  1//1
```

```
E32_4 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 -(3*16)//2 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E43_4 = 4×4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1   0//1
 0//1  1//1  0//1   0//1
 0//1  0//1  1//1   0//1
 0//1  0//1  1//37  1//1
```

```
E43_4 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 0//1 9//333 1//1
]
```

```
U_4 = 4×4 Matrix{Rational{Int64}}:
 3//1  -4//1    6//1    -7//1
 0//1   2//3  -14//1    50//3
 0//1   0//1  333//1  -379//1
 0//1   0//1    0//1  -305//37
```

```
U_4 = E43_4*E32_4*E21_4*E31_4*P4231_4*D
```

```
L_4 = 4×4 Matrix{Rational{Int64}}:
 0//1   0//1  -1//37  1//1
 5//3   1//1   0//1   0//1
 2//1  24//1   1//1   0//1
 1//1   0//1   0//1   0//1
```

```
L_4 = [
0//1 0//1 -1//37 1//1;
5//3 1//1 0//1 0//1;
2//1 24//1 1//1 0//1;
1//1 0//1 0//1 0//1
    ]
```

```
true
```

```
L_4*U_4==D
```

Extra Credit: Find the LDU factorization of this 4×4 matrix, where:

- L is a lower triangular matrix with 1's on the main diagonal.
- D is a *diagonal matrix* with numbers on the main diagonal and 0's elsewhere.
- U is an upper triangular matrix with 1's on the main diagonal.

```
M = 4×4 Matrix{Rational}:
  8//1  -1//1   1//1  7//1
 -1//1  -6//1   0//1  1//1
  3//1  -1//1  -6//1  4//1
  3//1   3//1  -9//1  9//1
```

```
M = no_zero_pivots(4)
```

```
M = 4×4 Matrix{Rational{Int64}}:
      8//1  -1//1   1//1  7//1
     -1//1  -6//1   0//1  1//1
      3//1  -1//1  -6//1  4//1
      3//1   3//1  -9//1  9//1
```

```
M =
[
  8//1  -1//1   1//1  7//1;
 -1//1  -6//1   0//1  1//1;
  3//1  -1//1  -6//1  4//1;
  3//1   3//1  -9//1  9//1
]
```

```
E41_5 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  0//1
        0//1  0//1  1//1  0//1
        0//1  3//1  0//1  1//1
```

```
E41_5 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 3//1 0//1 1//1
]
```

```
E31_5 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        0//1  1//1  0//1  0//1
        0//1  3//1  1//1  0//1
        0//1  0//1  0//1  1//1
```

```
E31_5 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 3//1 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E21_5 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1  0//1  0//1
        1//8  1//1  0//1  0//1
        0//1  0//1  1//1  0//1
        0//1  0//1  0//1  1//1
```

```
E21_5 =
[
1//1 0//1 0//1 0//1;
1//8 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E32_5 = 4×4 Matrix{Rational{Int64}}:
        1//1     0//1   0//1  0//1
        0//1     1//1   0//1  0//1
        0//1  -152//49  1//1  0//1
        0//1     0//1   0//1  1//1
```

```
E32_5 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 -(8*19)//49 1//1 0//1;
0//1 0//1 0//1 1//1
]
```

```
E42_5 = 4×4 Matrix{Rational{Int64}}:
        1//1     0//1   0//1  0//1
        0//1     1//1   0//1  0//1
        0//1     0//1   1//1  0//1
        0//1  -120//49  0//1  1//1
```

```
E42_5 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 -(8*15)//49 0//1 1//1
]
```

```
E43_5 = 4×4 Matrix{Rational{Int64}}:
        1//1  0//1     0//1    0//1
        0//1  1//1     0//1    0//1
        0//1  0//1     1//1    0//1
        0//1  0//1  -456//313  1//1
```

```
E43_5 =
[
1//1 0//1 0//1 0//1;
0//1 1//1 0//1 0//1;
0//1 0//1 1//1 0//1;
0//1 0//1 -456//313 1//1
]
```

```
U_5 = 4×4 Matrix{Rational{Int64}}:
       8//1   -1//1     1//1      7//1
       0//1  -49//8     1//8     15//8
       0//1    0//1  -313//49    58//49
       0//1    0//1     0//1   1779//313
```

```
U_5 = E43_5*E42_5*E32_5*E21_5*E31_5*E41_5*M
```

```
D_5 = 4×4 Matrix{Rational{Int64}}:
       8//1    0//1     0//1      0//1
       0//1  -49//8     0//1      0//1
       0//1    0//1  -313//49     0//1
       0//1    0//1     0//1   1779//313
```

```
D_5=
    [
8//1 0//1 0//1 0//1;
0//1 -49//8 0//1 0//1;
0//1 0//1 -313//49 0//1;
0//1 0//1 0//1 1779//313
    ]
```

```
U_5v2 = 4×4 Matrix{Rational{Int64}}:
        1//1  -1//8   1//8     7//8
        0//1   1//1  -1//49  -15//49
        0//1   0//1   1//1   -58//313
        0//1   0//1   0//1     1//1
```

```
U_5v2 = [
1//1 -1//8 1//8 7//8
0//1 1//1 -1//49 -15//49
0//1 0//1 1//1 -58//313
0//1 0//1 0//1 1//1
]
```

```
true
```

```
U_5 == D_5*U_5v2
```

```
L_5 = 4×4 Matrix{Rational{Int64}}:
        1//1    0//1     0//1    0//1
       -1//8    1//1     0//1    0//1
        3//8    5//49    1//1    0//1
        3//8  -27//49  456//313  1//1
```

```
L_5 = [
  1//1    0//1     0//1    0//1;
 -1//8    1//1     0//1    0//1;
  3//8    5//49    1//1    0//1;
  3//8  -27//49  456//313  1//1
]
```

```
true
```

```
L_5*D_5*U_5v2==M
```