

sysEq (generic function with 1 method)

equations (generic function with 1 method)

Solving Systems of Equations with Augmented Matrices

For this assignment, you will use an augmented matrix to solve a system of linear equations. Refer to [the pdf copy of the Pluto notebook file](#) we went over in [class on Thursday, Sept. 28](#), where I showed you how it's done.

Be sure that your augmented matrix has rational number entries. Use row reduction matrices to get the augmented matrix into reduced row echelon form (the coefficient matrix part of the augmented matrix should reduce to the identity matrix).

After you get a solution to the system, use Julia to verify it as shown in that same Pluto notebook file referenced earlier.

The function *equations(n)*

I created a function *equations(n)* which returns a system of n equations in n unknowns x_1, x_2, \dots, x_n .

For this assignment, you'll be solving a system of three linear equations, so enter

```
equations(3)
```

in a cell and evaluate it to get your system of three equations. After you've created your augmented matrix of rational values, be sure to disable the cell containing the function. That way it becomes frozen and can't be accidentally changed.

Extra Credit: You can earn extra credit points by solving a system of four equations.

$$\begin{aligned} -2x_1 + 3x_2 - 2x_3 - 3x_4 &= -6 \\ -4x_2 + 1x_3 + 4x_4 &= -9 \\ 9x_1 + 1x_2 + 8x_3 + 7x_4 &= 1 \\ 1x_1 + 1x_3 + 9x_4 &= -3 \end{aligned}$$

```
1 equations(4)
```

First I will do a system of 3 equations

```
A_aug = 3x4 Matrix{Rational{Int64}}:
```

```
-8//1 -2//1 5//1 6//1
 7//1 2//1 -9//1 5//1
-8//1 9//1 1//1 9//1
```

```
1 A_aug = [-8//1 -2//1 5//1 6//1; 7//1 2//1 -9//1 5//1; -8//1 9//1 1//1 9//1]
```

$$\begin{bmatrix} -8 & -2 & 5 & 6 \\ 7 & 2 & -9 & 5 \\ -8 & 9 & 1 & 9 \end{bmatrix}$$

```
1 latexify(A_aug)
```

```
E1 = 3x3 Matrix{Rational{Int64}}:
```

```
1//1 0//1 0//1
0//1 1//1 0//1
-1//1 0//1 1//1
```

```
1 E1 =
2 [
3 1//1 0//1 0//1;
4 0//1 1//1 0//1;
5 -1//1 0//1 1//1
6 ]
```

```
E2 = 3x3 Matrix{Rational{Int64}}:
```

```
1//1 0//1 0//1
7//8 1//1 0//1
0//1 0//1 1//1
```

```
1 E2 =
2 [
3 1//1 0//1 0//1;
4 7//8 1//1 0//1;
5 0//1 0//1 1//1
6 ]
```

```
E3 = 3x3 Matrix{Rational{Int64}}:
```

```
1//1 0//1 0//1
0//1 1//1 0//1
0//1 -44//1 1//1
```

```
1 E3 =
2 [
3 1//1 0//1 0//1;
4 0//1 1//1 0//1;
5 0//1 -44//1 1//1
6 ]
```

```
E4 = 3x3 Matrix{Rational{Int64}}:
```

```
1//1 0//1 0//1
0//1 1//1 37//1596
0//1 0//1 1//1
```

```
1 E4 =
2 [
3 1//1 0//1 0//1;
4 0//1 1//1 74//3192;
5 0//1 0//1 1//1
6 ]
```

```
E5 = 3x3 Matrix{Rational{Int64}}:
  1//1  0//1  -10//399
  0//1  1//1   0//1
  0//1  0//1   1//1
```

```
1 E5 =
2 [
3 1//1 0//1 -10//399;
4 0//1 1//1 0//1;
5 0//1 0//1 1//1
6 ]
```

```
E6 = 3x3 Matrix{Rational{Int64}}:
  1//1  8//1  0//1
  0//1  1//1  0//1
  0//1  0//1  1//1
```

```
1 E6 =
2 [
3 1//1 8//1 0//1;
4 0//1 1//1 0//1;
5 0//1 0//1 1//1
6 ]
```

```
B = 3x3 Matrix{Rational{Int64}}:
 -1//8  0//1  0//1
  0//1  4//1  0//1
  0//1  0//1  2//399
```

```
1 B =
2 [
3 -1//8 0//1 0//1;
4 0//1 4//1 0//1;
5 0//1 0//1 2//399
6 ]
```

```
E = 3x3 Matrix{Rational{Int64}}:
 664//399  376//399  64//399
 -65//1596 -8//399  37//1596
 -79//2    -44//1    1//1
```

```
1 E = E6*E5*E4*E3*E2*E1
```

```
A_aug_rref = 3x4 Matrix{Rational{Int64}}:
  1//1  0//1  0//1  -115//57
  0//1  1//1  0//1  -31//57
  0//1  0//1  1//1  -128//57
```

```
1 A_aug_rref = B*E*A_aug
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-115}{57} \\ 0 & 1 & 0 & \frac{-31}{57} \\ 0 & 0 & 1 & \frac{-128}{57} \end{bmatrix}$$

```
1 latexify(A_aug_rref)
```

$$x_1 = \frac{-115}{57}$$

$$x_2 = \frac{-31}{57}$$

$$x_3 = \frac{-128}{57}$$

```
1 md"""
2 $x_1 = \frac{-115}{57}$
3 $x_2 = \frac{-31}{57}$
4 $x_3 = \frac{-128}{57}$
5 """
```

Now to verify the solution:

```
1 md"""
2 ##### Now to verify the solution:
3 """
```

$$\begin{bmatrix} 6 \\ 5 \\ 9 \end{bmatrix}$$

```
1 latexify((-115//57)*A_aug[:,1] + (-31//57)*A_aug[:,2] + (-128//57)*A_aug[:,3])
```

As you can see, the result matches the \vec{x} coefficient vector from the beginning

```
1 md"""
2 As you can see, the result matches the  $\vec{x}$  coefficient vector from the
3 beginning
4 """
```

EC: Now I will do a system of four equations

$$-7x_1 - 1x_2 + 3x_3 - 5x_4 = -3$$

$$3x_1 - 3x_2 - 5x_3 - 1x_4 = 7$$

$$-1x_1 + 6x_2 + 8x_3 - 5x_4 = 2$$

$$-6x_1 - 7x_2 - 9x_3 + 5x_4 = 9$$

```
1 equations(4)
```

```
R_aug = 4x5 Matrix{Rational{Int64}}:
  -7//1  -1//1   3//1  -5//1  -3//1
    3//1  -3//1  -5//1  -1//1   7//1
  -1//1   6//1   8//1  -5//1   2//1
  -6//1  -7//1  -9//1   5//1   9//1
```

```
1 R_aug =
2 [
3 -7//1 -1//1 3//1 -5//1 -3//1;
4 3//1 -3//1 -5//1 -1//1 7//1;
5 -1//1 6//1 8//1 -5//1 2//1;
6 -6//1 -7//1 -9//1 5//1 9//1
7 ]
```

```
e1 = 4x4 Matrix{Rational{Int64}}:
  1//1  0//1  0//1  0//1
  0//1  1//1  0//1  0//1
  0//1  0//1  1//1  0//1
  0//1  2//1  0//1  1//1
```

```
1 e1 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 2//1 0//1 1//1
7 ]
```

```
e2 = 4x4 Matrix{Rational{Int64}}:
  1//1  0//1  0//1  0//1
  0//1  1//1  0//1  0//1
  0//1  1//3  1//1  0//1
  0//1  0//1  0//1  1//1
```

```
1 e2 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 1//3 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e3 = 4x4 Matrix{Rational{Int64}}:
  1//1  0//1  0//1  0//1
  3//7  1//1  0//1  0//1
  0//1  0//1  1//1  0//1
  0//1  0//1  0//1  1//1
```

```
1 e3 =
2 [
3 1//1 0//1 0//1 0//1;
4 3//7 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e4 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  0//1  0//1
 0//1  0//1  1//1  0//1
 0//1  0//1  13//5  1//1
```

```
1 e4 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 13//5 1//1
7 ]
```

```
e5 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  0//1  0//1
 0//1  35//24 1//1  0//1
 0//1  0//1  0//1  1//1
```

```
1 e5 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 35//24 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e6 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  0//1  0//1
 0//1  0//1  1//1  0//1
 0//1  0//1  152//55 1//1
```

```
1 e6 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 (12*38)//(11*15) 1//1
7 ]
```

```
e7 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  0//1  0//1
 0//1  0//1  1//1  -1309//5052
 0//1  0//1  0//1  1//1
```

```
1 e7 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 (-11*119)//(421*12);
6 0//1 0//1 0//1 1//1
7 ]
```

```
e8 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  0//1 -242//2947
 0//1  0//1  1//1  0//1
 0//1  0//1  0//1  1//1
```

```
1 e8 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 0//1 (-11*22)//(421*7);
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e9 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1 -55//421
 0//1  1//1  0//1  0//1
 0//1  0//1  1//1  0//1
 0//1  0//1  0//1  1//1
```

```
1 e9 =
2 [
3 1//1 0//1 0//1 (-11*5)//(421);
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e10 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1
 0//1  1//1  312//77  0//1
 0//1  0//1  1//1  0//1
 0//1  0//1  0//1  1//1
```

```
1 e10 =
2 [
3 1//1 0//1 0//1 0//1;
4 0//1 1//1 (12*26)//(11*7) 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e11 = 4x4 Matrix{Rational{Int64}}:
 1//1  0//1 -36//11  0//1
 0//1  1//1  0//1  0//1
 0//1  0//1  1//1  0//1
 0//1  0//1  0//1  1//1
```

```
1 e11 =
2 [
3 1//1 0//1 (-12*3)//(11) 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e12 = 4x4 Matrix{Rational{Int64}}:
 1//1  -7//24  0//1  0//1
 0//1   1//1  0//1  0//1
 0//1   0//1  1//1  0//1
 0//1   0//1  0//1  1//1
```

```
1 e12 =
2 [
3 1//1 (-7)//(24) 0//1 0//1;
4 0//1 1//1 0//1 0//1;
5 0//1 0//1 1//1 0//1;
6 0//1 0//1 0//1 1//1
7 ]
```

```
e = 4x4 Matrix{Rational{Int64}}:
 -42//421  -35//421  490//421  441//421
2964//2947 -1740//2947 -5952//2947 -3336//2947
1793//10104 -2365//10104 -1969//5052 -1309//5052
 19//11      86//11      59//11      1//1
```

```
1 e = e12*e11*e10*e9*e8*e7*e6*e5*e4*e3*e2*e1
```

```
b = 4x4 Matrix{Rational{Int64}}:
 -1//7  0//1  0//1  0//1
 0//1  -7//24  0//1  0//1
 0//1  0//1  12//11  0//1
 0//1  0//1  0//1  -11//421
```

```
1 b =
2 [
3 -1//7 0//1 0//1 0//1;
4 0//1 -7//24 0//1 0//1;
5 0//1 0//1 12//11 0//1;
6 0//1 0//1 0//1 -11//421
7 ]
```

```
R_aug_rref = 4x5 Matrix{Rational{Int64}}:
 1//1  0//1  0//1  0//1  -690//421
 0//1  1//1  0//1  0//1  2625//421
 0//1  0//1  1//1  0//1  -2426//421
 0//1  0//1  0//1  1//1  -762//421
```

```
1 R_aug_rref = b * e * R_aug
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{690}{421} \\ 0 & 1 & 0 & 0 & \frac{2625}{421} \\ 0 & 0 & 1 & 0 & -\frac{2426}{421} \\ 0 & 0 & 0 & 1 & -\frac{762}{421} \end{bmatrix}$$

```
1 latexify(R_aug_rref)
```


$$x_1 = \frac{-690}{421}$$

$$x_2 = \frac{2625}{421}$$

$$x_3 = \frac{-2426}{421}$$

$$x_4 = \frac{-762}{421}$$

```
1 md"""
2 $x_1 = \frac{-690}{421}$
3 $x_2 = \frac{2625}{421}$
4 $x_3 = \frac{-2426}{421}$
5 $x_4 = \frac{-762}{421}$
6 """
```

Now to verify the solution:

```
1 md"""
2 ##### Now to verify the solution:
3 """
```

$$\begin{bmatrix} -3 \\ 7 \\ 2 \\ 9 \end{bmatrix}$$

```
1 latexify((-690//421)*R_aug[:,1] + (2625//421)*R_aug[:,2] +
(-2426//421)*R_aug[:,3] + (-762//421)*R_aug[:,4])
```

As you can see, the solution is correct as this matrix matches the original coefficient matrix

```
1 md"""
2 As you can see, the solution is correct as this matrix matches the original
  coefficient matrix
3 """
```