

Last time:

Prop I.2: Given a point A and a length BC. How do we move the length BC to the point A (even though our compass is collapsing)?

Tactic	Reason
1. Given points A, B and C. Want a point H so that $\text{dist } A H = \text{dist } B C$	Given
1'. If $A = B$ , then BC is already at A, and we're done. Assume $A \neq B$	Case split
2. Construct an equilateral triangle on AB (say), call it ABD.	Prop I.1
2'. If $B = C$ , then $\text{dist } A A = \text{dist } B C$ , so use A. Else $B \neq C$	Case split
3. Draw a circle $\beta$ with center B and passing through C	Postulate 3 and $B \neq C$
4. Draw a line L through B and D	Postulate 1
4'. By the way, $B \neq D$ because $A \neq B$ , and $\text{dist } AB = \text{dist } BD$	From Step 2
5. Draw a line M through D and A	Postulate 1
5'. B is interior of circle $\beta$	$\text{in\_Circ\_of\_center } B \beta$
6. Line DB and circle $\beta$ intersect	$\text{Line\_Circ\_inter\_iff}$ , Step5' Step4
7. There are two distinct points E and F on circle $\beta$ and line L	$\text{Pts\_of\_Line\_Circ\_inter}$ Step 6
8. Draw a circle $\delta$ centered at D passing through E (or is it F???)	Postulate 3
9. Circle $\delta$ intersects the line M through A and D	$\text{Line\_Circ\_inter\_iff}$ (D is the center of Circle $\delta$ , and hence inside)
10. Obtain points H and I where M and $\delta$ intersect	$\text{Pts\_of\_Line\_Circ\_inter}$
11. use H. then argue (using chain of transitivity and symmetry of the equal sign, together with definition of a circle) that $\text{dist } A H = \text{dist } B C$ as desired	As described.

Things we still need: A predicate for when lines intersect circles

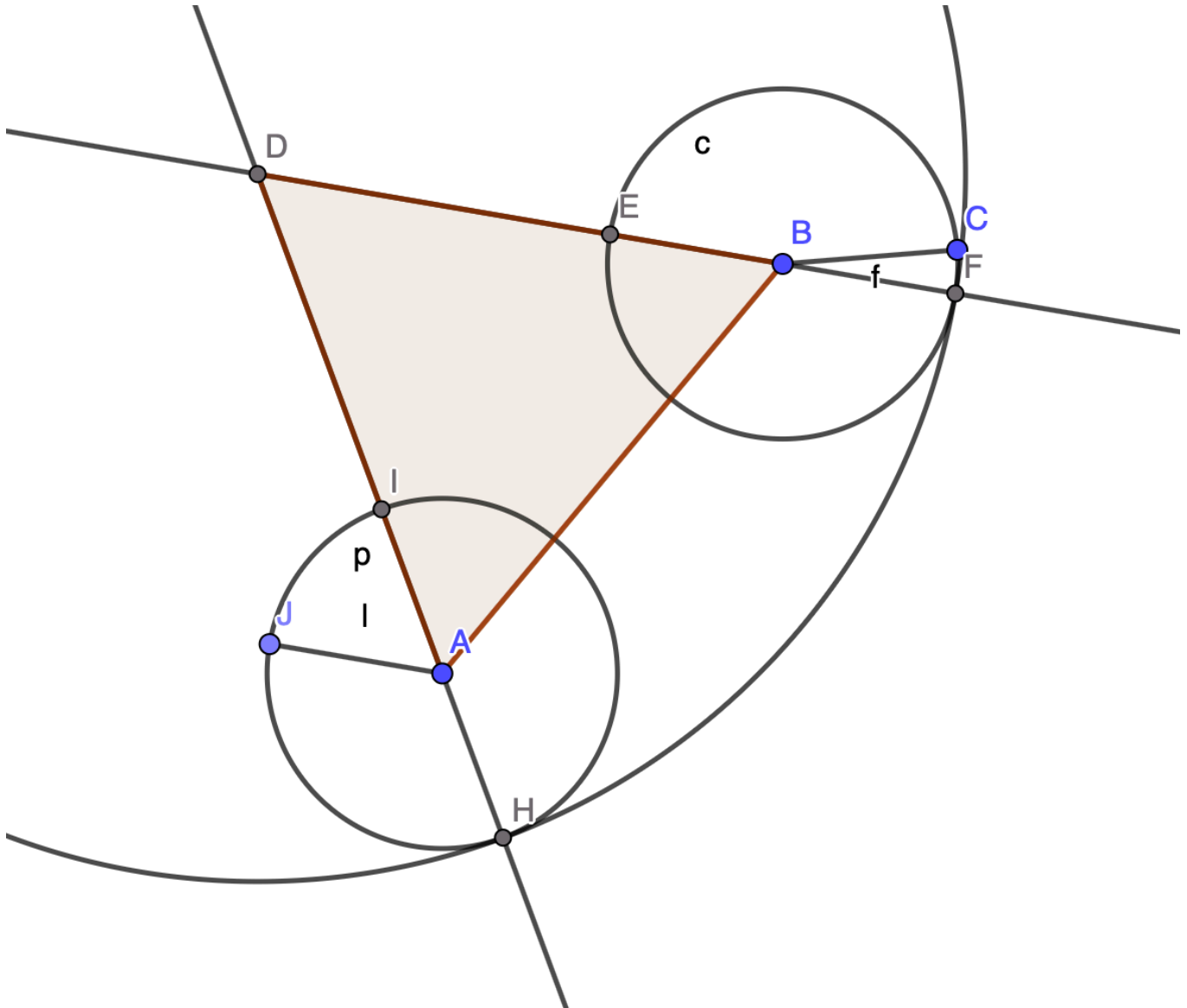
$\text{Line\_Circ\_inter} : \text{Line} \rightarrow \text{Circ} \rightarrow \text{Prop}$

Need a condition under which lines and circles do intersect, and then we need an axiom that says that if they do intersect, then they intersect at two distinct points.

```
Line_Circ_inter_iff : ∀ (L : Line) (α : Circ), Line_Circ_inter L α ↔ ∃ (a : Pt),
  Pt_in_Circ a α ∧ Pt_on_Line a L
```

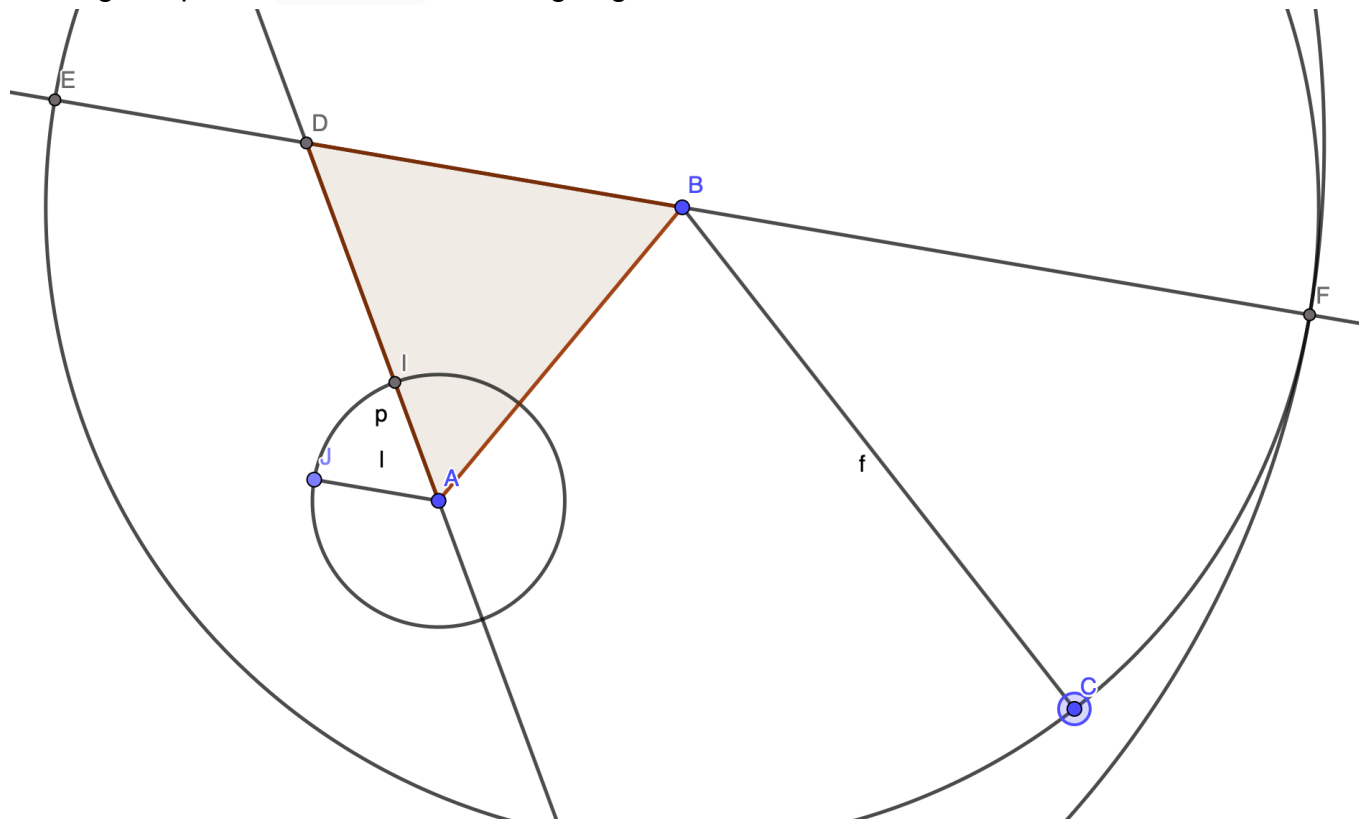
If a Line and Circle do intersect, then there are two distinct points of intersection

```
Pts_of_Line_Circ_inter : ∀ (L : Line) (α : Circ), Line_Circ_inter L α →
  ∃ (a b : Pt), Pt_on_Line a L ∧ Pt_on_Line b L ∧ Pt_on_Circ a α ∧
  Pt_on_Circ b α ∧ a ≠ b
```



But wait a second, if I pull C a little farther from B, then the configuration flips, and `dist A H` is

no longer equal to  $\text{dist } B \ C$  ! What's going on??!?



Homework: Think deeply about what is going on and how we can resolve things.

Big hint: Add a new predicate of betweenness

$\text{InOrder} : \text{Pt} \rightarrow \text{Pt} \rightarrow \text{Pt} \rightarrow \text{Prop}$

Where  $\text{InOrder } a \ b \ c$  is meant to represent the fact that all three points lie on the same line, and they occur on that line in that order,  $a$  then  $b$  then  $c$ .