

Data Science

Kamalini Ramdas & Kanishka Bhattacharya



Rules of Engagement



- Roomies keep your face covering on at all times while in the lecture theatre
- Roomies touch in to the attendance monitoring system SEAtS using the card readers before entering the lecture theatre.
- Zoomies your live attendance will be monitored through Zoom.
- All students (Roomies and Zoomies) should have laptops with webcams that run Zoom

To maximise your Zoom experience:

- Join the Zoom meeting without audio and if you have joined with audio, select "Leave Audio"
- Add "R" (for Roomie) in front of your name if you're in the LT and "Z" (for Zoomie) if you are remote
- Use a headset with a microphone (noise cancelling is better) for breakout sessions
- Turn on your camera
- · Open both the 'Participant' and 'Chat' windows and in the View Options menu select "Side-by-side" mode
- Zoomies should raise your hand (in Zoom) if you have a question and wait to be asked to speak
- For a better audio experience only 1 person should speak at any one time
- If you have a question you can also write it in the Zoom chat window
- Address technical issues in a private message to the Facilitator in the Zoom chat window

Kamalini Ramdas

BIO



- BSc. (Hons) in Mathematics, St. Stephens College, Delhi, MS in Operations Research, U. of Delaware, Ph.D in Operations Management, Wharton
- Faculty at UT Austin for 2 years, Darden, U. of Virginia for 12 years, LBS for 13 years
- Research: Operational innovation (design / evaluation of radical operational innovations in healthcare, ICT and other areas)
- Teaching: MBA, EMBA Operations Management, EMBA Entrepreneurship, Business Model Experiments (elective) and executive education.

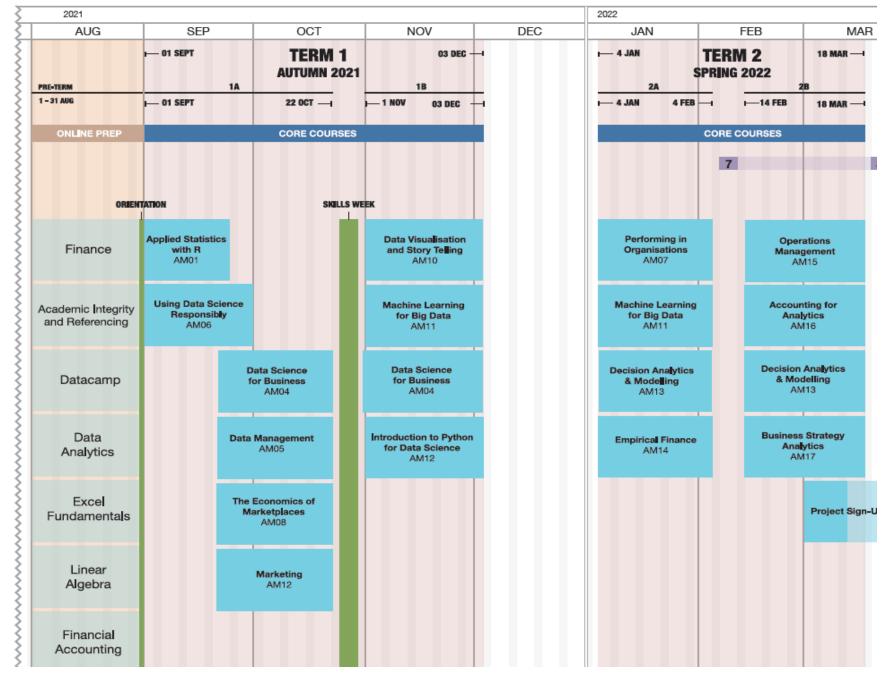
Kanishka Bhattacharya



BIO

- DPhil in Applied Statistics (in Statistical Genomics) and an MSc in Applied Statistics from Oxford University.
- research interests include building (novel) computationally efficient statistical algorithms for large scale compute challenges in the world of Statistical Genomics.
- With over 15 years of experience in statistical analysis of large scale, real life, and noisy datasets, Kanishka has developed cutting edge data science solutions in the domains of finance, retail, human genetics, epidemiology, media and advertising.
- worked as a Quant for two hedge funds and a digital advertising start up, alongside leading teams/practices and delivering data science programs for large global consulting firms.

MAM2022 Programme Overview





Skills Courses

- Intro to R
- Intro to Python
- Data Management
- Excel
- Interpersonal skills

Experiential Learning

- London LAB
- Global ImmersionField Trips (GIFTs)

Core Analytics courses

- Descriptive Analytics → What has happened and why?
 - Applied Statistics
 - Visualization
- Predictive Analytics → What will happen in the future?
 - Data Science
 - · Machine Learning for Big Data
- Prescriptive Analytics → What should we do about it?
 - Decision Analysis & Modelling

Management Applications

Marketing, Operations,
 Finance, Accounting

Management Background

- Using Data Science Responsibly
- Economics of Marketplaces
- Performing in Organizations
- Business Strategy



Course contents (first part of the course – Kamalini)

- Session 1: The Art & Science of Regression Models For Prediction
- Session 2: More on Using Linear Regression For Prediction
- Session 3: Workshop I Engineer an algorithm that sets interest rates for new Lending Club loans
 - Group assignment 1, due 6 days after the workshop
- Session 4: Classification using Logistic Regression
- Session 5: Workshop Invest in a portfolio of Lending Club loans
 - Individual project 1, due 13 days after the end of the workshop

Course contents (second part of the course – Kanishka)

See canvas syllabus

The Lending Club case

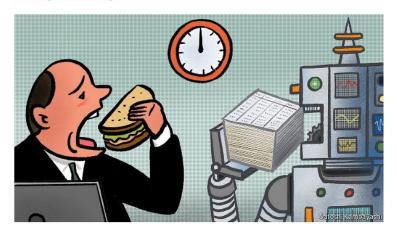
Engineer an algorithm that sets interest rates for new Lending Club loans

- Why would such an algorithm be useful?

Unshackled algorithms

Machine-learning promises to shake up large swathes of finance

In fields from trading to credit assessment to fraud prevention, machinelearning is advancing



Print edition | Finance and economics > May 25th 2017





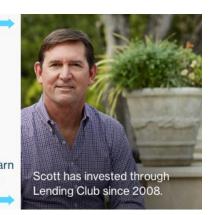






MACHINE-LEARNING is beginning to shake up finance. A subset of artificial intelligence (AI) that excels at finding patterns and making predictions, it used to be the preserve of technology firms. The financial





Innovation transforms lending

Lending Club is the world's largest marketplace connecting borrowers and investors, where consumers and small business owners lower the cost of their credit and enjoy a better experience than traditional bank lending, and investors earn attractive risk-adjusted returns.4

Here's how it works:

- Customers interested in a loan complete a simple application at LendingClub.com
- We leverage online data and technology to quickly assess risk, determine a credit rating and assign appropriate interest rates. Qualified applicants receive offers in just minutes and can evaluate loan options with no impact to their credit score
- Investors ranging from individuals to institutions select loans in which to invest and can earn monthly returns

The entire process is online, using technology to lower the cost of credit and pass the savings back in the form of lower rates for borrowers and solid returns for investors

The Art & Science of Using Linear Regression for Prediction

- ICE the data: Inspect, Clean, Explore
- Fit several reasonable models (iterative process!)
 - Ordinary Least Squares (OLS) estimation
 - Feature engineering: Non-linear terms, interactions, categorical variables, look beyond the data
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 - Sample size determination
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 - Regularization and LASSO regression
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Inspect

- Understand each variable definition
- Note the units of measurement (e.g., do not confuse lbs with Kg)
- Identify any data issues (missing values, incorrect entries, etc.)

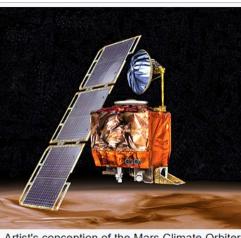
Clean

- Decide what to do with missing values
- Code variables correctly (e.g., numerical variables, factors, dates)
- Rearrange the data set if needed to be tidy (each row should be one unit of analysis, each column should be one feature associated with this unit of analysis)

Explore

- Articulate hypotheses as to what may be happening discuss with colleagues / experts
- Create correlation charts, histograms, scatter plots, etc

Most project failures are due to improper ICE-ing!

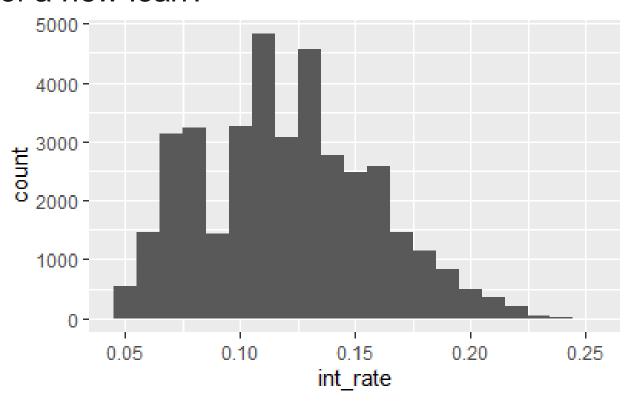


Artist's conception of the Mars Climate Orbiter



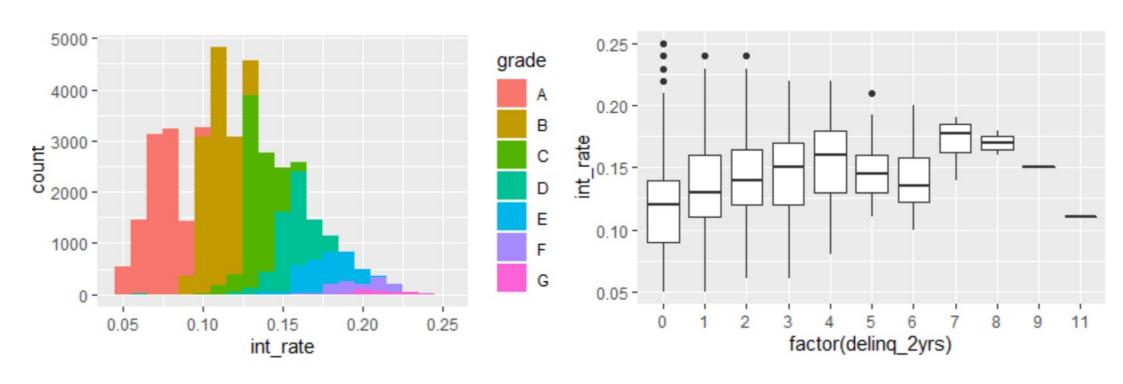
Lending Club – our goal is to predict interest rates of new loans

 Histogram: Based on this, what would be reasonable guess for the interest rate of a new loan?



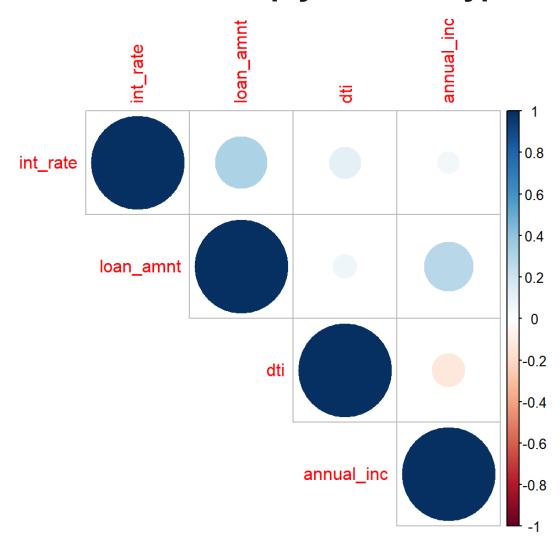
ICE the data

 Based on this, what would be reasonable guess for the interest rate of a new loan?



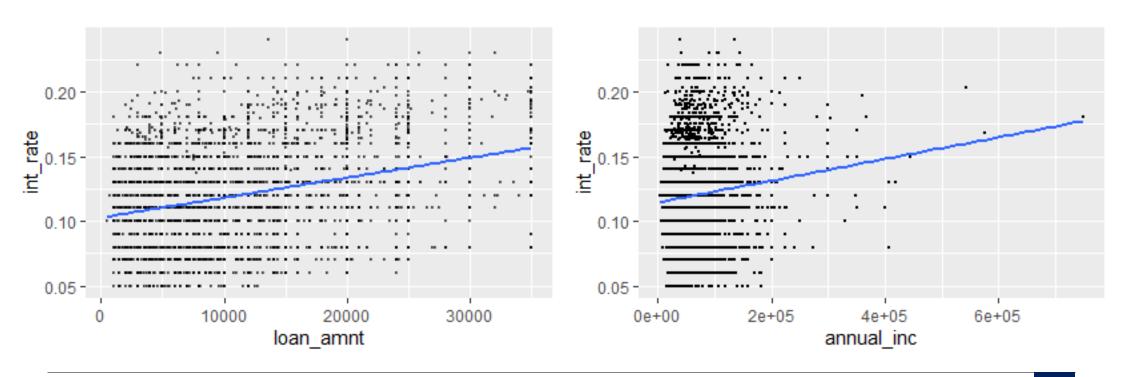
ICE the data

Correlation table: Does this help you form hypotheses?





 Based on this, what would be reasonable hypotheses for the drivers of interest rates?

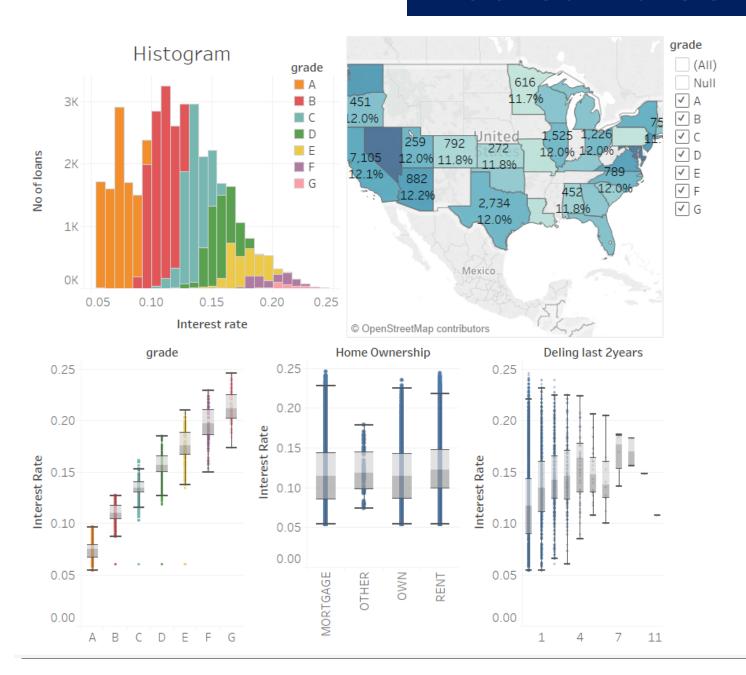




```
#histogram of interest rtes
                                                                        102
                                                                              #scatter plots
     ggplot(lc_clean, aes(x=int_rate))+
77
                                                                        103
       geom_histogram(binwidth=0.01)+scale_x_continuous(labels =
                                                                              x=loan_amnt)) +
78
     scales::percent) +labs(x="Interest Rate")
                                                                        104
79
                                                                        105
     #histogram with colour for different grades and term.
                                                                              Amount ($)")
80
     ggplot(lc_clean, aes(x=int_rate, fill=grade))+
                                                                        106
     geom_histogram(binwidth=0.01)+scale_x_continuous(labels =
                                                                        107
     scales::percent)+ labs(x="Interest Rate")
                                                                        108
82
                                                                              x=annual_inc) +
     ggplot(lc_clean, aes(x=int_rate, fill=term))+
                                                                        109
                                                                        110
     geom_histogram(binwidth=0.01)+scale_x_continuous(labels =
     scales::percent)+ labs(x="Interest Rate")
                                                                              Income ($)")
                                                                        111
84
     #density plot with colour for different grades.
85
                                                                        112
                                                                        113
     ggplot(lc_clean, aes(x=int_rate, fill=grade, alpha = 0.2))+
86
                                                                              delinq_2yrs)) +
87
       geom_density()+
                                                                                geom_boxplot()+
       facet_grid(rows = vars(grade))+
                                                                        114
88
       theme_bw()+
89
                                                                        115
                                                                                # geom_jitter()+
                                                                        116
       theme(legend.position = "none")+
                                                                                theme_bw()+
90
       scale_x_continuous(labels = scales::percent) + labs(x="Interest 117")
91
                                                                        118
     Rate")
92
                                                                        119
93
     #boxplot with colour for different home_ownerhsip
                                                                        120
     ggplot(lc_clean, aes(x=home_ownership, y=int_rate,
                                                                              rate charged?",
     colour=home_ownership))+
                                                                        121
       geom_boxplot()+
                                                                              Rate"
95
       theme bw()+
                                                                        122
96
       theme(legend.position = "none")+
97
       coord_flip()+ scale_y_continuous(labels=scales::percent)+
98
       labs(y="Interest Rate", x="Home Ownership")
99
```

```
ggplot(lc_clean[seg(1, nrow(lc_clean), 10), ] , aes(y=int_rate,
  geom_point(size=0.1, alpha=0.5)+
  geom_smooth(method="lm", se=0) + labs(y="Interest Rate", x="Loan
ggplot(lc_clean[seq(1, nrow(lc_clean), 10), ] , aes(y=int_rate,
  geom_point(size=0.1)+
  geom_smooth(method="lm", se=0) +labs(y="Interest Rate", x="Annual
#box plot for delinquencies
ggplot(lc_clean , aes(y=int_rate, x=deling_2yrs, colour=
   scale_y_continuous(labels=scales::percent)+
  theme(legend.position = "none")+
    title = "Do delinquencies in the last two years impact interest
    x= "Number of delinquecies in last two years", y="Interest
```

You can also use Tableau!



Wait until you've had the visualization course

Visualization is all I need...NOT

Visualization is great for

- Understanding the data
 - Get a sense of what different variables mean
 - Investigate if there are any data-quality issues
- Quickly (and very roughly!) testing intuition and generating hypotheses

Visualization is not so good at

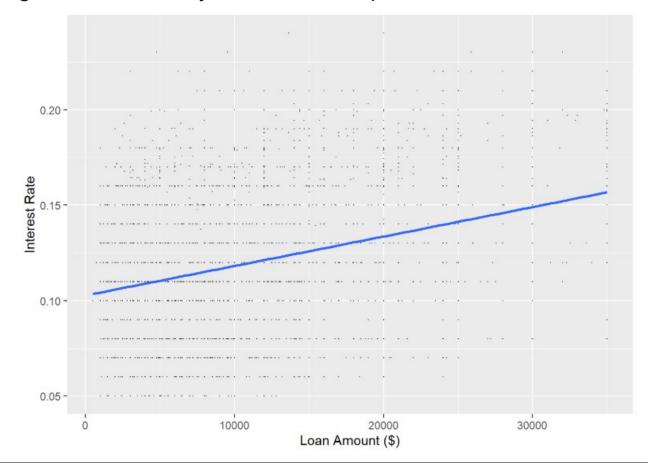
- Inference: Do interest rates increase due to past delinquencies once we control for credit rating and sampling error? Does higher income lead to higher interest rate once we control for loan amount and sampling error? How much would the interest rate decrease if you increased the loan amount by \$10K?
 - Here we care about the estimated coefficients and their errors
- Predicting the future: What interest rate should a new loan be charged? How likely is it that this estimate is off?
 - Here we care about the ability to make accurate predictions (out of sample prediction error)

The Art & Science of Using Linear Regression for Prediction

- ICE the data: Inspect, Clean, & Explore
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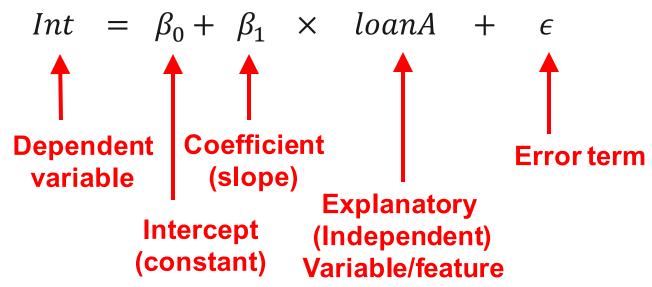
Let's start with a single variable

- Is there a relationship between interest rate and loan amount?
 - What would you expect this relationship to be?
 - How would investigate if there really is a relationship?



Setting up a model

 Express the relationship between the interest rate and the loan as a mathematical equation



- Intercept β_0 is the expected interest rate for a loan with 0 loan amount
- Slope β_1 is the expected increase in the interest rate when loan amount increases by 1 unit
- $-\epsilon$ is the part of the variation in interest rates that cannot be explained by loan amount. This is assumed to be a random variable with mean zero and variance σ^2

Estimate the model

$$Int = \beta_0 + \beta_1 \times loanA + \epsilon$$

- Estimation question: How do we choose the "best" line?
 - In other words, how do we choose the values of β_0 , β_1 , σ^2 that best describe the data?
- Since we are interested in forecasts, maybe choose the line that will
 - minimise sum (or average) forecast error? not useful
 - minimise sum of absolute deviations? possible
 - minimise sum of squared deviations? focuses on avoiding big forecast errors and minimises standard error of forecast (BLUE: Best Linear Unbiased Estimator)
- Can run Ordinary Least Squares (OLS) regression using software
 - Excel Data Analysis tool pack
 - R and the basic "Im" command or more likely using the caret library

Least squares error algorithm

$$Int = \beta_0 + \beta_1 \times loanA + \epsilon$$

- Assume we use our sample to estimate $\,eta_0$ and $\,eta_1$ as $b_0=0.12, b_1=0.01$
- For the first loan, the residual would be

$$-e_1 = 0.1095 - (0.12 + 0.01 \times 5) = -0.0605$$

- Similarly, we can estimate e_2 , e_3 , e_4 , ..., $e_{10,000}$
- The sum of squared residuals is $RSS = (e_1^2 + e_2^2 + \cdots + e_{10,000}^2)$

Loan	(K USD) Amount	Interest Rate	
1	5	10.95%	
2	2.5	14.27%	
3	2.4	15.96%	
4	10	13.49%	
5	3	11.69%	
6	5	15.00%	
7	7	15.96%	
8	8	18.64%	
9	5.6	21.28%	
10	5.375	12.69%	
11	6.5	14.65%	
12	12	17.69%	

12 randomly chosen observations

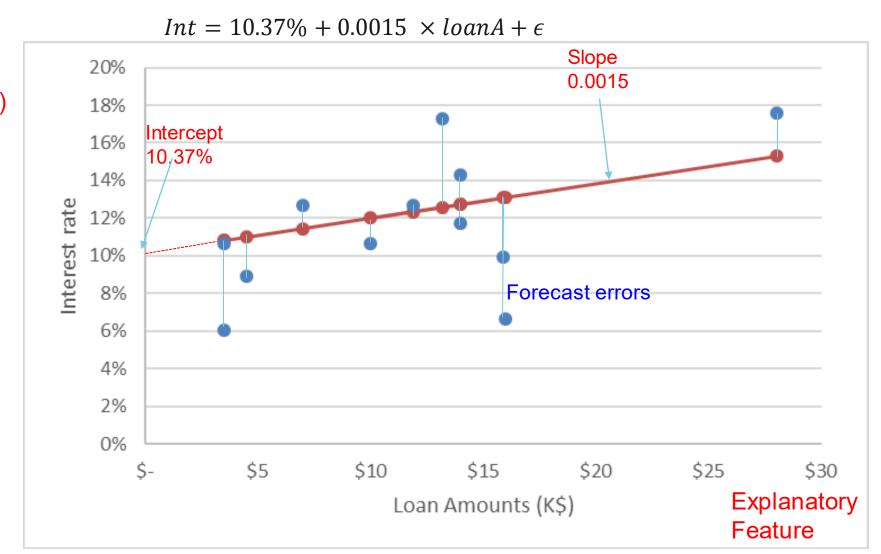
Mean interest rate =15.19% Standard Deviation = 2.97%

Watch Video

- The least squares error principle asks
 - What values of b_0 , b_1 make RSS the smallest?
 - Can use a "solver" to find these values
 - The problem turns out to be "simple" enough to solve analytically (i.e., we don't need to run a solver as there is a (complicated) formula we can use)

Line of best fit: Minimize sum of squared errors

Dependent Variable (interest rate)



Assess Goodness of Fit

Models fitted on the whole dataset using R

Model with only intercept

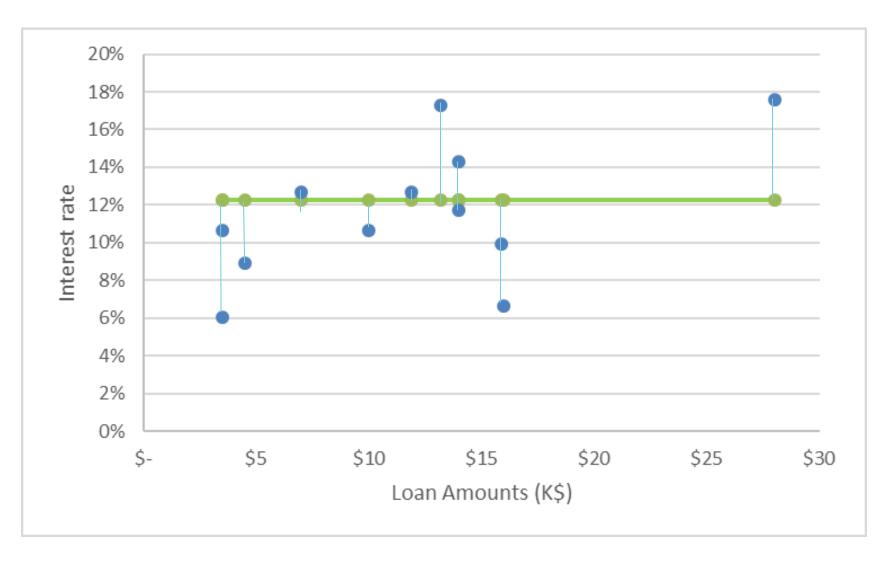
Model with intercept & Loan amount

```
Call:
lm(formula = int rate ~ loan amnt, data = lc clean)
Residuals:
     Min
                10 Median
-0.089412 -0.028316 -0.001426 0.024900 0.128370
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.028e-01 3.292e-04 312.15 <2e-16 ***
loan amnt 1.555e-06 2.438e-08
                                63.77 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.03541 on 37867 degrees of freedom
Multiple R-squared: 0.09698,
                               Adjusted R-squared: 0.09695
F-statistic: 4067 1 and 37867 DF, p-value: < 2.2e-16
```

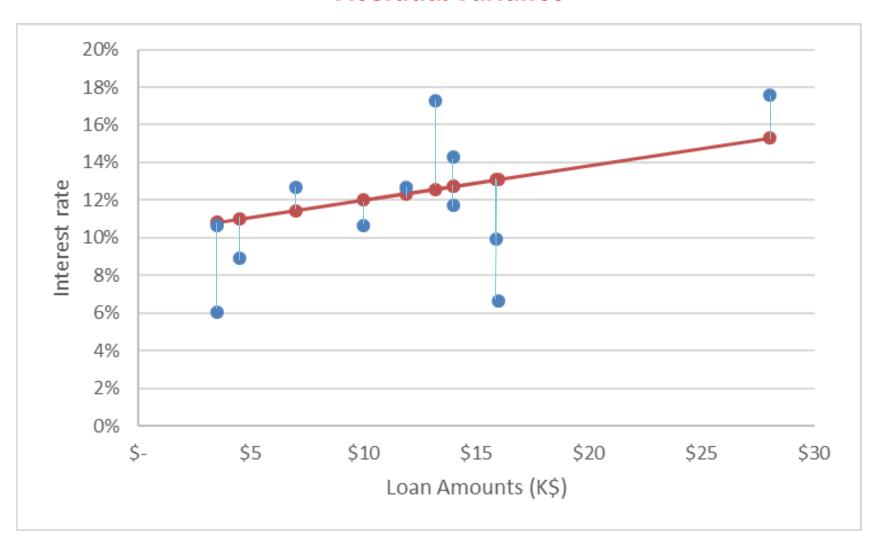
R-squared (explained variance / total variance)

- proportion of the total variance "explained" by the model
- adjusted: makes an adjustment for the number of variables in the model

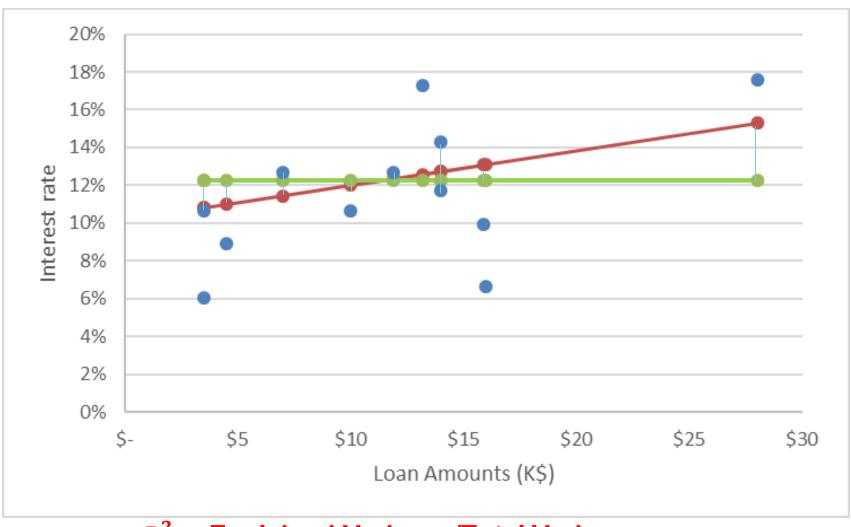
Total Variance
Model with only intercept



Model with intercept & Loan amount Residual Variance

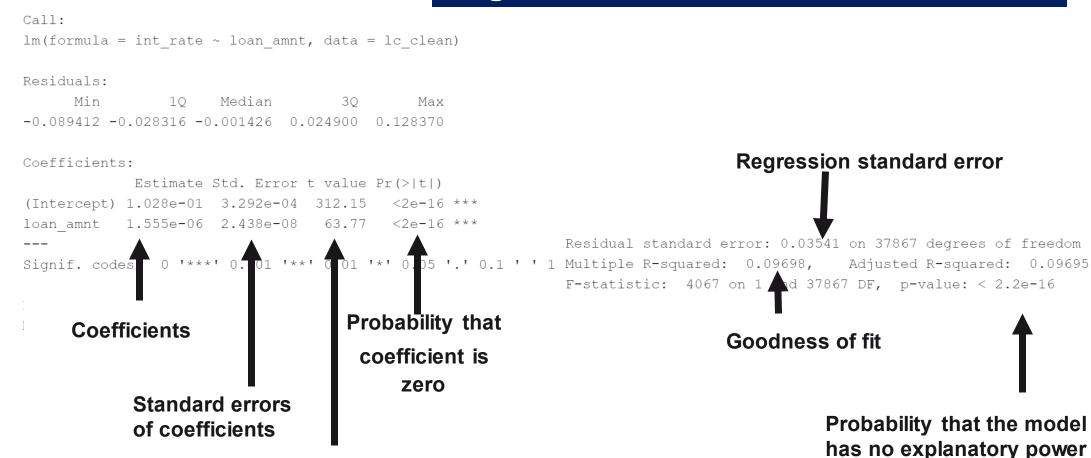


Explained Variance



 R^2 = Explained Variance/Total Variance

Significance of individual features

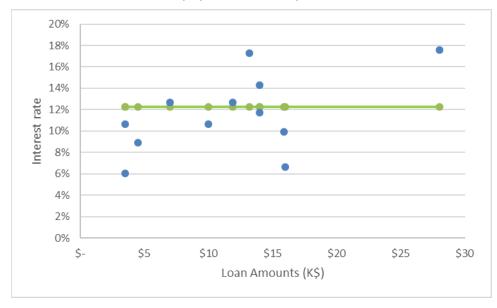


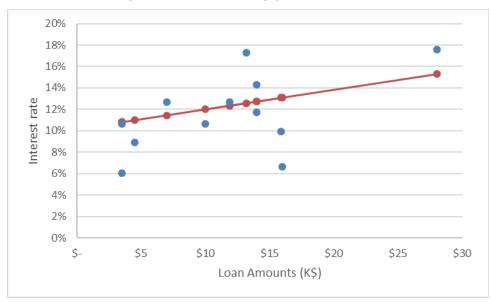
t-value: Coefficient divided by its Standard Error

Significance of individual feature

Is there a (linear) relationship between interest rate and loanA?

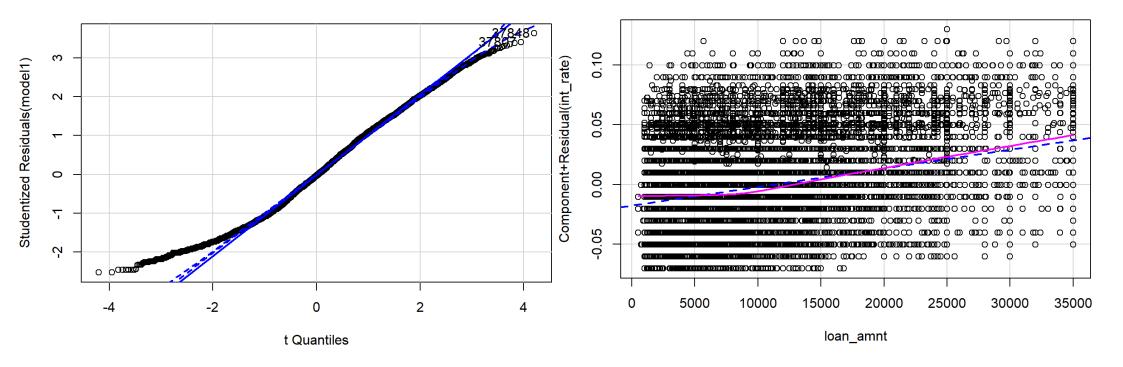
- Yes \Rightarrow slope (b) is different from zero
- No \Rightarrow slope (b) is zero (we obtained a non-zero slope by chance only)





- Test: Could b be equal to 0 ? (Could it be that there is no relationship ?)
 - compute t-statistic (b / standard error of b)
 - if t-statistic > 2, b is probably not zero (95% confident)
 - p-value = probability that b could be zero (if this is <5%, confident that $b \neq 0$)

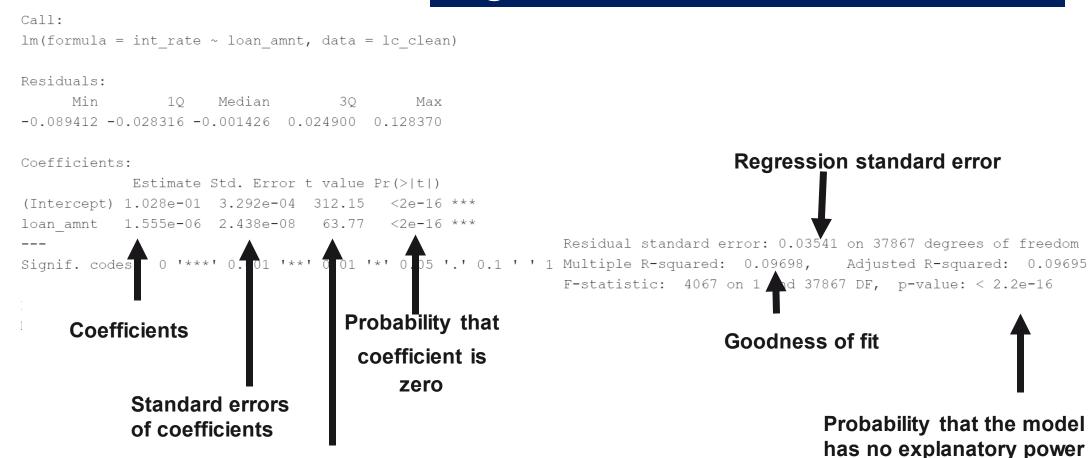
Goodness of fit Examining residuals



QQ plot (left) examines how well the residuals follow the normal distribution—if they don't then the standard errors estimated are not reliable

Residual Scatter plot (right) examines if the residuals are randomly distributed for different loan amounts – if they are not then perhaps there is a non-linear relationship \rightarrow investigate

Significance of individual features



t-value: Coefficient divided by its Standard Error

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Feature Engineering

Use information from the dataset

- Candidates: *Term, income, dti, grade*, number of delinquencies, employment length, etc
 - Some of these are numerical others are factors. How do we use factor variables?
- Interaction terms: Perhaps the loan amount affects 36-month loans differently than 60-month loans. How do you model this?
 - Interactions between two factor features, a factor and a numerical feature, two numerical features
- Non-linear terms: Perhaps a small increase in the loan amount doesn't affect interest rate so much but a large increase does. How would you model this?
 - **Polynomial terms** (powers of a feature) or any **other non-linear transformation** (better have a good reason for the non-linear transformation)
 - Dummy variable creation → converting a numerical variable into a factor variable (e.g., low, mid, high income, or *deciles of income*). This is a non-parametric way of modelling non-linear relationships
- Look for data outside your model
- Feature engineering is more of an art than science! Know your context (or work with people who do)!

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- If you have a question you can also write it in the Zoom chat window
- · Address technical issues in a private message to the Facilitator in the Zoom chat window

All workshops will be virtual (Kamalini & TA's will join virtually) however, we have booked the following rooms below for those who will be on campus.

STUDY GROUP	ROOM
1	AG01
2	AG02
3	AG03
4	AG04
5	AG05
6	AG06
7	AG07
8	AG08
9	AG09
10	AG10
11	AG11
12	AG12

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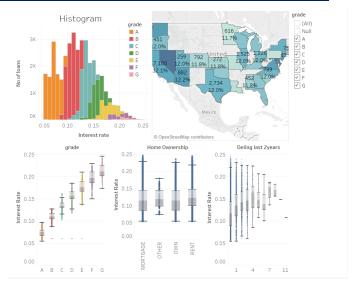
Office Hours- Virtual

Date & Time	Meeting Link	MeetingID	Passcode
13/10/21 16:30-17:30	https://zoom.us/j/97620086295	976 2008 6295	703473
20/10/21 16:00-17:00	https://zoom.us/j/91582880144	915 8288 0144	372574
27/10/21 16:00-17:00	https://zoom.us/j/91582880144	915 8288 0144	372574
03/11/21 16:00-17:00	https://zoom.us/j/91582880144	915 8288 0144	372574

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- Our goal is to come up with an algorithm that predict the interest rate that lending club will charge to new loans
 - We ICEd the Data: Inspect, Clean, Explore
 - We used the method of ordinary least squares to fit a model that uses loan amount as a predictor
 - This is the model that reduces the prediction error as much as possible
- How good is the model?
 - Is the loan amount statistically significant?
 - Does the model have any explanatory power?
 - How much error remains in our predictions?
- What can we do to improve predictions?

Recap from last time



Call:

lm(formula = int rate ~ loan amnt, data = lc clean)

Residuals:

Min 1Q Median 3Q Max -0.089412 -0.028316 -0.001426 0.024900 0.128370

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 1.028e-01 3.292e-04 312.15 <2e-16 ***

loan_amnt 1.555e-06 2.438e-08 63.77 <2e-16 ***

--
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03541 on 37867 degrees of freedom

Multiple R-squared: 0.09698, Adjusted R-squared: 0.09695

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Factor Features

- Have added dti and annual income as features. Was this a good idea?
- Loans can be taken for either 36 or 60 months
 - Create a dummy variable that takes the value 1 if coefficients: the loan is 60 months or 0 otherwise – add this variable to the model
 - No need to create another dummy that takes the value 1 if the loan is 36 months and zero otherwise (The two dummy variables convey they signif. codes: same information – they are colinear)
 - The coefficient of the "Term60" can be interpreted as the average interest rate difference between 60 month and 36 month loans: on average, a 60 month loan has a 3.22% higher interest rate than a 36 month loan!
 - Did you expect it to be positive? Does it have explanatory power?
- How much better is this model?

```
call:
lm(formula = int_rate ~ loan_amnt + term + dti + annual_inc,
    data = lc_clean)
```

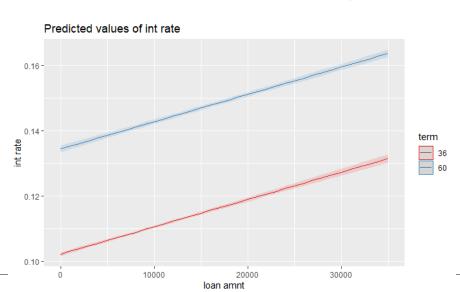
Residuals:

```
Median
     Min
-0.091963 -0.027528 0.000165
```

```
(Intercept)
             9.707e-02 4.713e-04 205.967
loan amnt
             8.384e-07
                        2.521e-08
                                   33.261
                                   79.269
term60
             3.226e-02
                        4.070e-04
                                  15.040
dti
             3.830e-04
                        2.546e-05
                                            <2e-16 ***
                        2.869e-09
annual_inc
            -1.840e-10
                                   -0.064
                                             0.949
                  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Estimate Std. Error t value Pr(>|t|)

Residual standard error: 0.03264 on 37864 degrees of freedom Multiple R-squared: 0.233, Adjusted R-squared: 0.2329 F-statistic: 2875 on 4 and 37864 DF, p-value: < 2.2e-16



call:

Interaction effects

 What if the loan amount influences differently the interest rate for 60 month loans than 36 month loans. How would you investigate this hypothesis?

```
lm(formula = int_rate ~ loan_amnt + term + dti + annual_inc +
              term:loan_amnt, data = lc_clean)
          Residuals:
                                                                                Predicted values of int rate
                Min
                                  Median
                            10
                                                 30
                                                           Max
Interaction_0.092635 -0.027663 0.000206 0.024082
term in R
                                                                             0.16
          Coefficients:
                             Estimate Std. Error t value Pr(>|t|)
          (Intercept)
                            9.806e-02 5.066e-04 193.575
                                       3.201e-08 22.924
                                                                            int rate
0.14
          loan_amnt
                            7.337e-07
          term60
                            2.879e-02 7.703e-04 37.377
                                                 15.012
                                       2.546e-05
          dti
                            3.822e-04
          annual_inc
                            2.614e-10
                                       2.869e-09
                                                    0.091
                                                              0.927
                                                                             0.12 -
          loan_amnt:term60 2.615e-07
                                       4.929e-08
                                                    5.306 1.12e-07 ***
          Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '. 0.1 ' 1
                                                                             0.10 -
                                                                                                                30000
          Residual standard error: 0.03263 on 37863 degrees of freedom
                                                                                                  loan amnt
          Multiple R-squared: 0.2336, Adjusted R-squared: 0.2334
```

 The coefficient of LoanA is 7.337e-07 – this implies than on average an additional \$10,000 of loan will increase the interest rate of a 36 month loan by 0.73%

F-statistic: 2308 on 5 and 37863 DF, p-value: < 2.2e-16

• The coefficient of interaction "Loan x Term60_dummy" is 2.615e-07 -- this implies that on average an additional \$10K of loan will increase the interest rate of a 60month loan by 0.73%+0.26%=0.99%

Feature Engineering

Use information from the dataset

- Candidates: *Term, income, dti, grade*, number of delinquencies, employment length, etc
 - Some of these are numerical others are factors. How do we use factor variables?
- Interaction terms: Perhaps the loan amount affects 36-month loans differently than 60-month loans. How do you model this?
 - Interactions between two factor features, a factor and a numerical feature, two numerical features
- Non-linear terms: Perhaps a small increase in the loan amount doesn't affect interest rate so much but a large increase does. How would you model this?
 - **Polynomial terms** (powers of a feature) or any **other non-linear transformation** (better have a good reason for the non-linear transformation)
 - Dummy variable creation → converting a numerical variable into a factor variable (e.g., low, mid, high income, or **deciles of income**). This is a non-parametric way of modelling non-linear relationships

Model Comparison

- Suppose I run multiple models. How do I choose between them?
 - Smaller standard error → leading to more precise forecasts
 - Higher R² /adjusted R² → more variation explained
 - All coefficients significant (p < 0.05)
 - Simple models / sensible relationships
- Let's see some examples

Model Comparison in R

```
Model 2:
                   Call:
                   lm(formula = int rate ~ loan amnt + term + dti + annual inc +
                       grade, data = lc clean)
                   Call:
Model 3:
                  lm(formula = int rate ~ loan amnt + term + dti + annual inc +
                      grade + loan amnt:grade, data = lc clean)
                                                                                   Polynomial term
                                                                                   of order 2
                   Call:
Model 4:
                   lm(formula = int rate ~ poly(loan amnt, 2) + term + term:grade +
                                                                                   Deciles of loan
                       I(1/(dti + 1)) + annual inc + grade, data = lc clean)
                                                                                   amount as factor
                   lc clean <- lc clean %>% mutate(loan amnt decile = as.factor(ntile(loan amnt, 10)))
Model 5:
                   Call:
                   lm(formula = int rate ~ loan amnt decile + term + dti + annual inc +
                       grade, data = lc clean)
                     Res.Df
                            RSS Df Sum of Sq
                                                           Pr (>F)
                                                F
                   1 37867 47.485
ANOVA
                                        43.266 43157.40 < 2.2e-16 ***
                   2 37858 4.219 9
                   3 37852 4.186 6 0.033 49.60 < 2.2e-16 ***
                   4 37851 4.106 1 0.080
                                                719.22 < 2.2e-16 ***
                   5 37850 4.216 1
                                        -0.110
                   Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Feature Engineering & Multicollinearity

- What if one feature is perfectly correlated with another feature (or a linear combination of other features)?
 - For example, this would be the case if
 - We estimate a model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$, and
 - It happens to be the case that $x_1 = \alpha x_2 + \gamma x_3$
 - Not possible to estimate the model as the feature matrix is not full rank
- What if some features are highly correlated but not perfectly so?
 - Model is still consistent
 - Predictions unbiased, prediction errors estimated correctly; R-squared can be interpreted as usual
 - Estimated coefficients can become unreliable (large confidence intervals, large p-values)
 - This is not a problem for predictions, only a problem for inference!
 - If you are interested in inference you can calculate variance inflation factors (VIF) to assess the problem

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 - **Polynomial terms** (powers of a feature) or any **other non-linear transformation** (better have a good reason for the non-linear transformation)
 - Dummy variable creation → converting a numerical variable into a factor variable (e.g., low, mid, high income, or *deciles of income*). This is a non-parametric way of modelling non-linear relationships
- Look for data outside your model
- Feature engineering is more of an art than science! Know your context (or work with people who do)!

- ICE the data: Inspect, Clean, & Explore
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Out of sample testing

- Assessing how good the predictions of a model are in-simple is helpful but could be misleading → problem of overfitting
- Gold standard out-of-sample goodness of fit
- Hold out method: Randomly partition the data in two sets
 - Training set (~75% of the data): Data in this dataset is used to fit the model
 - Testing set (~25% of the data): Data in this dataset is used to assess how good is the model
 - Check RMSE and R² of the validation set and compare them to the training set. If difference is small then overfitting is not a problem. In any case, report the out of sample RMSE and R²
 - Typically, the fit out-of-sample is worse than in-sample and more representative of the model's true predictive value -- it does not suffer from overfitting

Out-of-Sample testing in R

• For model 2

```
265
      set.seed(4444)
266
      train_test_split <- initial_split(lc_clean, prop = 0.75)
267
      training <- training(train_test_split)</pre>
268
      testing <- testing(train_test_split)</pre>
269
270
      #Fit model2 to the training set
271
      model2<-lm(int_rate ~ loan_amnt + term+ dti + annual_inc + grade ,
      data = training)
272
273
      #Calculate the in sample RMSE of the model
274
      rmse_training<-sqrt(mean(residuals(model2)\(^2\))
275
276
      #USe the model to make predictions out of sample in the testing set
277
      pred<-predict(model2.testing)</pre>
278
279
      # Calculate the out of sample RMSE of the model
280
      rmse_testing<- RMSE(pred,testing$int_rate)</pre>
```

- [1] "RMSE in sample: 0.0105306346945014"
- [1] "RMSE out of sample: 0.0106305940013813"
- [1] "Increase in error: 0.9492%."

k-fold Cross-validation

- Splitting the data in training and validation means that
 - we reduce the data used for training which may be a problem if we have a relatively small dataset
 - we don't get to use every data point for training and for testing (only one of the two)
- K-fold cross-validation overcomes this problem:
 - Randomly divide data into K equal-size groups (referred to as folds)
 - Use the first fold as validation and the other K-1 for training and compute RMSE and R²
 - Repeat this K times; each time a different fold is treated as a validation set
 - Use the average RMSE and average R² to assess goodness of fit
 - Estimate the model again using all of the data and report these coefficients but report RMSE and R² estimated out of sample
- Typically set K = 5 or 10
 - The more folds the more accurate the out of sample estimation but the longer it takes to run

Data

Validation	Train	Train	Train	Train
1	2	3	4	5

k-fold cross validation in R

```
#the method "cv" stands for cross validation. We re going to create 10
      folds.
285
286
      control <- trainControl (
          method="cv".
287
288
          number=10.
          verboseIter=TRUE) #by setting this to true the model will report its
289
      progress after each estimation
290
     #we are going to train the model and report the results using k-fold cross
      validation
      plsFit<-train(
292
          int_rate ~ loan_amnt + term+ dti + annual_inc + grade ,
293
294
          lc_clean,
         method = "lm".
295
          trControl = control
296
297
298
299
      summary(plsFit)
300
```

The coefficients reported are based on the whole dataset.

RMSE and R squared are based on the average of the 10 out-of-of sample validations

```
## Call:
## lm(formula = .outcome ~ ., data = dat)
## Residuals:
        Min
                         Median
                   10
                                       30
                                                Max
## -0.118827 -0.007035 -0.000342 0.006828 0.035081
## Coefficients: (1 not defined because of singularities)
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.072e-01 6.347e-04 326.404 < 2e-16 ***
## loan amnt
               1.475e-07 8.284e-09
                                      17.809
## term60
               3.608e-03 1.419e-04
                                      25.431 < 2e-16 ***
## dti
               4.328e-05 8.269e-06
                                       5.234 1.66e-07 ***
## annual inc -9.734e-10 9.283e-10
                                      -1.049
                                                0.294
## gradeA
              -1.355e-01 6.208e-04 -218.245 < 2e-16 ***
              -9.994e-02 6.142e-04 -162.713 < 2e-16 ***
## gradeB
## gradeC
              -7.533e-02 6.172e-04 -122.039
## gradeD
              -5.376e-02 6.213e-04 -86.534 < 2e-16 ***
## gradeE
              -3.550e-02 6.328e-04 -56.091
                                             < 2e-16 ***
## gradeF
              -1.594e-02 6.846e-04 -23.288
                                             < 2e-16 ***
  gradeG
                                                   NΑ
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.01056 on 37858 degrees of freedom
## Multiple R-squared: 0.9198, Adjusted R-squared: 0.9197
## F-statistic: 4.34e+04 on 10 and 37858 DF, p-value: < 2.2e-16
```

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Sample size determination

Remember, any predictions we make come with error

$$Int = \beta_0 + \beta_1 \times loanA + \epsilon$$

- Any prediction will be subject to two sources of error
 - Estimation error \rightarrow because we estimate the coefficients β_0 , β_1 with error
 - Stochastic (random) error \rightarrow because of the error term ϵ
- Having more data (i.e., more rows) helps us reduce estimation error

 → more accurate estimates of the coefficients → more accurate
 forecasts
- Having more datapoints does not reduce the second source of error
 - What can we do to reduce the second source of error?
- How much data is enough?

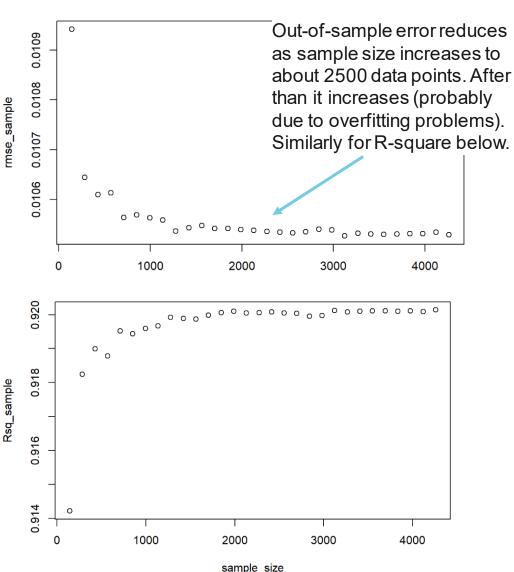
Sample size determination Learning curves

- We can check if collecting more rows of data would improve the out-of-sample performance of a model
 - Fix a testing data set (say 25% of the data randomly drown)
 - From the remaining 75% of the data draw training sets of different size. Start small and progressively increase the sample size.
 - Estimate the model on this training set and evaluate the performance of the model on the fixed testing dataset. Record how the performance changes as we increase the sample size of the training set.
 - If the performance stabilizes then estimation error is not a big deal and we have enough data
 - If we use all the training data and we are still in the "steep" part of the curve the we do not have enough data → Need to collect more and/or run a LASSO regression instead (more on this later)

Learning Curves in R

```
302 • ```{r, learning curves}
      #select a testing dataset (25% of all data)
304
      set.seed(102)
305
      train_test_split <- initial_split(lc_clean, prop = 0.75)</pre>
306
      testing <- testing(train_test_split)</pre>
      remaining <- training(train_test_split)</pre>
308
309
      #We are now going to run 30 models starting from a tiny training set and
310
      progressively increasing the size of the training set. The testing set
      remains the same in all iterations.
311
312
      #define some variables
313
      rmse sample <- 0
      sample_size<-0
314
315
      Rsg sample<-0
316
317
      #start a for loop
318 - for(i in 1:30) {
      #from the remaining dataset select a smaller subset to traing the data
320
321
      train_test_split <- initial_split(remaining, prop = 0.005+(i-1)/200)
      training <- training(train_test_split)</pre>
322
323
324
      sample_size[i]=nrow(training)
325
326
      #train the model on the small dataset
327
      model<-lm(int_rate ~ loan_amnt + term+ dti + annual_inc + grade , training)</pre>
328
      #test the performance of the model on the large testing dataset
      pred1<-predict(model.testing)</pre>
329
      rmse_sample[i]<-RMSE(pred1,testing$int_rate)</pre>
331
      Rsq_sample[i]<-R2(pred1,testing$int_rate)</pre>
332
333
      plot(sample_size,rmse_sample)
334
      plot(sample_size,Rsq_sample)
```

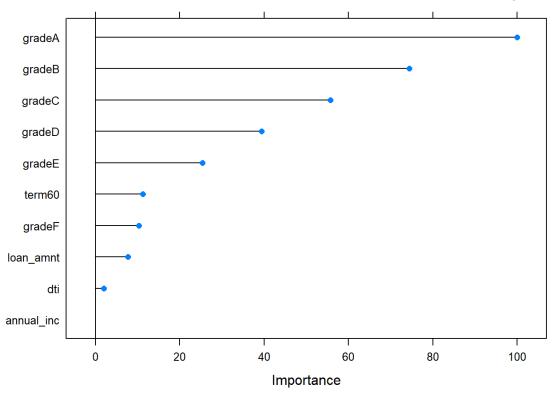
For Model 2



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Not all features are equally important

- For model 2, graphical representation of the relative importance of different features in explaining interest rates
- Clearly, the annual income and dti have tiny importance compared to Grade A
- Can we use a plot such as this to select a subset of important features?
 - Not so easy because off correlations! We need to check all subsets to be sure which combination produces the best results.



Feature selection

- Let say we have 10 candidate features to select from. How many possible subsets are there? What if it was 100 candidate features?
- Automated feature selection algorithms such as stepwise regression try to identify
 a sensible combination of features that performs well out of sample without having
 to check all possible combinations
 - Backward step: Start with the full model and remove variables one at a time based on explanatory power
 - Forward step: Start with a null model and add variables one at a time based on their correlation with the dependent variable (or some other measure)
 - Mixed step: A combination of the above
- **Stepwise regression** selects the best model in terms of RMSE or R². It can (and should be) combined with some out-of-sample validation method (e.g., k-fold cross validation) to avoid overfitting the data
- These are easy to implement but in general they do not guarantee to find the best model. They can be slow for large models and are a bit of a black box... Use them cautiously!

Stepwise regression in R

```
391 ▼ ```{r, automated variable selection}
392
393
      #set the out-of-sample validation method
394
      control <- trainControl (
          method="CV",
395
396
          number=5,
          verboseIter=FALSE)
397
398
      #Find the best model with 10 to 16 variables with backward induction
399
      BackFit <- train(int_rate ~ loan_amnt + term+ dti + annual_inc + grade
400
      +grade:loan_amnt,
          lc_clean,
401
          method = "leapBackward". #can chance method to "leapSeg". "leapForward"
402
          tuneGrid = data.frame(nvmax = 10:16), #Will find the best model with
403
      10:16 variables
          trControl = control
404
405
406
      #show the results of all models
407
      BackFit$results
408
      #simmarize the model of best fit and its coefficients
409
410
      summary(BackFit$finalModel) #depending on the number of models estimated,
      the output of this command could be long
      coef(BackFit$finalModel,BackFit$bestTune$nvmax)
411
412
      . . .
413
```

The only difference in the train command is the "method."

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- All methods of choosing between models we have discussed (e.g., manual comparisons, stepwise regression) relied on estimating a bunch of models using the Ordinary Least Squares (OLS) algorithm and then comparing their performance based on some (out-of-sample) performance measure (e.g., RMSE)
- As such they are passive if the algorithm overfits the data we find out but we cannot do anything to correct it!
- An alternative method that tries to actively avoid overfitting is regularization
 - Main idea is to modify the OLS algorithm so that the estimated model becomes less sensitive to the training set
 - By changing the least squares model the estimated coefficients and the predictions become biased
 - But the bias is something worth tolerating as the model's predictions become less variable (lower error)
- There are two popular regularization methods, Ridge regression and LASSO
 - They work much better than ordinary linear regression especially for small datasets ("steep part" of the learning curve)
 - LASSO regression is considered better for models with a lot of potentially irrelevant variables as it forces the
 estimated coefficient of irrelevant variables to be zero more so than ridge regression
 - LASSO stands for Least Absolute Shrinkage and Selection Operator. We will focus on this.
 - For more information I recommend these you tube clips from StatQuest Part 1 & Part 2

LASSO Regression

- Set up a linear model as usual, e.g., $Int = \beta_0 + \beta_1 \times loanA + \epsilon$
- OLS Regression finds the coefficients that minimize the sum of squared errors
- LASSO regression finds the coefficients that minimize the following objective:

Sum of Squared Errors + λ [sum of absolute value of estimated coefficients]

- The parameter $\lambda \geq 0$ (pronounced *lambda*) is called a *hyper-parameter* and it is user specified
 - If $\lambda = 0$ the model reduces to the OLS algorithm
 - If $\lambda > 0$ the model penalizes the objective for any coefficient that is different to zero
 - Therefore, for a coefficient to be different from zero by 1 unit it needs to reduce the sum of square errors by at least λ units
 - The larger the λ the more coefficients will move towards zero or become zero → this is the notion of shrinkage in LASSO
- We typically estimate the model using several values for λ and choose the best one using out of sample predictions based on k-fold cross validation.
 - This is called hyper-parameter optimization.

LASSO Regression

- Since coefficients of different variables are measured in different units, it is important to *standardize* any continuous variable (subtract the mean and divide by standard deviation). Otherwise, results will be misleading!
- Unlike linear regression, LASSO regression
 - Allows us to estimate a model even if we have more parameters to estimate than data points. Useful in a world of big data (e.g., detecting combinations of genes that may be associated with specific phenotypes / disease)
 - Allows us to estimate the model even if there are multicollinearity problems (even perfect collinearity)
 - Generates biased estimates for variable coefficients. So do not use if your goal is inference instead of prediction
- Most modern data-science applications working on big data would be using a LASSO model for prediction!



```
437 * ```{r, LASSO compared to OLS, warning=FALSE, message=FALSE}
     #split the data in testing and training. The training test is really small.
439
                                                                                 463
     set.seed(1234)
                                                                                        # Make predictions
                                                                                 464
     train_test_split <- initial_split(lc_clean, prop = 0.01)
                                                                                        predictions <- predict(lasso.testing)</pre>
                                                                                 465
     training <- training(train_test_split)
                                                                                 466
     testing <- testing(train_test_split)</pre>
444
                                                                                 467
                                                                                        # Model prediction performance
445
     #we will look for the optimal lambda in this sequence
                                                                                        LASSO_results<-data.frame(
                                                                                                                      RMSE = RMSE(predictions, testing$int_rate),
     lambda\_seg <- seg(0, 0.01, length = 1000)
                                                                                                                      Rsquare = R2(predictions, testing$int_rate)
                                                                                 469
     #lasso regression with using k-fold cross validation to select the best
                                                                                 470
      lambda
                                                                                 471
                                                                                       LASSO_results
448
     lasso <- train(
                                                                                       #compare the out of sample performance of the lasso regression to a linear
      int_rate ~ poly(loan_amnt,3) + term+ dti + annual_inc + grade
                                                                                        model's predictions on the same training/testing datasets
      +grade:poly(loan_amnt,3):term +poly(loan_amnt,3):term +grade:term,
                                                                                        model_lm<-lm(int_rate ~ poly(loan_amnt,3) + term+ dti + annual_inc + grade</pre>
450
      data = training,
      method = "glmnet",
                                                                                        +grade:poly(loan_amnt,3):term +poly(loan_amnt,3):term +grade:term,
451
       preProc = c("center", "scale"), #This option standardizes the data before
                                                                                        training)
     running the LASSO regression
                                                                                 474
                                                                                        predictions <- predict(model_lm,testing)</pre>
453
       trControl = control,
                                                                                 475
       tuneGrid = expand.grid(alpha = 1, lambda = lambda_seq) #alpha=1 specifies
                                                                                        # Model prediction performance
     to run a LASSO regression. If alpha=0 the model would run ridge regression.
                                                                                 477
                                                                                        OLS_results<-data.frame(
455
                                                                                 478
                                                                                          RMSE = RMSE(predictions, testing\sint_rate),
456
     # Model coefficients
                                                                                 479
                                                                                          Rsquare = R2(predictions, testing\sint_rate)
     coef(lasso$finalModel, lasso$bestTune$lambda)
457
                                                                                 480
458
     #Best lambda
     lasso$bestTune$lambda
                                                                                 481
     # Count of how many coefficients are greater than zero and how many are
                                                                                 482
                                                                                        OLS_results
                                                                                 483
     sum(coef(lasso$finalModel, lasso$bestTune$lambda)!=0)
                                                                                 484
     sum(coef(lasso$finalModel, lasso$bestTune$lambda)==0)
463
```

- We select a really small training set with only 380 loans (1% of the dataset)
- The model I try to fit has 66 coefficients (multiple interaction terms)
 - OLS model out of sample estimation results

RMSE	Rsquare
0.0446	0.352

LASSO out of sample results using 5-fold cross validation to determine the best λ=0.0007. Only 20 coefficients are not zero.

 RMSE
 Rsquare

 0.0111
 0.916

 Remember RMSE was 0.0105 when we used 75% of the data to train the model! So LASSO with 1% of data performs almost as well as linear regression with 75% of the data (error is only 5.9% higher from 0.0111 to 0.0105)

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Forecast the interest rate of a new loan

- Model 1:
$$Int = \beta_0 + \beta_1 \times loanA + \epsilon$$

loan_amnt (K\$)	term (months)	dti	delinq_2yrs	annual_inc
25	60	25.74	0	78000

- Interest rate prediction: 14.17%
- 95% Confidence interval (+/- 2 x standard error) = [7.27%-21.08%]
- Model 2: $Int = b_1 + b_2 loanA + other explanatory variables + \epsilon$
 - Interest rete prediction = 16.00%
 - 95% Confidence Interval (+/- 2 x standard error) = [9.74%-22.25%]
- Best model I could come up with (using more features)
 - Interest rate prediction = 11.66%
 - 95% Confidence Interval (+/- 2 x standard error) = [10.10%-13.22%]
- Actual value 11.99%
- Always report C.I. using out-of-sample validation statistics
- Continuously monitor and review your models!

Next Lecture

Course contents (first part of the course - Kamalini)

- Session 1: The Art & Science of Regression Models For Prediction
- Session 2: More on Using Linear Regression For Prediction
- Session 3: Workshop I Engineer an algorithm that sets interest rates for new Lending Club loans
 - Group assignment 1, due 6 days after the workshop
- Session 4: Classification using Logistic Regression
- Session 5: Workshop Invest in a portfolio of Lending Club loans
 - Individual project 1, due 13 days after the end of the workshop

Course contents (second part of the course – Kanishka)

See canvas syllabus



Innovation transforms lending

Lending Club is the world's largest marketplace connecting borrowers and investors, who consumers and small business owners lower the cost of their credit and enjoy a better experience than traditional bank lending, and investors earn attractive risk-adjusted returns.

Here's how it works:

- Customers interested in a loan complete a simple application at LendingClub.com
- We leverage online data and technology to quickly assess risk, determine a credit ratio
 appropriate interest rates. Qualified applicants receive offers in just minutes and can e
 with no impact to their credit score
- Investors ranging from individuals to institutions select loans in which to invest and c returns

The entire process is online, using technology to lower the cost of credit and pass the savings back in borrowers and solid returns for investors.