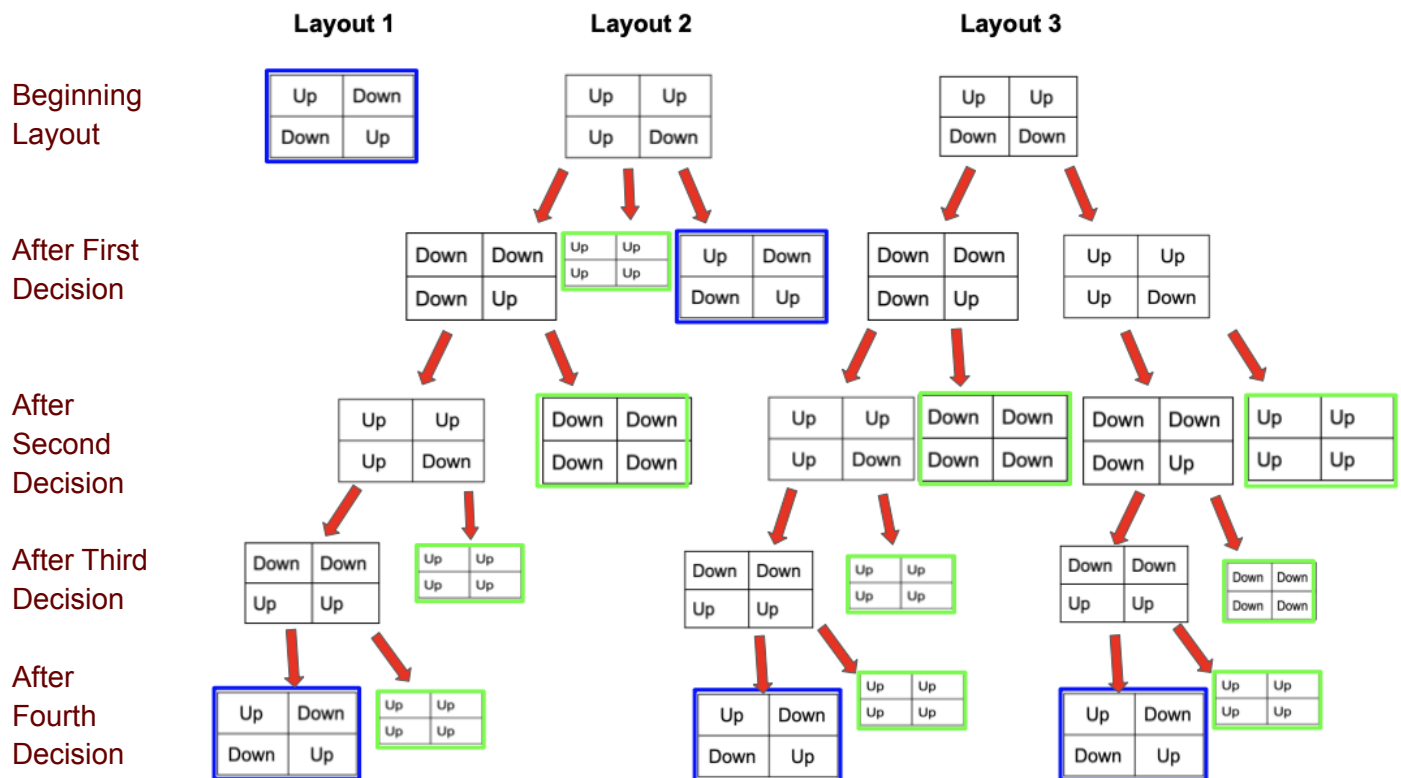


Etude 7: Up or Down

Mathew Shields (2419874) and Alex Lake-Smith (5400306)

1. Devise a method by which you can guarantee to solve this puzzle in a bounded number of steps.

In the diagram below, boxes in green are puzzles solved, and boxes in blue are puzzles in a configuration that will be solved in the next turn by choosing two diagonal containers and flipping both cups.



Beginning

At the beginning of the game, after the platform has been spun for the first time, up to symmetry under rotation, and the specific orientation of the cups, there are three effectively different cup configurations (which don't result in the bell ringing immediately).

First Step

Pick two diagonal containers to check. Flip both cups if they have the same orientation, or flip one of the cups if they have different orientations, so that they are now the same. Remember what way you flipped the cups. After this, you will have either solved the puzzle, have a layout 1 configuration which will be solved in the next step, or a 3-1 layout (three up cups and one down cup or three down cups and one up cup).

Second Step

Pick two diagonal containers to check. Flip both cups if they have the same orientation, or, if they have different orientations, flip one cup in the same way that you flipped the one cup in the previous step to solve the puzzle.

If the bell hasn't rung, you will be able to guarantee that you have a 3-1 layout, and know which 3-1 layout it is based on your previous decisions.

Third Step

Pick two diagonal containers to check again and you will see either:

- Two cups with the same orientation. Choose one of these cups to flip the opposite way and this will leave you with a two up and two down cup configuration adjacent to one another (like seen in the above layout 3 table).
- Two cups with different orientations. Knowing which 3-1 layout you have, flip the final cup the same way to complete the puzzle.

Fourth Step

Pick two adjacent containers. You will see either:

- Two up cups or two down cups. Flip both to solve the puzzle.
- One up cup and one down cup. Flip both of them around, leaving you with the cups laid out like layer 1, two up cups and two down cups diagonally.

Fifth Step

Pick two diagonal containers. You will see two cups with the same orientation. Flip both to solve the puzzle.

2. What is the maximum number of steps your method might require? Can you guarantee that no method can succeed more quickly (in all cases)?

Our method will require a maximum of five steps to solve the puzzle. We guarantee that this is the quickest method, because there is no way to be certain of what layout you have and that all layouts are the same in less than two steps, and no way to solve the puzzle from here in less than three steps, whilst also being certain of the cup configuration in each step.

3. Write a report describing and justifying your method that also addresses the second item above. You may assume that the intended reader of this report has read this document but has not thought about how to solve the puzzle. Coming up with a clear, attractive, and easy to understand, way to describe the possible configurations of the puzzle is a key part of this report.

Our method aims to turn every starting configuration of the cups into a layout 1 type layout, with two up cups and two down cups diagonal from each other. From this layout, when you choose two diagonal containers and flip them both, you are guaranteed to solve the puzzle.

The objective of the first two steps is to solve any puzzles already in that layout, or get the cups into a 3-1 layout, regardless of what layout the puzzle started in. The third step is used to either solve the puzzle, or prepare the layout to be changed to a layout 1 type layout in the following step. After setting up the cups in a layout 3 type layout, by picking two adjacent containers to check in the fourth step, you will either see two of the same cups you can flip both of to win, or two different cups, which can both be flipped into a layout 1 type layout. In the fifth step two diagonal containers are checked, which we can now guarantee will always contain two cups of the same orientation that can be flipped to solve the puzzle.