Up or Down

Imagine a square platform that can rotate about its centre. On each of the four corners of the platform is an opaque container with a lid. The containers are unmarked and indistinguishable. Inside each container is a cup. Each cup can be either upside down, or right way up.

This set-up is part of a puzzle that works as follows:

- The containers with cups are locked and you cannot look inside.
- When you press '*start*' the cups in the containers are shaken up, so that they are randomly placed in a certain position, either upside down or right way up.
- When you press '*continue*' the platform turns around its centre and stops at a random point (like a roulette wheel). It spins so quickly that you have no hope of keeping track of the position of any container.
- The containers unlock and you are now allowed to look inside any two containers (as soon as you have opened two the others lock again)[1].
- You may change the way the two cups you can see are placed, from upside down to right side up or vice versa (you may change none, one, or two).
- You close the containers and press '*continue*'; the previous three steps are repeated until you have solved the puzzle.
- The puzzle is solved when all the cups are turned the same way, either all upside down *or* all right way up. When this state has been reached a bell will ring to tell you that you have solved the puzzle successfully.

[1] Note that you are not allowed to open one, look inside and on the basis of what you see decide which other container to open. You must decide which two containers to open before you open either one.

Task

1. Devise a method by which you can guarantee to solve this puzzle in a bounded number of steps.
2. What is the maximum number of steps your method might require? Can you guarantee that no method can succeed more quickly (in all cases)?
3. Write a report describing and justifying your method that also addresses the second item above. You may assume that the intended reader of this report has read this document but has not thought about how to solve the puzzle. Coming up with a clear, attractive, and easy to understand, way to describe the possible configurations of the puzzle is a key part of this report.

Relates to Objectives

1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.9, 2.10, 4.1, 4.2, 4.7, 4.8

(Pair)