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Etude 10: Epidemic

The minimum number of sick people you need to infect a group of people made up of only vulnerable people when both the number of cols and number of rows are odd is to have every second person along the top row sick and then two of the either side edges as sick as well. As seen in the example below. You must have one sick in either corner of the universe however to ensure that all corner cases get sick. The example on the left uses the correct layout, the one on the right shows what happens if there is not a sick person in both corners of the table.

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When there is an even amount of rows and an even amount of columns you must have two sick people on the edges of one side and then have another sick person in the opposite corner of the table to ensure all the edge cases get sick as seen in the example below.

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The formula for working out the minimum amount of sick people needed to infect a group of vulnerable people is $x/2 + y/2$. X is equal to the number of rows and Y is equal to the number of columns. If the number of columns is even but the number of rows is odd then you must round up the number of sick people. E.g in the bottom example . Number of cols is $14/2 = 7 + 5/2 = 2.5 = 9.5$ so you round the number of sick people needed up to 10.

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If both row and column lengths are even and the immune cell is put in the corner which needs the sick cell, you will need one more sick cell in the opposite corner to get the edge cases sick. If there is a wall of immune cells down the columns then you just need to treat each side of the wall as a separate universe and apply the formula to each side.

For the cases with immune cells that are randomly placed amongst the universe you could also use an exhaustive search which is a computational solution as it is hard to mathematically solve the number of minimum cells needed without looking at different possible combinations.. You first must count the number of cells that are immune and then minus that number from the total number of people in the universe. For example if there was a universe of size 50 and you had 7 immune cells the number of vulnerable cells would be $43 = n$ (either sick or not sick). You then times 2 to the n so in this case it is 2^{43} as there are 2 possible states that those 43 cells could be in. This is the number of possible combinations that the universe could be in once it is filled with either sick or non sick individuals. You then iteratively go through all the possible cases of these different combinations. You could keep track of whether that combination has resulted in a person in that universe becoming sick or being healthy and if in that case one person is not sick you throw away that result as it does not reach the ideal solution of everyone being sick. Once you find a combination that results in making all the vulnerable cells sick, you can then find the minimum number of sick cells needed that reaches this desired outcome. Whichever solution that has the minimum number of sick cells needed to be placed to spread through the universe is the optimal solution and the one that should be chosen.