# Non-Linear Slip Control For Anti-lock Braking System

Conference Paper · September 2012  DOI: 10.2316/P.2012.761-014					
CITATIONS 0	;	READS 96			
1 author:					
	Samuel John The Namibia University of Science and Technology 22 PUBLICATIONS 277 CITATIONS SEE PROFILE				
Some of the authors of this publication are also working on these related projects:					
Project	Development of Wheel Slip Control Schemes for Antilock Braking Systems View project				
Project	Network of Energy Excellence for Development View project				

# NON-LINEAR SLIP CONTROL FOR ANTI-LOCK BRAKING SYSTEM

Samuel John
Department of Mechanical Engineering
The Polytechnic of Namibia
P. Bag 13388
Windhoek, Namibia
email: sjohn@polytechnic.edu.na

Jimoh O. Pedro
School of Mechanical, Industrial and Aeronautical Engineering
University of the Witwatersrand
P. Bag 3 Wits 2050
Johannesburg, South Africa
email: jimoh.pedro@wits.ac.za

#### **ABSTRACT**

The Anti-lock braking system (ABS) is an active safety device in road vehicles, which during hard braking maximizes the braking force between the tyre and the road irrespective of the road condition. This is accomplished by regulating the wheel slip around its optimum value. Due to the high non-linearity of the tyre and road interaction, and uncertainties from vehicle dynamics, linear controllers will not suffice. This paper, therefore proposes a more robust non-linear control design using input-output state feedback linearization approach. To enhance the robustness of the non-linear controller, an integral feedback method was employed. The stability of the controller is analysed using a Lyapunov method. To demonstrate the robustness of the proposed method, simulations were conducted for two different road conditions. The results of the proposed controller were found to be superior to results obtained using a standard feedback linearisation controller.

#### **KEY WORDS**

Non-linear control, feedback linearization, quarter-car model, ABS, wheel slip, Lyapunov function.

#### 1. Introduction

The operation of the ABS is briefly described as follows: during emergency braking, usually the driver suddenly applies the brakes in panic and this leads to the locking of the wheels while the vehicles body momentum is still high, this causes the car to skid. During skidding, the driver loses the control of the steering and the outcome could be disastrous. The anti-lock braking system, is a device that senses when the wheels of the vehicle are about to lock during braking and it releases the brakes so that locking does not occur. This operation results in the improving of the longitudinal stability and hence driver's control of the steering, thereby improving the driver's ability to avoid obstacles. The ABS also aims at maximising the frictional forces between the tyres and the road, consequently minimising the braking distance.

Most commercial ABS have a design objective of maximising the friction force between the tyres and the road surface to achieve shorter braking distance and

steering control [1, 2]. They are implemented using an algorithm that is based on complicated logic rules (table rules), which attempt to capture all possible operating scenarios. These rules are executed by a control computer that switches on and off solenoid valves to ensure the right pressures are delivered to the wheels while avoiding slippage [3]. Current ABS research is based on slip control. The aim of the controller is to continuously monitor the slip value ( $\lambda$ ) and by manipulating the braking pressure  $(P_b)$ , it is possible to avoid a slip value of 100% (wheel lock) and maintain the slip at about the desired  $(\lambda_d)$  value, which is estimated for most road conditions to be about 18% to 20% [4, 5]. The braking and traction of road vehicles are greatly influenced by the frictional forces developed between the tyre and the road surface. This tyre-road friction in wheeled vehicles is a complex non-linear problem which has attracted a lot of research work in the eighties to nineties [6, 7, 8, 9]. When the rotation of the wheel around its axle is free, partly or fully locked, three phenomena are likely to take place; these are free rolling, skidding and full locking. The available maximum acceleration / deceleration of the vehicle body is determined by the maximum friction coefficient describing the contact of the road and the wheels. For this reason the behaviour of various tyres under various environmental conditions are extensively studied [6, 9, 10, 11, 12]. This non-linear nature of the wheel dynamics requires non-linear or robust control design approach.

The sliding mode control (SMC) is one of the proposed robust control method for ABS in the literature. The SMC consists of a robust controller, an equivalent controller and a sliding surface estimator. The robust controller compensates for broad range of uncertainties while the equivalent controller tracks the desired slip. The robustness of the SMC is its main strength in ABS controller application. However, the major drawback with the SMC is the chattering caused by the non-linearity in the ABS model, which could shorten the life-span of the ABS elements. According to a study by Austin and Morrey [13], some researches have tried solving the chattering problem by introducing a saturation function in place of the sign function for switching control for different road conditions. The introduction of the saturation function

eliminates the chattering, however, it introduces a steady state error [13, 14].

Another modified SMC method proposed by Jiang et al [15] is the moving sliding surface, based on global sliding mode control (GSMC) strategy. In this method, unlike in the conventional SMC, the sliding surface moves to the desired sliding surface from the initial condition, thus achieving fast tracking of the desired slip. This strategy aimed at eliminating the reaching phase that causes chattering in the conventional SMC method. In addition, the radial basis neural network functions are used for the sliding mode controller. Simulation results on a quarter-car model comparing the proposed method and the conventional SMC method, indicates that the proposed method reduced the chattering.

Lin and Ting [16] proposes a backstepping control design scheme, for a non-linear anti-lock braking system (ABS) assisted with active suspension system. In an emergency braking, the ABS leads to shorter braking distance, and provides steering control. However Lin et al [16] proposes to take advantage of ABS combined with active suspensions to further reduce vehicle braking time and stopping distance. The goal is to utilise the vertical normal force that increases during braking, leading to increased frictional forces between the tyre and the road to achieve more stopping distance than using just ABS. The integrated system combines the active suspension controller with the ABS controller and this integrated controller co-ordinates the two sub-systems. Simulation results on a quarter-car shows that the integrated system achieved a 12% improvement in stopping distance.

The feedback linearisation control method as applied to non-linear systems is one of the proposed methods for solving the slip control problem. Park and Lim [17] presented simulation results of a wheel slip control employing the feedback linearisation control method with an adaptive sliding mode control. The novelty of this work is the introduction of a time delay to the input. Park and Lim [17] claim that the time delay is necessary to compensate for the actuators time delay. To compensate for the time delay, the sliding mode controller is incorporated to bound the uncertainties, using a method proposed by Shin et al [18]. The simulation results presented show some improvement of this new method.

The current work proposes a combination of feedback linearization method with a PID controller. The goal of this combination is to reduce the chattering effect on the braking torque. Chattering of the braking torque is observed when using the traditional method of feedback linearization with pole placement scheme. The performance of the proposed controller is tested in simulations on two road conditions. The results from the proposed method is compared with the performance of the standard feedback linearization

method.

# 2. Mathematical Model of a Quarter-Car

A quarter-car model is used to develop the longitudinal braking dynamics. It consists of a single wheel carrying a quarter mass m of the vehicle and at any given time t, the vehicle is moving with a longitudinal velocity v(t). Before brakes are applied, the wheel moves with an angular velocity of  $\omega(t)$ , driven by the mass m in the direction of the longitudinal motion. Due to the friction between the tyre and the road surface, a tractive force  $F_x$  is generated. When the driver applies the braking torque  $T_b$ , it will cause the wheel to decelerate until it comes to a stop. A two degree of freedom quarter-car model is shown in Figure 1.

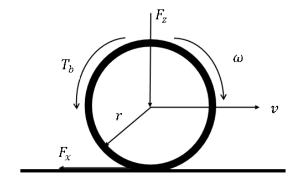


Figure 1. Quarter-car model

Applying Newton's second law of motion, the equation describing the wheel rotational dynamics is given by:

$$\dot{\omega} = \frac{1}{J}(r\mu(\lambda)F_z - B\omega - T_b(sign(\omega)) \tag{1}$$

where  $\omega$  is the angular velocity of the wheel, J is the rotational inertia of the wheel, r is the radius of the tyre, B is the viscous friction coefficient of the wheel bearings and  $T_b$  is the effective braking torque, which is dependant on the direction of the angular velocity.

The equation describing the vehicle longitudinal dynamics is given by:

$$\dot{v}_x = -\frac{1}{m}(\mu_x(\lambda_x)F_z + Cv_x^2) \tag{2}$$

where  $v_x$  is the longitudinal velocity of the vehicle, C is the vehicle's aerodynamic friction coefficient,  $\mu_x$  is the longitudinal friction coefficient between the tyre and the road surface  $\lambda_x$  is the longitudinal type slip and  $F_z$  is the normal force exerted on the wheel.

The hydraulic brake actuator dynamics is modeled as a first order system given by:

$$\dot{T}_b = \frac{1}{\tau} (-T_b + k_b P_b) \tag{3}$$

where  $k_b$  is the braking gain; which is a function of the brake radius, brake pad friction coefficient, brake temperature and the number of pads [19],  $P_b$  is the braking pressure from the action of the brake pedal which is converted to torque by the gain  $k_b$ . The hydraulic time constant  $\tau$  accounts for the brake cylinder's filling and dumping of the brake fluid [19].

The friction coefficient between the road and the tyre has influence on the braking or traction of the vehicle. The wheel slip results in the deformation and sliding of tread elements in the tyre/road patch. A simple definition of the longitudinal slip  $(\lambda_x)$  is given by:

$$\lambda_x = \frac{v_x - r\omega}{v_x} \tag{4}$$

The wheel slip is a critical parameter on which the available maximal friction coefficient depends as shown in Figure 2. The development of a practical friction model may enhance the performance of the ABS controller.

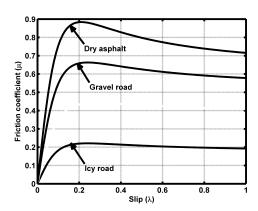


Figure 2.  $\mu - \lambda$  Curves for different road conditions

In this light, this work adopts a practical tyre-road friction model proposed by Lin and Ting [16] given as:

$$\mu(\lambda) = 2\mu_0 \frac{\lambda_0 \lambda}{\lambda_0^2 + \lambda^2} \tag{5}$$

where  $\mu_0$  and  $\lambda_0$  are the available maximum friction coefficient and optimum slip respectively. The advantage of this model is that it gives good result when  $\lambda \to \pm \infty$  and its sign modification can also be physically interpreted as turning from braking to accelerating phase and vice versa.

# 3. ABS Controller Design

#### 3.1 Controller Specifications

The following performance criteria are used for the evaluation of the controllers;

- stopping distance  $\leq 50m$  from initial speed of 80km/h (22.23m/s)
- the integral squared error [ISE] of the slip  $\int\limits_0^{t_f} \left(\lambda \lambda_d\right)^2 dt,$
- the integral squared control input  $\int_{0}^{t_f} P_b^2 dt$  [20]

The desired performance will therefore be; smaller ISE, using less braking effort, to achieve a shorter stopping distance.

#### 3.2 Wheel Slip Controller Design

The feedback linearization method is adopted for the current work, which is suitable for the affine non-linear single input single output (SISO) systems [21, 22, 23, 24]. The general affine non-linear system can be represented as:

$$\dot{\mathbf{x}} = \mathbf{f}(x) + \mathbf{g}(x)u \tag{6}$$

$$y = \mathbf{h}(x) \tag{7}$$

where the state variables  $\mathbf{x} = [x_1, x_2]^T$  are the wheel angular velocity  $\omega$  and the vehicle longitudinal velocity v respectively,  $f, g: \mathbf{R}^n \to \mathbf{R}^n$  are smooth functions and y is the output slip function.

The wheel slip dynamics is obtained by taking the derivative of the longitudinal wheel slip (Equation 4) with respect to time, assuming that the radius of the tyre remains constant.

$$\frac{d\lambda}{dt} = \frac{\partial \lambda}{\partial v} \frac{dv}{dt} + \frac{\partial \lambda}{\partial \omega} \frac{d\omega}{dt} + \frac{\partial \lambda}{\partial r} \frac{dr}{dt}$$
 (8)

$$\dot{\lambda} = \frac{\omega r}{v^2} \dot{v} - \frac{r}{v} \dot{\omega} \tag{9}$$

Substituting (1) and (2) into (9) yields the following:

$$\dot{\lambda} = -\frac{r}{v} \left( \frac{rF_x - T_b}{J} \right) - \frac{\omega r}{v^2} \left( \frac{F_x}{m} \right) \tag{10}$$

Rearranging (10) and knowing that  $F_x = \mu F_z(\lambda, \mu_0)$  yields the slip dynamics as;

$$\dot{\lambda} = -\frac{1}{v} \left( \frac{\omega}{mv} + \frac{r^2}{J} \right) \mu F_z(\lambda, \mu_0) + \frac{r}{Jv} T_b$$
 (11)

This can be written in the form:

$$\dot{\lambda} = -\frac{1}{v} \left( \frac{1}{m} (1 - \lambda) + \frac{r^2}{J} \right) F_z \mu(\lambda, \mu_0) + \frac{r}{Jv} T_b \quad (12)$$

From the slip dynamics (12), it can be seen that as  $v \to 0$  the slip dynamics  $\dot{\lambda} \to \infty$ , which occurs during a wheel lock-up, and must be avoided by switching off the ABS controller at low velocities.

The slip control problem can be described by Equation (6); where  $f(x) = -\frac{1}{v} \left(\frac{\omega}{mv} + \frac{r^2}{J}\right) \mu F_z(\lambda, \mu_0)$ ,  $g(x) = \frac{r}{Jx_2}$  and  $u = T_b$ . In this case  $f(\cdot)$  and  $g(\cdot)$  are non-linear dynamic functions. The goal of the ABS is to track a predetermined slip set-point  $(\lambda_d)$ . At this operating point, it is safely assumed that  $g(x) \neq 0$ , and hence the control input can be chosen as:

$$u = \frac{1}{g(x)} \left[ \nu - f(x) \right] \tag{13}$$

where  $\nu$  is a virtual input.

The non-linearity in (12) is therefore cancelled and a simplified relationship between the integral of the output  $\dot{\lambda}$  and the new input  $\nu$  can be presented as:

$$\dot{\lambda} = \nu \tag{14}$$

Let the tracking error (e) be given as:

$$e = \lambda(t) - \lambda_d(t) \tag{15}$$

and let the virtual input be chosen as:

$$\nu = \dot{\lambda}_d - \kappa e \tag{16}$$

From (14) and (16) the tracking error for the closed loop system is given as:

$$\dot{e} + \kappa e = 0 \tag{17}$$

The next step involves the investigation of the stability of the controller.

#### 3.3 Stability Analysis

If the virtual input is define as:

$$\nu = \kappa_v r + \Lambda_1 e^{n-1} + \dots + \Lambda_{n-1} e + \dots + \dot{x}_{nd}$$
 (18)

where r is the filtered error given by:

$$r = \frac{d^{n-1}e}{dt^{n-1}} + \Lambda_1 \frac{d^{n-2}e}{dt^{n-2}} + \dots + \Lambda_{n-1}e$$
 (19)

where the design parameters  $\kappa_v$  and  $\Lambda s$  are chosen heuristically. The derivative of r will therefore be given by:

$$\dot{r} = -\kappa_v r \tag{20}$$

Considering the following Lyapunov function:

$$V = \frac{1}{2}r^2 \tag{21}$$

Table 1. System parameters and numerical values

Symbol	Description	Value	Unit
$\overline{m}$	Quarter car mass	395	kg
J	Moment of inertia	1.6	$Nms^2/rad$
r	Radius of wheel	0.3	m
C	Vehicle viscous friction	0.856	kg/m
B	Wheel viscous friction	0.08	$Nkgm^2/s$
$\mid \tau \mid$	Hydraulic time constant	0.3	sec
$k_b$	Hydraulic gain	0.8	constant
$\mid g \mid$	Gravitational acceleration	9.81	$m/s^2$
$\lambda_d$	Desired slip ratio	0.18	Ratio

The derivative of (21) will yield:

$$\dot{V} = r = \frac{d^{n-1}e}{dt} + \Lambda_1 \frac{d^{n-2}e}{dt} + \dots + \Lambda_{n-1}e$$
 (22)

It can be seen that  $r\to 0$  with time, while the design parameters  $\Lambda_1\dots\Lambda_{n-1}$  are chosen so that the system is stable.

#### 4. Simulation Results and Discussions

Simulations are carried out on a straight-line braking operation. Braking commenced at an initial longitudinal velocity of 80km/h (22.23m/s) [25], and the braking torque was limited to 1200Nm. In order to impose a desired slip trajectory, the following reference model was adopted [26].

$$\dot{\lambda}_d(t) + 10\lambda_d(t) = 10\lambda_c(t) \tag{23}$$

where the slip command is chosen to be  $\lambda_c = 0.18$ .

The parameters and numerical values used are presented in Table 1

Simulations are conducted for a high and low friction surfaces with friction coefficients of  $\mu=0.85$  and  $\mu=0.2$  respectively. These friction coefficients correspond to dry asphalt and icy road conditions respectively [27]. The simulations are terminated at speeds of 1m/s (3.6km/h), this is because as the speed of the wheel approaches zero, the slip becomes unstable, therefore the ABS should disengage at low speed to allow the vehicle to come to a stop.

Simulation results for the vehicle and wheel deceleration, slip tracking and the braking torque results for high and low friction surface conditions are presented for both the standard FBL controller and the proposed controller.

The summarised performance results are presented in Tables 2 and 3.

Both controllers achieved the required stopping distances on the high friction coefficient road condition. As expected, a longer stopping distance is recorded for the low friction coefficient road condition with the standard FBL

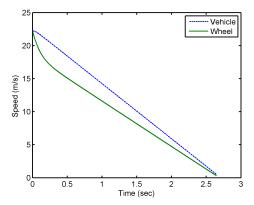


Figure 3. Vehicle and wheel deceleration on high friction surface ( $\mu=0.85$ ) using FBL controller

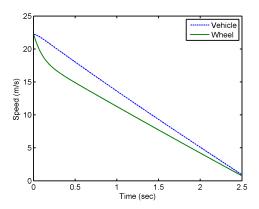


Figure 4. Vehicle and wheel deceleration on high friction surface ( $\mu=0.85$ ) using proposed controller

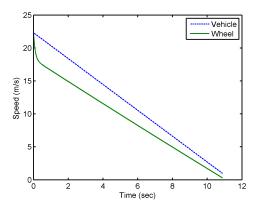


Figure 5. Vehicle and wheel deceleration on low friction surface ( $\mu = 0.2$ ) using FBL controller

recording a higher value than the proposed method. Both controllers have good slip response time with no obvious overshoots. Therefore it can be observed that both controllers meet the desired performance criteria. It can be

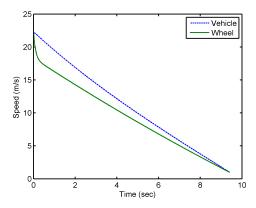


Figure 6. Vehicle and wheel deceleration on low friction surface ( $\mu=0.2$ ) using proposed controller

Table 2. Simulation results for  $\mu=0.85$  road condition

Performance parameters	Specs	FBL	Proposed-FBL
$\int_{0}^{t_f} (\lambda - \lambda_d)^2 dt (10^{-6})$	min	65.990	0.2113
$\int_{0}^{\tilde{t}_f} T_b^2 dt (Nm)^2 (10^5)$	min	2.411	2.083
$\int_{0}^{t_f} v dt(m)$	$\leq 50$	31	29

Table 3. Simulation results for  $\mu = 0.2$  road condition

Performance parameters	Specs	FBL	Proposed-FBL
$\int_{0}^{t_f} (\lambda - \lambda_d)^2 dt (10^{-6})$	min	1.412	4.148
$\int_{0}^{t_{f}} T_{b}^{2} dt (Nm)^{2} (10^{5})$	min	0.645	0.481
$\int\limits_{0}^{\widetilde{t}_{f}}vdt(m)$	$\leq 50$	127	103

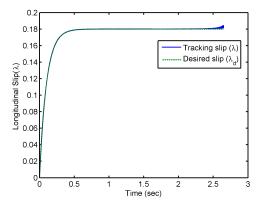


Figure 7. Slip tracking on high friction surface ( $\mu=0.85$ ) using FBL controller

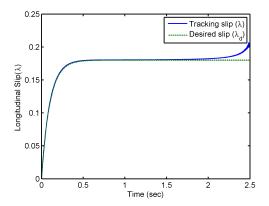


Figure 8. Slip tracking on high friction surface ( $\mu=0.85$ ) using proposed controller

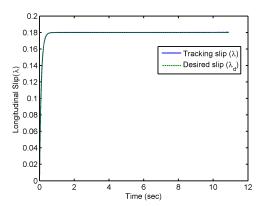


Figure 9. Slip tracking on low friction surface ( $\mu=0.2$ ) using FBL controller

seen that the high chattering effect observed in the braking torque plots for the standard FBL in Figures 11 and 13 have been drastically reduced through the proposed method as can be seen in Figures 12 and 14.

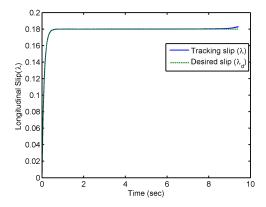


Figure 10. Slip tracking on low friction surface ( $\mu=0.2$ ) using proposed controller

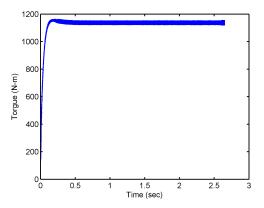


Figure 11. Braking torque on high friction surface ( $\mu=0.85$ ) using FBL controller

#### 5. Conclusion and Future Work

This paper proposes a feedback linearisation method with an integral feedback to solve the chattering problem observed in application of the standard feedback linearisation to the ABS control problem. The over-all performance of the proposed method demonstrates a more superior performance over the standard FBL controller for the ABS. Correlating the plots with the performance results presented in Tables 2 and 3 confirm this assertion. The robustness of both controllers however, can be seen in their performances at low friction road conditions where both the transient and steady state conditions of the slip behaved quite well, without excessive slippage.

The scope of this paper covers vehicle dynamics modeling, controller design and analysis and implementation in simulations using the Matlab  $^{\circledR}$  / Simulink  $^{\circledR}$  simulation environment. Future work will investigate the application of intelligent-based FBL control scheme.

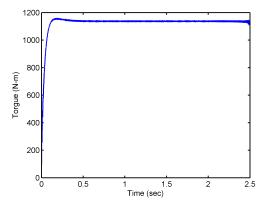


Figure 12. Braking torque on high friction surface ( $\mu = 0.85$ ) using proposed controller

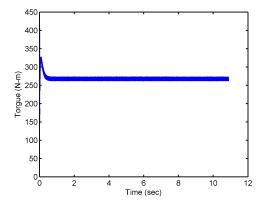


Figure 13. Braking torque on low friction surface ( $\mu=0.2$ ) using FBL controller

### References

- [1] Solyom S. Synthesis of a model-based tire slip controller. PhD Thesis, Department of Automatic Control, Lund Institute of Technology, SE-221 00 Lund Sweden 2002.
- [2] Petersen I. Wheel slip control in abs brakes using gain scheduled optimal control with constraints. PhD Thesis, Department of Enginnering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway 2003.
- [3] Wellstead P, Pettit N. Analysis and redesign of an antilock brake system controller. *IEE Proceedings on Control Theory and Applications* sep 1997; **144**(5):413 –426, doi:10.1049/ip-cta:19971441.
- [4] Chikhi F, El Hadri A, Cadiou J. ABS control design based on wheel-slip peak localization. *Proceedings of the Fifth International Workshop on Robot Motion and Control RoMoCo*, 2005; 73 77.

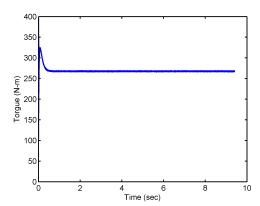


Figure 14. Braking torque on low friction surface ( $\mu=0.2$ ) using proposed controller

- [5] Xu C, Cheng K, Sha L, Ting W, Ding K. Simulation of the integrated controller of the anti-lock braking system. *Proceedings of the 3rd International Conference on Power Electronics Systems and Applications*, 2009; 1–3.
- [6] Bakker E, Pacejka H, Lidner L. A new tire model with an application in vehicle dynamics studies. *SAE Technical Series* 1989; (890087):83 95.
- [7] Pacejka H, Bakker E. The magic formula tyre model. Proceedings of 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis (Supplement to Vehicle System Dynamics Vol. 21), vol. 21, Pacejka HB (ed.), SWETS & ZEITLINGER B.V. AMSTER-DAM / LISSE, 1993; 1–18.
- [8] Oosten JV, Bakker E. Detarmination of magic tyre model parameters. Proceedings of 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis (Supplement to Vehicle System Dynamics Vol. 21), vol. 21, Pacejka HB (ed.), SWETS & ZEITLINGER B.V. AMSTERDAM / LISSE, 1993; 19–29.
- [9] Lidner L. Experience with the magic formula tyre model. Proceedings of 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis (Supplement to Vehicle System Dynamics), vol. 21, Pacejka HB (ed.), SWETS & ZEITLINGER B.V. AMS-TERDAM / LISSE, 1993; 30–46.
- [10] Canudas-de Wit C, Tsiotras P. Dynamic tire friction models for vehicle traction control. *Proceedings of the 38th IEEE Conference on Decision and Control*, vol. 4, 1999; 3746 –3751 vol.4.
- [11] Lacombe J. Tire model for simulations of vehicle motion on high and low friction road surfaces. *Simulation Conference Proceedings*, vol. 1, 2000; 1025 1034 vol.1, doi:10.1109/WSC.2000.899907.

- [12] Olson BJ. Nonlinear dynamics of longitudinal ground vehicle traction. Master's Thesis, Michigan State University 2001.
- [13] Austin L, Morrey D. Recent advances in antilock braking systems and traction control systems. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering January 2000; 214(6):625–638.
- [14] Buckholtz KR. Reference input wheel slip tracking using sliding mode control. *SAE Technical Series* 2002; doi:10.4271/2002-01-0301.
- [15] Jing Y, e Mao Y, Dimirovski G, Zheng Y, Zhang S. Adaptive global sliding mode control strategy for the vehicle antilock braking systems. *Proceedings of the American Control Conference (ACC '09)*, 2009; 769 –773, doi:10.1109/ACC.2009.5160357.
- [16] Lin JS, Ting WE. Nonlinear control design of antilock braking systems with assistance of active suspension. *Control Theory Applications, IET* january 2007; 1(1):343 –348, doi:10.1049/iet-cta:20050218.
- [17] Park KS, Lim JT. Wheel slip control for ABS with time delay input using feedback linearization and adaptive sliding mode control. *Proceedings of the International Conference on Control, Automation and Systems (ICCAS)*, 2008; 290 –295, doi: 10.1109/ICCAS.2008.4694658.
- [18] Shin HS, Choi HL, Lim JT. Feedback linearisation of uncertain nonlinear systems with time delay. *IEE Proceedings on Control Theory and Applications* nov 2006; **153**(6):732 –736.
- [19] Alleyne A. Improved vehicle performance using combined suspension and braking forces. *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility* 1997; **27**(4):235–265.
- [20] Mirzaeinejad H, Mirzaei M. A novel method for non-linear control of wheel slip in anti-lock braking systems. *Control Engineering Practice* 2010; 18(8):918 926, doi:10.1016/j.conengprac.2010.03.015.
- [21] Behera L, Kar I. *Intelligent systems and control principles and applications*. Oxford University Press, 2009.
- [22] Ball S, Barany E, Schaffer S, Wedeward K. Nonlinear control of power network models using feedback linearization. *Proceedings of the Circuits, Signals and Systems*, Oklobdzija VG (ed.), 2005; 493 800.
- [23] Yeşildirek A, Lewis FL. Adaptive feedback linearization using efficient neural networks. *J. Intell. Robotics Syst.* May 2001; **31**:253–281, doi: 10.1023/A:1012011226385.

- [24] Slotine JJ, Li W. *Applied Nonlinear Control*. Prentice-Hall, New Jersey, USA, 1991.
- [25] Dietsche K, Klingebiel M. Automotive Handbook. 7 edn., Robert Bosch GmbH, 2007.
- [26] Poursamad A. Adaptive feedback linearization control of antilock braking systems using neural networks. *Mechatronics* 2009; 19(5):767 773, doi: DOI: 10.1016/j.mechatronics.2009.03.003.
- [27] MacIsaac Jr JD, Garrot WR. Preliminary findings of the effect of tire inflation pressure on the peak and slide coefficients of friction. *Technical Report*, National Highway Traffic Safety Administration, 400 Seventh St., S.W. Washington, D.C. 20590, USA 2002.