

Analyzing risk and return in stock and portfolio investments with CAPM and Fama-French three-factor model

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1 Introduction

In financial data science, there are many ways to analyze the risk and the expected return of a portfolio. The capital asset pricing model (CAPM) is one of the most prominent models to analyze portfolios, and it builds on Markowitz (1952) theory stating that investors can achieve the optimal portfolio by considering the trade-off between expected return and risk, while also introducing the concepts of systematic risk (non-diversifiable) and unsystematic risk (diversifiable). In this analysis, we will focus on the CAPM by Sharpe (1964) and Lintner (1965) and the Fama-French (1992) three-factor model, which extends the CAPM. Our analysis focuses on two stocks, Colgate-Palmolive and Etsy, as well as a portfolio with both. Then we will analyze these through a descriptive analysis, the CAPM, and the Fama-French three-factor model. Ultimately, we aim to understand the risk and returns profiles of the stocks and the portfolio, and how the models explain the behavior of the stocks and the portfolio in comparison to the market.

2 Data

For this analysis, we utilize two datasets: the SP500 dataset and the FFfactors dataset. The SP500 dataset contains the daily closing prices of all the stocks listed in the S&P 500 index, while the FFfactors dataset includes daily market excess returns and the daily returns of a risk-free asset, necessary for the CAPM, and also the Fama-French factors SMB (Small Minus Big) and HML (High Minus Low).

Our analysis focuses on two specific stocks: Colgate-Palmolive and Etsy. We extracted their daily closing prices from the SP500 dataset. Since the data for the Etsy stock began on 16 April 2015, we set it as the starting point for our analysis, which reached to 26 March 2024. We did this to ensure that we were able to have calculations based on the same time-frame for both stocks, without any missing data for certain dates, and to make them comparable to each other.

Next, we calculated the daily returns for both Colgate-Palmolive and Etsy, using the following formula

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

where R_t is the return on day t , P_t is the closing price on day t , and P_{t-1} is the closing price on the previous day. We then constructed a portfolio that weights both stocks equally, assigning 50% to Colgate-Palmolive and 50% to Etsy.

We also ensured that all the data were expressed in percentages to avoid any issues with further calculations or regressions.

	Colgate-Palmolive	Etsy	PF	Market
mean	0.0185	0.1084	0.0635	0.0533
median	0.0419	0.0000	0.0608	0.0600
var	1.4799	14.4041	4.2750	1.3939
sd	1.2165	3.7953	2.0676	1.1806
skew	0.1629	0.4544	0.2699	-0.5794
kurt	14.4861	10.7464	9.9839	16.0868
min	-9.7829	-28.3411	-13.7104	-11.9940
max	12.6083	30.6001	15.2113	9.3460

Table 1: Summary statistics of the daily returns

3 Data analysis

3.1 Descriptive analysis

Observing table 1 we can see that Etsy's mean return (0.1084) is significantly higher than that of Colgate-Palmolive (0.0185), the portfolio (0.0635), and the market (0.0533), indicating Etsy's potential for higher returns. Nevertheless, Etsy also exhibits the highest variance (14.4041) and standard deviation (3.7953), indicating greater volatility and risk compared to the other assets. This volatility reflects the greater uncertainty in Etsy's returns, which could result in substantial gains or losses. In contrast, Colgate-Palmolive shows a significantly lower mean return but also lower variance (1.4799) and standard deviation (1.2165), indicating more stable and predictable returns suitable for risk-averse investors seeking consistent performance.

The portfolio's a mean return (0.0635) and variance (4.2750), fall between those of the two individual stocks. This demonstrates the principle of diversification: by combining assets with different risk profiles, the portfolio reduces overall risk while balancing the mean returns, but is not diversified enough to have a lower variance than the market (1.3939). There is a clear relationship between variance and mean returns highlighted in the data: higher variance generally also implies higher mean returns. This can be seen with Etsy's high mean and variance in comparison to Colgate-Palmolive's low mean and variance.

The minimum and maximum daily returns further illustrate the volatility of each asset. Etsy has both the highest maximum return (30.6001) and the lowest minimum return (-28.3411), reflecting its high volatility. Colgate-Palmolive, on the other hand, has a lower maximum return (12.6083) and a minimum return (-9.7829), indicating lower volatility. Despite Colgate-Palmolive's higher variance compared to the market variance (1.4799 vs. 1.3939), its minimum return was less extreme than that of the market (-11.9940). This indicates, that while Colgate-Palmolive experiences slightly more volatility than the market, it may offer slightly better protection against extreme negative returns compared to the market.

In terms of distribution, Colgate-Palmolive's skewness (0.1629) indicates a slight tendency towards positive outliers, while Etsy's skewness (0.4544) shows a stronger tendency. The portfolio's skewness (0.2699) balances both stocks. In contrast, the market's skewness (-0.5794) indicates a tendency towards negative outliers. Both stocks present frequent extreme

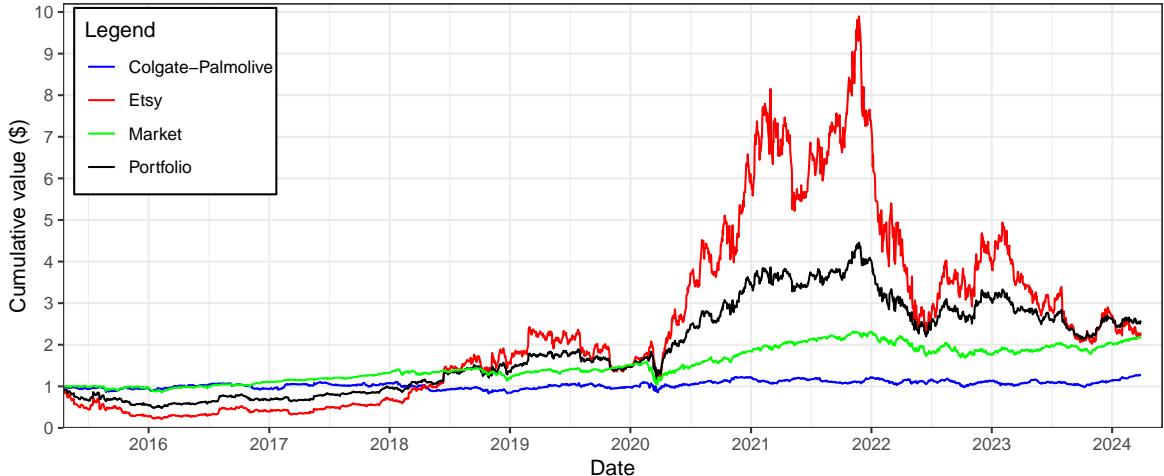


Figure 1: Cumulative value of a 1\$ investment over time

returns, as indicated by their high kurtosis values, with the portfolio displaying less frequent but still heavy tails, while The market’s kurtosis (16.0868) has the highest peak in heavy tails.

Figure 1 shows a visual representation of the growth of an initial \$1 investment in Colgate-Palmolive, Etsy, the portfolio, and the market over time, highlighting performance and volatility of each asset. Etsy shows the most significant growth, especially in 2020, followed by a rapid decline, reflecting the volatile nature of the stock. Colgate-Palmolive has instead a steady but moderate growth. The portfolio’s performance lies between the two stocks, while the market provides a reference point. If we compare Etsy with the market, we can see that Etsy behaves similar to it, but accentuates considerably all the fluctuations.

In summary, the descriptive statistics and the cumulative value plot shows that Etsy offers higher returns with greater risk, Colgate-Palmolive provides more stable but lower returns, the portfolio offers a balance between the two, while the market gives us a benchmark to compare it to.³

3.2 Estimation of the Capital Asset Pricing Model

The capital asset pricing model (CAPM) describes the relationship between an asset’s expected return and its systematic risk. In the following section we estimated the CAPM for Etsy, Colgate-Palmolive and the equally weighted portfolio.

3.2.1 Methodology

In our estimation of the capital asset pricing model (CAPM) for each stock and portfolio, we first calculate the excess returns of our assets by subtracting the return of the risk-free asset from the returns. Where R_i^e is the excess return, R_i is the returns of the asset, and R_f is the risk-free rate. This step ensures that we are accounting for the risk-free rate in our analysis, isolating the returns attributable to the risk taken by investors.

$$R_i^e = R_i - R_f$$

After that, we calculate α and β of the stocks and the portfolio by linear regression.

$$R_{i,t}^e = \alpha_{i,m} + \beta_{i,m} R_{m,t}^e$$

In this model $\alpha_{i,m}$ is the intercept term, which indicates the ability of an asset to beat the market, $\beta_{i,m}$ is the coefficient which measures the sensitivity of the asset's returns to the market returns, and $R_{m,t}^e$ is the excess return of the market portfolio. Additionally, we applied heteroskedasticity and autocorrelation consistent (HAC) standard errors to the regression estimates to ensure robust inference.

Etsy					Colgate-Palmolive				
	Estimate	Std. Error	t value	Pr(> t)		Estimate	Std. Error	t value	Pr(> t)
α	0.0324	0.0732	0.443	0.6578	α	-0.0111	0.0216	-0.5164	0.6056
β	1.4770	0.0800	18.465	<2e-16	β	0.5035	0.0541	9.3023	<2e-16

Portfolio				
	Estimate	Std. Error	t value	Pr(> t)
α	0.0106	0.0378	0.2814	0.7784
β	0.9903	0.0382	25.9324	<2e-16

Figure 2: Regression Analysis Results of the CAPM for Etsy, Colgate-Palmolive, and portfolio regression coefficients

3.2.2 CAPM estimation results

The results of our estimation are all observable in figure 2. Esty's estimated α of 0.0324 indicates outperformance against the market, but it is not statistically significant (p-value: 0.6578), indicating this result may be due to random variation. On the other hand, β is highly significant, and also greater than 1, which implies that Etsy's stock is more volatile than the market, moving 1.477 times the market movements. This higher volatility signifies greater risk for investors relative to the market, however in a growing market, this leads also to higher returns. Colgate-Palmolive has a negative α , meaning that it slightly underperformed the market, but also statistically insignificant. It's β is highly significant, but while Etsy's β indicates an aggressive investment, Colgate-Palmolive's β indicates, with a value of 0.5035, a defensive investment with less volatility than the market, and therefore it is a lower-risk investment compared to the overall market. The portfolio with both stocks has again a slightly positive, but not statistically significant α . With a β close to 1 it has a similar volatility to the market, so that the returns of the portfolio move almost in tandem with the market, reflecting market-level risk and performance. In summary, Etsy stands out with its higher β , indicating greater volatility and risk compared to the market, and is a more appealing investment for investors with a higher risk tolerance. In contrast, Colgate-Palmolive with its significantly lower β , points to lower risk and less sensitivity to market movements better for investors with a lower risk tolerance. The portfolio, with its β close to 1, demonstrates risk and performance characteristics that balance the two and that are very in line with the market. None of the assets show a significant α , suggesting that their returns do not consistently outperform or underperform the market after adjusting for risk.

3.3 Estimation of the Fama-French three-factor model

The Fama-French three-factor model is an asset pricing model that expands on the capital asset pricing model (CAPM) by incorporating two additional factors beyond just market risk. The first factor is the size premium or size effect (SMB), which refers to the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks. The second factor is the value premium or book-to-market value effect (HML), which refers to the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio.

3.3.1 Methodology

Using Linear Regression, we estimated the different coefficients for this model.

$$R_{i,t}^e = \alpha_{i,m} + \beta_{i,m} R_{m,t}^e + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t$$

To ensure robust inference, we additionally applied heteroskedasticity and autocorrelation consistent (HAC) standard errors to the regression estimates

Etsy					Colgate-Palmolive				
	Estimate	Std. Error	t value	Pr(> t)		Estimate	Std. Error	t value	Pr(> t)
α	0.0405	0.0363	1.1166	0.2643	α	-0.0171	0.0363	-0.4705	0.6381
β	1.3312	0.0418	31.8642	<2e-16	β	0.5575	0.0418	13.3437	<2.2e-16
β_{SMB}	0.8743	0.0720	12.1385	<2.2e-16	β_{SMB}	-0.4597	0.0720	-6.3820	2.115e-10
β_{HML}	-1.0622	0.0456	-23.3046	<2.2e-16	β_{HML}	0.0372	0.0455	0.8153	0.4150

Portfolio				
	Estimate	Std. Error	t value	Pr(> t)
α	0.0117	0.0363	0.3231	0.746675
β	0.9444	0.041779	22.6040	< 2.2e-16
β_{SMB}	0.2073	0.0720	2.8782	0.0040
β_{HML}	-0.5125	0.0456	-11.2447	< 2.2e-16

Figure 3: Regression Analysis Results of the Fama-French three-factor model for Etsy, Colgate-Palmolive, and portfolio regression coefficients.

3.3.2 Fama-French Three-Factor Model estimation results

Figure 3 shows the results for the three-factors model. The results for α and β of the market are pretty similar to the ones of the CAPM estimation. The α values are still not statistically significant, not allowing to give any definitive conclusions. Etsy has still a similar β to the one of the CAPM estimation, indicating an aggressive investment, while Colgate-Palmolive's β remains pretty low, indicating a defensive investment. The portfolio keeps a β near 1, but this time a little bit lower, reflecting a market-level risk and performance. For Etsy, the SMB coefficient β_{SMB} is estimated at 0.8743, and it has high statistical significance. This implies that it behaves more like a small-cap stock. The β_{HML} coefficient is negative (-1.0622), implying that Etsy behaves more like a growth stock rather than a value stock. In contrast to Etsy, Colgate-Palmolive's β_{SMB} coefficient indicates, with a value of -0.4597, that Colgate-Palmolive behaves more like a large-cap stock. However, the value of the β_{HML}

(0.0372) is rather small and not statistically significant, suggesting that Colgate-Palmolive doesn't show a strong value effect. The portfolio, with a statistically significant β_{SMB} coefficient of -0.4597, tilts slightly towards small-cap stocks. In addition, it has a statistically significant but also negative value of the β_{HML} coefficient (-0.5125), therefore it behaves more like a growth-oriented investment. In summary, the Fama-French three-factor model results align closely with the CAPM estimation for α and β , maintaining their statistical insignificance/significance and expected risk profiles. Etsy continues to show characteristics of a small-cap, growth stock with high volatility, while Colgate-Palmolive remains a defensive, large-cap stock with lower volatility. The portfolio reflects market-level risk with slight tilts towards small-cap and growth stocks. Overall, the three-factor model, with its additional insights of the size and value effects, enhances our understanding of the risk and return characteristics of these investments.

4 Conclusion

In this report we performed a detailed analysis of the daily returns of Colgate-Palmolive and Etsy stocks, and their equally weighted portfolio using both the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model. This analysis spanned from 16 April 2015 to 26 March 2024, using datasets containing daily closing prices of S&P 500 stocks and Fama-French factors. Our findings revealed distinct risk-return profiles for the two stocks. Etsy, with a higher mean return and greater volatility, behaves like a growth stock, making it suitable for investors with a higher risk tolerance. This is further supported by its significant β and β_{SMB} coefficients, indicating high market sensitivity and characteristics akin to small-cap stocks. In contrary, Colgate-Palmolive has lower returns, but also lower volatility, aligning with the characteristics of a large-cap value stock. Its lower β indicates it is a more stable investment, suitable for risk-averse investors. While the portfolio's risk and return profile was intermediate between the two stocks, it provided a balance that reduced overall risk compared to holding Etsy alone, while still offering higher returns than Colgate-Palmolive, making it a more viable option for investors seeking a balance between risk and return.

However, several limitations should be acknowledged. The analysis relied on historical data, which may not fully capture future market conditions. Additionally, the focus on only two specific stocks means the findings are not broadly generalizable to other stocks or portfolios.

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