I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties. (Signature) _____

3a) $P_x(x) = \begin{cases} \frac{1}{6} & \text{for } x = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$

$P_Y(y) = \begin{cases} \frac{1}{6} & \text{for } y = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$

$P_{XY}(x,y) = \begin{cases} \frac{1}{36} & \text{for } x = 1, 2, 3, 4, 5, 6 \\ & \quad y = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$

b) 1,1 1,2 1,3 1,4 1,5 1,6

2,1 2,2 2,3 2,4 2,5 2,6

⋮

and so on

$P(X \le Y) = \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{2}{3} + \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times 0$

$= \frac{5}{12}$ //

c)

Z	
0	(2,2) (3,3) (1,1) (4,4) (5,5) (6,6)
1	(1,2) (2,1) (2,3) (3,2) (3,4) (4,3) (5,4) (4,5) (5,6) (6,5)
2	(2,4) (4,2) (6,4) (4,6) (5,3) (3,5) (1,3) (3,1)
3	(5,2) (2,5) (3,6) (6,3) (1,4) (4,1)
4	(2,6) (6,2) (1,5) (5,1)
5	(1,6) (6,1)

$$P(Z)= \begin{cases} \frac{1}{6}, & Z=0 \\ \frac{5}{18}, & Z=1 \\ \frac{2}{9}, & Z=2 \\ \frac{1}{6}, & Z=3 \\ \frac{1}{9}, & Z=4 \\ \frac{1}{18}, & Z=5 \end{cases}$$

d)

Z \ Y	1	2	3	4	5	6
0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
1	$\frac{5}{108}$	$\frac{5}{108}$	$\frac{5}{108}$	$\frac{5}{108}$	$\frac{5}{108}$	$\frac{5}{108}$
2	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$
3	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{54}$
5	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{108}$

$$P_{ZY}(z,y)\begin{cases} \frac{1}{36}, z=0,3, x=1,2,3,4,5,6 \\ \frac{5}{108}, z=1, x=1,2,3,4,5,6 \\ \frac{1}{27}, z=2, x=1,2,3,4,5,6 \\ \frac{1}{54}, z=4, x=1,2,3,4,5,6 \\ \frac{1}{108}, z=5, x=1,2,3,4,5,6 \\ 0 \quad otherwise \end{cases}$$

e) $E\{Z\} = \left(\frac{1}{6}x0\right) + \left(\frac{5}{18}\right)x1 + \left(\frac{2}{9}\right)x2 + \left(\frac{1}{6}\right)x3 + \left(\frac{1}{9}\right)x4$

$+ \left(\frac{1}{18}\right)x5$

$= 1.94$ //

$E\{Z^3\} = \sum_{z=0}^{z} p(z)z^3 = 20.61$ //

$Var\{Z\} = E\{Z^2\} - E\{Z\}^2$

$= \sum_{z=0}^{z} p(z)z^2 - 3.7636$

$= 2.0697$ //