Ex 3-4)

$$x(t) = t^2 \quad , \quad -\pi < t < \pi, \quad , \quad T_0 = 2\pi, \quad \omega_0 = \frac{2\pi}{T_0} = 1$$

$$C_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{t^2}_{u} \cdot \underbrace{e^{-jkt}}_{v'} \, dt.$$

$$\Rightarrow \frac{1}{2\pi} \left[\cancel{\frac{t^2 \cdot e^{-jkt}}{(-jk)} \Bigg|_{-\pi}^{\pi}} \rightarrow 0 \; - \; \frac{2}{(-jk)} \int_{-\pi}^{\pi} t \cdot e^{-jkt} \, dt \right]$$

$$e^{-jk\pi} = (e^{-j\pi})^k = (-1)^k$$

$$e^{jk\pi} = (e^{j\pi})^k = (-1)$$

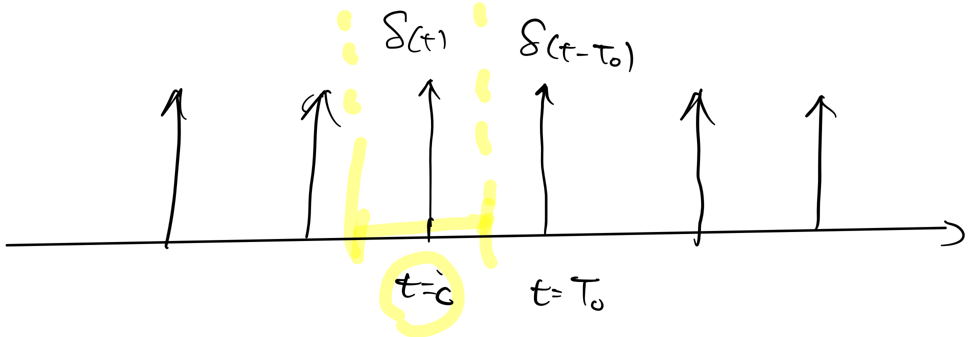
$$\Rightarrow \frac{1}{jk\pi} \left[\left(\frac{t}{(-jk)} - \frac{1}{(-jk)^2} \right) e^{-jkt} \Bigg|_{-\pi}^{\pi} \right]$$

$$= \frac{1}{jk\pi} \left[\left(\frac{\pi}{(-jk)} - \cancel{\frac{1}{(-jk)^2}} \right) (-1)^k + \left(\frac{\pi}{(-jk)} - \cancel{\frac{1}{(-jk)^2}} \right) (-1)^k \right]$$

$$= \frac{1}{jk\cancel{\pi}} \cdot \frac{2\cancel{\pi}}{(-jk)} (-1)^k = \frac{2(-1)^k}{k^2} = C_k.$$

Ex 3-3) Impulse train $\delta_{T_0}(t)$

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$



$$C_0 = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\,dt = \frac{1}{T_0}$$

$$C_k = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta_{T_0}(t)\, e^{-jk\omega_0 t}\,dt$$

$$\delta_{T_0}(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_0).$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \underbrace{\delta(t)}_{t=0}\, \underbrace{e^{-jk\omega_0 t}}_{t=0}\,dt$$

$$= \frac{1}{T_0}$$

$a_0 = 2C_0 = \frac{2}{T_0}$. $\quad a_k = 2\mathrm{Re}(C_k) = \frac{2}{T_0}$

$$\boxed{b_k = -2\,\mathrm{Im}(C_k) = \phi}$$

$$\boxed{\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)}$$

$$= \frac{1}{T_0} \sum_{k=-\infty}^{\infty} e^{-jk\omega_0 t}$$

$$= \frac{1}{T_0} + \frac{2}{T_0} \sum_{k=-\infty}^{\infty} \cos(k\omega_0 t)$$