$\int f(x)\,dx$ or $\int f(y)\,dy$ → Area

$\iint f(x,y)\,dx\,dy$ → 2D
→ Area
→ volume of surface.

$\iiint f(x,y,z)\,dx\,dy\,dz$ → 3D
→ volume of shape

Cylindrical
- change bounds
- Jacobian
  Det |2x2|

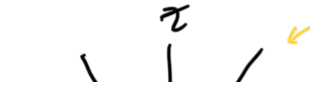
Spherical
- change bounds
- Jacobian
  Det |3x3|

Ex. $\int_{-1}^{1}\left[\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\left[\int_{\sqrt{x^2+y^2}}^{1} 1\,dz\right]dy\right]dx$

Ans.

$\sqrt{x^2+y^2} \le z \le 1$

$z = \sqrt{x^2+y^2}$

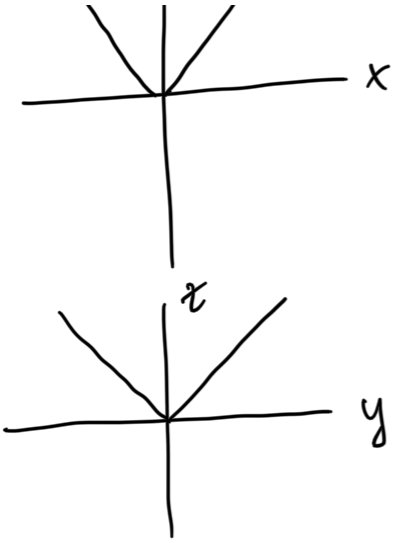


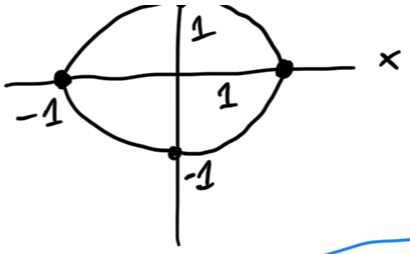
$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

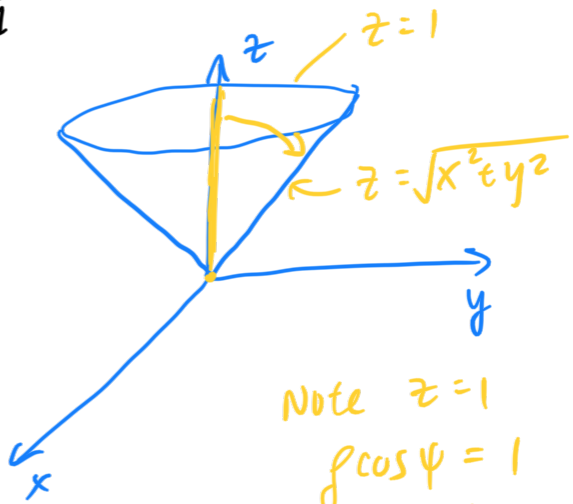
$y^2 = 1-x^2$

$x^2+y^2 = 1$

$-1 \le x \le 1$







$0 \le \theta \le 2\pi$

$0 \le \rho \le \sec\psi$

$0 \le \psi \le \pi/4$

Note $z = 1$

$\rho\cos\psi = 1$

$\rho = \frac{1}{\cos\psi}$

$\rho = \sec\psi$

Note. $z = \sqrt{x^2+y^2}$

$\rho\cos\psi = \sqrt{\rho^2\cos^2\theta\sin^2\psi + \rho^2\sin^2\theta\sin^2\psi}$

$\rho\cos\psi = \sqrt{\rho^2\sin^2\psi\,(\underbrace{\cos^2\theta + \sin^2\theta}_{1})}$

$\rho\cos\psi = \sqrt{\rho^2\sin^2\psi}$

$\rho\cos\psi = \rho\sin\psi$

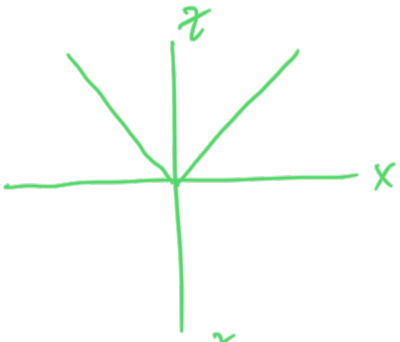
$1 = \frac{\sin\psi}{\cos\psi}$

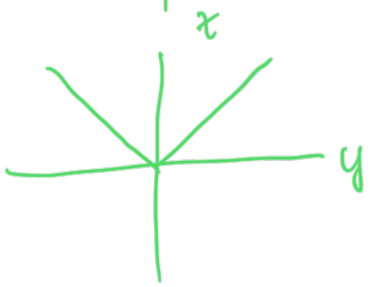
$1 = \tan\psi$

$\pi/4 = \psi$

EX $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{3(x^2+y^2)}}^{\sqrt{16-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dx\,dy$

$\sqrt{3(x^2+y^2)} \le z \le \sqrt{16-x^2-y^2}$





$0 \le \theta \le \frac{\pi}{2}$

$0 \le \rho \le 4$

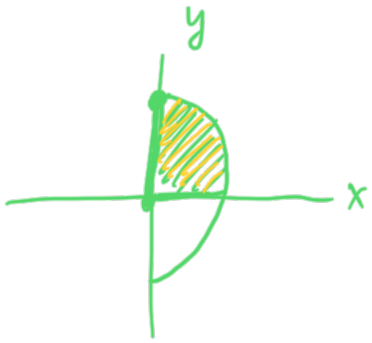
$0 \le \psi \le \pi/6$

$0 \le x \le \sqrt{4-y^2}$

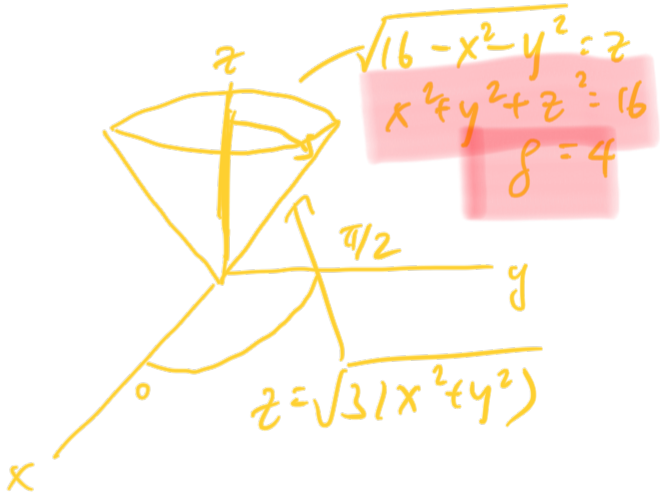
$x^2 = 4-y^2$

$x^2+y^2 = 4$

$r = 2$



$0 \le y \le 2$



$z = \sqrt{3(x^2+y^2)}$

$\rho\cos\psi = \sqrt{3(\rho^2\cos^2\theta\sin^2\psi)+(\rho^2\sin^2\theta\sin^2\psi)}$

$\rho\cos\psi = \sqrt{3\rho^2\sin^2\psi(\cos^2\theta+\sin^2\theta)}$

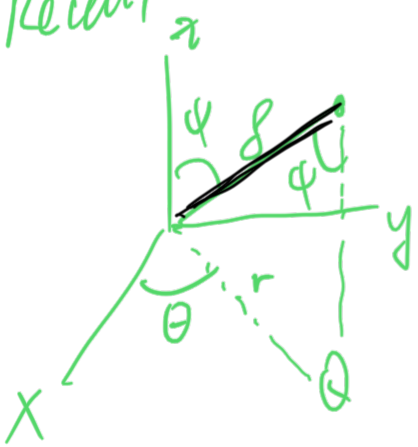
$\rho\cos\psi = \sqrt{3}\rho\sin\psi$

$\frac{1}{\sqrt{3}} = \frac{\sin\psi}{\cos\psi}$

$\frac{\pi}{6} = \psi$

Recall

*



pg 19. $x = \rho\sin\psi\cos\theta$

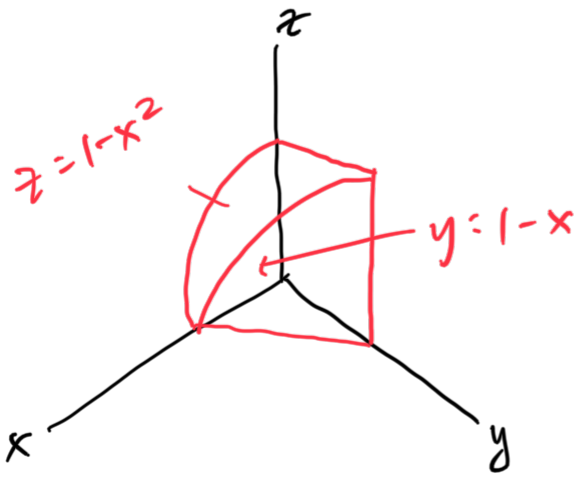
$y = \rho \sin\varphi \sin\theta$

$z = \rho \cos\varphi$

Conclusion

Ex Let E be the solid in the first octant bounded by the parabolic cylinder $z = 1 - x^2$ and by the plane $y = 1 - x$.

Ans.



$$\int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z)\, dz\, dy\, dx$$

$0 \le x \le 1$

$0 = 1 - x$

$x = 1$

$0 \le y \le \underbrace{1-x}_{y = 1-x}$

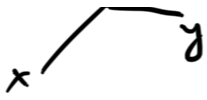
$0 \le z \le \underbrace{1-x^2}_{z = 1-x^2}$

Ex. $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$

HW.

Hint step 1) Draw diagram

2) $\left[\right.^{z}$ * green ↑



3) ρ, θ, ψ

4) Jacobian.