

# **Tutorial 1**

1. Consider an experiment of rolling 3 fair dice where the sum of the faces is recorded.
  - (a) Determine the sample space.
  - (b) Determine the number of possible events.
  - (c) Find the probability that the sum is (i) 3; (ii) 10; and (iii) 9.
  
2. Consider an experiment of tossing a fair coin. Find the probability that a tail will be resulted in 10 trials. For example, it includes the cases that a tail is obtained in the 1st and 10th trials. Also, when a tail appears in the 1st trial, then the experiment stops.

3. Two events  $A$  and  $B$  are related as  $B \subset A$ . Prove

$$P(B) \leq P(A)$$

4. For any events  $A$  and  $B$ , prove

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

5. 3 fair dice are rolled. What is the probability of obtaining at least one “6”?

6. Extend the result of  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  to the case of 3 events  $A$ ,  $B$ , and  $C$ :  $P(A \cup B \cup C)$ .

## **Solution**

1(a)

Since the minimum and maximum number are 3 and 18, respectively, the sample space is:

$$\{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$$

1(b)

Since there are 16 possible outcomes in the sample space, the number of possible events is  $2^{16} = 65536$ .

1(c)(i)

The chance of having 3 only when all faces are 1. Since each face has the same probability of  $1/6$ , and there is only one combination for  $1+1+1$ , we have:

$$P(\{3\}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

1(c)(ii)

Recall:

$$10 = 6+3+1 = 6+2+2 = 5+4+1 = 5+3+2 = 4+4+2 = 4+3+3$$

For "6+3+1", there are in fact 6 combinations with "6+1+3", "3+1+6", "3+6+1", "1+3+6", "1+6+3".

There are also 6 combinations for "5+4+1" and "5+3+2".

While for "6+2+2", "4+4+2" and "4+3+3", there are only 3 combinations. As a result,

$$P(\{10\}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot [6 + 3 + 6 + 6 + 3 + 3] = \frac{27}{216} = \frac{1}{8}$$

1(c)(iii)

$$9 = 6+2+1 = 5+3+1 = 5+2+2 = 4+4+1 = 4+3+2 = 3+3+3$$

$$P(\{9\}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot [6 + 6 + 3 + 3 + 6 + 1] = \frac{25}{216}$$

2.

Denote head and tail as H and T. For fair coin, we know:

$$P(H) = P(T) = \frac{1}{2}$$

From Example 1.2, possible outcomes are:

$x_1$ : T

$x_2$ : H, T

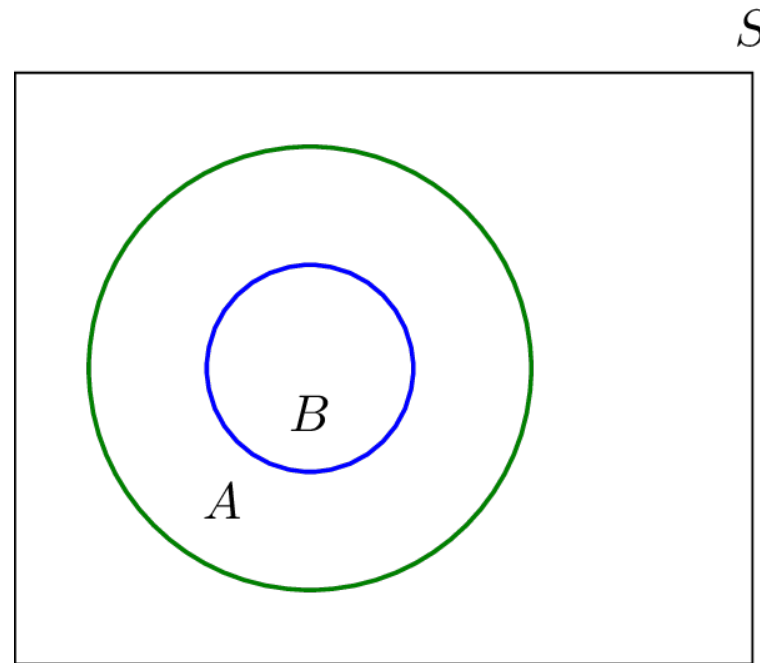
$x_3$ : H, H, T

...

The required probability should be:

$$\sum_{n=1}^{10} P(x_n) = \sum_{n=1}^{10} 0.5^n = 0.5 \frac{1 - 0.5^{10}}{1 - 0.5} = 0.999$$

3.



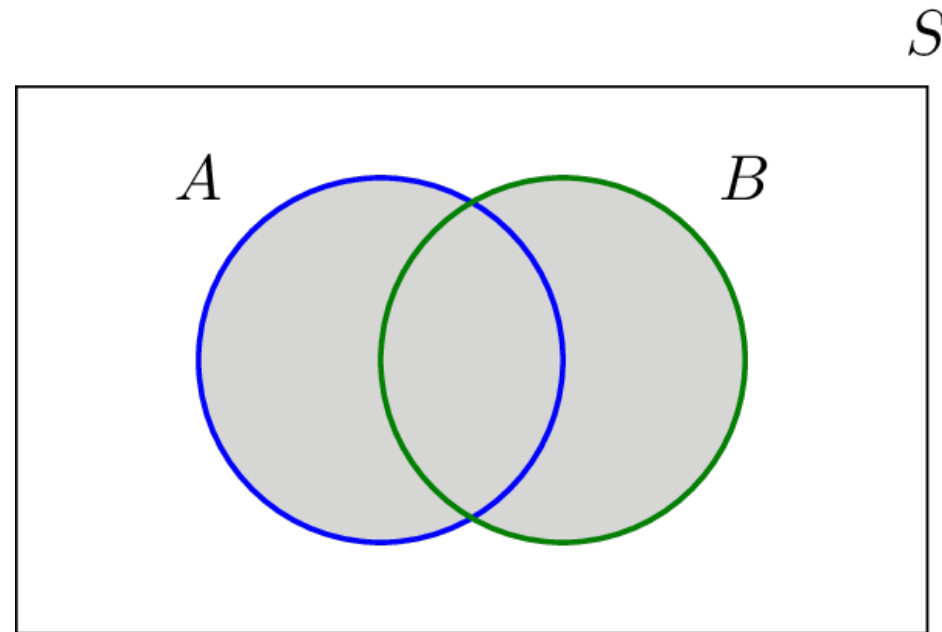
As  $B \subset A$ , we can express

$$A = B \cup (\overline{B} \cap A), \quad B \cap (\overline{B} \cap A) = \emptyset$$

Applying Axioms 1 and 3 yields:

$$P(A) = P(B) + P(\overline{B} \cap A) \geq P(B)$$

4.



$$A \cup B = A \cup (\bar{A} \cap B), \quad A \cap (\bar{A} \cap B) = \emptyset$$

$$B = (A \cap B) \cup (\bar{A} \cap B), \quad (A \cap B) \cap (\bar{A} \cap B) = \emptyset$$

Applying Axiom 3 yields:

$$P(A \cup B) = P(A) + P(\bar{A} \cap B) \quad \text{and} \quad P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

The proof is then obtained from the two equations.

5.

At least one “6” can mean one, two or three “6”. One direction is to compute the probabilities of these 3 cases and sum together. On the other hand, we can make use of the complement rule in (1.1):

$$\overline{\bigcup_{n=1}^N A_n} = \overline{A_1 \cup A_2 \cup \dots \cup A_N} = \overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_N} = \bigcap_{n=1}^N \overline{A_n}$$

That is, we first compute the probability that there is no “6”, and then subtract this probability from 1:

$$P(\geq 1 \text{ "6"}) = 1 - \left(\frac{5}{6}\right)^3 = \frac{91}{216}$$



6.

With the use of the Venn diagram or by other means, we get:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

