CITY UNIVERSITY OF HONG KONG

Course code and title	:	MA1200 Calculus and Basic Linear Algebra I
Session	:	Semester B, 2013/2014
Time allowed	:	Two hours

This paper has **SIX** pages (including this cover page).

A brief table of derivatives is attached on pages 5 and 6.

Instructions to candidates:

1. This paper has **TEN** questions.
2. Attempt **ALL** questions in Section A and B.
3. Each question in Section A carries 9 marks.
4. Each question in Section B carries 15 marks.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

Section A

Answer **ALL** questions in this section. Each question carries 9 marks.

Question 1

(a) Let

$$f(x)=\begin{cases}\dfrac{x^3-8}{x-2}, & \text{if } x\neq 2\\ c, & \text{if } x=2.\end{cases}$$

Find the value of c for which $f(x)$ is continuous at $x=2$. Give your reason.

(5 marks)

(b) Evaluate

$$\lim_{x\to 0}\frac{x-\sin x}{\tan^2 x}.$$

(4 marks)

Question 2

Express $\dfrac{x+13}{(x+3)(x^2+x-1)}$ in partial fractions. (9 marks)

Question 3

Differentiate with respect to x:

(a) $x^3\log_e x$; (3 marks)

(b) $\sin^2 x+2\sinh x$; (3 marks)

(c) $\sqrt[3]{\dfrac{(2x-1)(3x+5)}{x^2+1}}$. (3 marks)

Question 4

(a) If $y=\cos x$, find $\dfrac{dy}{dx},\dfrac{d^2y}{dx^2},\dfrac{d^3y}{dx^3}$ and then conjecture the formula for $\dfrac{d^ny}{dx^n}, n\in\mathbb{N}$.

(6 marks)

(b) Find the value of $\dfrac{d^{20}}{dx^{20}}(\cos x)$

when $x=\dfrac{\pi}{4}$. (3 marks)

Question 5

$P(2, \frac{2+\sqrt{3}}{2})$ is a point on the curve $x^2+4y^2-6x-8y+9=0$.

(a) Find the slope of the tangent to the curve at P. (4 marks)

(b) Find the equation of the normal to the curve at P. (5 marks)

Question 6

A curve is given parametrically by the equations, $x = 4\sin t$, $y = 3\cos t$, where t is the parameter and $0 \le t \le 2\pi$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t. (4 marks)

(b) Eliminate t from the given equations to find the Cartesian equation of the curve. (3 marks)

(c) What is the geometrical shape of the curve represented by this equation? (2 marks)

Question 7

Solve the equation

$$\sinh^2 x - \cosh x - 1 = 0.$$

(9 marks)

(Hint: $\sinh x = \frac{1}{2}(e^x - e^{-x})$,

$\cosh x = \frac{1}{2}(e^x + e^{-x})$,

$\cosh^2 x - \sinh^2 x = 1$.)

Question 8

Find the coordinates of the local maximum and minimum points of the curve $y = \frac{x^3}{1-x^2}$ and show that there is a value of x for which $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is zero. (9 marks)

Section B
Answer **ALL** questions in this section. Each question carries 15 marks.

Question 9

If $y = e^{\sin^{-1} x}$, show that

$$(1-x^2)\frac{d^2y}{dt^2} - x\frac{dy}{dx} - y = 0$$

By further differentiation of this result, or otherwise, find the values of $\frac{d^3y}{dx^3}, \frac{d^4y}{dx^4}$ and $\frac{d^5y}{dx^5}$ at $x = 0$.

Hence, find the Maclaurin series for $e^{\sin^{-1} x}$ as far as the term in x^5. (15 marks)

Question 10

(a) A spherical balloon is being inflated, the volume increasing at the constant rate of 20 $cm^3 s^{-1}$. Find the rate of increase of its surface area when its radius is 15cm long. (8 marks)

(Hint: The formulae for the volume of a sphere of radius r units and for its surface area are $V = \frac{4\pi r^3}{3}$ units^3 and $S = 4\pi r^2$ units^2 respectively.)

(b) A brick in the shape of a cuboid with dimensions x units by $3x$ units by y units. The total surface area of the brick is 1800 units^2.

(i) Show that $y = \frac{900 - 3x^2}{4x}$.

(ii) Find the dimensions of the brick that will yield the largest volume. (7 marks)

Short Table of Derivatives of $y=f(u)$ with respect to x, where u is a function of x

Functions, $y=f(u)$	**Derivative of y with respect to x**
$y=c$, where c is a constant.	$\frac{dy}{dx}=0$
$y=cu$, where c is a constant.	$\frac{dy}{dx}=c\frac{du}{dx}$
$y=u^p$, where p is a constant.	$\frac{dy}{dx}=pu^{p-1}\frac{du}{dx}$
$y=u+v$	$\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$
$y=uv$	$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$
$y=\frac{u}{v}$	$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
$y=f(u)$, where u is a function of x.	$\frac{dy}{dx}=\frac{d f(u)}{du}\cdot\frac{du}{dx}$, the chain rule
$y=\log_a u$, $a>0$.	$\frac{dy}{dx}=\frac{1}{u}\log_a e\frac{du}{dx}$
$y=a^u$, $a>0$.	$\frac{dy}{dx}=a^u\log_e a\frac{du}{dx}$
$y=e^u$	$\frac{dy}{dx}=e^u\frac{du}{dx}$
$y=u^v$	$\frac{dy}{dx}=vu^{v-1}\frac{du}{dx}+u^v\log_e u\frac{dv}{dx}$
$y=\sin u$	$\frac{dy}{dx}=\cos u\frac{du}{dx}$
$y=\cos u$	$\frac{dy}{dx}=-\sin u\frac{du}{dx}$
$y=\tan u$	$\frac{dy}{dx}=\sec^2 u\frac{du}{dx}$
$y=\cot u$	$\frac{dy}{dx}=-\mathrm{cosec}^2 u\frac{du}{dx}$
$y=\sec u$	$\frac{dy}{dx}=\sec u\tan u\frac{du}{dx}$
$y=\mathrm{cosec}\, u$	$\frac{dy}{dx}=-\mathrm{cosec}\, u\cot u\frac{du}{dx}$
$y=\sin^{-1}u$	$\frac{dy}{dx}=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
$y=\cos^{-1}u$	$\frac{dy}{dx}=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$
$y=\tan^{-1}u$	$\frac{dy}{dx}=\frac{1}{1+u^2}\frac{du}{dx}$

Functions, $y = f(u)$	Derivative of y with respect to x
$y = \cot^{-1} u$	$\frac{dy}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$
$y = \sec^{-1} u$	$\frac{dy}{dx} = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$
$y = \operatorname{cosec}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$
$y = \sinh u$	$\frac{dy}{dx} = \cosh u \frac{du}{dx}$
$y = \cosh u$	$\frac{dy}{dx} = \sinh u \frac{du}{dx}$
$y = \tanh u$	$\frac{dy}{dx} = \operatorname{sech}^2 u \frac{du}{dx}$
$y = \coth u$	$\frac{dy}{dx} = -\operatorname{cosech}^2 u \frac{du}{dx}$
$y = \operatorname{sech} u$	$\frac{dy}{dx} = -\operatorname{sech} u \tanh u \frac{du}{dx}$
$y = \operatorname{cosech} u$	$\frac{dy}{dx} = -\operatorname{cosech} u \coth u \frac{du}{dx}$
$y = \sinh^{-1} u$	$\frac{dy}{dx} = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$
$y = \cosh^{-1} u$	$\frac{dy}{dx} = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$
$y = \tanh^{-1} u$	$\frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$
$y = \coth^{-1} u$	$\frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$
$y = \operatorname{sech}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$
$y = \operatorname{cosech}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{\lvert u\rvert\sqrt{u^2+1}}\frac{du}{dx}$

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