

Exercise on Positive Definite

1. Determine whether each of the following quadratic forms in two variables is positive or negative definite or semidefinite, or indefinite.
 - a. $x^2 + 2xy$.
 - b. $-x^2 + 4xy - 4y^2$
 - c. $-x^2 + 2xy - 3y^2$.
 - d. $4x^2 + 8xy + 5y^2$.
 - e. $-x^2 + xy - 3y^2$.
 - f. $x^2 - 6xy + 9y^2$.
 - g. $4x^2 - y^2$.
 - h. $(1/2)x^2 - xy + (1/4)y^2$.
 - i. $6xy - 9y^2 - x^2$.
2. Determine whether each of the following quadratic forms in three variables is positive or negative definite or semidefinite, or indefinite.
 - a. $-x^2 - y^2 - 2z^2 + 2xy$
 - b. $x^2 - 2xy + xz + 2yz + 2z^2 + 3zx$
 - c. $-4x^2 - y^2 + 4xz - 2z^2 + 2yz$
 - d. $-x^2 - y^2 + 2xz + 4yz + 2z^2$
 - e. $-x^2 + 2xy - 2y^2 + 2xz - 5z^2 + 2yz$
 - f. $y^2 + xy + 2xz$
 - g. $-3x^2 + 2xy - y^2 + 4yz - 8z^2$
 - h. $2x^2 + 2xy + 2y^2 + 4z^2$
3. Consider the quadratic form $2x^2 + 2xz + 2ayz + 2z^2$, where a is a constant. Determine the definiteness of this quadratic form for each possible value of a .
4. Determine the values of a for which the quadratic form $x^2 + 2axy + 2xz + z^2$ is positive definite, negative definite, positive semidefinite, negative semidefinite, and indefinite.
5. Consider the matrix

$$\begin{Bmatrix} a & 1 & b \\ 1 & -1 & 0 \\ b & 0 & -2 \end{Bmatrix}.$$

Find conditions on a and b under which this matrix is negative definite, negative semidefinite, positive definite, positive semidefinite, and indefinite. (There may be no values of a and b for which the matrix satisfies some of these conditions.)

6. Show that the matrix

$$\begin{Bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{Bmatrix}$$

is not positive definite.