EE3331 Probability Models in Information Engineering

Semester B 2021 – 2022

Test 1

12:00 p.m. – 1:30 p.m.

Answer **ALL SIX** questions:

Question 1

Given three events A, B and C with probabilities $P(A) = 0.25$, $P(B) = 0.4$, $P(C) = 0.45$, and $P(A \cap B \cap C) = 0.1$. It is also known that A and B are independent, B and C are independent, and $A \cap \overline{B} \cap C = \emptyset$.

(a)   Draw the Venn diagram with indicating all probabilities.   **(10 marks)**

(b)   Find $P((A \cup B) \cap C)$ and $P(\overline{A} \cap (\overline{B} \cup C))$.   **(6 marks)**

(c)   Are A and C independent? Briefly explain your answers.   **(4 marks)**

Question 2

(a)   Determine the number of possible arrangements for the letters in the word PROBABILITY.
                                                                                                          (4 marks)

(b)   A password consists of 6 characters which can only include upper-case English alphabet letters "A" to "Z" and digits "0" to "9".
    (i)   If repeated characters are allowed, determine the total number of possible passwords.
                                                                                                          (4 marks)
    (ii)   If repeated characters are not allowed, determine the total number of possible passwords.   **(4 marks)**
    (iii)   Repeat (b)(i) and (b)(ii) with the restriction that the first two characters must be digits.
                                                                                                          (8 marks)

Question 3

Discrete random variable X is an angle in degrees, and its probability mass function (PMF) is:

$$p(x) = \begin{cases} \alpha, & x = 90° \\ \beta, & x = 180° \\ 0.4, & x = 270° \\ 0, & \text{otherwise} \end{cases}$$

The random variable X is transformed to another random variable Y as $Y = \sin(X)$.

(a)   Is Y a discrete random variable?   **(1 marks)**

(b)   Determine the range of possible values of $\mathbb{E}\{Y\}$.   **(6 marks)**

(c)   If $\mathbb{E}\{Y\} = 0.1$, compute the values of α and β.   **(3 marks)**

Question 4
A Gaussian random variable $X \sim \mathcal{N}(2,2)$ is transformed to another random variable Y as $Y = aX + b$ where a and b are constants. If the mean and power of Y are 10 and 200, respectively, determine the values of a and b. **(15 marks)**

Question 5
An online test consists of 10 multiple choice (MC) questions where each question has 4 choices with only one correct answer. To pass the test, at least 40 marks are needed. A student randomly chooses one choice for each question.

(a) Suppose for each correct MC answer, 10 marks will be received, while no marks will be deducted for an incorrect choice.
 (i) Compute the mean marks for one MC question. **(3 marks)**
 (ii) Find the probability that this student can pass the test. **(5 marks)**

(b) Suppose for each correct MC answer, 10 marks will be received, while 3 marks will be deducted for an incorrect choice.
 (i) Compute the mean marks for one MC question. **(3 marks)**
 (ii) Find the probability that this student can pass the test. **(5 marks)**

(c) Let X be the random variable representing the total marks for (b). Determine the possible values of X. **(4 marks)**

Question 6
A company has 4 machines, namely, B_1, B_2, B_3, and B_4, which manufacture resistors. A resistor with gold tolerance band means that its actual resistance is within $\pm 5\%$ of the nominal value. It is known that 90%, 80%, 70%, and 60% of the resistors produced by B_1, B_2, B_3, and B_4, respectively, correspond to the gold tolerance band. Furthermore, the production rates of B_1, B_2, B_3, and B_4 are 300, 400, 500, and 600 resistors per minute. All of the produced resistors are randomly mixed together in one bin and packed for shipment.

(a) What is the probability that the company ships a resistor with gold tolerance band? **(8 marks)**

(b) What is the probability that a resistor with gold tolerance band comes from B_4? **(7 marks)**