EE 4211 Computer Vision

Lecture 3A: Image enhancement (Frequency)

Semester B, 2021-2022

Schedules

Week	Date	Topics
1	Jan. 11 (face to face)	Introduction/Imaging
2	Jan. 18 (online)	Image enhancement in spatial domain
3	Jan. 25 (online)	Image enhancement in frequency domain (HW1 out) publish online on Jan. 26 noon.
4	Feb. 8	Morphological processing
5	Feb. 15	Image restoration (HW1 due)
6	Feb. 22	Midterm (no tutorials this week)
7	Mar. 1	Edge detection (HW2 out)
8	Mar. 8	Image segmentation
9	Mar. 15	Face recognition with PCA, LDA (tutorial on segmentation) (HW2 due)
10	Mar. 22	Face recognition based on deep learning Image segmentation based on deep learning (tutorial on detection)
11	Mar. 29	Object detection with traditional methods (Quiz) Object detection based on deep learning
12	Apr. 5	Events / Public Holidays
13	Apr. 12	Invited project presentation and Summary

Project information

- Topics: Image segmentation and object detection
- Four students form a group (file sharing, finalized before Mar. 8)

https://docs.google.com/spreadsheets/d/1b9LiFO4XwFJCpn-eZOOUU18IEQRobP4cRcJid1JtZlI/edit?usp=sharing

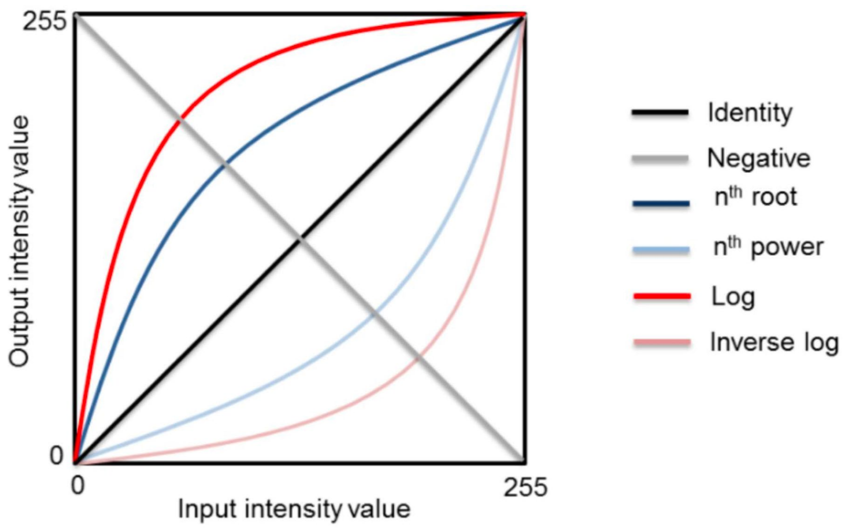
- Images: will be provided through Kaggle and Kaggle link will be published Mar. 8
- Codes:
  - Basic codes including the traditional ones and deep learning ones will be provided.
  - Please modify these codes to achieve better performance
- Submission and Evaluation:
  - Codes and reports should be submitted
  - Report writing(will illustrate the writing in tutorial session), results
  - Excellent ones will be invited to present their work in the last lecture and get extra bonus for the marks

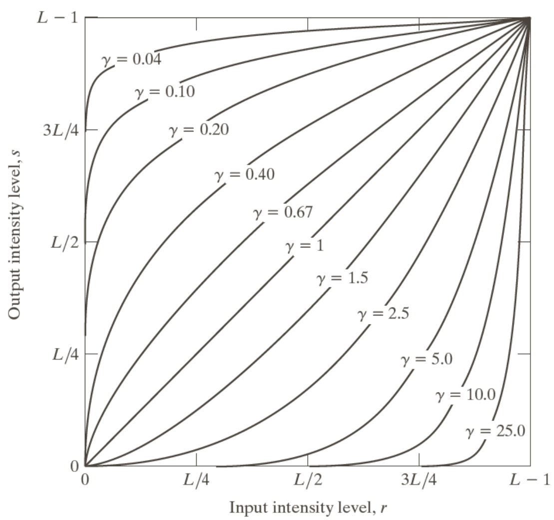
Spatial Domain vs. Frequency Domain

- Spatial Domain (image plane)
  - Techniques are based on direct manipulation of pixels in an image
- Frequency Domain
  - Techniques are based on modifying the spectral transform (in our course, we'll use Fourier transform) of an image
- There are some enhancement techniques based on various combinations of methods from these 2 domains

Spatial Domain Topics

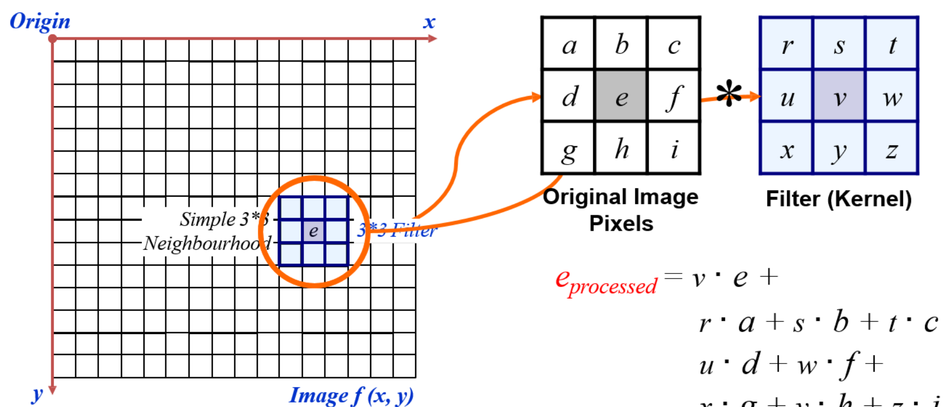
- Point processing – Gray values change without any knowledge of its surroundings (Part I)
  - Log (spectrum), power-law, linear (choose suitable transforms）
  - Histogram Equalization (pdf->cdf->transformation) (calculation)





Spatial Domain Topics

- Neighborhood processing – Gray values change depending on the gray values in a small neighborhood of pixels around the given pixel (Part II)
  - Smoothing filters
  - Median filters
  - Sharpening



$$e_{processed} = v \cdot e + r \cdot a + s \cdot b + t \cdot c + u \cdot d + w \cdot f + x \cdot g + y \cdot h + z \cdot i$$

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

0	1	0
1	-4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

Spatial Domain Topics

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Has clear physical meaning: Calculate four direction gradients and also preserve the original value

$$= 5f(x,y) - f(x+1,y) - f(x-1,y) - f(x,y+1) - f(x,y-1)$$

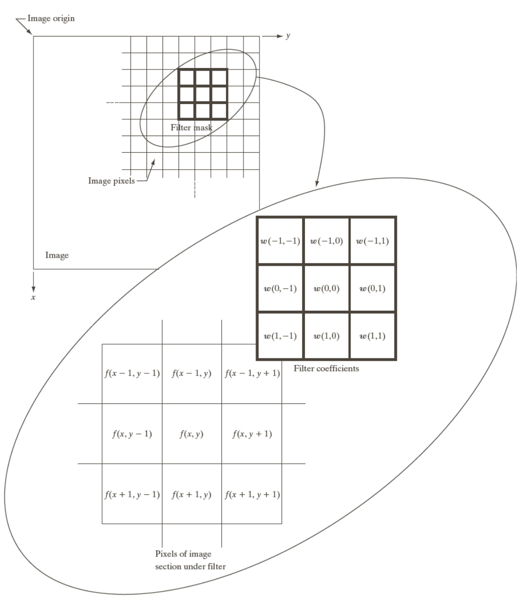
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Spatial Domain Topics

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

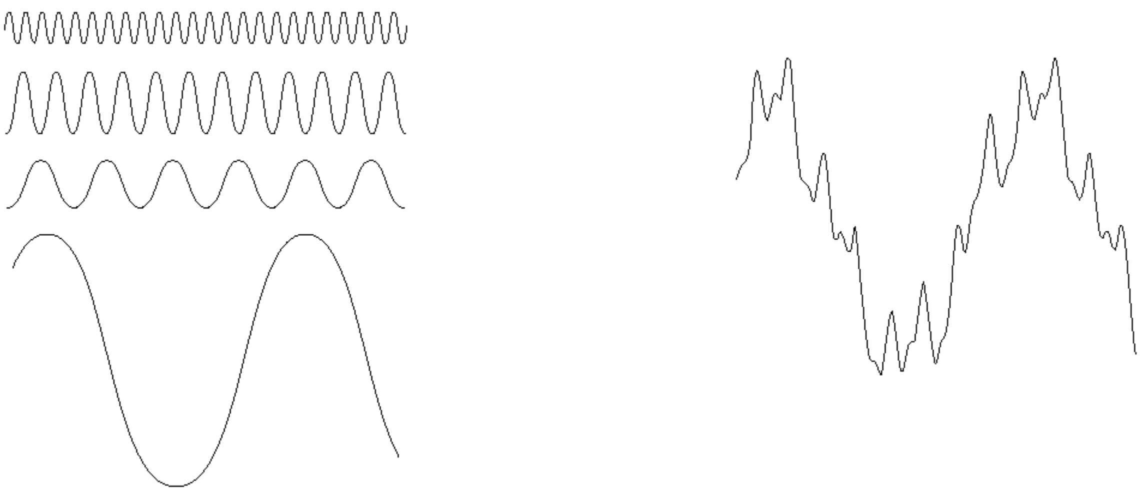
Has clear physical meaning: Calculate eight direction gradients and also preserve the original value

$$\begin{aligned} &= 9f(x,y) \\ &- f(x-1,y-1) - f(x-1,y) - f(x\text{-}1,y+1) \\ &- f(x,y-1) - f(x,y) - f(x,y+1) \\ &- f(x+1,y-1) - f(x+1,y) - f(x+1,y+1) \end{aligned}$$



What is a Transform?

- Transforms are decompositions of a function f(x) into some basis functions Φ(x,u)
- Fourier's Idea in 1807: Periodic functions could be represented as a weighted sum of sines and cosines



The right one can be represented by a weighted sum of the left ones

1D Fourier Transform

- Forward Transform

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} \, dx$$

- Inverse Transform

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} \, du$$

Discrete Fourier Transform (DFT)

- The Fourier Transform of a discrete function (DFT) of one variable, f(x), x=0,1,...M-1, is given by

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}$$

- The Inverse DFT is

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M}$$

Discrete Fourier Transform (DFT)

- The Euler's formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\boxed{F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M}}$$

- By substituting the Euler formula, we can write the DFT as:

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos(2\pi ux/M) - j\sin(2\pi ux/M)]$$

- The domain of values of *u* is the frequency domain

Example: 1D FT

$\int \cos(x)dx$	$\sin(x)$
$\int \sin(x)dx$	$-\cos(x)$
$\int x\cos(x)dx$	$\cos(x) + x\sin(x)$
$\int x\sin(x)dx$	$\sin(x) - x\cos(x)$
$\int x^2\cos(x)dx$	$2x\cos(x) + (x^2 - 2)\sin(x)$
$\int x^2\sin(x)dx$	$2x\sin(x) - (x^2 - 2)\cos(x)$
$\int e^{\alpha x}dx$	$\frac{e^{\alpha x}}{a}$
$\int xe^{\alpha x}dx$	$e^{\alpha x}\left[\frac{x}{a} - \frac{1}{a^2}\right]$
$\int x^2e^{\alpha x}dx$	$e^{\alpha x}\left[\frac{x^2}{a} - \frac{2x}{a^2} - \frac{2}{a^3}\right]$
$\int \frac{dx}{\alpha + \beta x}$	$\frac{1}{\beta}\ln\lvert\alpha + \beta x\rvert$
$\int \frac{dx}{\alpha^2 + \beta^2 x^2}$	$\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\beta x}{\alpha}\right)$

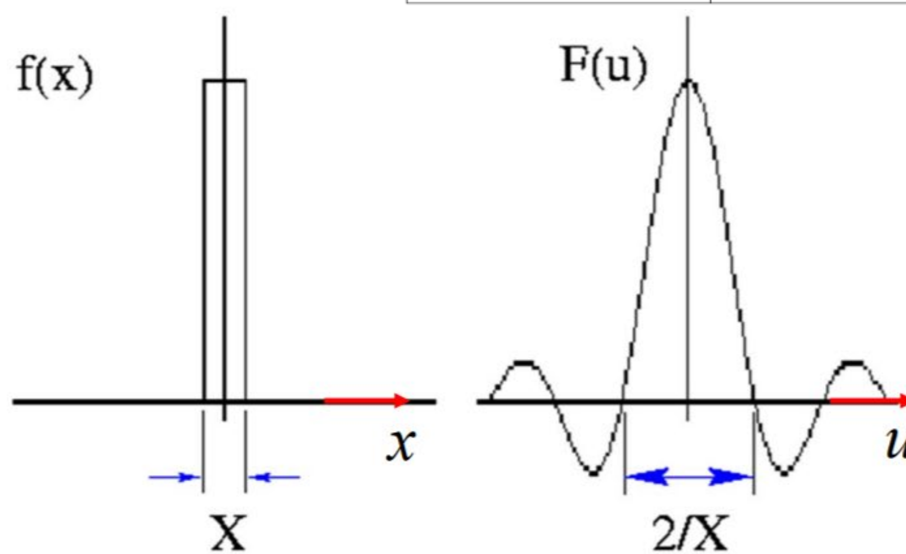
- Reminder transform pair – definition

$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}\,dx$$
$$f(x) = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux}\,du$$

- Example

$$f(x) = \begin{cases} 1, & |x| < \frac{X}{2}, \\ 0, & |x| \geq \frac{X}{2}. \end{cases}$$

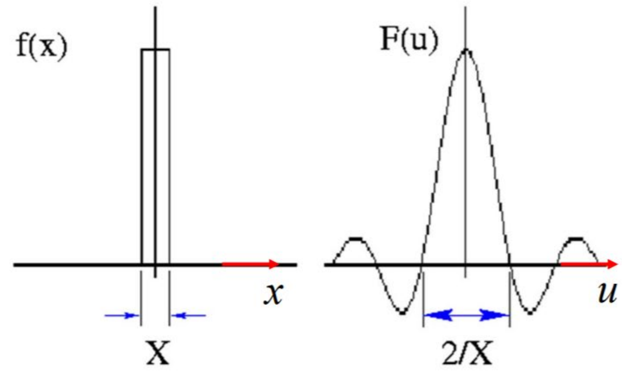
$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx$$
$$= \int_{-X/2}^{X/2} e^{-j2\pi ux}dx \qquad \left[\frac{e^{-j2\pi ux}}{-j2\pi u}\right]_{-X/2}^{X/2}$$
$$= \frac{1}{-j2\pi u}[e^{-j2\pi uX/2} - e^{j2\pi uX/2}]$$
$$= X\frac{\sin(\pi Xu)}{(\pi Xu)} = X\mathrm{sinc}(\pi \mathrm{X}u).$$



Example: 1D FT

$$f(x) = \begin{cases} 1, & |x| < \frac{X}{2}, \\ 0, & |x| \geq \frac{X}{2}. \end{cases}$$

$$\begin{aligned} F(u) &= \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \\ &= \int_{-X/2}^{X/2} e^{-j2\pi ux} dx \\ &= \frac{1}{-j2\pi u}[e^{-j2\pi uX/2} - e^{j2\pi uX/2}] \\ &= X\frac{\sin(\pi X u)}{(\pi X u)} = X\text{sinc}(\pi \text{Xu}). \end{aligned}$$



parameter

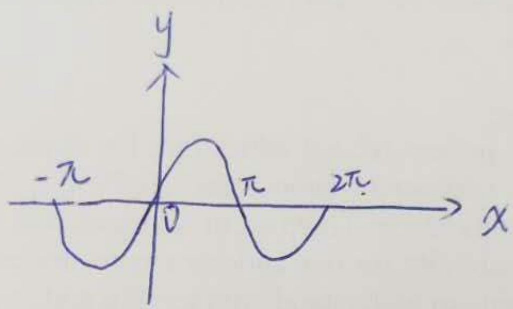
$$X \frac{\sin(\pi X u)}{\pi X u} = 0$$

$$\Downarrow$$

$$\sin(\pi X u) = 0$$

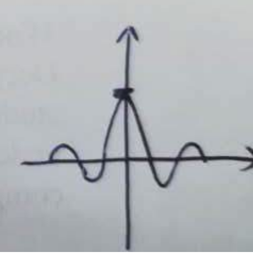
$$\pi X u = k\pi \qquad k = \cdots -1, 0, 1, \cdots$$

$$u = \frac{k}{X}$$



$$\frac{1}{X} - \frac{-1}{X} = \frac{2}{X}$$

/ When $k=0$, $u \doteq 0$, $\frac{\sin(\pi X u)}{\pi X u} \to \infty$



2-D Fourier Transform

1-D Fourier Transform

$$\boxed{\begin{aligned} F(u) &= \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux}\, dx \\ f(x) &= \int_{-\infty}^{\infty} F(u) e^{j2\pi ux}\, du \end{aligned}}$$

- Forward Transform

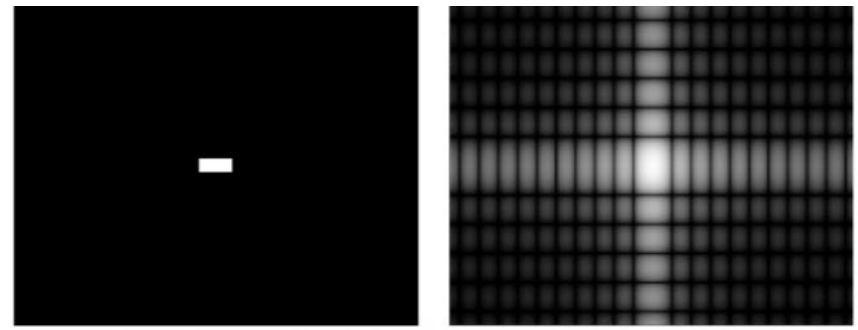
$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)}\, dx\, dy$$

- Inverse Transform

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)}\, du\, dv$$

Important Fourier Transform Pairs

rectangle centred at origin
with sides of length X and Y



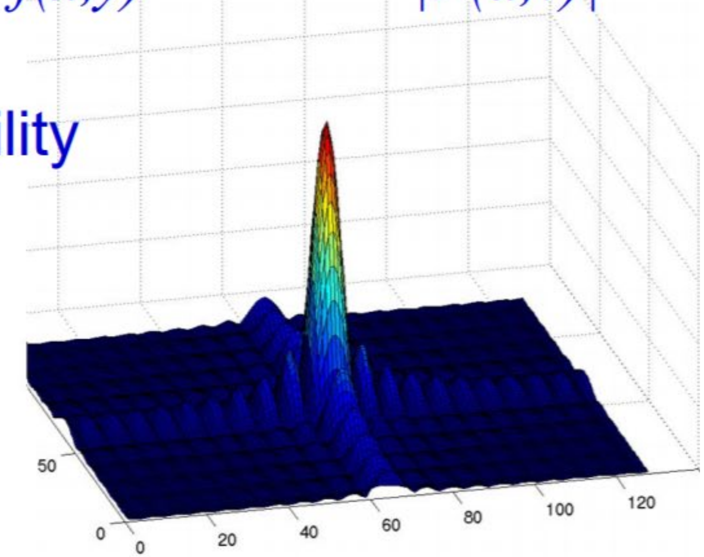
f(x,y)

|F(u,v)|

When X is larger than Y, then the value

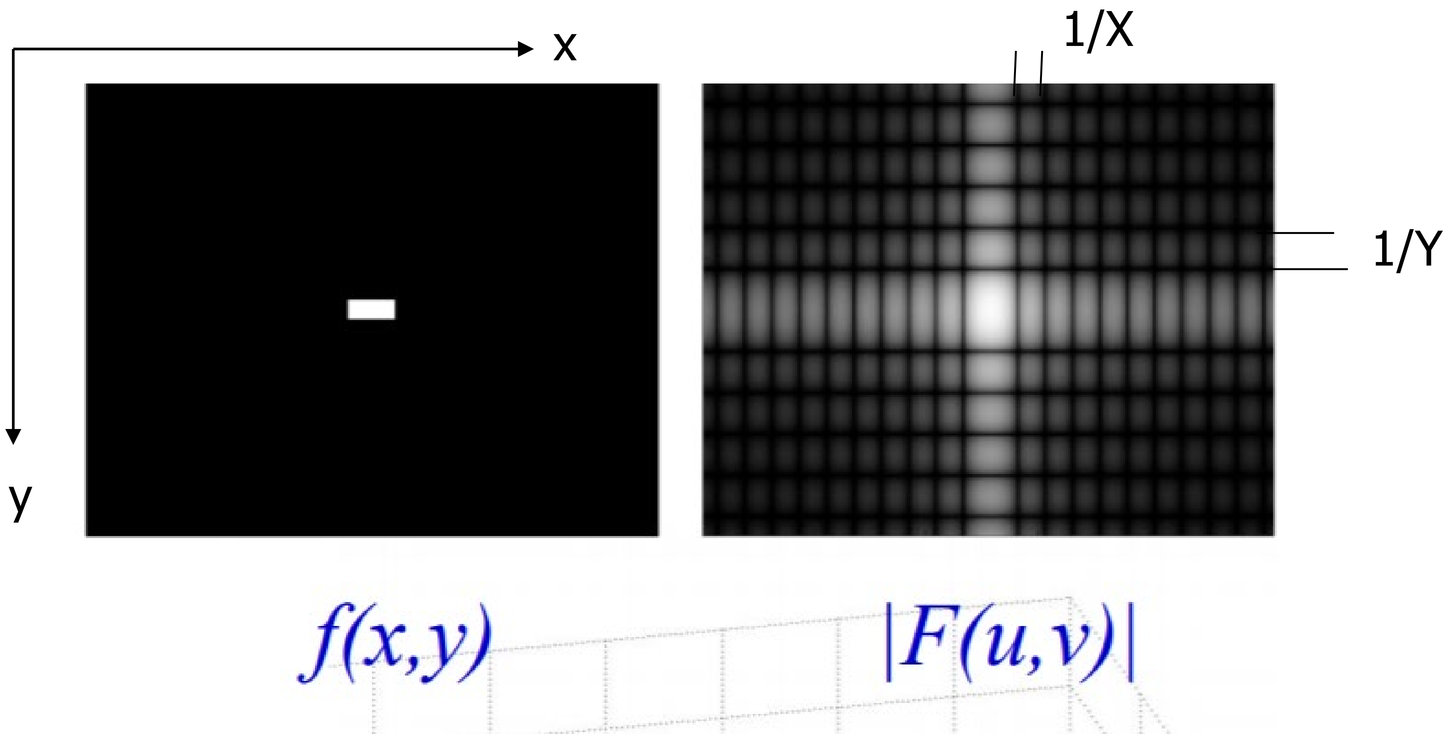
$$
\begin{aligned}
F(u,v) &= \int\int f(x,y)e^{-j2\pi(ux+vy)}dxdy, \\
&= \int_{-X/2}^{X/2} e^{-j2\pi ux}dx \int_{-Y/2}^{Y/2} e^{-j2\pi vy}dy, \\
&= \left[\frac{e^{-j2\pi ux}}{-j2\pi u}\right]_{-X/2}^{X/2} \left[\frac{e^{-j2\pi vy}}{-j2\pi v}\right]_{-Y/2}^{Y/2}, \\
&= \frac{1}{-j2\pi u}\left[e^{-juX} - e^{juX}\right]\frac{1}{-j2\pi v}\left[e^{-jvY} - e^{jvY}\right], \\
&= XY\left[\frac{sin(\pi Xu)}{\pi Xu}\right]\left[\frac{sin(2\pi Yv)}{\pi Yv}\right] \\
&= XY\mathrm{sinc}(\pi \mathrm{Xu})\mathrm{sinc}(\pi \mathrm{Yv}).
\end{aligned}
$$

separability



|F(u,v)|

Important Fourier Transform Pairs

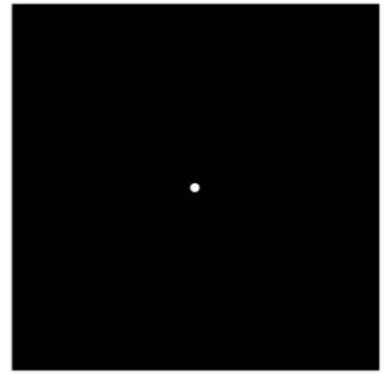


Important Fourier Transform Pairs

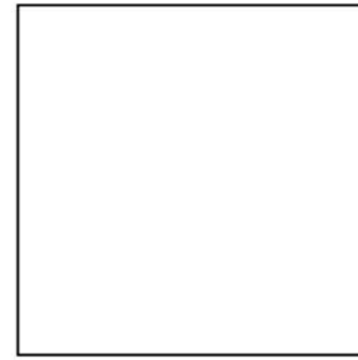
$$f(x,y) = \delta(x,y) = \delta(x)\delta(y)$$

$$F(u,v) \;=\; \int\int \delta(x,y) e^{-j2\pi(ux+vy)}\, dxdy$$

$$=\; 1$$



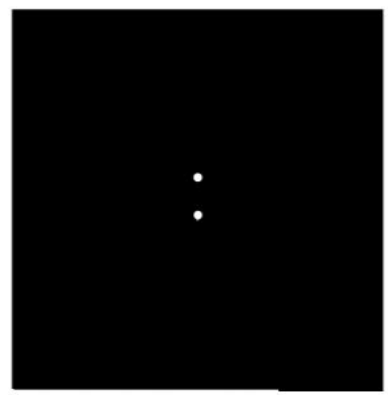
f(x,y)



F(u,v)

$$\boxed{\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0). \qquad \int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)}$$

$$f(x,y) = \frac{1}{2}(\delta(x,y-a) + \delta(x,y+a))$$



$$F(u,v) \;=\; \frac{1}{2}\int\int (\delta(x,y-a) + \delta(x,y+a))\, e^{-j2\pi(ux+vy)}\, dxdy$$

$$=\; \frac{1}{2}\left(e^{-j2\pi av} + e^{j2\pi av}\right) = \cos 2\pi av$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

2-D Discrete Fourier Transform

- The Fourier Transform of a discrete function (DFT) of two variables, f(x, y), an image of size MxN, where x=0,1,...M-1 and y=0,1,...N-1 is given by

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

- The Inverse 2-D DFT is

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

2-D Discrete Fourier Transform

$$F(u,v) = R(u,v) + jI(u,v) = |F(u,v)|e^{j\phi(u,v)}$$

- Fourier Spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v) \right]^{1/2}$$

- Phase

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)} \right]$$

- Power Spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

2D DFT Example (1)

- Calculation

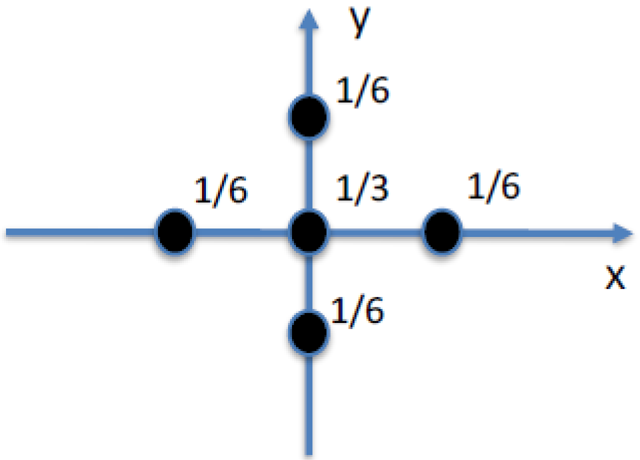
$$H(u,v) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} h(x,y)e^{-j2\pi(xu+yv)}$$

$$= h(0,0) + h(-1,0)e^{j2\pi u} + h(1,0)e^{-j2\pi u} + h(0,-1)e^{j2\pi v}$$
$$+ h(0,1)e^{-j2\pi v}$$

$$= \frac{1}{3} + \frac{1}{6}e^{j2\pi u} + \frac{1}{6}e^{-j2\pi u} + \frac{1}{6}e^{j2\pi v} + \frac{1}{6}e^{-j2\pi v}$$

$$= \frac{1}{3} + \frac{1}{6} \cdot 2cos2\pi u + \frac{1}{6} \cdot 2cos2\pi v$$

$$= \frac{1}{3}(1 + cos2\pi u + cos2\pi v)$$



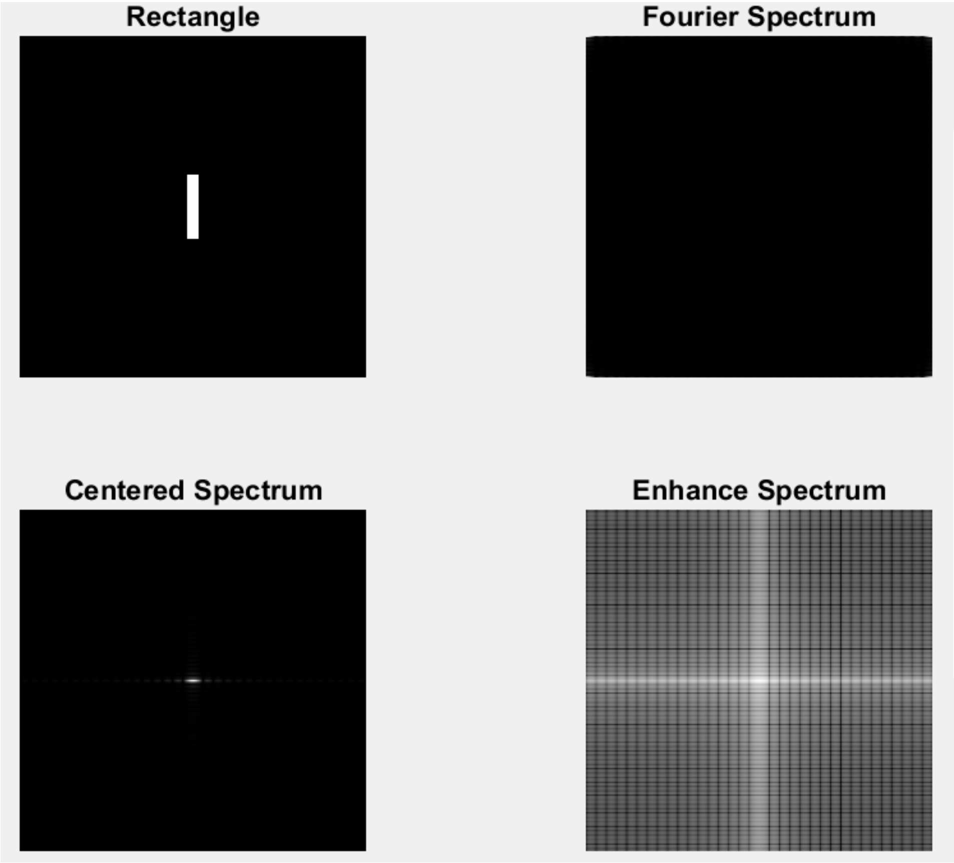
2D DFT Example (2)

$$h(x,y) = \begin{bmatrix} 0.075 & 0.124 & 0.075 \\ 0.124 & 0.204 & 0.124 \\ 0.075 & 0.124 & 0.075 \end{bmatrix}$$

$$H(u,v) = 0.204 + 0.124 \cdot 2 \cdot cos2\pi u + 0.124 \cdot 2 \cdot cos2\pi v + 0.075 \cdot 2 \cdot \cos(2\pi u + 2\pi v) + + 0.075 \cdot 2 \cdot \cos(2\pi u - 2\pi v)$$

Example of Fourier Transform Spectrum

- In Matlab, the relevant functions for 2-D DFT are
  - fft2: take the DFT of a 2-D matrix
  - ifft2: take the inverse DFT of a 2-D matrix
  - fftshift: shift a transform to place the coefficient in the center



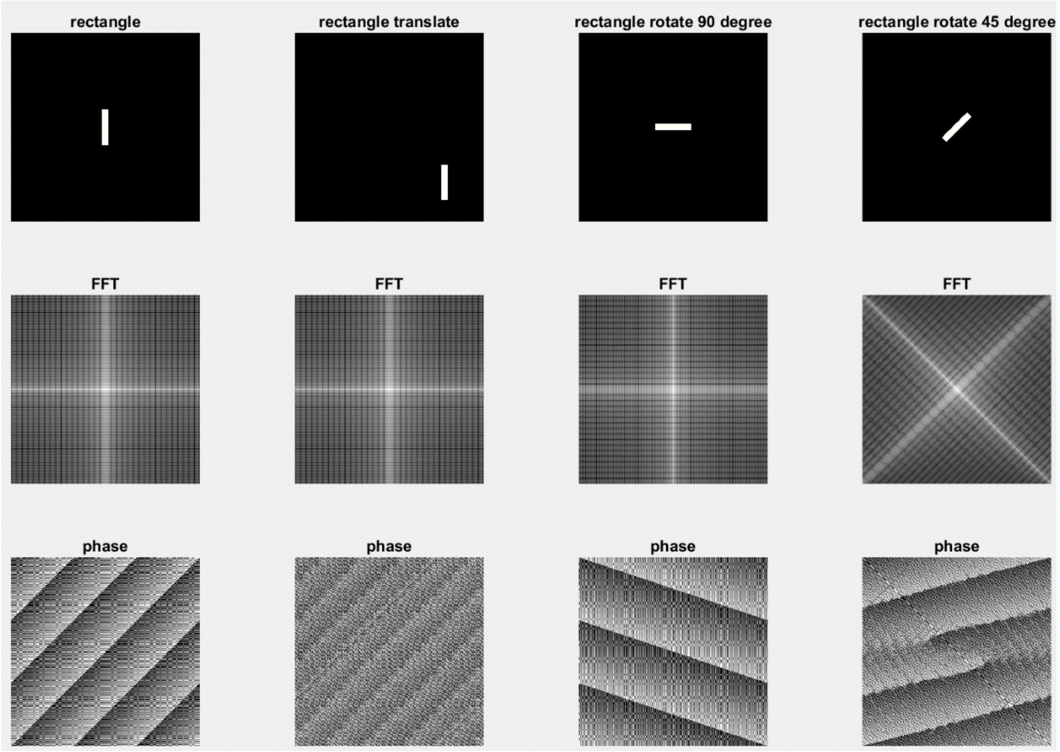

```
%% phase and magnitute
clc;
clear;
close all;
I1=imread('rectangle.tif');
F1=fft2(I1);

F2=fftshift(F1);

S1=abs(F1);
S2=abs(F2);
S3= log(1+ abs(F2));

subplot(2,2,1);imshow(I1);title('Rectangle');
subplot(2,2,2);imshow(S1,[]);title('Fourier Spectrum');
subplot(2,2,3);imshow(S2,[]);title('Centered Spectrum');
subplot(2,2,4); imshow(S3,[]); title('Enhance Spectrum');
```

Example of Fourier Transform Spectrum

- Edges and shape transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform
- The boundary of spectrum represents high frequency information
- Regions of relatively uniform gray values in an image contribute to low frequency content of its Fourier transform
- The center of spectrum represents low frequency information

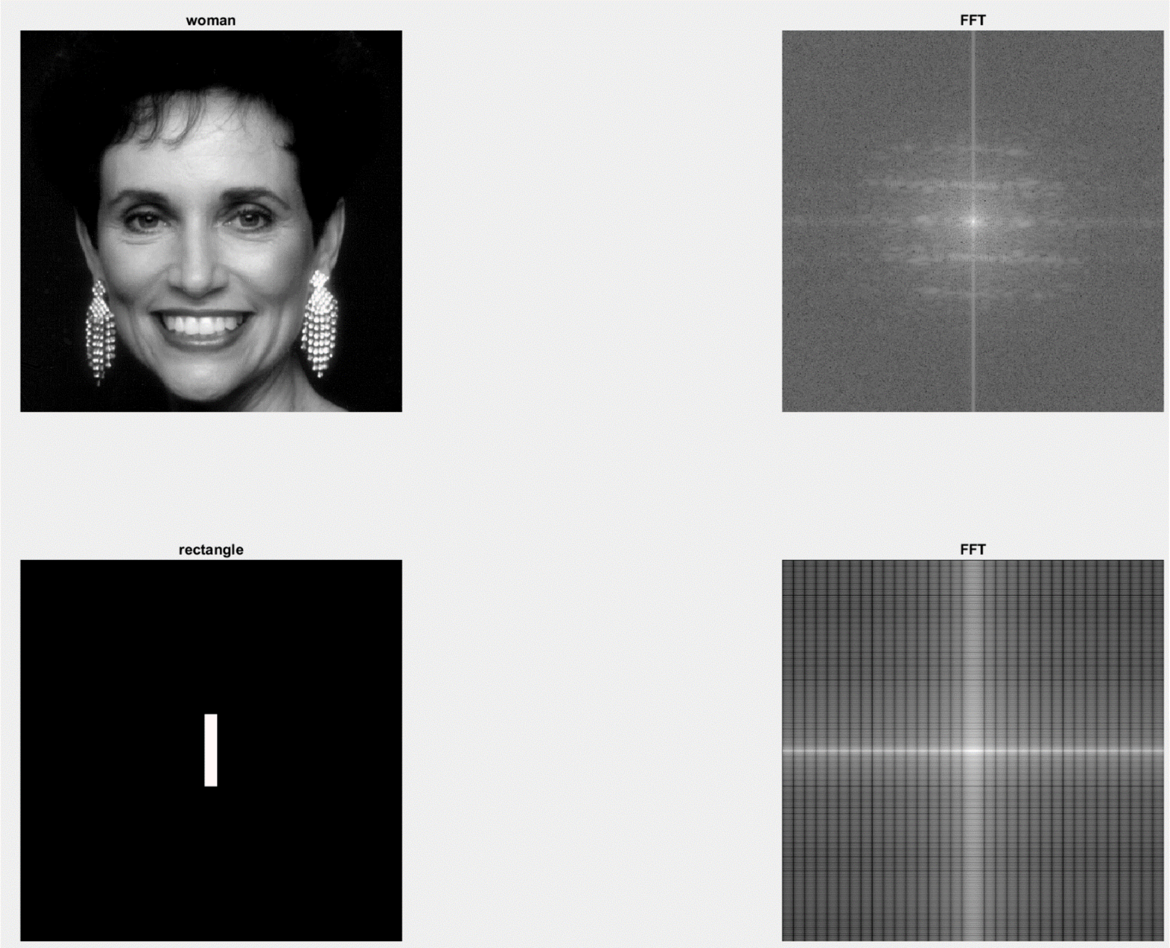
Example of Fourier Transform Spectrum

- Translation will not change spectrum, but change the phase
- Rotation will change spectrum and the phase
- The zero crossing of the spectrum is closer in the vertical direction because the rectangle is longer in that direction




```
%% translation and rotation
I1=imread('rectangle.tif');
figure;
subplot(3,4,1);imshow(I1);title('rectangle');

I2=fft2(I1);
spectrum =fftshift(I2);
temp= log(1+ abs(spectrum) );
phase=angle(I2);
subplot(3,4,5);imshow(temp,[]);title('FFT');
subplot(3,4,9);imshow(phase,[]);title('phase');

se=translate(strel(1),[300 300]);
T1=imdilate(I1,se);
subplot(3,4,2);imshow(T1);title('rectangle translate');

I2=fft2(T1);
spectrum =fftshift(I2);
temp= log(1+ abs(spectrum) );
phase=angle(I2);
subplot(3,4,6);imshow(temp,[]);title('FFT');
subplot(3,4,10);imshow(phase,[]);title('phase');

% R1 = imrotate(I1,-90,'bilinear','crop');
R1 = imrotate(I1,-90);
subplot(3,4,3);imshow(R1);title('rectangle rotate 90 degree');

I2=fft2(R1);
spectrum =fftshift(I2);
temp= log(1+ abs(spectrum) );
phase=angle(I2);

subplot(3,4,7); imshow(temp,[]);title('FFT')
subplot(3,4,11);imshow(phase,[]);title('phase');

R1 = imrotate(I1,-45,'bilinear','crop');
subplot(3,4,4);imshow(R1);title('rectangle rotate 45 degree');
```

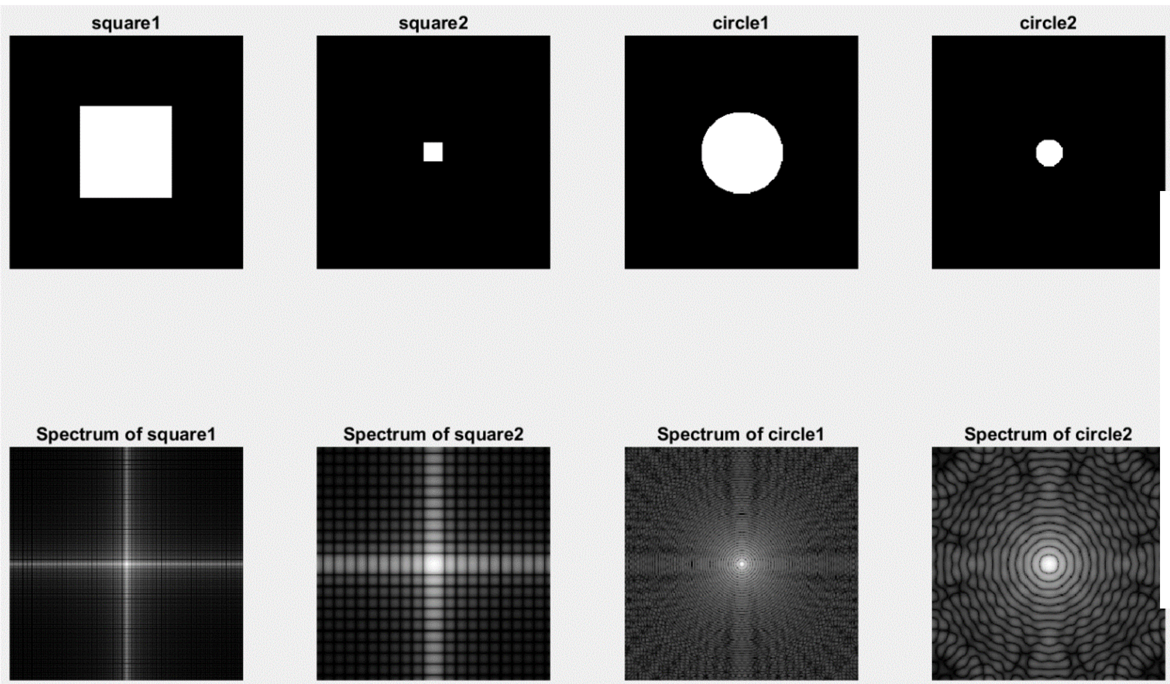
Example of Fourier Transform Spectrum




```
%% FFT

clc;
clear;
close all;
I1=imread('woman.tif');
subplot(2,2,1);imshow(I1);title('woman');

I2=fft2(I1);
spectrum =fftshift(I2);
temp= log(1+ abs(spectrum) );
subplot(2,2,2);imshow(temp,[]);title('FFT');

I1=imread('rectangle.tif');
subplot(2,2,3);imshow(I1);title('rectangle');

I2=fft2(I1);
spectrum =fftshift(I2);
temp= log(1+ abs(spectrum) );
subplot(2,2,4); imshow(temp,[]);title('FFT')
```

Example of Fourier Transform Spectrum

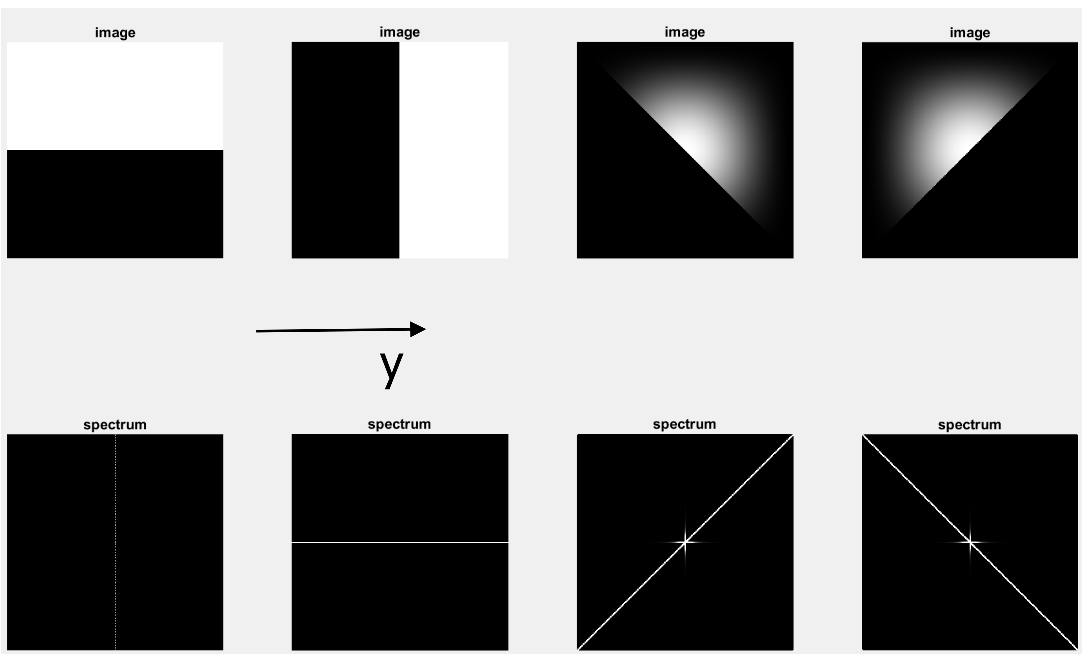
- High frequencies correspond to small image detail, shape edges.
- A shape edge will result in high energy perpendicular to the edge.




```
%% square and circle
I1=zeros(256,256);
I1(78:178,78:178)=1;

I2=zeros(256,256);
I2(118:138,118:138)=1;

[x,y]=meshgrid(-128:127,-128:127);
z=sqrt(x.^2+y.^2);
I3=(z<45);

I4=(z<15);
figure
subplot(2,4,1),imshow(I1,[]),title('square1'),
subplot(2,4,2),imshow(I2,[]),title('square2'),
subplot(2,4,3),imshow(I3,[]),title('circle1'),
subplot(2,4,4),imshow(I4,[]),title('circle2'),

subplot(2,4,5),imshow(log(abs(fftshift(fft2(I1)))+1),[]),title('Spectrum of square1'),
subplot(2,4,6),imshow(log(abs(fftshift(fft2(I2)))+1),[]),title('Spectrum of square2'),
subplot(2,4,7),imshow(log(abs(fftshift(fft2(I3)))+1),[]),title('Spectrum of circle1');
subplot(2,4,8),imshow(abs(fftshift(fft2(I4))),[]),title('Spectrum of circle2');
```

Example of Fourier Transform Spectrum

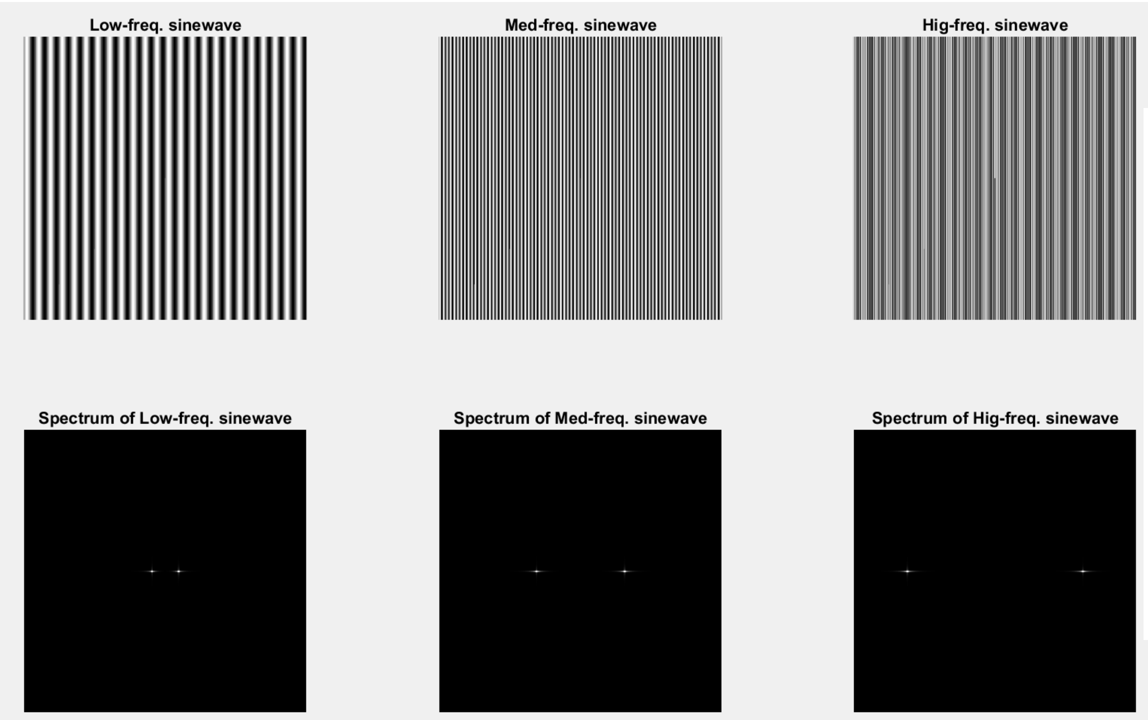
- The longitudinal periodic changes in the spatial domain are reflected on the X-axis of the spectrum
- The periodic change across horizon in the spatial domain is reflected on the Y-axis of the spectrum




```
%% direction
%black&white
Isize = 512;
Iblackwhite = zeros(Isize);
Iblackwhite(1:Isize/2,:) = 1;
figure;
subplot(2,4,1);imshow(Iblackwhite); title('image');
subplot(2,4,5);imshow(log(abs(fftshift(fft2(Iblackwhite)))+1));title('spectrum');

Iwhiteblack = zeros(Isize);
Iwhiteblack(:,Isize/2:end) = 1;
subplot(2,4,2);imshow(Iwhiteblack); title('image');
subplot(2,4,6);imshow(log(abs(fftshift(fft2(Iwhiteblack)))+1));title('spectrum');

% black&white triangle
Hanning = hann(Isize)*hann(Isize)';
ItriangleUpper = triu(ones(Isize),0);
ItriangleUpper = ItriangleUpper.*Hanning;
subplot(2,4,3);imshow(ItriangleUpper); title('image');
subplot(2,4,7);imshow(log(abs(fftshift(fft2(ItriangleUpper)))+1));title('spectrum');

% flip the upper triangle matrix
ItriangleUpperMirror = fliplr(triu(ones(Isize),0));
ItriangleUpperMirror = ItriangleUpperMirror.*Hanning;
subplot(2,4,4);imshow(ItriangleUpperMirror); title('image');
subplot(2,4,8);imshow(log(abs(fftshift(fft2(ItriangleUpperMirror)))+1));title('spectrum');
```

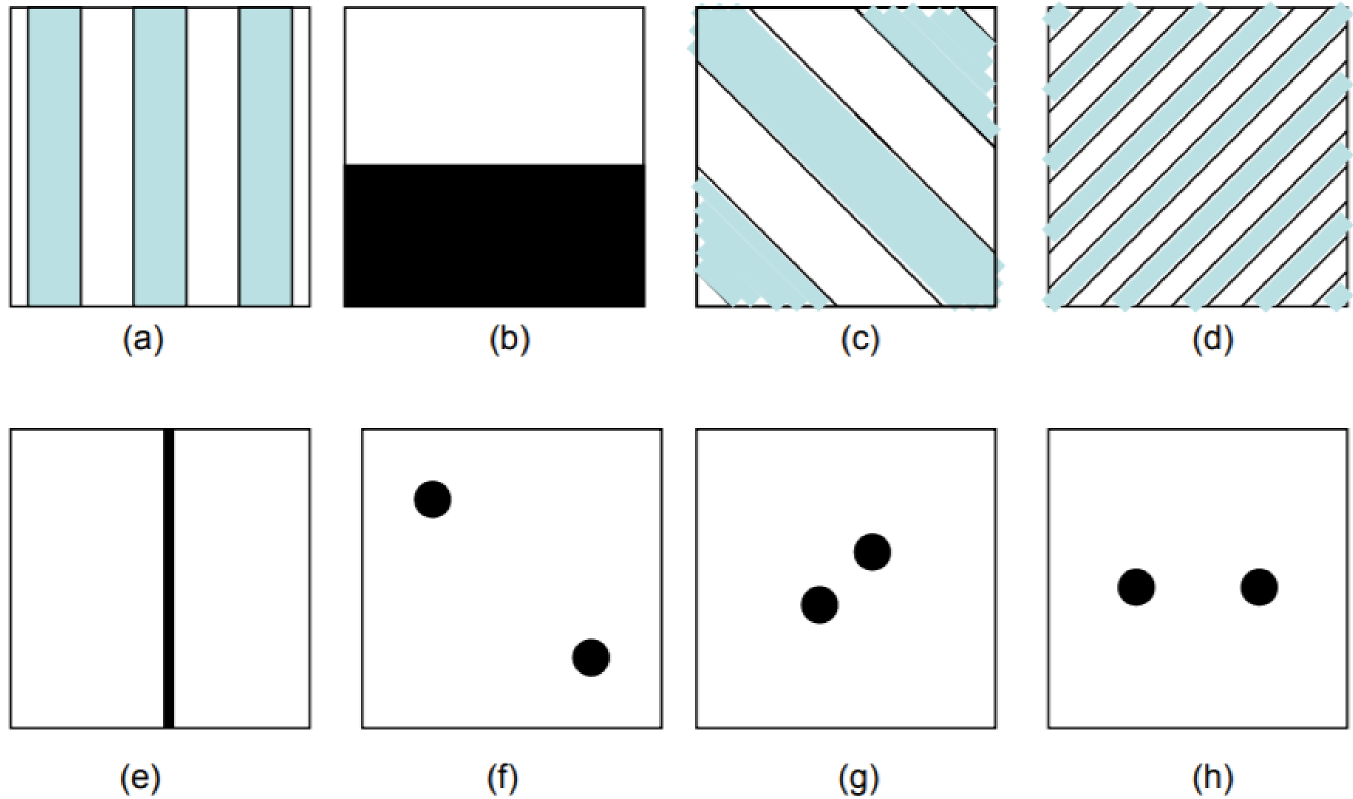
Example of Fourier Transform Spectrum

- The closer in the center of the spectrum, the lower the frequency in spatial domain (slower changes in the spatial domain)
- The farther in the center of the spectrum, the higher the frequency
- The changes of gray values are faster, then the spectrum are higher




```
%% specturm and frequency
% % % Length of signal
L = 512;
% Sampling frequency
FsLow = 150; FsMid = 500; FsHigh = 1000;
% Form sampling vectors
IndexLow = linspace(0,FsLow,L);
IndexMid = linspace(0,FsMid,L);
IndexHigh = linspace(0,FsHigh,L);

% build 1D sinewave
SineLow = sin(IndexLow);
SineMid = sin(IndexMid);
SineHigh = sin(IndexHigh);

% translate 1D wave into 2D sinewave
SinewaveL = repmat(SineLow,[L,1]);
SinewaveM = repmat(SineMid,[L,1]);
SinewaveH = repmat(SineHigh,[L,1]);

% Window function
Window = hann(L)*hann(L)';
SinewaveLwindowed = SinewaveL.*Window;
SinewaveMwindowed = SinewaveM.*Window;
SinewaveHwindowed = SinewaveH.*Window;
figure;
subplot(2,3,1),imshow(SinewaveL,[]),title('Low-freq. sinewave'),
subplot(2,3,2),imshow(SinewaveM,[]),title('Med-freq. sinewave'),
subplot(2,3,3),imshow(SinewaveH,[]),title('Hig-freq. sinewave'),
subplot(2,3,4),imshow(log(abs(fftshift(fft2(SinewaveLwindowed)))+1),[]),title('Spectrum of Low-freq.
subplot(2,3,5),imshow(log(abs(fftshift(fft2(SinewaveMwindowed)))+1),[]),title('Spectrum of Med-freq.
```

Practice question 1



a-h b-e c-g d-f

Practice question 2

- Match the spatial domain image to the Fourier magnitude image