Pledge I Agree

1a) $\begin{pmatrix} 6 & -2 & -1 \\ 2 & a & -1 \\ -1 & -1 & b \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ a-3 \\ b-2 \end{pmatrix} \Leftarrow$ L.H.S.

$\begin{pmatrix} \lambda \\ \lambda \\ \lambda \end{pmatrix} \Leftarrow$ R.H.S.

RHS = LHS

$\lambda = 3$

$\lambda = a-3$

$\lambda = b-2$

$\therefore \lambda = 3,\ a = 6,\ b = 5$

b) $\begin{pmatrix} 6-\lambda & -2 & -1 \\ -2 & 6-\lambda & -1 \\ -1 & -1 & 5-\lambda \end{pmatrix}$

$\det A = 6-\lambda \cdot \begin{vmatrix} 6-\lambda & -1 \\ -1 & 5-\lambda \end{vmatrix} - 6-\lambda \cdot \begin{vmatrix} -2 & -1 \\ -1 & 5-\lambda \end{vmatrix} + 5-\lambda \begin{vmatrix} -2 & 6-\lambda \\ -1 & -1 \end{vmatrix}$

$= -\lambda^3 + 17\lambda^2 - 90\lambda + 144 = -(\lambda-8)(\lambda-6)(\lambda-3)$

at $\lambda = 8$

$\begin{pmatrix} 6-8 & -2 & -1 \\ -2 & 6-8 & -1 \\ -1 & -1 & 5-8 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -1 \\ -2 & -2 & -1 \\ -1 & -1 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

at $\lambda = 3$, $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

at $\lambda = 6$

$\begin{pmatrix} 0 & -2 & -1 \\ -2 & 0 & -1 \\ -1 & -1 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{v}_2 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$

2 a) $f(x,y) = x^2 - y^3$ $\quad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial x}\Big|_{(1,2)} = 2$

$f(1,2) = 7$ $\quad \frac{\partial f}{\partial y} = -3y^2 \quad \frac{\partial f}{\partial y}\Big|_{(1,2)} = -12$

$$L(x,y) = -7 + 2(x-1) - 12(y-2) = 2x - 12y + 15$$

b) let $f_x = 0$ $\quad$ let $f_y = 0$ $\quad \frac{\partial^2 f}{\partial x^2} = 2 \quad \frac{\partial^2 f}{\partial y^2} = -6y \quad \frac{\partial^2 f}{\partial x \partial y} = 0$

$2x = 0$ $\quad y = 0$

$x = 0$

$$\Delta(0,0) = 2(-6y) - 0^2 = -12y$$

$\therefore$ Cannot be determind by second derivative test

3) a) $f_z(x, y, z) \equiv 0 + 4\left(3y^2 \frac{\partial}{\partial z} y\right) + 6z = 116$

$$\frac{\partial y}{\partial z} = -\frac{z}{2y^2}$$

$$f_x(x, y, z) \equiv \quad \frac{\partial y}{\partial x} = -\frac{3z}{x}$$

4) $\dfrac{3x^2}{2x^2+y^2}$

$$\lim_{x \to 0} f(x, 0) = \lim_{x \to 0} \frac{3x^2}{2x^2+y^2} = \frac{0}{0+y} = 0$$

$$\lim_{y \to 0} f(0, y) = \lim_{y \to 0} \frac{3x^2}{2x^2+y^2} = \frac{3x^2}{2x^2+0} = \frac{3}{2}$$

No lim does not exist

5) $z = 2x^5 + y^5 \qquad x = \sin t + \cos s \qquad y = \cos t - \sin s$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \qquad \frac{\partial z}{\partial x} = 10x^4 \qquad \frac{\partial z}{\partial y} = 5y^4$$

$$\frac{\partial x}{\partial t} = \cos t \qquad \frac{\partial y}{\partial t} = -\sin t$$

$$\frac{dz}{dt} = 10x^4\cos t + 5y^4(-\sin t) = 10(\sin t - \cos s)^4 \cdot \cos t + 5(\cos t - \sin s)^4(-\sin t)$$

sub $t=0$ & $s=0$

$$\left.\frac{\partial z}{\partial t}\right|_{\substack{t=0\\ s=0}} = 10(1)^4(1) + 5(1)^4(0) = 10$$

$$\frac{d^2 z}{\partial t\, \partial s} = \frac{\partial}{\partial s}\frac{dz}{dt} = \frac{d}{\partial s}\left(10(\sin t - \cos s)^4 \cdot \cos t + 5(\cos t - \sin s)^4(-\sin t)\right)$$

$$= 40(\cos t)(\sin t)(\sin t - \cos s)^3 + 20(\cos t)(\sin t)(\cos t - \sin s)^3$$

$$\left.\frac{d^2 z}{\partial t\, \partial s}\right|_{t=0,\, s=0} = 40(1)(0)(-1)^3 + 20(1)(0)(1)^3 = 0$$