Slope of $L = -\frac{1}{2}$

Let the required line be M

$$m_L m_M = -1$$
$$\left(-\tfrac{1}{2}\right)(m_M) = -1$$
$$m_M = 2$$
$$4 = 2(-1) + c$$
$$c = 6$$

required line : $y = 2x + 6$ //

2a) $x^2 - 9y^2 + 2x + 36y - 44 = 0$

$$(x^2 + 2x + 1) - 9(y^2 - 4y + 2^2) = 44 + 1 - 36$$
$$\frac{(x+1)^2}{3^2} - \frac{(y-2)^2}{1} = 1$$

$\therefore$ It's a hyperbola with center $(-1, 2)$ //

b) $c^2 = a^2 + b^2$

$$c = \sqrt{3^2 + 1^2} = \sqrt{10}$$

foci $= (-1 \pm \sqrt{10}, 2)$ $\therefore$ $F_1 = (-1 + \sqrt{10}, 2)$ //, $F_2 = (-1 - \sqrt{10}, 2)$ //

asymptotes : $y = \pm\left(\frac{1}{3}\right)(x - (-1)) + 2$

$y = \frac{1}{3}x + \frac{7}{3}$ // and $y = -\frac{1}{3}x + \frac{5}{3}$ //

$0 = \frac{1}{3}x + \frac{7}{3}$ $\qquad$ $0 = -\frac{1}{3}x + \frac{5}{3}$

$x = -7$ $\qquad$ $x = 5$

Intercept with x-axis at $(-7, 0)$ // and $(5, 0)$ //

3a) $9x^2 + 16y^2 - 36x - 32y - 92 = 0$

$$9(x^2 - 4x + 2^2) + 16(y^2 - 2y + 1) = 92 + 9(2^2) + 16(1)$$
$$\frac{(x-2)^2}{4^2} + \frac{(y-1)^2}{3^2} = 1$$

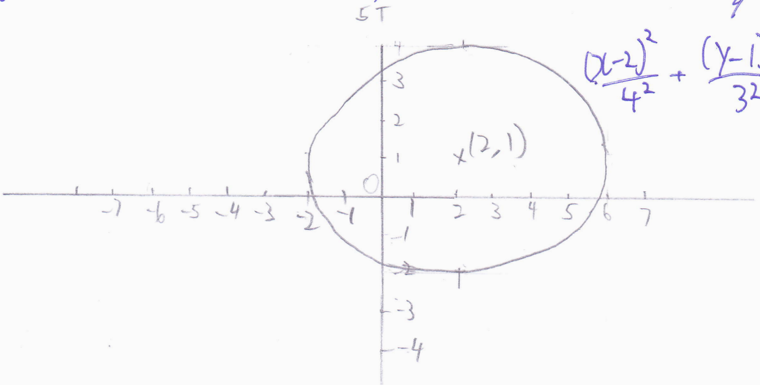
$\therefore$ It's an ellipse with centre $(2, 1)$ //

3b) $c^2 = |a^2 - b^2|$

$c = \sqrt{|4^2 - 3^2|} = \sqrt{7}$

$foci = (2 \pm \sqrt{7}, 1)$, $\therefore F_1 = (2+\sqrt{7}, 1)$ // , $F_2 = (2-\sqrt{7}, 1)$ //

c)



$$\frac{(x-2)^2}{4^2} + \frac{(y-1)^2}{3^2} = 1$$

4a) $f(x) = 3x - 2$

Let $y = 3x - 2$

$x = \frac{y+2}{3}$

$f^{-1}(x) = \frac{x+2}{3}$ //

$Dom(f) = \mathbb{R}$

$Ran(f^{-1}) = Dom(f)$

$= \mathbb{R}$ //

$Ran(f) = \mathbb{R}$

$Dom(f^{-1}) = Ran(f)$

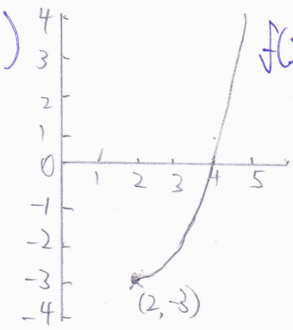
$= \mathbb{R}$ //

b) $(g \circ f)(x) = \frac{1}{(3x-2)-2} = \frac{1}{3x-4}$ //

$Dom(g \circ f) = \mathbb{R} \setminus \{\frac{4}{3}\}$ //

$Ran(g \circ f) = \mathbb{R} \setminus \{0\}$ //

5a)



$f(x) = (x-2)^2 - 3$

$x \in [2, \infty)$

b) Let $y = (x-2)^2 - 3$

$\sqrt{y+3} = x - 2$

$x = \sqrt{y+3} + 2$

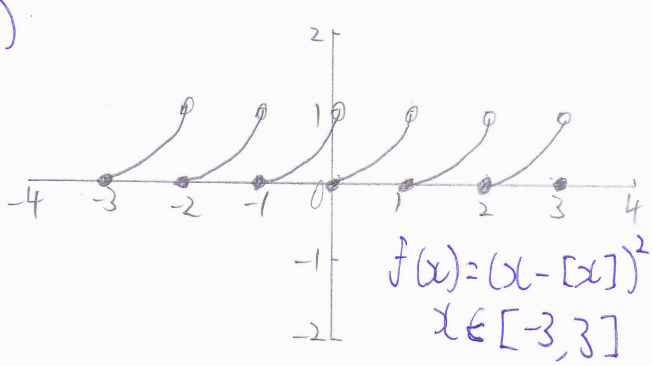
$f^{-1}(x) = \sqrt{x+3} + 2$ //

$Ran(f) = [-3, \infty)$

$Dom(f^{-1}) = Ran(f)$

$= [-3, \infty)$ //

6a)



$f(x) = (x - [x])^2$

$x \in [-3, 3]$

b) $Ran(f) = [0, 1)$ //

c) Yes //