EE 4211 Computer Vision

Lecture 4B: Morphology

Semester B, 2021-2022

Lecture Outline

- Morphological Algorithms
  - Hit or Miss Transform
  - Boundary Extraction
  - Hole Filling
  - Connected Components
  - Skeletons

Hit-or-Miss Transform

- Hit-or-Miss Transform is a powerful method for finding shapes, and their locations in images
- Can be defined entirely in terms of erosion only

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$$B = (B_1, B_2)$$

B_1 : object $\qquad B_2$: background

- Useful for detecting specific shapes that are intended to extract, e.g. squares, triangles, ridges, corners, junctions, etc.

Hit-or-Miss Transform

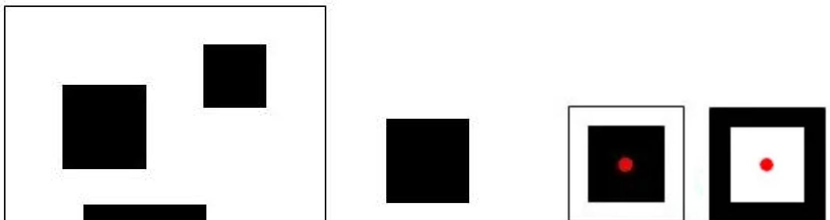
- Steps

Step 1: Perform an erosion $A \Theta B_1$ with B_1 being the SE shape that we intend to find.

Step 2: Next, erode the complement of A with B_2, a SE that is the border that encloses around the shape B_1.

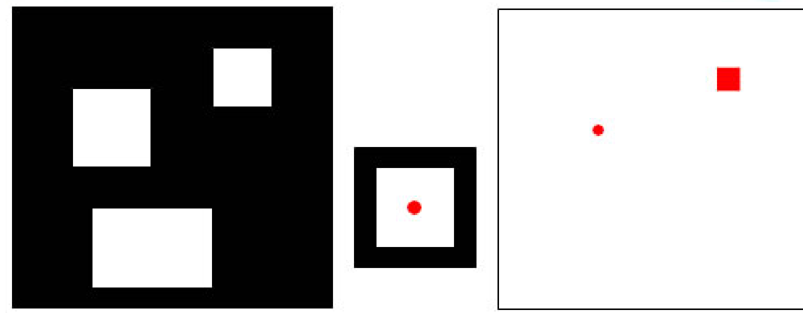
Step 3: The intersection of the two erosion operations would produce just one pixel at the center position of the found, shape, resulting in a "hit". Other parts of set A which did not return anything are considered "miss".

Hit-or-Miss Transform



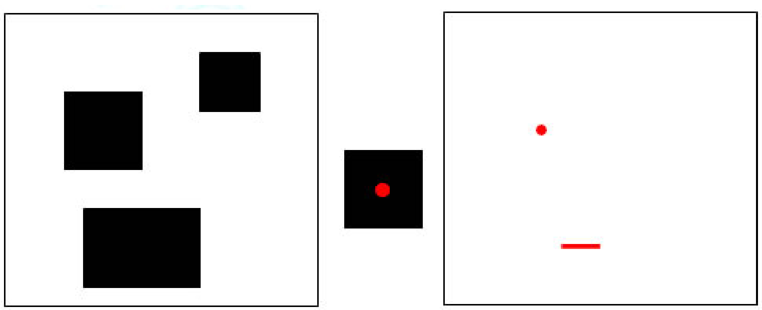
A B B_1 B_2

Step 1:



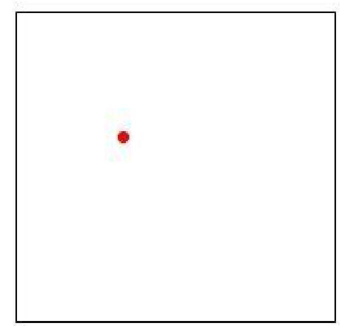
A^c B_2 $A^c \ominus B_2$

Step 2:



A B_1 $A\Theta B_1$

Step 3:



Y

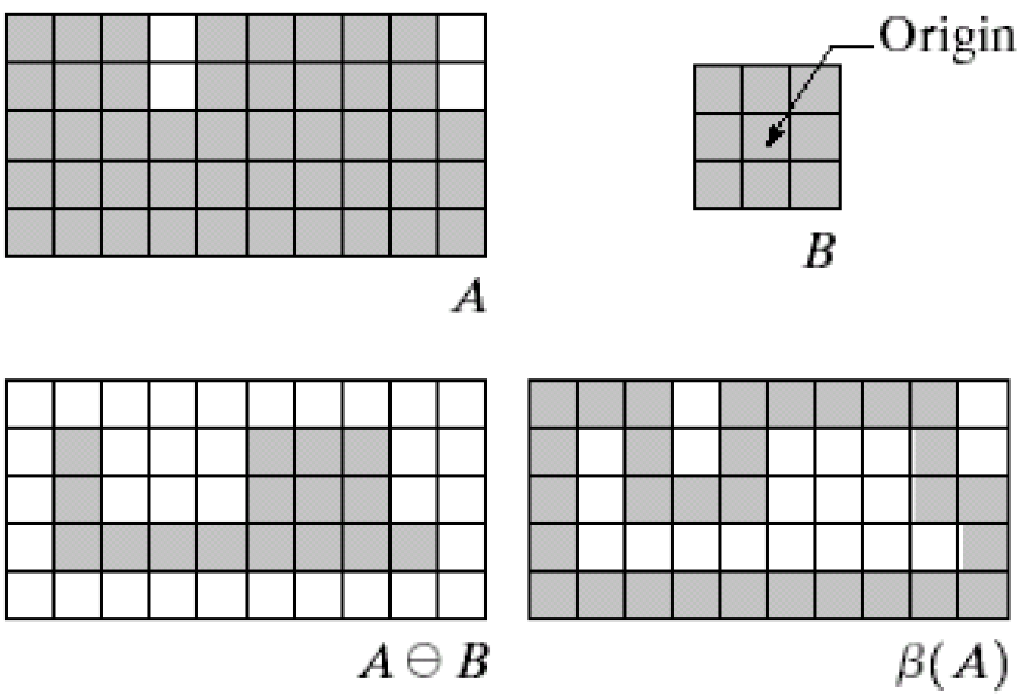
Lecture Outline

- Morphological Algorithms
  - Hit or Miss Transform
  - Boundary Extraction
  - Hole Filling
  - Connected Components
  - Skeletons

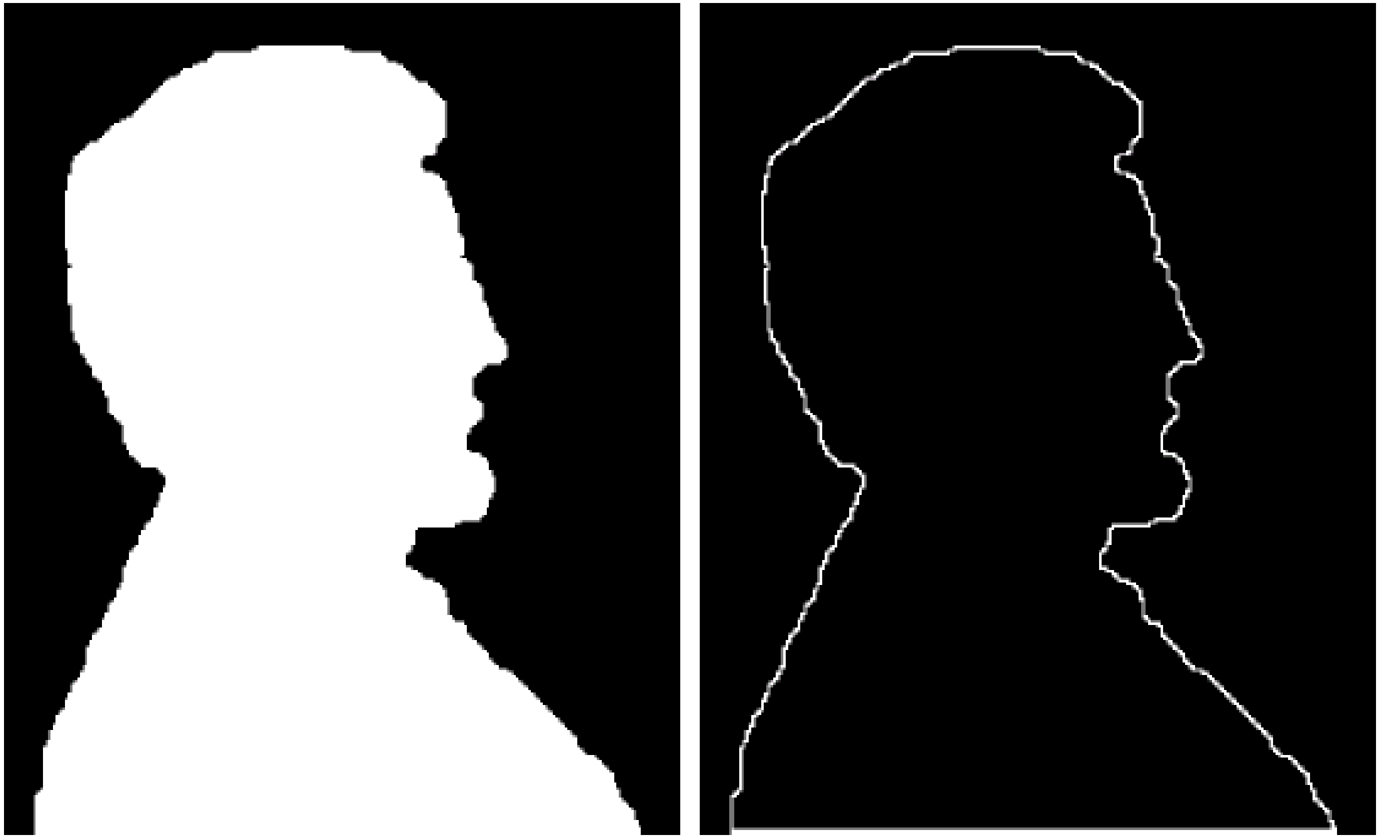
Boundary Extraction

- A boundary of a set A, denoted by β(A), is obtained by eroding A by B, then perform the set difference between A and its erosion:

$$\beta(A) = A - (A \Theta B)$$



Boundary Extraction



Lecture Outline

- Morphological Algorithms
  - Hit or Miss Transform
  - Boundary Extraction
  - Hole Filling
  - Connected Components
  - Skeletons

Hole Filling

- Sometimes also referred to as Region Filling
- Hole: A background region surrounded by a connected border of foreground pixels
- Let A denote a set whose elements are 8-connected boundaries – each boundary encloses a background region.
- **Objective:** to fill all holes in set A with 1s

Region Filling

- Start with a point inside the region
- Repeatedly dilate
- At each step, set to zero the points corresponding to the region

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c, \quad k = 1,2,3,\ldots$$

- A^c is the complement of A
- Stop when no more changes

Region Filling

Form an array, X_0 of 0s (same size as array containing A), except at the locations in X_0 corresponding to the points in each hole, which are set to 1. The following procedure fills all the holes with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1,2,3\ldots$$

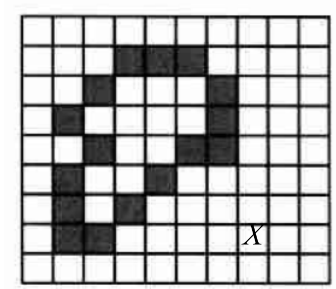
where B is a symmetric SE.

The algorithm terminates at iteration step *k* if $X_k = X_{k-1}$.

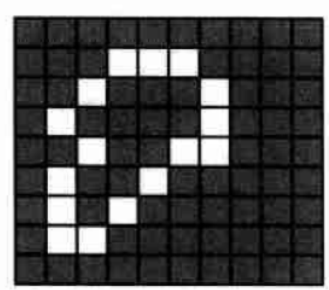
Set X_k contains all the filled holes.

The set union, $X_k \cup A$ contains all filled holes and their boundaries.

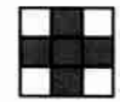
Region Filling



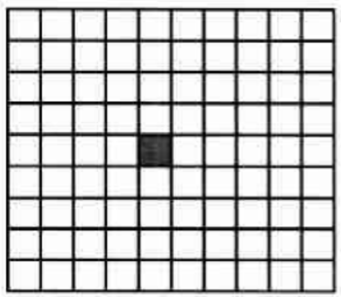
A



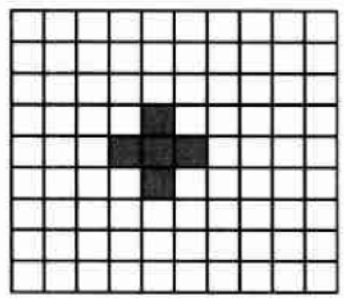
A^C



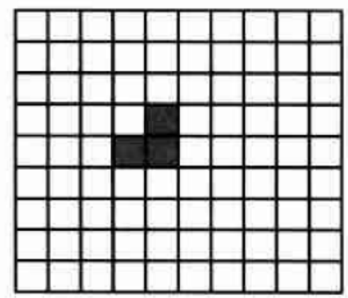
B



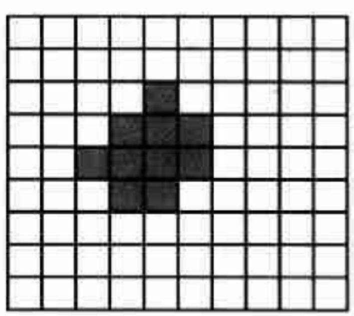
X_0



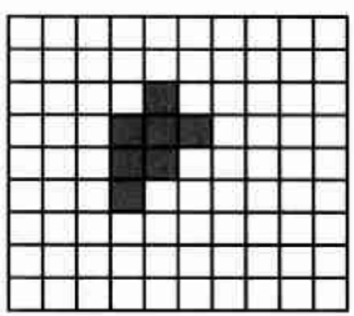
$X_0 \oplus B$



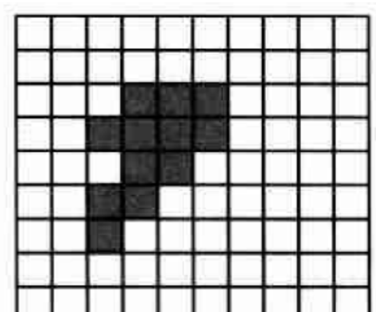
X_1



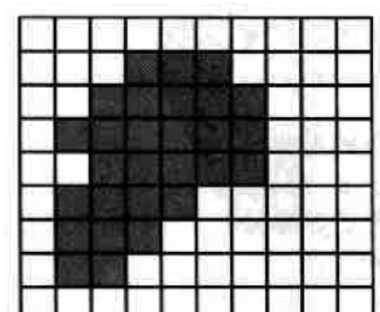
$X_1 \oplus B$



X_2

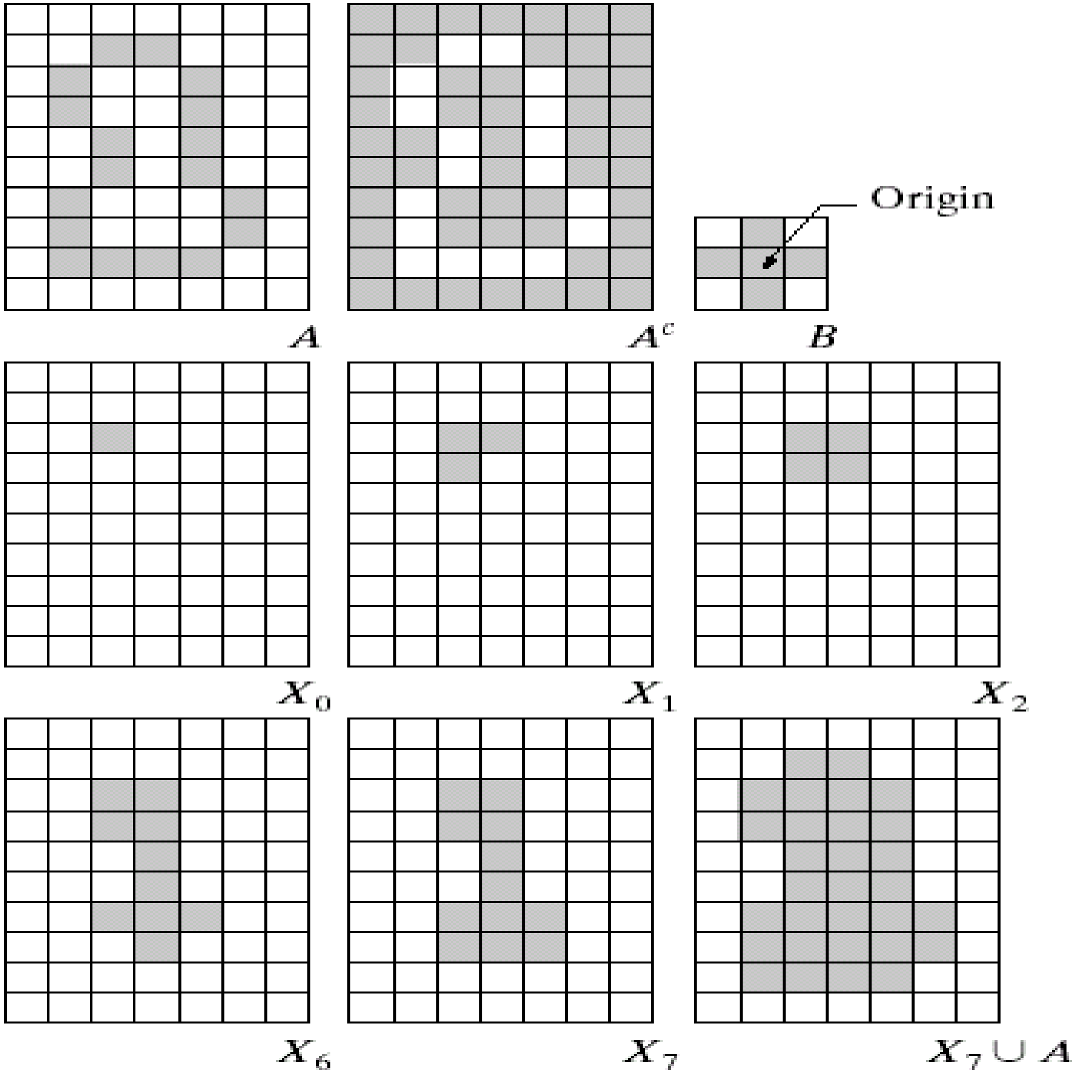


$X_2 \oplus B$



Final

Region Filling

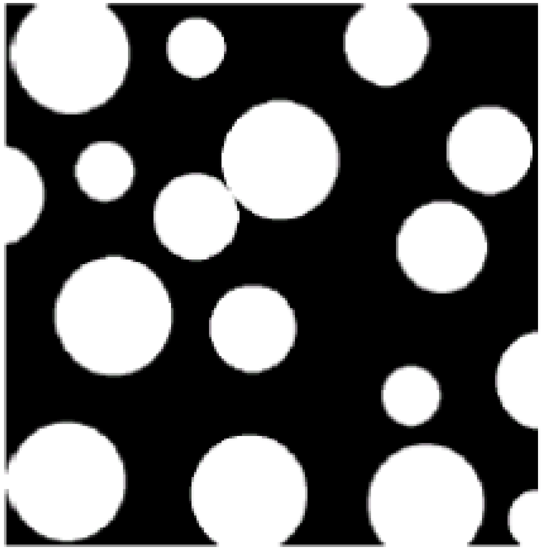


Lecture Outline

- Morphological Algorithms
  - Hit or Miss Transform
  - Boundary Extraction
  - Hole Filling
  - Connected Components
  - Skeletons

Extraction of Connected Components

- Process that is central to many automated image analysis applications
- Connected components require connectivity to be specified (4-connected, 8-connected)
- Labelling: How many “connected components” are there in this image?



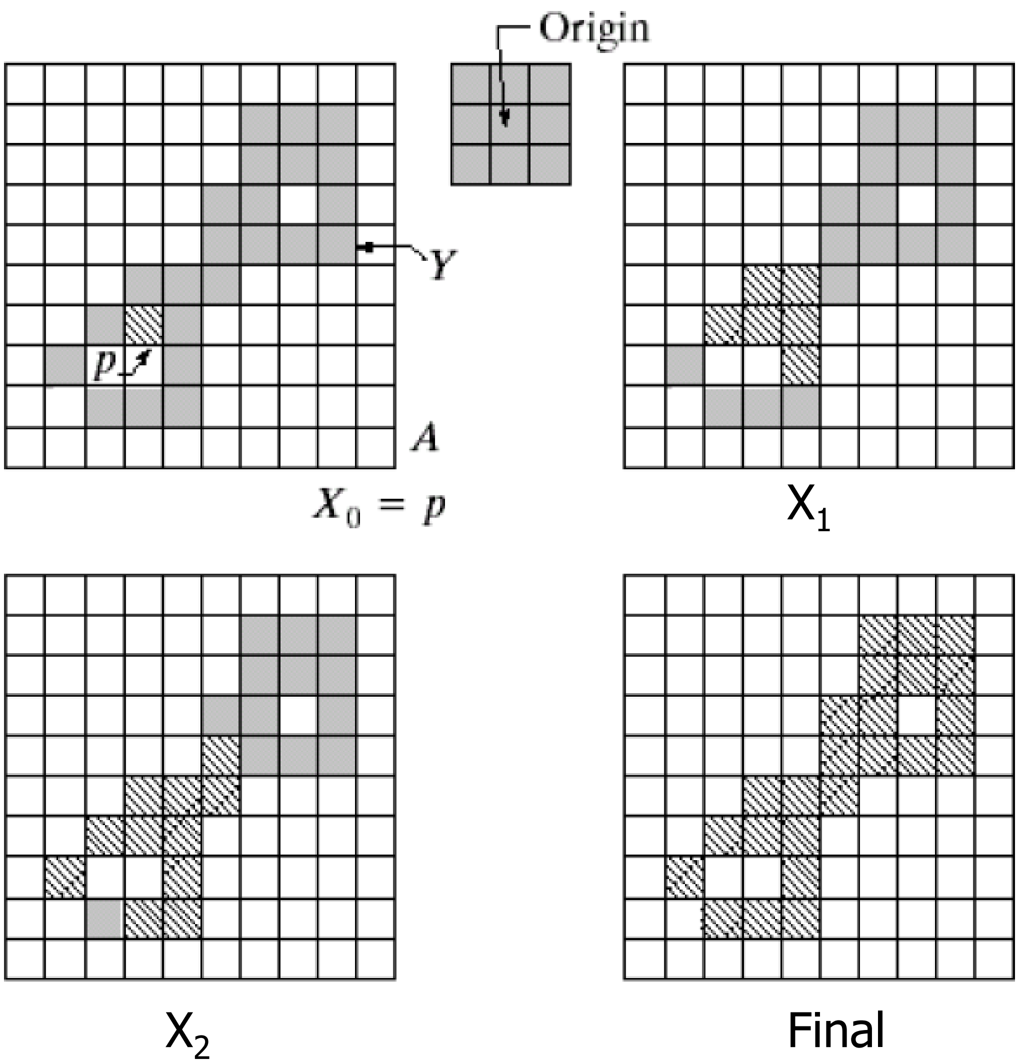
Extraction of Connected Components

- Let A be the set of 8 connected boundary points of a region
- Start with a point inside the region, Repeatedly dilate
- At each step, set to zero the points corresponding to the region boundary

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1,2,3\ldots$$

- Stop when no more changes
- Note its similarity to Hole Filling algorithm

Extraction of Connected Components



Y: connected component in set A,

p: a known point in Y

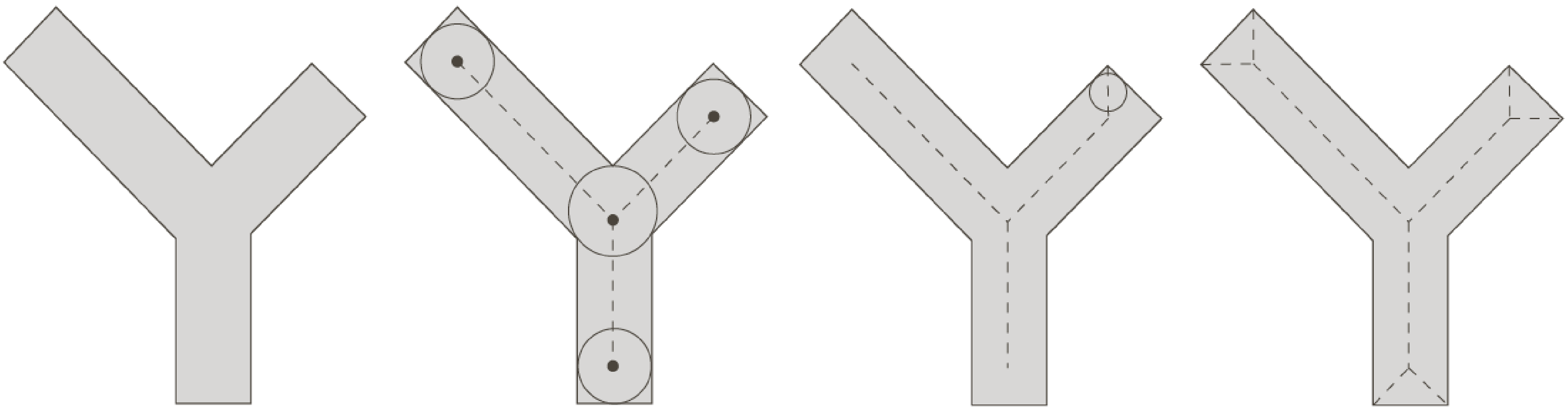
$$X_0 = p$$
$$X_k = \left(X_{k-1} \oplus B\right) \cap A$$
$$\text{if } X_k = X_{k-1}$$
$$\text{then } Y = X_k$$

Lecture Outline

- Morphological Algorithms
  - Hit or Miss Transform
  - Boundary Extraction
  - Hole Filling
  - Connected Components
  - Skeletons

Skeletons (Skeletonizing)

The notion of a skeleton, S(A), of a set A:



(a) (b) (c) (d)

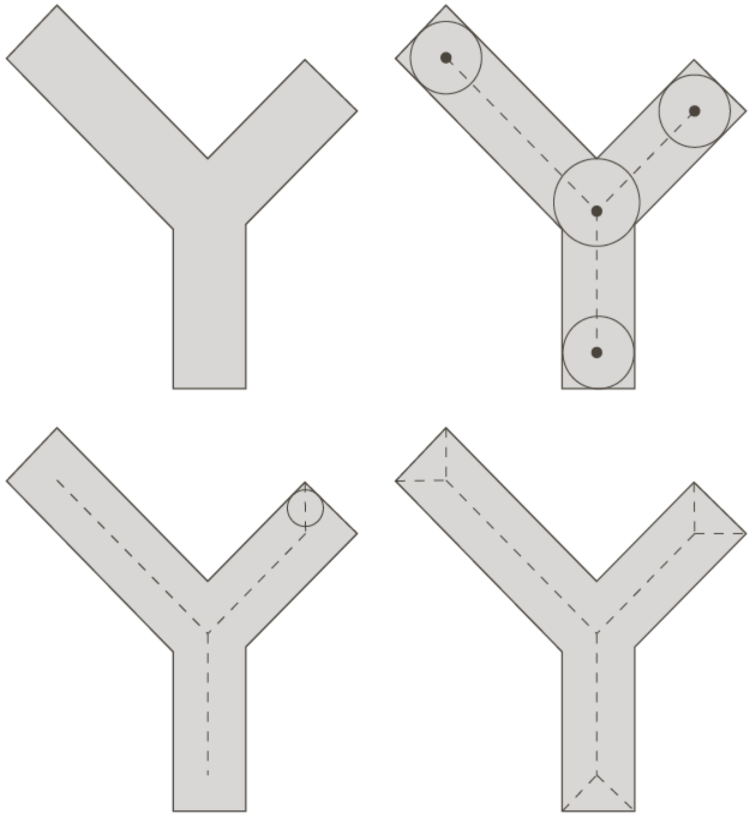
(a) Set A
(b) Various positions of maximum disks with centers on the skeleton of A
(c) Another maximum disk on a different segment of the skeleton of A
(d) Complete Skeleton

Skeletons (Skeletonizing)

The notion of a skeleton, $S(A)$, of a set A:

If z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. The disk $(D)_z$ is a maximum disk.

The disk $(D)_z$ touches the boundary of A at two or more different places



Skeletons (Skeletonizing)

The skeleton of A can be expressed in terms of erosions and openings:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

where

$$S_k(A) = (A \Theta kB) - \left[(A \Theta kB) \circ B\right]$$

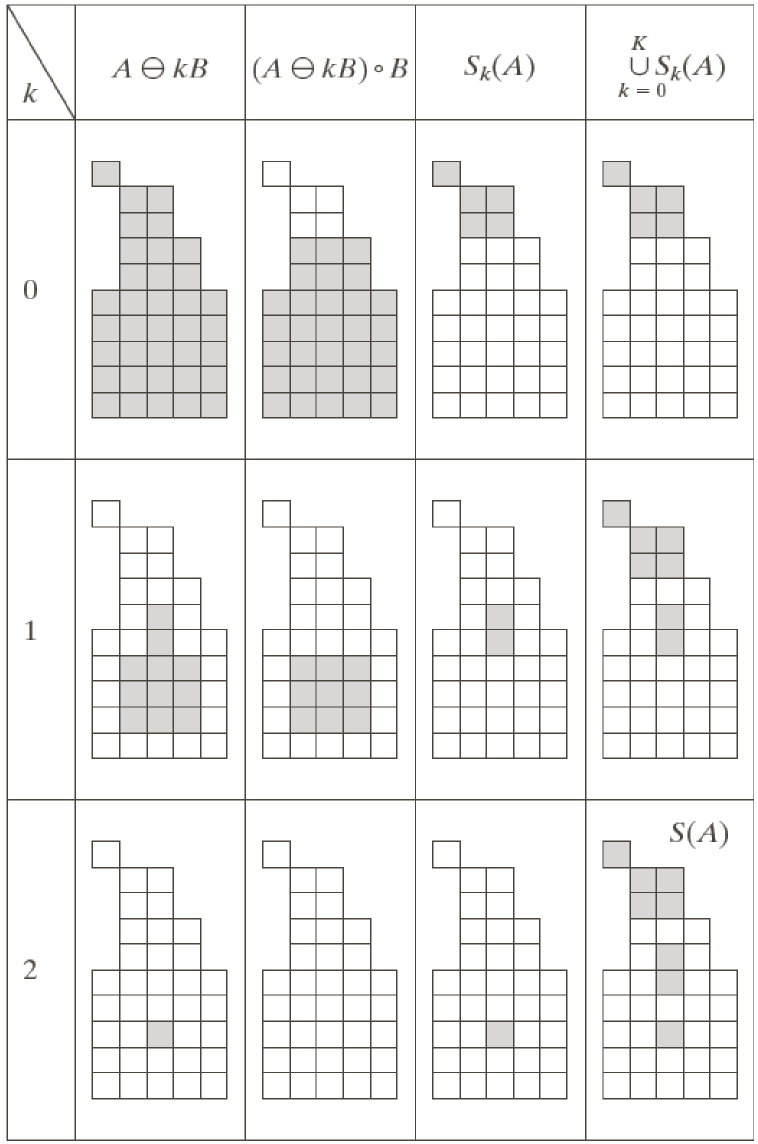
$A \circ B = (A \ominus B) \oplus B$
Opening: Erosion followed by Dilation

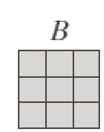
where B is the SE, and (AΘ*k*B) indicates *k* successive erosions of A:

$$(A \Theta kB) = ((\ldots((A \Theta B) \Theta B) \Theta \ldots) \Theta B)$$

k times, and K is the last iterative step before A erodes to an empty set.

Skeletons (Skeletonizing)





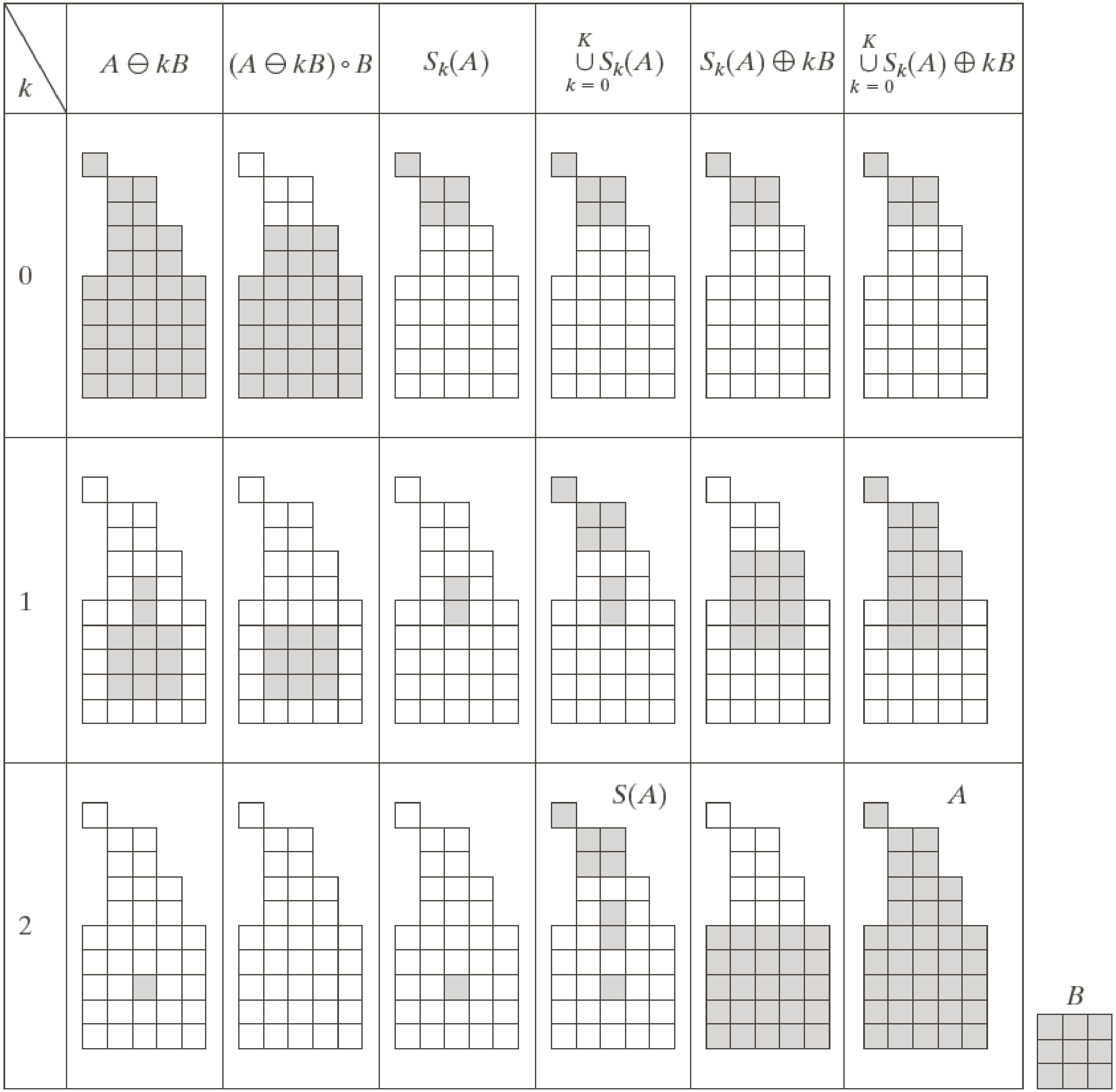
Skeletons (Skeletonizing)

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B)\ldots \oplus B)$$



Skeleton

Reconstructed set