EE3331 Probability Models in Information Engineering

Semester B 2021-2022

Assignment 3

Due Date: **30 March 2022**

1. The joint probability distribution function (PDF) of two random variables X and Y has the form of:

$$p(x,y) = \begin{cases} Ae^{-(3x+4y)}, & x > 0,\ y > 0, \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of A.
   (b) Determine the joint cumulative distribution function of X and Y.
   (c) Find $P(0 \le X < 1,\ 0 \le Y < 2)$.

2. Consider two independent Gaussian random variables $X \sim \mathcal{N}(0, 1/2)$ and $Y \sim \mathcal{N}(0, 1/2)$. With the use of $\int_0^\infty ue^{-u^2/2}du = 1$, compute the variance of $|X - Y|$. Note that a linear combination of Gaussian random variables is also a Gaussian random variable.

3. The joint probability mass function (PMF) of random variables N and K is given as:

$$P_{NK}(n,k) = \begin{cases} \dfrac{100^n e^{-100}}{n!} \dbinom{100}{k} p^k (1-p)^{100-k}, & n = 0, 1, \cdots,\ k = 0, 1, \cdots, 100 \\ 0, & \text{otherwise} \end{cases}$$

   Find the marginal PMFs of N and K.

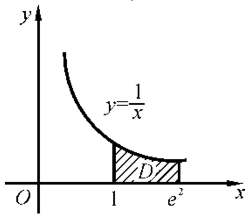
4. Given two independent uniform random variables $X \sim \mathcal{U}(0,3)$ and $Y \sim \mathcal{U}(0,3)$, determine $P(\max\{X, Y\} \le 1)$.

5. Suppose random variables X and Y are independent of each other where $X \sim \mathcal{U}(0, 0.2)$, and the PDF of Y is:

$$p_Y(y) = \begin{cases} 5e^{-5y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

   Find $P(Y \le X)$.

6. Suppose the plane region D is bounded by the curve $y = 1/x$ and 3 straight lines, namely, $y = 0$, $x = 1$, and $x = e^2$, which is shown as follows:



The joint PDF of X and Y obeys a uniform distribution in the region D.
(a) Find the marginal PDFs of X and Y.
(b) Are X and Y independent? Briefly explain your answer.

7. Consider the problem of estimating a constant A from N observations:

$$r_n = A + w_n, \quad n = 1, 2, \cdots, N$$

where w_n is a white noise with mean 0 and variance σ_w^2. It is suggested to estimate A using $\hat{A}$ which is given by:

$$\hat{A} = \frac{1}{N-1} \sum_{n=1}^{N} r_n$$

Compute the mean, variance and mean square error of $\hat{A}$.

8. The joint PMF of random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} 0.01, & x = 1, 2, \cdots, 10, \; y = 1, 2, \cdots, 10 \\ 0, & \text{otherwise} \end{cases}$$

Let A be the event that $\min(X, Y) > 5$. Find the conditional PMF $P_{XY|A}(x, y)$.

9. The joint PDF of random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} 6e^{-(2x+3y)}, & x \geq 0, \; y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Let A be the event that $X + Y \leq 1$. Find the conditional PDF $P_{XY|A}(x, y)$.

10. The joint PDF of random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, \; 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional PDFs $P_{X|Y}(x|y)$ and $P_{Y|X}(y|x)$.