DT convolution.

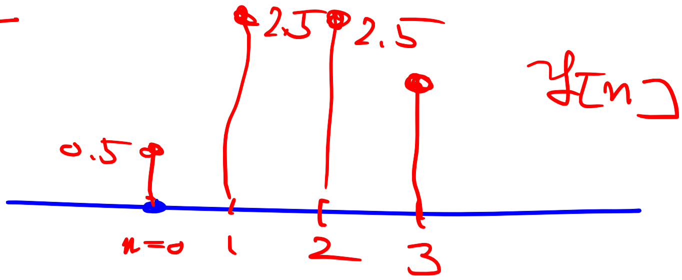
$$z[n] = x[n] * y[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, y[n-k]$$

$$= \sum_{k=-\infty}^{\infty} y[k]\, x[n-k]$$

Ex 1) $x[n] = \{ \underset{\uparrow n=0}{\boxed{0.5}},\ 2 \}$

$h[n] = \{ \underset{\uparrow n=0}{1},\ 1,\ 1 \} \leftarrow h[0,1,2]$



$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= x[0]\, h[n-0] + x[1]\, h[n-1]$$

$$= 0.5\, h[n] + 2\, h[n-1]$$

$$y[0] = 0.5 \cdot 1 + 2 \cdot \cancel{0} = 0.5$$

$$y[1] = 0.5 \cdot 1 + 2 \cdot 1 = 2.5$$

$$y[2] = 0.5 \times 1 + 2 \cdot 1 = 2.5$$

$$y[3] = 0.5 \cdot \cancel{0} + 2 \cdot 1 = 2.$$

$y[n] = 0 \quad$ if $\quad n \geq 4$ or
$n \leq -1$

Ex 2)

$x[n] = \{ 2, 1, 3 \}$

$\boxed{h[n] = \frac{1}{2^n} u[n]}$ ← if $n \geq 0$, $h[n] = \frac{1}{2^n}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= x[0] h[n] + x[1] h[n-1] + x[2] h[n-2]$$

$\frac{1}{2^{(n-2)}} u[n-2]$

$\frac{1}{2^{(n-1)}} u[n-1]$

$\Rightarrow$

$$y[n] = 2 \cdot \underbrace{h[n]}_{n \geq 0} + \underbrace{h[n-1]}_{n \geq 1} + 3 \cdot \underbrace{h[n-2]}_{n \geq 2}$$

$y[0] = 2 \cdot \frac{1}{2^n} u[n] \Big|_{n=0} = 2$

$y[1] = 2 \cdot \frac{1}{2} + \frac{1}{2^{1-1}} = 1 + 1 = 2$

$y[2] = \frac{4}{2^{2-2}} = 4$

$y[n] = 2 \cdot \frac{1}{2^n} + \frac{1}{2^{n-1}} + 3 \cdot \frac{1}{2^{n-2}}$ if $n \geq 2$

$$= \frac{1}{2^{n-2}} + \frac{3}{2^{n-2}} = \boxed{\frac{4}{2^{n-2}}}$$

$$h[n] \Rightarrow n = \underbrace{0, 1, \ldots, N-1}_{N \text{ terms}}$$

$$\boxed{x[n] \Rightarrow n = \underbrace{0, 1, \ldots, M-1}_{M \text{ terms.}}}$$

$$\underline{y[n]} = x[n] * h[n]$$

$\hookrightarrow N+M-1$ terms

$$n = 0, 1, \ldots, N+M-2$$

$$\underbrace{\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[N+M-2] \end{bmatrix}}_{(N+M-1)\times 1} = \underbrace{\overset{x}{\begin{bmatrix} x[0] & 0 & & 0 \\ x[1] & x[0] & 0 & 0 \\ \vdots & x[1] & x[0] & x[0] \\ x[M-1] & \vdots & x[1] & x[1] \\ 0 & x[M-1] & & \vdots \\ 0 & 0 & x[M-1] & \\ 0 & 0 & 0 & x[M-1] \end{bmatrix}}}_{(N+M-1)\times N} \underbrace{\overset{h}{\begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[N-1] \end{bmatrix}}}_{N\times 1}$$

$$= \begin{bmatrix} h[0] & 0 & & 0 \\ h[1] & h[0] & & 0 \\ \vdots & h[1] & & 0 \\ h[N-1] & \vdots & \cdots & 0 \\ 0 & h[N-1] & & h[0] \\ 0 & 0 & & h[1] \\ 0 & 0 & & \vdots \\ 0 & 0 & & h[N-1] \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[M-1] \end{bmatrix}$$

$(N+\boxed{M-1}) \times M.$

$M \times 1.$

Ex 3)

$x[n] = \{4, 2\}$ $\quad N = 2$

$h[n] = \{4, 3, 2, 4\}$ $\quad M = 4$

$y[n] = \{ \quad \}$ $\quad N+M-1 = 5$

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \end{bmatrix} = \begin{bmatrix} \cancel{4} & \cancel{0} \\ \cancel{3} & \cancel{4} \\ 2 & 3 \\ 4 & 2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

x

$5 \times 1 \qquad 5 \times 2 \qquad 2 \times 1$

$$
= \begin{bmatrix} 16 + \cancel{0 \times 2} \\ \underbrace{3 \times 4}_{} \overset{12}{} + \underbrace{4 \times 2}_{} \overset{8}{} \\ \underbrace{2 \times 4}_{8} + \underbrace{3 \times 2}_{6} \\ \underbrace{4 \times 4}_{16} + \underbrace{2 \times 2}_{4} \\ \cancel{0 \times 4} + \underbrace{4 \times 2}_{8} \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 14 \\ 20 \\ 8 \end{bmatrix}
$$

$$
\begin{bmatrix} y[0] \\ \vdots \\ y[4] \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 4 \end{bmatrix}
$$

$5 \times 1 \qquad 5 \times 4 \qquad 4 \times 1$