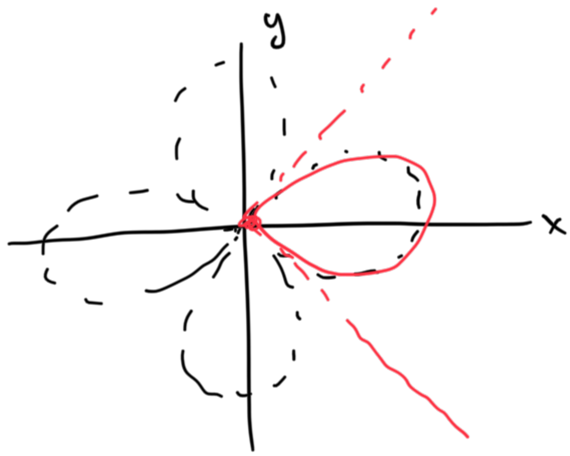
recall: Find the area inside one loop of the four-leaved rose

$$\boxed{r = \cos 2\theta}$$



$$\iint_D 1\,dA = \iint_D r\,dr\,d\theta$$

$$\int_{-\pi/4}^{\pi/4}\left[\int_0^{\cos 2\theta} r\,dr\right]d\theta$$

$$\int_{-\pi/4}^{\pi/4} \frac{r^2}{2}\Big|_0^{\cos 2\theta} d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \frac{1}{2}\cos^2 2\theta\, d\theta$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} (1+\cos 4\theta)\, d\theta$$

$\vdots$

$$\pi/8$$

$\cos 2\theta = 0$ let $a = 2\theta$

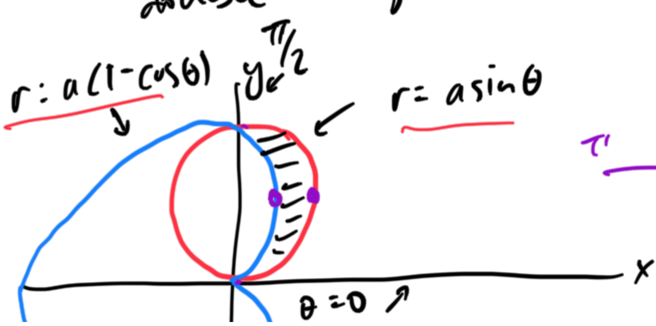
$\cos a = 0$

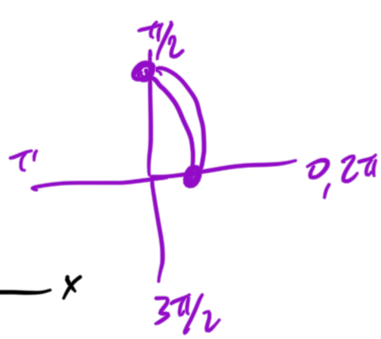
$a = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \ldots$

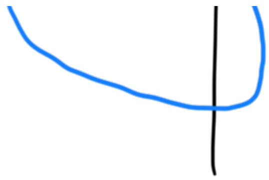
$2\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

$\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

Ex. Find the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$ using double integration







Ans

$$\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a\sin\theta} r\,dr\,d\theta = \int_0^{\boxed{\pi/2}} \frac{r^2}{2}\Bigg|_{a(1-\cos\theta)}^{a\sin\theta} d\theta$$

$$= \int_0^{\pi/2} \frac{(a\sin\theta)^2}{2} - \frac{(a(1-\cos\theta))^2}{2}\, d\theta$$

$\vdots$

$$= a^2\left(1-\frac{\pi}{4}\right)$$

$\cancel{a}\sin\theta = \cancel{a}(1-\cos\theta)$

$\sin\theta = 1-\cos\theta$

$\sin\theta + \cos\theta = 1$

$\vdots$

$1+2\sin\theta = 1$ or $\sin 2\theta = 0$

$2\sin\theta = 0$ | let $a = 2\theta$

$\sin\theta = 0$ | $\sin a = 0$

$\boxed{\theta = \pi/2}$ | $\boxed{\theta = \frac{\pi}{2}}$

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change of variable in 3D

If $x = g(u,v,w)$

$y = h(u,v,w)$

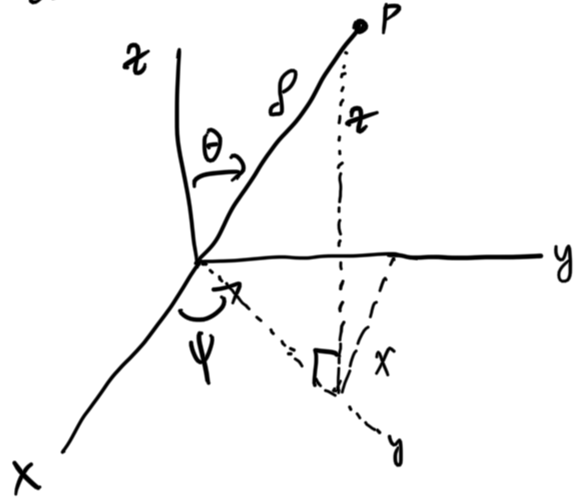
$z = j(u,v,w)$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_G f(g(u,v,w), h(u,v,w), j(u,v,w)) \cdot |J(u,v,w)|\,du\,dv\,dw$$

$$J(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \end{vmatrix}$$

$$\left| \frac{\partial z}{\partial y} \quad \frac{\partial y}{\partial v} \quad \frac{\partial z}{\partial w} \right|$$

Ex. compute Jacobian for spherical coordinates



$$\begin{cases} x = \rho \cos\theta \sin\psi \\ y = \rho \sin\theta \sin\psi \\ z = \rho \cos\psi \end{cases}$$

HW.