

# EE3210 Signals & Systems

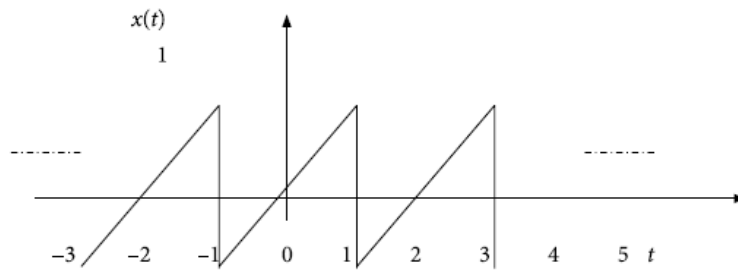
Due on 11:59 PM, December 7th, 2021

## Homework #2, 3

1. Total mark is 200 points ( $= 20$  points per problem  $\times 10$  problems)
2. Submission due by December 7th, Mid-night, 2021. **We will not accept late submission.**
3. Online submission through Canvas
  - Scan or taking a photo of your answer sheet, then upload to Canvas

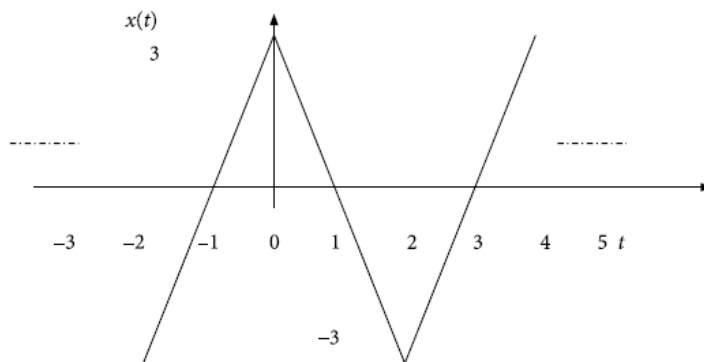
## Problem 1

Derive the complex and trigonometric FS representation of the following periodic signal  $x(t)$ .



## Problem 2

a) Derive the complex and trigonometric FS representation of the following periodic signal  $x(t)$ .



b) Use the trigonometric FS representation of  $x(t)$  to prove the following equality

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \dots$$

### Problem 3

- a) Derive the FT of the following signal  $x(t)$ , which is given by

$$x(t) = e^{(1+2t)} u(-t+2)$$

- b) Find the inverse FT of  $X(f)$ , which is given by

$$X(f) = \begin{cases} 2 \cos(2\pi f) & \text{for } |f| \leq \frac{1}{2} \\ 0 & \text{for } |f| > \frac{1}{2} \end{cases}$$

### Problem 4

Consider a continuous time LTI systems described by the following differential equation.

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

Derive the frequency response  $H(f)$  and the corresponding impulse response  $h(t)$ . Furthermore, derive the system output for input signal  $x(t) = e^{-4t} u(t)$ .

### Problem 5

Consider the following filters with impulse response  $h(t)$ .

$$h(t) = w_0 e^{-w_0 t} u(t)$$

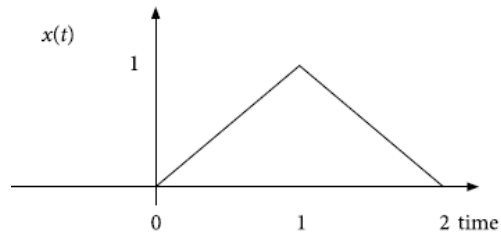
For the given filter, derive the 3-dB bandwidth  $f_{3 \text{ dB}}$ , equivalent bandwidth  $f_{\text{eq}}$ , and 80 percent energy containment bandwidth  $f_{80\%}$ , respectively.

## Problem 6

- a) Find the Laplace Transform (LT) and its corresponding ROC of the following signal.

$$x(t) = \cos^2(t) u(t)$$

- b) Find the LT and its corresponding ROC of the following signal.



- c) Find the LT of the following signal.

$$x(t) = e^{2t} u(t) * tu(t)$$

- d) Find the inverse LT of the following  $X(s)$  for ROC given by  $-2 < \text{Re}(s) < -1$ .

$$X(s) = \frac{-5s - 7}{(s+1)(s-1)(s+2)}$$

## Problem 7

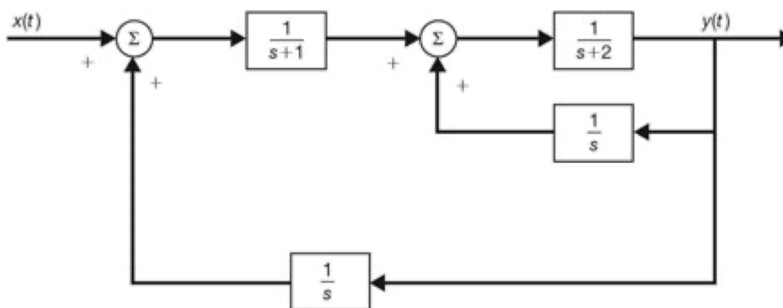
Consider a causal, stable LTI system described by the following expression.

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = \frac{d^2 x(t)}{dt^2} + 8 \frac{dx(t)}{dt} + 13x(t)$$

Use LT to find the transfer function  $H(s)$  and its corresponding impulse response  $h(t)$ .

## Problem 8

Determine the overall transfer function  $H(s)$  for the system diagram illustrated below.



## Problem 9

- a) Find the Z-transform of the following sequence and its corresponding ROC.

$$x[n] = \left\{ 5, 3, \underset{\uparrow}{-2}, 0, 4, -3 \right\}$$

- b) Find the Z-transform and its corresponding ROC for the following sequences

$$x_1[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1],$$

$$x_2[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[-n-1]$$

- c) Find the inverse Z-transform of the following  $X(z)$

$$X(z) = Z^2 \left( 1 - \frac{1}{2}z^{-1} \right) (1 - z^{-1}) (1 + 2z^{-1}), \quad 0 < |z| < \infty$$

**Problem 10**

- a) Find the Z-transform of the following sequence and its corresponding ROC.

$$x[n] = \begin{cases} 1 & \text{for } n < 0, \\ 0.5^n & \text{for } n \geq 0 \end{cases}$$

- b) Calculate the convolution of the two sequences using Z-transform

$$x_1[n] = \left\{ \underset{\uparrow}{1}, -2, 1 \right\}, \quad x_2[n] = \left\{ \underset{\uparrow}{1}, 1, 1, 1, 1, 1 \right\}$$

- c) Find the inverse Z-transform of the following  $X(z)$  for three different ROCs.

$$X(z) = \frac{Z + 1}{3Z^2 - 4Z + 1}$$

- (i) Find  $x[n]$  if ROC is  $|Z| > 1$
- (ii) Find  $x[n]$  if ROC is  $|Z| < \frac{1}{3}$
- (iii) Find  $x[n]$  if ROC is  $\frac{1}{3} < |Z| < 1$