EE 4211 Computer Vision

Lecture 4A: Morphology

Semester B, 2021-2022

Schedules

Week	Date	Topics
1	Jan. 11 (face to face)	Introduction/Imaging
2	Jan. 18 (online)	Image enhancement in spatial domain
3	Jan. 25 (online)	Image enhancement in frequency domain (HW1 out) publish online on Jan. 26 noon.
4	Feb. 8 (online)	Morphological processing
5	Feb. 15 (online)	Image restoration (HW1 due)
6	Feb. 22 (online)	Midterm (no tutorials this week)
7	Mar. 1 (online)	Edge detection (HW2 out)
8	Mar. 8 (online)	Image segmentation
9	Mar. 15	Face recognition with PCA, LDA (tutorial on segmentation) (HW2 due)
10	Mar. 22	Face recognition based on deep learning (Quiz) Image segmentation based on deep learning
11	Mar. 29	Object detection with traditional methods Object detection based on deep learning (tutorial on detection)
12	Apr. 5	Events / Public Holidays
13	Apr. 12	Invited project presentation and Summary

Lecture Outline

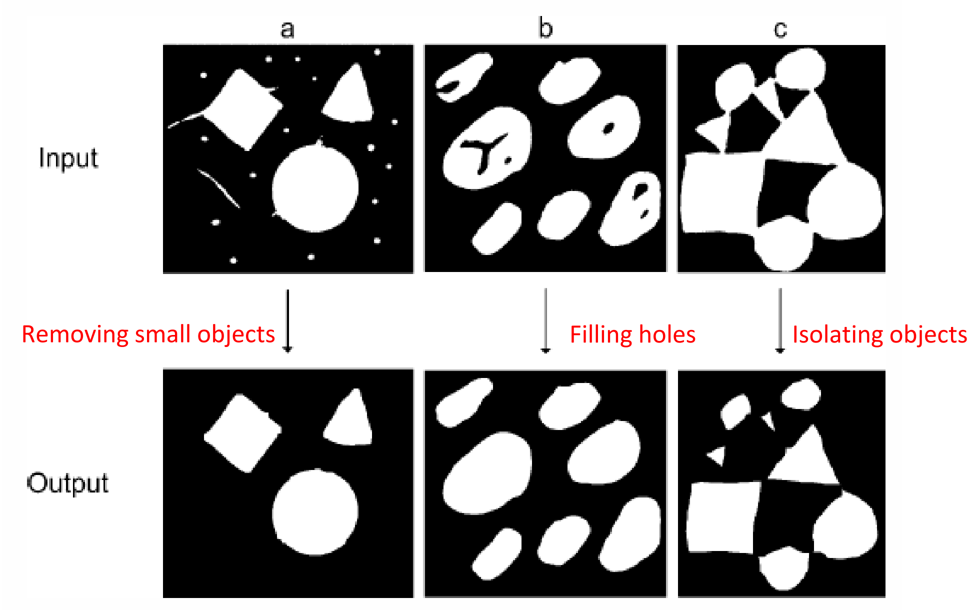
- Mathematical Morphology
- Review on Set Theory
- Structure Elements
- Basic Morphological operations
  - Hit
  - Fit
  - Dilation
  - Erosion
  - Opening
  - Closing

Morphology

- Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons
- Usually applied to binary (black & white, or 0 & 1) images but it is also applicable to gray-scale images
- Based on set theory

Major Applications of Morphology

- Removing Small Objects
  - Remove noise as a side effect of thresholding
  - Tackle over-segmentation in the form of the small objects
- Filling Holes
  - Remove holes inside the object due to under-segmentation
- Isolating Objects
  - Ensure that the objects are separated from each others



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Review of Set Theory

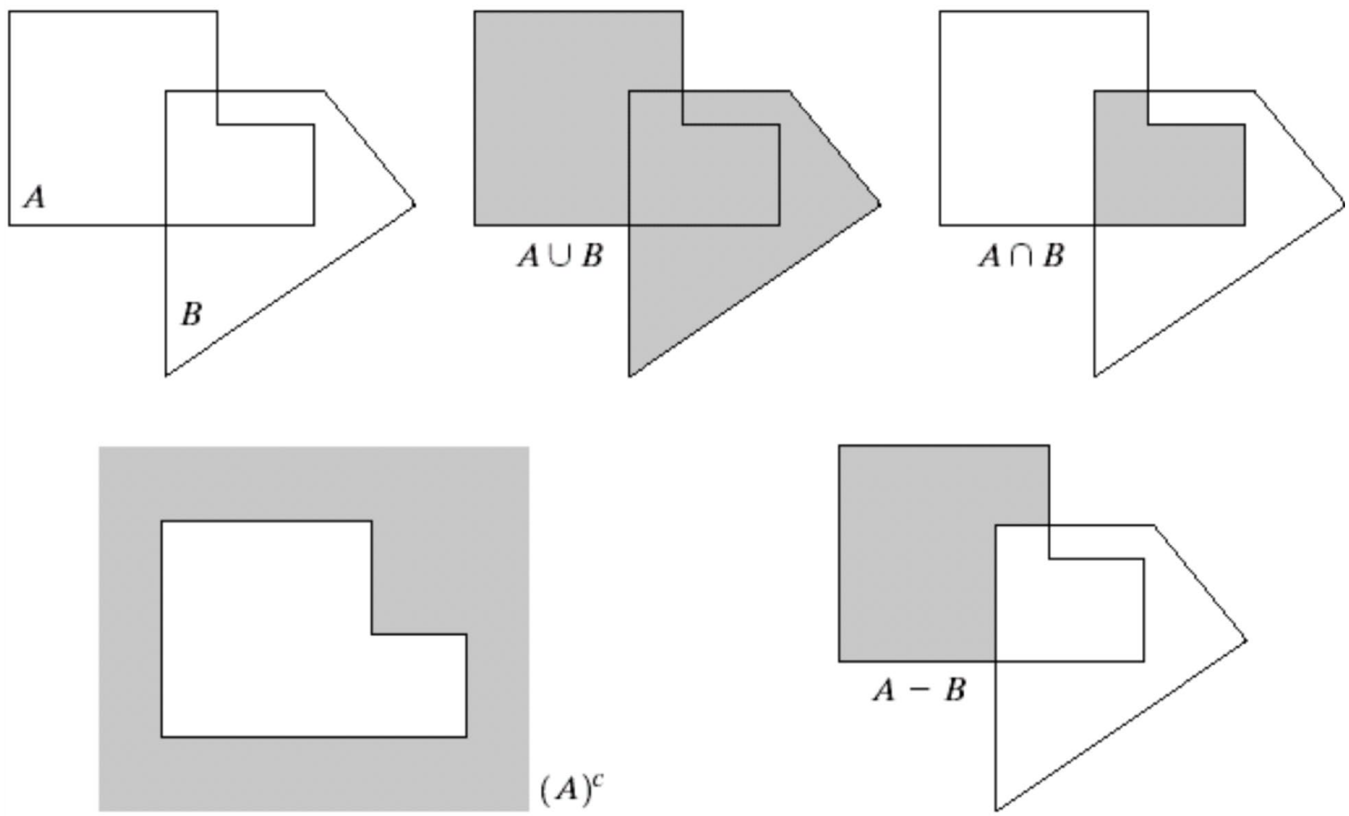
- Set (Ω)
  - A collection of objects (elements)
- Membership ($\in$)
  - If ω is an element (member) of a set Ω, we write $\omega \in \Omega$
- Subset ($\subset$)
  - Let A, B are two sets. If for every $a \in A$, we also have $a \in B$, then the set A is a *subset* of B, that is, $A \subset B$
  - If $A \subset B$ and $B \subset A$, then $A = B$.
- Empty set ($\varnothing$)

Review of Set Theory

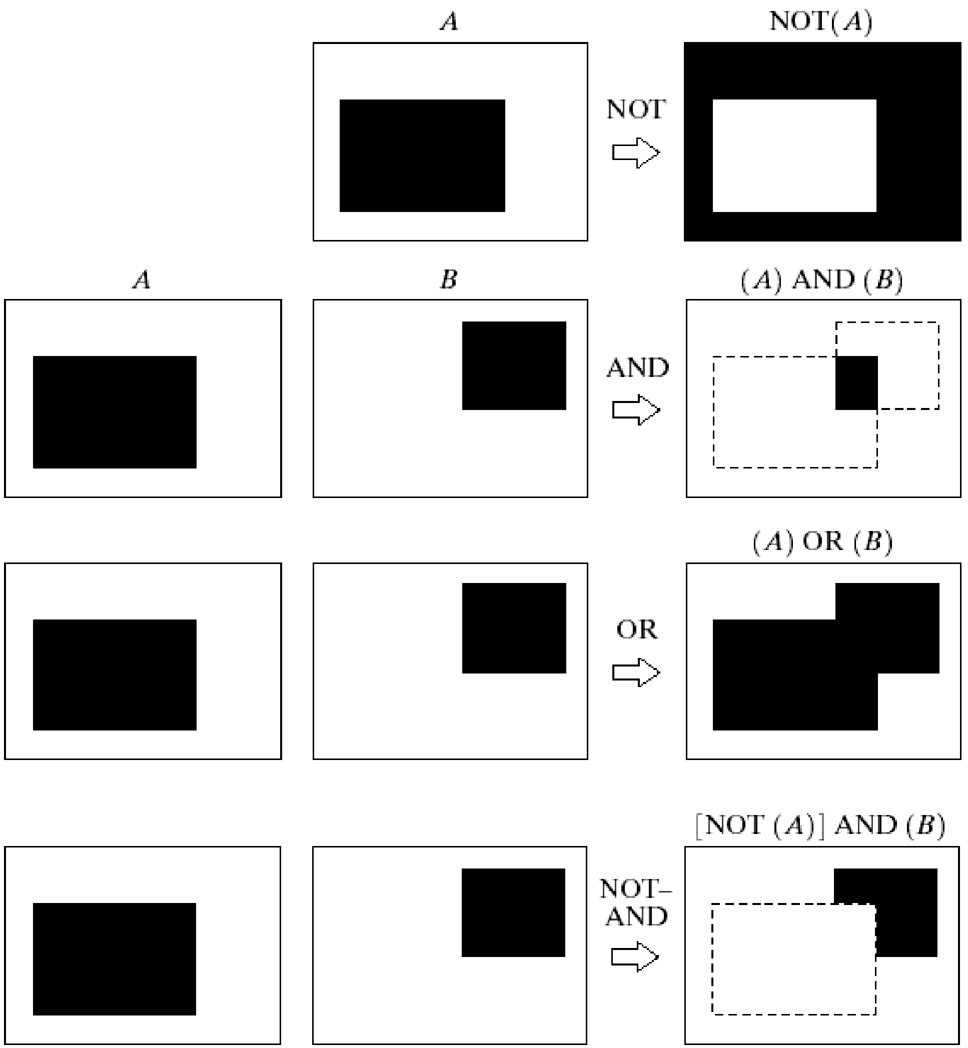
- Complement set
  - If $A \subset \Omega$, then its complement set $A^c = \{\omega \mid \omega \in \Omega, \text{ and } \omega \notin A\}$
- Union ($\cup$)
  - $A \cup B = \{\omega \mid \omega \in A \text{ or } \omega \in B\}$
- Intersection ($\cap$)
  - $A \cap B = \{\omega \mid \omega \in A \text{ and } \omega \in B\}$
- Disjoint set
  - A and B are disjoint (mutually exclusive) if $A \cap B = \varnothing$

Set Operations

$$A = \{(x, y) \mid I_A(x, y) = 1\}, \quad B = \{(x, y) \mid I_B(x, y) = 1\}$$



Logic Operations Between Binary Image

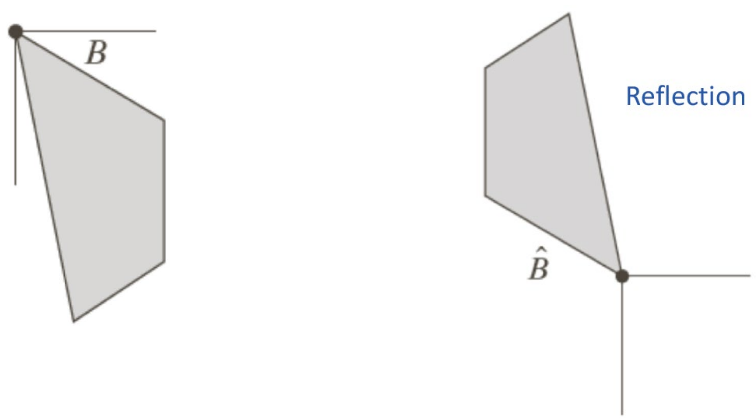


Set Relations

- Reflection

The reflection of a set B, denoted $\hat{B}$, is defined as

$$\hat{B} = \{ w \mid w = -b, \text{for } b \in B \}$$

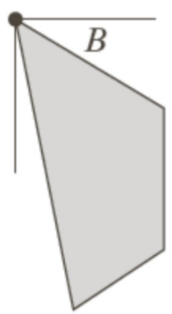


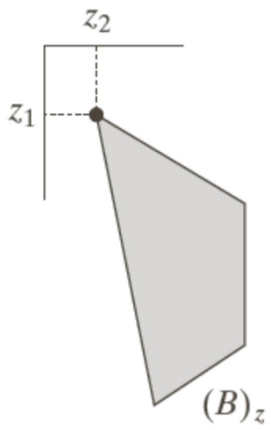
Logic Operations Between Binary Image

- Translation

  The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$





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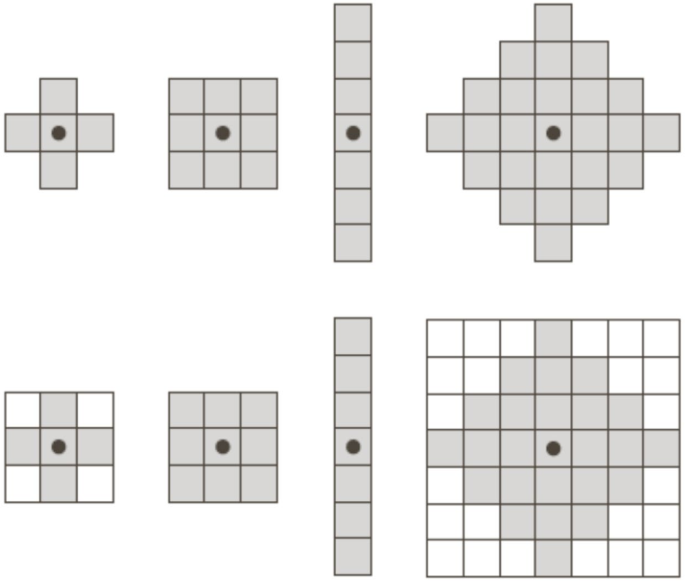
Structure elements

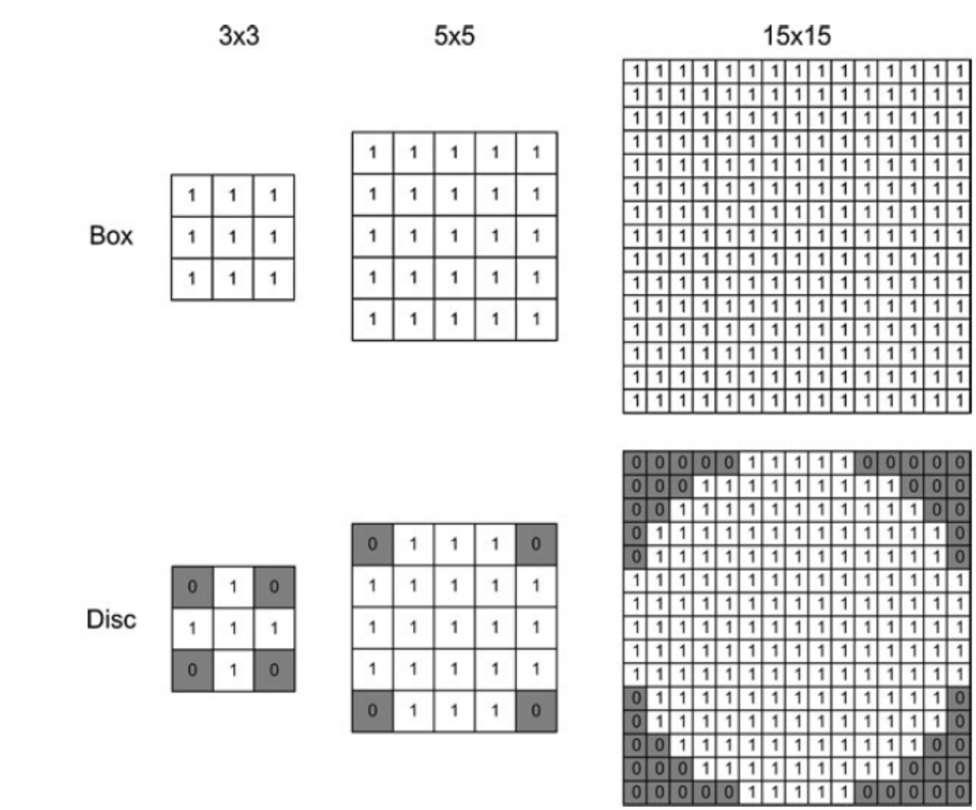
- Morphological operations are defined based on "structuring elements"
- A structuring element is a small set or subimage, used to probe for structure



The dot denotes the origins of the SEs

Type and Size of SE

- Type and size to use is up to the designer
- Box-shaped SE tends to preserve sharp object corners
- Disk-shaped SE tends to round the corners of the objects



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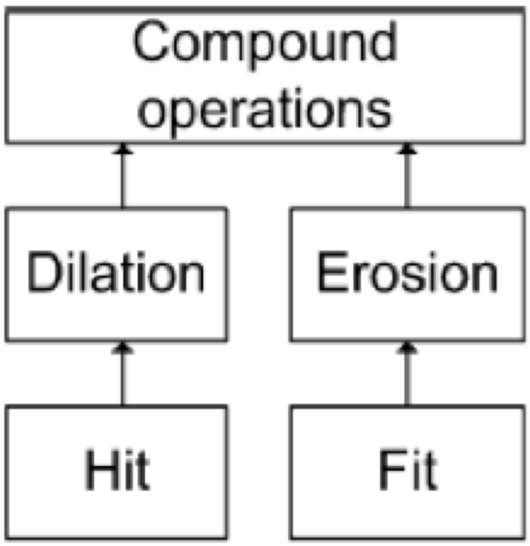
Morphology Processing

- A SE is applied using either a Fit or a Hit operation
- Applying one of these operations to each pixel in an image is denoted Erosion and Dilation, respectively
- Combining these two methods are known as Compound Operations: Opening and Closing



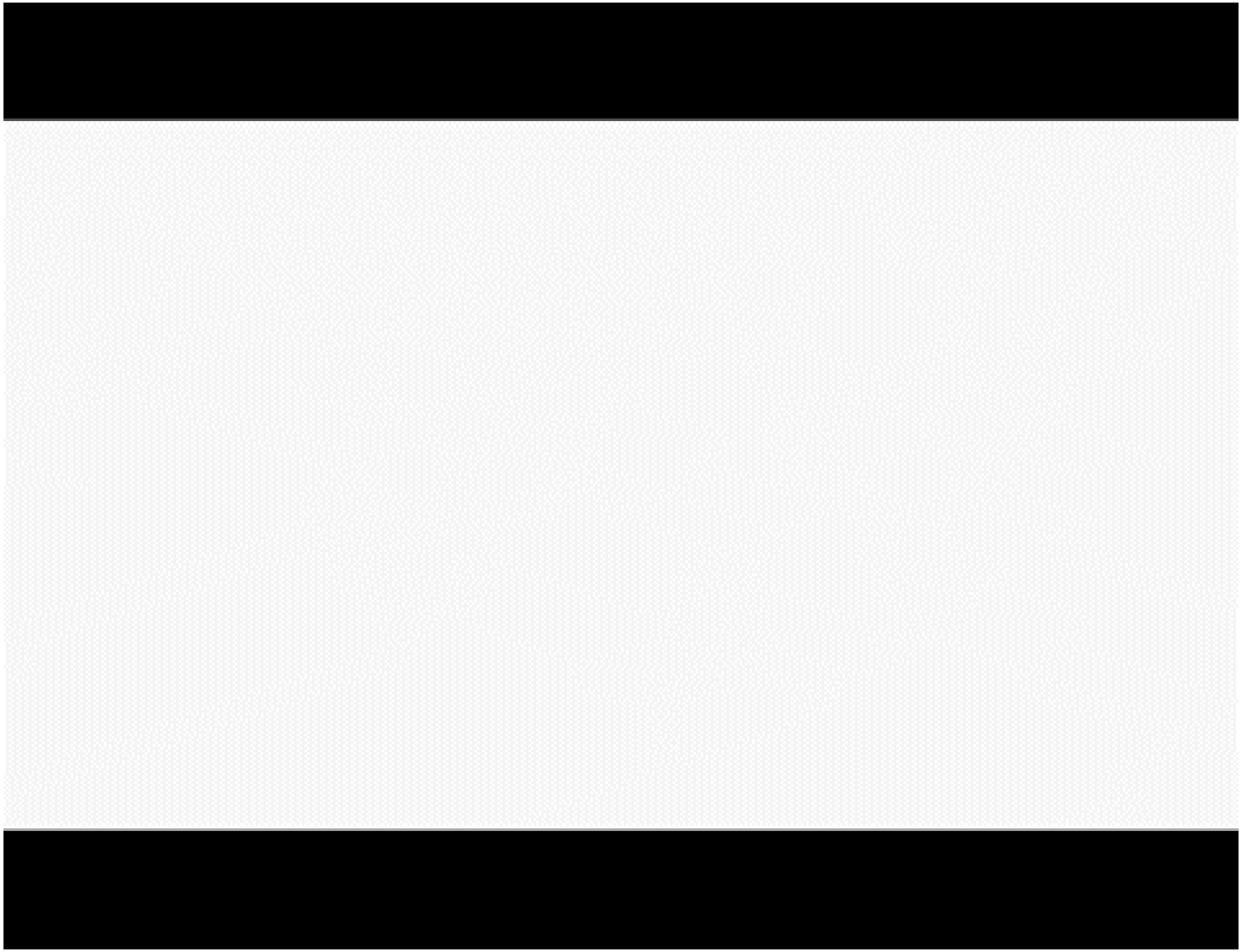
Fit

- For each ‘1’ in the SE, we investigate whether the pixel at the same position in the image is also a ‘1’.
- If **ALL** of the ‘1’s in the SE are covered by the image, we say that the SE **fits** the image at the pixel position (the one on which the SE is centered).
- This pixel is set to ‘1’ in the output image.
- Otherwise, it is set to ‘0’ in the output image.

Hit

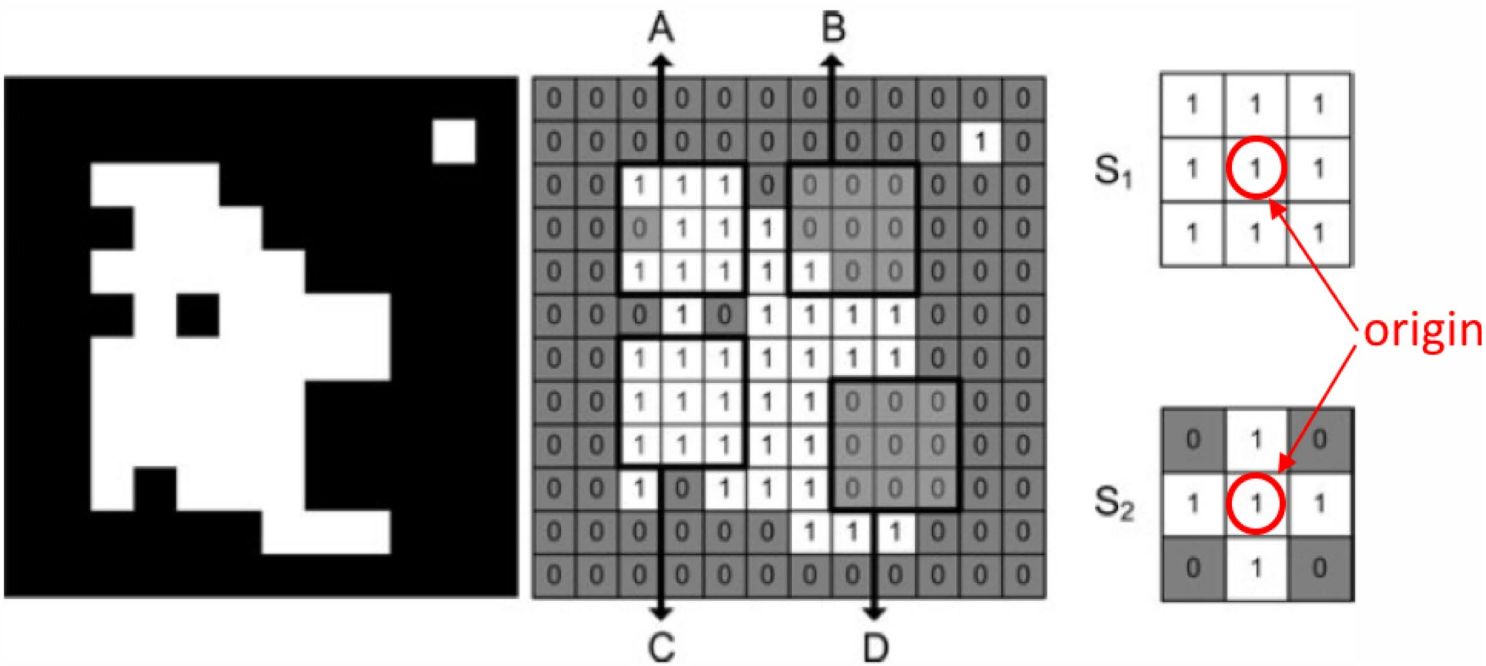
- For each ‘1’ in the SE, we investigate whether the pixel at the same position in the image is also a ‘1’.
- If any **ONE** of the ‘1’s in the SE is covered by the image, We say that the SE **hits** the image at the pixel position (the one on which the SE is centered).
- This pixel is set to ‘1’ in the output image.
- Otherwise, it is set to ‘0’ in the output image.

Example



https://www.youtube.com/watch?v=vWWpZtQxlzA

Example



Position	*SE*	Fit	Hit
A	S_1	No	Yes
A	S_2	No	Yes
B	S_1	No	Yes
B	S_2	No	No
C	S_1	Yes	Yes
C	S_2	Yes	Yes
D	S_1	No	No
D	S_2	No	No

Basic morphological operations

- Erosion 

- Dilation 

- Opening -> object
- Closing -> background

Erosion and Dilation

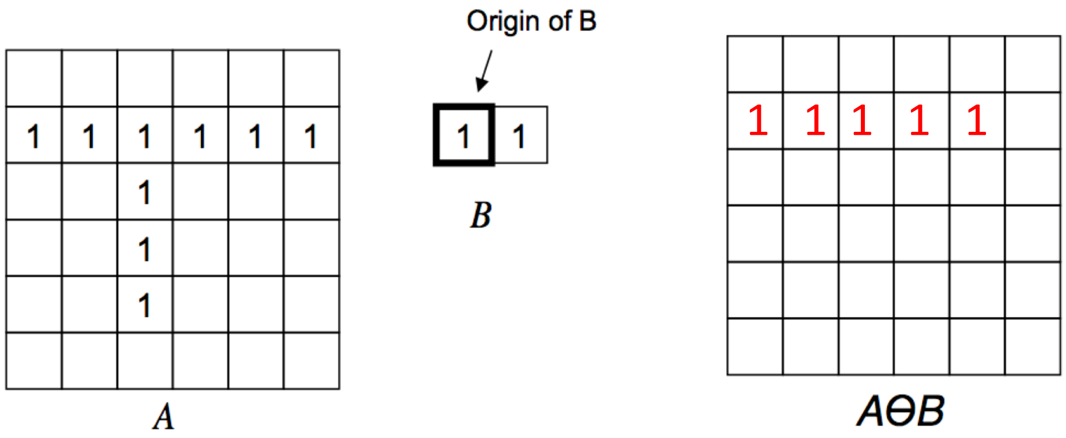
- Hit or Fit is applied to every single pixel by scanning through the image
- The size of the SE in these operations has the same importance as the kernel size of the spatial filtering
- The bigger the SE, the bigger the effect in the image

Erosion

- **Applying Fit to an entire image is denoted Erosion**
- The erosion of set A by set B (structuring element) is

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- Interpretation: shift B by z, if it is completely inside A, output a 1

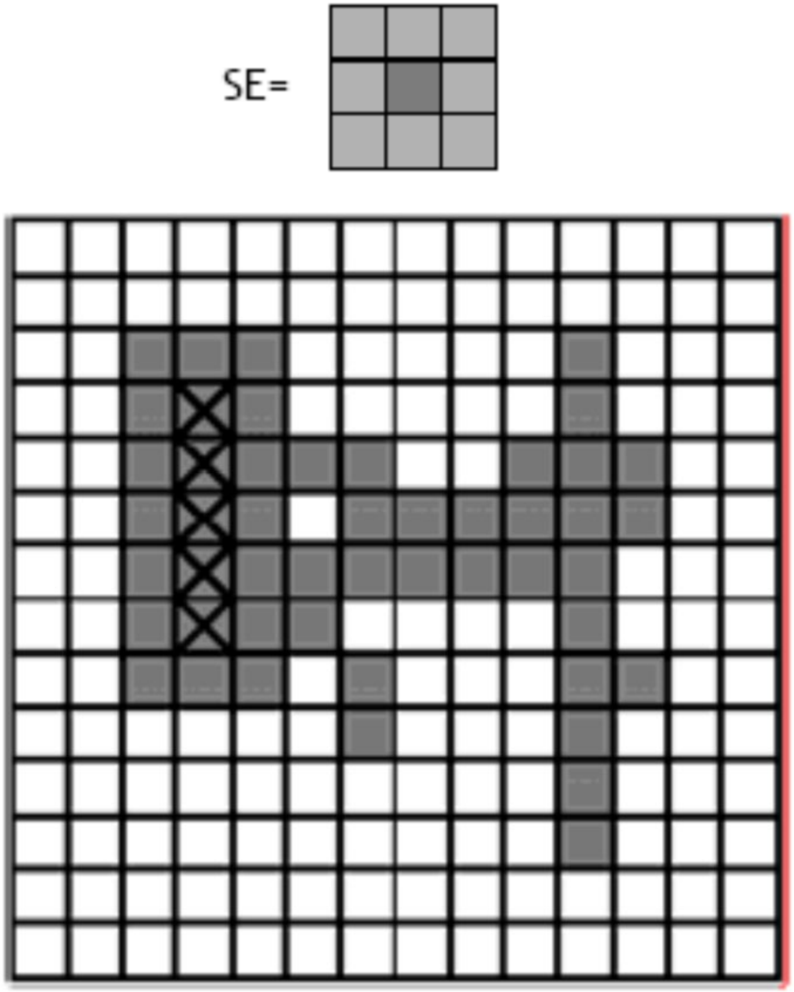


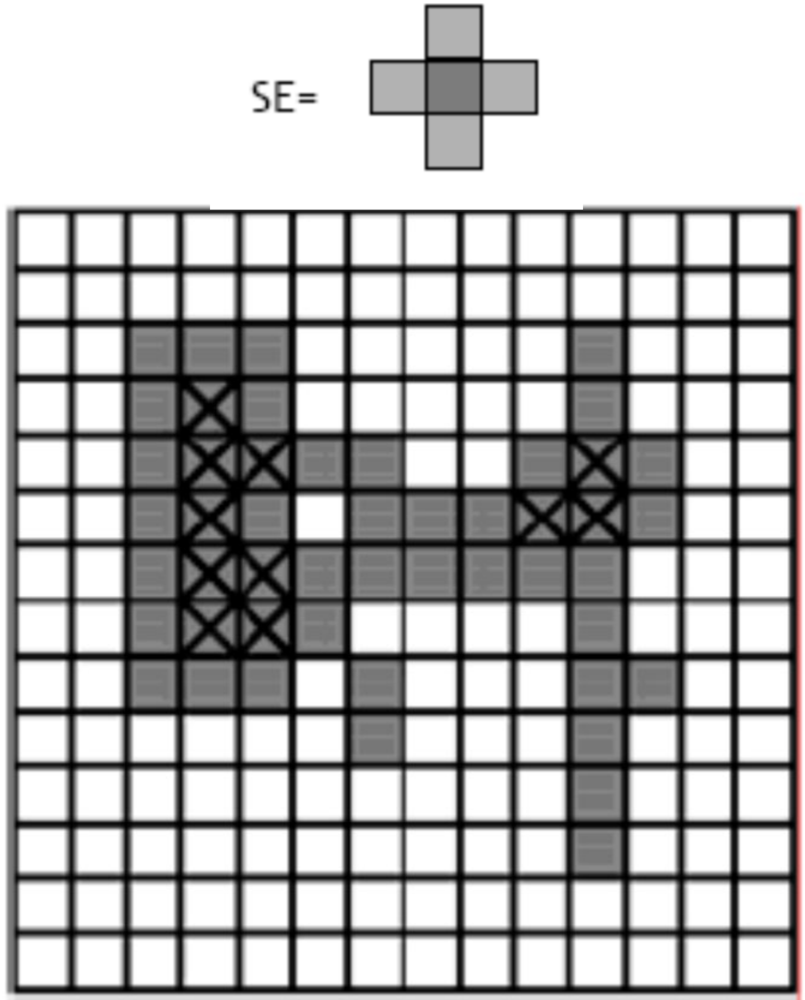
Example: Erosion

https://www.youtube.com/watch?v=fmyE7DiaIYQ

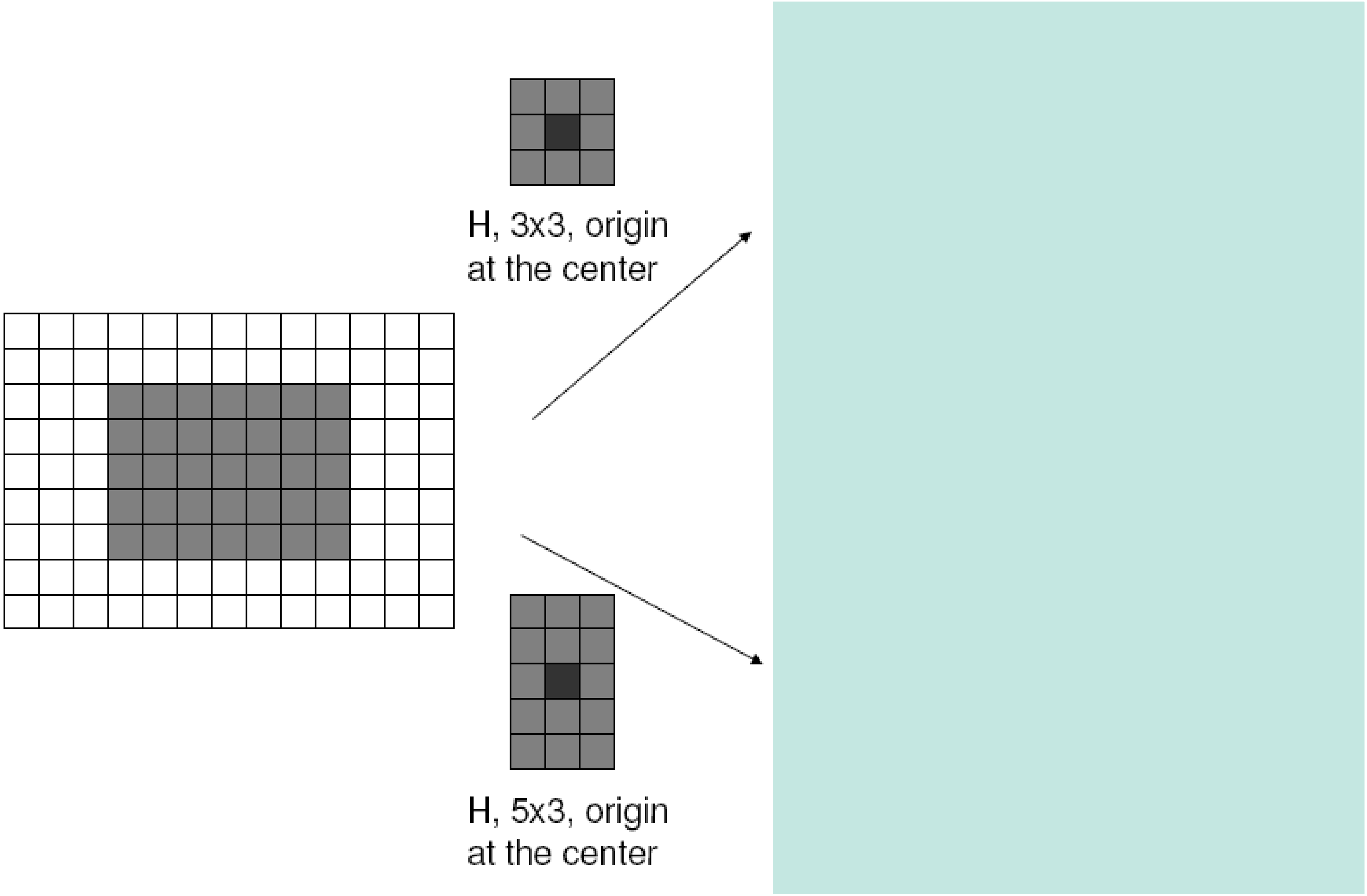
Example: Erosion

- Different SE with the same image will lead to different results

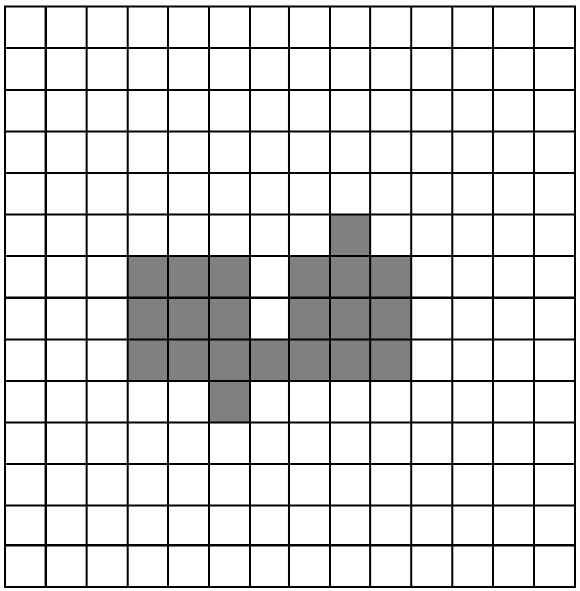




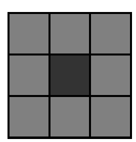
Example: Erosion



Example: Erosion



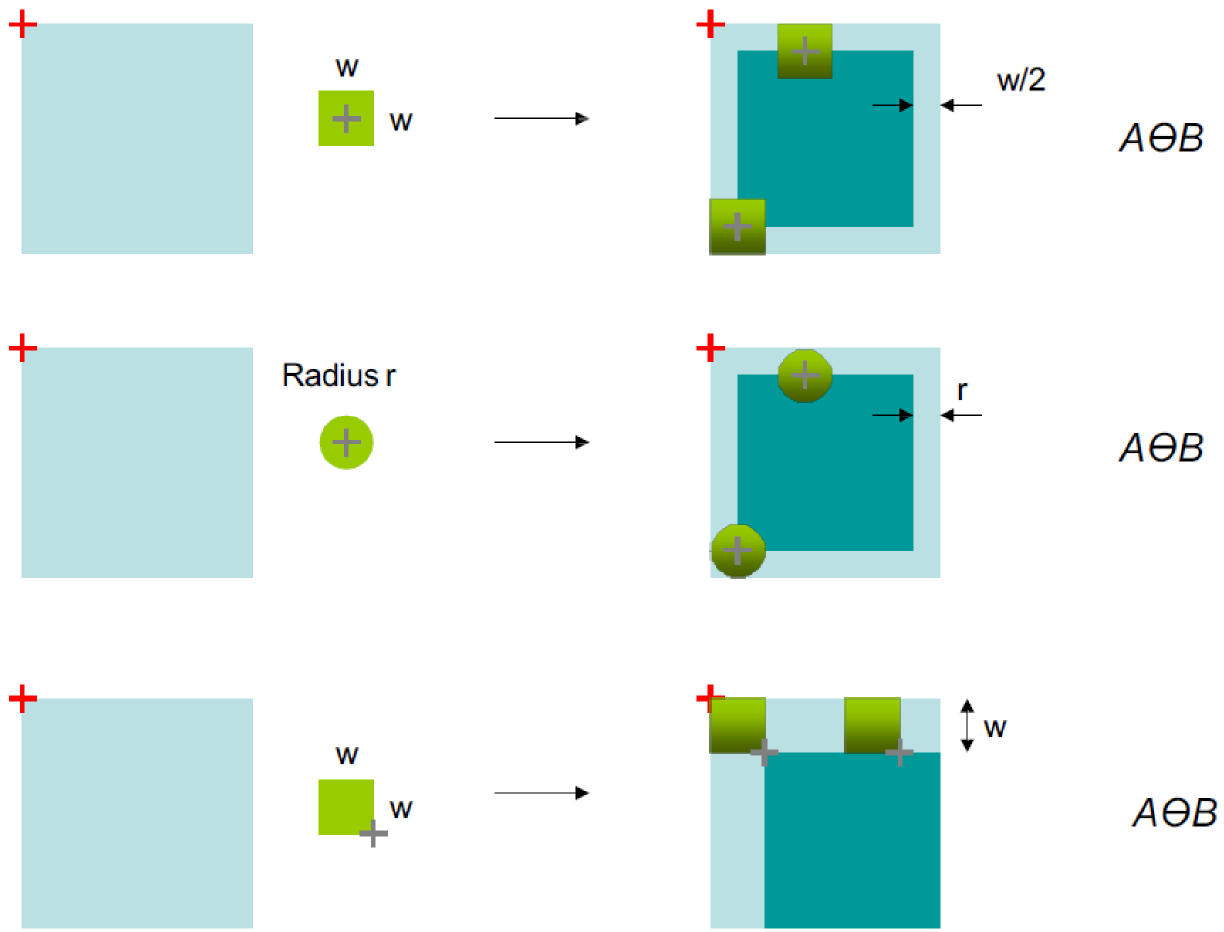
F



H, 3x3, origin at the center

Example: Erosion

- SE with different centers for the same image will lead to different results

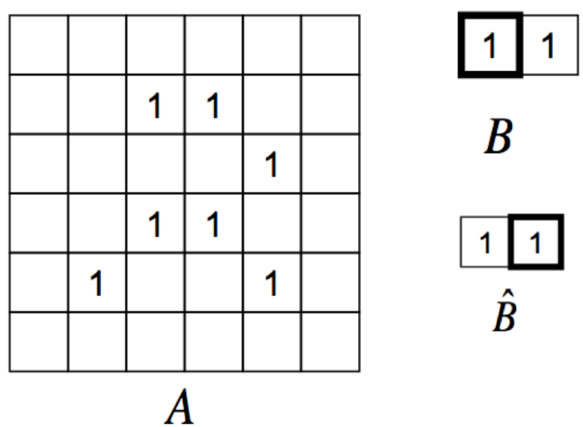


Dilation

- Applying **Hit** to an entire image is denoted Dilation
- The dilation of set A by set (structuring element) B is

$$A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$$

- Interpretation: reflect B, shift by z, if it overlaps with A, output a 1 at the center of B

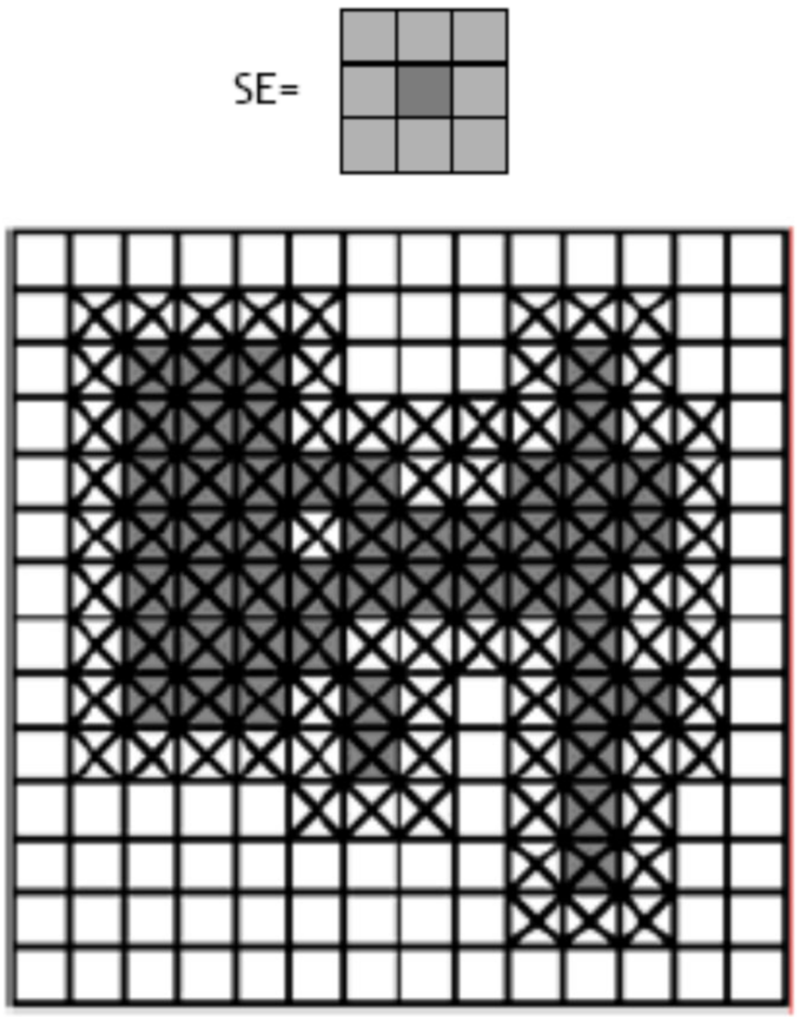


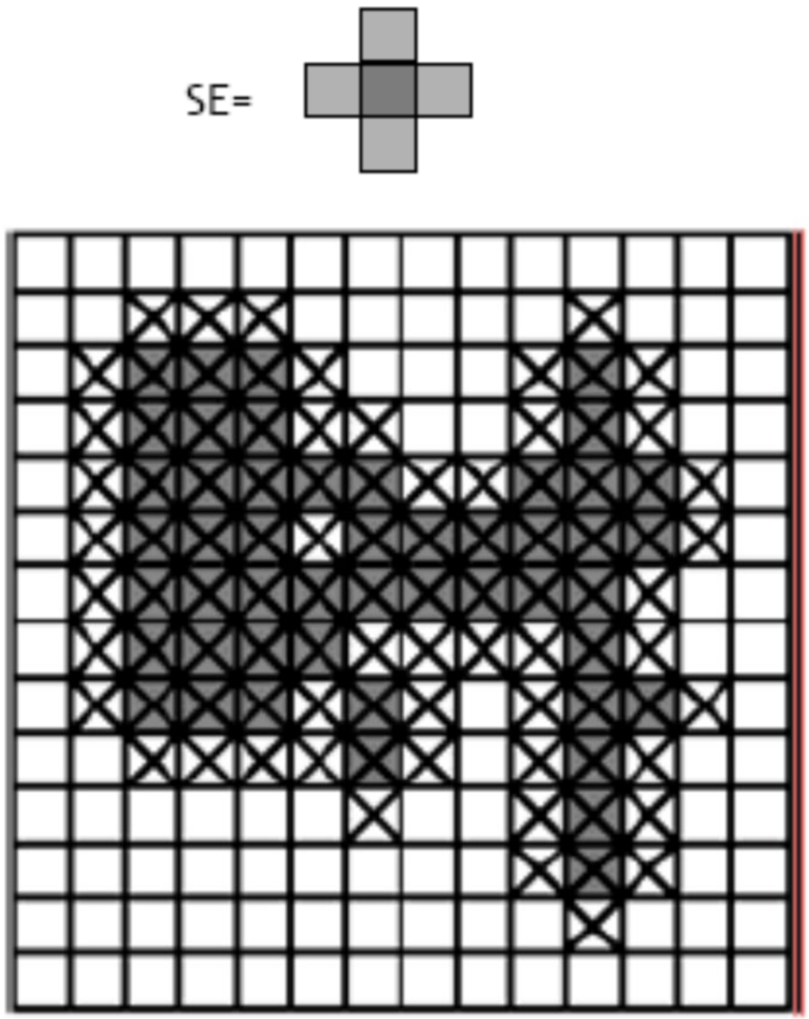
Example: Dilation

https://www.youtube.com/watch?v=xO3ED27rMHs

Example: Dilation

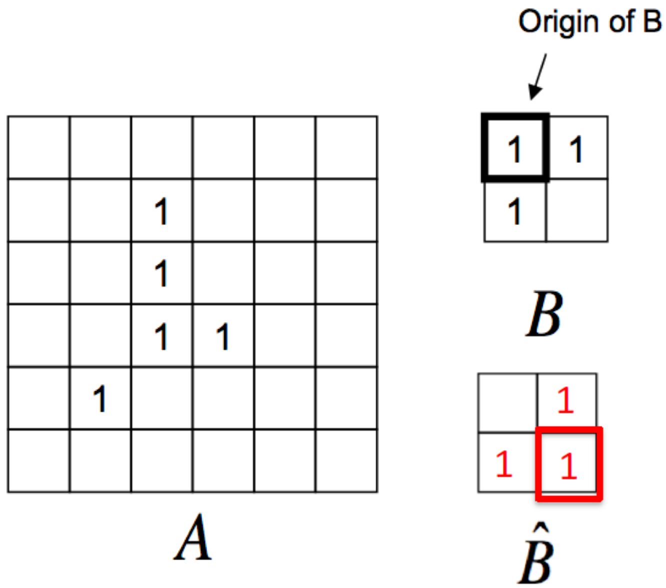
- Different SE with the same image will lead to different results





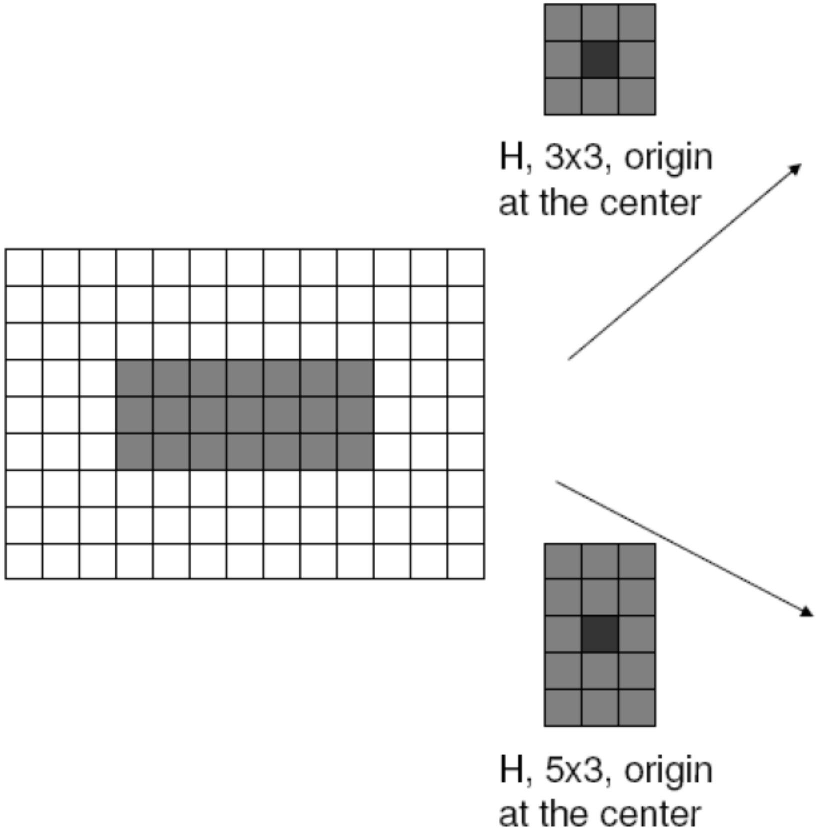
Example: Dilation

- Different SE with the same image will lead to different results

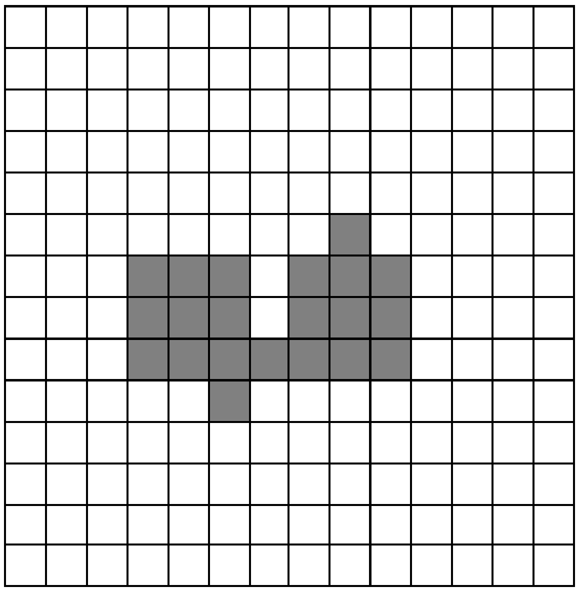


Example: Dilation

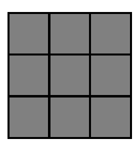
- Different SE with the same image will lead to different results



Example: Dilation



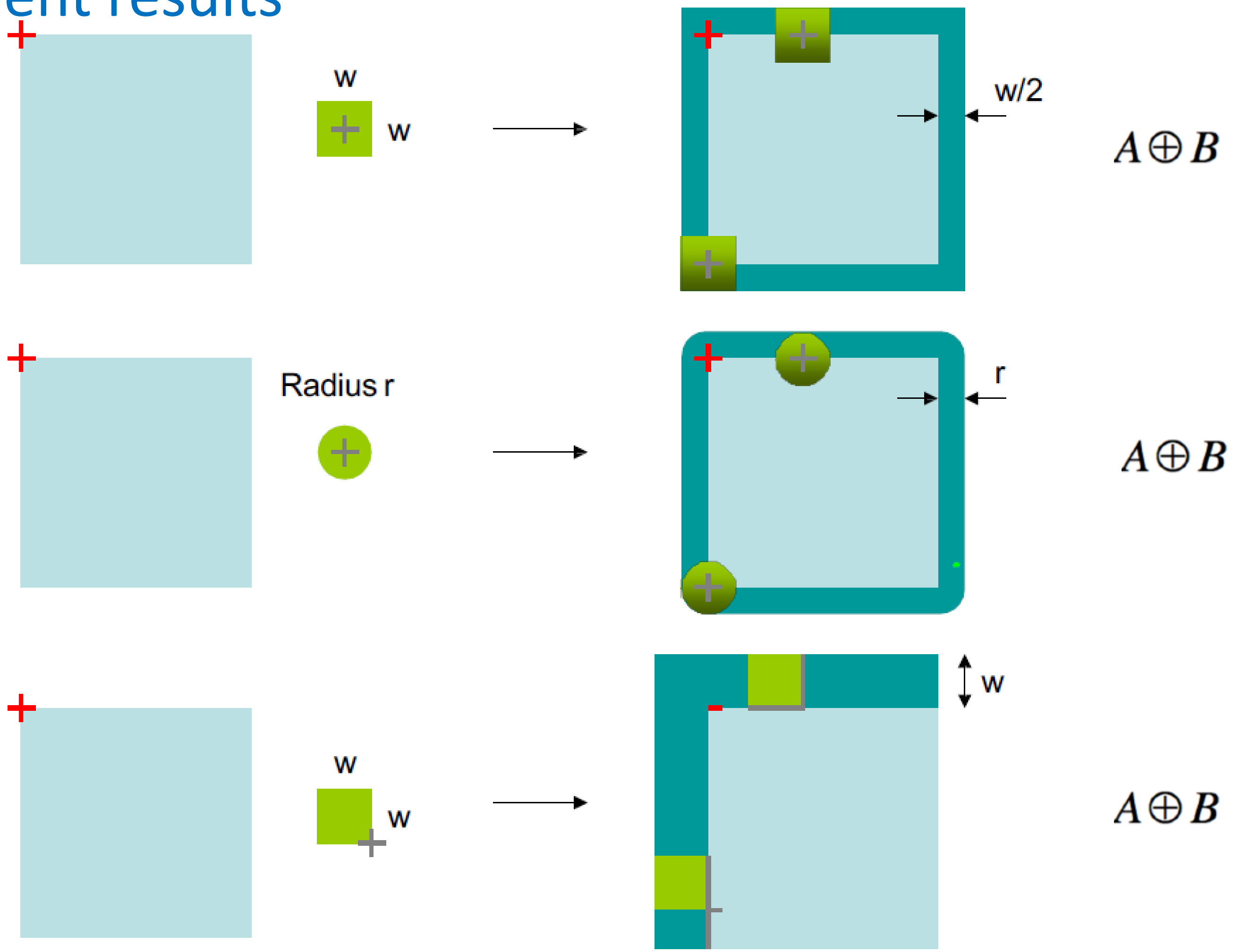
F



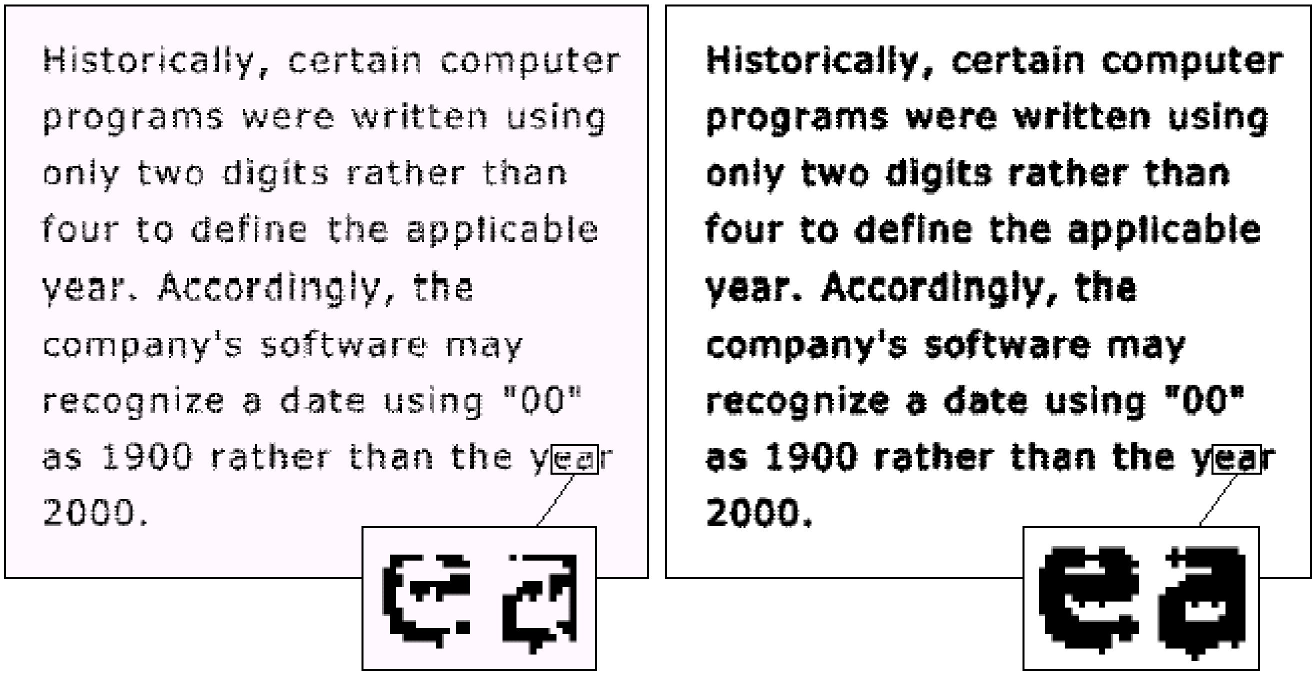
H, 3x3, origin at the center

Example: Dilation

- SE with different centers for the same image will lead to different results



Example: Dilation



0	1	0
1	1	1
0	1	0

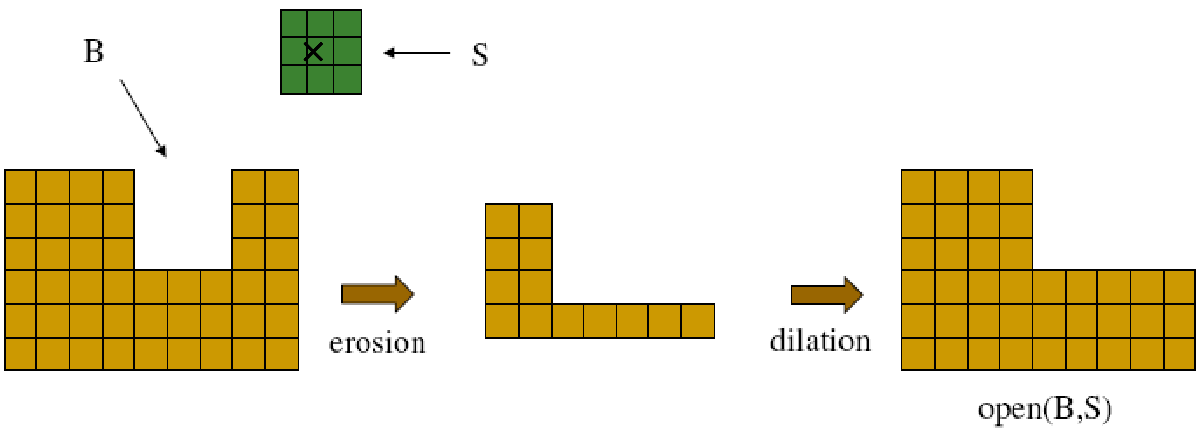
Opening & Closing: Intuitive Interpretation

- Dilation expands an object
- Erosion contracts an object
- Opening
  - Smoothens contours, enlarges narrow gaps, eliminates thin protrusions and ridges
- Closing
  - Fills narrow gaps, holes and small breaks

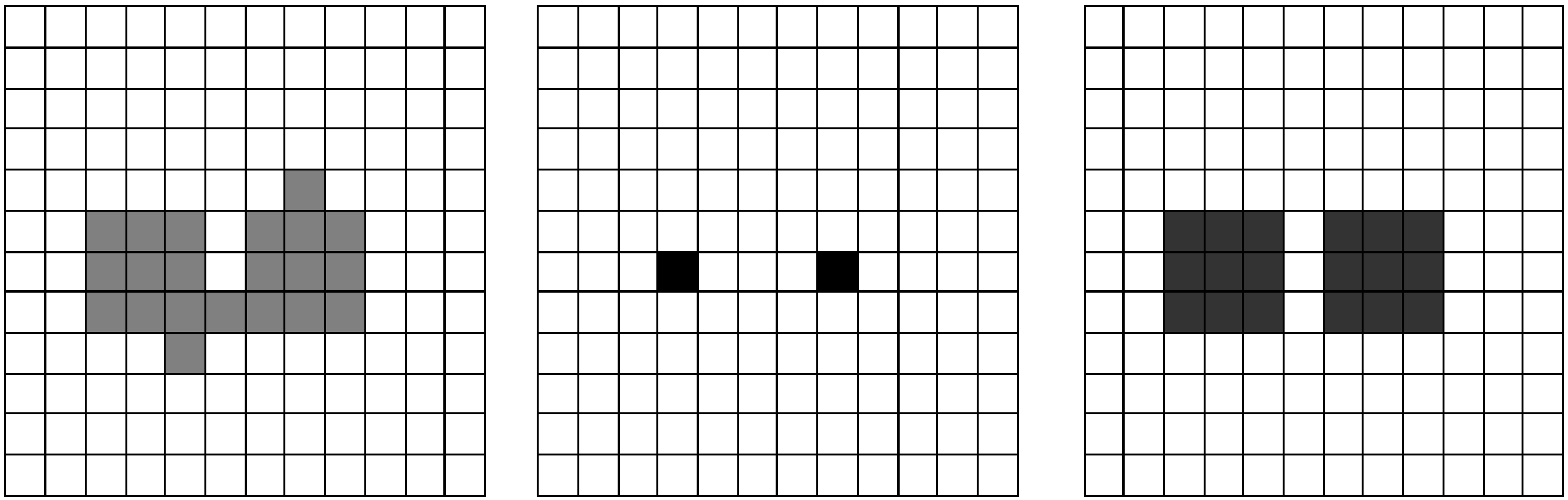
Opening

- Opening: Like “smoothing from the inside”
- Erosion followed by Dilation

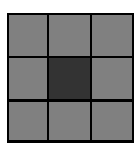
$$A \circ B = (A \ominus B) \oplus B \;=\; \cup\,((B)_z \mid (B)_z \subseteq A\}$$



Example: Opening

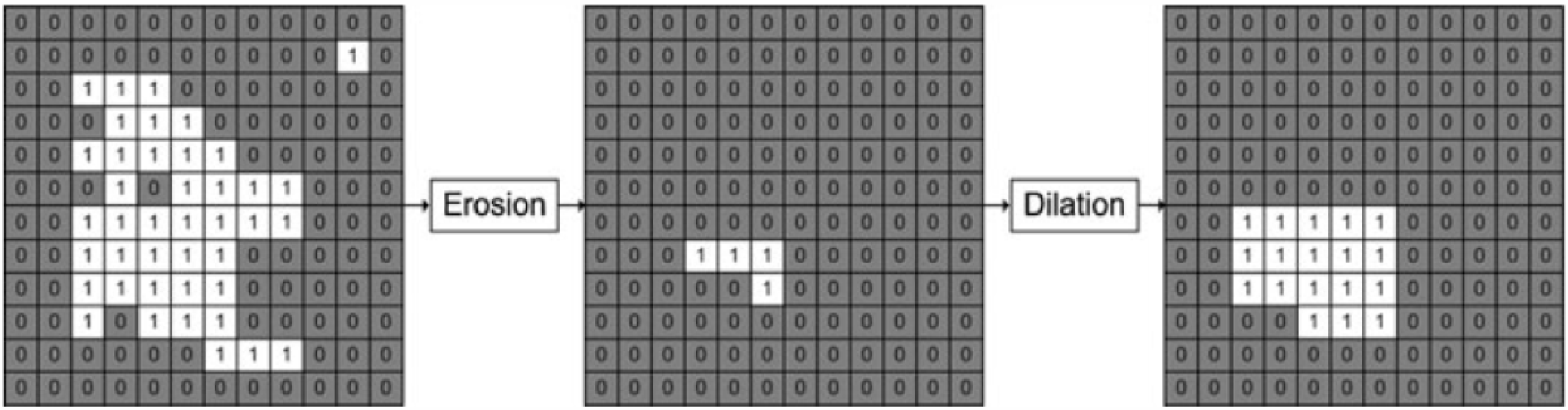


F $\qquad F \Theta H \qquad (F \Theta H) \oplus H$



H, 3x3, origin at the center

Example: Opening



$$S_1 \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Example: Opening

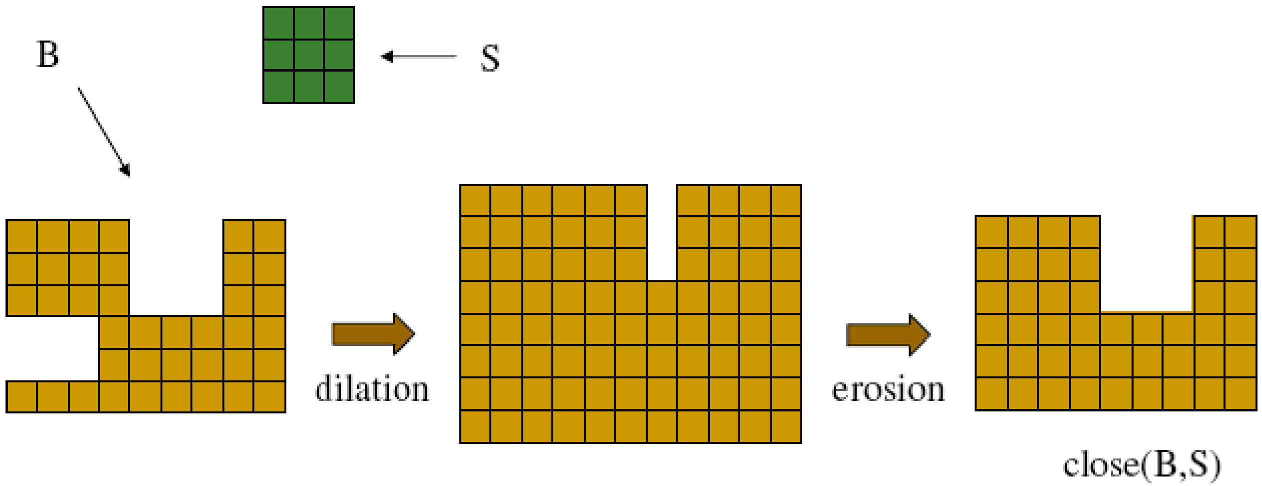
a b c

FIGURE (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

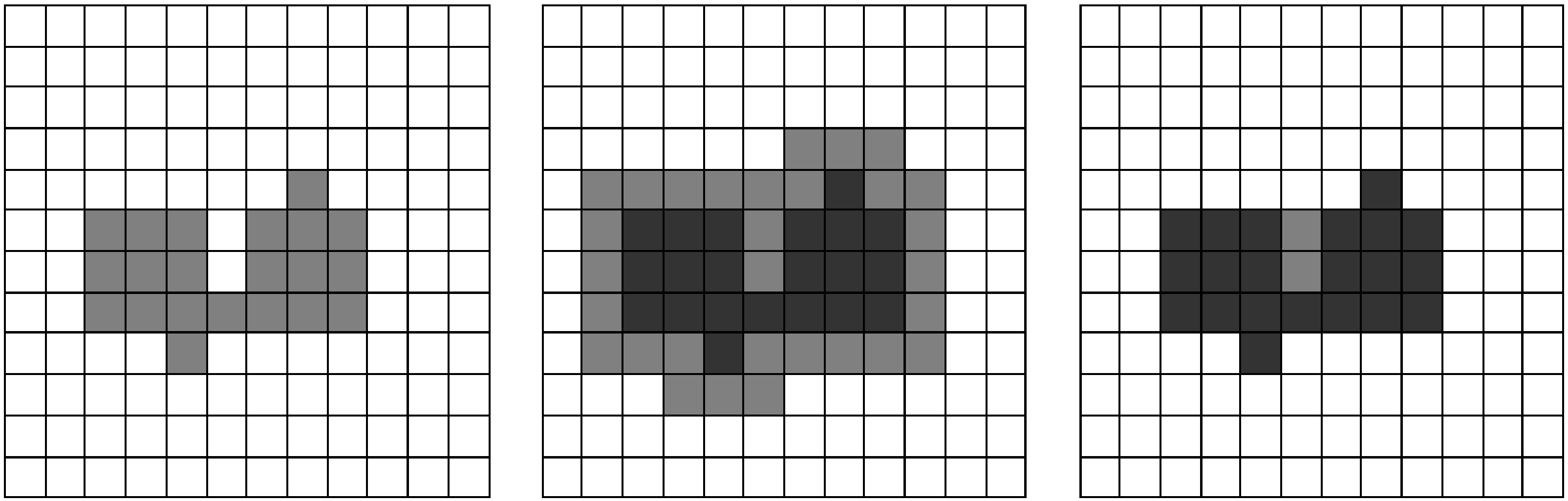
Closing

- Closing: Like "smoothing from the outside"
- Dilation followed by Erosion

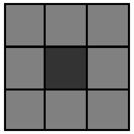
$$A \bullet B = (A \oplus B) \ominus B$$



Example: Closing



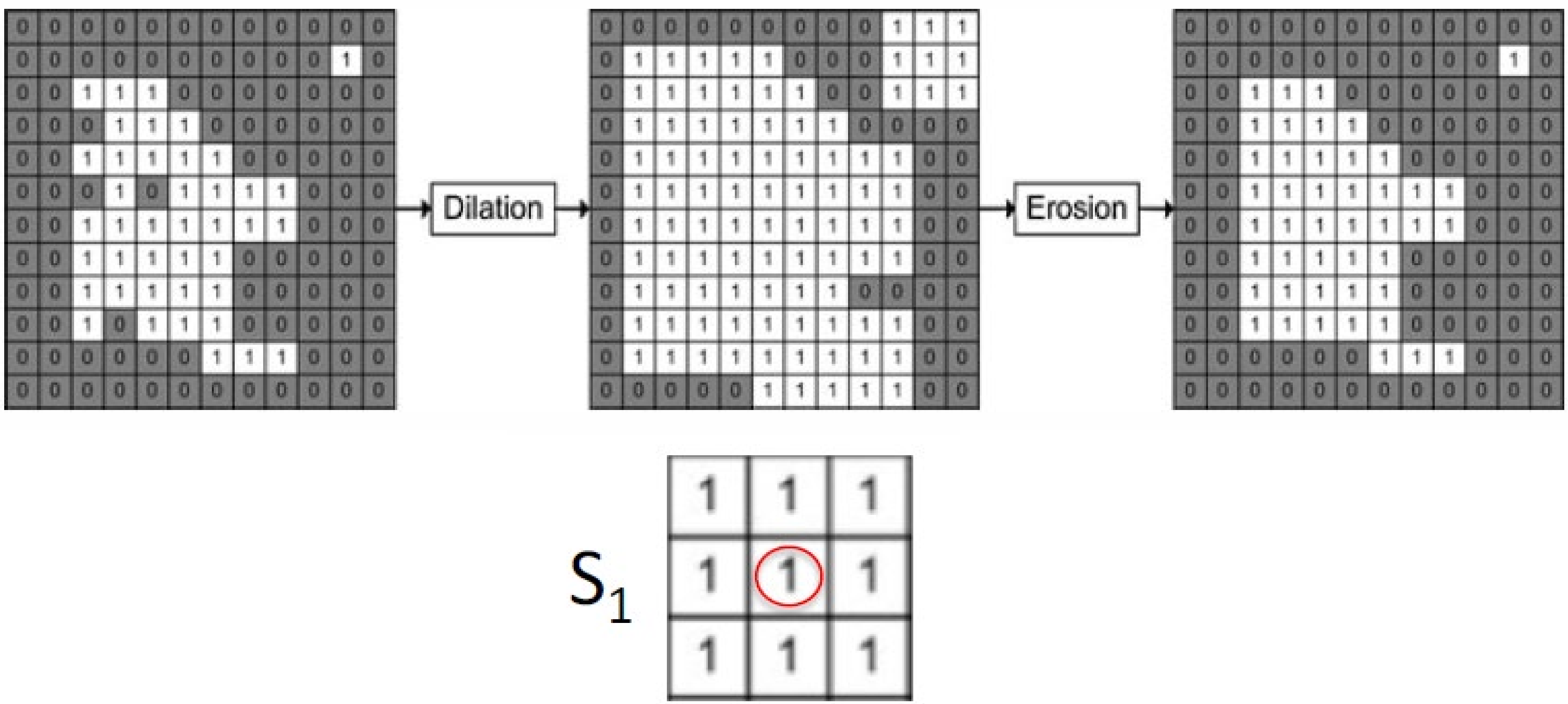
F $\quad\quad F \oplus H \quad\quad (F \oplus H)\Theta H$



H, 3x3, origin at the center

Example: Closing

- Holes and convex structures are filled
- The object preserves its size



Combining Opening and Closing

- In some situations, we need to apply both opening and closing to an image
- For example, in case when both holes inside the main object and small noisy objects
- Note that the SEs used in the opening and the closing operations need not be the same

Combination of opening and closing