EE3331 Probability Models in Information Engineering

Semester B 2021-2022

Assignment 2

Due Date: 23 February 2022

1. Consider the experiment of rolling 2 fair dice and the outcome is two times the absolute difference of the two faces. For example, if the face values are 1 and 2, then the outcome is 2. Assign a random variable (RV) for this experiment and then compute the probability mass function (PMF) for all admissible values of the RV. Then determine the probability that the outcome is greater than 5.

2. Consider the experiment of rolling 2 fair dice and the outcome is the sum of the two faces. We define an event of sum equaling 10. Compute the probability when this event occurs at the fifth trial.

3. With the use of the binomial distribution, compute the probability of obtaining 49, 50 or 51 heads, when flipping 100 fair coins. Then use Poisson distribution to obtain the approximate probability. Is the Poisson approximation accurate enough? Briefly explain your answer.

4. Let X be a discrete random variable with values 1, 2, 3, …, and its probability mass function (PMF) has the form of:

$$p(x) = \begin{cases} \frac{5}{6}, & x = 1 \\ \frac{1}{\alpha^x}, & x = 2, 3, \cdots \end{cases}$$

   (a) Find the value of α.
   (b) Determine the cumulative distribution function (CDF) of X.

5. We assume that the height of Hong Kong people obeys Gaussian distribution with $\mu = 1.75$m and $\sigma = 0.05$m. In door design of public buses, it is required that the door's height should be no less than the height of 95% Hong Kong people. Determine the height of the door.

6. Suppose the arrival time of Peter at MTR platform is uniformly distributed between 8:31:00 and 8:34:00. During this time interval, two MTR trains arrive exactly at 8:32:00 and 8:34:00.
   (a) Derive the probability distribution function (PDF) of MTR train waiting time for Peter in terms of seconds.
   (b) Determine the mean MTR train waiting time for Peter.

7. A discrete random variable X obeys the following probability function:

$$P(X=i)=\begin{cases}\frac{1}{5}, & i=-2,-1,0,1,2\\ 0, & \text{otherwise}\end{cases}$$

(a) Compute $\mathbb{E}\{X\}$.
(b) Define $Z=|2X|$. Compute $\mathbb{E}\{Z\}$.

8. At a party there are 100 identical boxes in a big basket and only one box contains a diamond ring. Each participant is given a chance to randomly pick a box and open it: if the diamond is inside, the game is over; if not, to close the box and return it to the basket. What is the probability that the diamond ring will not be chosen in the first 20 draws?

9. A discrete random variable X has zero variance, i.e., $\text{var}(X)=0$. Prove that X must be a constant.

10. The cumulative distribution function (CDF) of a continuous random variable X is expressed as:

$$F(x)=\begin{cases}0, & x<-1\\ \frac{x+1}{2}, & -1\le x\le 1\\ 1, & x>1\end{cases}$$

(a) Compute $\mathbb{E}\{X\}$.
(b) Compute $\mathbb{E}\{X^2\}$.