MA2001 **Assignment for Multiple Integrals**

1. Change the order of the integration in $\int_0^1 \left[\int_0^{2y} f(x,y)dx \right] dy + \int_1^3 \left[\int_0^{3-y} f(x,y)dx \right] dy$.

2. Evaluate the following double integrals:
   (a) $\iint_S xy\, dxdy$, where S is the region bounded by the lines $x = 0,\ x = 1,\ y = x^2$ and $y = 4$.
   (b) $\iint_S x^2\, dxdy$, where S is the region bounded by $y = 2x$ and $x^2 + y = 8$.

3. Evaluate $\iint_S xy\, dxdy$, where S is the region enclosed by the 4 parabolas $y^2 = x$, $y^2 = 2x$, $x^2 = y$, $x^2 = 2y$ using the change of variable $u = \dfrac{x^2}{y}, v = \dfrac{y^2}{x}$.

4. Evaluate $E_z = \dfrac{\sigma_0 z}{4\pi\varepsilon_0} \iint_S \dfrac{1}{(x^2 + y^2 + z^2)^{3/2}}\, dxdy$, where S is the disc $x^2 + y^2 \le a^2$, which represents the z-component of the electric field at the point $(0,0,z)$ due to a uniformly charged circular disc lying in $x^2 + y^2 \le a^2,\ z = 0$.

5. Let R be the region bounded by $x + y = 1,\ x = 0,\ y = 0$. Show that $\iint_R \cos\left(\dfrac{x-y}{x+y}\right) dx\, dy = \dfrac{\sin 1}{2}$, using the substitution $x - y = u$, $x + y = v$.

6. Evaluate $\iint_S e^{xy}\ dxdy$, where S is the region enclosed by $xy = 1$, $xy = 2$, $y = x$, $y = 4x$ using the change of variable $xy = u,\ \dfrac{y}{x} = v$.

7. Use the change of variables $x + y = u, x - y = v$ to evaluate $\iint_{|x|+|y|\le 1} e^{(x-y)} dxdy$.

8. An iterated integral like $\int_0^1 \left[\int_0^{\frac{1-x}{2}} \left(\int_0^{1-x-2y} f(x,y,z)dz \right) dy \right] dx$ is called an **iterated integral** with order $dzdydx$. Change the order of the iterated integral $\int_0^1 \left[\int_0^{\frac{1-x}{2}} \left(\int_0^{1-x-2y} f(x,y,z)dz \right) dy \right] dx$ to an equivalent iterated integral with order $dxdzdy$.

9. Let V be the region in the first octant, where $x, y, z \ge 0$ bounded by $x^2 + y^2 = 1$, $x = 0,\ y = 0,\ z = 0,\ z = 1$. Using cylindrical polar coordinate, compute $\iiint_V xydxdydz$.

10. (Optional)

In a sample model of the charge distribution around the positively charged (Q) nucleus of the hydrogen atom the charge density at the point (x, y, z) in the electron cloud is $f(x,y,z) = \frac{-Q}{\pi\, a^3} e^{-\frac{2\sqrt{x^2+y^2+z^2}}{a}}$, where a is the Bohr radius. Determine the total charge in theelectron cloud.

11. (Optional)

Let V be the region enclosed by both the surfaces: $\begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z \geq 1 \end{cases}$ and $x^2 + y^2 = z^2$.

Using spherical coordinate, compute $\iiint\limits_V z dxdydz$.

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