EE3331 Probability Models in Information Engineering

Semester B 2021-2022

Assignment 1

Due Date: 9 February 2022

1. Inside a non-transparent box, there are two red balls and seven black balls.
   (a) Consider taking out three balls one by one with replacement from the box. That is, we select one from nine each time. Determine the sample space.
   (b) Consider taking out three balls at one time from the box. What is the probability that there are two red balls and one black ball?

2. Suppose there are four traffic lights on the way from your home to campus. It is assumed that when you encounter a red signal, you must stop 50s, 60s, 30s and 30s for the first, second, third, and fourth light, respectively. In addition, the probabilities of encountering a red signal and green signal (no waiting time) are both 0.5.
   (a) Determine the sample space of the waiting time due to the four traffic lights.
   (b) Compute the probabilities of all possible outcomes in (a).
   (c) Consider an event when you encounter two red signals at the first and second lights, and you spend a total waiting time of 140s due to the four traffic lights. Calculate the probability of this event.

3. Given $P(\overline{A}) = 0.2, P(B) = 0.4, P(A \cap \overline{B}) = 0.5$, find $P(B \mid A \cup \overline{B})$.

4. Prove that two events A and B are independent if $P(A \mid B) = P(A \mid \overline{B})$.

5. Consider 20 people eating in a restaurant. What is the probability that at least 2 of them have the same birthday (same day of the year while the year may differ)? Assume a year consists of 365 days and ignore leap years. If we desire the probability that 2 people or more have the same birthday is at least 0.95, determine the minimum number of people needed.

6. A game-show host offers Peter the choices of 3 doors. Behind one of these doors is an iPhone 13 Pro Max, and behind the other two are balloons. The host, who knows what is behind each of the doors, announces that after Peter selects a door without opening it, he will open one of the other two doors corresponding to the balloons.

   Suppose Peter selected a door, and the host then opened one of the other doors and showed the balloons. Now the host offers Peter the chance to switch his choice to the remaining door. If Peter does not switch his choice, what is the probability of getting the prize? How about the probability of the getting the iPhone 13 Pro Max if Peter switches his choice? Explain your answers.

7. Three dices are rolled. What is the probability that the sum of three dices is 6?

8. Prove

$$\sum_{k=0}^{N} C(N,k) = 2^{N}$$

9. Box A contains 2 red balls and 3 green balls, while box B contains 2 red balls and 1 green ball. You firstly pick one of the boxes at random, then draw a ball multiple times, with replacement of balls after each draw. If red ball is continuously drawn k times, what is the probability that you picked box B?

10. According to the previous analysis of the examination results in a class, students who study hard pass the course with probability of 95%, while students who do not study hard fail the course with probability of 90%. Moreover, 80% of students study hard.
    (a) If a student who passes the course is chosen from the class, what is the probability that he/she does not study hard?
    (b) If a student who fails the course is chosen from the class, what is the probability that he/she is hard-working?