MA1200 TAKE HOME PROBLEM SET 2

The following is the second take-home assignment of MA1200, which counts 3 points of total 100 of your final score of the course.

Please submit it via canvas in a pdf file (you can handwrite the answers and take photos by your phone, then make it into a pdf file, see for example, https://www.wikihow.com/Convert-JPG-to-PDF) for how to combine jpg files to a pdf; you can also do it by note-taking apps on an iPad or an Surface)

Q1. $f(x) = 3x^2 + 18x + 8$, find the range, and sketch the grapgh.

Q2. Factorize $3x^3 + 4x^2 - 17x - 6$ into product of three linear factors (hint, try $x = 2$)

Q3. Express $\dfrac{x^2 + 11x + 20}{(x-1)(x+3)^2}$ into partial fractions.

Q4. Rewrite $3\cos(x) - 4\sin(x)$ as $r\cos(x + \alpha)$, where $r > 0$ and $\alpha \in (0, \pi/2)$. (hint, $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$)

Q5. Solve $3\cos(x) - 4\sin(x) = 5/2$

Q6. Solve $\sin(3\theta) = \cos(2\theta)$

The assignment is due on 23:59 of Oct 23, Friday.

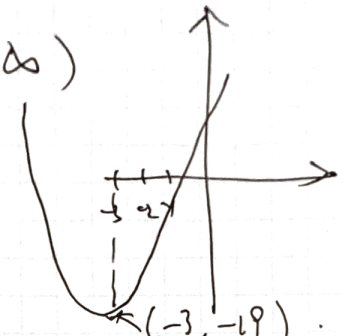
You will lose 1 point for each day of late submission. All submissions after the midnight of Oct 26 will be marked as 0.

Date: October 14, 2020.

MA200. HW2.

Q1. $f(x) = 3x^2+18x+8 = 3(x+3)^2-19$.

range $[-19, \infty)$



axis of sym

$x = -3$.

Q2. $f(x) = 3x^3+4x^2-17x-6$

$f(2)=0 \Rightarrow f(x) = (x-2)(3x^2+10x+3)$

$= (x-2)(x+3)(3x+1)$.

Q3. $\dfrac{x^2+11x+20}{(x-1)(x+3)^2} = \dfrac{A}{x-1} + \dfrac{B}{x+3} + \dfrac{C}{(x+3)^2}$

$x^2+11x+20 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$

$x=1 \Rightarrow 1+11+20 = 16A \Rightarrow A=2$

$x=-3 \Rightarrow 9-33+20 = -4C \Rightarrow C=1$

$x=0 \Rightarrow 20 = 9A - 3B - C = 18 - (-3B)$

$B=-1$

$\Rightarrow \dfrac{2}{x-1} - \dfrac{1}{x+3} + \dfrac{1}{(x+3)^2}$

Q4. $3\cos x - 4\sin x = 5\left(\frac{3}{5}\cos x - \frac{4}{5}\sin x\right)$

let $A = \cos^{-1}\left(\frac{3}{5}\right)$. then $\cos A = 3/5$, $\sin A = 4/5$

$\Rightarrow 3\cos x - 4\sin x = 5(\cos A\cos x - \sin A\sin x)$

$= 5\cos(x+A) = 5\cos\left(x + \cos^{-1}\left(\frac{3}{5}\right)\right)$

or $= 5\cos\left(x + \sin^{-1}\left(\frac{4}{5}\right)\right)$

Q5. $3\cos x - 4\sin x = 5/2$

$\Rightarrow 5\cos\left(x + \cos^{-1}\left(\frac{3}{5}\right)\right) = 5/2$

$\Rightarrow \cos\left(x + \cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$x + \cos^{-1}\left(\frac{3}{5}\right) = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi - \cos^{-1}\left(\frac{3}{5}\right) \pm \frac{\pi}{3}$, $n \in \mathbb{Z}$.

$\sin^{-1}\left(\frac{4}{5}\right)$

Q6 $\sin(3\theta) = \cos(2\theta)$

method 1

$\cos\left(\frac{\pi}{2} - 3\theta\right) = \cos(2\theta)$

$\frac{\pi}{2} - 3\theta = 2n\pi \pm 2\theta$.

$+$: $5\theta = -2n\pi + \pi/2$

$\theta = \frac{2}{5}n\pi + \frac{\pi}{10}$. $n \in \mathbb{Z}$

$-$: $\theta = 2n\pi + \frac{\pi}{2}$

method 2.

$\sin(3\theta) = \sin\left(\frac{\pi}{2} - 2\theta\right)$

or $3\theta = 2n\pi + \frac{\pi}{2} - 2\theta$

or

$3\theta = 2n\pi + \pi - \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \theta = \frac{2}{5}n\pi + \frac{\pi}{10}$

or $\theta = 2n\pi + \pi/2$ $n \in \mathbb{Z}$.