Take Home Assignment MA2001 #3

Due: 2th Dec (Wednesday), 5pm

Submit online via Canvas

Q5-Q7 are optional related to Chapter 5 that won't be counted into assessment.

Make a copy of the assignment before your submission. The marking of this assignment will not be returned to you. Solutions of the assignment will be released in Canvas.

For each of the following questions, write down your solution with details of steps. Marks will not given if only final answers are provided.

1. Evaluate $\iint_S e^{xy}dxdy$, where S is the region enclosed by $xy = 1, xy = 2, y = x, y = 4x$ using the change of variable $xy = u$, $\frac{y}{x} = v$.

2. Compute the following multiple integrals using suitable method.

   (a) $\displaystyle\iint_R x^3dxdy$, where R is the region bounded by x-axis, y-axis, $x = 2$, $y = 1 + x$, and $y = 3 - x$.

   (b) $\displaystyle\iiint_V \frac{1}{\sqrt{4 - x^2 - y^2}}dxdydz$, where V is the region which is bounded above by a sphere $x^2 + y^2 + z^2 = 4$ and is bounded below by a plane $z = 1$.

3. Find $grad f = \nabla f$ for $f(x, y, z) = x^2 + y^2 + z^2$. Hence calculate

   (a) the directional derivative of f at $(1, 1, 1)$ in the direction of the unit vector $\frac{1}{3}(2, 2, 1)$;

   (b) the maximum rate of change of the function at $(1, 1, 1)$ and its direction.

4. Let $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - 2z)\vec{j} + (4x + cy + 2z)\vec{k}$ be a vector field on $\mathbb{R}^3$, where a, b, and c are real constants.

   (a) Find the values of a, b, and c such that $\vec{F}$ is irrotational.

   (b) With the values of a, b, and c obtained in (a), determine a potential function φ on $\mathbb{R}^3$ for which $\nabla\varphi = \vec{F}$.

5. (optional) Compute the following line integrals using suitable method.

   (a) $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 3x\vec{i} + 4xy\vec{j}$ and C is the boundary curve of the region in the first quadrant bounded by x-axis, $y = x$, and a circle $x^2 + y^2 = 1$.

   (b) $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = [2xz^2 \cos(1 + x^2 + 3y^3)]\vec{i} + [9y^2z^2 \cos(1 + x^2 + 3y^3)]\vec{j} + [2z \sin(1 + x^2 + 3y^3)]\vec{k}$ and C is the path moving from a point $(0, 1, 2)$ and $(3, 4, 7)$ along a straight line.

6. (optional) Compute the following surface integrals using suitable method.

   (a) $\iint_S (x^2+y^2)dS$, where S is the part of the surface $z = 9 - y$ lying inside the cylinder $x^2 + y^2 = 1$.

   (b) $\iint_S \vec{F}\cdot\vec{n}dS$, where $\vec{F} = y\vec{i} + x\vec{j}$ and S is the part of the cone $z = \sqrt{x^2+y^2}$ lying inside the cylinder $x^2 + y^2 = 9$. (Here, $\vec{n}$ is upward pointing normal).

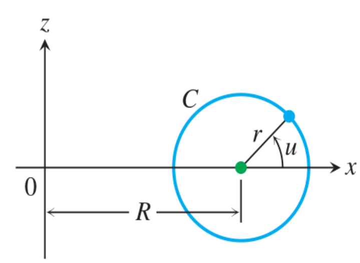
   (c) $\iint_S \vec{F}\cdot\vec{n}dS$, where $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S is the boundary surface of the region bounded by the cone $z = \sqrt{\frac{x^2+y^2}{3}}$ and the upper-half sphere $x^2 + y^2 + z^2 = 8$. (Here, $\vec{n}$ is outward-pointing normal).

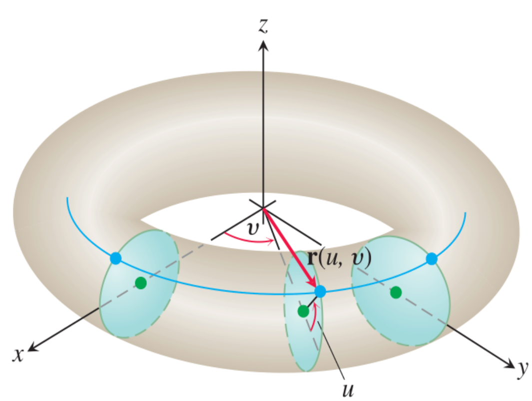
7. (*Discovery Question) Read Lecture Note Chapter 5 Section 3 on Surface Given Parametrically by Three Equations or Chapter 16.5 of the book [Thomas's Calculus.(13th ed.) Wesley, 2014]. Do the following exercise.

   A *torus of revolution* (doughnut) is obtained by rotating a circle C in the xz-plane about the z-axis in the space (See Figure). If C has radius $r > 0$ and center $(R, 0, 0)$ $(R > r)$, show that a parameterization of the torus is

$$\vec{r}(u,v) = ((R + r\cos u)\cos v)\vec{i} + ((R + r\cos u)\sin v)\vec{j} + (r\sin u)\vec{k},$$

   where $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$ are the angles in the figure, and show that the surface area of the torus is $A = 4\pi^2 Rr$.





8. (*Discovery Question) Determine if the vector field shown in the figure is conservative or solenoidal? (For a reference, see Section 16.3 of *CALCULUS-Early Transcendentals* 6th edition by James Stewart)