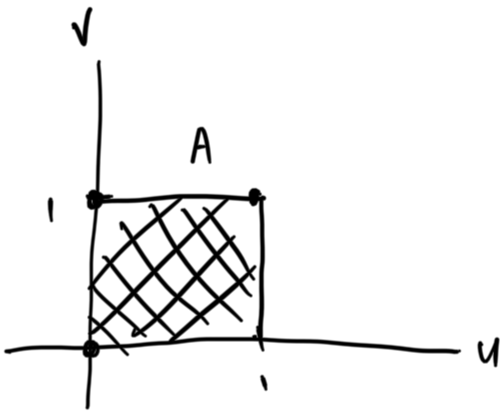
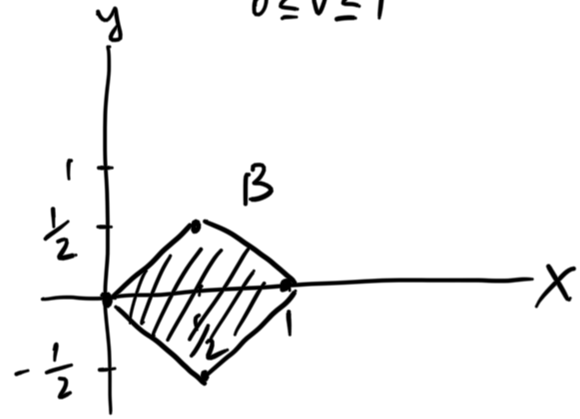
(change of variable)

Ex. 1. $\boxed{x = \frac{u+v}{2} \quad y = \frac{u-v}{2}}$ The rectangle A is given by $0 \le u \le 1$, $0 \le v \le 1$





(u,v)	(x,y)
$(0,0)$	$(0,0)$
$(0,1)$	$(\frac{1}{2},-\frac{1}{2})$
$(1,1)$	$(1,0)$
$(1,0)$	$(\frac{1}{2},\frac{1}{2})$

From (u,v) to (x,y)

① $x = \frac{u+v}{2}$ ② $y = \frac{u-v}{2}$

From (x,y) to (u,v)

$$2x = u+v$$
$$2y = u-v$$
$$2x+2y = 2u$$
$$\boxed{u = x+y}$$
$$v = 2x-u = 2x-(x+y)$$
$$\boxed{v = x-y}$$

abs value.

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v), h(u,v))\,|J(u,v)|\,du\,dv$$

From (x,y) to

where $J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \underbrace{\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}}_{\text{Just determinant}}$ (u,v)

Ex 2. polar coordinates $\underline{x = r\cos\theta}$ $y = r\sin\theta$ what is $J(r,\theta)$?

$$J(r,\theta) = \begin{vmatrix} \frac{\partial x}{\partial r} & \boxed{\frac{\partial x}{\partial \theta}} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta - (-r\sin^2\theta)$$

$$= r(\cos^2\theta + \sin^2\theta)$$

$$= r$$

$\therefore$ $dxdy$ exchanged for $r\,dr\,d\theta$ for integration

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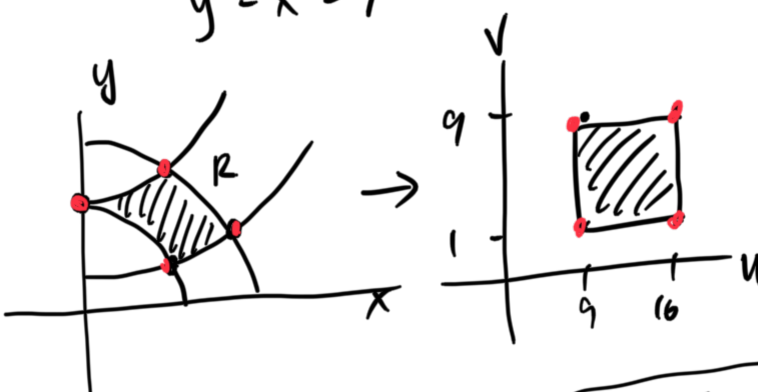
Ex. $\iint_R 5(\underline{x^2+y^2})\,dx\,dy$ where the Region in Quadrant I and bounded by $x^2+y^2=9$, $x^2+y^2=16$, $y^2-x^2=1$ and $y^2-x^2=9$

Ans:



$$\begin{aligned} (+)\quad u &= x^2+y^2 \\ v &= -x^2+y^2 \\ \hline u+v &= 2y^2 \end{aligned}$$

$\frac{u+v}{2} = y^2$ $\sqrt{\frac{u+v}{2}}$

$\boxed{u = x^2+y^2}$ $v = y^2-x^2$

$x = \sqrt{\frac{u-v}{2}}$ $y = \sqrt{\frac{u+v}{2}}$

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\left| \frac{\partial}{\partial u} \quad \frac{\partial}{\partial v} \right|$$

$$= \begin{vmatrix} \frac{1}{2\sqrt{2(u-v)}} & -\frac{1}{2\sqrt{2(u-v)}} \\ \frac{1}{2\sqrt{2(u+v)}} & \frac{1}{2\sqrt{2(u+v)}} \end{vmatrix} = \frac{1}{4\sqrt{u^2-v^2}}$$

$$= \iint_R 5(x^2+y^2)\,dx\,dy$$

we let $u = x^2+y^2$
$v = y^2-x^2$

$$= \iint_A 5u\left(\frac{1}{4\sqrt{u^2-v^2}}\right) du\,dv$$

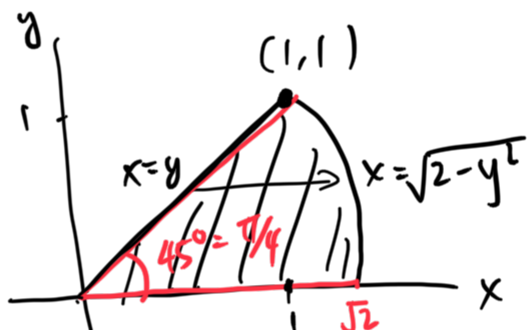
$$= \frac{5}{4}\int_1^9\int_9^{16} \frac{u}{\sqrt{u^2-v^2}}\,du\,dv \;\star$$

⋮ ← Finish at home

$$= \frac{5}{4}\left(-\frac{256}{2}\cos^{-1}\left(\frac{v}{16}\right) + \frac{v}{2}\sqrt{256-v^2} - \left(-\frac{81}{2}\cos^{-1}\left(\frac{v}{9}\right) + \frac{v}{2}\sqrt{81-v^2}\right)\right)\Bigg|_1^9$$

Ex. $\displaystyle\int_0^1\int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$

$x = r\cos\theta$
$y = r\sin\theta$



$x = \sqrt{2-y^2}$

← refer ex 2.

$$\int^{\pi/4}\int^{\sqrt{2}} \cdots)\,r\,dr\,d\theta$$

$$\int_0 \int_0 (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

$$\int_0^{\pi/4} \int_0^{\sqrt{2}} r(\cos\theta + \sin\theta)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/4} \int_0^{\sqrt{2}} r^2 (\cos\theta + \sin\theta)\, dr\, d\theta$$

$$= \int_0^{\pi/4} (\cos\theta + \sin\theta)\, d\theta \int_0^{\sqrt{2}} r^2\, dr$$

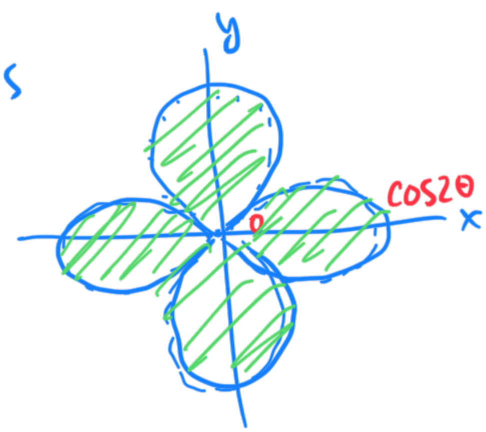
⋮ do at home !

$$= \frac{2\sqrt{2}}{3}$$

4:05

Ex. Use a double Integral to find the area inside one loop of the four-leaved rose $r = \cos 2\theta$

Ans



$$\text{Area} = \iint 1\, dA$$

$$= \iint r\, dr\, d\theta$$

$$\text{Area} = \int_{-}^{-} \int_0^{\cos 2\theta} r\, dr\, d\theta$$