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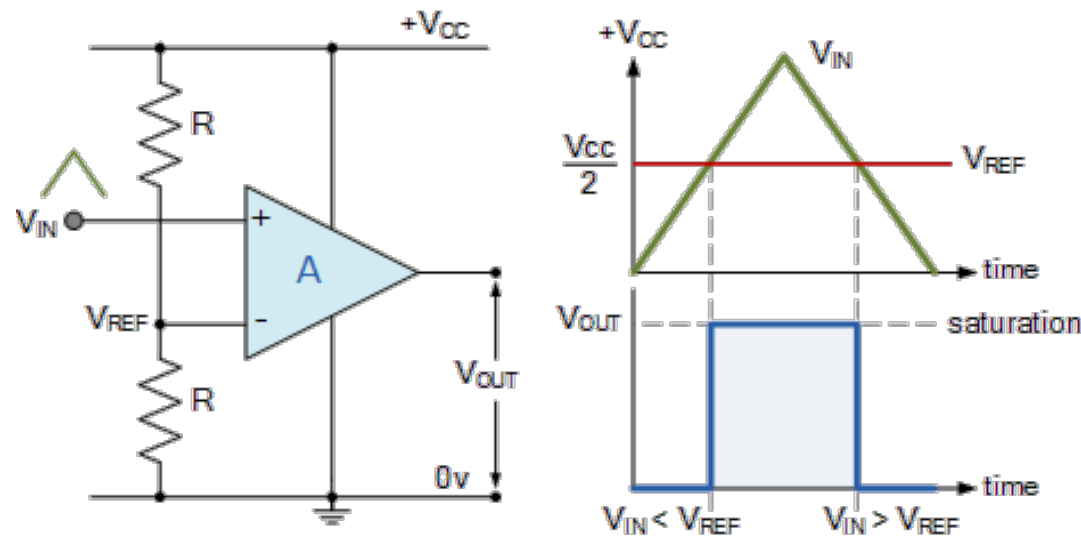
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- > Review of Op Amps with Examples
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# Open-Loop Op Amp Circuit : Non-inverting Comparator

A comparator is an example of an op amp operated in open loop  
Open loop means that no feedback is applied



$$V_{out} = A(V_{+} - V_{-})$$

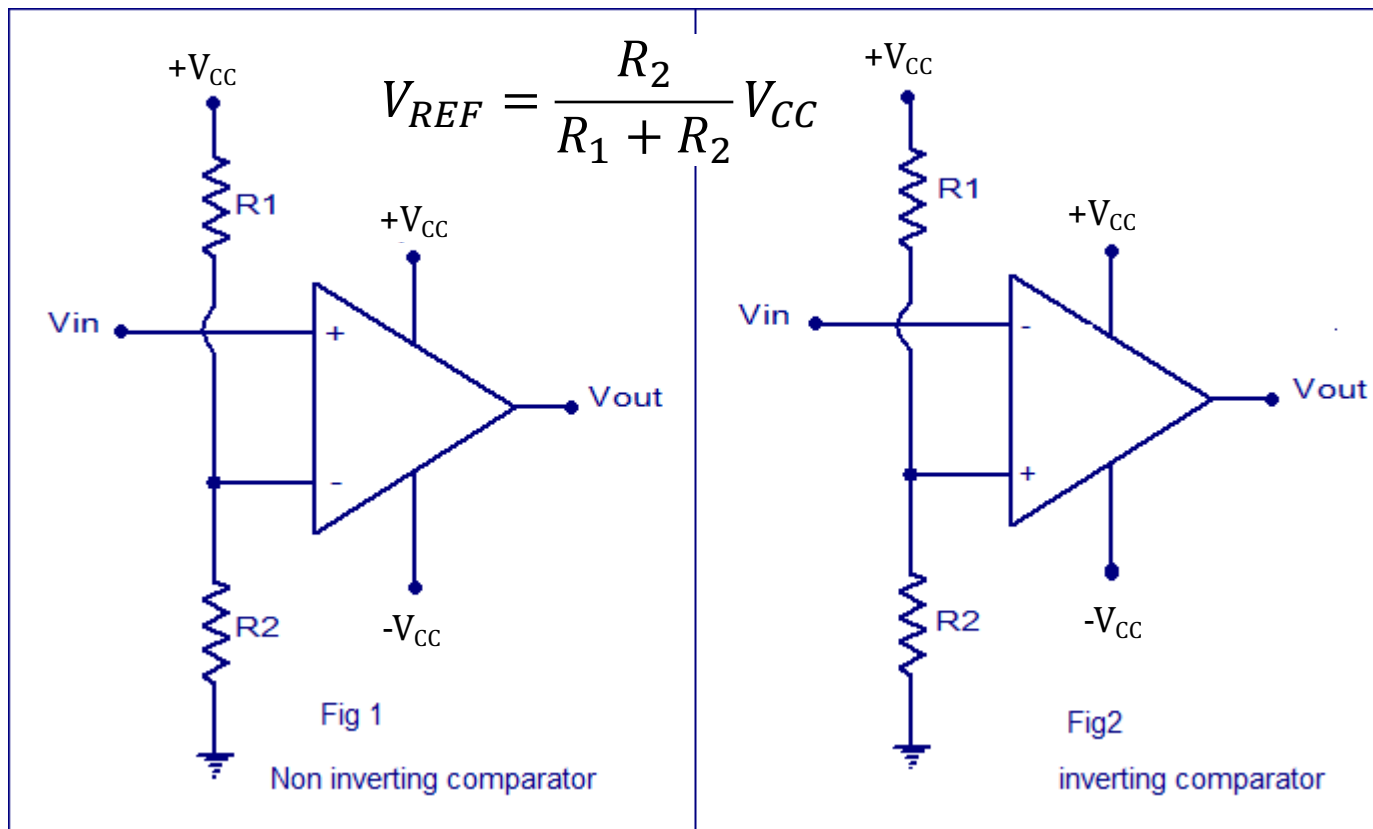
For an ideal op amp ( $A \rightarrow \infty$ ),  
given there is no feedback  
between inputs and output:

When  $V_{IN} > V_{REF}$ :  $V_{out} = +V_{CC}$ ;

When  $V_{IN} < V_{REF}$ :  $V_{out} = 0$

In the above example,  $V_{REF} = V_{CC}/2$  (no input current into op amp) Note  
that  $V_{REF}$  can be set to any desired value by the choice of resistors

# Open-Loop Op Amp Circuit : Inverting Comparator



When  $V_{IN} > V_{REF}$ :  $V_{out} = +V_{CC}$ ;

When  $V_{IN} < V_{REF}$ :  $V_{out} = -V_{CC}$

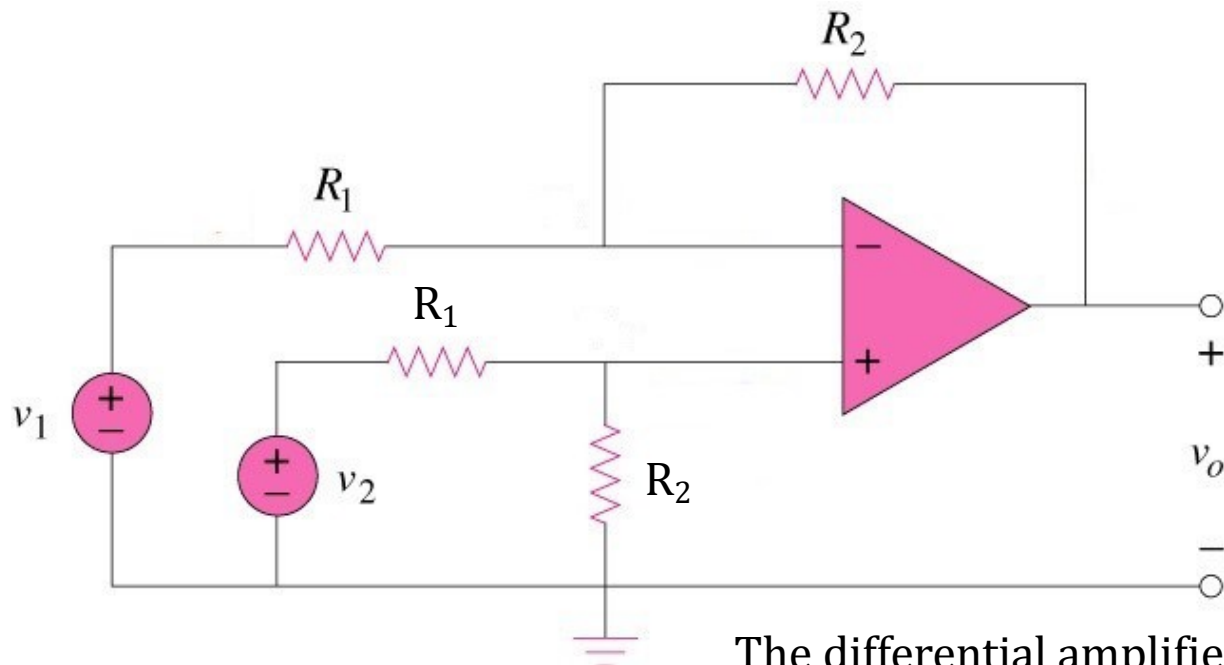
When  $V_{IN} > V_{REF}$ :  $V_{out} = -V_{CC}$ ;

When  $V_{IN} < V_{REF}$ :  $V_{out} = +V_{CC}$

# Closed-Loop Application: Ideal differential op amp

We look at several op amp circuits that use negative feedback, starting with the differential amplifier from the previous lecture.

Negative feedback: Taking part or all of the output and returning it back to the input out of phase. The aim is to stabilize the output.

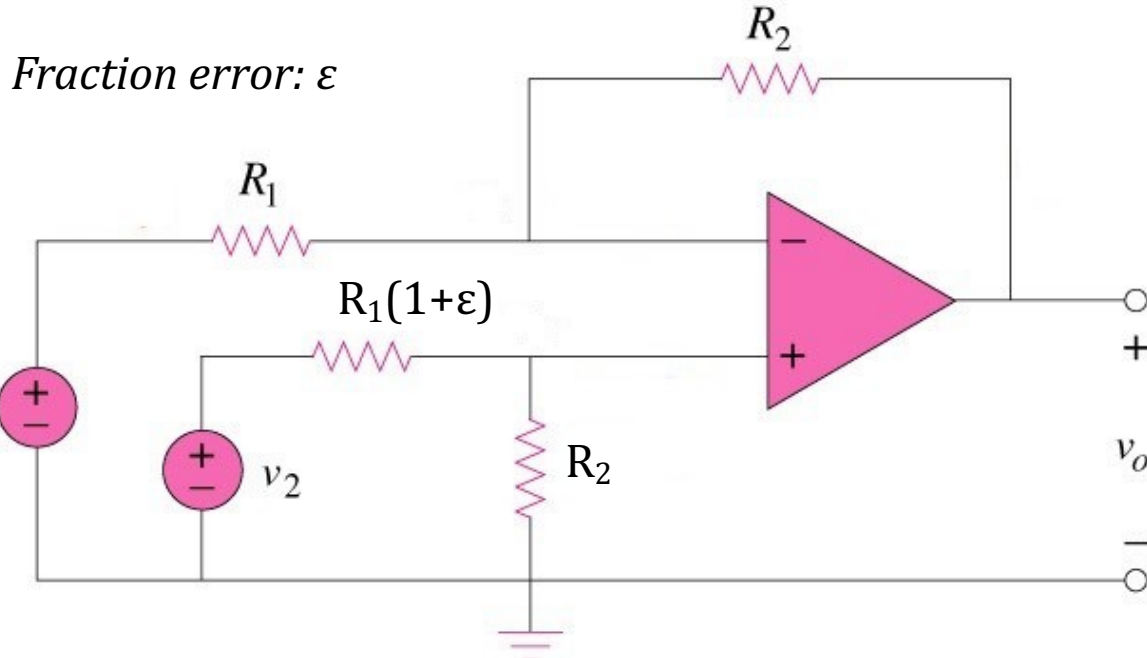


$$V_o = \frac{R_2}{R_1} (V_2 - V_1)$$

The differential amplifier is supposed to amplify only differences between the inputs.

# Differential op amp in practice

In practice, it is not possible for two resistors to be exactly equal. So let us consider what happens when there is some error between just one of the resistors.

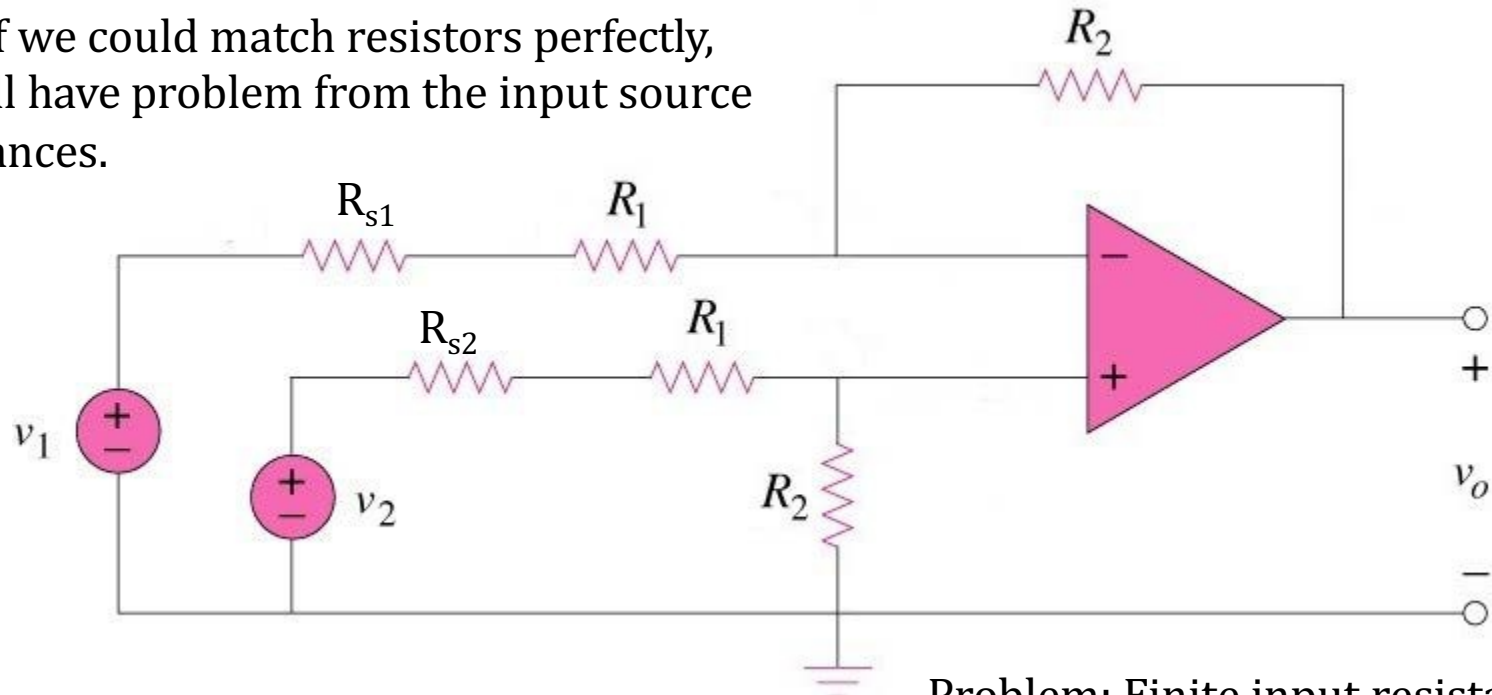


$$V_o = \frac{R_2}{R_1} (V_2 - V_1) - \frac{\varepsilon R_2}{R_1(1 + \varepsilon) + R_2} V_2$$

We can see that  $V_o$  no longer depends on only the difference between the inputs; the value of the inputs also matters now.

# Effect of source resistance

Even if we could match resistors perfectly, we will have problem from the input source resistances.



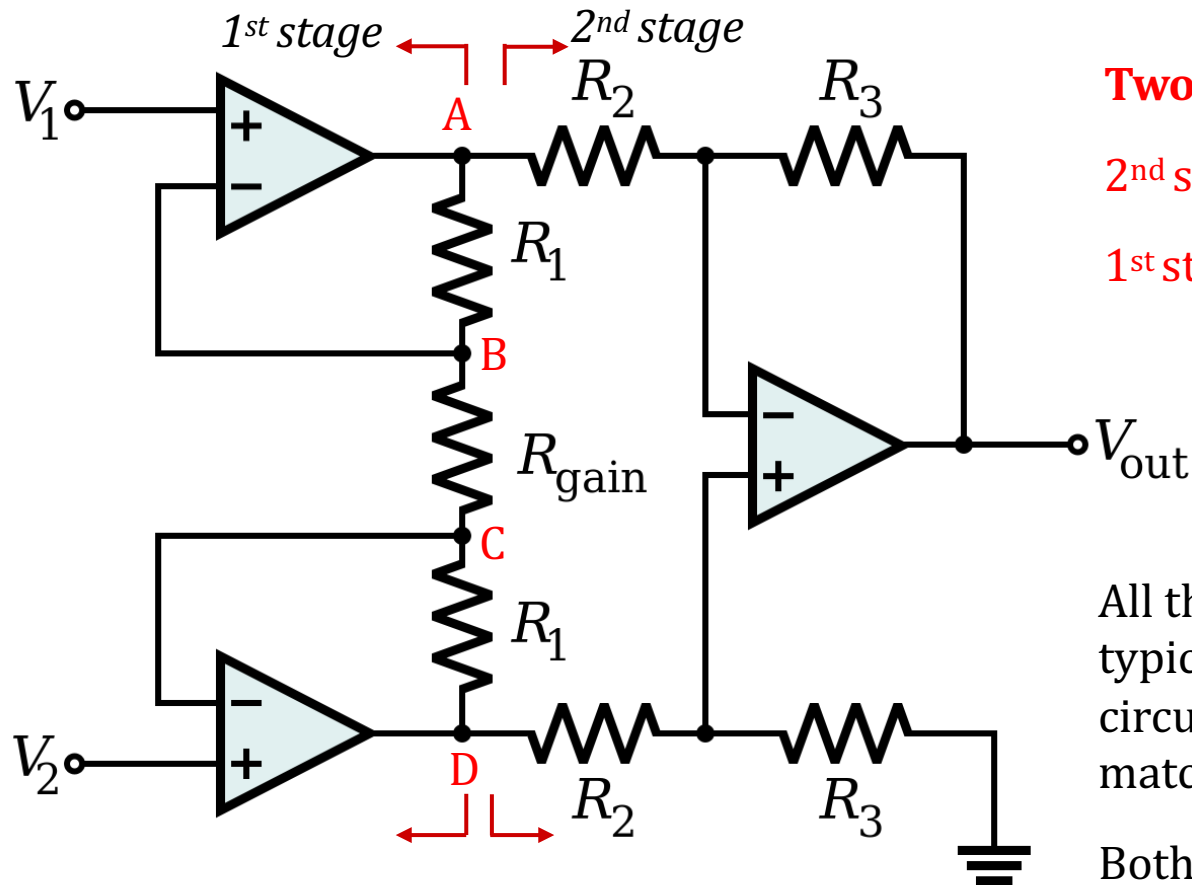
Problem: Finite input resistance

$$V_o = \left( \frac{R_2}{R_1 + R_{s1}} \right) \left[ \left( \frac{R_2 + R_1 + R_{s1}}{R_1 + R_2 + R_{s2}} \right) V_2 - V_1 \right]$$

Clearly  $V_o$  does not depend just on the difference between the inputs.

If  $V_2 = V_1$ ,  $V_o$  would not be zero.

# Instrumentation amplifier



**Two stage cascaded amplifier**

2<sup>nd</sup> stage: Differential amplifier

1<sup>st</sup> stage: Non-inverting amplifier

All the resistors (except  $R_{gain}$ ) are typically built into the integrated circuit and thus can be precisely matched.

Both inputs see the large input resistance of the op amp

⇒ Source resistance has little effect

$$V_{out} = \left(1 + \frac{2R_1}{R_{gain}}\right) \left(\frac{R_3}{R_2}\right) (V_2 - V_1)$$

# Instrumentation amplifier: Analysis

Current from A  $\rightarrow$  B = Current from B  $\rightarrow$  C = Current from C  $\rightarrow$  D

Node A:  $V_{o1}$ ; Node B:  $V_1$ ; Node C:  $V_2$ ; Node D:  $V_{o2}$

*1<sup>st</sup> stage*

$$\frac{V_{o1} - V_1}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) V_1 - \left(\frac{R_1}{R_{gain}}\right) V_2$$
$$\frac{V_2 - V_{o2}}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o2} = \left(1 + \frac{R_1}{R_{gain}}\right) V_2 - \left(\frac{R_1}{R_{gain}}\right) V_1$$

None of the above intermediate output voltages are affected by the source resistance of the inputs

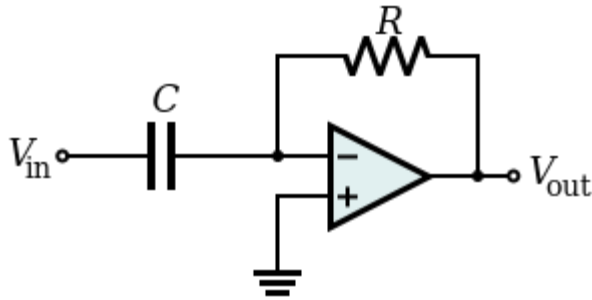
*2<sup>nd</sup> stage*

$$V_{out} = \frac{R_3}{R_2} (V_{o2} - V_{o1}) \quad V_{o2} - V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) (V_2 - V_1) - \left(\frac{R_1}{R_{gain}}\right) (V_1 - V_2)$$

$$\Rightarrow V_{out} = \frac{R_3}{R_2} \left(1 + \frac{2R_1}{R_{gain}}\right) (V_2 - V_1)$$



# Differentiating amplifier



KCL at inverting input:

$$I_S = I_F = -V_{out}/R \quad \dots (1)$$

Voltage across C =  $V_{in}$ :

$$I_S = C \left( \frac{dV_{in}}{dt} \right) \quad \dots (2)$$

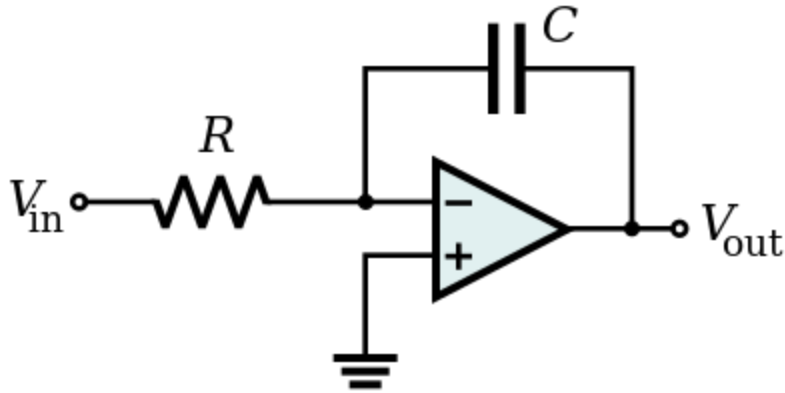
Sub (1) into (2):

$$V_{out} = -RC \left( \frac{dV_{in}}{dt} \right)$$

Output is the time-differential of the input

Ref: [http://www.allaboutcircuits.com/vol\\_3/chpt\\_8/11.html](http://www.allaboutcircuits.com/vol_3/chpt_8/11.html)

# Integrating amplifier



KCL at inverting input:

$$I_S = I_F = V_{in}/R \quad \dots (1)$$

Voltage across C =  $-V_{out}$ :

$$I_F = -C \left( \frac{dV_{out}}{dt} \right) \quad \dots (2)$$

Sub (1) into (2):

$$\frac{V_{in}}{R} = -C \left( \frac{dV_{out}}{dt} \right)$$

$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

Output is the time-integral of the input

Ref: [http://www.allaboutcircuits.com/vol\\_3/chpt\\_8/11.html](http://www.allaboutcircuits.com/vol_3/chpt_8/11.html)

# Review: Passive Filters

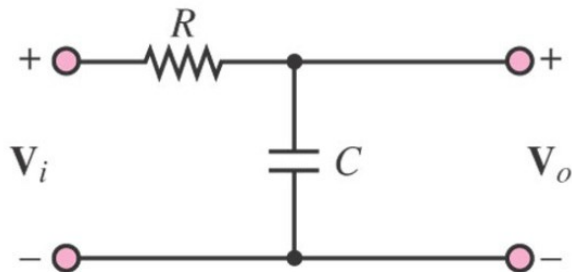
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- Made up of passive components only, e.g. R, L, C
- No amplifying elements, e.g. transistors, op amps
- Load dependent
- No power supply needed
- Scale better to large signals

# Passive 1<sup>st</sup> order low pass filter

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + j(\omega/\omega_c)}$$

$$\omega_c = \frac{1}{RC}$$

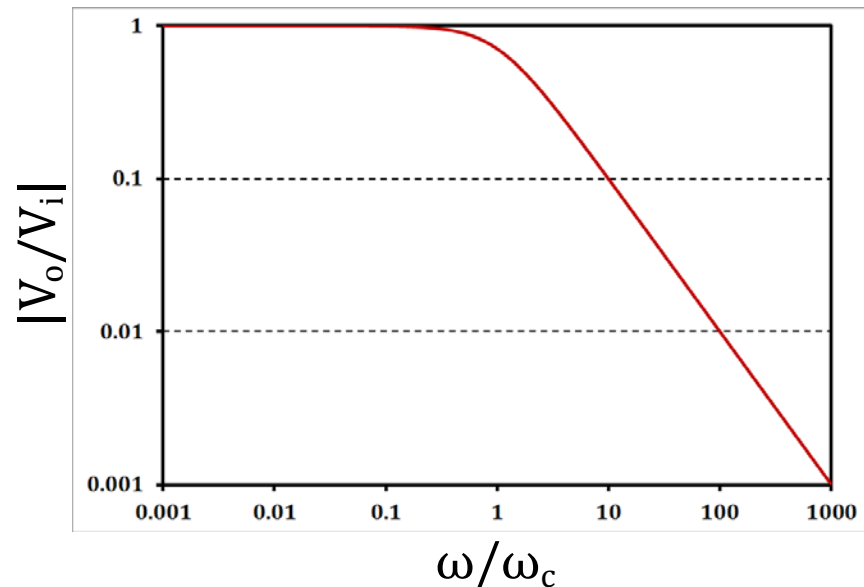


Recall that  $\omega_c$  is a constant that is determined by the circuit components, referred to as the corner frequency.

For an input at a frequency of  $\omega_c$ ,  $V_o = V_i/\sqrt{2}$

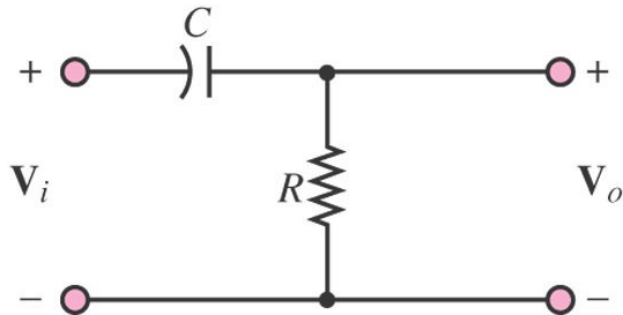
For frequencies  $< \omega_c$ ,  $|V_o| \approx |V_i|$  (i.e. nearly constant with frequency)

For frequencies  $> \omega_c$ ,  $|V_o| \approx |V_i| \omega_c/\omega$  (i.e. inversely proportional to the frequency)



# Passive 1<sup>st</sup> order high pass filter

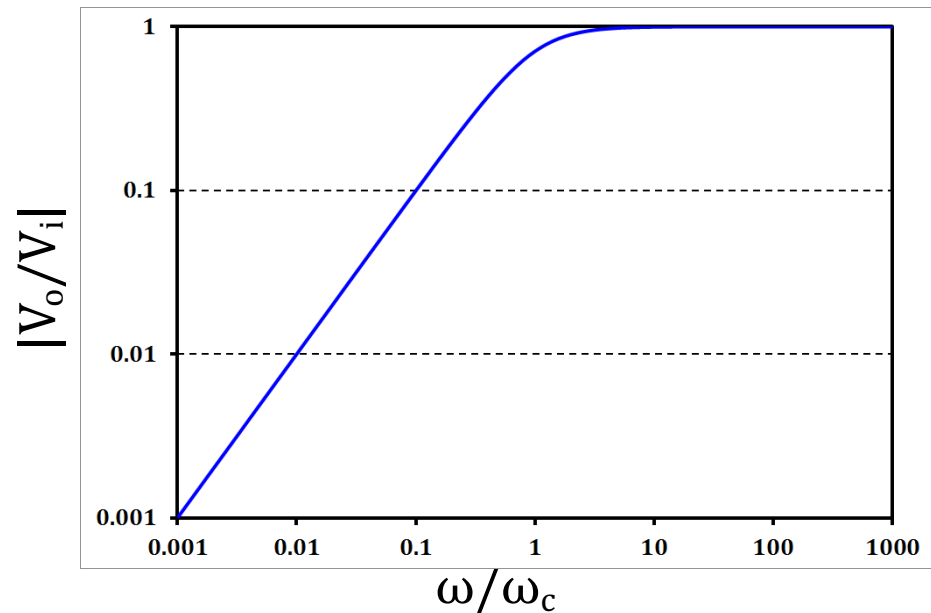
$$\frac{V_o}{V_i}(j\omega) = \frac{j(\frac{\omega}{\omega_c})}{1 + j(\frac{\omega}{\omega_c})}$$
$$\omega_c = \frac{1}{RC}$$



For an input at a frequency of  $\omega_c$ ,  $V_o = V_i/\sqrt{2}$

For frequencies  $> \omega_c$   $|V_o| \approx |V_i|$  (i.e. nearly constant with frequency)

For frequencies  $< \omega_c$   $|V_o| \approx |V_i| \omega/\omega_c$  (i.e. proportional to the frequency)



# Decibels (dB)

$$\text{Gain in decibels (dB)} = 10 \log \frac{P_{out}}{P_{in}}$$

$$\text{Gain in decibels (dB)} = 20 \log \frac{V_{out}}{V_{in}}$$

$$\text{Gain} = 1 \Rightarrow 0\text{dB}$$

$$\text{Gain} = 1/\sqrt{2} \Rightarrow -3\text{dB}$$

$$\text{Gain} = 0.1 \Rightarrow -20\text{dB}$$

$$\text{Gain} = 0.01 \Rightarrow -40\text{dB}$$

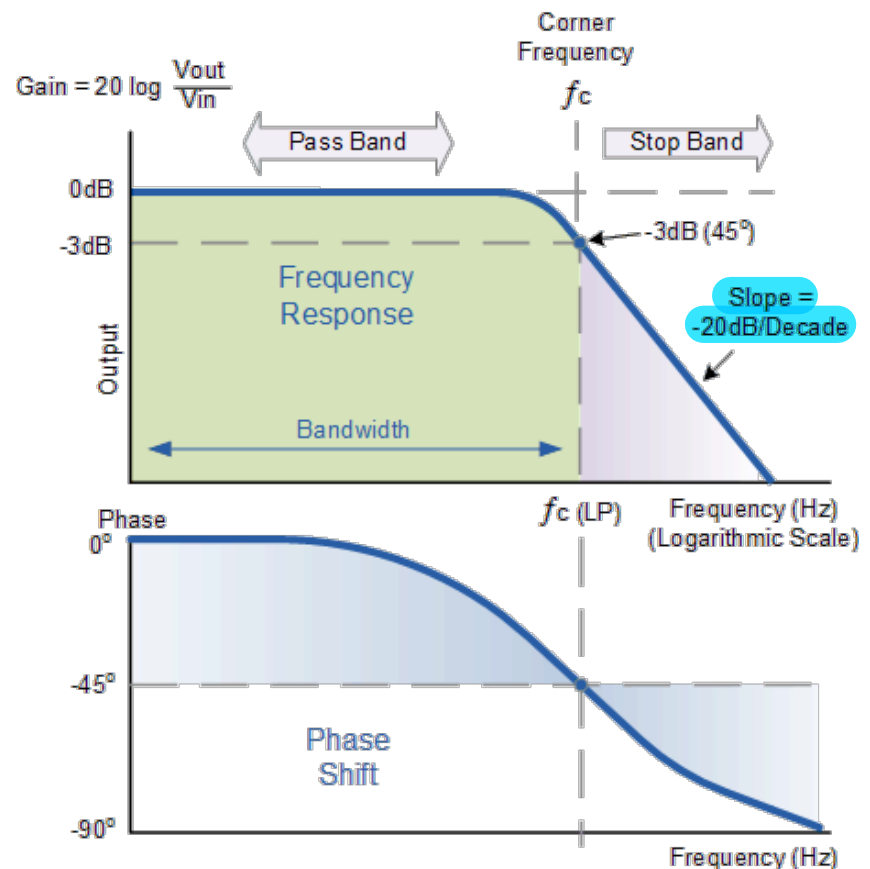
Every factor of 10 corresponds to a change of 20dB

Times 10  $\Rightarrow$  +20dB

Divide by 10  $\Rightarrow$  -20dB

Recall:  $P = V^2/R$

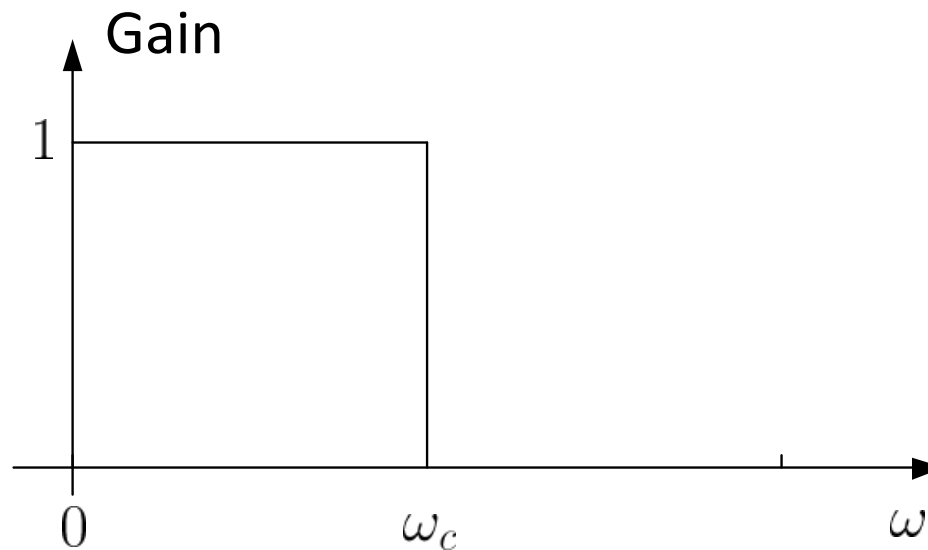
At the corner frequency ( $f_c$ ), gain drops by 3dB relative to the passband



# Ideal Low-Pass Filter Specification

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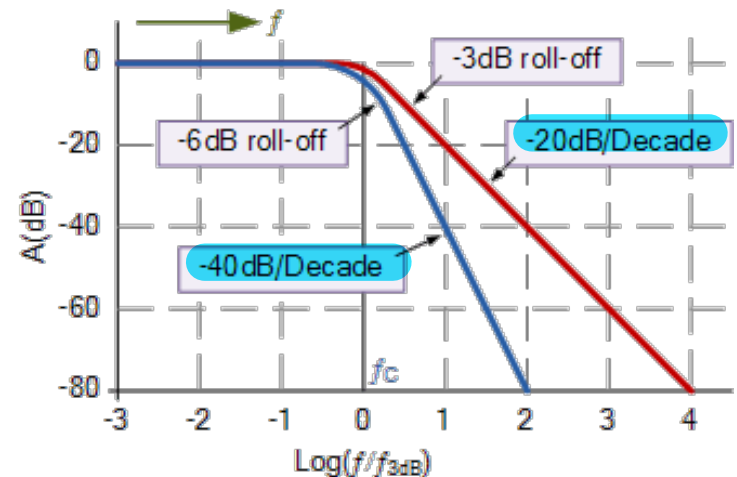
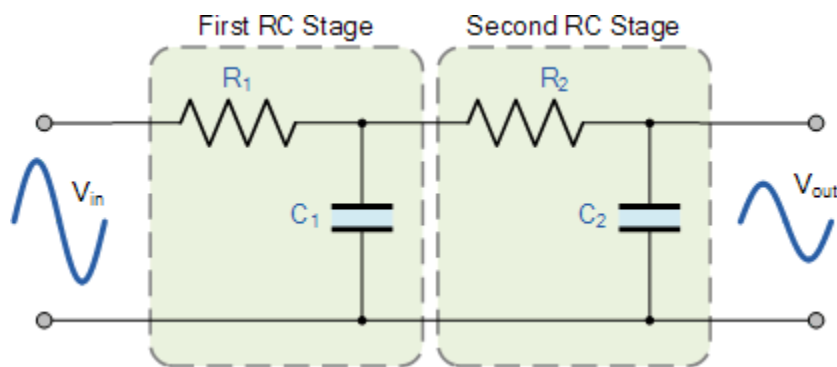
- Unity gain for the whole range of  $\omega \in (0, \omega_c)$
- Complete suppression for  $\omega > \omega_c$
- Step change in frequency response at  $\omega = \omega_c$



# Passive 2<sup>nd</sup> order low pass filter

We can increase the slope of the roll off beyond the cut-off frequency by cascading several stages of low pass filters.

The example here shows a two stage passive low pass filter.



By choice of values, if  $(R_2 + 1/j\omega C_2) \gg 1/j\omega C_1$ :

$$\frac{V_o}{V_i}(j\omega) = \left( \frac{1}{1 + j(\omega/\omega_{c1})} \right) \left( \frac{1}{1 + j(\omega/\omega_{c2})} \right)$$

$$\omega_{c1} = \frac{1}{R_1 C_1} \quad \omega_{c2} = \frac{1}{R_2 C_2}$$

If we chose the values of components so that  $\omega_{c1} = \omega_{c2} = \omega_c$ :

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{[1 + j(\omega/\omega_c)]^2}$$



# Passive 2<sup>nd</sup> order low pass filter

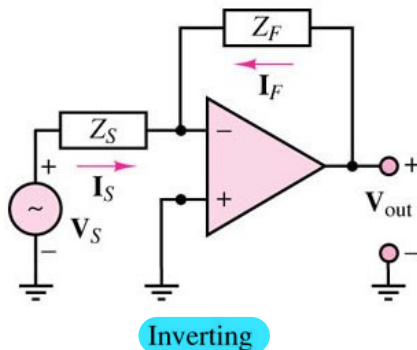
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- > Slope:
  - 2<sup>nd</sup> order filter: -40dB/decade ( $|V_o/V_i|$  drops by 100 times for 10 times increase in frequency)
  - 1<sup>st</sup> order filter: -20dB/decade ( $|V_o/V_i|$  drops by 10 times for 10 times increase in frequency)
- > At cut-off frequency ( $f_c$ ),
  - 2<sup>nd</sup> order filter: gain drops by -6dB
  - 1<sup>st</sup> order filter: gain drops by -3dB
- > Coupling issues: Impedance of stage 2 chosen to be much larger than load impedance of stage 1
  - What happens as we increase the number of stages in the cascade for higher orders?
- > Passband gain: Passband gain is still less than 1

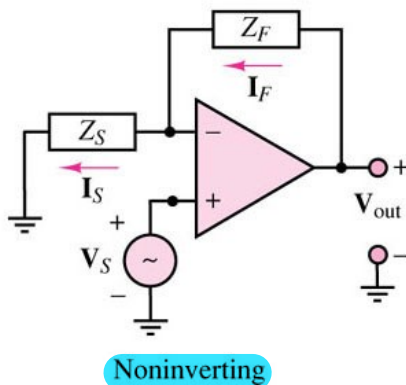
# Active filters

- Range of applications is greatly expanded if reactive components are used
- Addition of reactive components allows us to shape the frequency response
- **Active filters:** Op-amp provides amplification (gain) in addition to filtering effects

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$$I_S = \frac{V_S}{Z_S}; I_F = \frac{V_{out}}{Z_F}; I_S + I_F = 0 \Rightarrow \frac{V_S}{Z_S} + \frac{V_{out}}{Z_F} = 0 \Rightarrow \frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

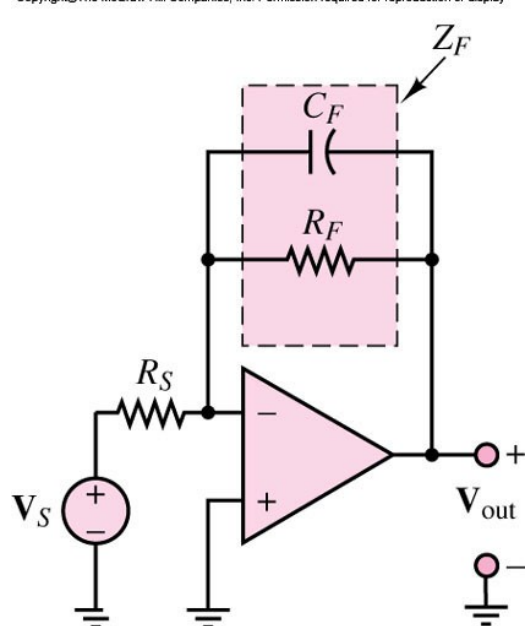


$$I_S = \frac{V_S}{Z_S}; I_F = \frac{V_{out} - V_S}{Z_F}; I_F = I_S \Rightarrow \frac{V_S}{Z_S} = \frac{V_{out} - V_S}{Z_F} \Rightarrow Z_F V_S = Z_S V_{out} - Z_S V_S \Rightarrow V_S (Z_F + Z_S) = Z_S V_{out} \Rightarrow \frac{V_{out}}{V_S}(j\omega) = \frac{Z_F + Z_S}{Z_S} = 1 + \frac{Z_F}{Z_S}$$

$Z_F$  and  $Z_S$  can be arbitrary (i.e. any) complex impedance

# Active 1<sup>st</sup> order low pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

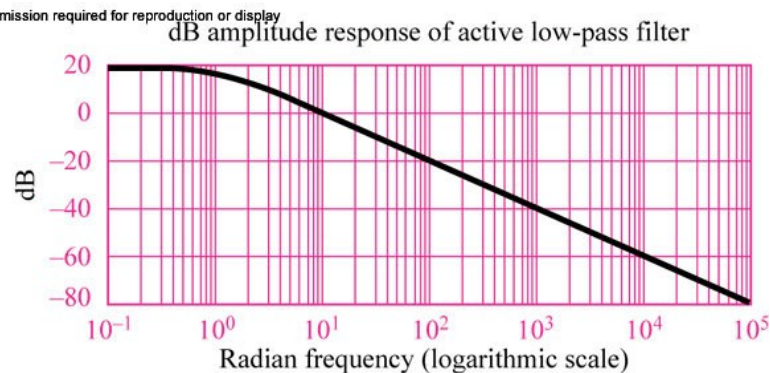
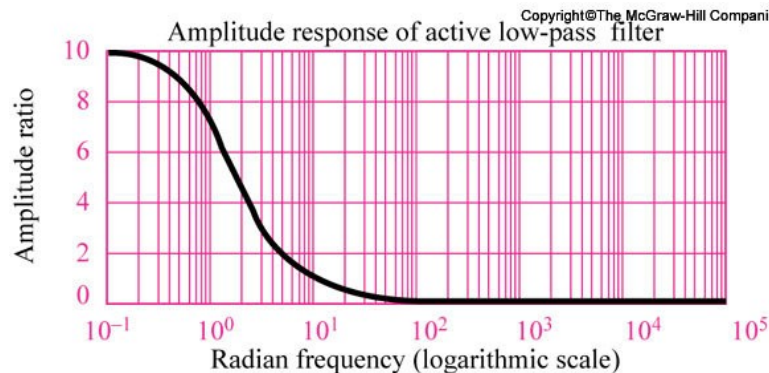
$$Z_S = R_S$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F / R_S}{1 + j\omega C_F R_F}$$

Passband gain is  $R_F/R_S$

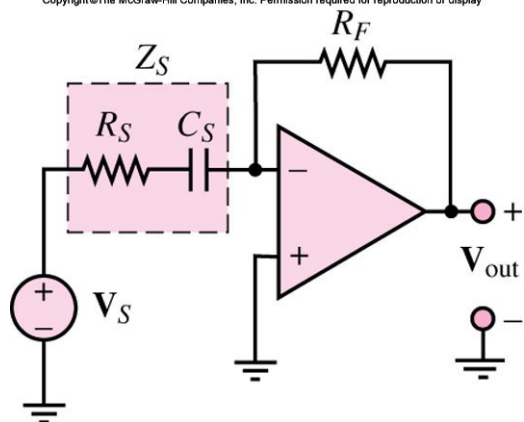
$$\omega_c = \frac{1}{R_F C_F}$$

If we choose  $R_F/R_S = 10$ , frequency response becomes:



# Active 1<sup>st</sup> order high pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F$$

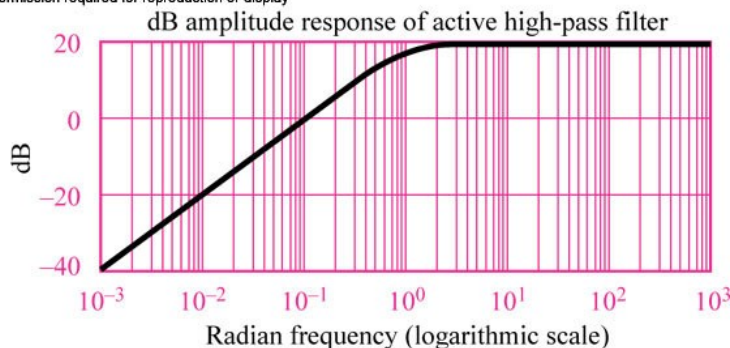
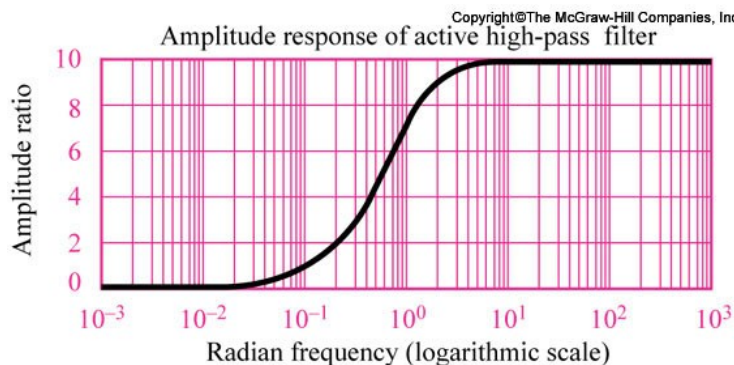
$$Z_S = R_S + 1/j\omega C_S$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F/R_S}{1 + (1/j\omega C_S R_S)}$$

Passband gain is  $R_F/R_S$

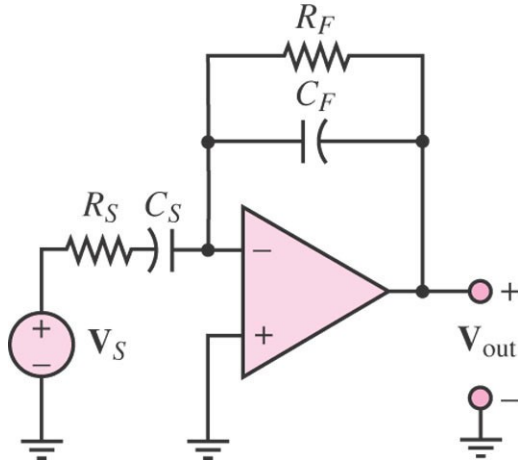
$$\omega_c = \frac{1}{R_S C_S}$$

If we choose  $R_F/R_S = 10$ , frequency response becomes:



# Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S + \frac{1}{j\omega C_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F / R_S}{(1 + 1/j\omega C_S R_S)(1 + j\omega C_F R_F)}$$

$Z_S$ :  $C_S$  blocks low frequency inputs but lets high frequency inputs through

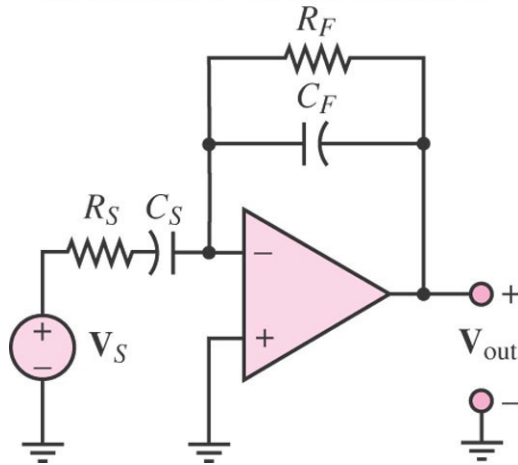
⇒ High pass filter

$Z_F$ :  $C_F$  shorts  $R_F$  at high frequency (reducing the gain), but otherwise looks just like a low pass filter at lower frequencies

⇒ Low pass filter

# Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F/R_S}{(1 + \omega_{HP}/j\omega)(1 + j\omega/\omega_{LP})}$$

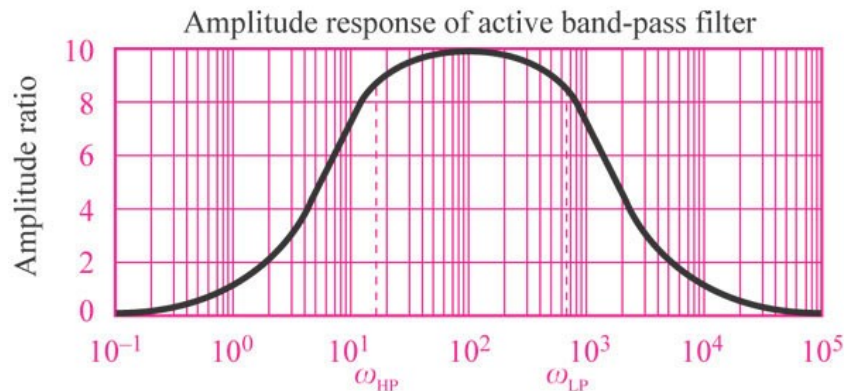
$\omega_{HP} = 1/(C_S R_S)$  – Lower cut-off frequency

$\omega_{LP} = 1/(C_F R_F)$  – Upper cut-off frequency

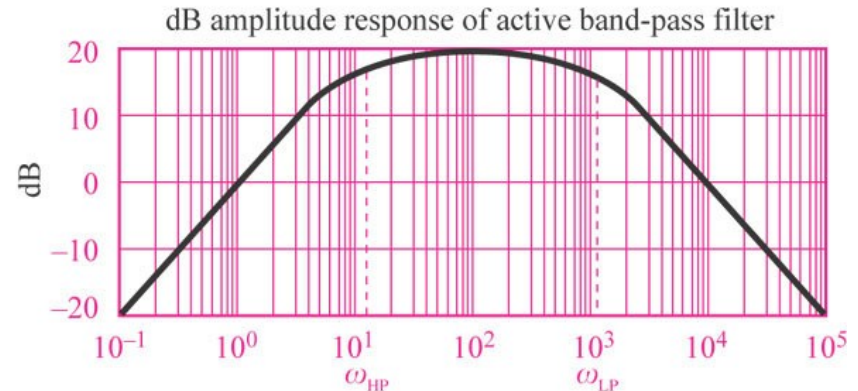
Passband gain is  $R_F/R_S$

For  $\omega_{HP} < \omega_{LP}$  and  $R_F/R_S = 10$ : Frequency response curve is shown below

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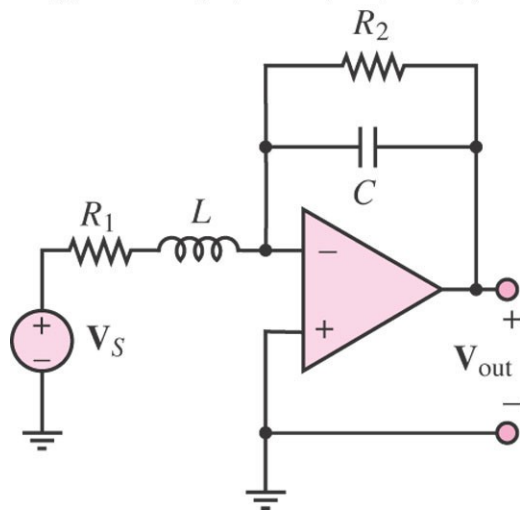
Radian frequency (logarithmic scale)



Radian frequency (logarithmic scale)

# Active 2<sup>nd</sup> order low pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_2 \parallel 1/j\omega C = \frac{R_2}{1 + j\omega CR_2} \quad Z_S = R_1 + j\omega L$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_2/R_1}{(1 + j\omega CR_2)(1 + j\omega L/R_1)}$$

If  $R_1$ ,  $R_2$ ,  $C$  and  $L$  are chosen so that:  $\omega_c = 1/(CR_2) = R_1/L$ .

The frequency response function then simplifies to:

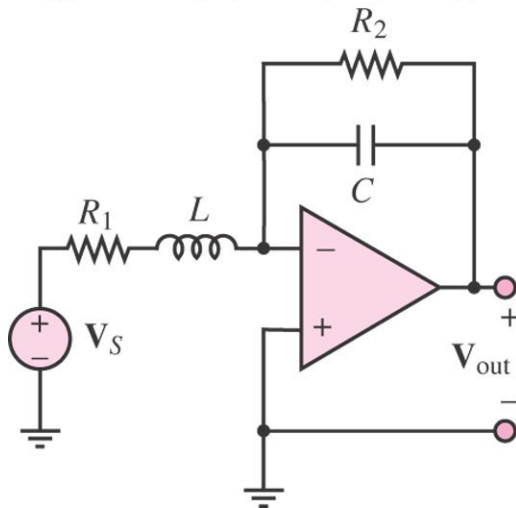
$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_2/R_1}{(1 + j\omega/\omega_c)^2}$$

Passband gain is  $R_F/R_S$



# Active 2<sup>nd</sup> order low pass filter

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$$H_v(j\omega) = -\frac{R_2/R_1}{(1+j\omega/\omega_c)^2}$$

Above  $\omega_c$ ,  $H_v$  is reduced by a factor of **100** for a ten fold increase in  $\omega$  (40dB drop per decade)

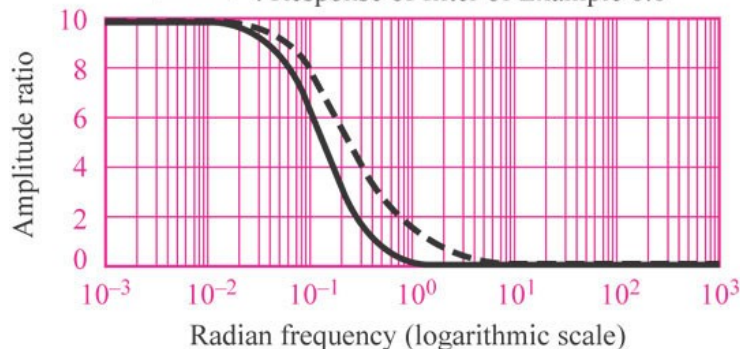
Recall for 1<sup>st</sup> order filter:  $H_v$  is reduced by a factor of **10** for a ten fold increase in  $\omega$  (20dB drop per decade)

If  $R_2/R_1 = 10$ : Frequency response curve is shown below

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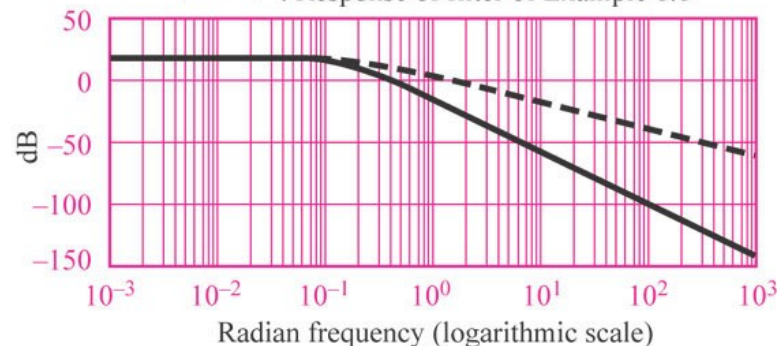
Comparison of active low-pass filters

--- : Response of filter of Figure 8.19  
— : Response of filter of Example 8.6



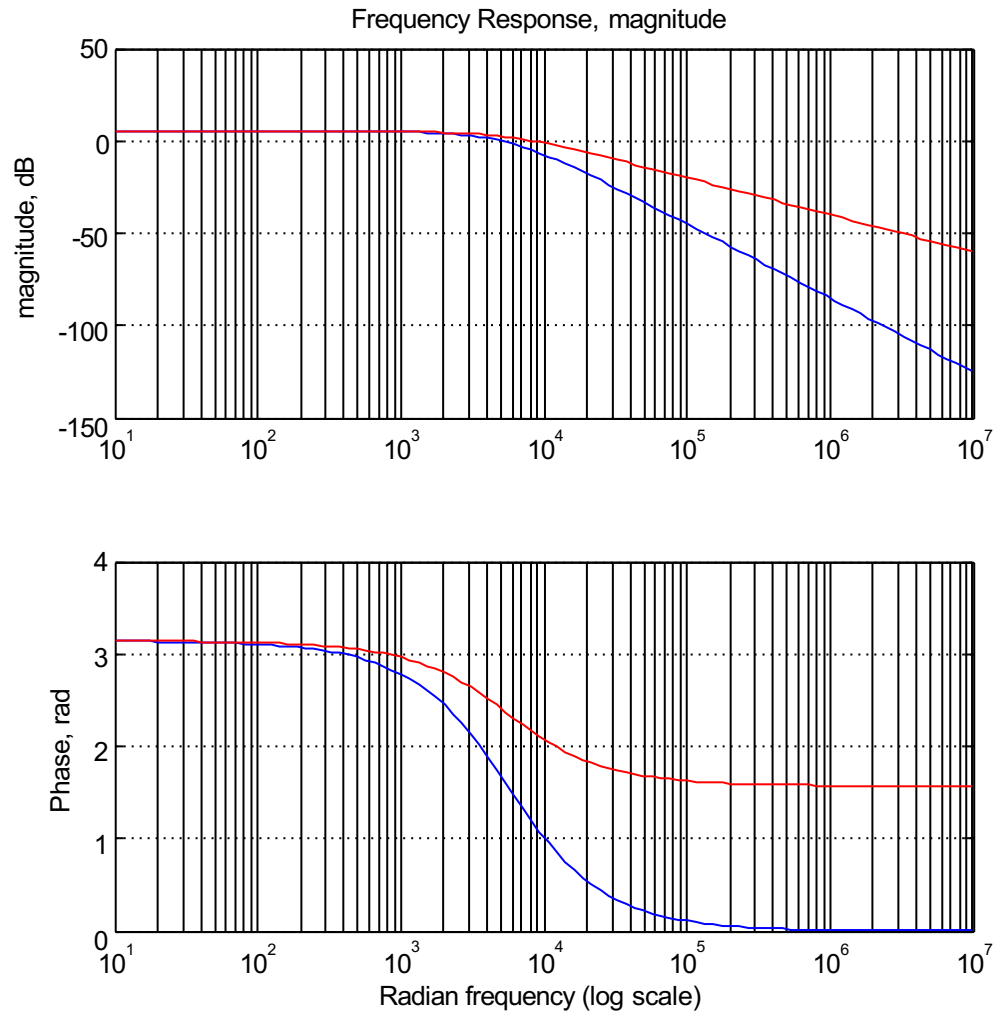
Comparison of active low-pass filters (dB plot)

--- : Response of filter of Figure 8.19  
— : Response of filter of Example 8.6

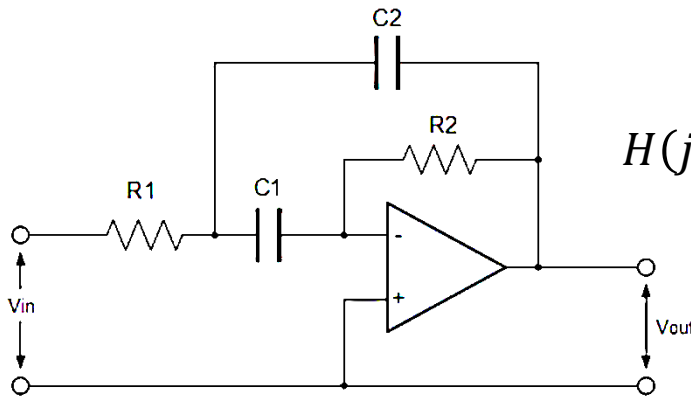




# Bode plot of 2<sup>nd</sup> order low pass filter



# Multiple Feedback Narrow Band Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left( \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \right) (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

1. Assuming  $C_1 = C_2 = C$ , and convert the above equation to the following form

$$H(s) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left( \frac{\omega_o}{Q} \right) (j\omega) + \omega_o^2}$$

2. The center frequency  $\omega_o$  is given by

$$\omega_o = \frac{1}{C \sqrt{R_1 R_2}}$$

$$C_1 = C_2$$

3. The Q and K factor are respectively given by

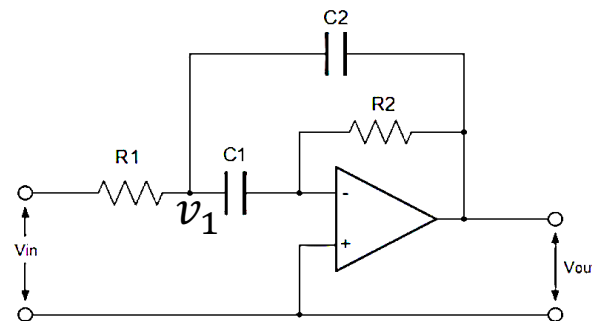
$$Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}}$$

$$K = -2Q^2 = -\frac{R_2}{2R_1}$$

# Derivation of $H(j\omega)$ for Multiple Feedback Narrow Band Filter

$$v_- = 0 \text{ V}$$

$$\frac{v_1}{\frac{1}{j\omega C_1}} = -\frac{v_{out}}{R_2} \Rightarrow v_1 = \frac{-v_{out}}{j\omega C_1 R_2}$$



$$\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{\frac{1}{j\omega C_2}} + \frac{v_1}{\frac{1}{j\omega C_1}} \Rightarrow \frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{C_1} + \frac{v_1}{C_2}$$

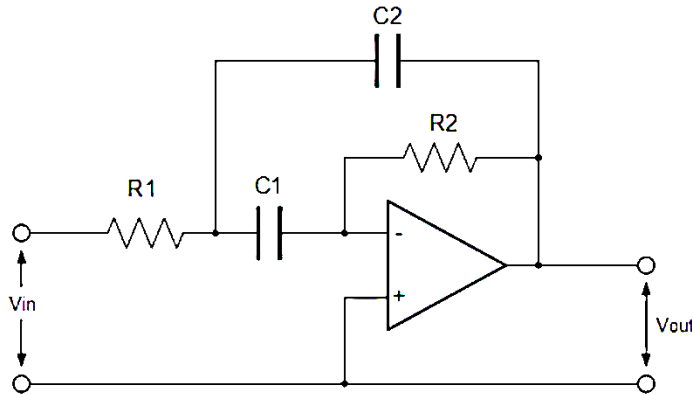
$$\Rightarrow \frac{v_{in}}{j\omega R_1 C_1 C_2} = \left( \frac{-1}{j\omega C_1^2 R_2} - \frac{1}{C_1} - \frac{1}{j\omega C_1 C_2 R_2} - \frac{1}{(j\omega)^2 R_1 R_2 C_1^2 C_2} \right) v_{out}$$

$$\Rightarrow v_{in} = -v_{out} \left( \frac{R_1 C_2}{R_2 C_1} + j\omega R_1 C_2 + \frac{R_1}{R_2} + \frac{1}{j\omega R_2 C_1} \right)$$

$$= -v_{out} \left( \frac{j\omega R_1 (C_1 + C_2) + (j\omega)^2 R_1 R_2 C_1 C_2 + 1}{j\omega R_2 C_1} \right)$$

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left( \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \right) (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

# Multiple Feedback Narrow Band Filter



$$H(j\omega) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) (j\omega) + \omega_o^2}$$

$$\omega_o = \frac{1}{C \sqrt{R_1 R_2}}$$

$$Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}}$$

$$K = -\frac{R_2}{2R_1}$$

$$f_0 = 1.125\text{kHz}$$

Given

1.  $R_1 = 100\Omega$ ;  $R_2 = 20\text{k}\Omega$ ;  $C_1 = C_2 = 100\text{nF}$ , calculate  $\omega_0$  and  $Q$ .  $K = -100$ ,  $Q = 7.07$

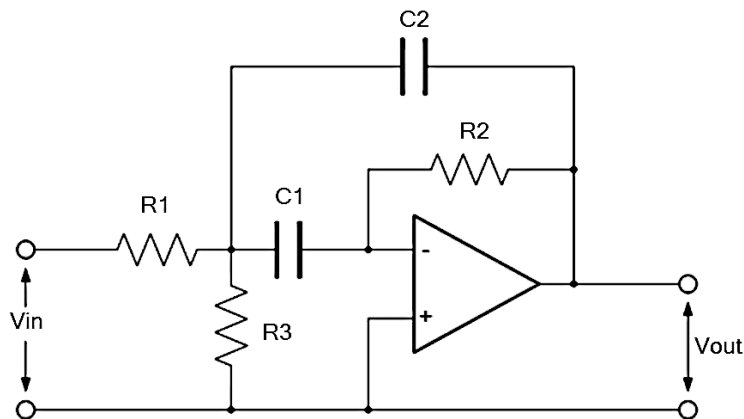
2.  $R_1 = 1\text{k}\Omega$ ;  $R_2 = 2\text{k}\Omega$ ;  $C_1 = C_2 = 100\text{nF}$ , calculate  $\omega_0$  and  $Q$ .  $K = -1$ ,  $Q = 0.707$

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>

# Multiple Feedback Narrow Band Filter

## (A more General Form)



$$H(j\omega) = \frac{v_{out}}{v_{in}}$$

$$= \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right) (j\omega) + \frac{1}{R_2 C_1 C_2} \left(\frac{1}{R_1} + \frac{1}{R_3}\right)}$$

1. Assuming  $C_1 = C_2 = C$ , and convert the above equation to the following form

$$H(j\omega) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) (j\omega) + \omega_o^2}$$

2. The center frequency  $\omega_0$  is given by

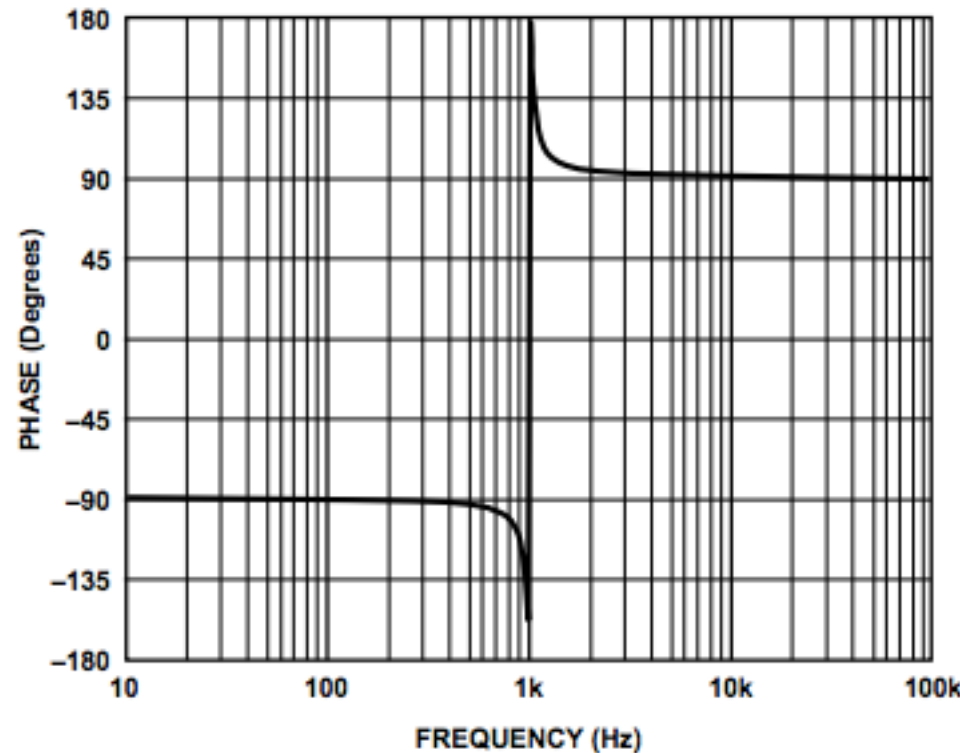
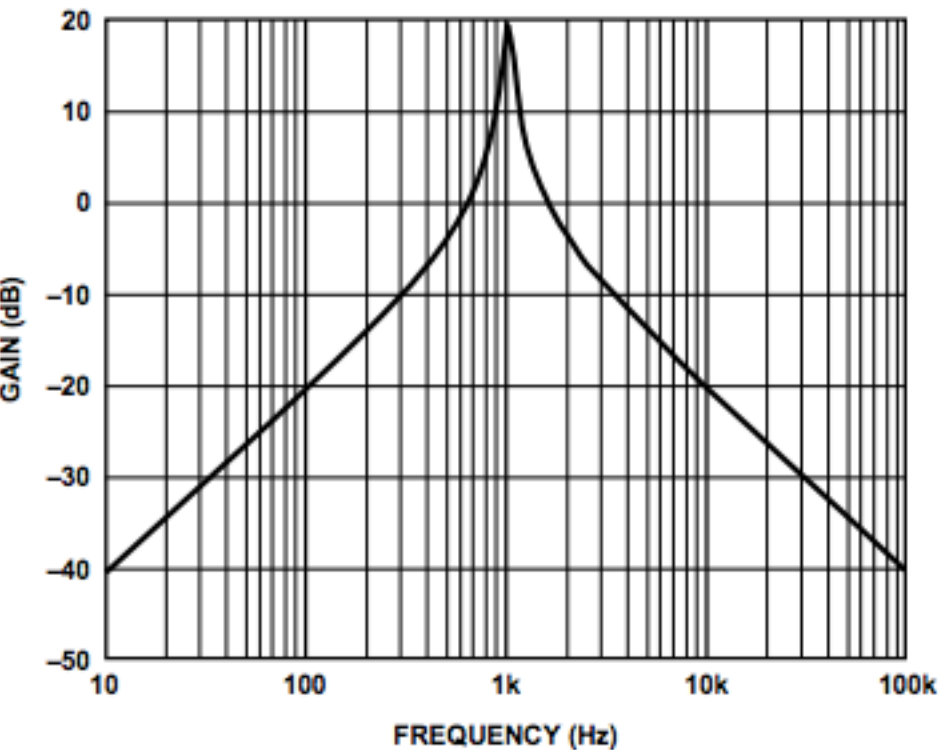
$$\omega_o = \frac{1}{C} \sqrt{\frac{1}{R_2} \left(\frac{1}{R_1} + \frac{1}{R_3}\right)}$$

3. The Q and K factor are respectively given by

$$Q = \frac{1}{2} \sqrt{R_2 \left(\frac{1}{R_1} + \frac{1}{R_3}\right)} \quad K = -\frac{R_2}{2R_1}$$

# Frequency Response of a Multiple Feedback Band-Pass Filter

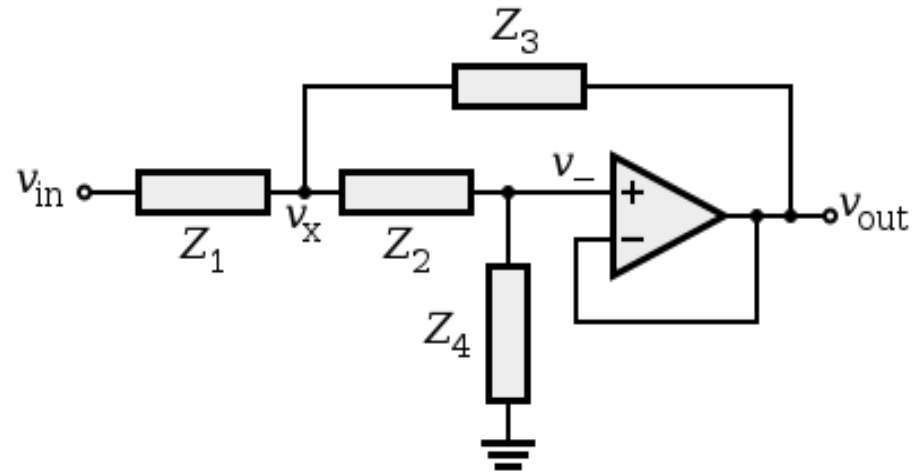
Simple and reliable band-pass implementation



<https://www.analog.com/media/en/training-seminars/tutorials/MT-218.pdf>

# Sallen-Key Filter Topology

Assuming that the op-amp is ideal, transfer function of this filter is given by



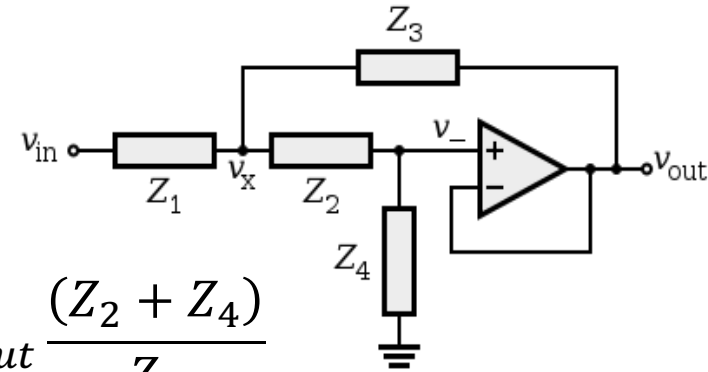
$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

different types of R, L and C for Z's components  
→ different kinds of filter responses.

# Derivation of $H(j\omega)$ in Sallen-Key Filter Topology

$$V_+ = v_- = v_{out}$$

$$v_x \times \frac{Z_4}{Z_2 + Z_4} = v_- = v_{out} \Rightarrow v_x = v_{out} \frac{(Z_2 + Z_4)}{Z_4}$$



$$\frac{v_{in} - v_x}{Z_1} + \frac{v_{out} - v_x}{Z_3} = \frac{v_x}{Z_2 + Z_4}$$

$$\Rightarrow \frac{Z_3 v_{in} + Z_1 v_{out} - (Z_1 + Z_3) v_x}{Z_1 Z_3} = \frac{v_{out}}{Z_4}$$

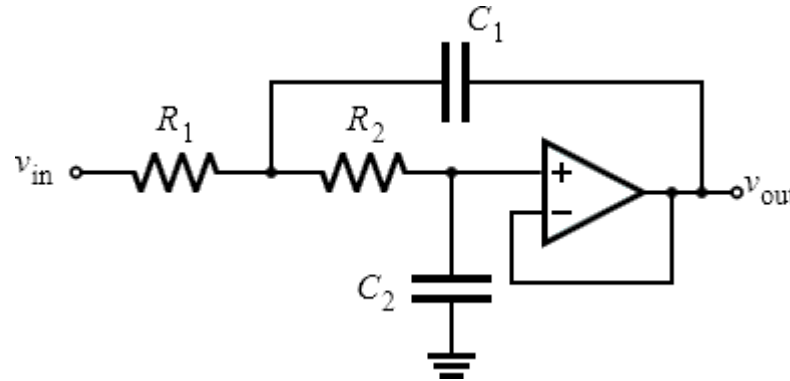
$$\Rightarrow Z_3 v_{in} + Z_1 v_{out} - \frac{(Z_1 + Z_3)(Z_2 + Z_4)}{Z_4} v_{out} = \frac{Z_1 Z_3}{Z_4} v_{out}$$

$$\Rightarrow Z_3 Z_4 v_{in} = (Z_1 Z_3 + Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_3 + Z_3 Z_4 - Z_1 Z_4) v_{out}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$



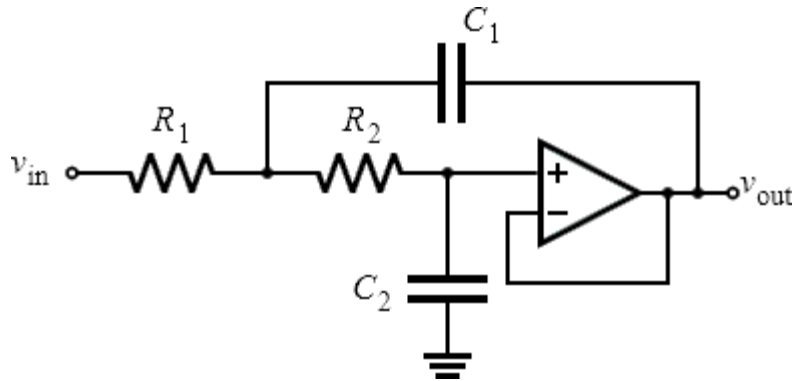
# Operations of Sallen-Key Low-Pass Filter



- At low frequencies,  $C_1$  and  $C_2$  appear as open circuits  $\rightarrow v_{out} \approx v_{in}$
- At high frequencies,  $C_1$  and  $C_2$  appear as short circuits  $\rightarrow v_{out} \approx 0\text{ V}$
- Near cut-off frequencies, impedance of  $C_1$  and  $C_2$  is on the same order as  $R_1$  and  $R_2$   $\rightarrow$  positive feedback through  $C_1$  provides Q enhancement of the signal

<https://www.ti.com/lit/an/sloa024b/sloa024b.pdf>

# Sallen-Key Filter Topology: Low Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

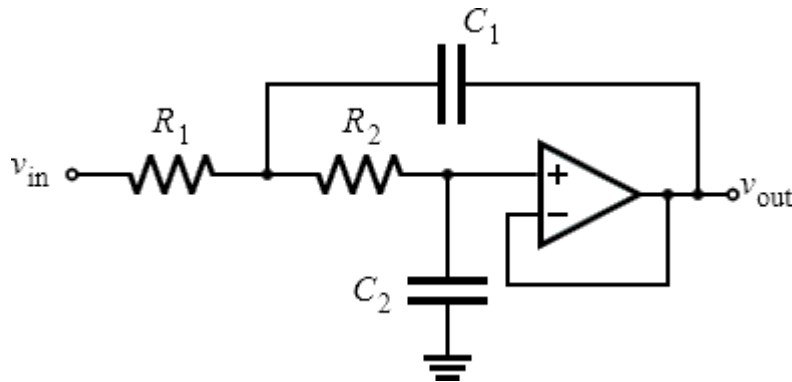
1. It can be formed by choosing  $Z_1=R_1$ ,  $Z_2=R_2$ ,  $Z_3=1/j\omega C_1$ ,  $Z_4=1/j\omega C_2$  ;

2. Convert the above equation to the following form  $H(s) = \frac{j\omega \omega_o^2}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right)(j\omega) + \omega_o^2}$

3. The undamped natural frequency  $\omega_o$  is given by  $\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

4. The Q factor is given by  $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$

# Sallen-Key Filter Topology: Low Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$$

$$f_0 = 999\text{Hz}$$

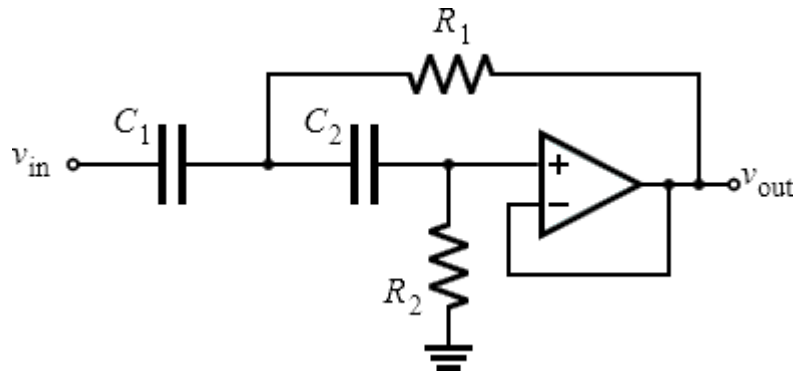
Given:

1.  $R_1 = 30\text{k}\Omega$ ;  $R_2 = 18\text{k}\Omega$ ;  $C_1 = 10\text{nF}$ ;  $C_2 = 4.7\text{nF}$ , calculate  $\omega_o$  and  $Q$ . ( $Q = 0.7$ )
2.  $R_1 = 30\text{k}\Omega$ ;  $R_2 = 18\text{k}\Omega$ ;  $C_1 = 20\text{nF}$ ;  $C_2 = 2.35\text{nF}$ , calculate  $\omega_o$  and  $Q$ . ( $Q = 1.4$ )

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>

# Sallen-Key Filter Topology: High Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

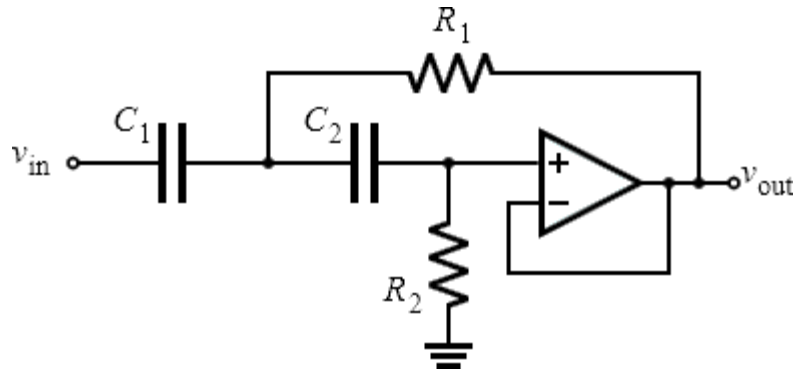
1. It can be formed by choosing  $Z_1=1/j\omega C_1$ ,  $Z_2=1/j\omega C_2$ ,  $Z_3=R_1$ ,  $Z_4=R_2$  ;

2. Convert the above equation to the following form  $H(s) = \frac{j\omega}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right)(j\omega) + \omega_o^2}$

3. The undamped natural frequency  $\omega_o$  is given by  $\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

4. The Q factor is given by  $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}$

# Sallen-Key Filter Topology: High Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}$$

$$f_0 = 999\text{Hz}$$

Given

1.  $R_1 = 7.5\text{k}\Omega$ ;  $R_2 = 72\text{k}\Omega$ ;  $C_1 = 10\text{nF}$ ;  $C_2 = 4.7\text{nF}$ , calculate  $\omega_o$  and  $Q$ . ( $Q = 1.44$ )
2.  $R_1 = 15\text{k}\Omega$ ;  $R_2 = 36\text{k}\Omega$ ;  $C_1 = 10\text{nF}$ ;  $C_2 = 4.7\text{nF}$ , calculate  $\omega_o$  and  $Q$ . ( $Q = 0.72$ )

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>