EE3331 Probability Models in Information Engineering

Project 1

MATLAB Simulations for Basic Probability and Random Variables

Intended Learning Outcomes:

On completion of this project, you should be able to

- Utilize MATLAB to handle basic mathematical operations
- Use MATLAB to compute experimental probabilities
- Use MATLAB to generate, transform and process random variables

Deliverable:

- Each student is required to submit a **short report** which includes your answers for questions and findings for this project work on or before **16 February 2022.**

Procedure:

1. If you are not familiar with MATLAB, you can read the materials at "Getting Started" after typing `help` at the MATLAB prompt. Or you can explore the MATLAB help capability at the command line, e.g., `help rand`. Apart from a number of MATLAB reference books such as [1]-[2] which can be found in City University's library, many on-line MATLAB resources, including [3]-[4], are available.

2. Consider examining the probability of a head outcome from tossing a fair coin by creating a file named "Phead.m" with the following MATLAB code:

```
clear
close all
maxNum = 10000;
stepSize = 100;
%% generate experimental outcomes
for i = 1 : maxNum
H(i) = binornd(1, 0.5, 1);
end
%% plot results
i = 1;
for totalNum = 10 : stepSize: maxNum
p(i) = head(totalNum,H);
i = i + 1;
end
plot(10 : stepSize: maxNum, p)
xlabel('number of experiments')
ylabel('probability')
%% function for probability computation
function y = head(totalNum,H)
y = sum(H(1:totalNum))/totalNum;
end
```

(a) Try to understand the above MATLAB code and then run `Phead`. You will see that a plot is produced. What does this plot show?

(b) Now suppose that the coin is a biased coin and the probability of getting a tail is 0.6. Describe how to modify the above code and attach the corresponding plot in your report.

3. Consider the experiment of rolling 3 independent dice and the outcome is the sum of the faces. Computer simulations are performed to study the expected value and probabilities of several outcomes versus number of experiments. The minimum and maximum numbers of experiments are 10 and 10000, respectively.

   (a) Assume all 3 dice are fair. Develop MATLAB code to plot the average outcome value versus number of experiments. Attach the corresponding plot in your report. Does the result align with the expected value of the outcome? Briefly explain your answer.

   (b) Develop MATLAB code to plot the experimental probabilities of getting 9 and 10 versus number of experiments. Do the experimental probabilities align with the theoretical probabilities? Briefly explain your answer.

   (c) Repeat (a) and (b) for unfair dice if each of them has probabilities of 0, 0.1, 0.15, 0.2, 0.25, and 0.3 for the outcomes of 1, 2, 3, 4, 5, and 6, respectively. To assign distinct probabilities for more than two outcomes, the MATLAB command `randsrc` can be used.

4. Create a file named “rand_u.m” with the following MATLAB code:

```
N=100;                        %sequence length is 100
rand('state',studentno);      %initialized with your ID
u=4*rand([1,N])-1;
```

   where `studentno` is your student ID number. Each element in `u` represents a realization of a random variable denoted by U.

   (a) What are the minimum and maximum possible values of U?

   (b) With the use of the command `mean` or by other means, compute the sample mean, power and variance of U based on `u`, defined as:

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} u_n$$

$$\hat{P} = \frac{1}{N}\sum_{n=1}^{N} u_n^2$$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N} (u_n - \hat{\mu})^2$$

   respectively, where u_n represents the nth element of `u`.

   (c) Repeat (b) by using a larger value of `N`, i.e., `N=100000`.

   (d) Compare the results of (b) and (c) with the expected values, namely, mean, power and variance of U. What are your findings?

(e) Transform U to another random variable $V = U^3$. Using the 100000 values of u_n in (c) and the command `histogram`, plot the probability density function (PDF) of V based on u_n and attach it in the report.

(f) Derive the PDF of V, and compare with the result obtained in (e). What are your findings?

5. Generate N standard uniform random numbers using `rand`. Apply the command `rand('state',studentno)` so that each student will work on a distinct number sequence.

   (a) Set $N = 100$. Use these 100 standard uniform random numbers to produce 100 Bernoulli random variables with $p = 0.3$. Plot the probability mass function (PMF) of the produced Bernoulli random variables using the 100 samples, and include it in the report.

   (b) Repeat (a) by using a larger value of N, namely, $N = 100000$. Compare with the results obtained in (a). What are your findings?

   (c) Extend the Bernoulli random variable generation to binomial random variables with $n = 4$ and $p = 0.3$. Write down the steps of binomial random variable generation using uniform random numbers clearly. Implement your steps using MATLAB and show the corresponding PMF plot in the report. What are your findings?

References:

[1] S. Attaway, *MATLAB: A Practical Introduction to Programming and Problem Solving*, 4th Edition, Butterworth-Heinemann, 2017

[2] A. Gilat, *MATLAB: An Introduction with Applications*, 5th Edition, John Wiley & Sons, 2015

[3] http://www-h.eng.cam.ac.uk/help/tpl/programs/matlab.html

[4] https://www.mathworks.com/help/matlab/getting-started-with-matlab.html