Interest Rates

What Do Interest Rates Mean? What is Their Role in Valuation?

Acknowledgement: This set of slides is modified from Dr. Alan Yeung's slides which based on the reference books as listed in the course document.

Present Value Introduction

- Different debt instruments (e.g. bonds, debentures, leases, etc.) have very different streams of cash payments to the holder (known as **cash flows**), with very different timing.

- All else being equal, debt instruments are evaluated against one another based on the ***amount*** of each cash flow and the ***timing*** of each cash flow.

- This evaluation, where the analysis of the ***amount*** and ***timing*** of a debt instrument's cash flows lead to its *yield to* ***maturity*** *or* ***interest rate***, is called *present value* analysis.

Present Value

- The concept of **present value** (or **present discounted value**) is based on the commonsense notion that a dollar of cash flow paid to you one year from now is less valuable to you than a dollar paid to you today.

- This notion is true because you could invest the dollar in a savings account that earns interest and have more than a dollar in one year.

- The term present value (PV) can be extended to mean the PV of a single cash flow or the *sum* of a sequence or group of cash flows.

Present Value Applications

There are four basic types of credit instruments which incorporate present value concepts:

1. Simple Loan
2. Fixed Payment Loan
3. Coupon Bond
4. Discount Bond

Q. What is the difference between loan and bond?

Present Value Concept: Simple Loan Terms

- *Loan Principal:* the amount of funds the lender provides to the borrower.
- *Maturity Date:* the date the loan must be repaid; the *Loan Term* is from initiation to maturity date.
- *Interest Payment:* the cash amount that the borrower must pay the lender for the use of the loan principal.
- *Simple Interest Rate:* the interest payment divided by the loan principal; the percentage of principal that must be paid as interest to the lender. Convention is to express on an annual basis, irrespective of the loan term.

Present Value (PV) Concept: Simple Loan (1 of 2)

Simple loan of $100

Year:	**0**	**1**	**2**	**3**	***n***
	$100	$110	$121	133	$100\times(1+i)^n$

Present Value (PV) Formula

$$PV\ of\ future\ \$1 = \frac{\$1}{(1+i)^n}$$

where i – annual interest rate and n = no of year

Present Value Concept: Simple Loan (2 of 2)

- The previous example reinforces the concept that $100 today is preferable to $100 a year from now since today's $100 could be lent (or deposited) at 10% interest to be worth $110 one year from now, or $121 in two years or $133 in three years.

Graphic illustration of the concept: https://www.mathsisfun.com/money/net-present-value.html#:~:text=Example%3A%20Let%20us%20say%20you,a%20Present%20Value%20of%20%241%2C000.

Yield to Maturity: Loans (1 of 2)

Yield to maturity = interest rate that equates today's value with present value of all future payments

Simple Loan Interest Rate (i = 10%)

$$\$100 = \$110 / (1 + i), \text{ or } I = 10\%$$

Remarks:

$100 – Presence Value; $110 – Future Value;
What is the interest Rate required? ➔ 10% (Yield to Maturity)

Example Simple Present Value

What is the present value of $250 to be paid in two years if the interest rate is 15%?

Present Value (PV) Formula

$$PV\ of\ future\ \$1 = \frac{\$1}{(1+i)^n}$$

where i – annual interest rate and n = no of year

Apply above formula, we have
$250 / (1 + 0.15)^2 = $250 / 1.3225 = $189.04

Remark: If Future Value (FV) (after 2 years) = $250 and i = 15%
➔PV (now) = $189.04

Present Value Concept: Fixed-Payment Loan Terms (1 of 2)

- *Simple Loans* require payment of one amount which equals the loan principal plus the interest.

- *Fixed-Payment Loans* are loans where the loan principal and interest are repaid in several payments, often monthly, in equal dollar amounts over the loan term.

Present Value Concept: Fixed-Payment Loan Terms (2 of 2)

- *Installment Loans,* such as auto loans and home mortgages are frequently of the fixed-payment type.

Table 3.1 Yields to Maturity on a 10% Coupon Rate Bond Maturing in 10 Years (Face Value = $1,000)

Price of Bond($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three interesting facts in Table 3.1

1. When bond is at par, yield equals coupon rate
2. Price and yield are negatively related
3. Yield greater than coupon rate when bond price is below par value

Relationship Between Price and Yield to Maturity

- It's also straight-forward to show that the value of a bond (price) and yield to maturity (YTM) are negatively related.
- If the interest rate *i* increases (YTM increases), the PV of any given cash flow is lower; hence, the price of the bond must be lower.