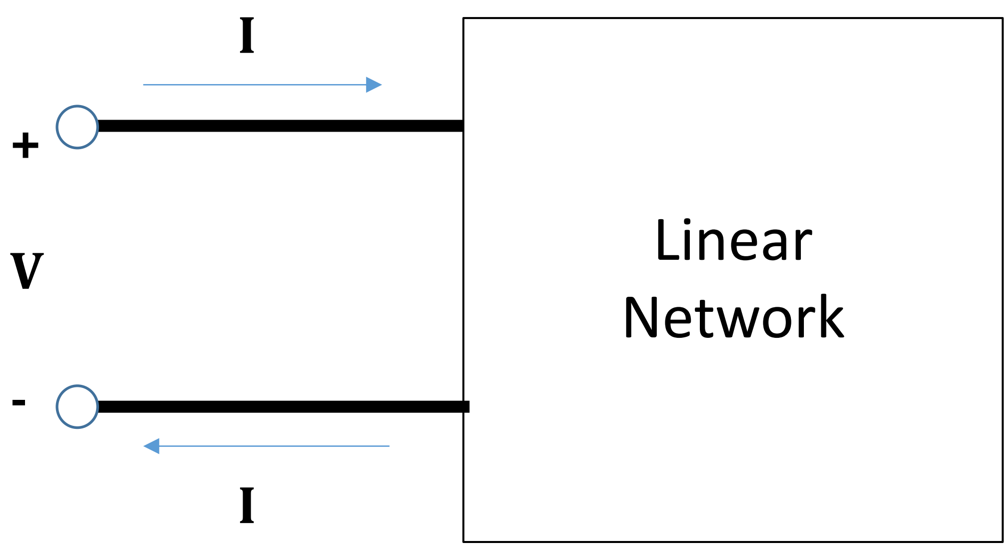
Two-Port Representations

1) Impedance parameters z
2) Admittance parameters y
3) Hybrid parameters h
4) Transmission parameters T

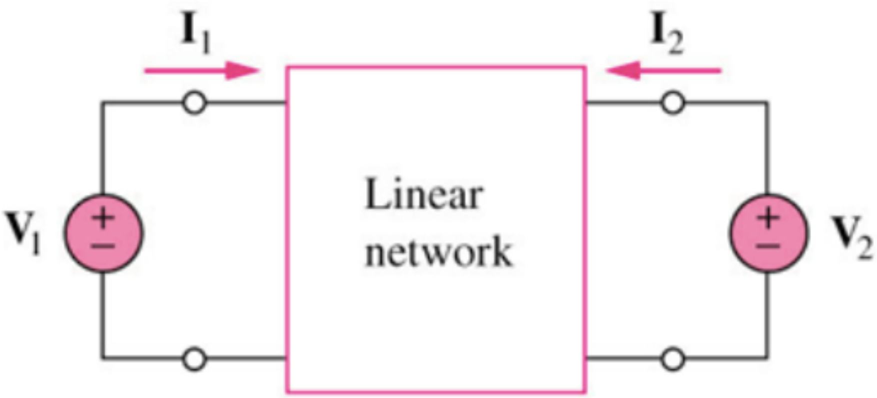
One-Port Network

- One-port corresponds to a pair of terminals associated with only one current and one voltage.
- e.g. Thevenin and Norton equivalent circuits

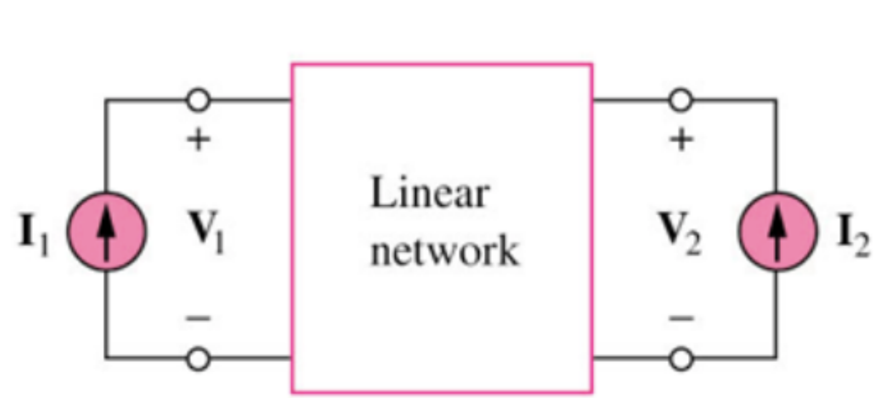


Two-Port Network

- A two-port network is an electrical network with two separate ports for inputs and outputs



driven by voltage sources



driven by current sources

- We consider circuits with no internal independent sources.
- 4 variables $\{V_1, I_1, V_2, I_2\}$: only 2 of the 4 are independent
  ➔ the other 2 can be found using terminal equations

Summary: Sets of Terminal Equations

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix};$$ **z** are the *impedance* parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix};$$ **y** are the *admittance* parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix};$$ **h** are the *hybrid* parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix};$$ **A, B, C, D** are *transmission* parameters

Impedance Parameters (1)

Assume: no independent source in the network

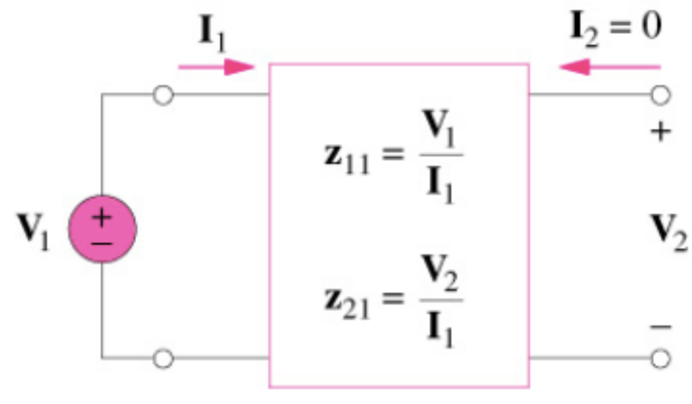
$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

z terms are called impedance parameters in unit of Ohm

Impedance Parameters (2)

open-circuit port-2

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ (open-circuit input impedance)



(a)

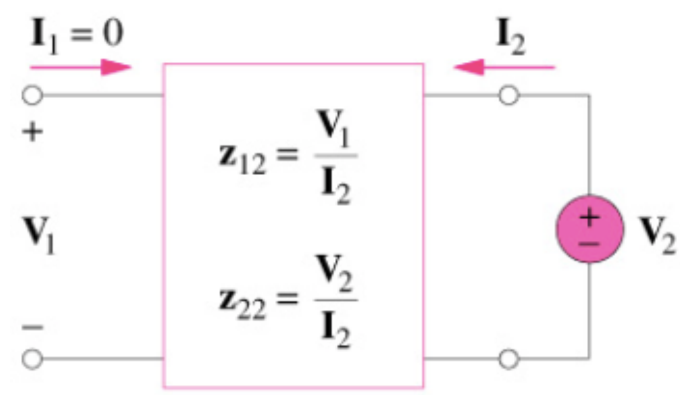
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

(open-circuit transfer impedance from port 2 to port 1)

open-circuit port-1

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

(open-circuit transfer impedance from port 1 to port 2)

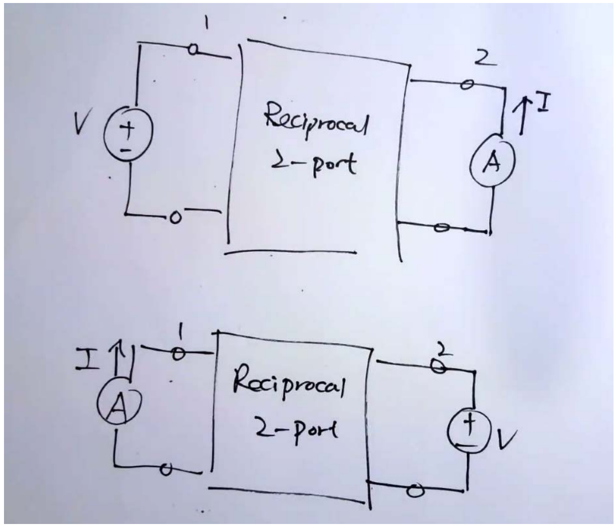


(b)

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$ (open-circuit output impedance)

Symmetry and Reciprocity (z-parameters)

- When $z_{11} = z_{22}$, the two-port network is said to be **symmetrical**.
- When the two-port network is **linear** and has **no dependent sources**, the transfer impedances are equal ($z_{12} = z_{21}$), and the two-port is said to be **reciprocal**.

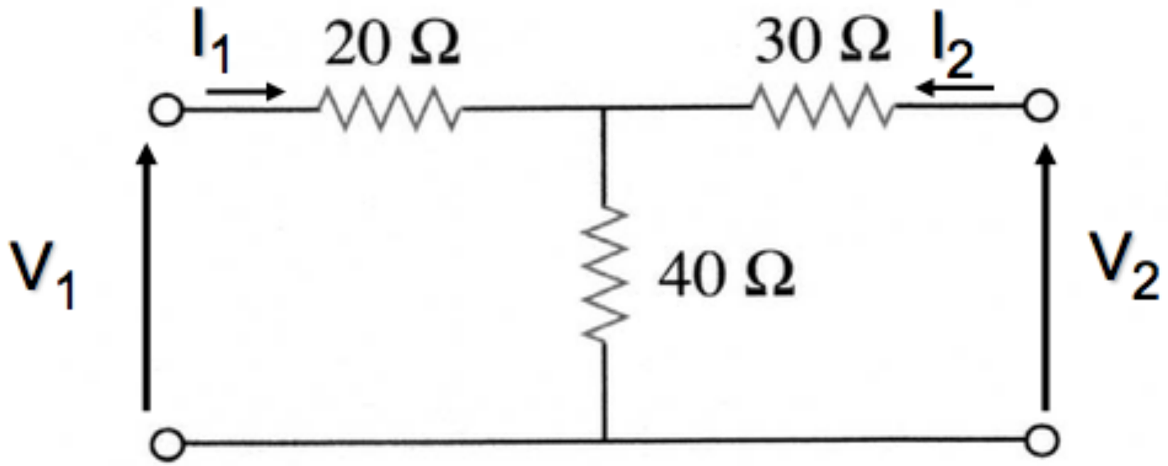


Example 1: Impedance Parameters

Determine the z parameters of the following circuit

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$



$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

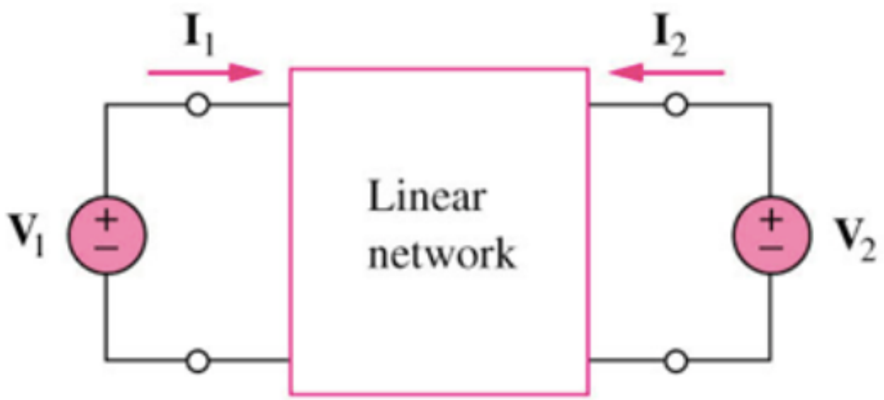
Solution:

$z_{11} = 20\Omega + 40\Omega = 60\Omega$; $z_{22} = 30\Omega + 40\Omega = 70\Omega$

$z_{21} = \frac{40I_1}{I_1} = 40\Omega$; $z_{12} = \frac{40I_2}{I_2} = 40\Omega$

Impedance matrix $\mathbf{Z} = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$

Admittance Parameters (1)



Assume: no independent source in the network

$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + z_{22}V_2 \end{cases} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

y terms are called admittance parameters in unit of Siemens

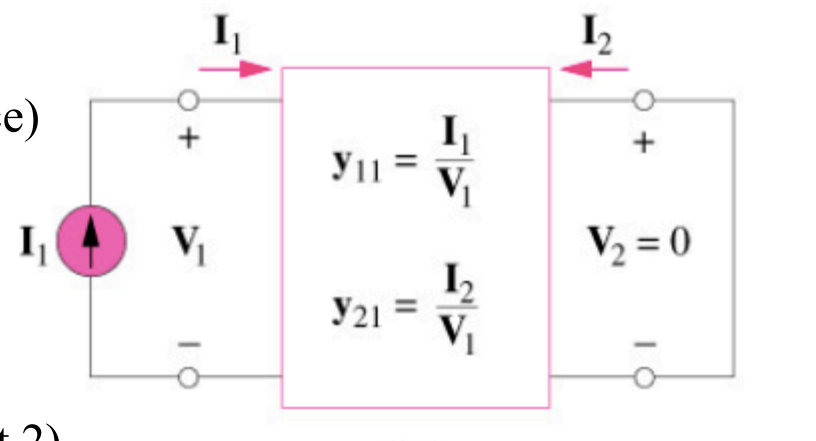
Admittance Parameters (2)

short-circuit port-2

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$ (short-circuit input admittance)

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

(short-circuit transfer admittance from port 1 to port 2)

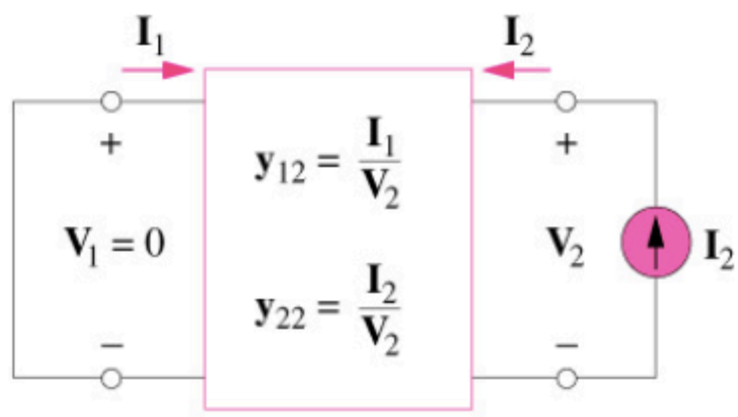


(a)

short-circuit port-1

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

(short-circuit transfer admittance from port 2 to port 1)



(b)

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$ (short-circuit output admittance)

Symmetry and Reciprocity (y-parameters)

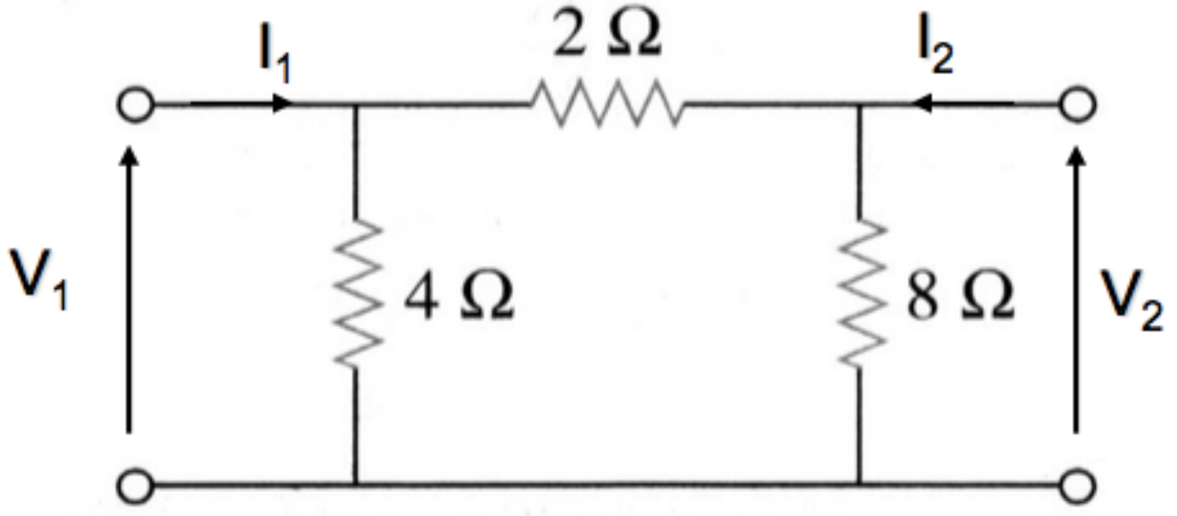
- When $y_{11} = y_{22}$, the two-port network is said to be **symmetrical**.
- When the two-port network is **linear** and has **no dependent sources**, the transfer admittance are equal ($y_{12} = y_{21}$), and the two-port is said to be **reciprocal**.

Example 2: Admittance Parameters

Determine the y-parameters of the following circuit.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



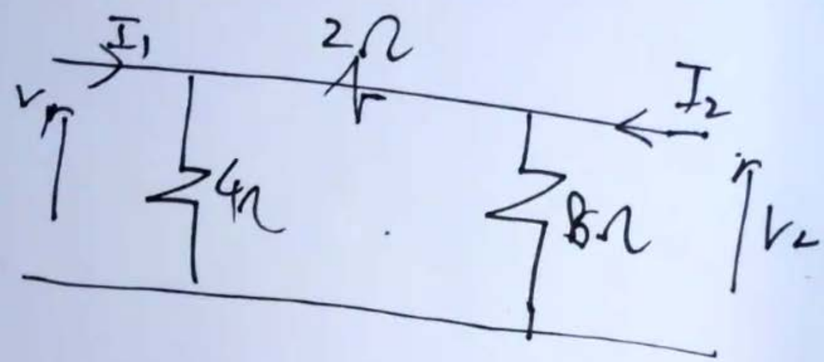
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Answer:

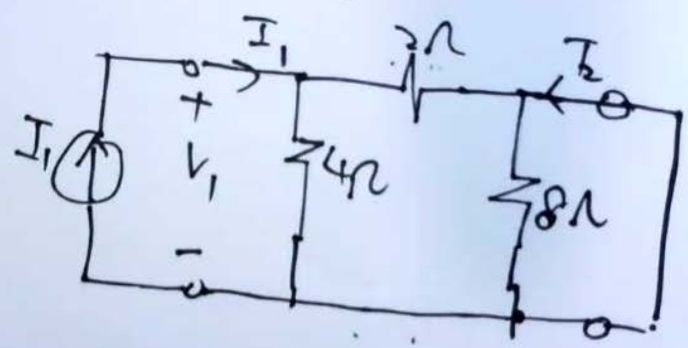
Admittance matrix $\mathbf{Y} = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$

Example 2: Admittance Parameters



$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



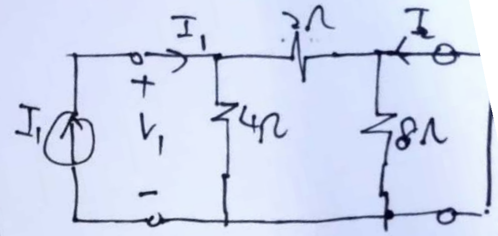
$$V_1 = I_1(4\|2) = \frac{4}{3} I_1$$

$$\Rightarrow y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = 0.75\ S$$

Example 2: Admittance Parameters

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$

$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

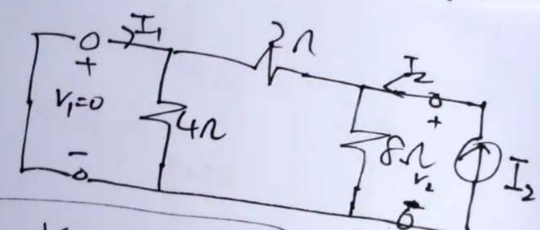


$V_1 = I_1(4\|2) = \frac{4}{3}I_1$

$\Rightarrow y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = 0.75\ S$

current division: $-I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1$

$\Rightarrow y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1} = -0.5\ S$



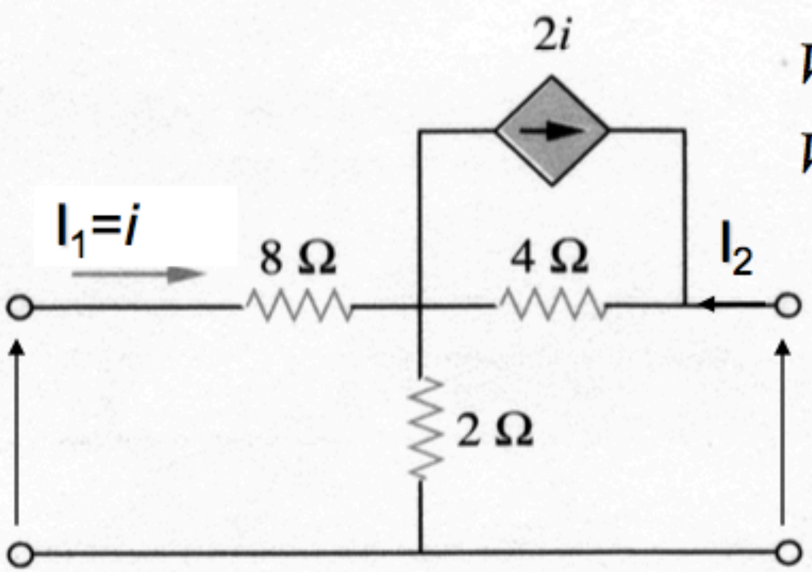
$V_2 = I_2(8\|2) = \frac{8}{5}I_2 \Rightarrow y_{22} = \frac{I_2}{V_2} = 0.625\ S$

Current division $-I_1 = \frac{8}{8+2}I_2 = \frac{4}{5}I_2 \Rightarrow y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5}I_2}{\frac{8}{5}I_2}$

$= -0.5\ S$

Example 3: Admittance parameters

Determine the y-parameters of the following circuit.



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

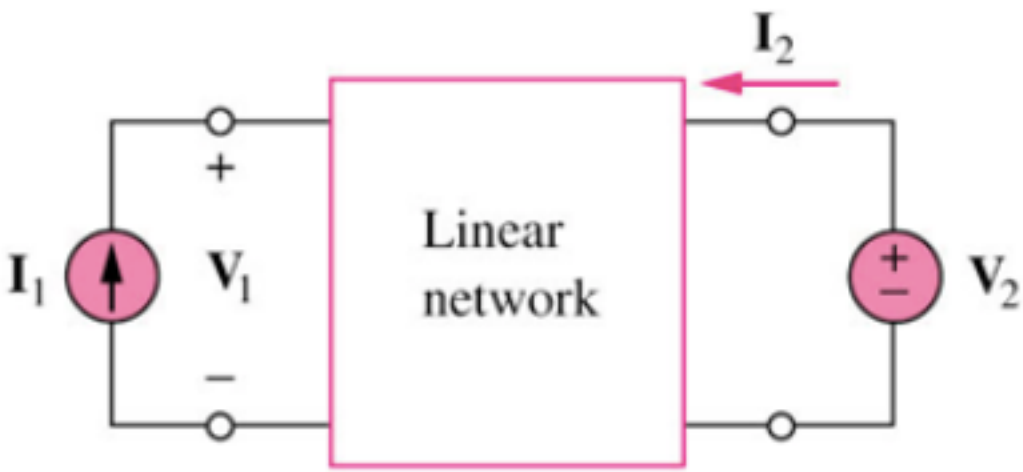
Apply KVL

$$V_1 = 8I_1 + 2(I_1 + I_2)$$
$$V_2 = 4(2i + I_2) + 2(I_1 + I_2)$$

$$I_1 = 0.15V_1 - 0.05V_2$$
$$I_2 = -0.25V_1 + 0.25V_2$$

Answer: $y = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} S$

Hybrid Parameters (1)

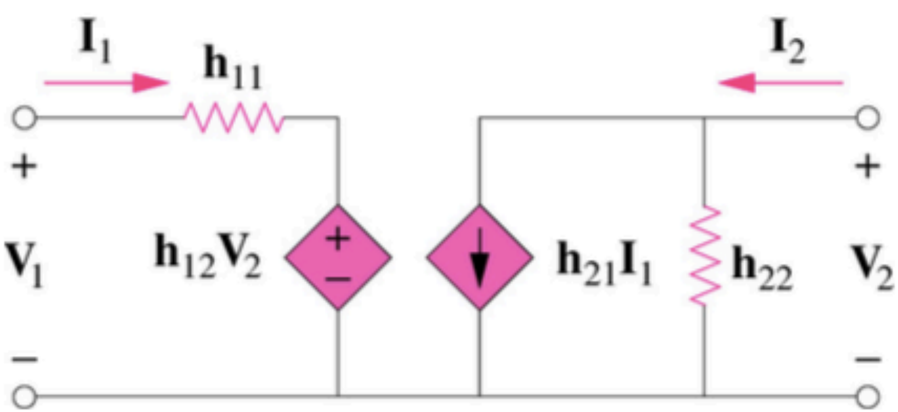


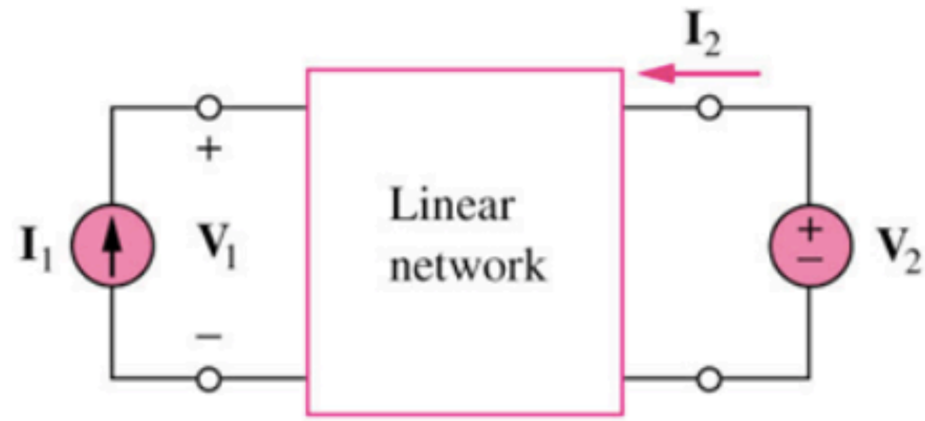
Assume: no independent source in the network

$$\begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases} \Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h terms are called hybrid parameters

Hybrid Parameters (2)





$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11}: short-circuit input impedance (Ω)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

h_{12}: open-circuit reverse voltage-gain

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

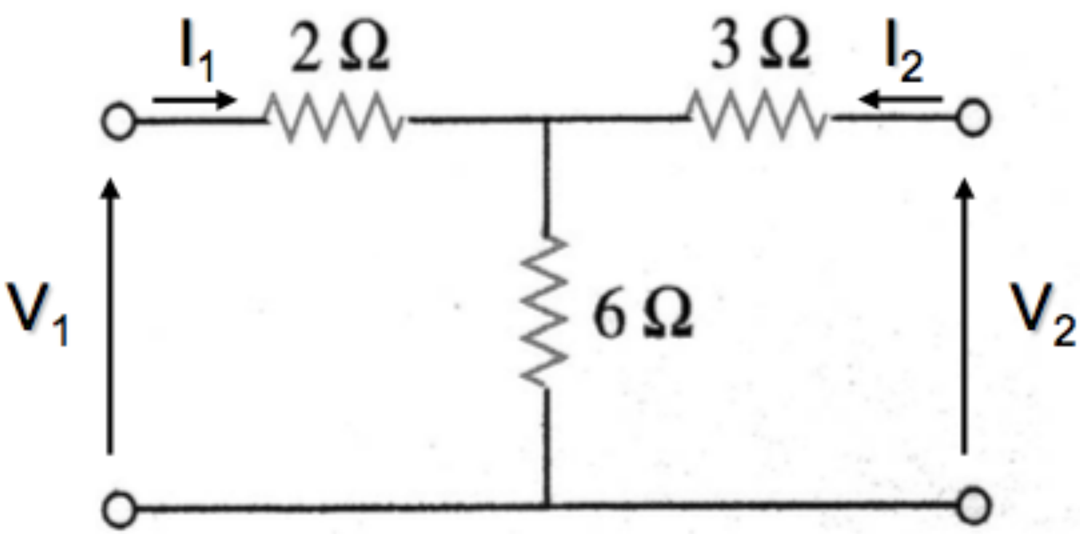
h_{21}: short-circuit forward current gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22}: open-circuit output admittance (S)

Example 4: Hybrid Parameters

Determine the h-parameters of the following circuit.

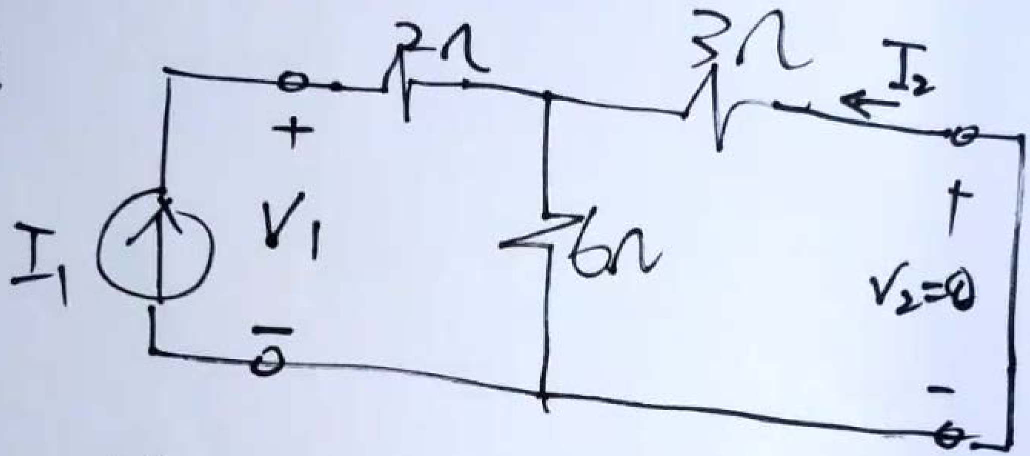


Answer:

Hybrid matrix $\mathbf{H} = \begin{bmatrix} 4\Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

Example 4: Hybrid Parameters

$\boxed{V_2=0}$



$$V_1 = I_1(2+3\|6) = 4I_1$$

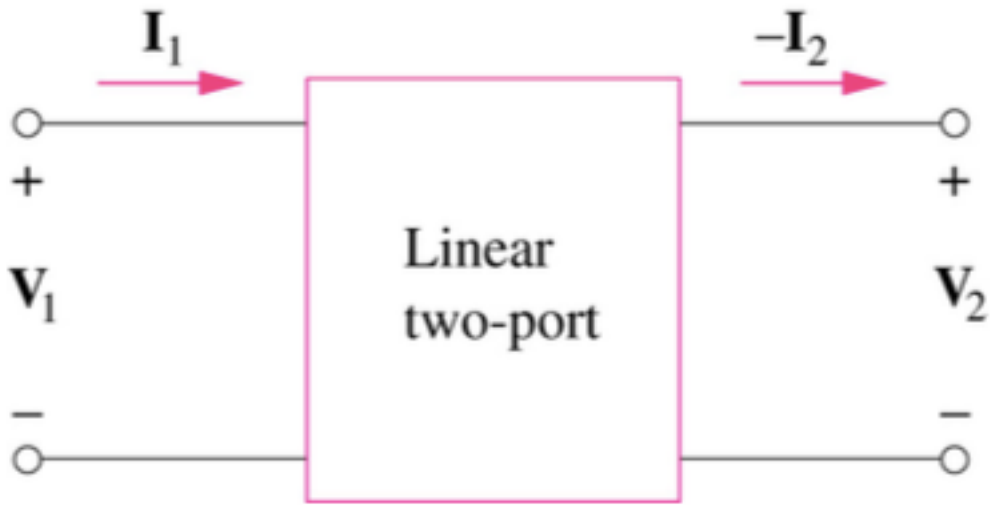
$$\Rightarrow h_{11} = \frac{V_1}{I_1} = 4\Omega$$

Current division

$$-I_2 = \frac{6}{6+3} I_1 = \frac{2}{3} I_1$$

$$\Rightarrow h_{21} = \frac{I_2}{I_1} = -\frac{2}{3}$$

Transmission Parameters (1)

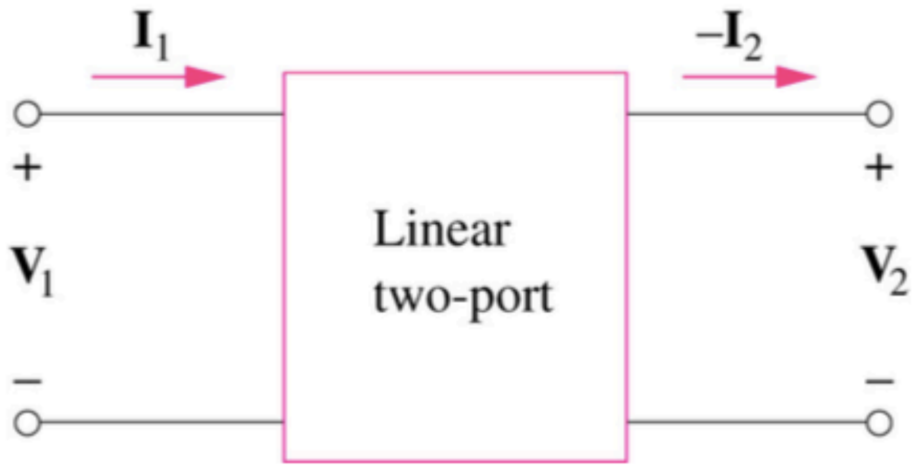


Assume: no independent source in the network

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases} \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For cascading applications

Transmission Parameters (2)



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

A: open-circuit voltage ratio

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

B: negative short-circuit transfer impedance (Ω)

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

C: open-circuit transfer admittance (S)

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

D: negative short-circuit current ratio

A network is reciprocal if **AD-BC = 1**

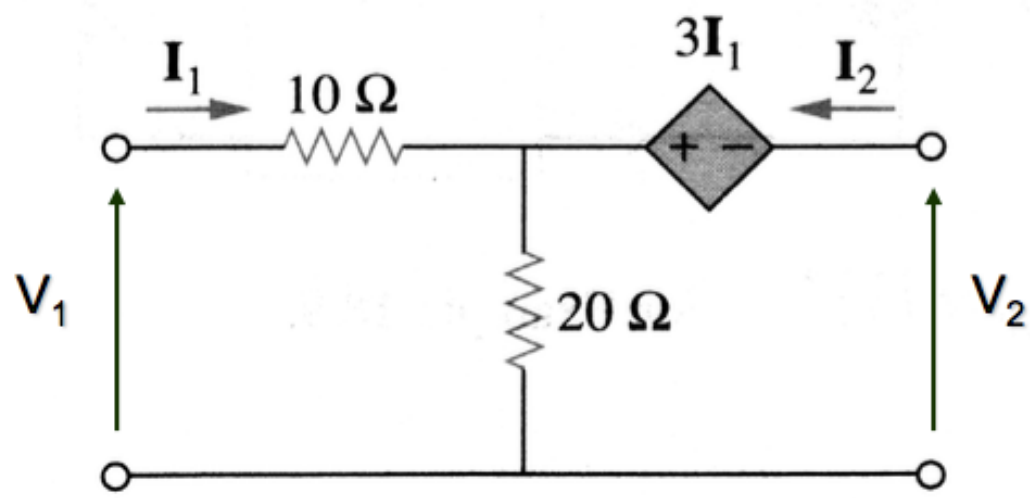
Example 5: Transmission Parameters

Determine the transmission parameters of the following circuit

<u>*Solution:*</u>

Apply KVL

$$\begin{cases} V_1 = 10I_1 + 20(I_1 + I_2) \\ V_2 = 20(I_1 + I_2) - 3I_1 \end{cases}$$

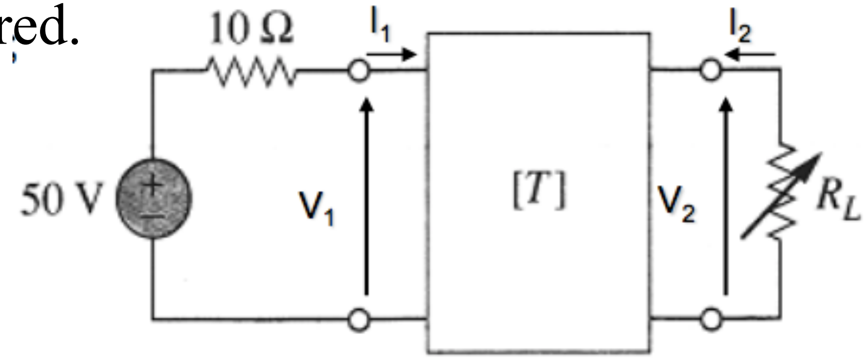


$$\begin{cases} V_1 = \frac{30}{17} V_2 - \frac{260}{17} I_2 \\ I_1 = \frac{1}{17} V_2 - \frac{20}{17} I_2 \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059S & 1.176 \end{bmatrix}$$

Example 6: Transmission Parameters

The ABCD parameters of the following two-port network are $\begin{bmatrix} 4 & 20\ \Omega \\ 0.1\ S & 2 \end{bmatrix}$.
The output port is connected to a variable load for maximum power transfer.
Find R_L and the maximum power transferred.



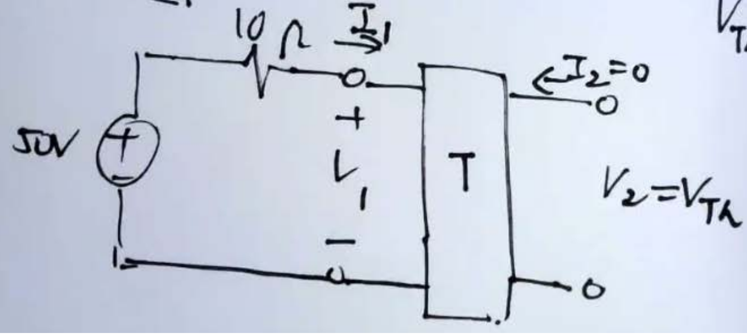
Answer:

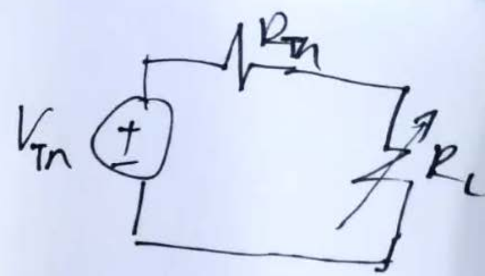
$R_L = 8\Omega$

$P_{max} = 3.125\ W$

Example 6: Transmission Parameters

$\boxed{\text{Find } V_{Th}}$





$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4 & 20\Omega \\ 0.1S & 2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{cases} V_1 = 4V_2 - 20I_2 \\ I_1 = 0.1V_2 - 2I_2 \end{cases}$$

Given

$I_2 = 0$

$V_1 = 50 - 10I_1$

$$\Rightarrow \boxed{\begin{matrix} 50 - 10I_1 = 4V_2 \\ I_1 = 0.1V_2 \end{matrix}}$$

$$\Downarrow$$

$$50 - V_2 = 4V_2$$

$$\Downarrow$$

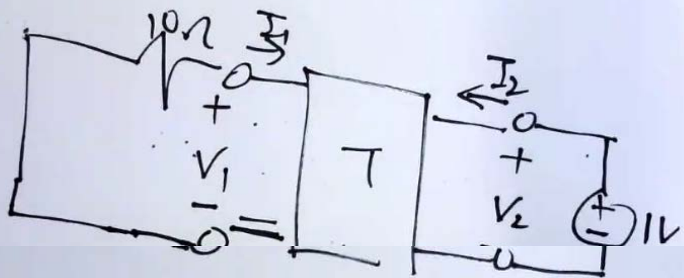
$$V_2 = 10$$

$$\Downarrow$$

$$V_{Th} = V_2 = 10V$$

Example 6: Transmission Parameters

Find R_{Th}



@INPUT PORT : $V_1 = -10I_1$

$-10I_1 = 4V_2 - 20I_2 \Rightarrow I_1 = -0.4V_2 + 2I_2$

$0.1V_2 - 2I_2 = -0.4V_2 + 2I_2$

$\Rightarrow 0.5V_2 = 4I_2$

$$R_{Th} = \frac{V_2}{I_2} = \frac{4}{0.5} = 8\Omega$$

For max power transfer

$R_L = R_{Th} = 8\Omega$

Max power transfer

$$P = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}} = \frac{100}{4\times 8} \Rightarrow 3.125W$$