

CITY UNIVERSITY OF HONG KONG

Course code & title : MA2001 Multivariable Calculus and Linear Algebra

Session : Semester A, 2021-20222

Time Allowed : 90 minutes for writing, 10 minutes for scanning and uploading.

Instructions:

1. This is a **closed** book Mid-term Test .
 2. Non-programmable portable battery operated calculator is permitted.
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Q1. If you are given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & a & -1 \\ -1 & -1 & b \end{bmatrix}$,

- (a) [10 marks] find the constants a and b in A ;
- (b) [15 marks] construct a matrix P such that $A = PDP^T$ with diagonal matrix D ;
- (c) [5 marks] determine A is positive or negative definite or semidefinite, or indefinite, and explain.
- (d) [5 marks] deduce the eigenvalues and eigenvectors of $C = A^2 - 3A + I$, where I is the identity matrix. [Note: Do not attempt to compute the matrix C].

Q2. Consider the function $f(x, y) = x^2 - y^3$.

- (a) [10 marks] Find the **linear** approximation of f at $P(1, 2)$.
- (b) [15 marks] Find the stationary points of f and determine their nature.

Q3. Suppose $x^2 + 4y^3 + 3z^2 = 116$ determines $z = f(x, y)$ as a function of x, y locally around $(x, y, z) = (2, 1, -6)$. Using implicit differentiation,

(a) [10 marks] find the directional derivative of f at $(2, 1)$ along $\vec{v} = (-1, 1)$;

(b) [10 marks] determine the direction \vec{u} along which f increases most rapidly at $(2, 1)$;

Q4. [10 marks] Let $f(x, y) = \frac{3x^2}{2x^2 + y^2}$. Determine if

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y)$$

exists or not, and explain.

Q5. [10 marks] Suppose that $z = 2x^5 + y^5$ and $x = \sin t + \cos s$ and $y = \cos t - \sin s$, use the Chain Rule to find

$$\frac{\partial z}{\partial t} \quad \text{and} \quad \frac{\partial^2 z}{\partial t \partial s}$$

when $t = s = 0$.

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