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1) a) $\int \frac{\tan^{-1}\sqrt{x}}{\sqrt{x}(1+x)}$ let $u=\sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$

$= 2\int \frac{\tan^{-1}u}{1+u^2}$ let $v = \tan^{-1}u$, $dv = \frac{1}{1+u^2}du$

$= 2\int v$

$= \cancel{2}\cdot\frac{v^2}{\cancel{2}} + C$

$= \tan^{-2}\sqrt{x} + C$ //

b) $\int_{\ln 2}^{\ln 3} \frac{1}{e^x - e^{-x}} = \int_{\ln 2}^{\ln 3} \frac{e^x}{e^{2x}-1}$ let $u=e^x$, $du=e^x dx$

$= \int \frac{1}{u^2-1} = \int \frac{1}{(u-1)(u+1)}$

$= \frac{\ln|u-1|}{2} - \frac{\ln|u+1|}{2} + C$

$= \left[\frac{\ln|e^x-1| - \ln|e^x+1|}{2}\right]_{\ln 2}^{\ln 3}$

$= \frac{\ln(3) - \ln(4)}{2} + \frac{\ln(3)}{2}$

$= \ln(3) - \frac{\ln(4)}{2}$ //

1 d) $\frac{3x^2+3x-1}{(x-1)(x^2+2x+2)} = \frac{Ax+B}{(x^2+2x+2)} + \frac{C}{x-1}$

$3x^2+3x-1 = (A+C)x^2+(B-A+2C)x+(-B+2C)$

$A+C=3$

$B-A+2C=3$

$2C-B=-1$

when solved $A=2, B=3, C=1$

$\int \frac{2x+3}{x^2+2x+2} + \int \frac{1}{x-1}$ (let $u=x+1$)

$= \int \frac{2u}{u^2+1} + \int \frac{1}{u^2+1} + \int \frac{1}{x-1}$

$= \ln|u^2+1| + \tan^{-1}|u| + \ln|x-1|$

$= \ln|x^2+2x+2| + \ln|x-1| + \tan^{-1}|x+1|$ //

2. a)

$$= \pi \int_0^{\pi} \sin^2 x$$

$$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^{\pi}$$

$$= \frac{\pi^2}{2} \quad //$$

b) $x = \sin^{-1} y$

$$\pi \int_0^1 \sin^{-1} y$$

$$= \pi\left(y\sin^{-1} y - \int \frac{y}{\sqrt{1-y^2}} \right)$$

$$= \pi\left[y\sin^{-1} y + \sqrt{1-y^2} \right]_0^1$$

$$= \pi\left(\frac{\pi}{2} - 1\right) \quad //$$

c) $\because$ Since $y=0$ is a bound that made the area from π to 2π incalculable or equal to zero

$\therefore \pi\left(\frac{\pi}{2} - 1\right) \quad //$

3.a) $y = x^2$

$y = ax$

$\therefore x^2 - ax = 0$

$x(x-a) = 0$

$x = 0$ or $\cancel{x} = a$

$y = 0$ or $y = a^2$

$(0,0)$ & (a, a^2)

$S_1 = \int_0^a (ax - x^2)$ $\because$ Since the are bound by $y=ax, y=x^2$ & $x=1$ will never go past $x=1$ $(0<a<1)$

$= \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$ $\therefore S_2 = S_1 = \frac{a^3}{2} - \frac{a^3}{3}$

$= \frac{a^3}{2} - \frac{a^3}{3}$

$S_1 + S_2 = 0$ $a = 0$, $a = \frac{2}{3}$

$2\left(\frac{a^3}{2} - \frac{a^3}{3}\right) = 0$ $a = \frac{2}{3}$ //

$a^2\left(\frac{a}{2} - \frac{1}{3}\right) = 0$

3b) $V_x = 2\pi \int_0^a 2\left(\frac{a^3}{2} - \frac{a^3}{3}\right) dx$

$= 4\pi \int_0^a \frac{a^3}{2} - \frac{a^3}{3}\, dx$

$= 4\pi \left[\frac{8}{54}x - \frac{8}{81}x\right]_0^{\frac{2}{3}}$

$= \frac{32}{243}\pi$ //

4) $L = \int_0^a \sqrt{1+[f'(x)]^2}\, dx$ $\quad f'(x) = \frac{1}{\cos x} \cdot -\sin(x)$

$= -\tan(x)$

$= \int_0^a \sqrt{1+\tan^2 x}$

$(\sec^2(x) - \tan^2(x) = 1)$

$= \int_0^a \sec(x)$

$= \int_0^a \frac{\sec(x)(\tan x + \sec x)}{(\tan x + \sec(x))}$ $\quad$ let $u = \tan x + \sec x$

$du = \sec(x)(\tan(x) + \sec(x))\,dx$

$= \int_0^a \frac{1}{u}$

$= \left[\ln|\tan x + \sec x|\right]_0^a$

$= \ln|\tan a + \sec a| + \ln|1|$

$= \ln(\tan a + \sec a)$ //