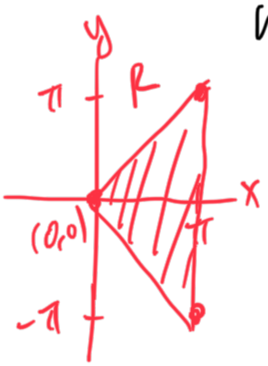
Evaluate $\iint_R \cos(x-y)\sin(x+y)\,dA$

where R is the triangle in the xy-plane with vertices at $(0,0)$, $(\pi,-\pi)$ and (π,π).

Use the change of variables to answer this question.



Ans.

① $dA = dx\,dy$ or $dy\,dx$

② $u = x-y \quad v = x+y$ ☆☆☆☆

step1

$$(+)\begin{cases} u = x-y \\ v = x+y \end{cases}$$

$u+v = 2x$

$$\boxed{x = \frac{u+v}{2}}$$

$y =$

$u = \frac{u+v}{2} - y$

$2u = u+v-2y$

$u-v = -2y$

$$\boxed{\frac{v-u}{2} = y}$$

$\pi = \frac{u+v}{2}$

$2\pi = u+v$

$\frac{v-u}{2} = \pi$

$v-u = 2\pi$

step2

$dx\,dy = |J(u,v)|\,du\,dv$

determinant

$$J(u,v) = \begin{vmatrix} \boxed{\frac{\partial x}{\partial u}} & \boxed{\frac{\partial x}{\partial v}} \\ \boxed{\frac{\partial y}{\partial u}} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{4} + \frac{1}{4}$$

$- \frac{}{4} = \frac{}{2}$

$\boxed{x} = \frac{u+v}{2}$ $\qquad \frac{\partial x}{\partial u} = \frac{1}{2}$

$x = \boxed{\frac{1}{2}u} + \underbrace{\frac{1}{2}v}_{\text{constant}}$

$\boxed{x} = \underbrace{\frac{u}{2}}_{\text{constant}} + \boxed{\frac{v}{2}}$ $\qquad \frac{1}{2}$

$y = \frac{v-u}{2}$ $\qquad \frac{\partial y}{\partial u} = -\frac{1}{2}$

$\boxed{y} = \frac{v}{2} - \boxed{\frac{u}{2}}$

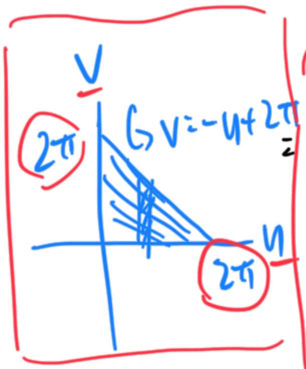
constant

Step 3

$\iint_R \cos(x-y)\sin(x+y)\,dx\,dy$

Recall step ① #2

$$\boxed{\begin{aligned} u &= x-y \\ v &= x+y \end{aligned}}$$



$= \iint_G \cos(u)\sin(v)\,\frac{1}{2}\,du\,dv$

$= \int_0^{2\pi}\int_0^{2\pi-u} \frac{\cos u \sin v}{2}\,dv\,du$

$= \int_0^{2\pi} \boxed{\frac{1}{2}} \cos u \left(\boxed{-} \cos v \Big|_0^{2\pi - u} \right) du$

$= -\frac{1}{2}\int^{2\pi} \cos u \left(\cos(2\pi - u) - \cos(1) \right) du$

$$= -\frac{1}{2}\int_0^{2\pi} \cos u\,(\cos(2\pi-u)-1)\,du$$

$$= -\frac{1}{2}\int_0^{2\pi} \cos u\cos(2\pi-u) - \cos u\,du$$

$$= -\frac{1}{2}\int_0^{2\pi} \cos^2 u\,du + \frac{1}{2}\int_0^{2\pi} \cos u\,du$$

$$= -\frac{1}{2}\int_0^{2\pi} \frac{1+\cos(2u)}{2}\,du + \frac{1}{2}\sin u\Big|_0^{2\pi}$$

$$= -\frac{1}{4}\left(u + \frac{\sin 2u}{2}\right)\Big|_0^{2\pi}$$

$$= -\frac{1}{4}\left[\left(2\pi + \frac{\sin(4\pi)}{2}\right) - \left(0 + \frac{\sin(2(0))}{2}\right)\right] = -\frac{\pi}{2}$$