EE3210 Signals & Systems

Due on 11:00 AM, May 15, 2021

Homework #2, 3

1. Total mark is 200 points (= 20 points per problem × 10 problems)
2. Submission due by 11:00 AM, May 15, 2021. We will not accept late submission.
3. Online submission through Canvas
    - Scan or taking a photo of your answer sheet, then upload to Canvas

Problem 1

Let's consider LTI systems described by the following differential equations. Derive the frequency responses $H(f)$, the corresponding impulse response $h(t)$, and the step responses using Fourier transform.

a)

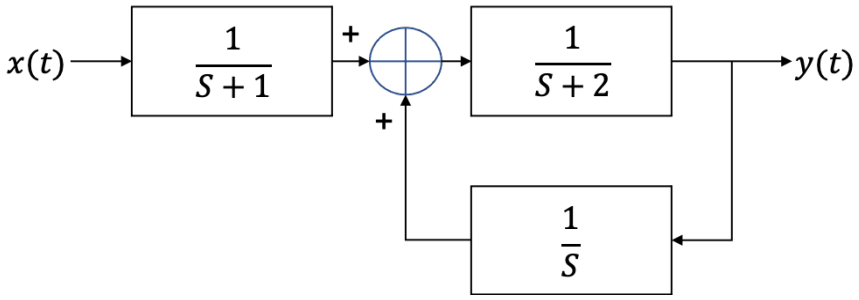
$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}$$

b)

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

Problem 2

Determine the transfer function (or system function) $H(s)$ of the following system model.



Problem 3

Let us consider LTI system with a following transfer function $H(s)$

$$H(s) = \frac{4}{(S+5)(S+3)(S-2)}.$$

a) Indicate all possible ROC that can be associated with the given $H(s)$.

b) For each ROC, specify whether the associated system is stable and/or causal.

Problem 4

Find the discrete time Fourier transform of the following signals

a)

$$x[n] = -a^n u\left[-n-1\right], \quad a \text{ is real}$$

b)

$$x[n] = u\left[n\right] - u\left[n-N\right]$$

Problem 5

Prove the following four properties of discrete time Fourier transform based on the definition of DTFT

$$X(f) = \sum_{n=-\infty}^{n=\infty} x[n]e^{-j2\pi fn}$$

a) Time shifting property

$$x\left[n - n_0\right] \leftrightarrow e^{-j2\pi f n_0} X(f)$$

b) Frequency shifting property

$$e^{j2\pi f_0 n} x\left[n\right] \leftrightarrow X\left(f - f_0\right)$$

c) Differentiation

$$nx[n] \leftrightarrow \frac{j}{2\pi} \cdot \frac{dX(f)}{df}$$

Problem 6

Derive the Fourier transform of the following signals.

a)

$$x(t) = \text{sinc}^2(4t)$$

b)

$$x(t) = \text{sgn}(t)$$

c)

$$x(t) = e^{-2t}u(t)$$

d)

$$x(t) = \text{rect}\left(\frac{t}{3}\right)$$

Problem 7

a) Derive (bilateral) Laplace transform of $x(t) = e^{-5t}u(t)$ and define the ROS.

b) Derive (bilateral) Laplace transform of $x(t) = tu(t)$ and define the ROS.

c) Derive (bilateral) Laplace transform of $x(t) = x_1(t-4)$ where $x_1(t) = e^{-3t}u(t)$

d) Find the inverse Laplace transform of the following $X(s)$

$$X(s) = \frac{S^2 + 2S + 5}{(S+3)(S+5)^2}, \quad \mathrm{Re}(S) > -3$$

Problem 8

Derive the Z-transform of $x[n] = r^n \sin(w_0 n)\, u[n]$.

Problem 9

a) Find the Z-transform of the following sequence

$$x[n] = \left\{5, 3, \underset{\uparrow}{4}, 1, 0, 2\right\}$$

b) Find the Z-transform of the following signal

$$x[n] = \frac{1}{3} \cdot \frac{1}{4^n}\ u[n] - \frac{2}{2^n}\ u[n] + \frac{8}{3}\ u[n]$$

c) Find the inverse Z-transform of the following $X(z)$

$$X(z) = \log\left(\frac{1}{1-Z}\right), \quad |Z| < 1$$

d) Find the inverse Z-transform of the following $X(z)$

$$X(z) = \frac{3}{Z-4}, \quad |Z| > 4$$

Problem 10

Solve the following difference equation using Unilateral Z-transform

$$y[n] - 5y\left[n-1\right] + 6y\left[n-2\right] = 3x\left[n-1\right] + 5x\left[n-2\right],$$

where the initial conditions are given by

$$y\left[-1\right] = \frac{11}{6}, \quad y\left[-2\right] = \frac{37}{36},$$

and the input is $x[n] = 2^{-n}u[n]$.