

Modular I/O Reasoning in DimSum

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int echo () :=  
  let c := getc();  
  putc(c);  
  return 0;
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reads from increasing sequence

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echo
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$$\left\{ \lambda es, \lceil es = [] \rceil * \left\{ \lambda es, \lceil es = [] \rceil \right\} \text{getc} \left\{ \lambda v, \left\{ \lambda es, \lceil es = v \rceil \right\} \text{putc} \{ _ \} \right\} \right\} \\ \text{echo} \\ \left\{ \lambda v, \lceil v = 0 \rceil \right\}$$

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$$\{\lambda es, \ulcorner es = [] \urcorner * \exists v, P \ v * (getc_spec \ P) * (putc_spec \ P)\}$$

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$$\{\lambda es, \lceil es = [] \rceil * \exists v, P\ v\} \text{getc} \{\lambda ret, \lceil ret = v \rceil * P\ (v + 1)\}$$
$$\{\lambda es, \exists v, \lceil es = v \rceil * P\ (v + 1)\} \text{putc} \{\lambda ret, P\ (v + 1)\}$$
$$\{\lambda es, \lceil es = [] \rceil * \exists v, P\ v * (\text{getc_spec}\ P) * (\text{putc_spec}\ P)\}$$

echo

$$\{\lambda v, \lceil v = 0 \rceil\}$$

- Decentralized/language-agnostic multi-language semantics
 - No fixed source
 - No fixed set of languages
 - No fixed memory model
 - No fixed notion of linking
- Notion of semantic linking: \oplus
 - Link semantic components (modules) rather than syntactic
 - Link programs with specifications (abstract program)
- Program semantics as LTS, interaction via synchronization
- Reason locally in terms of interaction of two modules

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$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \sqsubseteq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

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$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket$ \preceq $\llbracket \text{echo}_{\text{spec}} \rrbracket$

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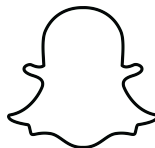
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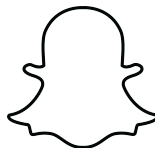


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$$\gamma_{\sigma_1} \rightsquigarrow \sigma_1, \gamma_{\sigma_2} \rightsquigarrow \sigma_2, \gamma_{\sigma_{\oplus}} \rightsquigarrow \sigma_{\oplus}, \gamma_{\sigma_{\text{spec}}} \rightsquigarrow \sigma_{\text{spec}}$$

- Get familiar with DimSum (and Iris)
- Experiment with new reasoning style
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int echo () :=  
  let c := getc();  
  putc(c);  
  return 0;
```

```
int getc (l) :=  
  let _ := read(l);  
  return *l;
```

```
void echo () :=  
  let c := getc();  
  putc(c);  
  echo ();
```

```
global pos = 0;  
read (l, c) :=  
  if (c <= 0) {  
    return 0;  
  } else {  
    l <- *pos;  
    pos <- *pos + 1;  
    ret = read(l + 1, c - 1);  
    return ret + 1;  
  }
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TGT Call "echo" es @  $\Pi$ 
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      PRE  $|-*: es \text{ POST}, \vdash es = [] \vdash *$ 
      POST  $|*: ret,$ 
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          POST  $|*: \_,$ 
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```
TGT Call "getc" es @ Π
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```
PRE |-*: es POST,  $\exists v, P\ v * \lceil es = [] \rceil *$ 
```

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POST |*: ret,  $\lceil ret = v \rceil * P\ (v + 1)$ .
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$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

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```
Lemma sim_getc_spec `(!specGS) Π Φ :
  switch Π
  PRE |-*: κ σ1 POST,
    ∃ f es h, ⌈κ = Some (Incoming, ERCall f es h)⌉ *
  POST Tgt _ _ |-*: σ' Π',
    ∃ v, ⌈f = "getc"⌉ * ⌈es = []⌉ * spec_state v * ⌈σ' = σ1⌉ (*)
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  PRE |-*: κ σ POST,
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Lemma `sim_getc_spec` `{!specGS}` $\Pi \Phi$:

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POST Tgt `_ _ |*:` $\sigma' \Pi'$,

$\exists v, \lceil f = \text{"getc"} \rceil * \lceil \text{es} = [] \rceil * \text{spec_state } v * \lceil \sigma' = \sigma1 \rceil (*)$

switch Π'

PRE `|-*: κ σ POST,`

$\lceil \kappa = \text{Some } (\text{Outgoing}, \text{ERReturn } (\text{ValNum } v) h) \rceil * \text{spec_state } (v + 1) *$

POST Tgt `_ _ |*:` $\sigma' \Pi''$,

$\lceil \sigma' = \sigma \rceil * \lceil \Pi'' = \Pi \rceil * \text{TGT } \text{getc_spec } @ \Pi \{ \{ \Phi \} \} \text{ } -*$

TGT `getc_spec @ Π { { Φ } }`.

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    ⌈κ = Some (Outgoing, ERReturn (ValNum v) h)⌉ * spec_state (v + 1) *
    POST Tgt _ _ |-*: σ' Π'',
      ⌈σ' = σ⌉ * ⌈Π'' = Π⌉ * TGT getc_spec @ Π {{ Φ }} -*
  TGT getc_spec @ Π {{ Φ }}.
```

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

```
getc_spec :=
  Spec.forever(
    TExists '(f, vs, h);
    TVis (In, Call f vs h);;
    TAssume (f = "getc");;
    TAssume (vs = []);;
    v ← TGet;
    TPut (v + 1);;
    TVis (Out, Return v h)).
```

```
Lemma sim_getc_spec `(!specGS) Π Φ :
  switch Π
  PRE |-*: κ σ1 POST,
    ∃ f es h, ⌈κ = Some (Incoming, ERCall f es h)⌉ *
  POST Tgt _ _ |-*: σ' Π',
    ∃ v, ⌈f = "getc"⌉ * ⌈es = []⌉ * spec_state v * ⌈σ' = σ1⌉ (*)
  switch Π'
  PRE |-*: κ σ POST,
    ⌈κ = Some (Outgoing, ERReturn (ValNum v) h)⌉ * spec_state (v + 1) *
  POST Tgt _ _ |-*: σ' Π'',
    ⌈σ' = σ⌉ * ⌈Π'' = Π⌉ * TGT getc_spec @ Π {{ Φ }} -*
  TGT getc_spec @ Π {{ Φ }}.
```

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

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getc_spec :=
  Spec.forever(
    TExists '(f, vs, h);
    TVis (In, Call f vs h);;
    TAssume (f = "getc");;
    TAssume (vs = []);;
    v ← TGet;
    TPut (v + 1);;
    TVis (Out, Return v h)).
```

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Lemma sim_getc_spec `(!specGS) Π Φ :
  switch Π
  PRE |-*: κ σ1 POST,
    ∃ f es h, ⌈κ = Some (Incoming, ERCall f es h)⌉ *
  POST Tgt _ _ |*: σ' Π',
    ∃ v, ⌈f = "getc"⌉ * ⌈es = []⌉ * spec_state v * ⌈σ' = σ1⌉ (*)
  switch Π'
  PRE |-*: κ σ POST,
    ⌈κ = Some (Outgoing, ERReturn (ValNum v) h)⌉ * spec_state (v + 1) *
  POST Tgt _ _ |*: σ' Π'',
    ⌈σ' = σ⌉ * ⌈Π'' = Π⌉ * TGT getc_spec @ Π {{ Φ }} -*
  TGT getc_spec @ Π {{ Φ }}.
```

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

Lemma `sim_getc fns Π_l Π_r PL σ_i :`

`"getc" \hookrightarrow None -*`

`PL σ_i -*`

`$\lceil \sigma_i.1 \equiv \text{getc_spec} \rceil$ -*`

`$\lceil \sigma_i.2 = 0 \rceil$ -*`

`\square switch_linked_fixed Tgt Π_l Π_r`

`PRE $|$ -*: σ_l POST, \exists h v σ_g , PL σ_g *`

`POST (ERCall "getc" \square h) σ_g $|$ *: σ_r Π_r' ,`

`switch_link Tgt Π_r'`

`Pre $|$ -*: σ_r' POST, \exists h'`

`POST (ERReturn (ValNum v) h') $_$ σ_l $|$ *: $_$ Π_l' ,`

`$\lceil \Pi_l' = \Pi_l \rceil$ * PL σ_r' ==*`

`\exists P, P 0 * \square rec_fn_spec_hoare Tgt Π_l "getc" (getc_fn_spec P).`

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

```

Lemma sim_getc fns  $\Pi_l \Pi_r$  PL  $\sigma_i$  :
  "getc"  $\hookrightarrow$  None -*
  PL  $\sigma_i$  -*
   $\ulcorner \sigma_i.1 \equiv \text{getc\_spec} \urcorner$  -*
   $\ulcorner \sigma_i.2 = 0 \urcorner$  -*
   $\square$  switch_linked_fixed Tgt  $\Pi_l \Pi_r$ 
    PRE  $|-*: \sigma_l$  POST,  $\exists h v \sigma_g, \text{PL } \sigma_g *$ 
    POST (ERCall "getc"  $\square h$ )  $\sigma_g |-*: \sigma_r \Pi_r'$ ,
    switch_link Tgt  $\Pi_r'$ 
    Pre  $|-*: \sigma_r'$  POST,  $\exists h'$ 
    POST (ERReturn (ValNum v)  $h'$ )  $_ \sigma_l |-*: _ \Pi_l'$ ,
     $\ulcorner \Pi_l' = \Pi_l \urcorner * \text{PL } \sigma_r' ==*$ 
     $\exists P, P 0 * \square \text{rec\_fn\_spec\_hoare Tgt } \Pi_l \text{"getc" (getc\_fn\_spec P)}.$ 

```

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

```

Lemma sim_getc fns  $\Pi_l$   $\Pi_r$  PL  $\sigma_i$  :
  "getc"  $\hookrightarrow$  None -*
  PL  $\sigma_i$  -*
   $\ulcorner \sigma_i.1 \equiv \text{getc\_spec} \urcorner$  -*
   $\ulcorner \sigma_i.2 = 0 \urcorner$  -*
   $\square$  switch_linked_fixed Tgt  $\Pi_l$   $\Pi_r$ 
    PRE  $\mid$  -:  $\sigma_l$  POST,  $\exists$  h v  $\sigma_g$ , PL  $\sigma_g$  *
    POST (ERCall "getc"  $\square$  h)  $\sigma_g$   $\mid$  -:  $\sigma_r$   $\Pi_r'$ ,
    switch_link Tgt  $\Pi_r'$ 
      Pre  $\mid$  -:  $\sigma_r'$  POST,  $\exists$  h'
      POST (ERReturn (ValNum v) h')  $\_$   $\sigma_l$   $\mid$  -:  $\_$   $\Pi_l'$ ,
       $\ulcorner \Pi_l' = \Pi_l \urcorner$  * PL  $\sigma_r'$  ==*
   $\exists$  P, P 0 *  $\square$  rec_fn_spec_hoare Tgt  $\Pi_l$  "getc" (getc_fn_spec P).
  
```

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

```

Lemma sim_getc fns  $\Pi_l \Pi_r$  PL  $\sigma_i$  :
  "getc"  $\hookrightarrow$  None -*
  PL  $\sigma_i$  -*
   $\lceil \sigma_i.1 \equiv \text{getc\_spec} \rceil$  -*
   $\lceil \sigma_i.2 = 0 \rceil$  -*
   $\square$  switch_linked_fixed Tgt  $\Pi_l \Pi_r$ 
    PRE  $|-*: \sigma_l$  POST,  $\exists h v \sigma_g, \text{PL } \sigma_g *$ 
    POST (ERCall "getc"  $\square h$ )  $\sigma_g |-*: \sigma_r \Pi_r'$ ,
    switch_link Tgt  $\Pi_r'$ 
      Pre  $|-*: \sigma_r'$  POST,  $\exists h'$ 
      POST (ERReturn (ValNum v)  $h'$ )  $_ \sigma_l |-*: _ \Pi_l'$ ,
       $\lceil \Pi_l' = \Pi_l \rceil * \text{PL } \sigma_r' ==*$ 
     $\exists P, P 0 * \square \text{rec\_fn\_spec\_hoare Tgt } \Pi_l \text{"getc"} (\text{getc\_fn\_spec } P).$ 

```



```

Lemma sim_getc fns  $\Pi_l \Pi_r$  PL  $\sigma_i$  :
  "getc"  $\hookrightarrow$  None -*
  PL  $\sigma_i$  -*
   $\lceil \sigma_i.1 \equiv \text{getc\_spec} \rceil$  -*
   $\lceil \sigma_i.2 = 0 \rceil$  -*
   $\square$  switch  $\Pi_l$ 
    PRE  $| -: \kappa \ \sigma_0 \text{ POST}, \exists h \ v \ \sigma_g, \text{PL } \sigma_g *$ 
    POST Tgt _ _  $| -: \sigma_{i0} \ \Pi_i, \lceil \sigma_{i0} = \sigma_g \rceil * \lceil \Pi_i = \Pi_r \rceil *$ 
    switch  $\Pi_i$ 
      PRE  $| -: \kappa' \ \sigma \text{ POST}_0, \exists e' : \text{rec\_ev}, \lceil \kappa' = \text{Some (Incoming, } e') \rceil *$ 
      POST0 Tgt _ _  $| -: \sigma_r \ \Pi_r, \lceil \sigma_r = \sigma \rceil * \lceil e' = \text{ERCall "getc" [] } h \rceil *$ 
      switch  $\Pi_r$ 
        PRE  $| -: \kappa_0 \ \sigma_1 \text{ POST}_1, \exists h', \lceil \kappa_0 = \text{Some (Outgoing, ERReturn } v \ h') \rceil *$ 
        POST1 Tgt _ _  $| -: \sigma_{i1} \ \Pi_{i0}, \lceil \sigma_{i1} = \sigma_0 \rceil *$ 
        switch  $\Pi_{i0}$ 
          PRE  $| -: \kappa'_0 \ \sigma_2 \text{ POST}_2, \exists e'_0, \lceil \kappa'_0 = \text{Some (Incoming, } e'_0) \rceil *$ 
          POST2 Tgt _ _  $| -: \sigma_{r0} \ \Pi_{r0},$ 
             $\lceil \sigma_{r0} = \sigma_2 \rceil * \lceil e'_0 = \text{ERReturn } v \ h' \rceil * \lceil \Pi_{r0} = \Pi_l \rceil * \text{PL } \sigma_1 == *$ 
         $\exists P, P \ 0 * \square \text{ rec\_fn\_spec\_hoare Tgt } \Pi_l \text{ "getc" (getc\_fn\_spec } P).$ 

```

- Lemma for TCallRet
- Keep Π s the same - new lemmas for linking
- Balance between Abstraction and Information
- Balance between Hacking and Thinking