Modular I/O Reasoning in DimSum

Alex Loitzl[1]

[1]Institute of Science and Technology Austria (ISTA)

March, 2025

Modular I/O Reasoning

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

{…} `echo` {…}

Modular I/O Reasoning

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$$\{\lambda\ es,\ \ulcorner es = [\ ] \urcorner\}\ \texttt{echo}\ \{\lambda\ v,\ \ulcorner v = 0 \urcorner\}$$

Modular I/O Reasoning

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$$\left\{\lambda\ \mathsf{es},\ \ulcorner \mathsf{es} = [\,] \urcorner * \{\lambda\ \mathsf{es},\ \ulcorner \mathsf{es} = [\,]\urcorner\}\texttt{getc}\{\lambda\ \mathsf{v},\ \{\lambda\ \mathsf{es},\ \ulcorner \mathsf{es} = \mathsf{v}\urcorner\}\texttt{putc}\{\_\}\ \}\right\}$$
$$\texttt{echo}$$
$$\left\{\lambda\ \mathsf{v},\ \ulcorner \mathsf{v} = 0\urcorner\right\}$$

Modular I/O Reasoning

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$$\{\lambda\ \mathsf{es},\ \ulcorner \mathsf{es} = [\ ] \urcorner * \exists\ \mathsf{v},\ \mathsf{P}\ \mathsf{v} * (\texttt{getc\_spec}\ \mathsf{P}) * (\texttt{putc\_spec}\ \mathsf{P})\}$$
$$\texttt{echo}$$
$$\{\lambda\ \mathsf{v},\ \ulcorner \mathsf{v} = 0 \urcorner\}$$

Modular I/O Reasoning

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$$\{\lambda\ \mathsf{es}, \ulcorner \mathsf{es} = [\,] \urcorner * \exists\ \mathsf{v}, \mathsf{P}\ \mathsf{v}\}\ \texttt{getc}\ \{\lambda\ \mathsf{ret}, \ulcorner \mathsf{ret} = \mathsf{v} \urcorner * \mathsf{P}\ (\mathsf{v} + 1)\}$$

$$\{\lambda\ \mathsf{es}, \exists\ \mathsf{v}, \ulcorner \mathsf{es} = \mathsf{v} \urcorner * \mathsf{P}\ (\mathsf{v} + 1)\}\ \texttt{putc}\ \{\lambda\ \mathsf{ret}, \mathsf{P}\ (\mathsf{v} + 1)\}$$

$$\{\lambda\ \mathsf{es}, \ulcorner \mathsf{es} = [\,] \urcorner * \exists\ \mathsf{v}, \mathsf{P}\ \mathsf{v} * (\texttt{getc\_spec}\ \mathsf{P}) * (\texttt{putc\_spec}\ \mathsf{P})\}$$
$$\texttt{echo}$$
$$\{\lambda\ \mathsf{v}, \ulcorner \mathsf{v} = 0 \urcorner\}$$

Multi-language Reasoning in DimSum

Rotation Project

Summary

- Formally verified compiler
  - Proof covers all optimizations
  - Correct w.r.t. the modeled semantics
- Discrepancies between hardware and model
  - Cannot implement correct calling conventions
  - Cannot support TriCore architecture
- Suboptimal code generation
  - Inserted moves
  - Higher register pressure

Example

$$\boxed{[\![\mathsf{echo}_{\mathsf{rec}} \quad \oplus \quad \mathsf{getc}_{\mathsf{spec}}]\!]} \preceq \quad [\![\mathsf{echo}_{\mathsf{spec}}]\!]$$

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

(Call f vs h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
Call f vs h
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

0

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

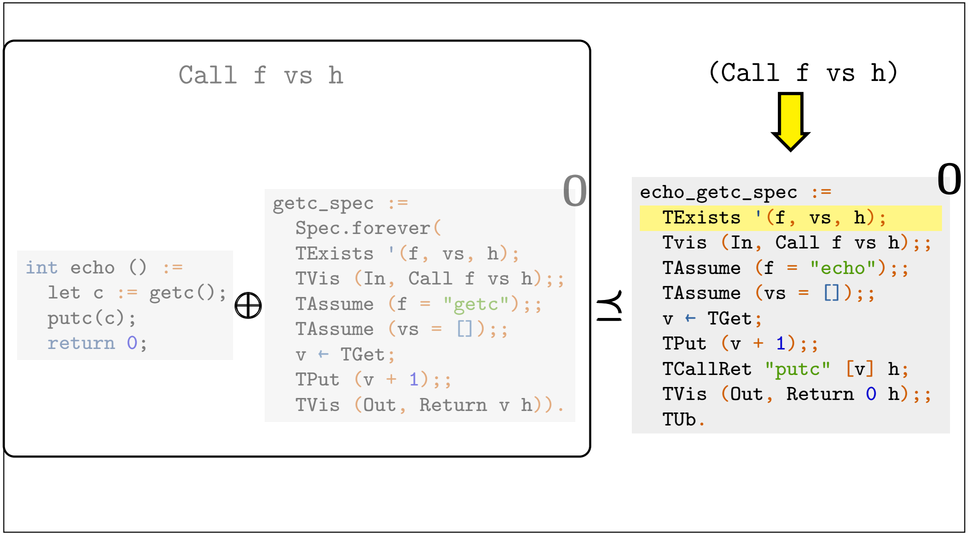
```
(Call f vs h)
```

$\Downarrow$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example



```
Call f vs h
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

```
(Call f vs h)
```

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

Call f vs h

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

(Call f vs h)

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
(Call "echo" vs h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
(Call "echo" [] h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
all "getc" [] h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

0

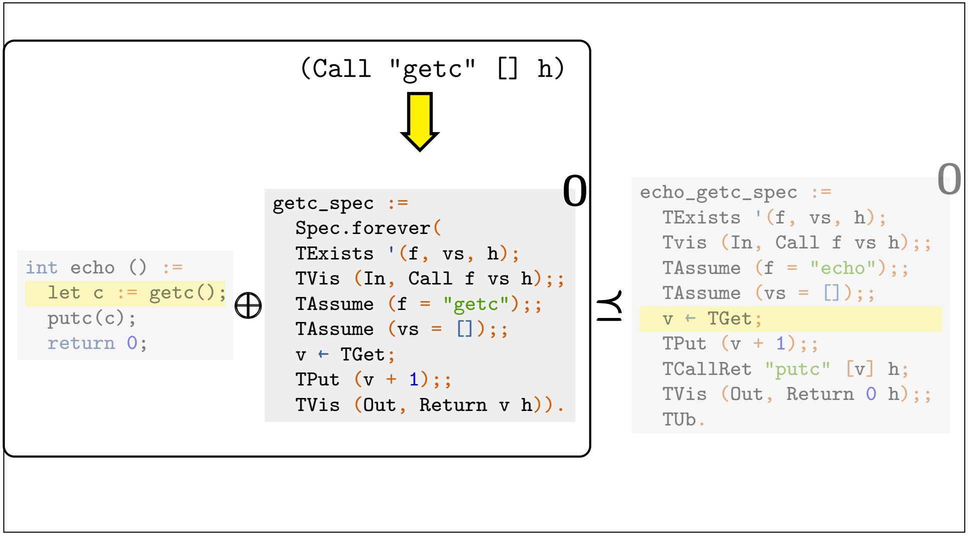
```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example



(Call "getc" [] h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

0
```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0
```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
(Call "getc" [] h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

$\mho$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

$\mho$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
(Call "getc" [] h)
```

↓

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

0

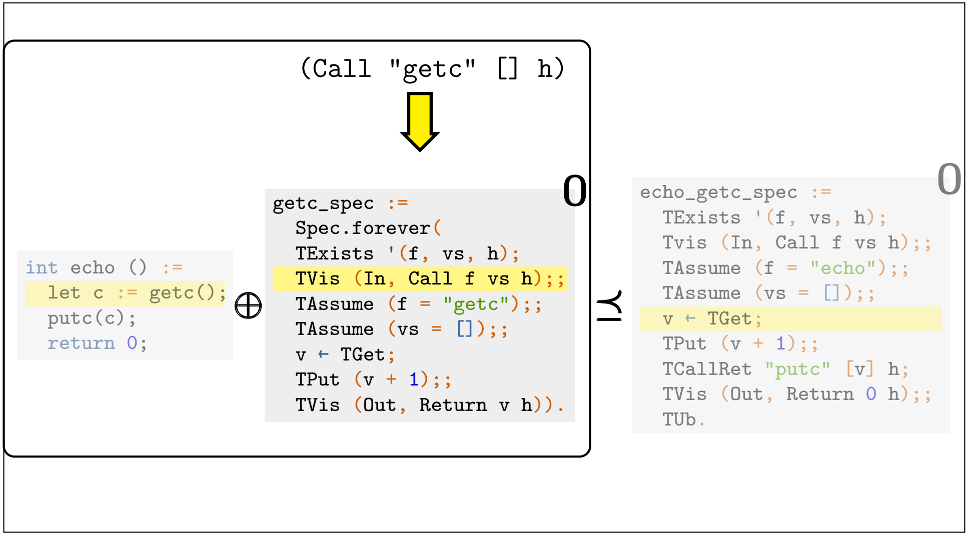
```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example



```
(Call "getc" [] h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

(Call "getc" [] h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

0

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
(Call "getc" [] h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

0

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

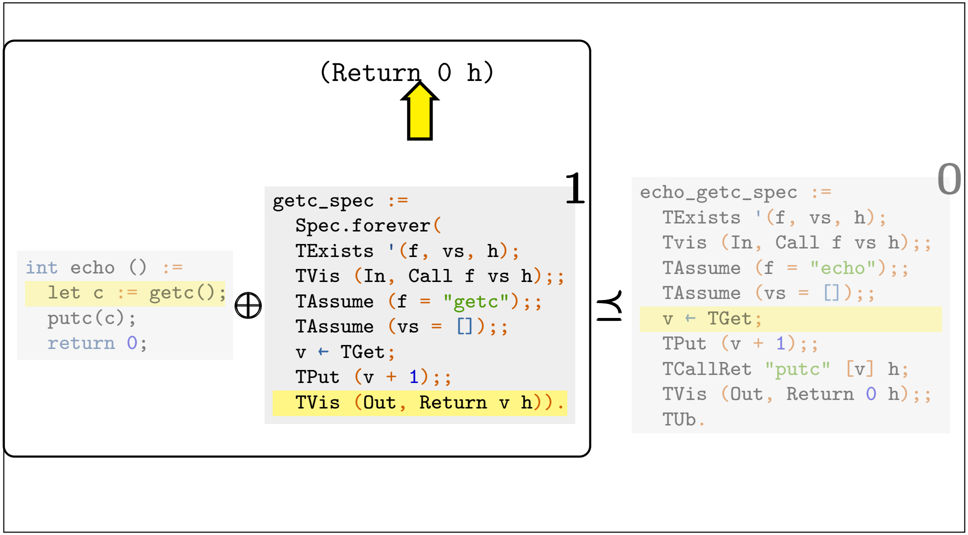
1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example



```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

(Return 0 h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
(Return 0 h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
Call "putc" 0 h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

0

Example

```
(Call "putc" 0 h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

```
(Call "putc" 0 h)
```

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

```
(Call "putc" 0 h)
```

$\Downarrow$

0

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
(Call "putc" 0 h)
```

$\Downarrow$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1
```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

```
(Call "putc" 0 h)
```

⇓

1
```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
(Call "putc" 0 h)
```

$\Downarrow$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Example

(Return v h)

⇑

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

⊕

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Example

(Return v h)

$\Downarrow$

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

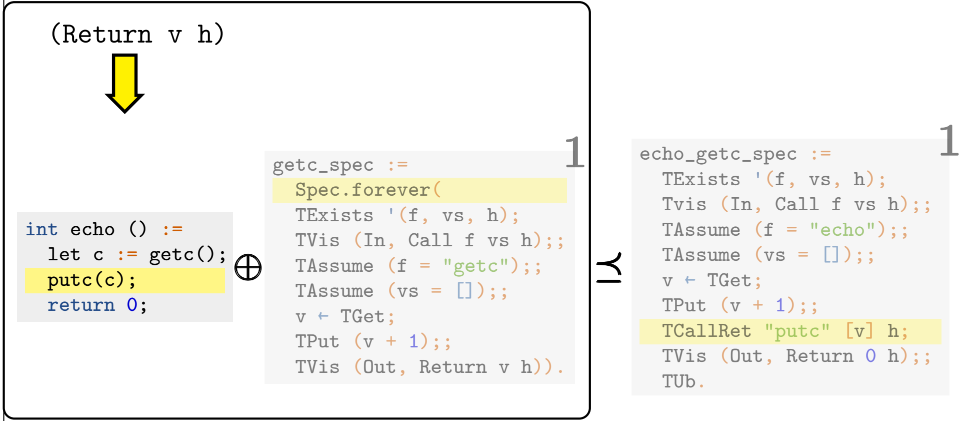
$\oplus$

1
```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

1
```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example



(Return v h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Example

```
(Return 0 h)
```

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

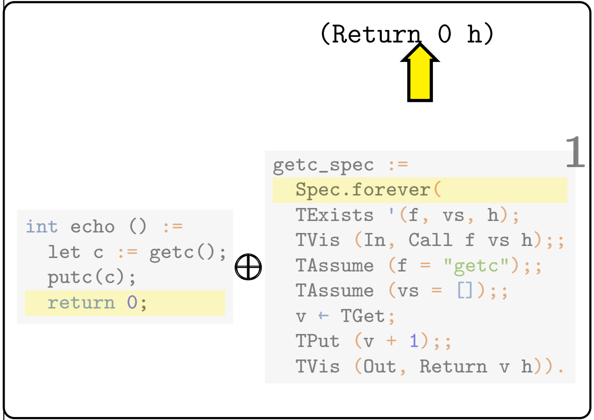
1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Example



```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

1

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

(Return 0 h)

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

1

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

$\preceq$

1

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

Example

```
int echo () :=
  let c := getc();
  putc(c);
  return 0;
```

$\oplus$

```
getc_spec :=
  Spec.forever(
  TExists '(f, vs, h);
  TVis (In, Call f vs h);;
  TAssume (f = "getc");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TVis (Out, Return v h)).
```

1

$\preceq$

```
echo_getc_spec :=
  TExists '(f, vs, h);
  Tvis (In, Call f vs h);;
  TAssume (f = "echo");;
  TAssume (vs = []);;
  v ← TGet;
  TPut (v + 1);;
  TCallRet "putc" [v] h;
  TVis (Out, Return 0 h);;
  TUb.
```

1

Towards Hoare-style Reasoning

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \quad \preceq \quad [\![\mathsf{echo}_{\mathsf{spec}}]\!]$$

Towards Hoare-style Reasoning

$\llbracket \mathsf{echo}_{\mathsf{rec}} \oplus \mathrm{getc}_{\mathsf{spec}} \rrbracket \approx\!> (\boldsymbol{\lambda} \kappa_t \sigma_t,$

Towards Hoare-style Reasoning

$[\![\mathsf{echo_{rec}} \oplus \mathsf{getc_{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_t \sigma_t, [\![\mathsf{echo_{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_s \sigma_s,$

Towards Hoare-style Reasoning

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_t \sigma_t, [\![\mathsf{echo}_{\mathsf{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_s \sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

Towards Hoare-style Reasoning

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \approx> (\lambda \kappa_t \sigma_t, \llbracket \text{echo}_{\text{spec}} \rrbracket \approx> (\lambda \kappa_s \sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \approx> \prod_s$$

Towards Hoare-style Reasoning

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_t \sigma_t, [\![\mathsf{echo}_{\mathsf{spec}}]\!] \approx\!> (\boldsymbol{\lambda} \kappa_s \sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \approx\!> \Pi_s$$

$$[\![\mathsf{echo}_{\mathsf{rec}}]\!] \approx\!> \Pi_{\oplus}(\Pi_s)$$

Towards Hoare-style Reasoning

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_t \sigma_t, [\![\text{echo}_{\text{spec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_s \sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \approx> \prod_s$$

$$[\![\text{echo}_{\text{rec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_l \sigma_l, \texttt{if} \_ \texttt{then} \; [\![\text{getc}_{\text{spec}}]\!] \approx> \ldots \; \texttt{else} \; \Pi_s \kappa_l \sigma)$$

$$[\![\text{echo}_{\text{rec}}]\!] \approx> \prod_{\oplus} (\Pi_s)$$

Towards Hoare-style Reasoning

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_t \sigma_t, [\![\mathsf{echo}_{\mathsf{spec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_s \sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \approx> \prod_s$$

$$[\![\mathsf{echo}_{\mathsf{rec}}]\!] \approx> (\boldsymbol{\lambda} \kappa_l \sigma_l, \texttt{if \_ then } [\![\mathsf{getc}_{\mathsf{spec}}]\!] \approx> \ldots \texttt{ else } \Pi_s \kappa_l \sigma)$$

$$[\![\mathsf{echo}_{\mathsf{rec}}]\!] \approx> \prod_{\oplus}(\Pi_s)$$

$$[\![\mathsf{echo}_{\mathsf{rec}}]\!] \approx> \prod$$

Towards Hoare-style Reasoning

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \approx\!> (\boldsymbol{\lambda}\kappa_t\sigma_t, [\![\text{echo}_{\text{spec}}]\!] \approx\!> (\boldsymbol{\lambda}\kappa_s\sigma_s, \kappa_t = \kappa_s * \sigma_t \preceq \sigma_s))$$

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \approx\!> \Pi_s$$

$$[\![\text{echo}_{\text{rec}}]\!] \approx\!> (\boldsymbol{\lambda}\kappa_l\sigma_l, \texttt{if \_ then } [\![\text{getc}_{\text{spec}}]\!] \approx\!> \ldots \texttt{ else } \Pi_s\kappa_l\sigma)$$

$$[\![\text{echo}_{\text{rec}}]\!] \approx\!> \Pi_\oplus(\Pi_s)$$

$$[\![\text{echo}_{\text{rec}}]\!] \approx\!> \Pi$$

$$\text{TGT Call “echo” es } @\ \Pi\ \{\{\Phi\}\}$$

(Modular) Hoare-style Reasoning

```
TGT Call "echo" es @ Π
  PRE |-∗: es POST_e, ⌜es = []⌝ ∗
    TGT Call "getc" es @ Π
      PRE |-∗: es POST, ⌜es = []⌝ ∗
      POST |∗: ret,
        TGT Call "putc" es @ Π
          PRE |-∗: es POST, ⌜es = [v]⌝ ∗
          POST |∗: _,
  POST_e |∗: ret, ⌜ret = 0⌝.
```

(Modular) Hoare-style Reasoning

```
TGT Call "getc" es @ Π
  PRE |-∗: es POST, ∃ v, P v ∗ ⌜es = []⌝ ∗
  POST |∗: ret, ⌜ret = v⌝ ∗ P (v + 1).
```

```
TGT Call "echo" es @ Π
  PRE |-∗: es POST_e, ⌜es = []⌝ ∗
    TGT Call "getc" es @ Π
      PRE |-∗: es POST, ⌜es = []⌝ ∗
      POST |∗: ret,
        TGT Call "putc" es @ Π
          PRE |-∗: es POST, ⌜es = [v]⌝ ∗
          POST |∗: _,
  POST_e |∗: ret, ⌜ret = 0⌝.
```

(Modular) Hoare-style Reasoning

```
TGT Call "getc" es @ Π
  PRE |-∗: es POST, ∃ v, P v ∗ ⌜es = []⌝ ∗
  POST |∗: ret, ⌜ret = v⌝ ∗ P (v + 1).
```

```
TGT Call "echo" es @ Π
  PRE |-∗: es POST_e, ∃ v, P v ∗ ⌜es = []⌝ ∗
    TGT Call "putc" es @ Π
      PRE |-∗: es POST, P (v + 1) ∗ ⌜es = [v]⌝ ∗
      POST |∗: _, P (v + 1) (∗)
  POST_e |∗: ret, ⌜ret = 0⌝ ∗ P (v + 1).
```

Example 1

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \preceq [\![\text{echo}_{\text{spec}}]\!]$$

```
Lemma sim_getc_spec `{!specGS} Π Φ :
  switch Π
    PRE |-∗: κ σ1 POST,
      ∃ f es h, ⌜κ = Some (Incoming, ERCall f es h)⌝ ∗
    POST Tgt _ _ |∗: σ' Π',
      ∃ v, ⌜f = "getc"⌝ ∗ ⌜es = []⌝ ∗ spec_state v ∗ ⌜σ' = σ1⌝ (∗)
  switch Π'
    PRE |-∗: κ σ POST,
      ⌜κ = Some (Outgoing, ERReturn (ValNum v) h)⌝ ∗ spec_state (v + 1) ∗
    POST Tgt _ _ |∗: σ' Π'',
      ⌜σ' = σ⌝ ∗ ⌜Π'' = Π⌝ ∗ TGT getc_spec @ Π {{ Φ }} -∗
  TGT getc_spec @ Π {{ Φ }}.
```

Example 1

$$\llbracket \text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}} \rrbracket \preceq \llbracket \text{echo}_{\text{spec}} \rrbracket$$

```
Lemma sim_getc_spec `{!specGS} Π Φ :
  switch Π
    PRE |-∗: κ σ1 POST,
      ∃ f es h, ⌜κ = Some (Incoming, ERCall f es h)⌝ ∗
    POST Tgt _ _ |∗: σ' Π',
      ∃ v, ⌜f = "getc"⌝ ∗ ⌜es = []⌝ ∗ spec_state v ∗ ⌜σ' = σ1⌝ (∗)
  switch Π'
    PRE |-∗: κ σ POST,
      ⌜κ = Some (Outgoing, ERReturn (ValNum v) h)⌝ ∗ spec_state (v + 1) ∗
    POST Tgt _ _ |∗: σ' Π'',
      ⌜σ' = σ⌝ ∗ ⌜Π'' = Π⌝ ∗ TGT getc_spec @ Π {{ Φ }} -∗
  TGT getc_spec @ Π {{ Φ }}.
```

Example 1

$$[\![\mathrm{echo}_{\mathrm{rec}} \oplus \mathrm{getc}_{\mathrm{spec}}]\!] \preceq [\![\mathrm{echo}_{\mathrm{spec}}]\!]$$

```
Lemma sim_getc_spec `{!specGS} Π Φ :
  switch Π
    PRE |-∗: κ σ1 POST,
      ∃ f es h, ⌜κ = Some (Incoming, ERCall f es h)⌝ ∗
    POST Tgt _ _ |∗: σ' Π',
      ∃ v, ⌜f = "getc"⌝ ∗ ⌜es = []⌝ ∗ spec_state v ∗ ⌜σ' = σ1⌝ (∗)
  switch Π'
    PRE |-∗: κ σ POST,
      ⌜κ = Some (Outgoing, ERReturn (ValNum v) h)⌝ ∗ spec_state (v + 1) ∗
    POST Tgt _ _ |∗: σ' Π'',
      ⌜σ' = σ⌝ ∗ ⌜Π'' = Π⌝ ∗ TGT getc_spec @ Π {{ Φ }} -∗
  TGT getc_spec @ Π {{ Φ }}.
```

Example 1

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \preceq [\![\text{echo}_{\text{spec}}]\!]$$

```
Lemma sim_getc_spec `{!specGS} Π Φ :
  switch Π
    PRE |-∗: κ σ1 POST,
      ∃ f es h, ⌜κ = Some (Incoming, ERCall f es h)⌝ ∗
    POST Tgt _ _ |∗: σ' Π',
      ∃ v, ⌜f = "getc"⌝ ∗ ⌜es = []⌝ ∗ spec_state v ∗ ⌜σ' = σ1⌝ (∗)
  switch Π'
    PRE |-∗: κ σ POST,
      ⌜κ = Some (Outgoing, ERReturn (ValNum v) h)⌝ ∗ spec_state (v + 1) ∗
    POST Tgt _ _ |∗: σ' Π'',
      ⌜σ' = σ⌝ ∗ ⌜Π'' = Π⌝ ∗ TGT getc_spec @ Π {{ Φ }} -∗
  TGT getc_spec @ Π {{ Φ }}.
```

Example 1

$$[\![\text{echo}_{\text{rec}} \oplus \text{getc}_{\text{spec}}]\!] \preceq [\![\text{echo}_{\text{spec}}]\!]$$

```
Lemma sim_getc_spec `{!specGS} Π Φ :
  switch Π
    PRE |-∗: κ σ1 POST,
      ∃ f es h, ⌜κ = Some (Incoming, ERCall f es h)⌝ ∗
    POST Tgt _ _ |∗: σ' Π',
      ∃ v, ⌜f = "getc"⌝ ∗ ⌜es = []⌝ ∗ spec_state v ∗ ⌜σ' = σ1⌝ (∗)
  switch Π'
    PRE |-∗: κ σ POST,
      ⌜κ = Some (Outgoing, ERReturn (ValNum v) h)⌝ ∗ spec_state (v + 1) ∗
    POST Tgt _ _ |∗: σ' Π'',
      ⌜σ' = σ⌝ ∗ ⌜Π'' = Π⌝ ∗ TGT getc_spec @ Π {{ Φ }} -∗
  TGT getc_spec @ Π {{ Φ }}.
```

Example 2

$$[\![\mathsf{echo}_{\mathsf{rec}} \oplus \mathsf{getc}_{\mathsf{spec}}]\!] \preceq [\![\mathsf{echo}_{\mathsf{spec}}]\!]$$

```
Lemma sim_getc fns Π_l Π_r PL σi :
  "getc" ↪ None -∗
  PL σi -∗
  ⌜σi.1 ≡ getc_spec⌝ -∗
  ⌜σi.2 = 0⌝ -∗
  □ switch_linked_fixed Tgt Π_l Π_r
    PRE |-∗: σ_l POST, ∃ h v σg, PL σg ∗
    POST (ERCall "getc" [] h) σg |∗: σr Π_r',
  switch_link Tgt Π_r'
    Pre |-∗: σ_r' POST, ∃ h'
    POST (ERReturn (ValNum v) h') _ σ_l |∗: _ Π_l',
      ⌜Π_l' = Π_l⌝ ∗ PL σ_r' ==∗
  ∃ P, P 0 ∗ □ rec_fn_spec_hoare Tgt Π_l "getc" (getc_fn_spec P).
```

Example 2

```
Lemma sim_getc fns Π_l Π_r PL σi :
  "getc" ↪ None -∗
  PL σi -∗
  ⌜σi.1 ≡ getc_spec⌝ -∗
  ⌜σi.2 = 0⌝ -∗
  □ switch Π_l
      PRE |-∗: κ σ0 POST, ∃ h v σg, PL σg ∗
      POST Tgt _ _ |∗:  σi0 Πi, ⌜σi0 = σg⌝ ∗ ⌜Πi = Π_r⌝ ∗
    switch Πi
      PRE |-∗: κ' σ POST0, ∃ e' : rec_ev, ⌜κ' = Some (Incoming, e')⌝ ∗
      POST0 Tgt _ _ |∗: σr Πr, ⌜σr = σ⌝ ∗ ⌜e' = ERCall "getc" [] h⌝ ∗
    switch Πr
      PRE |-∗: κ0 σ1 POST1, ∃ h', ⌜κ0 = Some (Outgoing, ERReturn v h')⌝ ∗
      POST1 Tgt _ _ |∗: σi1 Πi0, ⌜σi1 = σ0⌝ ∗
    switch Πi0
      PRE |-∗: κ'0 σ2 POST2, ∃ e'0, ⌜κ'0 = Some (Incoming, e'0)⌝ ∗
      POST2 Tgt _ _ |∗: σr0 Πr0,
        ⌜σr0 = σ2⌝ ∗ ⌜e'0 = ERReturn v h'⌝ ∗ ⌜Πr0 = Π_l⌝ ∗ PL σ1 ==∗
  ∃ P, P 0 ∗ □ rec_fn_spec_hoare Tgt Π_l "getc" (getc_fn_spec P).
```

Outcome & Takeaways

- Lemma for `TCallRet`
- Keep Πs the same - new lemmas for linking
- Balance between Abstraction and Information
- Balance between Hacking and Thinking