Compton Scattering Densities vs Pion Photoproduction Densities

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1 Translating Densities

The reaction of interest for pion photoproduction is $\gamma A \to \pi^0 A$ and for Compton scattering we have $\gamma A \to \gamma A$.
In both cases the incoming photon has 4 momentum: $(\omega, 0, 0, \omega)$ however there is a difference in the form of the outgoing.

Outgoing for Compton:

$$k' = (\omega, \omega\, R(\theta)\boldsymbol{e}_z) \implies t = (k' - k)^2 = q^2 = -2\omega^2(1 - \cos\theta) \tag{1}$$

Outgoing for threshold pion photoproduction, since the pion is at rest

$$k' = (m_{\pi^0}, 0, 0, 0) \implies t = (k' - k)^2 = q^2 = m_{\pi^0}^2 - 2\omega_{th} m_{\pi^0} \tag{2}$$

Where ω_{th} is the threshold energy of the pion, note the lack of angle dependence.

The densities are dependent on the mandalstam variables that is:

$$t = q^2 = -(140\text{MeV})^2 \neq m_{\pi^0}^2 - 2\omega_{th} m_{\pi^0} \tag{3}$$

In order to calculate pion photoproduction at threshold we choose the angle θ, such that:

$$m_{\pi^0}^2 - 2\omega_{th} m_{\pi^0} = -2\omega_{th}^2(1 - \cos\theta_{\text{Compton}}) \tag{4}$$

First the calculation of ω_{th}

$$\begin{aligned}
s = (k+p)^2 &= \left[\left(\sqrt{M^2 + \omega_{th}^2}, 0, 0, -\omega_{th}\right) + (\omega_{th}, 0, 0, \omega_{th})\right]^2 && (5)\\
&= [(M, 0, 0, 0) + (m_{\pi^0}, 0, 0, 0)]^2 && (6)\\
&\implies (\sqrt{M^2 + \omega_{th}^2} + \omega, 0, 0, 0)^2 = (M + m_{\pi^0})^2 && (7)\\
&\implies \omega_{th} = \frac{m_{\pi^0}(m_{\pi^0} + 2M)}{2(m_{\pi^0} + M} = 131.875 \text{ MeV} && (8)
\end{aligned}$$

Where M is the mass of the nucleus.
For ^{3}He, $M = 2808.4$MeV, also $m_{\pi^0} = 139.97$MeV $\implies \omega_{th} = 138.21$MeV So then from eq.4 we have:

$$\begin{aligned}
\theta &= \arccos\left(\frac{m_{\pi^0}^2 - 2m_{\pi^0}\omega_{th} + 2\omega_{th}^2}{2\omega_{th}^2}\right) && (9)\\
&= \arccos\left(\frac{m_{\pi^0}^2 + 2m_{\pi^0}M + 2M^2}{(m_{\pi^0} + 2M)^2}\right) && (10)\\
&= 59.98^\circ && (11)
\end{aligned}$$

2 General Kinematics

Define all parameters, use CM frame

$$\text{Incoming Photon Momenta:}\quad k = (\omega, \omega \boldsymbol{e}_z) \tag{12}$$

$$\text{Incoming Nucleon Momenta:}\quad -k = \left(\sqrt{M^2+\omega^2},\ -\omega \boldsymbol{e}_z\right) \tag{13}$$

From N. Rijneveen - "Pion photoproduction in chiral perturbation theory...", we can express the center of mass energy in terms of the Mandalstam variables as the following:

$$\omega = \frac{s - M^2}{2\sqrt{s}}, \quad E_\pi = \frac{s + m^2 - M^2}{2\sqrt{s}} \tag{14}$$

Where m is the pion mass and M is the Nucleon mass. We already can calculate the Mandalstam variable s, so now its easy to find the pion momentum k' by just inverting:

$$E_\pi = \frac{s + m^2 - M^2}{2\sqrt{s}} = \sqrt{m^2 + \vec{k}'^2} \tag{15}$$

$$\Longrightarrow \vec{k}'^2 = \frac{(s + m^2 - M^2)^2}{4s} - m^2 = \frac{1}{4s}\left[m^4\left(M^2 - s\right)^2 - 2m^2\left(M^2 + s\right)\right] \tag{16}$$

So now we have k' in terms of variables we have access to. For the outgoing nucleon, we have four momentum conservation which gives $\omega + \sqrt{M^2+\omega^2} = E_\pi + E_N'$, and the momentum of the outgoing nucleon is $-k'$

$$E_N' = \omega + \sqrt{M^2+\omega^2} - E_\pi \tag{17}$$

$$E_\pi = \frac{s + m^2 - M^2}{2\sqrt{s}} = \sqrt{m^2 + \vec{k}'^2} \tag{18}$$

$$\text{Outgoing pion momenta:}\ (E_\pi, \vec{k}') = (E_\pi, |k'| R(\theta) \boldsymbol{e}_z) = (E_\pi, 0, |k'|\sin\theta, |k'|\cos\theta) \tag{19}$$

$$\text{Outgoing nucleon momenta:}\ (E_N', -\vec{k}') = (E_N', -|k'| R(\theta) \boldsymbol{e}_z) = (E_N', 0, -|k'|\sin\theta, -|k'|\cos\theta) \tag{20}$$

Where without loss of generality the scattering is in the y plane. And in the code as a check we can verify:

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \quad \text{Along with the other Mandalstam variables} \tag{21}$$

$$\frac{s + m^2 - M^2}{2\sqrt{s}} = \sqrt{m^2 + \vec{k}'^2} \quad \text{Pion energy} \tag{22}$$

$$\omega + \sqrt{M^2+\omega^2} = E_\pi + E_N' \quad \text{Energy conservation} \tag{23}$$

$$\tag{24}$$

We don't need to check the 3 momentum conservation because the total three momentum is zero by construction. Additionally we can check we get the same as the Compton case when we set $m_\pi = 0$.