

side the deuteron, which is in conflict with general properties of field theories. In Ref. [22] both problems were solved: once the  $\Delta$ -isobar is included as an explicit degree of freedom, simultaneously the residual boost contribution becomes sufficiently small to be in line with Weinberg counting and appears at an order where there is also a counter term.

Finally, we analyze the contribution of the triple scattering diagram of Fig. 2 in Sec. 5.3. In Weinberg counting this diagram is estimated to contribute only at order  $(m_\pi/m_N)^2$  relative to the leading two-nucleon operator. On the other hand numerical studies revealed that its actual value exceeds this estimate by about an order of magnitude. In Sec. 5.3 we will demonstrate that this enhancement is not due to the smallness of the deuteron binding energy, contrary to the claim in Ref. [20], but due to an integrable singularity in the corresponding expression that produces an enhancement by a factor of  $\pi^2$ . In view of this, also item *ii*) of the list that was used to argue in favour of  $Q$ -counting does not apply anymore.

We are led to conclude that Weinberg counting gives a more reliable power counting scheme than  $Q$ -counting.

With these remarks, we now want to look at the possible few-nucleon contributions in more detail.

### 3 Few-nucleon contributions

Our main concern here are the few-nucleon corrections. For an evaluation of such contributions, expectation values of the basic pion-few-nucleon ( $\pi$ - $A$ ) amplitudes with the few-nucleon wave functions need to be calculated. Such wave functions can be obtained using standard methods to solve the few-nucleon Schrödinger equation based on nucleon-nucleon (NN) interactions [23]. Whereas in the first attempts to understand such contributions, wave functions based on phenomenological NN interactions were used [18,19], it is by now standard to also employ wave functions generated with nuclear interactions based on ChPT. The basic idea is to use naive dimensional analysis for a potential, which is then used to solve a Schrödinger equation [24,25]. Several groups have developed NN interactions based on this approach [26,27,28] and also three- (3N) and four-nucleon (4N) interactions have been formulated [29,30] and employed [31], see also Ref. [32] for a recent review. Obviously, employing chiral interactions is preferable to calculate the pion-nucleus scattering length since nuclear interactions and  $\pi$ - $A$  amplitudes will be consistent.

In this work, we will present results on both kinds of interactions. We still show results based on phenomenological interactions, namely AV18 [33], Nijmegen 93 [34], and CD-Bonn [35]. For  $^3\text{He}$  and  $^4\text{He}$ , we augment the nuclear Hamiltonian by a 3N interaction, so that the binding energies of these nuclei are reasonably well reproduced [36]. These results may serve as benchmark and might give indications on the size of the model-dependence of older calculations. We also employ wave functions based on chiral nuclear interactions between the leading order (LO, or-

der 0 in the chiral expansion) and next-to-next-to-leading order ( $\text{N}^2\text{LO}$ , order 3 in the chiral expansion).

Chiral interactions require a regularization scheme in order to obtain a well-defined Schrödinger equation. Most realizations use a momentum cutoff of the order of 500 MeV to this aim. For the leading order, at least when restricting oneself to  $S$ -wave interactions, it is possible to obtain fits for a much larger range of cutoffs [37]. These attempts have triggered a controversy in the community [38]. It should be noted that an inconsistency becomes apparent for large cutoffs, when higher order NN interactions are used [39]. Here, we use a wide range of cutoffs only for LO interactions, so that also this inconsistency does not apply. This allows us to use results for a wide range of cutoffs to estimate the size of leading counter terms. Parameter sets for the  $S$ -wave contact interactions are given in Tables 3, 4, and 5. Since these calculations are by no means high precision ones, we have neglected the minor contribution of higher partial waves in this case.

For the higher order chiral forces, we have employed order 2 (NLO) and order 3 ( $\text{N}^2\text{LO}$ ) ones of Ref. [28]. In the  $\text{N}^2\text{LO}$  case, we added, as required by power counting, 3N forces, which were tuned to reproduce the  $^3\text{He}$  binding energy and  $\text{N-}^2\text{H}$  scattering lengths.

Since we restrict ourselves to leading, tree-level  $\pi$ - $A$  operators, we may derive them using Feynman diagrams as done below. The reduction from four-dimensional quantities to three-dimensional ones can be easily performed using the on-shell energies of nucleons and pions. The pertinent integrals involving the wave functions and operators will be given below since their form depends on the number of nucleons involved.

#### 3.1 Leading two-nucleon contributions

The leading two-nucleon contributions are known for many years [18,19,20]. In the following, we call the numerically most important contribution, depicted in Fig. 1(a), “Coulombian” because of its  $\frac{1}{q^2}$  pion propagator. The explicit expression for the amplitude is

$$i\mathcal{M}^{(1a)} = i \frac{m_\pi^2}{4f_\pi^4 q^2} \{ 2\delta^{ab} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) - \tau_1^b \tau_2^a - \tau_1^a \tau_2^b \} \quad (6)$$

where  $\mathbf{q}$  is the momentum transfer between the nucleons, the  $\boldsymbol{\tau}_i$  are usual Pauli matrices acting in the isospin space of nucleon  $i$  and  $f_\pi$  the pion decay constant. Throughout this work, we employ  $f_\pi = 92.4$  MeV. The small latin letters refer to isospin indices of the pions as given in the figure.

The amplitudes of Figs. 1(b) and 1(c) are individually dependent on the parametrization of the pion field. The sum of both, however, is independent of this choice as it should. Therefore, we will only show results for the sum of both contributions for which the amplitude reads

$$i\mathcal{M}^{(1b+1c)} = -i \frac{g_A^2 m_\pi^2}{4f_\pi^4} \frac{1}{(q^2 + m_\pi^2)^2} (\boldsymbol{\sigma}_1 \cdot \mathbf{q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \times \{ \delta^{ab} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) - (\tau_1^a \tau_2^b + \tau_1^b \tau_2^a) \}. \quad (7)$$