

Neutral pion photoproduction off ^3H and ^3He in chiral perturbation theory

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Abstract

We calculate electromagnetic neutral pion production off three-nucleon bound states (^3H , ^3He) at threshold to leading one-loop order in the framework of chiral nuclear effective field theory. In addition, we analyze the dependence of the nuclear S-wave amplitude E_{0+} on the elementary neutron amplitude $E_{0+}^{\pi^0 n}$ which in the case of ^3He provides a stringent test of the prediction based on chiral perturbation theory. Uncertainties from higher order corrections are estimated.

Keywords: Pion production, Chiral Lagrangians, Neutron properties

PACS: 13.75.Gx, 12.39.Fe, 13.40.Ks

1. Introduction

Threshold neutral pion photo- and electroproduction off the nucleon is one of the finest reactions to test the chiral QCD dynamics, see [1] for a recent review. While elementary proton targets are accessible directly in experiment, pion production off neutrons requires the use of nuclear targets like the deuteron or three-nucleon bound states like ^3H (triton) or ^3He . For a recent review on revealing the neutron structure from electron or photon scattering off light nuclei, see [2]. Of particular interest is to test the counterintuitive chiral perturbation theory prediction (CHPT) that the elementary neutron S-wave multipole $E_{0+}^{\pi^0 n}$ is larger in magnitude than the corresponding one of the proton, $E_{0+}^{\pi^0 p}$ [3, 4]. This prediction was already successfully tested in neutral pion photo- [5] and electroproduction off the deuteron [6]. However, given the scarcity and precision of the corresponding data, it is mandatory to study also pion production off three-nucleon bound states, that can be calculated nowadays to high precision based on chiral nuclear effective field theory (EFT), that extends CHPT to nuclear physics (for a recent review, see [7]). ^3He appears to be a particularly promising target to extract the information about the neutron amplitude. Its wave function is strongly dominated by the principal “s-state” component which suggests that the spin of ^3He is largely driven by the one of the neu-

tron. Consequently, in this letter we calculate threshold pion photo- and electroproduction based on chiral 3N wave functions at next-to-leading order in the chiral expansion. Experimentally, neutral pion photoproduction off light nuclei has so far only been studied at Saclay [8, 9] and at Saskatoon [10, 11].

In general, one has three different topologies for pion production off a three-nucleon bound state as shown in Fig. 1. While the single-nucleon contribution (a) features the elementary neutron and proton production amplitudes, the nuclear corrections are given by two-body (b) and three-body (c) terms. Based on the power counting developed in [12], at next-to-leading order (NLO), only the topologies (a) and (b) contribute. Here, we will specifically consider threshold photo- and electroproduction parameterized in terms of the electric E_{0+} and longitudinal L_{0+} S-wave multipoles. In particular, we will study the sensitivity of the three-body S-wave multipoles to the elementary $E_{0+}^{\pi^0 n}$ multipole, taking the proton amplitude $E_{0+}^{\pi^0 p}$ from CHPT (as this value is consistent with the data [13] and a recent study based on a chiral unitary approach [14]).

2. Anatomy of the Calculation

To analyze the process under consideration, we calculate the the nuclear matrix element of the given

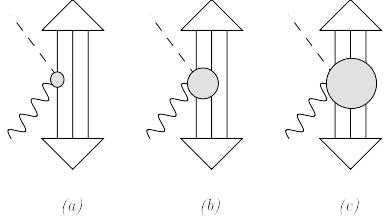


Figure 1: Different topologies contributing to pion production off the three-nucleon bound state (triangle). (a), (b) and (c) represent the single-, two- and three-nucleon contributions, respectively. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order. Topology (c) does not contribute to the order considered here (NLO).

transition operator \hat{O} as:

$$\langle M'_J | \hat{O} | M_J \rangle_\psi := \langle \psi M'_J \vec{P}'_{3N} \vec{q}_\pi | \hat{O} | \psi M_J \vec{P}_{3N} \vec{k}_\gamma \rangle, \quad (1)$$

where ψ refers to the three-nucleon wave function and \vec{k}_γ , \vec{q}_π , \vec{P}_{3N} and \vec{P}'_{3N} denote the momentum of the exchanged (virtual) photon, produced pion and the initial and final momentum of the 3N nucleus, respectively. The 3N bound state has total nuclear angular momentum $J = 1/2$ with magnetic quantum numbers M_J for the initial and M'_J for the final nuclear state. J can be decomposed in total spin $S = 1/2, 3/2$ and total orbital angular momentum $L = 0, 1, 2$. The total isospin is a mixture of two components, $T = 1/2$ and $3/2$. While the $T = 1/2$ component is large, the small $T = 3/2$ component emerges due to isospin breaking and is neglected in our calculation. Here, we consider neutral pion production by real or virtual photons of a spin-1/2 particle - either the nucleon or the ${}^3\text{H}$ and ${}^3\text{He}$ nuclei. At threshold, the corresponding transition matrix takes the form

$$\mathcal{M}_\lambda = 2i E_{0+} (\vec{\epsilon}_{\lambda,T} \cdot \vec{S}) + 2i L_{0+} (\vec{\epsilon}_{\lambda,L} \cdot \vec{S}), \quad (2)$$

with $\vec{\epsilon}_{\lambda,T} = \vec{\epsilon}_\lambda - (\vec{\epsilon}_\lambda \cdot \hat{k}_\gamma) \hat{k}_\gamma$ and $\vec{\epsilon}_{\lambda,L} = (\vec{\epsilon}_\lambda \cdot \hat{k}_\gamma) \hat{k}_\gamma$ the transverse and longitudinal photon polarization vectors. The transverse and longitudinal S-wave multipoles are denoted by E_{0+} and L_{0+} , respectively. Note that L_{0+} contributes only for virtual photons.

As explained before, the matrix element Eq. (1) receives contributions from one- and two-nucleon operators at the order we are working. Consider first the single nucleon contribution, given in terms of the 1-body transition operator \hat{O}^{1N} . After some algebra, one finds

$$\begin{aligned} & \langle M'_J | \hat{O}^{1N} | M_J \rangle_\psi \\ &= i \vec{\epsilon}_{\lambda,T} \cdot \vec{S}_{M'_J M_J} (E_{0+}^{\pi^0 p} F_T^{S+V} + E_{0+}^{\pi^0 n} F_T^{S-V}) \end{aligned}$$

$$+ i \vec{\epsilon}_{\lambda,L} \cdot \vec{S}_{M'_J M_J} (L_{0+}^{\pi^0 p} F_L^{S+V} + L_{0+}^{\pi^0 n} F_L^{S-V}). \quad (3)$$

where $F_{T/L}^{S\pm V} \equiv F_{T/L}^S \pm F_{T/L}^V$ and $F_{T/L}^{S,V}$ denote the corresponding form factors of the 3N bound state,

$$\begin{aligned} F_{T/L}^S \vec{\epsilon}_{\lambda,T/L} \cdot \vec{S}_{M'_J M_J} &= \frac{3}{2} \langle M'_J | \vec{\epsilon}_{\lambda,T/L} \cdot \vec{\sigma}_1 | M_J \rangle_\psi, \quad (4) \\ F_{T/L}^V \vec{\epsilon}_{\lambda,T/L} \cdot \vec{S}_{M'_J M_J} &= \frac{3}{2} \langle M'_J | \vec{\epsilon}_{\lambda,T/L} \cdot \vec{\sigma}_1 \tau_1^z | M_J \rangle_\psi, \end{aligned}$$

which parametrize the overall normalization of the response of the composite system to the excitation by photons in spin-isospin space. In the above equation, $\vec{\sigma}_i$ ($\vec{\tau}_i$) denote the spin (isospin) Pauli matrices corresponding to the nucleon i . Furthermore, z refers to the isospin quantization axis.

Using the 3N wave functions from chiral nuclear EFT at the appropriate order, the pertinent matrix elements in Eq. (4) can be evaluated. Here, we use chiral 3N wave functions obtained from the N²LO interaction in the Weinberg power counting [15, 16].¹ In order to estimate the error from higher order corrections, we use wave functions for five different combinations of the cutoff $\tilde{\Lambda}$ in the spectral function regularization of the two-pion exchange and the cutoff Λ used to regularize the Lippmann-Schwinger equation for the two-body T-matrix. The wave functions are taken from Ref. [17, 18] and the corresponding cutoff combinations in units of MeV are $(\tilde{\Lambda}, \Lambda) = (450, 500)$, $(600, 500)$, $(550, 600)$, $(450, 700)$, $(600, 700)$. All five sets describe the binding energies of the ${}^3\text{He}$ and ${}^3\text{H}$ nuclei equally well.

The one-body contributions to the 3N multipoles are given by

$$\begin{aligned} E_{0+}^{1N} &= \frac{K_{1N}}{2} (E_{0+}^{\pi^0 p} F_T^{S+V} + E_{0+}^{\pi^0 n} F_T^{S-V}), \\ L_{0+}^{1N} &= \frac{K_{1N}}{2} (L_{0+}^{\pi^0 p} F_L^{S+V} + L_{0+}^{\pi^0 n} F_L^{S-V}). \end{aligned} \quad (5)$$

Here, K_{1N} is the kinematical factor to account for the change in phase space from the 1N to the 3N system,

$$K_{1N} = \frac{m_N + M_\pi}{m_{3N} + M_\pi} \frac{m_{3N}}{m_N} \approx 1.092, \quad (6)$$

with m_N being the nucleon mass and m_{3N} the mass of the three-nucleon bound state.

¹The consistency of the Weinberg counting for short-range operators and the non-perturbative renormalization of chiral EFT are currently under discussion, see the review [7] for more details. A real alternative to the Weinberg approach for practical calculations, however, is not available.

nucleus	^3He	^3H
F_T^{S+V}	0.017(13)(3)	1.493(25)(3)
F_T^{S-V}	1.480(26)(3)	0.012(13)(3)
F_L^{S+V}	-0.079(14)(8)	1.487(27)(8)
F_L^{S-V}	1.479(26)(8)	-0.083(14)(8)

Table 1: Numerical results for the form factors $F_{T/L}^{S\pm V}$. The first error is an estimate of the theory error from higher orders in chiral EFT while the second error is the statistical error from the Monte Carlo integration.

We evaluate the matrix elements for the one-body contribution in Eq. (4) numerically with Monte Carlo integration using the VEGAS algorithm [19]. The results for the form factors $F_{T/L}^{S\pm V}$ are given in Table 1. The first error represents the theoretical uncertainty estimated from the cutoff variation in the wave functions. We take the central value defined by the five different cutoff sets as our prediction and estimate the theory error from higher-order corrections from the spread of the calculated values. Strictly speaking, this procedure gives a lower bound on the error, but in practice it generates a reasonable estimate.

We stress that we follow the nuclear EFT formulation of Lepage, in which the whole effective potential is iterated to all orders when solving the Schrödinger equation for the nuclear states. As discussed in Ref. [20], the cutoff should be kept of the order of the breakdown scale or below in order to avoid unnatural scaling of the coefficients of higher order terms. Indeed, using larger cutoffs can lead to a violation of certain low-energy theorems as demonstrated in Ref. [21] for an exactly solvable model.

The error related to the expansion of the production operator is difficult to estimate given that the convergence in the expansion for the single nucleon S-wave multipoles is known to be slow, see Ref. [3] for an extended discussion. We therefore give here only a rough estimate of this uncertainty. The extractions of the proton S-wave photoproduction amplitude based on CHPT using various approximations [22] lead to an uncertainty $\Delta E_{0+}^{\pi^0 p} \approx \pm 0.05 \times 10^{-3}/M_{\pi^+}$, which is about 5%. Similarly, we estimate the uncertainty of the neutron S-wave threshold amplitude to be the same. Consequently, our estimate of the error on the single nucleon amplitude is 5%.

The statistical error from the evaluation of the integrals is typically one order of magnitude smaller

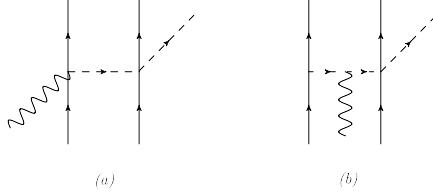


Figure 2: Leading two-nucleon contributions to the nuclear pion production matrix element at threshold. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order.

than the estimated theory error and can be neglected.

We now switch to the two-nucleon contribution. In Coulomb gauge, only the two Feynman diagrams shown in Fig. 2 contribute at threshold to the order we are working [5]. Their contribution to the multipoles can be written as

$$\begin{aligned} E_{0+}^{2N} &= K_{2N} (F_T^{(a)} - F_T^{(b)}) , \\ L_{0+}^{2N} &= K_{2N} (F_L^{(a)} - F_L^{(b)}) , \end{aligned} \quad (7)$$

with the prefactor

$$\begin{aligned} K_{2N} &= \frac{M_\pi e g_A m_{3N}}{16\pi(m_{3N} + M_\pi)(2\pi)^3 F_\pi^3} \\ &\approx 0.135 \text{ fm} \times 10^{-3}/M_{\pi^+} . \end{aligned} \quad (8)$$

The numerical value for K_{2N} was obtained using $g_A = 1.26$ for the axial coupling constant, $F_\pi = 93$ MeV for the pion decay constant, and the neutral pion mass $M_\pi = 135$ MeV. The transverse and longitudinal form factors $F_{T/L}^{(a)}$ and $F_{T/L}^{(b)}$ corresponding to diagrams (a) and (b), respectively, are

$$F_{T/L}^{(a)} \vec{\epsilon}_{\lambda,T/L} \cdot \vec{S}_{M'_J M_J} = \frac{3}{2} \langle M'_J | \frac{\vec{\epsilon}_{\lambda,T/L} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) (\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^z \tau_2^z)}{(\vec{p}_{12} - \vec{p}'_{12} + \vec{k}_\gamma/2)^2} | M_J \rangle_\psi , \quad (9)$$

and

$$F_{T/L}^{(b)} \vec{\epsilon}_{\lambda,T/L} \cdot \vec{S}_{M'_J M_J} = 3 \langle M'_J | (\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^z \tau_2^z) \times \frac{[(\vec{p}_{12} - \vec{p}'_{12} - \vec{k}_\gamma/2) \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)]}{[(\vec{p}_{12} - \vec{p}'_{12} - \vec{k}_\gamma/2)^2 + M_\pi^2][\vec{p}_{12} - \vec{p}'_{12} + \vec{k}_\gamma/2]^2} \times [\vec{\epsilon}_{\lambda,T/L} \cdot (\vec{p}_{12} - \vec{p}'_{12})] | M_J \rangle_\psi , \quad (10)$$

where $\vec{p}_{12} = (\vec{k}_1 - \vec{k}_2)/2$ and $\vec{p}'_{12} = (\vec{k}'_1 - \vec{k}'_2)/2$ are the initial and final Jacobi momenta of nucleons 1 and 2, respectively. The integral for the form factors $F_{T/L}^{(a)}$ contains an integrable singularity which can be removed by an appropriate variable transformation. Then, the form factors can be evaluated using Monte Carlo integration in the same way

nucleus	${}^3\text{He}$	${}^3\text{H}$
$F_T^{(a)} - F_T^{(b)} [\text{fm}^{-1}]$	-29.3(2)(1)	-29.7(2)(1)
$F_L^{(a)} - F_L^{(b)} [\text{fm}^{-1}]$	-22.9(2)(1)	-23.2(1)(1)

Table 2: Numerical results for the form factors $F_{T/L}^{(a)} - F_{T/L}^{(b)}$ parametrizing two-body contributions in units of fm^{-1} . The first error is an estimate of the theory error from higher orders in chiral EFT while the second error is the statistical error from the Monte Carlo integration.

as the form factors for the single-nucleon contribution. Our results for $F_{T/L}^{(a)} - F_{T/L}^{(b)}$ are given in Table 2. The first error is again the theory error estimated from the cutoff variation in the chiral interaction as described above. The second error is the statistical error from the Monte Carlo integration which is about half the size of the theory error.

3. Results and Discussion

We are now in the position to evaluate the nuclear S-wave multipoles. They are given as the sum of the one- and the two-nucleon contributions given in Eqs. (5, 7) in the previous section,

$$\begin{aligned} E_{0+} &= E_{0+}^{1N} + E_{0+}^{2N}, \\ L_{0+} &= L_{0+}^{1N} + L_{0+}^{2N}. \end{aligned} \quad (11)$$

Using the values for the one- and two-body form factors in Tables 1 and 2 together with the subleading chiral perturbation theory results for the single-nucleon multipoles at $\mathcal{O}(p^4)$ [3, 4]

$$\begin{aligned} E_{0+}^{\pi^0 p} &= -1.16 \times 10^{-3}/M_{\pi^+}, \\ E_{0+}^{\pi^0 n} &= +2.13 \times 10^{-3}/M_{\pi^+}, \\ L_{0+}^{\pi^0 p} &= -1.35 \times 10^{-3}/M_{\pi^+}, \\ L_{0+}^{\pi^0 n} &= -2.41 \times 10^{-3}/M_{\pi^+}, \end{aligned} \quad (12)$$

we obtain for the threshold multipoles on ${}^3\text{He}$ and on ${}^3\text{H}$ the values in Table 3. For the 1N contribution, the first error is the theory error from higher orders in chiral EFT estimated from the cutoff variation as explained above, while the second error is from the 5% uncertainty of the one-nucleon amplitudes. In the case of the 2N contribution, only the theory error is given. The total error is obtained by adding the theory error and the uncertainty of the one-nucleon amplitudes in quadrature. As noted before, the total error is dominated by the uncertainty in the single nucleon amplitudes. We stress that our estimate of the theory error is only a lower bound.

One observes that the multipoles get a large contribution from the two-body terms. This behavior is similar to the deuteron case [5]. For example, in the case of E_{0+} for ${}^3\text{He}$, the proton contribution is -0.01, the neutron one is 1.72 while the two-body contribution is -3.95 in the canonical units of $10^{-3}/M_{\pi^+}$. There is, however, still a large sensitivity to the single-neutron contribution.

The corresponding threshold S-wave cross section for pion photoproduction a_0 is given by

$$a_0 = \left. \frac{|\vec{k}_\gamma|}{|\vec{q}_\pi|} \frac{d\sigma}{d\Omega} \right|_{\vec{q}_\pi=0} = |E_{0+}|^2. \quad (13)$$

The longitudinal multipole L_{0+} contributes only in electro-production. The corresponding threshold cross section contains an extra term $\sim |L_{0+}|^2$. From here on, we will, however, concentrate on photoproduction. In Fig. 3, we illustrate the sensitivity of a_0 to the single-neutron multipole $E_{0+}^{\pi^0 n}$. The inner

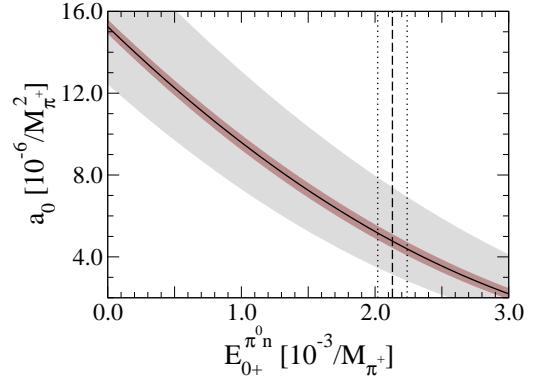


Figure 3: Sensitivity of a_0 for ${}^3\text{He}$ in units of $10^{-6}/M_{\pi^+}^2$ to the single-neutron multipole $E_{0+}^{\pi^0 n}$ in units of $10^{-3}/M_{\pi^+}$. The vertical dashed line gives the CHPT prediction for $E_{0+}^{\pi^0 n}$ and the vertical dotted lines indicate the 5% error in the prediction. The inner shaded band indicates the theory error estimated from the cutoff variation as described in the text. The outer shaded band corresponds to a 10% uncertainty in the 2N contribution.

shaded band indicates the theory error estimated from the cutoff variation as described above. The outer shaded band illustrates the effect of a 10% uncertainty in the 2N contribution. This corresponds to the size of the correction to the 2N contribution at the next order in the deuteron case [5]. The vertical dashed line indicates the CHPT prediction $E_{0+}^{\pi^0 n} = 2.13 \times 10^{-3}/M_{\pi^+}$. Changing this value by $\pm 20\%$ leads to changes in a_0 of about $\pm 30\%$. Thus, the ${}^3\text{He}$ nucleus appears to be a very promising target to test the CHPT prediction for $E_{0+}^{\pi^0 n}$. On the contrary, neutral pion production on ${}^3\text{H}$ is rather

nucleus	multipole	1N	2N	total
^3He	$E_{0+} [10^{-3}/M_{\pi^+}]$	+1.71(4)(9)	-3.95(3)	-2.24(11)
	$L_{0+} [10^{-3}/M_{\pi^+}]$	-1.89(4)(9)	-3.09(2)	-4.98(12)
^3H	$E_{0+} [10^{-3}/M_{\pi^+}]$	-0.93(3)(5)	-4.01(3)	-4.94(7)
	$L_{0+} [10^{-3}/M_{\pi^+}]$	-0.99(4)(5)	-3.13(1)	-4.12(7)

Table 3: Numerical results for the threshold multipoles E_{0+} and L_{0+} on ^3He and ^3H . The error estimates are explained in the text.

insensitive to $E_{0+}^{\pi^0 n}$: a variation of $E_{0+}^{\pi^0 n}$ from 0 to 3 changes a_0 only by 1%.

Next we compare our predictions with the available data. The consistency of the CHPT prediction for the single-neutron multipole with the measured S-wave threshold amplitude on the deuteron from Saclay and Saskatoon is well established, see Refs. [1, 5]. The reanalyzed measurement of the S-wave amplitude for ^3He at Saclay gives $E_3 = (-3.5 \pm 0.3) \times 10^{-3}/M_{\pi^+}$ [8, 9], which is related to a_0 according to

$$|E_{0+}|^2 = |E_3|^2 \left| \frac{F_T^{S-V}}{2} \right|^2 \left(\frac{1 + M_\pi/m_N}{1 + M_\pi/3m_N} \right)^2. \quad (14)$$

Here, we have approximated the $A = 3$ body form factor \mathcal{F}_A of Argan et al. [9] by the numerically dominant form factor F_T^{S-V} for ^3He , cf. Tab. 1. This results in

$$E_{0+} = (-2.8 \pm 0.2) \times 10^{-3}/M_{\pi^+}, \quad (15)$$

assuming the same sign as for our ^3He prediction in Table 3. In magnitude, the extracted value is about 25% above the predicted one. Given the model-dependence that is inherent to the analysis of Ref. [9], it is obvious that a more precise measurement using CW beams and modern detectors is very much called for.

4. Summary and Outlook

In this letter, we have presented a calculation of neutral pion production off ^3H and ^3He at threshold to leading one-loop order for the production operator in the framework of chiral nuclear effective field theory. We used the chiral wave functions of Refs. [15, 16] which are consistent with the pion production operator to calculate the S-wave 3N multipoles E_{0+} and L_{0+} . To this order, the production operator gets both one- and two-body contributions. Our calculation shows that the two-body contributions are of the same order of magnitude as

the one-body contributions. A similar behavior was observed in the deuteron case [5].

The theoretical uncertainty resulting from the cutoff variation in the employed wave functions appears to be small (of the order of 3%). The dominant theoretical error at this order stems from the threshold pion production amplitude off the proton and the neutron, which is estimated to be about 5%.

We explored the sensitivity of neutral pion photoproduction on ^3He to the elementary neutron multipole $E_{0+}^{\pi^0 n}$ and found a large sensitivity. This makes ^3He a promising target to test the counterintuitive CHPT prediction for $E_{0+}^{\pi^0 n}$ [3, 4]. The cutoff variation estimate leads to a very small error for the 2N contribution. If the error of this contribution is artificially enlarged by a factor of 10, the extraction of the neutron multipole is still feasible experimentally.

We have shown that our prediction for the ^3He S-wave multipole E_{0+} is roughly consistent with the value deduced from the old Saclay measurement of the threshold cross section [9]. A new measurement using modern technology and better methods to deal with few-body dynamics is urgently called for.

There are many natural extensions of this work. They include investigating higher orders, pion production above threshold, the extension to virtual photons and pion electroproduction, production of charged pions, and considering heavier nuclear targets such as ^4He . Further work in these directions is in progress.

Acknowledgements

We thank Andreas Nogga for providing us with the chiral 3N wave functions and Hermann Krebs for discussions. Financial support by the Deutsche Forschungsgemeinschaft (SFB/TR 16, “Subnuclear Structure of Matter”), by the European Community Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (acronym Hadron-Physics2, Grant Agreement n. 227431) under the

7th Framework Programme of the EU and by the European Research Council (acronym NuclearEFT, ERC-2010-StG 259218) is gratefully acknowledged.

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