factors represent neutron and proton properties inside the trinucleus. The nucleons are considered to be independently coupling to the electro-magnetic current, which is conventionally called impulse approximation. Corrections to the impulse approximation due to relative motion of the nucleons, nuclear interactions and recoil effects are considered in the following sections. Cross section and conventions can be found in appendix A and B, in order.

5.2.1 Impulse Approximation to Order q^4

The single nucleon contribution to threshold neutral pion photo- and electroproduction off the tri-nucleon takes the generic form (Eq. (4.3))

$$\begin{aligned}
\mathcal{M}^{\lambda}_{1N} =& \sum_{i=1}^{3} \mathcal{M}^{\lambda}_{i} = \vec{\epsilon}^{\lambda} \cdot \left(\vec{J}_1 + \vec{J}_2 + \vec{J}_3\right) \\
\mathcal{M}^{\lambda}_{i} =& \frac{1+\tau^z_i}{2} 2i \left\{ E^{\pi^0 p}_{0+} \left(\vec{\epsilon}^{\lambda}_{\mathrm{T}} \cdot \vec{s}_i\right) + L^{\pi^0 p}_{0+} \left(\vec{\epsilon}^{\lambda}_{\mathrm{L}} \cdot \vec{s}_i\right) \right\} + \frac{1-\tau^z_i}{2} 2i \left\{ E^{\pi^0 n}_{0+} \left(\vec{\epsilon}^{\lambda}_{\mathrm{T}} \cdot \vec{s}_i\right) + L^{\pi^0 n}_{0+} \left(\vec{\epsilon}^{\lambda}_{\mathrm{L}} \cdot \vec{s}_i\right) \right\} \\
=& 2i \left\{ \frac{1+\tau^z_i}{2} E^{\pi^0 p}_{0+} + \frac{1-\tau^z_i}{2} E^{\pi^0 n}_{0+} \right\} \left(\vec{\epsilon}^{\lambda}_{\mathrm{T}} \cdot \vec{s}_i\right) + 2i \left\{ \frac{1+\tau^z_i}{2} L^{\pi^0 p}_{0+} + \frac{1-\tau^z_i}{2} L^{\pi^0 n}_{0+} \right\} \left(\vec{\epsilon}^{\lambda}_{\mathrm{L}} \cdot \vec{s}_i\right) \\
=& 2i \left\{ E^{\pi^0 S}_{0+} + \tau^z_i E^{\pi^0 V}_{0+} \right\} \left(\vec{\epsilon}^{\lambda}_{\mathrm{T}} \cdot \vec{s}_i\right) + 2i \left\{ L^{\pi^0 S}_{0+} + \tau^z_i L^{\pi^0 V}_{0+} \right\} \left(\vec{\epsilon}^{\lambda}_{\mathrm{L}} \cdot \vec{s}_i\right) \qquad (5.22)
\end{aligned}$$

where

$$E^{\pi^0 p}_{0+} = -1.16 \times 10^{-3}/M_{\pi^+}, \qquad E^{\pi^0 n}_{0+} = +2.13 \times 10^{-3}/M_{\pi^+} \qquad (5.23)$$

are the electric pion-production amplitudes off proton and neutron calculated in [3, 4, 75] and given in the usual units or equivalently

$$E^{\pi^0 S}_{0+} = \frac{E^{\pi^0 p}_{0+} + E^{\pi^0 n}_{0+}}{2} = +0.49 \times 10^{-3}/M_{\pi^+} \qquad (5.24)$$

$$E^{\pi^0 V}_{0+} = \frac{E^{\pi^0 p}_{0+} - E^{\pi^0 n}_{0+}}{2} = -1.65 \times 10^{-3}/M_{\pi^+} \qquad (5.25)$$

the isoscalar and isovector nucleonic electric pion-production amplitudes and

$$L^{\pi^0 p}_{0+} = -1.35 \times 10^{-3}/M_{\pi^+}, \qquad L^{\pi^0 n}_{0+} = -2.41 \times 10^{-3}/M_{\pi^+} \qquad (5.26)$$

$$L^{\pi^0 S}_{0+} = -1.88 \times 10^{-3}/M_{\pi^+}, \qquad L^{\pi^0 V}_{0+} = +0.53 \times 10^{-3}/M_{\pi^+} \qquad (5.27)$$

their longitudinal equivalents at threshold to order q^4 [3, 4, 75]. Due to symmetry properties,

$$\langle M_{J'} | \mathcal{M}^{\lambda}_{1N} | M_J \rangle_{\psi} = \langle M_{J'} | \mathcal{M}^{\lambda}_{1} + \mathcal{M}^{\lambda}_{2} + \mathcal{M}^{\lambda}_{3} | M_J \rangle_{\psi} = \langle M_{J'} | 3\mathcal{M}^{\lambda}_{1} | M_J \rangle_{\psi}. \qquad (5.28)$$

With the transversal and the longitudinal form factors

$$\left(\vec{\epsilon}_{\lambda,\mathrm{T}} \cdot \vec{S}_{M'_J M_J}\right) F_T^{S\pm V} = \langle M_{J'} \,|\, 3\vec{\epsilon}_{\lambda,\mathrm{T}} \cdot \vec{s}_1 (1 \pm \tau_1^z) \,|\, M_J \rangle_\psi \,, \tag{5.29}$$

$$\left(\vec{\epsilon}_{\lambda,\mathrm{L}} \cdot \vec{S}_{M'_J M_J}\right) F_L^{S\pm V} = \langle M_{J'} \,|\, 3\vec{\epsilon}_{\lambda,\mathrm{L}} \cdot \vec{s}_1 (1 \pm \tau_1^z) \,|\, M_J \rangle_\psi \,, \tag{5.30}$$

$$\tag{5.31}$$

we can write down the general result

$$\begin{aligned}\left\langle M_{J'} \left|\mathcal{M}_{1N}^\lambda\right| M_J\right\rangle_\psi =& 2i \left\{ \frac{F_T^{S+V}}{2} E_{0+}^{\pi^0 p} + \frac{F_T^{S-V}}{2} E_{0+}^{\pi^0 n} \right\} \left(\vec{\epsilon}_{\lambda,\mathrm{T}} \cdot \vec{S}_{M'_J M_J}\right) \\ &+2i \left\{ \frac{F_L^{S+V}}{2} L_{0+}^{\pi^0 p} + \frac{F_L^{S-V}}{2} L_{0+}^{\pi^0 n} \right\} \left(\vec{\epsilon}_{\lambda,\mathrm{L}} \cdot \vec{S}_{M'_J M_J}\right) \\ =& 2i\tilde{E}_{0+}^{1N} \left(\vec{\epsilon}_{\lambda,\mathrm{T}} \cdot \vec{S}_{M'_J M_J}\right) + 2i\tilde{L}_{0+}^{1N} \left(\vec{\epsilon}_{\lambda,\mathrm{L}} \cdot \vec{S}_{M'_J M_J}\right) . \end{aligned} \tag{5.32}$$

To account for the change in phase space, we have to multiply the amplitudes with the phase factor from Eq. (B.21),

$$K_{1N} = \frac{m_N + M_\pi}{m_{3N} + M_\pi} \frac{m_{3N}}{m_N} \approx 1.092 : \tag{5.33}$$

$$E_{0+}^{1N} = K_{1N} \tilde{E}_{0+}^{1N} = \frac{K_{1N}}{2} \left(F_T^{S+V} E_{0+}^{\pi^0 p} + F_T^{S-V} E_{0+}^{\pi^0 n} \right) \tag{5.34}$$

$$L_{0+}^{1N} = K_{1N} \tilde{L}_{0+}^{1N} = \frac{K_{1N}}{2} \left(F_L^{S+V} L_{0+}^{\pi^0 p} + F_L^{S-V} L_{0+}^{\pi^0 n} \right) . \tag{5.35}$$

The results for photo-production off ^{3}He and ^{3}H are collected in table 5.3 and for electro-production with $k_\mu k^\mu = -0.1$ GeV^2 in table 5.4.
The error related to the expansion of the production operator is difficult to estimate given that the convergence in the expansion for the single nucleon S-wave multipoles is known to be slow, see Ref. [3] for an extended discussion. We therefore give only a rough estimate of this uncertainty. The extractions of the proton S-wave photoproduction amplitude based on CHPT using various approximations [76] lead to an uncertainty $\Delta E_{0+}^{\pi^0 p} \approx \pm 0.05 \times 10^{-3}/M_{\pi^+}$, which is about 5%. The uncertainty of the neutron S-wave threshold amplitude is estimated to be the same. Consequently, our estimate of the error on the single nucleon amplitude is 5%.

5.2.2 Boost Corrections to q^3-Contributions (at q^4)

Boost corrections arise in reactions involving composite nuclei and emerge from the relative motion of the nucleons to the center of mass of the nucleus. Due to the relative motion of the nucleons the reaction threshold is lowered with respect to the threshold of a reaction

nucleus ψ	^{3}He	^{3}H
F_T^{S+V}	0.017(13)(3)	1.493(25)(3)
F_T^{S-V}	1.480(26)(3)	0.012(13)(3)
F_L^{S+V}	−0.079(14)(8)	1.487(27)(8)
F_L^{S-V}	1.479(26)(8)	−0.083(14)(8)

Table 5.3: Numerical results for the form factors $F_{T/L}^{S\pm V}$. The first error is our estimation of the theoretical uncertainty resulting from the truncation of the chiral expansion while the second one is the statistical error from the Monte Carlo integration.

nucleus ψ	^{3}He	^{3}H
F_T^{S+V}	0.119(6)(1)	0.602(14)(1)
F_T^{S-V}	0.588(13)(1)	0.118(6)(1)
F_L^{S+V}	−0.131(14)(2)	0.589(17)(1)
F_L^{S-V}	0.577(15)(2)	−0.136(14)(1)

Table 5.4: Numerical results for the form factors $F_{T/L}^{S\pm V}$ for $k_\mu k^\mu = -0.1$ GeV^2. The first error is our estimation of the theoretical uncertainty resulting from the truncation of the chiral expansion while the second one is the statistical error from the Monte Carlo integration.

involving a single nucleon. In other words, threshold pion production off a composite nucleus entails production off single nucleons above threshold. The part of the phase space parametrizing the relative momentum of the nucleons is integrated out to merge the nucleons to the chosen asymptotic states, e.g. the tri-nucleon, masking the origin of the correction and leaving an effective remnant contribution, which is of the form of a correction to the impulse approximation. In this sense boost corrections are multi-nucleon effects, but corrections to the one-nucleon sector. We decide to associate this effect to the 1N sector. The boost corrections start to contribute at order q^4.
The proton and neutron production amplitudes are calculated in (N,γ)-cms. The boost of a $(3N,\gamma)$-cms 4-vector p to (N,γ)-cms 4-vector p^* has the general form

$$\begin{pmatrix} p^{0*} \\ \vec{p}^{*} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\vec{\beta} \\ -\gamma\vec{\beta} & (\mathbb{1}_3 - P_{\vec{\beta}}) + \gamma P_{\vec{\beta}} \end{pmatrix} \begin{pmatrix} p^0 \\ \vec{p} \end{pmatrix} = \begin{pmatrix} \gamma(p^0 - \vec{\beta}\cdot\vec{p}) \\ -\gamma\vec{\beta}p^0 + \vec{p}_\perp + \gamma\vec{p}_\parallel \end{pmatrix}, \tag{5.36}$$

where $P_{\vec{\beta}}$ is the projection operator onto the $\vec{\beta}$-direction, i.e. $P_{\vec{\beta}}\,\vec{x} = (\hat{\beta}\cdot\vec{x})\hat{\beta}$, and $\vec{p}_\parallel = P_{\vec{\beta}}\,\vec{p}$ is the parallel part and $\vec{p}_\perp = (1 - P_{\vec{\beta}})\vec{p}$ the perpendicular part of $\vec{p} = \vec{p}_\parallel + \vec{p}_\perp$ with respect to $\vec{\beta}$. To determine $\vec{\beta}$, consider $\vec{k}_1 + \vec{k}_\gamma$. In the (N,γ)-cms this combination has to vanish, i.e. $\vec{k}_1^* + \vec{k}_\gamma^* = \vec{0}$. We have

$$\vec{k}_1^* + \vec{k}_\gamma^* = \gamma\left(-\vec{\beta}(k_1^0 + k_\gamma^0) + P_{\vec{\beta}}(\vec{k}_1 + \vec{k}_\gamma)\right) + (1 - P_{\vec{\beta}})(\vec{k}_1 + \vec{k}_\gamma) \stackrel{!}{=} \vec{0}. \tag{5.37}$$

Because linear independent (even orthogonal) vectors have to vanish separately, i.e.

$$\begin{aligned}(1 - P_{\vec{\beta}})(\vec{k}_1 + \vec{k}_\gamma) &= \vec{0}\\ -\vec{\beta}(k_1^0 + k_\gamma^0) + P_{\vec{\beta}}(\vec{k}_1 + \vec{k}_\gamma) &= \vec{0},\end{aligned}$$

we conclude

$$\vec{\beta} = \frac{\vec{k}_1 + \vec{k}_\gamma}{k_1^0 + k_\gamma^0} = \frac{\vec{k}_1' + \vec{q}}{k_1'^0 + q^0} = \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{\sqrt{(\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3})^2 + m_N^2} + q^0}. \tag{5.38}$$

Near the static limit we have

$$\begin{aligned}\vec{\beta} &= \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{m_N}\left(\sqrt{\frac{(\vec{p}_{12}' - \frac{\vec{p}_3'}{2} - \frac{\vec{q}}{3})^2}{m_N^2} + 1} + \frac{\sqrt{\vec{q}^2 + M_{\pi^0}^2}}{m_N}\right)^{-1}\\ &= \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{m_N}\left\{1 - \frac{\sqrt{\vec{q}^2 + M_{\pi^0}^2}}{m_N} + \mathcal{O}\left(\left(\frac{1}{m_N}\right)^2\right)\right\}\\ &= \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{m_N} + \mathcal{O}\left(\left(\frac{1}{m_N}\right)^2\right) \xrightarrow{\text{threshold}} \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2}}{m_N} + \mathcal{O}\left(\left(\frac{1}{m_N}\right)^2\right),\end{aligned} \tag{5.39}$$

$$\gamma = \left(1 - \beta^2\right)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \mathcal{O}(\beta^4) = 1 + \mathcal{O}\left(\left(\frac{1}{m_N}\right)^2\right). \tag{5.40}$$

Correspondingly, a general $(3N, \gamma)$-cms 4-vector p^μ transforms to the (N, γ)-cms as

$$p^{0*} = \gamma(p^0 - \vec{\beta}\cdot\vec{p}) = p^0 - \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{m_N}\cdot\vec{p} \xrightarrow{\text{threshold}} p^0 - \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2}}{m_N}\cdot\vec{p}, \tag{5.41}$$

$$\vec{p}^* = -\gamma\vec{\beta}p^0 + \vec{p}_\perp + \gamma\vec{p}_\parallel = \vec{p} - \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2} + \frac{2\vec{q}}{3}}{m_N}p^0 \xrightarrow{\text{threshold}} \vec{p} - \frac{\vec{p}_{12}' - \frac{\vec{p}_3'}{2}}{m_N}p^0, \tag{5.42}$$

given up to first order in one over the nucleon mass. At threshold point we find:

$$\begin{aligned}k^{0*} &= k^0 - \frac{\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right)\cdot\vec{k}}{m_N}, & \vec{k}^* &= \vec{k} - \frac{k^0}{m_N}\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right),\\ q^{0*} &= q^0 - \frac{\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right)\cdot\vec{q}}{m_N} = M_\pi, & \vec{q}^* &= \vec{q} - \frac{q^0}{m_N}\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right) = -\frac{M_\pi}{m_N}\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right),\\ \epsilon_\lambda^{0*} &= \epsilon_\lambda^0 - \frac{\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right)\cdot\vec{\epsilon}_\lambda}{m_N}, & \vec{\epsilon}_\lambda^* &= \vec{\epsilon}_\lambda - \frac{\epsilon^0}{m_N}\left(\vec{p}_{12}' - \frac{\vec{p}_3'}{2}\right) = \vec{\epsilon}_\lambda.\end{aligned} \tag{5.43}$$

The polarization vector does not change except for the time component. To adopt the results for pion production off single nucleons in the (N,γ)-cms to the tri-nucleon case, the pion momentum in the $(3N,\gamma)$-cms at threshold, $\vec{q}=\vec{0}$, is boosted to the (N,γ)-cms value $\vec{q}^{\,*}=-\frac{M_\pi}{m_N}\left(\vec{p}^{\,\prime}_{12}-\frac{\vec{p}^{\,\prime}_3}{2}\right)=:-\mu\left(\vec{p}^{\,\prime}_{12}-\frac{\vec{p}^{\,\prime}_3}{2}\right)$ above threshold. The corresponding P-wave contribution off the nucleon with spin $\vec{s}$ reads (using the notation from Ref. [69])

$$
\begin{aligned}
\mathcal{M}^\lambda =&2i(\vec{\epsilon}^{\,\lambda}\cdot\vec{s})(\hat{q}^*\cdot\hat{k})P_1+2i(\vec{\epsilon}^{\,\lambda}\cdot\hat{q}^*)(\hat{k}\cdot\vec{s})P_2+(\vec{\epsilon}^{\,\lambda}\cdot[\hat{q}^*\times\hat{k}])P_3\\
&+2i(\vec{\epsilon}^{\,\lambda}\cdot\hat{k})(\hat{k}\cdot\vec{s})(\hat{q}^*\cdot\hat{k})(P_4-P_5-P_1-P_2)+2i(\vec{\epsilon}^{\,\lambda}\cdot\hat{k})(\hat{q}^*\cdot\vec{s})P_5.\\
=:&2i(\vec{\epsilon}^{\,\lambda}_{\mathrm{T}}\cdot\vec{s})(\hat{q}^*\cdot\hat{k})P_1+2i(\vec{\epsilon}^{\,\lambda}_{\mathrm{T}}\cdot\hat{q}^*)(\hat{k}\cdot\vec{s})P_2)+(\vec{\epsilon}^{\,\lambda}_{\mathrm{T}}\cdot[\hat{q}^*\times\hat{k}])P_3\\
&+2i(\vec{\epsilon}^{\,\lambda}_{\mathrm{L}}\cdot\vec{s})(\hat{q}^*\cdot\hat{k})P_4+2i(\vec{\epsilon}^{\,\lambda}_{\mathrm{L}}\cdot\hat{k})(\hat{q}^*_{\mathrm{T}}\cdot\vec{s})P_5.
\end{aligned}
$$

Close to threshold, the P-wave multipoles P_i behave as $P_i\approx\bar{P}_i|\vec{q}^{\,*}|=\mu\bar{P}_i|\vec{p}^{\,\prime}_{12}-\frac{\vec{p}^{\,\prime}_3}{2}|$ with

$$
\begin{aligned}
\bar{P}^p_1&=+0.01872\mathrm{fm}^2, & \bar{P}^p_3&=+0.02395\mathrm{fm}^2, & \bar{P}^p_4&=+0.00129\mathrm{fm}^2,\\
\bar{P}^n_1&=+0.01342\mathrm{fm}^2, & \bar{P}^n_3&=+0.02336\mathrm{fm}^2, & \bar{P}^n_4&=+0.00027\mathrm{fm}^2,
\end{aligned}
$$

where the numerical values refer to the P-wave low-energy theorems for pion photo- [3] and electroproduction [77]. Corrections to these theorems are beyond the accuracy of our calculation.
In analogy to the S-wave case discussed above, we define the P-wave form factors

$$
\begin{aligned}
\left(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot\vec{S}_{M'_JM_J}\right)F_1^{S\pm V}&=\left\langle M_{J'}\left|3(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot\vec{s}_1)\left((\vec{p}^{\,\prime}_{12}-\vec{p}^{\,\prime}_3/2)\cdot\hat{k}\right)\{1\pm\tau^z_1\}\right|M_J\right\rangle_\psi,\\
\left(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot\vec{S}_{M'_JM_J}\right)F_2^{S\pm V}&=\left\langle M_{J'}\left|3\left(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot(\vec{p}^{\,\prime}_{12}-\vec{p}^{\,\prime}_3/2)\right)(\hat{k}\cdot\vec{s}_1)\{1\pm\tau^z_1\}\right|M_J\right\rangle_\psi,\\
\left(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot\vec{S}_{M'_JM_J}\right)F_3^{S\pm V}&=\left\langle M_{J'}\left|-3i(\vec{\epsilon}_{\lambda,\mathrm{T}}\cdot\left[(\vec{p}^{\,\prime}_{12}-\vec{p}^{\,\prime}_3/2)\times\hat{k}\right])\{1\pm\tau^z_1\}\right|M_J\right\rangle_\psi,\\
\left(\vec{\epsilon}_{\lambda,\mathrm{L}}\cdot\vec{S}_{M'_JM_J}\right)F_4^{S\pm V}&=\left\langle M_{J'}\left|3(\vec{\epsilon}_{\lambda,\mathrm{L}}\cdot\vec{s}_1)\left((\vec{p}^{\,\prime}_{12}-\vec{p}^{\,\prime}_3/2)\cdot\hat{k}\right)\{1\pm\tau^z_1\}\right|M_J\right\rangle_\psi,\\
\left(\vec{\epsilon}_{\lambda,\mathrm{L}}\cdot\vec{S}_{M'_JM_J}\right)F_5^{S\pm V}&=\left\langle M_{J'}\left|3(\vec{\epsilon}_{\lambda,\mathrm{L}}\cdot\hat{k})\left((\vec{p}^{\,\prime}_{12}-\vec{p}^{\,\prime}_3/2)_{\mathrm{T}}\cdot\vec{s}_1\right)\{1\pm\tau^z_1\}\right|M_J\right\rangle_\psi,
\end{aligned}
$$

where the spin and isospin operators refer to nucleon 1. The contributions from the other nucleons are accounted for by the overall factor of three as before.
In terms of these form factors, the P-wave contribution to the 3N-production amplitude takes the form:

$$
E^{1N}_{0+}=K_{1N}\tilde{E}^{1N}_{0+}=-\frac{K_{1N}}{2}\mu\sum_{i=1}^{3}\left(F_i^{S+V}\bar{P}^p_i+F_i^{S-V}\bar{P}^n_i\right), \tag{5.44}
$$

$$
L^{1N}_{0+}=K_{1N}\tilde{L}^{1N}_{0+}=-\frac{K_{1N}}{2}\mu\sum_{i=4}^{5}\left(F_i^{S+V}\bar{P}^p_i+F_i^{S-V}\bar{P}^n_i\right). \tag{5.45}
$$

These form factors are evaluated using the same Monte Carlo method as employed for the S-waves. The numerical values are collected in Tab. 5.5. Note that F_2 and F_5 come out to be consistent with zero and are therefore not listed in the table. As before, the proton contribution is dominant in ^{3}H, whereas the neutron one features prominently in ^{3}He.

nucleus ψ	^{3}He	^{3}H
F_1^{S+V}	$+0.004(3)(1)$	$+0.339(6)(1)$
F_1^{S-V}	$+0.338(5)(1)$	$+0.002(3)(1)$
F_3^{S+V}	$-0.015(2)(0)$	$-0.011(2)(0)$
F_3^{S-V}	$-0.011(2)(0)$	$-0.015(2)(0)$
F_4^{S+V}	$-0.019(5)(4)$	$+0.339(6)(4)$
F_4^{S-V}	$+0.337(6)(4)$	$-0.021(3)(4)$

Table 5.5: Numerical results for the boost correction form factors $F_i^{S\pm V}$ in units of [fm^{-1}]. The first error is our estimation of the theoretical uncertainty resulting from the truncation of the chiral expansion while the second one is the statistical error from the Monte Carlo integration. $F_2^{S\pm V}$ and $F_5^{S\pm V}$ are not shown here, because they are consistent with zero.

Notice that in contrast to the single-nucleon corrections, we do not need to employ a special treatment for boost corrections to the leading two-nucleon contributions at the order we are working. All $1/m_N$-corrections to the leading three-body contributions to the production operator needed in the calculations are treated on the same footing as described in section 5.3.2.

5.3 Two-Nucleon Contributions

Two-nucleon contributions of an excitation process parametrize the leading nuclear corrections to the integral kernel, which in ChPT are mediated by one-pion-exchange and nucleon contact interactions. In neutral pion production processes, two-nucleon terms are known to be dominant with respect to one-nucleon effects in case of the deuteron [6]. We expect this behaviour also for the trinucleus. The lowest order only consists of one-pion-exchange diagrams, because all possible contact terms vanish in Coulomb gauge at LO (q^3) and at threshold even at NLO (q^4).

5.3.1 Leading Contributions (q^3)

In Coulomb gauge, only the two Feynman diagrams shown in Fig. 5.2 contribute at threshold to third order [6], labeled by (a) and (b). Their contribution exclusively consists of

^{3}He	1N (q^4)	2N (q^3)	1N-boost	2N-static (q^4)	2N-recoil (q^4)	total
$E_{0+}^{\pi^0\psi}$	+1.71(4)(9)	−3.95(3)	−0.23(1)	−0.02(0)(1)	+0.01(2)(1)	−2.48(11)
$L_{0+}^{\pi^0\psi}$	−1.89(4)(9)	−3.09(2)	−0.00(0)	−0.07(1)(1)	+0.07(7)(0)	−4.98(12)
^{3}H	1N (q^4)	2N (q^3)	1N-boost	2N-static (q^4)	2N-recoil (q^4)	total
$E_{0+}^{\pi^0\psi}$	−0.93(3)(5)	−4.01(3)	−0.35(1)	−0.02(1)(1)	+0.01(2)(0)	−5.28(7)
$L_{0+}^{\pi^0\psi}$	−0.99(4)(5)	−3.13(1)	−0.02(0)	−0.07(0)(1)	+0.07(7)(0)	−4.14(10)

Table 5.9: Numerical results for the 3N multipoles in $[10^{-3}/M_{\pi^+}]$. The first error is our estimation of the theoretical uncertainty resulting from the truncation of the chiral expansion while the second one is the statistical error from the Monte Carlo integration.

Notice that the statistical error is negligible compared to theory error. The 5% error from the single-nucleon amplitudes discussed above is not included in the numbers for the theory error, but appears as the second error of the single nucleon contribution in the table. For the total result only the combined error is given. Overall, we find that these fourth order corrections for the electric dipole amplitude $E_{0+}^{\pi^0\psi}$ for both tri-nuclear systems come out to be very small, much smaller than in case of the deuteron. This can be, in part, traced back to the smaller values of the various form factors (for the boost corrections) and also to the small prefactor K_{2N}^{q4}, cf. Eq. (5.53), for the two-nucleon contributions. For the longitudinal amplitude $L_{0+}^{\pi^0\psi}$, the sum of the fourth order corrections is consistent with zero within the uncertainties. This can be understood as follows: First, the boost corrections are proportional to the P-wave multipole P_4, which is much smaller than the corresponding multipoles P_1, P_3 that appear in the electric dipole amplitude, cf. Eq. (5.44). Second, there are almost perfect cancellations between the static and the recoil contributions for both tri-nucleon systems. These cancellations are accidental in the sense that they cannot be traced back to any symmetry or small prefactor. The process seems to ignore longitudinal degrees of freedom inside the trinucleus represented by boost, static and recoil corrections. In summary, we find that the chiral expansion for the S-wave multipoles at threshold converges fast and that the largest uncertainty remains in the single nucleon production amplitudes. We also remark that the fourth order corrections to the electric dipole amplitudes of the tri-nucleon systems are sizably smaller than for the deuteron.
Now, let us concentrate on photoproduction. The threshold S-wave cross section for pion photoproduction a_0 is given in terms of the photon momentum $\vec{k}$ and the pion momentum $\vec{q}$ by

$$a_0 = \frac{|\vec{k}\,|}{|\vec{q}\,|} \frac{d\sigma}{d\Omega}\bigg|_{\vec{q}=0} = \left|E_{0+}^{\pi^0\psi}\right|^2 . \tag{5.60}$$

In Fig. 5.8, we illustrate the sensitivity of a_0 to the single-neutron multipole $E_{0+}^{\pi^0 n}$.
The shaded band indicates the theory error estimated from the cutoff variation as described above and a 5% error in $E_{0+}^{\pi^0 p}$. As shown above, the uncertainties related to the nuclear