Scattering Observables from Few-Body Densities and Application in Light Nuclei

Alexander Phillips Long

June 1, 2022

A Dissertation Proposal submitted to

The Faculty of
The Columbian College of Arts and Sciences
of The George Washington University
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

Dissertation directed by

Harald Griesshammer
Professor of Physics

The Columbian College of Arts and Sciences of The George Washington University certifies that Alexander Phillips Long has passed the Final Examination for the degree of Doctor of Philosophy as of date of dissertation proposal defense. This is the final and approved form of the dissertation proposal.

Scattering Observables from Few-Body Densities and Application in Light Nuclei

Alexander Phillips Long

Dissertation Research Director
Dr. Harald Griesshammer, Professor of Physics

Readers
Dr. Evangeline Downie, Professor of Physics
Dr. Michael Doring, Associate Professor of Physics

Abstract

Scattering Observables from Few-Body Densities and Application in Light Nuclei

The dynamics of scattering on light nuclei is well understood, but calculation is numerically difficult using standard methods. Fortunately using recent developments, the relevant quantities can be factored into a product of the n-body transition densities and the interaction kernel of a chosen probe. These transition densities depend only on the target, and not the probe; they are calculated once and stored. The kernels depend on only the probe and not the target. Using the general formalism developed by Griesshammer *et al.* an extension is proposed, wherein we calculate scattering amplitudes for Compton scattering, $(\gamma X \to \gamma X)$, pion photo-production, $(\gamma X \to \pi^0 X)$, pion scattering $(\pi X \to \pi X)$, and dark matter scattering $(\Theta X \to \Theta X)$, where Θ is the dark matter particle. With the targets $X = {}^3\text{He}$, $X = {}^2\text{H}$, $X = {}^4\text{He}$, and $X = {}^6\text{Li}$.

Contents

1 Introduction

Nuclear physics is study of atomic nuclei, their interactions with each other, and how their structure arises from the Standard Model. Compared to orbital mechanics, where we can detect the location of celestial bodies, or electrodynamics, where we can insert a probe to quantify the electric and magnetic fields, the microscopic nature of nuclear physics means we cannot observe interactions as they occur. To overcome this we conduct experiments wherein one particle is scattered off another, often at relativistic speeds. The distribution of outgoing particles allows for quantitative descriptions of the nature of the particles and their general properties.

2 Effective Field Theories

In all fields of physics the energy and length scales at play dictate what theory we use to describe a system. When first semester physics students learn of Newtonian gravity, they do not have to worry about quantum gravity, or the dynamics of general relativity. Nonetheless, they can describe the motion of a satellite around the Earth with remarkable precision because at the energies and length scales of the satellite-Earth system, Newtonian gravity describes the system accurately as

$$V(r) = -\frac{GMm}{r} \ , \tag{1}$$

where G is Newton's gravitational constant and r is the distance between masses m and M. Often, we do not even need this form. For length scales on the order of a meter and when standing on the surface of the Earth radius R, it suffices to use the first order Taylor expansion around $r = R$:

$$V(r) = \frac{GMmz}{R^2} + const. + \mathcal{O}(z^2) \ , \tag{2}$$

where $z = r - R$ is the distance from the surface of the earth. Problems arise when one considers the Taylor series:

$$V(z) = \frac{GMm}{R} \sum_{n=0}^{N} \left(\frac{-z}{R} \right)^n , \tag{3}$$

where N is the order at which we truncate our calculation. Now consider the situation where eq. (3) is known up to some finite order, but eq. (1) is not. This is the kind of scenario we often find ourselves in when studying nuclear physics, and often N is much smaller than we would like. Eq. (3) only operates inside a specific domain, which we characterize by the expansion parameter $Q = z/R$. When $z \sim R$, this approximation no longer makes sense. Fortunately, this does not mean we need to throw away our theory all together, it simply means we have found the breakdown scale of our theory, beyond which we must resort to something more fundamental.

Both eq. (1) and eq. (2) can be considered Effective Field Theories (EFTs) for gravity and exhibit a property found more generally. Long distance, low-energy behavior, is insensitive to high energy, short distance behavior. We did not have to solve quantum gravity to solve celestial motion with eq. (1), and we did not have to solve celestial motion to solve gravitation on the surface of the Earth with eq. (2).

Generally, an EFT is a low-energy approximation of a more general theory; they are useful for their predictive ability and their tendency to provide a more efficient analysis. By analogy, one could use general relativity to calculate how an apple falls from a tree, but this is a waste of computational resources; eq. (2) is predictive in this regime so it is a better choice.

Perhaps most importantly, the EFT method gives a prescription of how to develop a theory that accurately characterizes a system at a given scale, rather than just hoping for a moment of creative genius, where the correct Lagrangian is picked out of nowhere. To construct an EFT one starts by identifying the scales in the problem, namely the typical low momenta of the incoming and outgoing particles, and a breakdown scale $\bar{\Lambda}_{\mathrm{EFT}}$ where new, higher energy physics enters. The expansion parameter is then

$$Q = \frac{p_{\mathrm{typ}}}{\bar{\Lambda}_{\mathrm{EFT}}} \ll 1, \tag{4}$$

and when $p_{\mathrm{typ}} \sim \bar{\Lambda}_{\mathrm{EFT}}$, then $Q \sim 1$ so our expansion no longer makes sense.

Continuing to construct our EFT, one then identifies all the possible interactions the Lagrangian could have. For example, the interaction of two real scalar fields ϕ and ψ can be described completely generally by:

$$\begin{aligned}\mathscr{L} = c_0\psi + c_1\phi + c_2\left(\partial_\mu\psi\right)\left(\partial^\mu\phi\right) + c_3\phi^2 + c_4\phi^2 + c_6\left(\partial_\mu\psi^2\right)\left(\partial^\mu\phi^2\right) \\ + c_7\partial^2\phi + c_8\partial^2\phi + c_9\left(\partial^2\partial_\mu\psi\right)\left(\partial^2\partial^\mu\phi\right) + \dots ,\end{aligned} \tag{5}$$

where c_i are coupling constants. $\mathscr{L}$ must be a scalar, so we have immediately removed any term of the form $\partial_\mu\phi$ by itself. In addition overall derivatives have no result on the final result, so we may dismiss these terms. Note the momenta enter through the representation of the Lagrangian in momentum space, $\partial_\mu \to -ip_\mu$. In principle, there is an infinite number of terms in our EFT Lagrangian such as in eq. (3) with $N = \infty$, but we can use our hierarchy of scales, $\bar{\Lambda}_{EFT} \gg p_{\mathrm{typ}}$, to order this Lagrangian in terms of the expansion parameter Q. In turn, this allows for truncation of the infinite series, and will provide for the discussion of error estimates in section 5.1.

Furthermore, by identifying the symmetries in the problem, we may eliminate even more terms, for example if ψ and ϕ in eq. (5) are parity even, they cannot be raised to an odd power by themselves, that is $c_i\psi^{2k+1}$ is not allowed. Lorentz invariance stipulates further constraints. These symmetries decrease the number of terms in the theory's Lagrangian. One then fits the coefficients c_i to data, or to computational results (like lattice QCD). Importantly, as long as the correct degrees of freedom of the theory have been identified and the correct symmetries imposed, then we have parameterized all possible dependence, and it is impossible for the EFT to be wrong. In one version of this theory, we find eq. (5) reduces to [1]:

$$\mathscr{L}_{int} = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - m_1^2\phi^2 + \partial_\mu\psi\partial^\mu\psi - m_2^2\psi^2\right) - \frac{\lambda_1}{4!}\phi^4 - \frac{\lambda_2}{4!}\psi^4 - \frac{\lambda_3}{4}\phi^2\psi^2 + ... \quad (6)$$

3 QCD and χEFT

Quantum chromodynamics (QCD) describes the strong interaction of quarks, mediated by gluons; it is not an EFT. Its Lagrangian is given by

$$\mathscr{L}_{QCD} = \bar{q}\left(i\gamma^\mu D_\mu - \mathcal{M}\right) q - \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}^{\mu\nu}_a\ . \quad (7)$$

Here D_μ is the Gauge covariant derivative, $G^a_{\mu,\nu}$ is the gluon field strength tensor, the quantity q contains the quark fields, γ^μ are the Dirac matrices, and $\mathcal{M} = \text{diag}(m_u, m_d ... m_t)$ is quark mass matrix [2, 3]. The mass of the up and down quarks are small compared to the mass of typical hadrons ($\sim$ 1GeV), so one often considers the "chiral limit" of vanishing quark masses with $\mathcal{M} \to 0$ along with only the two lightest quark fields $q = (u(x), d(x))^T$. Pions are composed of up and down quarks, and are therefore described QCD in the chiral limit. At low energy, we consider only the pion and nucleon degrees of freedom which simplifies our analysis. The details are lengthy, but the pion pion Lagrangian of chiral effective field theory (χEFT) is given by [4]

$$\mathscr{L}_{\pi\pi} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi\vec{\pi}^2 + \frac{1-4\alpha}{2f_\pi^2}\left(\vec{\pi}\cdot\partial_\mu\vec{\pi}\right)\left(\vec{\pi}\cdot\partial^\mu\vec{\pi}\right) + \mathcal{O}(\pi^3). \quad (8)$$

Here $\vec{\pi} = (\pi^-, \pi^0, \pi^+)$, $f_\pi = 93$MeV is the pion decay constant, and α is non-physical and depends on the parameterization, frequently $\alpha = \frac{1}{6}$. Of course this proposal is also concerned with the interaction of nucleons, so higher order terms are required, but those details are not illuminating. χEFT can be used to describe the interactions between nucleons via a potential. This particular application was pioneered by van Kolk *et al.* who developed a two and three nucleon potential [5]. Later, this was expanded by many groups, but we use the version from Reinert *et al.* who developed the "chiral semi-local momentum space" (chiral SMS) potential [6]. It has constants fit to 2 and 3 nucleon bound states, along with two nucleon phase shifts, but has shown predictive validity up to 12 nucleons. This proposed work does not address this potential directly, but it is used to create the few nucleon "transition densities" which play a central role. We will discuss these densities shortly.

4 Previous Work and Summary of Problem

In order to properly understand the problem, we must summarize results from previous publications. It is emphasized that being a proposal, the mathematics and conventions presented here come almost entirely from Griesshammer *et al.* which considers the three nucleon system and gives Compton scattering as a concrete example [7]. We continue using their description of the problem; the extension from $\gamma^3\text{He} \to \gamma^3\text{He}$ to a more general case is the subject of this thesis proposal. A diagram of the possible interactions and relevant kinematics of scattering on a three nucleon state ($A = 3$) can be seen in fig. 1.

All of our analysis is done in the center of mass frame, where the incoming probe has momentum $\vec{k}$ and the nucleus has momentum $-\vec{k}$. We label all outgoing quantities with a prime; the outgoing probe momentum is $\vec{k}'$. We choose $\vec{q} = \vec{k} - \vec{k}'$ to be the momentum transfer dumped onto the nucleons, namely the momentum difference between the incoming and outgoing particles.

In fig. 1 the external probe is in blue and interacts with the nucleus wavefunction represented in green. The $A = 3$ nucleons are represented as black lines. We define

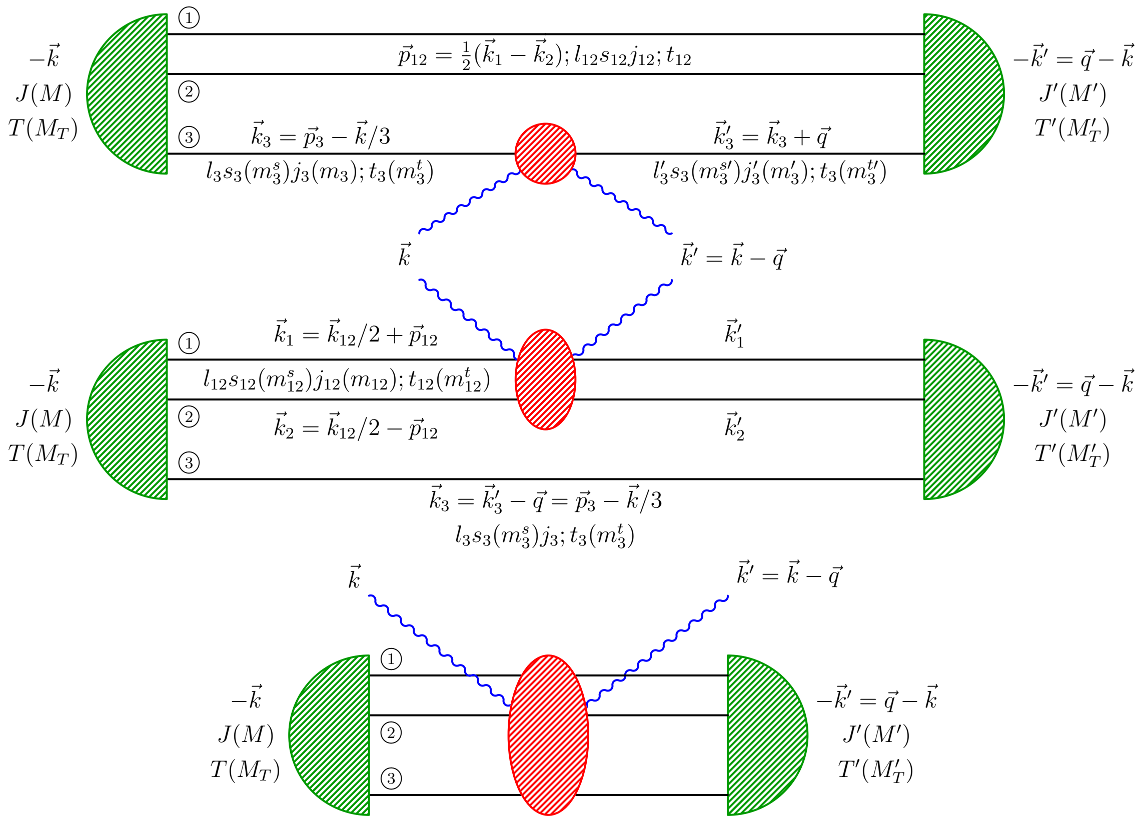

Figure 1: Kinematics for a three nucleon bound state interacting with an external probe, figure from Griesshammer *et al.* [7]

$n = 1, 2, 3, ..., A$ to be the number of *active* nucleons which interact with the probe inside an A body nucleus; the *spectators* are the $A-n$ nucleons which are not involved. The properties of the spectators are then subsumed into a *n-body transition density amplitude*, and the interaction between the active nucleons and the probes is described by an *n-body interaction kernel*.

The top diagram of fig. 1 showcases the interaction of the external probe with a single nucleon ($n = 1$) in an $A = 3$ nucleus and requires a *one-body kernel*. This also demands, its *one-body transition density amplitude*. This is the transition amplitude that a nucleon labeled 3 with momentum $\vec{k}_3$, absorbs momentum $\vec{q}$, and in addition has its quantum numbers change, while the nucleus remains in its ground state and isospin is conserved. The details of this will be provided shortly.

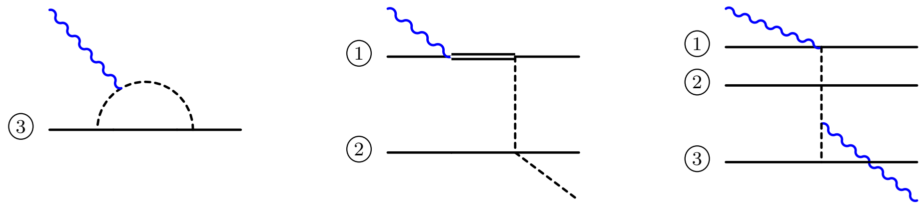

Figure 2: Examples of forms of the red blob "interaction kernels" above, from [7]

Likewise, the middle diagram proceeds with a *two-body kernel* and contains the *two-body transition density amplitude.* These $n = 2$ densities are the transition amplitude for a two body state having two-body CM momentum $\vec{k}_{12}$ and relative momentum $\vec{p}_{12}$ between the two active nucleons, to absorb another momentum $\vec{q}$, and have its quantum numbers changed, while the nucleus remains in its ground state and isospin is conserved. For densities with $n \geq 3$, we would continue in the same manner. The $n = 3$ case can be seen at the bottom of the diagram. To calculate the scattering problem in full for a nucleus with A nucleons, we must in principle calculate all transition densities for $n = 1, 2...A$ active nucleons, but χEFT predicts contributions for $n \geq 3$ are generally suppressed by at least one power in the expansion parameter. This truncation is paramount; the transition densities become unfeasibly large and the integrations become non-trivial even in the $n = 3$ case.

5 Analysis

We now write the wave function in the three body system as the partial-wave decomposition of Jacobi momenta p_i and the relevant quantum numbers α. The wave function in momentum space is given by

$$\psi_\alpha(p_{12},\, p_3) = \langle p_{12} p_3 \alpha | M \rangle \,. \tag{9}$$

The nucleus being described has total angular momentum J and spin-projection M. The momenta are $\vec{p}_{12} = \frac{1}{2}\left(\vec{k}_1 - \vec{k}_2\right)$ where $\vec{p}_3 = \vec{k} + \frac{1}{3}\vec{k}_3$, and $p_{12} = |\vec{p}_{12}|$ and $p_1 = |\vec{p}_1|$. Recall $\vec{k}_i$ are the individual nucleon momenta, and $\vec{k}$ is the probe momentum in the CM frame. The quantity α represents all the quantum numbers of the nucleons inside the nucleus [7]:

$$|\alpha\rangle = |[(l_{12}s_{12})\, j_{12}\,(l_3 s_3)\, j_3]\, JM, (t_{12}t_3)\, TM_T\rangle\,, \tag{10}$$

where s, l and j are the spin, orbital and total angular momentum respectively. The quantum number s_3 simply represent the spin of nucleon 3, whereas s_{12} represents the total spin of the 1-2 subsystem; the quantities l_{12} and j_{12} combine the 1-2 subsystem similarly. Furthermore, s_{12} and l_{12} combine to j_{12} and likewise for l_3, s_3 and j_3. Finally j_{12} and j_3 combine to the total nucleus spin J. The same combinations are done for t_{12}, t_3 and T, with isospin projection M_T. Here t_3 and t_{12} are the isospin of the nucleon labeled 3 and the 1-2 subsystem respectively, T is the isospin of the entire nucleus and M_T is the number of protons minus neutrons over 2. We now seek to describe the scattering amplitudes.

We restrict ourselves to elastic processes, which simplifies the following discussion by requiring that the probe does not change the charge of any of the nucleons. For the one body density, the *one-body kernel* of the probe interaction with nucleon 3 is O_3. Leaving nucleons 1 and 2 as spectators [7],

$$\begin{aligned}\left\langle \vec{k}_3' \left| \left\langle s_3 m_3^{s\prime} \left| \left\langle t_3 m_3^{t\prime} \left| \hat{O}_3(\vec{k},\vec{q}) \right| t_3 m_3^t \right\rangle \right| s_3 m_3^s \right\rangle \right| \vec{k}_3 \right\rangle & \\ \equiv \delta_{m_3^{t\prime} m_3^t}\, \delta^{(3)}\left(\vec{k}_3' - \vec{k}_3 - \vec{q}\right) O_3 \left(m_3^{s\prime} m_3^s m_3^t; \vec{k}_3; \vec{k}, \vec{q}\right), &\end{aligned} \tag{11}$$

where m_t and m_t' are the isospin of the active nucleon before and after the interaction (recall $m_t = \pm\frac{1}{2}$ is the proton/neutron). Symbolically, the matrix element $\hat{O}_3$ is:

$$\begin{aligned}\left\langle M' \left| \hat{O}_3(\vec{k},\vec{q}) \right| M \right\rangle = \sum_{\alpha\alpha'} \int \mathrm{d}p_{12}p_{12}^2 \; \mathrm{d}p_3 p_3^2 \; \mathrm{d}p'_{12}p'^2_{12} \; \mathrm{d}p'_3 p'^2_3 \psi^\dagger_{\alpha'}\left(p'_{12}p'_3\right) \psi_\alpha\left(p_{12}p_3\right) \\ \times \left\langle p'_{12}p'_3 \left[\left(l'_{12}s'_{12}\right) j'_{12} \left(l'_3 s_3\right) j'_3\right] J'M' \left(t'_{12}t_3\right) T'M_T \right| \hat{O}_3(\vec{k},\vec{q}) \\ \left| p_{12}p_3 \left[\left(l_{12}s_{12}\right) j_{12} \left(l_3 s_3\right) j_3\right] JM \left(t_{12}t_3\right) TM_T \right\rangle .\end{aligned} \tag{12}$$

The central result is that up to relativistic corrections, this can be written as:

$$\left\langle M' \left| \hat{O}_3(\vec{k},\vec{q}) \right| M \right\rangle = \sum_{\substack{m_3^{s\prime}\, m_3^s \\ m_3^t}} \hat{O}_3\left(m_3^{s\prime} m_3^s, m_3^t; \vec{k}, \vec{q}\right) \rho^{m_3^t M_T, M'M}_{m_3^{s\prime} m_3^s}(\vec{k},\vec{q}) \,. \tag{13}$$

Here ρ, is the *one-body transition density amplitude* for the nucleus which was discussed previously and can truly be interpreted as the probability amplitude that nucleon m_3^t absorbs momentum $\vec{q}$, changes its spin projection from m_s^3 to $m_s^{3'}$ and changes the spin-projection of the nucleus from M to M'. Its operator form is

$$\rho^{m_3^t M_T, M'M}_{m'_3 m_3^s}(\vec{k},\vec{q}) = \left\langle M' \left| s_3 m_3^{s\prime}, t_3 m_3^t \right\rangle \mathrm{e}^{\mathrm{i}\frac{2}{3}\vec{q}\cdot\vec{r}_3} \left\langle s_3 m_3^s, t_3 m_3^t \right| M \right\rangle . \tag{14}$$

Importantly, ρ can be computed generically from a nuclear potential without reference to the kernel $\hat{O}_3$ in eq. (13). The two body case works similarly, and results in

$$\left\langle M' \left| \hat{O}_{12} \right| M \right\rangle = \sum_{\alpha'_{11},\alpha_{12}} \int \mathrm{d}p_{12}\, p_{12}^2 \; \mathrm{d}p'_{12}\, p'^2_{12} \; O_{12}^{\alpha'_{12}\alpha_{12}}\left(p'_{12}, p_{12}\right) \rho^{M_T,M'M}_{\alpha'_{12}\alpha_{12}}\left(p'_{12}, p_{12}; \vec{q}\right) \;. \tag{15}$$

This two-body density $\rho^{M_T,M'M}_{\alpha'_{12}\alpha_{12}}$ is of course completely distinct from the one-body density and has a more involved expression equivalent to (14). It is dependent on the incoming and outgoing quantum numbers α_{12} and α'_{12} of the 1-2 system, and also on the relative momenta of the two nucleons which are integrated over. As a result, the total file size for the two nucleon densities are on the order of a few GB per energy and angle, whereas those of the one nucleon densities are on the order of a few KB. The additional integration as well as the larger file size results in significantly increased computational processing. The final scattering amplitude is given by

$$A_M^{M'}(\vec{k},\vec{q}) = 3\,\langle M'|\,\hat{O}_3(\vec{k},\vec{q})\,|M\rangle + 3\,\langle M'|\,\hat{O}_{12}(\vec{k},\vec{q})\,|M\rangle\,, \tag{16}$$

the symmetry factors arise because the probe can either hit $\binom{3}{1} = 3$ indistinguishable nucleons, or $\binom{3}{2} = 3$ indistinguishable nucleon pairs. Note we may also have to sum over the incoming/outgoing quantum numbers.

The important result is that the densities can be computed completely separately from the incoming probe kernel. The reaction $\gamma^3\mathrm{He} \to \gamma^3\mathrm{He}$ uses the same kernel as $\gamma\,^6\mathrm{Li} \to \gamma\,^6\mathrm{Li}$, as do all other $\gamma X \to \gamma X$ processes. On the other hand, $\pi\,^3\mathrm{He} \to \pi\,^3\mathrm{He}$ uses the same density as $\gamma\,^3\mathrm{He} \to \gamma\,^3\mathrm{He}$ as do all other $Y\,^3\mathrm{He} \to Y\,^3\mathrm{He}$ processes. The target specifies only the density, and the reaction specifies only the kernel. As a result, the densities can be calculated in advance and applied for any interaction one wishes to analyze. In the case of the targets listed here, our collaborator Andras Nogga has calculated the relevant ρ using a chiral SMS potential with various cutoffs [7]. We now seek to apply this to more interactions.

My main role will be the calculation of the kernels, $\hat{O}_3$ and $\hat{O}_{12}$ and convolution of the kernels with the densities for various reactions analogous to eq. (15) and eq. (13). The physics community has published many of the kernels that will be required, but some of them will need to be improved upon, and all of them will need to be implemented in our scheme. For Compton scattering, we will use the same kernels as Griesshammer *et al.* [7]. The pion-photoproduction kernels in essence come from Beane *et al.* 1997 but will need to be expanded on [8]. Beane *et al.* 2002 provides a kernel for $\pi\,^3\mathrm{H} \to \pi\,^3\mathrm{H}$, but again may need to be developed further [9]. Many assumptions have to be made about dark matter scattering, but the foundation for this was provided by Korber *et al.* who provide a tentative kernel [10].

As is exemplified in fig. 3 all of the processes have very similar topologies. This is because, in all of these systems the lowest order contributions come without loops, with one particle going in, and one particle going out. Importantly this greatly simplifies our analysis, and allows for the reuse of code for different processes.

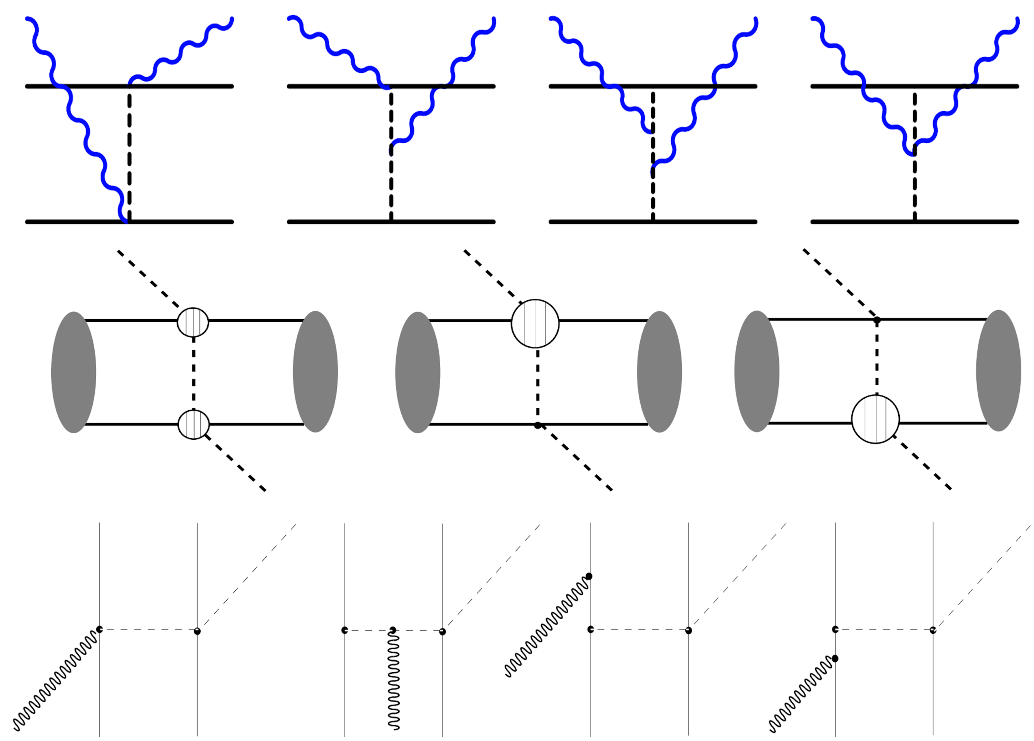

Figure 3: Top: γ^3He $\to \gamma^3$He [7]. Middle: $\pi^2 H \to \pi^2 H$ [9]. Bottom $\gamma X \to \pi X$ [11]

5.1 Uncertainty

Serious theories in nuclear physics come with uncertainty estimates; there are many sources of uncertainty in this problem, but we will briefly discuss three. First, the truncation of the amplitude calculation at the two-body considerations $\hat{O}_{12}$, that is, from not including $\hat{O}_{123}$ and higher terms. We handle this by estimating the general expansion parameter for the system, which gives an upper bound on the effects of $\hat{O}_{123}$

and other higher order terms. Second, there is uncertainty in O_3 and O_{12} because at the moment we consider only one loops contributions. When developing this work, we may deem it prudent to include higher order loop corrections, but truncation will still occur at some finite loop number. The third source of uncertainty comes comes from the chiral SMS potential; it is a pairwise potential only, that is three body forces are neglected. Additionally it is truncated at a given order in the expansion parameter Q. This truncation results in different results for each cutoff energy of the potential, which in turn manifests as different values for the densities ρ, for each value of the cutoff. The uncertainty will be estimated by calculation of the amplitude using many different cutoffs.

6 Proposal

"Scattering Observables from One- and Two-Body Densities..." by Griesshammer *et al.* describes the density formalism for the calculation of scattering cross sections. This technique makes the computation of scattering cross sections and other observables possible on a standard computer. In addition to greatly improving productivity of researchers, this also brings heavier nuclei into reach. It allows us to apply tested techniques to new problems, provides a framework by which the densities can rapidly be produced, and overall simplifies the numeric considerations.

I will study pion photoproduction $\gamma X \to \pi^0 X$, Compton scattering $\gamma X \to \gamma X$, pion scattering $\pi^0 X \to \pi^0 X$ and dark matter scattering $\Theta X \to \Theta X$, for the targets $X = {}^3\text{He}, {}^4\text{He}, {}^3\text{H}$ and $X = {}^6\text{Li}$, with the exception of $\gamma^3\text{He} \to \gamma^3\text{He}$, and $\gamma^4\text{He} \to \gamma^4\text{He}$ since this was already calculated by the Griesshammer group. Note that I may improve it by including higher order terms. This calculation will allow me to interpret the results of past, ongoing, and future experiments.

7 Relevance to the Community

Scattering on light nuclei has been the subject of much research over the past few decades. In particular, Compton scattering allows for the extraction of electromagnetic polarizabilities α_{E1} and β_{M1} of the proton and neutron which have been identified as significant by the Long Range Plan for Nuclear Science [12]. These quantities describe the induced electric and magnetic dipole moment of nucleons induced by a external field and also enter in the electromagnetic Hamiltonian of the system $\mathcal{H} = -4\pi \left(\frac{1}{2}\alpha_{E1}\vec{E}^2 + \frac{1}{2}\beta_{M1}\vec{H}^2\right)$. With this motivation, George Washington University professor Gerry Feldman has worked extensively on resource intensive experimental projects seeking to characterize Compton scattering on lithium 6, $\gamma^6\mathrm{Li} \to \gamma^6\mathrm{Li}$ [13, 14]. All of this was done without the aid of theoretical prediction, or ways to extract the quantities α_{E1} and β_{M1} without resorting to phenomenological only models; this proposal addresses this gap. Additionally this theoretical prediction can be leveraged to select energies for future Compton scattering experiments in order to extract α_{E1} and β_{M1} with maximum accuracy. Furthermore, there has been extensive work at the MAMI facility on pion-nucleon scattering as well as pion-photoproduction on the proton and neutron [15]. This work will lay the foundation for future extension of these experiments to light nuclei targets.

As described in the Long Range Plan for Nuclear Science, dark matter is currently outside of the standard model, and therefore is the subject of intense scrutiny. There are attempts to conduct dark matter scattering, in particular through the observation of "dark photons" [16], but this remains difficult due to the low rate of interaction with standard model particles. As a result, quantitative description of dark matter is rare. This proposal attempts to fill in some of this required description.

In addition to the experimental relevance, my work will be valuable from a theoretical standpoint since it serves as a test of chiral symmetry. This entire work functions in the chiral EFT framework, which confines the theory by mandating chiral symmetry. If this theoretical work matches experiment, this increases our confidence that nature does truly follow chiral symmetry.

8 Relevant Skills and Skills to Acquire

In order to successfully pursue this thesis topic I must come to a complete understanding of the mathematical framework the system is embedded in, including developing a state of the art understanding of Effective Field Theories. I have already completed Nuclear and Particle Physics I and II, as well as studied Effective Field Theories in a summer project. Additionally, I have enrolled in "TALENT School @MITP, Effective Field Theories in Light Nuclei: From Structure to Reactions" in Mainz Germany during summer of 2022. This program will allow me to greatly expand my knowledge base.

I will be inheriting a code base written by Griesshammer *et al.* in Fortran upon which I will expand. I do not know how to program in Fortran at the moment, so I will have to learn this skill. I may choose to write some of my code in Rust, which is a modern, compiled alternative to C, which requires interfacing between the Fortran code and Rust. Rust has been chosen for this project due to its robust feature set, and support for functional programing, including passing functions as arguments to other functions. Prior to this thesis project I have been working with Dr. Griesshammer on programing solutions to scattering problems for use in a different experiment. In addition, all of my research during my undergraduate years used numerical analysis to quantify chaotic systems. Having to learn Fortran and Rust will not fundamentally impede the work on this thesis.

9 Conclusion, Timeline and Fallbacks

Developing and applyinDeveloping and applyingnts are distinct tasks. Often, taking time away from a programming task allows one to look at it with a new perspective when returning; therefore these tasks will be worked on in parallel to the extent that is possible. A tentative timeline is as follows:

	Task 1	Task 2
September 2022	Reproduce Griesshammer *et. al.* Result	-
Spring 2023	Develop kernels for $\gamma X \to \pi^0 X$	Complete Compton scattering on other targets
Summer 2023	Develop kernels for $\pi^0 X \to \pi^0 X$	Complete Analysis of $\gamma X \to \pi^0 X$ on all targets
Fall 2023	Develop kernels for $\Theta X \to \Theta X$	Complete Analysis of $\pi^0 X \to \pi^0 X$ on all targets
Jan./Feb. 2024	Begin compiling previous writing into thesis	Complete Analysis of $\Theta X \to \Theta X$ on all targets
Summer 2024	Finish writing thesis	-

Each topic under "Task 2" will also include the comparison of my results to experiment. Note if a certain subproject turns out to be too time consuming, or if the densities for a certain nucleus are not ready in time, then we can shift focus away from these areas into more productive ones. I could, for example, completely strip the Θ scattering analysis if we ran into issues with those kernels. In addition I could entirely drop the $\gamma X \to \pi X$ if this kernel proved too technically challenging. The final publication and defense of my thesis will occur at Dr. Griesshammer's discretion on some (non-empty) subset of the items described above.

References

[1] S. R. Juárez Wysozka et al. *Two interacting scalar fields: practical renormalization.* 2021. DOI: 10.48550/ARXIV.2104.09681. URL: https://arxiv.org/abs/2104.09681.

[2] R. Machleidt and D.R. Entem. "Chiral effective field theory and nuclear forces". In: *Physics Reports* 503.1 (June 2011), pp. 1–75. DOI: 10.1016/j.physrep.2011.02.001. URL: https://doi.org/10.1016%2Fj.physrep.2011.02.001.

[3] Stefan Scherer and Matthias R. Schindler. "A Chiral Perturbation Theory Primer". In: (2005). DOI: 10.48550/ARXIV.HEP-PH/0505265. URL: https://arxiv.org/abs/hep-ph/0505265.

[4] J. Gasser and H. Leutwyler. "Chiral perturbation theory: Expansions in the mass of the strange quark". In: *Nuclear Physics B* 250.1 (1985), pp. 465–516. ISSN: 0550-3213. DOI: https://doi.org/10.1016/0550-3213(85)90492-4. URL: https://www.sciencedirect.com/science/article/pii/0550321385904924.

[5] U. van Kolck. "Few-nucleon forces from chiral Lagrangians". In: *Phys. Rev. C* 49 (6 June 1994), pp. 2932–2941. DOI: 10.1103/PhysRevC.49.2932. URL: https://link.aps.org/doi/10.1103/PhysRevC.49.2932.

[6] H. Krebs P. Reinert and E. Epelbaum. "Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order". In: (2018). DOI: 10.1140/epja/i2018-12516-4. URL: https://link.springer.com/article/10.1140/epja/i2018-12516-4.

[7] Harald W. Grießhammer et al. "Scattering Observables from One- and Two-body Densities: Formalism and Application to γ^3 Scattering". In: *Few-Body Systems* 61.4 (Nov. 2020). DOI: 10.1007/s00601-020-01578-w. URL: https://doi.org/10.1007%2Fs00601-020-01578-w.

[8] S.R. Beane et al. "Neutral pion photoproduction on deuterium in baryon chiral perturbation theory to order q4". In: *Nuclear Physics A* 618.4 (June 1997), pp. 381–401. DOI: 10.1016/s0375-9474(97)00133-4. URL: https://doi.org/10.1016%2Fs0375-9474%2897%2900133-4.

[9] S.R. Beane et al. "The S-wave pion–nucleon scattering lengths from pionic atoms using effective field theory". In: *Nuclear Physics A* 720.3-4 (June 2002), pp. 399–415. DOI: 10.1016/s0375-9474(03)01008-x. URL: https://doi.org/10.1016%2Fs0375-9474%2803%2901008-x.

[10] C. Korber, A. Nogga, and J. de Vries. "First-principle calculations of dark matter scattering off light nuclei". In: *Phys. Rev. C* 96 (3 Sept. 2017), p. 035805. DOI: 10.1103/PhysRevC.96.035805. URL: https://link.aps.org/doi/10.1103/PhysRevC.96.035805.

[11] S. R. Beane, C. Y. Lee, and U. van Kolck. "Neutral pion photoproduction on nuclei in baryon chiral perturbation theory". In: *Physical Review C* 52.6 (Dec. 1995), pp. 2914–2924. DOI: 10.1103/physrevc.52.2914. URL: https://doi.org/10.1103%2Fphysrevc.52.2914.

[12] "Reaching for the Horizon: The 2015 Long Range Plan for Nuclear Science". In: (Sept. 2015). URL: https://www.osti.gov/biblio/1296778.

[13] L. S. Myers et al. "Compton scattering from ^{6}Li at 60 MeV". In: *Phys. Rev. C* 86 (4 Oct. 2012), p. 044614. DOI: 10.1103/PhysRevC.86.044614. URL: https://link.aps.org/doi/10.1103/PhysRevC.86.044614.

[14] L. S. Myers et al. "Compton scattering from ^{6}Li at 86 MeV". In: *Phys. Rev. C* 90 (2 Aug. 2014), p. 027603. DOI: 10.1103/PhysRevC.90.027603. URL: https://link.aps.org/doi/10.1103/PhysRevC.90.027603.

[15] D. Ashery et al. “True absorption and scattering of pions on nuclei”. In: *Phys. Rev. C* 23 (5 May 1981), pp. 2173–2185. DOI: 10.1103/PhysRevC.23.2173. URL: https://link.aps.org/doi/10.1103/PhysRevC.23.2173.

[16] Liangliang Su, Lei Wu, and Bin Zhu. “Probing for an ultralight dark photon from inverse Compton-like scattering”. In: *Physical Review D* 105.5 (Mar. 2022). DOI: 10.1103/physrevd.105.055021. URL: https://doi.org/10.1103%2Fphysrevd.105.055021.