Scattering Observables from Few-Body Densities and Application in Light Nuclei

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1. Introduction
Motivation

Transition Density Method:

Factor into probe interaction with active nucleons (kernel) and spectator nucleon behavior (density)

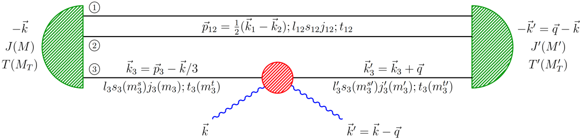
Allows interaction kernel to be recycled for different targets

Allows nucleus description to be recycled for different interactions

Allows code to be recycled for different interactions and nuclei

"Scattering Observables from One- and Two-Body Densities Griesshammer et. al. arXiv:2005.12207

Overview



Probe interacts with n active nucleons $\Longrightarrow$ *n body kernel* $\Longrightarrow$ *n body transition density*. Density independent of probe, kernel independent of density

Transition density ρ is the probability amplitude of a nucleus with quantum numbers $|M_J\rangle$ to absorb the momentum $\vec{q} = \vec{k}_2 - \vec{k}_1$ and change into quantum numbers $|M_J'\rangle$

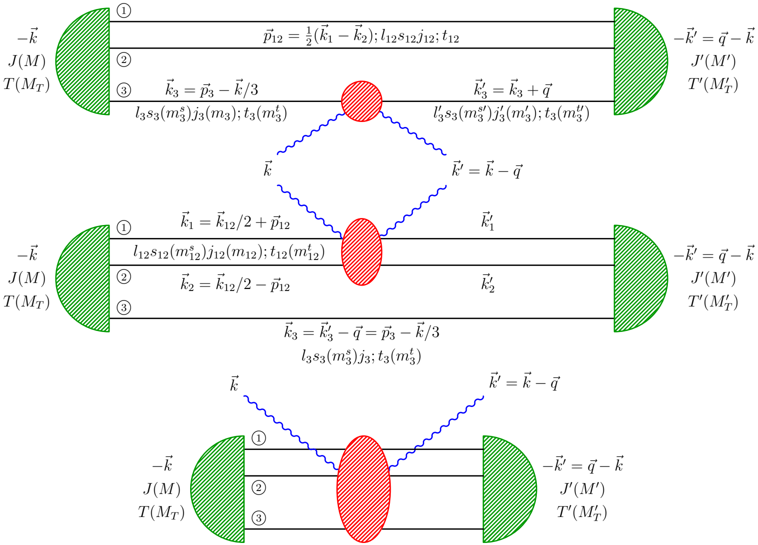


Figure: Griesshammer/... Few-B Sys 61 (2020) 48

Total Contribution

For an n body system, total scattering amplitude is:

$$\begin{aligned} A_M^{M'}(\vec{k},\vec{q}) = & \binom{A}{1} \langle M' | \hat{O}_3(\vec{k},\vec{q}) | M \rangle + \binom{A}{2} \langle M' | \hat{O}_{12}(\vec{k},\vec{q}) | M \rangle \\ & + \binom{A}{3} \langle M' | \hat{O}_{123}(\vec{k},\vec{q}) | M \rangle + \binom{A}{4} \langle M' | \hat{O}_{1234}(\vec{k},\vec{q}) | M \rangle \\ & + ... + \binom{A}{A} \langle M' | \hat{O}_{1...A}(\vec{k},\vec{q}) | M \rangle \end{aligned}$$

$\binom{A}{i}$ ways to hit i nucleons in a nucleus with A nuclei

Truncated Result

We use only the first two terms

$$A_M^{M'}(\vec{k},\vec{q}) = \binom{A}{1} \langle M' | \hat{O}_3(\vec{k},\vec{q}) | M \rangle + \binom{A}{2} \langle M' | \hat{O}_{12}(\vec{k},\vec{q}) | M \rangle$$

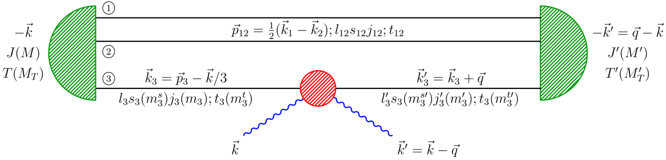
Higher order density suppressed by Q^i in this case. χEFT provides this ordering scheme.

- Depends on quantum numbers of probe (helicity), and quantum numbers of nucleus $M, M^{'}$

Uncertainties of Kernels and Densities

- Numerical integration uncertainty is negligible
- One and two body densities only
- Convergence pattern, truncation error
  - $\mathcal{O} = \mathcal{O}_0 Q^0 + \mathcal{O}_1 Q^1 + \mathcal{O}_2 Q^2 + ...$
- Finite order in expansion parameter Q
- Different cutoffs Λ in densities, estimate of residual dependence
- Expect $< 10\%$ theory uncertainty, analysis will determine

Details - 1 body

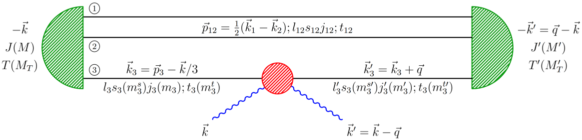


1 body contribution with $|\alpha\rangle = |[(l_{12}s_{12})j_{12}(l_3s_3)j_3]JM, (t_{12}t_3)TM_T\rangle$

$$\langle M'|\,\hat{O}_3(\vec{k},\vec{q})\,|M\rangle = \sum_{\alpha\alpha'} \int \mathrm{d}p_{12}\mathrm{d}p_3 p_3^2 \mathrm{d}p_{12}' p_{12}'^2 \mathrm{d}p_3' p_3'^2 \;\psi_{\alpha'}^{\dagger}(p_{12}'p_3')\psi_\alpha(p_{12}p_3)$$
$$\times \left\langle p_{12}'p_3' \left[(l_{12}'s_{12}')j_{12}'(l_3's_3)j_3'\right] J'M'(t_{12}'t_3)T'M_T \,\middle|\, \hat{O}_3(\vec{k},\vec{q}) \right.$$
$$\left| \, p_{12}p_3 \left[(l_{12}s_{12})j_{12}(l_3s_3)j_3\right] JM(t_{12}t_3)TM_T \right\rangle$$

Probe kernel: $\hat{O}_3$ changes quantum numbers of active nucleons

Details - 1 body



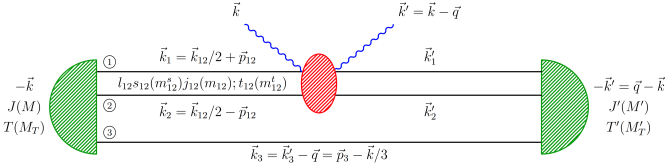
1 body contribution:

$$\left\langle M' \left| \hat{O}_3(\vec{k},\vec{q}) \right| M \right\rangle = \sum_{\substack{m_3^{s\prime}\, m_3^s \\ m_3^t}} \hat{O}_3\left(m_3^{s\prime} m_3^s, m_3^t; \vec{k}, \vec{q}\right) \rho^{m_3^t M_T, M'M}_{m_3^{s\prime} m_3^s}(\vec{k},\vec{q})$$

Example: Pion Photoproduction: $\hat{O}_3 = \frac{1}{2}\vec{\varepsilon}\cdot\vec{\sigma}_1$

```
do i=1, maxI
    rho=readRho(i)
    Result(Mz,Mzp)+= 0.5* rho*sigmax(m1p,m1)
end do
```

Details - 2 Body



$$\left\langle M' \left| \hat{O}_{12} \right| M \right\rangle = \sum_{\alpha'_{11},\alpha_{12}} \int \mathrm{d}p_{12}\, p_{12}^2\, \mathrm{d}p'_{12}\, p'^2_{12}\, O_{12}^{\alpha'_{12}\alpha_{12}} \left(p'_{12}, p_{12}\right)$$
$$\times\, \rho_{\alpha'_{12}\alpha_{12}}^{M_T,M'M} \left(p'_{12}, p_{12}; \vec{q}\right)$$

Relative angular momentum ℓ_{12} goes into α_{12}
Probe kernel: $\hat{O}_{12}$ changes quantum numbers of active nucleons

Density: $\rho_{\alpha'_{12}\alpha_{12}}^{M_T,M'M}$ involves only spectator nucleons

Same kernel convolution code can be used with different target densities

Swapping out densities of different targets is trivial

Potentials

Use potentials to calculate densities

We use χSMS potential with NN at N4LO and 3N at N2LO and with cutoffs of 400 MeV and 550 MeV

H. Krebs P. Reinert and E. Epelbaum. "Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order"

SRG - Similarity Renormalization Group

My work: $A \leq 6$, with ^{6}Li, many body interactions much more complicated

$\implies$ Density calculation more efficient with SRG evolution.

See reviews:

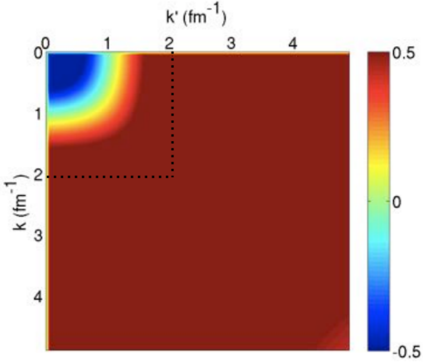
Kai Hebeler "Momentum space evolution of chiral three-nucleon forces" arXiv:1201.0169

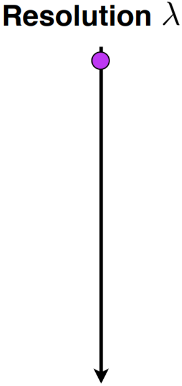
Sergio Szpigel "The Similarity Renormalization Group" arXiv:hep-ph/0009071

Defining an SRG transform

Strong dependence on high momentum

$\implies$ Difficult numerics for $A > 3$

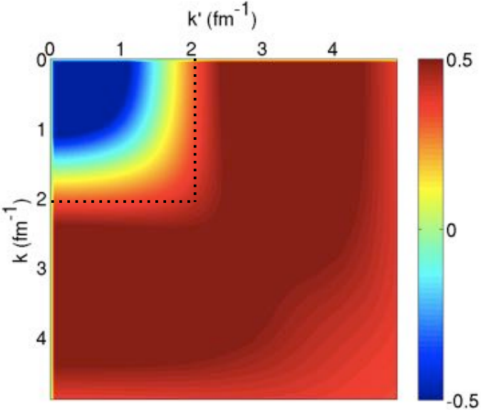


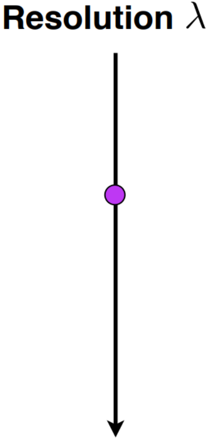


Figures from Kai Hebeler: "Chiral Effective Field Theory and Nuclear Forces: overview and applications" presentation at TALENT school at MITP 2022

SRG - shovels all dependence into lower momenta

Medium resolution





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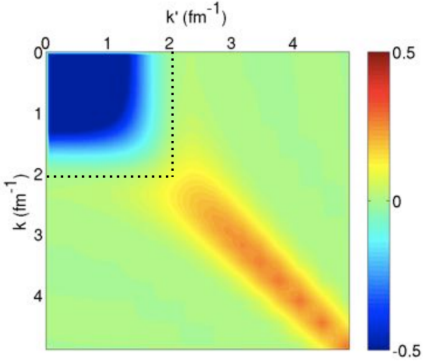
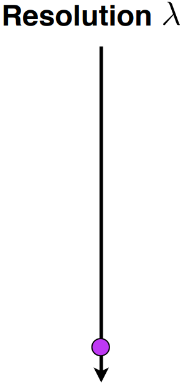
Low resolution





Approximate - cut the potential, only use dependence with $k < 2\text{fm}^{-1}$.
Neglects higher order contributions (4 and 5 nucleon etc).
Allows for calculation with less “area” of the potential used
Allows ^{6}Li calculation

SRG vs Fourier Transform

Fourier transforms are discrete unitary transformations

$$
\begin{aligned}
V(\vec{r}, \vec{r}\,') &= \langle r' | V | r \rangle \\
&= \int \mathrm{d}^3 p \, \mathrm{d}^3 p' \langle r' | p' \rangle \langle p' | V | p \rangle \langle p | r \rangle \\
&= V(\vec{p}, \vec{p}\,')
\end{aligned}
$$

After the transform our free variables have different physical meaning.
SRG is similar and it creates problems

SRG Variables

Any unitary transform, also transforms the coordinates

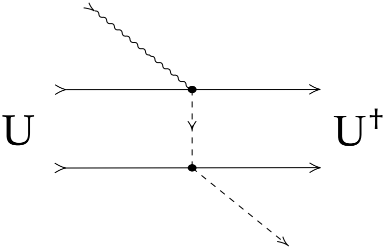
$$
\begin{aligned}
\langle p'|V|p\rangle &= \langle p'|\mathbb{1}V\mathbb{1}|p\rangle \\
&= \langle p'|U^\dagger U V U^\dagger U|p\rangle \\
&= \left(\langle p'|U^\dagger\right)\,\left(UVU^\dagger\right)\,\left(U|p\rangle\right) \\
&= \langle\widetilde{p}'|V_{eff}|\widetilde{p}\rangle
\end{aligned}
$$

Calling the free parameters in the SRG potential "momenta" is abuse of notation. They are not physical momenta.

Density Calculation

SRG changes Hamiltonian $\Longrightarrow$ changes Lagrangian $\Longrightarrow$ diagram contribution changes, and momenta aren't physical

One option: do a unitary transform of diagrams and kernels

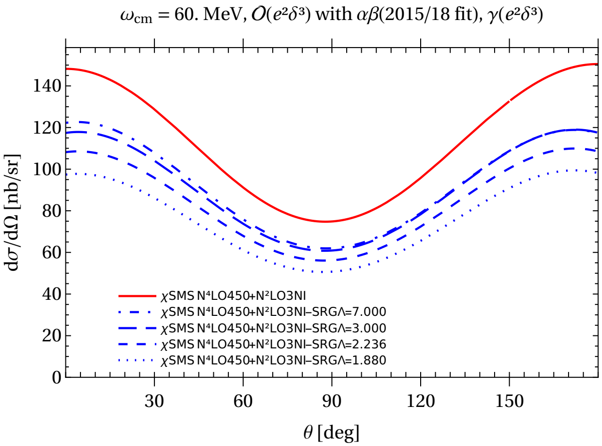


Problem: This breaks kernel - target (density) independence. Would have to introduce SRG λ dependence into the code

- Effectively would give SRG dependence on Lagrangian and therefore the diagrams.

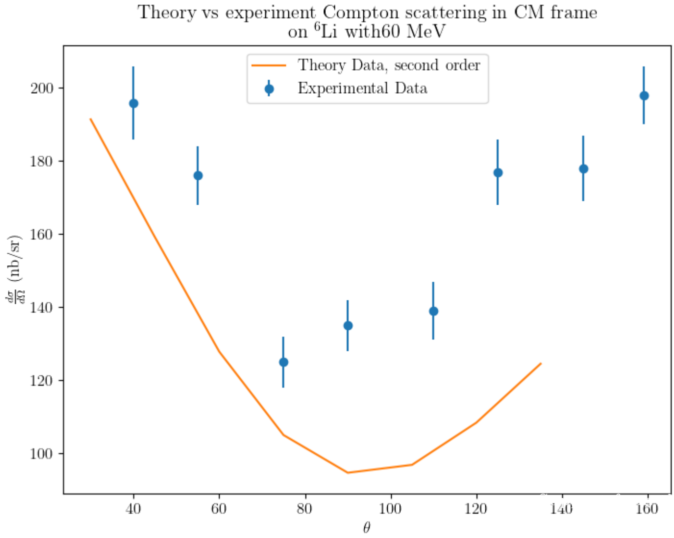
3. Specific Systems
Initial ^{4}He

^{4}He Compton scattering, with and without SRG



Smaller $\lambda \Rightarrow$ more change $\Rightarrow$ further deviation from true value

Initial ^{6}Li



Theory vs experiment Compton scattering in CM frame
on ^{6}Li with60 MeV
Theory Data, second order
Experimental Data
200
180
160
140
120
100
$\frac{d\sigma}{d\Omega}$ (nb/sr)
40
60
80
100
120
140
160
θ

Density Calculation

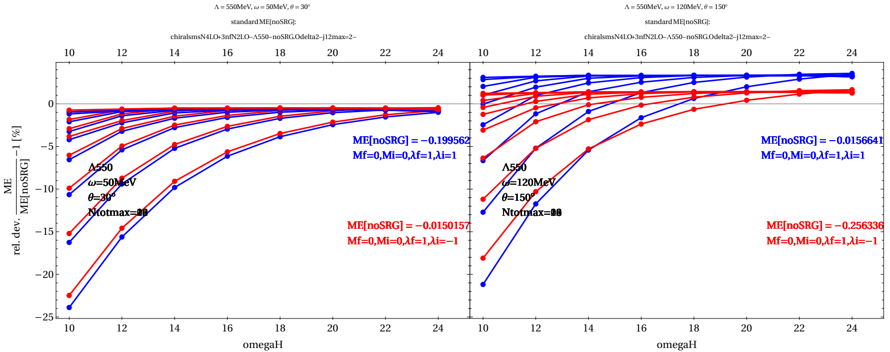
- Transforming the diagrams against the philosophy of separating the kernel from the target
- Instead: Back transform the densities

Xiang-Xiang Sun and Andreas Nogga have completed the back transform, with our collaboration (to be published).

Have ^{4}He with and without SRG back transform for comparison

Gained confidence moving ^{6}Li

Comparison - Compton Scattering Results on ^{4}He



SRG calculation uses harmonic oscillator basis, non-SRG uses Fadeev basis

Differences come from SRG induced many body forces and difference in basis

Each line: different maximum of number of states

x-axis: width of harmonic oscillator potential

Reactions

$\gamma X \to \gamma X$
Already implemented, can do new targets

$\gamma X \to \pi^0 X$
Technical limitation $\implies$ require π^0
Initial kernel: Beane/... NPA 618(1997) 381

$\pi X \to \pi X$
Charged pions $\implies$ easier for experiment
Initial considerations: Beane/... NPA 720(2003) 399

Kernel Similarity

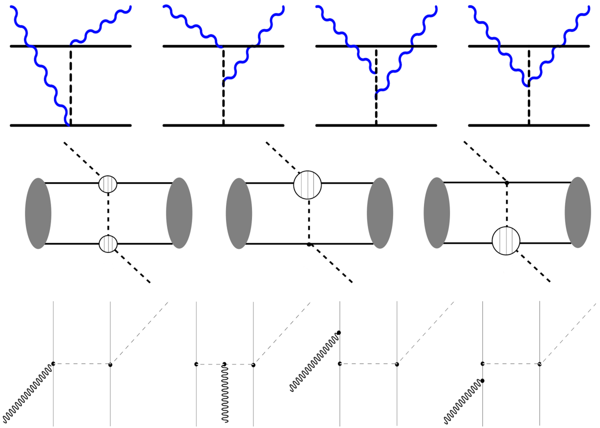


Figure: Two body kernels for: Top - Compton scattering; Middle - Pion scattering; Bottom - pion photoproduction

Specific Systems

Technical limitation $\implies$ nucleus doesn't change

Reactions are $yX \to zX$ with

$$X = {}^3\text{He}, {}^4\text{He}, {}^3\text{H}, {}^6\text{Li}$$

Extraction - Compton Scattering $\gamma X \to \gamma X$

Feldman, Downie: $\gamma^6\text{Li} \to \gamma^6\text{Li}$

Extract nucleon polarizabilities α_{E1} and β_{M1} (stiffness)

$$\mathcal{H} = -4\pi\left(\frac{1}{2}\alpha_{E1}\vec{E}^2 + \frac{1}{2}\beta_{M1}\vec{H}^2\right).$$

Many experiments on ^{6}Li, no theory prediction

Kernel exists and is implemented

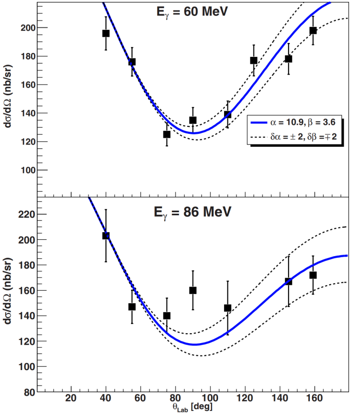
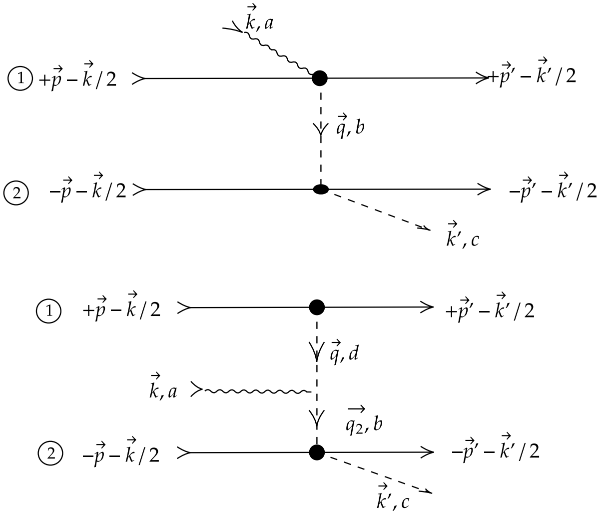


Figure: Myers/... PRC 90(2014) 027603

Two body Pion Photoproduction



Pion Photoproduction Two Body Result - At threshold

Lenkewitz Result arXiv:1103.3400

$$
\begin{array}{lrr}
 & \text{Lenkewitz (AV18+UIX)} & \text{My Result (CHSMS)} \\
x, y \text{ polarization} : & -29.3 \text{ fm}^{-1} & \sim -31 \text{ fm}^{-1} \\
z \text{ polarization} : & -22.9 \text{ fm}^{-1} & \sim -24 \text{ fm}^{-1}
\end{array}
$$

%6.6 and %5.3 difference

My result is currently numerically unstable

Conclusion

- Kernel, for $\gamma X \to \gamma X$, $\gamma X \to \pi X$, $\pi X \to \pi X$
  - Development
  - Coding
  - Convolution
- Extract, predict, and parameterize scattering processes
- Fill in theory gap for experiment
- Lay groundwork for future work with densities
  - So far has only been used with Compton, and dark matter scattering
  - Trigger interest: J. de Vries et. al. "Dark matter scattering off ^{4}He in chiral effective field theory"
    arxiv:2310.11343