

Scattering Observables from Few-Body Densities and Application in Light Nuclei

Alexander Long* and Harald W. Griesshammer

*Institute for Nuclear Studies, Department of Physics,
George Washington University, Washington DC 20052, USA*

E-mail: alexlong@gwu.edu, hgrie@gwu.edu

The dynamics of scattering on light nuclei is numerically expensive using standard methods. Fortunately, using recent developments, the relevant quantities can be factored into a product of the n -body transition density amplitude (TDA) and the interaction kernel of a chosen probe. These TDAs depend only on the target, and not the probe; they are calculated once and stored. The kernels depend on only the probe and not the target; they can be reused for different targets. The calculation of transition densities becomes numerically difficult for $n \geq 4$, but we discuss a solution through use of a similarity renormalization group transformation, and back transformation. This technique allows for extending the TDA method to ${}^6\text{Li}$. We present preliminary results for Compton scattering on ${}^6\text{Li}$ and compare with available data, anticipating an upcoming thorough study [18]. We also discuss ongoing extensions to pion-photoproduction and other reactions on $A \leq 6$ nuclei.

*The 11th International Workshop on Chiral Dynamics (CD2024)
26-30 August 2024
Ruhr University Bochum, Germany*

*Speaker

This is a comment, (for Dr.Griesshammer's use) a^2 This is a question (for Alex's use)

1. Introduction

my proc will be very similar to EPJ Web Conf. 303 (2024) 04002 MENU 2023 e-Print: 2401.15673 [nucl-th] – you might want to have a look. Effective Field Theories (EFTs) in nuclear physics make precise predictions by employing only those degrees of freedom that are most pertinent to the physical system under consideration, rather than relying on the complete set of intrinsic degrees of freedom present in the underlying theory (typically quarks and gluons in nuclear and particle physics). In this work, we utilize Chiral Effective Field Theory (χ EFT) which adopts nucleons and pions as its fundamental degrees of freedom. The present study is concerned with scattering probes off light nuclei. To this end, the Transition Density Amplitude (TDA) method was developed by Griesshammer *et al.* and de Vries *et al.* [1, 17]. The TDA formalism describes the interaction of a probe with an A -body target. The process is factored into the interaction with n nucleons which are therefore called *active* and the background $A - n$ nucleons which do not directly interact with the probe which are called *spectators*. The former enters in description of the kernel (along with the probe description) whereas the latter constitutes the TDA. Figure 1 provides an illustrative example for the case $A = 3$. The mathematical treatment of these two components is entirely distinct therefore if one has access to a distinct kernels (e.g. Compton pion and scattering) and b distinct TDAs (e.g. ^3He , ^4He , ^6Li), then a total of ab different outcomes may be generated (provided certain details about the kinematics hold). *I want to omit the detail that the one and two body kernels are different here, otherwise it gets too wordy. If it really needs it then I think its best to leave this part out. Maybe we leave it out anyway, I only get 10 pages.* The n -body kernel characterizes the interaction in the reduced case where interacts exclusively with the n -body system. For example, the one-body kernel in Compton scattering encompasses the same contributions as those arising in Compton scattering off a single nucleon.

For scattering off an A -body nucleus, the total scattering amplitude is given by

$$A_M^{M'}(\vec{k}, \vec{q}) = \left\langle M' \left| \binom{A}{1} \hat{O}_1(\vec{k}, \vec{q}) + \binom{A}{2} \hat{O}_2(\vec{k}, \vec{q}) + \dots + \binom{A}{A} \hat{O}_A(\vec{k}, \vec{q}) \right| M \right\rangle \quad (1)$$

where \hat{O}_i is the i -body kernel, M, M' is the spin projection of the target nucleus, and there are $\binom{A}{i}$ ways for a probe to hit i nucleons. Fortunately, χ EFT provides small dimensionless expansion parameter $\delta \approx 0.4$ [3] which predicts a hierarchy of scales for probe energies greater than $\gtrsim 40\text{MeV}$ [20–22]. Therefore, the 3-body contribution and higher are negligible at this order, and we simply use

$$A_M^{M'}(\vec{k}, \vec{q}) = \binom{A}{1} \langle M' | \hat{O}_3(\vec{k}, \vec{q}) | M \rangle + \binom{A}{2} \langle M' | \hat{O}_2(\vec{k}, \vec{q}) | M \rangle$$

In practice, this is enough for accuracy on roughly the 5% level [1].

2. Kernels and Densities

The one-body and two-body kernel must be considered separately. Their form is different, and they require a one- and two-body density respectively. Symbolically, the matrix element \hat{O}_1 is

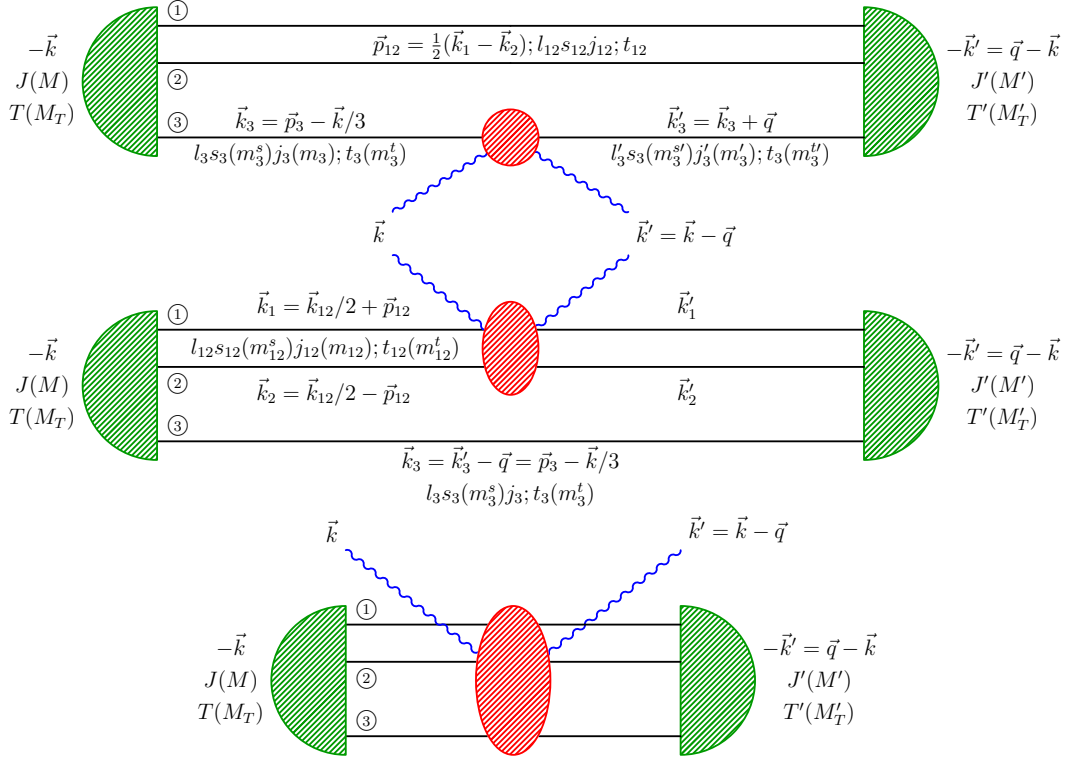


Figure 1: Kinematics in the center of mass frame and quantum numbers for an $A = 3$ system in the case of Compton scattering. Generalization to other reactions only changes the ingoing/outgoing probe. Generalization to $A > 3$ would result in more internal lines representing the nucleons. Top: one-body processes \hat{O}_1 (one active nucleon, two spectators), center: two-body processes \hat{O}_2 (two active nucleons, one spectator), bottom: three-body processes \hat{O}_3 (all nucleons active, no spectators). Red represents the kernels; everything else is represented by the densities. Green represents the wavefunction of the nucleons. $J(M)$ is the spin (projection) and $T(M_T)$ is the isospin (projection) of the nucleus. $l_i, s_i(m_i^s), j_i(m_i), t_i(m_i^t)$ refer to the angular momentum, spin angular momentum, total angular momentum, isospin (and their projections where appropriate) of the specific subsystem in the kernel being considered. From Griebhammer *et al.*[1].

This is getting kind of wordy...

rather involved and can be found in Griebhammer *et al.*[1, 3]. The central result is that up to boost corrections it can be written as:

$$\langle M' | \hat{O}_1(\vec{k}, \vec{q}) | M \rangle = \sum_{\substack{m_3^{s'} m_3^s \\ m_3^t}} \hat{O}_1(m_3^{s'} m_3^s, m_3^t; \vec{k}, \vec{q}) \rho_{m_3^{s'} m_3^s}^{m_3^t M_T, M' M}(\vec{k}, \vec{q}). \quad (2)$$

Here ρ , is the *one-body transition density amplitude* (TDA) for the nucleus and can be interpreted as the probability amplitude that nucleon with isospin projection m_3^t absorbs momentum \vec{q} , changes its spin projection from m_3^s to $m_3^{s'}$ and changes the spin-projection of the nucleus from M to M' , hence the name "Transition Density Amplitude". Additionally, M_T is the isospin projection of the entire nucleus and \vec{k} is the momentum of the incoming probe. The two-body case works similarly,

and results in

$$\langle M' | \hat{O}_2 | M \rangle = \sum_{\alpha'_{11}, \alpha_{12}} \int dp_{12} p_{12}^2 dp'_{12} p_{12}'^2 \hat{O}_2^{\alpha'_{12} \alpha_{12}}(p'_{12}, p_{12}) \rho_{\alpha'_{12} \alpha_{12}}^{M_T, M' M}(p'_{12}, p_{12}; \vec{q}) \quad (3)$$

Where

$$|\alpha\rangle = |[(l_{12}s_{12})j_{12}(l_3s_3)j_3]JM, (t_{12}t_3)TM_T\rangle \quad (4)$$

This is the two-body equivalent to (2). The two-body density $\rho_{\alpha'_{12} \alpha_{12}}^{M_T, M' M}$ is of course distinct from the one-body density. Moreover, just like the one-body case, it can be interpreted as a transition probability density amplitude. It depends on the incoming and outgoing quantum numbers α_{12} and α'_{12} of the system of the two active nucleons, and also on their initial and final relative momenta p_{12} and p'_{12} (also of the two nucleons) which are integrated over **it's not the 2 nucleons which are integrated over but their rel momenta. Need better formulation!** . *I do not understand what you mean, isn't this what I am already saying?* As a result, the file size for the two nucleon TDAs is approximately 20 MB per energy and angle, whereas those of the one nucleon TDAs are on the order of a few KB. Importantly, the densities ρ can for a given momentum transfer \vec{q} be computed directly from a nuclear potential, such as the chiral Semilocal Momentum-Space (χ SMS) potential [2] without reference to the kernel \hat{O}_1 or \hat{O}_2 .

3. SRG Transformation

Previous work using the TDA formalism has analyzed ^3He and ^4He [1, 3], but to extend this to ^6Li involves more interactions and as a result the calculation is more complicated and computationally expensive. To make the calculation of a TDA feasible for $A > 6$, a *Similarity Renormalization Group* (SRG) transformation is employed [4, 15]. This is of much experimental interest since ^6Li is a stable solid at room temperature and is therefore relatively simple to conduct an experiment on. There has been some experiments on ^6Li [10, 11], yet to date there is no theory prediction. We seek to fill in this gap. When using nuclear potentials, we approximate the nucleon-nucleon potential to be zero beyond a certain cutoff Λ_{NN} , and consequently neglect contributions above this cutoff in our calculations. In general, a nuclear potential, such as the χ SMS potential [2] does not fall off rapidly at high momenta. As a result we would have to extend the cutoff Λ_{NN} which in turn significantly increases the computational cost. The SRG transformation is a unitary transformation that shifts the relevant physics into the low-momentum region, thereby lowering minimum effective Λ_{NN} in the SRG evolved space. This, in turn, significantly improves the convergence rate of calculations for $A = 6$ making them actually possible.

The SRG transformation can be thought of as a local averaging or smoothing of the potential, resulting in decreased “resolution” as the SRG is applied, however it does this without losing any of the underlying information or compromising the physics.

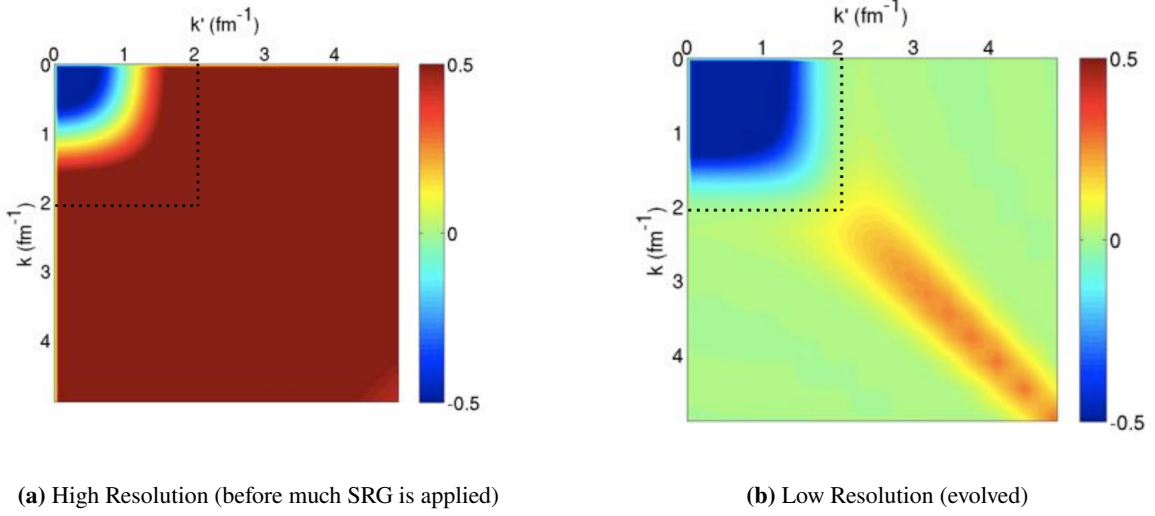


Figure 2: Nuclear potentials $V(k, k')$. Figures from Kai Hebeler: “Chiral Effective Field Theory and Nuclear Forces: overview and applications” presentation at TALENT school at MITP 2022, and modified with permission from Furnstahl *et al.*[15].

In the under-evolved, high resolution panel, fig. 2a, the potential does not go to zero rapidly at large momenta, whereas it does once the transformation is applied in the right panel, fig. 2b. As a result, a cutoff can be made at $\Lambda_{\text{NN}} = 2\text{fm}^{-1}$ without losing much accuracy, whereas the under-evolved potential required at least $\Lambda_{\text{NN}} = 5\text{fm}^{-1}$. The time complexity is at a minimum proportional to the number of array elements present, therefore we gain at least a factor of $(5/2)^2 = 6.25$ in efficiency; in practice the gains are even higher because near-zero values of the potential at large momenta means sparser grids can be used there.

where are you talking about the NCSM as a method to compute 6Li? *I don't, I didn't feel it was relevant here. We will include it in the next paper.*

The SRG transformation is essential, but it also creates a change in the physical meaning of the free variables. In fact, any unitary transformation ($U^\dagger U = \mathbb{1}$) also transforms the coordinates:

$$\langle p' | V | p \rangle = \langle p' | U^\dagger U V U^\dagger U | p \rangle = \langle p' | U^\dagger (U V U^\dagger) U | p \rangle = \langle \tilde{p}' | V_{eff} | \tilde{p} \rangle = V_{eff}(\tilde{p}, \tilde{p}') \quad (5)$$

So referring to the free variables in an SRG-transformed potential as “momenta” is, to some extent, incorrect. They do not represent physical states in the sense that they are not eigenstates to physical momenta. The Lagrangeans that generate the Feynman diagrams in the kernel, however, depend on physical momenta, and therefore we cannot directly use an SRG evolved potential in the non-SRG evolved kernel. To solve this, previous work with SRG transformations has transformed the Lagrangeans - and therefore the kernels - into the SRG evolved space as well *get citation*. However, in the context of the density formalism this would mean adding SRG dependence into the kernel, thereby breaking kernel-density independence. This is undesirable since it means more work for the group developing the kernel; in particular it would mean one would have to transform the kernel with the SRG corresponding to the density every time a new SRG evolved density is applied. Additionally, the SRG transformation can take many different forms [4, 15]; we wish to allow for these developments without having to re-write the kernel code. Therefore, we developed a method

whereby we first perform an SRG, then compute the densities via the SRG evolved potential, and then apply an inverse SRG transformation to the densities [16].

The process of completing the SRG evolution of the potential, solving for the wavefunction, and then applying the inverse transformation has parameters that must be fine-tuned, but this allows for uncertainty estimation. To solve for the nucleus wavefunction an expansion in the harmonic oscillator basis is used *I want to avoid referencing the basis used as much as possible. I only do it here because we need to motivate ω_H . It's fine for the next paper, but too much detail here in my opinion*. When expanded to infinite order, this basis forms a complete set, however, we truncate this expansion by including harmonic oscillator excitations up to N_{tot} . Additionally, the harmonic oscillator basis has a characteristic width, denoted by ω_H , and finally, the parameter Λ_{SRG} represents the SRG evolution of the potential, as seen in figure 2. $\Lambda_{\text{SRG}} = \infty$ corresponds to no evolution. As we will see all of these parameters affect the resulting cross-section. We note that above a certain minimum value uncertainty decreases with increasing N_{tot} , and at $N_{\text{tot}} = \infty$ the associated uncertainty goes to zero.

Unfortunately the application of the cutoff in the SRG transformation results in it no longer being strictly unitary. Our method neglects the resulting induced many-body forces, and it is essential to test its impact. The ${}^4\text{He}$ system is the highest nucleon number system we can calculate without using the SRG evolution, therefore in order to prepare for ${}^6\text{Li}$ where we cannot preform the raw calculation, we first assess this method's viability on ${}^4\text{He}$. Fortunately figure 3 shows the uncertainty associated with this approach is small.

During the TDA calculation, we obtain the binding energy of the simulated system, which we can use calibrate our parameters through comparison to the known experimental value. We determine an optimal ω_H that yields a good binding energy, while N_{tot} is taken as large as feasible. With this benchmark established, we have extended our analysis to ${}^6\text{Li}$, for which only the SRG evolved form is accessible. Future investigations [18] will carefully assess the extent to which different Λ_{SRG} values extrapolate to compatible results.

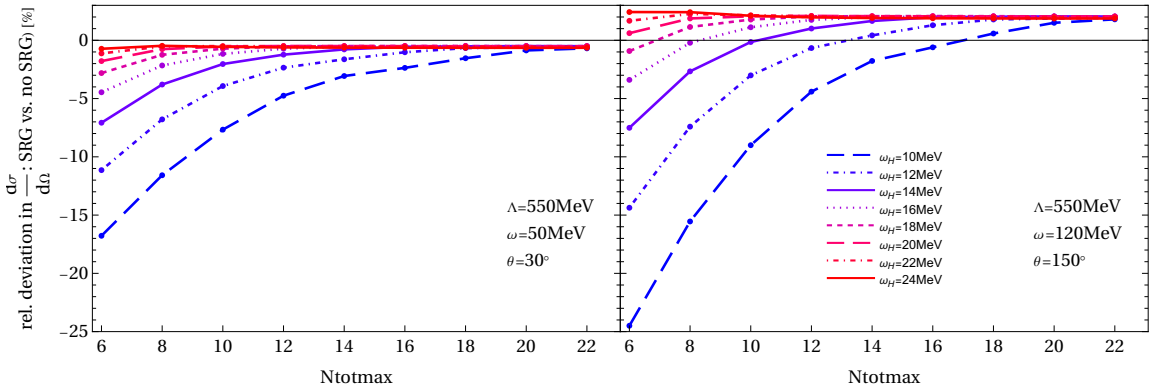


Figure 3: ${}^4\text{He}$ Compton scattering SRG convergence, comparing the exact, vs SRG evolved approaches. Deviations are due to induced manybody forces. “Relative deviation, (Rel. deviation) of A from M ”:= $\frac{A}{M} - 1$

In figure 3, we see the effectiveness of the results in the ${}^4\text{He}$ case. We expect the deviation to decrease as N increases, and importantly for our analysis, this shows what value of N is required.

The small error present at high N_{tot} is present do to the induced many body forces.

from here on, presentation needs a lot more verbiage and detail – but I guess you know that

discuss input: kernels same as in ${}^3\text{He}$ (reference), central values of polarisabilities same as there (aae central values of most recent extractions, reference). Comparison to HIGS data is good/bad/undecided. Will study convergence in detail. Here assumed overall 10% error as in ${}^3\text{He}$ from potential/cutoff variations plus order-by-order convergence plus numerics plus extrapolations in LambdaSRG/ N_{totmax} /omegaH,... get inspired by 1-paragraph summary in ${}^4\text{He}$ paper.

3.1 Results for ${}^6\text{Li}$

The kernels used for ${}^6\text{Li}$ Compton scattering are the same as Griebhammer *et al.* uses in ${}^3\text{He}$ and ${}^4\text{He}$ Compton scattering [1, 3]. Long *et al.*[18] will provide a full analysis of uncertainty. For this analysis an overall 10% uncertainty is assumed in agreement with Griebhammer *et al.*[1, 3] which accounts for the cutoff Λ_{NN} and Λ_{SRG} variation, along with ω_H selection. We include an additional, independent, contribution from N_{tot} which is easy to calculate, but is negligible compared to other sources; the methodology behind this will be discussed in Long *et al.*[18].

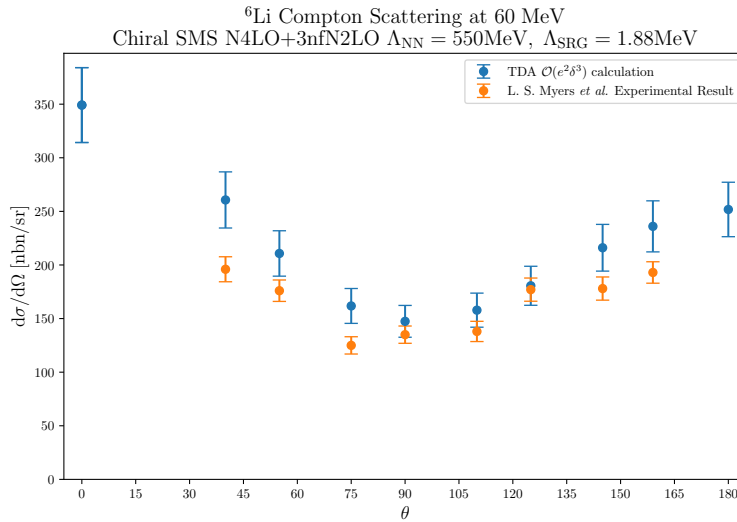


Figure 4: Compton scattering on ${}^6\text{Li}$ as compared to Myers *et al.*[11]

The data present does not match the experimental results. *What do we say about this? They did have a broken panel. One/two sentences wraps up this section.*

4. Using TDAs in different processes

With the TDAs calculated for Compton scattering, we now wish to recycle them for new processes. In particular pion-photoproduction, and pion scattering are of interest. Fortunately their kernels share remarkable similarity since if one ignores the type of incoming/outgoing particle the processes are topologically identical.

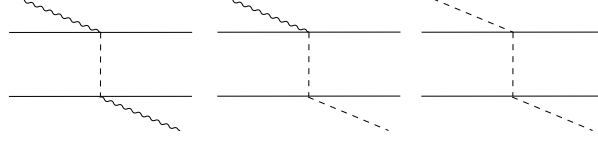


Figure 5: Topologically identical diagrams in Compton scattering, pion-photoproduction, and pion scattering
two-body only – add onebody? I'd rather not

4.1 Pion-Photoproduction

For the pion-photoproduction one-body kernel, we use the results from single-nucleon scattering, $\gamma N \rightarrow \pi N$ which has been studied extensively both in χ EFT and phenomenologically [5, 7, 13, 14]. Its differential cross section can be decomposed into the electric and magnetic multipoles $E_{l\pm}, M_{l\pm}$ [5]. Over the years, many experiments have measured these multipoles to high order and with good precision [6]. The resulting scattering matrices \mathcal{M} are exactly what enters as $\hat{\mathcal{O}}_1$ in equation (2). This approach solves a significant problem since the calculation of the one-body pion-photoproduction kernel to high accuracy directly from Feynman diagrams requires including many terms in the chiral expansion due to the proximity of the $\Delta(1232)$ resonance at ~ 200 MeV. A theoretical prediction of these multipoles is given by Rijnneeven et al [7], which we intend to compare to experimental data.

The two-body contributions do not easily decompose into multipoles, so we perform the calculation through expansions in the chiral Lagrangean via calculation of Feynman diagrams. To this end Beane *et al.* [23] provides the kernel for the reaction on the deuteron at threshold and since we only go up to the two-body kernel this is sufficient for our needs. Additionally this reaction kernel has been analyzed by Lenkewitz *et al.* [8, 9] whose result for ^3He we now compare to. In particular, we consider the commonly referred to form factors $F_S^{S\pm V}, F_L^{S\pm V}, F_{T/L}^{(a)} - F_T^{(b)}$, a full description of these formfactors is present in Lenkewitz *et al.* [8]. Note that here we present only the central values, but upcoming work will provide a full analysis [19]. We use the χ SMS potential, whereas Lenkewitz used the AV-18 potential, so it is expected our values will differ slightly.

^3He Compton Scattering	Lenkewitz <i>et al.</i> AV-18 [8]	χ SMS TDA calculation [19]
One-body		
F_T^{S+V}	0.017(13)(3)	-0.017
F_T^{S-V}	1.480(26)(3)	1.48
F_L^{S+V}	-0.079(14)(8)	-0.005
F_L^{S-V}	1.479(26)(8)	1.48
Two-body		
$F_T^{(a)} - F_T^{(b)}$	-29.3 fm $^{-1}$	-29.4 fm $^{-1}$
$F_L^{(a)} - F_L^{(b)}$	-22.9 fm $^{-1}$	-22.9 fm $^{-1}$

4.2 Pion scattering and other reactions

Beane *et al.* have developed the pion scattering kernel at threshold for both one-body and two-body interactions [12]. We anticipate extending this analysis to finite energy targets ^3H , ^3He , ^4He , ^6Li . Once the pion-photoproduction and pion scattering kernels have successfully been developed, we will be able to calculate all of these reactions on previously analyzed targets in the density formalism since we already have produced most of the TDAs required. This will be the first time chiral symmetry has been tested with heavier nuclei targets. *What else is there to say here? The Braun thesis makes it hard to justify... why is this interesting? first study (?) of ChiSym in these processes at heavier nuclei. But beware Braun PhD thesis: maybe cite to reflect literature?*

5. Conclusion

We have described a comprehensive framework for computing scattering observables in light nuclei by factorizing the full amplitude into target-dependent few-body transition density amplitudes (TDAs) and probe-dependent interaction kernels. This work expands on work in refs, as well as work on few body targets [1, 3, 9]. This separation allows us to treat the nuclear structure and the reaction mechanism independently, thereby streamlining the calculation of observables. The central thrust of this work is the successful extension of the density formalism to heavier targets like ^6Li by incorporating a similarity renormalization group (SRG) transformation. The SRG not only accelerates the convergence of our calculations by lowering the effective momentum cutoff, but—when combined with an appropriate inverse transformation of the densities—also preserves the kernel–density independence that is crucial for the versatility of our approach. Furthermore, we have presented preliminary results for Compton scattering on ^6Li which agrees *check this* agrees well with data, and we have outlined the ongoing extension of the formalism to other reactions, such as pion-photoproduction and pion scattering on light nuclei [18] *You have a note saying to I cite my proposal, but how? We never published it.* The ability to plug in different reaction kernels into the same TDA framework and vice versa not only enhances the predictive power of our approach but also paves the way for a unified treatment of various scattering processes in a wide range of few-body systems. Ultimately, this framework provides a promising route toward high-precision theoretical predictions, deepening our understanding of nuclear dynamics in light nuclei.

6. Acknowledgements

acknowledgments: Andreas+, supercomputer usage, DOE and DFG grants – see 4He paper We thank Andreas Nogga and Xiang-Xiang Sun of FZ Jülich, whose work on produces the required TDAs we deeply appreciate. *Below is a copy paste of all of the acknowledgements from 4He that might be relevant, this obviously needs to be fixed for this paper*

We appreciate the warm hospitality and financial support for stays which were instrumental for this research: HWG at the University of Manchester, Ohio University and FZ Jülich; and DRP at George Washington University and Chalmers University of Technology. HWG is grateful to the organisers and participants of the meeting of MAMI's A2 collaboration in Mainz for the stimulating atmosphere and financial support. He also thanks the organisers and participants of MENU 2023 in Mainz for the opportunity to present preliminary results and for a delightful atmosphere. This work

was supported in part by the US Department of Energy under contract DE-SC0015393 (HWG, JL) and DE-FG02-93ER-40756 (DRP), by the UK Science and Technology Facilities Council grant ST/P004423/1 (JMcG), by the Deutsche Forschungsgemeinschaft and the Chinese National Natural Science Foundation through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” (AN; DFG grant TRR 110; NSFC grant 11621131001), by the Ministerium für Kultur und Wissenschaft NordrheinWestphalen (MKW-NW) under funding code NW21-024-A (AN) and by a Tage Erlander Professorship from the Swedish Research Council, grant 2022-00215 (DRP). Additional funds for HWG were provided by an award of the High Intensity Gamma-Ray Source HIγS of the Triangle Universities Nuclear Laboratory TUNL in concert with the Department of Physics of Duke University, and by George Washington University: by the Office of the Vice President for Research and the Dean of the Columbian College of Arts and Sciences; by an Enhanced Faculty Travel Award of the Columbian College of Arts and Sciences. His research was conducted in part in GW’s Campus in the Closet. The computations of nuclear densities were performed on Jureca of the Jülich Supercomputing Centre (Jülich, Germany).

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