Chapter 6

Work and Kinetic Energy

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Introduction

- Newton's 2nd law is a second-order differential equation that can be integrated to determine an object's trajectory as a function of time. However, if we are only interested in changes to certain quantities along this trajectory there are often simpler methods of obtaining them.

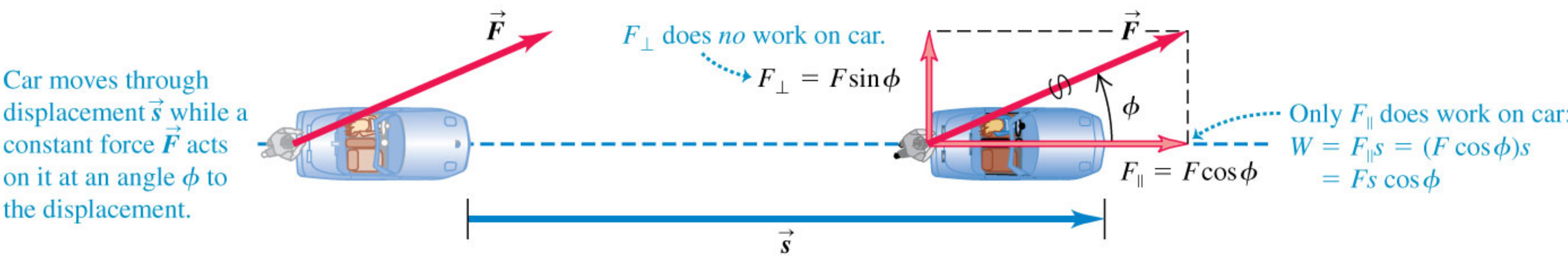
- In this chapter, the introduction of the new concepts of *work*, *energy*, and the *conservation of energy* will allow us to deal with such problems.

Work done by a constant force

- We define the *work W* done on an object experiencing a displacement $\boldsymbol{d}$ by a constant force $\boldsymbol{F}$ to be

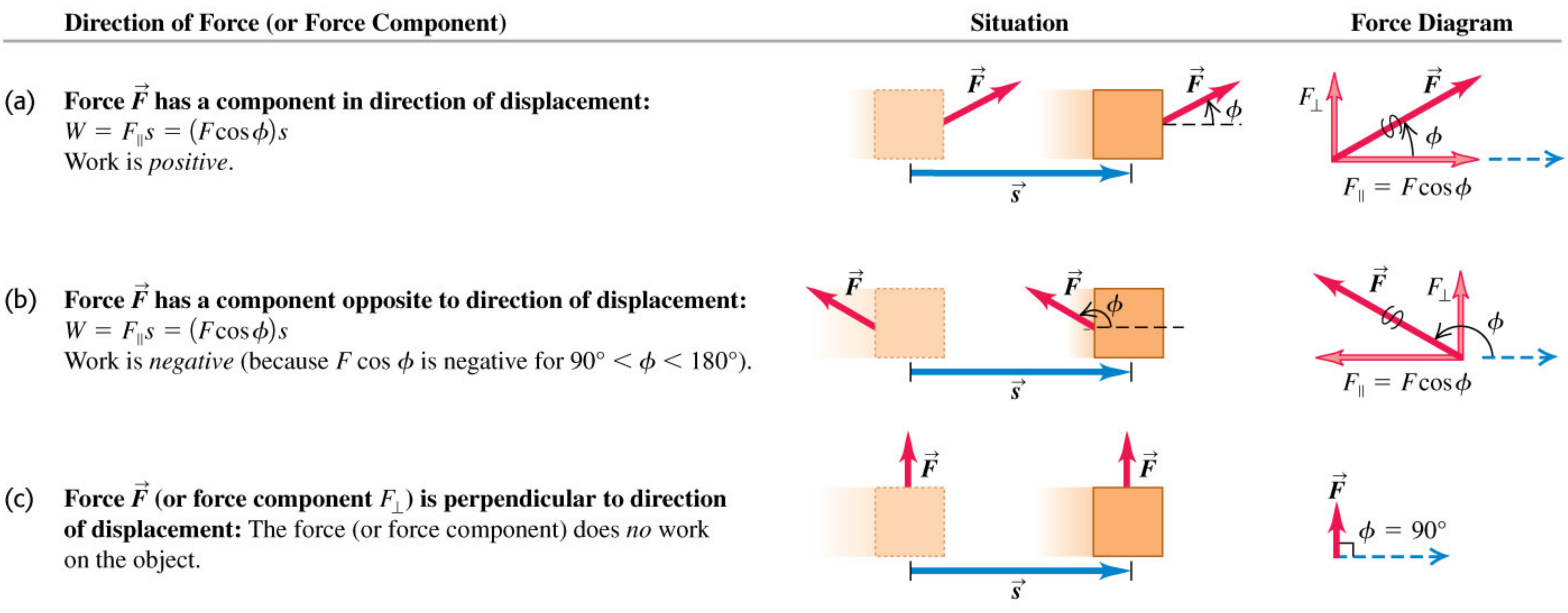
$$W = \boldsymbol{F} \bullet \boldsymbol{d}$$

- Work is a scalar with units N • m ≡ J (Joules).
- Only the parallel component of the force $F_{\|} = F \cos \phi$ does work.



Positive, negative, and zero work

- Work, like all scalar products, can be positive, negative, or zero depending on the angle ϕ between the force ***F*** and displacement ***d***.



Work done by several forces

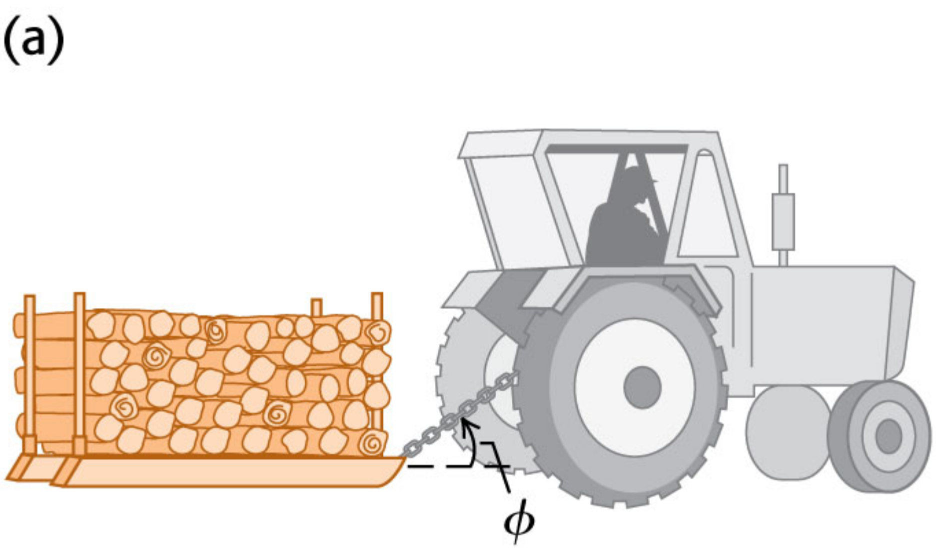
- Since the work is linear in the applied forces, the total work on a systems can be calculated in two ways:

$$W = \Sigma \boldsymbol{F}_i \bullet \boldsymbol{d} = (\Sigma \boldsymbol{F}_i) \bullet \boldsymbol{d} = \boldsymbol{F}_{\text{net}} \bullet \boldsymbol{d} = \Sigma(\boldsymbol{F}_i \bullet \boldsymbol{d}) = \Sigma W_i$$

- Let's see an example!

Work done by several forces

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground. The total weight of the sled and load is 14,700 N. The tractor exerts a constant 5,000 N force at an angle of 36.9° above the horizontal. A 3500 N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.



Solution

The normal force and weight perform no work since they are perpendicular to the displacement (normal forces never do work when motion is along the surface).

The work done by the tractor is:

$\boldsymbol{F}_T \bullet \boldsymbol{d} = (5000\text{ N})(20\text{ m}) \cos 36.9° = 80\text{ kJ}$

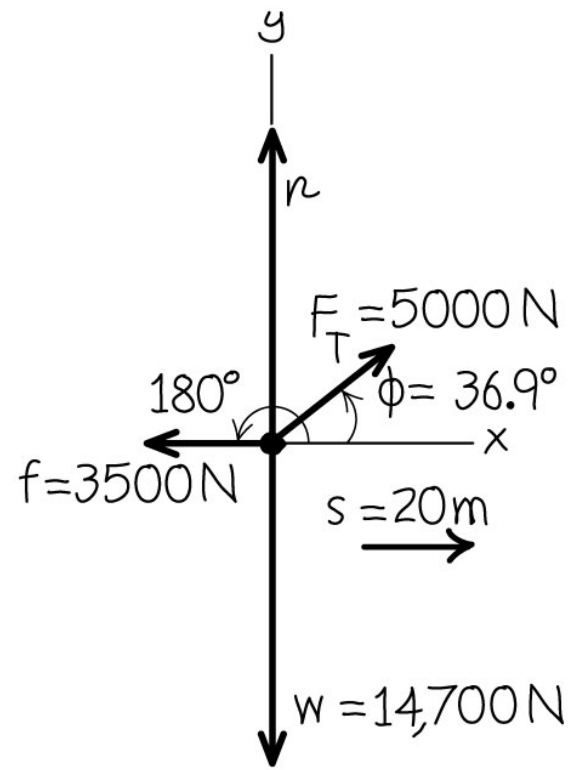
The work done by friction is:

$\boldsymbol{f} \bullet \boldsymbol{d} = (3500\text{ N})(20\text{ m}) \cos 180° = -70\text{ kJ}$

The total work is 80 kJ + (-70 kJ) = 10 kJ.

$\Sigma F_x = F_{Tx} + f_x = (5000\text{ N}) \cos 36.9° - 3500\text{ N} = 500\text{ N} \Longrightarrow W_{\text{tot}} = (\Sigma F_x)d = (500\text{ N})(20\text{ m}) = 10\text{ kJ}$

(b) Free-body diagram for sled



Kinetic energy and the work-energy theorem

- We learned when studying motion that component of the force parallel to the instantaneous displacement (velocity) is also responsible for changes in the speed.
- According to an equation of motion for a constant acceleration:

$$v_2^2 = v_1^2 + 2ad \Longrightarrow$$

$$W = Fd = mad = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \Delta K$$

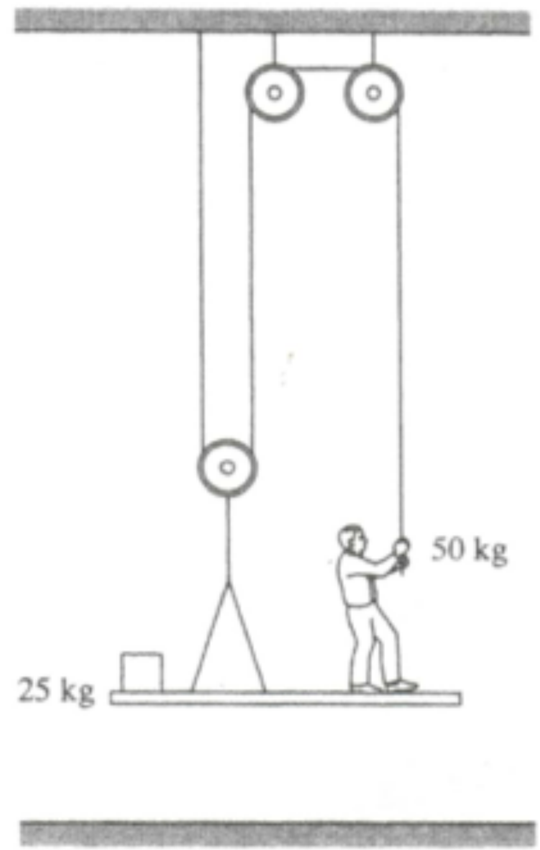
where $K = \tfrac{1}{2}mv^2$ is the *kinetic energy*

- The work done on an object equals the change in its kinetic energy, which also has units $\mathrm{kg} \cdot (\mathrm{m/s})^2 = \mathrm{J}$

ConcepTest #1

A 50 kg person stands on a 25 kg platform. He pulls on the rope that is attached to the platform via the frictionless pulley system shown here. If he pulls the platform up at a steady rate, with how much force is he pulling on the rope? Ignore friction and assume g = 10 m/s^2.

1. 750 N
2. 625 N
3. 500 N
4. 250 N
5. 75 N
6. 50 N
7. 25 N
8. not enough info



Solution

Method 1: Newton's 1st law

Steady rate $\Longrightarrow$ equilibrium

$\Sigma F_y = 3T - m_{tot}g = 0 \Longrightarrow T = ⅓(75\ \text{kg})(10\ \text{m/s}^2) = 250\ \text{N}$

Method 2: Work

Steady rate $\Longrightarrow$ no change in kinetic energy $\Longrightarrow$ no total work

If the platform rises 1 m, gravity does:

$W_g = (75\ \text{kg})(10\ \text{m/s}^2) \cos 180° = -750\ \text{J}$

Man pulls down on rope which moves down 3 m:

$W_T = T(3\ \text{m}) = -W_g = 750\ \text{J} \Longrightarrow T = 250\ \text{N}$

Example #2

You and your bicycle have a combined mass of 80.0 kg. When you reach the base of a bridge, you are traveling along the road at 5.00 m/s. At the top of the bridge, you have climbed a vertical distance of 5.20 m and slowed to 1.50 m/s. Ignore the work done by friction.

a. What is the total work done on you and your bicycle?

b. How much work have you done with the force you apply to the petals?

Solution #2

a. The total work done equals the change in kinetic energy

$$W_{tot} = \Delta K = \tfrac{1}{2}m(v_2^2 - v_1^2) = \tfrac{1}{2}(80.0\text{ kg})[(1.5\text{ m/s})^2 - (5.0\text{ m/s})^2]$$

$$= -910\text{ J}$$

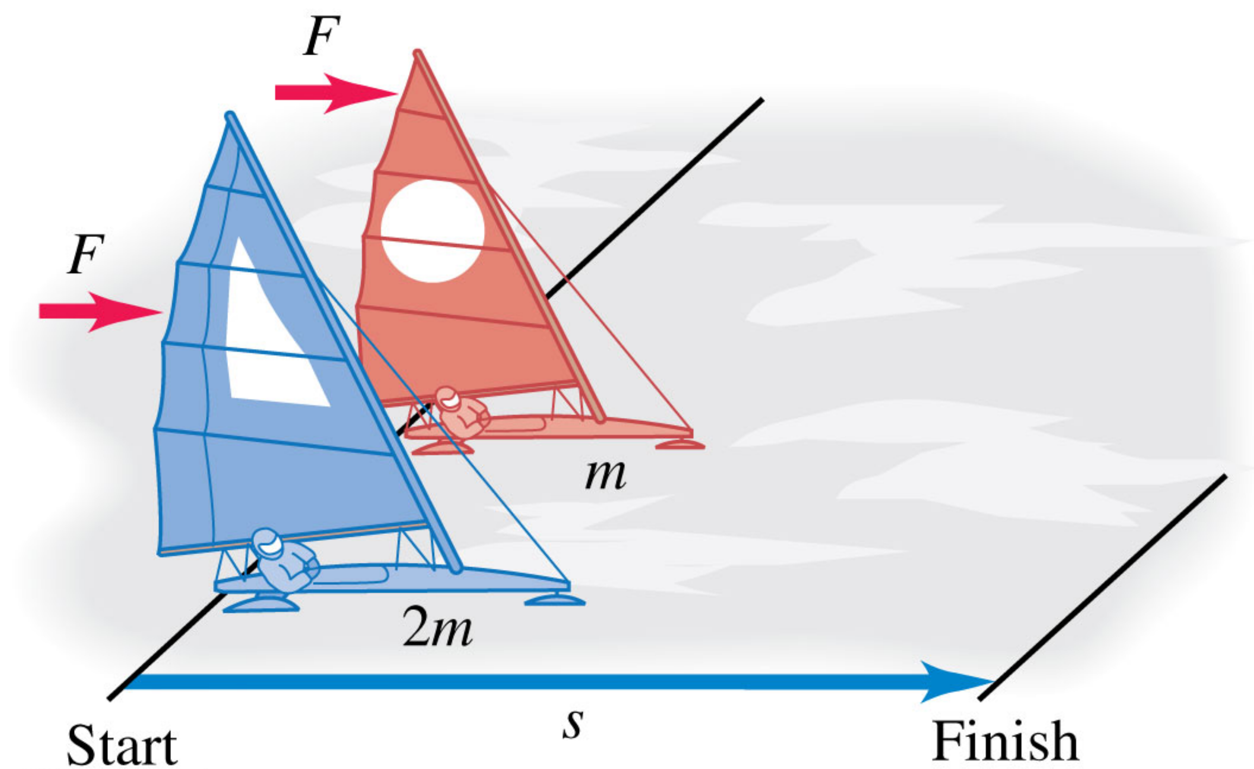
b. The work done by pedaling is the total work minus that done by gravity:

$$W_g = \boldsymbol{w} \bullet \boldsymbol{d} = -(80.0\text{ kg})(9.8\text{ m/s}^2)(5.20\text{ m}) = -4077\text{ J}$$

$$W_p = W_{tot} - W_g = -910\text{ J} - (-4077\text{ J}) = 3167\text{ J}$$

ConcepTest #2

Two iceboats hold a race on a frictionless horizontal lake. The two boats have masses m and $2m$. The boats have identical sails, so the wind exerts the same force $\boldsymbol{F}$ on each boat. They start from rest and cross the finish line a distance s away. Which boat wins? Which boat cross the finish line with more kinetic energy?



Solution

By Newton's 2nd law, the boat with a smaller mass will experience a greater acceleration and cross the finish line first. Since the boats experience the same force over the same distance, the work $W = \boldsymbol{F} \bullet \boldsymbol{d}$ done by the wind is the same.

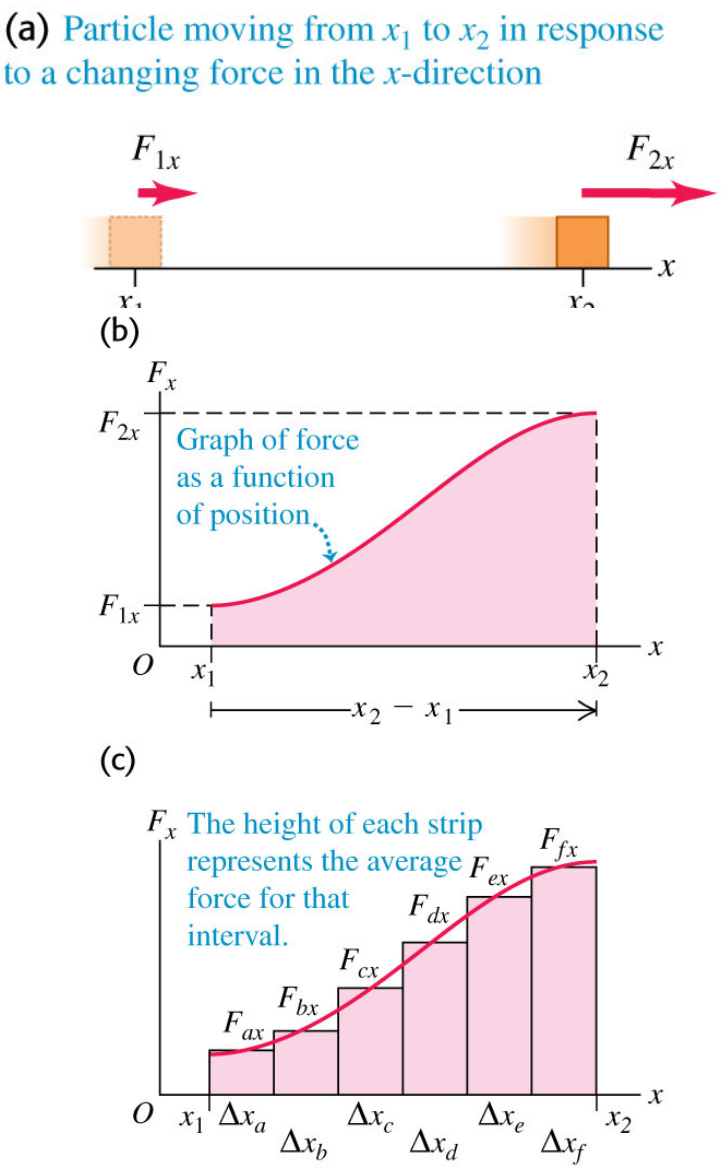
Work and energy with varying forces

- Many forces are not constant.
- If we approximate the work by dividing the total displacement into many small segments, the total work is the sum:

$$W = \Sigma F_i \, \Delta x_i$$

- In the limit the segments are infinitesimal, this becomes an integral:

$$W = \int_{x_1}^{x_2} F_x \, dx$$



Stretching a spring

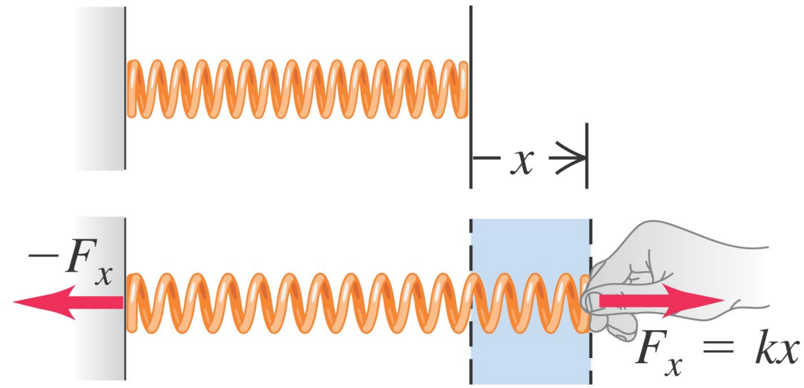
- The force required to stretch a spring a distance x is proportional to x (Hooke's law):

$$F_x = kx$$

- k is the *spring constant* and has units of N/m.

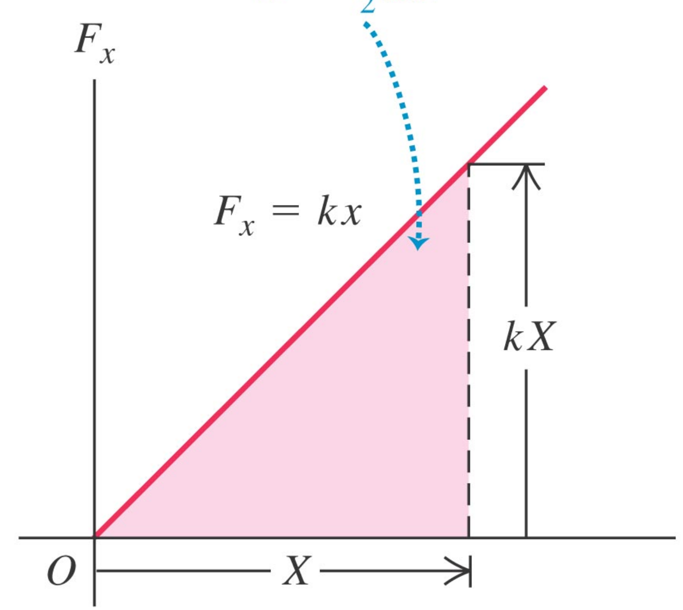
- The area under the graph represents the work done on the spring to stretch it a distance X:

$$W = \int_{x_1}^{x_2} F_x \, dx = \int_0^X kx \, dx = \frac{1}{2}kX^2$$



The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:

$W = \frac{1}{2}kX^2$

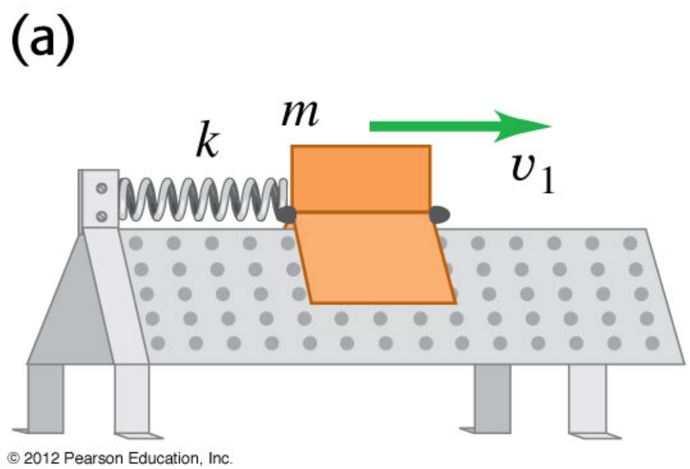


Motion with a varying force

An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant 20.0 N/m. Initially the spring is unstretched and the glider moves at 1.50 m/s to the right. Find the maximum distance *d* that the glider moves to the right

a) if the air track is turned on, so there is no friction

b) If the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.47$.



Solution

a) Without friction, only the spring does work on the glider which equals the change in kinetic energy:

$W_s = -\frac{1}{2}kd^2 = \Delta K = 0 - \frac{1}{2}mv^2 \Longrightarrow$

$d = v(m/k)^{1/2}$ = (1.5 m/s)[(0.1 kg)/(20 N/m)]$^{1/2}$ = 0.106 m

b) With friction, both friction and the spring do work:

$W_s + W_f = -\frac{1}{2}kd^2 - \mu_k mgd = \Delta K = -\frac{1}{2}mv^2$

Solving this quadratic equation for $d \Longrightarrow d$ = 0.0855 m

$$d = -\frac{\mu_k mg}{k}\left[-1 + \sqrt{1 + \frac{k}{m}\left(\frac{v}{\mu_k g}\right)^2}\,\right]$$

Motion on a curved path

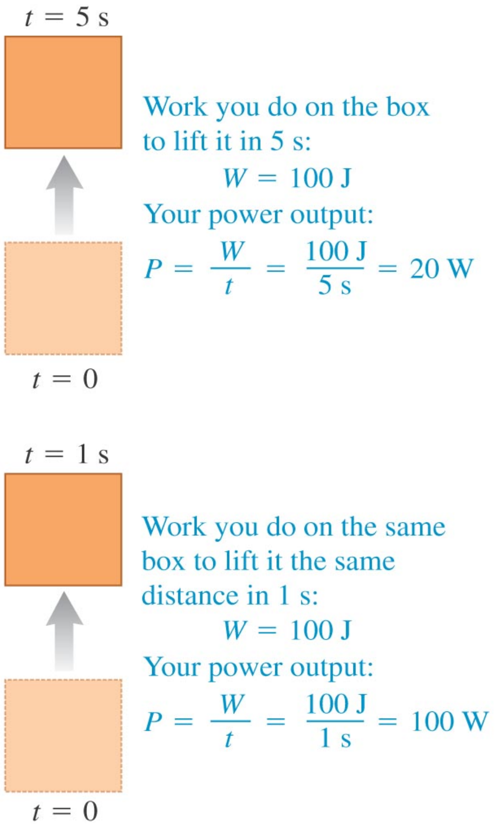
- A force $\boldsymbol{F}$ performs work $dW = \boldsymbol{F} \bullet \boldsymbol{dx}$ over a small displacement $\boldsymbol{dx}$.
- In 1D this becomes $dW = \boldsymbol{F} \bullet \boldsymbol{dx} = F_x\, dx$ and can be integrated, but what if the object's path is not a straight line like the x axis?
- For curved paths, we can write $\boldsymbol{dx} = \boldsymbol{v}\, dt$ to express the work as an integral with respect to a scalar variable

$$W = \int dW = \int \mathbf{F} \cdot \mathbf{dx} = \int \mathbf{F} \cdot \mathbf{v}\, dt$$

- This is an example of a *line integral* which you will see often in physics.

Power

- *Power* is the rate at which work is done.
- *Average power* is $P_{av} = \Delta W/\Delta t$ and *instantaneous power* is $P = dW/dt$.
- Our previous expression for work along curved paths implies that $P = \boldsymbol{F} \bullet \boldsymbol{v}$.
- The SI unit of power is the *watt* (1 W = 1 J/s), but other familiar units are the *horsepower* and the *kilowatt-hour*.



Example

A ski tow operates on a 15.0° slope of length 300 m. The rope moves at 12.0 km/h and provides power for 50 riders at one time, with an average mass per rider of 70.0 kg. Estimate the power required to operate the tow.

Solution

The ski tow must exert a force $F = mg \sin \theta$ to pull each rider up the slope. The total power required is therefore:

$P = \boldsymbol{F} \bullet \boldsymbol{v} = Nmgv \sin \theta =$

$(50)(70.0 \text{ kg})(9.8 \text{ m/s}^2)(12.0 \text{ km/h})(10^3 \text{m/km})(1\text{h}/3600\text{s}) \sin 15°$
$= 114 \text{ kW}$