Chapter 1

Units, Physical Quantities, and Vectors

PowerPoint® Lectures for
University Physics, Thirteenth Edition
– Hugh D. Young and Roger A. Freedman

Lectures by Wayne Anderson

What is physics?

- Physics is the science concerned with the most *fundamental* building blocks of the universe and their interactions.
- Scientists construct idealized models governed by *physical theories* to explain natural phenomena, then perform experiments to test the predictions of these models.
- Successful theories come to be called *physical laws*, but they can never be proven, only supported or contradicted by new data.
- Classical mechanics is the study of the motion of macroscopic (big) objects moving slowly compared to the speed of light.
- Most of the principles we will learn in this course originated with Galileo and Newton in the 17th century but remain invaluable today for scientists and engineers.

Physical quantities and units

- Experiments measure *physical quanties* that must be expressed in a particular choice of units.
- The three *fundamental* physical quantities we will encounter in this course are *distance, mass,* and *time*; all other quantities (such as velocity, force, energy) can be derived from these.
- In the *Système International* (SI), distance is measured in *meters*, time in *seconds*, and mass in *kilograms*.

Scientific notation and prefixes

- Scientific notation is used to express large and small numbers. All numbers can be written as a number $(1 < A < 10) \times 10^{n}$ where n is an integer exponent.
- Prefixes corresponding to power of 10^3 are often used to modify SI units:
- milli (m) = 10^{-3}, micro (μ) = 10^{-6}, nano (n) = 10^{-9}, pico (p) = 10^{-12}, femto (f) = 10^{-15}
- kilo (k) = 10^{3}, Mega (M) = 10^{6}, Giga (G) = 10^{9}, Tera (T) = 10^{12}

Unit consistency and conversions

- An equation must be *dimensionally consistent*. Terms to be added or equated must *always* have the same units.
- Always carry units through calculations to make sure that the final answer has the desired units.
- Example: How much work is done by a force that accelerates a 5 kg mass at 5 m/s^2 across the floor over a distance of 4 m?

$$W = Fd = mad = (5\ \mathrm{kg})(5\ \mathrm{m/s^2})(4\ \mathrm{m}) = 100\ \mathrm{kg \cdot m^2/s^2} = 100\ \mathrm{J}$$

Error propagation and significant figures

- Physics is an experimental science, and all measured data is both inaccurate (wrong) and imprecise (only known within certain margins of error). We can never eliminate errors entirely, but we strive to make sure that they are smaller than the estimated margins of error (known unknowns).
- A measured estimate should be quoted with an error bar: $\langle X \rangle_{est} = 7.3 \pm 0.5$. The number of digits after the decimal point and the number of *significant figures* is determined by the error.
- For example, $(4.132 \pm 0.015) \times 10^{-8}$ has 4 significant figures. You will learn about error propagation; we will only track the number of significant figures.
- For multiplication and division, the answer has the same number of significant figures as the *smallest* factor: $3.7 \times 4.0198 = 15$.
- For addition and subtraction, the number of significant figures is determined by the term having the fewest digits to the right of the decimal point: $17.01 + 3.2 = 20.2$, $15 + 0.1 = 15$.

Estimates and orders of magnitude

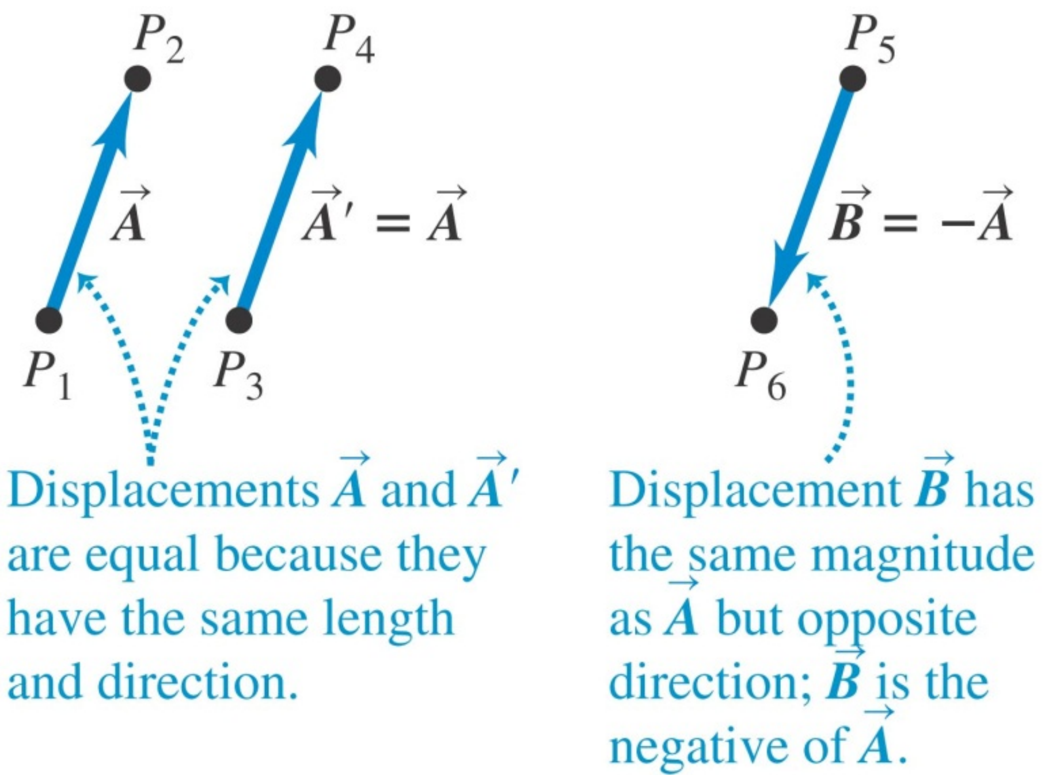
- An *order-of-magnitude estimate* of a quantity gives a rough idea of its magnitude (at most one significant figure).
- Try to break the problem down into factors that can be estimated separately.
- Provides a guess than can be used to check more sophisticated calculations (common sense).
- What should I estimate?

Scalars and vectors

- A *scalar quantity* is invariant under a transformation such as a spatial rotation (in special relativity we consider spacetime Lorentz transformations). Mass is an example of a scalar.
- A *vector quantity* has both a *magnitude* and a *direction* in space. A rotation will change the vectors direction, but not its magnitude. Velocity and acceleration are examples of vectors.
- In Young and Freedman, a vector quantity is represented in boldface italic type with an arrow over it: $\vec{\boldsymbol{A}}$. Many scientific references use boldface but not the arrow.
- The magnitude of $\vec{\boldsymbol{A}}$ is written as A or $|\vec{\boldsymbol{A}}|$.

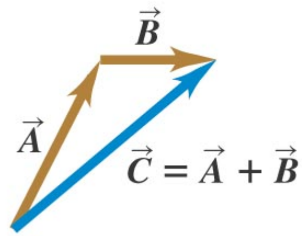
Drawing vectors

- Draw a vector as a line with an arrowhead at its tip.
- The *length* of the line shows the vector's *magnitude*.
- The *direction* of the line shows the vector's *direction*.

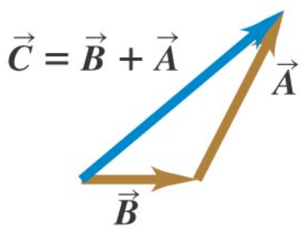


Adding two vectors graphically

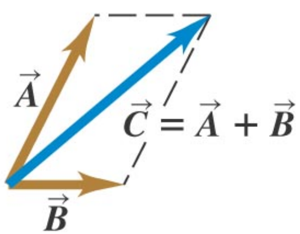
(a) We can add two vectors by placing them head to tail.



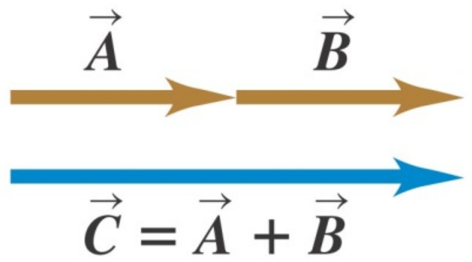
(b) Adding them in reverse order gives the same result.



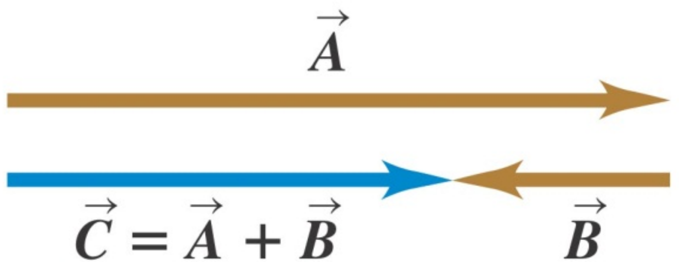
(c) We can also add them by constructing a parallelogram.



(a) The sum of two parallel vectors

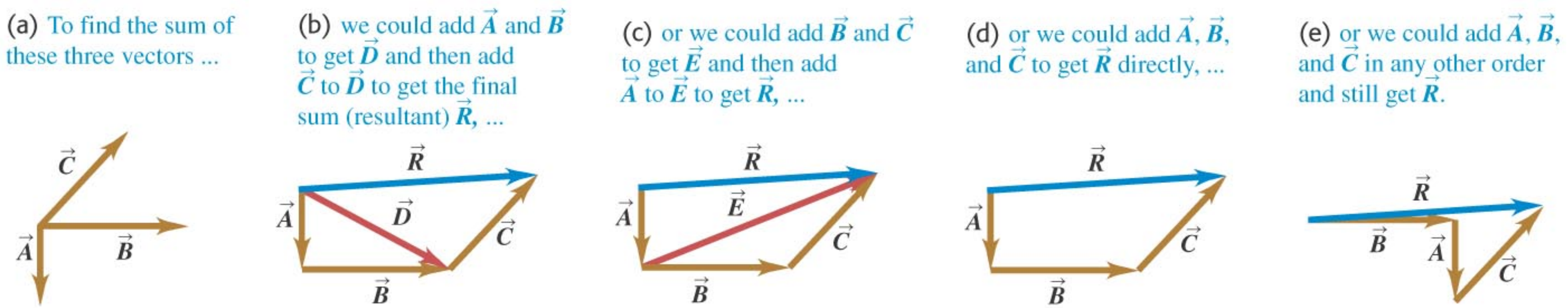


(b) The sum of two antiparallel vectors



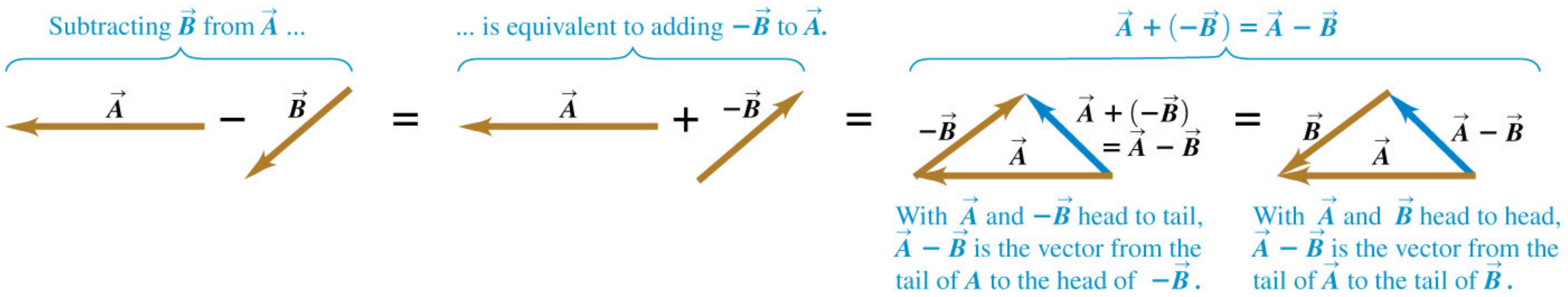
Adding more than two vectors graphically

- Vector addition is *commutative:* $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$.
- Vector addition is *associative:* $\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$.
- This implies that multiple vectors can be added in any order.



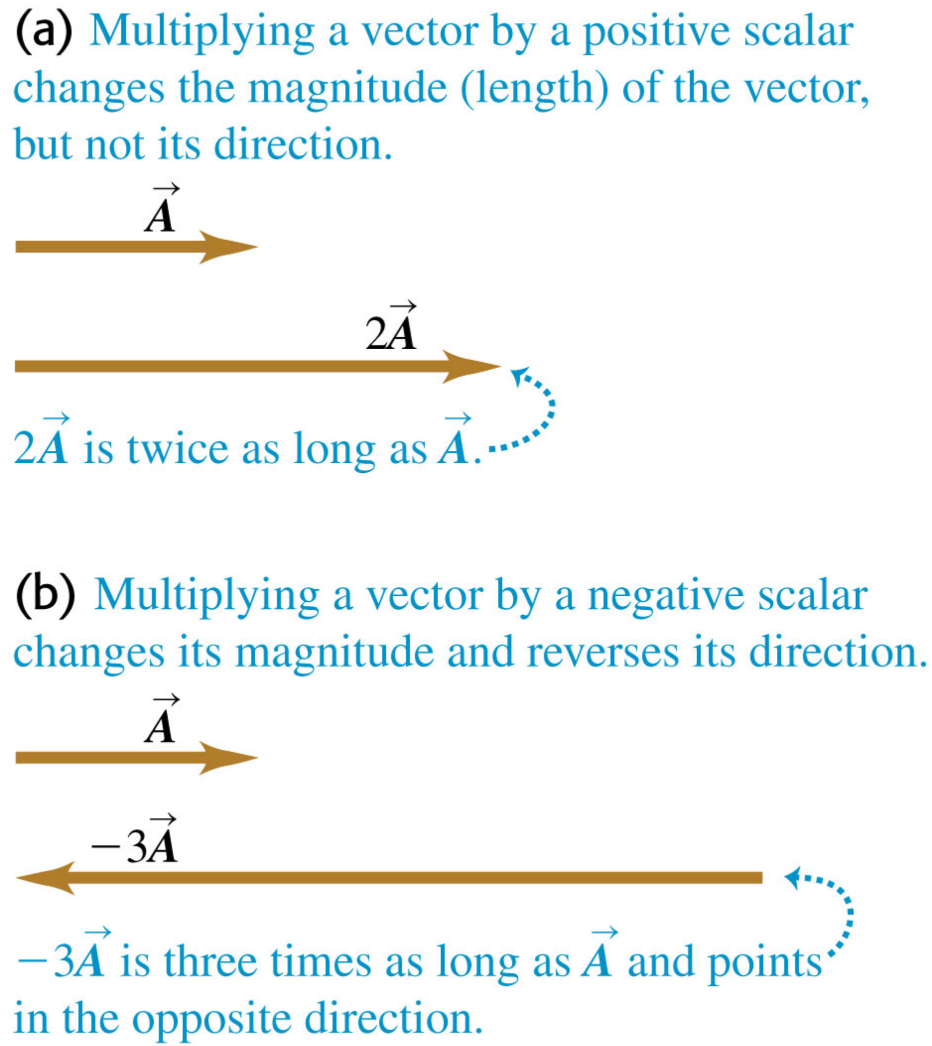
Subtracting vectors

- We can subtract vectors by recognizing that subtracting a vector is the same as adding its negative: $\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$.



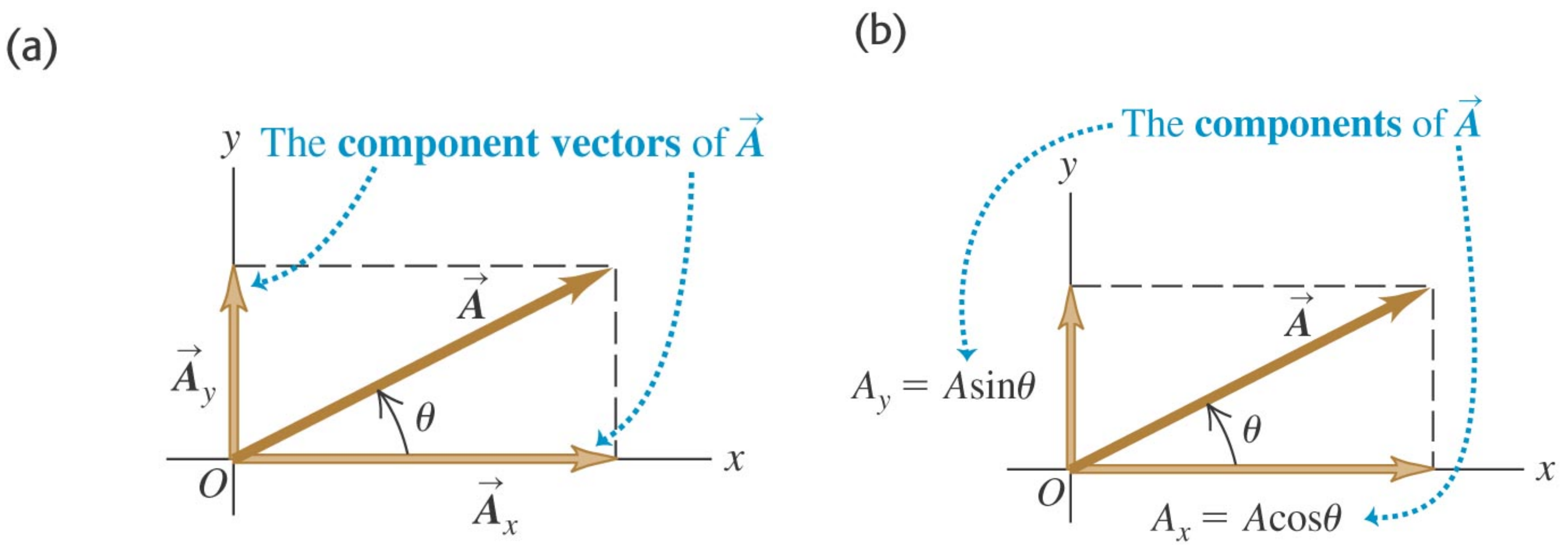
Multiplying a vector by a scalar

- If c is a scalar, the product $c\vec{A}$ has magnitude $|c|A$ and points in the direction $(\mathrm{sgn}\ c)\vec{A}$.



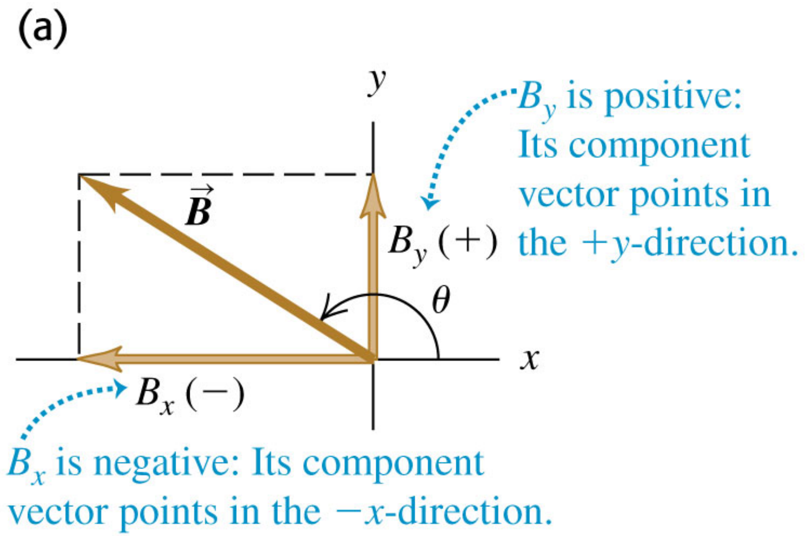
Components of a vector

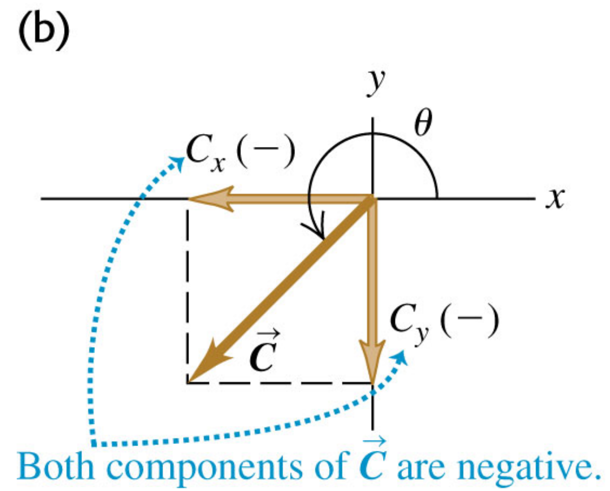
- Any vector ***A*** can be projected a set or perpendicular coordinate axes to yield an *x*-component A_x and a *y*-component A_y (and A_z in 3D).
- Use trigonometry to find the components of a vector: $A_x = A\cos\theta$ and $A_y = A\sin\theta$, where θ is measured from the +*x*-axis toward the +*y*-axis.
- In 3D, we can generalize using spherical coordinates (latitude and longitude): $A_x = A \sin\theta \cos\phi$, $A_y = A \sin\theta \sin\phi$, $A_z = A \cos\theta$



Positive and negative components

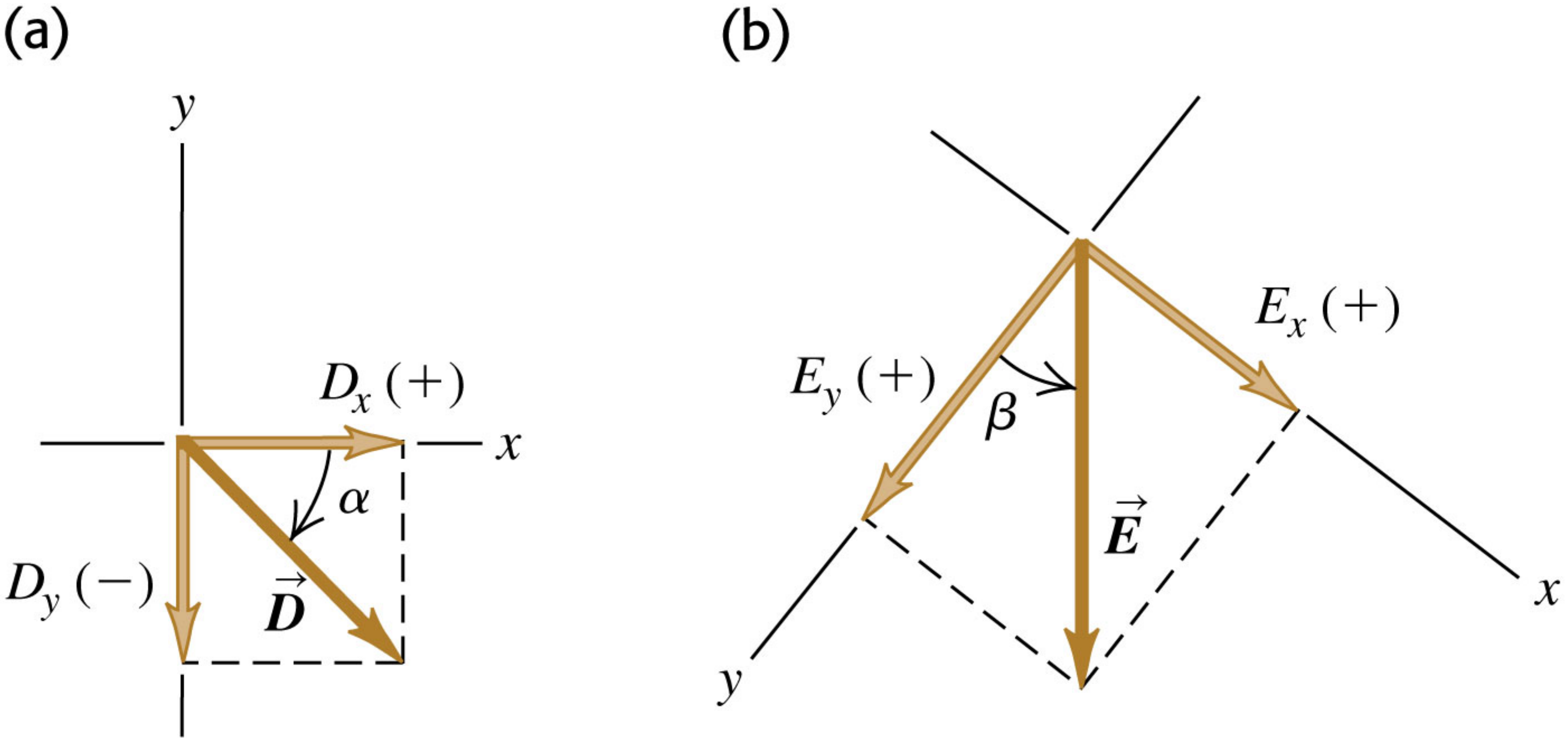
- The components of a vector can be positive or negative numbers, depending in whether the component of the vector along that axis points in the direction of increasing or decreasing values.





Finding components from magnitude and direction

- We can calculate the components of a vector from its magnitude and direction using trigonometry.



Calculations using components

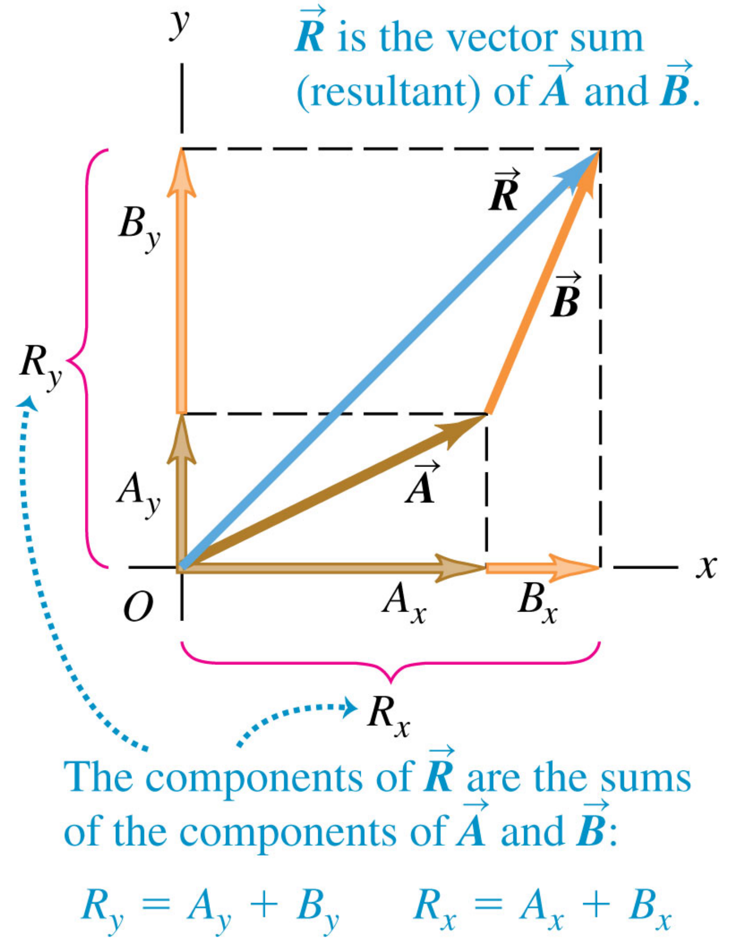
- We can use the components of a vector to find its magnitude and direction using the Pythagorean theorem and trigonometry:

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \tan\theta = \frac{A_y}{A_x}$$

- We can use the components of a set of vectors to find the components of their sum:

$$R_x = A_x + B_x \qquad R_y = A_y + B_y$$

- Let's try a numerical example:



Adding two vectors using their components

$\boldsymbol{A}$ has magnitude $A = 5.0$ and makes an angle $\theta_A = 31°$ with the x axis.

$\boldsymbol{B}$ has magnitude $B = 3.7$ and makes an angle $\theta_B = -74°$ with the x axis.

What is the magnitude and direction of their sum **C**?

First let's find their components:

$A_x = A \cos \theta_A = 4.2858$, $A_y = A \sin \theta_A = 2.5752$

$B_x = B \cos \theta_B = 1.0199$, $B_y = B \sin \theta_B = -3.5567$

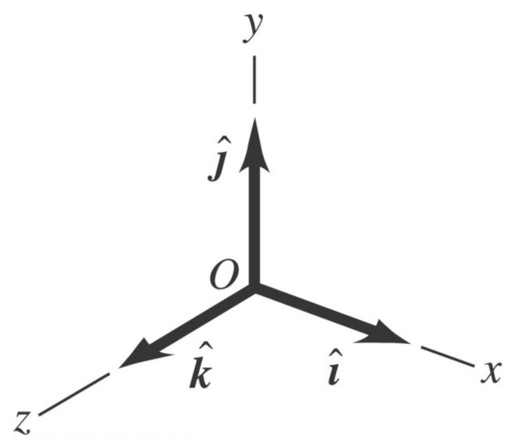
$C_x = A_x + B_x = 5.3057$, $C_y = A_y + B_y = -0.9815$

$C = \sqrt{(C_x^2 + C_y^2)} = 5.4$ (only 2 significant figures)

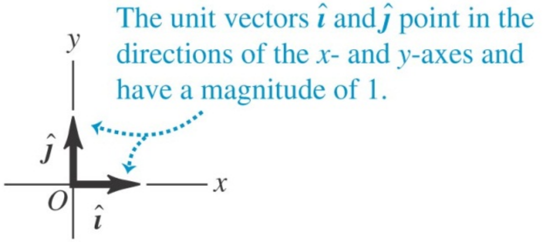
$\theta_C = \tan^{-1}(C_y/C_x) = -10°$ (only 2 significant figures)

Unit vectors

- A *unit vector* has a magnitude of 1 with no units.
- Unit vectors:
  - $\boldsymbol{i}$ points in the $+x$-direction
  - $\boldsymbol{j}$ points in the $+y$-direction
  - $\boldsymbol{k}$ points in the $+z$-direction
- Any vector can be expressed in terms of its components as $\boldsymbol{A} = A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$.
- Components denote length, unit vectors denote direction.



(a)



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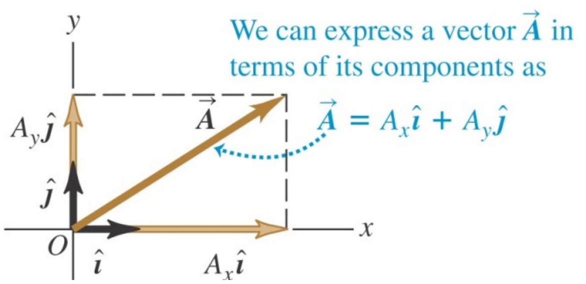


The scalar or dot product

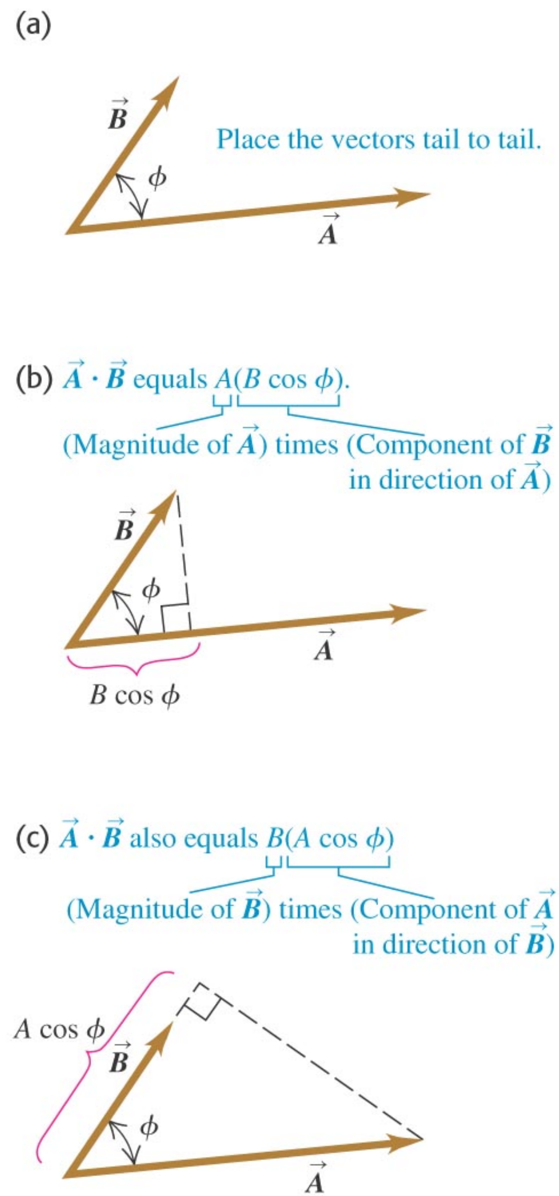
- The *scalar product* of two vectors is a scalar:

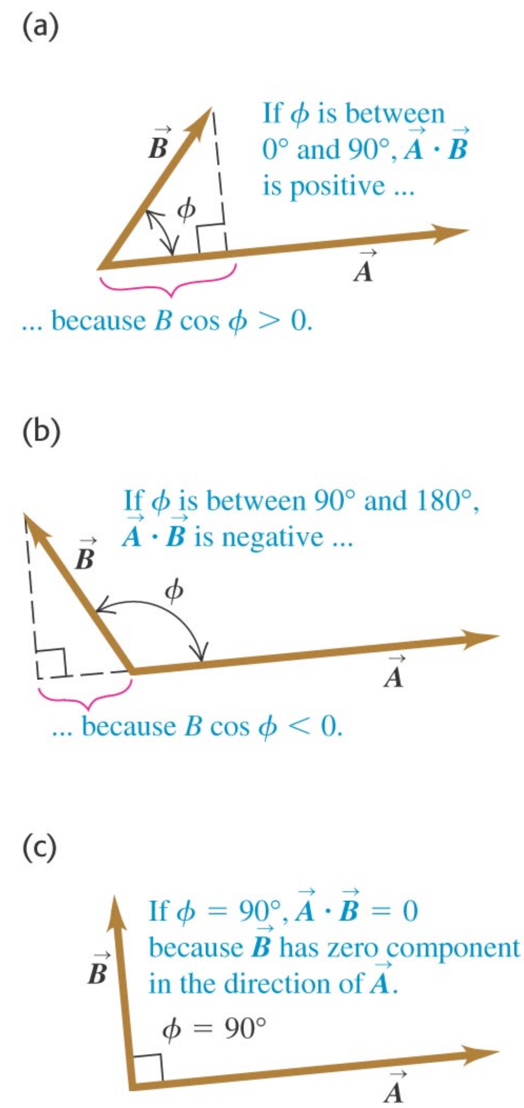
$$\boldsymbol{A} \bullet \boldsymbol{B} = AB \cos \phi$$

$$= (A \cos \phi)B$$

$$= (B \cos \phi)A$$

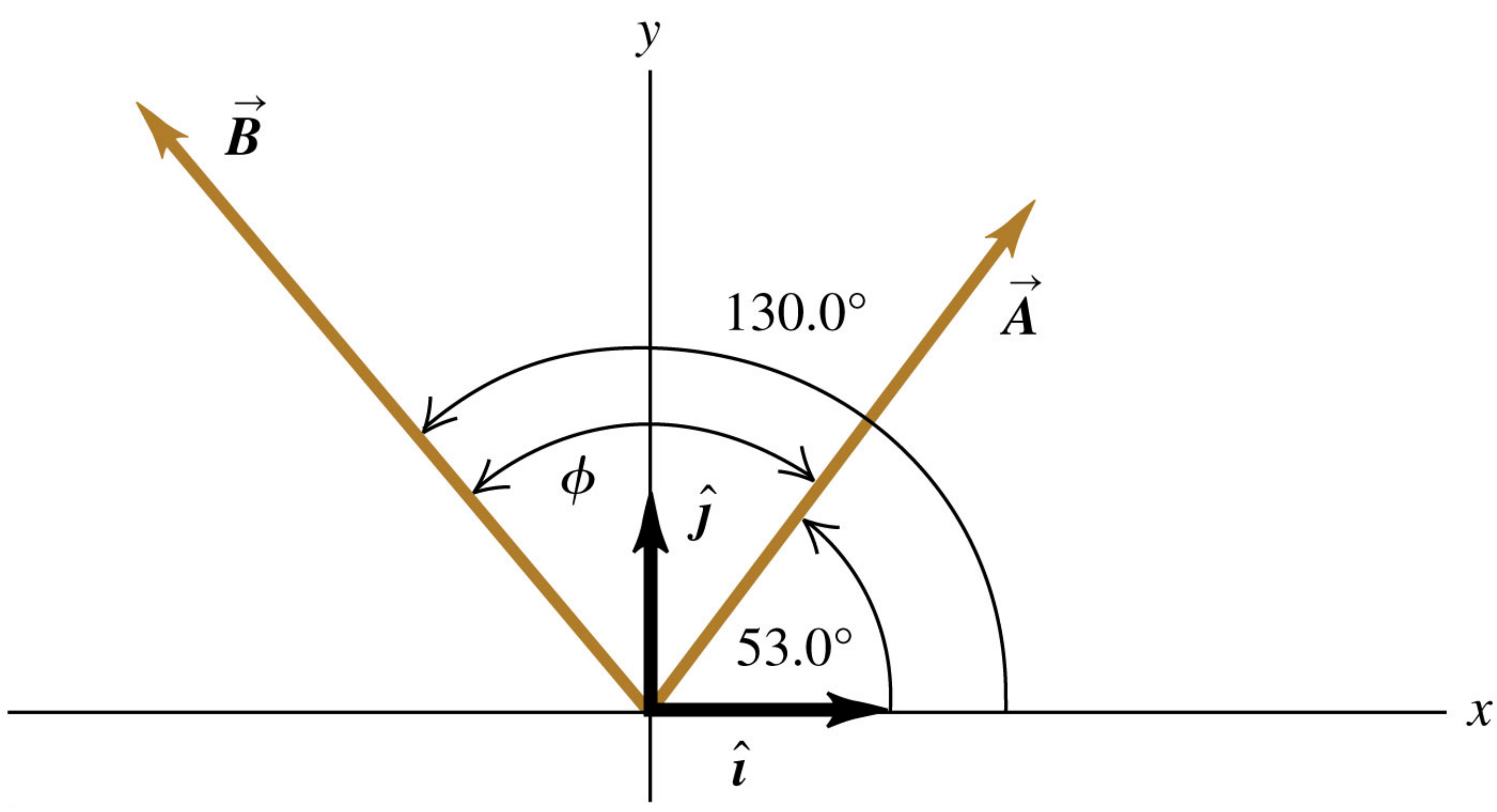
- Scalar product is:
  - positive if $\phi < 90°$
  - 0 if $\phi = 90°$
  - negative if $\phi > 90°$





Calculating a scalar product

- In terms of components, $\boldsymbol{A} \bullet \boldsymbol{B} = A_x B_x + A_y B_y + A_z B_z$.
- Let's try this example if $A = 3.78$ and $B = 5.61$



Two different ways to calculate the scalar product

- From the definition:

$\boldsymbol{A} \bullet \boldsymbol{B} = AB \cos(\phi_A - \phi_B) = (3.78)(5.61) \cos(130° - 53°) = 4.77$

- Using components:

$\boldsymbol{A} \bullet \boldsymbol{B} = A_x B_x + A_y B_y = AB(\cos\phi_A \cos\phi_B + \sin\phi_A \sin\phi_B)$

$= AB \cos(\phi_A - \phi_B) = 4.77$

The 3rd equality comes from the trigonometric identity for the cosine of the difference of two angles.

- The scalar product can also be used to find the angle between two vectors from their components:

$\cos(\phi_A - \phi_B) = (\boldsymbol{A} \bullet \boldsymbol{B})/(AB) = (A_x B_x + A_y B_y)/(AB)$

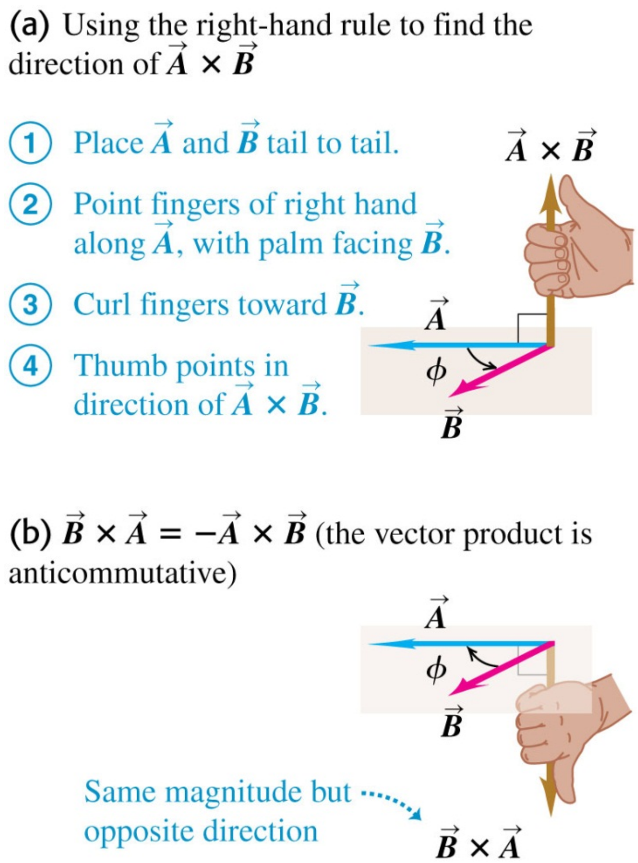
The vector or cross product

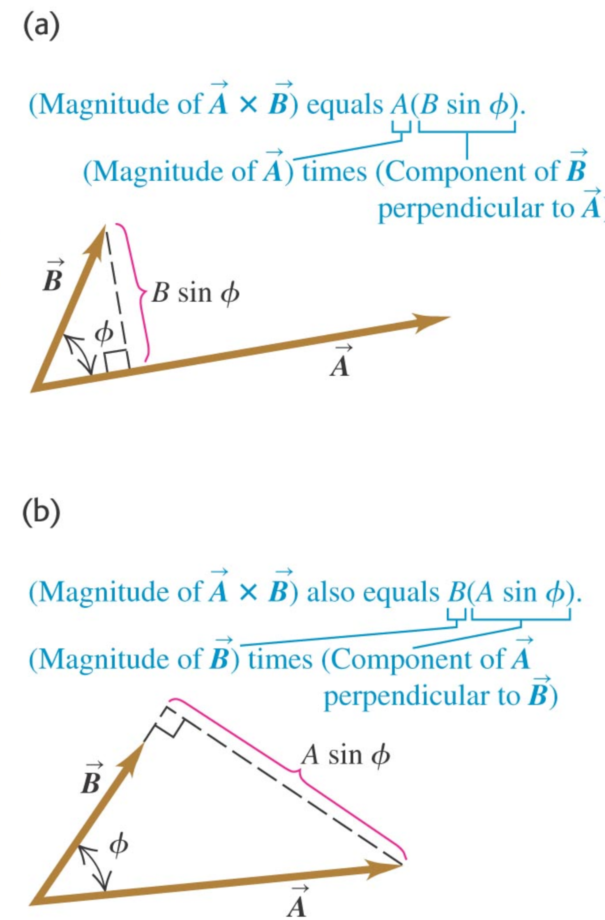
- The vector product of two vectors is itself a vector: $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$.
- Its magnitude is:

$$|\boldsymbol{A} \times \boldsymbol{B}| = AB\,|\sin(\phi_A - \phi_B)|$$

- Its direction is perpendicular to the plane spanned by $\boldsymbol{A}$ and $\boldsymbol{B}$ and is given by the “right-hand rule”.
- This implies that the cross product anti-commutes:

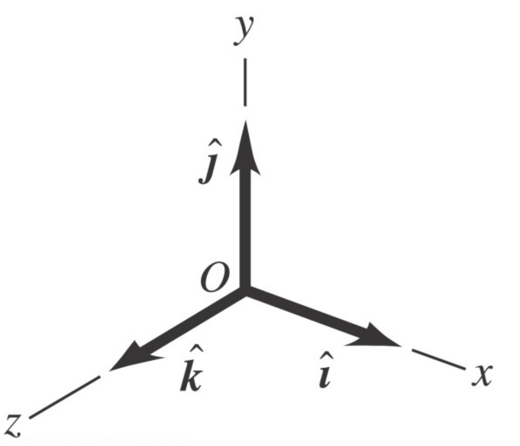
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$





Calculating cross products

- Unit vector cross products:



$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k} \qquad \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i} \qquad \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}$$

$$\boldsymbol{i} \times \boldsymbol{i} = 0 \qquad \boldsymbol{j} \times \boldsymbol{j} = 0 \qquad \boldsymbol{k} \times \boldsymbol{k} = 0$$

$$\boldsymbol{A} = A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}$$

$$\boldsymbol{B} = B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k}$$

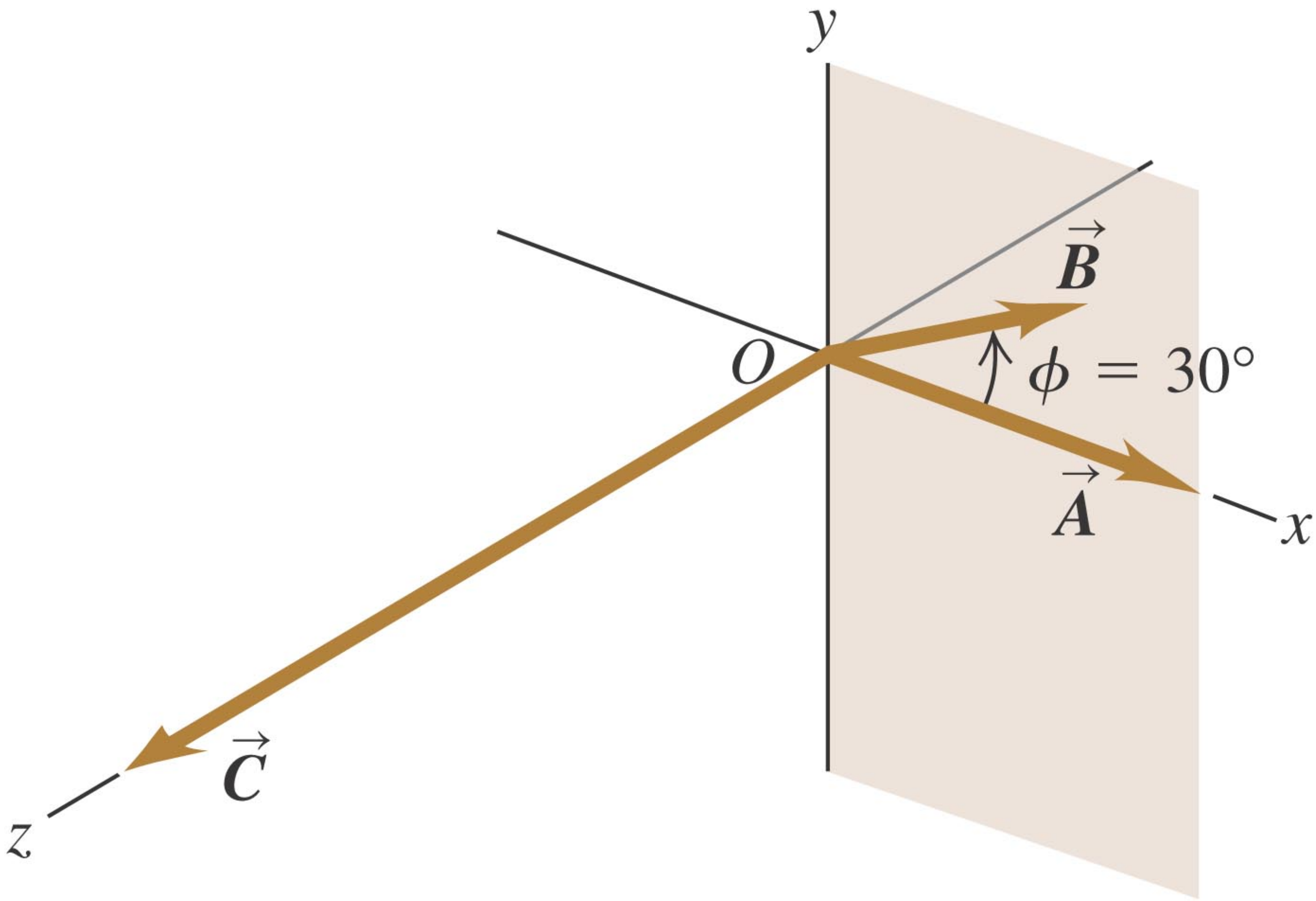
$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$

$$= (A_yB_z - A_zB_y)\boldsymbol{i} + (A_zB_x - A_xB_z)\boldsymbol{j}$$

$$+ (A_xB_y - A_yB_x)\boldsymbol{k}$$

Calculating the vector product

What are ***A***, ***B***, and ***C*** in vector notation if A = 5.0 and B = 4.0?



Answer to previous problem

$\boldsymbol{A} = 5.0\ \boldsymbol{i} \quad \boldsymbol{B} = 4.0(\cos 30°\ \boldsymbol{i} + \sin 30°\ \boldsymbol{j})$

$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = 20 \sin 30°\ \boldsymbol{k} = 10\ \boldsymbol{k}$

Sample problem #1

A plane leaves the airport in Dallas and flies 170 km at 68.0° east of north, then changes direction to fly 230 km at 36.0° south of east after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, where should it fly to go directly to the plane?

Answer to sample problem #1

We need to find the sum of these two displacements. Let's choose the x axis to point east. The two displacements are

$\boldsymbol{A}$ = 170 km (sin 68.0° $\boldsymbol{i}$ + cos 68.0° $\boldsymbol{j}$)

= 157.6 km $\boldsymbol{i}$ + 63.7 km $\boldsymbol{j}$

$\boldsymbol{B}$ = 230 km (cos 36.0° $\boldsymbol{i}$ – sin 36.0° $\boldsymbol{j}$)

= 186.1 km $\boldsymbol{i}$ – 135.2 km $\boldsymbol{j}$

$\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$ = 343.7 km $\boldsymbol{i}$ – 71.5 km $\boldsymbol{j}$

The rescuers should travel $C = \sqrt{(C_x^2 + C_y^2)}$ = 350 km at an angle $\theta = \tan^{-1}$ (71.5 km/343.7 km) = 12° south of east.

Sample problem #2

Two workers pull horizontally on a heavy box, but one pulls twice as hard as the other. The smaller pull is directed at 21.0° west of north, and the resultant of these two pulls is 460.0 N directly northward. What are the two forces?

Answer to sample problem #2

Let's again choose the x axis to point east. Then:

$\boldsymbol{F}_1 = F_1(-\sin 21.0° \, \boldsymbol{i} + \cos 21.0° \, \boldsymbol{j})$

$= F_1(-0.358 \, \boldsymbol{i} + 0.934 \, \boldsymbol{j})$

$\boldsymbol{F}_2 = 2F_1(\cos \theta_2 \, \boldsymbol{i} + \sin \theta_2 \, \boldsymbol{j})$

Their sum is:

$F_1[(2\cos \theta_2 - 0.358)\boldsymbol{i} + (2\sin \theta_2 + 0.934)\boldsymbol{j}] = 460.0 \text{ N } \boldsymbol{j}$

These two equations imply $\theta_2 = \cos^{-1}(0.358/2) = 79.7°$ and

$F_1 = (460.0 \text{ N})/(2\sin \theta_2 + 0.934) = 159 \text{ N}$

Sample problem #3

In the methane molecule CH_4, each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the center. In coordinates for which one of the the C–H bonds is in the direction of $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, an adjacent bond is in the $\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ direction. Calculate the angle between the two bonds.

Answer to sample problem #3

The scalar product of two vectors is the product of their magnitudes times the cosine of the angle between them:

$$\theta = \cos^{-1}[\boldsymbol{A} \cdot \boldsymbol{B}/(AB)] = \cos^{-1}[(1 - 1 - 1)/(\sqrt{3}\sqrt{3})]$$

$$= 109°$$

Sample problem #4

The Lorentz force $\boldsymbol{F}$ on a charge q in magnetic field $\boldsymbol{B}$ is $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$. If $q = 2.0$ and:

$\boldsymbol{v} = 2.0\,\boldsymbol{i} + 4.0\,\boldsymbol{j} + 6.0\,\boldsymbol{k}$

$\boldsymbol{F} = 4.0\,\boldsymbol{i} - 20\,\boldsymbol{j} + 12\,\boldsymbol{k}$

What is $\boldsymbol{B}$ if $B_x = B_y$?

Answer to sample problem #4

If $\boldsymbol{B} = B_x\,\boldsymbol{i} + B_x\,\boldsymbol{j} + B_z\,\boldsymbol{k}$, then:

$\boldsymbol{F} = 2.0[(4B_z - 6B_x)\boldsymbol{i} + (6B_x - 2B_z)\boldsymbol{j} + (2B_x - 4B_x)\boldsymbol{k}]$

Equating this to the given expression for $\boldsymbol{F}$ yields:

$B_x = B_y = -3.0$, $B_z = -4.0$

from F_z and F_x.

We can use F_y to check these results.

Challenge

Prove that the diagonals of an equilateral parallelogram are perpendicular.