Lab 4: Gravity and 2-D Ballistic Motion

1 Introduction

This lab focuses on **ballistic motion**, or the path of a body moving only under the influence of gravity. In particular, we will measure the value of **gravitational acceleration** and analyze the trajectories of launched projectiles. Newton's laws are important here, since they provide the theoretical foundation for describing such motion mathematically. This experiment is intended to take two class periods, and you will write a full laboratory report for it. For guidance, refer to the file on eLearning called Writing Lab Reports, which has a description of all the information you will need to include. Remember that this lab report is 25% of your final grade.

2 Key Concepts

- Gravitational acceleration
- Projectile motion
- Newton's laws
- Full laboratory write-up

3 Theory

3.1 Newton's Laws

1. *Law of Inertia:* Every object in uniform motion—that is, moving with constant velocity—will stay in uniform motion unless a net external force acts on it.
2. $\vec{\mathbf{F}}_{net} = m\,\vec{\mathbf{a}}$. The net force acting on an object equals the mass of the object times its acceleration. Remember that force and acceleration are vector quantities, while mass is a scalar.
3. When one object exerts a force on a second object, the second object also exerts a force on the first object that is equal in magnitude and opposite in direction. This is often phrased as "for every action, there is an equal and opposite reaction."

The second law is most useful to us in this lab, because we know that the force of gravity is, to very good approximation, constant and pointing downward near the surface of the earth. The second law says that a constant force acting *on* an object translates into constant acceleration *of* the object, and under the assumption of constant acceleration, the following **kinematic equations** can be derived. See your text or the appendix of Lab 3 for more details.

$$\Delta \vec{\mathbf{x}} = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\, t^2\,, \tag{1}$$

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\, t\,, \tag{2}$$

$$\vec{\mathbf{v}}^2 = \vec{\mathbf{v}}_0^2 + 2\vec{\mathbf{a}} \cdot \Delta \vec{\mathbf{x}}\,. \tag{3}$$

Time t is the independent variable, $\Delta \vec{\mathbf{x}}$ is the change in position, $\vec{\mathbf{v}}_0$ is the initial velocity, $\vec{\mathbf{v}}$ is the final velocity (at time t), and $\vec{\mathbf{a}}$ is the acceleration. Note that in the third equation, there is a dot product between $\vec{\mathbf{a}}$ and $\Delta \vec{\mathbf{x}}$, and the notation $\vec{\mathbf{v}}^2$ means $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$, the square of the magnitude of $\vec{\mathbf{v}}$.

The equations above are vector equations, which we can think of as describing relationships between geometrical arrows associated with our object moving in three-dimensional space. However, it is often more useful to use the equations in component form relative to a Cartesian coordinate system (the usual $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$), although other coordinate systems are possible. This means that Equations (1) and (2) each really stand for three equations—one for each coordinate of the motion. For example, from Equation (2), we have $v_x = v_{0x} + a_x t$, and likewise for y and z. We can therefore treat the motion along different coordinate directions separately in our analysis when the motion occurs in more than one dimension. Making a table of the components of each variable is a helpful tool in keeping track of things. Consider Figure 1 and Table 1 for a 2-D example of this process.

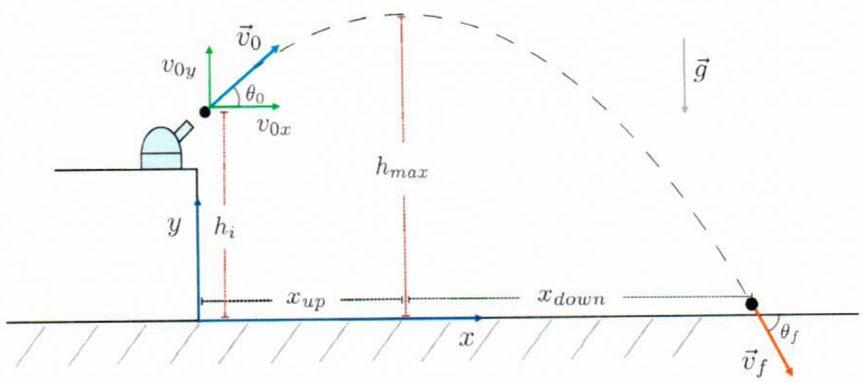

Figure 1: Diagram of general projectile motion.

3.2 Projectile Motion

We will be using the kinematic equations above to analyze the trajectory of a launched projectile. **Projectile motion** occurs when an object is subject to an initial force that propels it into the air, after which it follows an arcing path to the ground. In reality, projectile motion is complicated by the presence of air resistance and other **external forces** besides gravity. However, in this lab, we will neglect these other influences, and our equations will only reflect the effect of gravity on the motion. It is important to remember that we are applying the kinematic equations to the object only when it is actually in flight; the launcher just serves to give the projectile an initial velocity, and what happens inside the launcher is irrelevant to the object's trajectory.

Motion description		$\Delta\vec{x}$ (m)	$\vec{v}_0$ (m/s)	$\vec{v}$ (m/s)	$\vec{a}$ (m/s^2)	t (s)
Motion from h_i to h_{max}	x	x_{up}	v_{0x}	v_{0x}	0	t_{up}
	y	$h_{max} - h_i$	v_{0y}	0	-9.80	t_{up}
Motion from h_{max} to h_f	x	x_{down}	v_{0x}	$v_{fx} = v_{0x}$	0	t_{down}
	y	$h_f - h_{max}$	0	v_{fy}	-9.80	t_{down}

Table 1: Table for organizing components of constant acceleration motion problems. Have a set of x and y rows for each part of the motion, keeping in mind where the beginning and ending points of the motion are. This table is set up for a projectile where you want to know the maximum height (h_{max}) and range ($x_{up} + x_{down}$).

Calculations in trajectory problems can be lengthy, as there are many quantities of interest. The maximum height, the total flight time, the total horizontal range, and the initial and final velocities and angles are all variables in this type of problem that you might be asked to solve for. When solving these equations, keep in mind the following key points.

1. The projectile attains its maximum height when the vertical component of its velocity is zero.
2. The acceleration due to gravity points vertically downward and has magnitude 9.80 m/s^2.
3. All objects free fall at the same rate in the absence of air resistance.
4. Since time is a scalar, the time in the x direction is the same as the time in the y direction.
5. Be careful to take into account the difference in height between your initial and final positions.

For more information on Newton's laws and projectile motion, see the appropriate sections in your text. For more information on analyzing graphs, see Lab 3 and the file called Graphing Skills on eLearning.

3.3 Full Laboratory Report

why

A full laboratory report is a professional way to present all of the data you have obtained in a clear and logical format. Full laboratory reports are broken into the following sections.

- Abstract
- Theory
- Data
- Analysis
- Conclusion

Each of these sections has specific requirements, as outlined below. These are summaries that discuss the main points to include in each section, but you should refer to the Writing Lab Reports file on eLearning for full details.

3.3.1 Abstract

An abstract is where you will briefly describe the purpose of the paper in three or four sentences. This should include what you are trying to prove, how you are trying to prove it, what the results you have obtained are, and how these results compare to theory. The abstract should give the reader a clear concept of what the paper is about in compact form.

3.3.2 Theory

The theory of the paper describes which principles and concepts you are trying to prove or disprove. This is where you examine the equations used and explain the variables you are using. All equations needed in determining the results need to be presented here. Do the equations used have assumptions? A brief description of the lab setup with appropriate diagrams should be included here.

3.3.3 Data

Here is the section where you clearly present your data. Outline what the equipment does for the experiment and how you will obtain your data. What assumptions are made in order for the experiment to match the theory? In this section you can quantify the information you have experimentally derived in the form of graphs, tables, charts, and numerical results. It is also important to justify steps you have taken to reduce the amount of error in your results.

3.3.4 Analysis

The analysis section is a comparison between what was obtained experimentally and what was derived theoretically. Describe the discrepancies between the theory and experiment as different sources of error, and where the error has occurred in your experiment. Try to explain and defend your results as fully as possible. If all sources of error are taken into account, how would you amend the theory?

3.3.5 Conclusion

The conclusion section of the report is a summary of the experiment, a final statement of the results obtained, and a justification of the results with percent errors and percent differences. It is also customary to suggest specific improvements to the procedure to obtain either more accurate or more precise results. A discussion of whether the assumptions made in the theory section were sufficient is also useful. Also, state what to do next with the experiment, whether taking the measurements in a different way, or testing the capability of the equipment under different conditions.

3.3.6 Requirements

The report is due after the second week's experiment, but it is strongly recommended that you bring a draft covering the first week's experiment to class. A full report is graded on:

1. Full write-up format – be sure to include all sections outlined above.
2. Completeness – describe and evaluate all experiments. Do not leave one out!
3. Grammar and writing skills – use full sentences and proper English.
4. TA Discretion – your TA may also include specific requirements.

4 Experiment Week 1

We will first study the motion of a freely falling object by measuring its acceleration due to gravity. We will then use the steel ball and launcher to study horizontal and vertical ballistic motion.

4.1 Equipment

- Triple beam balance
- Set of masses
- Meter stick
- Bar tape with mass hanger
- Photogate with bar tape attachment
- Small table clamp
- Large ring stand
- Steel ball launcher and steel ball
- Plunger tube
- White box target
- Carbon paper, tape, and white paper

4.2 Freely Falling Object

4.2.1 Procedure

1. Open the Logger Pro file named Lab 4 Gravity Bar Tape. You should find a shortcut icon on the desktop.
2. With no mass on the hanger initially, thread the free end of the bar tape through the bar tape guide mounted on the photogate, and pull it up so that the hanger is near the photogate.
3. Hold the bar tape vertically, and make sure that the computer says the photogate is unblocked.
4. Click the Collect button, and after it turns red, release the bar tape.
5. Click Stop after the tape has fallen all the way through the photogate.
6. We want to find the acceleration from the data the computer took. To do this, click to select the velocity vs. time graph, which should be a straight line. Click the linear fit button (it has "R=" on it) to the left of the Collect button, and record the slope of this line as a_1 in Table 2. (Do you know why we are using the slope of this line?)
7. Repeat 3 more times to complete the 0 g row of the table.
8. Now put 20 g on the hanger, and do 4 trials as above for this mass.
9. Repeat for the remaining masses to complete the table.

mass (g)	a_1 (m/s^2)	a_2 (m/s^2)	a_3 (m/s^2)	a_4 (m/s^2)
0				
100				
300				

Table 2: Data table for accelerations of tape measurements.

4.2.2 Data

Find the average, standard deviation, and standard error of the mean (SEOM) of the accelerations for each mass, and record them in Table 3. Notice that the units of σ and $\sigma_{\bar{a}}$ are the same as for $\bar{a}$.

mass (g)	Mean $\bar{a}$ (m/s^2)	Standard deviation σ (m/s^2)	SEOM $\sigma_{\bar{a}}$ (m/s^2)
0			
100			
300			

Table 3: Data table for the statistics of the accelerations of tape measurements.

4.3 Projectile Motion: Horizontal Launch

4.3.1 Procedure

1. With the launcher on the ground, verify that the angle of the launcher is 0 degrees and that the height of the muzzle above the ground is equal to the height of the target box with the side labeled "TOP" facing up. Measure and record this height in the space below.

   Launcher height (m): ---------------- ± ----------------

2. Make sure that the photogate is positioned so that the ball will pass through the light beam when launched.
3. The launcher has three launch settings that correspond to the three possible compression amounts of the spring inside. Load the steel ball to the first setting using the plunger, and do a test launch to see how far it goes.
4. Tape down the white paper where the ball first hit the ground, and lay the carbon paper on top. DO NOT TAPE THE CARBON PAPER.
5. Connect the launcher's photogate to the computer (if it is not already) and open the Lab 4 Projectile Launcher file. Launch the ball at the first setting again, measuring its initial speed using the photogate.
6. The ball should now have landed on the carbon paper, leaving a visible dot on the white paper underneath. Label this dot with a 1 using your pen or pencil. There will be many launches, so numbering them will help you keep track of which dot goes with which trial.

7. Measure the *horizontal* distance Δx from the muzzle of the launcher to the ball's landing position.

8. Record the distance and initial speed in the appropriate boxes of Table 4.

9. Repeat for a total of 5 trials, numbering the dots as you go.

10. Repeat steps 3 through 9 for the second and third launcher settings.

Trial	Setting 1		Setting 2		Setting 3	
	Δx (m)	v_0 (m/s)	Δx (m)	v_0 (m/s)	Δx (m)	v_0 (m/s)
1						
2						
3						
4						
5						

Table 4: Data table for horizontal distance, and initial speed measurements.

4.3.2 Data

1. Calculate the theoretical time of flight for each setting by manipulating Equation (1), and record this value in Table 5. (Use the height of the launcher as Δy and 9.80 m/s^2 as acceleration.)

2. Compute the average initial (horizontal) speeds for all three settings from Table 4, and record in Table 5.

3. Calculate the initial speed for each launcher setting using the theoretical time and the average of your measured distances for that setting. Record in Table 5.

	Setting 1	Setting 2	Setting 3
t_{theory} (s)			
photogate $v_{0,\text{avg}}$ (m/s)			
calculated $v_{0,\text{avg}}$ (m/s)			
Δv_0 (m/s)			
v_0 % difference			

Table 5: Data table for horizontal motion times of flight, muzzle speeds, and errors.

4. Using the techniques from Lab 1, propagate the error in this calculation and record as Δv_0. Recall that this involves calculating the SEOM of the distances for each launcher setting—the SEOM serves as the uncertainty in the distance measurement.

5. Compare the average initial speed as measured by the photogate to that which you calculated from your measurements by finding the percent difference between them.

6. Note that v_0 is the initial speed of the ball regardless of the angle of the launcher. With this setup, though, it happens that $v_{0x} = v_0$ and $v_{0y} = 0$.

4.4 Projectile Motion: Vertical Launch

4.4.1 Procedure

1. Adjust the angle of the launcher so that it will launch directly upward.

2. Hold the meter stick vertically next to the launcher with the 0 cm end touching the floor.

3. Measure the launcher's muzzle height above the floor, and record in the space below.

   Launcher height (m): ________________ ± ________________

4. Load the ball into the launcher at the first setting.

5. Fire the ball, and measure the initial speed of the ball with the photogate.

6. Record the initial speed in Table 6.

7. Follow this procedure for 5 trial runs at this setting.

8. Repeat steps 4-7 for the other two launcher settings to complete the velocity portion of the data table.

Trial	Setting 1		Setting 2		Setting 3	
	Δy (m)	v_0 (m/s)	Δy (m)	v_0 (m/s)	Δy (m)	v_0 (m/s)
1						
2						
3						
4						
5						

Table 6: Data table for vertical motion distance and initial speed measurements.

9. Disconnect the photogate and move the launcher setup to the whitewall.

10. Obtain a dry-erase marker from the TA.

11. Fire the ball on the first setting, and put a small mark at the approximate height reached on the wall.

12. Fire the ball again, using the first mark as a guide for where you expect the maximum height to be. Mark the maximum height of the ball's trajectory on the wall, and label it with a 1.

13. Complete all 5 runs at this setting, enumerating the maximum height markings as you go.
14. For each trial, subtract the launcher height from the ball's maximum height marking, as measured from the ground, and record this value as Δy in Table 6.
15. Repeat steps 11-14 for the other two settings to complete the rest of the data table.

4.4.2 Data

1. Compute the average vertical distance and average initial speed for each setting of Table 6, and record in Table 7.
2. Using Equation (3), calculate the theoretical initial speed of the launcher for each setting. Use your average Δy values and $g = 9.80$ m/s^2.
3. Find the percent difference between the photogate's average measurement of v_0 and your calculated v_0.

	Setting 1	Setting 2	Setting 3
Δy_{avg} (m)			
photogate $v_{0,\text{avg}}$ (m/s)			
calculated $v_{0,\text{avg}}$ (m/s)			
v_0 % difference			

Table 7: Data table for vertical motion distances and muzzle speeds.

4.5 Predictions

Before coming to class next time, complete Table 8 below. In other words, use the initial speeds for the different launcher settings you found today to calculate theoretically how far the ball should travel for each of the specified launch angles. Assume that the ball launches and lands at the same height. You can choose which values to use for the initial speeds, as you will justify this choice in the analysis section. These calculations will serve as your predictions for what will happen in next week's part of the experiment.

	15°	30°	45°	60°	75°
Setting 1 Δx (m)					
Setting 2 Δx (m)					
Setting 3 Δx (m)					

Table 8: Data table for your prediction of distances at different firing angles.

5 Experiment Week 2

In this week's experiment, you'll test the predictions you made in Table 8 by launching the ball at different angles and measuring the actual horizontal distance traveled. You will also be given a target to hit, where you must first calculate the appropriate angle and launch speed to get the ball to land on it. Finally, you will examine the more general case of projectile motion where the ball is launched from a higher starting point than it lands.

5.1 Projectile Motion: Angled Launch on Ground

5.1.1 Procedure

1. With the launcher on the ground, adjust the angle to 15° with the muzzle height above the ground equal to the height of the box with the side labeled "TOP" facing up.
2. Place the box at the distance Δx you calculated in Table 8 for Setting 1 (low speed) at 15° so that the ball should land near the center of the box.
3. Load the steel ball to the first setting, and fire the launcher toward the box.
4. Adjust the position of the box so that the ball lands on the bullseye. Note that the center of the bullseye does not necessarily coincide with the center of the top of the box.
5. Tape down the white paper on top of the box (fold it if you need to), and lay the carbon paper on top. Again, do not tape the carbon paper.
6. Fire for a total of 3 trials, enumerating the marks and measuring the distances as before. Record the distances in Table 9 for Setting 1.
7. Compute the average Δx of the trials, and record in the last row.

Setting 1 (low speed)

Trial	Δx (m) at 15°	Δx (m) at 30°	Δx (m) at 45°	Δx (m) at 60°	Δx (m) at 75°
1					
2					
3					
Δx_{avg}					

Setting 2 (medium speed)

Trial	Δx (m) at 15°	Δx (m) at 30°	Δx (m) at 45°	Δx (m) at 60°	Δx (m) at 75°
1					
2					
3					
Δx_{avg}					

Setting 3 (high speed)

Trial	Δx (m) at 15°	Δx (m) at 30°	Δx (m) at 45°	Δx (m) at 60°	Δx (m) at 75°
1					
2					
3					
Δx_{avg}					

Table 9: Data tables for horizontal distance measurements at different firing angles.

8. Repeat for all angles and launcher settings to complete the three data tables. **Make sure that the end of the launcher barrel is at box-height above the ground for each new angle.** You might save time by taking all the measurements for the three settings at 15°, then moving on to 30°, and so on.

5.1.2 Data

Calculate the percent error in your measured average distances using the Table 8 predictions as your theoretical values. Record in Table 10.

	15°	30°	45°	60°	75°
Setting 1 % error					
Setting 2 % error					
Setting 3 % error					

Table 10: Data table for percent errors in your measured average horizontal distances compared to predicted values.

5.2 Projectile Motion: Hitting a Target

5.2.1 Procedure

1. Move the box to a distance of **1.50 m** away from the launcher as measured from the center of the box.

2. Using Equation (1), find an algebraic expression for Δx, the horizontal distance the projectile will travel in time Δt, in terms of the initial velocity v_0 and the launch angle θ.

   $\Delta x =$

3. Find an expression for the total flight time Δt_{flight} in terms of v_0, θ, and the acceleration due to gravity g.

   $\Delta t_{\text{flight}} =$

4. Using the above answers, find an expression for the horizontal distance traveled by the projectile in flight time Δt_{flight}.

   $\Delta x_{\text{flight}} =$

5. Find an expression for the maximum Δx for a given launch speed. This will correspond to a specific launch angle. (Hint: consider the ranges of $\sin\theta$ and $\cos\theta$.)

   $\Delta x_{\text{max}} =$

   Calculate Δx_{max} for the three launcher settings.

	Setting 1	Setting 2	Setting 3
Δx_{max} (m)			

6. Based on the above results and the values you found in Table 8, determine with which speed setting(s) you will be able to hit the target.

7. For this setting (choose one if more than one are possible), calculate a launch angle that will enable the ball to land on the target.

   Setting # ____ $v_0 =$ __________ m/s $\theta =$ __________

8. Adjust the launcher to these specifications, and launch the ball to hit the target. Prove to your TA that you have been successful in order to proceed.

5.3 Projectile Motion: Angled Launch off Table

5.3.1 Procedure

1. Use one of the two setups at the makeup lab station in the back of the room where the launcher is on the table. You will be firing the ball onto the large cardboard pieces lying on the floor.

2. Verify that the angle is set to 30°.

3. Measure the height of the barrel's end above the floor, and record in the space below.

   Launcher height (m): ---------------- ± ----------------

4. Following the same launching procedures as before, perform 5 trials at each of the three launch speed settings, and record the measured distances in Table 11.

5.3.2 Data

1. Compute the average distance for each setting, and record in the last row of Table 11.

Trial	Setting 1 Δx (m)	Setting 2 Δx (m)	Setting 3 Δx (m)
1			
2			
3			
4			
5			
Avg.			

Table 11: Data table for horizontal distances of launching off the table at 30°.

6 Analysis Questions

6.1 Freely Falling Object

1. Do your results support the idea that all objects fall at the same rate in the absence of air resistance?
2. For which mass on the bar tape hanger was the acceleration closest to the accepted value for gravity?
3. Name two sources of error in this part of the experiment.

6.2 Horizontal Launch

1. Once the ball leaves the launcher, what force(s) are acting on the ball and in what direction?
2. The initial launcher height was the same for all three settings. Compare the flight times. Is this what you would expect?
3. For which setting was the percent difference in v_0 the smallest?

6.3 Vertical Launch

1. What is the correlation, if any, between the different settings of the launcher and Δy_{avg}?
2. For which setting was the percent difference in v_0 the smallest?

6.4 Predictions

1. Which values did you choose to use for the initial speeds of the launcher to fill out Table 8? Why?

6.5 Angled Launch on Ground

1. Did you have any trouble getting the ball to land on the box for the different angles and settings? If so, was the problem in your theoretical predictions or in carrying out the experiment?
2. Which angle and setting gave the lowest percent error?

3. Where does the minimum *velocity* occur in the trajectory of a projectile launched this way?

6.6 Hitting a Target

1. For any launch speed, what launch angle will cause a projectile to go the farthest horizontally if it lands at the same height it is launched?
2. Were you able to hit the target on the first try? If not, what needed to be changed?
3. Find the other possible angle and initial speed combinations of your launcher for which the ball would hit the target. (Hint: it might help to use $\sin(2\theta) = 2\sin\theta\cos\theta$.)

6.7 Angled Launch off Table

In doing these calculations, you will need to take into account the initial height of the launcher. You might find Figure 1 and its accompanying table useful.

1. For each launcher setting, calculate the theoretical
   - maximum height of the ball in its trajectory
   - time of flight
   - horizontal distance traveled
   - components of the ball's final velocity vector as it hit the ground
   - magnitude and angle of the final velocity
2. Find the percent error of your measured average Δx values for each setting.