Lab 7: Rotational Motion

1 Introduction

The purpose of this lab is to understand rotational motion—its causes and its mathematical description. The kinematic and dynamic equations that describe rigid rotating bodies come at half price, since they essentially have the same form (just with new variables) as those we already know for translational motion under constant acceleration. We will also study the transformation of gravitational potential energy into kinetic energy of different parts of our experimental system.

2 Key Concepts

- Angular position, angular velocity, and angular acceleration
- Moment of inertia
- Torque
- Rotational kinetic energy
- Energy conservation

3 Theory

We have studied the motion of objects undergoing constant acceleration, describing their behavior with kinematic equations relating position $\vec{\mathbf{x}}$, velocity $\vec{\mathbf{v}}$, and acceleration $\vec{\mathbf{a}}$. We also know that the acceleration of an object is determined by its mass and the net force acting on it via Newton's second law, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. But what about objects that do not move from one place to another (translate), but instead rotate about an axis? It turns out the equations that describe such rotating objects are closely related to the ones we already know for translational motion.

For a rigid rotating object, instead of thinking about its spatial position x (e.g., in 1D), which is the distance it has moved from some starting point, we think about the angle it has rotated with respect to its starting orientation. This is called the **angular coordinate** or **angular position** θ, and it is the rotational analog of position. The rate of change of this angle in time is appropriately called the **angular velocity** $\vec{\omega}$, and the rate of change of the angular velocity (or the second rate of change of θ) is called the **angular acceleration** $\vec{\alpha}$. In other words, $\vec{\omega}$ and $\vec{\alpha}$ are the rotational versions of $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$, respectively.

You probably noticed that $\vec{\omega}$ and $\vec{\alpha}$ are vectors. Thought of as arrows attached to the rotating object, their lengths are just ω and α in whatever units we are using, but what direction do they point? By convention, you can use the right hand rule to determine their orientation: curl the fingers of your right hand in the direction of rotation, and your thumb points in the direction of $\vec{\omega}$, which is perpendicular to the rotation plane. If the body's rotation is speeding up, $\vec{\alpha}$ is parallel to $\vec{\omega}$; if it is slowing down, $\vec{\omega}$ and $\vec{\alpha}$ are anti-parallel.

By replacing translational variables with rotational ones in the kinematic equations we already know, we get the corresponding relationships between θ, ω, and α for motion with constant angular acceleration.

$$\begin{array}{ccc} \underline{\text{Translational}} & & \underline{\text{Rotational}} \\[1em] \Delta x = v_0 t + \frac{1}{2} a t^2 & & \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\[1em] v = v_0 + at & \rightarrow & \omega = \omega_0 + \alpha t \\[1em] v^2 = {v_0}^2 + 2a\Delta x & & \omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta \end{array}$$

The same way an object's mass is a measure of how hard it is to get it moving when you push on it, the **moment of inertia** I of an object is a measure of how hard it is to get it rotating. It depends not only on the total mass, but also on the distribution of the mass around the rotation axis. An object with most of its mass concentrated near the axis of rotation will be easier to start rotating (smaller moment of inertia) than one with the same *total* mass but where it is spread out far away from the axis.

A force applied to an object that tends to cause it to rotate is called a **torque** $\vec{\tau}$. Torque is a vector, and its definition is $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$. Here the vector $\vec{\mathbf{r}}$ stretches perpendicularly from the rotation axis to the point on the body where the force is applied, and $\vec{\mathbf{F}}$ is the rotation-inducing force. By properties of cross products, the magnitude $|\vec{\tau}| = \tau$ is $rF\sin\theta$, where θ is the angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$, and you can determine the direction using the right hand rule. It will always be perpendicular to the plane spanned by $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$. Since there are rotational analogues of force, mass, and acceleration, you might guess that there is a rotational version of Newton's second law. Sure enough, we can write

$$\sum \vec{\tau} = I\vec{\alpha},$$

or the sum of all the torques on a body equals its moment of inertia times its angular acceleration.

This experiment includes a rotating disk attached by a cable over a pulley to a hanging mass, the weight of which provides a torque on the disk. The mass hanger only moves vertically, and the disk just spins about its center, so there are only two relevant equations from Newton's second law.

$$\begin{array}{cc} \underline{\text{Hanger}} & \underline{\text{Disk}} \\[1em] \sum F = W_\text{h} - T = m_\text{h} a & \sum \tau = rT = I\alpha \end{array}$$

These two equations are connected through T, the tension in the string, and the acceleration a. For a rotating disk described by θ, ω, and α, a point located a distance r from the axis travels the equivalent linear distance

$$x = \theta \cdot r$$

with speed and acceleration

$$v = \omega \cdot r \qquad \text{and} \qquad a = \alpha \cdot r.$$

The latter relationship between a and α can then be used to solve the system of two equations from Newton's second law above. It is important to realize that although the acceleration of the falling hanger may be small, it will not be zero. The tension in the cable does not balance out the weight of the hanger, so assuming $T = W_\text{h}$ above will cause errors in your analysis.

The last thing to know about rotating bodies is that they have kinetic energy, which is given by

$$K_{\rm rot} = \frac{1}{2} I \omega^2.$$

Again, there's no need to memorize this equation if you already know $K_{\rm tr} = \frac{1}{2}mv^2$ for translational motion. One of the goals of this lab will be to determine how much of the initial gravitational potential energy of the system (the hanger starts at some height above the ground) gets converted into rotational kinetic energy of the disk and translational kinetic energy of the falling hanger.

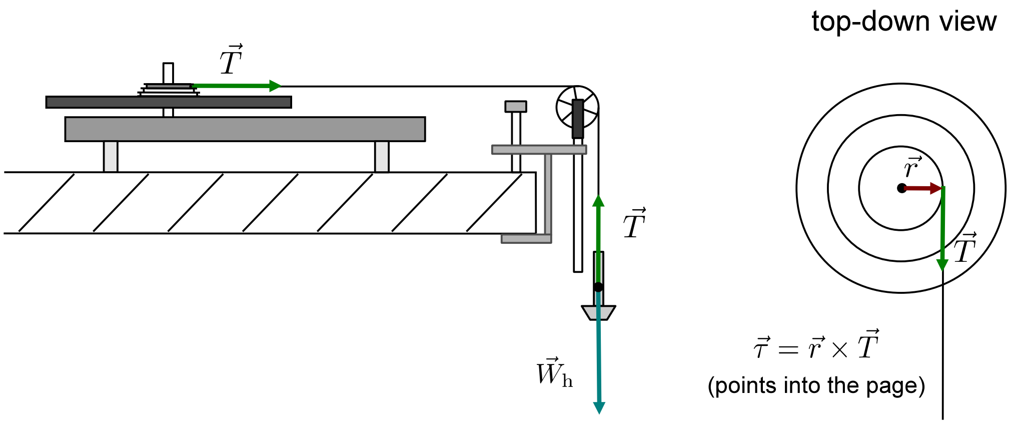

Figure 1: Diagram of rotational apparatus setup.

4 Experiment

4.1 Equipment

- Triple beam balance
- Set of masses
- Meter stick
- Vernier calipers
- Mass hanger
- Computer with Logger Pro software
- Rotational motion apparatus
- Rotary motion sensor
- Pulley
- Small table clamp

4.2 Procedure

You will be measuring the angular acceleration and final angular velocity of the rotating disk as the weight of the mass hanger unwinds it. You will also determine what percentage of initial potential energy gets converted to kinetic energy of the rotating disk and falling mass hanger.

4.2.1 Measuring α and ω_{f}

1. Verify that your setup looks like Figure 1. The rotary motion sensor will be recording the rotation of the large disk as it spins alongside it.

2. Open the Lab 8 Logger Pro file on the computer. There should be data columns on the left and three graphs on the right: angle (rad) vs. time (s), angular velocity (rad/s) vs. time (s), and angular acceleration (rad/s^2).

3. Measure the diameters of the three spindles on the top of the rotating disk using calipers. Make sure you are measuring the inner grooves where the cable will actually be wrapped. Record the *radii* in Table 1 as $r_\perp$, where trials 1-3, 4-6, and 7-9 correspond to the small, medium, and large spindles, respectively. (The $\perp$ on r is to remind you that this is the perpendicular distance between the applied force and the rotation axis.)

4. With the mass hanger attached to the cable, and the cable completely unwound, measure the distance between the bottom of the hanger and the floor at its lowest point. This should be some small distance above the floor. Record this distance as the final height h_{f} below.

5. Now spin the large disk to wind up the cable on the small radius spindle. Stop winding when the hanger gets near the pulley, and pick a reference spot on the hanger that lines up nicely with some point on the pulley attachment so that you can *always start the hanger at exactly this height*. Measure the distance between the bottom of the hanger and the floor, and record as the initial height h_{i} below.

6. Take the difference of these heights to find Δh, the distance traveled by the hanger.

   h_{i} = **initial height (m):** ________________________

   h_{f} = **final height (m):** ________________________

   Δh = **distance traveled (m):** ________________________

Trial	$r_\perp$ (m)	m_{h} (kg)	α (rad/s^2)	a (m/s^2)	ω_{f} (rad/s)	v_{f} (m/s)	$U = m_{\mathrm{h}}\, g \Delta h$ (J)
1							
2							
3							
4							
5							
6							
7							
8							
9							

Table 1: Data table for angular acceleration and final angular speed of disk and hanger system.

7. Detach the mass hanger, add 50 g to it, and measure the total mass. Record this as m_{h} for trial 1 of Table 1.
8. Reattach the hanger to the cable, and hold the disk steady with the cable wound on the smallest spindle. Click Collect, and when the software is ready to take data, release the disk.
9. Stop the data collection after the hanger reaches its lowest point and the cable starts to wind back up the other way.
10. Perform a linear fit on the angular velocity vs. time graph to obtain the angular acceleration. As usual, we are interested in the time during which the disk is rotating with *constant* angular acceleration, so highlight only this part of the data for the linear fit. Record α in Table 1.
11. Use the highlighted data columns to find the final angular velocity ω_{f} (really the angular speed) of the disk—that is, the maximum angular velocity when the hanger reached its lowest point. Record in Table 1.
12. Repeat steps 7-11 for trials 2 and 3 (still small spindle) using 75 g and 100 g on the hanger, respectively.
13. Repeat the above procedure for trials 4-6 using the medium spindle and trials 7-9 using the large spindle with the three different mass amounts on the hanger.

4.2.2 Data Calculations

For each row in Table 1, calculate the

- linear acceleration of the hanger for radius $r_\perp$ using $a = \alpha \cdot r_\perp$
- final linear speed of the hanger using $v_\mathrm{f} = \omega_\mathrm{f} \cdot r_\perp$
- decrease in potential energy of the hanger, $U = m_\mathrm{h}\, g \Delta h$

Trial	τ (Nm)	I (kg m^2)	K_rot (J)	K_tr (J)	K_final (J)	$\frac{K_\mathrm{final}}{m_\mathrm{h}\, g\Delta h} \cdot 100\%$
1						
2						
3						
4						
5						
6						
7						
8						
9						

Table 2: Data table for energy conservation analysis.

For each row in Table 2, calculate the

- torque on the disk, $\tau = r_\perp m_\mathrm{h}(g - a)$
- moment of inertia of the disk, $I = \tau/\alpha$; the value of I should be about the same for a given spindle size (i.e., applied torque)
- rotational kinetic energy of the disk, $K_\mathrm{rot} = \frac{1}{2} I {\omega_\mathrm{f}}^2$
- translational kinetic energy of the hanger, $K_\mathrm{tr} = \frac{1}{2} m_\mathrm{h} {v_\mathrm{f}}^2$
- total final kinetic energy of the system, $K_\mathrm{final} = K_\mathrm{rot} + K_\mathrm{tr}$
- ratio (as a percentage) of total final kinetic energy to the decrease in potential energy, $K_\mathrm{final}/(m_\mathrm{h}\, g\Delta h) \cdot 100\%$

Determine the average moment of inertia for each spindle size using the numbers you obtained in Table 2.

I_sm = ________________________ (kg m^2)

I_med = ________________________ (kg m^2)

I_lg = ________________________ (kg m^2)

5 Analysis

1. Using the Newton's second law equations for a rotating body in the Theory section, solve for the tension T in the cable. Substitute this into $\tau = r_{\perp}T$ to obtain an expression for the torque in terms of measured quantities.

2. Calculate the final velocity of the hanger for each trial by a different method: use the linear distance traveled by the hanger, the acceleration measured by the computer (from Table 1), and the equation ${v_\mathrm{f}}^2 = {v_0}^2 + 2a\Delta y$. Convert this to ω_f, and find the percent difference with the measured values from Table 1.

3. Looking at the last column of Table 2, was the total mechanical energy conserved for each trial? Use labels like *conserved*, *mostly conserved*, or *not conserved*, and say why. (If any of your trials ended up with >100%, explain what this means and how it could have happened.)

4. Name three possible sources of error (excluding human error) that could have limited the accuracy of your data and prevented the total mechanical energy from being perfectly conserved.

5. Describe any relationships you see between spindle size $r_{\perp}$ and the measured/calculated quantities of Tables 1 and 2.