Lab 5: Newton's Second Law

1 Introduction

The purpose of this lab is to study the acceleration of a mass as a function of the force applied to it. When a constant force $\vec{\mathbf{F}}$ acts on an object of mass m, the object will undergo uniformly accelerated motion with acceleration $\vec{\mathbf{a}}$. The relationship between these quantities is given by Newton's second law: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

2 Key Concepts

- Newton's second law
- Force
- Mass
- Acceleration
- Connected coordinate system

3 Theory

A **force** is an interaction of an object with its environment. An object's **mass** is a characteristic of the body that relates its acceleration to the forces acting on it. Sir Issac Newton postulated three laws that attempt to explain why an object behaves as it does when it experiences forces. These were first published in 1687 in his famous work, *Philosophiæ Naturalis Principia Mathematica*.

1. Assuming there are no net forces acting on a body ($\vec{\mathbf{F}}_{\text{net}} = 0$), then if it is at rest, it will remain at rest; if it is moving with constant velocity, it will remain moving with constant velocity.

2. The **acceleration** of a body is directly proportional to the sum of the forces acting on it: $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$. The constant of proportionality is the mass of the body.

3. For every action force there is an opposite reaction force that is equal in magnitude and opposite in direction: $\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$. It is important to realize here that these two forces act on different bodies.

Consider the second law. Mass can only be changed by changing the characteristics of a body, like its geometry or density. Therefore, this law tells us that changing the force on a body only affects its acceleration. It also says that $\vec{\mathbf{a}}$ is just a scaled version of $\vec{\mathbf{F}}$, and since $m > 0$, these vectors point in the same direction.

We often use Newton's second law to determine the acceleration of a body or system given its mass and the forces that act on it. $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$ is a vector equation, so in practice, we write out the components for each body individually. For three-dimensional motion in general, there will be 3 equations from Newton's second law for each body in the problem. Finding the acceleration then amounts to solving these (potentially many) equations simultaneously. The best method for this is the substitution method, which works for nonlinear equations as well as linear ones.

In a system of connected objects, like we have in this lab, the objects will move with the same acceleration. It is therefore convenient to choose a so-called **connected coordinate system** that reflects this. See the diagram in Figure 1. The air track glider of mass m_{atg} is attached to the hanger of mass m_{h} by a cord. The positive x direction is chosen to be in the direction of initial motion for both objects; $+x$ is to the right for the glider and downward for the hanger. This is shown by the curved arrow in the diagram.

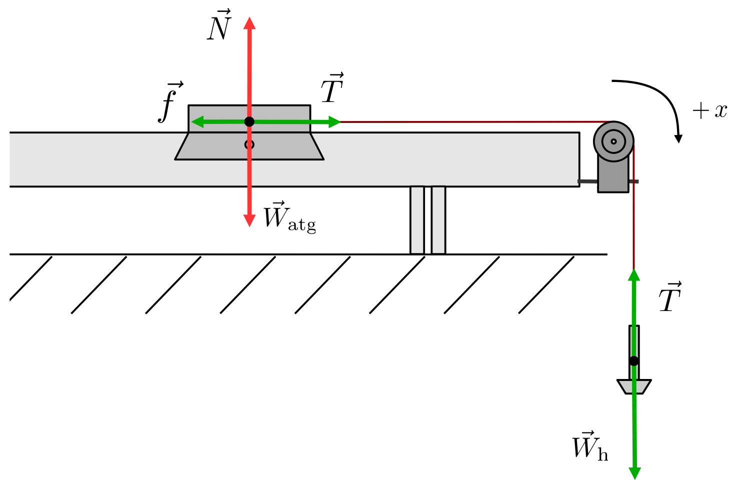

Figure 1: Diagram of the lab setup. The air track glider and the hanger are connected by a cord, so it is convenient to use a connected coordinate system. Let the positive x-axis lie in the direction of motion, which is to the right for the glider and downward for the hanger. The forces in the x direction are green, and the forces in the y direction are red.

The hanging mass pulls the glider with a force of magnitude $F = W_{\text{h}} = m_{\text{h}} \cdot g$. The experiment measures the acceleration for different combinations of m_{atg} and m_{h}, and the results are compared to what we expect from Newton's second law. Applying Newton's second law to the system gives us the three equations below—two for the glider and one for the hanging mass. $\vec{\mathbf{T}}$ is the tension in the cord, $\vec{\mathbf{N}}$ is the normal force of the air track on the glider, and a friction force $\vec{\mathbf{f}}$ has been included for generality, although the purpose of the air track is minimize the friction as much as possible.

	Glider	Hanger
$x:$	$\sum F_x = T - f = m_{\text{atg}} a$	$\sum F_x = W_{\text{h}} - T = m_{\text{h}} a$
$y:$	$\sum F_y = N - W_{\text{atg}} = 0$	—

4 Experiment

4.1 Equipment

- Triple beam balance
- Set of masses
- Meter stick
- Mass hanger
- Red cable
- Air track with air supply and glider
- Rotary motion sensor
- Computer with Logger Pro software
- Stopwatch

4.2 Procedure

We will first use a computer to determine the acceleration of the glider-hanger system. One experiment will keep the total system mass constant, varying both the glider and hanger masses, and the other will vary the total system mass but keep the hanger mass constant. The experiments will then be repeated using a stopwatch to measure travel times from which the accelerations can be calculated.

4.2.1 Part 1: Using Computer

1. Verify that your setup looks like Figure 1.
2. Add 250 g to the air track glider. Make sure the masses are distributed evenly so that the glider is balanced.
3. Measure m_{atg}, the total mass of the glider, and record in Table 1 for Trial 1.
4. Place a mass of 5 g on the mass hanger, and measure the total hanger mass m_{h}. Record in Table 1.
5. Add these two masses, and record the total mass of the system as m_{total}.
6. On the computer, open the Lab 5 Air Track file in Logger Pro. Three graphs will appear on the screen: position (m) vs. time (s), velocity (m/s) vs. time (s), and acceleration (m/s^2) vs. time (s).
7. Turn on the air supply, and hold the glider so that its leading edge (the side with the cord attached) lines up with the 130 cm mark on the air track as measured from the end with the rotary motion sensor.
8. Click the Collect button, wait for it to turn red, and then release the glider. Beware of the delay between clicking Collect and the actual start of data collection by the computer.
9. Click Stop after the hanger hits the floor and the glider has traveled its full possible range.

Trial	m_{atg} (kg)	m_{h} (kg)	m_{total} (kg)	a (slope) (m/s^2)	$F_{\text{app}} = m_{\text{h}} \cdot g$ (N)	$m_{\text{total}} \cdot a$ (N)
1						
2						
3						
4						
5						
6						

Table 1: Data table for **constant total mass** using computer.

Trial	m_{atg} (kg)	m_{h} (kg)	m_{total} (kg)	a (slope) (m/s^2)	$F_{\text{app}} = m_{\text{h}} \cdot g$ (N)	$m_{\text{total}} \cdot a$ (N)
1						
2						
3						
4						
5						
6						

Table 2: Data table for **constant hanger mass** using computer.

10. Examine your graphs. Since there is a constant net force on the glider (the tension in the cord provided by the weight of the hanger), we expect a constant acceleration, i.e., a horizontal line. The velocity graph should therefore be a straight (oblique) line.

11. Click to select the velocity graph, and highlight the section of data that represents when the glider was experiencing constant acceleration. Fit a straight line to it by clicking the linear fit button.

12. Record the acceleration a, the *slope* of the linear fit function, in Table 1.

13. Repeat the above steps for a total of 6 trials, where for each new trial, you **move 10 g from the glider to the hanger**. Make sure that the mass on the glider is always distributed evenly on each side.

14. For all trials, compute the applied force on the glider, which is the weight of the hanger, and the total system mass times the measured acceleration to complete Table 1.

15. Now put 25 g on the hanger, and record the total hanger mass as m_{h} in Table 2 for Trial 1. This will remain the same for the rest of this part of the experiment.

16. Put 160 g on the air track glider, and compute the total system mass as before. Record these values in Table 2.

17. Complete 6 trials following the same procedure as above to measure the acceleration of the glider, but where for each new trial you **decrease the mass on the glider by 10 g and keep the hanger mass the same**.

18. Compute F_{app} and $m_{\text{total}} \cdot a$ for each trial to complete Table 2.

4.2.2 Part 2: Using Stopwatch

1. Put 250 g on the air track glider, making sure the masses are distributed evenly.

2. Measure m_{atg}, and record in Table 3 for Trial 1.

3. Place 5 g on the mass hanger, and record m_{h} and m_{total}.

4. Hold the leading edge of the glider again at the 130 cm mark, and measure the distance from the *bottom* of the hanger to the floor. This is the distance Δx that the glider will travel. Record in the space below.

   $\Delta x =$ **distance from mass hanger to floor (m):** ________________ $\pm$ ________________

Trial	m_{atg} (kg)	m_{h} (kg)	m_{total} (kg)	t (s)	a (m/s^2)	$F_{\text{app}} = m_{\text{h}} \cdot g$ (N)	$m_{\text{total}} \cdot a$ (N)
1							
2							
3							
4							
5							
6							

Table 3: Data table for **constant total mass** using stopwatch.

Trial	m_{atg} (kg)	m_{h} (kg)	m_{total} (kg)	t (s)	a (m/s^2)	$F_{\text{app}} = m_{\text{h}} \cdot g$ (N)	$m_{\text{total}} \cdot a$ (N)
1							
2							
3							
4							
5							
6							

Table 4: Data table for **constant hanging mass** using stopwatch.

5. Release the glider and measure its total travel time t with the stopwatch—that is, the time until the hanger hits the floor.

6. Calculate the acceleration of the system using the kinematic equation $\Delta x = \frac{1}{2}at^2$. (We have set $v_0 = 0$, because the glider starts from rest.) Note that $a \neq g$, since there is tension in the cord, and the hanger is not in free fall.

7. Perform 6 trials, keeping m_{total} constant while varying m_{atg} and m_{h} as before by moving 10 g from the glider to the hanger each time.

8. Carry out the remaining calculations to complete Table 3.

9. Now leave m_{h} constant with 25 g on it and start with 160 g on the glider.

10. Perform 6 trials for Table 4 as you did for Table 2 by decreasing m_{atg} in 10 g increments.

11. Carry out the remaining calculations to complete Table 4.

5 Analysis

1. Write out the component equations that result from applying Newton's 2nd law ($\vec{\mathbf{F}} = m\vec{\mathbf{a}}$) to the glider and the hanger. (Hint: there will be two equations for the glider, since it has forces acting in the x and y directions, and the hanger will have one equation.) Include a friction force f on the glider for generality.

2. Eliminate the tension T from the set of equations to solve algebraically for the acceleration a. That is, find an expression for a in terms of m_{atg}, m_{h}, etc.. Remember that since the glider and hanger are connected by the cord, their accelerations are the same.

3. Obtain an expression for a in the absence of friction. The easy way to do this is to just let $f = 0$ in the expression you already found for a.

4. Draw free body diagrams for the glider and hanger, first including friction, and then ignoring friction.

5. Compare the expressions for acceleration that you get for the two cases ($a_{f\neq 0}$ and $a_{f=0}$) to each other and to the acceleration of gravity g. List the three accelerations in order of magnitude. You can do this just by examining the algebraic expressions; you don't need to plug in numbers.

5.1 Part 1: Using Computer

1. Plot the applied force F_{app} vs. acceleration a for the data in Table 1.

2. Examine your graph carefully. Should the data be linear? Explain your answer using the expression you got for the acceleration (ignoring friction). It might be helpful to consider the information from Lab 3 concerning graphs.

3. What physical quantity does the slope of this graph represent? (The expression for a will tell you this.)

4. Use the data in Table 2 to graph the total system mass m_{total} vs. the inverse of acceleration $1/a$. Note that you will actually need to compute $1/a$ for each trial and use these values when plotting. The units of $1/a$ are $1/(\text{m/s}^2)$, or s^2/m.

5. Examine your graph carefully. Should the data be linear? Explain your answer using the expression you got for the acceleration, which you will need to rearrange.
6. What physical quantity does the slope represent?
7. Find the percent difference between the slope of the line and the measured quantity it represents for both of the graphs above.
8. Based on your expression for the acceleration, do you expect the results in the last two columns of Table 1 to be the same? What about for Table 2?
9. Compare these two values for each trial by finding their percent difference.
10. What are the possible sources of error that might have limited the precision of your result?

5.2 Part 2: Using Stopwatch

1. Repeat questions 1 through 10 you did in Part 1 above for the data in Tables 3 and 4.
2. How did the human error from the stopwatch reaction times affect your results?