Lab 2: Vectors

1 Introduction

A vector is a quantity that has both a magnitude and a direction. This is in contrast to a scalar quantity, which is just a single number. In this lab we will learn to accurately describe and manipulate vectors, including resolving them into components and adding them together. We will add vectors graphically, componentwise, and also experimentally using the force table apparatus.

2 Key Concepts

- Vector addition
- Force
- Free body diagrams
- Equilibrium

3 Theory

3.1 Vectors

When completely describing a physical quantity, sometimes it is necessary to assign it a **magnitude** and **direction** rather than using only a magnitude. Quantities needing only a single value are **scalars**, and quantities requiring a magnitude and a direction are **vectors**. Vectors are given in boldface type and are capitalized letters with an arrow on top, $\vec{\mathbf{A}}$, while scalars are normal font without the arrow, A.

To identify and use a vector, first establish a **coordinate system**, or set of axes, that is appropriate for the system you are trying to describe. This will give you a frame of reference from which to make your measurements. An example is labeling North on a map. Figure 1 shows two angles possible in describing the vector $\vec{\mathbf{V}}$, one with respect to the x-axis (θ for standard use) and one with respect to the y-axis (ϕ occurs sometimes in problems).

In this lab manual, we will use the x- and y-axes. All angles given are measured counterclockwise from the x-axis. Remember, some problems are made easier by a clever choice of axes.

Vectors can be described in two ways: the magnitude and direction method or the **component method**. The magnitude and direction method describes the vector with a given magnitude and direction. But you must be careful to include enough information in giving the direction. You must say what the direction is with respect to, such as 5N @ 30° N of E, or above the x-axis. This is very intuitive to graph but mathematically more difficult to use. The component form of a vector breaks up the magnitude and direction into how much along the x-axis and how much along the y-axis. Vectors in component form look like

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \langle A_x, A_y \rangle.$$

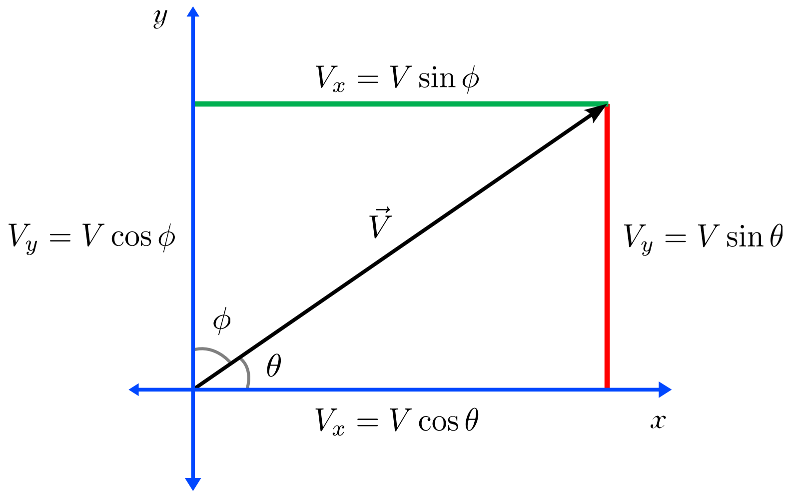

Figure 1: The components of a vector depend on the coordinate system used. Notice there are two angles indicated: θ is measured with respect to the x-axis, and ϕ with respect to the y-axis. How you determine the x and y components depends on which angle you are given.

We will stick to the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ unit vector notation in this manual, but the angle bracket notation is also common.

To find the components of vectors, you need to use the trig identities

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}\ , \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}.$$

Which equation you need to use depends on your coordinate system. For the angles used in this lab manual, A_x, the x component of 5 N @ 30 degrees, can be found from cosine.

$$A_x = 5\ \cos(30^\circ) = 4.33$$

Similarly for the y component,

$$A_y = 5\ \sin(30^\circ) = 2.5$$

Thus, $\vec{\mathbf{A}} = 4.33\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}}$, where $\hat{\mathbf{i}}$ is the unit vector along x, and $\hat{\mathbf{j}}$ is the unit vector along y.

Given component form, the angle is solved for by $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$. The magnitude of the vector is solved by $A = \sqrt{(A_x)^2 + (A_y)^2}$.

3.2 Adding Vectors

Adding two vectors results in a third called the **resultant vector**. Now that a vector is broken up into its components, adding multiple vectors is simply adding up all of the x components to find the x component of the resultant vector and adding all of the y components to find the y component of the resultant vector. For example, if $\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$, then $\vec{\mathbf{A}} + \vec{\mathbf{B}} = (3+2)\hat{\mathbf{i}} + (5-4)\hat{\mathbf{j}} = 5\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$. Notice that you need to be careful to take the sign of the components into account when adding vectors.

To **graphically** find the resultant vector, use the 'tail to tip' method. Once a coordinate axis is drawn, the 'tail' of the first vector begins at the origin. Measure the appropriate angle using a protractor, and draw the length of the vector to match the vector's magnitude. The ending point of the first vector is known as the 'tip,' and an arrow is drawn at this end. From the 'tip' of the first vector, start the 'tail' of the next vector. To measure the angle for the second vector, hold the protractor parallel to the original axis. Be sure to use the same scale as the first vector in drawing the length of the second vector. Continue this way until all vectors to be added have been drawn. The resultant vector begins at the origin and ends at the tip of the last vector added.

To **experimentally** find the resultant vector, it is easiest to first find the **equilibrant vector**, and the resultant is then equal in magnitude but in opposite direction to the equilibrant. Opposite direction for component form amounts to changing the sign of each component. When an angle and magnitude are given, the opposite direction is given by adding 180° to the angle. The equilibrant vector is the vector that balances out the system. It brings the system to equilibrium and therefore keeps the system stationary.

For more information on vectors, please see the sections on vector components and vector addition in your text. There are also some great figures to help you understand the techniques.

When you add two or more vectors, it can be very helpful to make a table for organizing the data. An example is given in Table 1. Another thing that is sometimes helpful in picturing these vectors and their components is to draw them all on the same coordinate system with their tails all at the origin.

Vector name	Vector magnitude	x component	y component
$\vec{\mathbf{A}}$	A	A_x	A_y
$\vec{\mathbf{B}}$	B	B_x	B_y
$\vec{\mathbf{C}}$	C	C_x	C_y
$\vec{\mathbf{R}}$	$R = \sqrt{R_x^2 + R_y^2}$	$R_x = A_x + B_x + C_x$	$R_y = A_y + B_y + C_y$

Table 1: Table that is helpful in organizing data to add multiple vectors.

3.3 Free Body Diagrams

It is often beneficial in a problem to be able to draw a picture. A **free body diagram** is a representation of all the vectors of the same type that are acting on the system. In this lab, these vectors will be **forces**. If the free body is stationary, that means the body is in equilibrium; the sum of all the forces acting on the body is zero. For simplicity, assume all objects are a single point at the center. In a free body diagram, all the 'tails' of the vectors acting on the body are located at this center point. Be sure to label each vector and that all vectors represent the same type of quantity. A simple check can be done with unit analysis. If all the vectors must add, they must have all the same units. Like the graphical method of finding vectors, each vector length must represent the magnitude of the vector, and must be labeled with its angle if its not on an axis.

For more information on free body diagrams, please see the section in your book on free body diagrams.

4 Experiment

The first part of the experiment examines force vectors acting on an air track glider. Since the glider is constrained to move only along the air track, its motion and the forces that cause it to move are one-dimensional. The forces are provided by strings (attached to mass hangers) pulling at either side of the glider as well as by the component of the glider's weight along the air track. By elevating one end of the air track, you will be determining experimentally what force is needed to bring the glider to equilibrium for different acting forces. In the second part of the experiment, you will be balancing forces on a ring, but this time in two dimensions using the force table.

4.1 Equipment

- Triple beam balance
- Set of masses
- Meter stick
- Mass hangers (5)
- Air track with air supply and glider
- Red cables (2)
- Rotary motion sensor
- Marble tiles (6) and aluminum squares (2)
- Force table with ring, pulleys (3), and black cables (3)

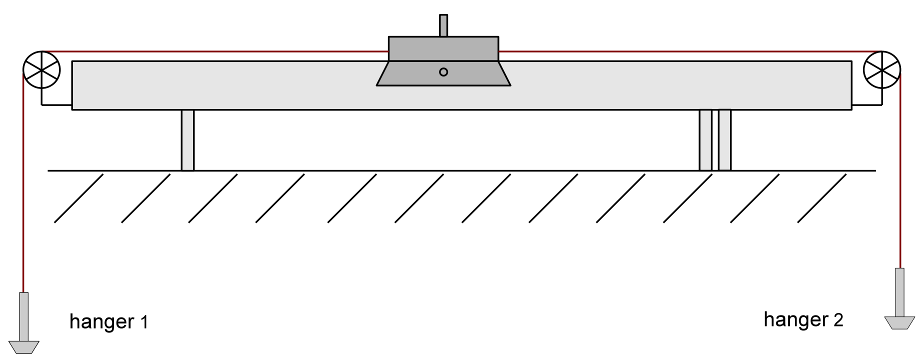

Figure 2: Diagram of air track setup.

4.2 Procedure

4.2.1 1D Vectors

1. Measure the distance between the feet of the air track with a meter stick. Record this length and its uncertainty in the space provided above Table 2. Be sure to measure from the centers of the feet, and pay attention to units.

2. Add 200 g to the air track glider (100 g on each side) and measure its total mass using the triple beam balance. Record this number and its uncertainty in the space provided above Table 2.

3. Measure and record the masses and uncertainties of the two hangers for the air track with no weights on them.

4. Place the glider on the air track near the middle, and attach the strings to either side. Thread the strings through the bumpers and over the pulleys at each end of the air track. Attach the mass hangers to the strings. Your setup should look like the diagram in Figure 2.

5. Turn on the air supply and verify that the air track is level. If it is, the glider will not move when placed at rest. If it is not, notify your TA for help in leveling the track. (The purpose of the air track is to reduce unwanted friction on the glider, and you might need to increase the power from the air supply so that the glider can move smoothly.)

6. When the track is level, add the specified mass to each hanger for trial 1 of Table 2.

7. Calculate the total mass in kg of each hanger and record in Table 2. $m_1 = m_{\text{hanger1}} + m_{\text{added1}}$ and $m_2 = m_{\text{hanger2}} + m_{\text{added2}}$.

8. If necessary, elevate the end of the track with one foot by placing marble/aluminum squares underneath it until the glider is in equilibrium (at rest) on the track. Make sure that the hangers are not swinging.

9. Record the elevation of the track in Table 2.

10. Calculate incline angle of the air track. (Note that if α is the incline angle, there is a right triangle whose hypotenuse is the distance between the feet of the air track and whose opposite side is the elevation. You can use trigonometry to find α.)

11. Repeat for trials 2-6 to complete Table 2.

4.2.2 2D Vectors

1. Measure the mass of one hanger used with the force table. We will assume that all three hangers are the same here. Record this measurement and its uncertainty in the space above Table 2. In all the work below, you will need to use the total mass (mass of the hanger plus the added mass) in your calculations.

2. Place the ring around the center peg. Make sure that all three cords are parallel to the force table—you might need to adjust the heights of the pulleys to achieve this.

3. Place the first two pulleys at the angles given for hangers 1 and 2 in trial 1 of Table 3, and add the specified masses to the hangers at each angle. You can choose which to call hanger 1 and hanger 2.

4. Calculate the magnitudes of the forces (in Newtons) provided by each hanger and record in Table 3.
5. Place the third pulley where you would guess the equilibrant vector to be, and hang a trial mass. (Note that you can put the cords of more than one hanger over the same pulley if you need to.)
6. Adjust both the angle and mass of the third hanger until the ring appears centered around the peg.
7. Test the equilibrium position by giving the system a small pull along one of the vectors. If the ring oscillates slightly but remains centered, you have found the correct equilibrant force. That is, you have found the force that balances out the net effect of the first two.
8. Record the magnitude and angle of this equilibrant force in Table 3. Also record the resultant force, which is the negative of the equilibrant, and repeat for the remaining trials.

5 Data

Distance between feet of air track (m): ___________ ± ___________

Mass of air track glider + added 200 g (kg): ___________ ± ___________

Mass of air track hanger 1 (kg): ___________ ± ___________

Mass of air track hanger 2 (kg): ___________ ± ___________

Mass of force table hanger (kg): ____________ ± __________

Trial	Mass on hanger 1 (g)	m_1 (kg)	Mass on hanger 2 (g)	m_2 (kg)	Elevation (m)	Incline angle (°)
1	10		10			
2	15		10			
3	20		10			
4	25		10			
5	30		10			
6	35		10			

Table 2: Data table for air track measurements.

					Equilibrant	Resultant
Trial	Hanger 1	F_1 (N)	Hanger 2	F_2 (N)	$\vec{\mathbf{F}}_\mathrm{E} =$ F_E (N) @ $\angle$	$\vec{\mathbf{F}}_\mathrm{R} =$ F_R (N) @ $\angle$
1	50 g @ 0°		100 g @ 180°			
2	30 g @ 0°		40 g @ 90°			
3	50 g @ 0°		100 g @ 120°			
4	30 g @ 30°		50 g @ 270°			

Table 3: Data table for force table measurements.

6 Analysis

6.0.3 1D Vectors

1. Is there a trend in Table 2 as m_1 increases? Describe your results.
2. Draw a free body diagram of the glider at equilibrium on the level track.
3. Draw a free body diagram of the glider at equilibrium on the inclined track.
4. Label the incline angle α.
5. Notice that the incline angle of the air track is the same angle between the negative of the normal force and the weight of the glider. Therefore, draw the negative of the normal force as a dashed line and label the angle between it and the weight as α also.
6. For each row in Table 2:
   (a) Calculate the x and y components of the weight of the glider. Consider the positive x-axis to be up the incline and the positive y-axis to lie along the normal force on the glider.
   (b) Calculate the net resultant force $\vec{\mathbf{F}}_\mathrm{R}$ on the glider due to m_1 and m_2. Recall that the magnitude of the gravitational force is $F = mg$, and use $g = 9.80$ m/s^2.
   (c) Find the percent difference between the magnitude of $\vec{\mathbf{F}}_\mathrm{R}$ and the magnitude of the x component of the glider's weight.

6.0.4 2D Vectors

For each row in Table 3:

1. Draw a free body diagram of the three forces acting on the ring. You can also draw in the resultant as a dashed line if you like, but realize that it is just the sum of the initial two forces, and not really a separate force acting on the ring.
2. Label x- and y-axes appropriate for the system.
3. Resolve each vector into its x and y components.

4. You found the resultant force experimentally already using the force table. Now find the resultant force in another experimental way, graphically, and remember that to add vectors graphically, you need to define a suitable scale and draw using a ruler and protractor.
5. Find the resultant force using component addition.
6. Taking the result of component addition to be the theoretical value, find the percent error of the magnitude of the resultant for both experimental methods.
7. Which method gave lower percent error?
8. Consider each step of the graphical method. How can you improve upon the sources of error?
9. Give two sources of error in determining the equilibrant/resultant vector on the force table.

6.0.5 Check your understanding

1. Give two examples of scalar quantities and two examples of vector quantities.
2. Label appropriate axes for the four free body diagrams in Figure 3. Note that you are free to choose new coordinate axes to make the problem simpler if you like.
3. For each diagram, resolve the vectors into component form.
4. Find the resultant vector in each case (a) graphically, and (b) by component addition. You will need to redraw the vectors yourself, since the diagrams are not drawn to a consistent scale. Your answers should be the components, magnitudes, and angles of the resultants relative to your coordinate system.

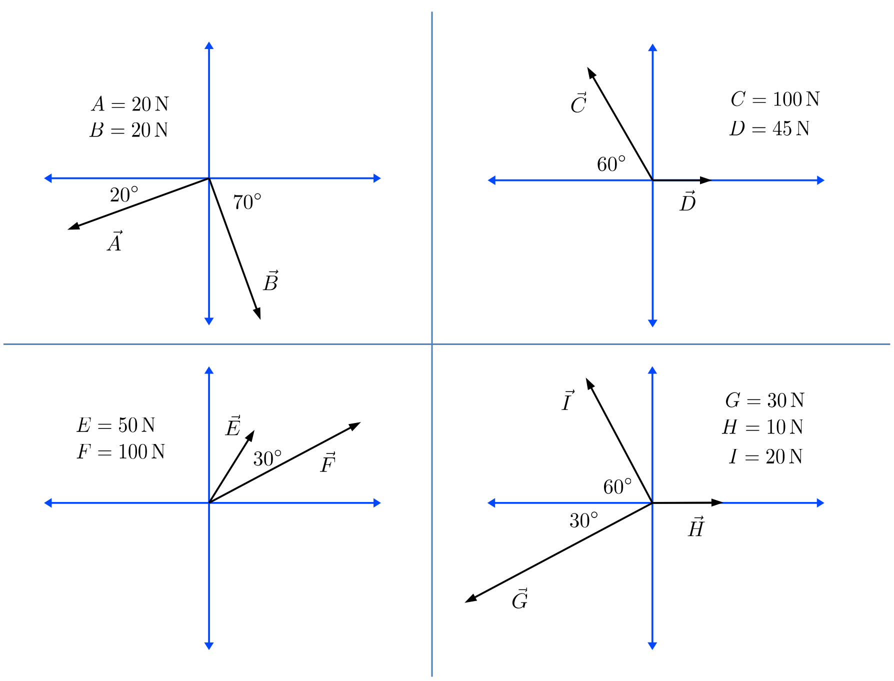

Figure 3: These are 4 cases to practice your vector addition. Vectors are not drawn to a consistent scale.