Graphing Skills

An important skill to master is analyzing experimental data in graphical form. Graphs can be very informative, because they allow us to see trends in the data and extract information about the measured quantities and their relationships to each other. The **experimental equation** is an equation that best represents the relationship between the variables graphed. The **linearization equation** is the equation of a best fit trendline that fits the linear graph of the data variables. The **linear graph** is the graph of a straight line obtained from the data variables. It is found by taking your data and making 3 graphs: y vs. x, log y vs. log x, ln y vs. x. The graph with a linear relationship (where the R^2 value of a linear trendline is closest to 1) is the linear graph. We will be finding the experimental equation of the data in several labs.

A plot of the ordinate (Y or vertical axis) vs. the abscissa (X or horizontal axis) needs to have an appropriate scale for each axis. You want the graph to be easy to read without it being bunched up in a corner. Use a scale so most of the graph is used. If using a program (like Excel), use an appropriate graph size so that it is legible. There should be no more than 3 graphs per page. Always include units and labels for the axes as well as a graph title that is informative and easy to understand.

There are *3 major types of graphs* used. They consist of <u>linear</u> graphs where both axes are linear, <u>power</u> graphs where both axes are logarithmic, and <u>exponential</u> graphs where the abscissa (X) is linear and the ordinate (Y) is logarithmic. Below I give you the basic equation that represents the relationship between the data points graphed (*experimental equation*) and the equation of a best fit trendline fitting the linear graph (*linearization equation*). You will need to know how to recognize each type, determine the slope and y intercept of the linearization equation, and find the experimental equation. Therefore you need to be able to go back and forth between the different equations.

Type	Experimental Equation	Linearization Equation
Linear	$y = mx + b$	already a line
Power	$y = bx^m$	$\log y = m \log x + \log b$
Exponential	$y = be^{mx}$	$\ln y = mx + \ln b$

The first is a straight line given by the equation $y = mx + b$ where the slope is $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$ and the y intercept (the point where the graph hits the y axis) is b. Determine these values by directing excel to add the linear trendline and ask it to display the equation and R^2 value in the chart. *Do not plug in data points into the formulas to get the slope or y intercept. This would be finding an average slope between the two points you use, not the line of best fit.*

Instructions for going from a linearization equation to an experimental equation

Functions of the form $y = bx^m$ can also be plotted as a straight line. The math to show this is possible is below.

$$\begin{aligned} y &= bx^m \\ \log y &= \log bx^m \\ &= \log b + \log x^m \\ &= \log b + m \log x \\ \therefore \log y &= m \log x + \log b \end{aligned}$$

$$y_t = m_t x_t + b_t$$
$$m_t = m$$
$$b_t = \log b \Rightarrow b = 10^{b_t}$$

Where m_t is the slope of the linear trendline and b_t is the y intercept of the linear trendline. But to get a linearization first you must manually find the log of the x and y values. Then plot log y vs. log x and add a linear trendline to the graph. Don't plot data that is undefined, only the points with a real value.

Using the trendline of the graph you can find m and b from m_t and b_t as shown to the right above. This gives you the experimental equation of the data for this type of graph.

Functions of the form $y = be^{mx}$ can also be plotted as a straight line. The math to show this is possible is below.

$$\begin{aligned} y &= be^{mx} \\ \ln y &= \ln be^{mx} \\ &= \ln b + \ln e^{mx} \\ &= \ln b + mx \\ \therefore \ln y &= mx + \ln b \end{aligned}$$

$$y_t = m_t x_t + b_t$$
$$m_t = m$$
$$b_t = \ln b \Rightarrow b = e^{b_t}$$

But to get a linearization first you must manually find the ln of the y values. Then plot ln y vs. x and add a linear trendline to the graph. Again don't plot data that is undefined, only the points with a real value. Using the trendline of the graph you can find m and b from m_t and b_t as shown to the right above. This gives you the experimental equation of the data for this type of graph.

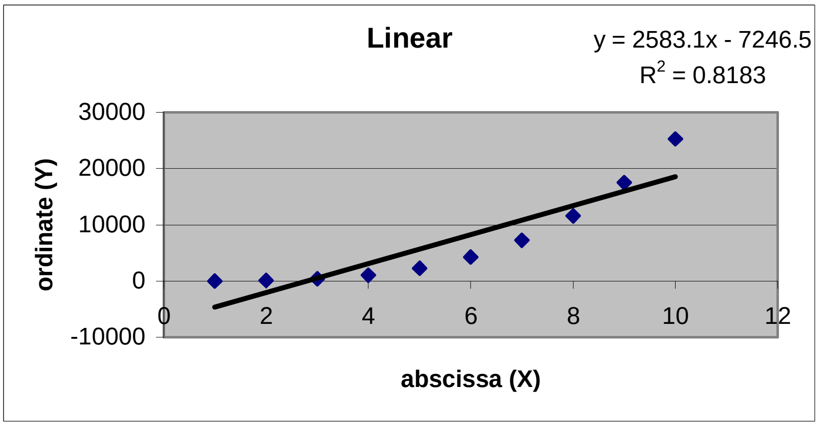
How to use Excel to make graphs

1) Type in your table (2 columns, x than y coordinates) *noting graphs will be asked for in y vs. x format*. To create a "log y vs. log x" or "ln y vs. x" graph, you will have to manually take the logs in your spreadsheet before creating each graph by using the log10() function to take logs of numbers and ln() function to take natural logs of numbers. You will then use each different set of columns to create the desired graph. Use the help guide to help you with anything you don't understand.
2) Select your data table (highlight it) that you want to graph
3) Go to the **Insert** menu and click on **Chart**
4) The chart wizard will pop up.
5) Click on the **XY (Scatter)** chart type
6) Then click on the appropriate Chart sub-type (with or without smoothed lines). It is best not to connect the data, because you will be adding a line of best fit later.
7) Click on **Next**. This will show a sample of how your data will be plotted. If your data is in rows rather than columns, then you will want to change the selection under the **Data Range** tab. *The chart source data should be fine if you selected your data table.*
8) Click on **Next. G**ive the chart an appropriate title and the x and y axis labels. The title should explain what you are graphing in the form "Dependent variable vs. Independent variable" when applicable (this won't work for the first lab). The X and Y axis labels should have what is being measured and its units (eg. Length (cm)). You can generally leave the other tabs untouched.
9) Click on **Next** and choose to create your chart **As a new sheet**.
10) Click on **Finish**
11) When the chart comes up you can double click on any part of it to change it.
12) Double click on the x axis to format it. This opens the **Format Axis** box. This box allows you to adjust the data range and other options. Make sure the Logarithmic scale is unchecked since we will not be using this option in this lab.
13) You can adjust the y axis the same way if necessary.
14) To add a best fit line, go to **Chart** and click on **Add Trendline**. You can also right click on one of the data points and click on **Add Trendline**. The **Add Trendline** box will appear. Choose the **linear** trendline line option. Go to the **Options** tab to display equation and R^2 value on the chart. The equation allows you to read off your slope and y intercept easily and **the R^2**

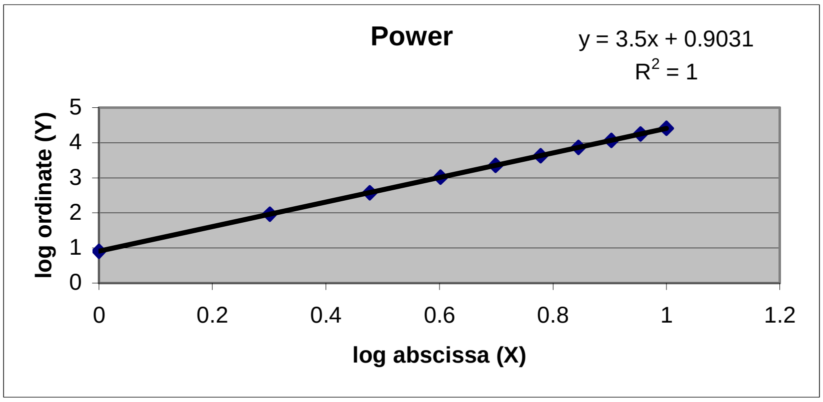
value gives you an idea of how well your data fits the line (the closer to 1 the better). ***NOTE**: For the way that we are finding equations of lines, you should only ever add linear trend lines. One of the things we are looking for is the power of the power function so we will never use quadratic trendlines.*

15) ***Caution:** It is a bad idea to calculate the slope or y intercept manually because you can not use actual data points from your lab in these calculations. You would have to find points on the line of best fit to manually calculate the slope or y intercept.*
16) Now use the labeled slope and y intercept to extrapolate back to the experimental equation of the data as shown in the previous section. If your data was originally linear, then the trendline is your experimental equation. However if your data was originally related by a power or exponential function, then you need to use the slope and y intercept from the linearized graphs (log x vs. log y or x vs. ln y) along with a little algebra to get back to the original power or exponential function.

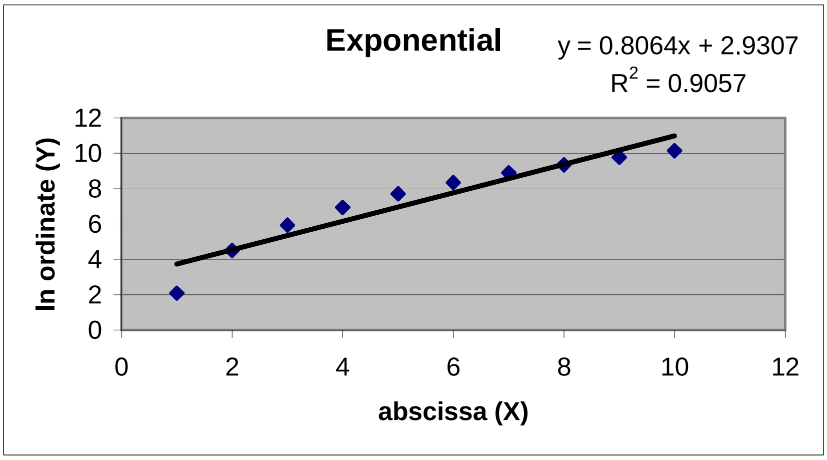
abscissa (X)	ordinate (Y)
1	8
2	90.509668
3	374.12297
4	1024
5	2236.068
6	4232.7183
7	7259.9416
8	11585.238
9	17496
10	25298.221



log(X)	log(Y)
0	0.90309
0.301029996	1.956695
0.477121255	2.5730144
0.602059991	3.0103
0.698970004	3.349485
0.77815125	3.6266194
0.84509804	3.8609331
0.903089987	4.0639049
0.954242509	4.2429388
1	4.40309



abscissa (X)	ln ordinate (Y)
1	2.0794415
2	4.5054567
3	5.9245846
4	6.9314718
5	7.7124742
6	8.3505997
7	8.8901271
8	9.3574869
9	9.7697276
10	10.138489



1) Put in the trendline equation and the R^2 values

2) Note which graph has the largest R^2 value

3) Use the information in the manual to determine b and m from b_t and m_t

EXAMPLE:

Power has largest R^2 value with $R^2 = 1$

linearization equation $y = m_t\, x + b_t \rightarrow y = 3.5\, x + .9031$

$m_t = m = 3.5$

$b_t = .9031$ $b = 10^{.9031} = 8$

experimental equation $y = b\, x^m \rightarrow y = 8\, x^{3.5}$

Plotting graphs:

1) Make 3 graphs of each of the three data tables below. Use the 3 graphs in the example above (y vs. x, log y vs. log x, and ln y vs. x) for each table of data. This will give you a total of 9 graphs. Find the trendline equation and R^2 value for each graph.
2) Print these graphs (9 in all) and turn them in with your lab report. (3 per page)
3) For the linear graph of each table (R^2 value closest to 1) record the slope and y intercept of the trendline (linearization equation) in Data Table III.
4) Determine which type of graph the three data tables below represent (linear, power, exponential). Put this in Data Table III. Determine the experimental equation of the data and put it in Data Table III. Show all of your work for these calculations in your Data and Calculations section of your lab report.
5) You should have all the data tables filled in before you leave. The computers in lab have Excel and you can use them to work on the graphs necessary. **Bring flash drives to save your work to take home and print.**

Table A		Table B		Table C	
y	x	y	x	y	x
-11	-5	5000	-5	1.52951E-06	-5
-7	-4	2048	-4	3.07211E-05	-4
-3	-3	648	-3	0.000617049	-3
1	-2	128	-2	0.012393761	-2
5	-1	8	-1	0.248935342	-1
9	0	0	0	5	0
13	1	8	1	100.4276846	1
17	2	128	2	2017.143967	2
21	3	648	3	40515.41964	3
25	4	2048	4	813773.9571	4
29	5	5000	5	16345086.86	5

Data Table III Types of Graphs

Table	Relationship of original data (linear, power, exponential)	Slope of Linearization Equation (trendline)	Y intercept of Linearization Equation (trendline)	Experimental Equation { $y = mx + b$ $y = bx^m$ }. $y = be^{mx}$
Table A				
Table B				
Table C				

Analysis Questions:

1.) For the three data tables given (Table A, B, and C) did you get experimental equations of each type of data (linear, power, and exponential)? Discuss your results .

(refer to the lab report guidelines as to how to include the answers to these questions in your report)