Chapter 7

Potential Energy and Energy Conservation

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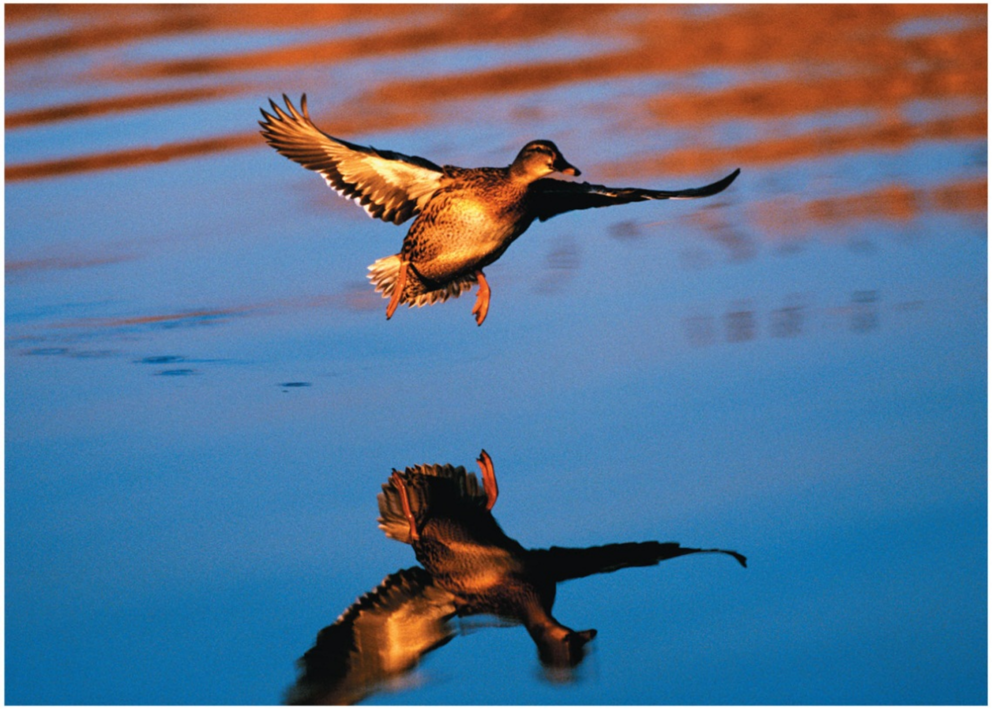
Introduction

- In the previous chapter, we learned how the work done on an object such as this duck changes its kinetic energy.
- In this chapter, we will show that the work done by conservative forces (like gravity) can define a potential energy such that the total energy (kinetic plus potential) is conserved by the force.



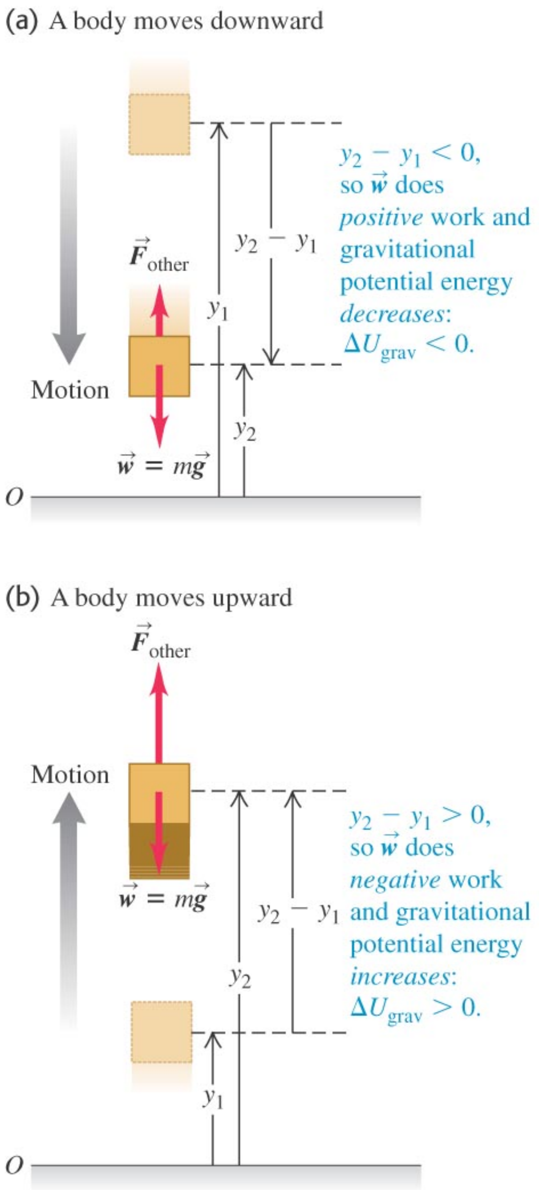
Gravitational potential energy

- If a mass *m* experiences a change in height $\Delta y = y_2 - y_1$, gravity does work

$$W_g = \boldsymbol{F}_g \bullet \boldsymbol{d} = -mg\Delta y$$

- We can define the *gravitational potential energy* $U_g = mgy$ such that

$$W_g = -\Delta U_g$$

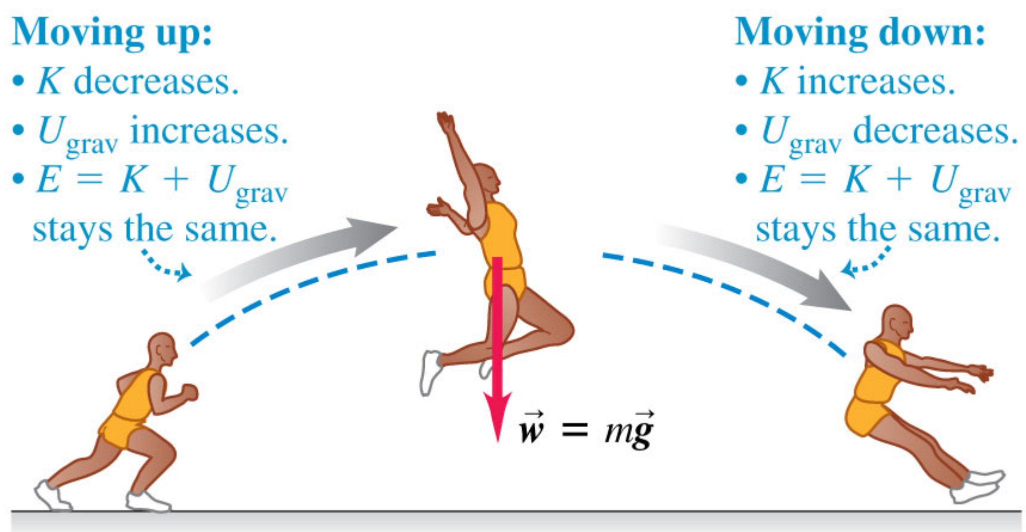


The conservation of mechanical energy

- The work $W_g = \boldsymbol{F}_g \bullet \boldsymbol{d} = -mg\Delta y$ done by gravity equals the change in kinetic energy $\Delta K = K_2 - K_1$ by the work-energy theorem.
- Using our newly defined *gravitational potential energy* $U_g = mgy$:

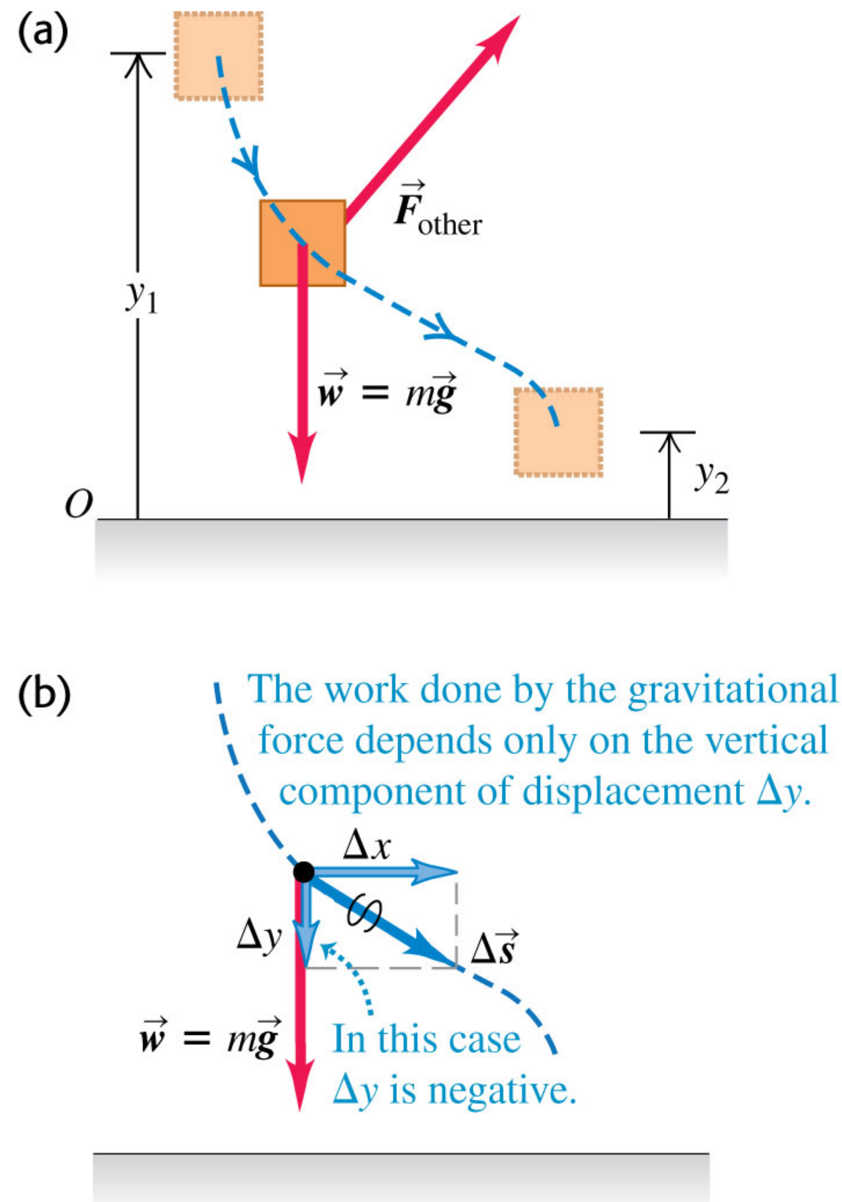
$$W_g = -\Delta U_g = \Delta K \Longrightarrow K_1 + U_1 = K_2 + U_2 \Longrightarrow E_1 = E_2$$

- The *total mechanical energy* $E = K + U_g$ is conserved unless the system is acted upon by another force $\boldsymbol{F}$: $\Delta E = W_F$.



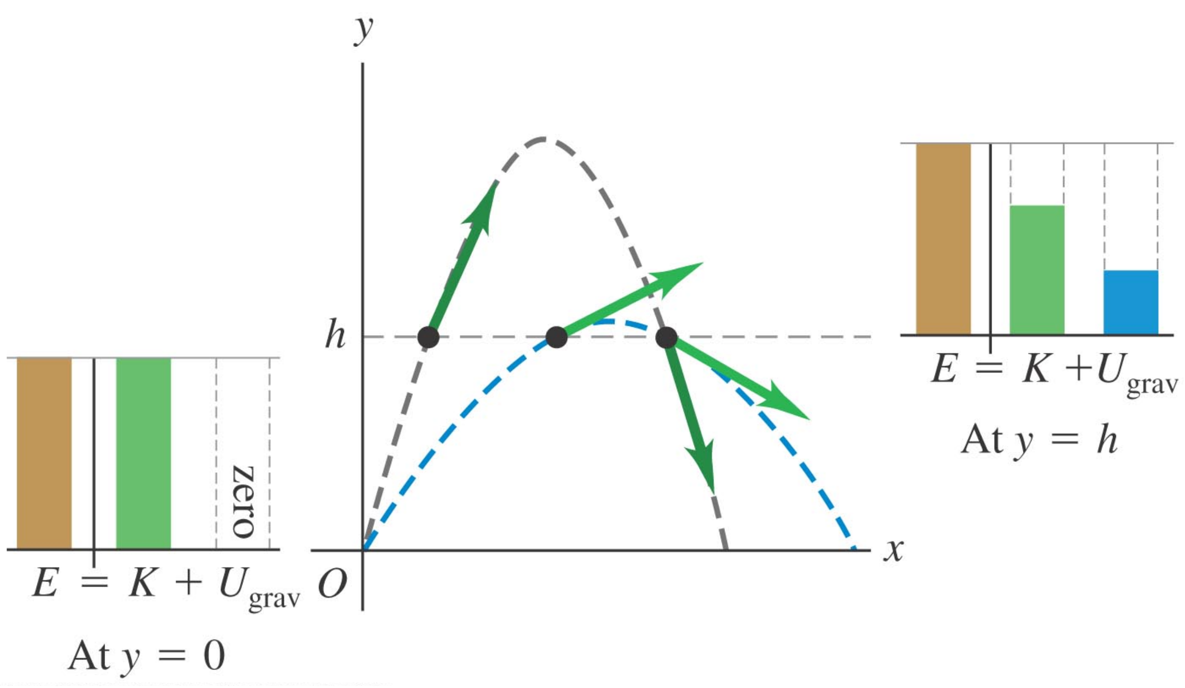
Work and energy along a curved path

- Since the gravitational acceleration **g** points down, the work it performs depends only on the vertical displacement $\Delta y = y_2 - y_1$
- We can use the same definition of potential energy $U_g = mgy$ even for a curved path.



Energy in projectile motion

- What happens if two identical balls leave from the same height with the same speed but at different angles?



Example #1

A baseball is thrown from the roof of a 22.0 m tall building with an initial velocity of magnitude 12.0 m/s and directed at an angle of 53.1° above the horizontal.

a. What is the speed of the ball just before it strikes the ground?

b. What is the answer to part (a) if the velocity is at an angle of 53.1° below the horizontal?

c. If the effects of air resistance are included, will part (a) or (b) give the higher speed?

Solution #1

a. From conservation of energy,

$K_2 = ½mv_2^2 = K_1 + U_1 – U_2 = ½mv_1^2 + mgh - 0$

$v_2 = [v_1^2 + 2gh]^{1/2}$ = [(12 m/s)^2 + 2(9.8 m/s^2)(22 m)]$^{1/2}$ = 24.0 m/s

b. The direction of the velocity doesn’t affect the kinetic energy, so the answer is still 24.0 m/s.

c. If air resistance is included, it will do more negative work on the longer upward path in part (a) implying that part (b) will give the higher final speed.

Example #2

A 10.0 kg microwave oven is pushed 6.00 m up the sloping surface of a loading ramp inclined at an angle of 36.9° above the horizontal by a constant force ***F*** with magnitude 110 N acting parallel to the ramp. The coefficient of kinetic friction between the oven and ramp is 0.250.

a. What is the work done on the oven by the force ***F***?

b. What is the work done by friction?

c. Compute the increase in potential energy for the oven?

d. Calculate the increase in kinetic energy using parts (a) – (c).

e. Use ***F*** = *m**a*** to calculate the oven's acceleration, then its change in speed. Compare to part (d).

Solution #2

a. $W_F = \boldsymbol{F} \cdot \boldsymbol{d}$ = (110 N)(6.00 m) = 660 J

b. If we choose the *x* axis up the ramp, in the *y* direction

$\Sigma F_y = n - mg \cos\theta = 0 \Rightarrow n = mg \cos\theta$

$W_f = \boldsymbol{f} \cdot \boldsymbol{d} = -\mu_k n d = -(0.25)(10 \text{ kg})(9.8 \text{ m/s}^2)(\cos 36.9°)(6 \text{ m}) = -118 \text{ J}$

c. $\Delta U_g = mg\Delta y = mgL \sin\theta = (10 \text{ kg})(9.8 \text{ m/s}^2)(6 \text{ m})(\sin 36.9°) = 353 \text{ J}$

d. $W_F + W_f = \Delta E = \Delta K + \Delta U_g \Rightarrow$

$$\Delta K = W_F + W_f - \Delta U_g = 660 \text{ J} - 118 \text{ J} - 353 \text{ J} = 189 \text{ J}$$

e. From Newton's 2nd law, $\Sigma F_x = F - f - mg \sin\theta = ma \Rightarrow$

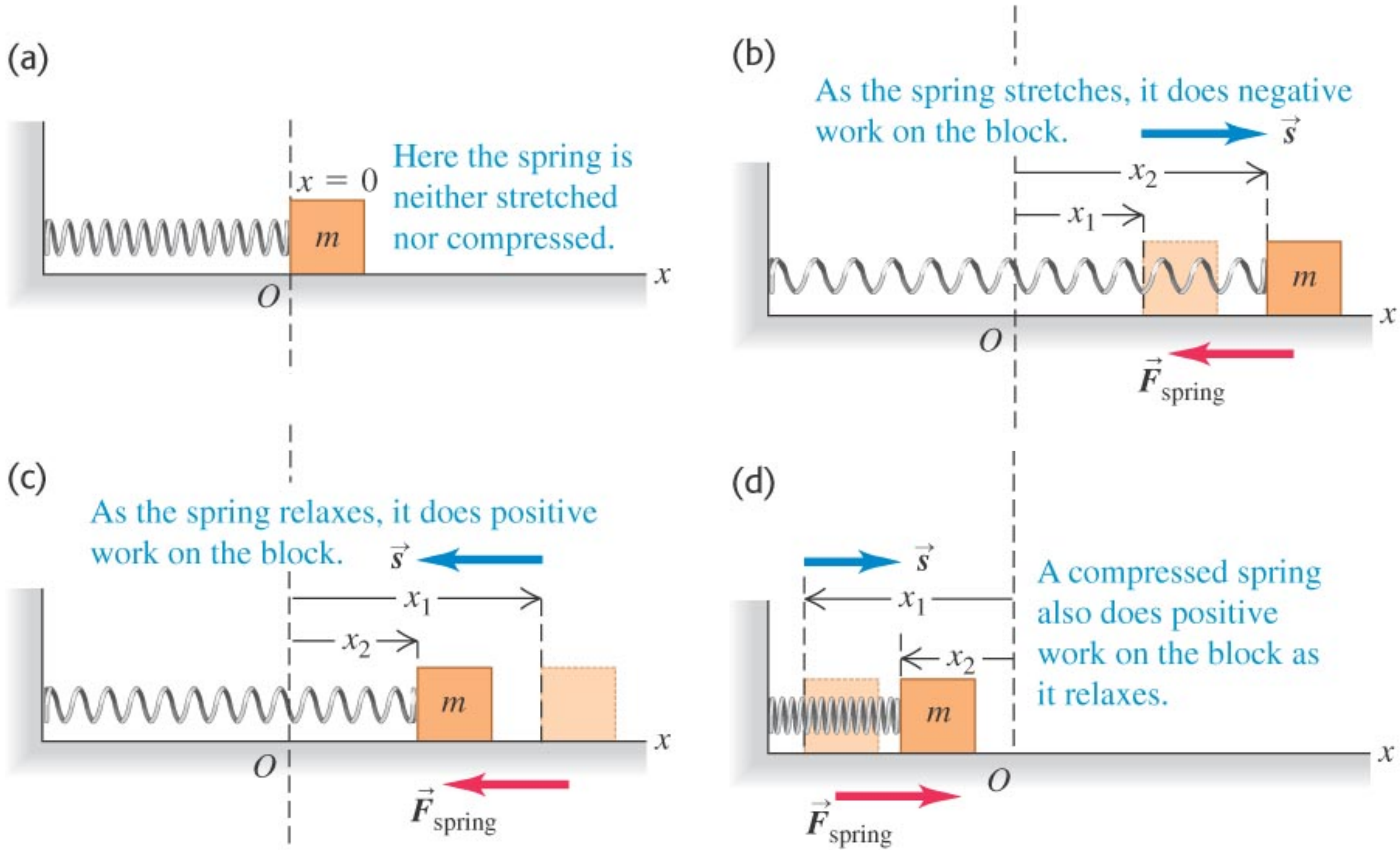
$a = F/m - \mu_k g \cos\theta - g \sin\theta$

$= (110 \text{ N})/(10 \text{ kg}) - (9.8 \text{ m/s}^2)[(0.25)(\cos 36.9°) + \sin 36.9°] = 3.16 \text{ m/s}^2$

$v_2^2 = v_1^2 + 2ad \Rightarrow \Delta K = \tfrac{1}{2}m(v_2^2 - v_1^2) = mad = (10 \text{ kg})(3.16 \text{ m/s}^2)(6 \text{ m}) = 189 \text{ J}$

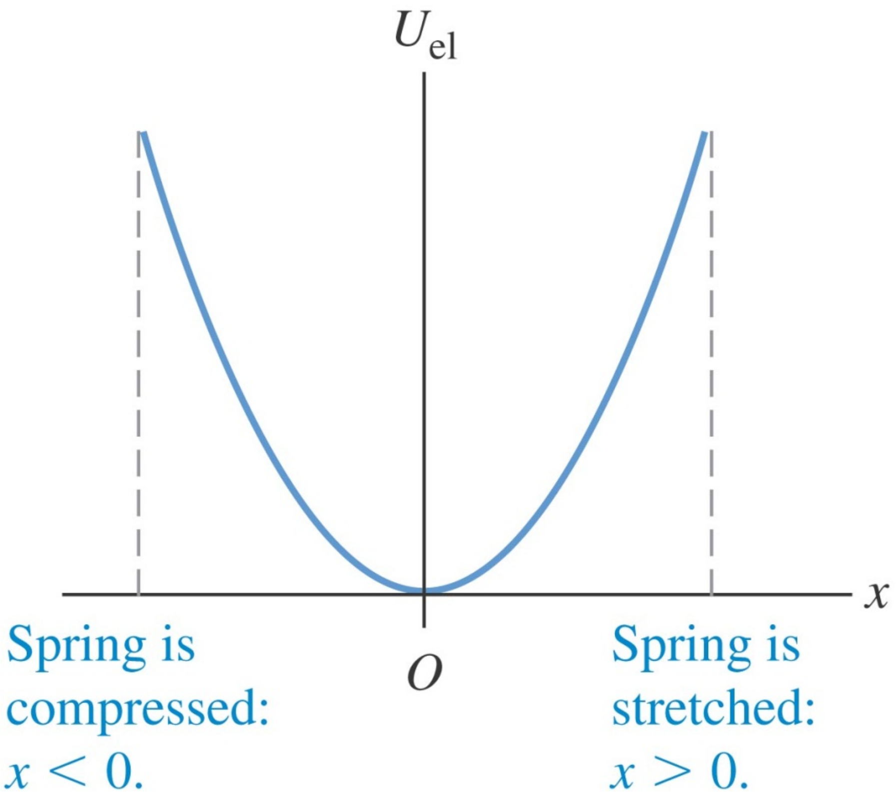
Work done by a spring

- Gravity is not the only force whose work can be represented as a potential energy.



Elastic potential energy

- A body is *elastic* if it returns to its original shape after being deformed.
- *Elastic potential energy* is the energy stored in an elastic body, such as a spring.
- The elastic potential energy stored in an ideal spring is $U_{\mathrm{el}} = \frac{1}{2}kx^2 = -W_{el}$.



Situations with both gravitational and elastic forces

- When a situation involves both gravitational and elastic forces, the total potential energy is the *sum* of the gravitational potential energy and the elastic potential energy: $U = U_g + U_{el}$ and the total mechanical energy becomes $E = K + U = K + U_g + U_{el}$.
- External forces (like friction) can still change the total energy:

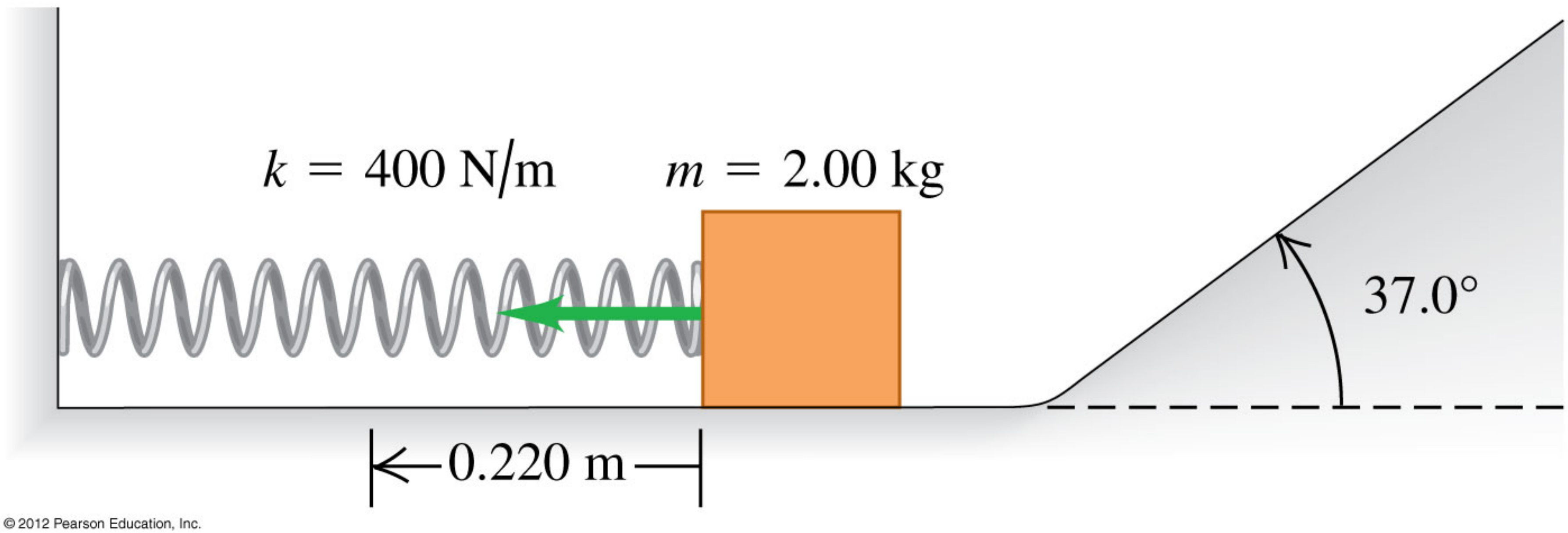
$$W_F = \Delta E = \Delta K + \Delta U_g + \Delta U_{el}$$

Example #3

A 2.00 kg block is pushed against a spring with negligible mass and force constant k = 400 N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up an incline with slope 37.0°.

a. What is the speed of the block after it leaves the spring?

b. How far does it go up the incline before it starts to slide down?



Solution #3

a. Since there is no friction, energy is conserved. It begins as elastic potential energy:

$U_{el} = \frac{1}{2}kx^2 = \frac{1}{2}(400\ \text{N/m})(0.22\ \text{m})^2 = 9.68\ \text{J}$

and is then converted to kinetic energy:

$K = U_{el} = \frac{1}{2}mv^2 \Longrightarrow v = (2K/m)^{1/2} = [2(9.68\ \text{J})/(2\ \text{kg})]^{1/2} = 3.11\ \text{m/s}$

b. As the block slides up the incline, kinetic energy is turned into gravitational potential energy:

$U_g = K = mgy = mgd \sin\theta \Longrightarrow$

$d = K/(mg \sin\theta) = (9.68\ \text{J})/[(2\ \text{kg})(9.8\ \text{m/s}^2)(\sin 37°)] = 0.821\ \text{m}$

Conservative and nonconservative forces

- Gravity and the spring force are examples of *conservative forces*.
- The work done between two points by any conservative force

  a) can be expressed in terms of a *potential energy function*.

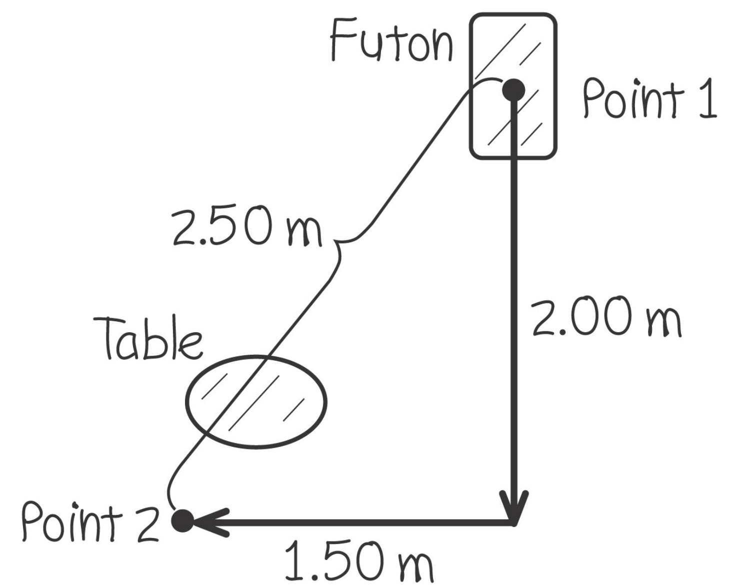
  b) is reversible.

  c) is independent of the path between the two points.

  d) is zero if the starting and ending points are the same.
- A force (such as friction) that is not conservative is called a *nonconservative force*, or a *dissipative force.*

Friction is nonconservative

- Friction violates all 4 of these properties:
  - There is no potential to store energy lost to friction.
  - Frictional losses don't reverse themselves (work done by friction is always negative).
  - Frictional work depends on the path taken, and doesn't vanish along a closed path.



Conservation of energy

- Nonconservative forces do not store potential energy, but they do change the *internal energy* U_{int} of a system.
- *The law of the conservation of energy* means that the total energy E_{tot} is never created or destroyed, it only changes form:

$$W_f = \Delta E = \Delta K + \Delta U = -\Delta U_{int} \Rightarrow \Delta E_{tot} = \Delta K + \Delta U + \Delta U_{int} = 0$$

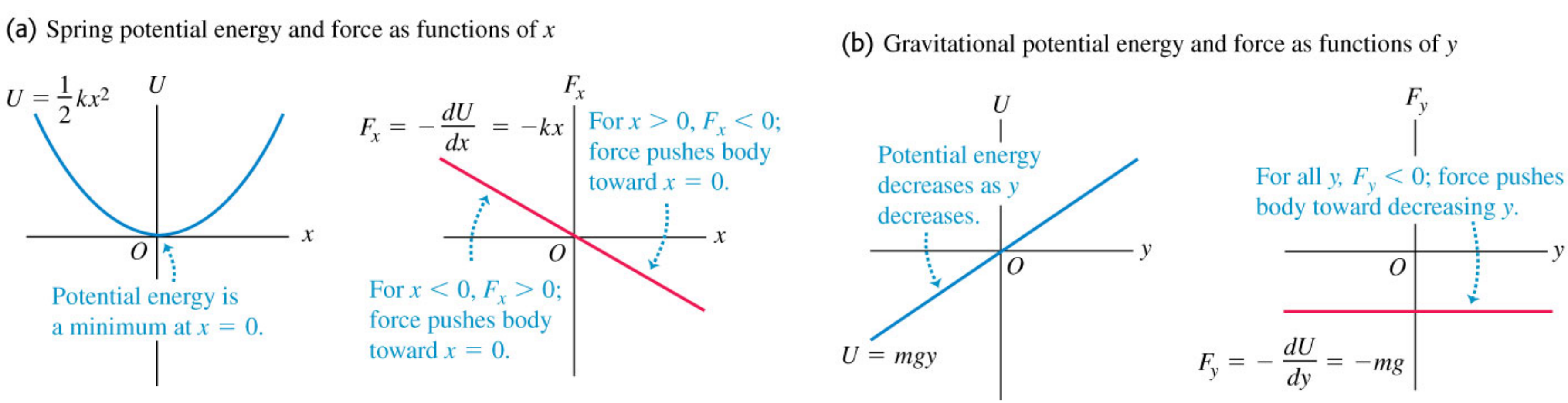
Force and potential energy in one dimension

- A conservative force can be obtained from its potential energy by inverting the definition of the potential:

$$U(x) = -\int^{x} F(x')\,dx' \Longleftrightarrow F(x) = -\frac{dU}{dx}$$

Examples include gravity: $U_g = mgy \Longrightarrow F_y = -dU/dy = -mg$

and springs: $U_{el} = ½kx^2 \Longrightarrow F_x = -kx$



Force and potential energy in higher dimensions

- In higher dimensions, the components of a conservative force are *partial derivatives* of the potential energy:

$F_x = -\partial U(x, y, z)/\partial x \quad F_y = -\partial U(x,y,z)/\partial y \quad F_z = -\partial U(x,y,z)/\partial z$

- The total force can be expressed as:

$$\mathbf{F} = -\frac{\partial U}{\partial x}\mathbf{i} - \frac{\partial U}{\partial y}\mathbf{j} - \frac{\partial U}{\partial z}\mathbf{k} = -\nabla U$$

- The potential can be derived from the force by a line integral:

$$U = -\int \mathbf{F} \cdot d\mathbf{x}$$

Conservative or nonconservative force?

A force $\boldsymbol{F} = -\alpha x^2 \boldsymbol{i}$ (α = 12 N/m^2) is exerted on a proton in an experiment.

a. How much work does $\boldsymbol{F}$ do when the proton moves along the straight-line path from (0.1 m, 0) to (0.1 m, 0.4 m)?

b. Along the straight-line path from (0.1 m, 0) to (0.3 m, 0)?

c. Along the straight-line path from (0.3 m, 0) to (0.1 m, 0)?

d. Is the force $\boldsymbol{F}$ conservative? Explain. If $\boldsymbol{F}$ is conservative, what is the associated potential-energy function? Let $U = 0$ when $x = 0$.

Solution

a. Since the displacement is in the *y* direction and the force is in the *x* direction, the work done is 0.

b. $$W = -\alpha \int_{x_1}^{x_2} x^2 \, dx = -\frac{\alpha}{3}(x_2^3 - x_1^3)$$
$$= -(4.0\mathrm{N/m^2})[(0.3\mathrm{m})^3 - (0.1\mathrm{m})^3] = -0.104\mathrm{J}$$

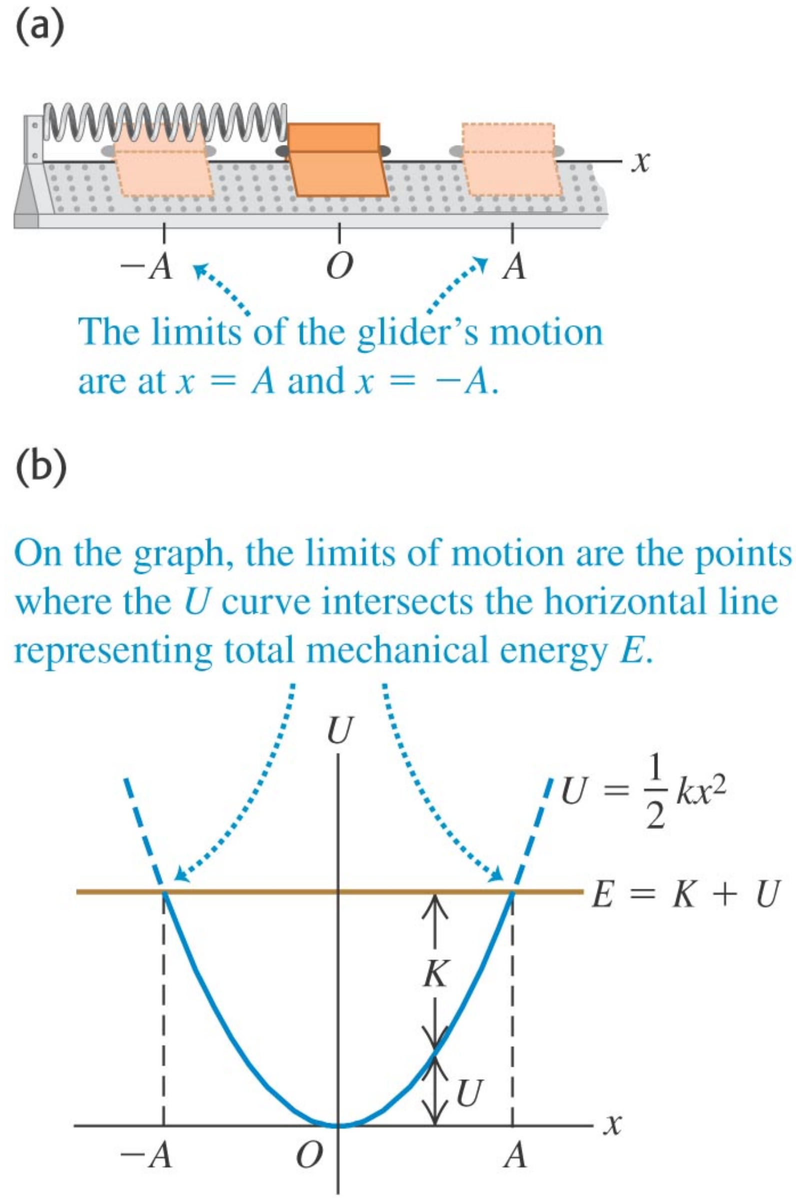
c. $$W = -\alpha \int_{x_1}^{x_2} x^2 \, dx = -\frac{\alpha}{3}(x_2^3 - x_1^3)$$
$$= -(4.0\mathrm{N/m^2})[(0.1\mathrm{m})^3 - (0.3\mathrm{m})^3] = 0.104\mathrm{J}$$

d. Since the work done is reversible, ***F*** is conservative. Since ***F*** is only in the *x* direction, *U* will only depend on *x*:

$$U(x) = -W = \tfrac{1}{3}\alpha x^3 \Longrightarrow \boldsymbol{F} = -(dU/dx)\boldsymbol{i} = -\alpha x^2\boldsymbol{i}$$

Energy diagrams

- An *energy diagram* is a graph that shows both the potential-energy function $U(x)$ and the total mechanical energy E.
- Here is the energy diagram for a glider attached to a spring on an air track.
- The energy determines the allowed regions and turning points of the motion.



Force and a graph of its potential-energy function