HW #7 - MATH 2418

1. A is in reduced row echelon form.

- $C(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

- $N(A) = \text{span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

- $R(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

- $N(A^T) = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

2 a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ the column space is the x-y plane in $\underline{\mathbb{R}^3}$.

b) $A\vec{x} = \vec{0} \Rightarrow \vec{x} = t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$.

the nullspace is the z-axis.

c) The row space is the x-y plane in $\mathbb{R}^3$.

3 a) $A \rightarrow \underset{m}{3} \times \underset{n}{3}$, nullspace is a line ($\dim = 1$).

<u>Answer:</u> No, $N(A) = 1 \Rightarrow \dim(C(A^T)) = \dim(C(A)) =$

$= 2 \Rightarrow$

$\Rightarrow C(A^T)$ and $C(A)$ must be planes through the origin.

b) • $N(A) = \{\vec{0}\} \Rightarrow \dim(N(A)) = 0$ //

• $\dim(N(A)) + \underset{\dim(C(A^T))}{r} = \underset{6}{n} \Rightarrow r = 6$ // (rank).

4 A, $\underset{\underset{m}{\uparrow}}{5} \times \underset{\underset{n}{\uparrow}}{7}$, $r = 4$.

a) $\dim(N(A)) = \underset{n}{7} - \underset{r}{4} = 3$

b) No. $C(A)$ is a subspace of $\mathbb{R}^5$ of dimension 4. Therefore there exist vectors $\vec{b} \in \mathbb{R}^5$ that are outside $C(A)$.

5 a) F. Ex. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

b) T. $A_{m \times n} \rightarrow$ If $m < n \Rightarrow$ columns in $\mathbb{R}^m$ must be lin. dependent.

If $m > n \Rightarrow$ rows in $\mathbb{R}^n$ must be lin. dependent.

Conclusion: $m = n$.

c) F. $\dim(N(A)) \leq n$. ($A_{m \times n}$)

d) F. For example, if the column has all zeros, adding it to a matrix does not change its rank.

e). T. $r \leq n-1 \Rightarrow N(A) = n - r \geq 1.$

f) F. $r < n \Rightarrow N(A) > 0.$

g) F. dim (row space) = dim (col. space).

h) F. rank (A^T) = rank(A) for any matrix.

i) T. $\dim(N(A)) + \dim(C(A^T)) = 3.$

j) F. Example: $n = 3.$

$$V = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \rightarrow V^{\perp} = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$W = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \rightarrow W^{\perp} = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$\underline{W^{\perp}}$ (the y-z plane) is <u>NOT</u> a subspace of $\underline{V^{\perp}}$.

(z-axis)