MATH 2418: Linear Algebra

Assignment 10 (sections 5.2 and 6.1)

Due: November 9, 2016 Term: Fall, 2016

Suggested problems(do not turn in): Section 5.2: 1, 2, 4, 5, 6, 7, 9, 12, 13, 16, 19, 20, 23, 24; Section 6.1: 1, 2, 3, 4, 5, 8, 9, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 30. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Compute the determinants of the matrices A, B and AB.

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & 0 \\ 0 & -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

2. [10 points] Compute the determinant of the matrix

$$B = \begin{bmatrix} 3 & 0 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ 7 & 0 & 2 & 0 \\ 6 & 3 & 0 & 0 \end{bmatrix}$$

3. [10 points] Find the determinant of the matrices P, P^2 and P^3.

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

4. [10 points] Compute the eigenvalues and the eigenvectors of matrices A and A^2:

$$A = \begin{bmatrix} 4 & 0 & -1 \\ 5 & -3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

5. [10 points] Find the maximal eigenvalue of the matrix A^{100} where

$$A = \begin{bmatrix} 1/6 & 0 & 1/3 \\ 1/3 & 1/2 & 1/3 \\ 1/2 & 1/2 & 1/3 \end{bmatrix}$$

6. [10 points] True or False

(a) (1 point) Determinant of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ equals 2.

(b) (1 point) Determinant of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ equals 0.

(c) (1 point) For any 2×2 matrices A, B, C and D one has

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det A \det D - \det B \det C$$

(d) (1 point) If matrix A has one of its diagonal elements equal to zero, then $\det(A) = 0$.

(e) (1 point) Absolute value of the determinant of the orthogonal matrix equals to 1.

(f) (1 point) If matrix A has eigenvalues 1 and 2 and matrix B has eigenvalues -1 and 1 then $A + B$ has eigenvalues 0 and 3.

(g) (1 point) Every projection matrix has 0 as an eigenvalue.

(h) (1 point) If determinant of a matrix A equals zero, then one of the eigenvalues of A is zero.

(i) (1 point) All the eigenvalues of singular matrix equal zero.

(j) (1 point) Matrix $\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}$ has no real eigenvalues.