Key

MATH 2418: Linear Algebra

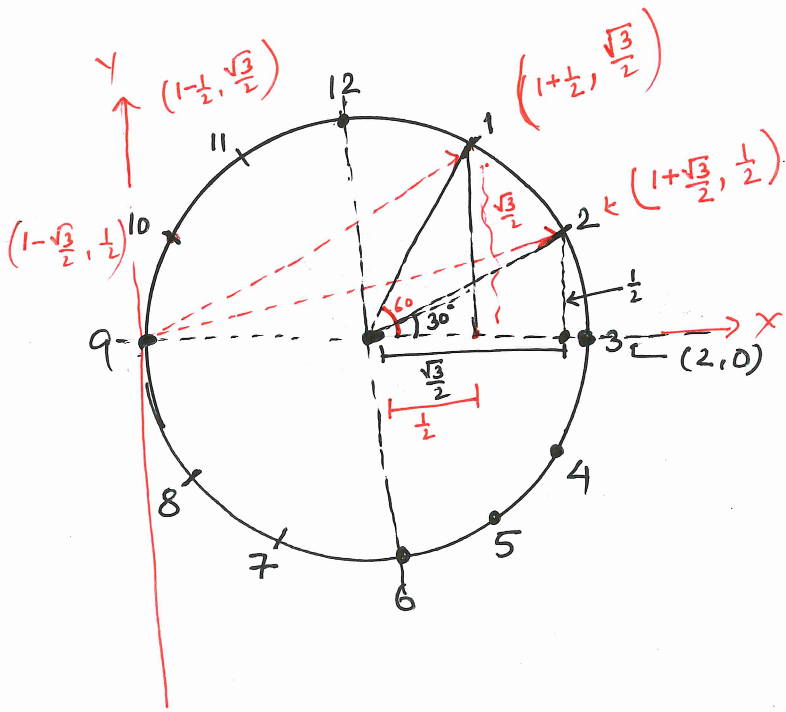
Assignment# 1

Due : 08/31 Wednesday Term Fall, 2016

Recommended Text Book Problems (do not turn in): [Sec 1.1: # 1, 2, 3, 5, 11, 19]; [Sec 1.2: # 1, 2, 6, 7, 13, 14, 29, 31];

1. Write down the 12 vectors that go from hour 9:00 of a clock to hours 1:00, 2:00, ..., 12:00.(For example: Hour 9:00 is $(0,0)$, hour 3:00 is $(2,0)$). Also, find the sum of these 12 vectors.

Hour	vector
9:00	$(0,0)$
3:00	$(2,0)$
2:00	$\left(1+\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
1:00	$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$
12:00	$(1, 1)$
11:00	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
10:00	$\left(1-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
9:00	$(0,0)$
8:00	$\left(1-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
7:00	$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
6:00	$(1,-1)$
5:00	$\left(\frac{3}{2}, -\frac{\sqrt{3}}{2}\right)$
4:00	$\left(1+\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$



Sum of 12 vectors

$= (12, 0)$.

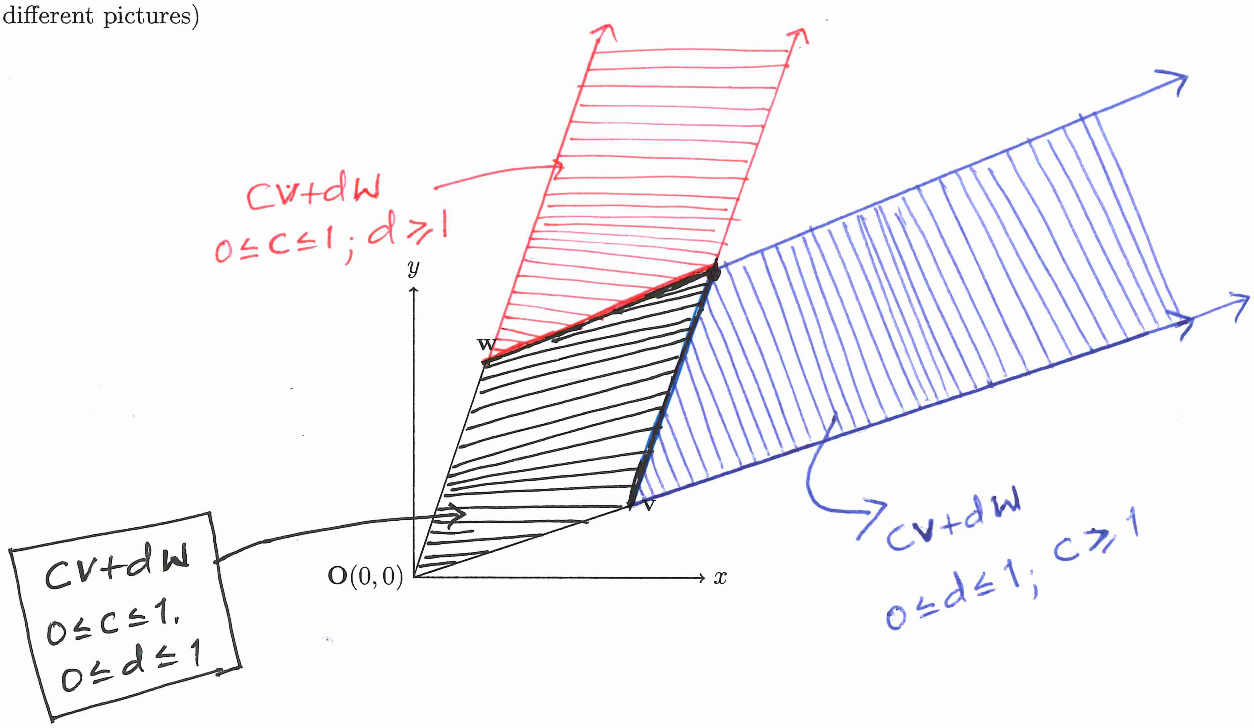
2. Given vectors $\mathbf{v}$ and $\mathbf{w}$ in diagram below, shade in all linear combinations $c\mathbf{v} + d\mathbf{w}$ for

(a) $0 \leq c \leq 1$ and $0 \leq d \leq 1$

(b) $0 \leq c \leq 1$ and $d \geq 1$

(c) $0 \leq d \leq 1$ and $c \geq 1$.

(You can use different shading styles in same picture for all three parts or can graph them separaetly in three different pictures)



3. Given $\mathbf{u} = (3, 1, 4)$ and $\mathbf{v} = (2, 2, -4)$,

(a) Calculate $\mathbf{u} \cdot \mathbf{v}$, $\|\mathbf{u}\|$, $\|\mathbf{v}\|$

(b) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$

(c) Find the unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ in the direction of $\mathbf{u}$ and $\mathbf{v}$ respectively.

(a) $u \cdot v = (3, 1, 4) \cdot (2, 2, -4) = 3 \cdot 2 + 1 \cdot 2 + 4 \cdot (-4) = 6 + 2 - 16 = -8$

(b) $\|u\| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{26}$

$\|v\| = \sqrt{2^2 + 2^2 + (-4)^2} = \sqrt{24}$

$$\cos\theta = \frac{u \cdot v}{\|u\| \, \|v\|} = \frac{-8}{\sqrt{26}\sqrt{24}} = \frac{-8}{\sqrt{26 \cdot 24}} = \frac{-8}{\sqrt{13 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}}$$

$$= -\frac{2}{\sqrt{39}}$$

$$\theta = \cos^{-1}\left(-\frac{2}{\sqrt{39}}\right) = 108.68^\circ$$

(c) $$\hat{u} = \frac{u}{\|u\|} = \frac{1}{\sqrt{26}}(3, 1, 4) = \left(\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$$

$$\hat{v} = \frac{v}{\|v\|} = \frac{1}{\sqrt{24}}(2, 2, -4) = \left(\frac{2}{\sqrt{24}}, \frac{2}{\sqrt{24}}, \frac{-4}{\sqrt{24}}\right)$$

$$= \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

4. (a) Use the triangle inequality: $\|\mathbf{u}+\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ to prove that

   (i) $\|\mathbf{u}-\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

   (ii) $\|\mathbf{u}\| - \|\mathbf{v}\| \leq \|\mathbf{u}-\mathbf{v}\|$

   (b) If $\|\mathbf{u}\| = 5$ and $\|\mathbf{v}\| = 3$, what are the smallest and largest possible values of $\|\mathbf{u}-\mathbf{v}\|$ and $\|\mathbf{v}-\mathbf{u}\|$?

a (i) $\|u-v\| = \|u+(-v)\| \leq \|u\| + \|-v\| = \|u\| + \|v\|$.

(ii) $\|u\| = \|(u-v)+v\| \leq \|u-v\| + \|v\|$

$\Rightarrow \|u\| - \|v\| \leq \|u-v\|$.

b By a (i) $\|u-v\| \leq \|u\| + \|v\|$.

$\Rightarrow \|u-v\| \leq 5+3 = 8$

By a (ii) $\|u\| - \|v\| \leq \|u-v\|$

$5-3 \leq \|u-v\|$.

$2 \leq \|u-v\|$.

Smallest possible value of $\|u-v\| = 2$

largest possible value of $\|u-v\| = 8$.

But $\|u-v\| = \|v-u\|$, so

Smallest possible value of $\|v-u\| = 2$

largest possible value of $\|v-u\|$ 8.

5. Given any two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, determine the scalar c so that $\mathbf{u} - c\mathbf{v}$ is perpendicular to $\mathbf{v}$. Let $\mathbf{v} = (3, 1, -2)$, find a non zero vector that is perpendicular to $\mathbf{v}$.

Sol:- $(u - cv)$ is perp. to $v \Leftrightarrow (u - cv) \cdot v = 0$

$$\Leftrightarrow u \cdot v - c\, v \cdot v = 0$$

$$\Leftrightarrow u \cdot v - c \|v\|^2 = 0$$

$$\Leftrightarrow u \cdot v = c \|v\|^2$$

$$\Leftrightarrow \frac{u \cdot v}{\|v\|^2} = c.$$

Let $u = (1, 1, 1)$ $\rightarrow$ choose any random non-zero vector, which is not a multiple of v.

Then $$c = \frac{u \cdot v}{\|v\|^2} = \frac{(1, 1, 1) \cdot (3, 1, -2)}{3^2 + 1^2 + (-2)^2} = \frac{3 + 1 - 2}{14} = \frac{2}{14} = \frac{1}{7}$$

So, vector perp. to v: $u - cv$

$$= (1, 1, 1) - \frac{1}{7}(3, 1, -2)$$

$$= (1, 1, 1) - \left(\frac{3}{7}, \frac{1}{7}, \frac{-2}{7}\right)$$

$$= \left(\frac{4}{7}, \frac{6}{7}, \frac{9}{7}\right)$$