MATH 2418: Linear Algebra

Assignment 5: Part 1 (sections 2.6 and 2.7)

Due: October 05, 2016 Term: Fall, 2016

Suggested problems(do not turn in): Section 2.6: 1, 2, 3, 4, 5, 6, 7, 8, 15, 16, 17; Section 2.7: 1, 2, 3, 4, 5, 6, 7, 8, 9 , 10 11, 20, 21, 22, 23. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 2 & -10 & 2 \end{bmatrix}.$$

   (a) (4 points) Compute a factorization $A = LU$.

   (b) (2 points) Given $\mathbf{b} = (1, 0, 1)$ solve $L\mathbf{c} = \mathbf{b}$ by forward substitution.

   (c) (2 points) Solve $U\mathbf{y} = \mathbf{c}$ by backwards substitution.

   (d) (2 points) What is the relation between the solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{y}$ computed above? Explain your answer.

2. [10 points] Find a LDU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}.$$

3. [10 points] Show that

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

has no LU decomposition.

4. [10 points] Consider the symmetric matrix

$$S = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 8 \\ 2 & 8 & 23 \end{bmatrix}.$$

Find a symmetric factorization of S in the form $S = LDL^T$ where L is lower diagonal and D is the diagonal matrix of pivots.

5. [10 points] Find the $PA = LU$ factorization of A and use it to solve $A\mathbf{x} = \mathbf{b}$ by solving $U\mathbf{x} = \mathbf{c}$ and $L\mathbf{c} = P\mathbf{b}$.

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}.$$