MATH 2418: Linear Algebra

Assignment# 4

Due : 09/21 Wednesday   Term Fall, 2016

Key

[First Name]   [Last Name]   [Net ID]

Recommended Text Book Problems (do not turn in): [Sec 2.4: # 3, 4, 6, 7, 13, 14, 15, 17, 26, 32, 36]; [Sec 2.5: # 1, 5, 6, 7, 10, 11, 12, 13, 18, 22, 25, 27, 29, 44];

1. Let $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 5 & 7 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 6 & -3 \\ 1 & 0 & 1 \end{bmatrix}$, compute the followings but nothing more:

   (a) The row 2 of AB.   (b) The column 3 of AB (c) The entries $(AB)_{12}$ and $(B^2)_{12}$.

Sol:

(a) Row 2 of AB = [row 2 of A] B $= \begin{bmatrix} -2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 4 & 6 & -3 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 22 & -4 \end{bmatrix}$

(b) Col. 3 of AB = A [Col 3 of B] $= \begin{bmatrix} 3 & -2 & 4 \\ -2 & 5 & 7 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

(c) $(AB)_{12}$ = (row 1 of A)$\cdot$(col. 2 of B) $= (3 \ \ -2 \ \ 4) \cdot (4 \ \ 6 \ \ 0) = 0$

$(B^2)_{12}$ = (row 1 of B)$\cdot$(col. 2 of B) $= (2 \ \ 4 \ \ -2) \cdot (4 \ \ 6 \ \ 0) = 32$

2. Compute the following products:

(a) $\begin{bmatrix} 2 & 4 & 1 \\ 0 & -2 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 14 & 15 \\ 0 & -6 & 11 \\ 0 & 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 3 & 6 \end{bmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 1 & 4 & 0 \\ 2 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 22 & 16 & 0 \\ 42 & 18 & 30 \end{bmatrix}$

(c) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 3 & 5 & -7 \\ 2 & 4 & 1 \\ -9 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 3a & 5a & -7a \\ 2b & 4b & b \\ -9c & 2c & 6c \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 5 & -7 \\ 2 & 4 & 1 \\ -9 & 2 & 6 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} 3a & 5b & -7c \\ 2a & 4b & c \\ -9a & 2b & 6c \end{bmatrix}$

3. (a) Let $A = \begin{bmatrix} x & c \\ 2 & (x+4) \end{bmatrix}$ find all $c \in \mathbb{R}$(if exist) so that matrix A is invertible for every $x \in \mathbb{R}$.

Sol:- For A to be invertible $x(x+4) - 2c \neq 0$

$\Leftrightarrow x^2+4x-2c \neq 0$

For $x^2+4x-2c=0$

$x = \dfrac{-4 \pm \sqrt{4^2-4.1.(-2c)}}{2 \cdot 1} = \dfrac{-4 \pm \sqrt{16+8c}}{2}$

For $x^2+4x-2c \neq 0$ $\quad 16+8c < 0 \Rightarrow 8c < -16$

$\Rightarrow \boxed{c < -2}$

(b) Let $B = \begin{bmatrix} x+4 & 0 & 0 \\ 0 & x^2+2x+7 & 0 \\ 0 & 0 & x^2+x-20 \end{bmatrix}$ find all $x \in \mathbb{R}$ so that B is non-singular.

$x+4=0 \Rightarrow x=-4$

$x^2+2x+7=0 \Rightarrow x = \dfrac{-2 \pm \sqrt{2^2-4.1.7}}{2 \cdot 1} = \dfrac{-2 \pm \sqrt{-24}}{2} \notin \mathbb{R}$

$x^2+x-20 \Rightarrow (x+5)(x-4)=0 \Rightarrow x=-5,\ x=4.$

So B will be non-singular if $\boxed{x \neq -4,\ x \neq -5,\ x \neq 4}$

(c) Is the matrix $D = \begin{bmatrix} 2 & 3 \\ -2 & 7 \end{bmatrix}$ invertible? If yes, find D^{-1}.

Sol:. $\mathrm{Det}(D) = 2 \cdot 7 - (-2) \cdot 3 = 20 \neq 0$. yes D is invertible.

$D^{-1} = \dfrac{1}{\mathrm{Det}(D)} \begin{bmatrix} 7 & -3 \\ 2 & 2 \end{bmatrix} = \dfrac{1}{20} \begin{bmatrix} 7 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{7}{20} & -\frac{3}{20} \\ \frac{2}{20} & \frac{2}{20} \end{bmatrix}$

4. Use the Gauss-Jordan method to find the inverse of $A = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

Sol:. $[A \quad I]$

$$\sim \left[\begin{array}{ccc|ccc} 4 & 3 & 2 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\frac{1}{4}R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 3R_1$

$R_3 - 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{5}{4} & \frac{1}{2} & -\frac{3}{4} & 1 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

$-\frac{4}{5}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{4}{5} & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

$R_3 - \frac{1}{2}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{4}{5} & 0 \\ 0 & 0 & -\frac{4}{5} & -\frac{4}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$-\frac{5}{4}R_3$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & \frac{3}{5} & -\frac{4}{5} & 0 \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & -\frac{5}{4} \end{array}\right]$$

$R_2 + \frac{2}{5}R_3$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & -\frac{5}{4} \end{array}\right]$$

$R_1 - \frac{1}{2}R_3$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{5}{8} \\ 0 & 1 & 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & -\frac{5}{4} \end{array}\right]$$

$R_1 - \frac{3}{4}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & -\frac{5}{4} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{5}{4} \end{bmatrix}$$

5. (a) Suppose $P^{-1} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $Q^{-1} = \begin{bmatrix} 0 & 3 \\ -4 & 2 \end{bmatrix}$, find the inverse of (PQ).

Sol:. $(PQ)^{-1} = Q^{-1}P^{-1} = \begin{bmatrix} 0 & 3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ -8 & -6 \end{bmatrix}$

(b) If $A^9 = I$, the identity matrix, what are the inverses of A, A^2, A^3, A^4, A^{20}?

$A^9 = I \Rightarrow AA^8 = I \Rightarrow A^{-1} = A^8$

$A^2 A^7 = I \Rightarrow (A^2)^{-1} = A^7$

$A^3 \cdot A^6 = I \Rightarrow (A^3)^{-1} = A^6$

$A^5 \cdot A^4 = I \Rightarrow (A^4)^{-1} = A^5$

$A^9 = I.$

$\Rightarrow (A^9)^3 = I \Rightarrow A^{27} = I$

$A^7 \cdot A^{20} = I \Rightarrow (A^{20})^{-1} = A^7.$

(c) Use the Gauss-Jordan method to find the inverse of the upper triangular matrix $U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$.

Sol:- $[U \; I]$

$$= \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 6R_3$
$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -6 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 - 3R_3$
$$\begin{bmatrix} 1 & 2 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -6 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 - 2R_2$
$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & 9 \\ 0 & 1 & 0 & 0 & 1 & -6 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

So $U^{-1} = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

6. True or False. Circle your answer.

(a) **T** **F**: The inverse of an upper triangular matrix is a lower triangular matrix.

(b) **T** **F**: Let A and B be square matrices of same size such that $AB = I$ then $A^{-1} = B$ and $B^{-1} = A$.

(c) **T** **F**: For any square matrices P and Q of same size $(P - Q)^2 = P^2 - 2PQ + Q^2$.

(d) **T** **F**: If A and B are invertible matrices of same size, then $A + B$, $A - B$, BA are all invertible.

(e) **T** **F**: If A^2 is not invertible, then A is not invertible.