MATH 2418: Linear Algebra

Assignment# 1

Due : <u>08/31 Wednesday</u>   Term <u>Fall, 2016</u>

Recommended Text Book Problems (do not turn in): [Sec 1.1: # 1, 2, 3, 5, 11, 19]; [Sec 1.2: # 1, 2, 6, 7, 13, 14, 29, 31];

1. Write down the 12 vectors that go from hour 9:00 of a clock to hours 1:00, 2:00, ..., 12:00.(For example: Hour 9:00 is $(0, 0)$, hour 3:00 is $(2, 0)$). Also, find the sum of these 12 vectors.
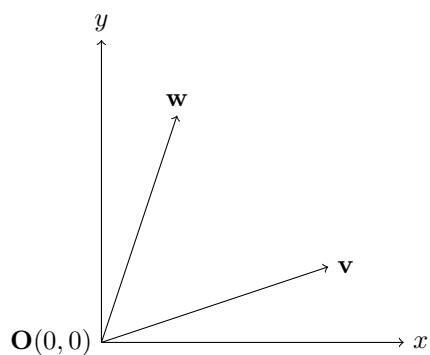
2. Given vectors $\mathbf{v}$ and $\mathbf{w}$ in diagram below, shade in all linear combinations $c\mathbf{v} + d\mathbf{w}$ for

   (a) $0 \le c \le 1$ and $0 \le d \le 1$
   (b) $0 \le c \le 1$ and $d \ge 1$
   (c) $0 \le d \le 1$ and $c \ge 1$.

   (You can use different shading styles in same picture for all three parts or can graph them separaetly in three different pictures)

3. Given $\mathbf{u} = (3, 1, 4)$ and $\mathbf{v} = (2, 2, -4)$,

    (a) Calculate $\mathbf{u} \cdot \mathbf{v}$, $\|\mathbf{u}\|$, $\|\mathbf{v}\|$

    (b) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$

    (c) Find the unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ in the direction of $\mathbf{u}$ and $\mathbf{v}$ respectively.

4. (a) Use the triangle inequality: $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ to prove that

    (i) $\|\mathbf{u} - \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

    (ii) $\|\mathbf{u}\| - \|\mathbf{v}\| \leq \|\mathbf{u} - \mathbf{v}\|$

(b) If $\|\mathbf{u}\| = 5$ and $\|\mathbf{v}\| = 3$, what are the smallest and largest possible values of $\|\mathbf{u} - \mathbf{v}\|$ and $\|\mathbf{v} - \mathbf{u}\|$?

5. Given any two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, determine the scalar c so that $\mathbf{u} - c\mathbf{v}$ is perpendicular to $\mathbf{v}$. Let $\mathbf{v} = (3, 1, -2)$, find a non zero vector that is perpendicular to $\mathbf{v}$.