

MATH 2418.

HW #2. Answers to the Recommended Questions:-

Ex # 1.3.

$$\textcircled{3} \begin{cases} y_1 = c_1 \\ y_2 = c_2 - c_1 \\ y_3 = c_3 - c_2 \end{cases} \text{ Solution.}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} c \leftarrow \text{solution in the form of } y = S^{-1}c$$

columns are independent.

$$\textcircled{5} (1, -2, 1) (2, -4, 2)$$

$$\textcircled{6} C = 3, C = -1, C = 0$$

$$\textcircled{8} \begin{aligned} x_1 &= b_1 & x_2 &= b_1 + b_2 \\ x_3 &= b_1 + b_2 + b_3 \\ x_4 &= b_1 + b_2 + b_3 + b_4 \end{aligned}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = A^{-1}b$$

$$\textcircled{9} C_4 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

sol. of $Cx = 0$ are $x = \begin{bmatrix} t \\ t \\ t \\ t \end{bmatrix}, t \in \mathbb{R}$

Ex # 2.1

$$\textcircled{29} u_2 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \quad u_3 = \begin{bmatrix} 0.65 \\ 0.35 \end{bmatrix}$$

sum of components = 1.

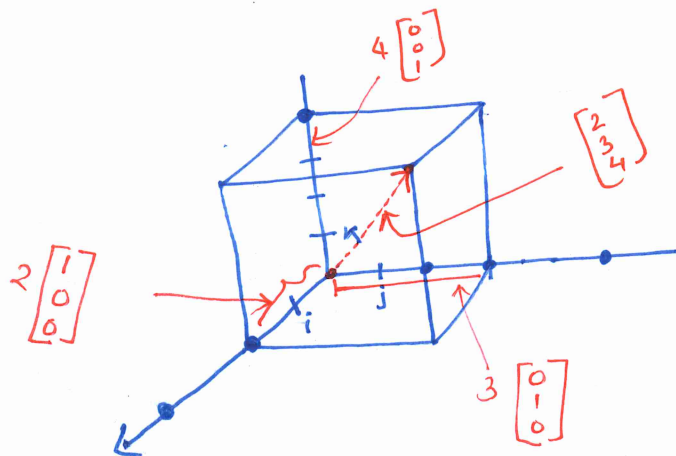
$$\textcircled{31} M_3 = \begin{bmatrix} 8 & 3 & 4 \\ 1 & 5 & 9 \\ 6 & 7 & 2 \end{bmatrix}$$

$$M_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix}$$

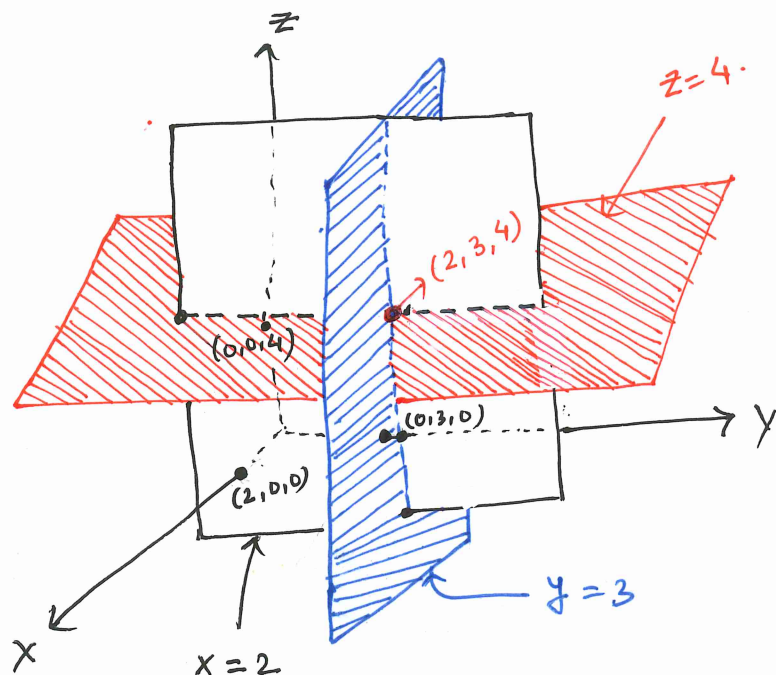
$$M_4 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 34 \\ 34 \\ 34 \\ 34 \end{bmatrix}$$

Ex# 2.1

$$\textcircled{1} \quad 2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$



Column picture



Row picture.

② Row picture doesnot change because, the equations are basically same.

Column picture will be dilated by factors 2, 3, 4 in x, y and z directions respectively. The columns will change but scalar coefficients remain same.

③ planes, vectors in column picture, coefficient matrix change. solution doesnot change.

$$\textcircled{9} \quad \textcircled{a} \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix} \quad \textcircled{b} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

$$\textcircled{10} \quad \textcircled{a} \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$$

$$\textcircled{b} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

$$\textcircled{16} \quad \textcircled{a} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \textcircled{b} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\textcircled{17} \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\textcircled{19} \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$E \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix} \quad E^{-1} E \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$