MATH 2418: Linear Algebra

Assignment 9 (sections 4.4 and 5.1)

Due: November 2, 2016 Term: Fall, 2016

Suggested problems(do not turn in): Section 4.4: 1, 3, 4, 5, 6, 9, 13, 15, 18, 19, 22, 24; Section 5.1: 1, 2, 3, 4, 5, 7, 8, 9, 12, 13, 15, 16, 17, 18, 21, 22, 23, 28. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Let $\mathbf{e}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 4/5 \\ 0 \\ 3/5 \end{bmatrix}$.

   (a) (5 points) Find vector $\mathbf{e}_3$ such that $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ form an orthonormal basis.

   (b) (1 point) How many different choices for $\mathbf{e}_3$ there is?

   (c) (4 points) Express vector $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.

2. [10 points] Find orthogonal vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ by Gram-Schmidt from the vectors $\mathbf{a} = (1, 1, 1, 0)$, $\mathbf{b} = (1, 1, 0, 1)$ and $\mathbf{c} = (1, 0, 1, 1)$.

3. [10 points]

(a) (5 points) Find an orthonormal basis in the subspace $\mathbf{S} \subset \mathbb{R}^4$ spanned by all solutions of $x_1 + 2x_2 + 3x_3 - 6x_4 = 0$.

(b) (4 points) Express vector $\mathbf{b} = (1, 1, 1, 1)$ in this basis.

(c) (1 point) Find a basis for $\mathbf{S}^{\perp}$.

4. [10 points] Let 3×3 matrices A and B have determinants -1 and 2 correspondingly.

   (a) (3 points) Find determinant of $(2A)B^{-1}$.

   (b) (3 points) Find determinant of B^2A^{-1}.

   (c) (3 points) Find a scalar $c \in \mathbb{R}$ such that $\det[(cA)B] = 1$.

   (d) (1 point) Find determinant of A^{2016}.

5. [10 points] Let $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix}$ be matrices of rotation around the z and x axis in $\mathbb{R}^3$ correspondingly. Find the determinant of the matrix AB.

6. [10 points] True or False

(a) (1 point) If three vectors in $\mathbb{R}^3$ are orthonormal then they form a basis.

(b) (1 point) If columns of matrix A are orthogonal then rows of A are independent.

(c) (1 point) If B is square orthogonal matrix then $B^{-1} = B^T$.

(d) (1 point) If Q is square orthogonal matrix such that $Q^{2016} = I$ then $Q^T = Q^{2015}$.

(e) (1 point) If for some basis $\{\mathbf{a}_1, \mathbf{a}_2\}$ for vector $\mathbf{b}$ one has $\mathbf{b} = x\mathbf{a}_1 + y\mathbf{a}_2$ then $\|\mathbf{b}\| = \sqrt{x^2 + y^2}$.

(f) (1 point) If $\det A = \det A^{-1}$ then $A = I$.

(g) (1 point) There are no matrix A such that $\det A^{-1} = 0$.

(h) (1 point) Row operations do not change the determinant of a matrix.

(i) (1 point) If $\det(2A) = \det(3A)$ then A is not invertible.

(j) (1 point) If $B = \begin{bmatrix} -1 & 4 & -6 \\ 0 & 2 & 5 \\ 0 & 0 & -3 \end{bmatrix}$ then $\det(-B^{-1}) = -6$.