MATH 2418: Linear Algebra

Assignment 7 (sections 4.4 and 5.1)

Due: `October 19, 2016` Term: `Fall, 2016`

Suggested problems(do not turn in): Section 3.5: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17; Section 4.1: 3, 8, 11, 12, 17, 18, 19, 24, 27, 28, 29. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Find bases for the four fundamental subspaces $(C(A^T), N(A), C(A), N(A^T))$ of the matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. [10 points] (a) (4 points) Give an example of a 3×3 matrix whose column space is a plane through the origin in R^3.

   (b) (3 points) What kind of geometric object is the nullspace of your matrix? Explain your answer.

   (c) (3 points) What kind of geometric object is the row space of your matrix? Explain your answer.

3. [10 points] (a) (5 points) Suppose that A is a 3×3 matrix whose nullspace is a line through the origin in R^3. Can the row and column space of A also be a line through the origin? Explain.

   (b) (5 points) Let A be a 7×6 matrix such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Find the rank and the dimension of the nullspace of A.

4. [10 points] Let A be 5×7 matrix with rank 4.

   (a) (5 points) What is the dimension of the solution space of $A\mathbf{x} = 0$?

   (b) (5 points) Is there a solution for $A\mathbf{x} = \mathbf{b}$ for all vectors $\mathbf{b}$ in R^5? Explain your answer.

5. True or False? (1 point each)

   (a) Either the row vectors or the column vectors of a square matrix are linearly independent.

   (b) A matrix with linearly independent row vectors and linearly independent column vectors is square.

   (c) The dimension of the nullspace of a nonzero $m \times n$ matrix is at most m.

   (d) Adding one additional column to a matrix increases its rank by one.

   (e) The dimension of the nullspace of a square matrix with linearly dependent rows is at least one.

   (f) If A is square and $A\mathbf{x} = \mathbf{b}$ does not have a solution for some vector $\mathbf{b}$, then the dimension of the nullspace of A is zero.

   (g) If a matrix A has more rows than columns, then the dimension of the row space is greater than the dimension of the column space.

   (h) If rank(A^T)=rank(A), then A is square.

   (i) There is no 3×3 matrix whose row space and nullspace are both lines in 3-space.

   (j) If V is a subspace of R^n and W a subspace of V, then $W^\perp$ is a subspace of $V^\perp$.