Assignment #3
Answers to Recommended problems.

Sec 2.2.

1. $\ell_{21} = 5$
   pivots: 2, −6
   solution: (2, −1)

2. (2, −1), (8, −4)

4. $\ell = \frac{c}{a}$

   2nd pivot: $\frac{ad-bc}{a}$

   $y = \frac{ag-cf}{ad-bc}$; $(ad-bc \neq 0)$

6. $b = 4$, $g = 32$

   (6, 1), (0, 4)

8. $k = 0, 3, -3$

   $k = 0 \rightarrow$ fixed by row exchange $\rightarrow$ 1 solution

   $k = 3 \rightarrow$ No solution

   $k = -3 \rightarrow \infty$ solutions.

12. $$\begin{bmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{bmatrix}$$

    $z = 1$, $y = 1$ $\quad x = 2$

14. $d = 10$

    $$\begin{bmatrix} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -\cancel{1} & 2 \end{bmatrix}$$

    Triangular system

14. contd.. $d = 11$ makes the System singular.

32. (a) 0 (b) 0 (c) $$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & & \vdots & \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{100 \times 100}$$

    or any 99 rows but 100th row is a linear combination of first 99 rows.

    (d) Row picture: 100 planes intersecting at a line through 0, the origin.

    column picture:- 100 vectors ~~hyperp~~ lying in a space of dimension ≤ 99.

Sec 2.3:-

(1) (a) $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix}$ (c) $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(3) $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 12 & -2 & 1 \end{bmatrix}$$

(4) $z = -5,\ y = \frac{1}{2},\ x = \frac{1}{2}$

(7) (a) Add 7 times row 1 to row 3

(b) $E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix}$

(9) (a) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

(b) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

(18) $EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$

$$FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b+ac & c & 1 \end{bmatrix}$$

(18) contd...

$$E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix}$$

$$F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix}$$

$$F^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 100c & 1 \end{bmatrix}$$

(30) (a) $E = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ (b) $F = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

(c) $EM = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ $FEM = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$EFEM = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $EEFEM = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = B$

$$M = E^{-1}F^{-1}E^{-1}E^{-1}B$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$