MATH 2418: Linear Algebra

Assignment# 3

Due : 09/14 Wednesday　　　　Term `Fall, 2016`

[First Name]　　　　[Last Name]　　　　[Net ID]

Recommended Text Book Problems (do not turn in): [Sec 2.2: # 1, 2, 4, 6, 8, 12, 14, 32]; [Sec 2.3: # 1, 3, 4, 7, 9, 18, 30];

1. Given linear system $\begin{cases} 2x + 4y - 2z = 2 \\ 4x + 9y - 3z = 4 \\ -2x - 3y + 7z = 10 \end{cases}$.

   (a) Solve by reducing into upper triangular form and back substitution.

   (b) List all multipliers used and circle all the pivots.

2. Consider the linear system $\begin{cases} ax \quad + 2y = -2 \\ \quad 3x + 6y = -6 \end{cases}$.

(a) For what values of a does the elimination break down (1) permanently (2) temporarily?

(b) Solve the system after fixing the temporary break down.

(c) Solve the system in case of permanent break down.

3. Consider a 3×3 matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

   (a) Reduce matrix A into upper triangular form using elimination steps.

   (b) Write down the matrices E_{21}, E_{31}, E_{32} that put A into triangular form.

   (c) Write down the matrix M such that MA is upper triangular.

4. Let $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 3 & 1 \\ 1 & 4 & 0 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 11 \\ 4 \\ 6 \end{bmatrix}$

(a) Write down the elementary matrices that reduce A into an upper triangular matrix.

(b) Write down the corresponding upper triangular system $U\mathbf{x} = \mathbf{c}$

(c) Solve the system

5. Let $E = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the products EF and FE. Is $EF = FE$?

(b) Write down the matrices E^{-1} and F^{-1}.

(c) Find the products $F^{-1}E^{-1}(EF)$ and $E^{-1}F^{-1}(EF)$.

6. True or False. Circle your answer.

(a) **T** **F**: Consider the augmented matrix $[A \;\; \mathbf{b}] = \begin{bmatrix} 4 & 5 & b_1 \\ 0 & r & b_2 \end{bmatrix}$. The existence and uniqueness of the solution depends on all of r, b_1, b_2.

(b) **T** **F**: If $AB = I$ and $BC = I$ then $A = C$.

(c) **T** **F**: If P is a permutation matrix(a matrix that switches two rows) then $P^{100} = I$.

(d) **T** **F**: If elimination fails permanently, the system has no solution.

(e) **T** **F**: There is only one possible linear system which reduces to $\begin{cases} x + \;\; y = 1 \\ \quad\;\; 2y = 3 \end{cases}$ after one elimination step.