MATH 2418: Linear Algebra

Assignment# 2

Due : 09/07 Wednesday Term `Fall, 2016`

[First Name] [Last Name] [Net ID]

Recommended Text Book Problems (do not turn in): [Sec 1.3: # 3, 5, 6, 8, 9]; [Sec 2.1: # 1, 2, 3, 9, 10, 16, 17, 19, 29, 31];

1. Given $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

   (a) Write the linear system corresponding to $A\mathbf{x} = \mathbf{b}$.

   (b) Solve the linear system.

   (c) Find A^{-1}, the inverse of matrix A.

2. (a) Prove that the vectors $\mathbf{u} = (1, 1, 0)$, $\mathbf{v} = (2, 0, -1)$, $\mathbf{w} = (0, 1, 1)$ are linearly independent.

   (b) Prove that the vectors $\mathbf{u} = (1, 2, 1)$, $\mathbf{v} = (3, -2, 1)$, $\mathbf{w} = (5, 2, 3)$ are linearly dependent.

3. Given linear system $\begin{cases} 2x + 3y = 1 \\ 5x + 2y = 8 \end{cases}$

    (a) Write down the matrix equation $A\mathbf{x} = \mathbf{b}$

    (b) Draw the row picture and the column picture

    (c) Write $\mathbf{b}$ as a linear combination of columns of A.

4. Find the matrix of following transformations:
   (a) Rotation of vectors in $\mathbb{R}^2$ by 60° counterclockwise.
   (b) Reflection of points in $\mathbb{R}^2$ across the line $y = -x$
   (c) Multiplication of the point (x, y, z) in $\mathbb{R}^3$ to give the point $(x + y, y, x + z)$

5. Given $A = \begin{bmatrix} 1 & 2 & 4 \\ -5 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -3 & 6 \\ 2 & 4 & -2 \\ 4 & 2 & -3 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 7 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 9 \\ 2 \\ -3 \end{bmatrix}$

(a) Calculate $A\mathbf{u}$ as a linear combination of columns of A

(b) Calculate $B\mathbf{v}$ as a dot products of rows of B and $\mathbf{v}$.

6. True or False. Circle your answer.

   (a) **T** **F**: If the equations in a linear system are multiplied by scalars, the row picture and the column picture both change.

   (b) **T** **F**: The set $\{(3,5,7),(2,2,9),(0,0,0)\}$ is linearly independent in $\mathbb{R}^3$.

   (c) **T** **F**: If matrix A is invertible, then $A\mathbf{x} = \mathbf{b}$ need not to have a unique solution.

   (d) **T** **F**: The matrix that projects the vector (x,y) onto $x-$axis to produce $(x,0)$ is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

   (e) **T** **F**: If $A\mathbf{u} = \mathbf{0}$, then $\mathbf{u}$ is orthogonal to each column of A.