MATH 2418: Linear Algebra

Assignment 5: Part 2 (sections 3.1 and 3.2)

Due: `October 05, 2016` Term: `Fall, 2016`

Suggested problems(do not turn in): Section 3.1: 1, 2, 4, 6, 7, 8, 10, 11, 12, 13, 19, 20, 22, 27; Section 3.2: 1, 2, 3, 5, 6, 7, 10, 15, 16, 17, 18, 19, 24. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Determine if the set consisting of:

    (a) (2 points) All vectors of the form (a, b, c) where $b = a + c$ is a subspace of R^3.

    (b) (2 points) All vectors of the form (a, b, c) where $b = a + c + 1$ is a subspace of R^3.

    (c) (3 points) All n x n matrices for which $A\mathbf{x} = \mathbf{0}$ has only the trivial solution is a subspace of M_{nn} (the vector space of all n x n matrices).

    (d) (3 points) All n x n matrices such that $A^T = -A$ is a subspace of M_{nn}.

2. [10 points] Determine whether the vectors $\mathbf{v_1} = (1, 1, 2)$, $\mathbf{v_2} = (1, 0, 1)$ and $\mathbf{v_3} = (2, 1, 3)$ span the vector space R^3.

3. [10 points] Let $A\mathbf{x} = \mathbf{b}$ be the linear system where $A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix}$ and $\mathbf{b} = (1, -9, -3)$. Show that $\mathbf{b}$ is in column space of A by expressing it as a linear combination of the column vectors of A.

4. [10 points] Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & 0 \end{bmatrix}.$$

(a) (4 points) Find the reduced row echelon form of A.

(b) (2 points) Find the nullspace $N(A)$.

(c) (2 points) Find 3 special solutions to $A\mathbf{x} = \mathbf{0}$.

(d) (2 points) Find the rank of A.

5. [10 points] Describe all possible reduced row echelon forms of

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

6. [10 points] Consider the system of equations: $\begin{cases} ax + by = 0 \\ cx + dy = 0 \\ ex + fy = 0 \end{cases}$

Discuss the row picture of the lines $ax + by = 0$, $cx + dy = 0$ and $ex + fy = 0$ when the system has only the trivial solution and when it has nontrivial solutions.

7. [10 points] (a) (5 points) Show that if $ad - bc \neq 0$ then the reduced row echelon form of $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(b) (5 points) Use the result in part (a) to show that if $ad - bc \neq 0$, then the linear system $\begin{cases} ax + by = k \\ cx + dy = l \end{cases}$ has exactly one solution.