# MATH 2418: Linear Algebra

## Assignment 6 (sections 3.3 and 3.4)

Due: `October 12, 2016` Term: `Fall, 2016`

**Suggested problems**(do not turn in): Section 3.3: 1, 2, 3, 4, 5, 7, 16, 17, 25, 26, 27, 28, 29; Section 3.4: 1, 2, 3, 4, 5, 11, 12, 13, 15, 16, 17, 18, 24, 26, 27, 35. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Solve the linear system below using the Gauss-Jordan elimination (the augmented matrix should be in reduced row echelon form):

$$\begin{cases} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \\ 3x - 3w = -3 \end{cases}$$

2. [10 points] Suppose that $x_1 = -1$, $x_2 = 2$, $x_3 = 4$, $x_4 = -3$ is a solution of $A\mathbf{x} = \mathbf{b}$ and that the solution set of $A\mathbf{x} = \mathbf{0}$ is given by $\begin{cases} x_1 = -3r + s \\ x_2 = r - s \\ x_3 = r \\ x_4 = s \end{cases}$ where $r$ and $s$ are parameters.

   (a) (5 points) Write the solution of $A\mathbf{x} = \mathbf{0}$ as a linear combination of special solutions;

   (b) (5 points) Write the complete solution of $A\mathbf{x} = \mathbf{b}$ as the sum of a particular solution ($x_p$) plus a linear combination of special solutions.

3. [10 points] Write the complete solution as $(x_p)$ plus any combination of special solutions in the nullspace:

(a) (5 points) $\begin{cases} x_1 - 3x_2 = 1 \\ 2x_1 - 6x_2 = 2 \end{cases}$

(b) (5 points) $\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ x_1 + x_3 = -2 \\ 2x_1 + x_2 + 3x_3 = 3 \end{cases}$

4. [10 points] Determine if the given vectors are basis of the specified vector spaces:

   (a) (3 points) $V = M_{22}$ (the vector space of all 2 x 2 matrices). Given vectors:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}.$$

   (b) (3 points) $V = M_{22}$. Given vectors:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

   (c) (4 points) $V = P_2$ (the space of all polynomials of degree $\leq 2$). Given vectors: $x^2 + 1$, $x^2 - 1$ and $2x - 1$.

5. [10 points] Find a basis for the column space of the matrix:

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

6. [10 points] Find a subset of the vectors $\mathbf{v_1} = (1, -2, 0, 3)$, $\mathbf{v_2} = (2, -5, -3, 6)$, $\mathbf{v_3} = (0, 1, 3, 0)$, $\mathbf{v_4} = (2, -1, 4, -7)$, $\mathbf{v_5} = (5, -8, 1, 2)$ that forms a basis for the subspace of $R^4$ spanned by these vectors.