MATH 2418: Linear Algebra

Assignment 8 (sections 4.2 and 4.3)

Due: `October 26, 2016` Term: `Fall, 2016`

Suggested problems(do not turn in): Section 4.2: 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 16, 17, 21, 23, 24; Section 4.3: 1, 2, 3, 4, 5, 7, 9, 10, 12, 13, 14, 17, 18, 19, 21, 22. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Find the projection onto the line through point $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   (a) (3 points) Find the projection matrix.

   (b) (3 points) Project the vectors $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

   (c) (1 point) Find the errors $\mathbf{e}_1 = \mathbf{b}_1 - \mathbf{p}_1$ and $\mathbf{e}_2 = \mathbf{b}_2 - \mathbf{p}_2$

2. [10 points] Find the projection matrix for the projection onto the null space of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & -3 \end{bmatrix}$

3. [10 points] Find the minimal distance from the point $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix}$ to the space of all linear combinations of the vectors $\mathbf{e}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

4. [10 points] Find the closest parabola At^2+Bt+C approximating the data set

t	-1	0	1	2	3
$b(t)$	5	2	1	2	5

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5. [10 points] Find the best line $At + B$ approximating the data set b at the times $t = 0, 1, 2, 3, 4$.

   (a) (5 points) $b = -1,\ -1, 2, 0, 0.$

   (b) (5 points) $b = -1, \quad 0, 2, 0, 0.$

6. [10 points] True or False

   (a) (2 points) If vector $\mathbf{b}$ is orthogonal to $\mathbf{a}$ then projection of $\mathbf{b}$ onto line through $\mathbf{a}$ has no errors.

   (b) (2 points) If P is a projection matrix then $P^3 = P$.

   (c) (2 points) If matrix A is a square matrix then $A(A^TA)^{-1}A^T = I$.

   (d) (2 points) Projection of the vector onto the subspace minimizes the length of the error vector.

   (e) (2 points) Least square approximation finds the line passing through all points in the data set.