

4.)

Prove:

$$(1)^3 + (2)^3 + (3)^3 + \dots + (n)^3 = \{ (n^2) * [(n + 1)^2] \} / 4$$

Base Case:

Pick some small, trivial, usually degenerate values to start process

Base cases start at minimal value

$$N = 1$$

$$(1)^3 = \{ (1^2) * [(1 + 1)^2] \} / 4$$

$$1 = 1$$

Inductive Hypothesis:

Theorem assumed true up until some value, usually $k + 1$

1.) Statement with k

$$(1)^3 + (2)^3 + (3)^3 + \dots (k)^3 = \{ ((k)^2) * [(k + 1)^2] \} / 4$$

2.) Proof of $k + 1$ by substitution of the right hand value of above equation for k

$$(1)^3 + (2)^3 + (3)^3 + \dots (k)^3 + (k+1)^3 = \{ ((k+1)^2) * [(k+1) + 1)^2] \} / 4$$

$$\{ \{ ((k)^2) * [(k + 1)^2] \} / 4 \} + (k+1)^3 = \quad \quad \quad // \text{ substitute statement with } k$$

$$\{ (k^2) * [(k + 1)^2] + 4[(k+1)^3] \} / 4 = \quad \quad \quad // \text{ common denominators to combine into one fraction}$$

$$\{ [(k + 1)^2] * [(k^2) + 4[(k+1)^3]] \} / 4 = \quad \quad \quad // \text{ factor out } (k + 1)^2$$

$$\{ [(k + 1)^2] * [(k^2) + 4k + 4] \} / 4 = \quad \quad \quad // \text{ foil inside}$$

$$\{ [(k + 1)^2] * [(k + 2)^2] \} / 4 = \quad \quad \quad // \text{ reverse foil inside}$$

$$\{ [(k + 1)^2] * [(k + 2)^2] \} / 4 = \quad \quad \quad // \text{ original statement, with } k + 1 \text{ substituted in for } k$$

$$\{ [(k + 1)^2] * [(k + 1) + 1)^2] \} / 4 = \quad \quad \quad // \text{ expanded the squared term to make it clean, this is} \\ // \text{ the exact original from the first line of this section}$$