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Assignment 1

1.)

input (BinaryString, index)

output (numberOfOnes)

```
BinaryRecursion (BinaryString, index, numberOfOnes){
```

```
    stringLength = BinaryString.length
```

```
    if(stringLength < 1){
```

```
        stringCharacter = charAtIndex (index)
```

```
        index + 1
```

```
        BinaryString = subString(index, stringLength)
```

```
        if(stringCharacter equals "1"){
```

```
            numberOfOnes + 1
```

```
        }
```

```
        BinaryRecursion (BinaryString, index, numberOfOnes)
```

```
    }
```

```
    else {
```

```
        return numberOfOnes
```

```
    }
```

```
}
```

2.)

a.)

$$\left[ \frac{1}{4^0} \right] + \left[ \frac{1}{4^1} \right] + \left[ \frac{1}{4^2} \right] + \left[ \frac{1}{4^3} \right] + \dots +$$
$$\left[ \frac{1}{1} \right] + \left[ \frac{1}{4} \right] + \left[ \frac{1}{16} \right] + \left[ \frac{1}{64} \right] + \dots +$$

Values approach but won't reach 2

b.)

$$\left[ \frac{0}{4^0} \right] + \left[ \frac{1}{4^1} \right] + \left[ \frac{2}{4^2} \right] + \left[ \frac{3}{4^3} \right] + \dots +$$
$$\left[ \frac{0}{1} \right] + \left[ \frac{1}{4} \right] + \left[ \frac{2}{16} \right] + \left[ \frac{3}{64} \right] + \dots +$$

Values approach but won't reach 1

### 3.) Fibonacci numbers

4.)

Prove:

$$(1)^3 + (2)^3 + (3)^3 + \dots + (n)^3 = \{ (n^2) * [(n + 1)^2] \} / 4$$

Base Case:

Pick some small, trivial, usually degenerate values to start process

Base cases start at minimal value

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$$N = 1$$

$$(1)^3 = \{ (1^2) * [(1 + 1)^2] \} / 4$$

$$1 = 1$$

---

Inductive Hypothesis:

Theorem assumed true up until some value, usually  $k + 1$

1.) Statement with  $k$

$$(1)^3 + (2)^3 + (3)^3 + \dots (k)^3 = \{ ((k)^2) * [(k + 1)^2] \} / 4$$

---

2.) Proof of  $k + 1$  by substitution of the right hand value of above equation for  $k$

$$(1)^3 + (2)^3 + (3)^3 + \dots (k)^3 + (k+1)^3 = \{ ((k+1)^2) * [(k+1) + 1)^2] \} / 4$$

$$\{ \{ ((k)^2) * [(k + 1)^2] \} / 4 \} + (k+1)^3 = \quad \quad \quad // \text{ substitute statement with } k$$

$$\{ (k^2) * [(k + 1)^2] + 4[(k+1)^3] \} / 4 = \quad \quad \quad // \text{ common denominators to combine into one fraction}$$

$$\{ [(k + 1)^2] * [ (k^2) + 4[(k+1)^3] ] \} / 4 = \quad \quad \quad // \text{ factor out } (k + 1)^2$$

$$\{ [(k + 1)^2] * [ (k^2) + 4k + 4 ] \} / 4 = \quad \quad \quad // \text{ foil inside}$$

$$\{ [(k + 1)^2] * [(k + 2)^2] \} / 4 = \quad \quad \quad // \text{ reverse foil inside}$$

$$\{ [(k + 1)^2] * [(k + 2)^2] \} / 4 = \quad \quad \quad // \text{ original statement, with } k + 1 \text{ substituted in for } k$$

$$\{ [(k + 1)^2] * [(k + 1) + 1)^2] \} / 4 = \quad \quad \quad // \text{ expanded the squared term to make it clean, this is} \\ // \text{ the exact original from the first line of this section}$$