Math 2414 Graded Homework 1 (Section 7.8, 8.1, 8.2) Spring 2015

Instructions:

1) **Due: Wednesday, February 18, 2015, during the first 10 minutes of class.**
2) **Assignments must be submitted in a blue book – no exceptions. Bluebooks are available at the on campus/off campus bookstore, student union, etc.**
3) **On the front cover, write the following at the top:**
   a) **TA name**
   b) **Your name**
   c) **Assignment name**
   d) **Problem session number**
4) **Write no more than one problem per page.**
5) **Solutions must be presented neatly, completely, and with logical flow.**
6) **15% will be deducted for assignments turned in after the first 10 minutes of class.**
7) **25% will be deducted for assignments which are not neat and orderly.**
8) **15% will be deducted for assignments without your TA's name.**
9) **Assignments will not be accepted after class.**

<u>Section 7.8</u>

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   a) $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$

   b) $\int_3^7 \frac{x}{(x^2-9)^2}\,dx$

   c) $\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+3}\,dx$

   d) Text problem #18 $\int_0^{\infty} \frac{1}{x^2+3x+2}\,dx$

2. Consider the integral $\int_2^{\infty} \frac{4}{x^2-4}\,dx$

   a) Rewrite the given integral as a sum of improper integrals of Type 1 and Type 2 needed to determine convergence or divergence of the given integral. Do not compute the integrals.

b) Compute the antiderivative $\int \frac{4}{x^2-4}dx$.

c) Determine whether the given integral $\int_2^{\infty} \frac{4}{x^2-4}dx$ converges or diverges.

d) It is necessary to compute only one of the integrals of Type 1 and Type 2 (from part (a)) to determine the convergence or divergence of the given integral. Indicate which one, and explain your answer.

3. Consider the function $F(s) = \int_0^{\infty} f(t)e^{-st}dt$, where s is treated as a constant inside the integral.

   Find the function $F(s)$ for $s > 0$ if $f(t) = t$.

4. Use the Comparison Test for Improper Integrals to determine whether the integral converges or diverges.

$$\int_0^{\infty} \frac{x\cos^2 x}{x^4+1}dx$$

Section 8.1

1. Find the exact length of the curve $x = \frac{1}{3}(y^2+2)^{3/2}$ on $1 \le y \le 2$.

2. Consider the arc length of the curve $y = \ln x$ on $1 \le x \le 2$.
   a) First, set up, but do not evaluate, an integral that represents the exact arc length.
   b) Next, rewrite your answer to part (a) in such a way that you can apply the trigonometric substitution, $x = \tan\theta$, then compute the integral.

Section 8.2

1. Consider the portion of a hyperbola, $y = \sqrt{x^2-1}$ on $\sqrt{5} \le x \le \sqrt{13}$, located in the first quadrant.

   a) Set up, but DO NOT evaluate, the integral with respect to x that represents the surface area obtained by rotating the curve about the y-axis.

   b) Set up, AND evaluate, the integral with respect to y that represents the surface area obtained by rotating the curve about the y-axis.

2. Consider the portion of the curve, $x = \ln(2y - 1)$ on $1 \le y \le 5$.

a) Set up, but DO NOT evaluate, the integral with respect to *x* that represents the surface area obtained by rotating the curve about the *x*-axis.

b) Set up, but DO NOT evaluate, the integral with respect to *y* that represents the surface area obtained by rotating the curve about the *x*-axis.