MATH 2418: Linear Algebra

Assignment 6

Due March 2, 2016 Term Spring, 2016

Recommended Text Book Problems (do not turn in): [Sec 4.2: # 1, 3, 9, 11, 19]; [Sec 4.3: # 3, 11, 13, 15, 17, 19];

1. Which of the followings are subspaces of $\mathbb{R}^3$? Show all of your work to receive full credit:

   (a) $W = \{(x, y, z) : x, y, z \in \mathbb{R}; x = y + z\}$.

   (b) $V = \{(x, y, 0) : x, y \in \mathbb{R}\}$

   (c) $U = \{(1, 1, z) : z \in \mathbb{R}\}$.

2. Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = (1, 6, 4), \mathbf{v}_2 = (2, 4, -1), \mathbf{v}_3 = (-1, 2, 5); \text{ and } \mathbf{w}_1 = (1, -2, -5), \mathbf{w}_2 = (0, 8, 9).$$

Prove that, $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \operatorname{Span}\{\mathbf{w}_1, \mathbf{w}_2\}$.

3. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^2$ be a matrix transformation, the multiplication by the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ and $\mathbf{u}_1 = (0,1,1), \mathbf{u}_2 = (2,-1,1), \mathbf{u}_3 = (1,1,-2)$ be vectors in $\mathbb{R}^3$. Determine if $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2), T_A(\mathbf{u}_3\}$ spans $\mathbb{R}^2$? Show all of your work to receive full credit.

4. Determine if the following set of vectors are linearly independent or dependent.

   (a) $(-3, 0, 4),\ (5, -1, 2),\ (1, 1, 3)$ in $\mathbb{R}^3$.

   (b) $\cos 2x,\ 3\sin^2 x,\ -4\cos^2 x$ in the space $F(-\infty, \infty)$ of all real valued functions defined on $(-\infty, \infty)$.

   (c) $1 + 3x + 3x^2,\ x + 4x^2,\ 5 + 6x + 3x^2$ in P_2, the vector space of all polynomials of degree ≤ 2.

5. (a) Determine if $(2,-2,0),(2,-1,4),(2,7,-6)$ lie on the same plane in $\mathbb{R}^3$.
   (b) Determine if $(-1,2,3),(2,-4,-6),(-7,14,21)$ lie on the same line on $\mathbb{R}^3$.

6. Use the **Wronskian** $W(x)$ to check if the following vectors are linearly independent in $F(-\infty, \infty)$.

   (a) $2,\ 2x+3,\ x^2-1.$

   (b) $5e^x,\ e^x \sin x,\ e^x \cos x.$

7. True or False.

   (a) **T** **F**: Let A and B be two subsets of a vector space V such that $\text{Span}\{A\} = \text{Span}\{B\}$, then $A = B$.

   (b) **T** **F**: Let A be an $m \times n$ matrix, then the solution set of $A\mathbf{x} = \mathbf{0}$ is a subspace of $\mathbb{R}^n$.

   (c) **T** **F**: Let A be an $m \times n$ matrix, then the solution set of $A\mathbf{x} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$.

   (d) **T** **F**: If the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent then $\{2\mathbf{u}, 3\mathbf{v}\}$ is also linearly independent.

   (e) **T** **F**: If three vectors in $\mathbb{R}^3$ are linearly dependent, then they must lie on the same line.

   (f) **T** **F**: The vectors $(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)$ in $\mathbb{R}^3$ are linearly dependent.