MATH 2418: Linear Algebra

Assignment 9

Due: March 30, 2016 Term: Spring, 2016

Suggested problems(do not turn in): Section 4.8: 1, 3, 5, 7, 9, 13, 15. Section 4.9: 13, 15, 17, 19, 21, 23, 39.

1. [10 points] (2 points each) Use matrix multiplication to:
   (a) Find the reflection of (a,b) about the y-axis.
   (b) Find the reflection of (a,b,c) about the xz-plane.
   (c) Find the orthogonal projection of (a,b) onto the the y-axis.
   (d) Find the orthogonal projection of (a,b,c) onto the yz-plane.
   (e) Find the image of the nonzero vector $\mathbf{v} = (v_1, v_2)$ when it is rotated about the origin through a negative angle $-\alpha$.

2. [10 points] Consider the following matrices, where R is the reduced row-echelon form of A:

$$A = \begin{bmatrix} 2 & 0 & -4 & 1 & 3 \\ 1 & 3 & 1 & 3 & 7 \\ 0 & 2 & 2 & -2 & 0 \\ -1 & 1 & 3 & 4 & 4 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R$$

Determine the following values without completing the solution to any system of equations.

(a) What is rank(A)?

(b) What is rank(A^{T})?

(c) What is nullity(A)?

(d) What is nullity(A^{T})?

(e) What is the dimension of the row space of A?

(f) What is the dimension of the row space of A^{T}?

(g) What is the dimension of the column space of A?

(h) What is the dimension of the column space of A^{T}?

(i) What is the dimension of the null space of A?

(j) What is the dimension of the null space of A^{T}?

3. [10 points] (5 + 5) In R^3 the **orthogonal projections** onto the x-axis, y-axis and z-axis are

$$T_1(x, y, z) = (x, 0, 0), \qquad T_2(x, y, z) = (0, y, 0), \qquad T_3(x, y, z) = (0, 0, z),$$

respectively.

(a) Find the standard matrices for T_1, T_2 and T_3.

(b) Show that if $T : R^3 \to R^3$ is one of these orthogonal projections, then for every vector $\mathbf{v} \in R^3$, $T(\mathbf{v})$ and $(\mathbf{v} - T(\mathbf{v}))$ are orthogonal.

4. [10 points] Find the rank (5 points) and nullity (5 points) of the standard matrix for T, where

$$T : \mathbb{R}^2 \to \mathbb{R}^3, \quad T(x_1, x_2) = (x_1 + 3x_2,\, x_1 - x_2,\, x_1).$$

5. [10 points] True or False.

   (a) **T** **F**: In a square matrix A, $\text{nullity}(A) = \text{nullity}(A^{\mathrm{T}})$.

   (b) **T** **F**: The nullity of an $m \times n$ matrix is at most m.

   (c) **T** **F**: If A has more rows than columns, the nullity of A^{T} is less than the nullity of A.

   (d) **T** **F**: If V is a subspace of $\mathbb{R}^n$ and W is a subspace of V then $V^{\perp}$ is a subspace of $W^{\perp}$.

   (e) **T** **F**: The kernel of the matrix transform $T_A : \mathbb{R}^n \to \mathbb{R}^m$ is the same as the null space of the corresponding $m \times n$ matrix A.