MATH 2418: Linear Algebra

Assignment 7

Due March 9, 2016 Term Spring, 2016

Recommended Text Book Problems (do not turn in): [Sec 4.4: # 1, 7, 13, 17, 19, 21, 25]; [Sec 4.5: # 3, 5, 7, 9, 13, 17, 19];

1. (a) Prove that $\mathcal{B} = \{(0,1,0),(2,2,0),(3,3,3)\}$ is a basis for $\mathbb{R}^3$.
   (b) Write the coordinate vector of $(5,-12,3)$ with respect to basis $\mathcal{B}$ of $\mathbb{R}^3$.

2. Let M_3^T be the vector space of all 3×3 symmetric matrices. Find a basis for M_3^T.

3. (a) Find a basis for the solution space of the given homogeneous linear system. State the dimension.

$$\begin{cases} 3x_1 + x_2 + x_3 + x_4 = 0, \\ 5x_1 - x_2 + x_3 - x_4 = 0. \end{cases}$$

(b) Find a basis for the subspace $W = \{(a, b, c, d) : d = a + b, c = a - b\}$ of $\mathbb{R}^4$. State the dimension.

4. (a) Let $\mathbf{v}_1 = (1, -1, 3)$, $\mathbf{v}_2 = (2, 2, 1)$. Find the standard basis vector of $\mathbb{R}^3$ that can be added to $\{\mathbf{v}_1, \mathbf{v}_2\}$ to produce a basis for $\mathbb{R}^3$. Show all of your work to receive full credit.

(b) Let $\mathbf{v}_1 = (0, 1, 1, 0)$, $\mathbf{v}_2 = (2, 2, 2, 0)$, $\mathbf{v}_3 = (0, 0, 0, 4)$, $\mathbf{v}_4 = (4, -3, -3, -3)$. Find a basis for the Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

5. True or False.

(a) **T** **F**: The set $S = \{(1,1),(2,3),(3,4)\}$ is linearly independent.

(b) **T** **F**: Every set consisting of 100 vectors that span $\mathbb{R}^{100}$ is a basis for $\mathbb{R}^{100}$.

(c) **T** **F**: The coordinate vector of $\mathbf{x} \in \mathbb{R}^n$ with respect to the standard basis of $\mathbb{R}^n$ is $\mathbf{x}$.

(d) **T** **F**: If $V = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2 \cdots, \mathbf{v}_n\}$, then $\{\mathbf{v}_1, \mathbf{v}_2 \cdots, \mathbf{v}_n\}$ is a basis of V.

(e) **T** **F**: There exists a basis of $M_{2\times 2}$ (the vector space of all 2×2 matrices) consisting of invertible matrices.