Discrete Mathematics for Computing

Ch 9.2 n-ary Relations and Their Applications

- Motivation:

- Relationships among elements of more than two sets

Flight: Airline, Flight number, starting point, destination, departure time, arrival time

n-ary Relations

- n-ary Relations
- Let A_1, A_2, …, A_n be sets
- An n-ary relation on these sets

 - is a subset of $A_1 \times A_2 \times \ldots \times A_n$

 - where A_i are the domains of the relation

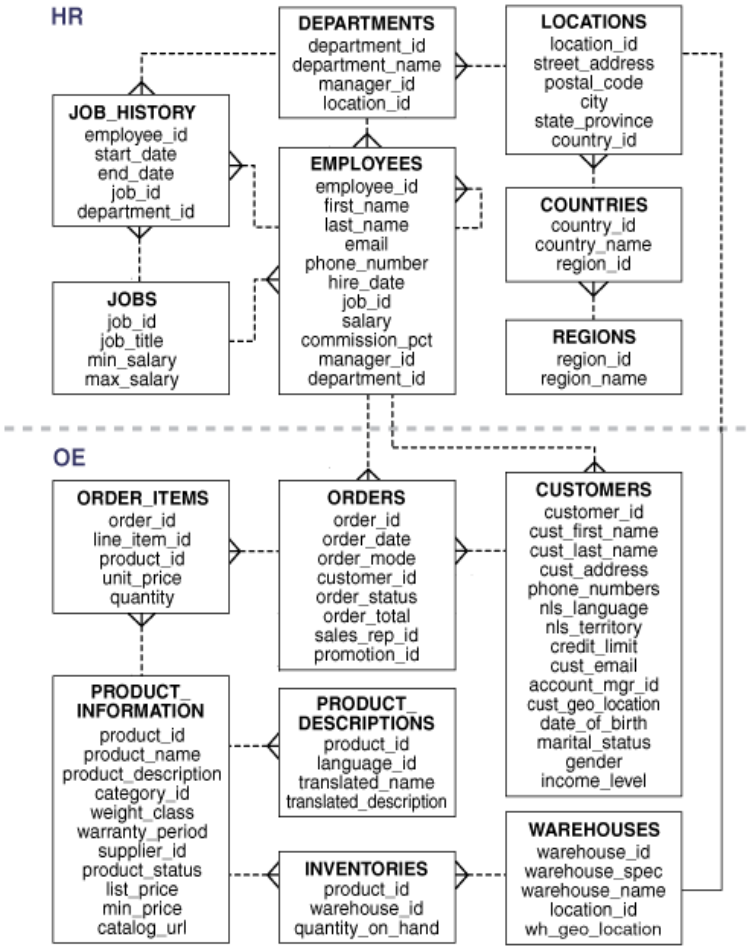
 - n is called its degree

UTD

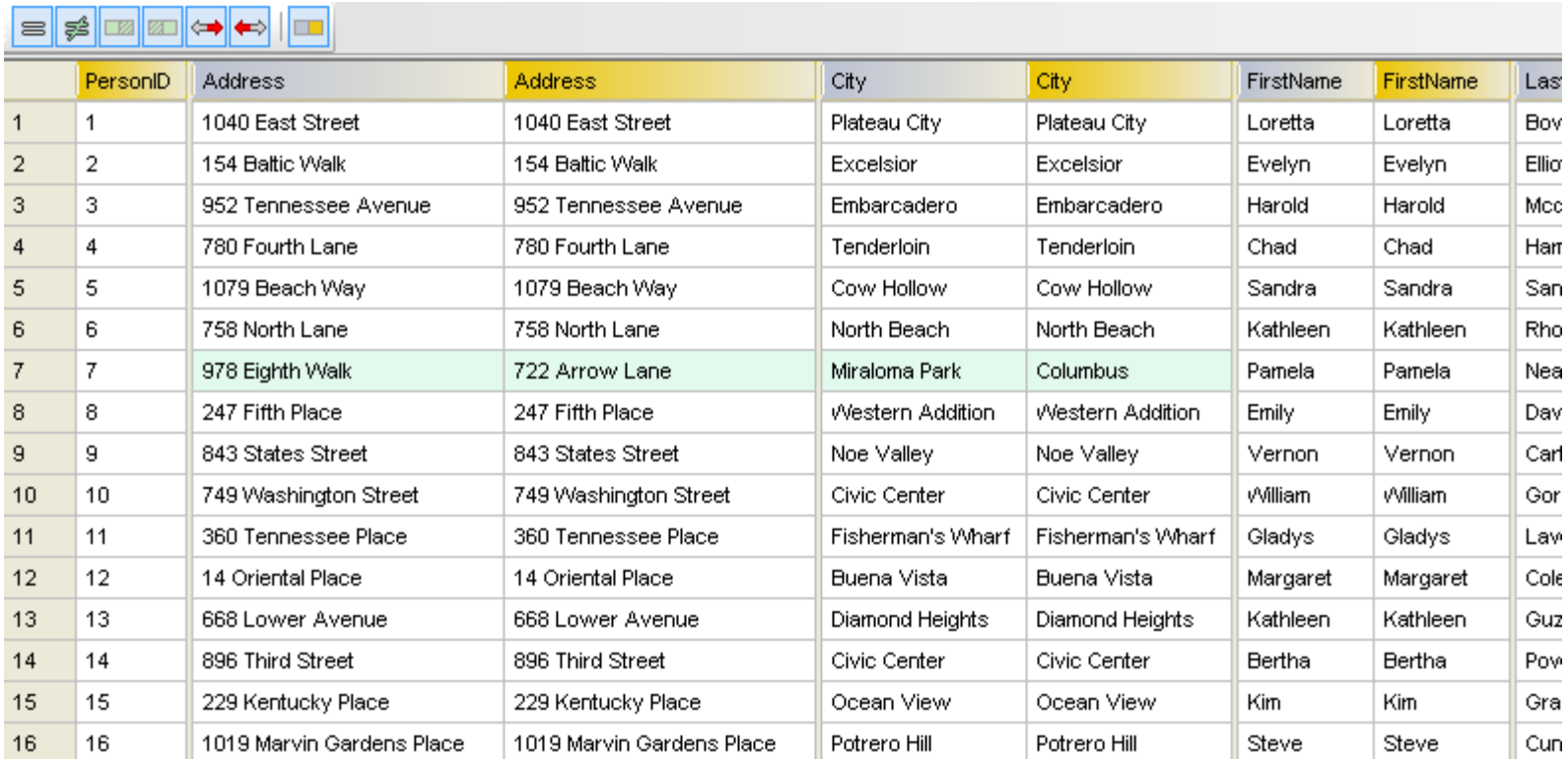
n-ary Relations

- Example: Let R be the relation on N * N * N consisting of triples (a, b, c) where a, b, and c are integers with a < b < c.
- Then (1,2,3) $\in$ R, but (2,4,3) $\notin$ R
- The degree of this relation is 3
- Its domains are equal to the set of integers **Z**

Practical Examples



Practical Examples



	PersonID	Address	Address	City	City	FirstName	FirstName	Las
1	1	1040 East Street	1040 East Street	Plateau City	Plateau City	Loretta	Loretta	Bov
2	2	154 Baltic Walk	154 Baltic Walk	Excelsior	Excelsior	Evelyn	Evelyn	Ellio
3	3	952 Tennessee Avenue	952 Tennessee Avenue	Embarcadero	Embarcadero	Harold	Harold	Mcc
4	4	780 Fourth Lane	780 Fourth Lane	Tenderloin	Tenderloin	Chad	Chad	Han
5	5	1079 Beach Way	1079 Beach Way	Cow Hollow	Cow Hollow	Sandra	Sandra	San
6	6	758 North Lane	758 North Lane	North Beach	North Beach	Kathleen	Kathleen	Rho
7	7	978 Eighth Walk	722 Arrow Lane	Miraloma Park	Columbus	Pamela	Pamela	Nea
8	8	247 Fifth Place	247 Fifth Place	Western Addition	Western Addition	Emily	Emily	Dav
9	9	843 States Street	843 States Street	Noe Valley	Noe Valley	Vernon	Vernon	Carl
10	10	749 Washington Street	749 Washington Street	Civic Center	Civic Center	William	William	Gor
11	11	360 Tennessee Place	360 Tennessee Place	Fisherman's Wharf	Fisherman's Wharf	Gladys	Gladys	Law
12	12	14 Oriental Place	14 Oriental Place	Buena Vista	Buena Vista	Margaret	Margaret	Cole
13	13	668 Lower Avenue	668 Lower Avenue	Diamond Heights	Diamond Heights	Kathleen	Kathleen	Guz
14	14	896 Third Street	896 Third Street	Civic Center	Civic Center	Bertha	Bertha	Pow
15	15	229 Kentucky Place	229 Kentucky Place	Ocean View	Ocean View	Kim	Kim	Gra
16	16	1019 Marvin Gardens Place	1019 Marvin Gardens Place	Potrero Hill	Potrero Hill	Steve	Steve	Cun

Databases and Relations

- Relational Data Model: developed for information processing
- Relations used to represent databases – Tables
- A table consists of records or rows
  - which are n-tuples made up of fields
- The fields contains information for the attributes

Databases and Relations

- Relational Data Model
- Example: Design a relational database table for "Students".

Attributes - Name, Student #, Major, Grade point average of the student

- The relation is R(Student-Name, Id-number, Major, GPA)

Databases and Relations

TABLE A: Students

Students Names	ID #	Major	GPA
Smith	3214	Mathematics	3.9
Stevens	1412	Computer Science	4.0
Rao	6633	Physics	3.5
Adams	1320	Biology	3.0
Lee	1030	Computer Science	3.7

Databases and Relations

- Operations on n-ary Relations

- Let R be an n-ary relation

- C a condition that elements in R may satisfy

- Then the selection operator s_C

- maps n-ary relation R to the n-ary relation of all n-tuples

- from R that satisfy the condition C

Databases and Relations

- Example:
- If s_C = “Major = “computer science” $\wedge$ GPA > 3.5”
- then the result of this selection consists of the 2 four-tuples
- (Stevens, 1412, Computer Science, 4.0)
- (Lee, 1030, Computer Science, 3.7)

Databases and Relations

- Projections
- Form new n-ary relations by deleting the same fields in every record of the relation
- The projection $P_{i_1,i_2,\ldots,i_m}$ deletes

  n – m of the components of n-tuple

  leaving the i_1th, i_2th, …, and i_mth components.

Databases and Relations

- Example: What relation results when the projection $P_{1,4}$ is applied to the relation in Table A?
- When the projection $P_{1,4}$ is used, the second and third columns of the table are deleted

Students Names	ID #	Major	GPA
Smith	3214	Mathematics	3.9
Stevens	1412	Computer Science	4.0
Rao	6633	Physics	3.5
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Lee	1030	Computer Science	3.7

Databases and Relations

Pairs representing student names and GPA are obtained

Table B displays the results of this projection

TABLE B: GPAs

Students Names	GPA
Smith	3.9
Stevens	4.0
Rao	3.5
Adams	3.0
Lee	3.7

Databases and Relations

- Join
- Let R be a relation of degree m
- S a relation of degree n
- The join $J_p(R,S)$, where $p \leq m$ and $p \leq n$
- is a relation of degree $m + n - p$
- that consists of all $(m + n - p)$-tuples $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$
- where the m-tuple $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p)$ belongs to R
- and the n-tuple $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$ belongs to S

TABLE C: Teaching Assignments

Professor	Dpt	Course #
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE D: Class Schedule

Dpt	Course #	Room	Time
Computer Science	518	N521	2:00 PM
Mathematics	575	N502	3:00 PM
Mathematics	611	N521	4:00 PM
Physics	544	B505	4:00 PM
Psychology	501	A100	3:00 PM
Psychology	617	A110	11:00 AM
Zoology	335	A100	9:00 AM
Zoology	412	A100	8:00 AM

Databases and Relations

- Solution: The join J_2 produces the relation shown in Table E

Table E: Teaching Schedule

Professor	Dpt	Course #	Room	Time
Cruz	Zoology	335	A100	9:00 AM
Cruz	Zoology	412	A100	8:00 AM
Farber	Psychology	501	A100	3:00 PM
Farber	Psychology	617	A110	11:00 AM
Grammer	Physics	544	B505	4:00 PM
Rosen	Computer Science	518	N521	2:00 PM
Rosen	Mathematics	575	N502	3:00 PM

Databases and Relations

- SQL:

- SQL (Structured Query Language) - express queries by showing how SQL can be employed to make a query about airline flights using Table F

Table F: Flights

Airline	Flight #	Gate	Destination	Departure time
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08”47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

Databases and Relations

- SQL statements:

```
SELECT departure_time
FROM Flights
WHERE destination = 'Detroit'
```

- Used to find the projection P_5 (on the departure_time attribute) of the selection of 5-tuples in the flights database that satisfy the condition: destination = 'Detroit'
- The output would be a list containing the times of flights that have Detroit as their destination, namely, 08:10, 08:47, and 9:44