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SE 3306

Assignment 02

1 a) the set of all students within one mile of school and walk to clsss

b) the set of students who either live within 1 mile or walk to clsss

c) the set of students who live within one mile of school but do not walk to class

d) the set of students who live more than 1 mile from school but walk to class

4 a) $A \cup B$
A intersect B = Set of all elements in both A & B
$A \cup B = \{a, b, c, d, e\}$

b) $A \cap B$
A union B = Set of all elements in A or B
$A \cap B = \{a, b, c, d, e, f, g, h\}$

c) $A - B$
The difference of A & B = $x \in A$ and $x \notin B$
$A - B = \{\emptyset\}$ no elements exist in A that are not in B

d) $B - A$
The difference of B & A = $x \in B$ & $x \notin A$
$B - A = \{f, g, h\}$

17 Show that if A, B and C are sets the
$\overline{A \wedge B \wedge C} = \bar{A} \cup \bar{B} \cup \bar{C}$

a) Show each side is a subset of eachother

Suppose $x \in \overline{A \cap B \cap C}$

$x \notin A \cap B \cap C$ Complement

$\neg((x \in A) \wedge (x \in B) \wedge (x \in C))$ Definition of Intersection

$\neg(x \in A) \vee \neg(x \in B) \vee \neg(x \in C)$ DeMorgans

$(x \notin A) \vee (x \notin B) \vee (x \notin C)$

$(x \in \bar{A}) \vee (x \in \bar{B}) \vee (x \in \bar{C})$

$x \in \bar{A} \cup \bar{B} \cup \bar{C}$ Definition Union

We have shown $\overline{A \cap B \cap C} \subseteq \bar{A} \cup \bar{B} \cup \bar{C}$

Suppose $x \in \bar{A} \cup \bar{B} \cup \bar{C}$

$x \notin A \cup B \cup C$ Complement

$(x \notin A) \vee (x \notin B) \vee (x \notin C)$

$\neg(x \in A) \vee \neg(x \in B) \cup \neg(x \in C)$ DeMorgans

$\neg((x \in A) \wedge (x \in B) \wedge (x \in C))$

$x \in \overline{A \cap B \cap C}$ Definition Intersection

We have shown $\bar{A} \cup \bar{B} \cup \bar{C} \subseteq \overline{A \cap B \cap C}$

19 Show that if A and B are sets, then

a) $A - B = A \cap \overline{B}$

Suppose $A \cap \overline{B}$

$\{x \in U \mid x \in A\} \cap \{x \in U \mid x \notin B\}$

$\{x \mid x \in A \wedge x \notin B\}$

$A - B = \{x \mid x \in A \wedge x \notin B\}$ $\boxed{\text{page } 128}$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

Suppose $(A \cap B) \cup (A \cap \overline{B})$

$x \in (A \cap B)$ or $x \in (A \cap \overline{B})$

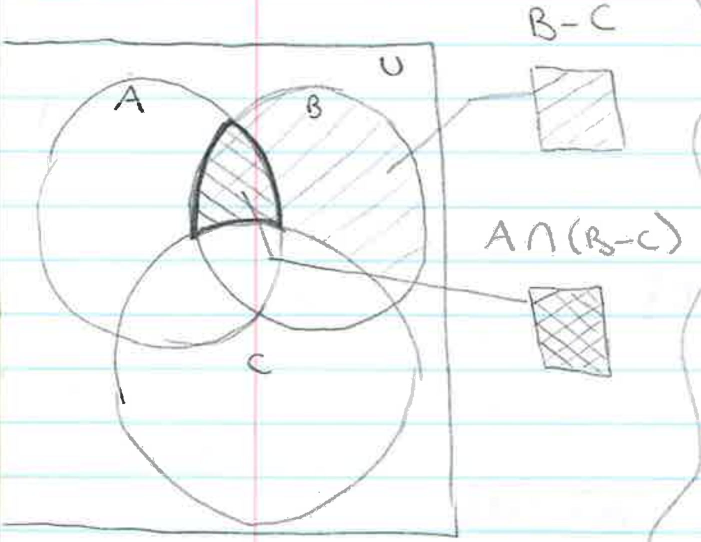
therefore $x \in A$

so $(A \cap B) \cup (A \cap \overline{B}) \subseteq A$

I did the wrong problem...

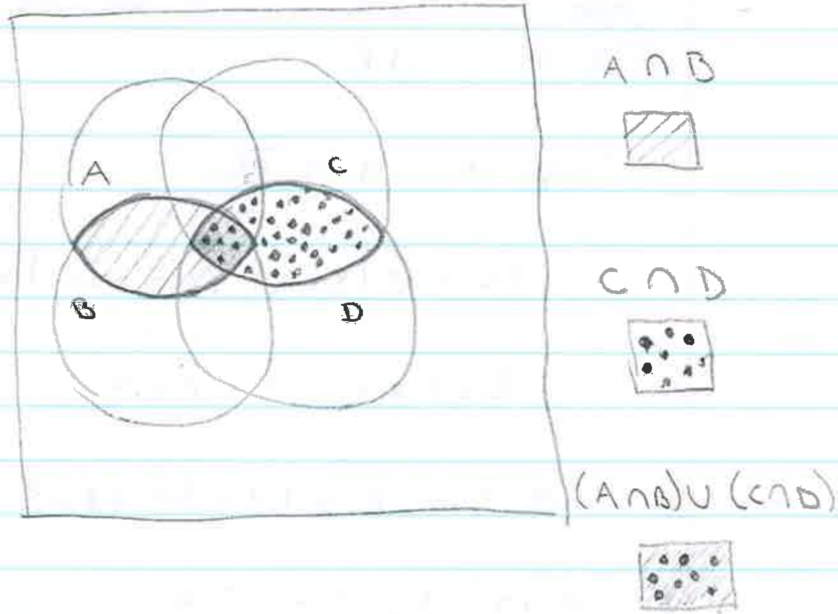
27)

a) $A \cap (B - C)$

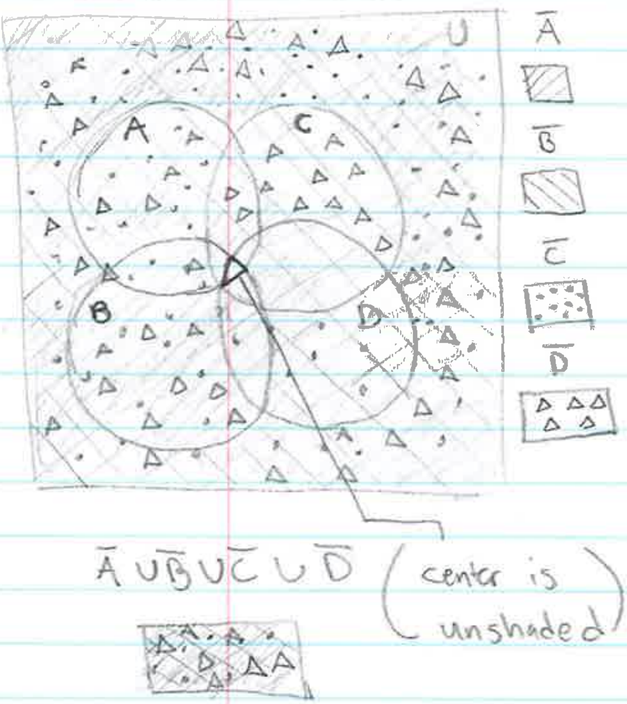


28)

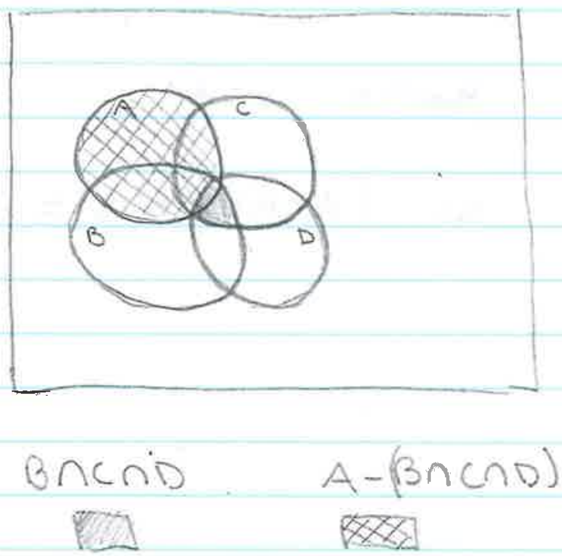
a) $(A \cap B) \cup (C \cap D)$



b) $\bar{A} \cup \bar{B} \cup \bar{C} \cup \bar{D}$



c) $A - (B \cap C \cap D)$



53) Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a) 11 1100 1111 each binary digit corresponds to an extry

10 9 8 7 4 3 2 1

$\{1, 2, 3, 4, 7, 8, 10\}$

c) 10 0000 0001

10 1

$\{1, 10\}$

b) 01 0111 1000

9 7 6 5 4

$\{4, 5, 6, 7, 9\}$

Ch 2.3 2, 8, 12, 22 a,b

2) Determine if f is a function from $\mathbb{Z}$ to $\mathbb{R}$

a) Not a function, each n yeilds two values (+ −)

b) Yes, n^2 will remove possibility of negatives under $\sqrt{}$

c) Undefinded at $n = -2, 2$ can't divide by 0

8) a) 1
b) 2
c) −1
d) 0
e) 3
f) −2
g) 1
h) 2

12) a) $f(n) = n - 1$
Yes, each n value yeilds a unique output

b) $f(n) = n^2 + 1$
No, $n = 1$ & $n = -1$ both yeild 2

c) $f(n) = n^3$
Yes, each n value yeilds a unique output

d) $f(n) = \lceil n/2 \rceil$
No $n = 1$ & $n = 2$ both yeild 1

22) Determine whether each of these functions is a bijection from R to R

a) $f(x) = -3x+4$

Yes, each x yeilds a unique result & this function has no restrictions on the codomain

b) $f(x) = -3x^2+7$

No, $x=1$ $x=-1$ both yeild 4 (not one-to-one)

$\boxed{\text{Ch 2.4 \quad 1, 8, 17abc, 32, 34}}$

1) a) $a_0 = 2\cdot(-3)^0 + 5^0 = 3$ c) $a_4 = 2\cdot(-3)^4 + 5^4 = 787$

b) $a_1 = 2\cdot(-3)^1 + 5^1 = -1$ d) $a_5 = 2\cdot(-3)^5 + 5^5 = 2639$

8) Find 3 sequences beginning w/ 3, 5, 7

1) $2n+1$ $n(1) = 2(1)+1 = 3$ $n(2) = 2(2)+1 = 5$ $n(3) = 2(3)+1 = 7$

2) $(a_{n+1})^2 - (a_n)^2$ $n(1) = (2)^2 - (1)^2 = 3$ $n(2) = 3^2 - 2^2 = 5$ $n(3) = 4^2 - 3^2 = 7$

17)

a) $a_n = 3a_{n-1}$

$= 3(3a_{n-2}) = 3^2 a_{n-2}$

$= 3^2(3a_{n-2}) = 3^3 a_{n-3}$

$\vdots$

$= 3^n a_{n-n} = 3^n a_0 = 3^n \cdot 2$

b) $a_n = 2a_{n-1}$

$= 2 + (2 + a_{n-2}) = (2+2) + a_{n-2} = (2 \cdot 2) + a_{n-2}$

$= (2 \cdot 2) + (2 + a_{n-3}) = (3 \cdot 2) + a_{n-3}$

$= (n \cdot 2) + a_{n-n} = (3 \cdot 2) + a_{n-3}$

c) $a_n = n + a_{n-1}$

$= n + ((n-1) + a_{n-2}) = (n + (n-1)) + a_{n-2}$

$= (n + (n-1)) + ((n-2) + a_{n-3}) = (n + (n-1) + (n-2)) + a_{n-3}$

$\vdots$

$= (n + (n-1) + (n-2) + \dots + (n-(n-1))) + a_{n-n}$

$= (n + (n-1) + (n-2) + \dots + 1) + a_0$

$= \frac{n(n+1)}{2} + 1 = \frac{n^2+n+2}{2}$

(32)

a) $$\sum_{j=0}^{8} (1+(-1)^j) = (1+(-1)^0) + (1+(-1)^1) + (1+(-1)^2) + (1+(-1)^3) + \dots +$$

$$= \overset{0}{(1+1)} + \overset{1}{(1-1)} + \overset{2}{(1+1)} + \overset{3}{(1-1)} + \overset{4}{(1+1)} +$$

$$\overset{5}{(1-1)} + \overset{6}{(1+1)} + \overset{7}{(1-1)} + \overset{8}{(1+1)}$$

$$= 5$$

b) $$\sum_{j=0}^{8} (3^j - 2^j) = (3^0-2^0) + (3^1-2^1) + (3^2-2^2) + (3^3-2^3) + (3^4-2^4)$$

$$+ (3^5-2^5) + (3^6-2^6) + (3^7-2^7) + (3^8-2^8)$$

$$= 0 + 1 + 5 + 19 + (81-16) + (243-32)$$

$$+ (729-64) + (2187-128) + (6561-256)$$

$$= 0+1+5+19+65+211+665+2059+6305$$

$$= 9330$$

c) $$\sum_{j=0}^{8} (2\cdot 3^j + 3\cdot 2^j) = (2\cdot 1 + 3\cdot 1) + (2\cdot 3 + 3\cdot 2) + (2\cdot 9 + 3\cdot 4) +$$

$$(2\cdot 27 + 3\cdot 8) + (2\cdot 81 + 3\cdot 16) + (2\cdot 243 + 3\cdot 32) +$$

$$(2\cdot 729 + 3\cdot 64) + (2\cdot 2187 + 3\cdot 128)$$

$$= (5) + (12) + (18+12) +$$

$$(54+24) + (162+48) + (486+96)$$

$$(1458+192) + (4374+256)$$

$$= 5 + 12 + (30) + (78) + (210) + (582) +$$

$$(1650) + (4630)$$

$$= 7197$$

d) $\sum_{j=0}^{8} (2^{j+1} - 2^{j}) = (2^1-2^0)+(2^2-2^1)+(2^3-2^2)+(2^4+2^3)+(2^5-2^4)$

$(2^6-2^5)+(2^7-2^6)+(2^8-2^7)$

$= (1)+(2)+(4)+(8)+(16)+(32)+(64)+(128)$

$= 255$

34) a) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j) = \sum_{i=1}^{2} (i+1)+(i+2)+(i+3)$

$=(2)+(3)+(4)+(3)+(4)+(5)$

$=21$

b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i+2j) = \sum_{i=0}^{3} (3i+2)(3i+4)$

$= (3(1)+2)+(3(2)+4)+(3(2)+2)+(3(2)+4)+(3(3)+2)+(3(3)+4$

$= 5+10+8+10+11+13$

$= 57$

c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j = \sum_{i=1}^{3} 0+1+2$

$= [0+0+0]+[1+1+1]+[2+2+2]$

$= 3+6$

$= 9$

d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^2 = \sum_{i=0}^{2} i^2 0^2 + i^2 1^2 + i^2 2^2 + i^2 3^2$

$= [0+0+0+0]+[0+1+4+9]+[0+4+16+36]$

$=70$

Ch 4.1 9abc, 13abc, 21, 23, 31

9 a) $19/7 = \boxed{2 \text{ r } 5}$

b) $11 \overline{)-111}$ = $\boxed{1 \text{ r } 1}$

1 R1

111

001

c) $23 \overline{)789}$ 34 R7 = $\boxed{34 \text{ r } 7}$

69

99

92

7

13 a) $c \equiv 9 * 4 \pmod{13} = 36 \bmod 13 = \boxed{10}$

$13 \overline{)36}$ 2 R10

26

10

b) $c \equiv 11 * 9 \bmod 13 = 99 \bmod 13 = \boxed{8}$

$13 \overline{)99}$ 7 R8

91

8

c) $c \equiv 4 \bmod 13 + 9 \bmod 13 \pmod{13}$

$= 4 + 9 \pmod{13}$

$= 13 \bmod 13$

$= \boxed{0}$

21) a) $13 \bmod 3$

$$\begin{array}{r} 4 \\ 3\overline{)13} \\ \underline{12} \\ 1 \end{array}$$

$\boxed{1}$

b) $-97 \bmod 11$

$-97 = 11 \cdot (-9) + 2$

$-97 \bmod 11 = \boxed{2}$

c) $155 = 19 \cdot 8 + 3$

$155 \bmod 19 = \boxed{3}$

d) $-221 \bmod 23$

$-221 = 23 \cdot (-10) + 9$

$-221 \bmod 23$ $\boxed{9}$

23) a) $228/119 = \boxed{1 \text{ R } 109}$

b) $9009/223 = \boxed{40 \text{ R } 89}$

c) $-10101/333 = \boxed{-31 \text{ R } 222}$

d) $-765432/38271 = \boxed{-21 \text{ R } 38259}$

33) a) $(99^2 \bmod 32)^3 \bmod 15 = (3^2 \bmod 32)^3 \bmod 15 = 9^3 \bmod 15 = 729 \bmod 15 = \boxed{9}$

$$\begin{array}{r} 4 \\ 17\overline{)81} \\ \underline{68} \\ 13 \end{array}$$

b) $(3^4 \bmod 17)^2 \bmod 11 = (81 \bmod 17)^2 \bmod 11 = 13^2 \bmod 11 = 2^2 \bmod 11 = 4$

c) $(19^3 \bmod 23)^2 \bmod 31 = ((-4)^3 \bmod 23)^2 \bmod 31 = (-64 \bmod 23)^2 \bmod 31$

$= 5^2 \bmod 31 = 25$

d) $(89^3 \bmod 79)^4 \bmod 26 = (10^3 \bmod 79)^4 \bmod 26$

$= (100 \bmod 79)^4 \bmod 26 = 52^4 \bmod 26 = 0 \bmod 26 = 0$