MATH 2418: Linear Algebra

Assignment 4

Due: February 10, 2016 Term: Spring, 2016

Recommended Text Book Problems (do not turn in): [Section 1.7: #1, 3, 5, 7, 9, 11, 13, 19, 23, 25, 27, 29, 31]; [Section 1.8: #1, 3, 5, 7, 9, 13, 15, 19, 21, 23, 27, 29, 31].

1. (a) [2 points] Let $A = \begin{bmatrix} 4-x & 0 & x^2-25 \\ 0 & 2x^2+1 & 0 \\ 0 & 0 & x-5 \end{bmatrix}$. Find all values of $x \in \mathbb{R}$ for which A is diagonal.

   (b) [4 points] Let $B = \begin{bmatrix} 1 & 0 & x \\ 0 & x^2+4 & 0 \\ 0 & 0 & x-x^3 \end{bmatrix}$. Find all values of $x \in \mathbb{R}$ for which B is diagonal and invertible.

   (c) [4 points] Let $A = \begin{bmatrix} x^2-x & 0 & 0 \\ 0 & x^2-9 & 0 \\ 0 & 0 & x^3-8 \end{bmatrix}$. Find all values of $x \in \mathbb{R}$ for which A is singular, i.e., non-invertible.

2. (a) [4 points] Let $Q = \begin{bmatrix} x & x^2 - 25 & 1 \\ x^2 - y & 2y & 4 \\ y & x - 5 & 3 + y \end{bmatrix}$. Find all values of $(x, y) \in \mathbb{R}^2$ for which Q is symmetric.

(b) [6 points] Let $A = \begin{bmatrix} 3 & a + 2b + c & 3a - 2c \\ 1 & 8 & b + 2c \\ -4 & 7 & -2 \end{bmatrix}$. Find all values of $(a, b, c) \in \mathbb{R}^3$ such that A is symmetric.

3. (a) [23 points] Consider the transformation $F(< x_1, x_2 >) =< 4x_1,\ -5x_2,\ x_1 - 2x_2,\ 8x_1 - 4x_2 >$. Is it linear? Find the domain and codomain of F.

(b) [4 points] Let $Q : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with $Q(\mathbf{e}_1) = (1, 2, 3)$ and $Q(\mathbf{e}_2) = (-3, -1, 4)$. Find $Q(< 5, -3 >)$. (Hint: recall that $\mathbf{e}_1$ and $\mathbf{e}_2$ form the standard basis for $\mathbb{R}^2$.)

(c) [4 points] Consider the linear transformation

$$T(x_1, x_2, x_3) = (2x_1 - 3x_3,\ 5x_2 + 7x_3,\ 9x_1 - 4x_2 + x_3,\ 8x_2 - 6x_3).$$

Find the standard matrix for T.

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation such that

$$T(2,1) = (2,-3,5), \quad \text{and} \quad T(1,1) = (4,7,2).$$

(a) [2 points] Find $T(-6,-3)$.

(b) [3 points] Find $T(3,2)$.

(c) [5 points] Find $T(3,-2)$.

5. [10 points] True or False.

(a) **T** **F**: If A and B are both diagonal $n \times n$ matrices, then so is AB.

(b) **T** **F**: If A and B are both symmetric $n \times n$ matrices, then so is AB.

(c) **T** **F**: If A and B are $n \times n$ matrices such that $A + B$ is symmetric, then A and B are also symmetric.

(d) **T** **F**: If A and B are $n \times n$ matrices such that $A + B$ is upper triangular, then A and B are also upper triangular.

(e) **T** **F**: For any diagonal matrix A, the linear system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

(f) **T** **F**: For *every* linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$, $T(\mathbf{0}) = \mathbf{0}$.

(g) **T** **F**: If $T_A\colon \mathbb{R}^3 \to \mathbb{R}^5$ is the matrix transformation associated with a matrix A, then A is a 3×5 matrix.

(h) **T** **F**: If a matrix transformation $T_A\colon \mathbb{R}^n \to \mathbb{R}^m$ satisfies $T_A(\mathbf{x}) = \mathbf{0}$ for every $\mathbf{x}$ in $\mathbb{R}^n$, then A is the $m \times n$ zero matrix.

(i) **T** **F**: There is at least one linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$ for which $T(3\mathbf{x}) = 5T(\mathbf{x})$ for **some** vector $\mathbf{x}$ in $\mathbb{R}^n$.

(j) **T** **F**: If the matrix transformation $T_A\colon \mathbb{R}^n \to \mathbb{R}^m$ associated with a matrix A satisfies $T_A(\mathbf{x}) = T_A(-\mathbf{x})$ for **every** vector $\mathbf{x}$ in $\mathbb{R}^n$, then A is the $m \times n$ zero matrix.