Discrete Mathematics for Computing

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Ch 1.3 Propositional Equivalences

- Motivation: In Mathematical Arguments replace a statement with another statement with the same truth value
- Compound Propositions with the same truth value
- Classification of Compound Propositions

  Always true – TAUTOLOGY

  Always false – CONTRADICTION

  Niether TAUTOLOGY , CONTRADICTION - CONTINGENCY

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Tautology and Contradiction

- Example
- TRUTH TABLE

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$

Tautology and Contradiction

- TRUTH TABLE

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F		
F	T		

Tautology and Contradiction

- TRUTH TABLE

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	
F	T	T	

Tautology and Contradiction

- TRUTH TABLE

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

$p \vee \neg p$ TAUTOLOGY

$p \wedge \neg p$ CONTRADICTION

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Logical Equivalences

- Logical Equivalence

  Compound Propositions p and q are equivalent

  $p \leftrightarrow q$ is a TAUTOLOGY

- Denoted by $\equiv$

Logical Equivalences

- Example 1: Verify the logical equivalence of Distributive Law of disjunction over Conjunction

$$\underbrace{p \vee (q \wedge r)}_{\text{Truth Table}} \equiv \underbrace{(p \vee q) \wedge (p \vee r)}_{\text{Truth Table}}$$

Logical Equivalences

- Distributive Law $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$

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Logical Equivalences

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Logical Equivalences

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T				
T	T	F	F				
T	F	T	F				
T	F	F	F				
F	T	T	T				
F	T	F	F				
F	F	T	F				
F	F	F	F				

Logical Equivalences

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T			
T	T	F	F	T			
T	F	T	F	T			
T	F	F	F	T			
F	T	T	T	T			
F	T	F	F	F			
F	F	T	F	F			
F	F	F	F	F			

Logical Equivalences

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T		
T	T	F	F	T	T		
T	F	T	F	T	T		
T	F	F	F	T	T		
F	T	T	T	T	T		
F	T	F	F	F	T		
F	F	T	F	F	F		
F	F	F	F	F	F		

Logical Equivalences

TRUTH TABLE

p	*q*	*r*	*q ^ r*	*p ˅ (q ^ r)*	*p ˅ q*	*p ˅ r*	*(p ˅ q) ^ (p ˅ r)*
T	T	T	T	T	T	T	
T	T	F	F	T	T	T	
T	F	T	F	T	T	T	
T	F	F	F	T	T	T	
F	T	T	T	T	T	T	
F	T	F	F	F	T	F	
F	F	T	F	F	F	T	
F	F	F	F	F	F	F	

Logical Equivalences

TRUTH TABLE

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

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Logical Equivalences

- Table 6, Page 27 -> More Logical Equivalences
- De Morgan’s Laws

English Mathematician Augustus De Morgan

$$\boxed{\neg(p \wedge q) \equiv \neg p \vee \neg q}$$

$$\boxed{\neg(p \vee q) \equiv \neg p \wedge \neg q}$$

De Morgan's Law

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- TRUTH TABLE

p	*q*	*p* ᵛ *q*	¬(*p* ᵛ *q*)	¬*p*	¬q	¬*p* ^ ¬q

De Morgan's Law

- TRUTH TABLE

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					

De Morgan's Law

- TRUTH TABLE

p	*q*	*p* ᵛ *q*	¬(*p* ᵛ *q*)	¬*p*	¬q	¬*p* ^ ¬q
T	T	T				
T	F	T				
F	T	T				
F	F	F				

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De Morgan's Law

- TRUTH TABLE

p	*q*	*p* ᵛ *q*	¬(*p* ᵛ *q*)	¬*p*	¬q	¬*p* ^ ¬q
T	T	T	F			
T	F	T	F			
F	T	T	F			
F	F	F	T			

De Morgan's Law

- TRUTH TABLE

p	*q*	*p* ᵛ *q*	¬(*p* ᵛ *q*)	¬*p*	¬q	¬*p* ^ ¬q
T	T	T	F	F		
T	F	T	F	F		
F	T	T	F	T		
F	F	F	T	T		

De Morgan's Law

- TRUTH TABLE

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	
T	F	T	F	F	T	
F	T	T	F	T	F	
F	F	F	T	T	T	

De Morgan's Law

- TRUTH TABLE

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Using De Morgan's Laws

- Example: Using De Morgan's Laws, express the negation of

  "Heather will go to the concert or Steve will go to the concert."

Using De Morgan's Laws

- Let p -> "Heather will go to the concert"
- Let q -> "Steve will go to the concert"
- Given English statement can be represented as

$$p \vee q$$

- By second De Morgan's law

$$\boxed{\neg(p \vee q) \equiv \neg p \wedge \neg q}$$

"Heather will not go to the concert and Steve will not go to the concert."

Constructing New Logical Equivalences

- Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.
- Using logical identities, build new logical identities
- Compound Propositions with a large number of variables

Constructing New Logical Equivalences

- $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$ from table 7

  $\equiv \neg(\neg p) \wedge \neg q$ by second De Morgan's law

  $\equiv p \wedge \neg q$ by double negation law

Logical Equivalences

- Show that this conditional statement is a tautology.

$$[\neg p \wedge (p \vee q)] \rightarrow q$$

p	*q*	¬*p*	*p* ∨ *q*	¬*p* ^ (*p* ∨ *q*)	[¬*p* ^ (*p* ∨ *q*)]→*q*

Logical Equivalences

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T				
T	F				
F	T				
F	F				

Logical Equivalences

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

Logical Equivalences

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T		
T	F	F	T		
F	T	T	T		
F	F	T	F		

Logical Equivalences

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	
T	F	F	T	F	
F	T	T	T	T	
F	F	T	F	F	

Logical Equivalences

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T