MATH 2418: Linear Algebra

Assignment 8

Due : $\pi + (0, 9, 0)$ Term `Spring, 2016`

Recommended Text Book Problems (do not turn in): [Sec 4.6: # 3, 5, 7, 9, 15]; [Sec 4.7: # 1, 3, 5, 7, 9, 11, 13, 17];

---

1. Consider the bases $\mathcal{B} = \{(-2, 1), (1, 2)\}$ and $\mathcal{B}' = \{(1, 1), (-1, 2)\}$ of $\mathbb{R}^2$,
   (a) Find the transition matrix $P_{\mathcal{B}' \to \mathcal{B}}$ from $\mathcal{B}'$ to $\mathcal{B}$.
   (b) Find the transition matrix $P_{\mathcal{B} \to \mathcal{B}'}$ from $\mathcal{B}$ to $\mathcal{B}'$.
   (c) For $\mathbf{v} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, compute $[\mathbf{v}]_{\mathcal{B}}$ and use part (a) or (b) to compute $[\mathbf{v}]_{\mathcal{B}'}$.
   (d) Compute $[\mathbf{v}]_{\mathcal{B}'}$ directly.
   (e) Prove that $\big(P_{\mathcal{B}' \to \mathcal{B}}\big)\big(P_{\mathcal{B} \to \mathcal{B}'}\big) = I$.

2. Let $\mathcal{B} = \{(1,3,1),(2,5,0),(3,0,8)\}$ and $\mathcal{B}' = \{(1,1,1),(2,3,0),(3,0,3)\}$ be two a bases of $\mathbb{R}^3$

   (a) Use the four step procedure to find the transition matrix $P_{\mathcal{B}\to\mathcal{B}'}$.

   (b) Find $P_{\mathcal{B}'\to\mathcal{B}}$.

   (c) If $[\mathbf{w}]_{\mathcal{B}} = (5,-3,1)$, find $[\mathbf{w}]_{\mathcal{B}'}$.

3. Let $\mathcal{S} = \{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$.

   (a) Find the basis $\mathcal{B}$ of $\mathbb{R}^3$ so that $P_{\mathcal{B}\to\mathcal{S}} = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 0 & 3 \\ 0 & 2 & 3 \end{bmatrix}$.

   (b) Find the basis $\mathcal{B}$ of $\mathbb{R}^3$ so that $P_{\mathcal{S}\to\mathcal{B}} = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 0 & 3 \\ 0 & 2 & 3 \end{bmatrix}$.

4. Given $A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & 7 & 9 & -1 & 8 & 2 \\ -3 & 9 & -12 & 7 & 9 & 7 \\ -11 & 33 & -44 & 22 & -55 & -44 \end{bmatrix}$.

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

5. Given the linear system $A\mathbf{x} = \mathbf{b}$:

$$\begin{cases} \quad x_1 + 2x_2 - 3x_3 + x_4 = 4, \\ -2x_1 + x_2 + 2x_3 + x_4 = -1, \\ \ -x_1 + 3x_2 - x_3 + 2x_4 = 3, \\ \ 4x_1 - 7x_2 \qquad - 5x_4 = -5. \end{cases}$$

(a) Find the vector form of the general solution of $A\mathbf{x} = \mathbf{b}$.

(b) Find the vector form of the general solution of $A\mathbf{x} = \mathbf{0}$.

6. Let $\mathbf{v}_1 = (1, -2, 3, 2), \mathbf{v}_2 = (-2, 5, 1, 0), \mathbf{v}_3 = (3, -9, 9, 4), \mathbf{v}_4 = (-5, 2, 3, 4), \mathbf{v}_5 = (0, 8, 2, -3)$ be vectors in $\mathbb{R}^4$. Find a basis for the Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$. Also express each vector as a linear combination of basis vectors.

7. True or False.

(a) **T** **F**: If $A\mathbf{x} = \mathbf{b}$ then $\mathbf{b}$ is in the column space of A.

(b) **T** **F**: The column space of matrix A is the solution set of $A\mathbf{x} = \mathbf{b}$.

(c) **T** **F**: The system $A^T\mathbf{x} = \mathbf{b}$ is inconsistent if and only if $\mathbf{b}$ is not in the row space of A.

(d) **T** **F**: Let A be an $n \times n$ square matrix with $\det(A) \neq 0$, then $A = P_{\mathcal{B}\to\mathcal{B}'}$ for some bases $\mathcal{B}$ and $\mathcal{B}'$ of $\mathbb{R}^n$.

(e) **T** **F**: If $P_{\mathcal{B}\to\mathcal{B}'}$ is a diagonal matrix then each vector in basis $\mathcal{B}$ is a multiple of some vector in $\mathcal{B}'$.

(f) **T** **F**: If A is a transition matrix, then so is A^k, for any integer k.

(g) **T** **F**: Let E be an $m \times m$ elementary matrix and A be an $m \times n$ matrix, then EA and A have same column space.