Discrete Mathematics for Computing

UTD

Chapter 1.1: Logic and Proofs

What are Propositions?

- Basic building blocks of logic
- Declarative sentence – either true or false, but not both

Propositions: Recap

- Examples

  What time is it?

Not a declarative sentence, hence not a proposition.

Propositions: Recap

- Examples

$x + 1 = 2$

Neither true or false, not a proposition

Propositional Logic

- Propositional Variables represent propositions
- Conventional Letters p, q, r, s, …

Propositional Logic

- Truth value of a proposition

  'T' - true

  'F' - false

- Truth table – inputs and outputs

Negation

TRUTH TABLE

p	$\neg p$
T	F
F	T

Conjunction

- TRUTH TABLE

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

- Example:

  “Today is Friday.”

  “ It is raining today.”

“ Today is Friday and it is raining today.”

When is this proposition true?

When is this proposition false?

Conjunction

- Example:

  "Today is Friday and it is raining today."

True: Rainy Fridays

False: Any day not a Friday, On Fridays when it does not rain.

Conjunction

- Practical Use:

In a programming environment, AND represented by ("**&&**").

```
int a = 7;
int b = 10;
if ( a > 4 && b < 20 ) {
 …
}
```

Disjunction

- Binary operator – Takes two operands
- represented by symbol $\vee$
- propositions p, q

  $p \vee q$ -> "p or q"
- false when both p and q are false
- true otherwise

Disjunction

- TRUTH TABLE

p q	$p \vee q$

Disjunction

- TRUTH TABLE

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

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Disjunction

- TRUTH TABLE

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

- Example:

  “Today is Friday.”

  “ It is raining today.”

Disjunction

- Example:

  " Today is Friday or it is raining today."

When is this proposition true?

When is this proposition false?

Disjunction

- Example:

  " Today is Friday or it is raining today."

True: Any day that is either a Friday or a rainy day (including rainy Fridays)

False: Days that are not Fridays when it also does not rain

Disjunction

- Practical Use:

In Java programming, OR represented by ("||").

```
int a = 10;
int b = 40;
if ( a == 7 || b > a ) {
  …
}
```

Exclusive Or

- Binary operator – Takes two operands
- represented by symbol $\oplus$
- propositions p, q

  $p \oplus q$ -> "p exclusive or q"
- true when exactly one of p and q is true
- false otherwise

Exclusive Or

- TRUTH TABLE

p	q	$p \oplus q$

Exclusive Or

- TRUTH TABLE

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Exclusive Or

- TRUTH TABLE

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Exclusive Or

- Practical Use:

In Java programming, EXCLUSIVE OR represented by (“**^**").

```
int a = 7;
int b = 10;
if ( b > a ^ b == 10 ) {
  …
}
```

Conditional Statements

- propositions p, q

  Conditional Statement

  $p \rightarrow q$ "if p, then q"

- q is true on the condition that p holds
- False: p is true, q is false
- True otherwise
- p is "hypothesis", q is "conclusion"

Conditional Statements

- TRUTH TABLE

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Statements

- Example:

  "I am elected."

  "I will lower taxes."

Conditional Statements

- Example:

"If I am elected, then I will lower taxes."

True: If the politician is not elected, then there is no expectation that this person will lower taxes.

False: If the politician is elected, but does not lower taxes. This person has broken his pledge.

Biconditionals

- propositions p, q

  Biconditional Statement denoted by $\leftrightarrow$

  $p \leftrightarrow q$ "p if and only if q"

- True

  p and q have the same truth values

  $p \rightarrow q$ is true and $q \rightarrow p$ is true

- False: otherwise

Biconditionals

- TRUTH TABLE

p	q	$p \leftrightarrow q$

Biconditionals

- TRUTH TABLE

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

Biconditionals

- TRUTH TABLE

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditionals

- Example:

  "You can take the flight."

  " You buy a ticket."

Biconditionals

- Example:

“You can take the flight if and only if you buy a ticket.”

True: If you buy a ticket and can take a flight

If you cannot buy a ticket and cannot take a flight

False: If you do not buy a ticket, but you can take the flight

If you buy a ticket and cannot take the flight

Converse, Contrapositive, Inverse

Given Conditional Statement $p \rightarrow q$

- Converse : proposition $q \rightarrow p$
- Contrapositive : proposition $\neg q \rightarrow \neg p$
- Inverse : proposition $\neg p \rightarrow \neg q$
- A conditional statement consists of two parts: a hypothesis in the "if" clause and a conclusion in the "then" clause.

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Converse

To form the converse of the conditional statement, interchange the hypothesis and the conclusion.

Statement: If a person is 18 years old, then he is a legal adult.

Converse: If a person is a legal adult, then he is 18 years old.

Converse

How to Form the Truth Table?

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	¬p	¬q	$\neg p \rightarrow \neg q$	$q \rightarrow p$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

Inverse

To form the inverse of the conditional statement, take the negation of both the hypothesis and the conclusion.

Statement: If a person is 18 years old, then he is a legal adult.

Inverse: If a person is not 18 years old, then he is not a legal adult.

Inverse

How to Form the Truth Table?

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	¬p	¬q	$\neg p \rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive

To form the contrapositive of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement.

Statement: If a person is 18 years old, then he is a legal adult.

Contrapositive: If a person is not a legal adult, then he is not 18 years old.

Contrapositive

How to Form the Truth Table?

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	¬q	¬p	$\neg q \rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Converse, Inverse, Contrapositive

Statement: If it snows tonight, then I will stay at home.

Converse: If I stay home, then it will snow tonight.

Contrapositive: If I do not stay at home, then it will not snow tonight.

Inverse: If it does not snow tonight, then I will not stay home.

Compound Propositions

- Important logical connectives

Conjunctions, Disjunctions, Conditional Statements, Biconditional Statements, Negations

- Compound Propositions – connectives, propositional variables

Compound Propositions

- Example: $(p \vee \neg q) \leftrightarrow (p \wedge q)$

Compound Propositions

- Example: $(p \vee \neg q) \leftrightarrow (p \wedge q)$

TRUTH TABLE

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \leftrightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Compound Propositions

- Find the truth table for $(p \oplus q) \vee (p \oplus \neg q)$
- Find the truth table for $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$

Compound Propositions

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T				
T	F				
F	T				
F	F				

Compound Propositions

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F			
T	F	T			
F	T	T			
F	F	F			

Compound Propositions

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	T		
T	F	T	F		
F	T	T	F		
F	F	F	T		

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Compound Propositions

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	T	T	
T	F	T	F	T	
F	T	T	F	T	
F	F	F	T	T	

Compound Propositions

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	T	T	F
T	F	T	F	T	F
F	T	T	F	T	F
F	F	F	T	T	F

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Compound Propositions

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$

Compound Propositions

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Compound Propositions

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$
T	T	T	T		
T	T	F	T		
T	F	T	T		
T	F	F	T		
F	T	T	T		
F	T	F	T		
F	F	T	F		
F	F	F	F		

Compound Propositions

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$
T	T	T	T	T	
T	T	F	T	T	
T	F	T	T	T	
T	F	F	T	T	
F	T	T	T	T	
F	T	F	T	T	
F	F	T	F	T	
F	F	F	F	F	

Compound Propositions

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	F
F	F	F	F	F	F

Logic and Bit Operations

- What is a bit?

Symbol with two possible values, 0 (zero) and 1(one)

Binary Digit

Can be used to represent a truth value

Logic and Bit Operations

Truth Value	Bit
T	1
F	0

Logic and Bit Operations

- Computer Bit Operations – Logical Connectives

Notation

OR

AND

XOR

Logic and Bit Operations

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Logic and Bit Operations

- Find the bitwise OR, bitwise AND and bitwise XOR of the pair of bit strings

00 0111 0001

10 0100 1000

----

10 0111 1001 **bitwise OR**

Logic and Bit Operations

- Find the bitwise OR, bitwise AND and bitwise XOR of the pair of bit strings

```
 00  0111  0001
 10  0100  1000
---------------------
 00  0100  0000     bitwise AND
```

Logic and Bit Operations

- Find the bitwise OR, bitwise AND and bitwise XOR of the pair of bit strings

00 0111 0001

10 0100 1000

----

10 0011 1001 **bitwise XOR**

Logic and Bit Operations

- Evaluate this expression

$$1\ 1000 \wedge \underbrace{(0\ 1011 \vee 1\ 1011)}$$

$$= \quad 1\ 1000 \wedge 1\ 1011$$

$$= \quad \mathbf{1\ 1000}$$