MATH 2418: Linear Algebra

Assignment 1

Due <u>January 20, 2016</u>   Term <u>Spring, 2016</u>

Recommended Text Book Problems (do not turn in): [Section 1.1: #1, 5, 7, 19, 21]; [Section 1.2: #1, 3, 9, 11, 27].

1. Suppose that the augmented matrix for a linear system has been reduced by row operations into the following matrix. For each of the matrices, i) determine whether or not the given matrix is in **reduced row echelon form**, whether or not in **row echelon form**; ii) solve each of the linear system. (Use x, y, z, u, v for unknowns if necessary)

a) $A = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

b) $B = \begin{bmatrix} 1 & 3 & 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 2 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

c) $C = \begin{bmatrix} 1 & 5 & 0 & 0 & -2 & 3 \\ 0 & 0 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

d) $D = \begin{bmatrix} 1 & 7 & 0 & 0 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2. Solve the following system of linear equations using Gauss-Jordan elimination.

$$\begin{cases} x + 2y - z = 2, \\ 2x + 5y + 2z = -1, \\ 7x + 17y + 5z = -1. \end{cases}$$

3. Solve the following system of linear equations using Gauss-Jordan elimination.

$$\begin{cases} 10z + x = 5, \\ 3x + y - 4z = -1, \\ 4x + y + 6z = 1. \end{cases}$$

4. Find all possible values of $k \in \mathbb{R}$ so that the system of linear equations

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ 2x_1 + 3x_2 + kx_3 = 3 \\ x_1 + kx_2 + 3x_3 = 2 \end{cases}$$

has

i) a unique solution;

ii) no solution;

iii) infinitely many solutions.

Note : The notation $k \in \mathbb{R}$ means that k is **in** the set $\mathbb{R}$ of all real numbers.

5. True or False.

   (a) **T**   **F**: If a matrix is in reduced row echelon form, it is also in row echelon form.

   (b) **T**   **F**: Every matrix has a unique row echelon form.

   (c) **T**   **F**: If a linear system has more unknowns than equations, then it has infinitely many solutions.

   (d) **T**   **F**: A homogeneous linear system is always consistent.

   (e) **T**   **F**: A homogeneous linear system with five unknowns and two nonzero rows in reduced echelon form has three free variables.

   (f) **T**   **F**: A homogeneous linear system with more unknowns than equations has infinitely many solutions.

   (g) **T**   **F**: If a linear system has the trivial solution, then it has no other solutions.

   (h) **T**   **F**: A linear system of two equations with two unknowns always has a unique solution.

   (i) **T**   **F**: The Gauss-Jordan elimination procedure requires only two elementary row operations.

   (j) **T**   **F**: Wilhelm Jordan popularized the method of elimination successfully used by Carl Friedrich Gauss for solving important systems of linear equations.