MATH 2418: Linear Algebra

Assignment 10 (sections 4.10, 5.1, 5,2 and 5.3)

Due April 13, 2016 Term Spring, 2016

Instructions: Submit your work to your TA during the problem session on Wednesday.

Suggested problems: Section 4.10: 1, 3, 5, 7, 11, 19. Section 5.1: 5, 7, 9, 15, 17. Section 5.2: 7, 9, 13, 19,23, 33. Section 5.3: 15, 17, 19, 21, 23, 25.

1. Determine whether the matrix operator $T : R^3 \to R^3$ defined by the equations is one-to-one; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2, w_3)$.

   (a) [5 points]
   $w_1 = x_1 - 2x_2 + 2x_3$
   $w_2 = 2x_1 + x_2 + x_3$
   $w_3 = x_1 + x_2$

   (b) [5 points]
   $w_1 = x_1 - 3x_2 + 4x_3$
   $w_2 = -x_1 + x_2 + x_3$
   $w_3 = -2x_2 + 5x_3$

2. Let T be multiplication by the matrix A:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 20 & 0 & 0 \end{bmatrix}$$

Find:

(a) [4 points] a basis for the range of T.

(b) [4 points] a basis for the kernel of T.

(c) [1 point] the rank and nulity of T.

(d) [1 point] the rank and nulity of A.

3. Find (i) the characteristic equation, (ii) the distinct eigenvalues, and (iii) basis for the eigenspaces of the following matrices

   (a) [4 points] $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

   (b) [6 points] $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

4. [10 points] Find a 3×3 matrix A that has eigenvalues $\lambda_1 = -2$, $\lambda_2 = 2$, $\lambda_3 = 0$, and for which $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ are their respective eigenvectors.

5. Find a matrix P that diagonalizes A.

(a) [4 points] $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

(b) [6 points] $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

6. Find the geometric and algebraic multiplicity of each eigenvalue of A, and determine whether A is diagonalizable. If A is diagonalizable, find P that diagonalizes A.

   (a) [5 points] $A = \begin{bmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{bmatrix}$

   (b) [5 points] $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$

7. [10 points] Compute A^5 for $A = \begin{bmatrix} 1 & 1 & -4 \\ 1 & 1 & -4 \\ 0 & 0 & -2 \end{bmatrix}$.

8. [10 points] Let $A = \begin{bmatrix} -1 & -5 \\ 4 & 7 \end{bmatrix}$. Find an invertible matrix P and $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ with $a, b \in \mathbb{R}$ such that $A = PCP^{-1}$. (Hint: use theorem 5.3.8.)

9. [10 points] True or False. For all subquestions below, assume that A is an $n \times n$ matrix.

(i) **T** **F**: λ is an eigenvalue of A then the linear system $(\lambda I - A)\mathbf{x} = 0$ has only the trivial solution.

(ii) **T** **F**: If λ is an eigenvalue of A, $\mathbf{x}$ is a corresponding eigenvector and s a scalar, then $\lambda - s$ is an eigenvalue of $(A - sI)$.

(iii) **T** **F**: Suppose λ and $\lambda_2 = \lambda/3$ are two distinct eigenvectors of A. Then $\mathbf{x}$ is an eigenvector corresponding to the eigenvalue λ if and only if $3\mathbf{x}$ is an eigenvector corresponding to the eigenvalue λ_2.

(iv) **T** **F**: If A is an $n \times n$ matrix and λ is one of its eigenvalues, then $\text{rank}(\lambda I_n - A) < n$.

(v) **T** **F**: If the column vectors of a square matrix A are linearly independent, then $\mathbf{0}$ is not an eigenvalue of A.

(vi) **T** **F**: Two eigenvectors of a *symmetric* matrix A corresponding to two distinct eigenvalues are orthogonal to each other.

(vii) **T** **F**: If a square matrix A is diagonalizable, then there is a unique matrix P such that $P^{-1}AP$ is diagonal.

(viii) **T** **F**: If a square matrix A is diagonalizable, then so is A^T.

(ix) **T** **F**: If λ is an eigenvalue of a square matrix A, then λ^k must be an eigenvalue of A^k for any positive integer k.

(x) **T** **F**: Two column vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{C}^n$ are complex orthogonal if and only if $\mathbf{u}^T\mathbf{v} = 0$.