Discrete Mathematics for Computing

Ch 10.1 Graphs

What are graphs?

In CS, discrete structures consisting of vertices and edges that connect these vertices.

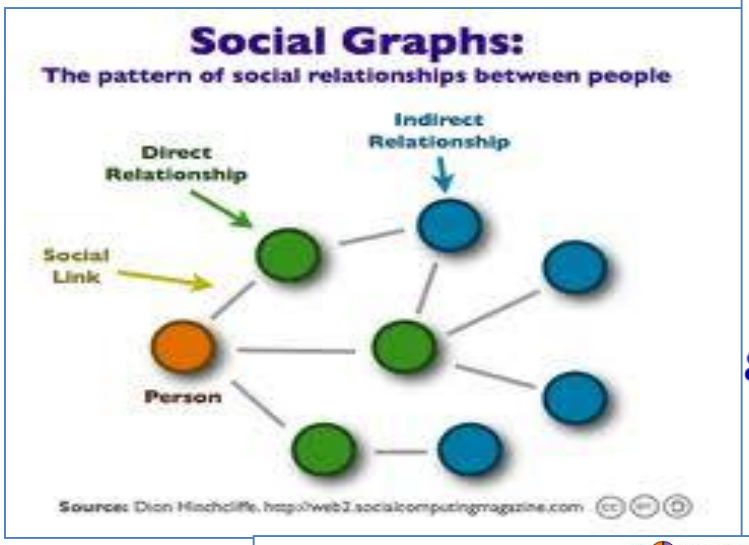
Used in modeling

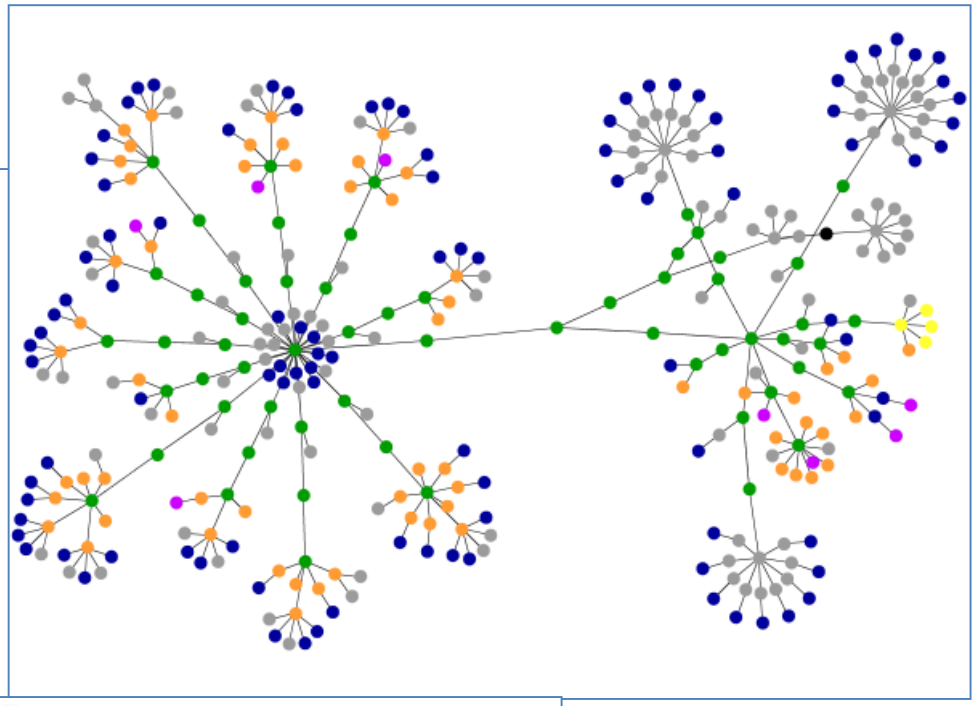
Communication networks

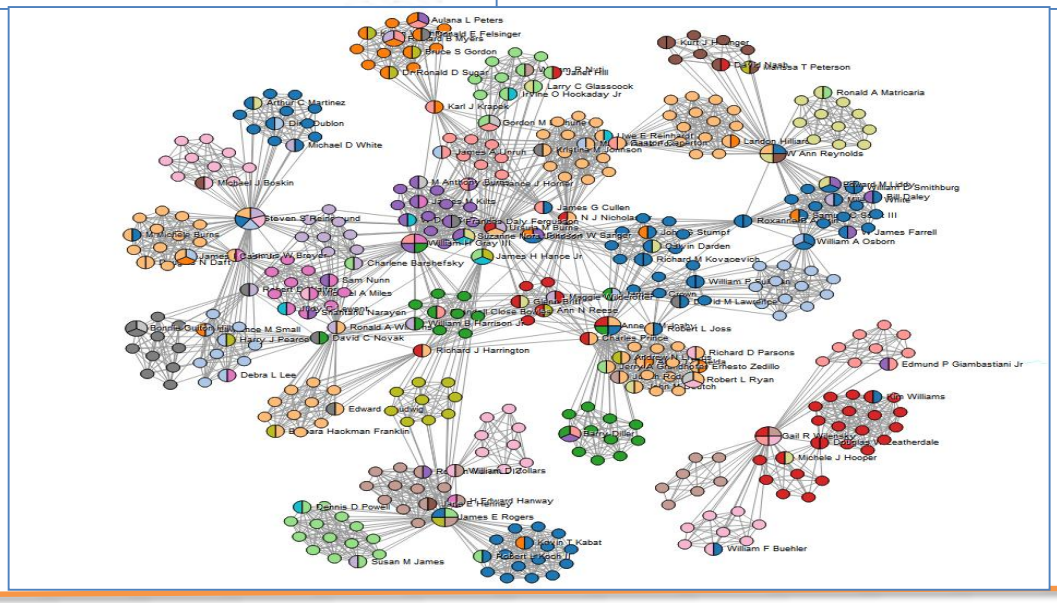
Data organization

Computational devices

Practical Examples







Graphs

- There are 5 main categories of graphs:
  - Simple graph
  - Multigraph
  - Pseudograph
  - Directed graph
  - Directed multigraph

Graphs

- Definition 1 : A simple graph G = (V,E)

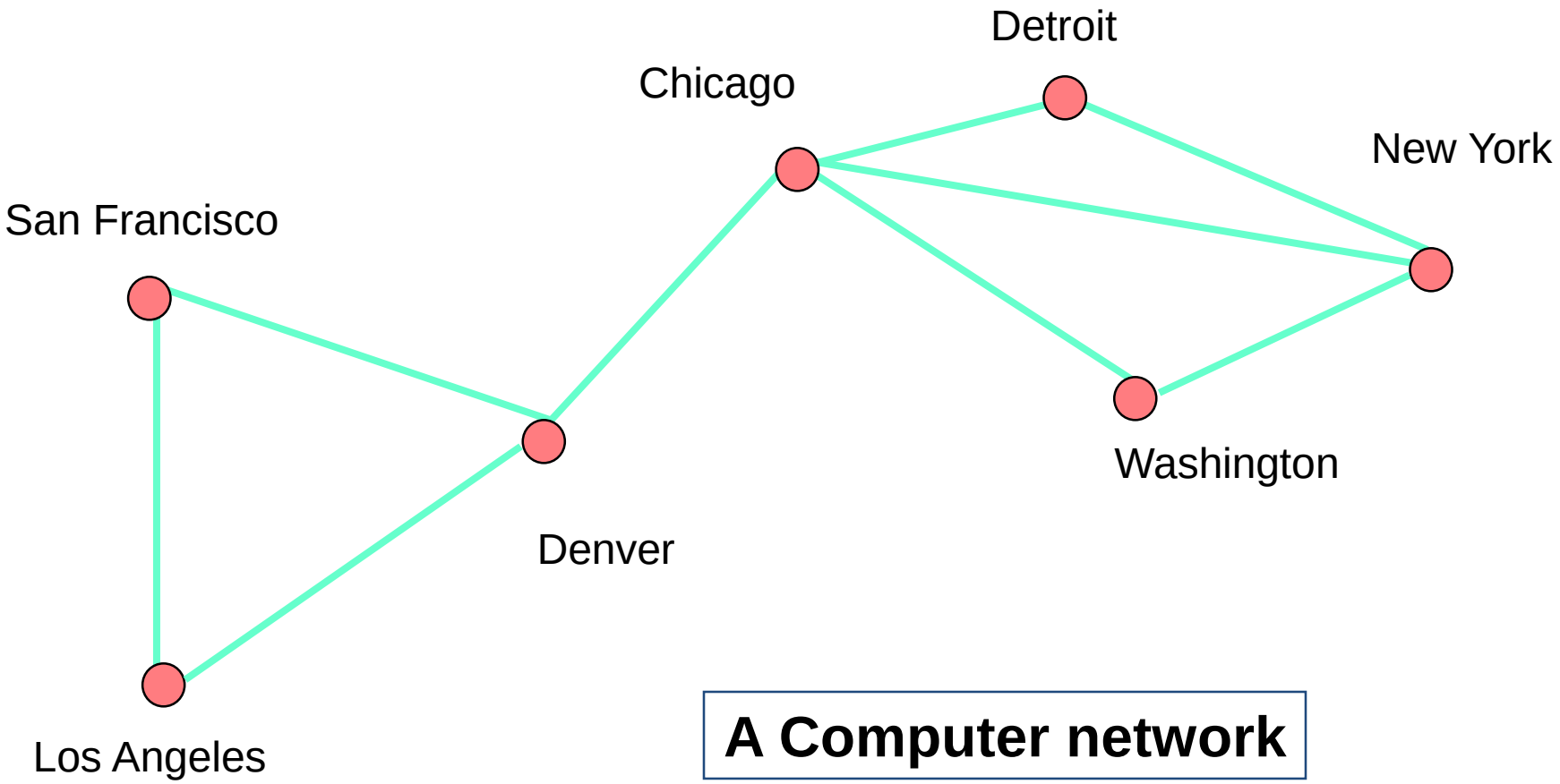
- consists of V, a nonempty set of vertices

- and E, a set of edges

- Each edge has one or two vertices called endpoints

Example: Telephone lines connecting computers in different cities

Graphs



A Computer network

Graphs

- Definition 2: A multi graph G = (V,E)

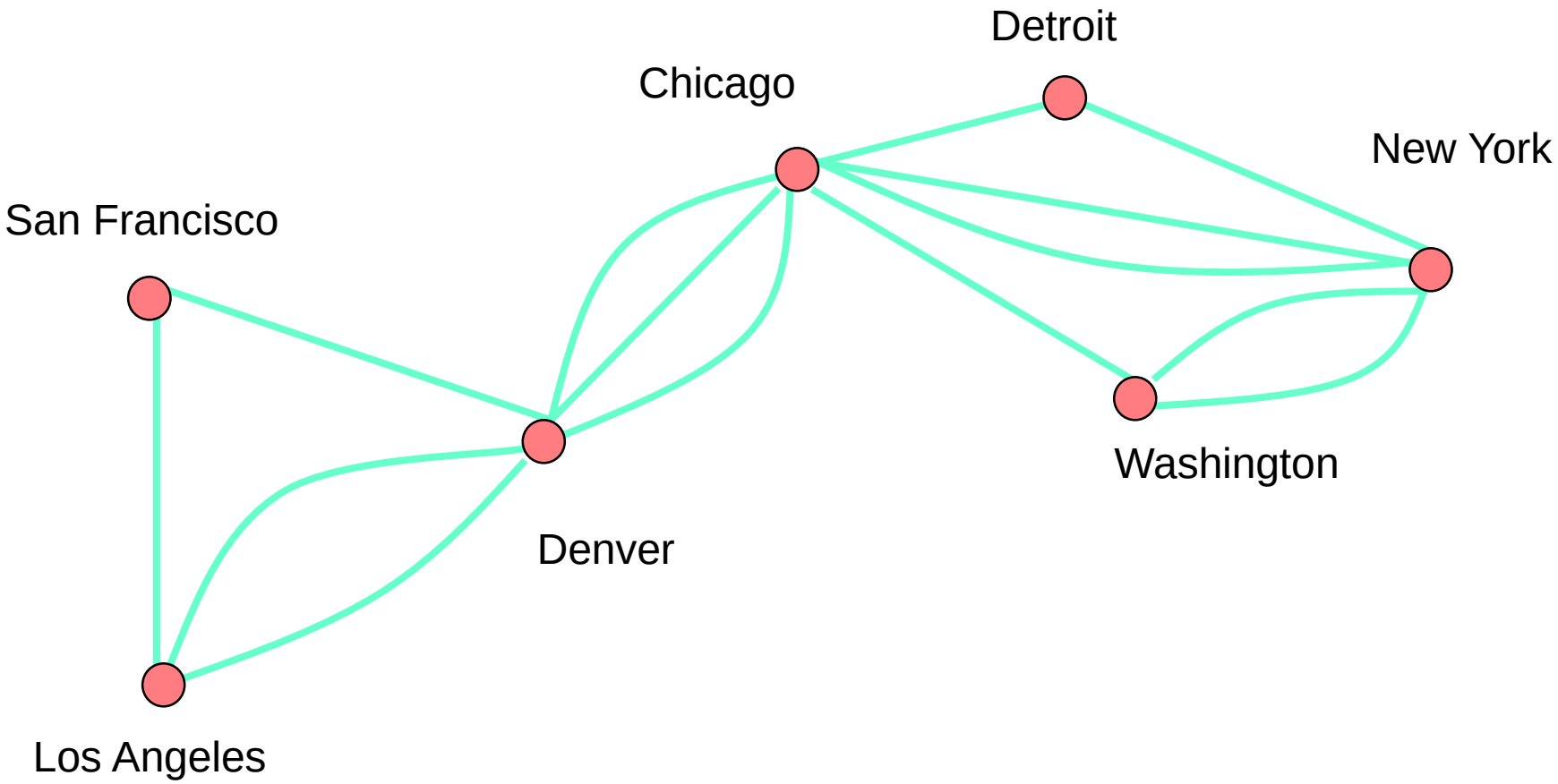
- consists of a set E of edges

- and a function f from E to {{u,v} | u, v $\in$ V, u $\neq$ v}

- The edges e_1 and e_2 are called multiple or parallel edges if $f(e_1) = f(e_2)$

Example: Multiple telephone lines connecting computers in different cities

Graphs



A Computer network with multiple lines

UTD

Graphs

- Definition 3: A pseudograph G = (V,E)

- consists of a set V of vertices

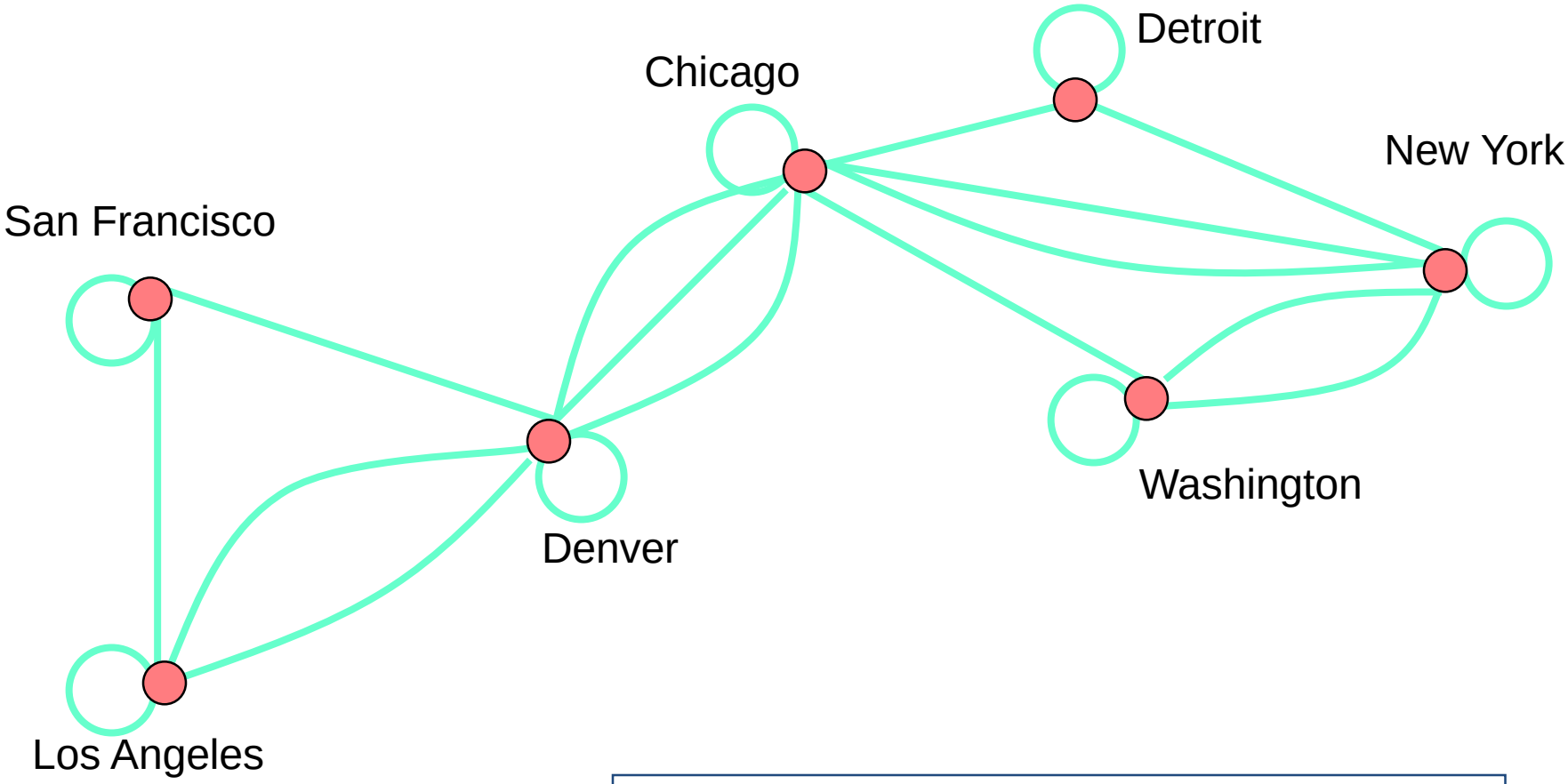
- a set E of edges

- and a function f from E to {{u,v} | u, v $\in$ V}

- An edge is a loop if f(e) = {u,u} = {u} for some u $\in$ V

Example: A computer network with diagnostic links

Graphs



A Computer network with diagnostic links

Graphs

- Definition 4: A directed graph (V,E)

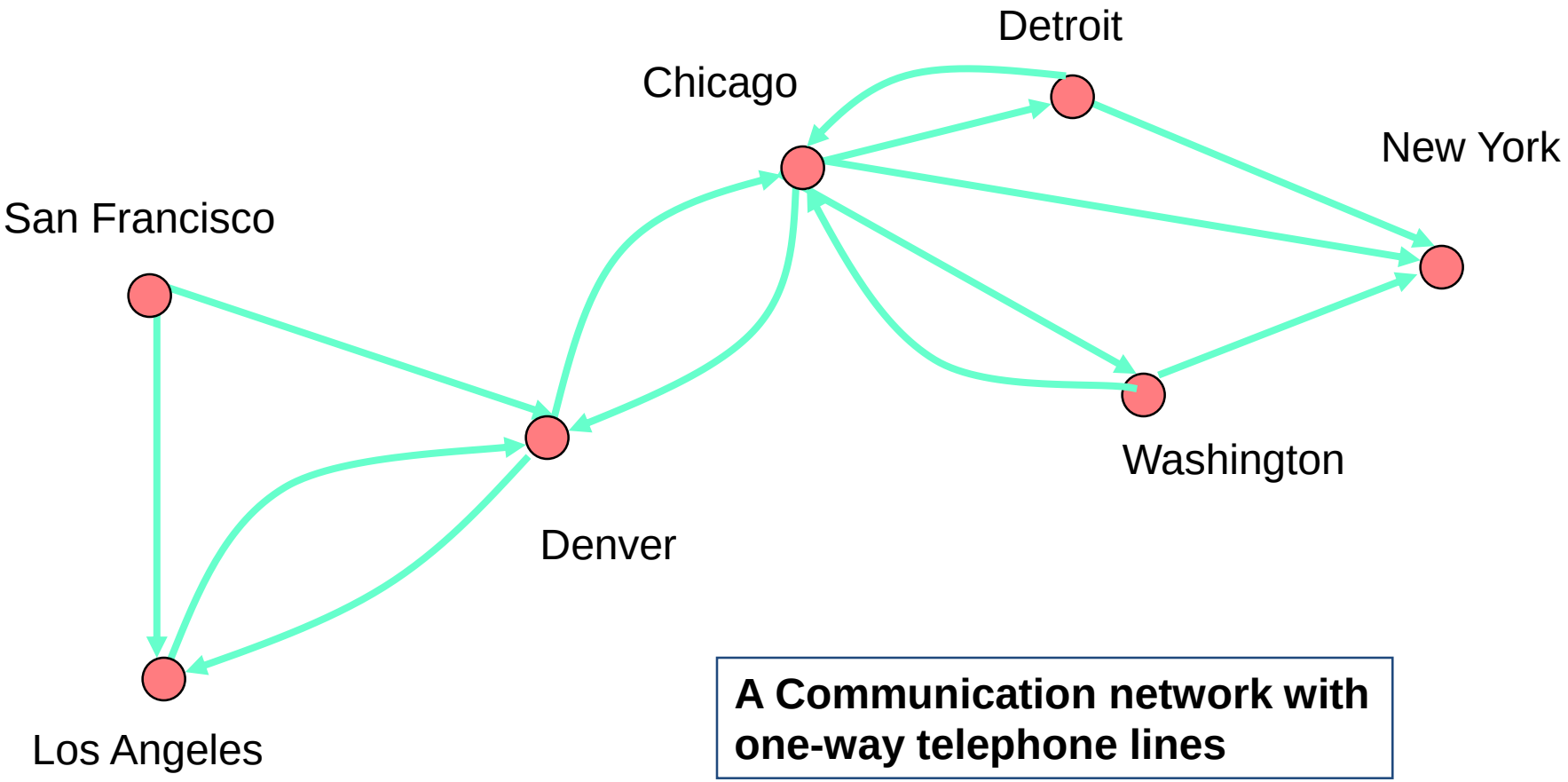
-consists of a set of vertices V

- and a set of edges E

- that are ordered pairs of elements of V

Example: A communications network with One-Way Communication Links

Graphs



A Communication network with one-way telephone lines

This example shows that the host computer can only receive data from other computer, it cannot emit

Graphs

- Definition 5: A directed multigraph G = (V,E)

- consists of a set V of vertices

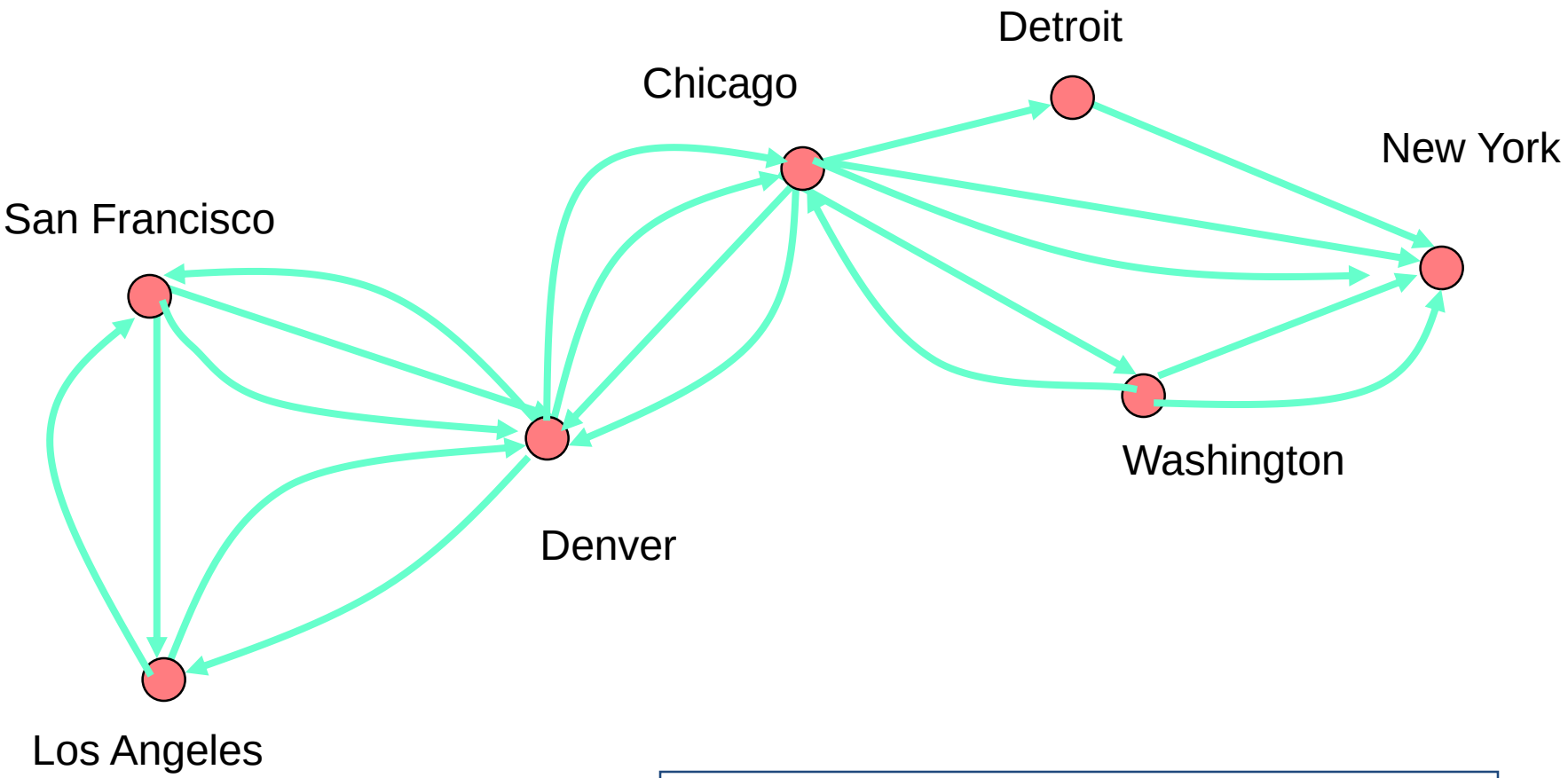
- a set E of edges

- and a function f from E to {{u,v} | u, v $\in$ V}

- The edges e_1 and e_2 are multiple edges if $f(e_1) = f(e_2)$

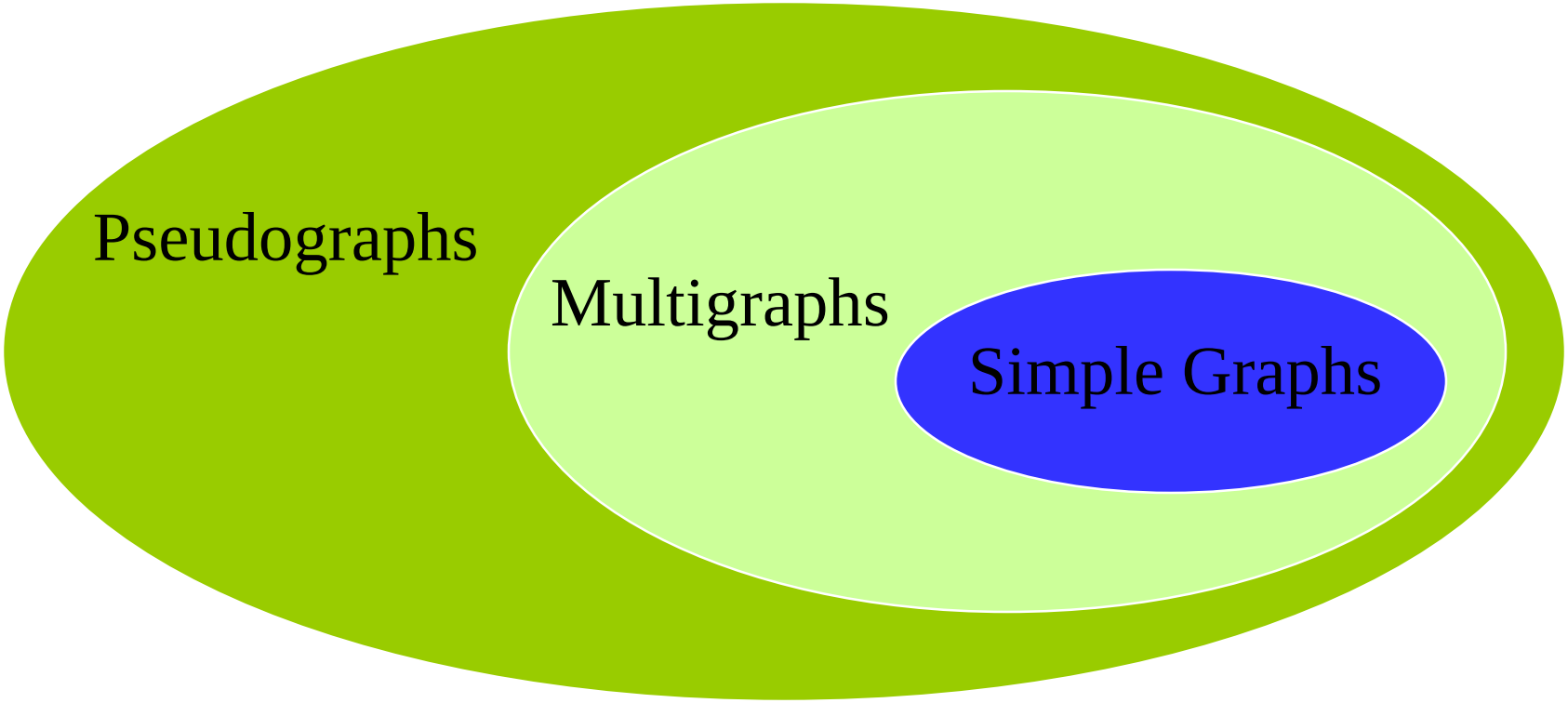
Example: A communications network with Multiple One-way Communication Links

Graphs

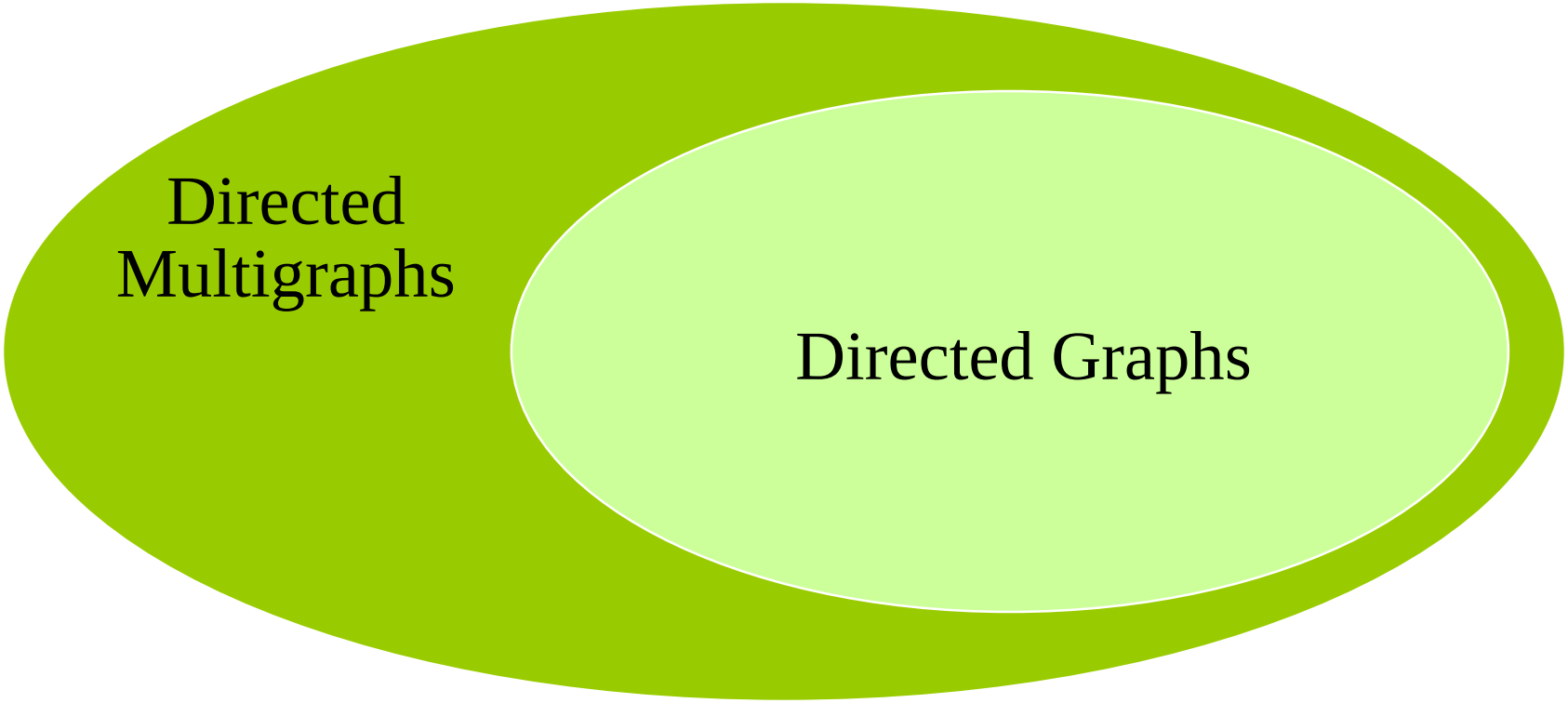


A Computer network with multiple one-way telephone lines

Types of Undirected Graphs



Types of Directed Graphs



Graph Terminology: Summary

- To understand the structure of a graph and to build a graph model, we ask these questions:
  - Are the edges of the graph undirected or directed (or both)?
  - If the edges are undirected, are multiple edges present that connect the same pair of vertices? If the edges are directed, are multiple directed edges present?
  - Are loops present?

TABLE 1 Graph Terminology.

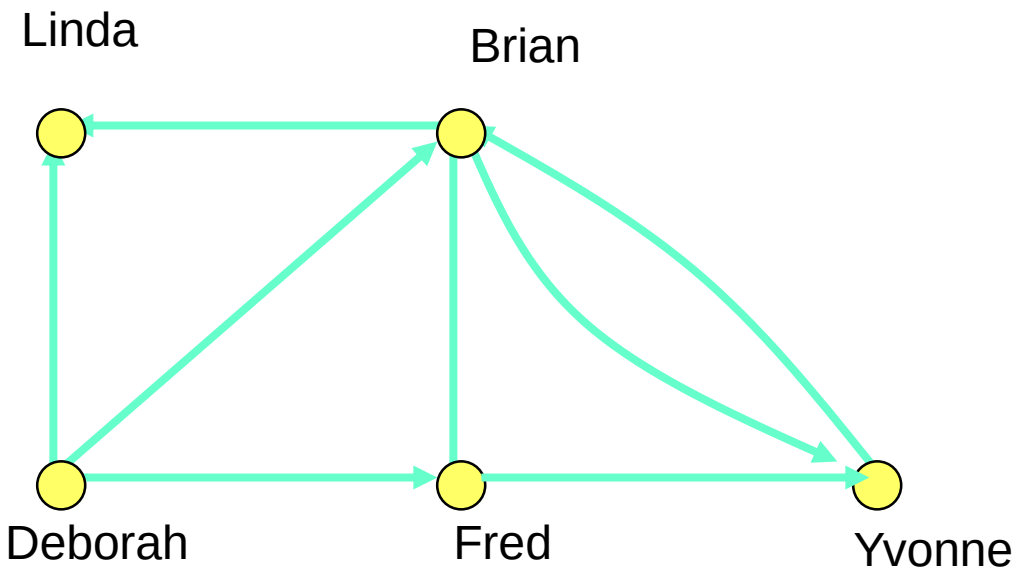
Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Graph Models: Social Networks

- Graphs can be used to model social structures
- *Social network*: vertices represent individuals or organizations, edges represent relationships between them

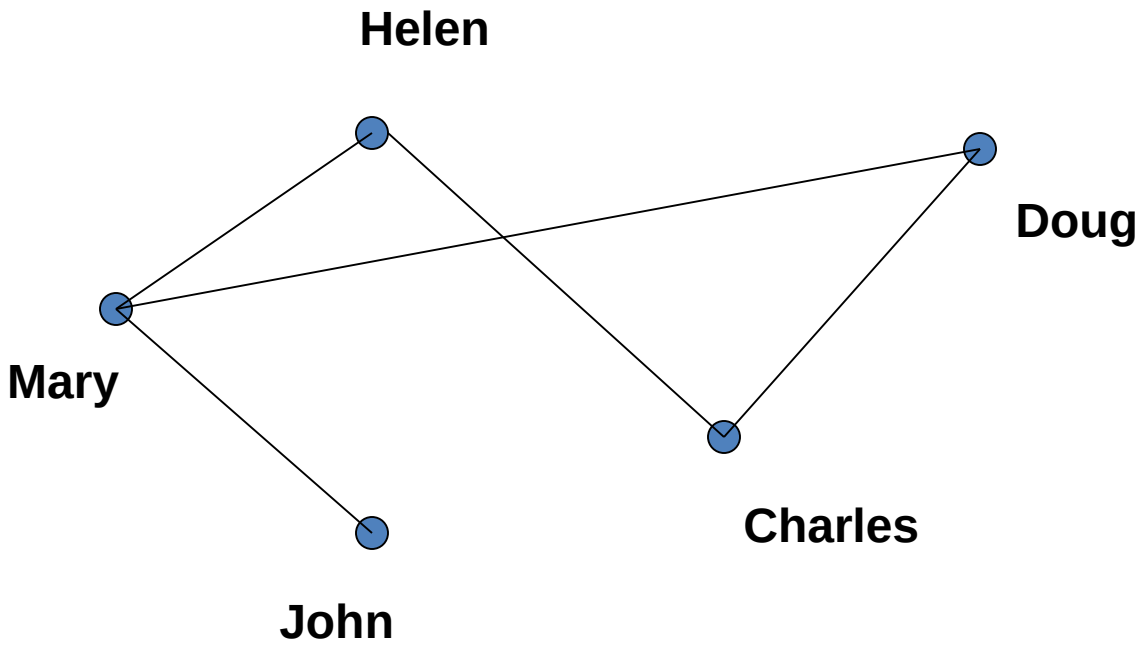
- Useful graph models of social networks include:
  - *friendship graphs* - undirected graphs where two people are connected if they are friends
  - *collaboration graphs* - undirected graphs where two people are connected if they collaborate in a specific way
  - *influence graphs* - directed graphs where there is an edge from one person to another if the first person can influence the second person

Graphs



An influence graph

Graphs



An acquaintanceship graph

Examples of Collaboration Graphs

- *Hollywood graph* models the collaboration of actors in films
  - Represent actors by vertices, connect two vertices if the actors they represent have appeared in the same movie
- *Academic collaboration graph* models the collaboration of researchers who have jointly written a paper in a particular subject
  - Represent researchers in a particular academic discipline using vertices
  - Connect the vertices representing two researchers in this discipline if they are coauthors of a paper

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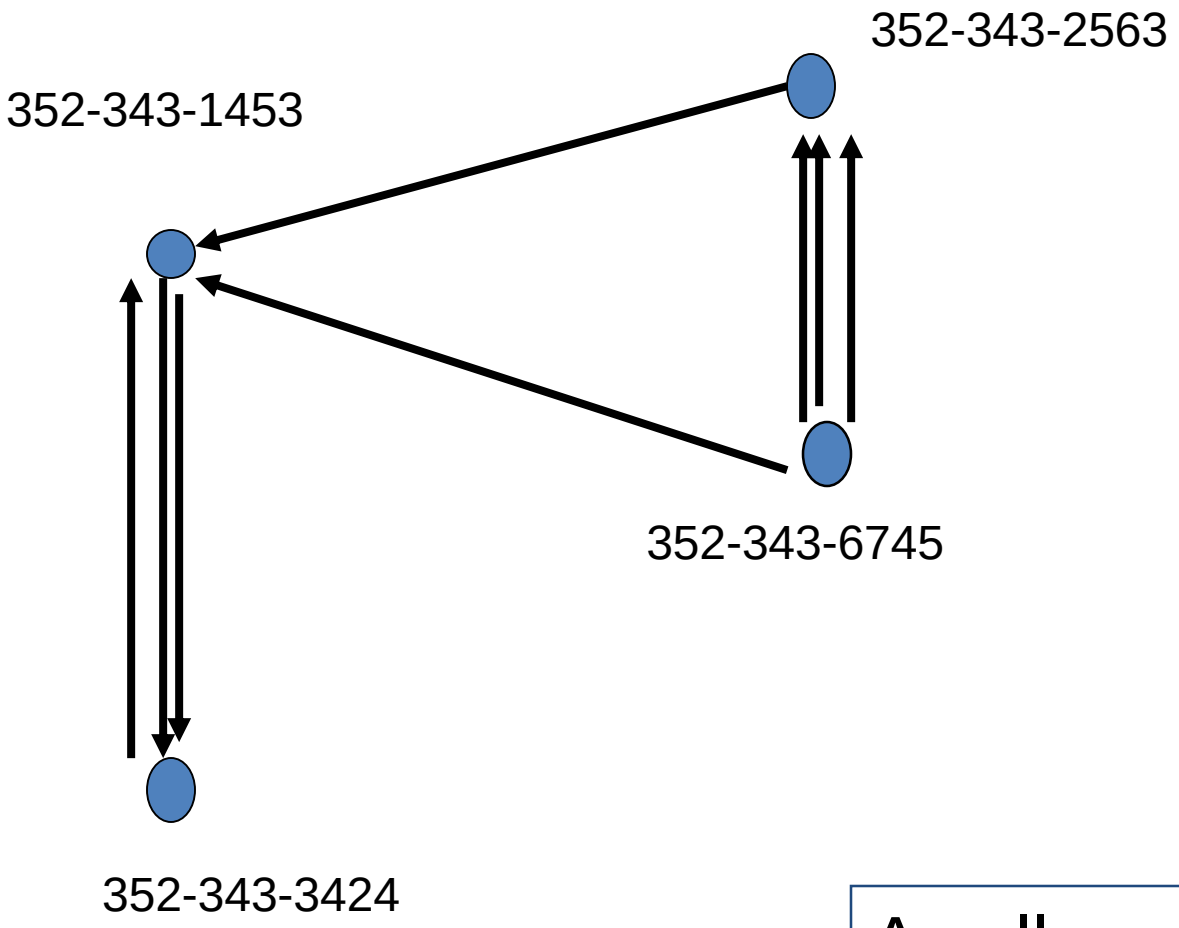
Applications to Information Networks

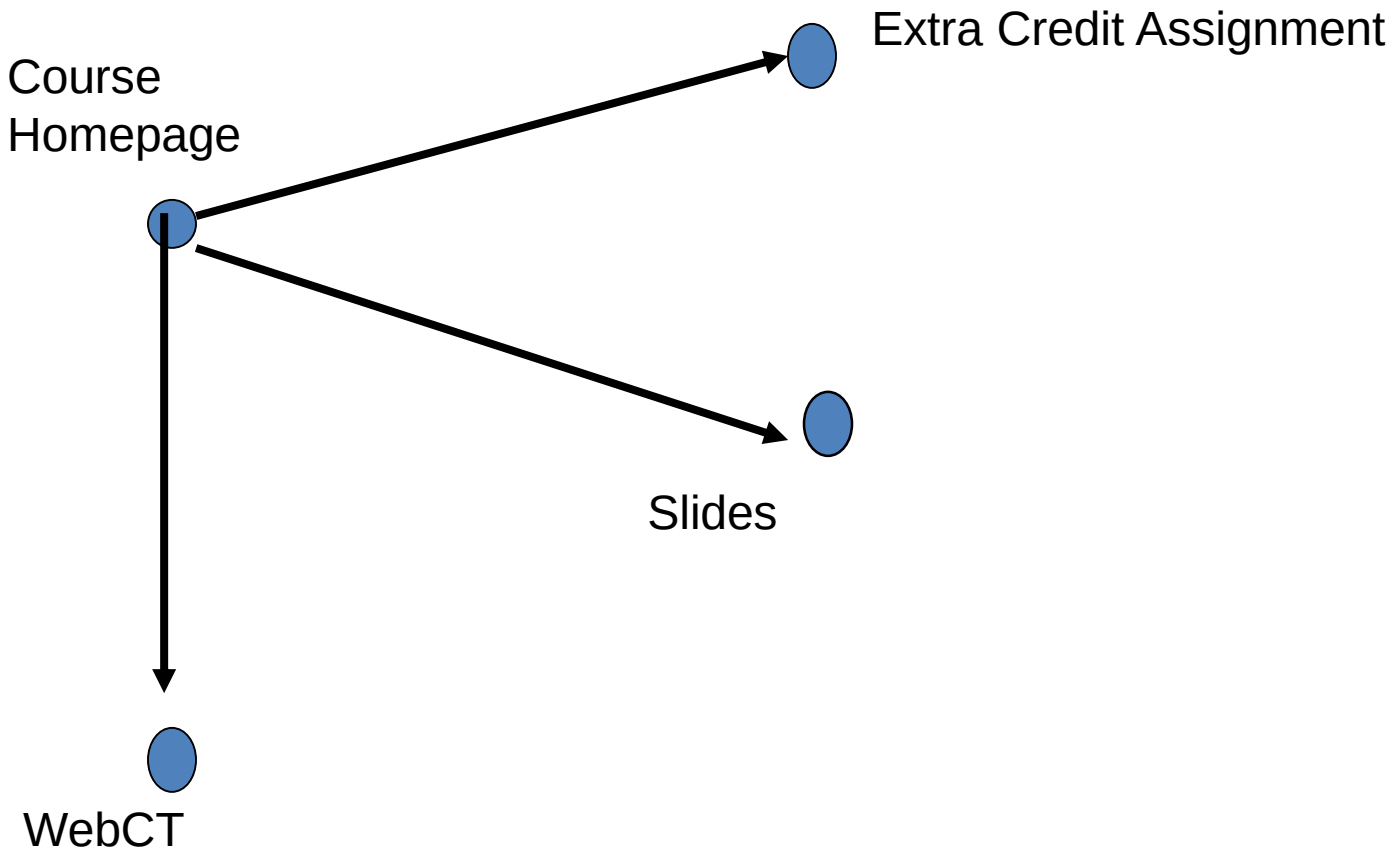
- ***Call Graph*:** Used to model telephone calls made in a network
- ***Web graph*:** Web pages are represented by vertices and links are represented by directed edges.
  - A web graph models the web at a particular time
- ***Citation network*:**
  - When a paper cites a second paper as a reference, there is an edge from the vertex representing this paper to the vertex representing the second paper

Graphs



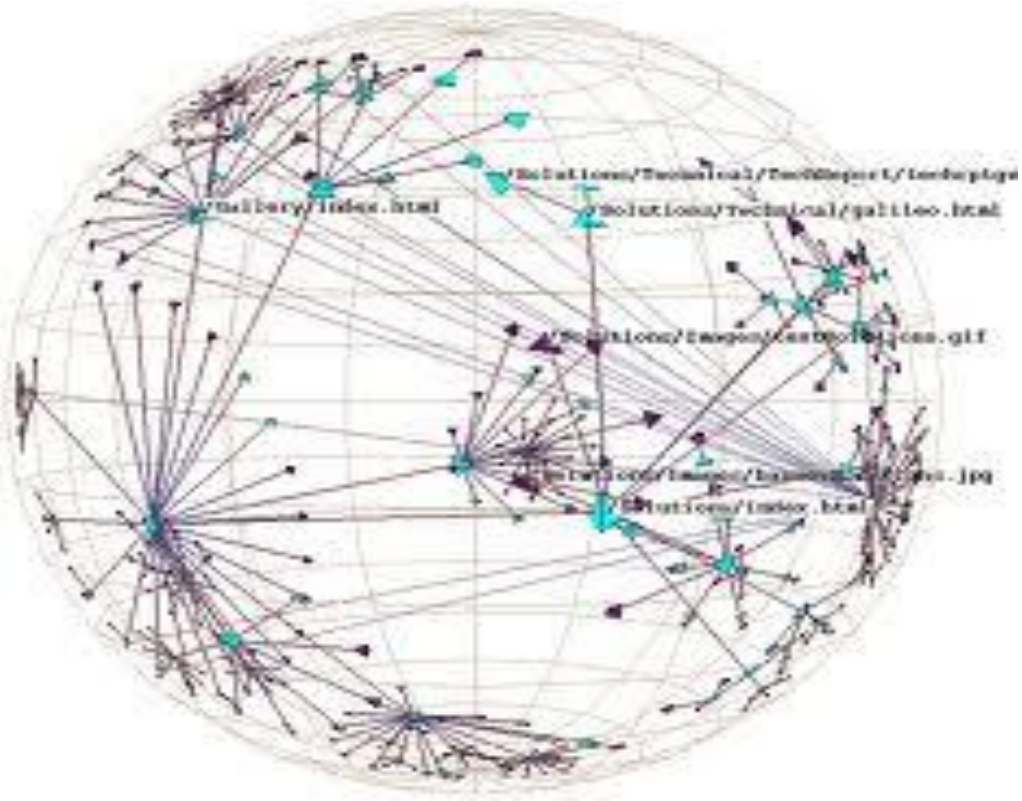
A call graph

Graphs



A web graph

Graphs



A world wide web graph

Graph Models: Transportation Graphs

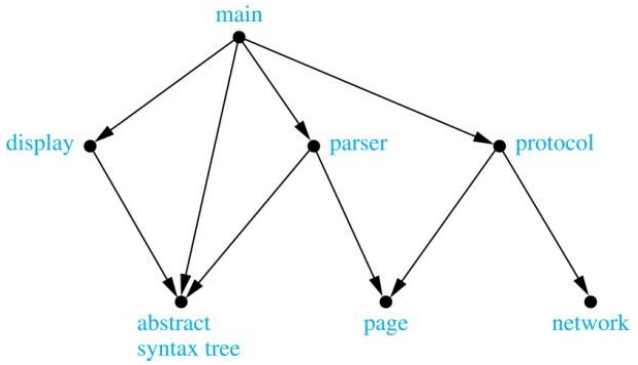
- Airline networks: can be modeled using directed multigraphs
  - directed edges from the vertex representing the departure airport to the vertex representing the destination airport
  - airports are represented by vertices

- Road networks: can be modeled using graphs
  - vertices represent intersections and edges represent roads
  - undirected edges represent two-way roads, directed edges represent one-way roads

Graph Models: Software Design Applications

- Top-down approach used to design software: system divided into modules, each performing a specific task
- ***Module dependency graph:*** represents the dependency between these modules, need to be understood before coding can be done
  - In a module dependency graph vertices represent software modules, edges from one module to another if the second module depends on the first

Example: The dependencies between the seven modules in the design of a web browser are represented by this module dependency graph



Software Design Applications

- ***Precedence graph*** – *directed graph* represents which statements must have already been executed before we execute each statement
  - Vertices represent statements in a computer program
  - There is a directed edge from a vertex to a second vertex if the second vertex cannot be executed before the first

Example: This precedence graph shows which statements must already have been executed before we can execute each of the six statements in the program

S_1 $a := 0$

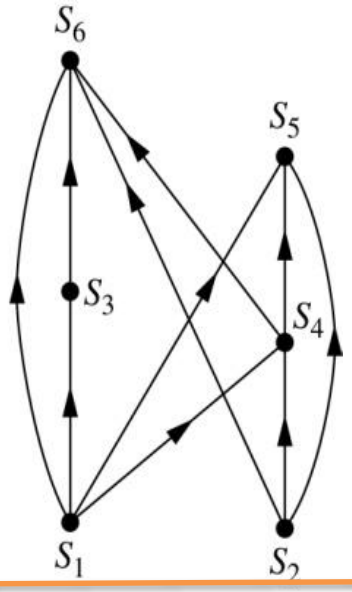
S_2 $b := 1$

S_3 $c := a + 1$

S_4 $d := b + a$

S_5 $e := d + 1$

S_6 $e := c + d$

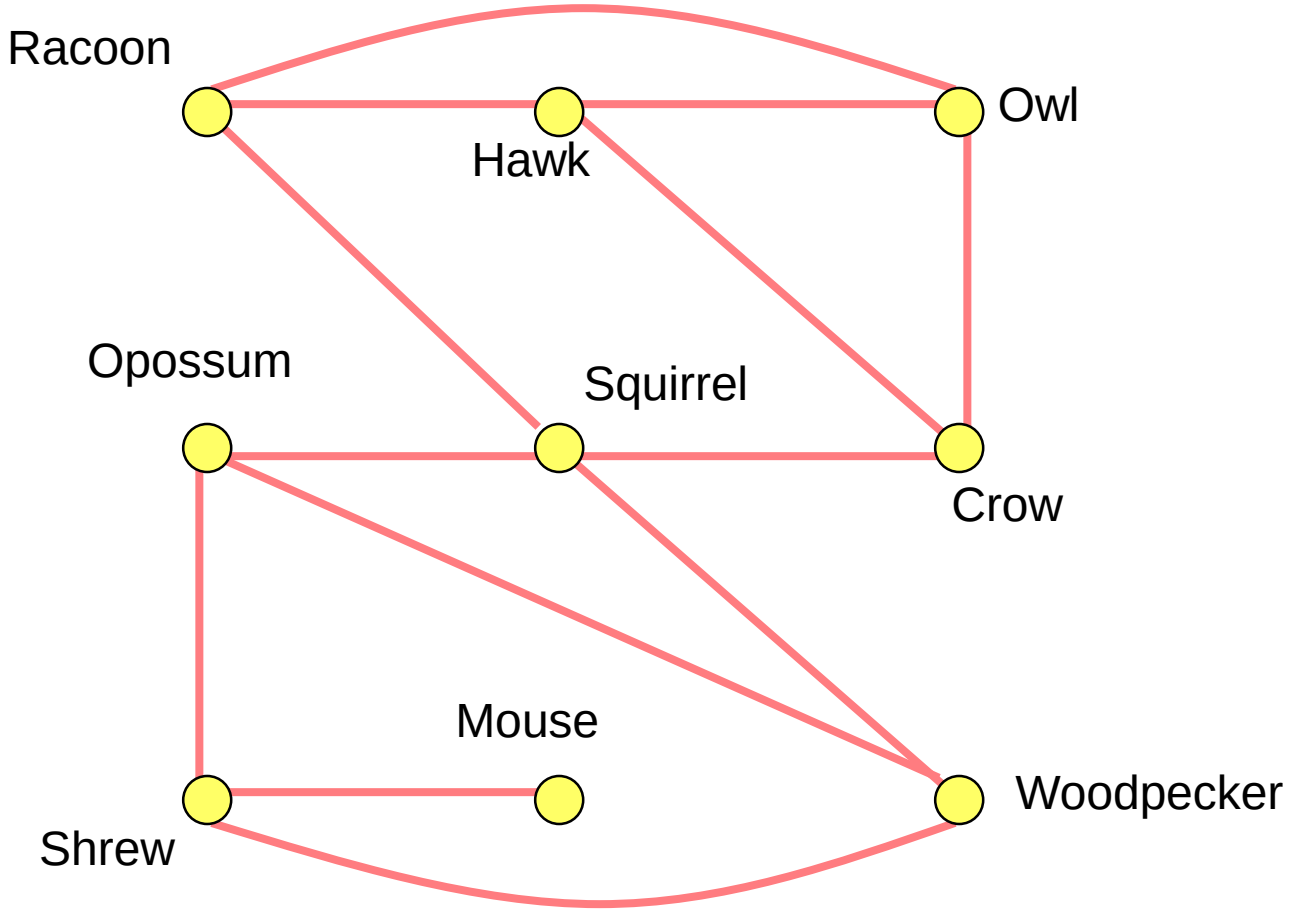


Graph Models: Niche Overlay graph

- Example: Competition between species in an ecological system can be modeled using a niche overlay graph.
- Each species is represented by a vertex
- An undirected edge connects two vertices

- if the two species represented by these vertices *compete (for food)*

Graphs



A niche overlay graph

Graph Models: Round-Robin Tournament Graph