# MATH 2418: Linear Algebra

## Assignment 5

Due February 24, 2016 Term Spring, 2016

**Recommended Text Book Problems (do not turn in):** [Sec 2.1: # 11, 23, 25, 29, 41]; [Sec 2.2: # 11, 13, 21, 25, 31]; [Sec 2.3: # 21, 23, 27, 31, 33]; [Sec 4.1: # 1, 3, 5, 7, 9]

1. Use cofactor expansion to evaluate the determinants:

a) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

b) $\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$

2. a) Find the determinants by inspection:

i) $\begin{vmatrix} 10 & 1 & 1 & 1 \\ 0 & 20 & 2 & 2 \\ 0 & 0 & 30 & 2 \\ 0 & 0 & 0 & 40 \end{vmatrix}$ ii) $\begin{vmatrix} 2+3k & 0 & 0 & 0 \\ 0 & 2-3k & 0 & 0 \\ 0 & 0 & 1-\sqrt{3}r & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$ where $k, r \in \mathbb{R}$

3. Use **arrow technique** to evaluate the determinant of $A = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 11 & 1 \\ 3 & -4 & 2 \end{bmatrix}$

4. Use the properties to evaluate the determinant( Do not use cofactor expansion along any row or column with more than one non-zero entry on any determinant of the matrices of size $3 \times 3$ or bigger.)

a) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

b) $\begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 6 & 8 & 6 & 1 \end{vmatrix}$

5. a) Without evaluating the determinant prove that: $\begin{vmatrix} a+bt & d+et & g+ht \\ at+b & dt+e & gt+h \\ c & f & i \end{vmatrix} = \left(1-t^2\right) \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$

b) Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -11$, find $\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g-4a & h-4b & i-4c \end{vmatrix}$.

6. Use the adjoint method to find the inverse of $A = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

7. Use Crammer's rule to solve the linear system:

$$\begin{cases} 3x+5y+ \quad 4z = 5, \\ \quad x+ \qquad\quad z = -2, \\ 2x+y+ \qquad z = -4. \end{cases}$$

8. Prove that the set $V = \{(3, x) : x \in \mathbb{R}\}$ with the addition $(3,\ x) + (3,\ y) = (3,\ x + y)$, and scalar multiplication $k(3,\ x) = (3,\ kx)$ is a vector space.

9. Let $W = \{(x,\ y) : x, y \in \mathbb{R}\}$ has addition and multiplication by scalars defined as:
For $\mathbf{u} = (u_1,\ u_2)$, $\mathbf{v} = (v_1,\ v_2)$ and any scalar $k$,

$$\mathbf{u} + \mathbf{v} = (u_1,\ u_2) + (v_1,\ v_2) = (u_1 + v_1 + 4,\ u_2 + v_2 - 3), \text{ and } k\mathbf{u} = k(u_1,\ u_2) = (ku_1,\ ku_2).$$

a) Compute $\mathbf{u} + \mathbf{v}$ and $k\mathbf{v}$ for $\mathbf{u} = (2, -3)$, $\mathbf{v} = (4, 2)$ and $k = 3$.

b) Find the element $\mathbf{0}$ of $W$.

c) Find $-\mathbf{u}$ and $-(3, 4)$.

d) Explain, why $W$ is not a vector space.

10. True or False.

(a) **T** **F**: Let $A$ be an $n \times n$ square matrix, then $\det(kA) = k\det(A)$.

(b) **T** **F**: For any $n \times n$ matrix $A$, $\det[A \cdot \operatorname{adj}(A)] = [\det(A)]^n$.

(c) **T** **F**: We can always find an $n \times n$ matrix $A$ such that $\det(A) = 7$.

(d) **T** **F**: If $\operatorname{adj}(A)$ has a row of zeros, then $A$ has a row of zeros.

(e) **T** **F**: If a linear system is consistent then the Crammer's rule is applicable.

(f) **T** **F**: A vector space must contain at least 2 elements.

(g) **T** **F**: For any vector $\mathbf{u}$ in any vector space, $(-1)\mathbf{u} = -\mathbf{u}$.

(h) **T** **F**: The zero element $\mathbf{0}$ of a vector space is unique.

(i) **T** **F**: Let $A$ and $B$ be $n \times n$ square matrices such that $[\det(A) \cdot \det(B)] = 1$, then $A^{-1} = B$.