MATH 2418: Linear Algebra

Assignment 2

Due `January 27, 2016` Term `Spring, 2016`

Recommended Text Book Problems (do not turn in): [Section 1.3: #1, 3, 5, 7, 9, 11, 13, 15, 17]; [Section 1.4: #1, 3, 5, 7, 11, 13, 15, 17, 19, 21, 23, 25].

1. For given matrices A and B, determine whether or not AB and $A - 2B$ exist. If yes, then find AB and $A - 2B$.

$$a) \qquad A = \begin{bmatrix} 3 & -5 & 0 & -3 \\ 2 & 1 & 0 & 0 \\ 8 & 6 & 1 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$b) \qquad A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 4 & -2 \\ 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & -1 & 5 \\ 1 & 4 & 2 \end{bmatrix}$$

$$c) \qquad A = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$$

$$d) \qquad A = \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}$$

2. (a) Let f be the quadratic polynomial of x, y defined by $f(x, y) = 2x^2 + 3y^2 - 4xy$. Verify that

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(b) Let g be the quadratic polynomial of x, y and z defined by $g(x, y, z) = x^2 + 3y^2 + 5z^2 - 4xy - 6yz + 8xz$. Verify that

$$g(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -2 & 4 \\ -2 & 3 & -3 \\ 4 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(c) Find all possible values of $k \in \mathbb{R}$ so that

$$\begin{bmatrix} 1 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix} = 0.$$

3. a) Let

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -3 & -7 \\ 7 & -3 & -2 & 9 \end{bmatrix}.$$

Find $(AB)_{13}$ and $(AB)_{24}$ without computing the whole matrix of AB.

b) Let C be a 100×200 matrix whose row 5 (5-th row) contains all -2's and row 51 contains all 2's, D a 200×201 matrix whose column 100 is all -2's, and column 101 is all 3's. Does CD exist? If yes, what is the value of the entry $(CD)_{5,101}$?

4. Find the inverses of the following matrices if they exist.

$$a) \qquad A = \begin{bmatrix} 3 & -5 \\ -1 & 1 \end{bmatrix}$$

$$b) \qquad B = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad \theta \in \mathbb{R}$$

$$c) \qquad C = \begin{bmatrix} \frac{1}{2}(e^{-x} - e^{x}) & \frac{1}{2}(e^{-x} + e^{x}) \\ \\ \frac{1}{2}(e^{-x} + e^{x}) & \frac{1}{2}(e^{-x} - e^{x}) \end{bmatrix}, \quad x \in \mathbb{R}.$$

5. True or False.

(a) **T** **F**: Let A be a square matrix. Then $\mathrm{tr}(A) = \mathrm{tr}(A^T)$.

(b) **T** **F**: Let A and B be square matrices. Then $\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B)$.

(c) **T** **F**: If A is of size 1×2, and B is of size 2×1. Then $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

(d) **T** **F**: If A and B are invertible, then we have $((AB)^T)^{-1} = (A^{-1})^T(B^{-1})^T$.

(e) **T** **F**: If A and B are square matrices of the same size then $A^2 + 2AB + B^2 = (A + B)^2$.

(f) **T** **F**: Let A, B and C be matrices of the same size. If A is invertible and $AB = AC$, then $B = C$.

(g) **T** **F**: Let A and I be $n \times n$ matrices. If $A^2 + 3A + I = 0$, then $A^{-1} = A + 3I$.

(h) **T** **F**: Let A, B and C be matrices of size $k \times n$, $n \times l$ and $l \times s$, respectively. Then ABC is a matrix with size $s \times k$.

(i) **T** **F**: There exists a 2×2 matrix A such that $A^2 = 0$ but $A \neq 0$, where 0 is the 2×2 zero matrix.

(j) **T** **F**: There exists a 2×2 matrix A such that $A^2 = I$ but $A \neq \pm I$, where I is the 2×2 identity matrix.