

Assignment 01 due 01/26/16

CH 1.1

1

- a.) T
- b.) F
- c.) T
- d.) F
- e.) Not a proposition
- f.) Not a proposition

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- a.) Mei does not have a MP3 player
- b.) There is pollution in New Jersey
- c.) $2 + 1 \neq 3$ (not equal to)
- d.) The summer in Maine is not hot or not sunny (DeMorgan's, negate and flip the and to or)

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- a.) F, ACME had the highest annual revenue
- b.) T, p is true, the or statement disregards q
- c.) T, p is true and q is true, so implication is true
- d.) T, p is false so the implication is true
- e.) T, p is true and q is true so the implication is true

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- a.) If it snows today, I will ski tomorrow

 $p \rightarrow q$ Converse: $q \rightarrow p$ If I ski tomorrow, it will snow today.Contrapositive: $\neg q \rightarrow \neg p$ If I do not ski tomorrow, it will not snow today.Inverse: $\neg p \rightarrow \neg q$ If it doesn't snow today, I will not ski tomorrow.

- b.) I come to class whenever there is going to be a quiz

If there is a quiz, then I come to class

 $p \rightarrow q$ Converse: $q \rightarrow p$ If I come to class then there will be a quiz.Contrapositive: $\neg q \rightarrow \neg p$ If do not come to class then there is not a quiz.Inverse: $\neg p \rightarrow \neg q$ If there is not a quiz, then I don't come to class.

- c.) A positive integer is a prime only if it has no divisors other than 1 and itself.
 “if p, q” “p only if q” (page 6)

$p \rightarrow q$

Converse: $q \rightarrow p$ A positive integer is a prime if it has no divisors other than 1 and itself

Contrapositive: $\neg q \rightarrow \neg p$ If positive integer has divisors other than 1 and itself, then it is not prime.

Inverse: $\neg p \rightarrow \neg q$ If a positive integer is not a prime then it has a divisor other than 1 and itself.

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Truth Table		$q \rightarrow p$				
p	q	r	s	$p \rightarrow q$	$((p \rightarrow q) \rightarrow r)$	$((p \rightarrow q) \rightarrow r) \rightarrow s$
F	F	F	F	T	F	T
F	F	F	T	T	F	T
F	F	T	F	T	T	F
F	F	T	T	T	T	T
F	T	F	F	T	F	T
F	T	F	T	T	F	T
F	T	T	F	T	T	F
F	T	T	T	T	T	T
T	F	F	F	F	T	F
T	F	F	T	F	T	T
T	F	T	F	F	T	F
T	F	T	T	F	T	T
T	T	F	F	T	F	T
T	T	F	T	T	F	T
T	T	T	F	T	T	F
T	T	T	T	T	T	T

CH 1.3

1

a) Truth Table $p \wedge T = p$

p	T	$p \wedge T$
F	T	F
T	T	T

From the truth table the p and $p \wedge T$ column match.

b) Truth Table $p \vee F = p$

p	F	$p \vee F$
F	F	F
T	F	T

From the truth table the p and $p \vee F$ column match.

c) Truth Table $p \wedge F = F$

p	F	$p \wedge F$
F	F	F
T	F	F

Anything anded with F will always be false.

d) Truth Table $p \vee T = T$

p	T	$p \vee T$
F	T	T
T	T	T

Anything or'ed with T will always be true.

e) Truth Table $p \vee p = p$

p	p	$p \vee p$
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F	F	F
T	T	T

Anything or'ed with itself will always be itself.

f) Truth Table $p \wedge p = p$

p	p	$p \wedge p$
F	F	F
T	T	T

Anything and'ed with itself will always be itself.

2

$$\neg(\neg p) = p$$

$$(p) = p \quad \text{double negation law}$$

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- a.) Jan is rich and happy
Jan is either, not rich or not happy
- b.) Carlos will bicycle or run tomorrow
Carlos will not bicycle and will not run tomorrow.
- c.) Mei walks or takes the bus to class
Mei walks and takes the bus to class
- d.) Ibrahim is smart and hard working
Ibrahim is not smart or Ibrahim is not hard working

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Truth Table $\neg(p \leftrightarrow q) = (p \leftrightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$
F	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
T	T	F	F	T	F	F

From the truth table we can see the last two columns are equivalent.

CH 1.4

1

$P(x)$ means " $x \leq 4$ "

- a.) $P(0) = "0 \leq 4" = T$
- b.) $P(4) = "4 \leq 4" = T$
- c.) $P(6) = "6 \leq 4" = f$

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$P(x)$ means " x can speak Russian"

$Q(x)$ means " x knows computer language C++"

- a.) There is a student in your school that can speak Russian and knows C++
A student is existential
 $\exists x (P(x) \wedge Q(x))$
- b.) There is a student in your school that can speak Russian and doesn't know C++
A student is existential
 $\exists x (P(x) \wedge \neg Q(x))$
- c.) Every student at your school either can speak Russian or knows C++
Every student is universal
 $\forall x (P(x) \vee Q(x))$
- d.) No student at your school either can speak Russian or knows C++
No student is negated universal
 $\neg \forall x (P(x) \vee Q(x))$

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$P(x)$ means " x is perfect"

$F(x)$ means " x is your friend"

- a.) No one is perfect
Everyone is not perfect
 $\forall x (\neg P(x))$
- b.) Not everyone is perfect
Negation of everyone is perfect
 $\neg \forall x (P(x))$
- c.) All your friends are perfect

$$\forall x(F(x) \rightarrow P(x))$$

d.) At least one of your friends is perfect

$$\exists x(F(x) \wedge P(x))$$

e.) Everyone is your friend and is perfect

$$\forall x(F(x) \wedge P(x))$$

f.) Not everyone is your friend or someone is not perfect

$$(\neg \forall x Q(x)) \vee (\exists x \neg P(x))$$

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- a.) None
- b.) 0 is neither greater than 0 or less than 0, $x = 0$
- c.) Any x other than 1 does not equal 1, $x = 2$

CH 1.5

1

- a.) For all real numbers x , for some real number y , we can choose any x we like and there will always be a y to choose that is greater
- b.) For all real numbers x , for some real number y ,
 x is greater than or equal to 0 AND y is greater than or equal to 0 IMPLIES x times y is greater than or equal to 0
 (non negative numbers multiplied together yield a non-negative result)
- c.) For all real numbers x , for all real numbers y , for some real number z ,
 x times y equals z
 (real numbers closed under multiplication)

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- a.) $\{2,3,4,\dots\}$ no $\{2\}$ is not a member
- b.) $\{4,16,25,36,\dots\}$ no $\{2\}$ is not a member
- c.) $\{2,\{2\}\}$ yes $\{2\}$ is a member
- d.) $\{\{2\},\{\{2\}\}\}$ yes $\{2\}$ is a member
- e.) $\{\{2\},\{2\{2\}\}\}$ yes $\{2\}$ is a member
- f.) $\{\{\{2\}\}\}$ no $\{2\}$ is not a member

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$M(x,y)$ "x has sent y a email message"

$\neg M(x,y)$ "x has no sent y a email message"

$T(x,y)$ "x has telephoned y"

$\neg T(x,y)$ "x has not telephoned y"

a.) $\neg M(\text{Chou, Koko})$

b.) $\neg (M(\text{Arlene, Sarah}) \vee T(\text{Arlene, Sarah}))$

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a.) $\neg \forall x \forall y P(x,y)$

$\exists x \exists y \neg P(x,y)$ bring negation inside quantifiers

b.) $\neg \forall y \exists x P(x,y)$

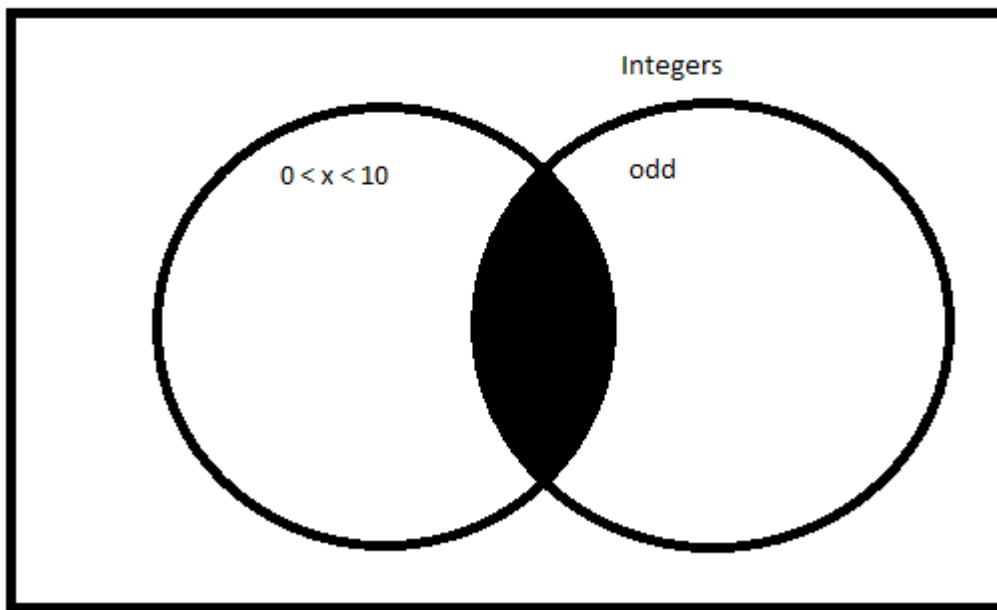
$\exists y \forall x \neg P(x,y)$ bring negation inside quantifiers

CH 2.1

2

- a.) $\{x \mid y \text{ is a non negative integer such that } y*3 = x\}$
- b.) $\{x \mid x \text{ is a integer such that } -3 \leq x \leq 3\}$
- c.) $\{x \mid x \text{ is a letter in the alphabet such that } m \leq x \leq p\}$

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- a.) 1
- b.) 1
- c.) 2
- d.) 3

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- a.) { null set , {a}}
- b.) { null set , {a}, {b}, {a,b}}
- c.) { null set , { null set }, {{ null set }}, { null set , { null set }}}

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A^2 is $A \times A$

So we match each n tuple from the first set with its corresponding element in the second set

- a.) {(0,0),(0,1),(0,3),(1,0),(1,1),(1,3),(3,0),(3,1),(3,3)}
- b.) {(1,1),(1,2),(1,a),(1,b),(2,1),(2,2),(2,a),(2,b),(a,1),(a,2),(a,a),(a,b),(b,1),(b,2),(b,a),(b,b)}