MATH 2418: Linear Algebra

Assignment 3

Due February 3, 2016 Term Spring, 2016

Recommended Text Book Problems (do not turn in): [Section 1.5: #1, 3, 5, 7, 9, 11, 13, 19, 23, 25, TF]; [Section 1.6: #1, 5, 9, 13, 15, 19, 21, TF].

1. Determine if the given matrix is elementary. If it is elementary, find the corresponding row operation and an elementary matrix that will restore the original matrix to the identity matrix.

   a) [2 pt] $B = \begin{bmatrix} 1 & 0 \\ 0 & \pi \end{bmatrix}$

   b) [2 pt] $C = \begin{bmatrix} 4 & 2 \\ 0 & 7 \end{bmatrix}$

   c) [2 pt] $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

   d) [2 pt] $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

   e) [2 pt] $G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 7 \\ 2 & -1 & 4 \end{bmatrix}$. Use row operations to determine the inverse of A if it exists.

a) [4 pt] Write elementary matrices corresponding to the following steps in Gauss-Jordan elimination process.

1) $R2 + (-1)R1$ $\qquad E_1 =$

2) $R3 + (-2)R1$ $\qquad E_2 =$

3) $R1 + (-1)R2$ $\qquad E_3 =$

4) $R3 + (3)R2$ $\qquad E_4 =$

b) [4 pt] Apply elementary matrices $E_1, \ldots, E_4$ to the matrix $[A : I_3] = \begin{bmatrix} 1 & 1 & 5 & 1 & 0 & 0 \\ 1 & 2 & 7 & 0 & 1 & 0 \\ 2 & -1 & 4 & 0 & 0 & 1 \end{bmatrix}$ to determine if A^{-1} exists.

c) [1 pt] If possible, write A as a product of elementary matrices. If not, explain why.

d) [1 pt] If possible, write A^{-1} as a product of elementary matrices. If not, explain why.

3. Let $A = \begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$. Use row operations to determine the inverse of A if it exists.

a) [4 pt] Write elementary matrices corresponding to the following steps in Gauss-Jordan elimination process.

1) $R3 + (-1)R1$ $\qquad E_1 =$

2) $(-1)R2$ $\qquad E_2 =$

3) $R1 + R2$ $\qquad E_3 =$

4) $R3 + (-2)R2$ $\qquad E_4 =$

5) $R1 + R3$ $\qquad E_5 =$

6) $R2 + (-1)R3$ $\qquad E_6 =$

b) [4 pt] Apply elementary matrices $E_1, \dots, E_6$ to the matrix $[A : I_3] = \begin{bmatrix} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ to determine if A^{-1} exists.

c) [1 pt] If possible, write A^{-1} as a product of elementary matrices. If not, explain why.

d) [1 pt] If possible, write A as a product of elementary matrices. If not, explain why.

4. (6+4 pts) 1) Solve the following system of linear equations $Qx = b$, where

$$Q = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

by determining the inverse of the coefficient matrix Q and then using matrix multiplication.

2) Is it true that b is a linear combination of the columns of Q? Justify your answer.

5. (10 pts) Verify whether or not the matrix $b = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$ is a linear combination of the matrices $A_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 3 \\ 6 \\ 8 \end{bmatrix}$.

6. (3+3+1 pts) 1) Use matrix partition to explain that if the system of linear equations $A_{m\times n}x = b_{m\times 1}$ is consistent, that is, $A_{m\times n}x = b_{m\times 1}$ has a solution $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, where $x_1, x_2, \cdots, x_n$ are scalars, then $b_{m\times 1}$ is a linear combination of the columns of $A_{m\times n}$;

2) Explain that if $b_{m\times 1}$ is a linear combination of the columns of $A_{m\times n} = [c_1 : c_2 : \cdots : c_n]$, i.e.,

$$b_{m\times 1} = x_1c_1 + x_2c_2 + \cdots + x_nc_n$$

where $x_1, x_2, \cdots, x_n$ are scalars, then the system of linear equations $A_{m\times n}x = b_{m\times 1}$ is consistent.

3) Is it true that the system of linear equations $A_{m\times n}x = b_{m\times 1}$ is consistent if and only if $b_{m\times 1}$ is a linear combination of the columns of $A_{m\times n}$?