

PHYS2326 Lecture #08

Prof. Fabiano Rodrigues

Department of Physics
The University of Texas at Dallas

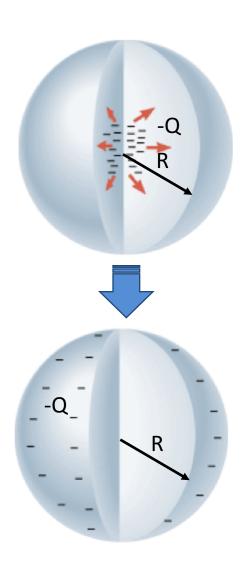
Goals for this lecture (Ch. 23)

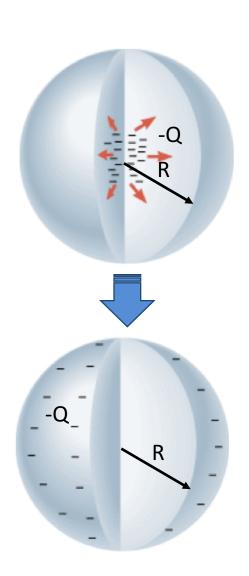
- Review/conclude:
 - Gauss's Law
 - Conductors in electrostatic equilibrium

- Understand electric potential energy
- Define and understand electric potential

Sections 23.1 and 23.2

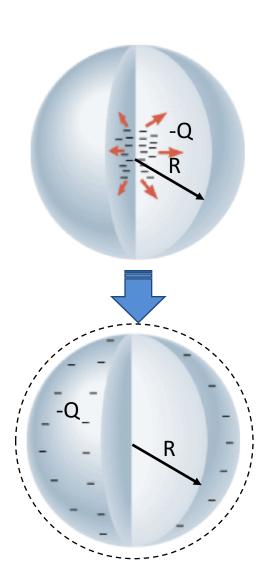
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$





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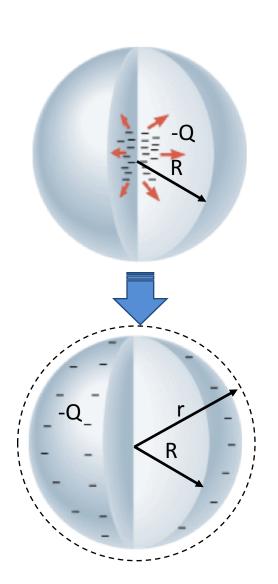
$$E(r < R) = 0$$



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$$E(r < R) = 0$$

$$E(r > R) = ?$$

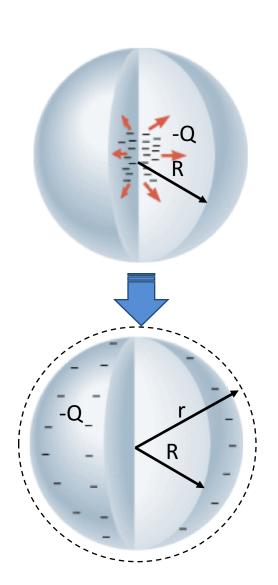


$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$E(r < R) = 0$$

$$E(r > R) = ?$$

$$E(r)A = \frac{-Q}{\epsilon_0}$$



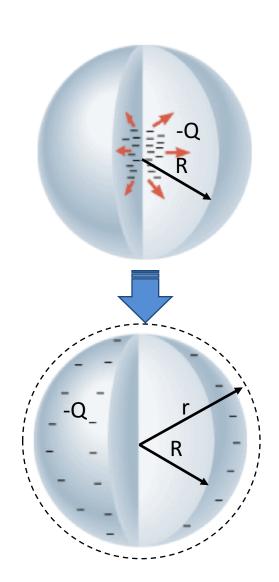
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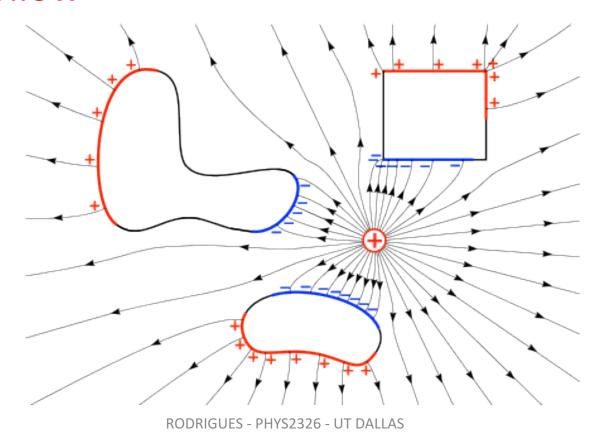
$$E(r) = \frac{-Q}{4\pi\epsilon_0 r^2} \quad for \ r > R$$

Conductors in Electrostatic Equilibrium

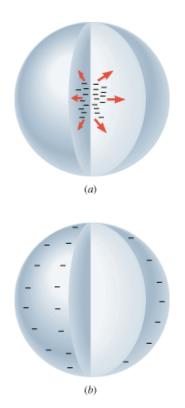
Properties

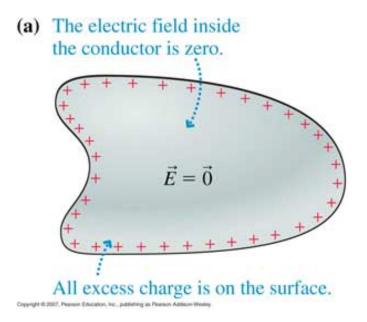
- E is zero anywhere inside the conductor, solid or hollow
- Net charge, if any, reside on the surface of a conductor
- 3. **E** just outside the conductor is perpendicular to its surface and has magnitude $|E| = \sigma/\epsilon_0$
- 4. On irregularly shaped conductors, σ is greatest where the radius of curvature is smallest

E is zero anywhere inside the conductor, solid or hollow

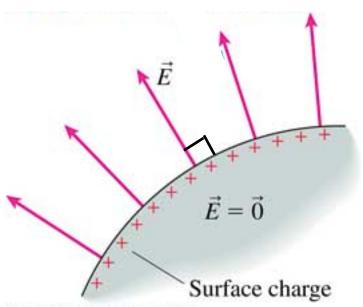


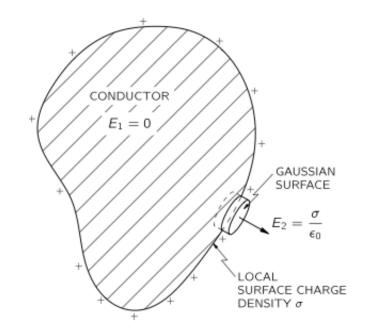
 Net charge, if any, will reside on the surface of a conductor





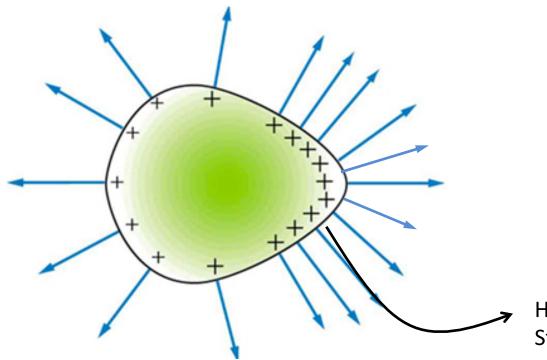
• **E** just outside the conductor is perpendicular to its surface and has magnitude $|E| = \sigma/\epsilon_0$





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 On irregularly shaped conductors, σ is greatest where the radius of curvature is smallest



Higher charge density Stronger electric field

Electric Potential Energy

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Work-Energy Theorem

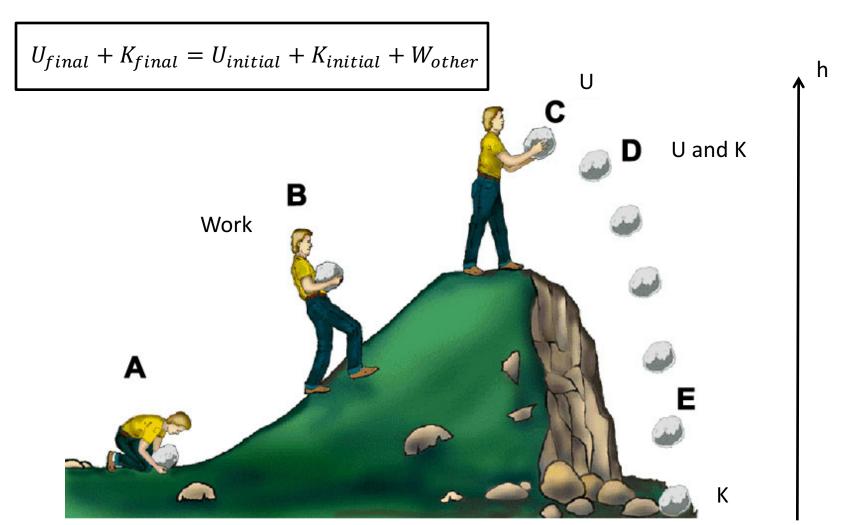
$$U_{final} + K_{final} = U_{initial} + K_{initial} + W_{other}$$

Potential Energy: U = mgh

Kinetic Energy: $K = \frac{1}{2}mv^2$

 $W_{other} = \int \vec{F}_{other} \cdot d\vec{s}$: Work done by forces other than gravity

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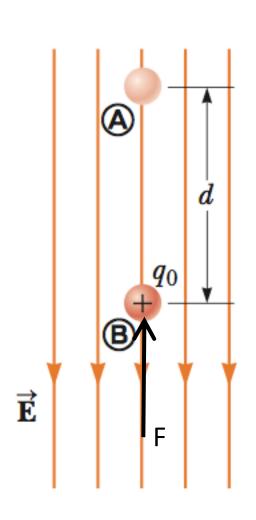


Electric Potential Energy

Electric Potential Energy

 Like in the gravitational field case, the electric field has potential energy U

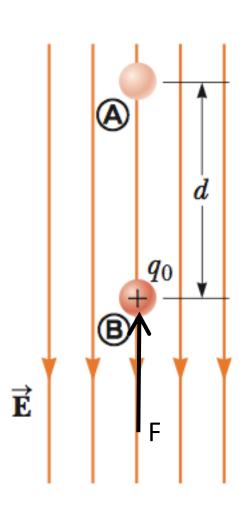
• Also, like in the gravitational case, we are interested in changes (or differences) in electric potential energy (ΔU)



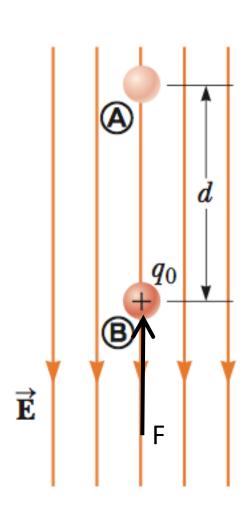
Lets consider the simple case:

Uniform E field

+Q moves along E from B to A



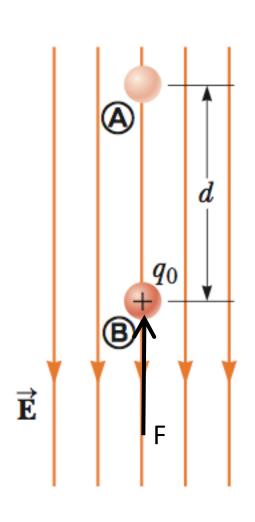
$$U_f + K_f = U_i + K_i + W_{other}$$



$$U_f + K_f = U_i + K_i + W_{other}$$

From B to A:

$$U_f + 0 = U_i + 0 + W_{other}$$

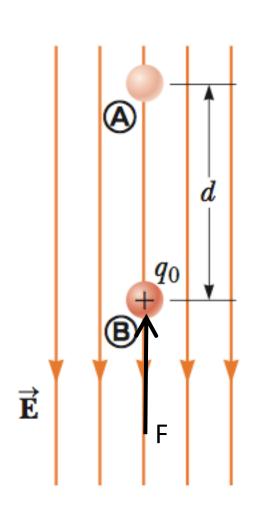


$$U_f + K_f = U_i + K_i + W_{other}$$

From B to A:

$$U_f + 0 = U_i + 0 + W_{other}$$

$$\Delta U = U_f - U_i = W_{other}$$



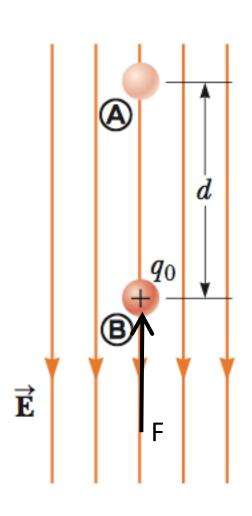
$$U_f + K_f = U_i + K_i + W_{other}$$

From B to A:

$$U_f + 0 = U_i + 0 + W_{other}$$

$$\Delta U = U_f - U_i = W_{other}$$

$$\Delta U = \int_{B}^{A} \vec{F} \cdot d\vec{s}$$



$$U_f + K_f = U_i + K_i + W_{other}$$

From B to A:

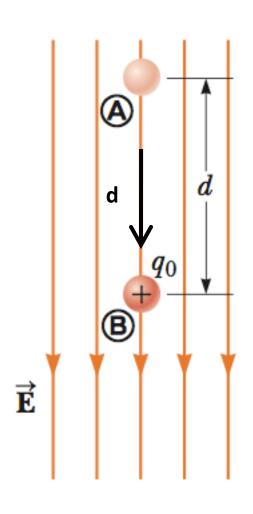
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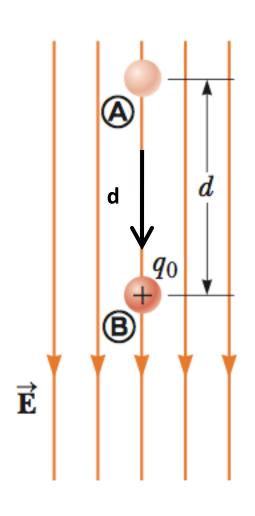
$$\Delta U = Fd$$

Uniform E, constant F



Now, from A to B

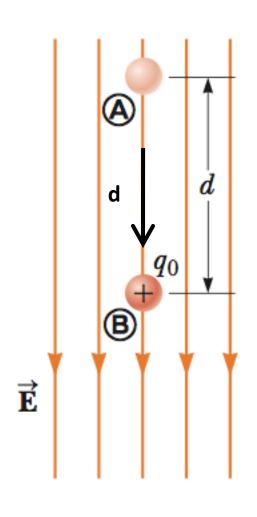
 $\Delta U = ?$



Now, from A to B

$$\Delta U = ?$$

$$\Delta U = -W_F$$

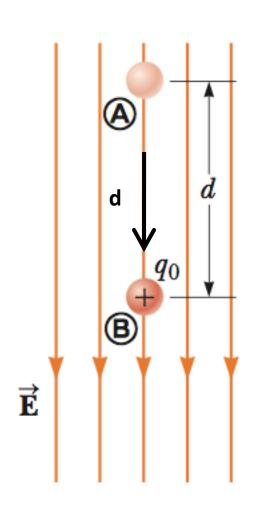


Now, from A to B

$$\Delta U = ?$$

$$\Delta U = -W_B$$

$$\Delta U = -\int_{A}^{B} \vec{F}_{E} \cdot d\vec{s}$$



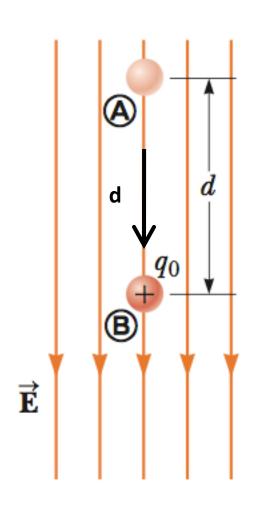
Now, from A to B

$$\Delta U = ?$$

$$\Delta U = -W_E$$

$$\Delta U = -\int_{A}^{B} \vec{F}_{E} \cdot d\vec{s}$$

$$\Delta U = -\vec{F}_E \cdot \vec{d}$$



Now, from A to B

$$\Delta U = ?$$

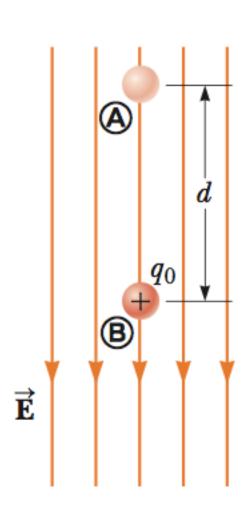
$$\Delta U = -W_E$$

$$\Delta U = -\int_{A}^{B} \vec{F}_{E} \cdot d\vec{s}$$

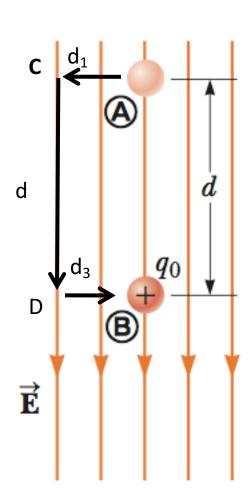
$$\Delta U = -\vec{F}_E \cdot \vec{d}$$

$$\Delta U = -q\vec{E} \cdot \vec{d}$$

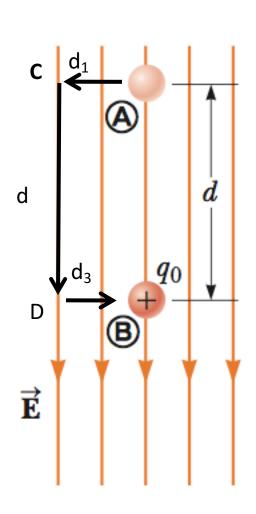
Uniform E, constant F



 What if we changed the path to get from A to B?

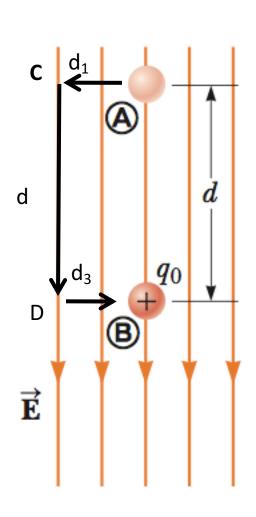


 What if we changed the path to get from A to B?



$$\Delta U = -W_E$$

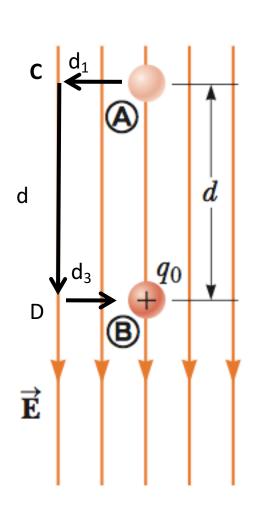
$$\Delta U = -\int_{A}^{B} \vec{F} \cdot d\vec{l}$$



$$\Delta U = -W_E$$

$$\Delta U = -\int_{A}^{B} \vec{F} \cdot d\vec{l}$$

$$\Delta U = -(\vec{F}_E \cdot \vec{d}_1 + \vec{F}_E \cdot \vec{d}_2 + \vec{F}_E \cdot \vec{d}_3)$$



$$\Delta U = -W_E$$

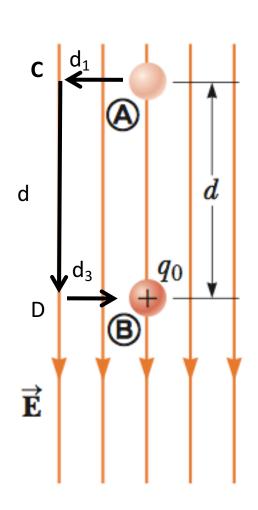
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$$\Delta U = -(\vec{F}_E \cdot \vec{d}_1 + \vec{F}_E \cdot \vec{d}_2 + \vec{F}_E \cdot \vec{d}_3)$$

$$\Delta U = -(0 + \vec{F}_E \cdot \vec{d} + 0)$$

$$\Delta U = -(q\vec{E}) \cdot \vec{d}$$

Potential Energy in Uniform E



$$\Delta U = -W_E$$

$$\Delta U = -\int_{A}^{B} \vec{F} \cdot d\vec{l}$$

$$\Delta U = -(\vec{F}_E \cdot \vec{d}_1 + \vec{F}_E \cdot \vec{d}_2 + \vec{F}_E \cdot \vec{d}_3)$$

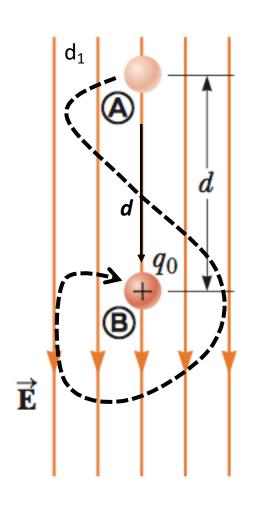
$$\Delta U = -(0 + \vec{F}_E \cdot \vec{d} + 0)$$

$$\Delta U = -(q\vec{E}) \cdot \vec{d}$$

$$\Delta U = -q\vec{E}\cdot\vec{d}$$

Same result of taking shortest path between A and B

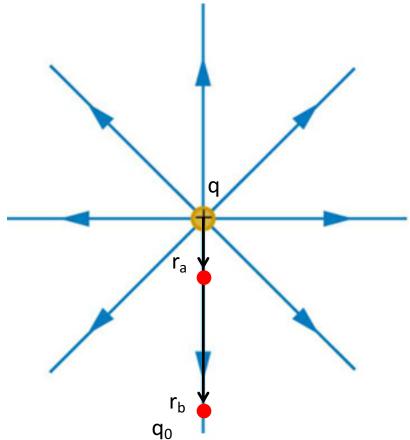
Potential Energy in Uniform E

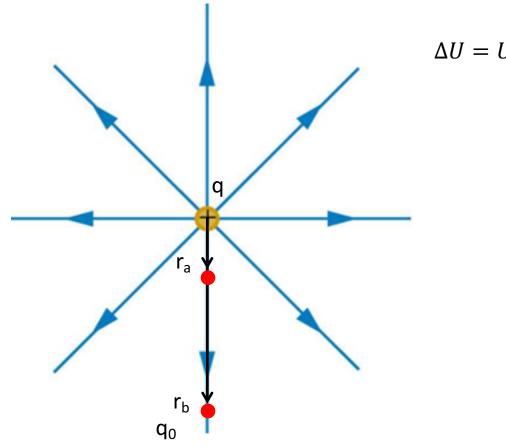


$$\Delta U = -q\vec{E} \cdot \vec{d}$$

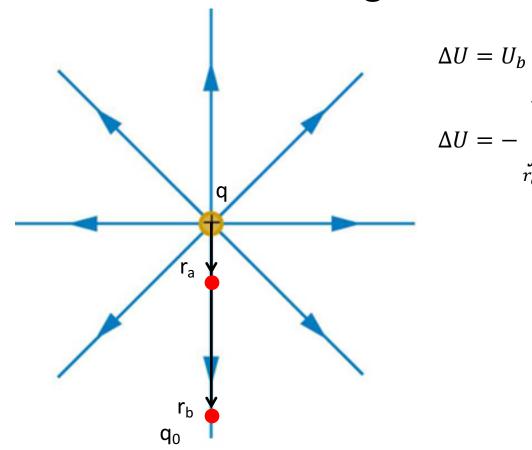
Result is independent of path taken

Potential Energy in Non-Uniform E



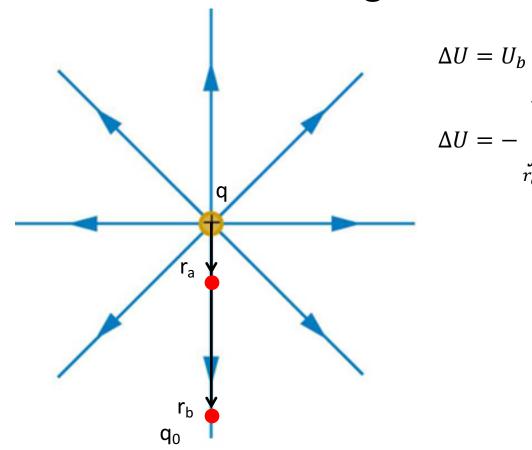


$$\Delta U = U_b - U_a = -W_E$$



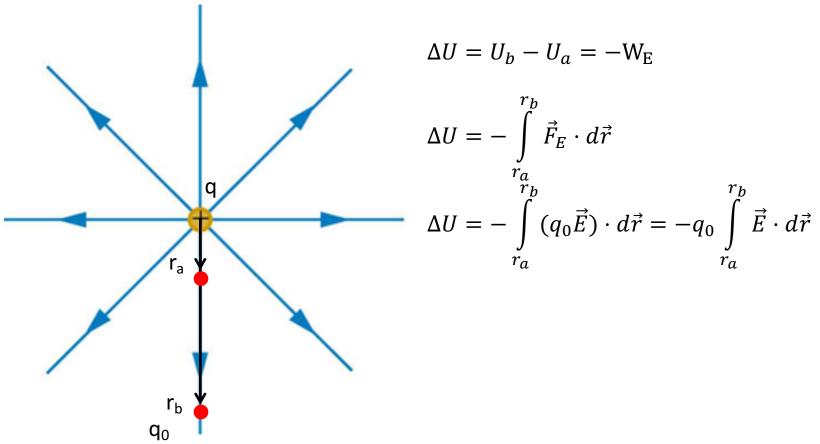
$$\Delta U = U_b - U_a = -W_E$$

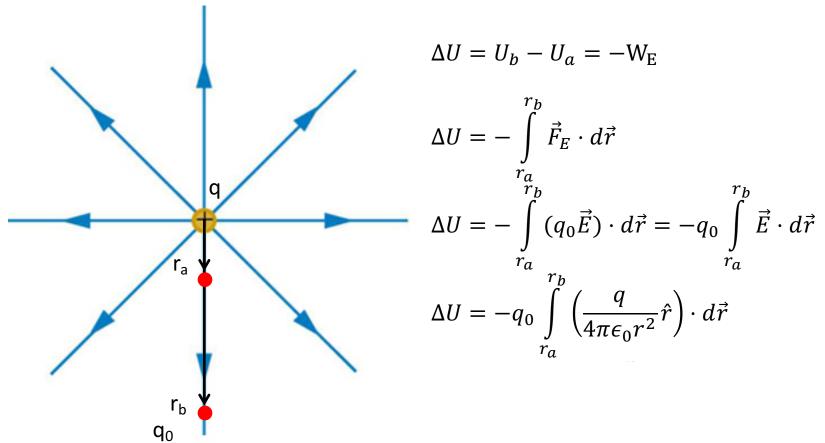
$$\Delta U = -\int_{r_a}^{r_b} \vec{F}_E \cdot d\vec{r}$$

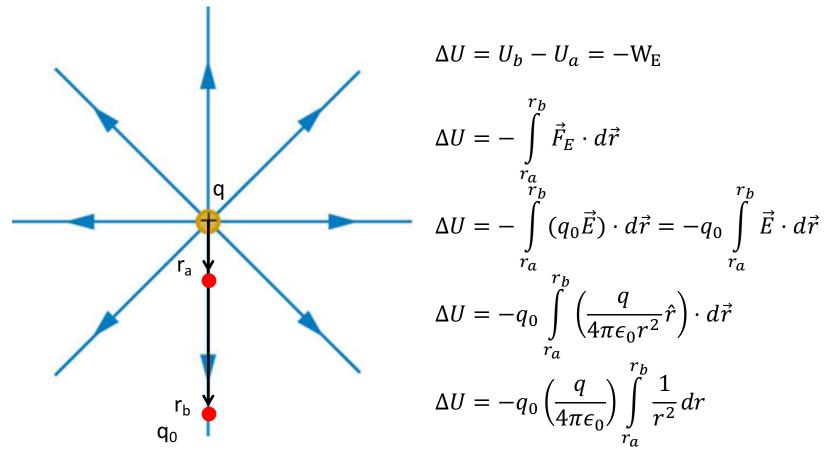


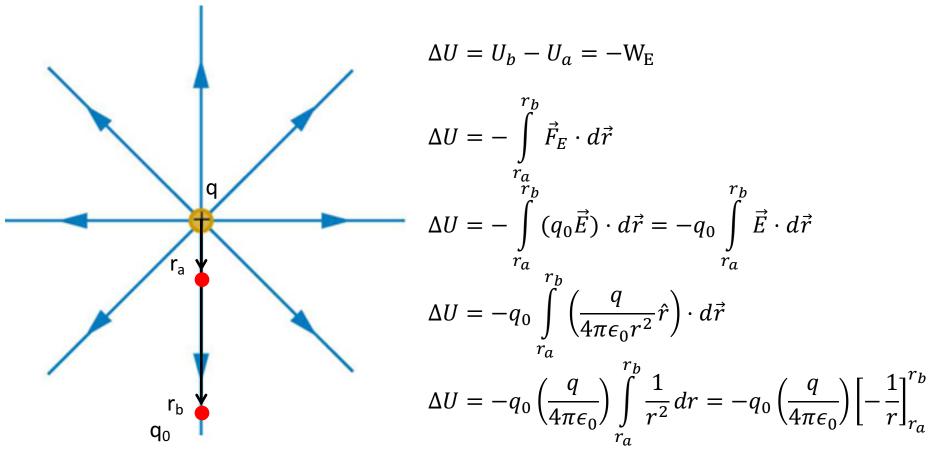
$$\Delta U = U_b - U_a = -W_E$$

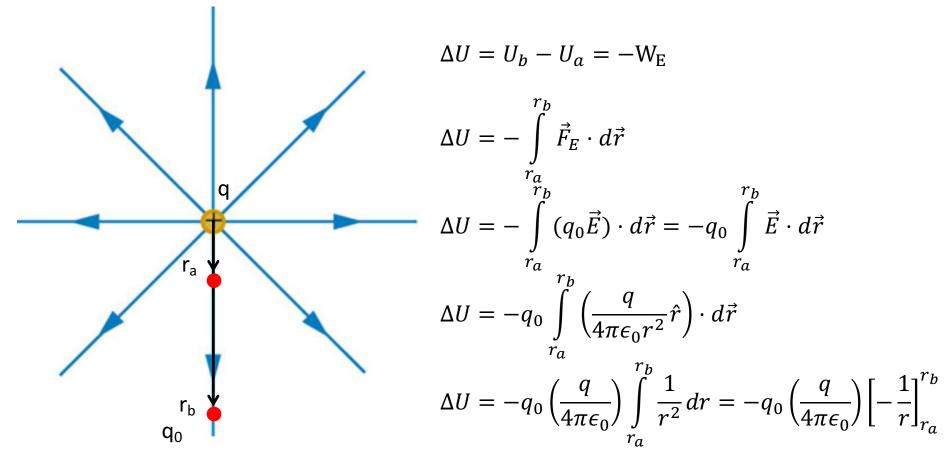
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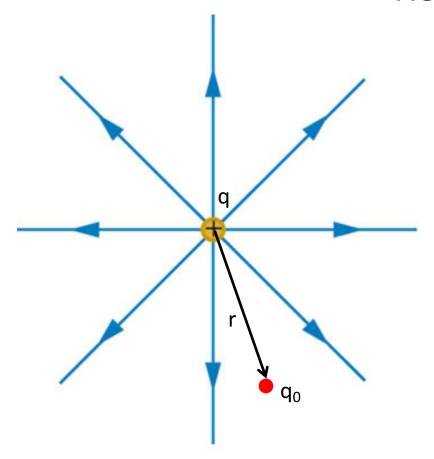




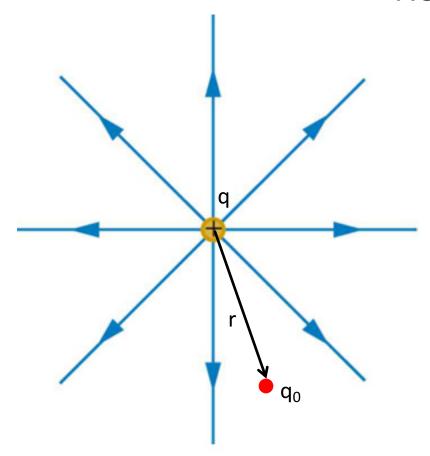




$$\Delta U = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

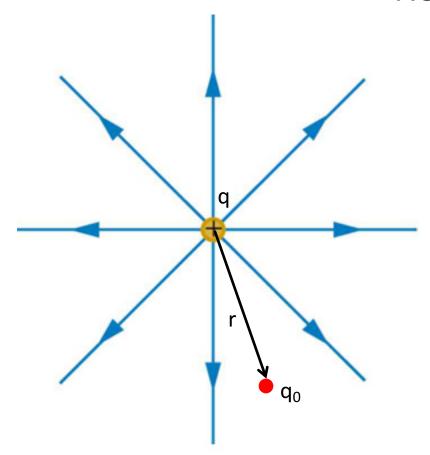


$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$



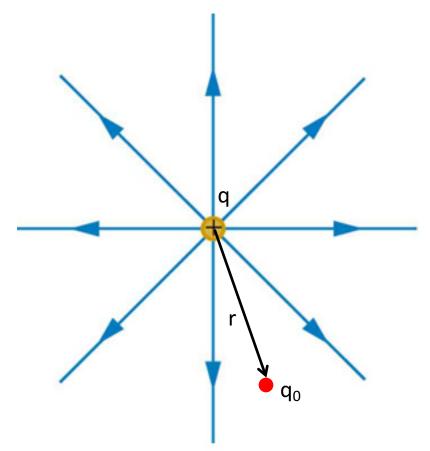
$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\Delta U = U(r) - U(r_{ref})$$



$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

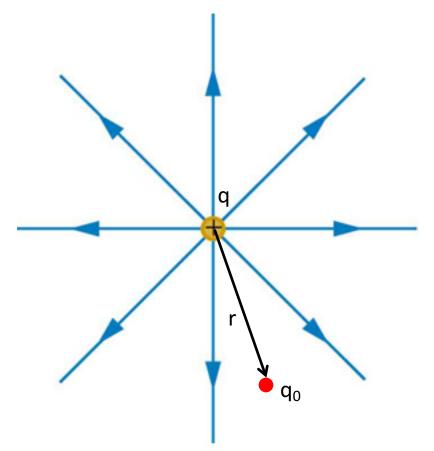
$$\Delta U = U(r) - U(r_{ref}) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{r_{ref}}\right]$$



$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\Delta U = U(r) - U(r_{ref}) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{r_{ref}}\right]$$

For the G-field we assumed U(h=0) = 0:

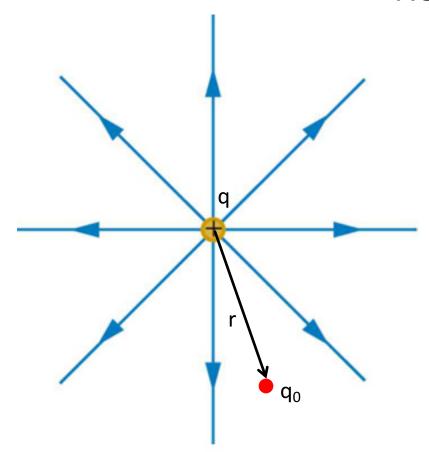


$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\Delta U = U(r) - U(r_{ref}) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{r_{ref}}\right]$$

For the G-field we assumed U(h=0) = 0:

For the E-field we assume $U(r=\infty) = 0$:



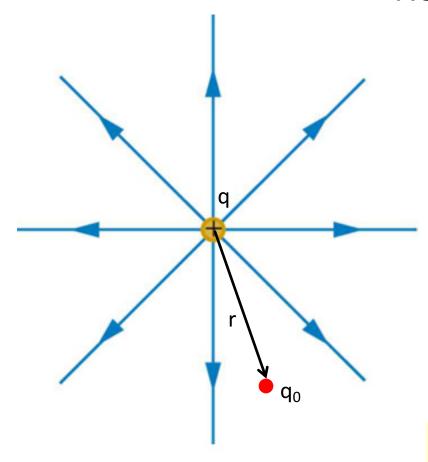
$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\Delta U = U(r) - U(r_{ref}) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{r_{ref}}\right]$$

For the G-field we assumed U(h=0) = 0:

For the E-field we assume $U(r=\infty) = 0$:

$$U(r) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{\infty}\right]$$



$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

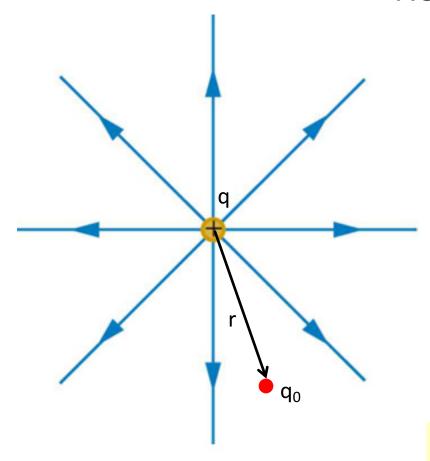
$$\Delta U = U(r) - U(r_{ref}) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{r_{ref}}\right]$$

For the G-field we assumed U(h=0) = 0:

For the E-field we assume $U(r=\infty) = 0$:

$$U(r) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$U = \frac{qq_0}{4\pi\epsilon_0 r}$$



$$\Delta U = U(r_b) - U(r_a) = \left(\frac{qq_0}{4\pi\epsilon_0}\right) \left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

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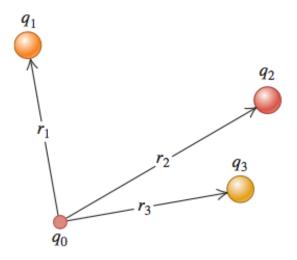
$$U = \frac{qq_0}{4\pi\epsilon_0 r}$$

U at a given point "r" is given with respect to $U(\infty) = 0$

Electric Potential Energy 2: Multiple Point Charges

Electric potential energy of q₀ due to field caused by point charge q₁

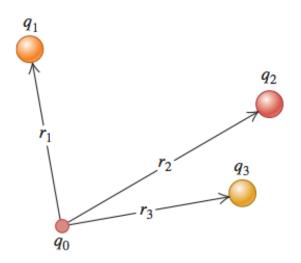
$$U=\frac{q_0q_1}{4\pi\epsilon_0r}$$



Electric Potential Energy 2: Multiple Point Charges

Electric potential energy of q_0 due to field caused by point charge q_1

$$U = \frac{q_0 q_1}{4\pi\epsilon_0 r}$$



Electric potential energy of q₀ due to field caused by multiple point charges

$$U = \frac{q_0}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \right) = \frac{q_0}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

 We found the potential energy of a "test" charge "q₀" within an electric field "E"

$$\Delta U = -q_0 \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

 We found the potential energy of a "test" charge "q₀" within an electric field "E"

$$\Delta U = -q_0 \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

• However, it is more useful to have a quantity that does not depend on q_0 . We start with:

$$\frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{\Delta U}{q_0} = -\int\limits_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{\Delta U}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{\Delta U}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

Defining the electric potential as $V = \frac{U}{q_0}$ we can write:

$$\frac{\Delta U}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

Defining the electric potential as $V = \frac{U}{q_0}$ we can write:

$$V_b - V_a = \Delta V = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{\Delta U}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

Defining the electric potential as $V = \frac{U}{q_0}$ we can write:

$$V_b - V_a = \Delta V = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r}$$

Electric potential (V): It is the amount of electric potential energy per unit charge, at a given point within an electric field.

Units:
$$\frac{Joules}{C} = \frac{Nm}{C} \equiv Volts$$

$$V = \frac{U}{q_0}$$

U for q_0 within E field of q_1 :

$$U = \frac{q_1 q_0}{4\pi\epsilon_0 r}$$

Therefore:

$$V = \frac{q_1}{4\pi\epsilon_0 r}$$



Electric Potential 2: Multiple Point Charges

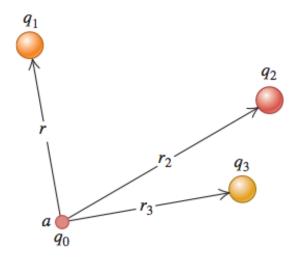
$$V = \frac{U}{q_0}$$

U for q₀ within E field created by multiple charges:

$$U = \frac{q_0}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

Therefore:

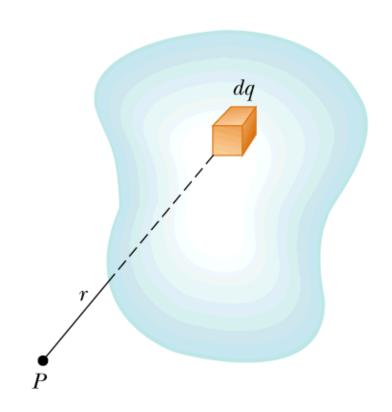
$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$



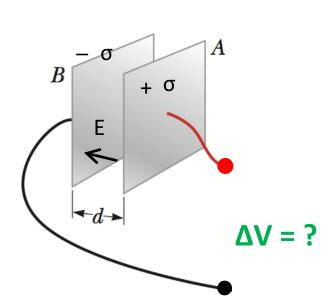
Electric Potential 3: Continuous Distribution of Charges

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

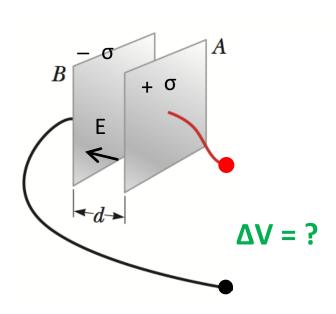


What is the electric potential difference between the plates?



$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

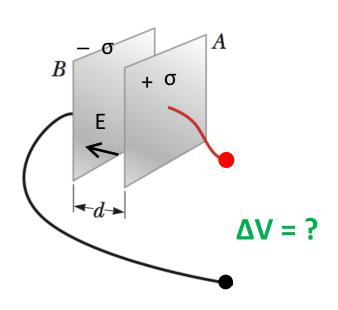
What is the electric potential difference between the plates?



$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

$$\Delta V = -\int_{a}^{b} \left(\frac{\sigma}{\epsilon_0}\right) ds = -\frac{\sigma}{\epsilon_0} \int_{a}^{b} ds$$

What is the electric potential difference between the plates?

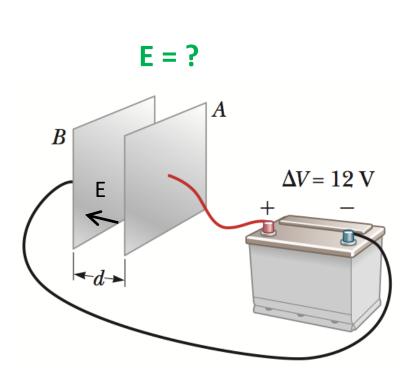


$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

$$\Delta V = -\int_{a}^{b} \left(\frac{\sigma}{\epsilon_0}\right) ds = -\frac{\sigma}{\epsilon_0} \int_{a}^{b} ds$$

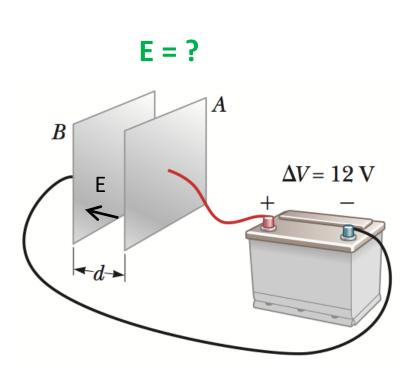
$$\Delta V = -\frac{\sigma}{\epsilon_0} d$$

• Now, what is the electric field (in terms of ΔV and d) between the plates?



$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

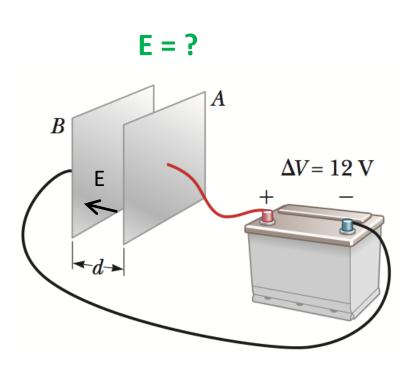
• Now, what is the electric field (in terms of ΔV and d) between the plates?



$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

$$\Delta V = -E \int_{a}^{b} ds$$

• Now, what is the electric field (in terms of ΔV and d) between the plates?

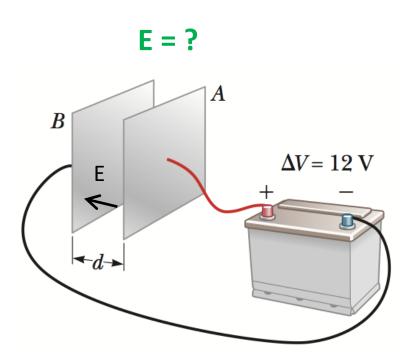


$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

$$\Delta V = -E \int_{a}^{b} ds$$

$$\Delta V = -Ed$$

 Now, what is the electric field (in terms of ΔV and d) between the plates?



$$\Delta V = -\int_{a}^{b} \vec{E} \cdot d\vec{s}$$

$$\Delta V = -E \int_{a}^{b} ds$$

$$\Delta V = -Ed$$

$$E = -\frac{\Delta V}{d}$$