Lecture 2. Basic Probability Rules

YULIA R. GEL

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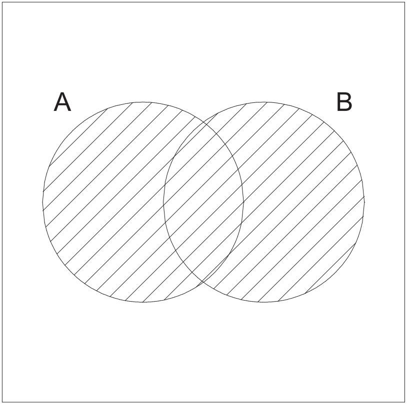
Basics

It is useful to interpret (and visualize!) events via algebra of sets. First, we start from basic notations:

- The statement that a possible outcome of an experiment s is a member of S is denoted symbolically by the relation $s \in S$.
- Any event E can be regarded as a certain subset of a sample space S, i.e. $E \subset S$ for all events E.
- An event A is said to be contained in another event B if every outcome that belongs to the subset defining the event A also belongs to the subset B, i.e. $A \subset B$.
- Some events are impossible! E.g., roll of a die cannot produce a negative number. Hence, such event of observing a negative number contains no outcomes. This subset of S is called the empty set and is denoted by the symbol $\emptyset$. Note that $\emptyset \subset A \subset S$ for all events A.

Unions

If A and B are any two events then the union of A and B is defined to be the event containing all outcomes that belong to A alone, to B alone or to both A and B. We denote the union of A and B by $A \cup B$.



Properties of a Union

For any events A and B such that $A \subset S$ and $B \subset S$, the following properties hold:

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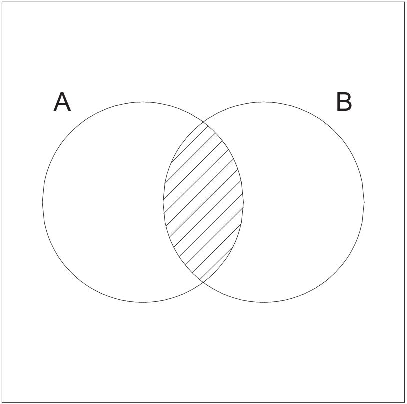
The union of n events $E_1, E_2, \ldots, E_n$ is defined to be the event that contains all outcomes which belong to at least one of the n events, and is denoted by $\cup_{i=1}^{n} E_i$.

Intersection

If A and B are any two events then the intersection of A and B is defined to be the event containing all outcomes that belong both to A and B. We denote the intersection of A and B by $A \cap B$.



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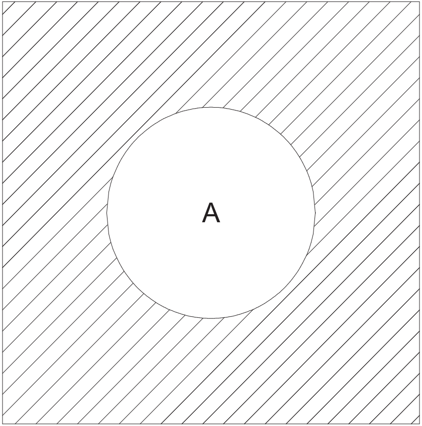
The intersection of n events $E_1, E_2, \ldots, E_n$ is defined to be the event that contains all outcomes which belong to all of the n events, and is denoted by $\cap_{i=1}^{n} E_i$.

Complement

The complement of an event A is defined to be event that contains all outcomes in the sample space S which do not belong to A. The complement of A is denoted as A^c.



Properties of a Compliment

For any events A and B such that $A \subset S$ and $B \subset S$, the following properties hold:

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Disjoint and Exhaustive Events

It is said that two events E_1 and E_2 are **disjoint** or **mutually exclusive** if E_1 and E_2 have no outcomes in common. It follows that E_1 and E_2 are disjoint if and only if $E_1 \cap E_2 = \emptyset$.

Events $E_1, E_2, \ldots$ are called **exhaustive** if $\cup_i E_i = S$ and at least one E_i occures for sure.

Example 1. E and E^c are disjoint and exhaustive. For instance, gender of a child, i.e. male and female are disjoint and exhaustive.

DeMorgan's Laws

Let A, B and C be subsets of S. Then,

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