Homework 7

Problem of the week.

A customer service phone line transitions every minute between "busy" and "free" states according to a Markov chain. If the line is busy, then it may become free within a minute with probability 0.4. Whereas if the line is free, then it may become busy within a minute with probability 0.8. Suppose you call the line at 10:00 and it was busy. You call again at 10:02. What is the chance it will be free this time?

Solution Let X_n denote the status ($1 = $ busy, $2 = $ free) of the phone line at minute $n = 1, 2, \ldots$. It follows a Markov chain with transition probability matrix

$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{pmatrix}$$

We want the probability $P(X_2 = 2 | X_0 = 1) = p_{12}^{(2)}$. The two-step transition probability matrix is

$$P^2 = P * P = \begin{pmatrix} 0.68 & 0.32 \\ 0.64 & 0.36 \end{pmatrix}$$

Therefore, the desired answer is $p_{11}^{(2)} = 0.32$.

This problem is similar, for example, to the practice problem # 6.2, 6.3, 6.4 in Chapter 3 of MB.