Homework 4

Problem of the week. Bits are sent over a communications channel in packets of 12.

(a) If the probability of a bit being corrupted over this channel is 0.1 and such errors are independent, what is the probability that no more than 2 bits in a packet are corrupted?

(b) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits?

Solution Let Y denote the number of corrupted bits in a packet, then Y follows a binomial distribution with $p = 0.1$ and $n = 12$. Thus in the first question, we would like to compute

$$\begin{aligned} P(Y \leq 2) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\ &= C(12, 0)(0.1)^0(0.9)^{12} + C(12, 1)(0.1)^1(0.9)^{11} + C(12, 2)(0.1)^2(0.9)^{10} \\ &= 0.282 + 0.377 + 0.230 \\ &= 0.889. \end{aligned}$$

Hence, the probability of a packet containing 3 or more corrupted bits is $1 - 0.889 = 0.111$. Let X be the number of packets containing 3 or more corrupted bits. Then X can be modelled with a binomial distribution with parameters $n = 6$, $p = 0.111$. The probability that at least one packet will contain 3 or more corrupted bits is:

$$1 - P(X = 0) = 1 - C(6, 0)(0.111)^0(1 - 0.111)^6 = 0.5064$$

This problem is similar, for example, to the practice problems 3.20–3.24 in Chapter 3 of MB.