PHYS2326
Lecture #23

Prof. Fabiano Rodrigues

Department of Physics
The University of Texas at Dallas

Reminders

- Exam #3 on Thursday
- Bring:
  - ID
  - Pen, pencil and scientific calculator
  - Formulas available on eLearning
- Arrive early

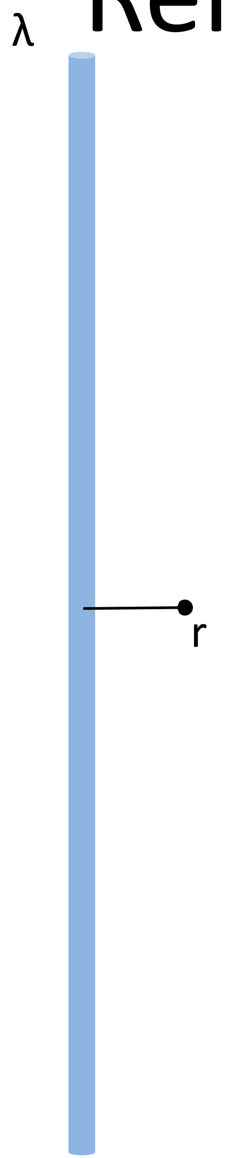
Goals for today's lecture

- Introduce Ampere's Law
- Applications of Ampere's Law

Chapter 28

Reminder: Electric Fields

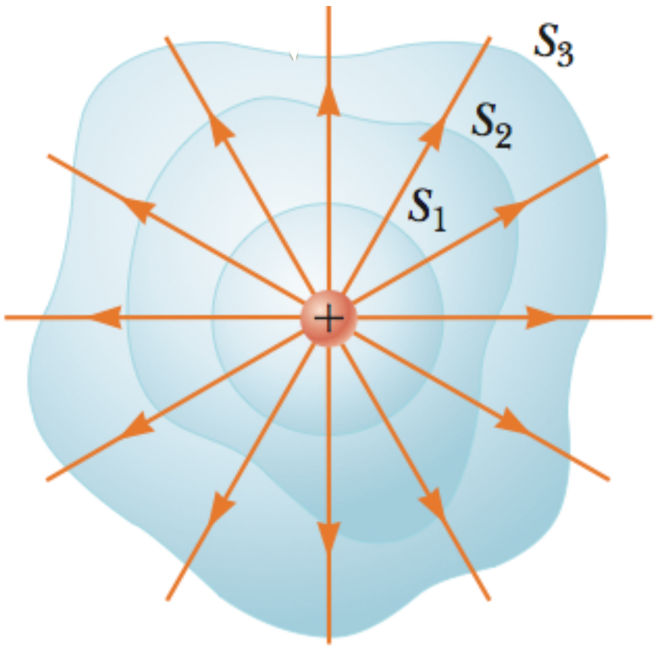
Reminder: Electric Fields



Electric field at point *r*?

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$

Reminder: Gauss's Law



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

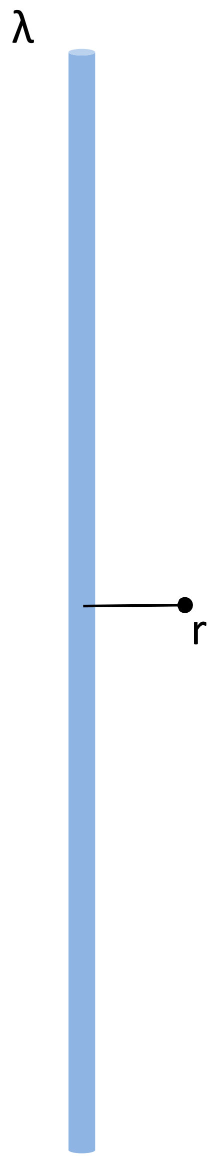
Applications of Gauss's Law

- Gauss's Law is particularly useful (analytically treatable) when the direction of the electric field is known from the symmetry of the charge distribution.

- Our task is to choose a Gaussian surface that simplifies the equation for Gauss's Law, so that electric field can be determined from:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

Example: Using Gauss's Law, find the magnitude of the electric field at a distance "r" from a infinite length line of positive charge with constant linear charge density "λ".



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$E(r)$=?

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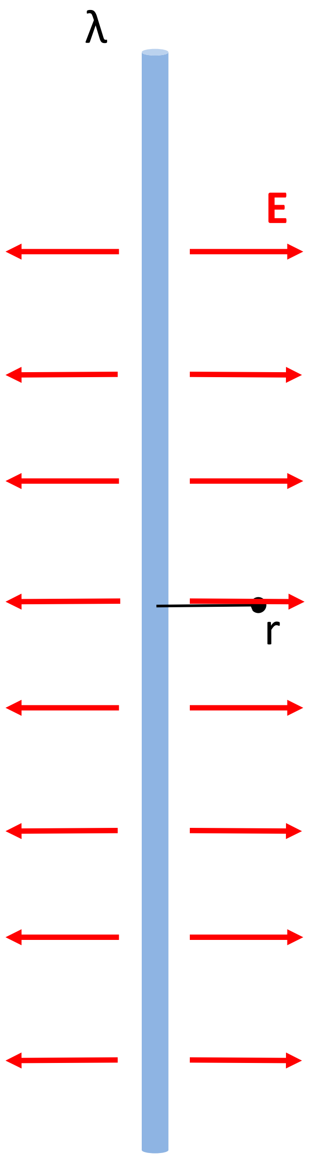

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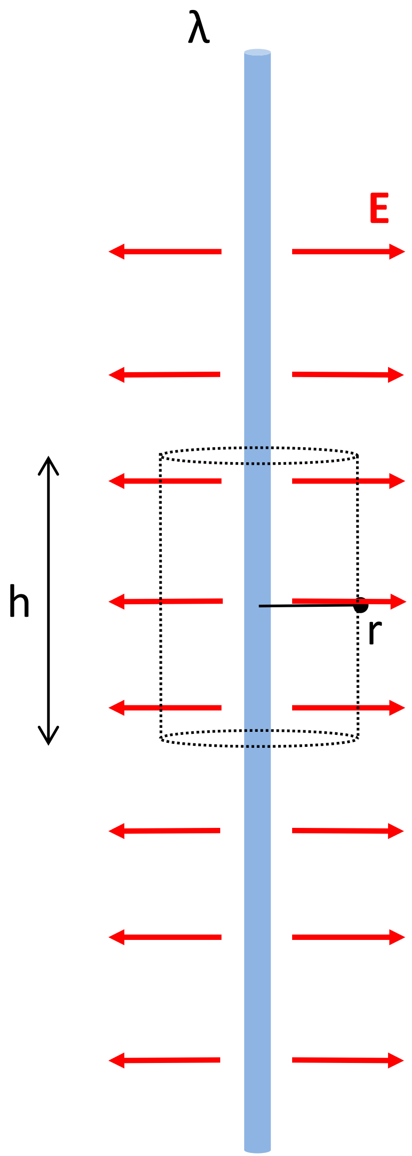

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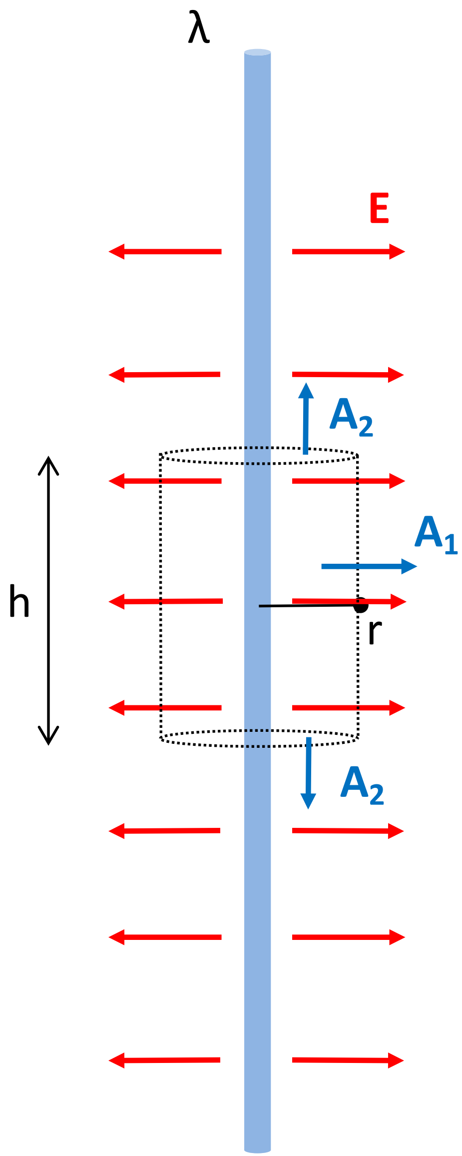

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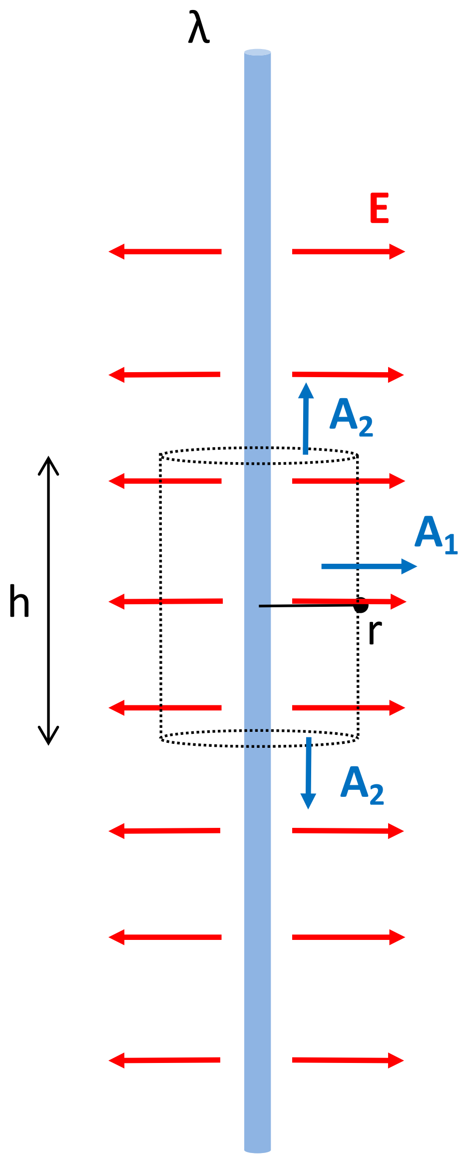

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$E(r)$=?

$$E(r)A_1 = \frac{\lambda h}{\epsilon_0}$$

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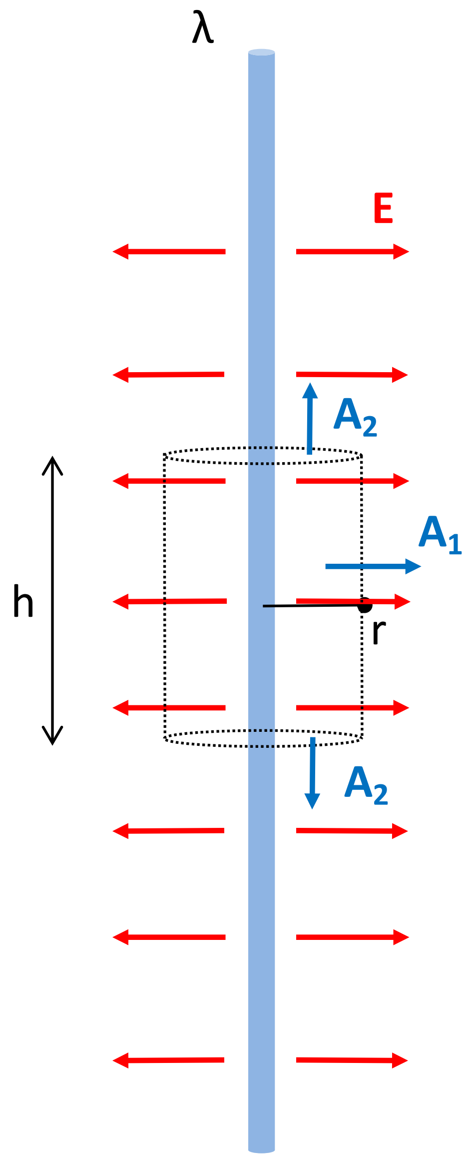

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$E(r)$=?

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$$E(r)(2\pi r)h = \frac{\lambda h}{\epsilon_0}$$

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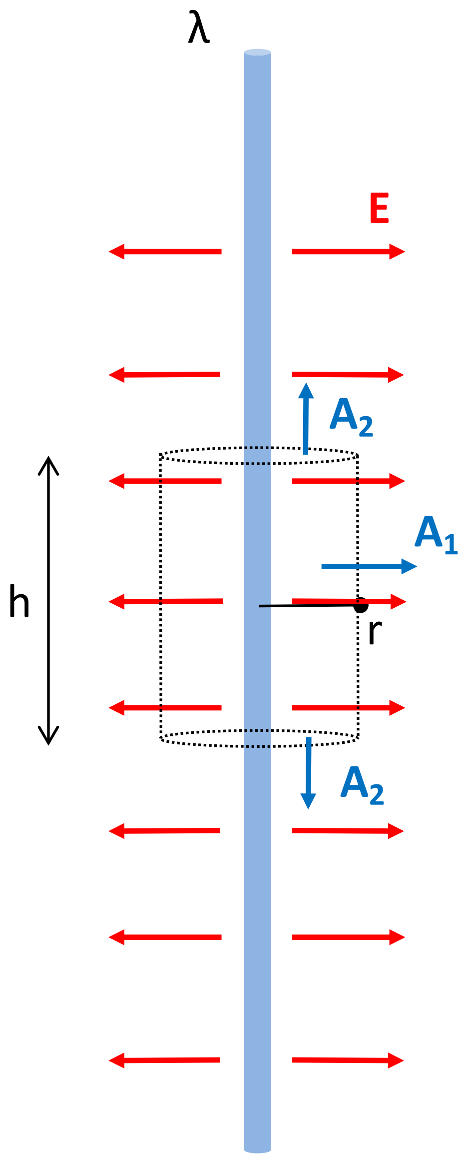


$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$E(r)$=?

$$E(r)A_1 = \frac{\lambda h}{\epsilon_0}$$

$$E(r)(2\pi r)h = \frac{\lambda h}{\epsilon_0}$$

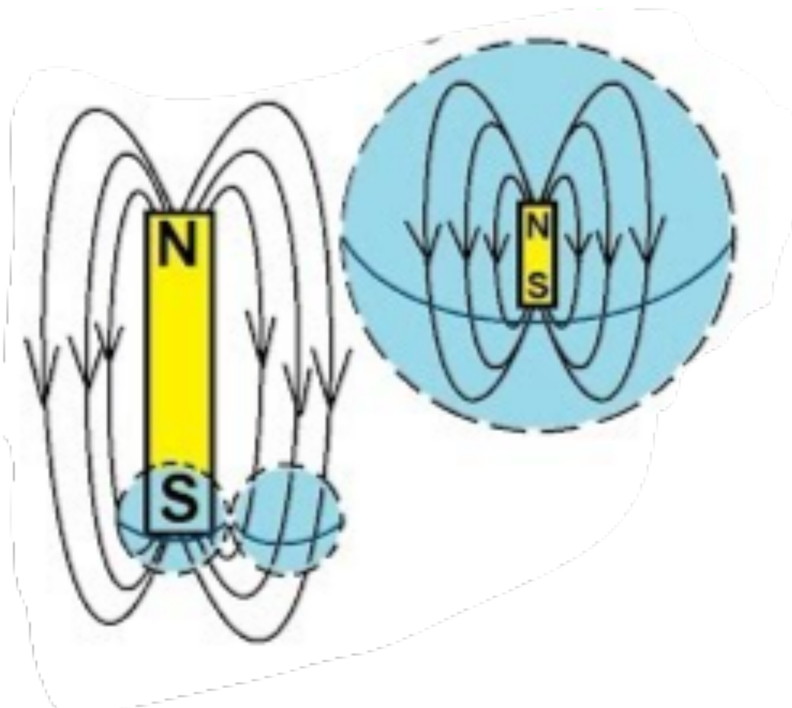
$$\boxed{E(r) = \frac{\lambda}{2\pi r \epsilon_0}}$$

Field of infinite line of charge

Gauss's Law (for Magnetism)

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law (for Magnetism)



$$\oint \vec{B} \cdot d\vec{A} = 0$$

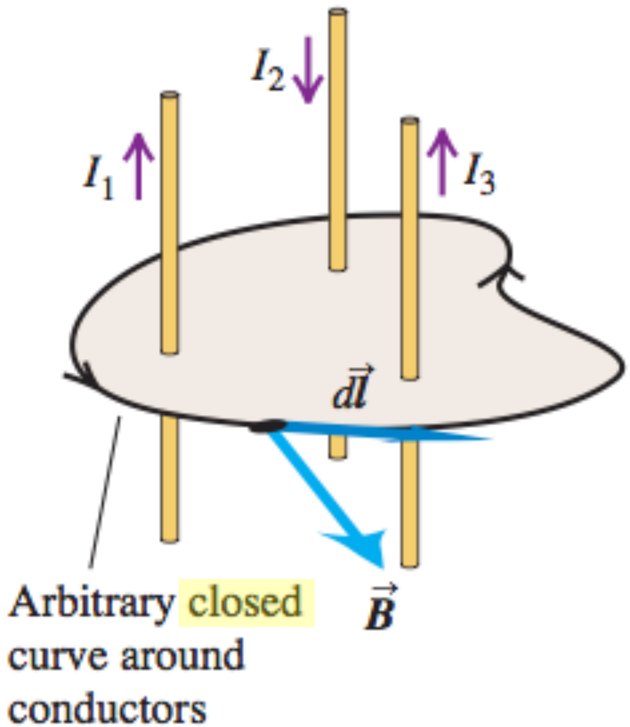
Maxwell's Equations

Gauss's Law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon}$
Gauss's Law for Magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$
Ampere's Law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$
Faraday's Law	$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_\mathrm{B}}{\partial t}$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

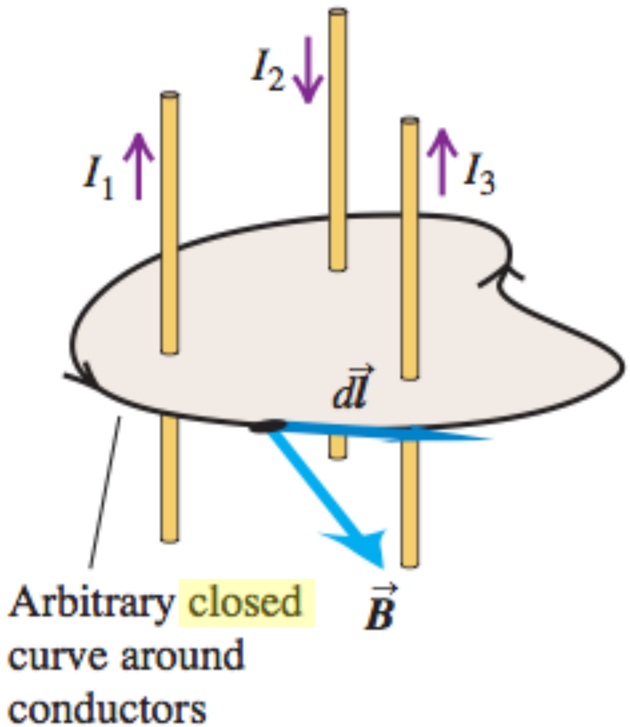
Ampere's Law



"Amperian loop"

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Ampere's Law



"Amperian loop"

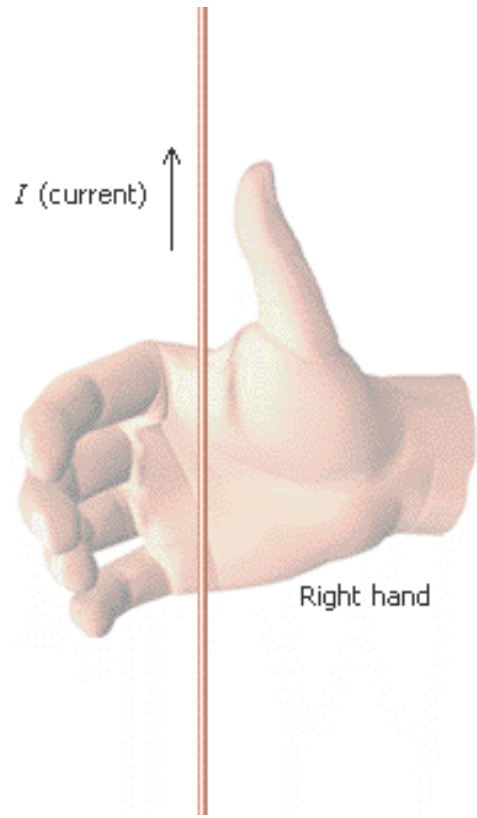
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Example:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(+I_1 - I_2 + I_3)$$

Application of Ampere's Law #1: Long Conductor

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$



Application of Ampere's Law #1: Long Conductor

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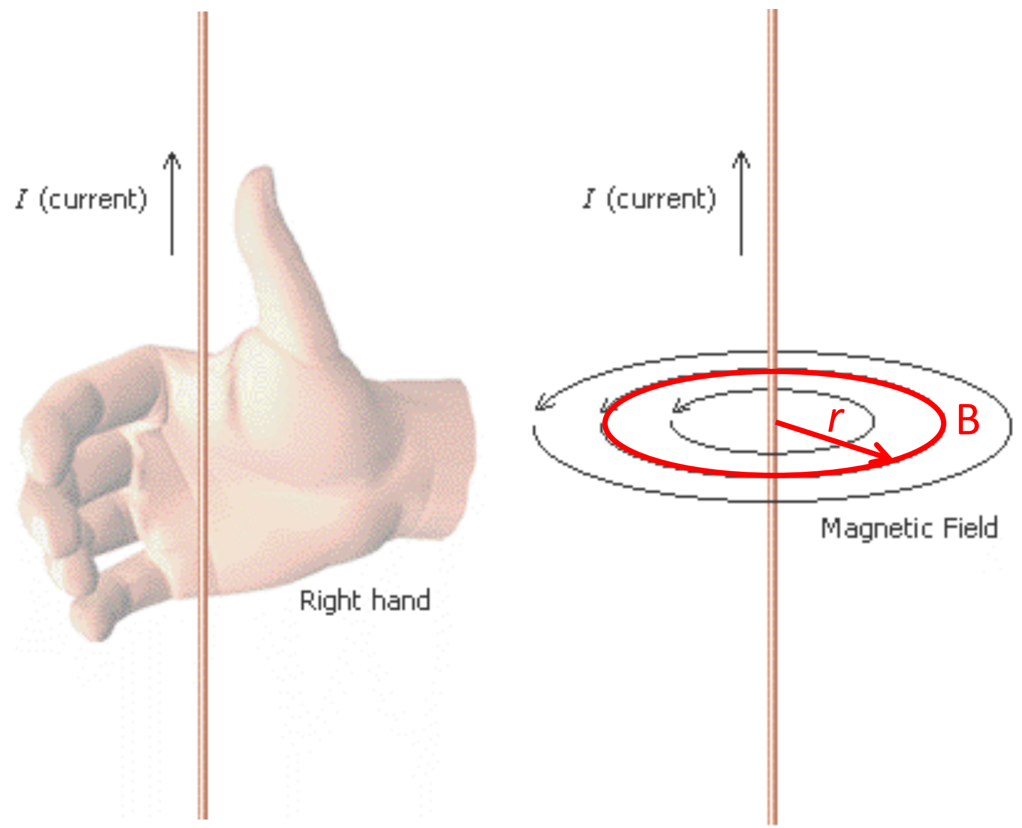
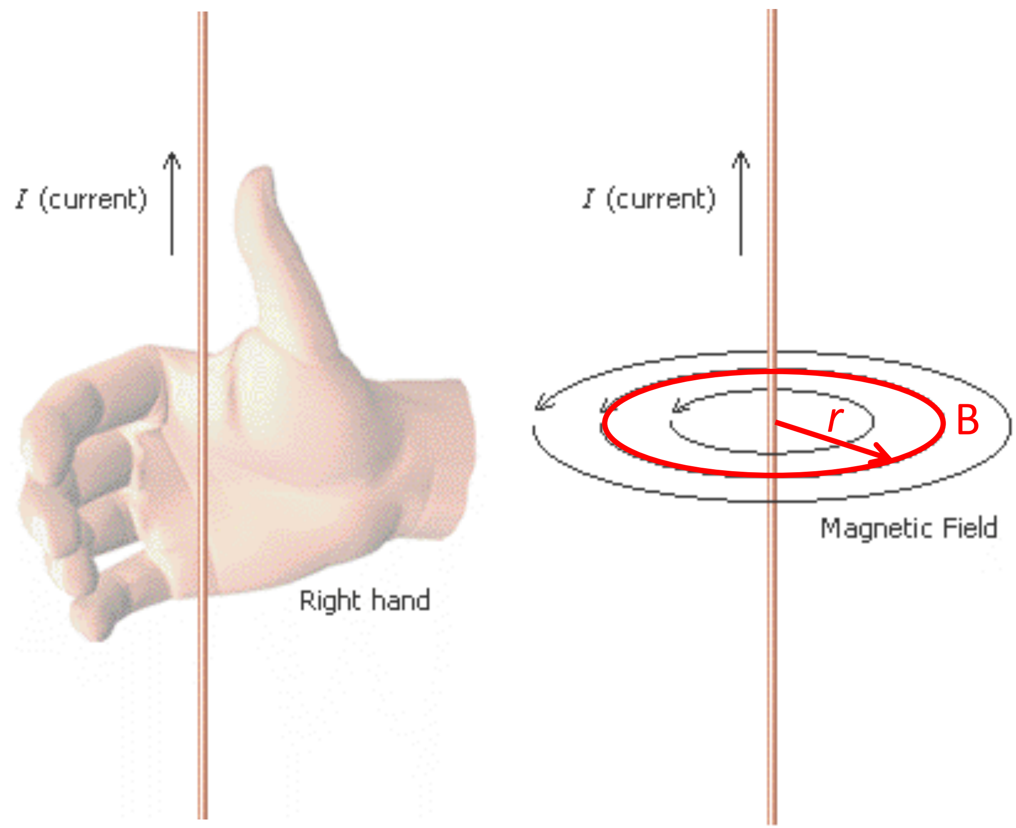


Application of Ampere's Law #1: Long Conductor

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B \int_0^{2\pi} r d\theta = \mu_0(+I)$$



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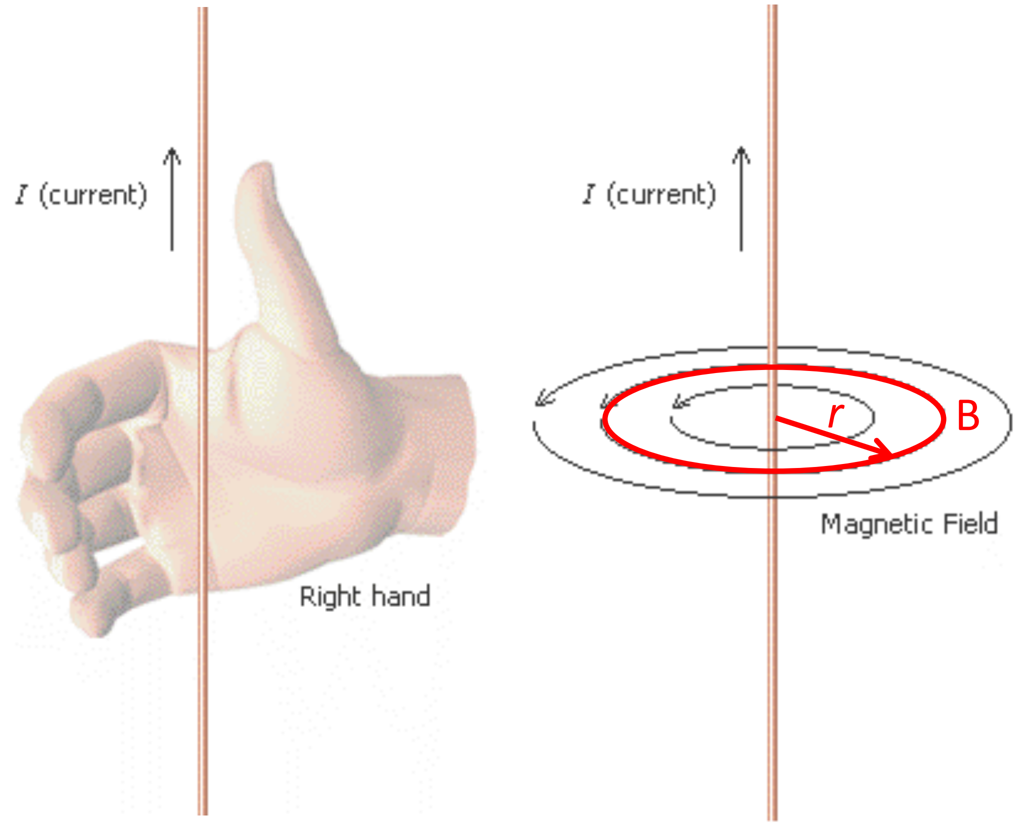
$$B2\pi r = \mu_0 I$$



Application of Ampere's Law #1: Long Conductor

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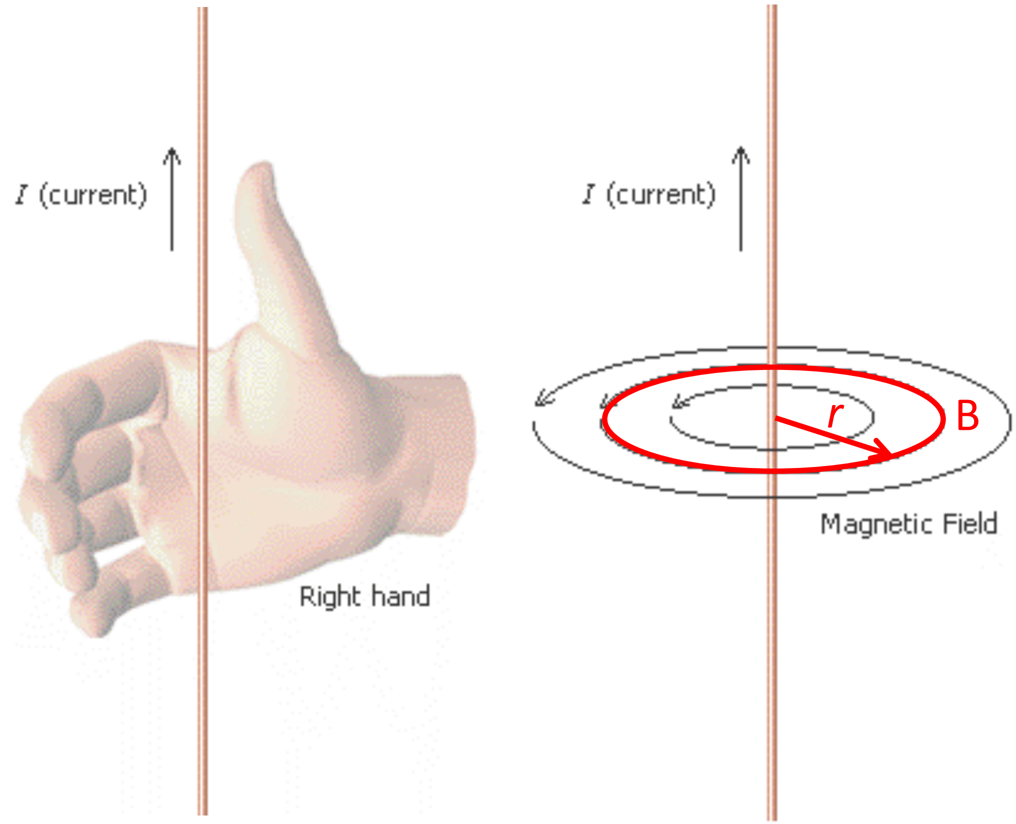
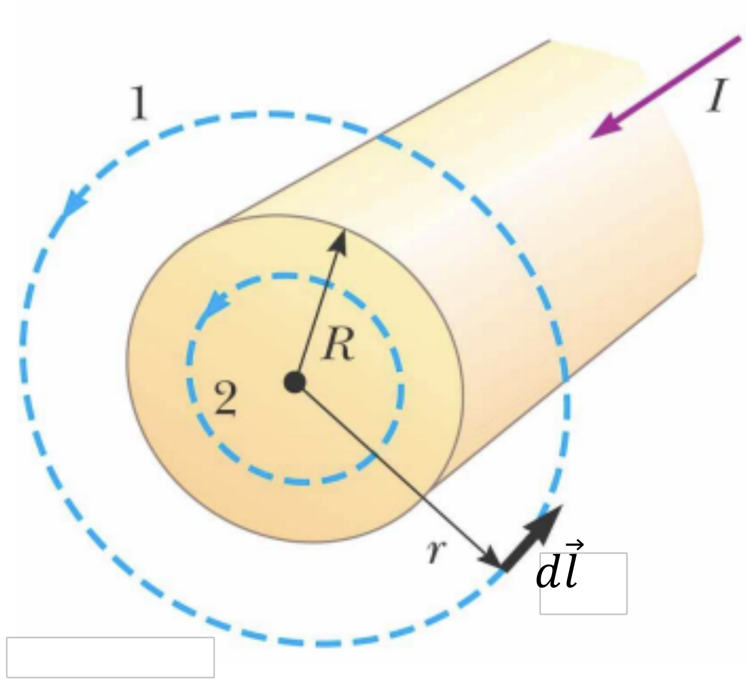
$$B 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

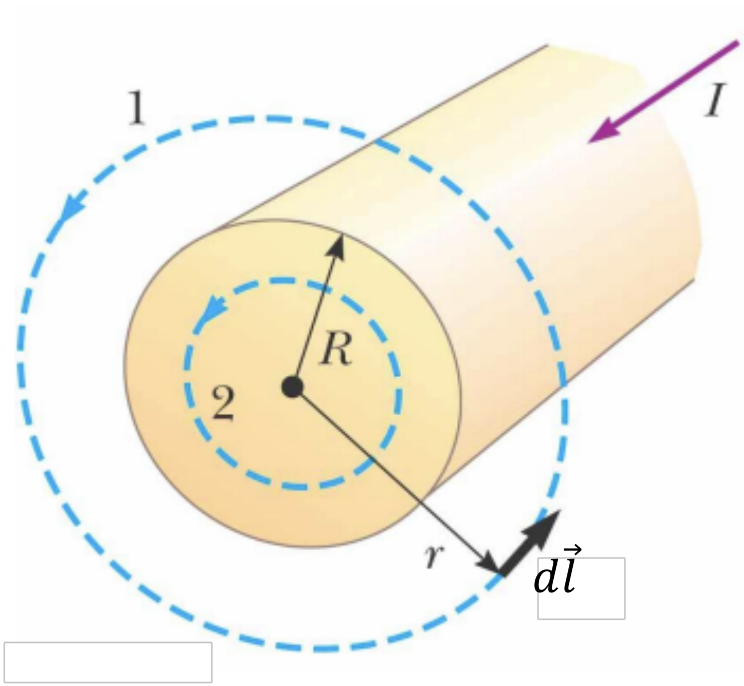


Application of Ampere's Law #2: Cylindrical Conductor

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$



Application of Ampere's Law #2: Cylindrical Conductor

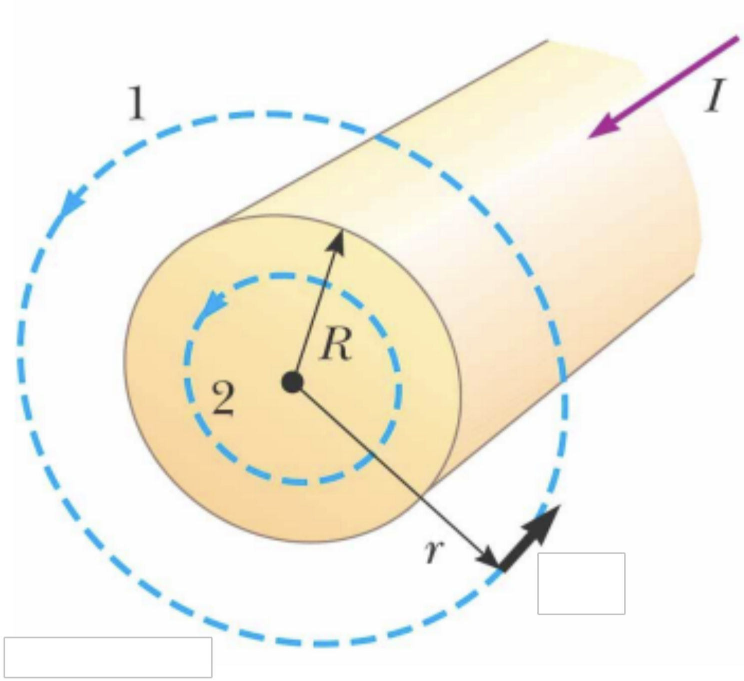


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$Two\ regions$:

$Region\ 1: r > R$
$Region\ 2: r < R$

Application of Ampere's Law #2: Cylindrical Conductor



$$Region\ 1\colon r > R$$

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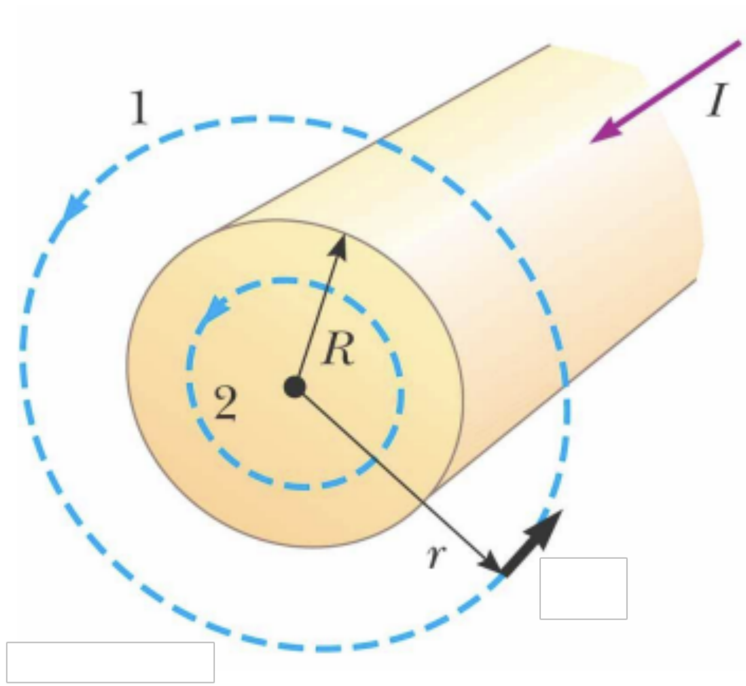
Application of Ampere's Law #2: Cylindrical Conductor



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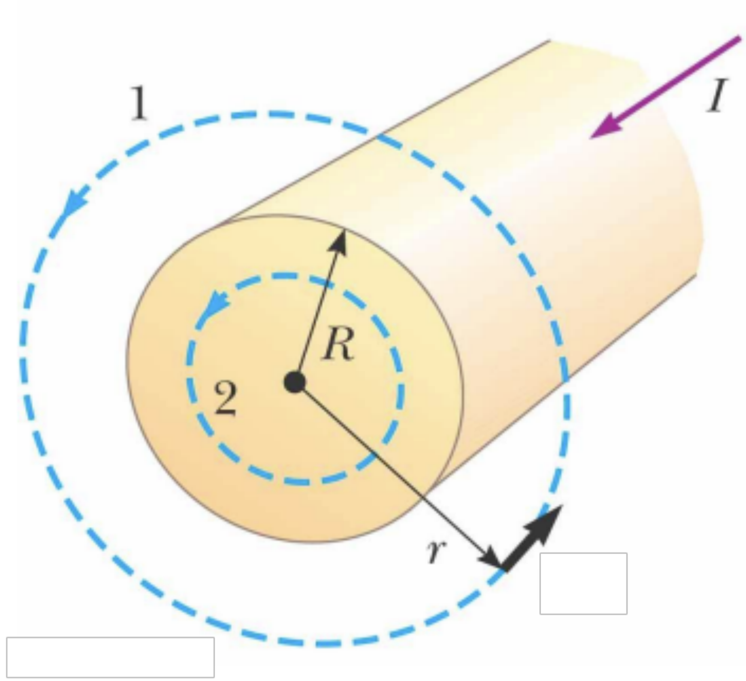

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$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

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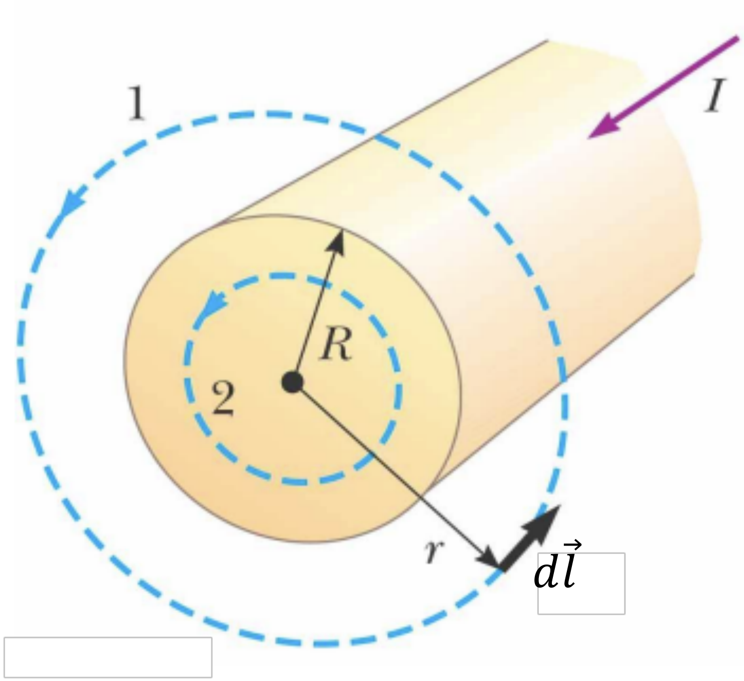
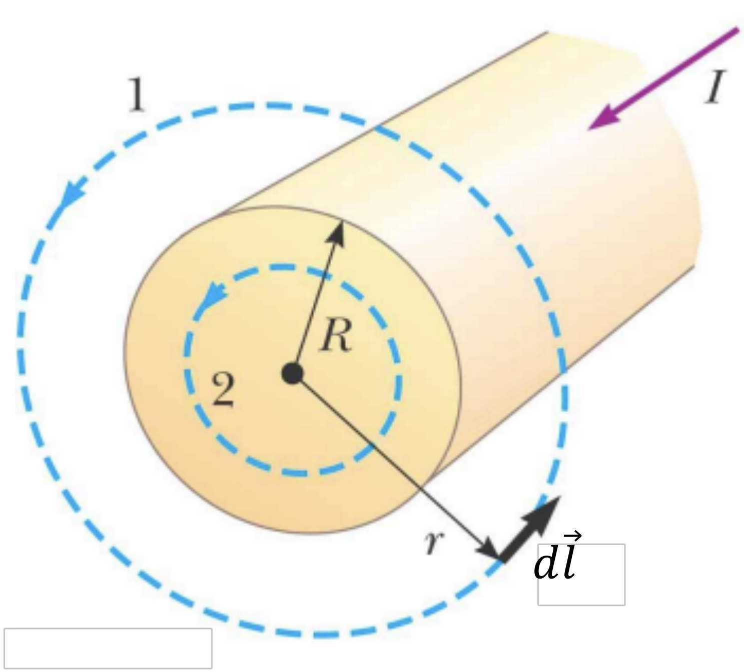


$Region\ 2: r < R$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Application of Ampere's Law #2: Cylindrical Conductor

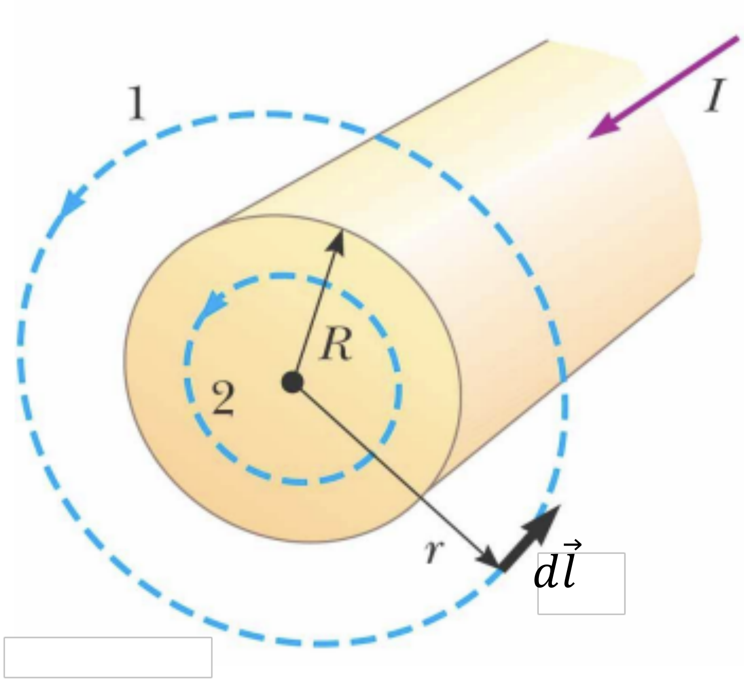


$Region\ 2: r < R$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B 2\pi r = \mu_0 (JA)$$

Application of Ampere's Law #2: Cylindrical Conductor



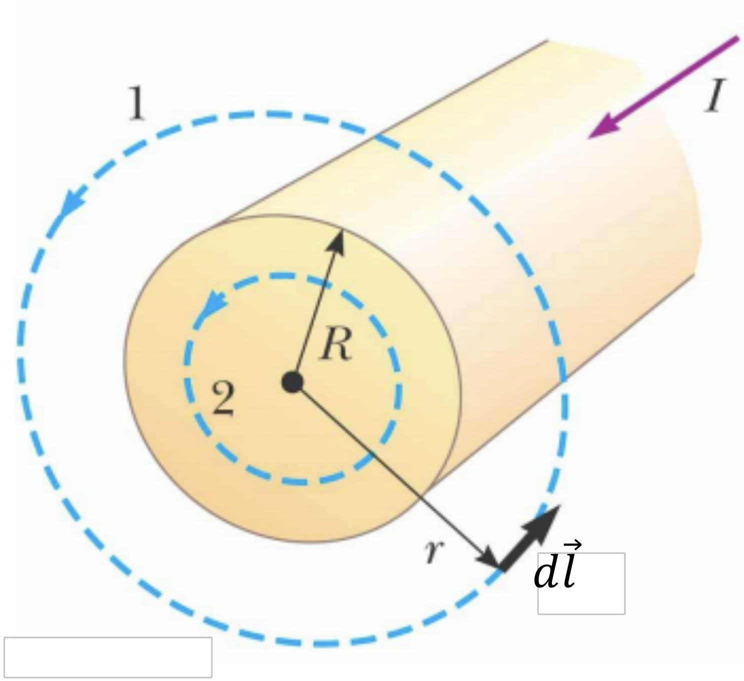
$Region\ 2\colon r < R$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B2\pi r = \mu_0 (JA)$$

$$B2\pi r = \mu_0 \left(\frac{I}{\pi R^2}\right) A$$

Application of Ampere's Law #2: Cylindrical Conductor



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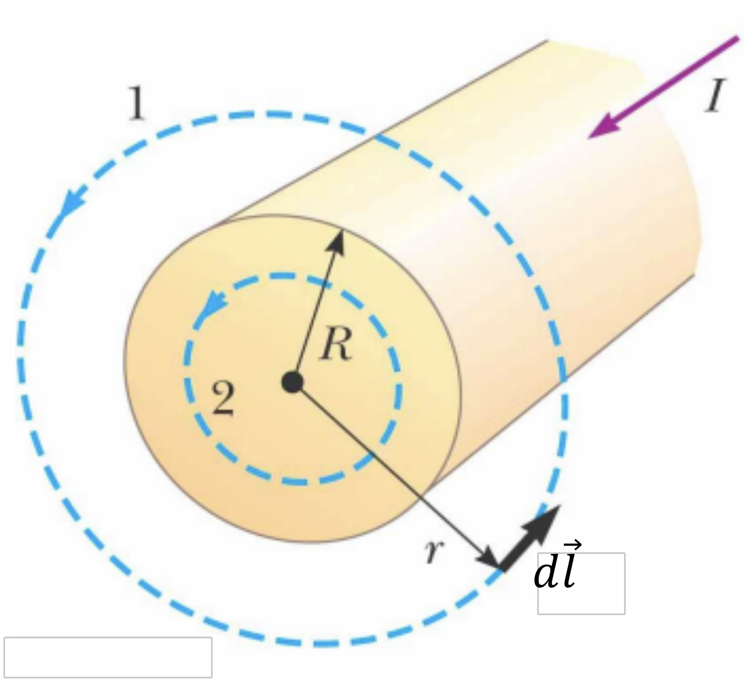
Application of Ampere's Law #2: Cylindrical Conductor



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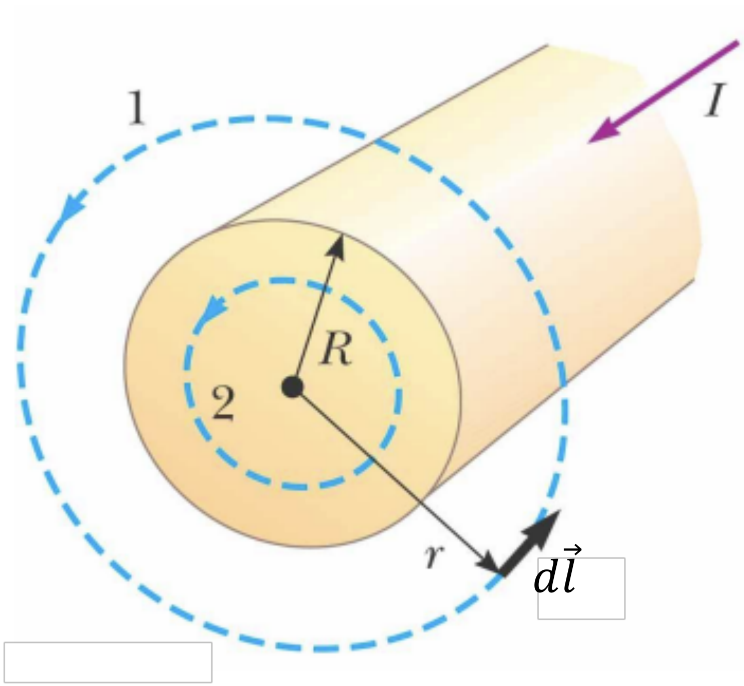
Application of Ampere's Law #2: Cylindrical Conductor



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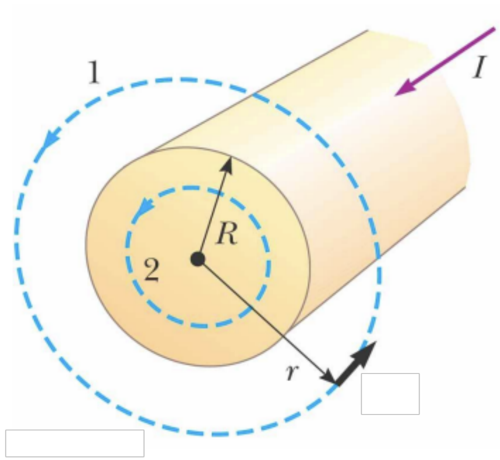
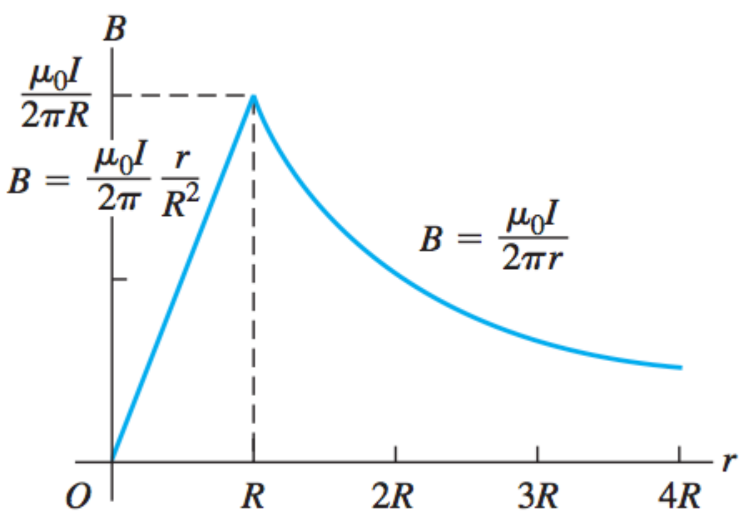
$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

Application of Ampere's Law #2: Cylindrical Conductor





$r < R$:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$r < R$:

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

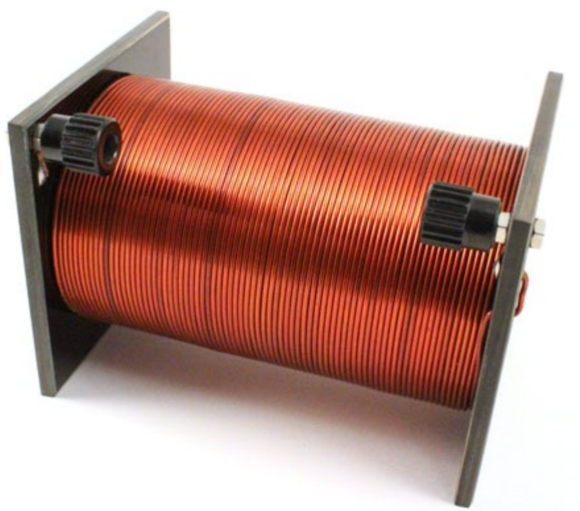
$r = R$:

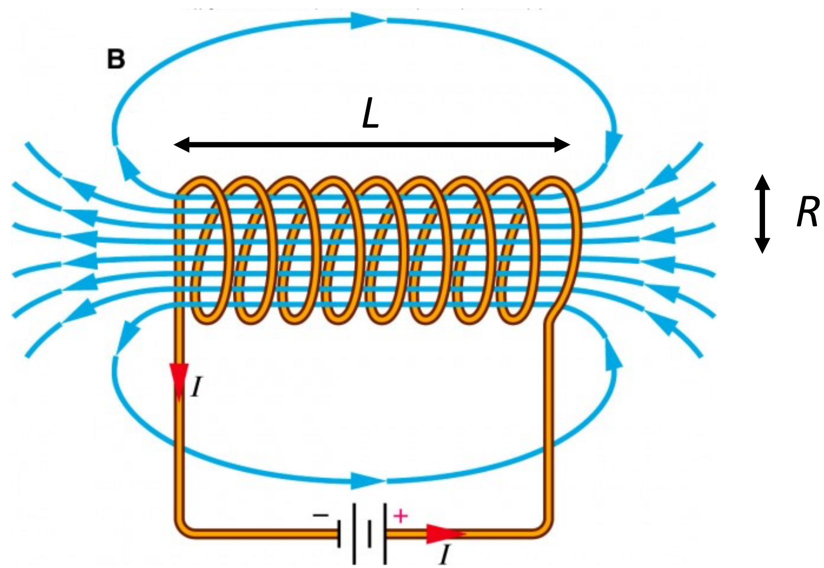
$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

Application of Ampere's Law #3: Ideal Solenoid

- A solenoid is a coil wound into a close-packed helix.

- Ideal: Solenoid has a large number of turns per unit length, and its length L is much greater than its radius R so that magnetic field inside can be considered uniform, and the magnetic field outside negligible.

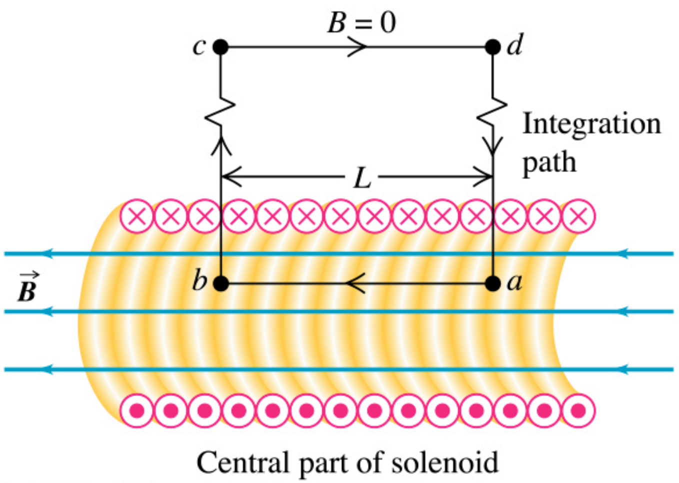




Application of Ampere's Law #3: Ideal Solenoid

$Solenoid \ \ with$
$"n" \ \ turns \ \ per \ \ unit \ \ lenght$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$



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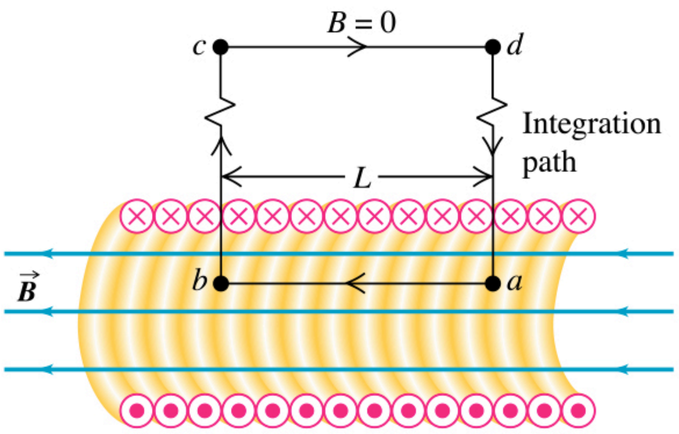


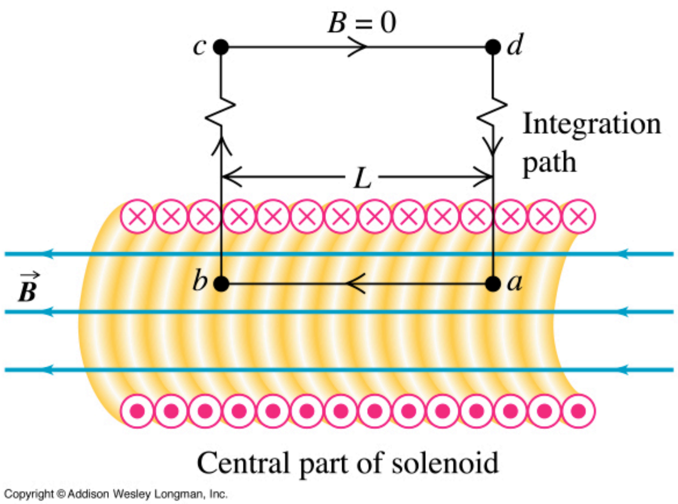
Central part of solenoid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l} = \mu_0 NI$$

Application of Ampere's Law #3: Ideal Solenoid

Solenoid with "n" turns per unit lenght

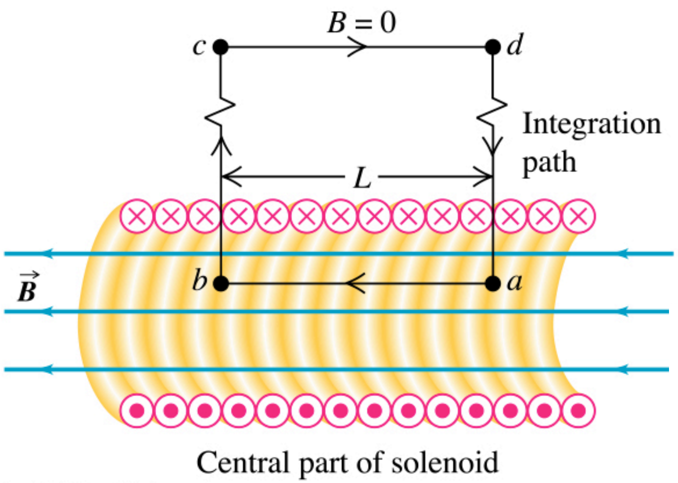


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\int_a^b \vec{B} \cdot d\vec{l} + \cancelto{0:\ \vec{B} \perp d\vec{l}}{\int_b^c \vec{B} \cdot d\vec{l}} + \cancelto{0:\ \vec{B} = 0}{\int_c^d \vec{B} \cdot d\vec{l}} + \cancelto{0:\ \vec{B} \perp d\vec{l}}{\int_d^a \vec{B} \cdot d\vec{l}} = \mu_0 N I$$

Application of Ampere's Law #3: Ideal Solenoid

$$Solenoid \;\; with$$
$$"n" \;\; turns \;\; per \;\; unit \;\; lenght$$



Central part of solenoid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

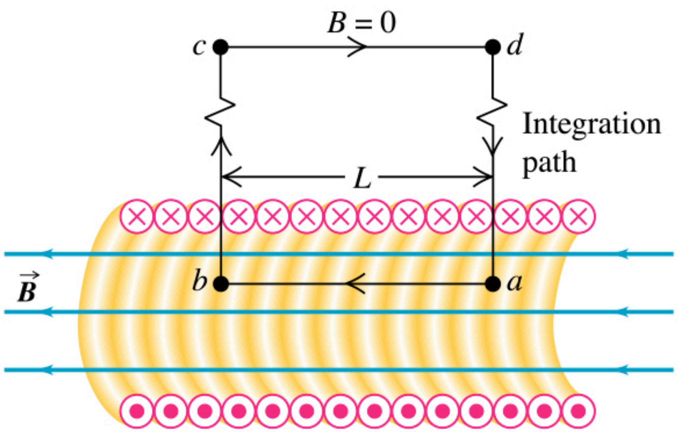
0: $\vec{B} \perp d\vec{l}$ $\quad$ 0: $\vec{B} = 0$ $\quad$ 0: $\vec{B} \perp d\vec{l}$

$$\int_a^b \vec{B} \cdot d\vec{l} + \cancelto{0}{\int_b^c \vec{B} \cdot d\vec{l}} + \cancelto{0}{\int_c^d \vec{B} \cdot d\vec{l}} + \cancelto{0}{\int_d^a \vec{B} \cdot d\vec{l}} = \mu_0 N I$$

$$\int_a^b \vec{B} \cdot d\vec{l} = \mu_0 N I$$

Application of Ampere’s Law #3: Ideal Solenoid

Solenoid with "n" turns per unit lenght



Central part of solenoid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \qquad 0:\ \vec{B} \perp d\vec{l} \qquad 0:\ \vec{B} = 0 \qquad 0:\ \vec{B} \perp d\vec{l}$$

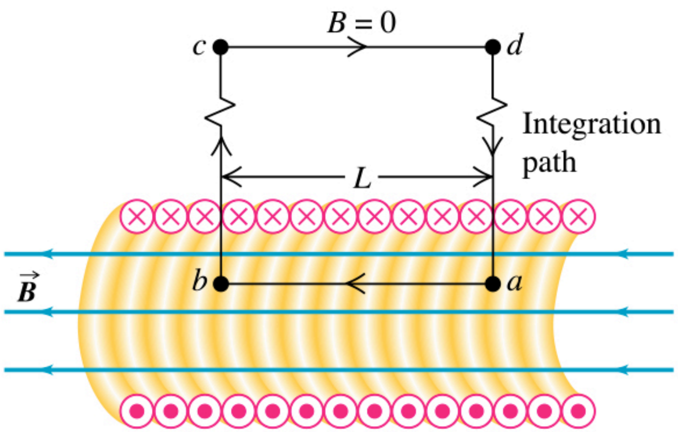
$$\int_a^b \vec{B} \cdot d\vec{l} + \cancel{\int_b^c \vec{B} \cdot d\vec{l}} + \cancel{\int_c^d \vec{B} \cdot d\vec{l}} + \cancel{\int_d^a \vec{B} \cdot d\vec{l}} = \mu_0 N I$$

$$\int_a^b \vec{B} \cdot d\vec{l} = \mu_0 N I$$

$$BL = \mu_0 N I$$

Application of Ampere’s Law #3: Ideal Solenoid

Solenoid with
"n" turns per unit lenght



Central part of solenoid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\int_a^b \vec{B} \cdot d\vec{l} + \overset{0:\ \vec{B} \perp d\vec{l}}{\cancel{\int_b^c \vec{B} \cdot d\vec{l}}} + \overset{0:\ \vec{B} = 0}{\cancel{\int_c^d \vec{B} \cdot d\vec{l}}} + \overset{0:\ \vec{B} \perp d\vec{l}}{\cancel{\int_d^a \vec{B} \cdot d\vec{l}}} = \mu_0 N I$$

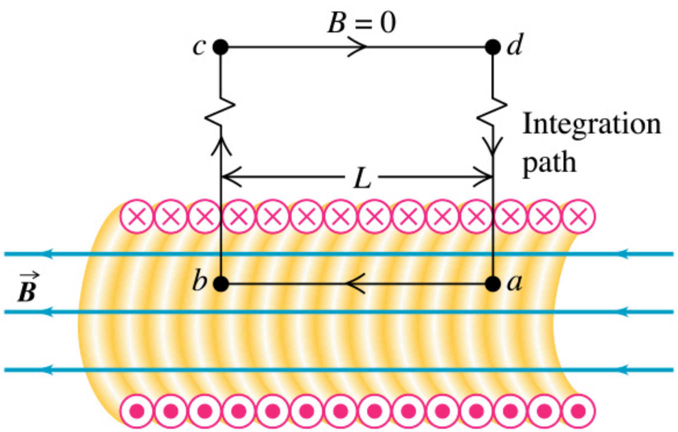
$$\int_a^b \vec{B} \cdot d\vec{l} = \mu_0 N I$$

$$BL = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{L}$$

Application of Ampere's Law #3: Ideal Solenoid

$$Solenoid\ \ with$$
$$"n"\ \ turns\ \ per\ \ unit\ \ lenght$$



Central part of solenoid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\int_a^b \vec{B} \cdot d\vec{l} + \overbrace{\cancel{\int_b^c \vec{B} \cdot d\vec{l}}}^{0:\ \vec{B} \perp d\vec{l}} + \overbrace{\cancel{\int_c^d \vec{B} \cdot d\vec{l}}}^{0:\ \vec{B} = 0} + \overbrace{\cancel{\int_d^a \vec{B} \cdot d\vec{l}}}^{0:\ \vec{B} \perp d\vec{l}} = \mu_0 NI$$

$$\int_a^b \vec{B} \cdot d\vec{l} = \mu_0 NI$$

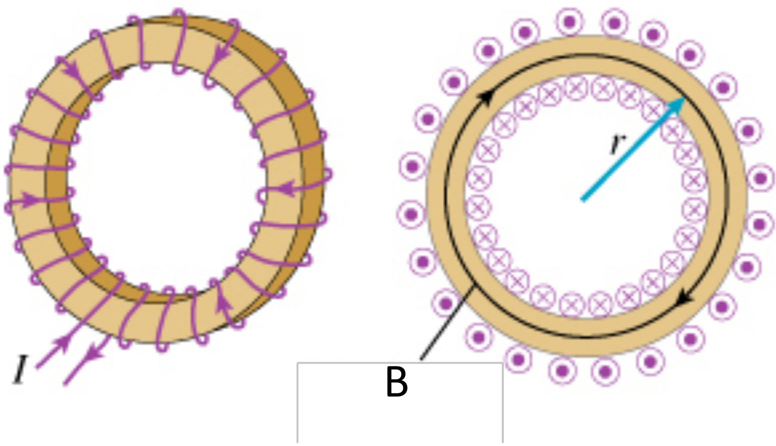
$$BL = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{L}$$

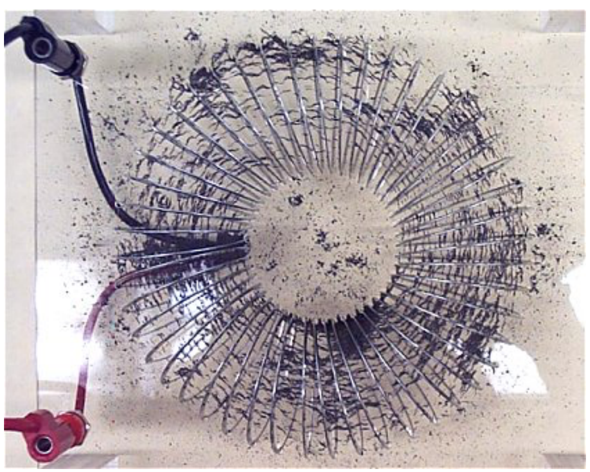
$$B = \mu_0 nI$$

$$Where\ n = \frac{N}{L}\ (turns\ per\ unit\ length)$$

Application of Ampere's Law #4: Toroid

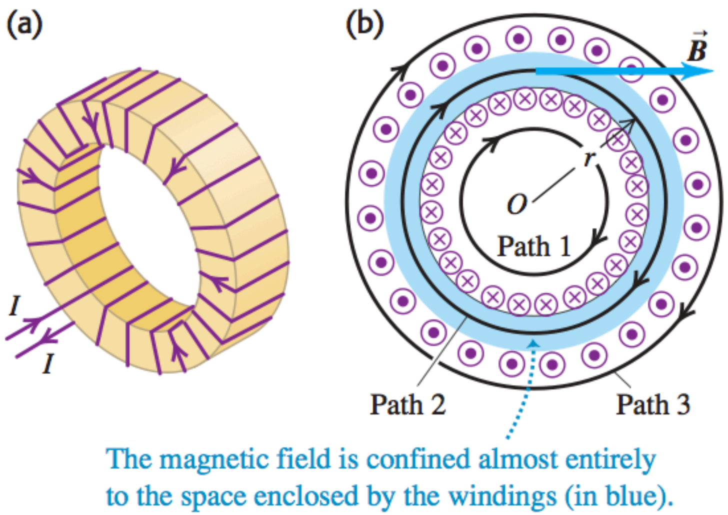


Toroid with N turns



- A toroid is a ring-shaped type of solenoid.
- Like the regular toroid the magnetic field is confined, almost entirely, to the space enclosed by the windings.

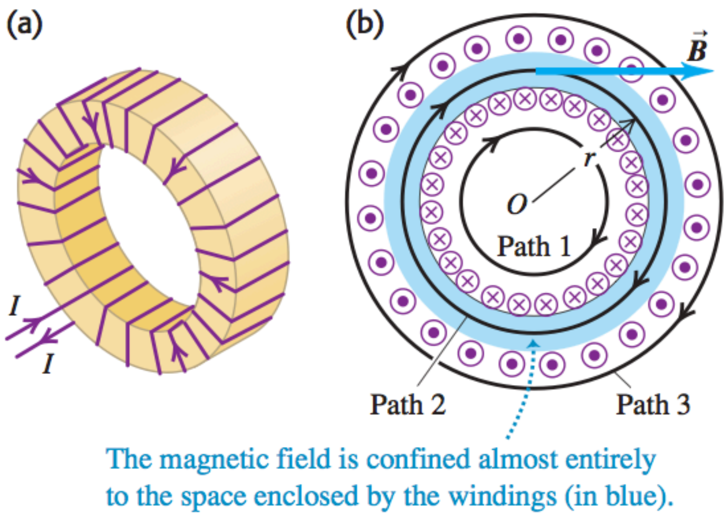
Application of Ampere's Law #4: Toroid



Toroid with N turns

Three regions (Amperian loops) possible

Application of Ampere's Law #4: Toroid

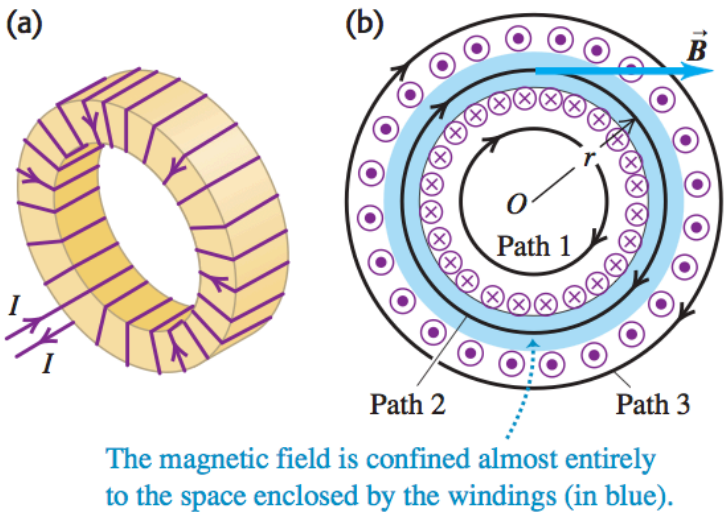


Toroid with N turns

$Region/Path$ 1:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Application of Ampere's Law #4: Toroid



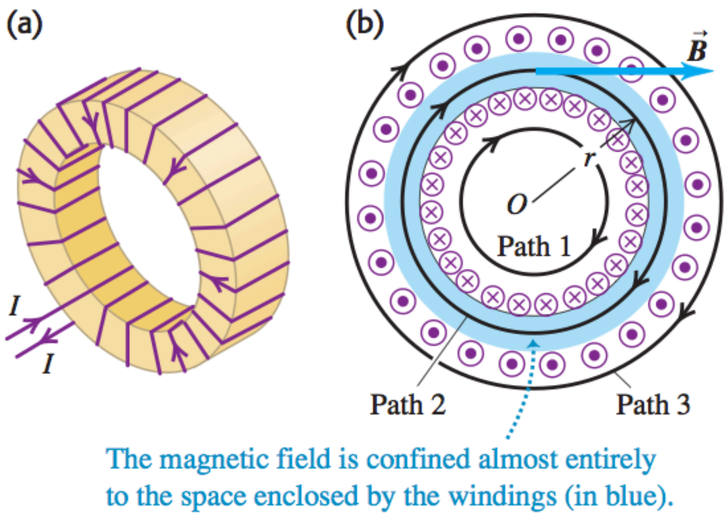
Toroid with N turns

$Region/Path\ 1$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B 2\pi r = \mu_0 (0)$$

Application of Ampere's Law #4: Toroid



Toroid with N turns

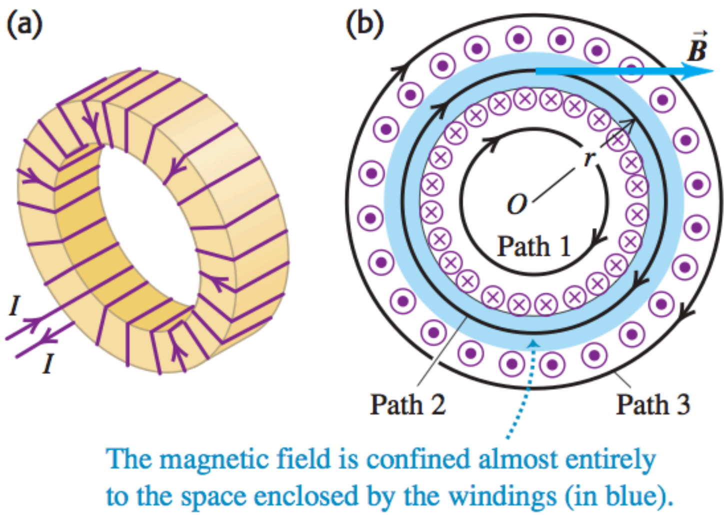
$Region/Path\ 1$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B2\pi r = \mu_0(0)$$

$$\boxed{B = 0}$$

Application of Ampere's Law #4: Toroid

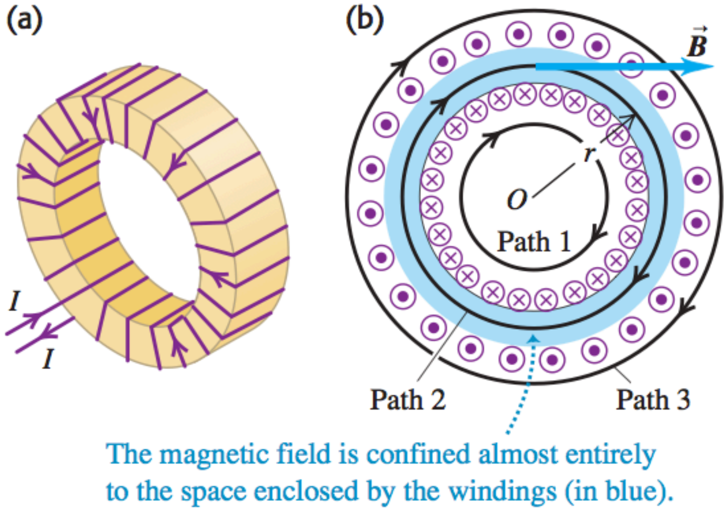


$Toroid \quad with \quad N \quad turns$

$Region/Path\ 2$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Application of Ampere's Law #4: Toroid



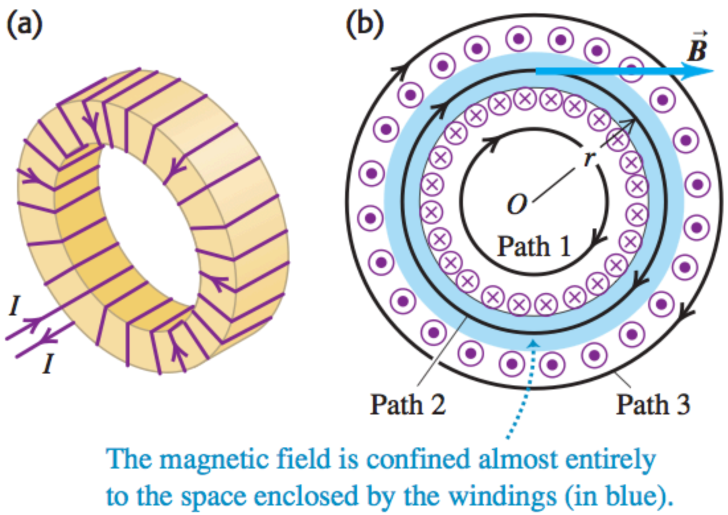
$Toroid \;\; with \;\; N \;\; turns$

$Region/Path\ 2$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B 2\pi r = \mu_0 N I$$

Application of Ampere's Law #4: Toroid



$Toroid \quad with \quad N \quad turns$

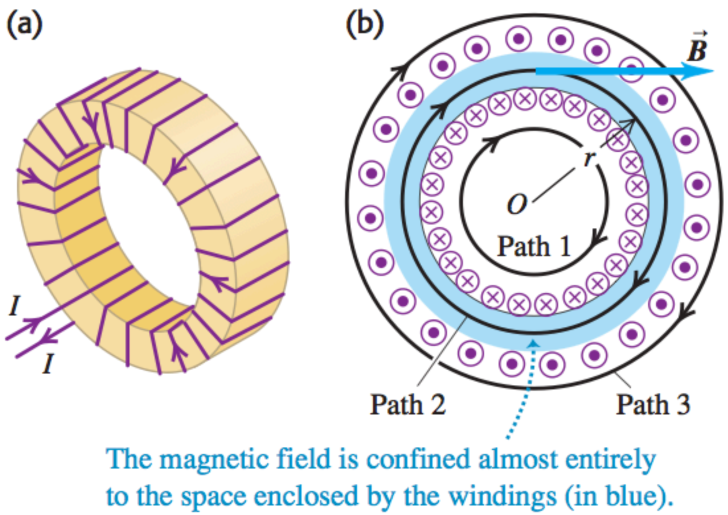
$Region/Path\ 2$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B 2\pi r = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Application of Ampere's Law #4: Toroid

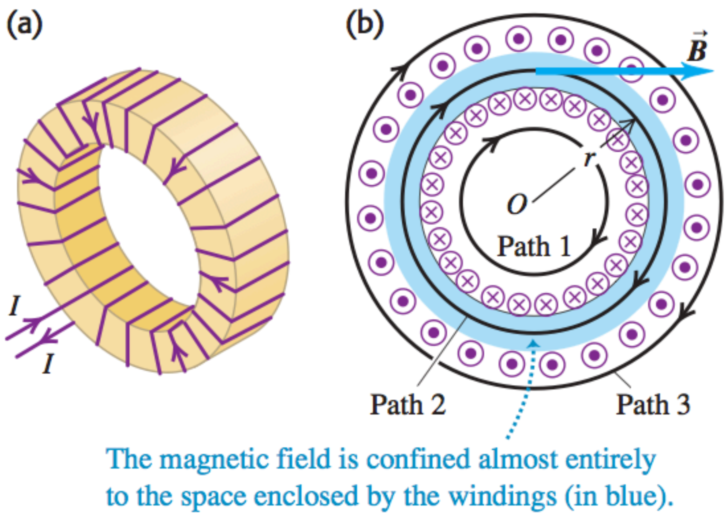


Toroid with N turns

$Region/Path\ 3$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Application of Ampere's Law #4: Toroid



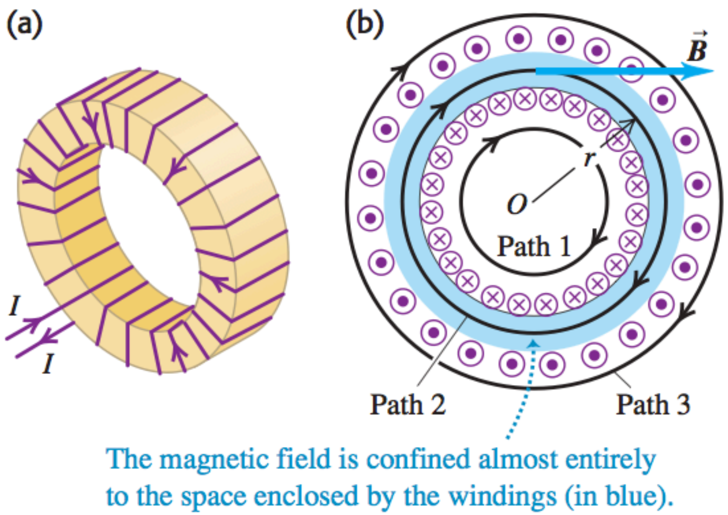
Toroid with N turns

$Region/Path\ 3$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B2\pi r = \mu_0(+NI - NI) = 0$$

Application of Ampere's Law #4: Toroid



Toroid with N turns

$Region/Path\ 3$:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B 2\pi r = \mu_0(+NI - NI) = 0$$

$$\boxed{B = 0}$$