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PHYS2326
Lecture #22

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The University of Texas at Dallas

Reminders / Tips

- Formula sheet for Exam 3 is available on eLearning
- Test covers Chapters/section 26.1 to 28.5

- 1 eV = 1.6 x 10^{-19} J

$$E = \frac{1}{2}mv^2$$

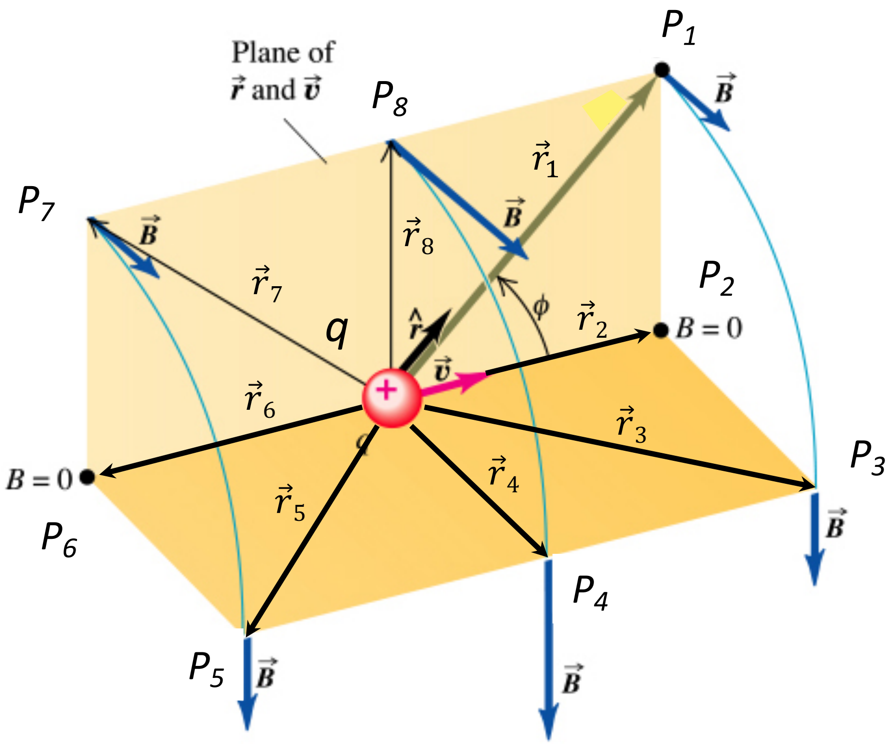
- 1 Tesla = 10,000 Gauss

Today's Lecture

- Understand the source of magnetic fields
  - Magnetic field of a moving charge
  - Magnetic field of a current element
  - Magnetic field of a straight current-carrying conductor
    - Force between parallel conductors
  - Magnetic field of a circular current loop

Chapter 28

B-field of a moving charge

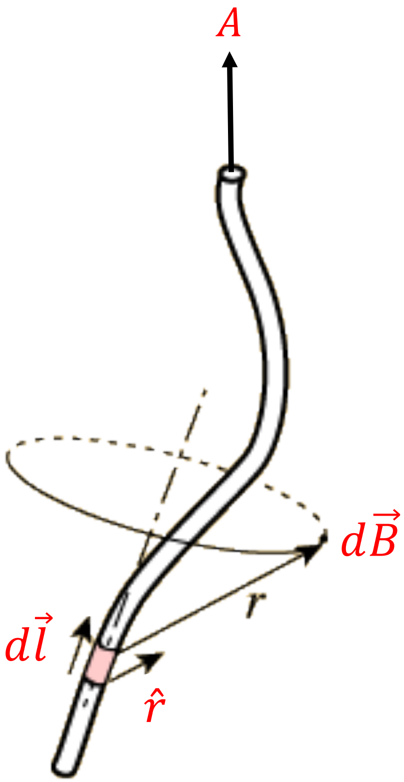


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic constant or
Permeability of free-space:

$$\mu_0 = 4\pi\times 10^7\ Tm/A$$

B-Field of a Current Element



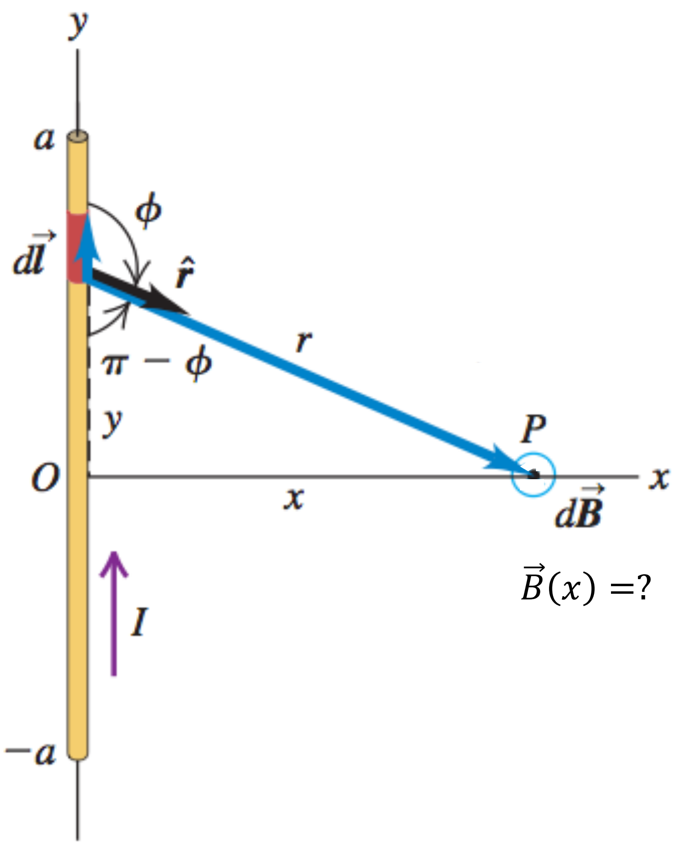
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{l}\times\hat{r}}{r^2}$$

Law of Biot and Savart

B-Field of a Straight Current-Carrying Conductor

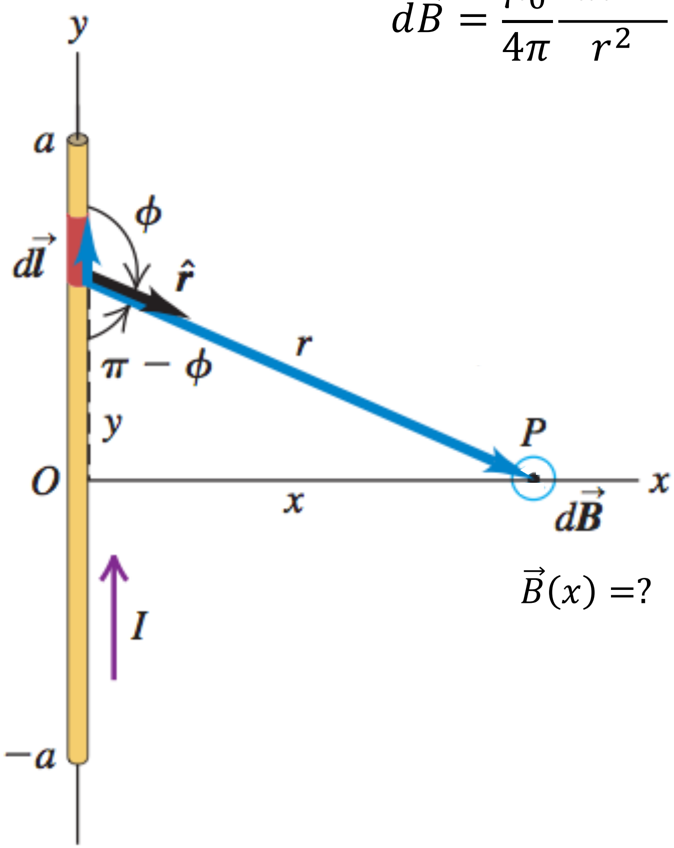
B-Field of a Straight Current-Carrying Conductor



$\vec{B}(x) = ?$

B-Field of a Straight Current-Carrying Conductor

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$



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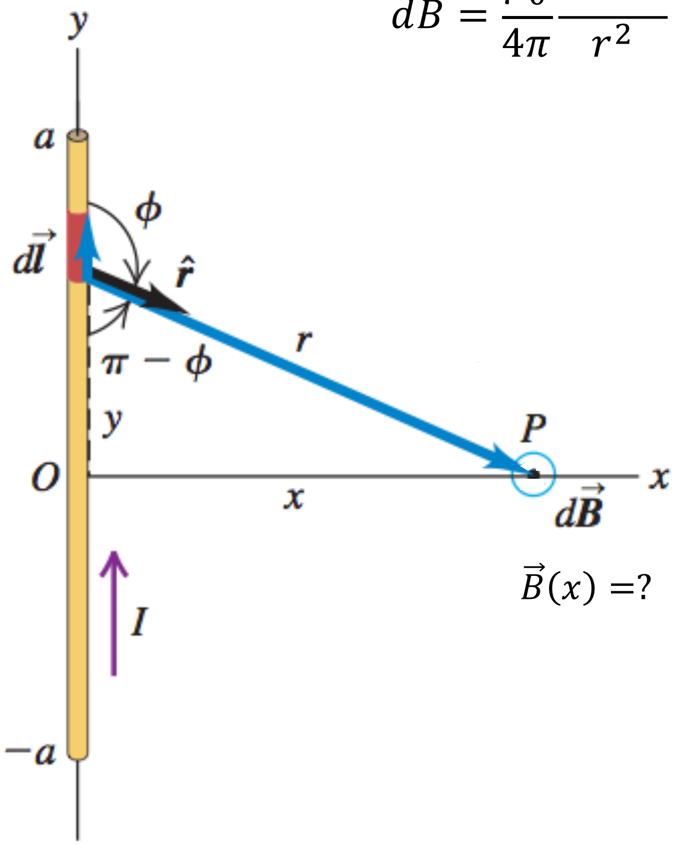
B-Field of a Straight Current-Carrying Conductor

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \rightarrow \vec{B} = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{Id\vec{l}\times\hat{r}}{r^2}$$



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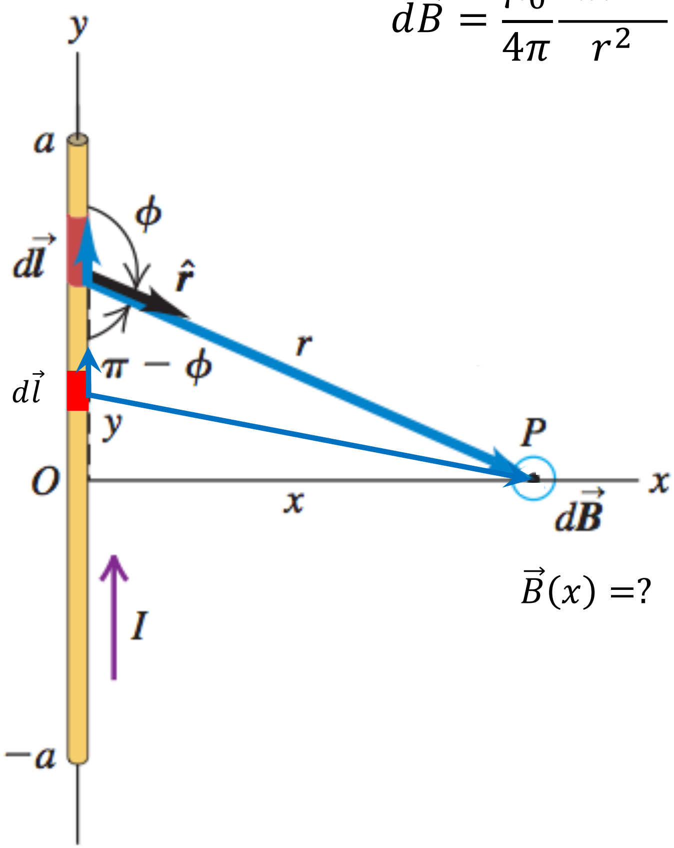

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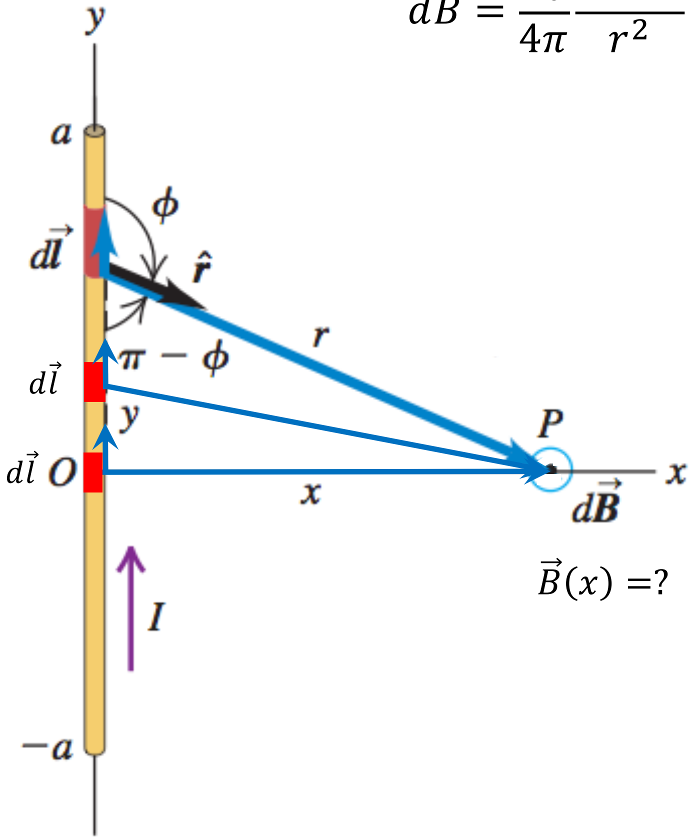
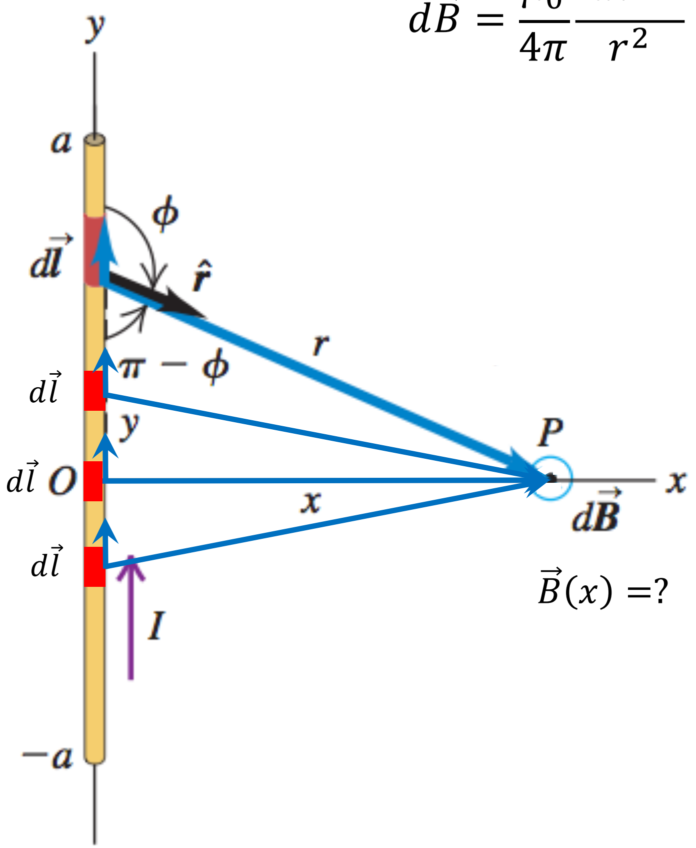


$\vec{B}(x) = ?$

B-Field of a Straight Current-Carrying Conductor

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2} \quad \color{red}{\rightarrow} \quad \vec{B} = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{I d\vec{l} \times \hat{r}}{r^2}$$



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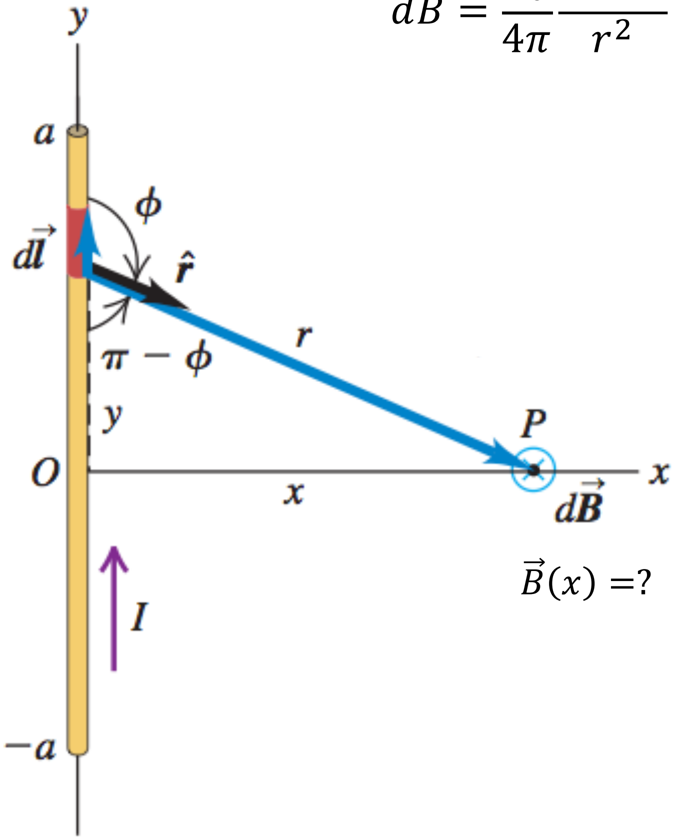

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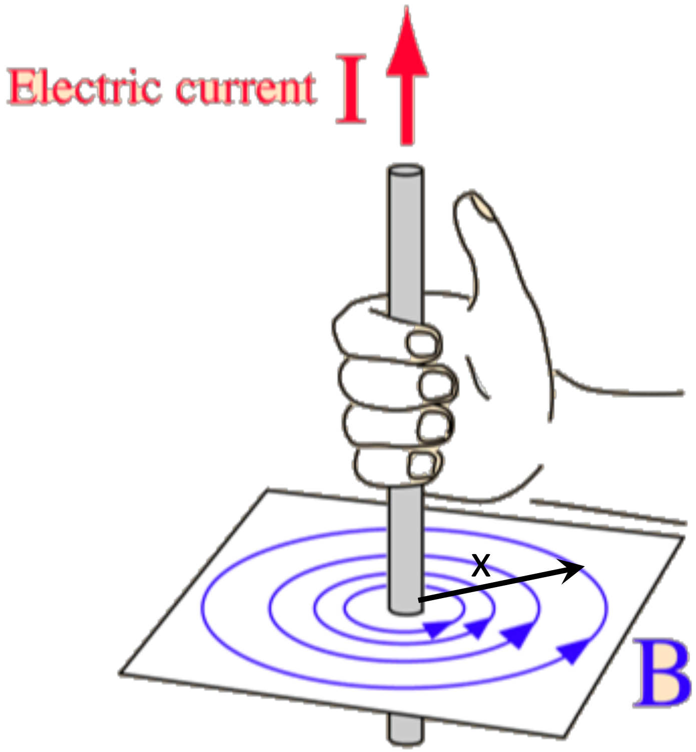
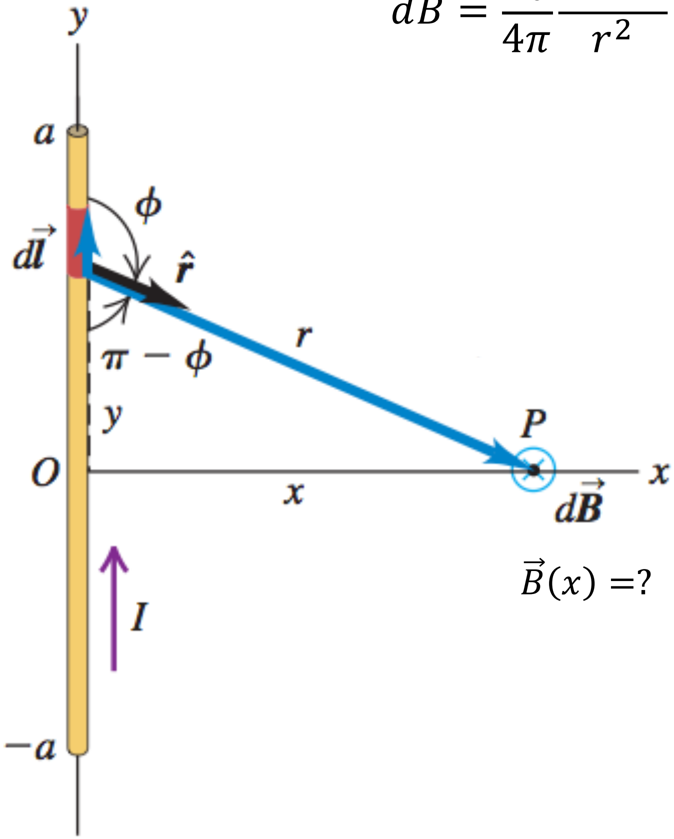


Direction of magnetic field due to a straight current carrying conductor

B-Field of a Straight Current-Carrying Conductor



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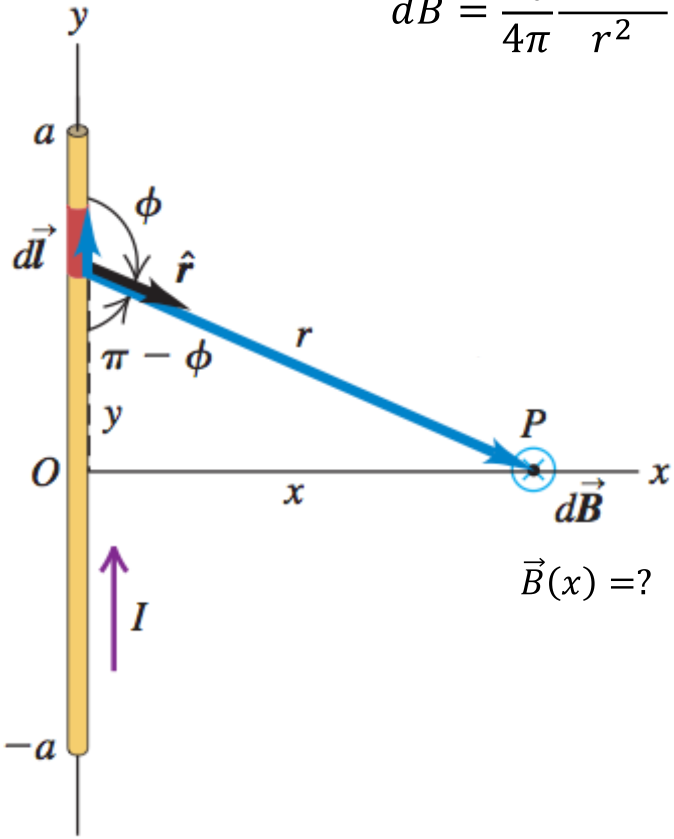
$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\phi)}{r^2}$$

B-Field of a Straight Current-Carrying Conductor



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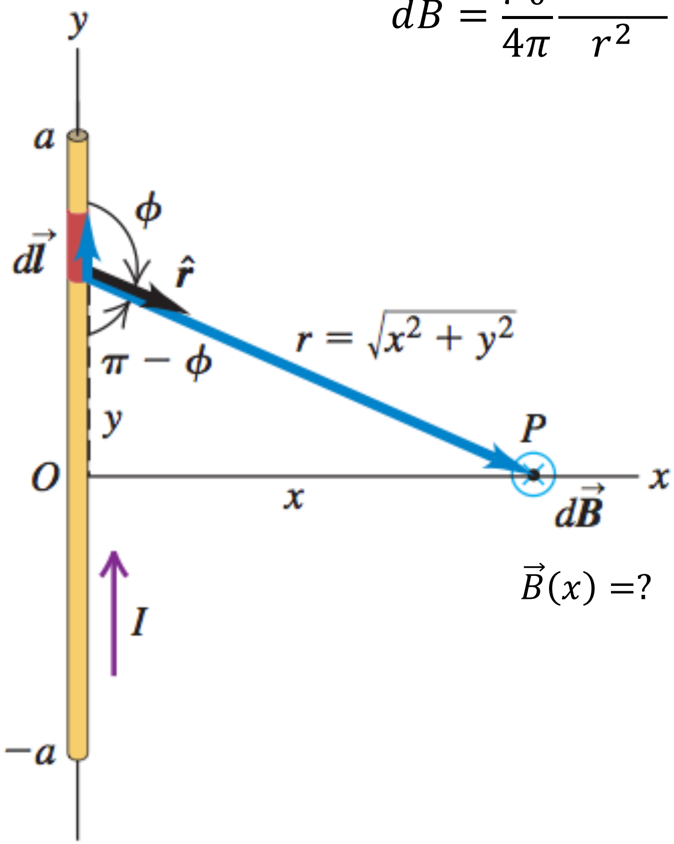

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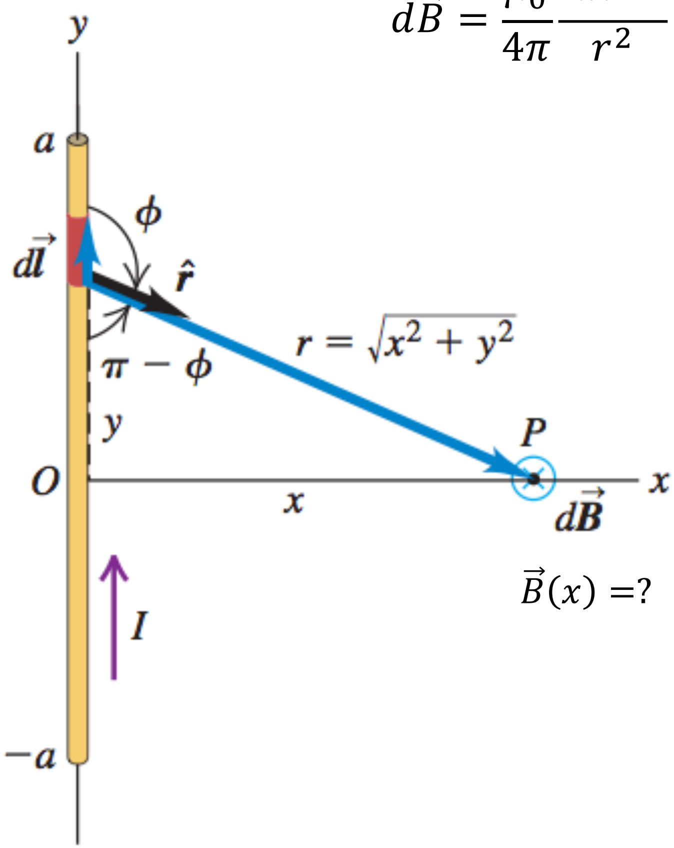
$$dl = dy$$
$$r = \sqrt{x^2 + y^2}$$

B-Field of a Straight Current-Carrying Conductor



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$sin(\phi) =$

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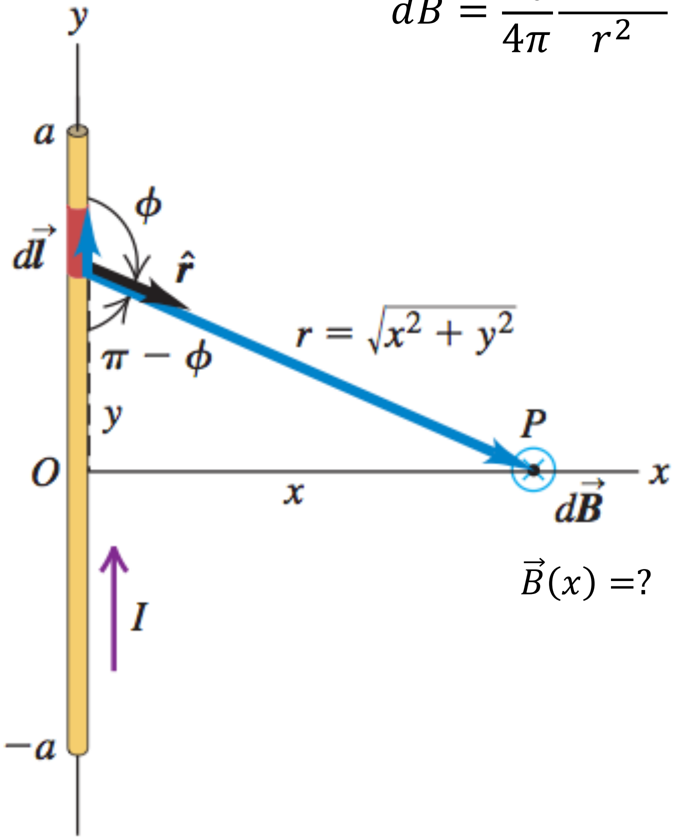

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$$sin(\phi) = \sin(\pi - \phi)$$

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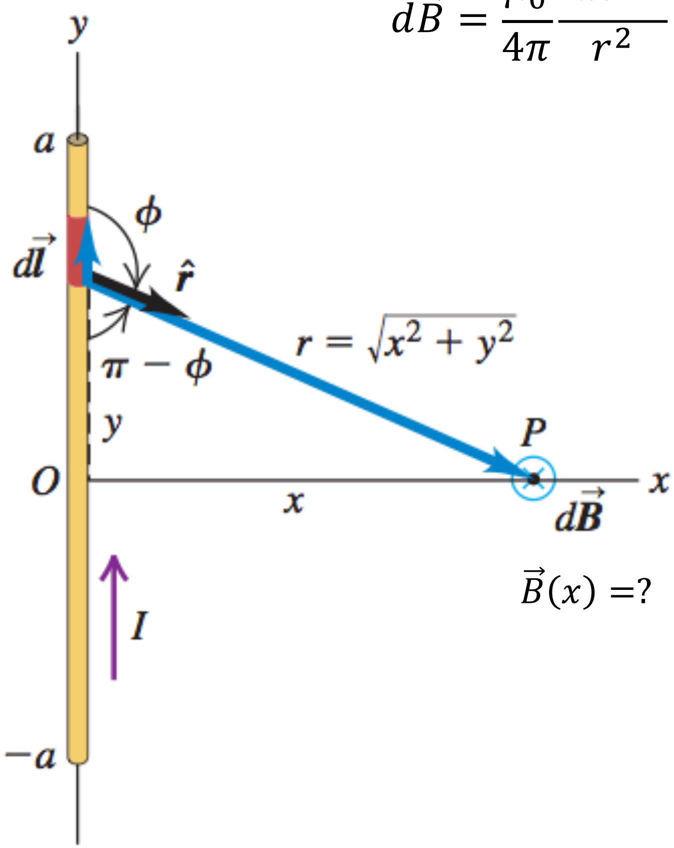

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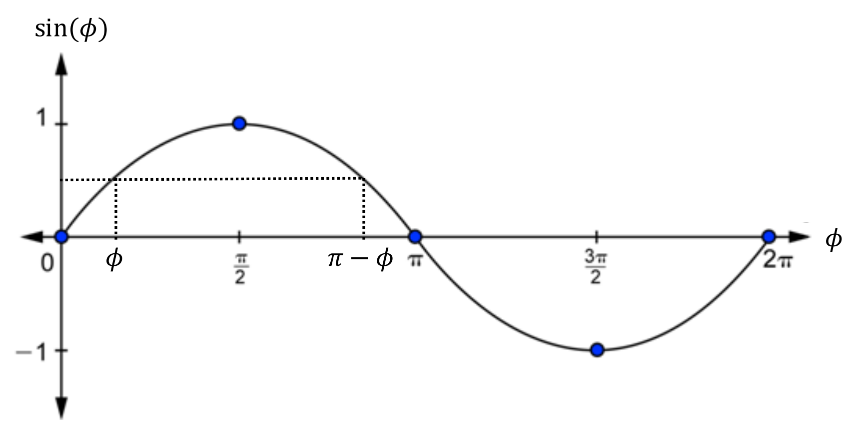

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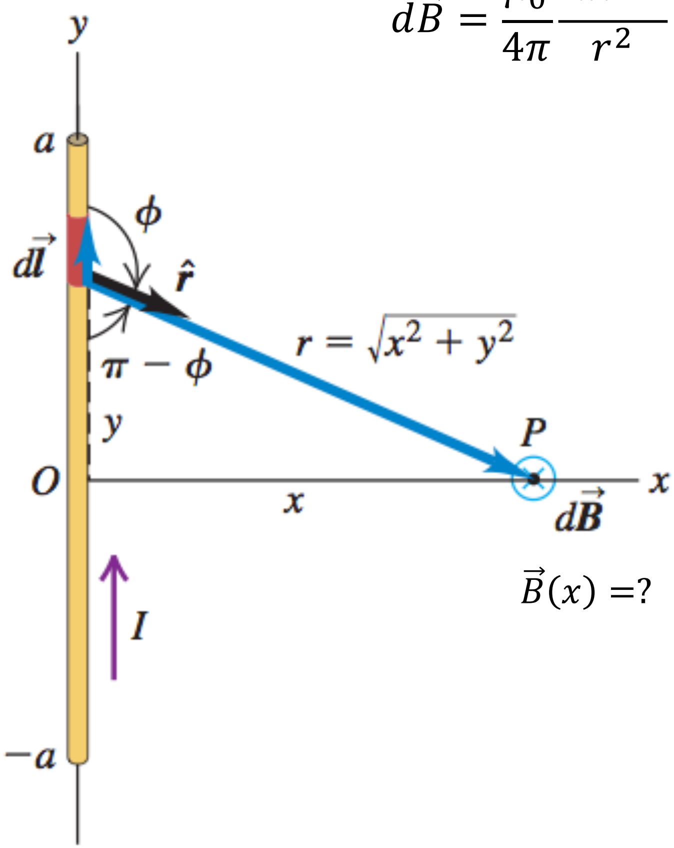

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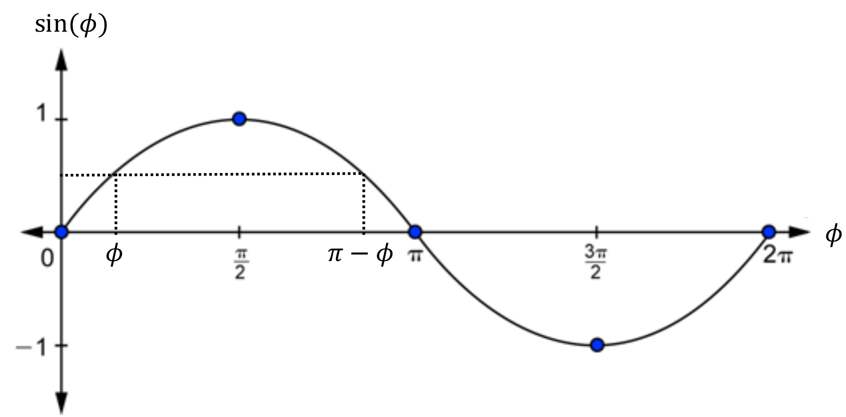

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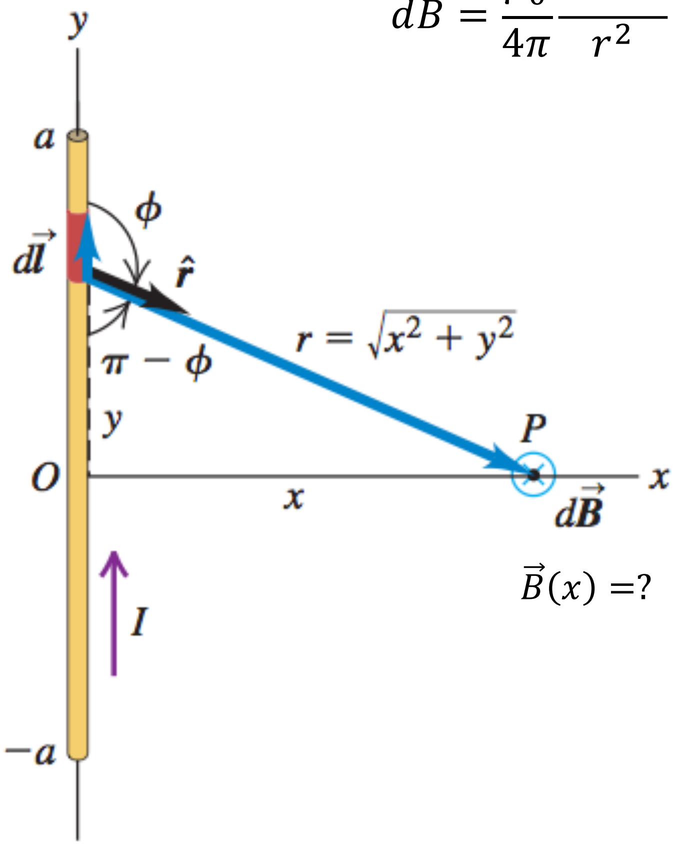

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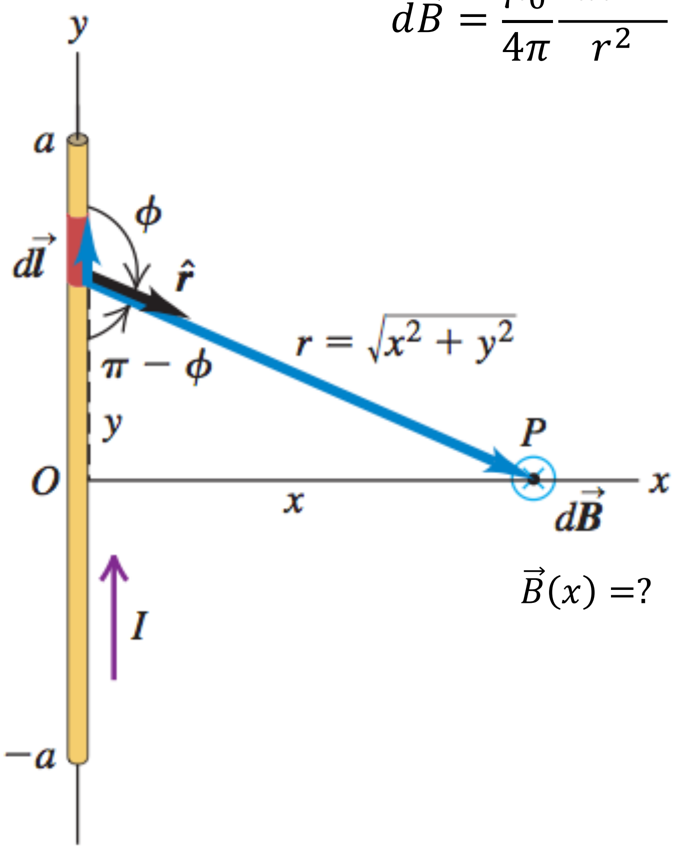

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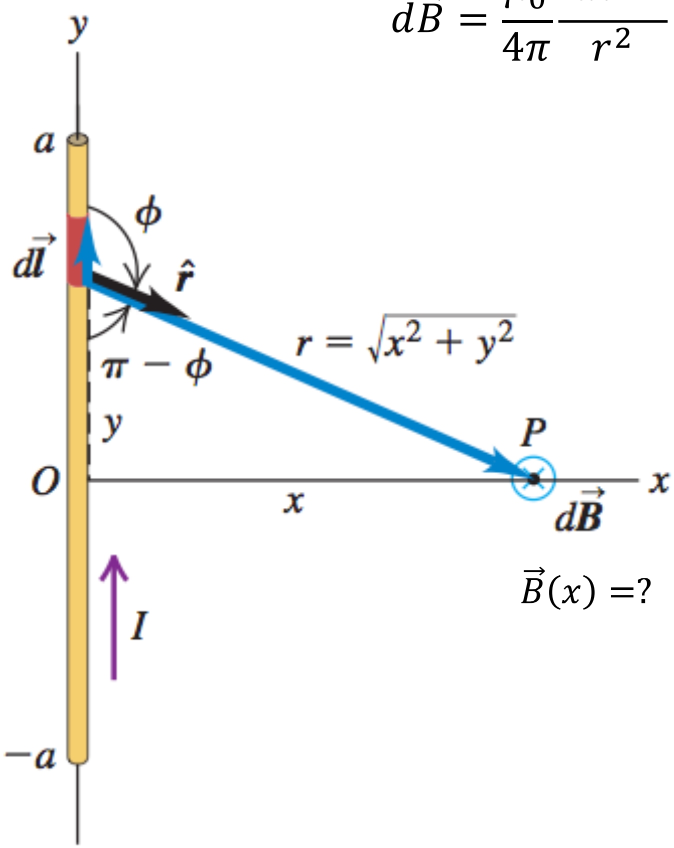
B-Field of a Straight Current-Carrying Conductor



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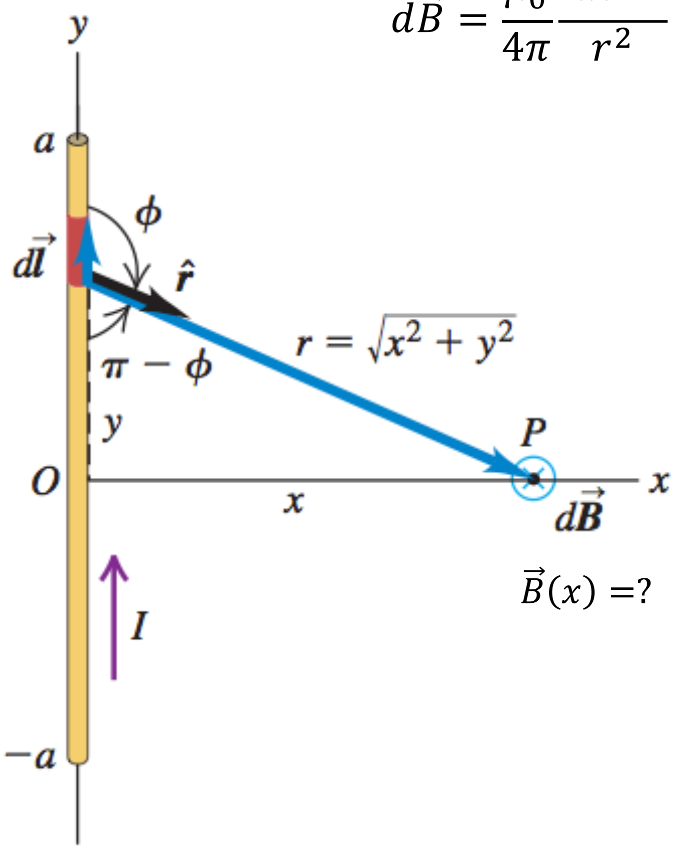
|**B**| at a distance x from wire

B-Field of a Straight Current-Carrying Conductor



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|**B**| at a distance x from wire

$$B = \frac{\mu_0 I}{2\pi x}$$

For a >> x (long wire)

Example: The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10μG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.00 cm from this current, how large is the current in the heart?

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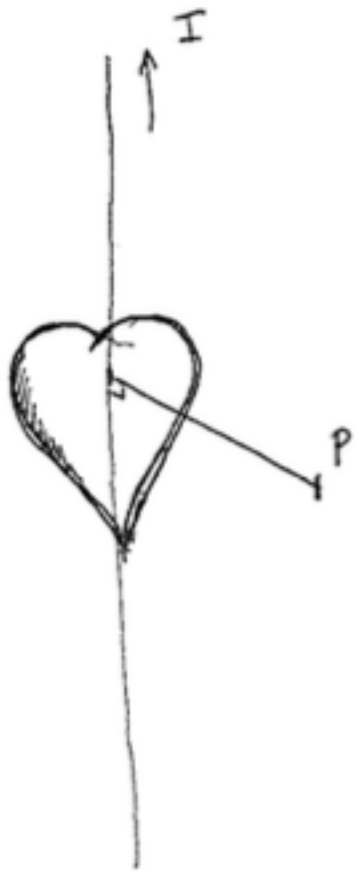

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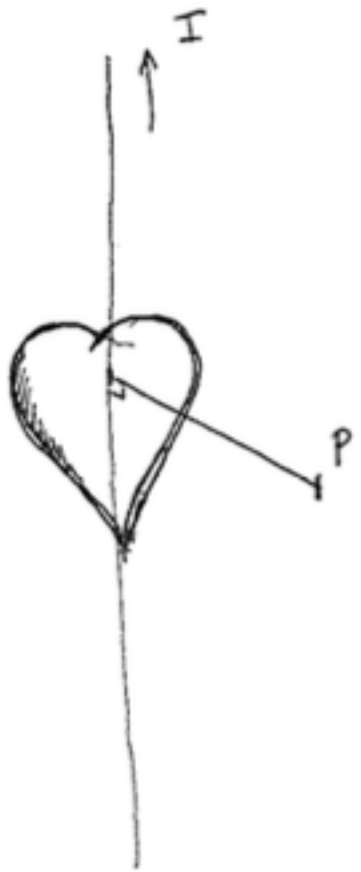
$I = ?$



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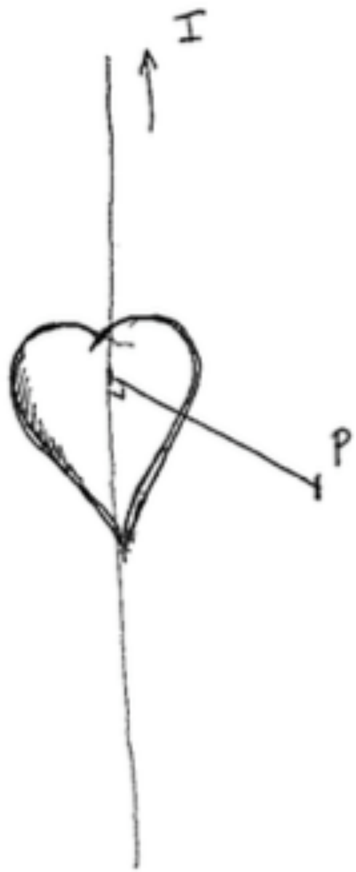

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$$B = \frac{\mu_0 I}{2\pi x} \Rightarrow I = \frac{2\pi x B}{\mu_0}$$

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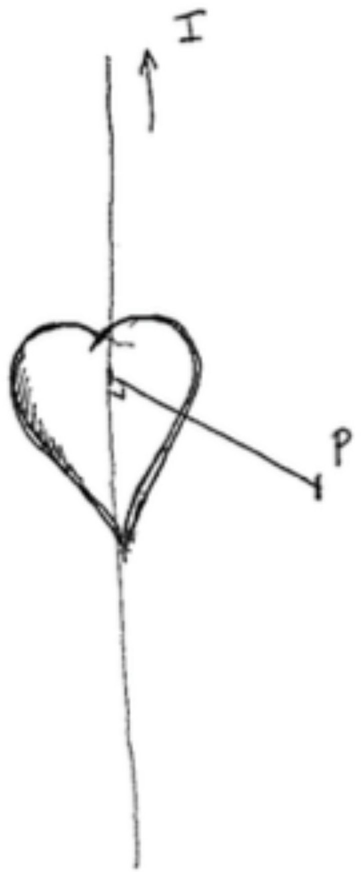

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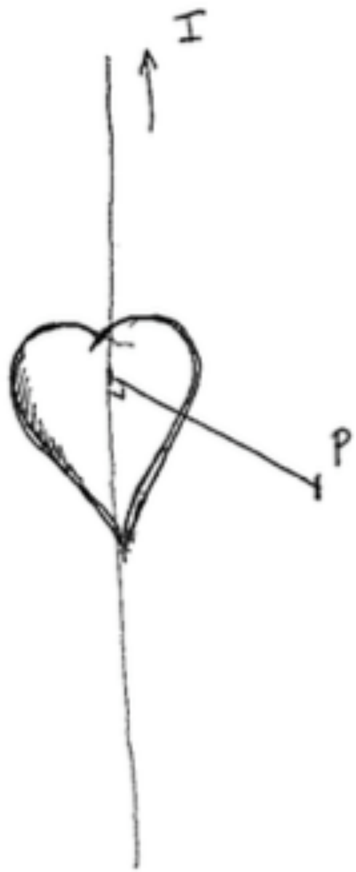
$$B = \frac{\mu_0 I}{2\pi x} \Rightarrow I = \frac{2\pi x B}{\mu_0}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$

$$B = 10 \times 10^{-6}\, G =$$

Example: The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10µG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.00 cm from this current, how large is the current in the heart?



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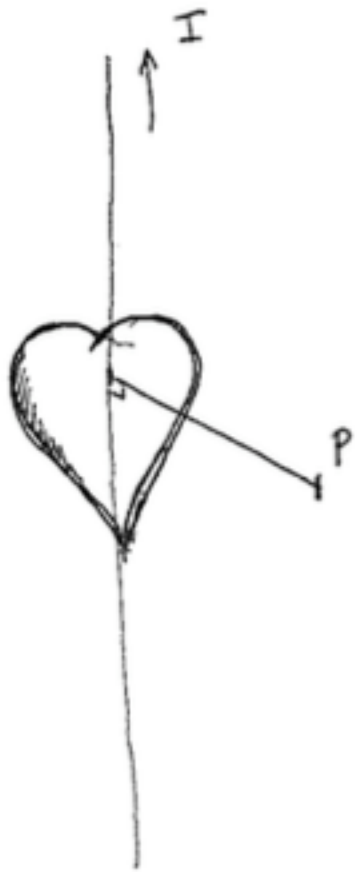
$$B = 10 \times 10^{-6}\, G = 10 \times 10^{-6} \left(\frac{1}{10{,}000}\right) T$$

$1\,T = 10{,}000\,G$

Example: The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10μG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.00 cm from this current, how large is the current in the heart?



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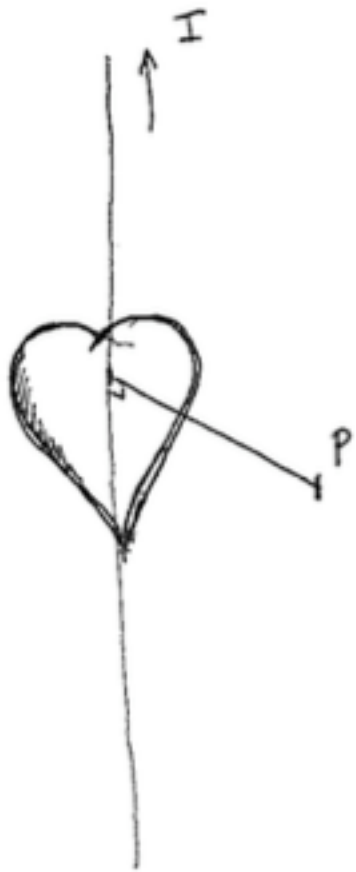

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$$B = 10 \times 10^{-6} G = 10 \times 10^{-6} \left(\frac{1}{10{,}000}\right) T = 1 \times 10^{-9} T$$

$$x = 0.05 \, m$$

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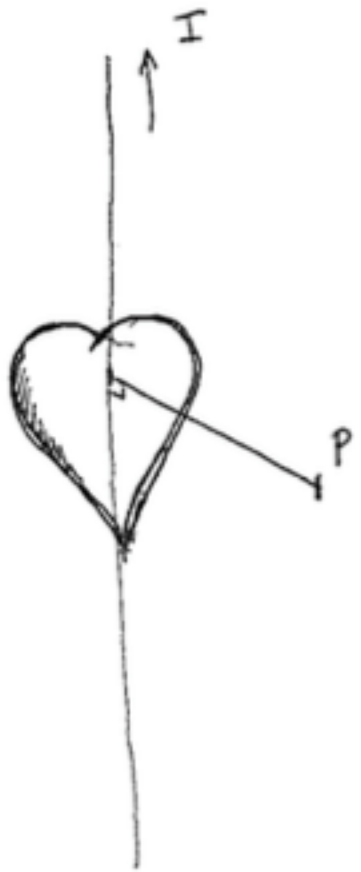

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$$B = \frac{\mu_0 I}{2\pi X} \Longrightarrow I = \frac{2\pi X B}{\mu_0}$$

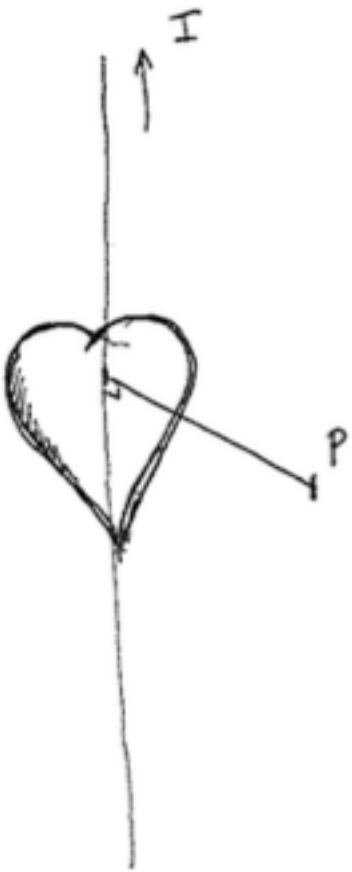
$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$

$$B = 10 \times 10^{-6} G = 10 \times 10^{-6} \left(\frac{1}{10{,}000}\right) T = 1 \times 10^{-9} T$$

$$X = 0.05 \text{ m}$$

$$I = \frac{2\pi (0.05)(1 \times 10^{-9})}{\mu_0}$$

Example: The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10µG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.00 cm from this current, how large is the current in the heart?



$I = ?$

$B = \frac{\mu_0 I}{2\pi x} \Longrightarrow I = \frac{2\pi x B}{\mu_0}$

$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

$B = 10 \times 10^{-6} G = 10 \times 10^{-6} \left(\frac{1}{10{,}000}\right) T = 1 \times 10^{-9} T$

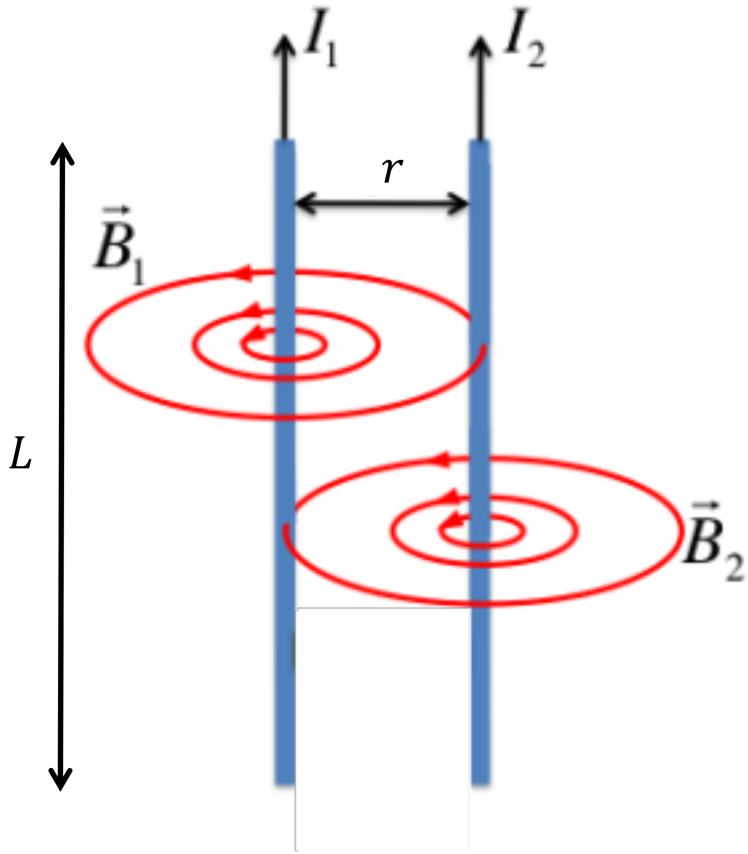
$x = 0.05 \, m$

$I = \frac{2\pi (0.05)(1 \times 10^{-9})}{\mu_0}$

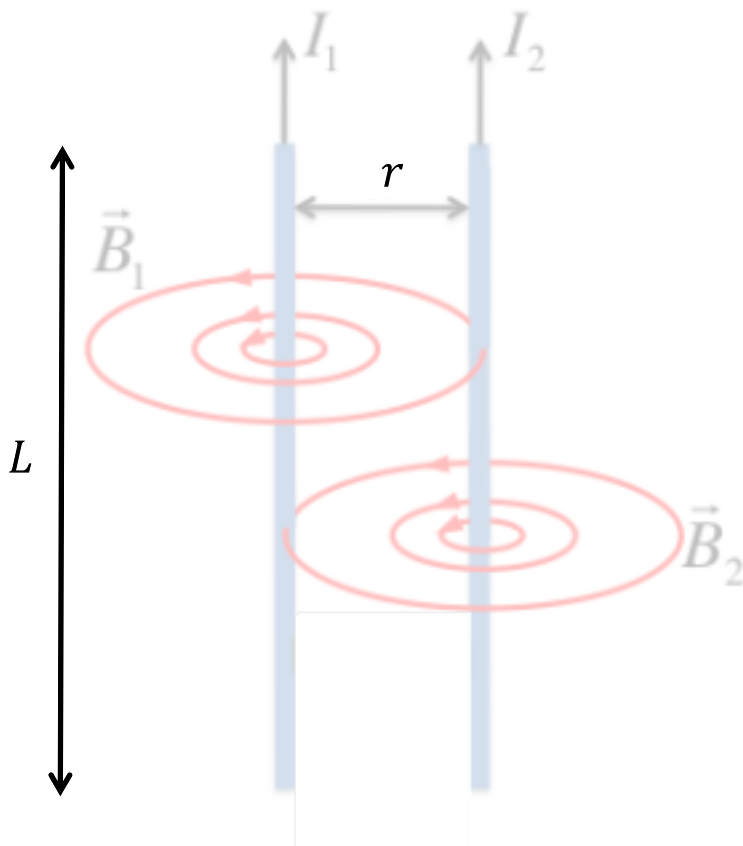
$I = \underline{250 \times 10^{-6}} \, A = 250 \mu A$

Force between parallel conductors

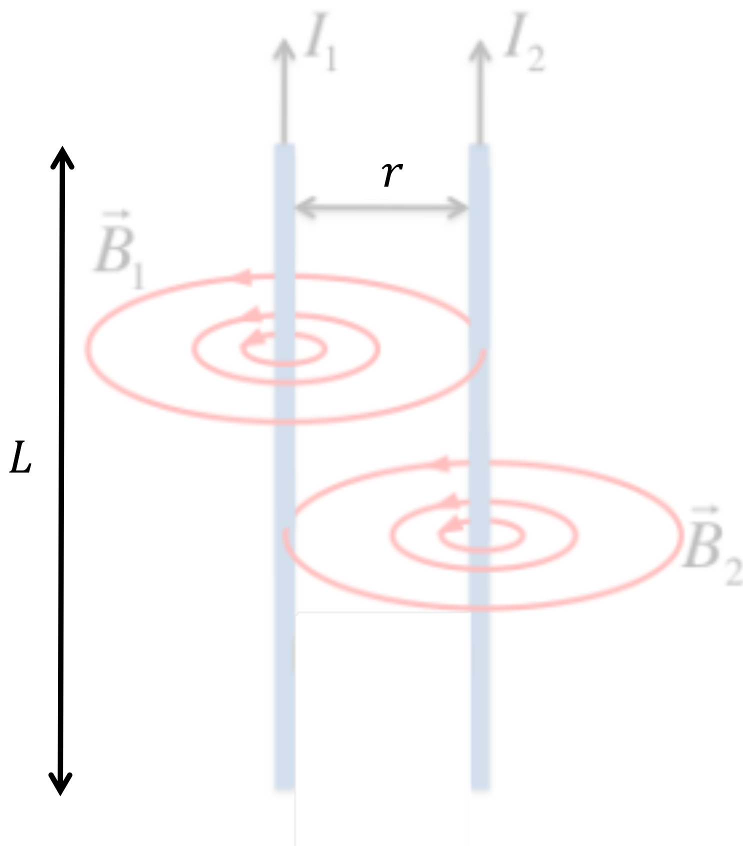
Force between parallel conductors



Force between parallel conductors

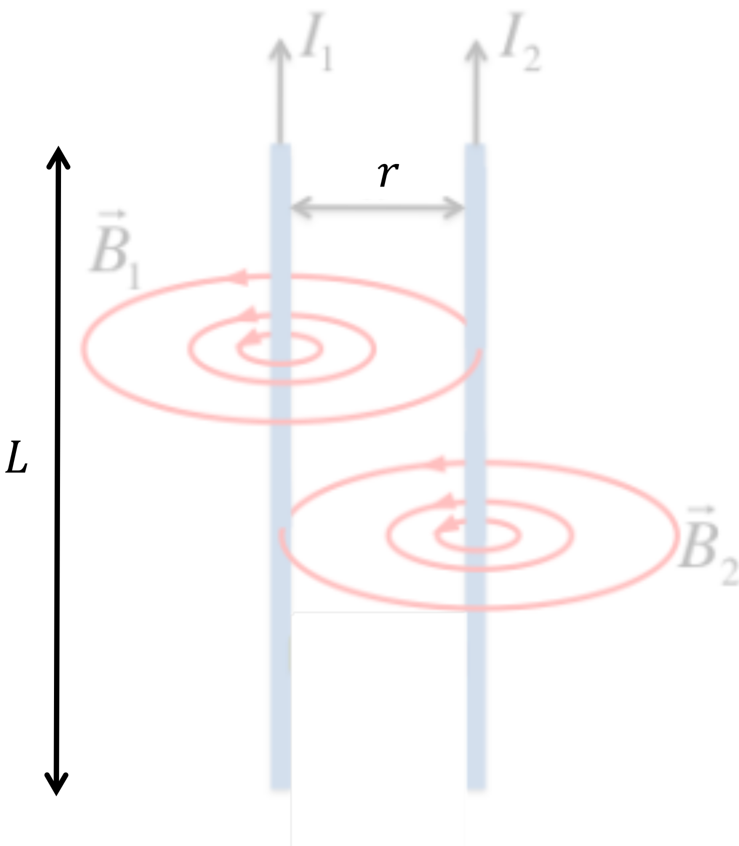


Force between parallel conductors



$F_{12} = Force\ in\ I_2\ due\ to\ I_1 = ?$

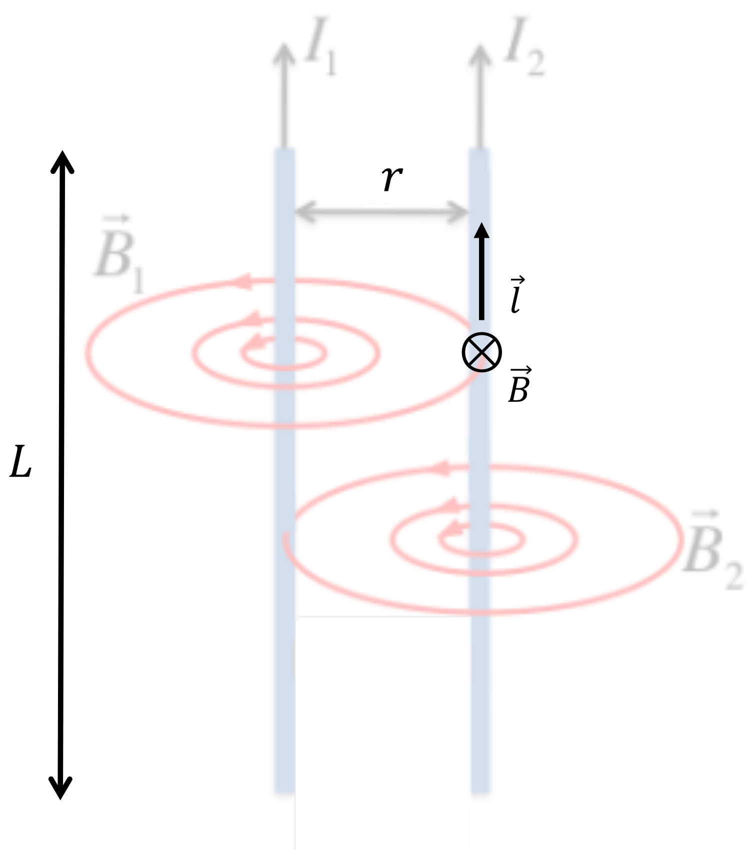
Force between parallel conductors



$F_{12} = Force\ in\ I_2\ due\ to\ I_1 =?$

$\vec{F} = I\vec{l}\times\vec{B}$
$\vec{F}_{12} = I_2\vec{l}_2\times\vec{B}_1$

Force between parallel conductors



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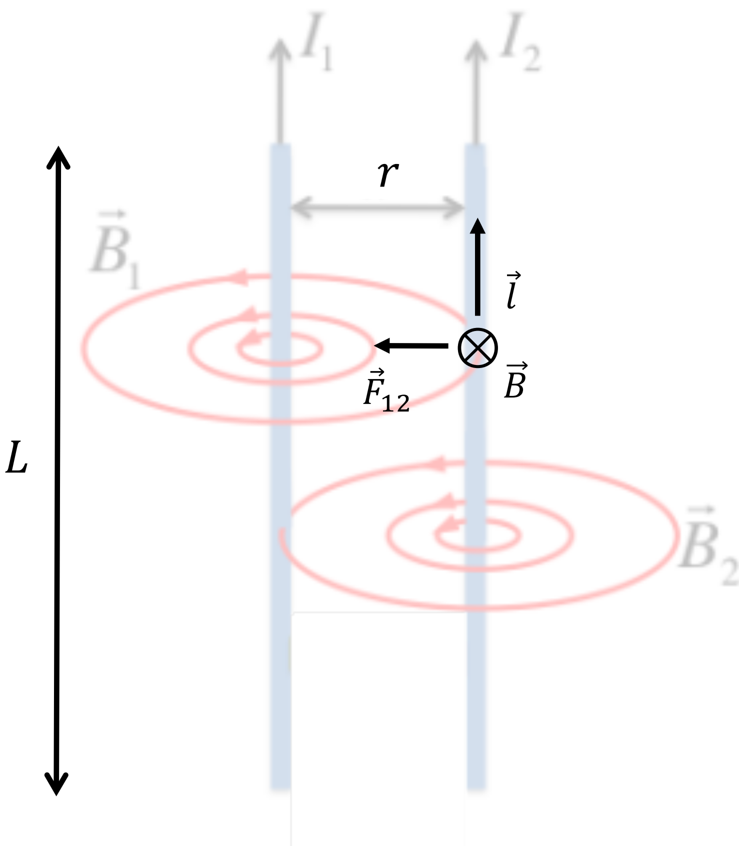

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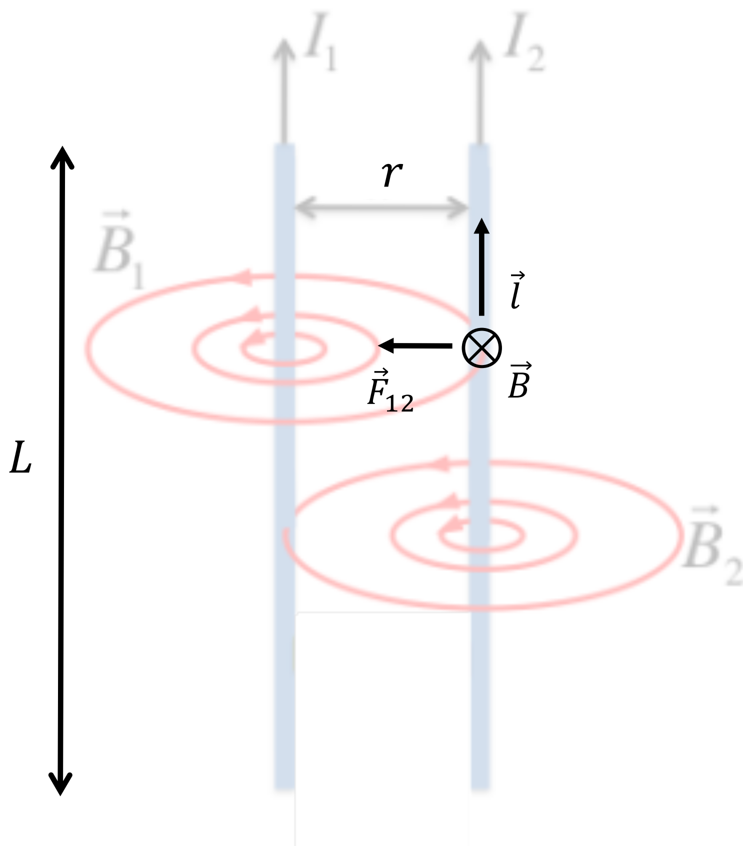


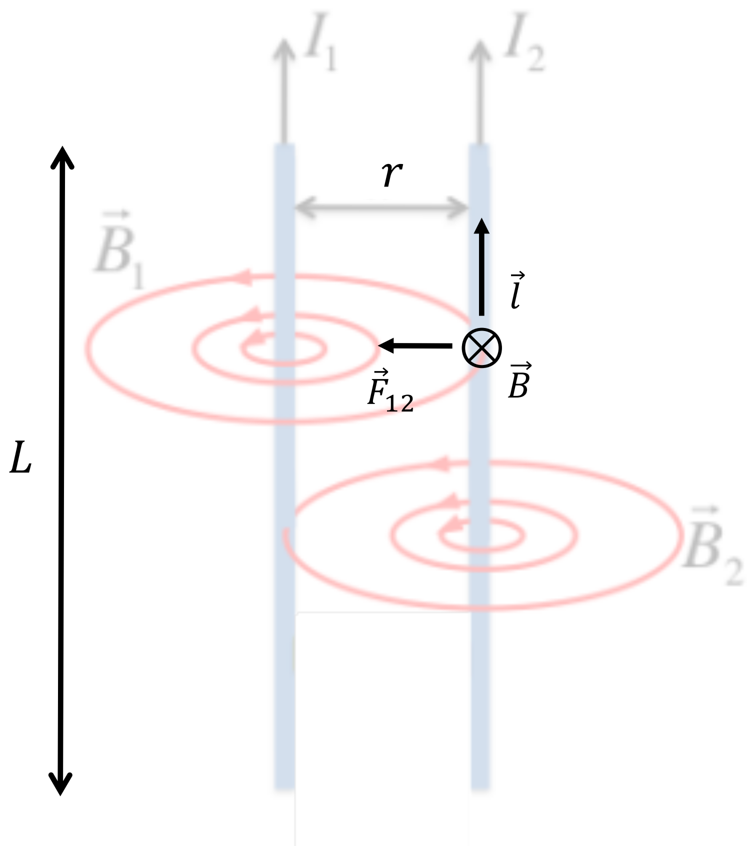
$F_{12} = Force\ in\ I_2\ due\ to\ I_1 = ?$

$\vec{F} = I\vec{l}\times\vec{B}$

$\vec{F}_{12} = I_2\vec{l}_2\times\vec{B}_1$

$F_{12} = I_2 L B_1$

Force between parallel conductors



$F_{12} = Force\ in\ I_2\ due\ to\ I_1 = ?$

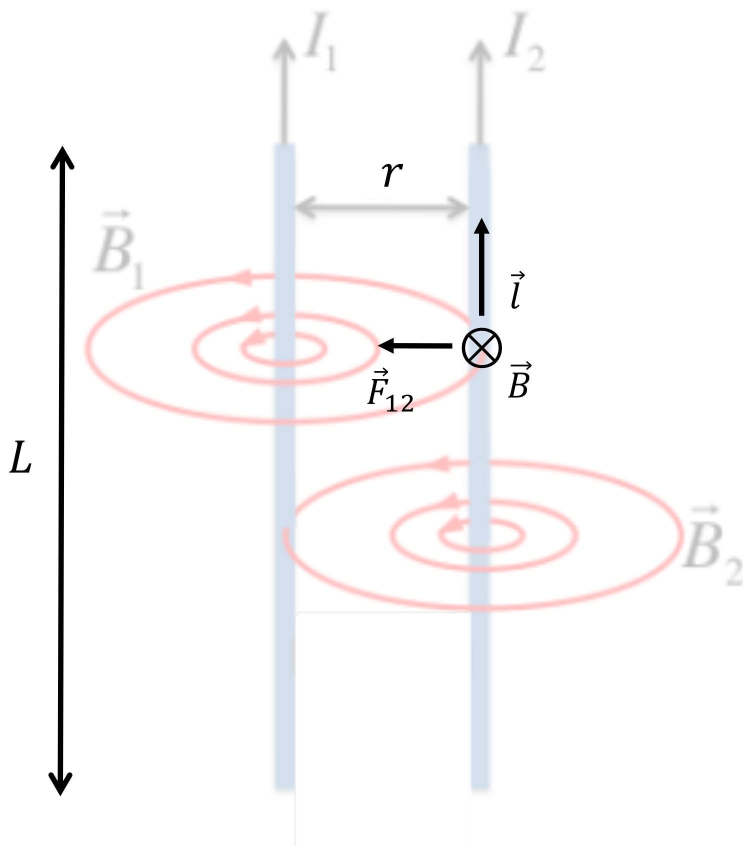
$\vec{F} = I\vec{l}\times\vec{B}$

$\vec{F}_{12} = I_2\vec{l}_2\times\vec{B}_1$

B-field due to long wire

$F_{12} = I_2 L B_1 = I_2 L\left(\dfrac{\mu_0 I_1}{2\pi r}\right)$

Force between parallel conductors



$F_{12} = Force\ in\ I_2\ due\ to\ I_1 = ?$

$$\vec{F} = I\vec{l} \times \vec{B}$$
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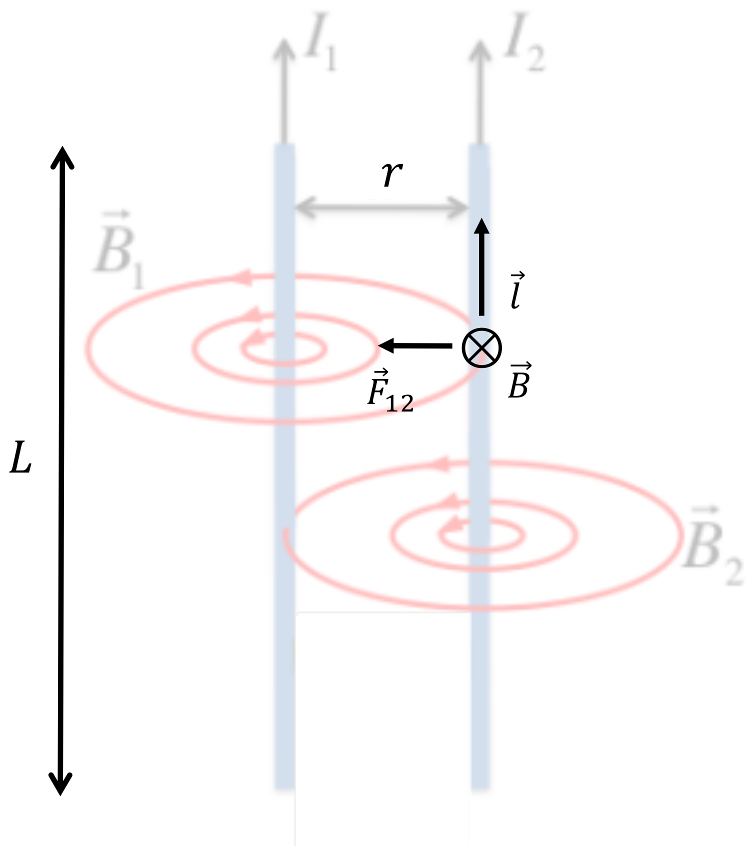
Force between parallel conductors



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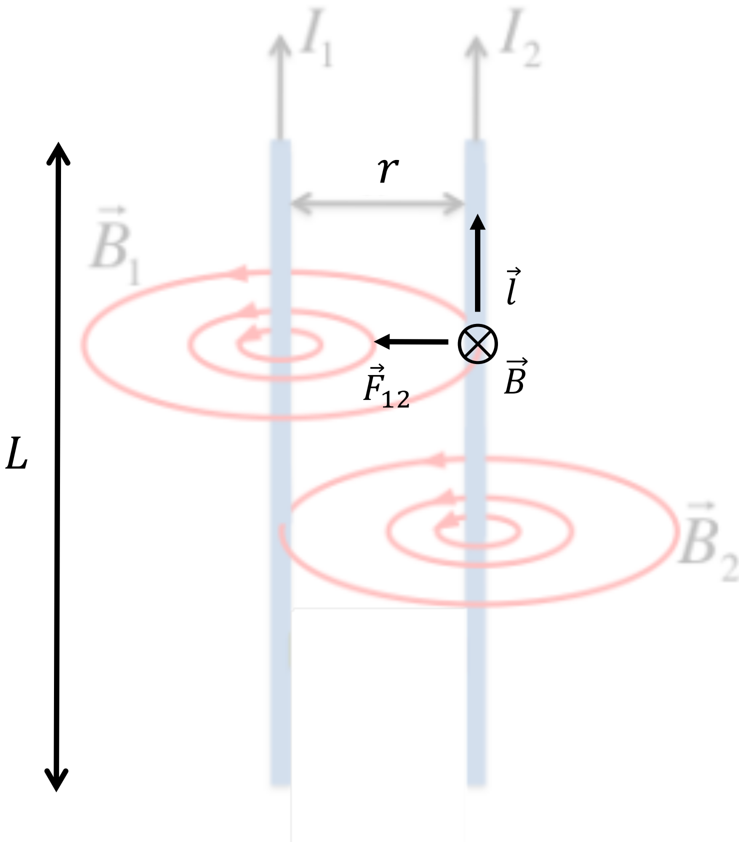
$$F_{12} = I_2LB_1 = I_2L\left(\frac{\mu_0 I_1}{2\pi r}\right)$$

$$F_{12} = \frac{\mu_0 L I_1 I_2}{2\pi r}$$

$F_{21} = Force\ in\ I_1\ due\ to\ I_2 = ?$

Force between parallel conductors



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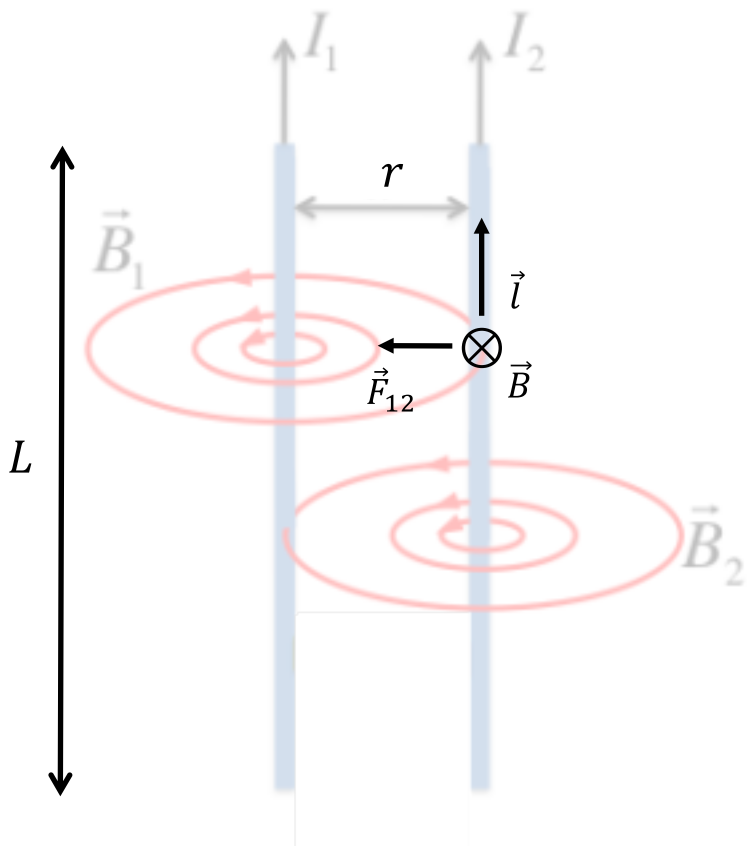
Force between parallel conductors



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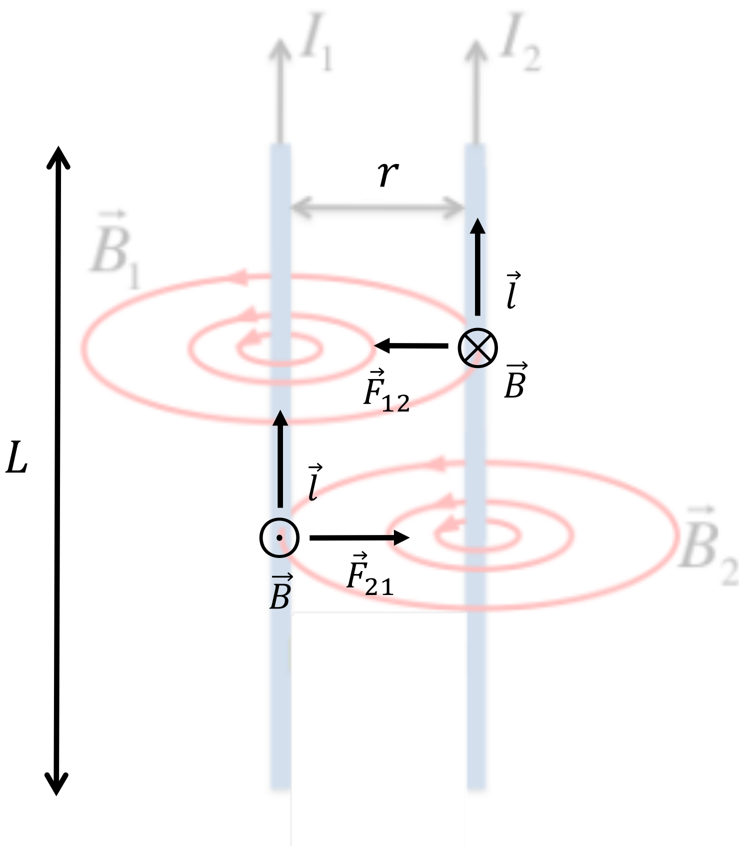
Force between parallel conductors



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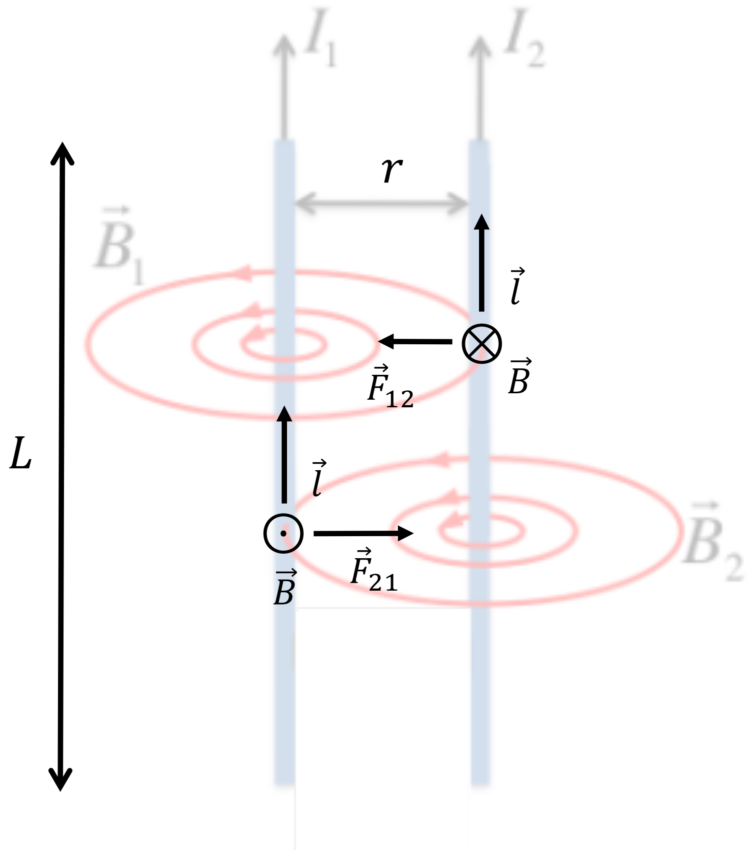
Force between parallel conductors



$F_{12} = Force\ in\ I_2\ due\ to\ I_1 = ?$

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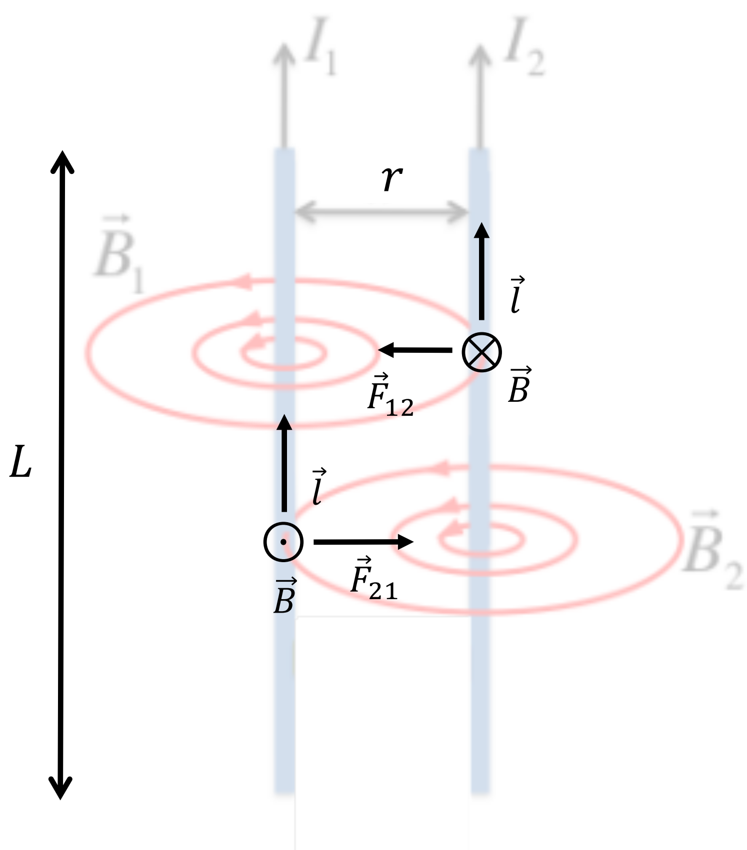
Force between parallel conductors



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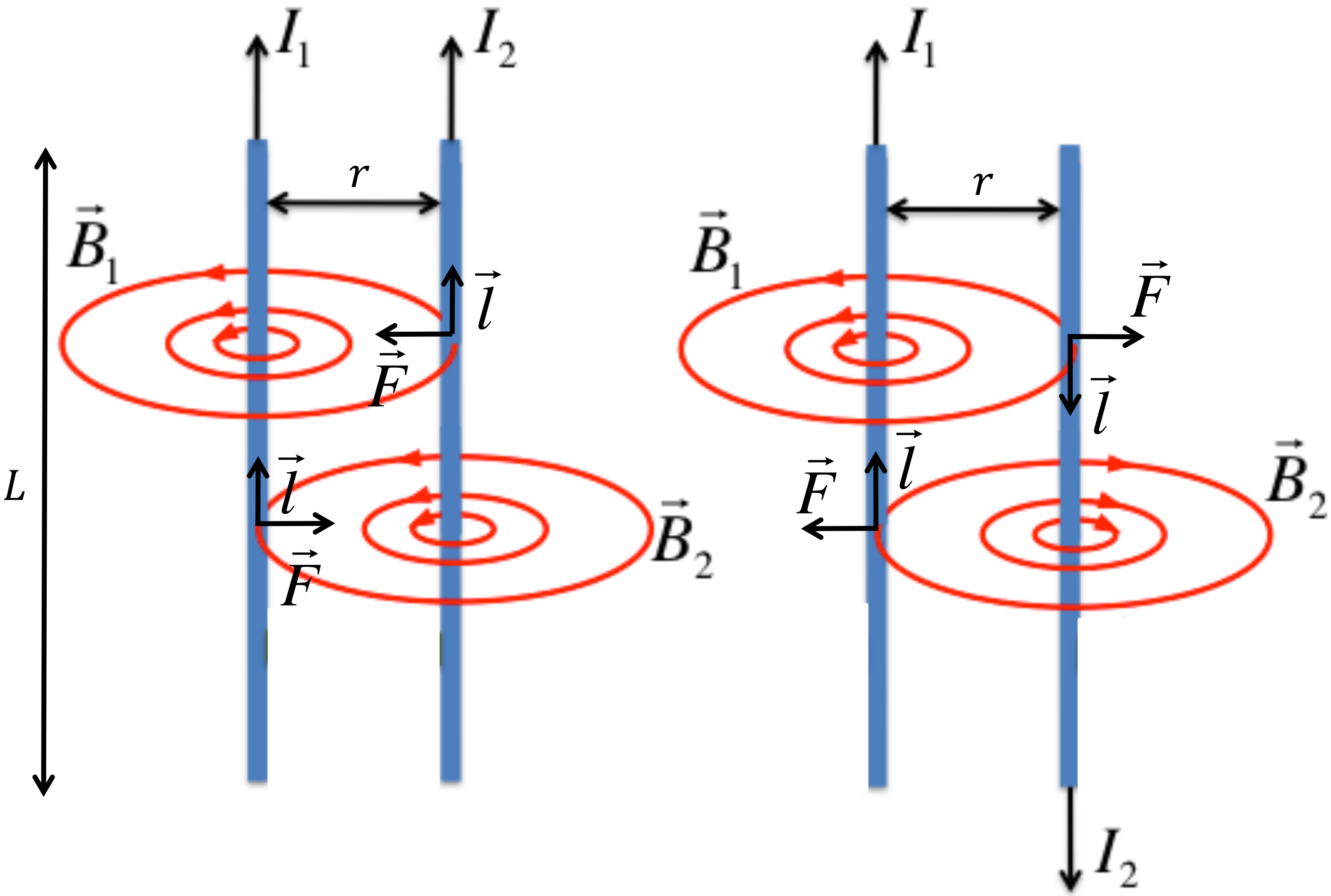
Summary



Currents in the same direction: ATTRACTION

Currents in the opposite direction: REPULSION

$$\vec{F} = I\vec{l}\times\vec{B}$$

$$F = \frac{\mu_0 L I_1 I_2}{2\pi r}$$

Example: The wires in a household lamp cord are typically 3.0 mm apart center to center and carry equal currents in opposite directions. (a) If the cord carries current to a 100−W light bulb connected across a 120−V potential difference, what force per meter does each wire of the cord exert on the other? (Model the lamp cord as a very long straight wire.). (b) Is the force attractive or repulsive?

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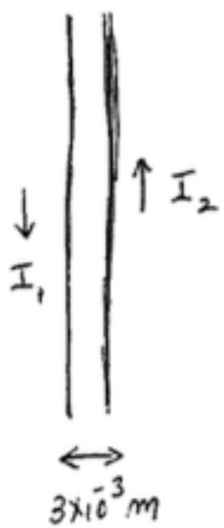

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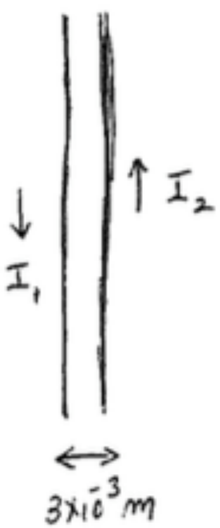
(a) $\frac{F}{L} = ?$



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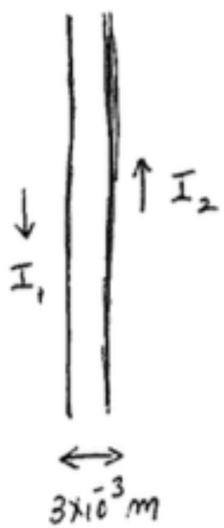

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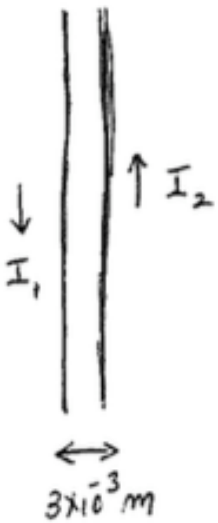

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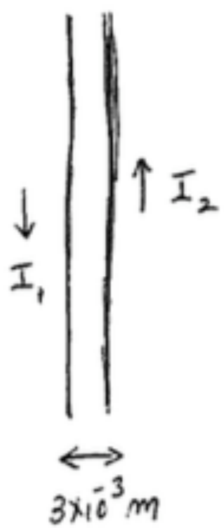

$$F = \frac{\mu_0 L I_1 I_2}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$I_1 = I_2 = I = ?$$

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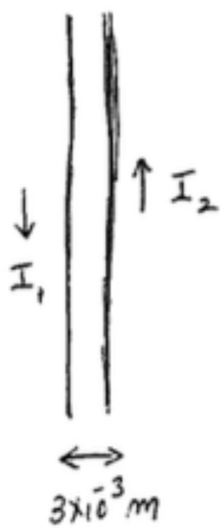

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$I_1 = I_2 = I = ?$

$P = VI$

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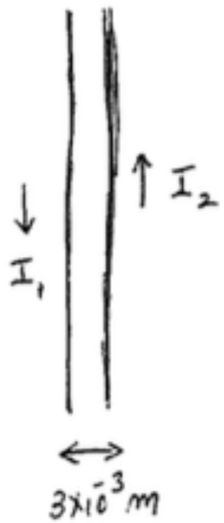

$$F = \frac{\mu_0 L I_1 I_2}{2\pi r}$$

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$$I_1 = I_2 = I = ?$$

$$P = VI \rightarrow I = \frac{P}{V}$$

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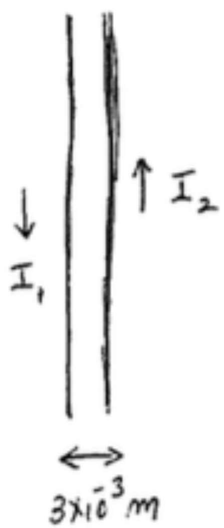

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$$I_1 = I_2 = I = ?$$

$$P = VI \rightarrow I = \frac{P}{V} = \frac{100}{120} = \underline{0.83}\ A$$

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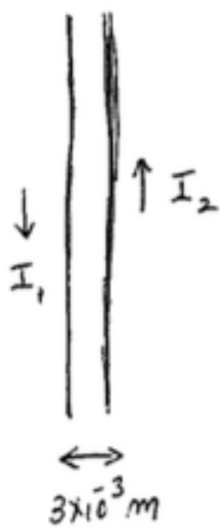

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$r = 3\times10^{-3}$ m

Example: The wires in a household lamp cord are typically 3.0 mm apart center to center and carry equal currents in opposite directions. (a) If the cord carries current to a 100−W light bulb connected across a 120−V potential difference, what force per meter does each wire of the cord exert on the other? (Model the lamp cord as a very long straight wire.). (b) Is the force attractive or repulsive?

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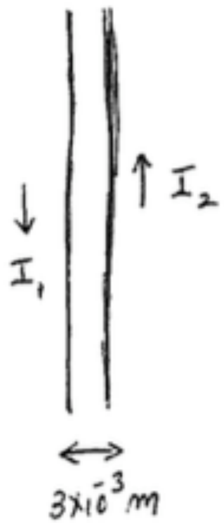

$$F = \frac{\mu_0 L I_1 I_2}{2\pi r}$$

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$$I_1 = I_2 = I = ?$$

$$P = VI \rightarrow I = \frac{P}{V} = \frac{100}{120} = \underline{0.83} \text{ A}$$

$$r = 3 \times 10^{-3} \text{ m}$$

$$\frac{F}{L} = \frac{\mu_0 (0.83)(0.83)}{2\pi (3 \times 10^{-3})} = 4.6 \times 10^{-5} \text{ N/m}$$

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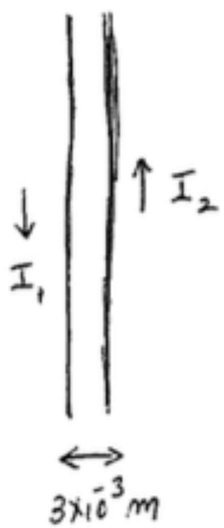

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(b) currents in opposite directions: REPULSION

B-field of a circular loop (28.5)

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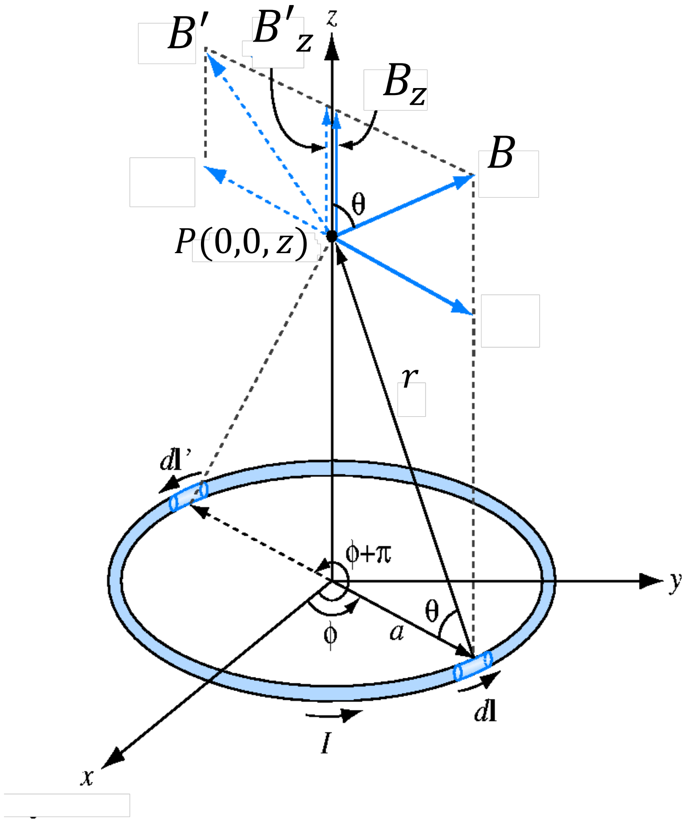
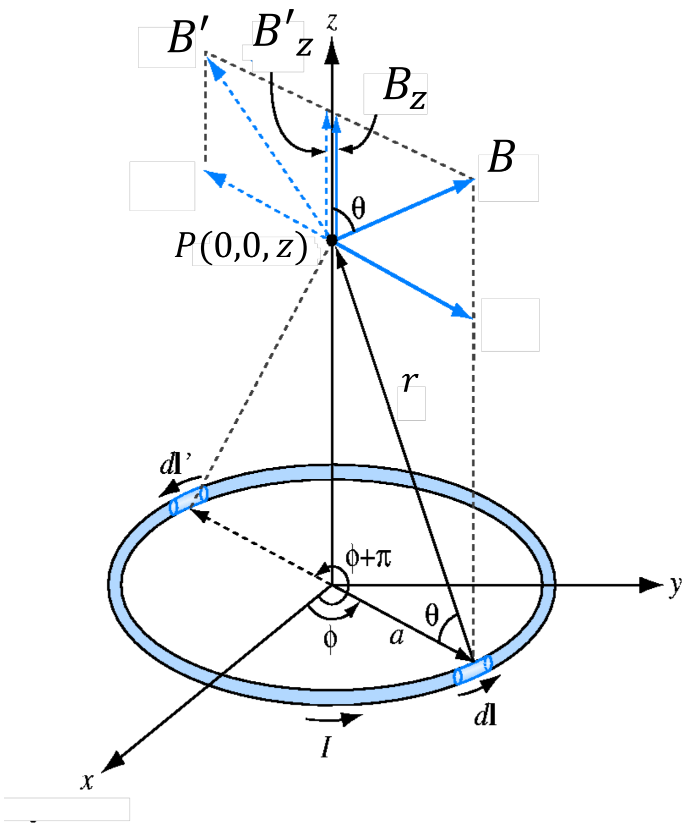


Direction of ***B*?**

B-field of a circular loop (28.5)

$\vec{B}(0,0,z) = ?$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l} \times \hat{r}}{r^2}$$



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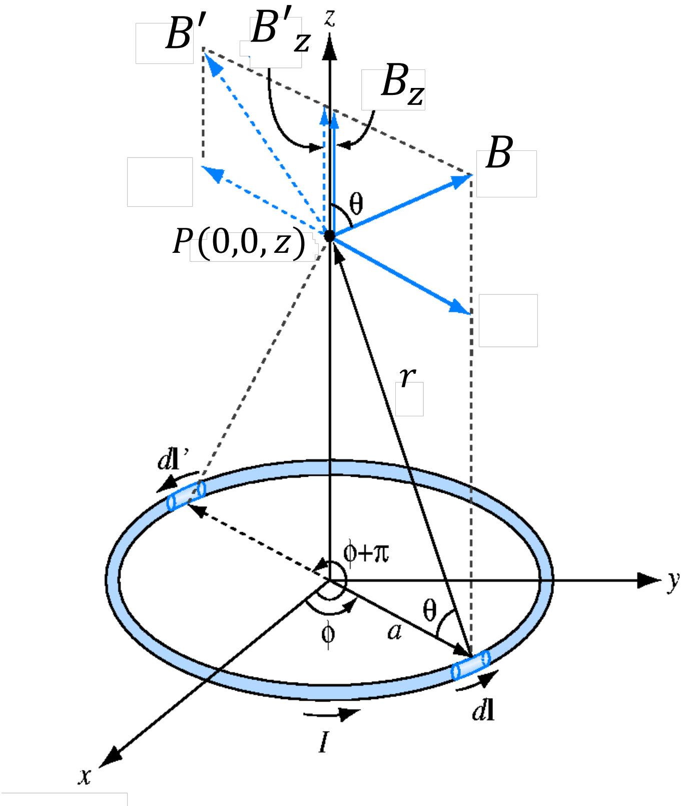
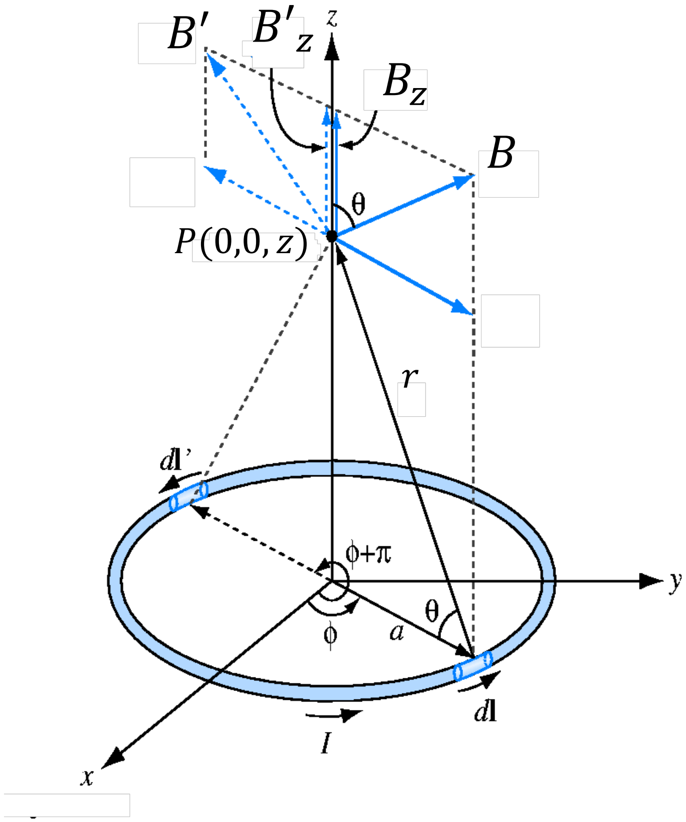


$\vec{B}(0,0,z) = ?$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l}\times\hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{(a d\phi)\cos(\theta)}{r^2}$$

B-field of a circular loop (28.5)



$\vec{B}(0,0,z) = ?$

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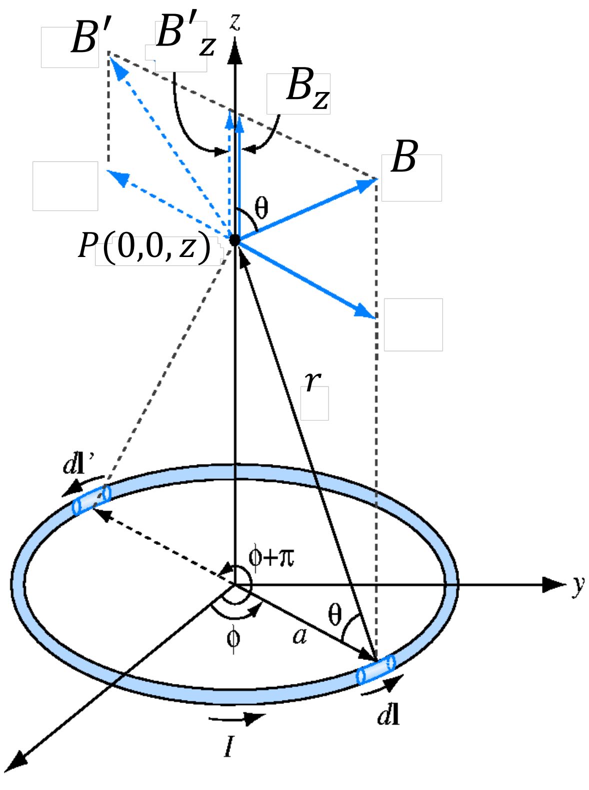

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$$B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{(ad\phi)\cos(\theta)}{r^2}$$

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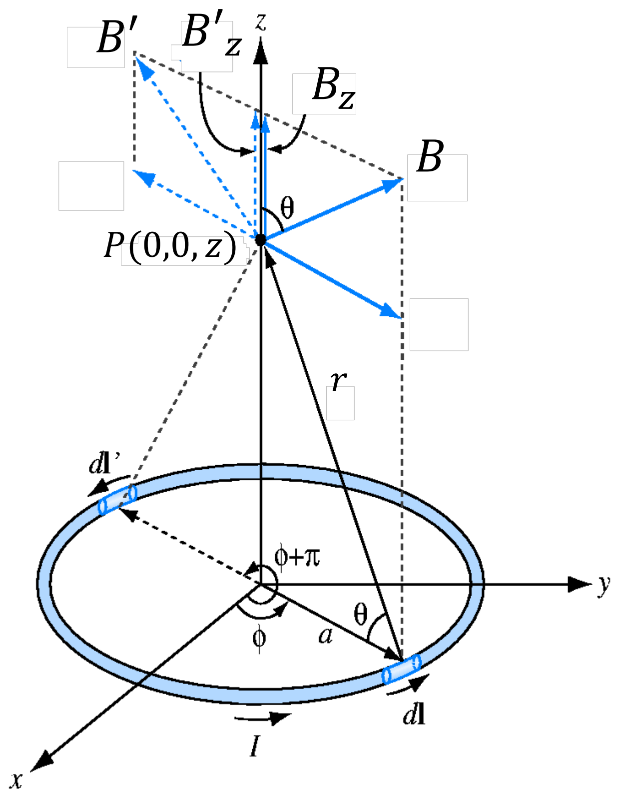

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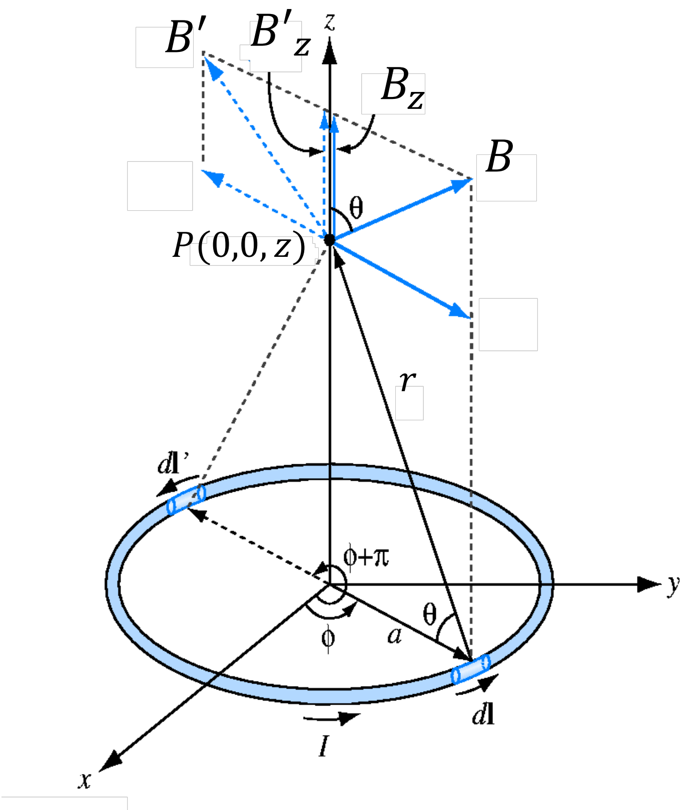

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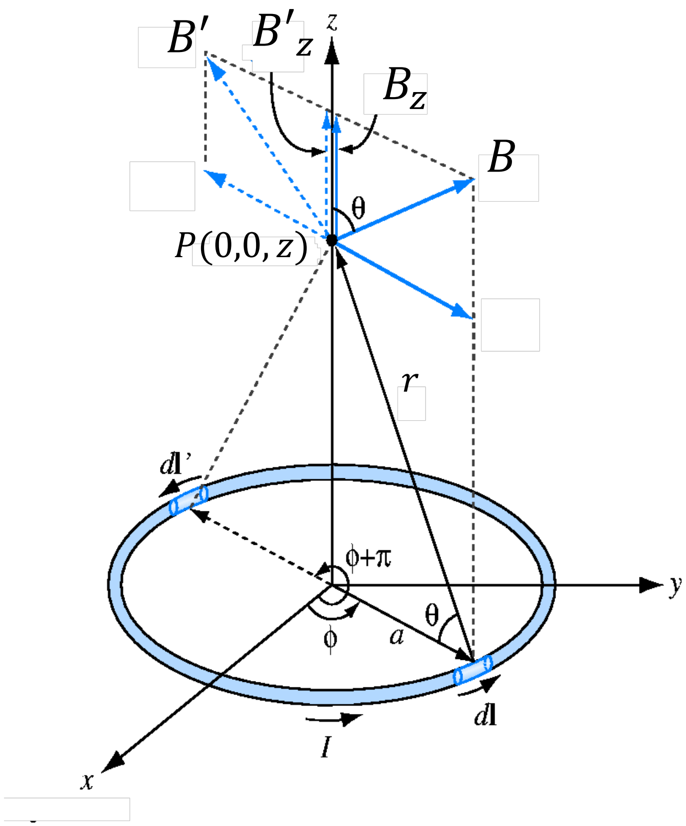

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Example: The magnetic field around the head has been measured to be approximately 3.00×10^{-8} G . Although the currents that cause this field are quite complicated, we can get a rough estimate of their size by modeling them as a single circular current loop 16.0 cm (the width of a typical head) in diameter. What is the current needed to produce such a field at the center of the loop?

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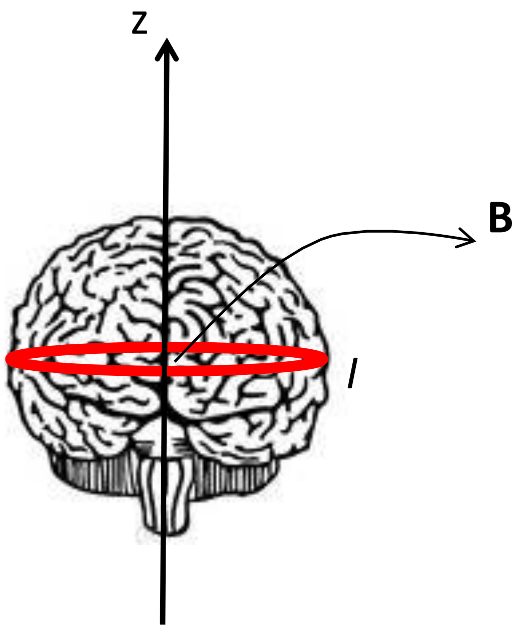

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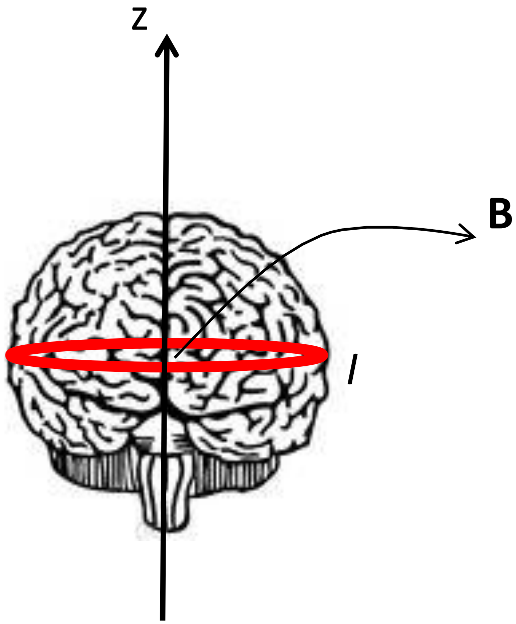

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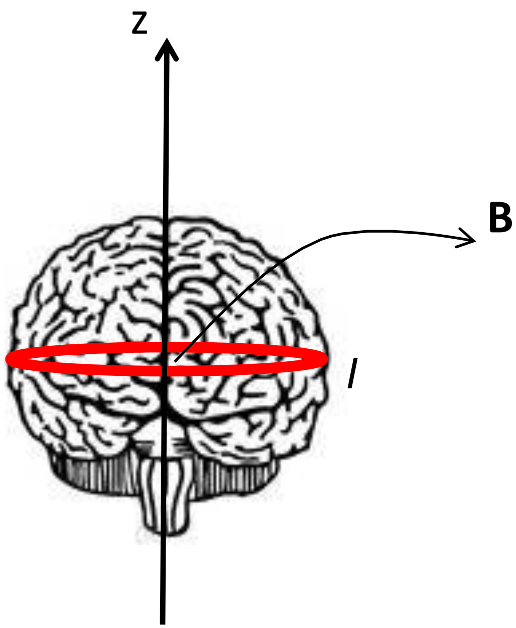

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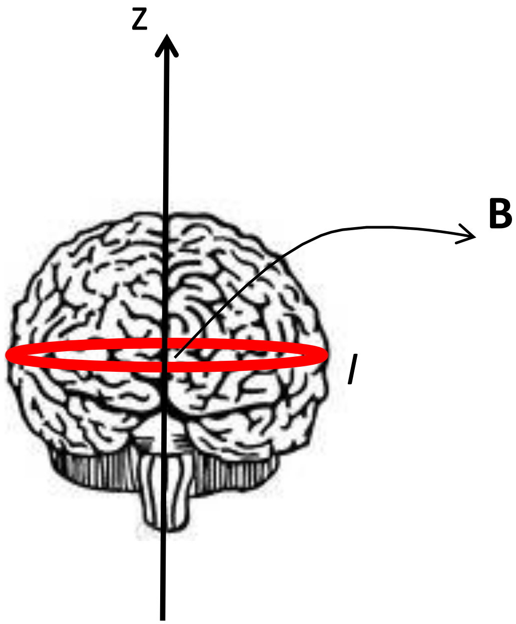

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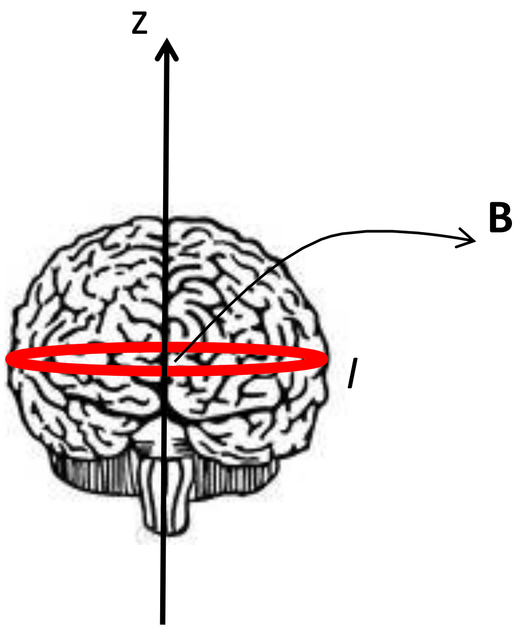

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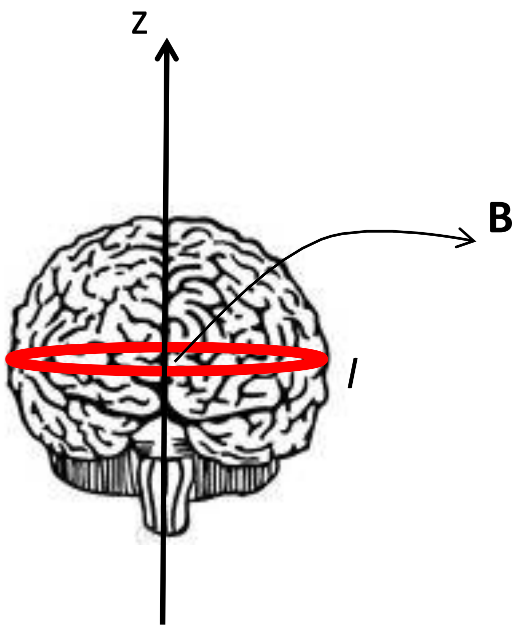

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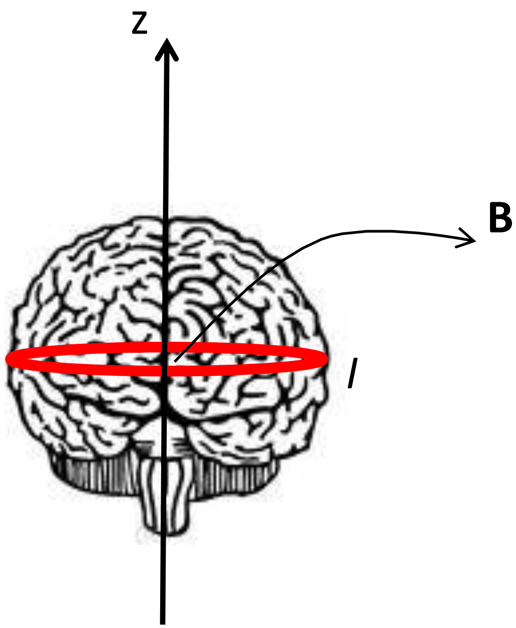

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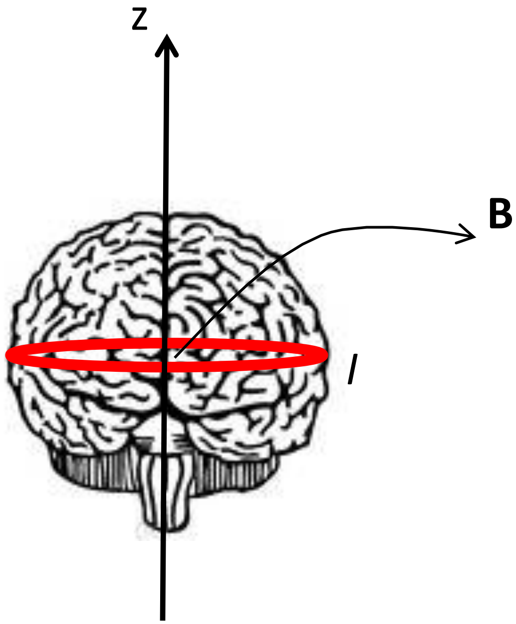

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$$I = \underline{3.82\times10^{-7}\text{ A}} = 0.382\ \mu\text{A}$$