UT DALLAS

PHYS2326
Lecture #09

Prof. Fabiano Rodrigues

Department of Physics
The University of Texas at Dallas

Announcements / Reminders

- Bring to exam:
  - Pen, pencil and scientific calculator
- Arrive early
- Final during last day of classes (April 27)?

Goals for this lecture

- Quick Recap
  - Electric Potential Energy
  - Electric Potential
- Examples
- Conductors and Electric Potential
- Potential gradient

Chapter 23

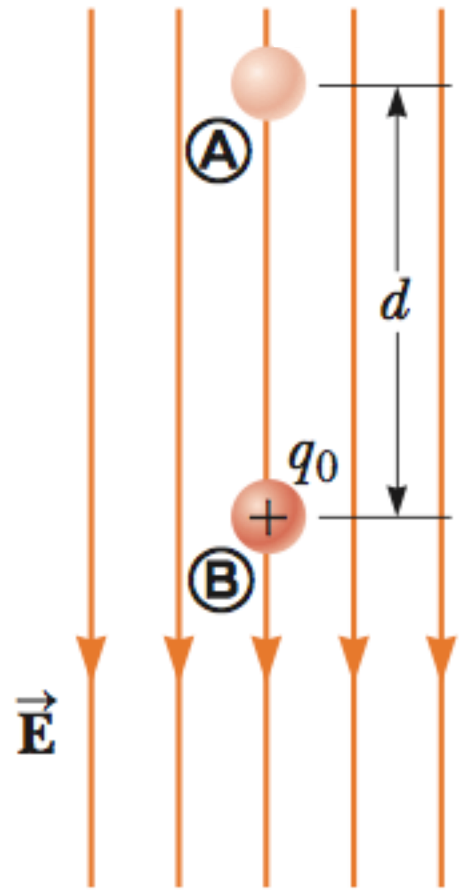
Review

Review

$\Delta U = -W_E$

Uniform electric field

$\Delta U = -\vec{F}_E \cdot \vec{d}$



Review

$$\Delta U = -W_E$$

Uniform electric field

$$\Delta U = -\vec{F}_E \cdot \vec{d}$$

Non-uniform electric field (two point charges)

$$\Delta U = -\int_{r_a}^{r_b} \vec{F}_E \cdot d\vec{r} = \left(\frac{q q_0}{4\pi\epsilon_0}\right)\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$U = \left(\frac{q q_0}{4\pi\epsilon_0}\right)\left[\frac{1}{r}\right] \qquad for\, U(\infty) = 0$$

$$V = \frac{U}{q_0}$$

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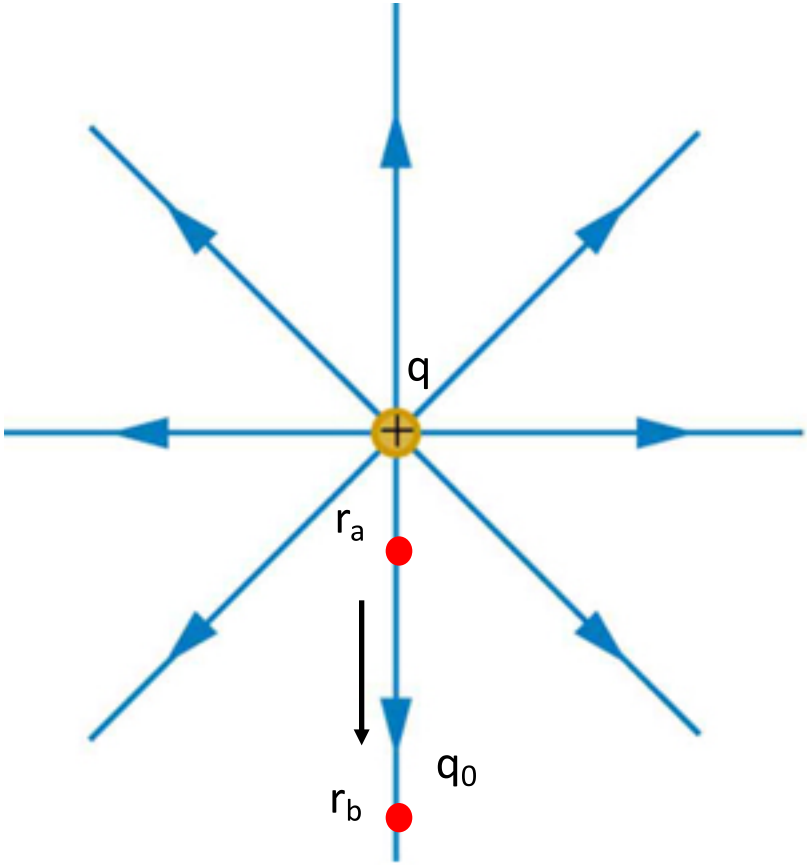
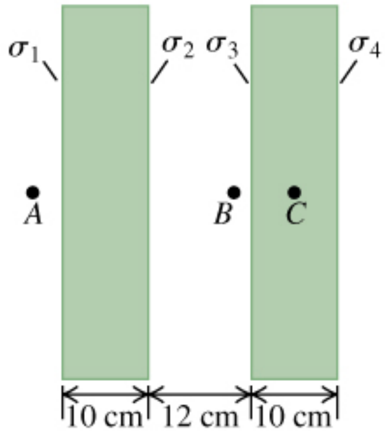


Example: Non-conducting sheets, each 10.0 cm think, carry uniform charge densities σ_1 = -6.00 μC, σ_2 = +5.00 μC, σ_3 = +2.00 μC, and σ_4 = +4.00 μC. Find the magnitude and direction of E at point A, 5.00 cm from the left face of the left-hand sheet.



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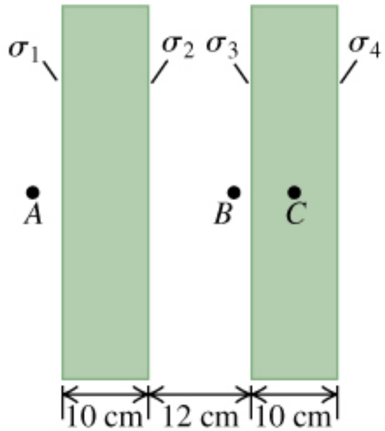

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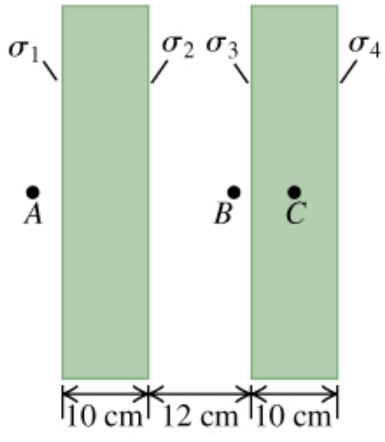

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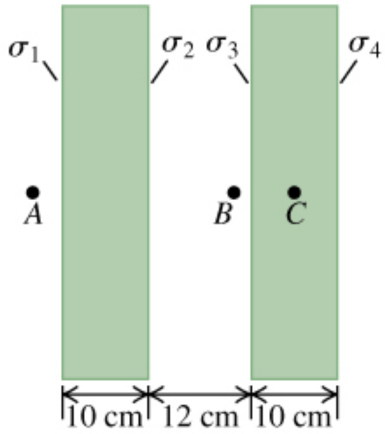

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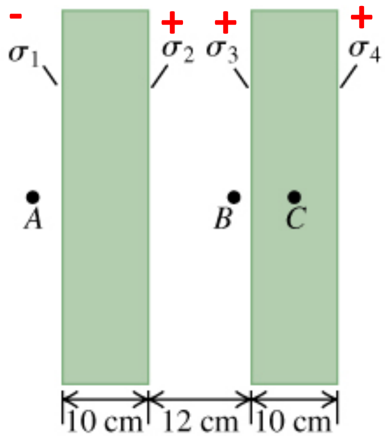

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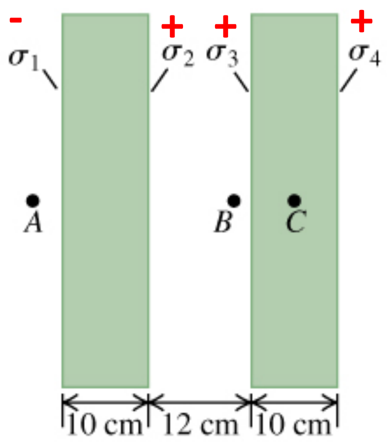

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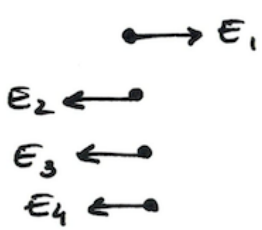

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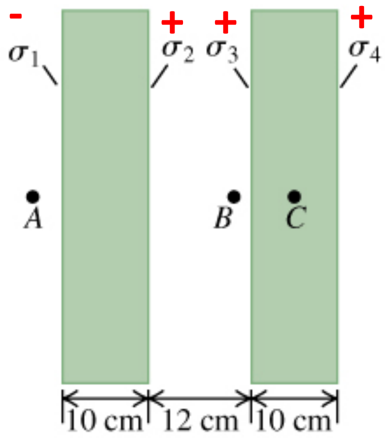

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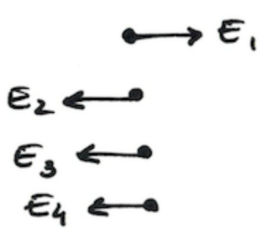

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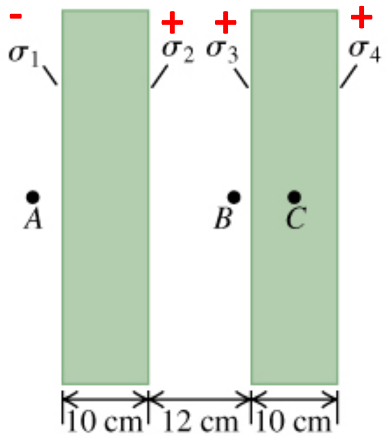

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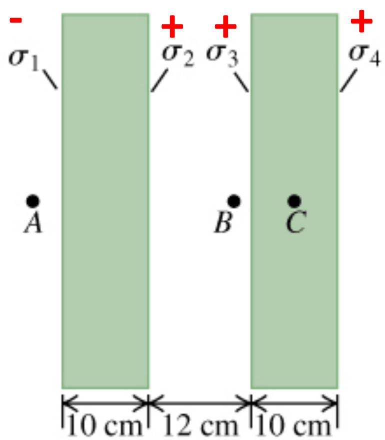

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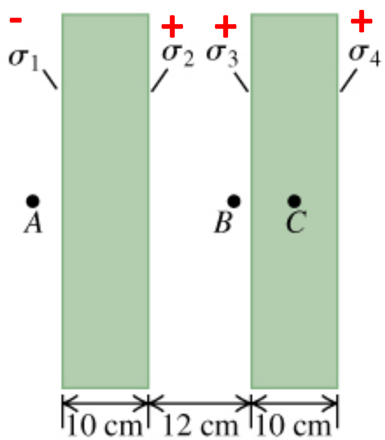

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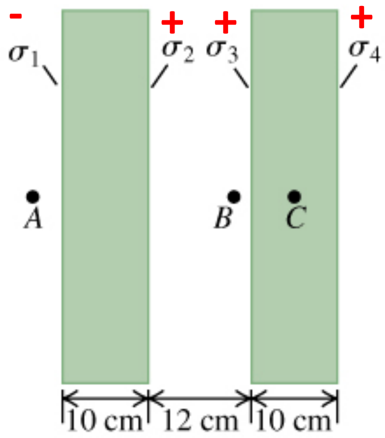

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Direction: to the left

Electric Potential Energy (U)

- Potential energy associated with electric fields
- Change in potential energy (ΔU) is what matters
- Change in $\Delta U = U_b - U_a$ for a charge that moved from “a” to “b” is determined from the work necessary to move the charge from “a” to “b”

$$\Delta U = -W_E$$

Example: A point charge q1 = 3.40μC is held stationary at the origin. A second point charge with charge q2 = -4.70 μC moves from the point x = 0.150 m, y = 0 m to the point x = 0.290 m, and y = 0.230 m. How much work is done by the electric force on q_2?

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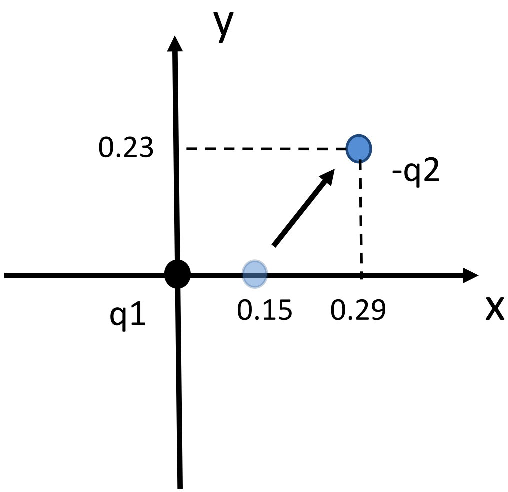

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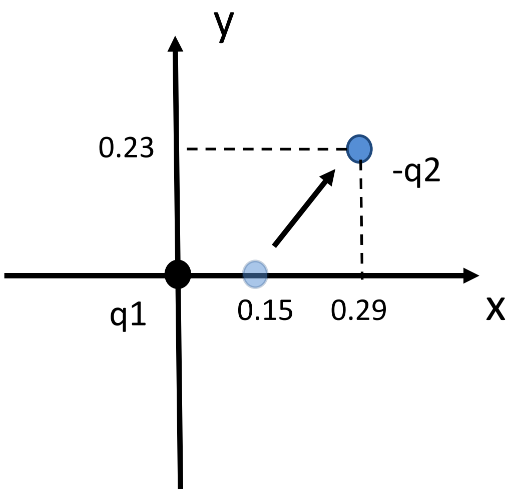

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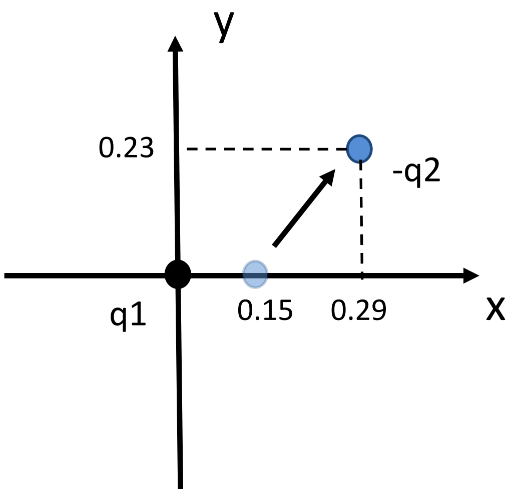

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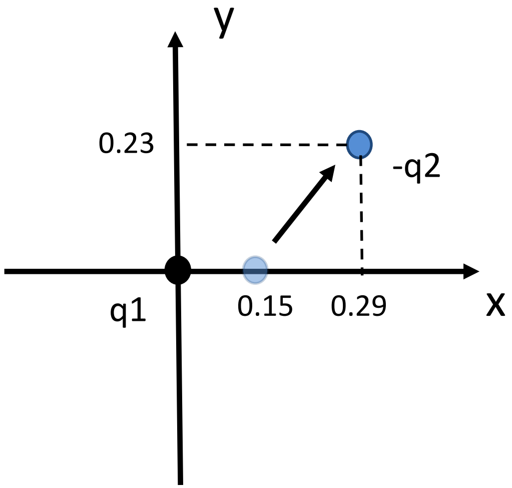

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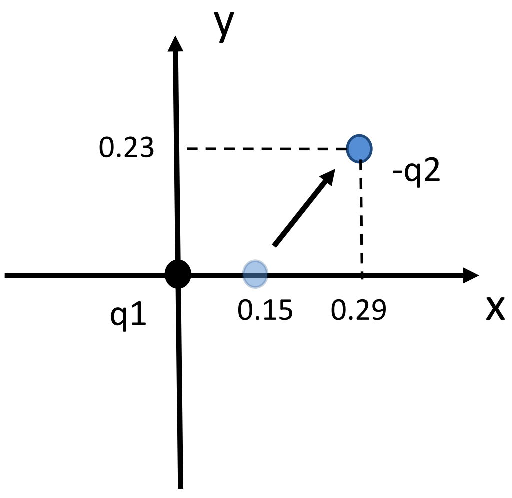

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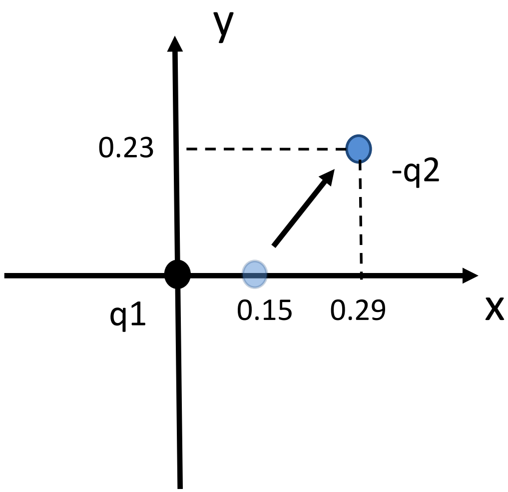

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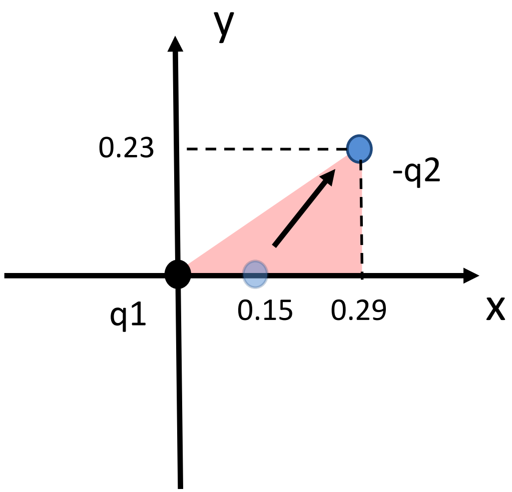

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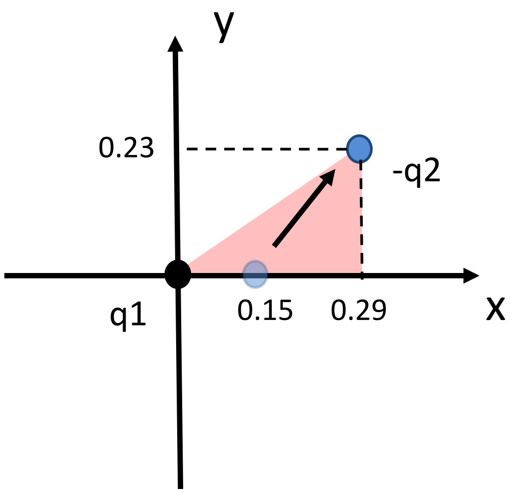

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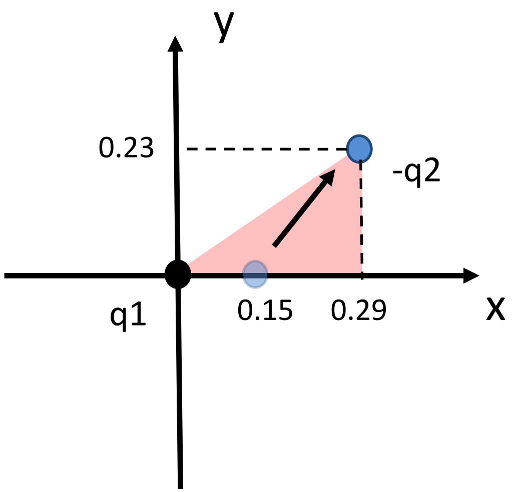

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$$\Delta U = 0.57\ J$$

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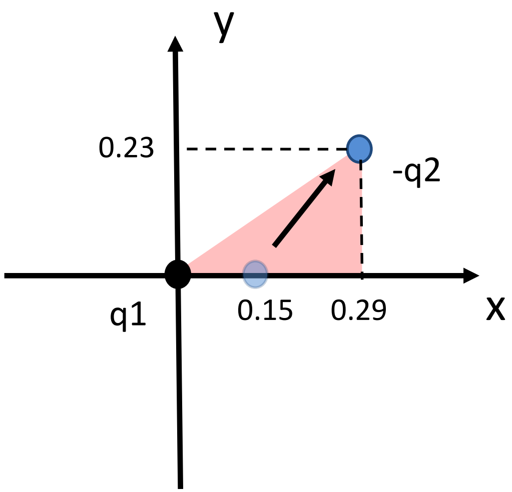

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$$\Delta U = 0.57\ J$$

$$\boxed{W_E = -0.57\ J}$$

Electric Potential (V)

- Electric potential refers to electric potential energy *per unit charge.*

$$U = \left(\frac{qq_0}{4\pi\epsilon_0}\right)\left[\frac{1}{r}\right] \qquad for\ U(\infty) = 0$$

$$V = \frac{U}{q_0}$$

$$V = \left(\frac{q}{4\pi\epsilon_0}\right)\left[\frac{1}{r}\right]$$

Determining the Potential (V)

Two routes:

1. Compute V from a charge distribution

2. Compute V from the electric field vector $\vec{E}(x, y, z)$

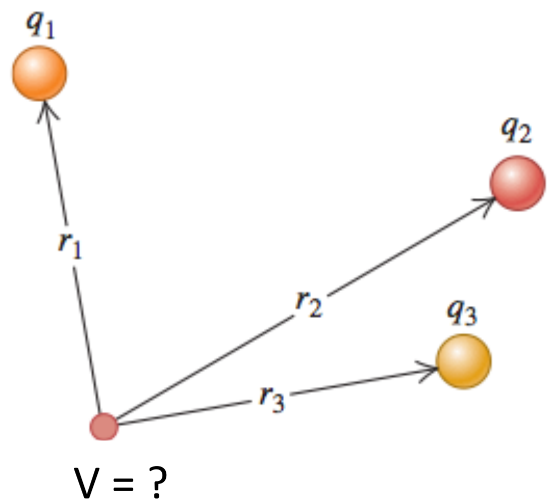
Determining the Potential (V)

- Potential from charge distribution:

$$V = \left(\frac{q}{4\pi\epsilon_0}\right)\left[\frac{1}{r}\right]$$ Single point charge

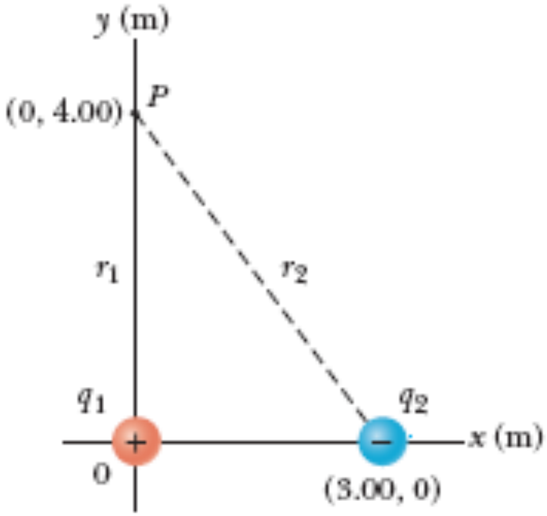
$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\sum_{i=1}^{N}\frac{q_i}{r_i}$$ Multiple point charges

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\int\frac{dq}{r}$$ Continuous charge



Example: Two point charges q1= 2.30nC and q2= -6.40nC are shown in the figure below. Take the electric potential to be zero at infinity. Find the potential at point P.

$$V = ?$$



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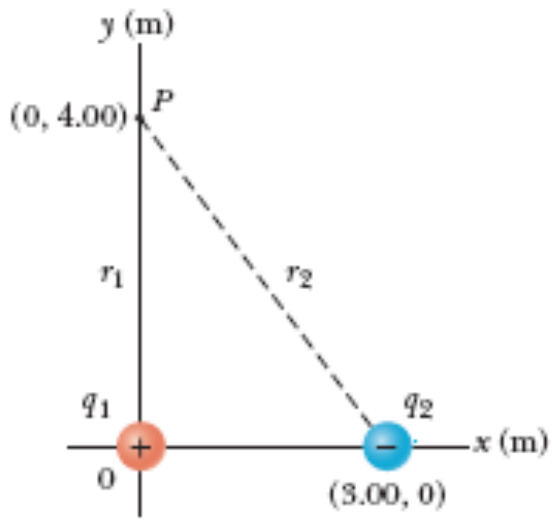

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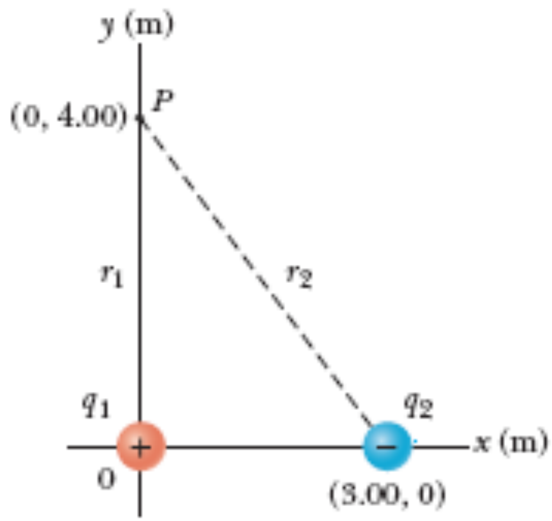

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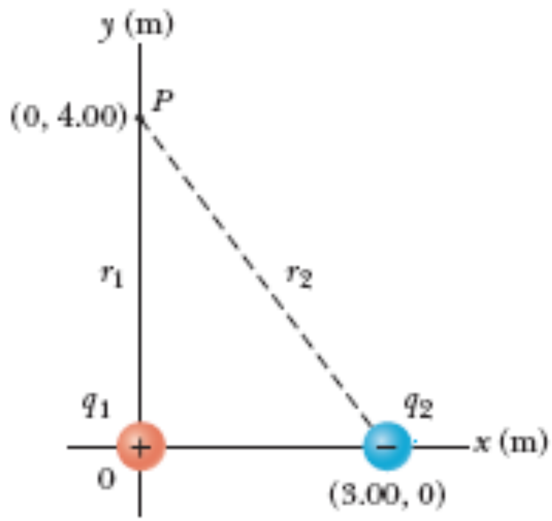

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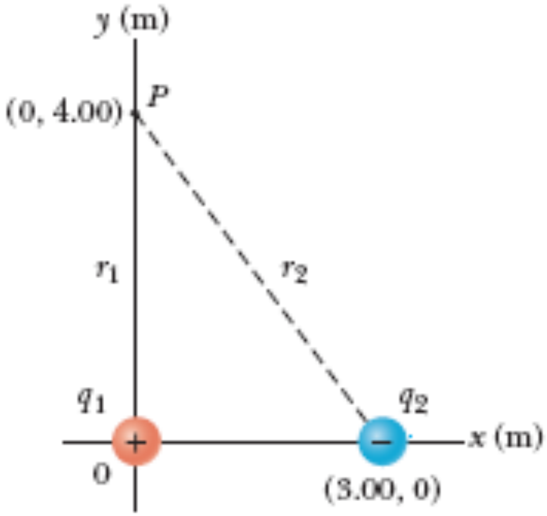

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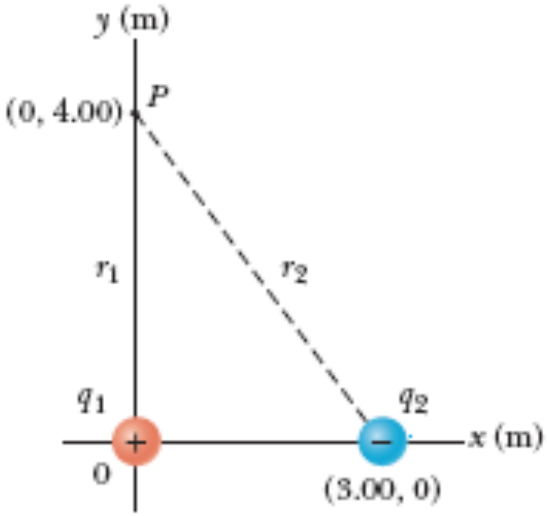

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Determining the Potential (V)

- Potential from electric vector field **E**(x,y,z)

$$V_b - V_a = \frac{\Delta U}{q_0} = -\frac{W_E}{q_0} = -\int_a^b \vec{E} \cdot d\vec{s}$$

Electron Volt (eV)

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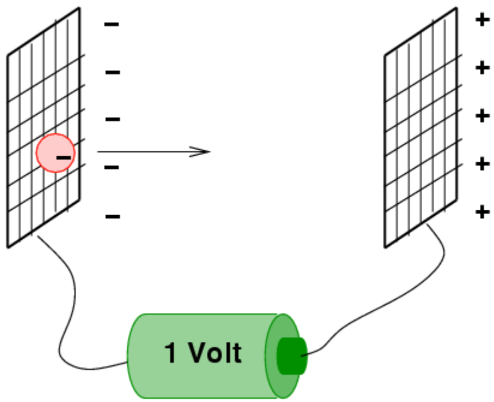
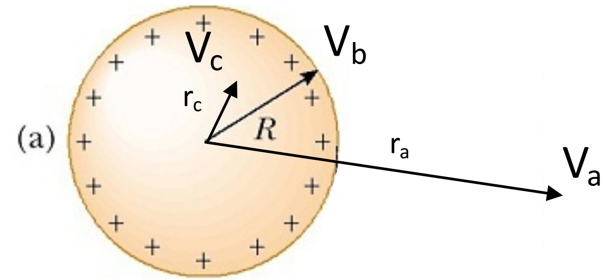


$$q\Delta V = q\left(\frac{\Delta U}{q}\right) = \Delta U \quad \text{(Energy)}$$

$$1\text{eV} = (1)(1.6\times10^{-19}\text{C})(1\text{V}) = 1.6\times10^{-19}\text{J}$$

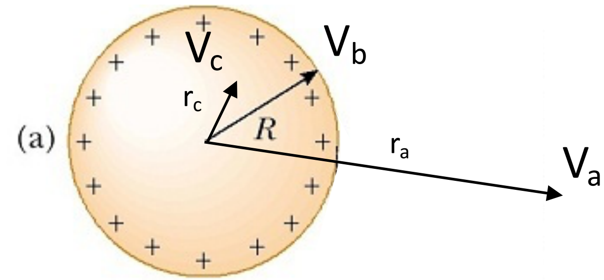
Conductors and Electric Potential

Conductors and Electric Potential



Conductors and Electric Potential

$$V_b - V_a = -\int_{r_a}^{R} \vec{E} \cdot d\vec{s}$$



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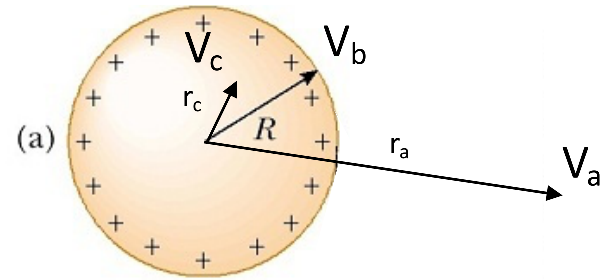

Now, inside conductor:

$$V_c - V_b = 0 \qquad or \qquad V_c = V_b$$

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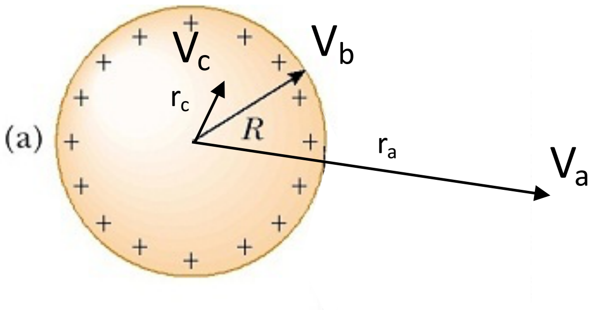

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V is the same within conductor!
Conductors are equipotential!

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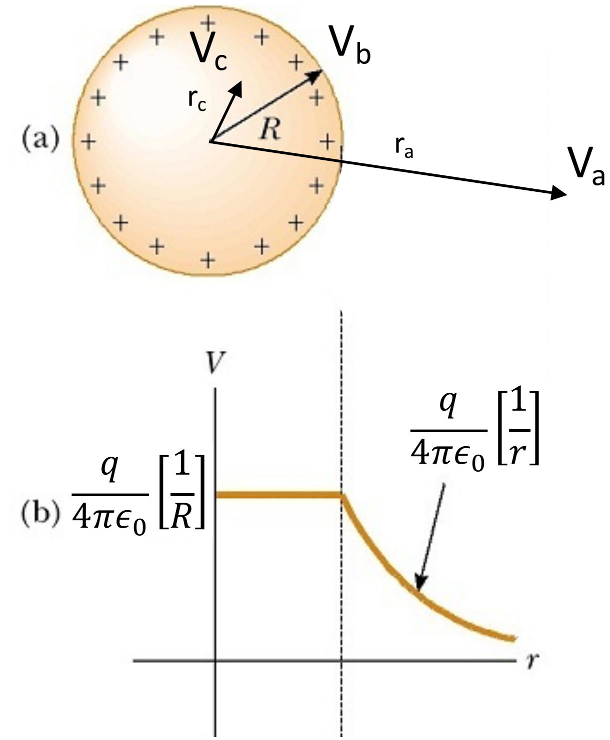

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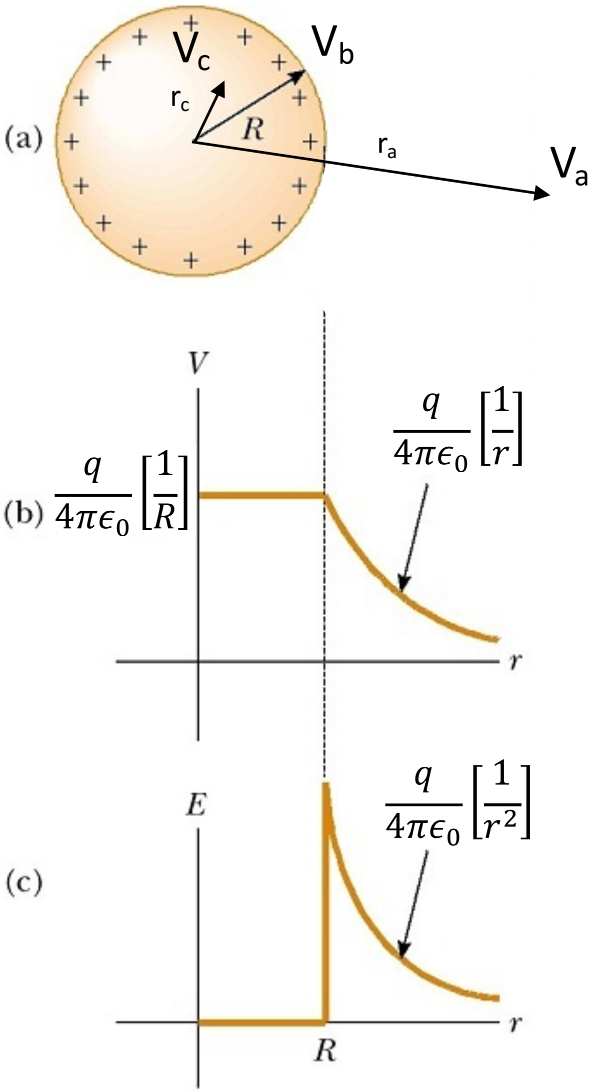
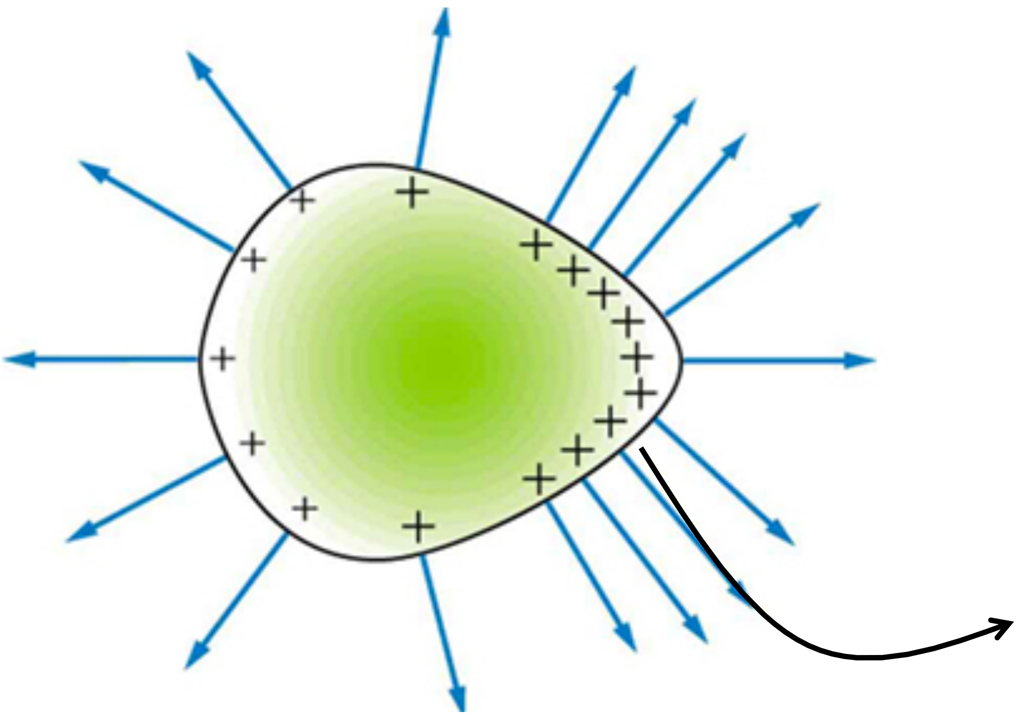


Conductors and Electric Potential

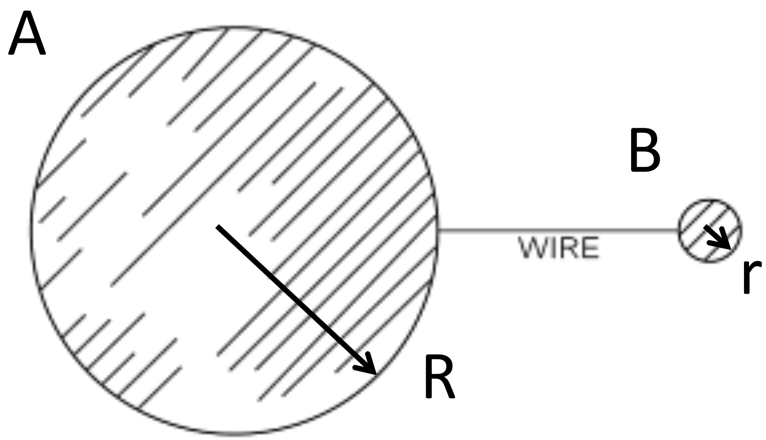
Conductors and Electric Potential

- On irregularly shaped conductors, σ is greatest where the radius of curvature is smallest



Conductors and Electric Potential

Conductors



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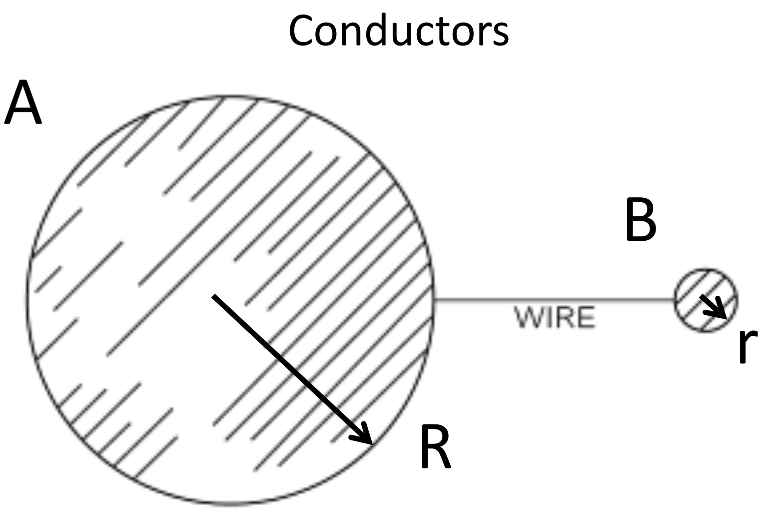


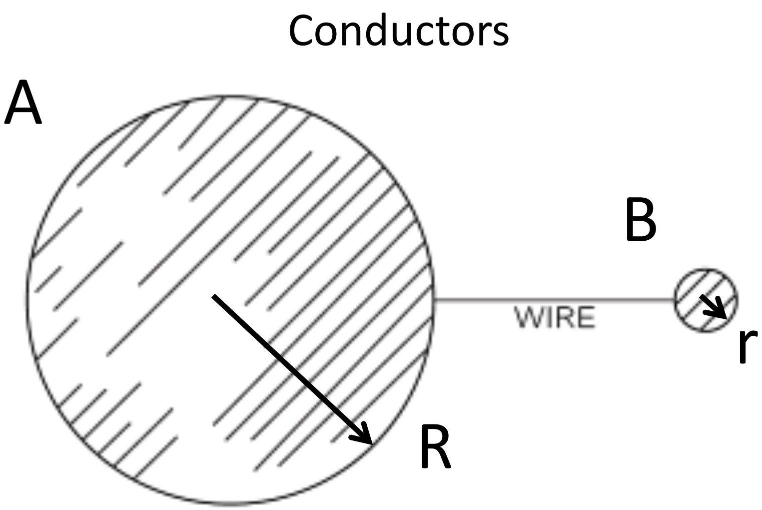
After charging:

$$\sigma_A = \frac{q_A}{4\pi R^2}$$

$$\sigma_B = \frac{q_B}{4\pi r^2}$$

$$\frac{\sigma_A}{\sigma_B} = ?$$

Conductors and Electric Potential



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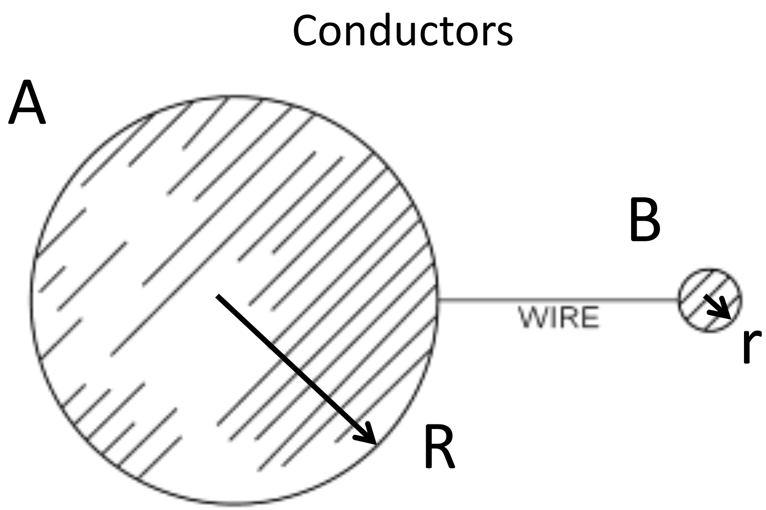

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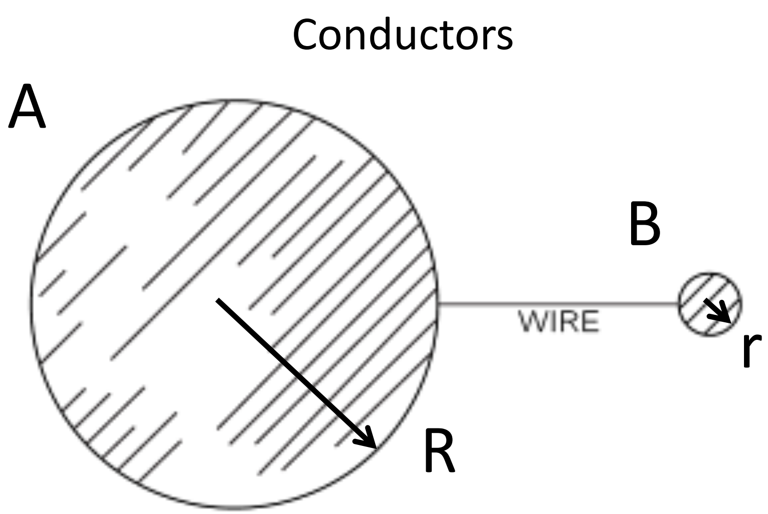

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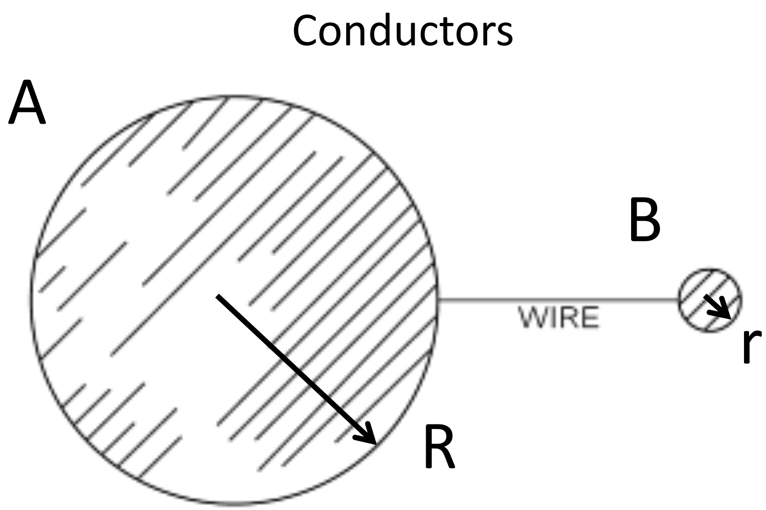

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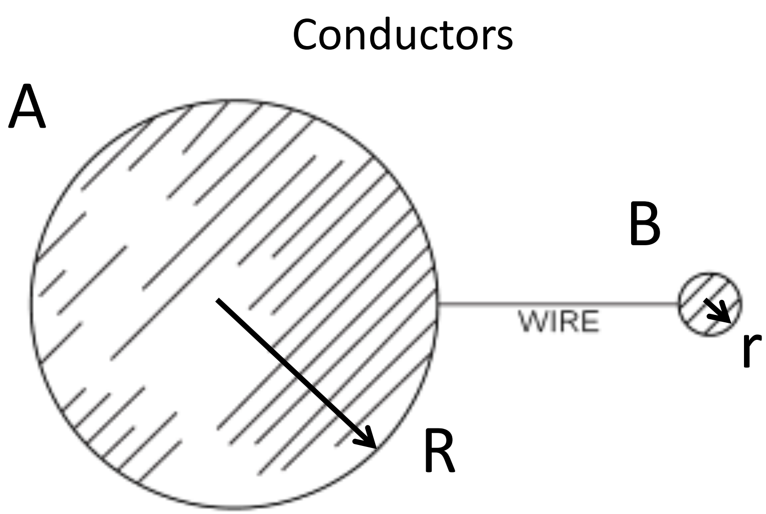

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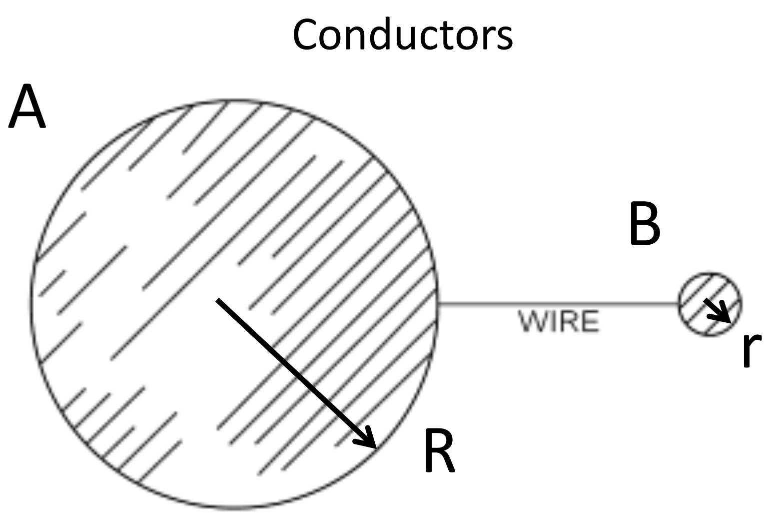

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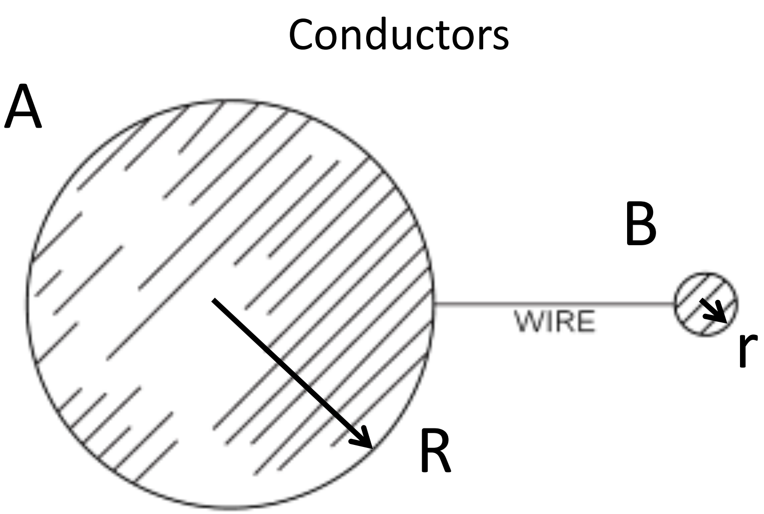

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Larger charge density (and E field) on the smaller radius sphere!

Potential Gradient

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$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\int_a^b dV = \int_a^b -\vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s} = -\left(E_x dx + E_y dy + E_z dz\right)$$

$$Assuming\ d\vec{s}\ only\ along\ x: dy = dz = 0$$

$$E_x = -\left.\frac{dV}{dx}\right|_{y=z=constant} = -\frac{\partial V}{\partial x}$$

Other components:

$$E_y = -\left.\frac{dV}{dy}\right|_{x=z=constant} = -\frac{\partial V}{\partial y}$$

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$$\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$$

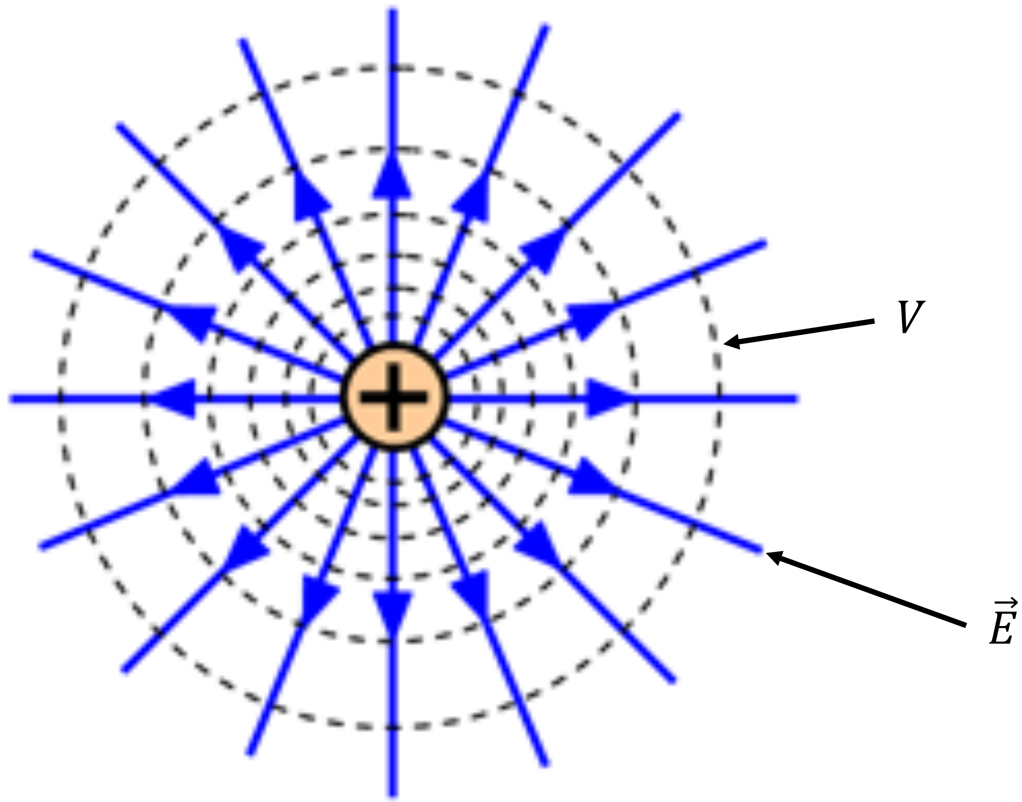
$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} - \frac{\partial}{\partial y}\hat{y} - \frac{\partial}{\partial z}\hat{z}$$

$$\vec{E} = -\vec{\nabla}V$$

1. $\vec{\nabla}V$ *is the potential gradient*

2. *If you know* $V(x, y, z)$ *you can compute* $\vec{E}(x, y, z)$

Potential Gradient



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