PHYS2326
Lecture #13

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Goals for this lecture

- Define and understand electric current
- Define and understand resistivity and resistance

Chapter 25

Charges in Motion

Charges in Motion

- So far we looked at situations where charges were in equilibrium and not in motion – **Electrostatics**

- We now start to look at charges moving within conductors.

Electric Current

Electric Current

$$\vec{E} = -\nabla V$$

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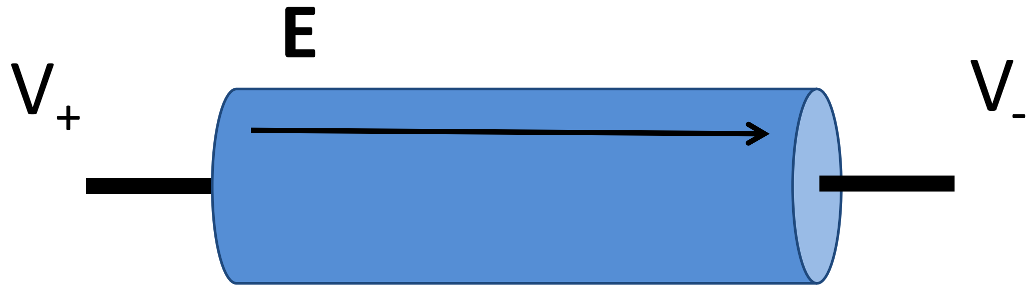

Electric Current

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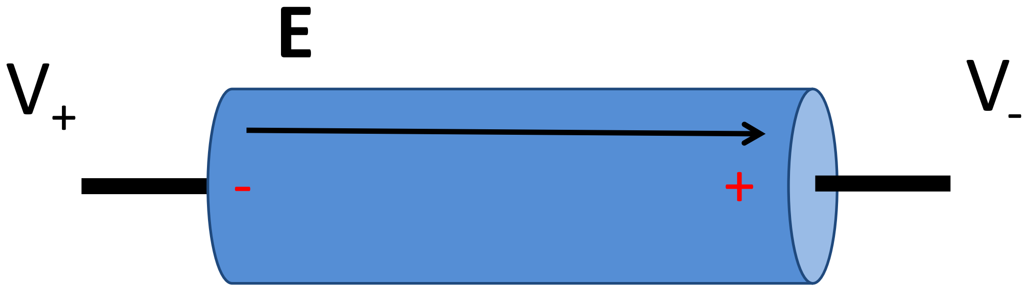

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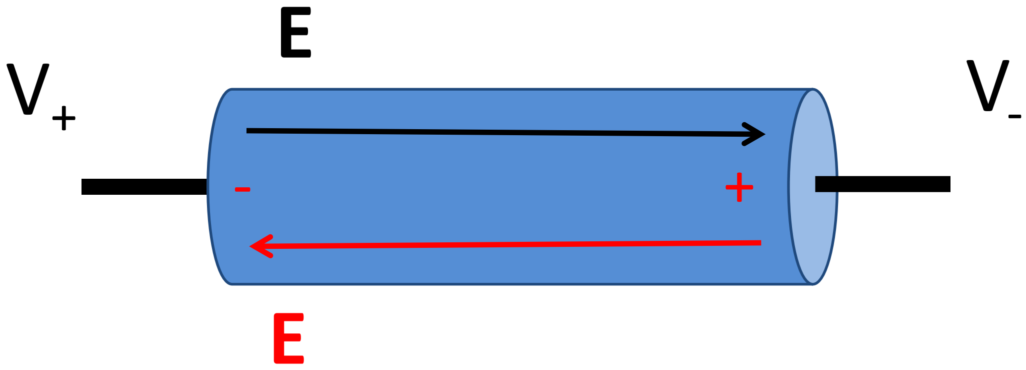

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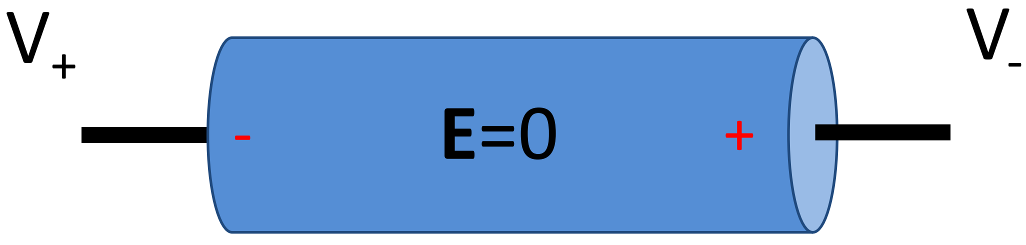
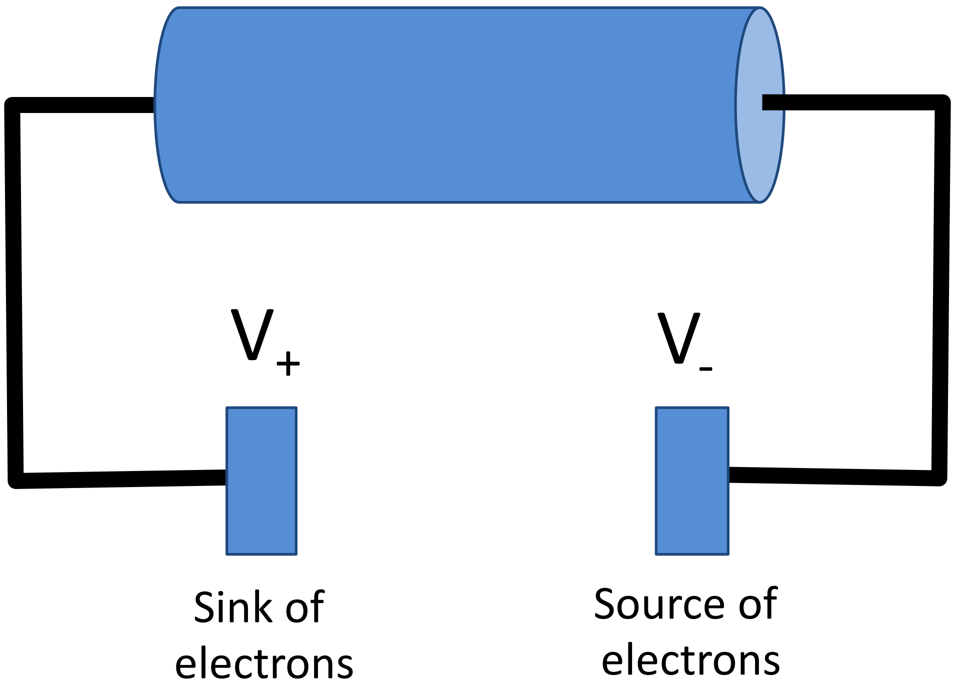


Electric Current



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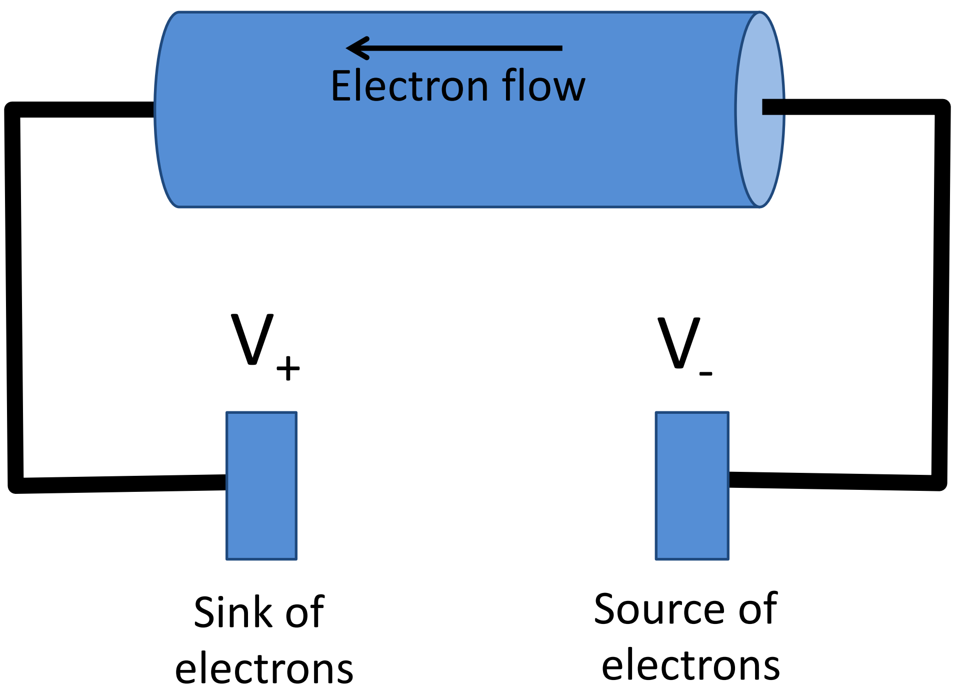
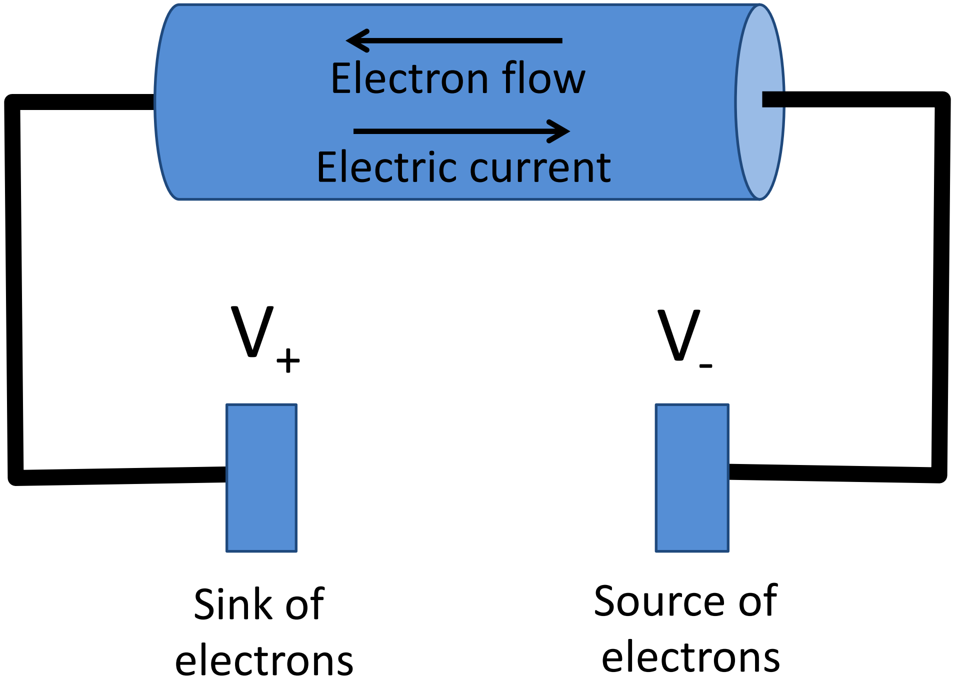
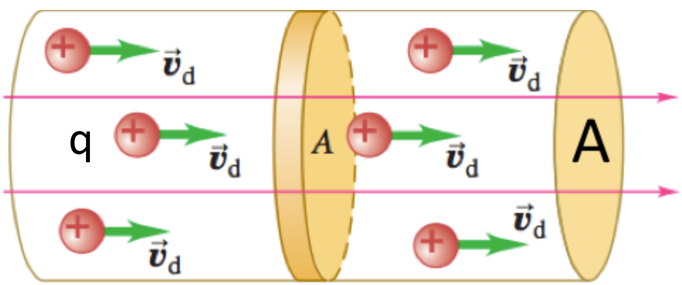


Electric Current



Electric Current: Definition

- Electric current (I) is the amount of charge flowing through an specific cross section area per unit of time.



$$\boxed{I = \frac{dQ}{dt}}$$

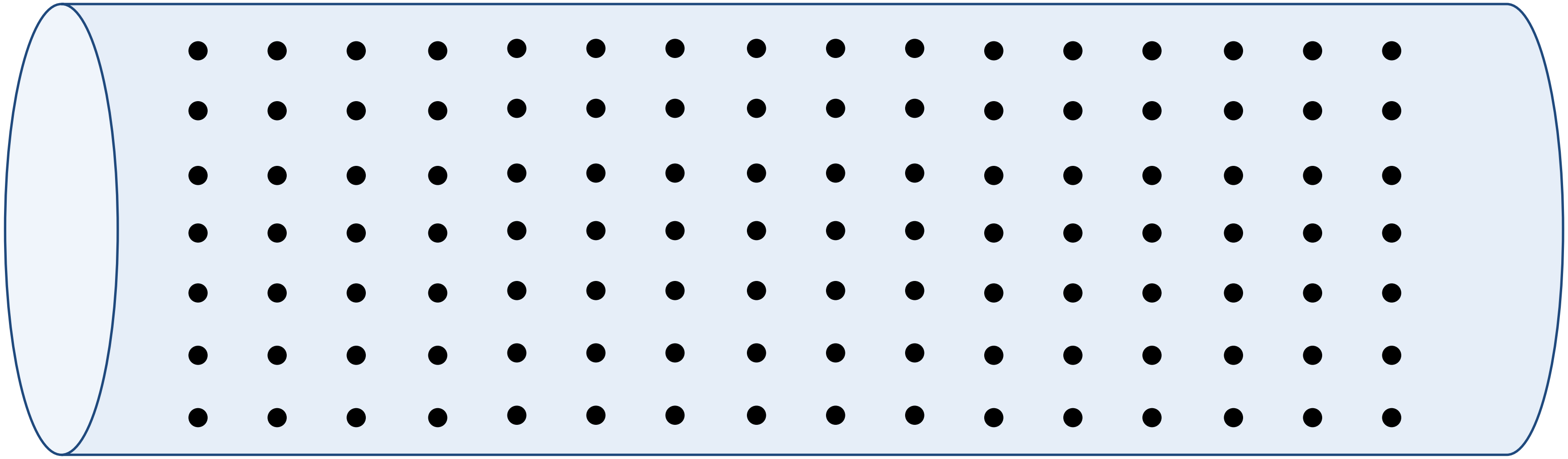
$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

Conductor

- Consider a conductor (free electrons available)

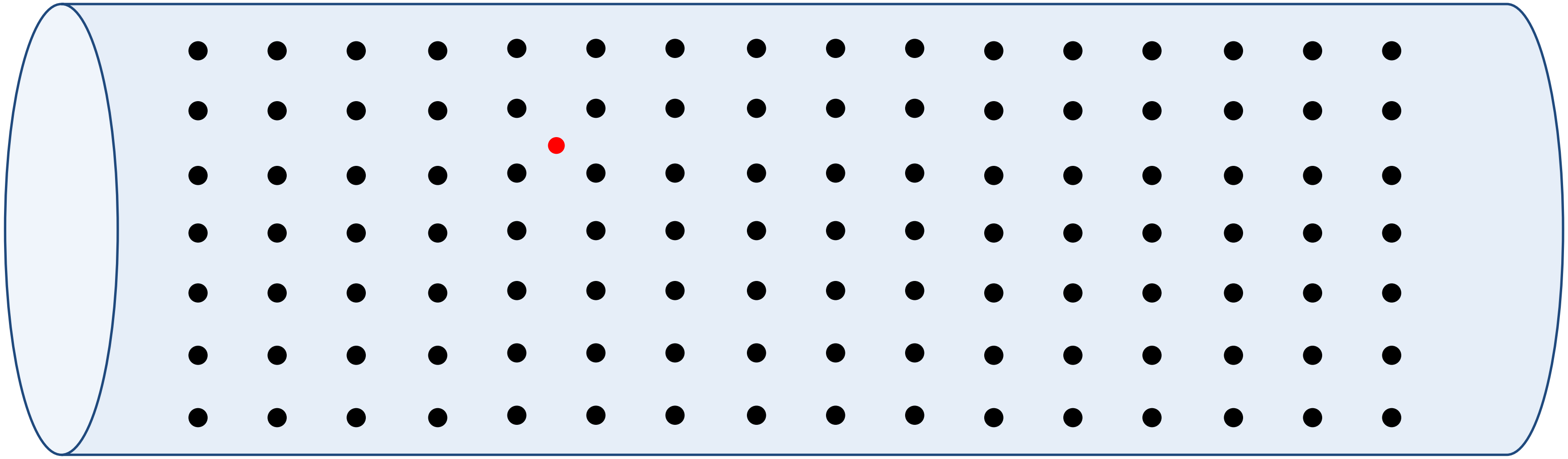
Conductor

- Now, consider its atomic structure



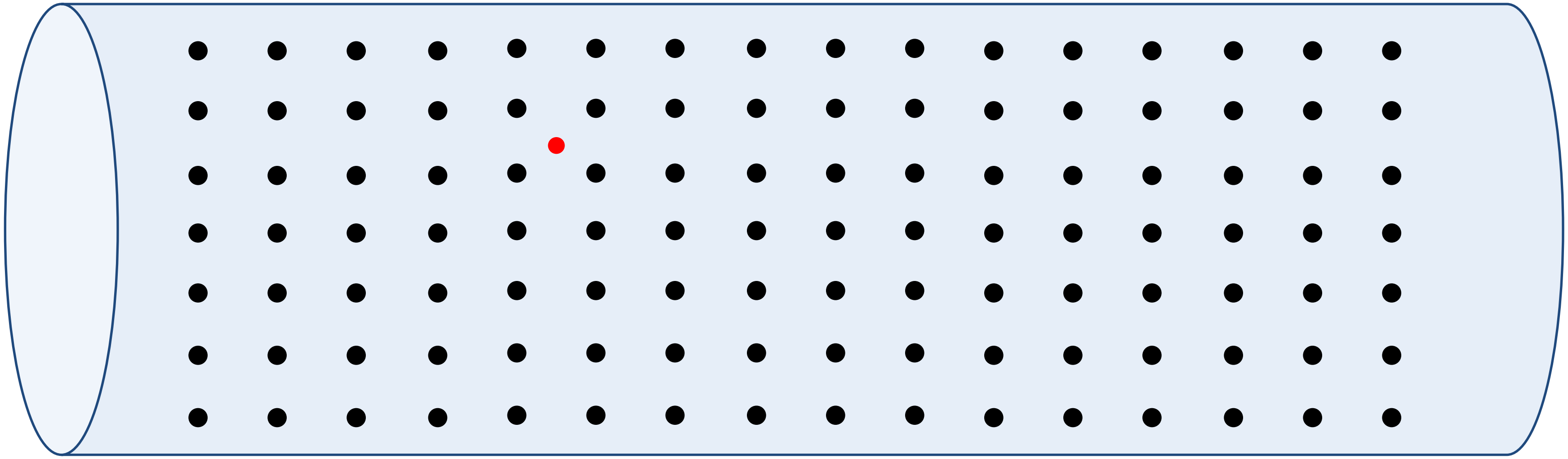
Conductor

- And pay attention to a single electron (out of many)



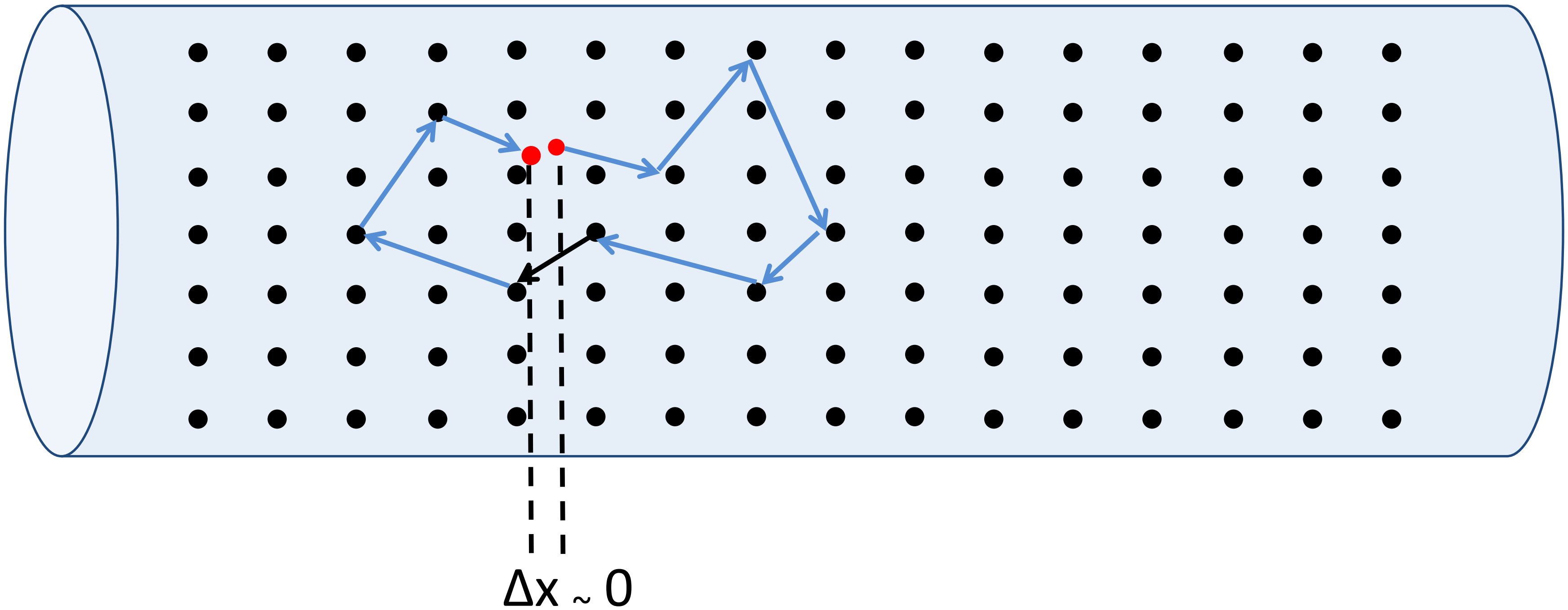
Conductor

- It will move due to thermal vibration



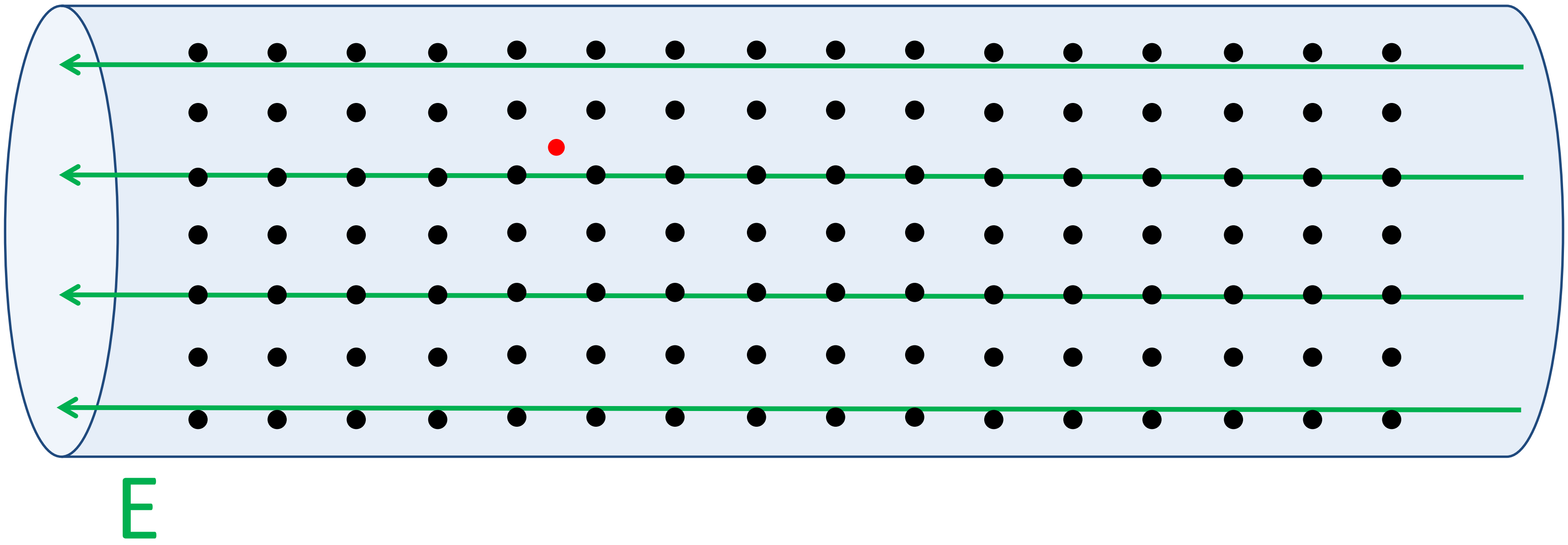
Conductor

- On average position does not change. Too many collisions. Random motion.



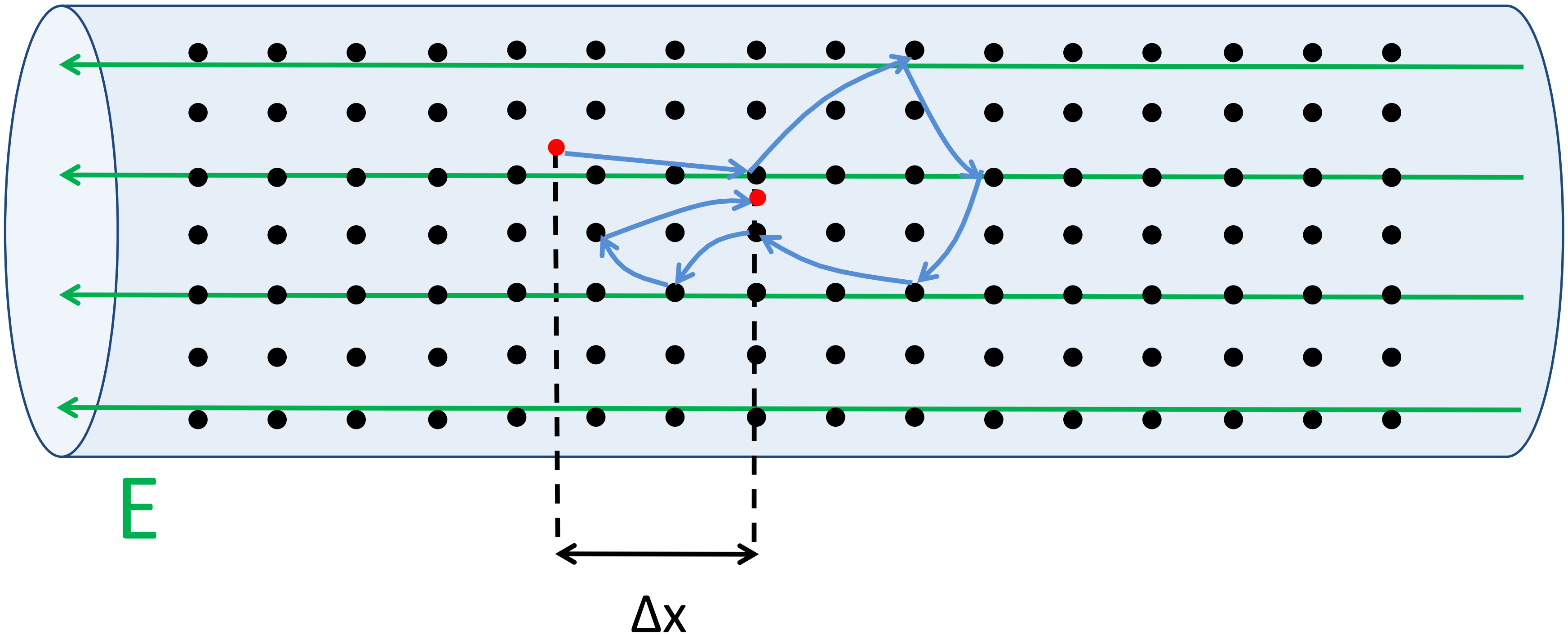
Conductor

- Now, consider that electron again! But also consider an applied electric field E.



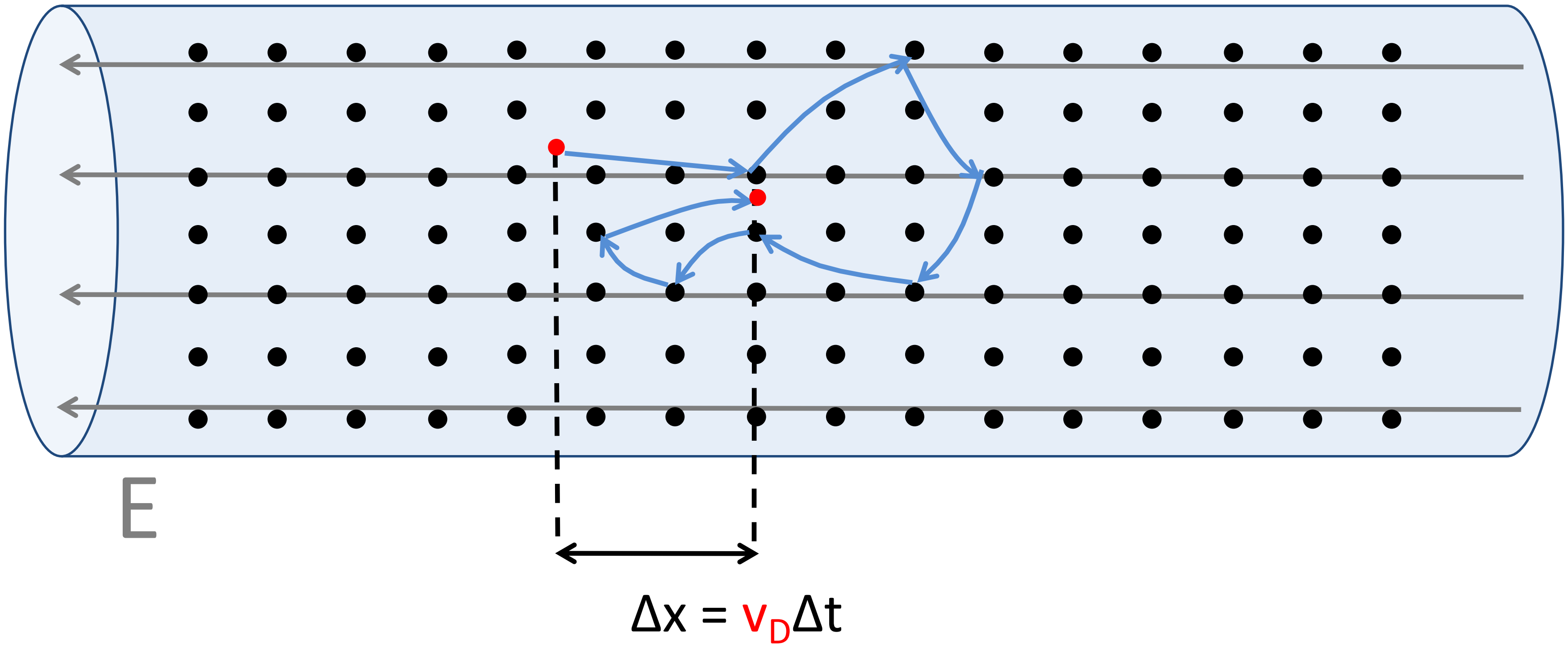
Conductor

- After same Δt, electron moves slightly to the right driven by the electric field



Conductor

- Electron is said to "drift" with velocity v_D

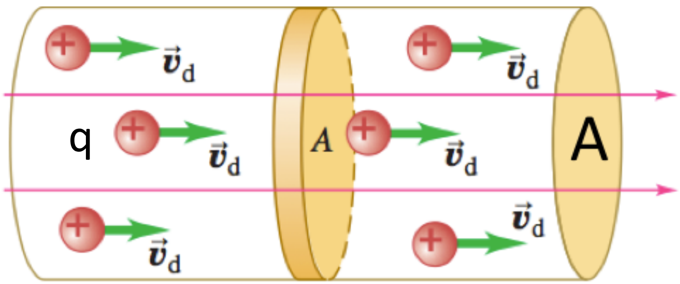


Electric Current (I)

- The "drift velocity – v_D", despite of being really tiny (~10^{-4} m/s), transfers charge from one point to another.

- Therefore, electric current occurs!

Electric Current (I)



$$I = \frac{dQ}{dt}$$

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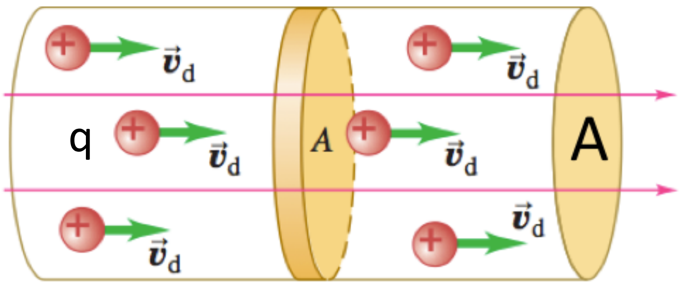
Electric Current (I)

$$I = \frac{dQ}{dt} = |q| n v_d A$$



$A = cross - section\ [m^2]$
$v_d = drift\ velocity\ [m/s]$
$q = charge\ [C]$
$n = volume\ charge\ density\ [m^{-3}]$

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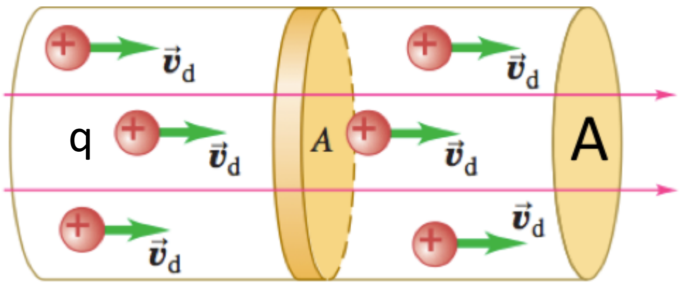
$$Unit: \frac{C}{s} = A = Ampere$$



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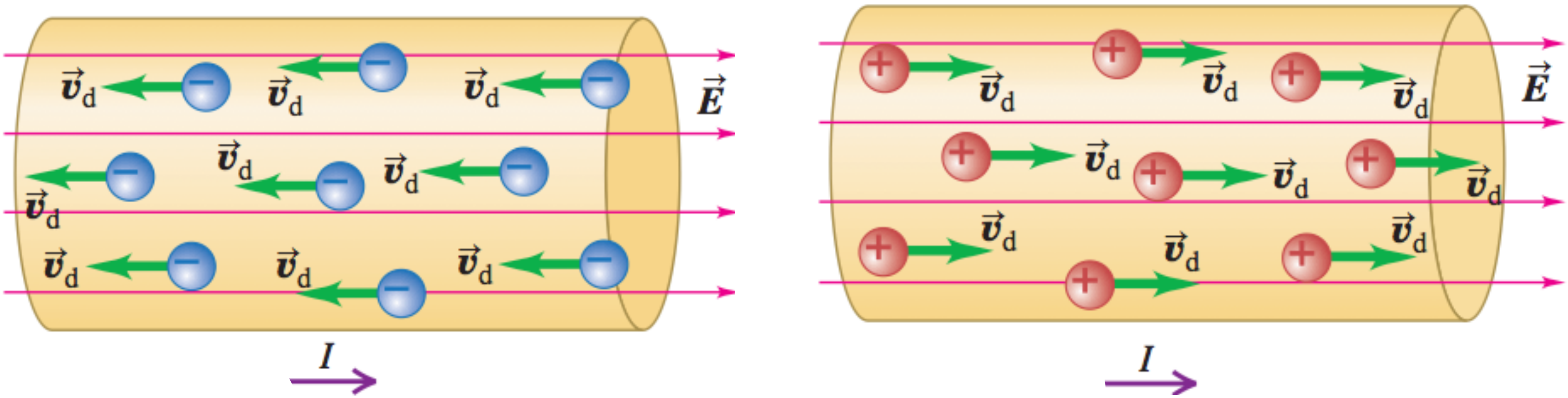
Direction of Electric Current

Direction of Electric Current

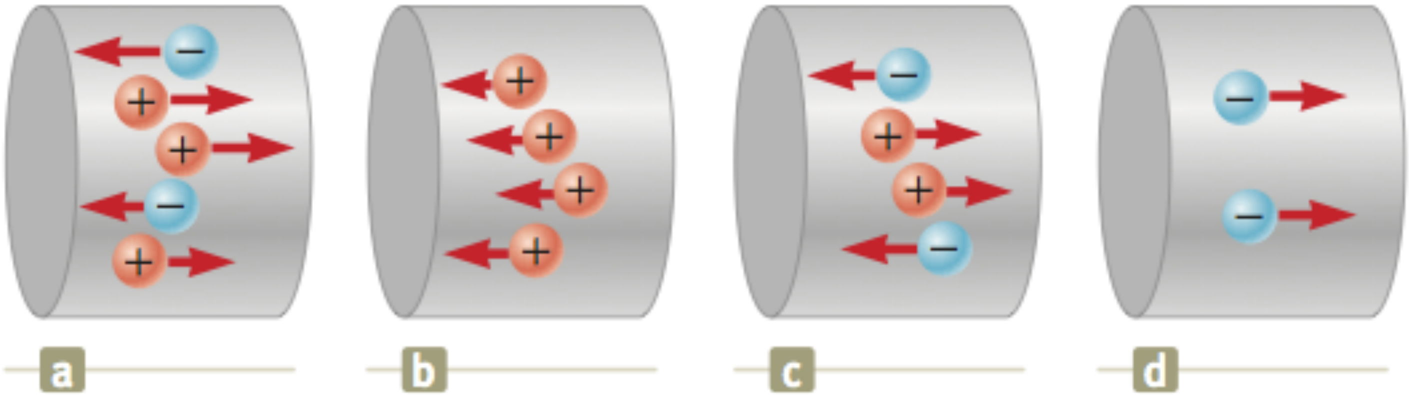
- In metals, the free, mobile charges are always the negative electrons.
- In plasmas (ionized gases) or ionic solutions, the charges can be (positive or negative) ions or electrons.
- Convention is that current points in the direction that positive charges would flow.

Direction of Electric Current



- **Direction of current**: Convention is that current points in the direction that positive charges would flow.

Analysis #1: What is the direction of the current in each of the following cases?



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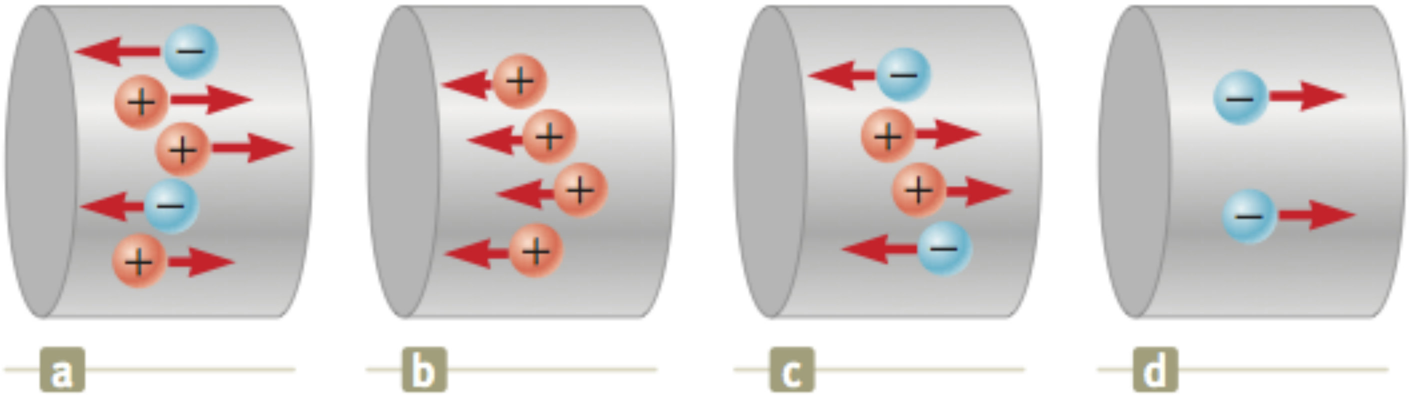

Right

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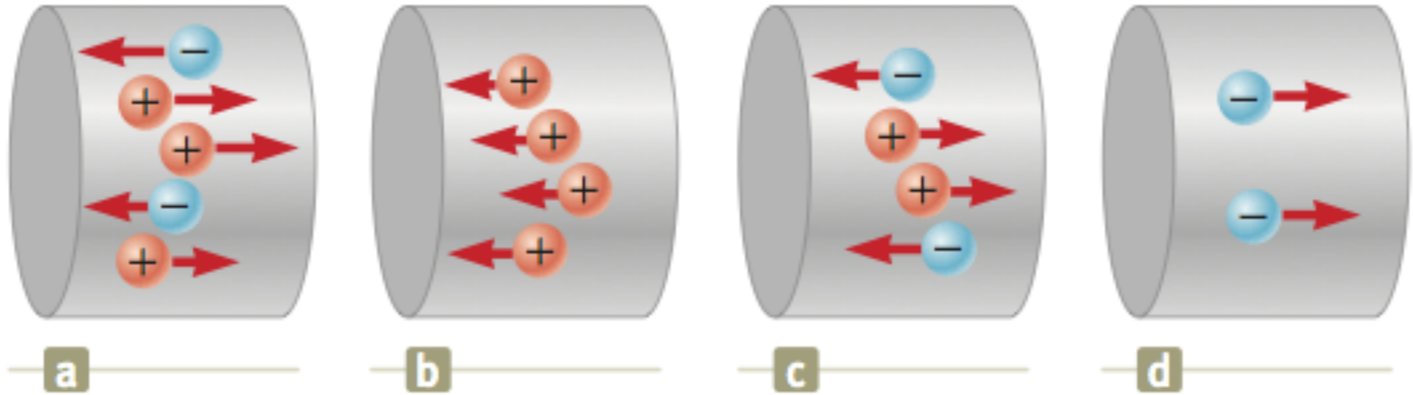

Right Left

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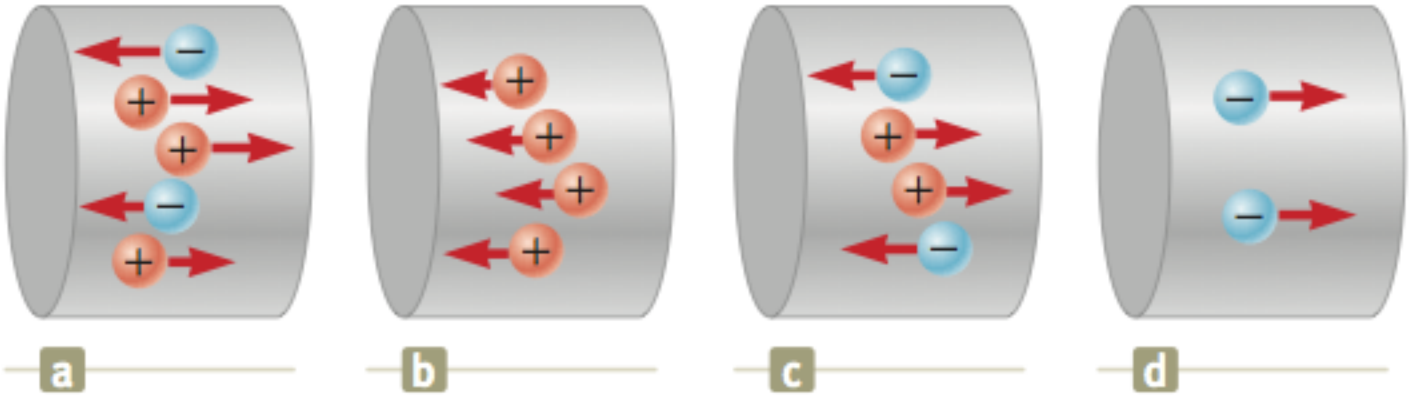

Right Left Right

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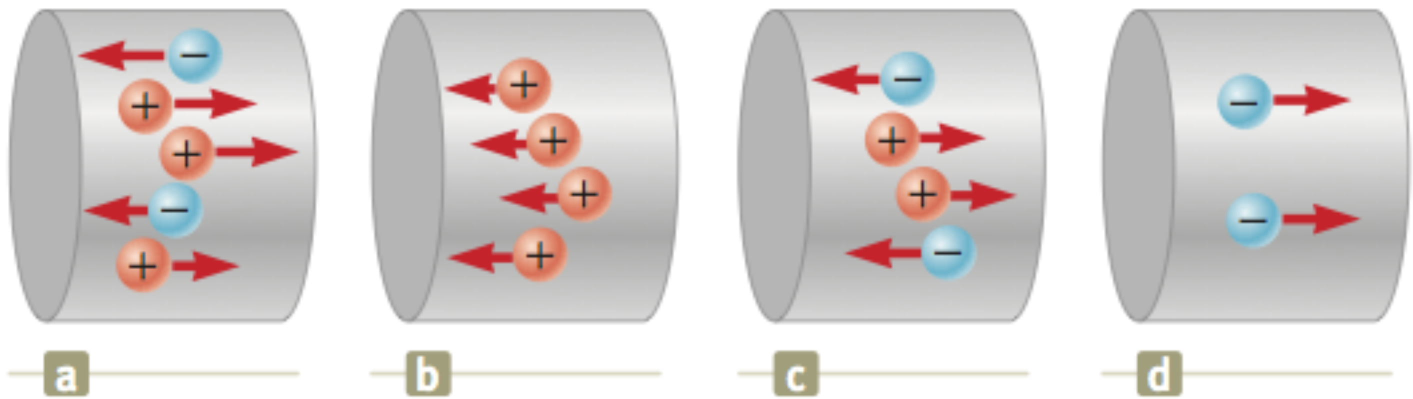

Right Left Right Left

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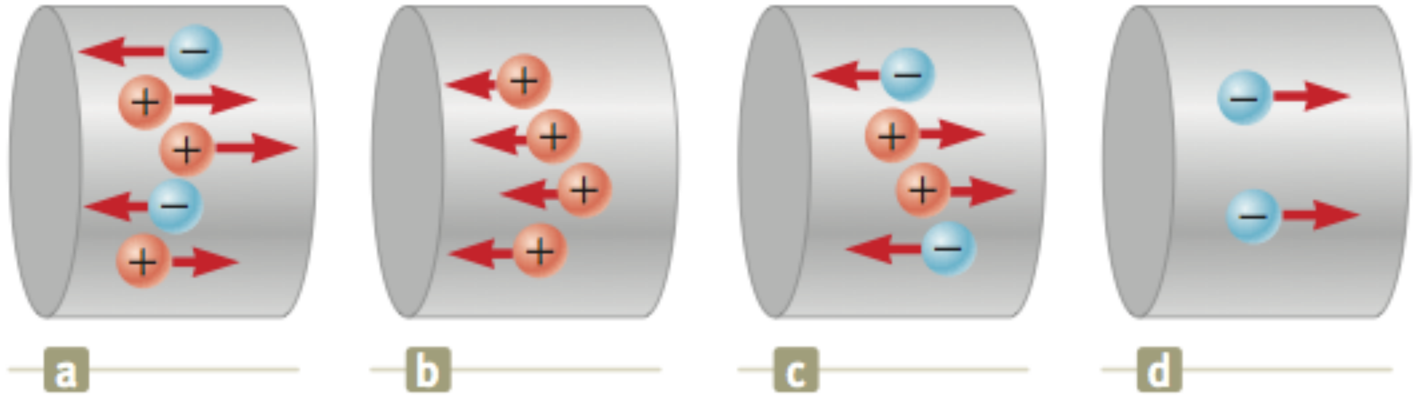
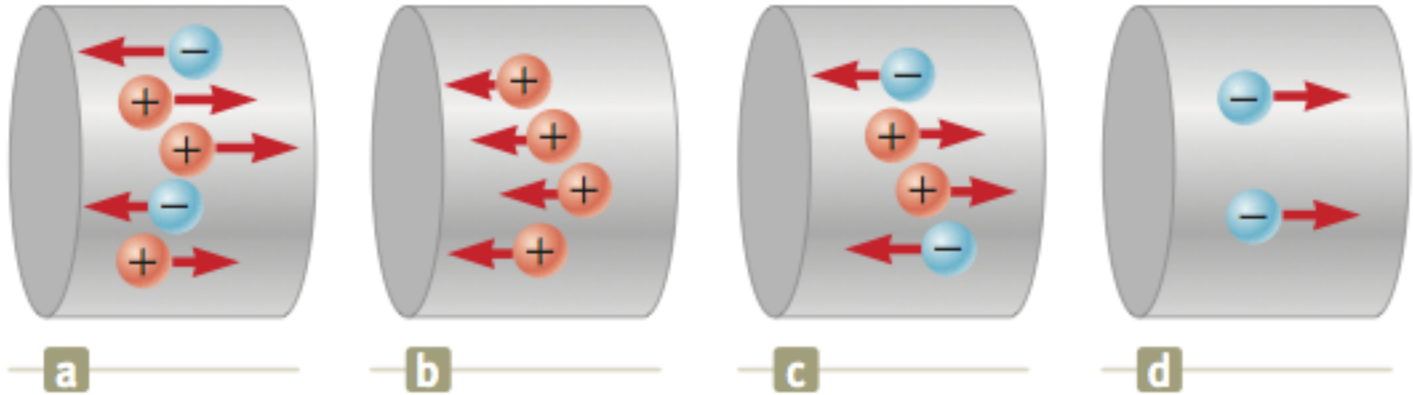


Right Left Right Left

Current direction: Direction positive charges would flow….

Analysis #2: Rank the current in these four situations from highest to lowest. Assume positive and negative carriers have the same drift velocity.



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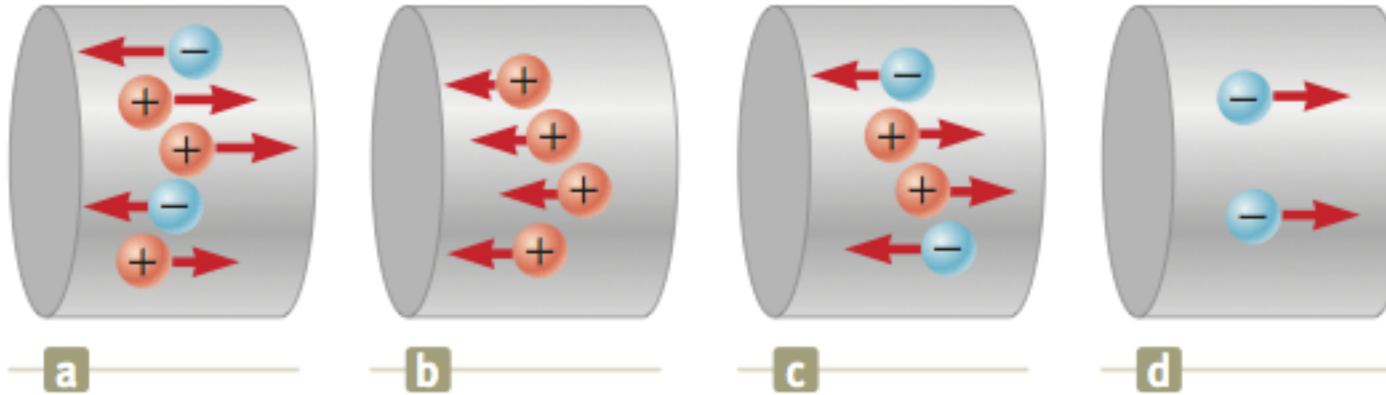

$$I = \frac{dQ}{dt}$$

1) a (Highest I)
2) b=c
3) d

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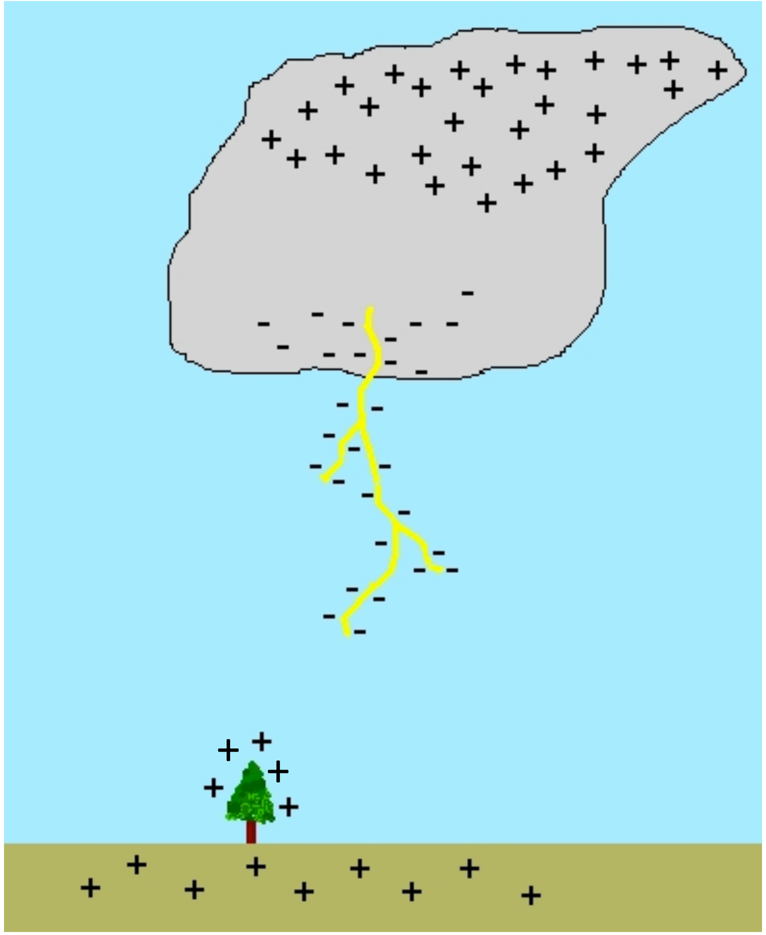

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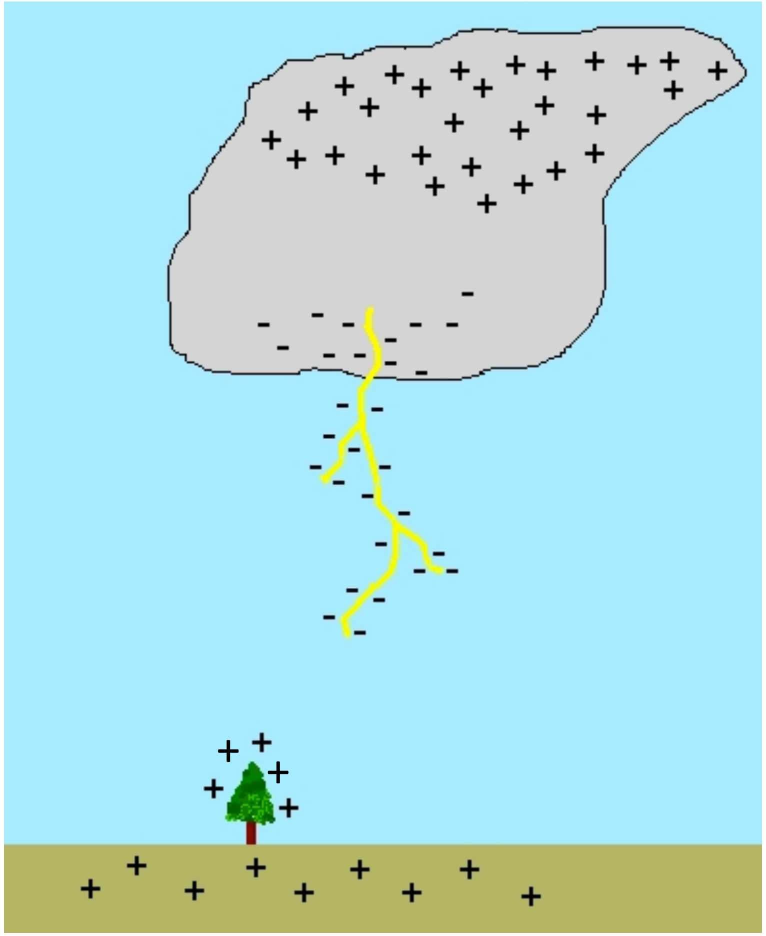
$$I = 25{,}000\ A$$
$$\Delta t = 40\times10^{-6} s$$



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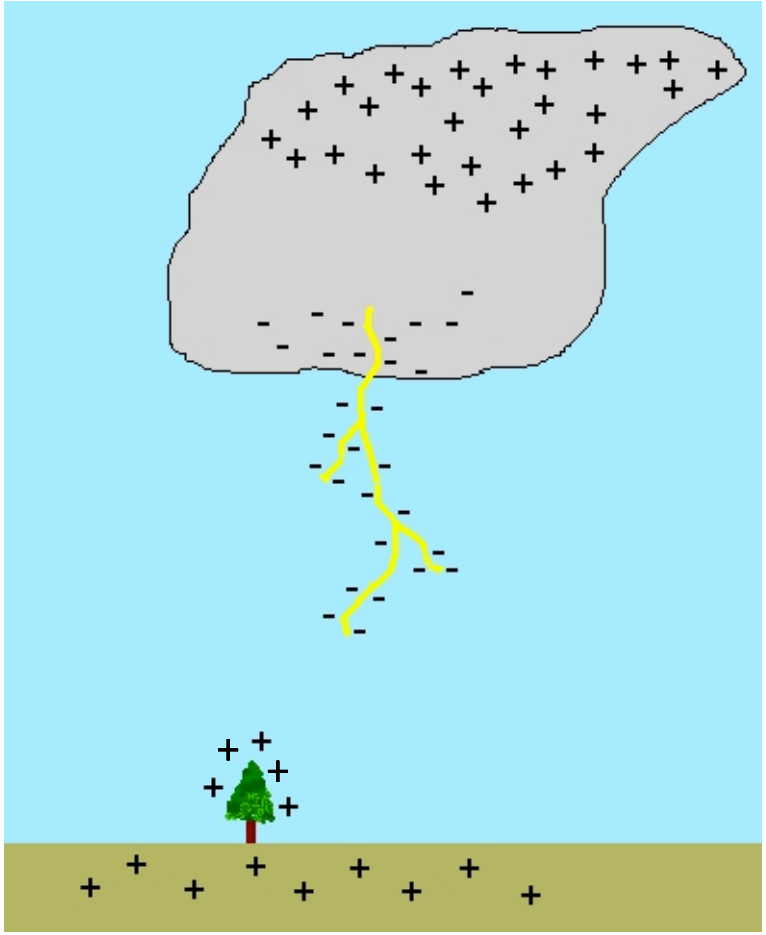

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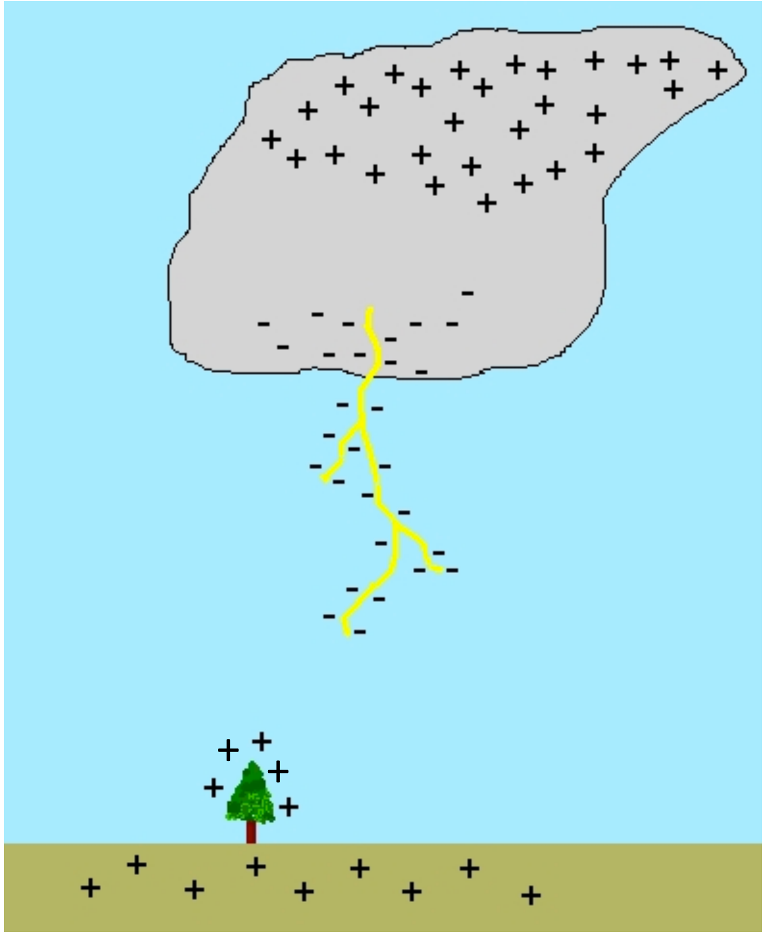

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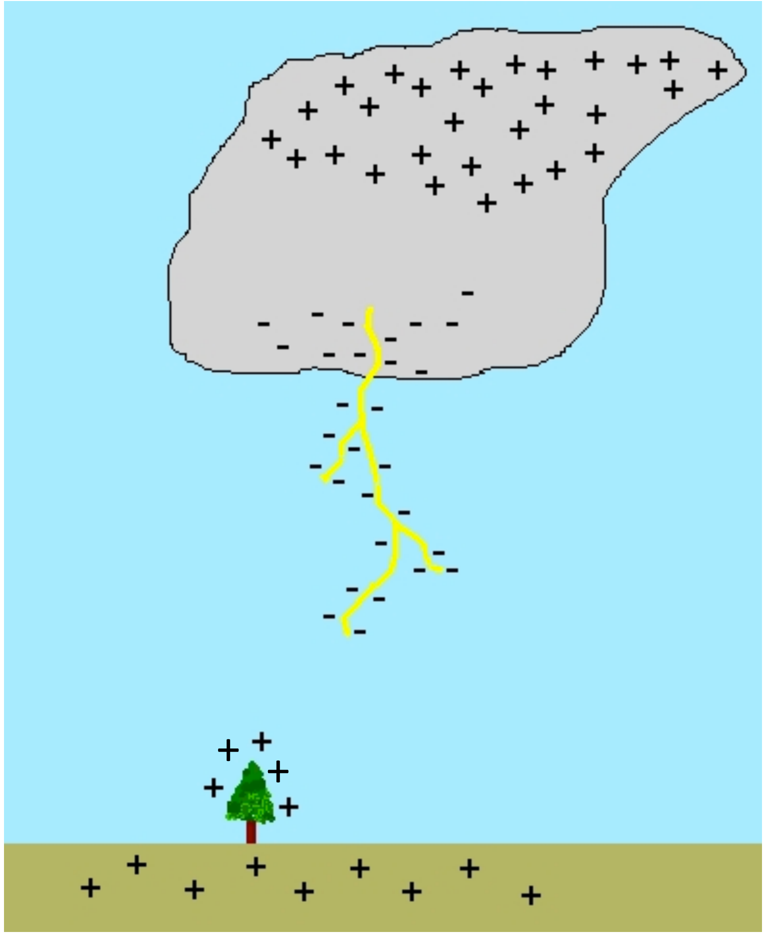

$$I = \frac{\Delta Q}{\Delta t}$$

$$\Delta Q = I\Delta t$$

$$\Delta Q = (25000)(40\times10^{-6})$$

$$\Delta Q = 1\ C$$

Electric Current Density (J)

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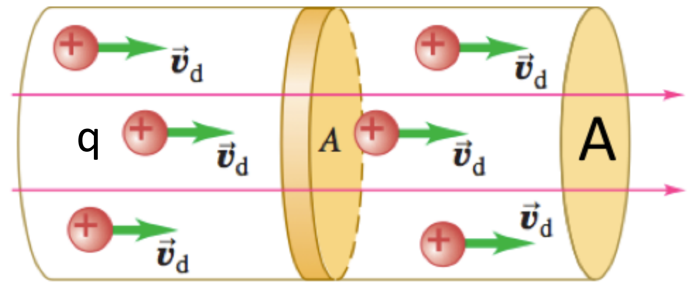
$$I = \frac{dQ}{dt} = |q|nAv_d$$



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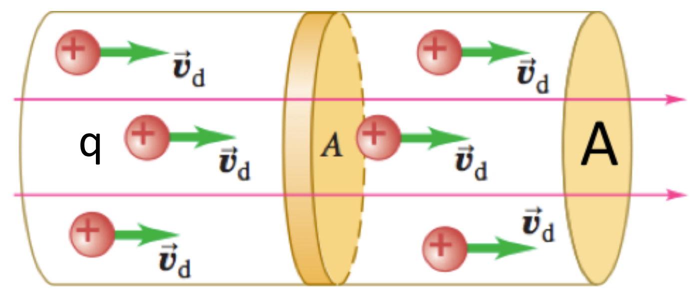
$$Units: C/(m^2 s) = A/m^2$$



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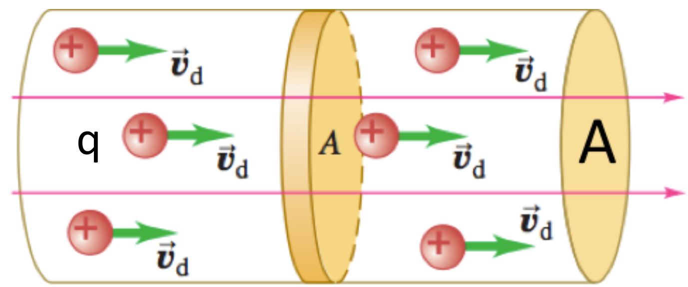

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Vector current density:

$$\vec{J} = qn\vec{v}_d$$

Note that q takes care of the right direction!

Example: An 18-gauge copper wire (diameter 1.02mm) carries a current with a current density of $1.60\times10^6 A/m^2$. Copper has 8.5×10^{28} free electrons per cubic meter. (a) Calculate the current in the wire. (b) Calculate the drift velocity of electrons in the wire.

$$A = \pi(0.51\times10^{-3})^2 = 8.17\times10^{-7}m^2$$

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$$q_e = 1.60\times10^{-19}\text{ C}$$

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$$(b)\ \ v_d = ?$$

$$J = |q| n v_d$$

$$v_d = \frac{J}{|q|n} = \frac{1.60\times10^6}{(1.60\times10^{-19})(8.50\times10^{28})}$$

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$$n = 8.50\times10^{28}\ m^{-3}$$

$$q_e = 1.60\times10^{-19}\ \mathrm{C}$$

$$I = \frac{dQ}{dt} = |q| n v_d A$$

$$J = \frac{I}{A} = |q| n v_d$$

$$(a)\ I = ?$$

$$I = JA$$

$$I = (1.60\times10^6)(8.17\times10^{-7})$$

$$I = 1.31\ A$$

$$(b)\ v_d = ?$$

$$J = |q| n v_d$$

$$v_d = \frac{J}{|q|n} = \frac{1.60\times10^6}{(1.60\times10^{-19})(8.50\times10^{28})}$$

$$v_d = 1.17\times10^{-4}\ \mathrm{m/s}$$

Resistivity

Resistivity

- We saw that:

$$\vec{J} = nq\vec{v}_d$$

- For many materials was also found, experimentally, that:

$$\vec{J} = \frac{\vec{E}}{\rho}$$

Ohm's Law

Ohm's Law

$$\vec{J} = \frac{\vec{E}}{\rho}$$


Georg Ohm
(1789-1854)

- ρ = resistivity of the conducting material
- Not really an universal law. Idealized model that is valid for some materials (ohmic or linear).
- Unit: Ωm (read ohm-meter)

  Where V/A = Ω = ohm
- Conductivity = 1/ρ

Resistivity

Table 25.1 Resistivities at Room Temperature (20°C)

	Substance	ρ ($\Omega \cdot m$)		Substance	ρ ($\Omega \cdot m$)
Conductors			**Semiconductors**		
Metals	Silver	1.47×10^{-8}		Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}		Pure germanium	0.60
	Gold	2.44×10^{-8}		Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**		
	Tungsten	5.25×10^{-8}		Amber	5×10^{14}
	Steel	20×10^{-8}		Glass	10^{10}–10^{14}
	Lead	22×10^{-8}		Lucite	$>10^{13}$
	Mercury	95×10^{-8}		Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}		Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}		Sulfur	10^{15}
	Nichrome	100×10^{-8}		Teflon	$>10^{13}$
				Wood	10^{8}–10^{11}

Resistance

Resistance

$$\left|\vec{J}\right| = \frac{|\vec{E}|}{\rho} \longrightarrow \text{Resistivity } [\Omega\text{m}]$$

Resistance

$$\left|\vec{J}\right| = \frac{|\vec{E}|}{\rho} \longrightarrow$$ Resistivity [Ωm]

$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{L}$$

Resistance

$$|\vec{J}| = \frac{|\vec{E}|}{\rho}$$ $\longrightarrow$ Resistivity [Ωm]

$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{L}$$

$$V = \left(\frac{\rho L}{A}\right) I$$

Resistance

$$|\vec{J}| = \frac{|\vec{E}|}{\rho}$$ Resistivity [Ωm]

$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{L}$$

$$V = \left(\frac{\rho L}{A}\right) I$$

We can then define the following:

$$R = \frac{\rho L}{A}$$ Resistance [Unit: ohms]

$$V = RI$$ Ohm's Law