PHYS2326
Lecture #5

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Department of Physics
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Goals for this Lecture

- Quick reminder
- Continue our tour on electric field calculation
  - Discrete charge distribution: electric dipole
  - Continuous (homogeneous) charge distribution
- Understand force and torque on electric dipole

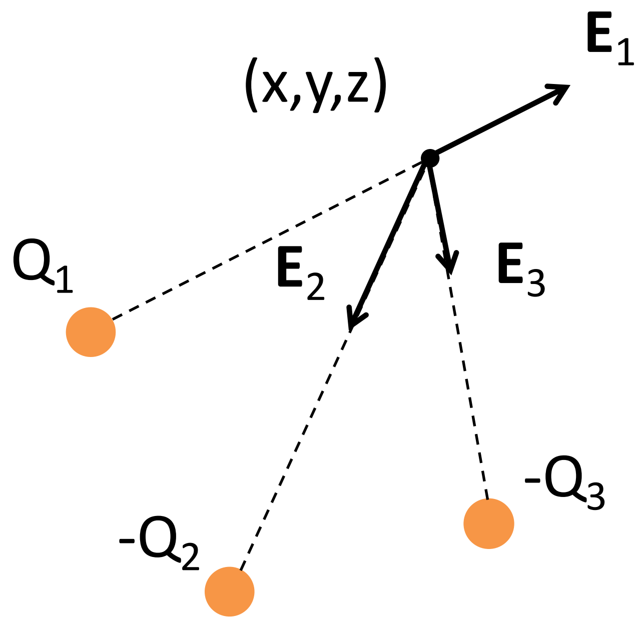
Reminder: E-Field Calculations

Electric Field Calculations

- Discrete distribution of charges
- Continuous distribution of charges

Electric Field Calculations

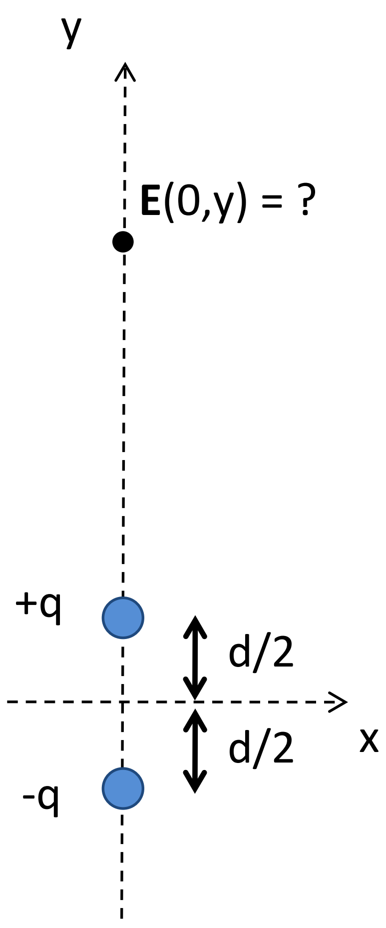
- Electric field produced by a discrete distribution of point charges (Superposition principle)



$$\vec{E}(x,y,z) = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

$$\vec{E}(x,y,z) = \sum_{i=1}^{N} \vec{E}_i$$

Electric Dipole



$$\vec{E}(y) = ?$$

$$\vec{E}(y) = \vec{E}_{-q}(y) + \vec{E}_{+q}(y)$$

$$\vec{E}(y) = \frac{1}{4\pi\epsilon_0}\frac{q}{r_{+q}^2}\hat{y} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_{-q}^2}\hat{y}$$

$$\vec{E}(y) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\left(y-\frac{d}{2}\right)^2} - \frac{1}{\left(y+\frac{d}{2}\right)^2}\right]\hat{y}$$

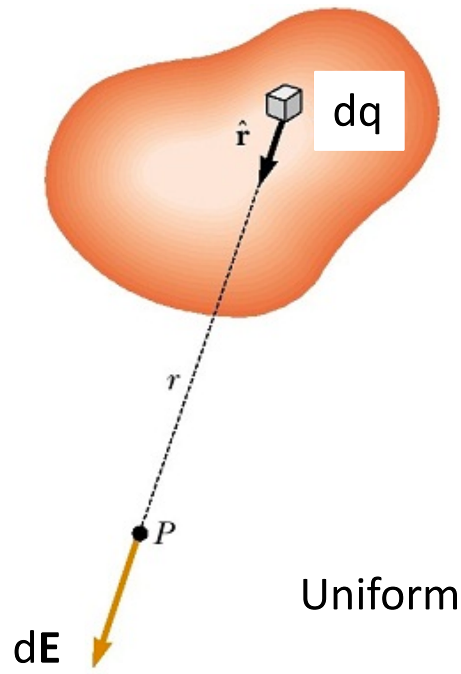
$$\vec{E}(y) = \frac{q}{4\pi\epsilon_0 y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]\hat{y}$$

$$for \quad y \gg d:$$

$$\vec{E}(y) = \frac{qd}{2\pi\epsilon_0 y^3}\hat{y}$$

Electric Field Calculations

- Electric field produced by a uniform distribution of point charges

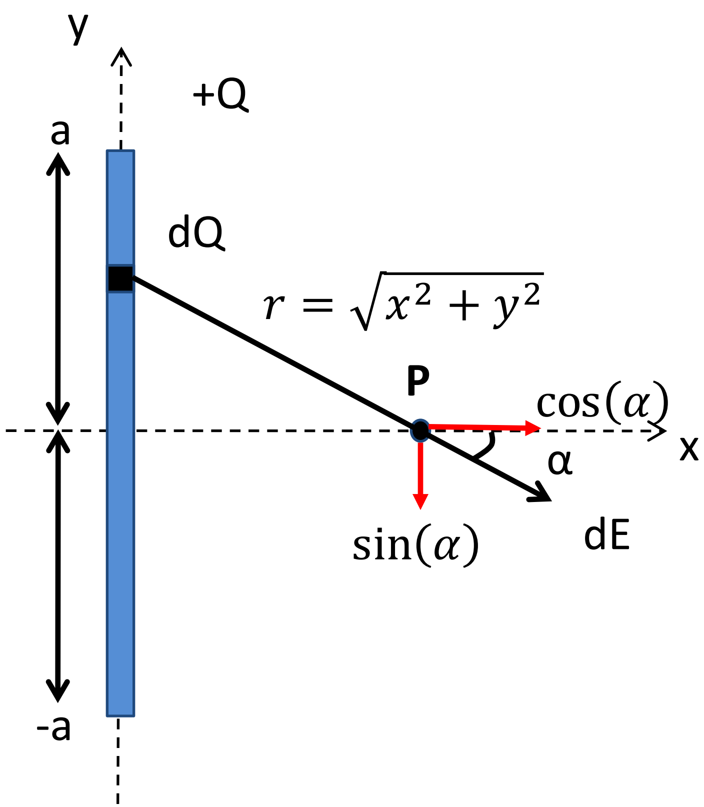


$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$

Uniform: The charge density is the same in any point of the object

Line of Charge



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$

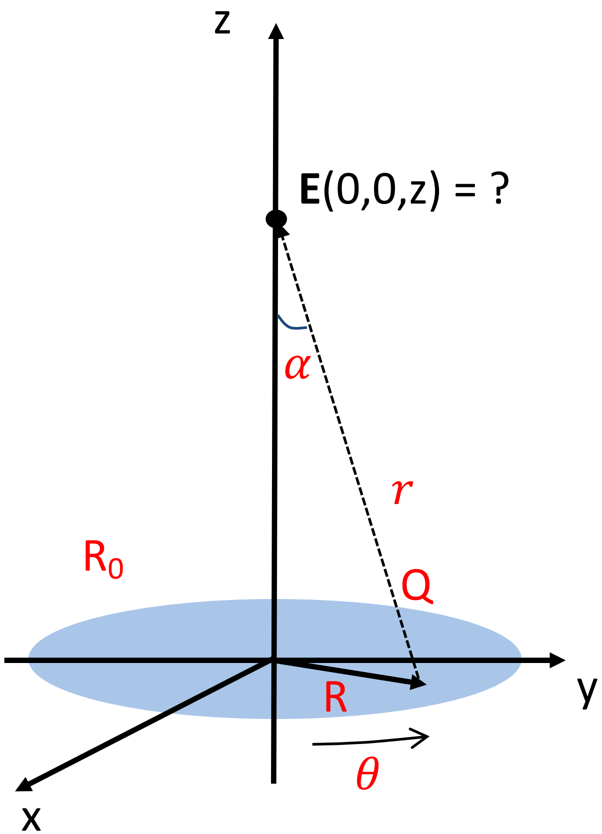
$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}\int_{-a}^{a}\frac{dy}{(x^2+y^2)}\left[\frac{x}{\sqrt{x^2+y^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2}}\hat{y}\right]$$

$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}\int_{-a}^{a}\frac{dy}{(x^2+y^2)^{3/2}}[x\hat{x} + y\hat{y}]$$

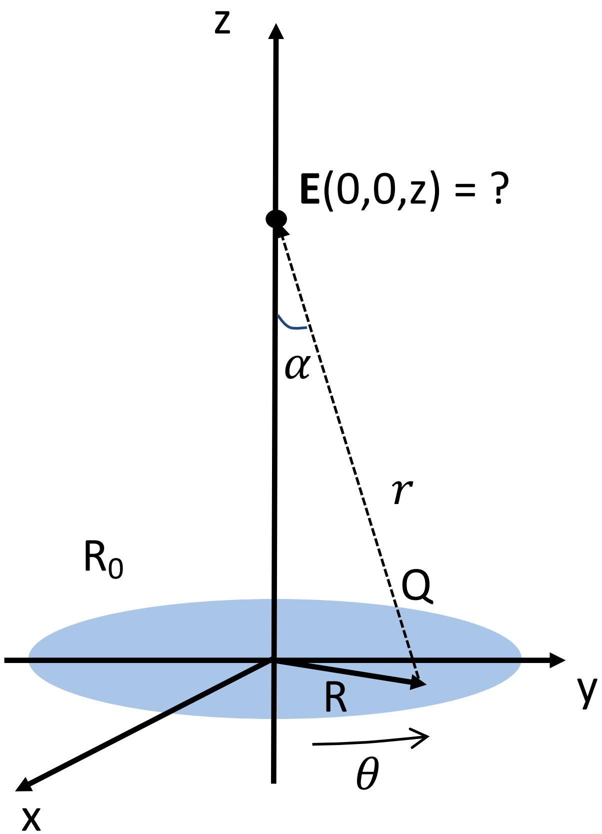
$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}\left[x\int_{-a}^{a}\frac{dy}{(x^2+y^2)^{3/2}}\hat{x} + \int_{-a}^{a}\frac{ydy}{(x^2+y^2)^{3/2}}\hat{y}\right]$$

$$\vec{E}(x,0) = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(a^2+x^2)^{1/2}}\hat{x}$$

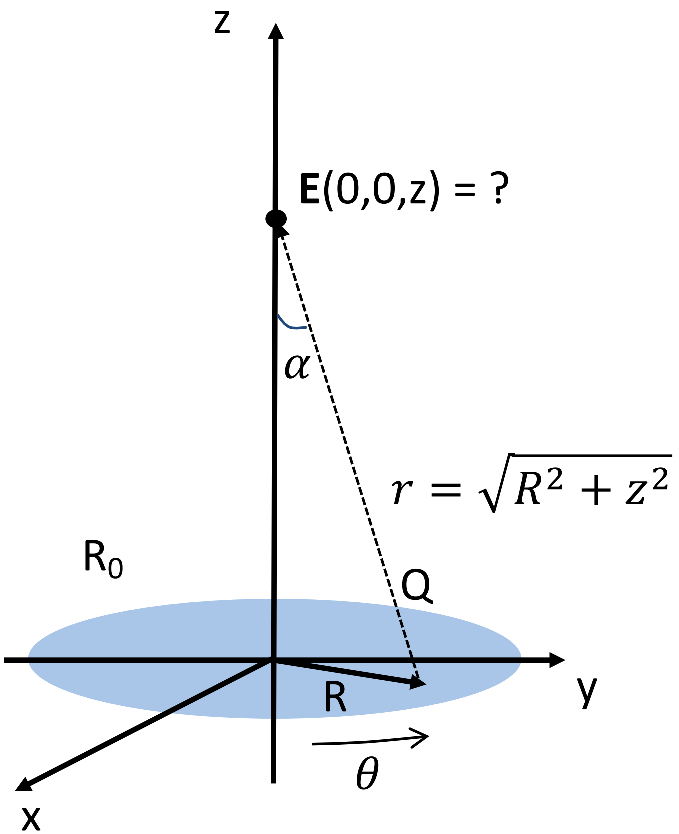
Uniformly Charged Disk



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\hat{r}$$



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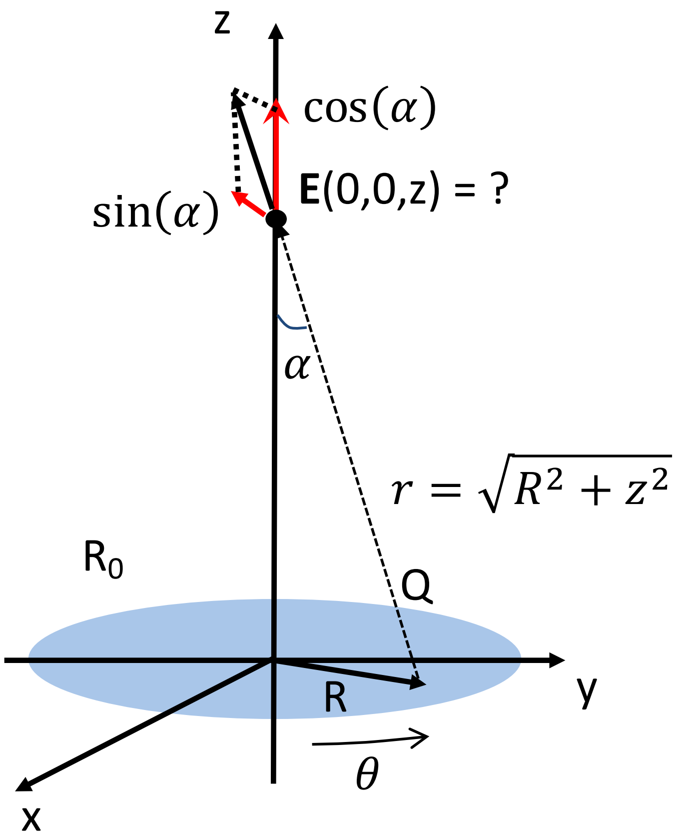

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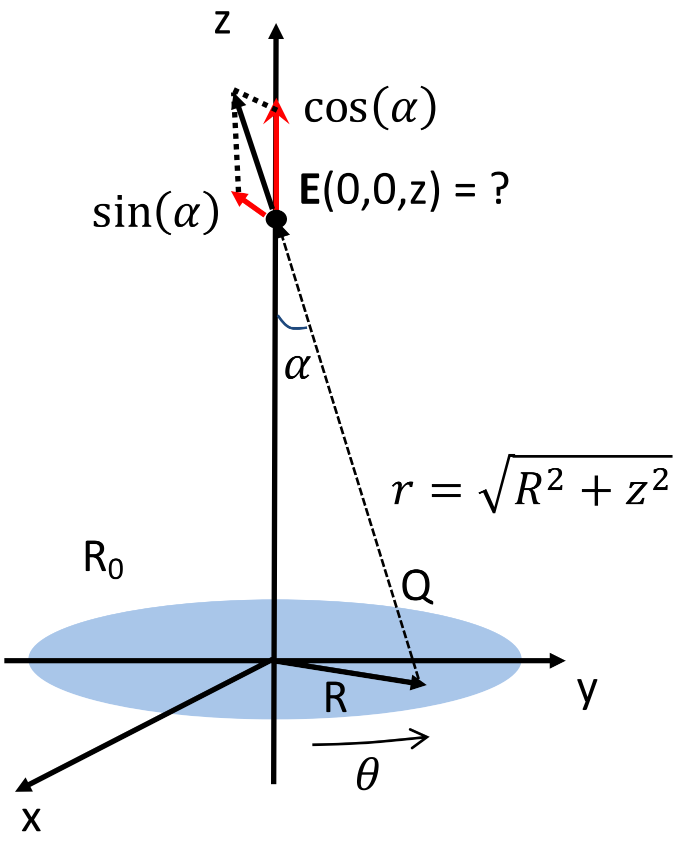

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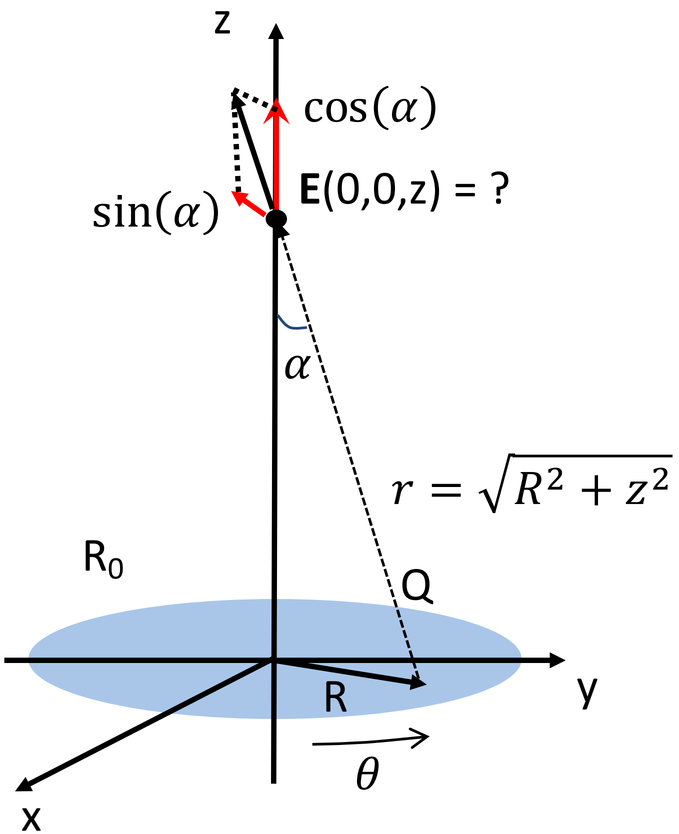
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\left[\frac{z}{(R^2+z^2)^{1/2}}\right]\hat{z}$$



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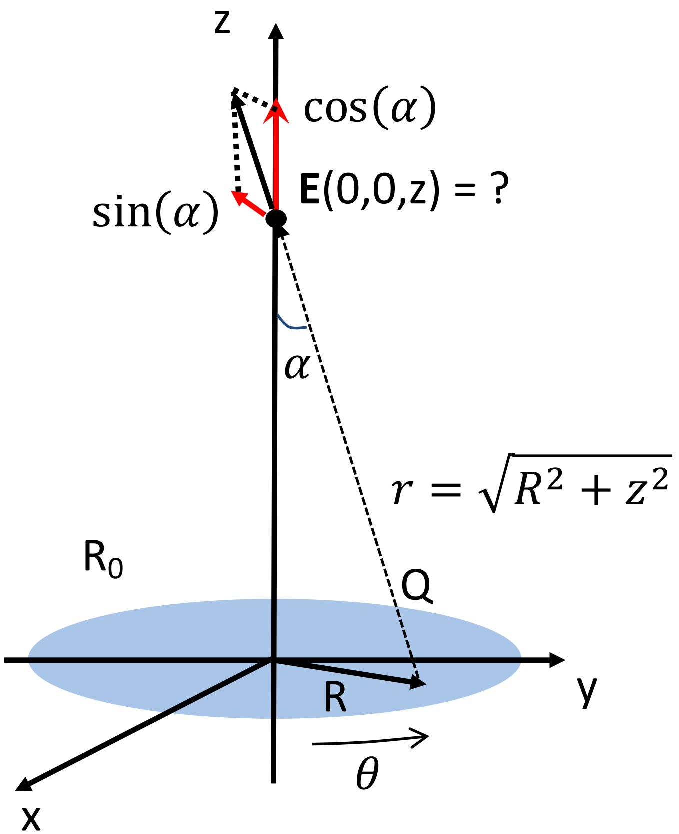

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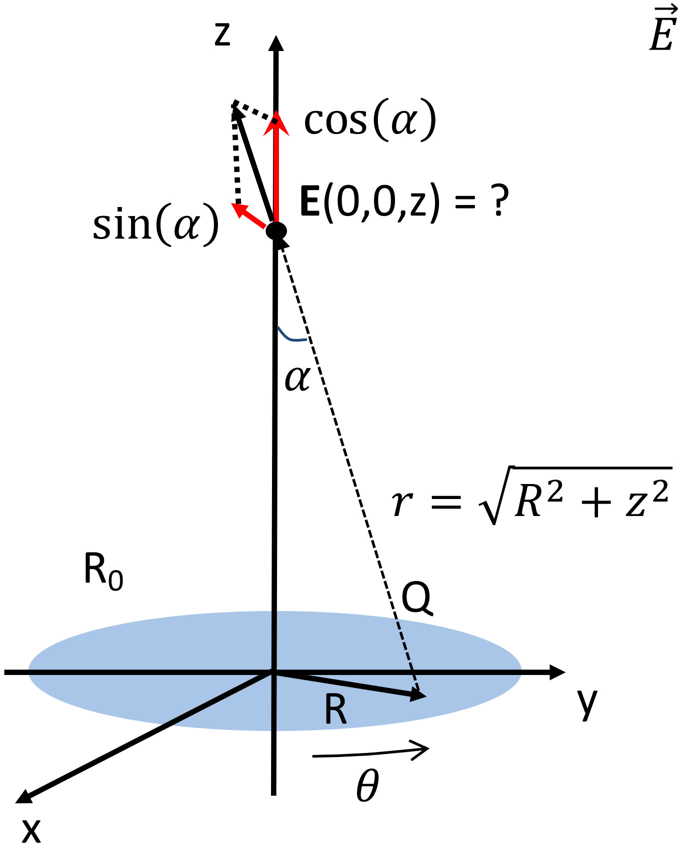
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{z\sigma dA}{(R^2+z^2)^{3/2}}$$



$\sigma = Q/A$

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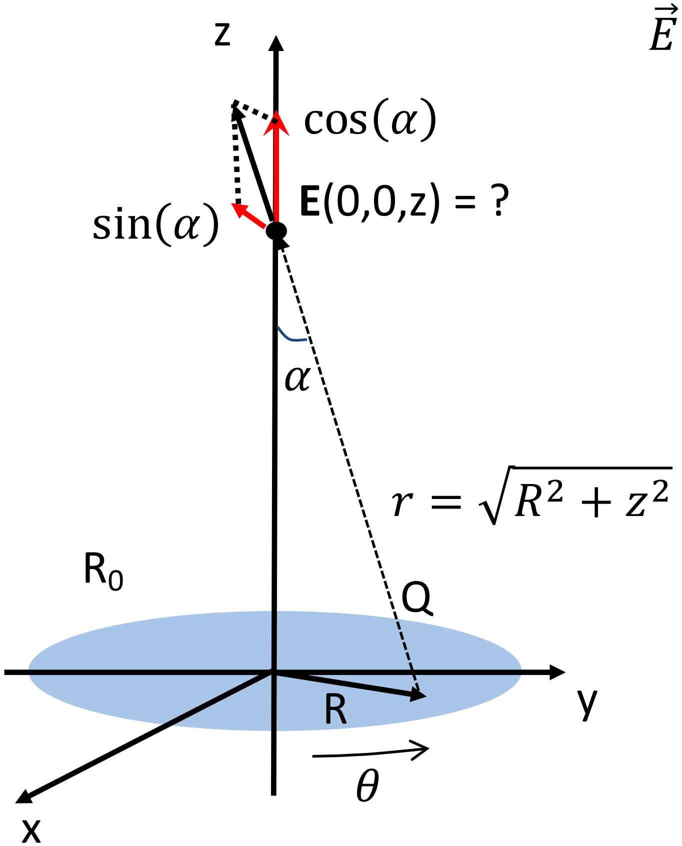

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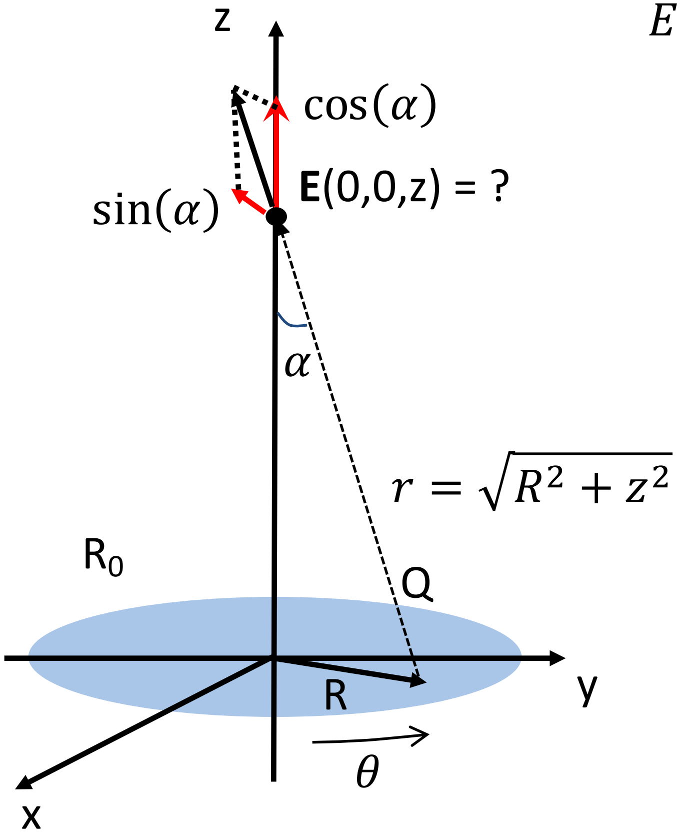

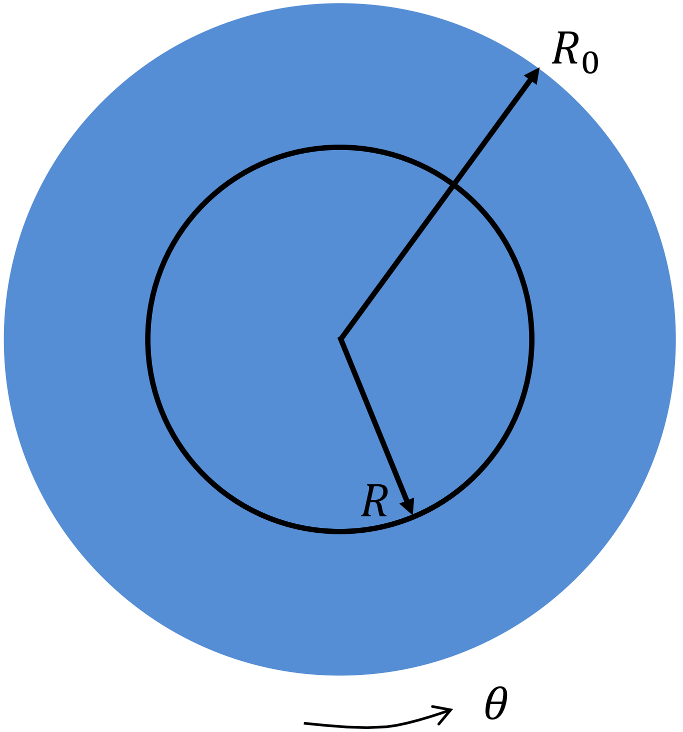

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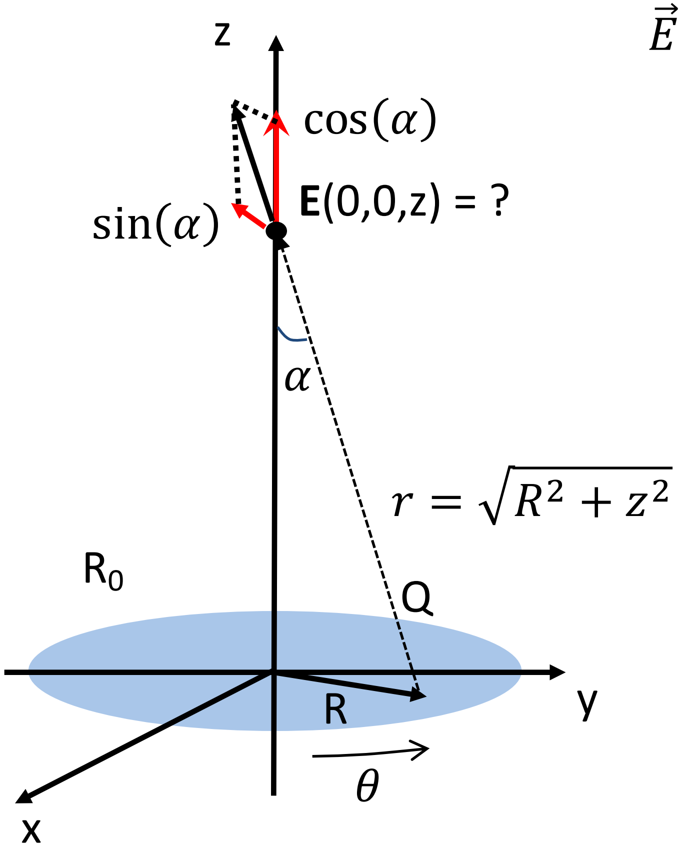

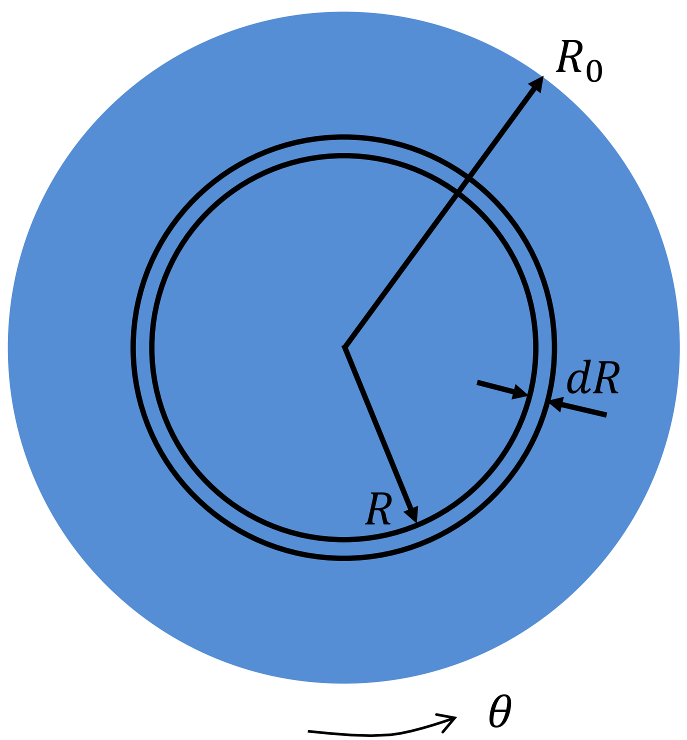

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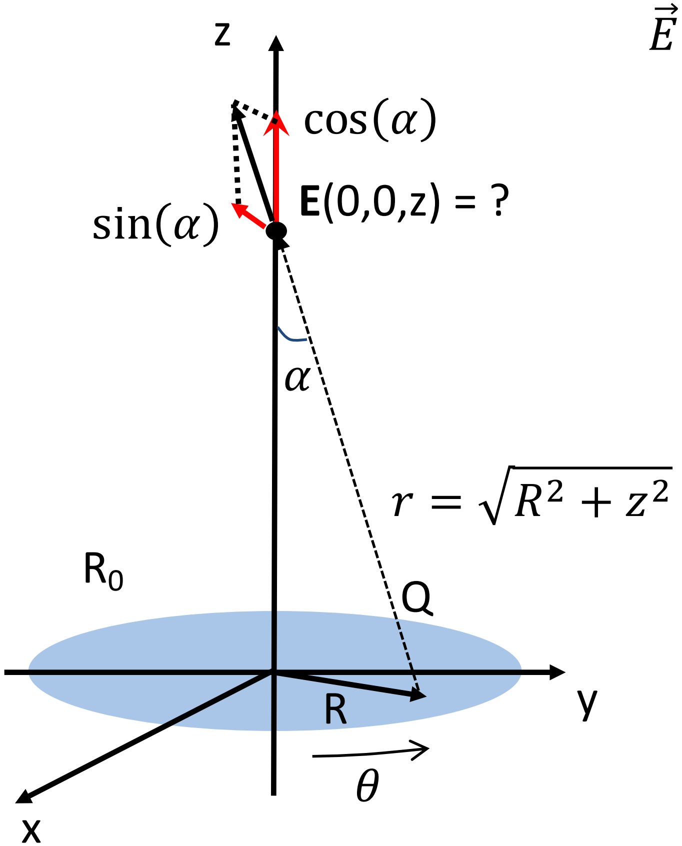

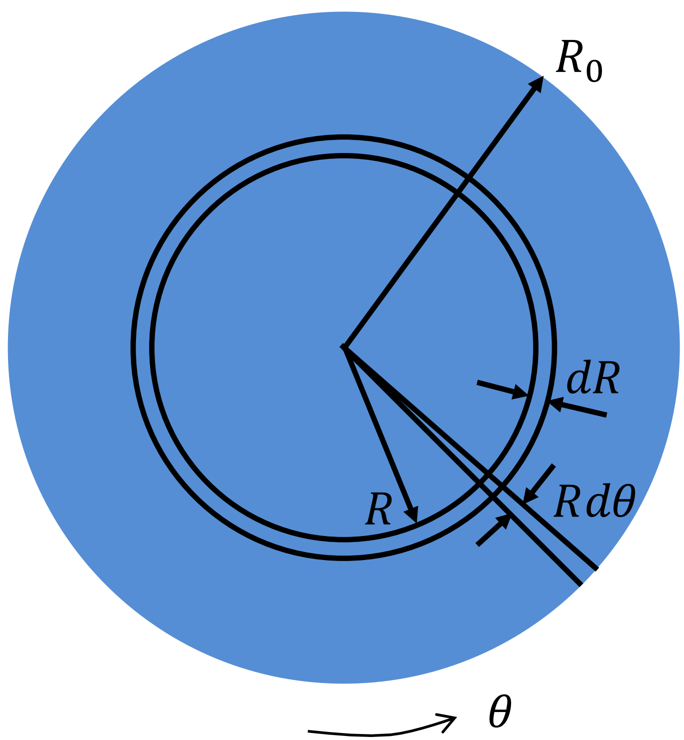

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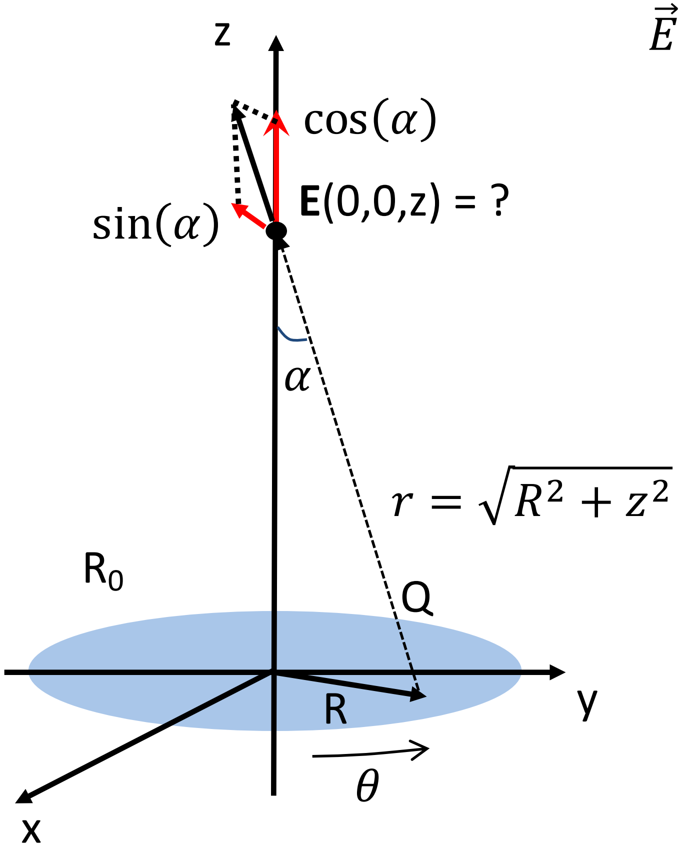

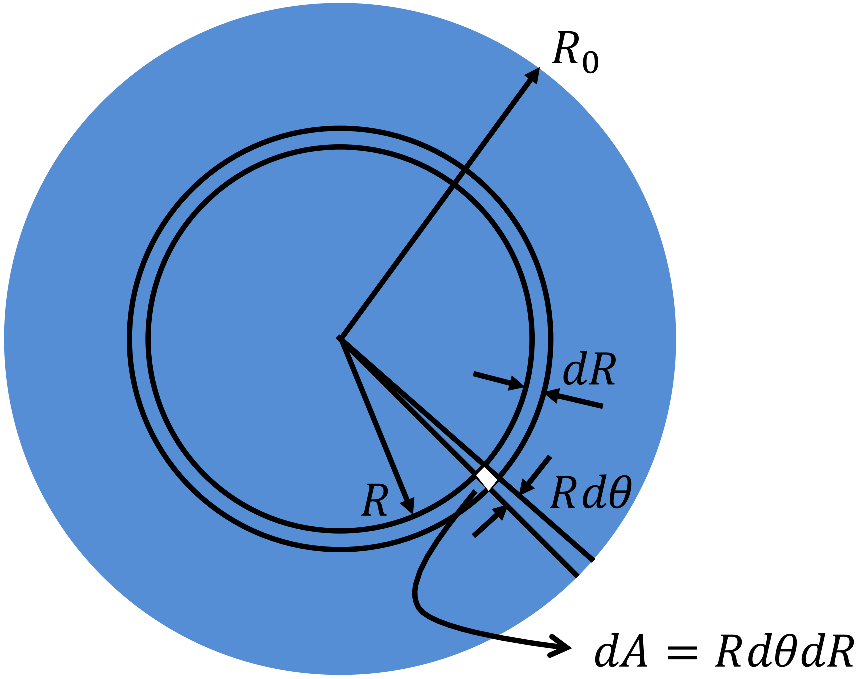

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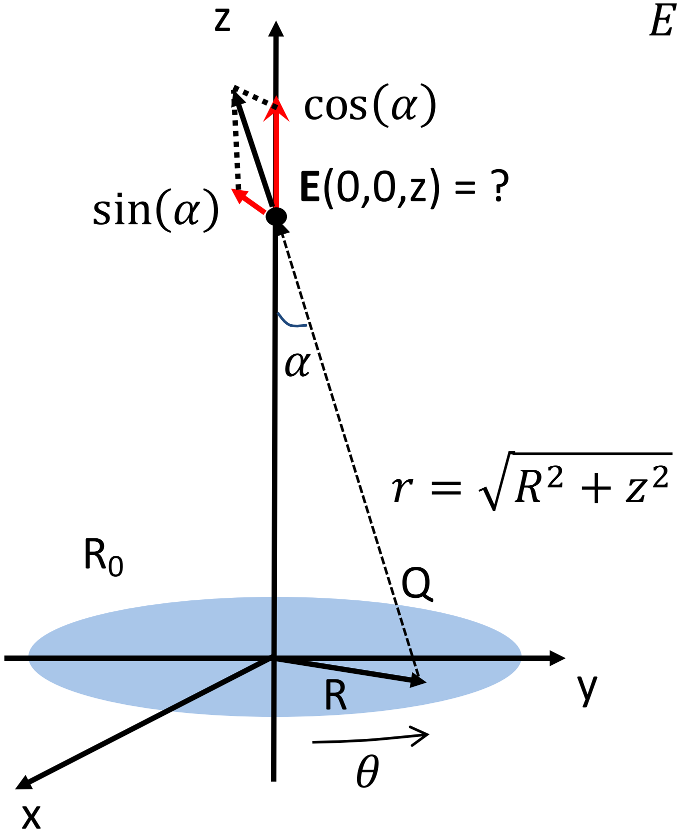

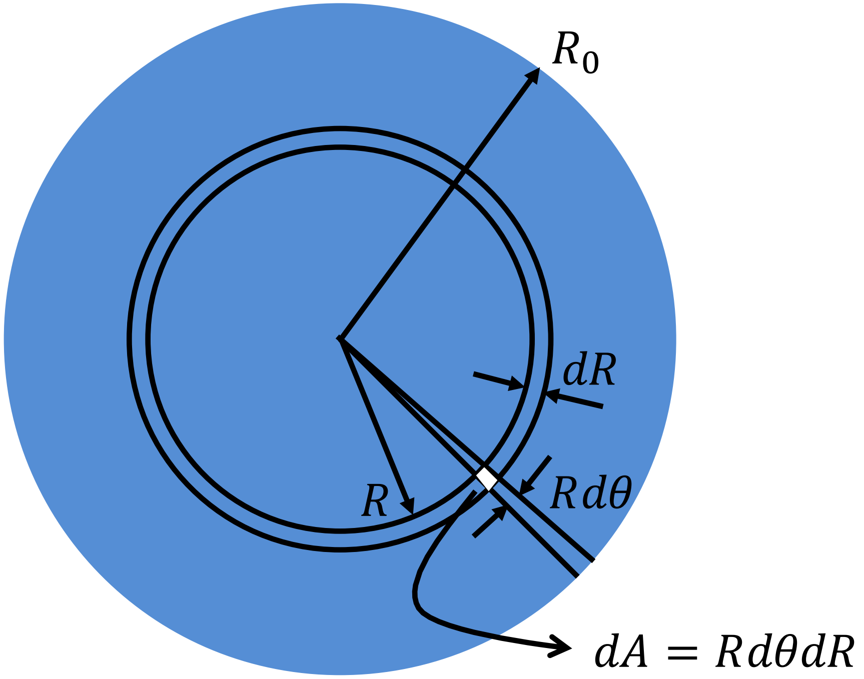

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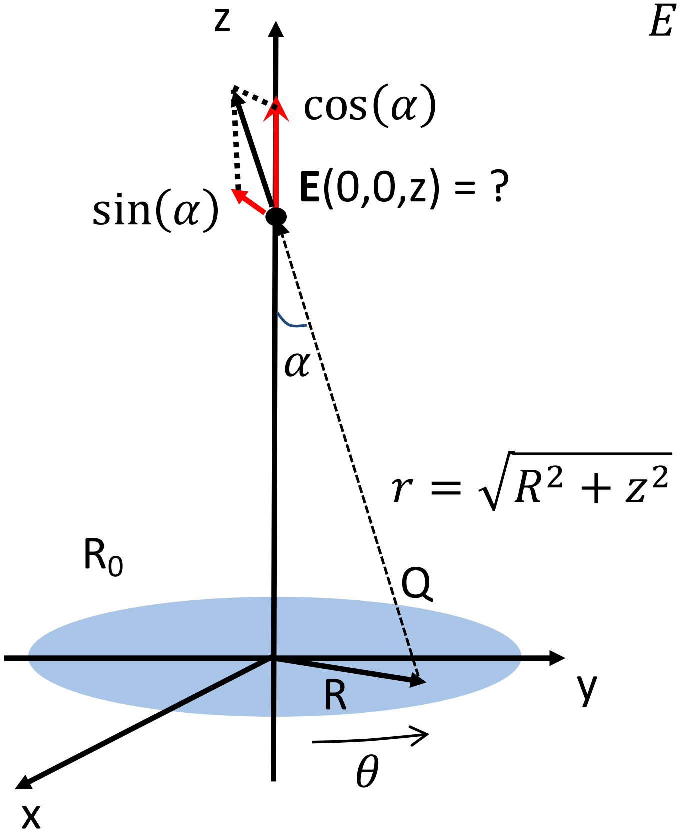

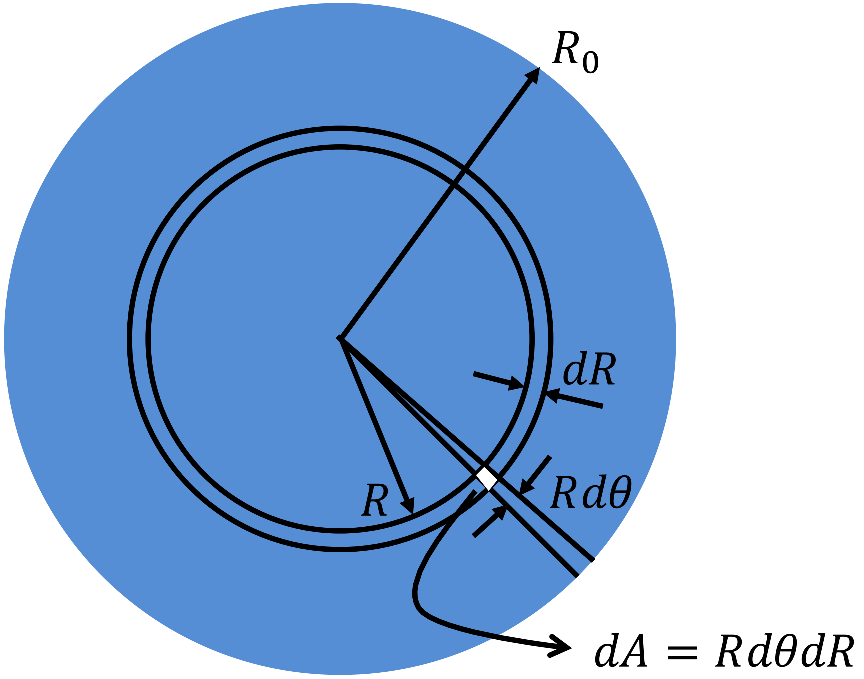

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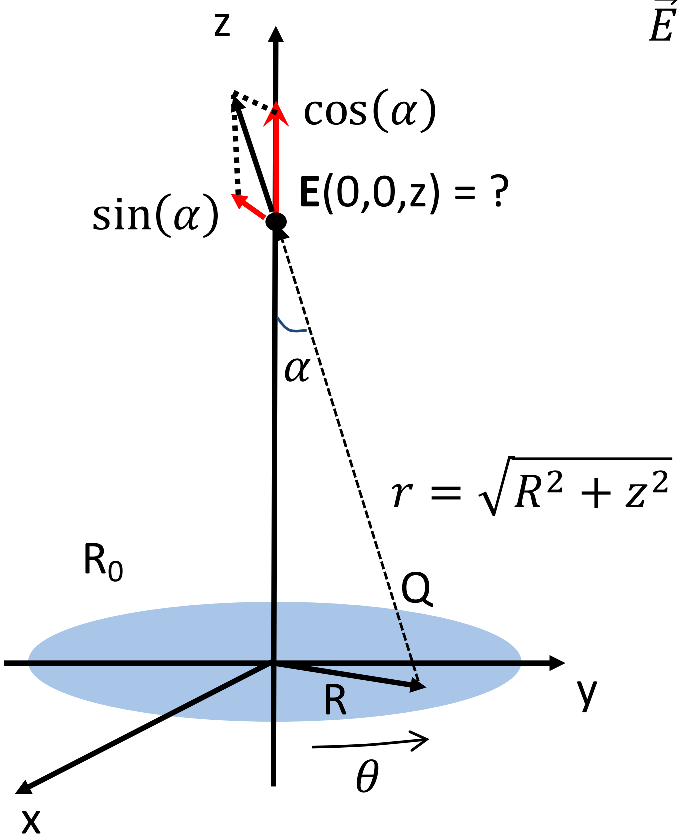

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{z\sigma\,dA}{(R^2+z^2)^{3/2}}\hat{z} = \frac{1}{4\pi\epsilon_0}\int_0^{R_0}\int_0^{2\pi} \frac{z\sigma R\,d\theta\,dR}{(R^2+z^2)^{3/2}}\hat{z}$$

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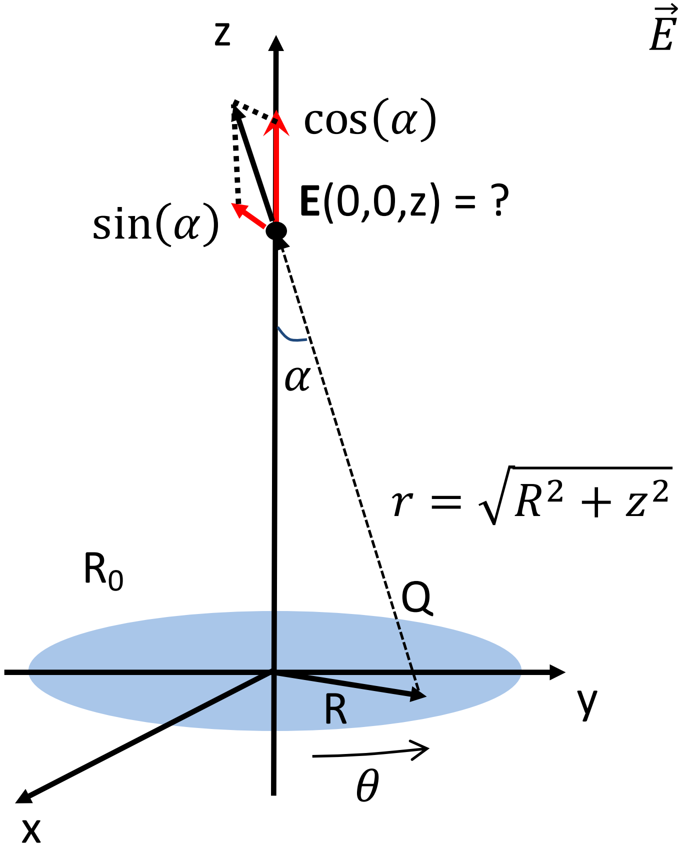

$$\vec{E} = \frac{z\sigma}{4\pi\epsilon_0}\int_0^{2\pi} d\theta \int_0^{R_0} \frac{RdR}{(R^2+z^2)^{3/2}}\hat{z}$$

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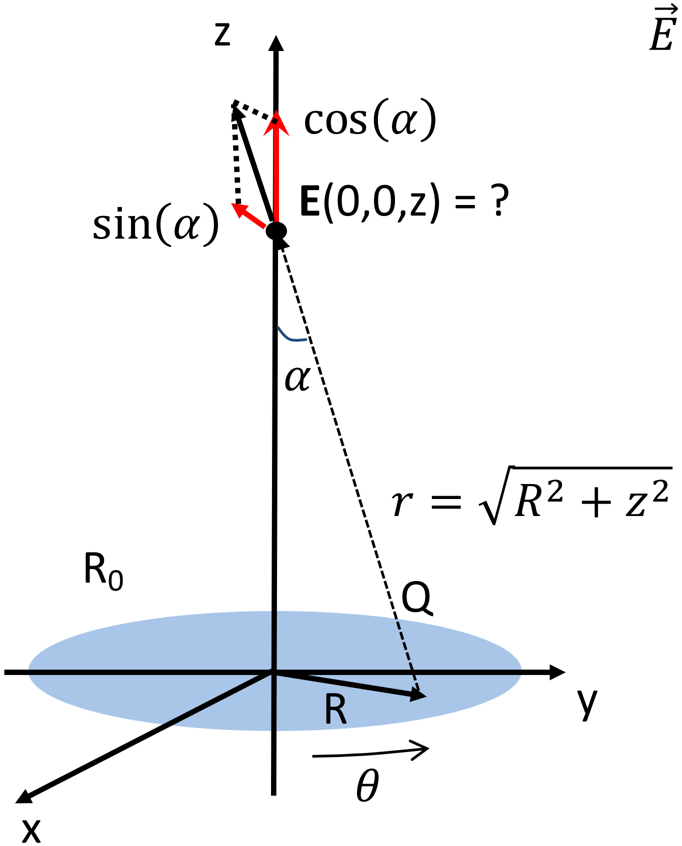

$$\vec{E} = \frac{z\sigma}{4\pi\epsilon_0}\int_0^{2\pi} d\theta \int_0^{R_0} \frac{RdR}{(R^2+z^2)^{3/2}}\hat{z}$$

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$$\int \frac{ydy}{(y^2+a^2)^{3/2}} = -\frac{1}{(y^2+a^2)^{1/2}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\cos(\alpha)\,\hat{z}$$

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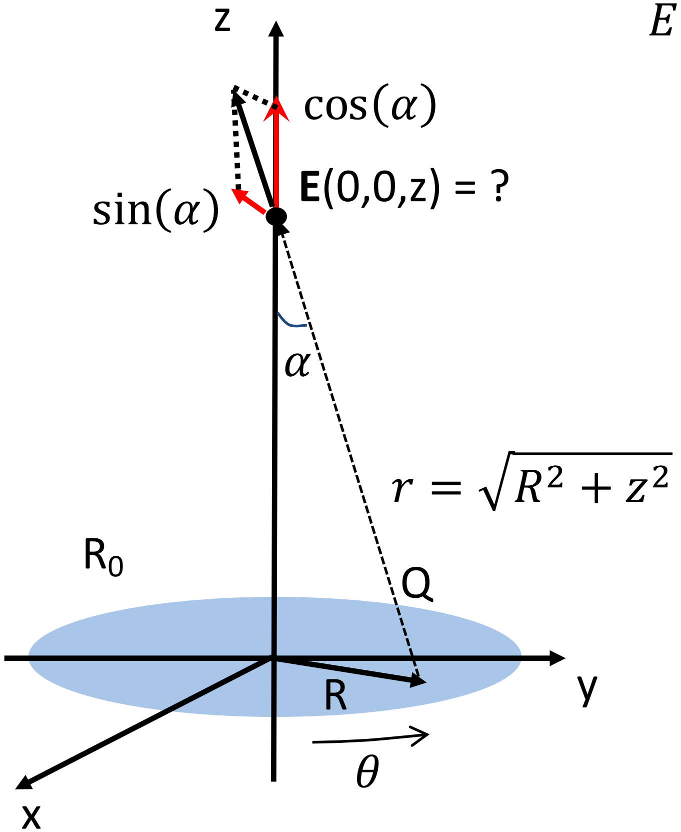

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{z\sigma dA}{(R^2+z^2)^{3/2}}\hat{z} = \frac{1}{4\pi\epsilon_0}\int_0^{R_0}\int_0^{2\pi} \frac{z\sigma R d\theta dR}{(R^2+z^2)^{3/2}}\hat{z}$$

$$\vec{E} = \frac{z\sigma}{4\pi\epsilon_0}\int_0^{2\pi} d\theta \int_0^{R_0} \frac{RdR}{(R^2+z^2)^{3/2}}\hat{z}$$

$$\vec{E} = \frac{z\sigma}{4\pi\epsilon_0}(2\pi)\int_0^{R_0} \frac{RdR}{(R^2+z^2)^{3/2}}\hat{z}$$

$$\vec{E} = \frac{z\sigma}{4\pi\epsilon_0}(2\pi)\left[-\frac{1}{(R^2+z^2)^{1/2}}\right]_0^{R_0}\hat{y}$$

$$\int \frac{ydy}{(y^2+a^2)^{3/2}} = -\frac{1}{(y^2+a^2)^{1/2}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\hat{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{(R^2+z^2)}\cos(\alpha)\,\hat{z}$$

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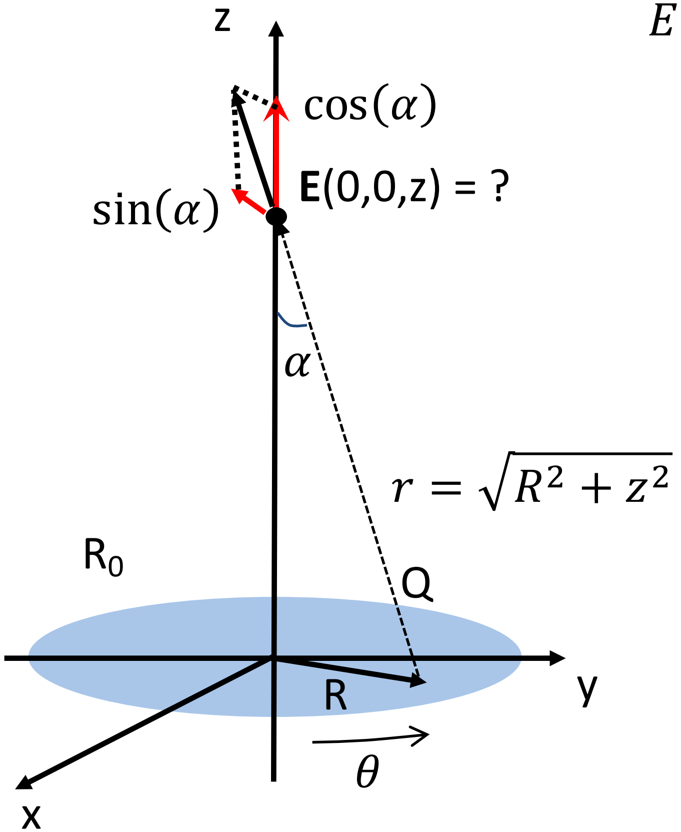

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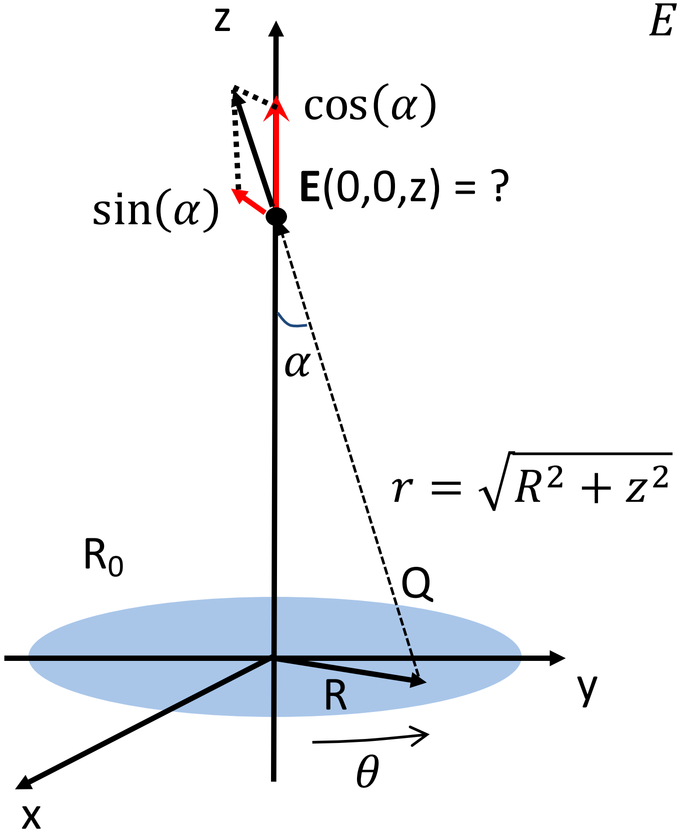

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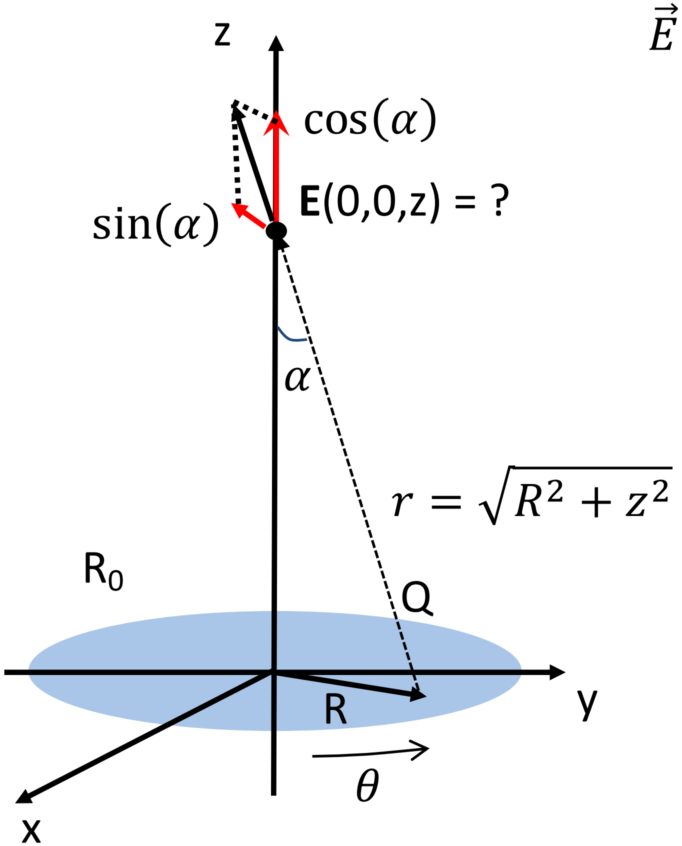

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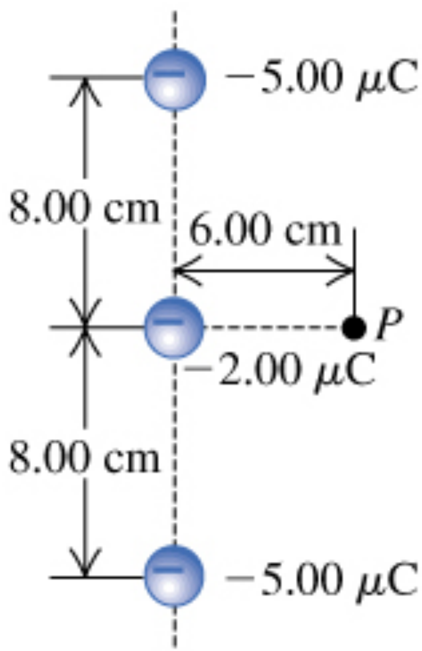
$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\left[1-\frac{z}{(R_0^2+z^2)^{1/2}}\right]\hat{y}}$$

Electric Field Calculations

Classical charge distributions:

- Line of charge
- Ring of charge (see book)
- Uniformly charged disk
- Parallel plates (see book)

Example: Three negative point charges lie along a line as shown in the figure. Find the magnitude of the electric field this combination of charges produces at point P, which lies 6.00 cm from the −2.00μC charge measured perpendicular to the line connecting the three charges. **Find the magnitude of the electric field this combination of charges produces at point P**.



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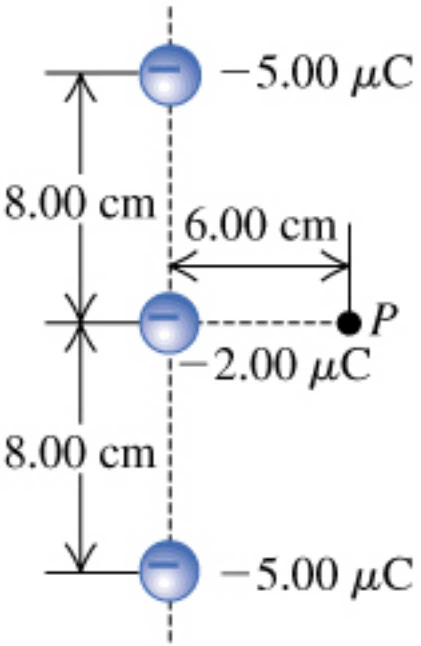

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$|\vec{E}| = ?$$

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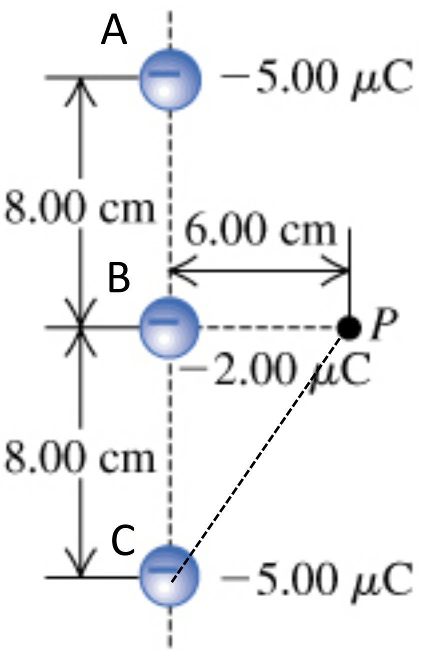

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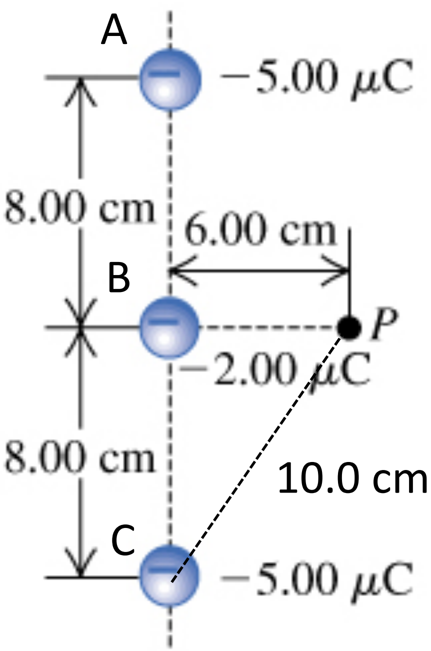

$$r = \sqrt{8.00^2 + 6.00^2} = 10.0\ cm$$

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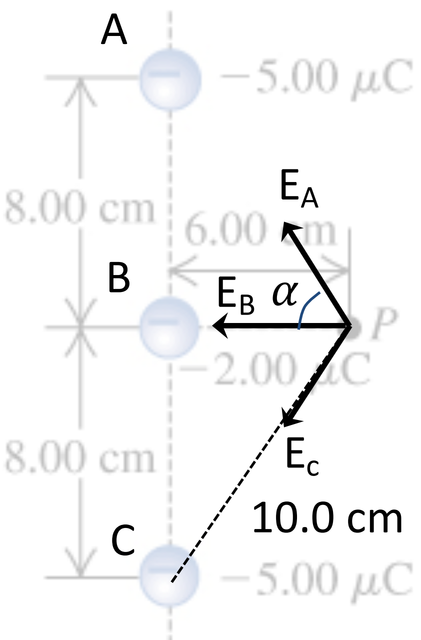

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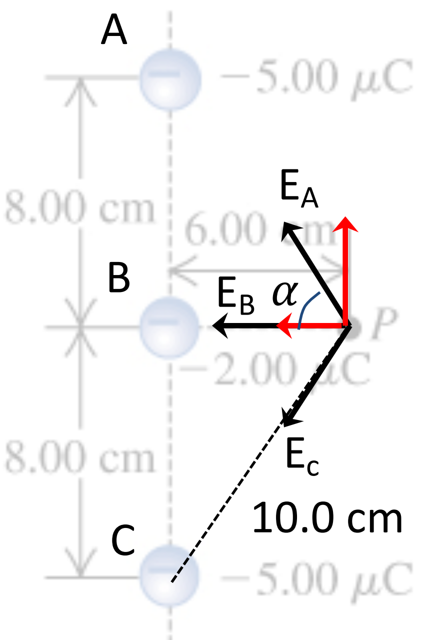

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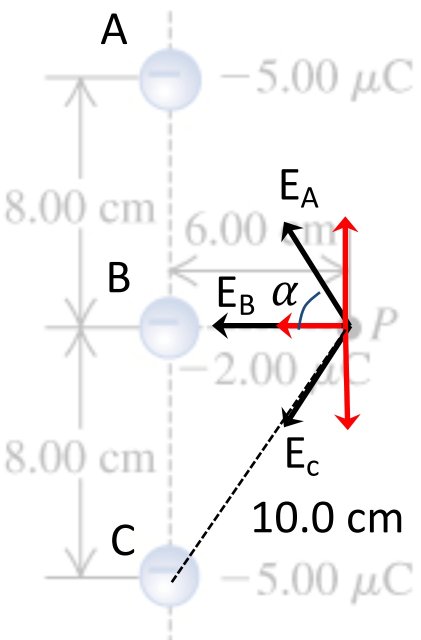

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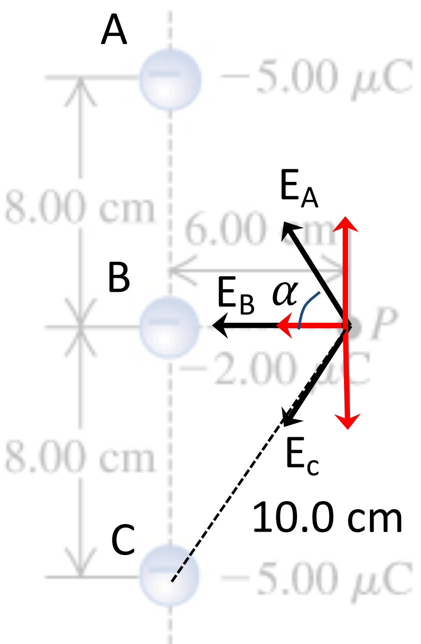

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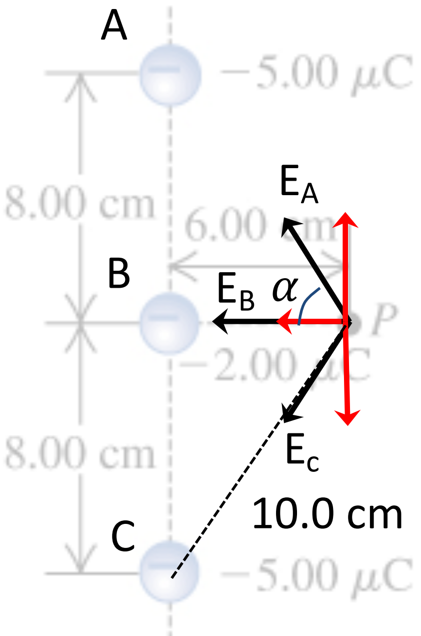

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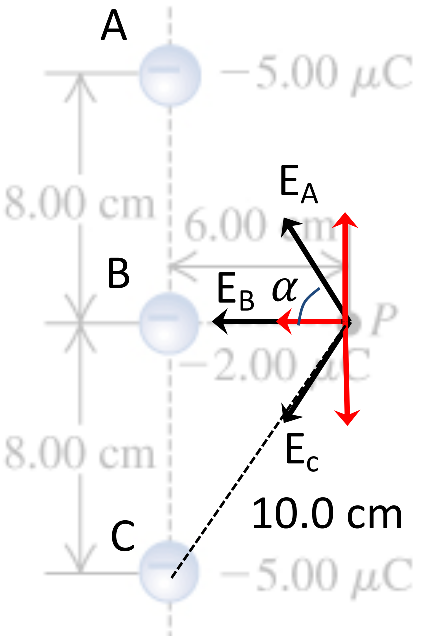

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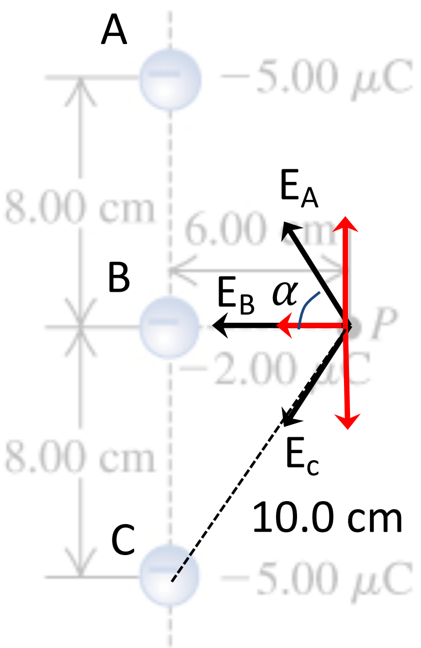

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$$\left|\vec{E}\right| = 2\left[\left(\frac{1}{4\pi\epsilon_0}\frac{q_A}{r^2}\right)\cos(\alpha)\right] + \left(\frac{1}{4\pi\epsilon_0}\frac{q_B}{r^2}\right)$$

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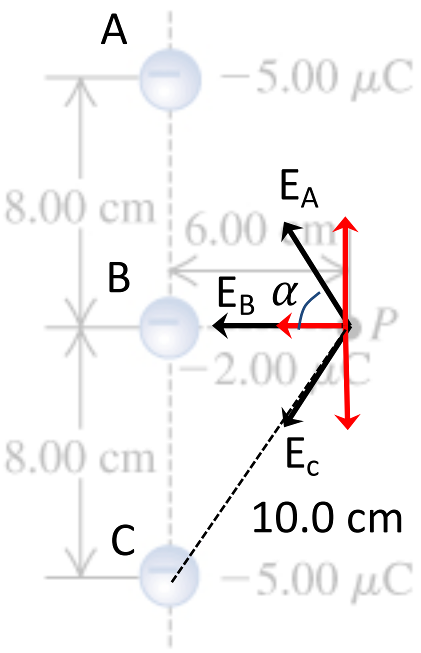

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$$|\vec{E}| = 2\left[\left(\frac{1}{4\pi\epsilon_0}\frac{5\times10^{-6}}{(0.1)^2}\right)\left(\frac{0.06}{0.10}\right)\right] + \left(\frac{1}{4\pi\epsilon_0}\frac{2\times10^{-6}}{(0.06)^2}\right)$$

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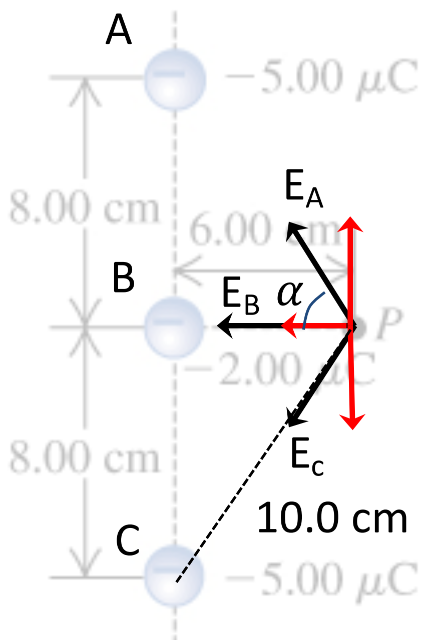

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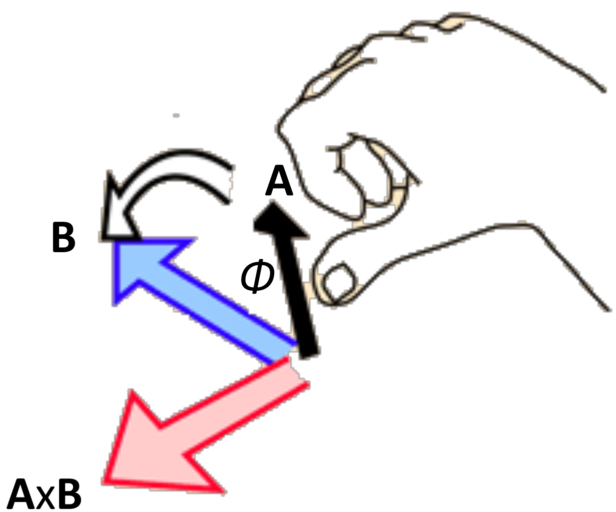
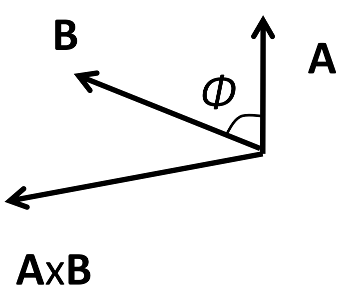
$$|\vec{E}| = 1.04 \times 10^7\ N/C$$

Electric Dipole: Force and Torque

Reminder: vector ops





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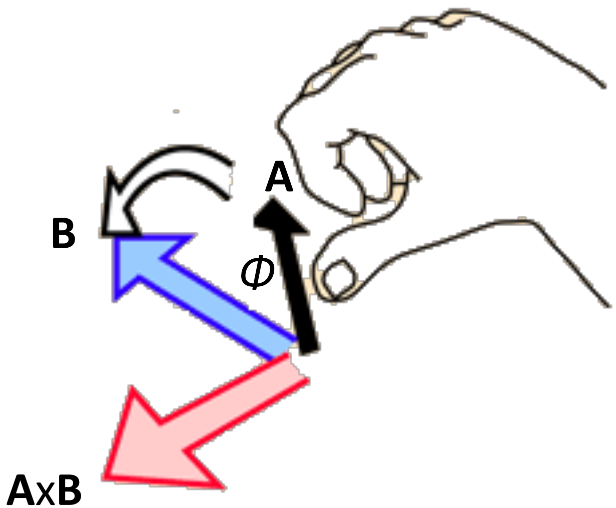
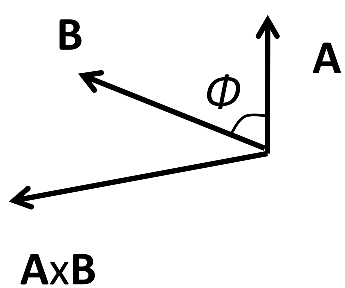


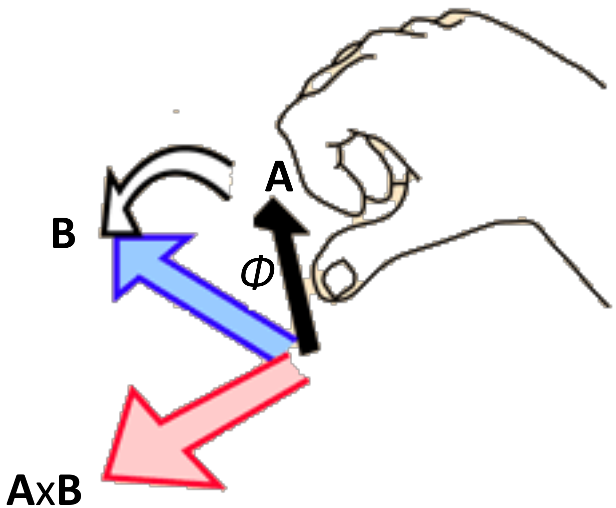


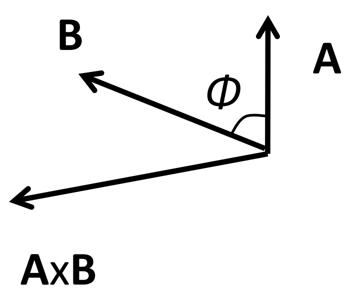
Cross product :

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin(\phi)$$

Reminder: vector ops





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$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin(\phi)$$

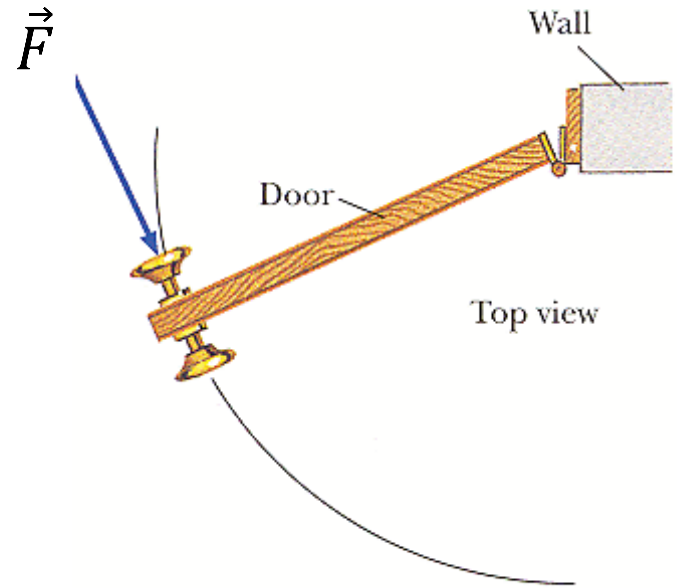
Scalar (dot) product :

$$\vec{A} \cdot \vec{B} = |A||B|\cos(\phi)$$

Reminder: Force and Torque

$$\sum \vec{F} = m\vec{a}$$

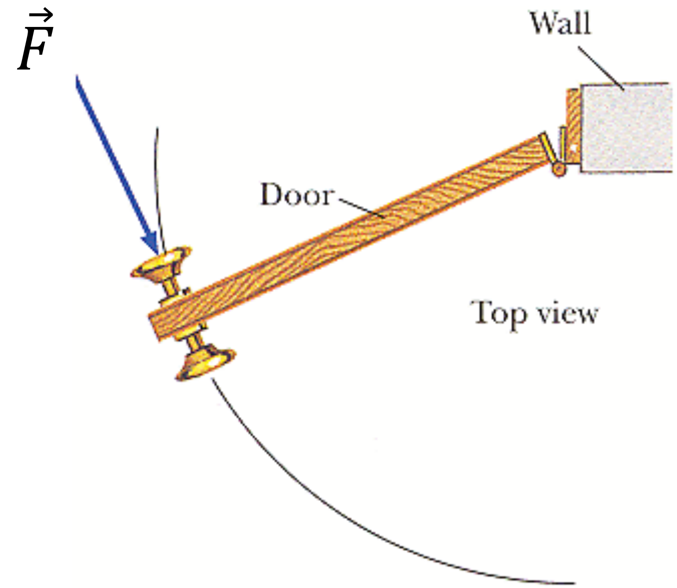
Reminder: Force and Torque



$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{\tau} = I\vec{\alpha}$$

Reminder: Force and Torque

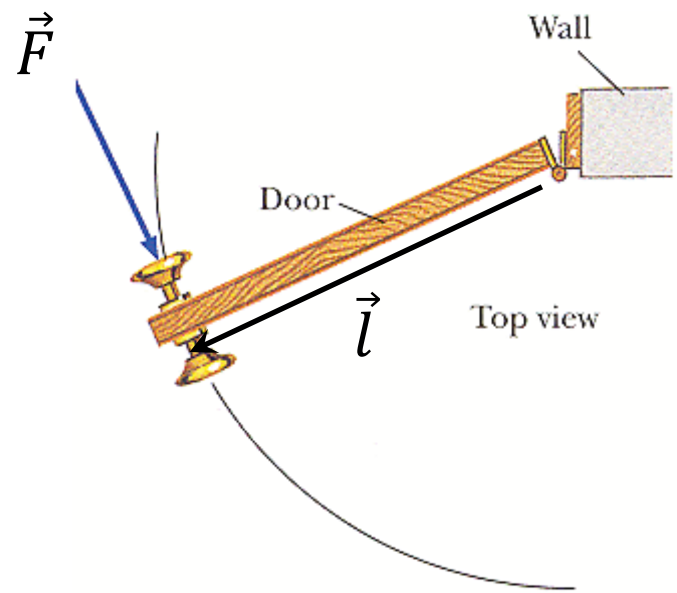


$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{\tau} = I\vec{\alpha}$$

I: *Moment of inertia*
α: *Angular acceleration*

Reminder: Force and Torque



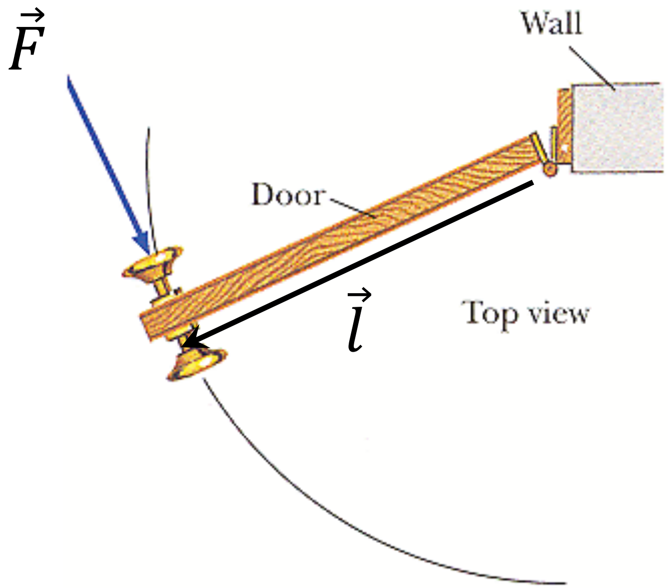
$$\sum \vec{F} = m\vec{a}$$

$$\vec{\tau} = \vec{l} \times \vec{F}$$

$\vec{l}$: *lever arm*

Direction of torque?

Reminder: Force and Torque



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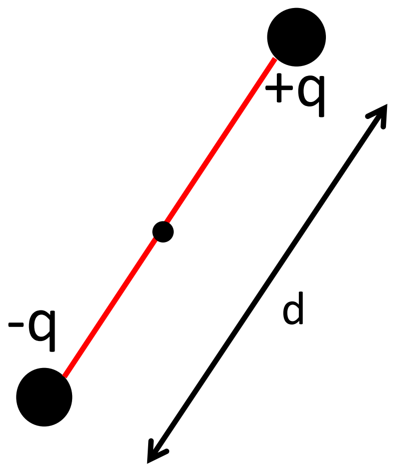
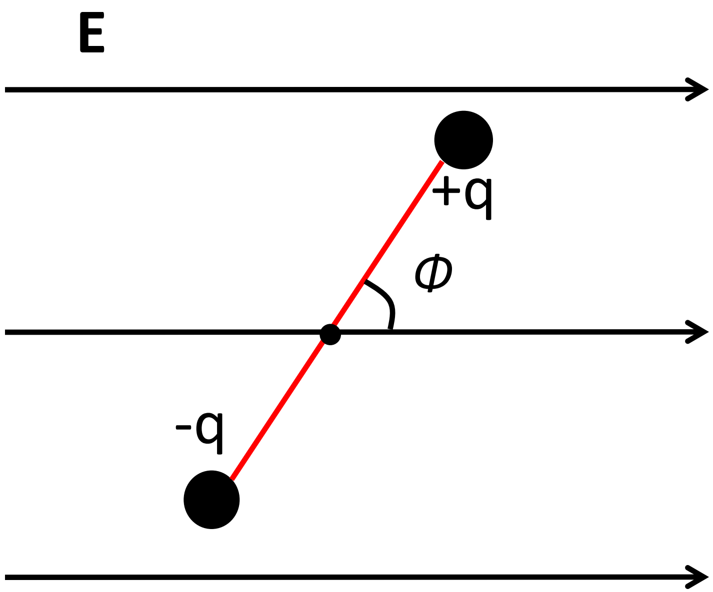
$\vec{l}$: *lever arm*

Direction of torque? Out of the page

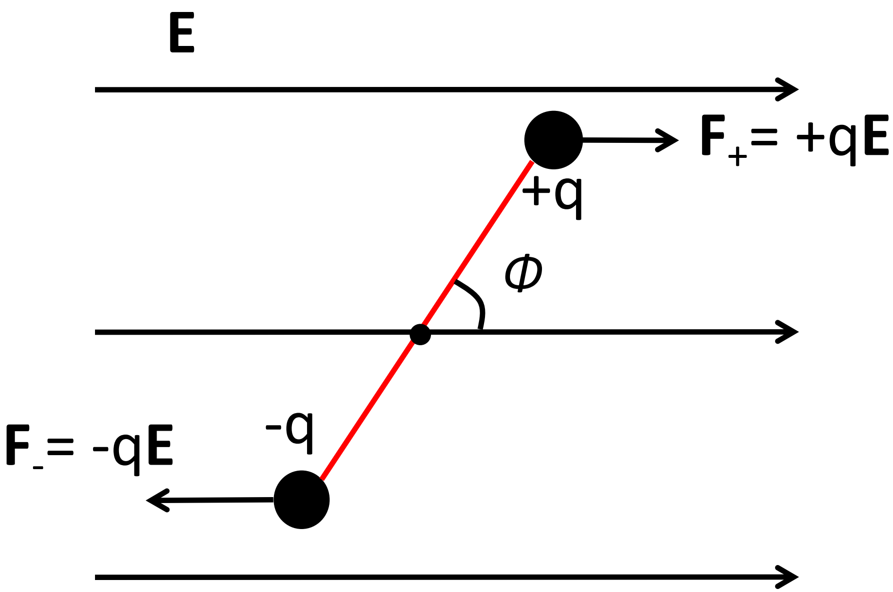
Electric Dipole: Force and Torque



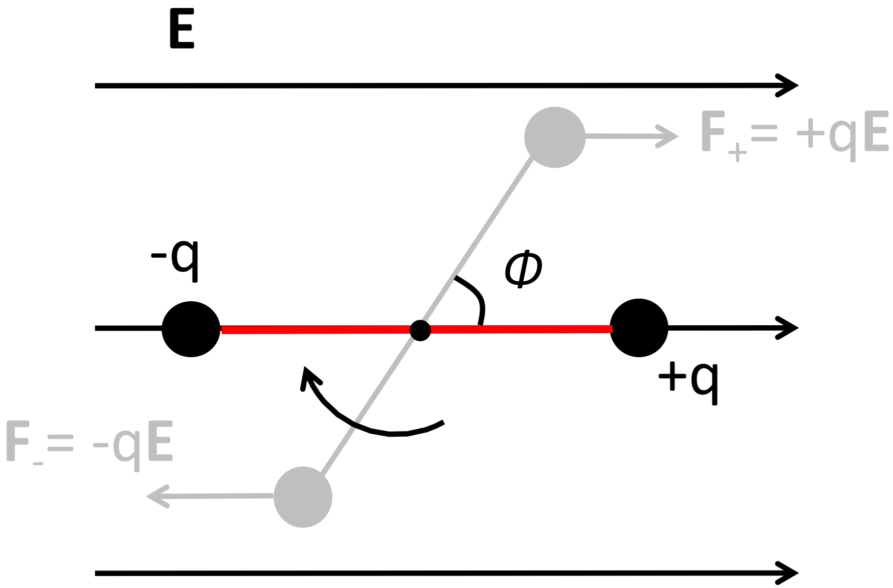
Electric Dipoles: Torque



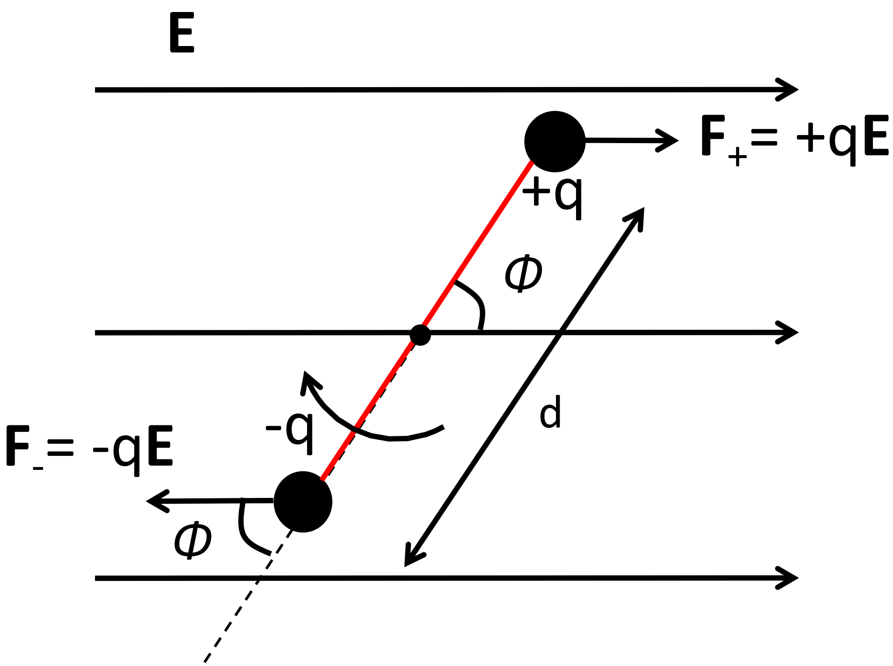
Electric Dipoles: Torque



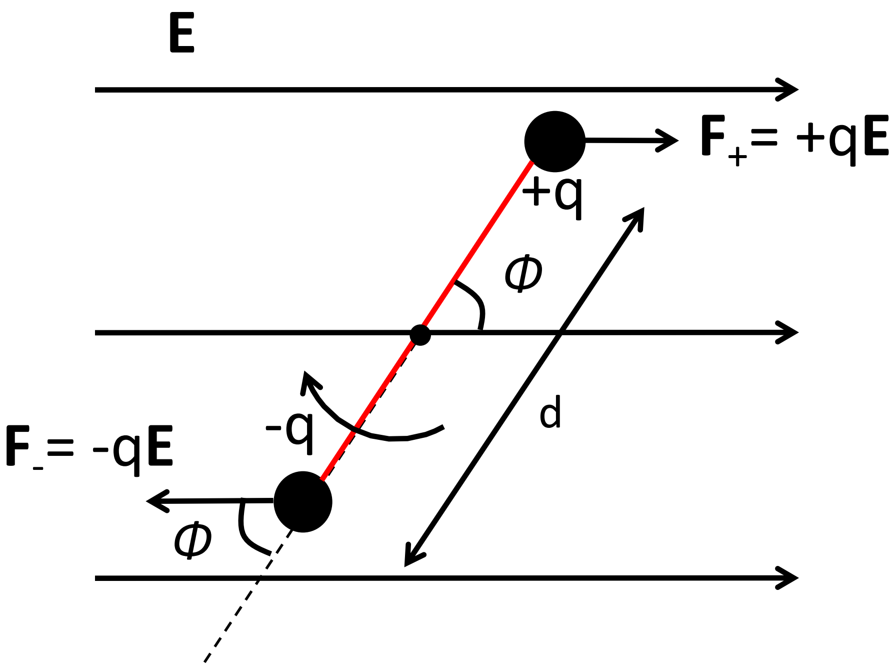
Electric Dipoles: Torque



$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$

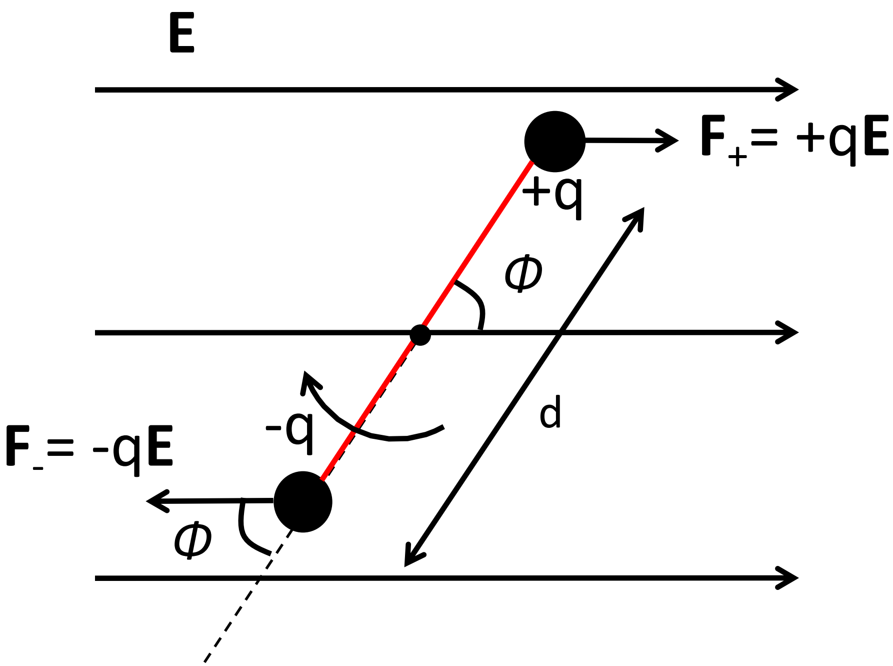


$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

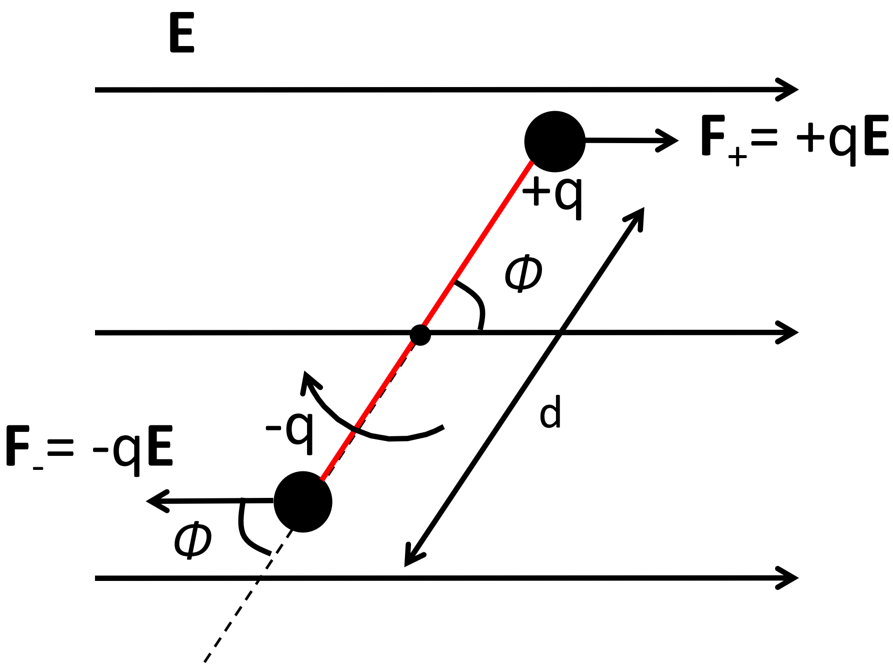
$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

$$|\vec{\tau}| = \left|\frac{d}{2}\right| |F_+| \sin(\phi) + \left|\frac{d}{2}\right| |F_-| \sin(\phi)$$

$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

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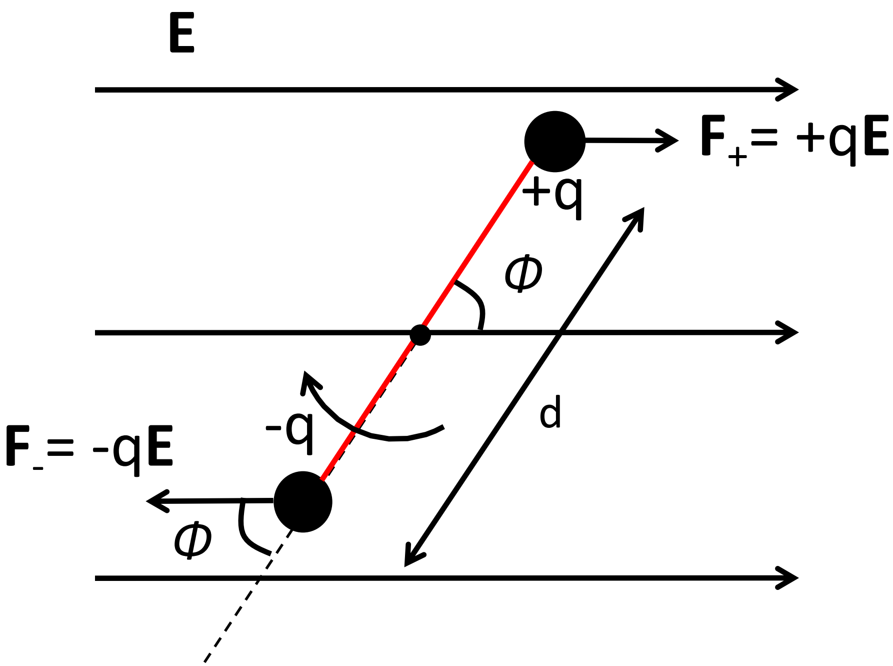
$$|\vec{\tau}| = \left|\frac{d}{2}\right| |qE|\sin(\phi) + \left|\frac{d}{2}\right| |qE|\sin(\phi)$$

$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

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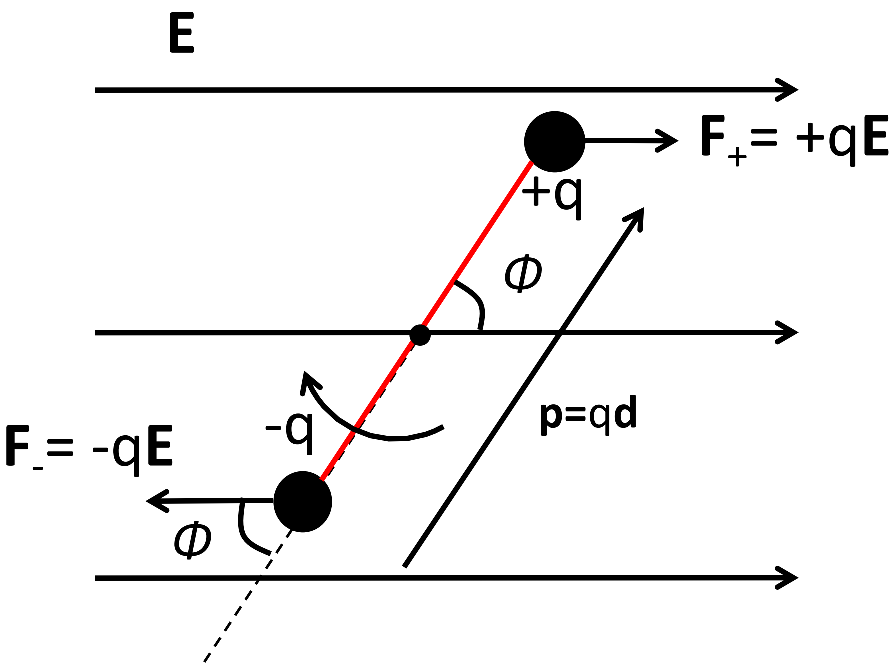
$$|\vec{\tau}| = \left|\frac{d}{2}\right| |qE|\sin(\phi) + \left|\frac{d}{2}\right| |qE|\sin(\phi)$$

$$|\vec{\tau}| = |d||q||E|\sin(\phi)$$

$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

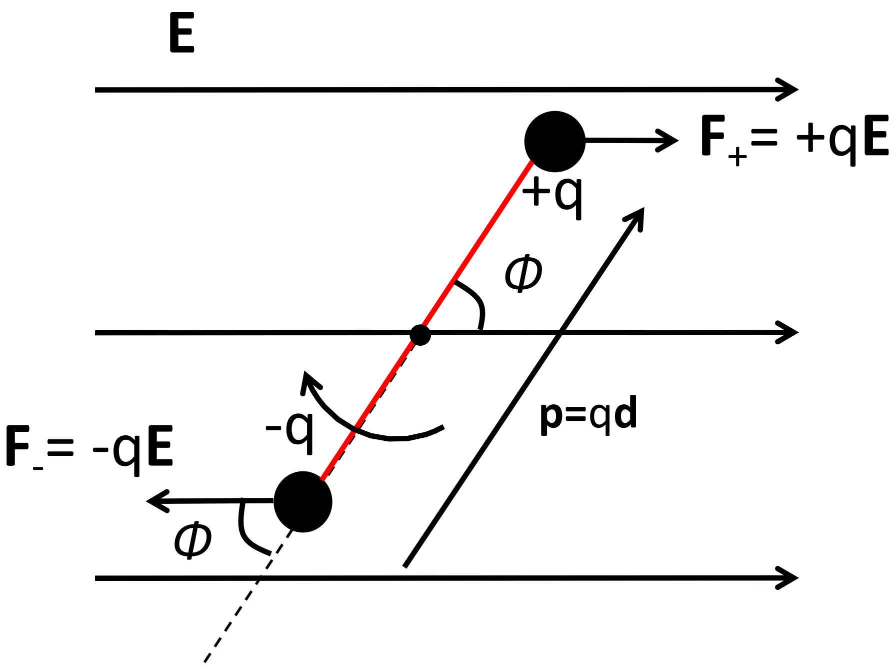
$$|\vec{\tau}| = \left|\frac{d}{2}\right| |F_+|\sin(\phi) + \left|\frac{d}{2}\right| |F_-|\sin(\phi)$$

$$|\vec{\tau}| = \left|\frac{d}{2}\right| |qE|\sin(\phi) + \left|\frac{d}{2}\right| |qE|\sin(\phi)$$

$$|\vec{\tau}| = |d||q||E|\sin(\phi)$$

$$Defining: \vec{p} = q\vec{d} \quad (dipole \quad moment)$$

$$\sum \vec{\tau} = \sum_{i=1}^{N} \vec{l}_i \times \vec{F}_i \quad (Torque)$$



$$|\vec{\tau}| = \sum_{i=1}^{N} |l_i||F_i|\sin(\phi)$$

$$|\vec{\tau}| = \left|\frac{d}{2}\right||F_+|\sin(\phi) + \left|\frac{d}{2}\right||F_-|\sin(\phi)$$

$$|\vec{\tau}| = \left|\frac{d}{2}\right||qE|\sin(\phi) + \left|\frac{d}{2}\right||qE|\sin(\phi)$$

$$|\vec{\tau}| = |d||q||E|\sin(\phi)$$

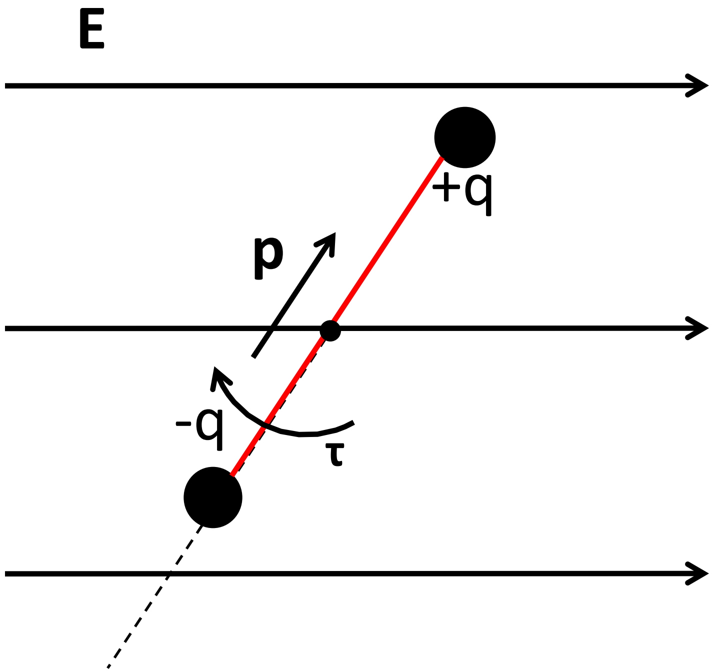
$$Defining : \vec{p} = q\vec{d} \quad (dipole \quad moment)$$

$$|\vec{\tau}| = |qd||E|\sin(\phi)$$

Torque on electric dipole

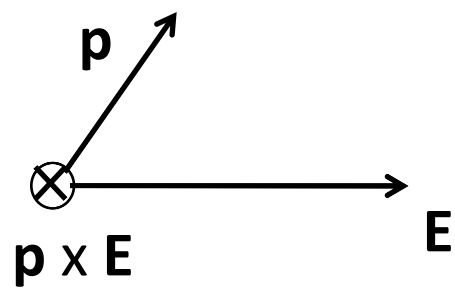
$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Electric Dipoles: Torque

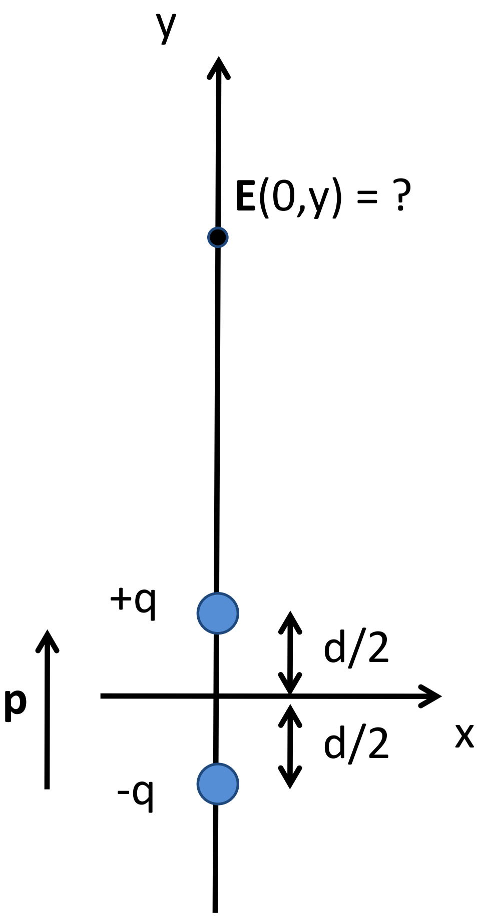


$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$



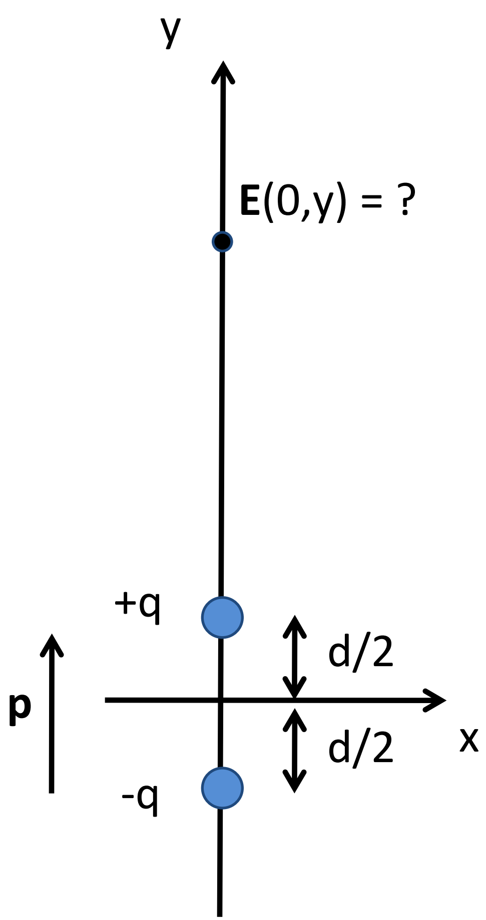
Electric Field: Torque



$$\text{Earlier, we found:} \quad \vec{E}(y) = \frac{qd}{2\pi\varepsilon_0 y^3}\hat{y}$$

$$\textit{And just now:} \quad \vec{p} = q\vec{d}$$

Electric Field: Torque



$$\text{Earlier, we found:} \quad \vec{E}(y) = \frac{qd}{2\pi\varepsilon_0 y^3}\hat{y}$$

$$\textit{And just now:} \quad \vec{p} = q\vec{d}$$

We can then write:

$$\boxed{\vec{E}(y) = \frac{\vec{p}}{2\pi\varepsilon_0 y^3}}$$