Lecture 1. Introduction and Basic Probability Concepts

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Where do we meet probability?

In our everyday life we deal with processes that cannot be described exactly. For example,

- how much time will it take to print 30 pages in Kinko? Will I be able to print these 30 pages and get on time before my project presentation?
- will you acquire a flu this season?
- how many cars will be on a freeway at 5pm?
- how likely there will be an electricity blackout during your final exam?

From our prior experience, our answers can be "usually", "typically", "unlikely", "about 5 min"...

Can we do better?

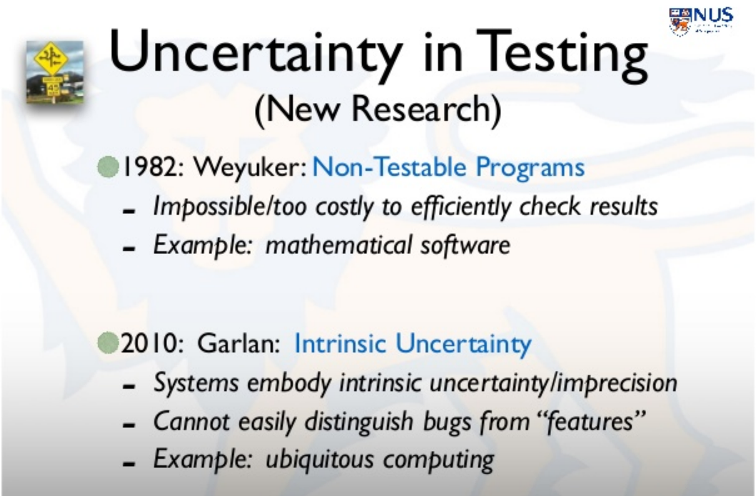
Ex 1. The Nurse in a hospital (Bertsekas & Tsitsiklis)

A patient is admitted to the hospital and a potentially life-saving drug is administered. The following dialog takes place between the nurse and a concerned relative.
RELATIVE: Nurse, what is the probability that the drug will work?
NURSE: I hope it works, well know tomorrow.
RELATIVE: Yes, but what is the probability that it will?
NURSE: Each case is different, we have to wait.
RELATIVE: But lets see, out of a hundred patients that are treated under similar conditions, how many times would you expect it to work?
NURSE (somewhat annoyed): I told you, every person is different, for some it works, for some it doesnt.
RELATIVE (insisting): Then tell me, if you had to bet whether it will work or not, which side of the bet would you take?
NURSE (cheering up for a moment): I'd bet it will work.
RELATIVE (somewhat relieved): OK, now, would you be willing to lose two dollars if it doesnt work, and gain one dollar if it does?
NURSE (exasperated): What a sick thought! You are wasting my time!

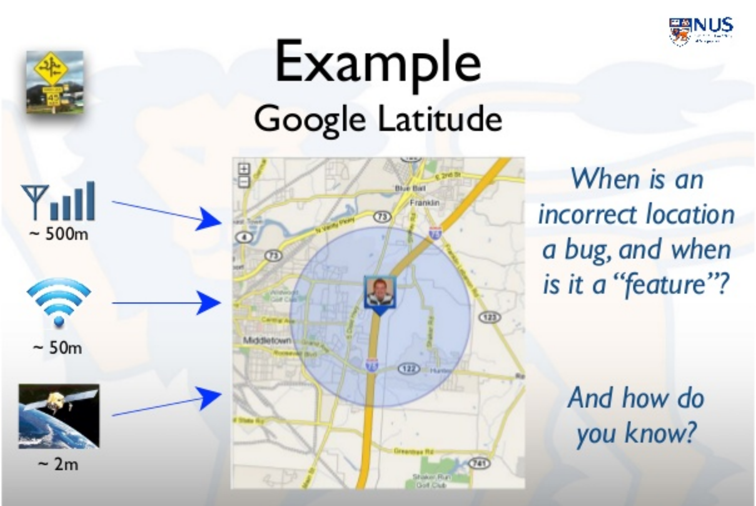
What can we observe from this example? Or the path to statistics?

- The relative tries to use the concept of probability to discuss an uncertain situation.
- The nurse's initial answer suggests that the concept of probability is not uniformly understood.
- The relative tries to make it more concrete and defines it in terms of frequency of occurrence, i.e. percentage of successes observed over a relatively large number of similar trials.
- The nurse refuses to use the concept of probability as frequency of occurrence. Is she wrong? Justify your answer.

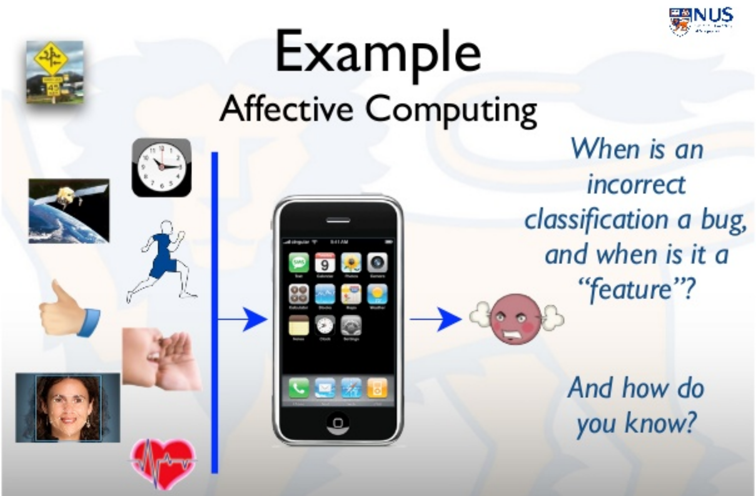
Ex. 2. Probability and Statistics in modern CS (Rosenblum, 2013)



Ex. 3. Probability and Statistics in modern CS (Rosenblum, 2013)



Ex. 4. Probability and Statistics in modern CS (Rosenblum, 2013)



Targets of CS/SE/STAT 3341

- Introduce probability theory (precise but not overly technical)
- Provide, via examples, applications of probability to computer science
- Provide some exposure to statistics and its role in modern computer science
- Lay a foundation for more advanced courses, e.g. data mining, artificial intelligence etc.
- **Last but not the least, have a fun time solving cool prob/stats problems!**

Sample Space, Events, and Probability

- **Probability theory** defines mathematical rules for assigning probabilities to outcomes of random experiments, e.g., coin flips, packet arrivals, noise voltage etc.
- **Sample space** is the set of all possible elementary or finest grain outcomes of the random experiment. It is usually denoted by Ω.

Example. You are expecting a child. The outcomes are either boy or a girl. Hence, $\Omega = \{Boy, Girl\}$.
Example. You submit a job on a cluster, and record job execution time. Then if you have no upper limit on your the execution time, $\Omega = \{(0, \infty)\}$.

Sample Space, Events, and Probability: Contd

- **Elementary events** are
  1. all *disjoint*,
  2. *collectively exhaustive*, i.e., together they make up the entire sample space.
- **Events** are subsets of the sample space. Example, flip coin 3 times and get exactly one T. This event consists of three possible combinations $\{$HHT, HTH, THH$\}$. Note that an event may be an empty set $\emptyset$.
- **Probability** is a function of an event, $P(E)$, and $0 \leq P(E) \leq 1$.
- **Probability law** assigns probabilities to events in a random experiment in a mathematically consistent way.

What's next?

- At home review Chapter 1.
- On Wednesday we will discuss the set algebra and basic probability rules.