PHYS2326
Lecture #3

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Goals for this lecture (Ch. 1)

- Quick review
- Remind ourselves of the principle of superposition
- Analyze the net electric force due to charges
- Understand the concept of electric field and electric field lines
- To understand how to derive the electric field for “typical” distribution of charges

Quick Review

- Electric charge
- Charging objects
- Coulomb's Law

Quick Review: Electric Charge

- Objects that have an excess or deficiency of electrons are said to be electrically charged
  - Number of electrons = protons: Neutral
  - Excess of electrons: negatively charged
  - Deficiency of electrons: Positively charged

Quick Review: Electric Charge

- Charged objects exert an "action-at-distance" or "non-contact" force
  - Same type of charge: Repulsion
  - Opposite type of charge: Attraction

Quick Review: Charging Objects

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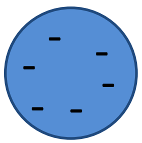
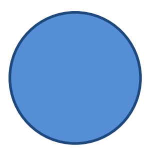
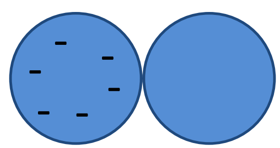


1. Two identical metal spheres
2. Sphere A initially charged with Q

3. What happens when spheres A and B touch?



Quick Review: Charging Objects



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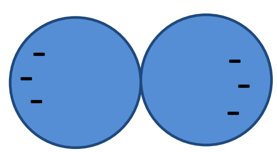

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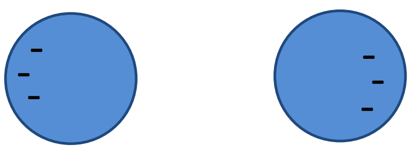

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2. Sphere A initially charged with Q

3. What happens when spheres A and B touch?

4. What is the final charge in B of sphere were separated?

Coulomb's Law

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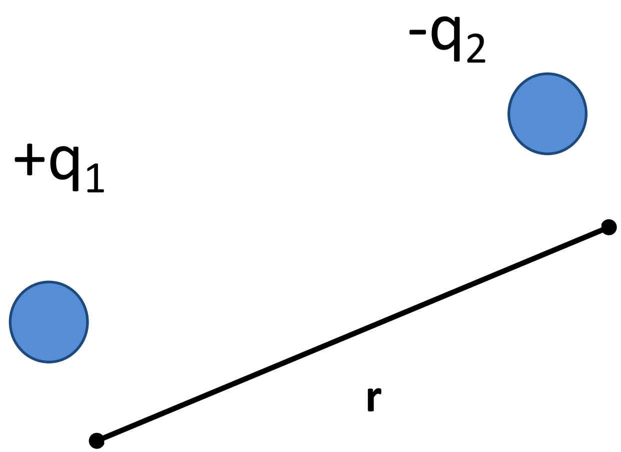

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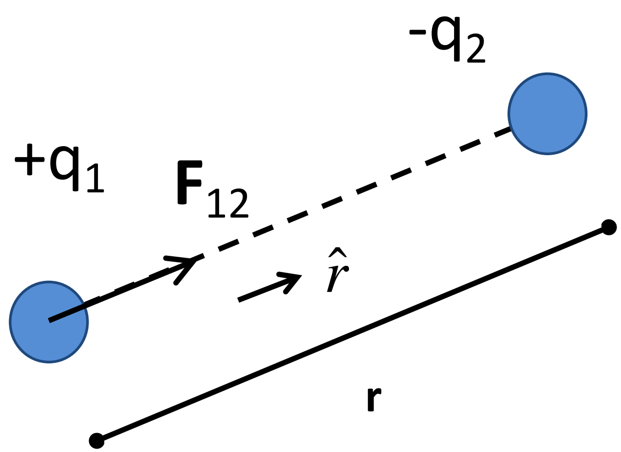

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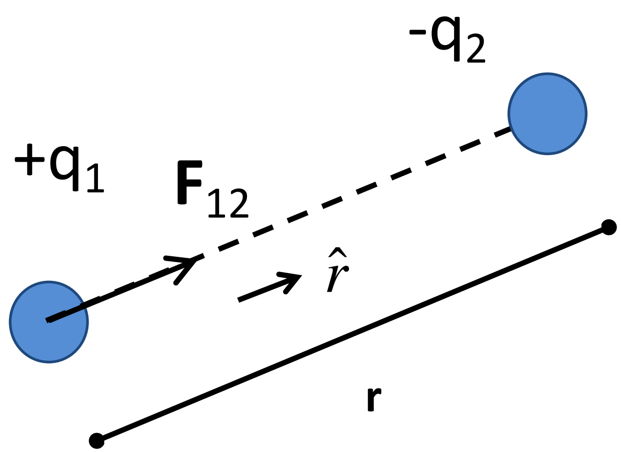
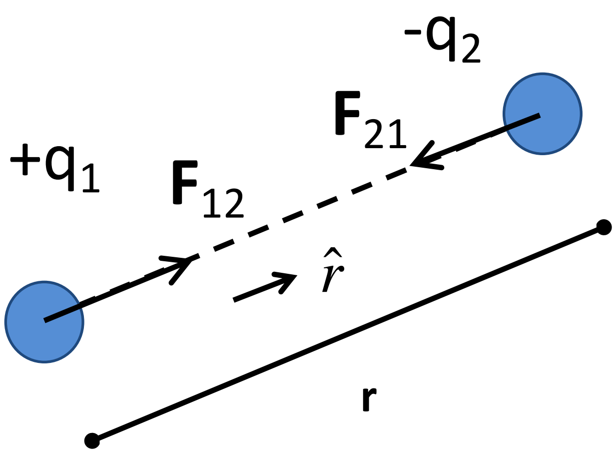


$$\vec{F}_{12} = k\ \frac{|q_1||q_2|}{r^2}\hat{r}$$

Example: F_{12} = Force in "1" due to "2"

Coulomb's Law



$$\vec{F}_{12} = k\ \frac{|q_1||q_2|}{r^2}\hat{r}$$

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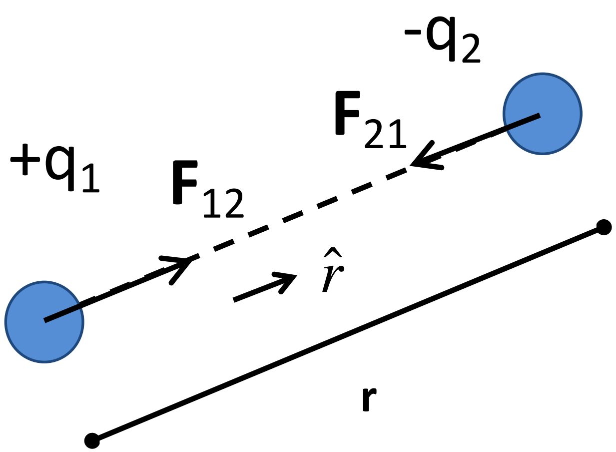

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$$\vec{F} = \vec{F}_{12} = -\vec{F}_{21}$$

$$k = \frac{1}{4\pi\epsilon_0} = constant\ \approx\ 9.0\times10^9 Nm^2/C^2$$

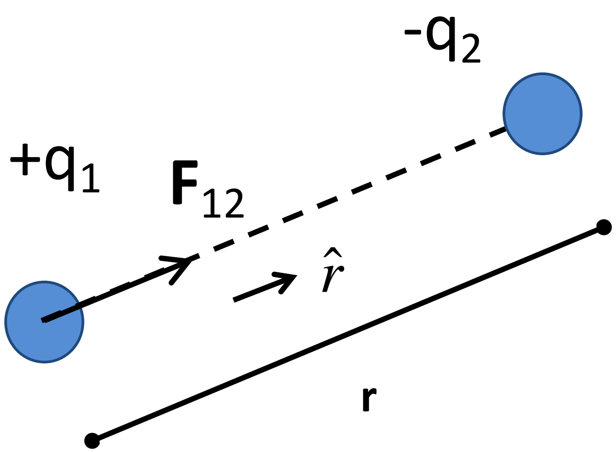
$\boldsymbol{\varepsilon}_0$ = Permittivity of free space (8.852×10^{-12} C/m^2)

q_1 = Amount of charge in object #1 (in C)

q_2 = Amount of charge in object #2 (in C)

r = distance between objects 1 and 2 (in m)

Coulomb's Law



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r = distance between objects 1 and 2 (in m)

Charge is Quantized

- The **electron charge (*e*)** is the elementary charge.

$$q_e = e = 1.602176565(35)\times10^{-19}\ \mathrm{C}$$

- The charge (Q) of any object is an integer multiple of the electron charge.

$$Q = nq_e$$

1.0 Coulomb

- What does 1.0 C represent?

1.0 Coulomb

- What does 1.0 C represent?

$$Q = 1.0 \;\; C$$

$$N_e = \frac{Q}{q_e} = \frac{1.0 \;\; C}{1.6 \times 10^{-19} \;\; C}$$

$$N_e = 6.25 \times 10^{18} \;\; electrons$$

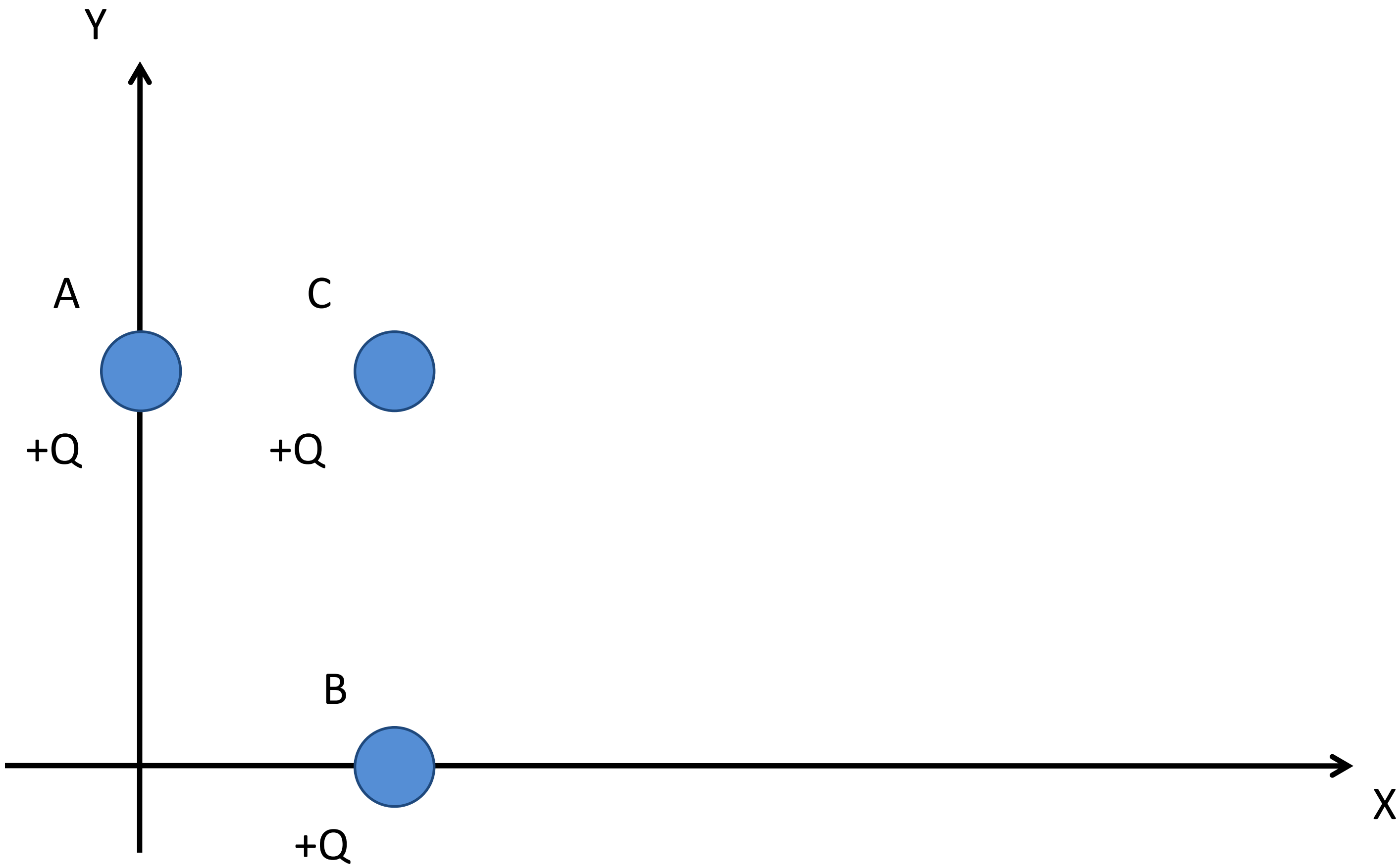
Principle of Superposition

Principle of Superposition

- The principle of superposition of forces states that the resulting (net) force on a given object is the vector summation of all individual forces.

- The principle of superposition applies to electric forces just like it does for gravitational forces.

Principle of Superposition



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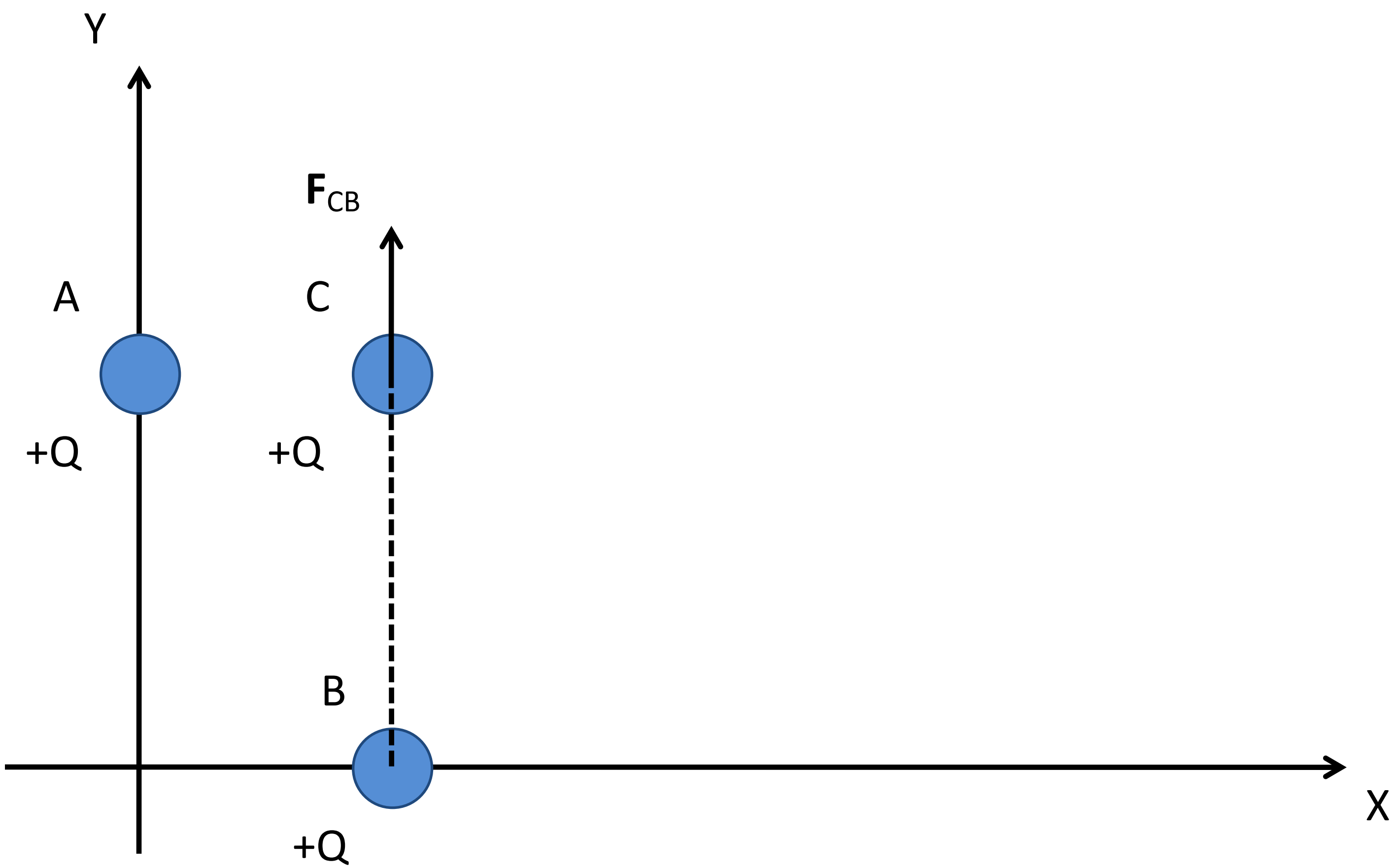

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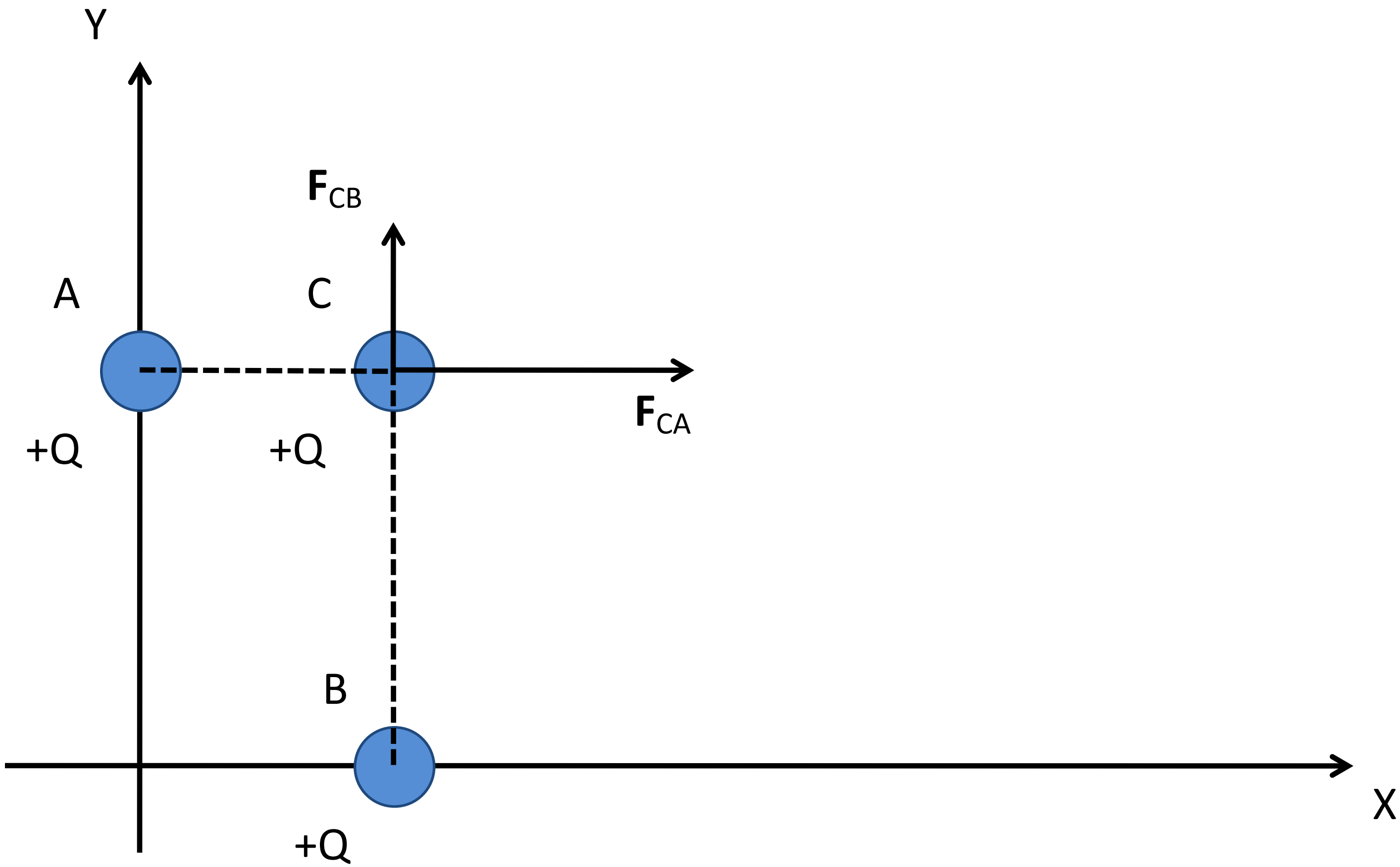

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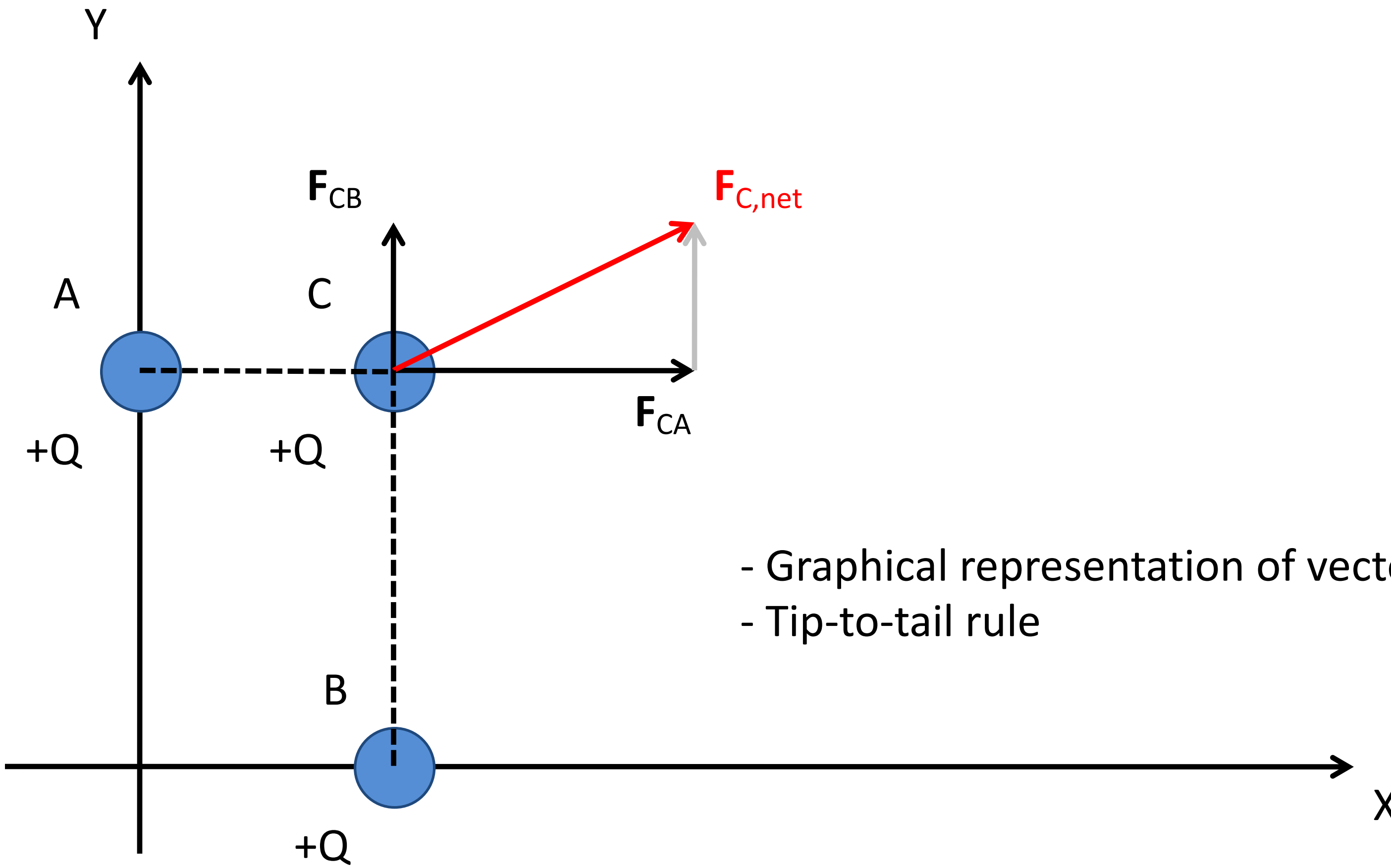
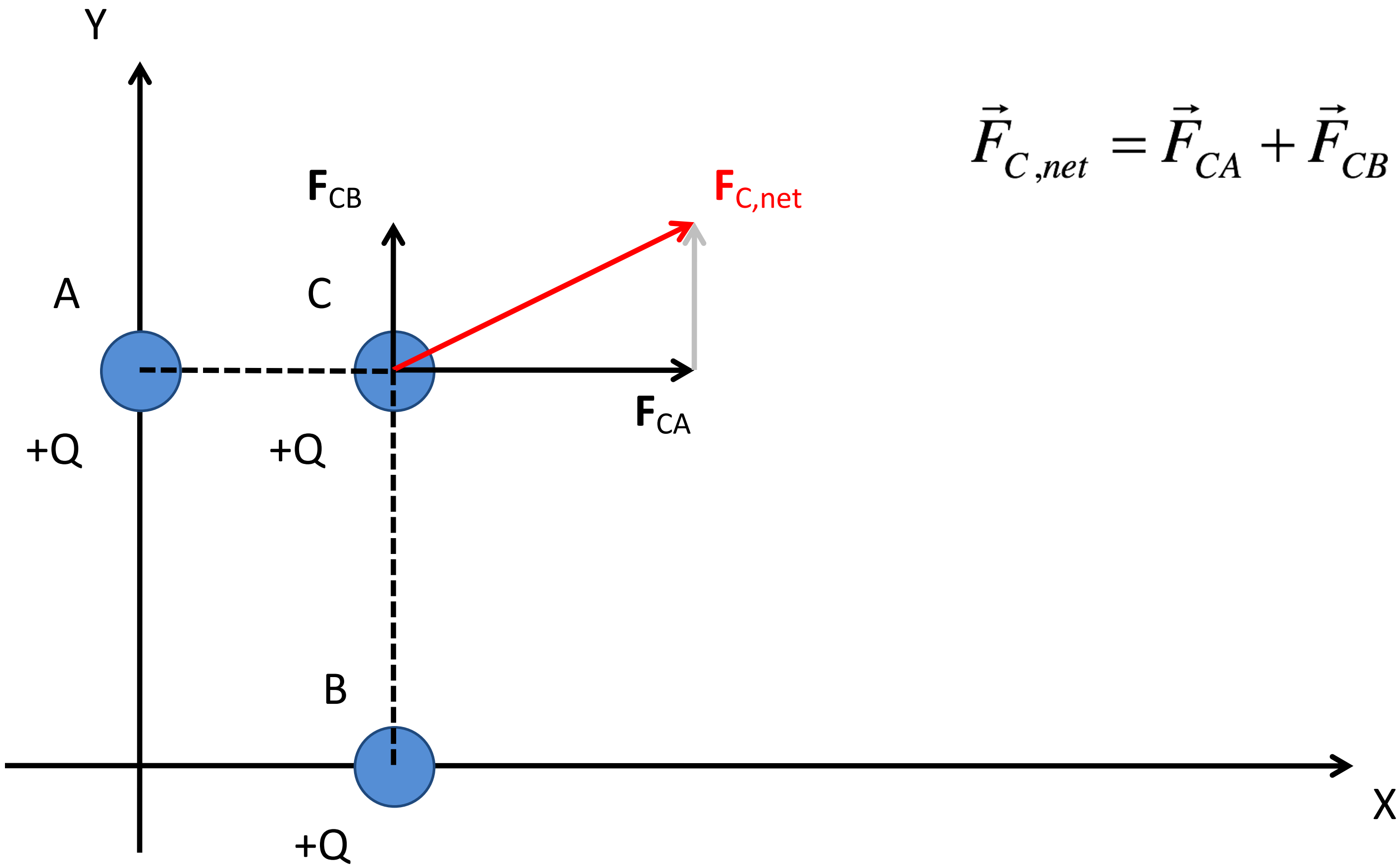


- Graphical representation of vectors
- Tip-to-tail rule

Principle of Superposition

$$\vec{F}_{C,net} = \vec{F}_{CA} + \vec{F}_{CB}$$



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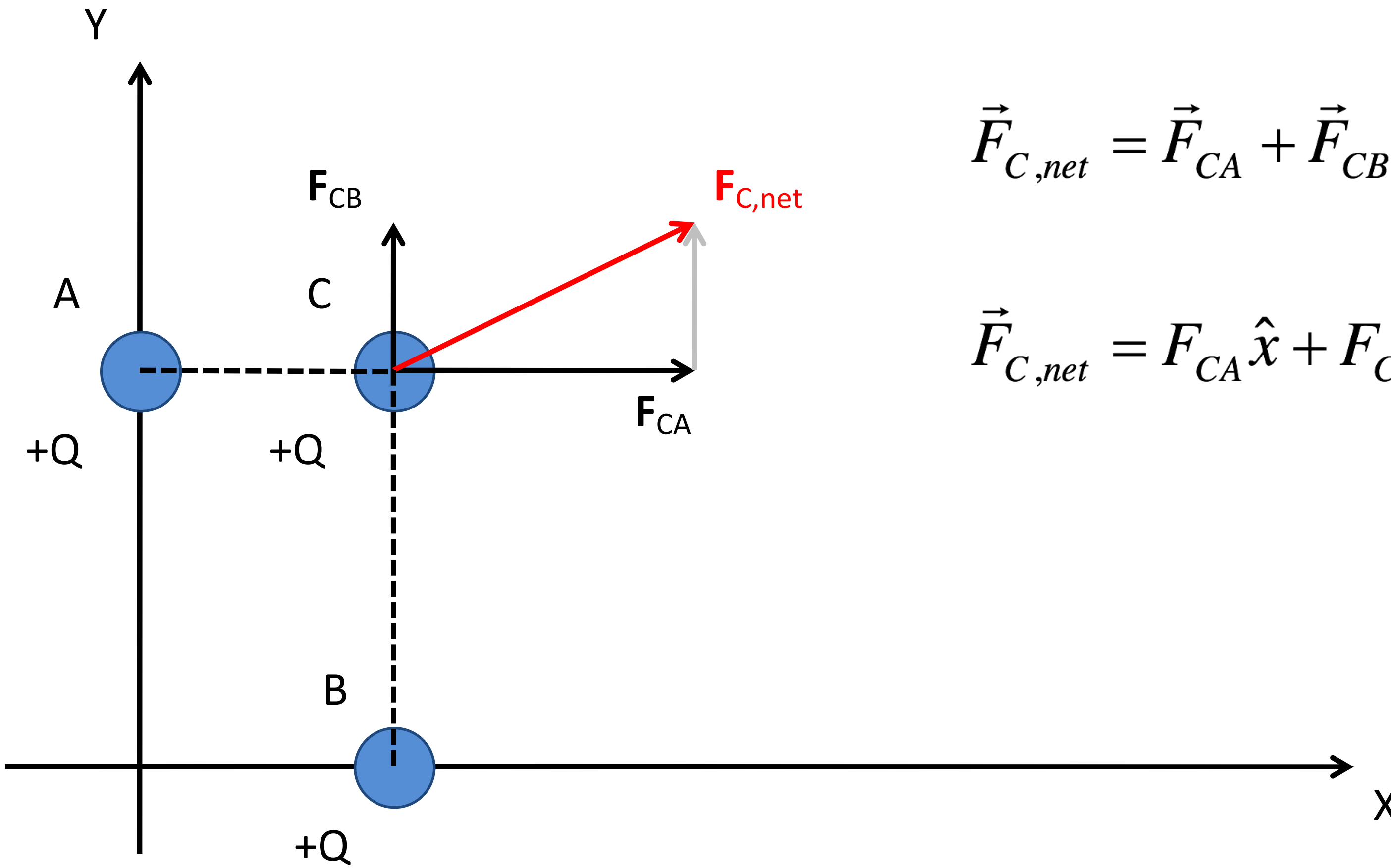

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$$\vec{F}_{C,net} = F_{CA}\hat{x} + F_{CB}\hat{y}$$

Example: Assume a proton ($q = q_e = +1.6 \times 10^{-19}$ C) is at the origin of a coordinate system, and an electron is at ($x=2, y=2$) m. (a) What is the magnitude of the electric force the proton exerts on the electron? (b) What is the direction (unit vector) of the force?

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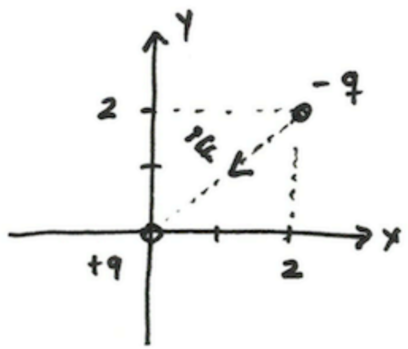

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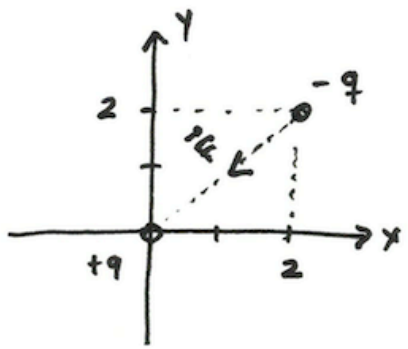

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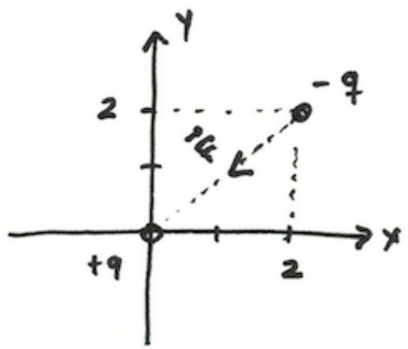

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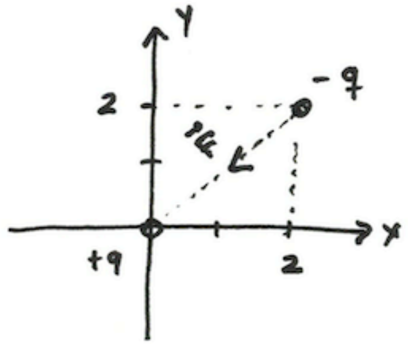

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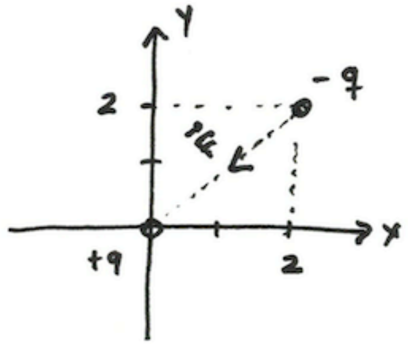

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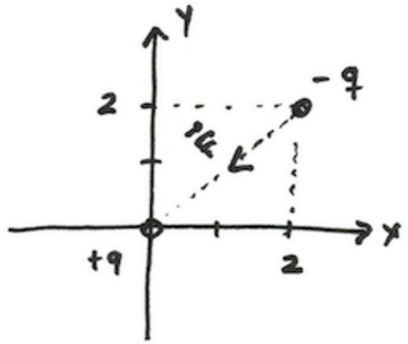

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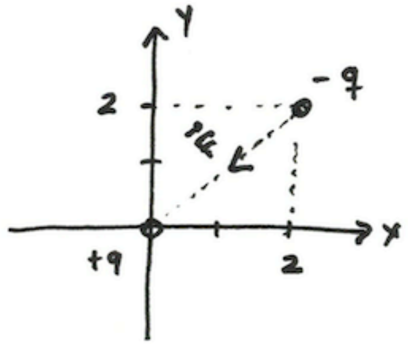

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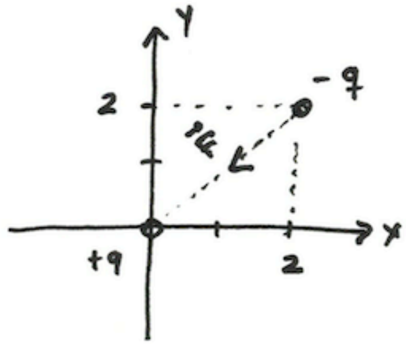

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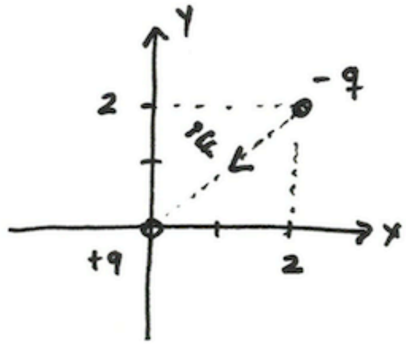

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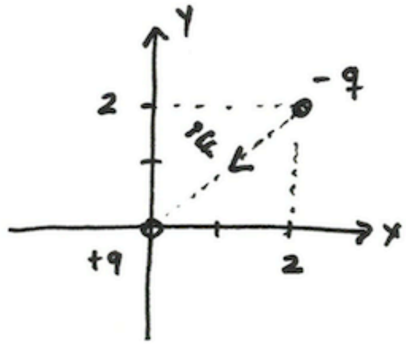

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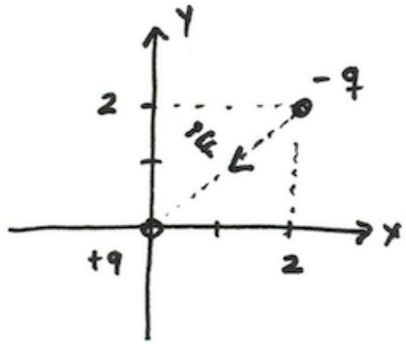

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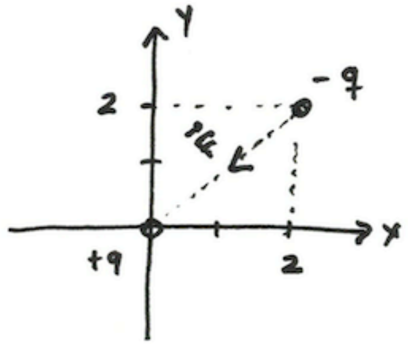

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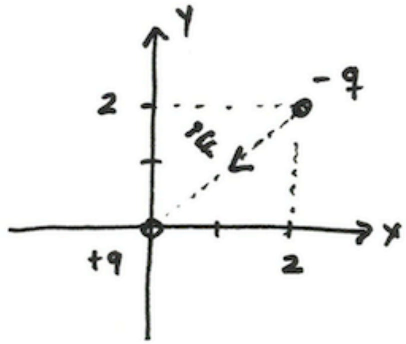

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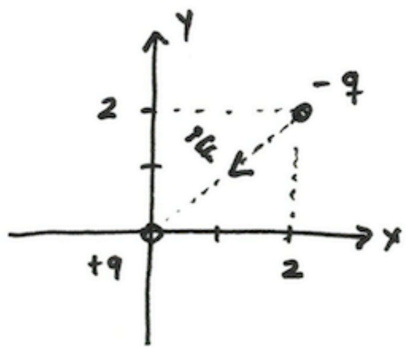

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$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}\hat{r}$$

$$|q_1| = 1.6\times10^{-19}\,C$$

$$|q_2| = 1.6\times10^{-19}\,C$$

$$r = ?$$

$$r = \sqrt{2^2+2^2} = \sqrt{8}$$

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\frac{(1.6\times10^{-19})^2}{(\sqrt{8})^2}$$

$$|\vec{F}| = 2.9\times10^{-29}\,N$$

(b)

$$\hat{r} = ?$$

$$\vec{r} = \vec{P}_2 - \vec{P}_1$$

$$\vec{r} = (0\hat{x} + 0\hat{y}) - (2\hat{x} + 2\hat{y})$$

$$\vec{r} = -2\hat{x} - 2\hat{y}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{-2\hat{x}-2\hat{y}}{\sqrt{2^2+2^2}}$$

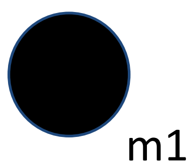
$$\hat{r} = -\frac{2}{\sqrt{8}}(\hat{x}+\hat{y})$$

Electric Field

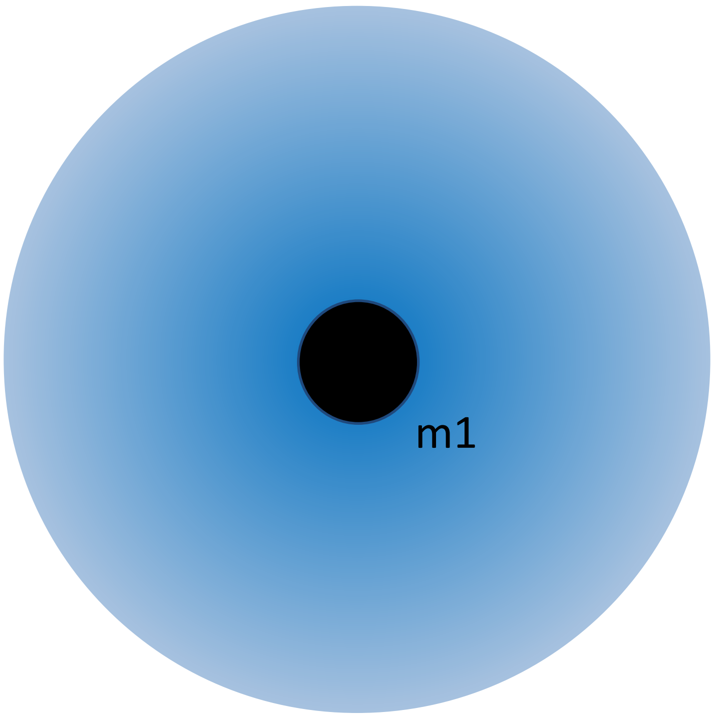
Electric Field

- Physically, the electric field can be defined as a vector force field due to charges / charged objects.

Gravitational ~~Electric~~ Field

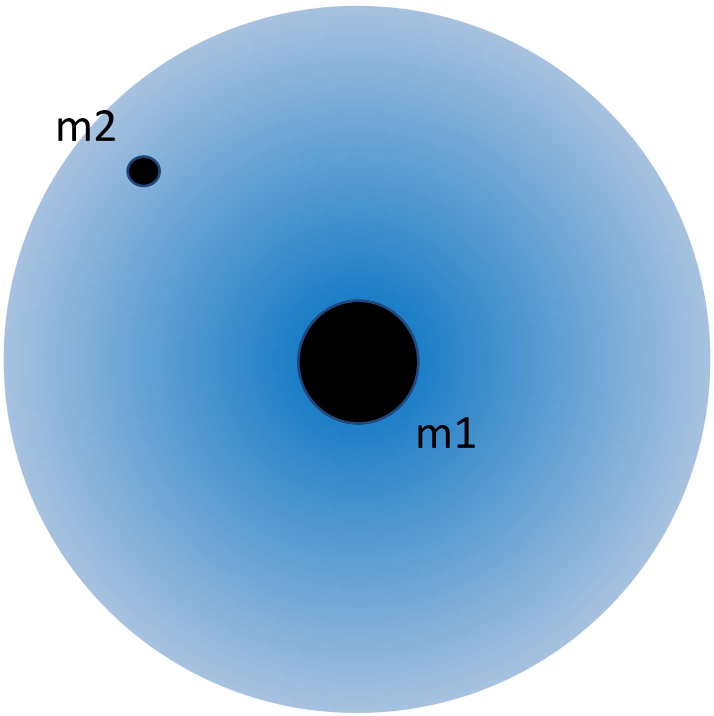


Gravitational
~~Electric~~ Field

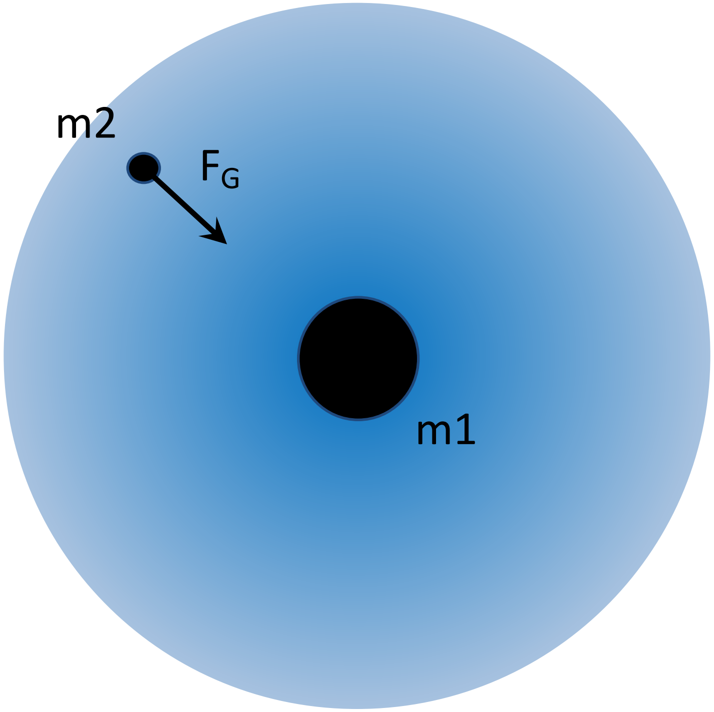


Gravitational

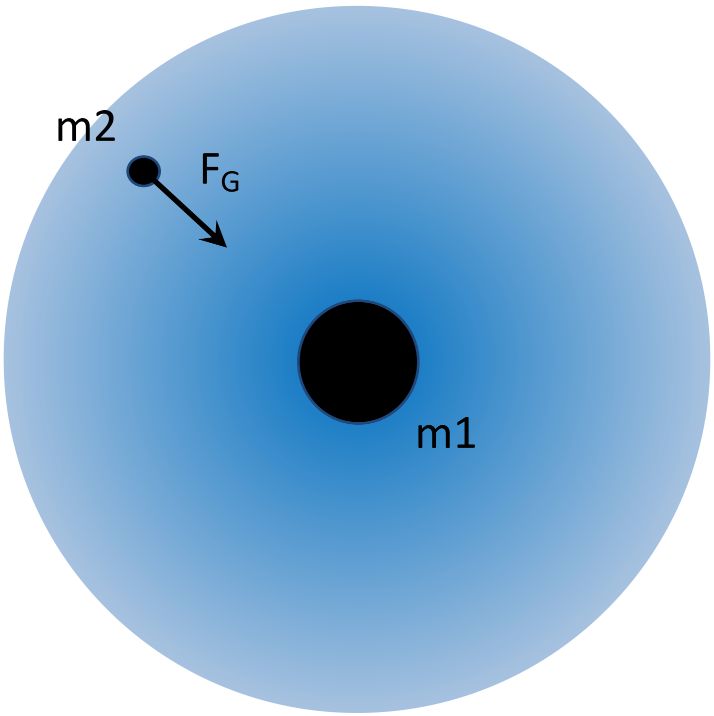
~~Electric~~ Field



Gravitational
~~Electric~~ Field

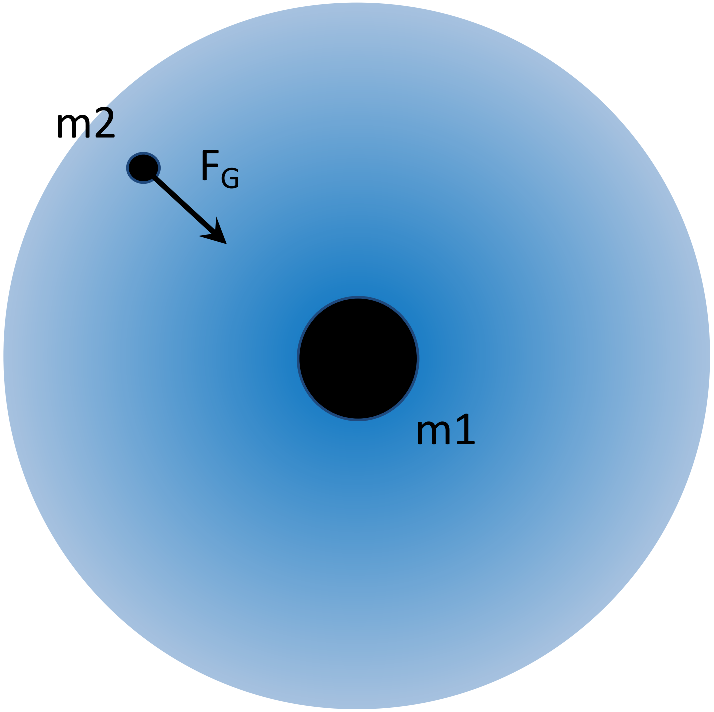


Gravitational
~~Electric~~ Field



$$\vec{F}_G = G\frac{m_1 m_2}{r^2}(-\hat{r})$$

Gravitational ~~Electric~~ Field

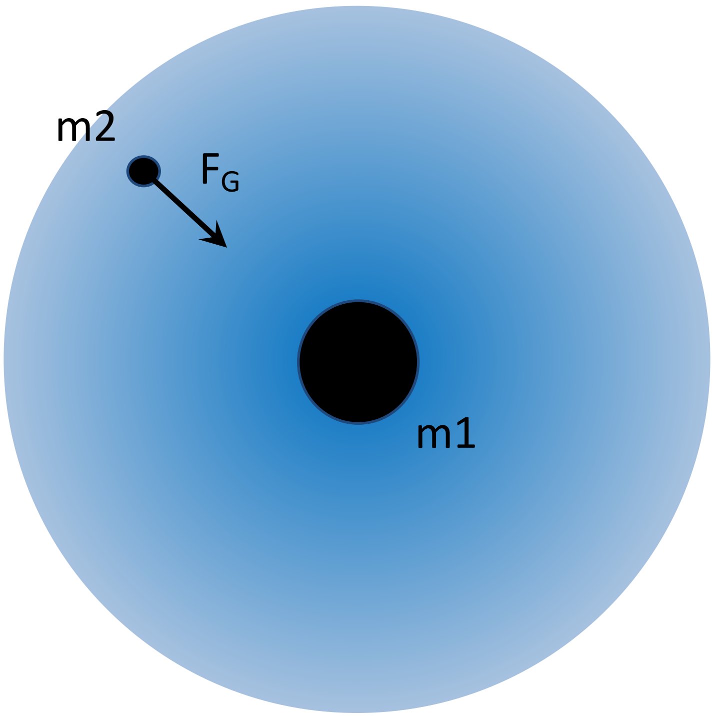


$$\vec{F}_G = G\frac{m_1 m_2}{r^2}(-\hat{r})$$

$$\vec{F}_G = m_2\left(G\frac{m_1}{r^2}\right)(-\hat{r})$$

Gravitational

~~Electric~~ Field

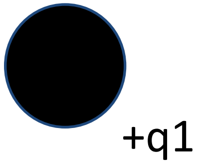


$$\vec{F}_G = G\frac{m_1 m_2}{r^2}(-\hat{r})$$

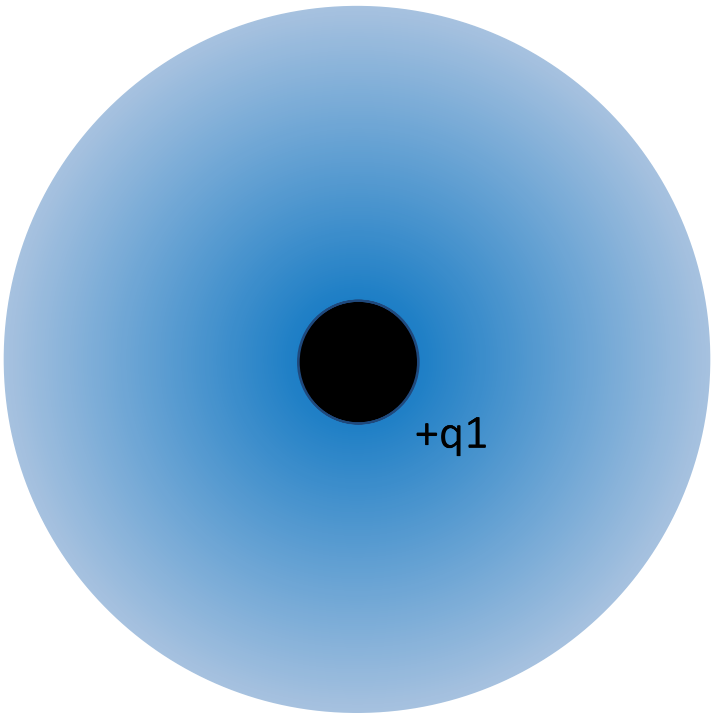
$$\vec{F}_G = m_2\left(G\frac{m_1}{r^2}\right)(-\hat{r})$$

$$\vec{F}_G = m_2\vec{g}$$

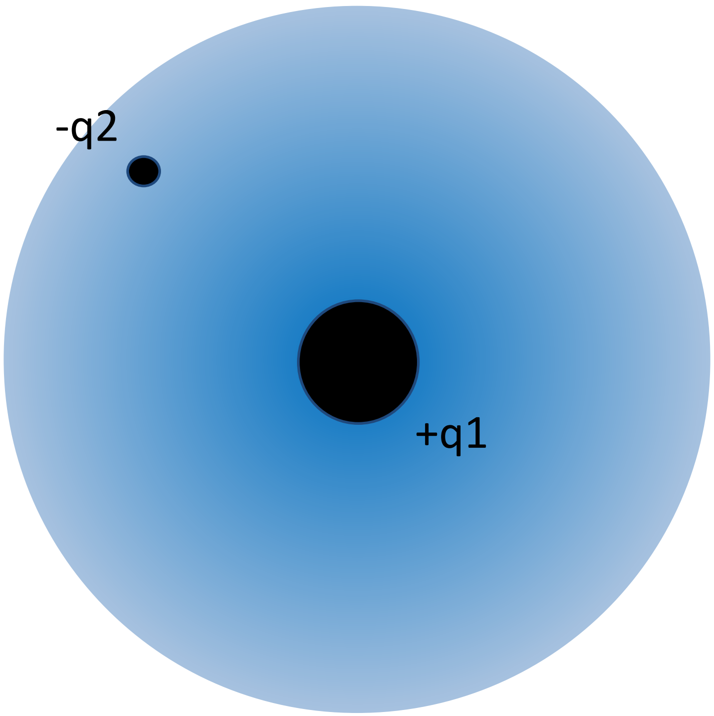
Electric Field



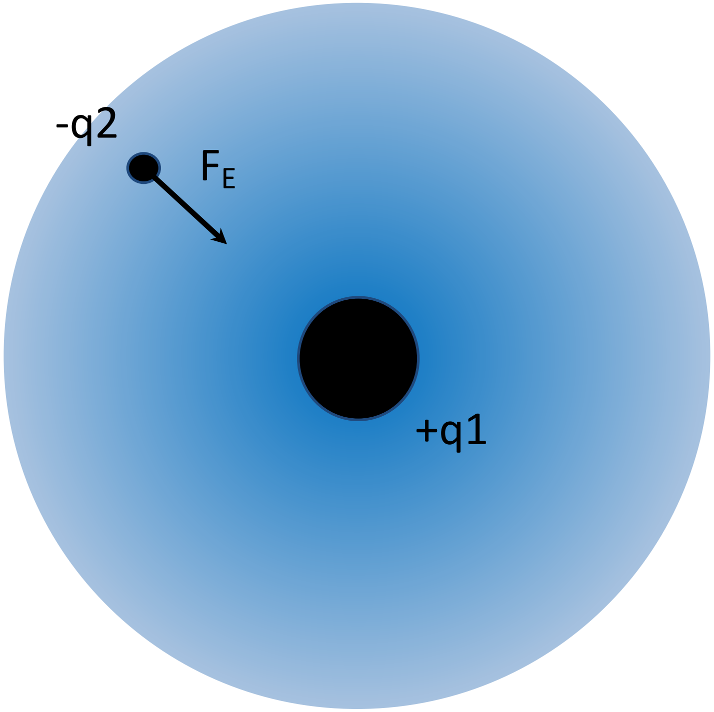
Electric Field



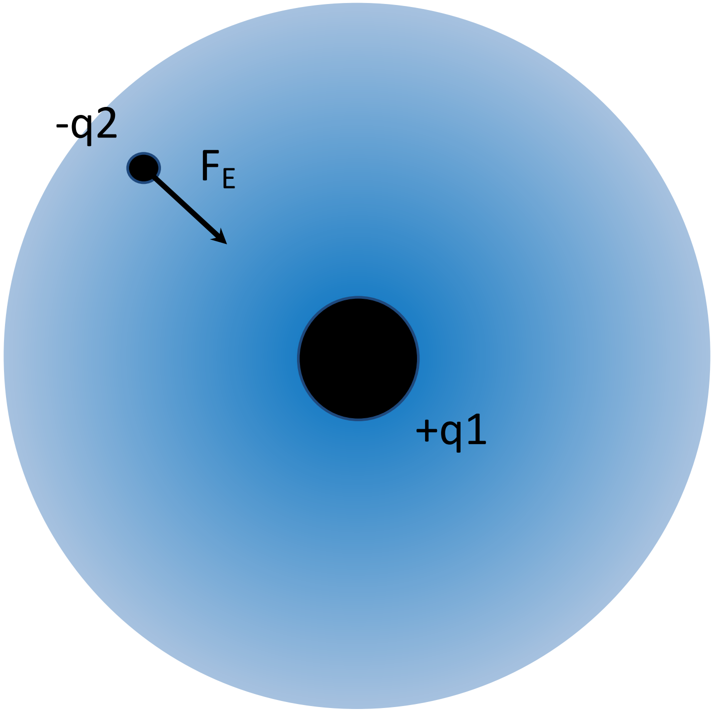
Electric Field



Electric Field

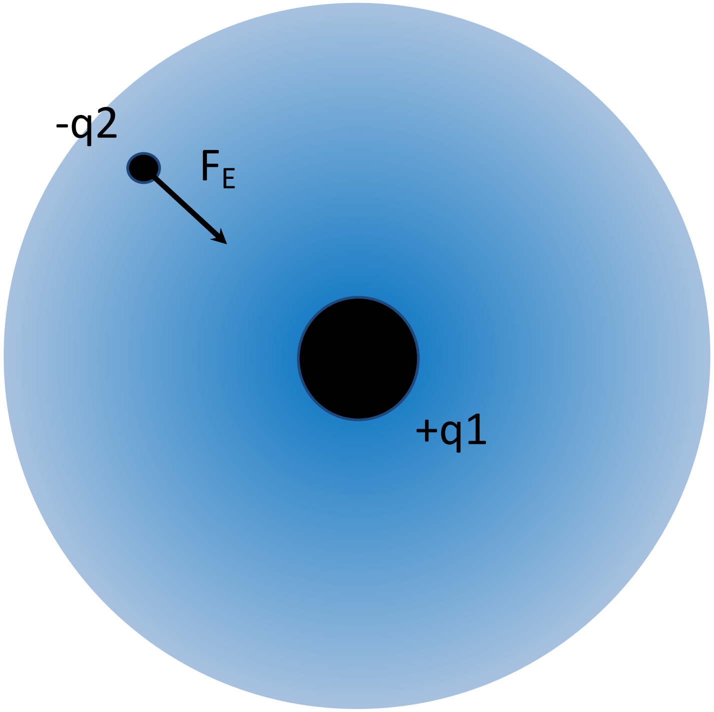


Electric Field



$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

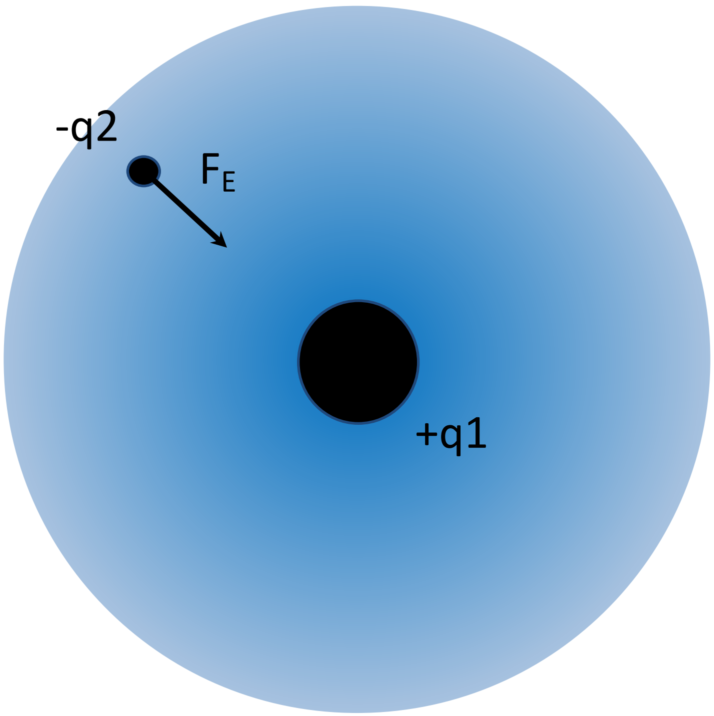
Electric Field



$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\vec{F}_E = q_2\left(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}\right)$$

Electric Field

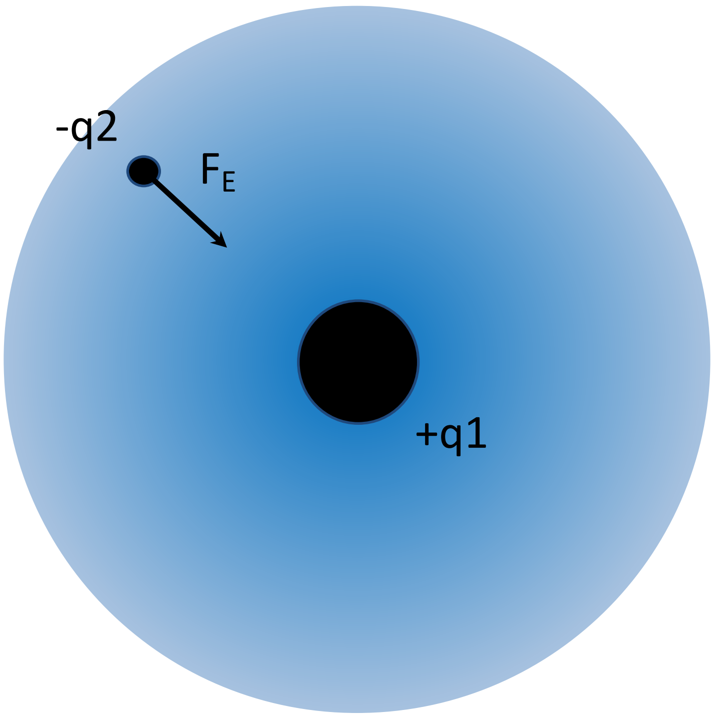


$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\vec{F}_E = q_2\left(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}\right)$$

$$\vec{F}_E = q_2\vec{E}$$

Electric Field



$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\vec{F}_E = q_2\left(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}\right)$$

$$\vec{F}_E = q_2\vec{E}$$

Where:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$$

Electric field due to a point charge

Electric Field

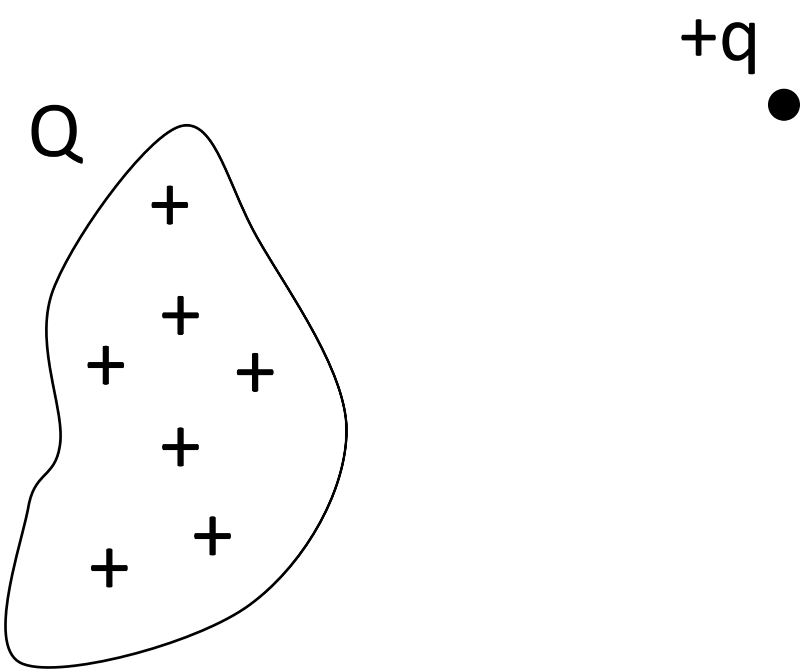
- Physically, the electric field can be defined as a vector force field due to charges / charged objects.

- Mathematically, the electric field can be defined as electric force per unit charge, that is:

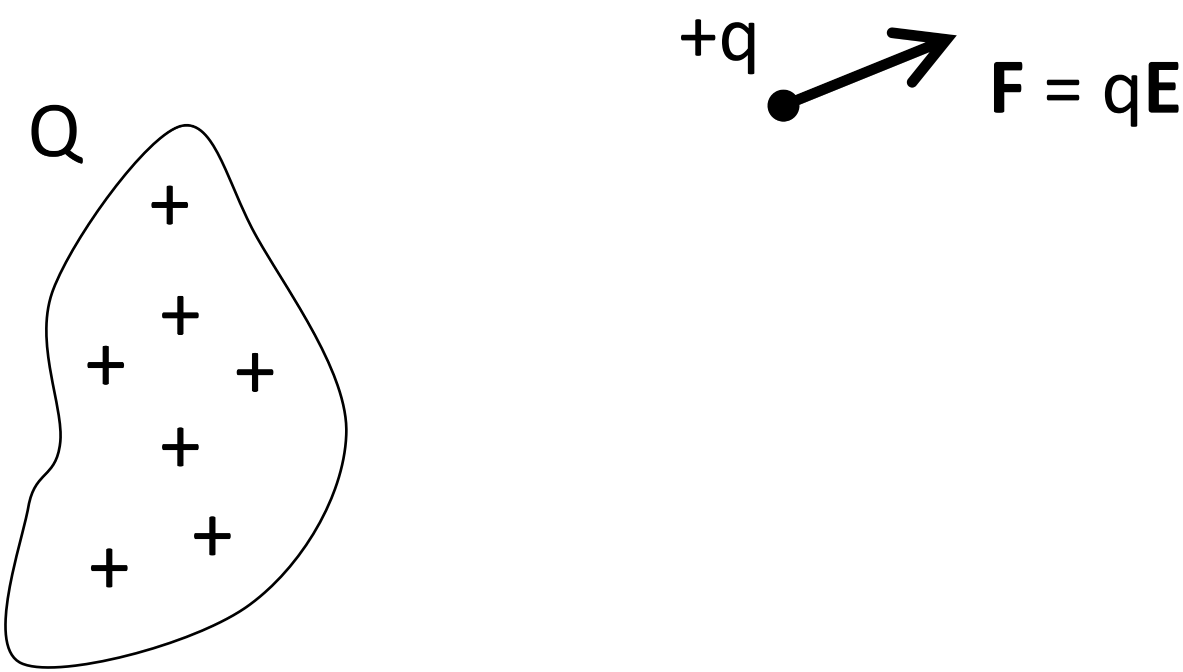
$$\boxed{\vec{E} = \frac{\vec{F}_E}{q}}$$

Electric Field Lines

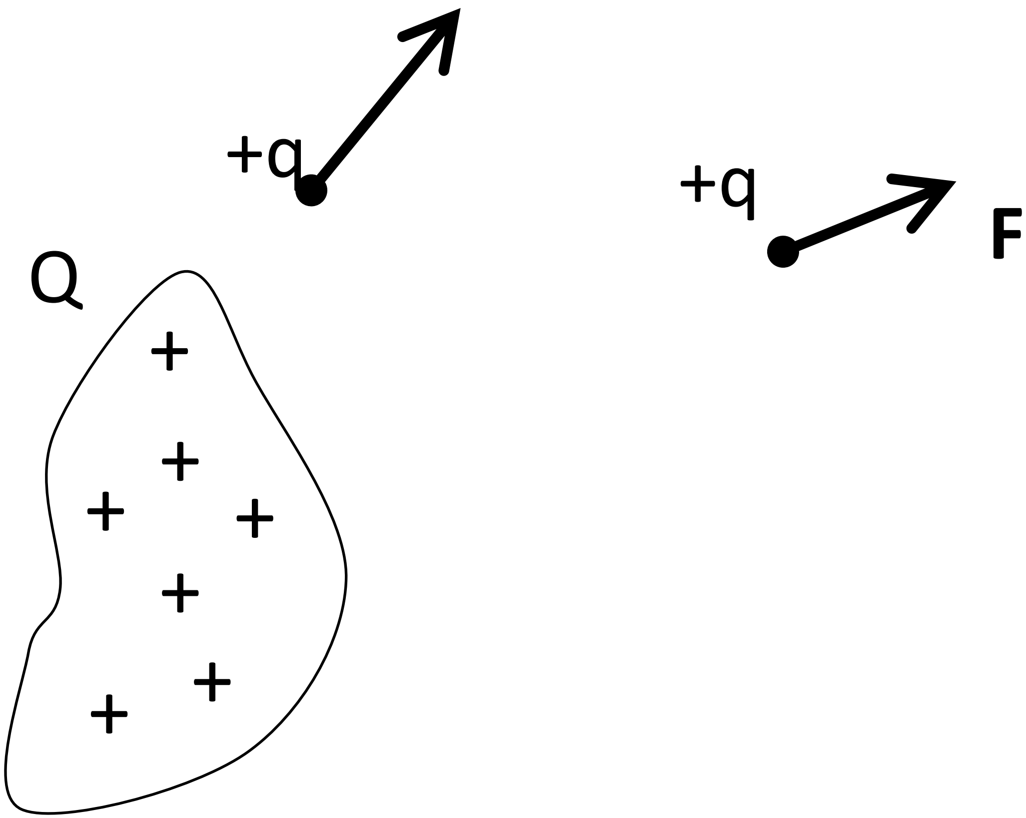
Electric Field Lines



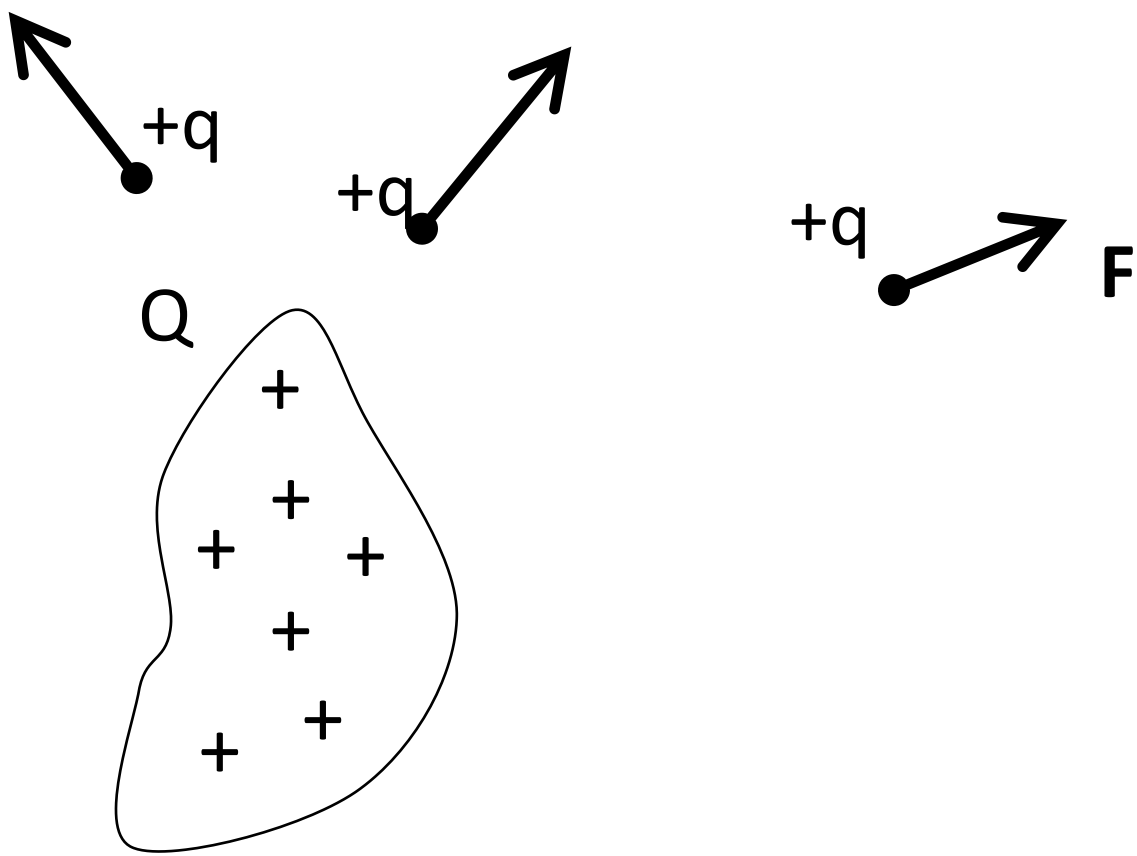
Electric Field Lines



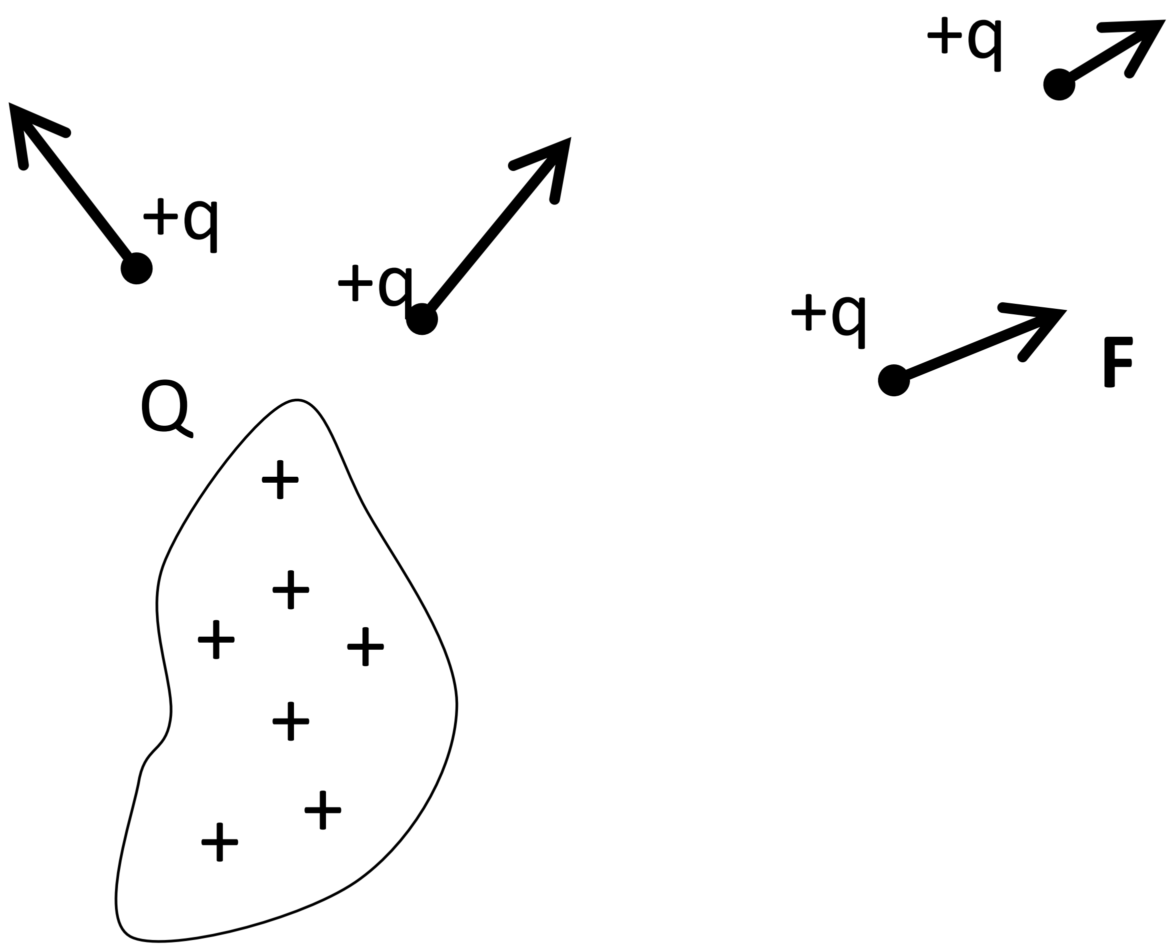
Electric Field Lines



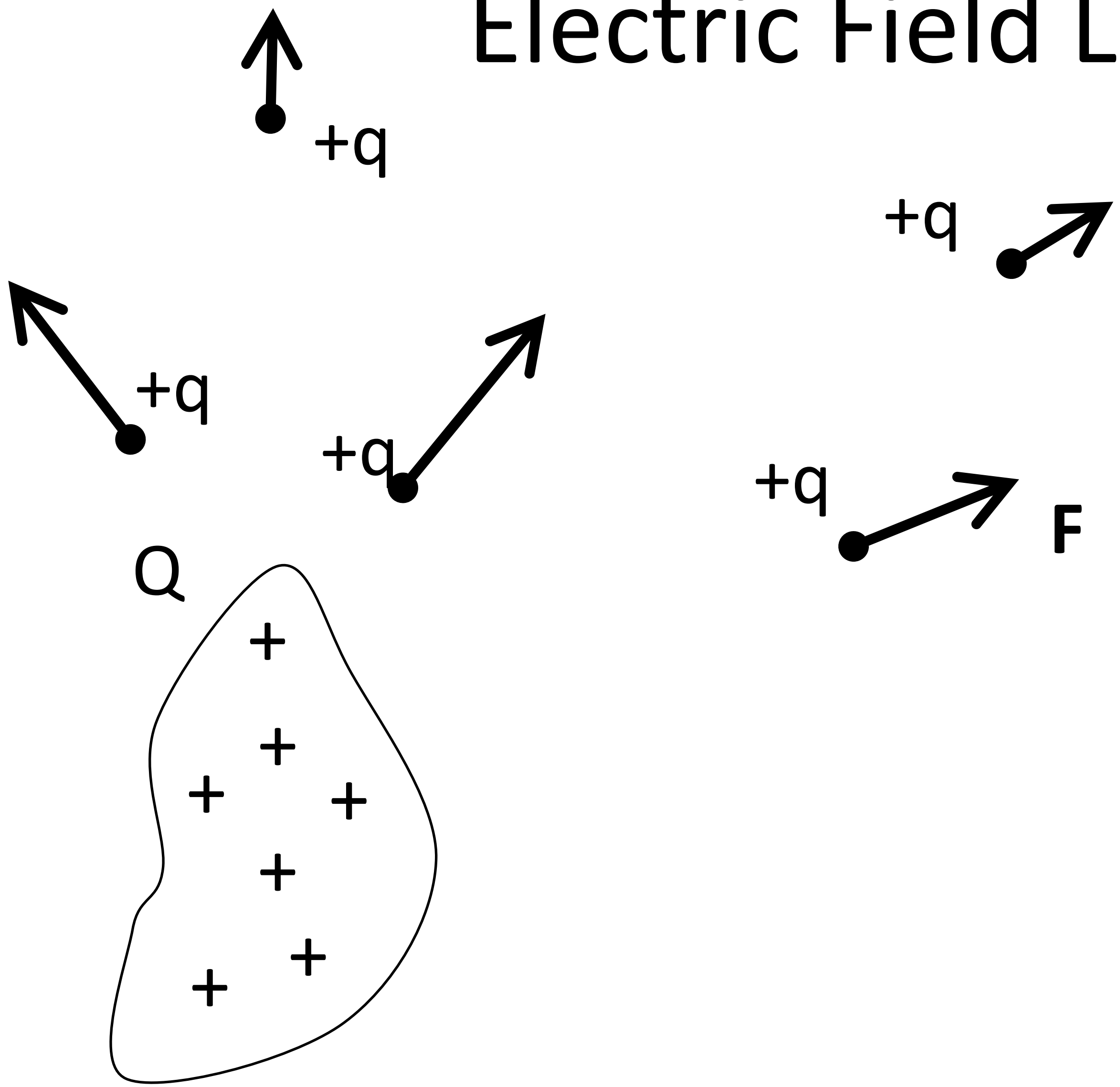
Electric Field Lines



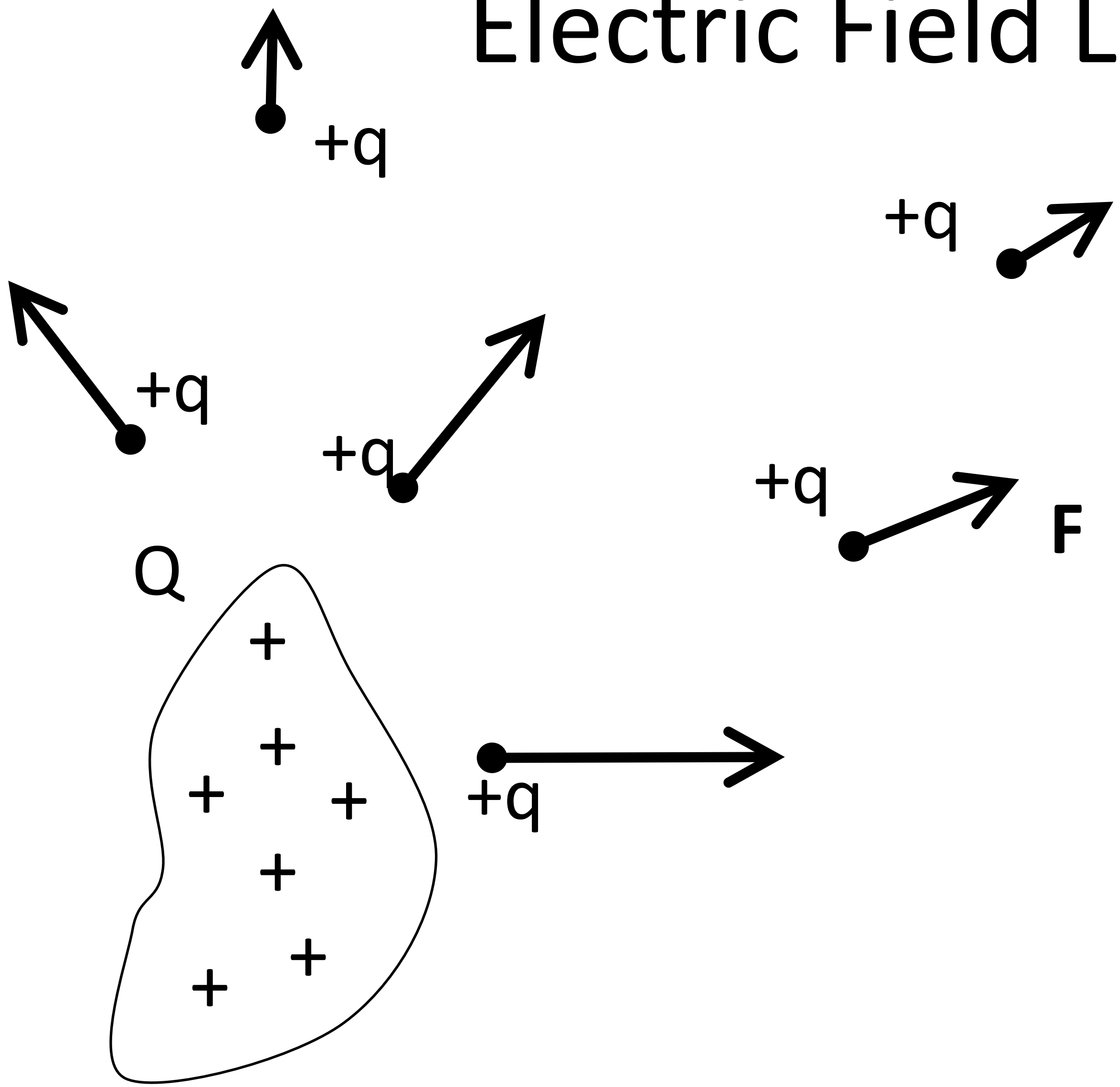
Electric Field Lines



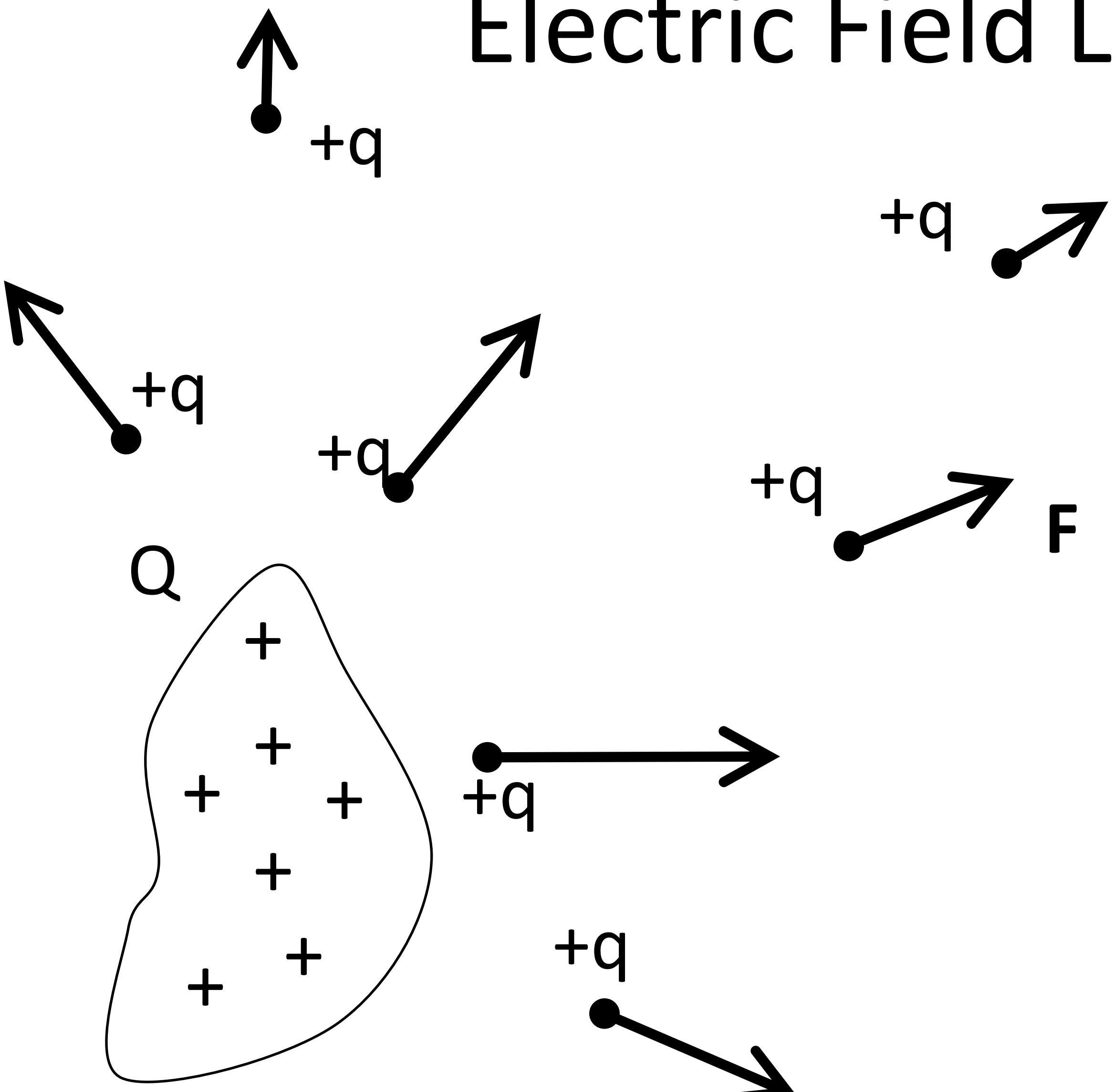
Electric Field Lines



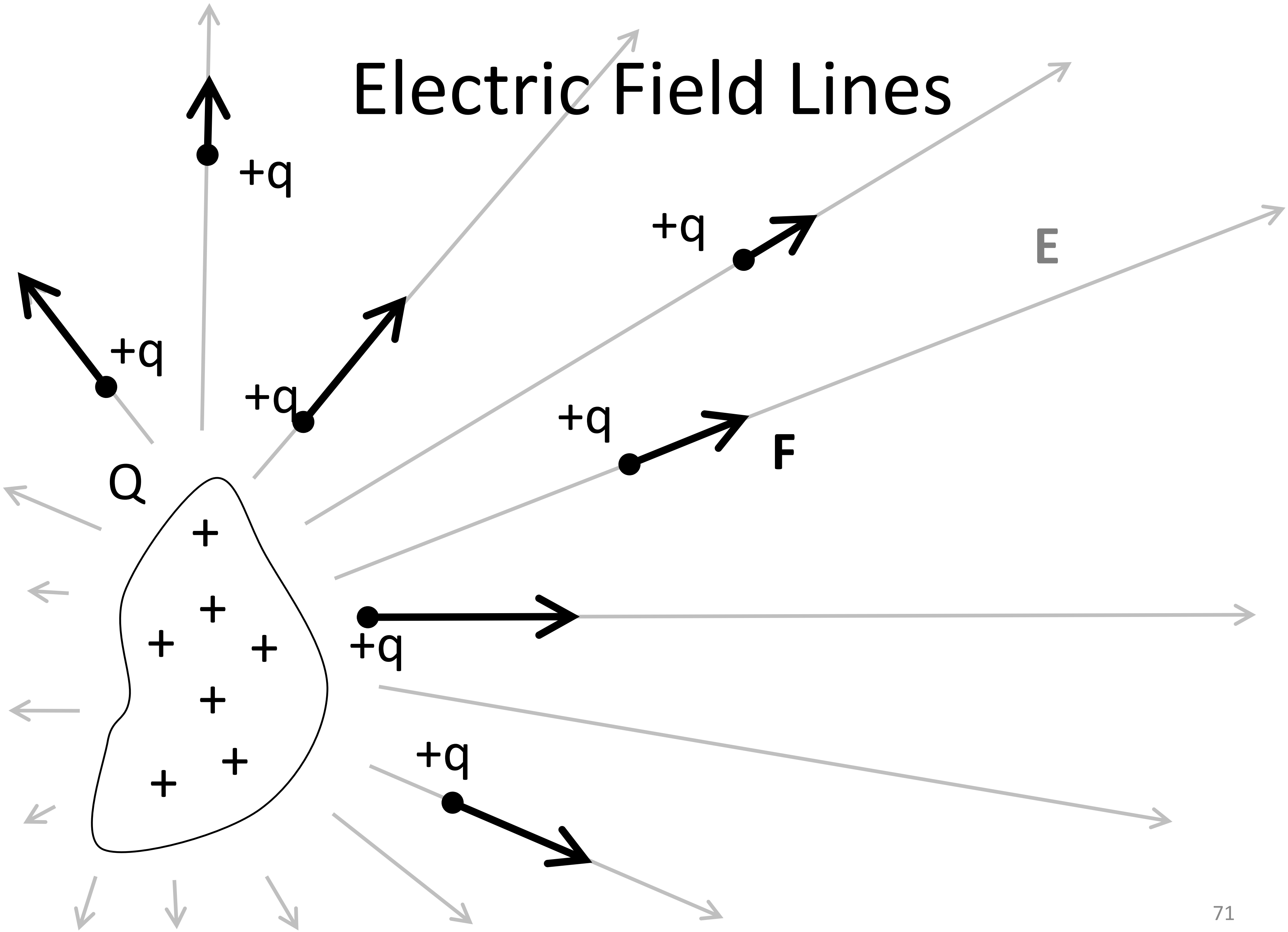
Electric Field Lines



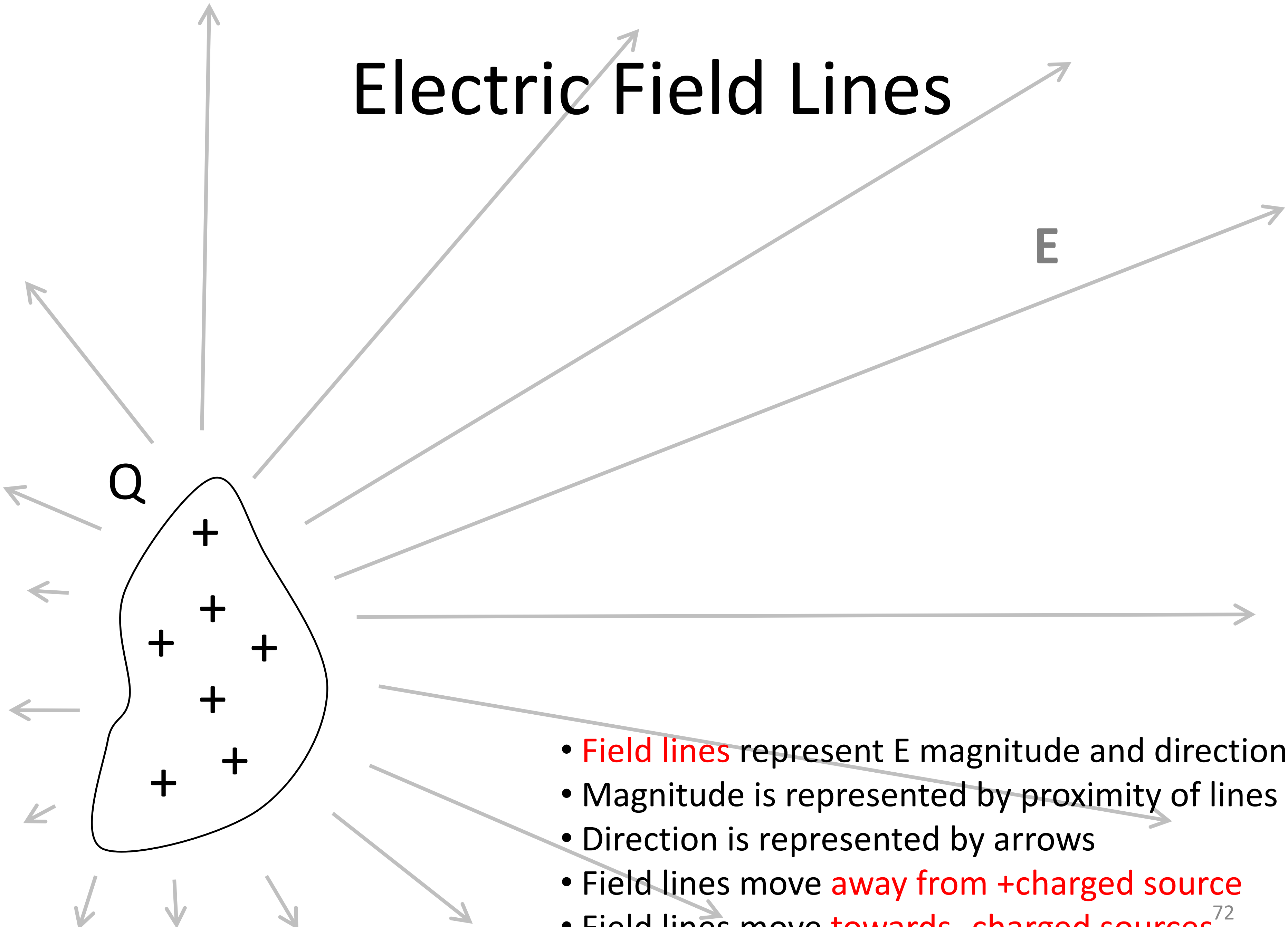
Electric Field Lines



Electric Field Lines

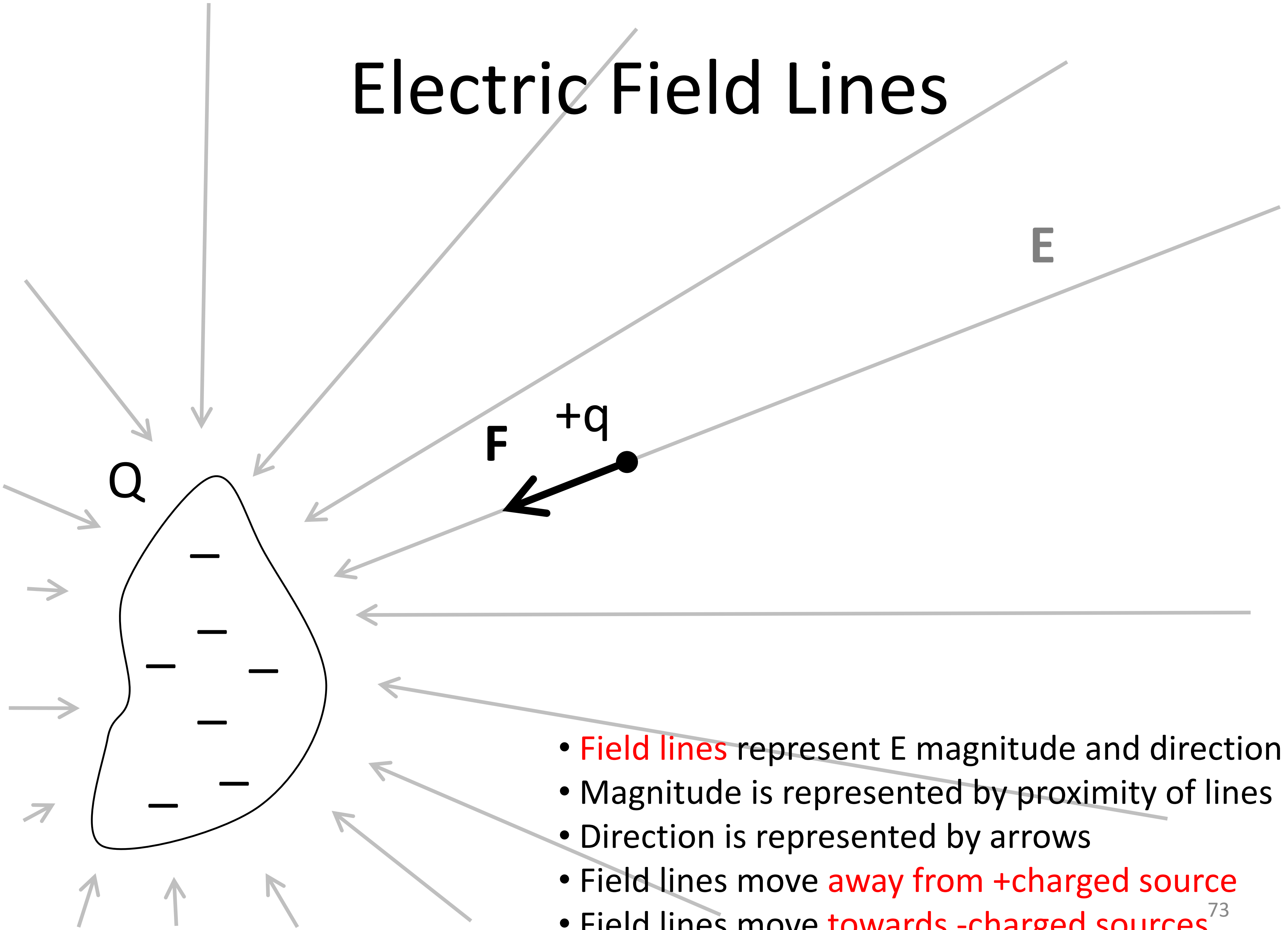


Electric Field Lines



- Field lines represent E magnitude and direction
- Magnitude is represented by proximity of lines
- Direction is represented by arrows
- Field lines move away from +charged source
- Field lines move towards -charged sources

Electric Field Lines



- Field lines represent E magnitude and direction
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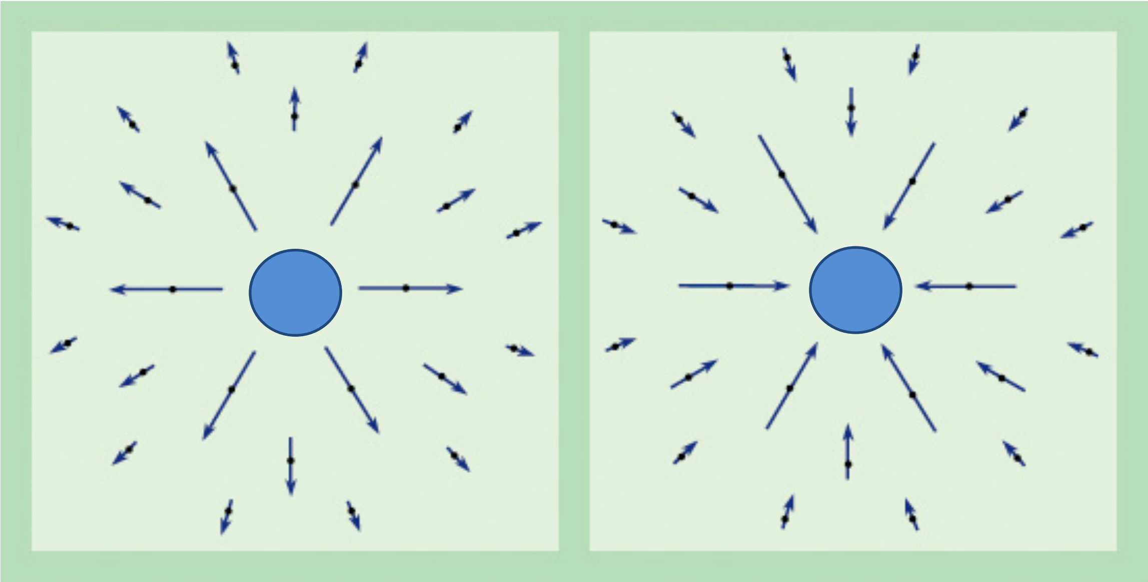
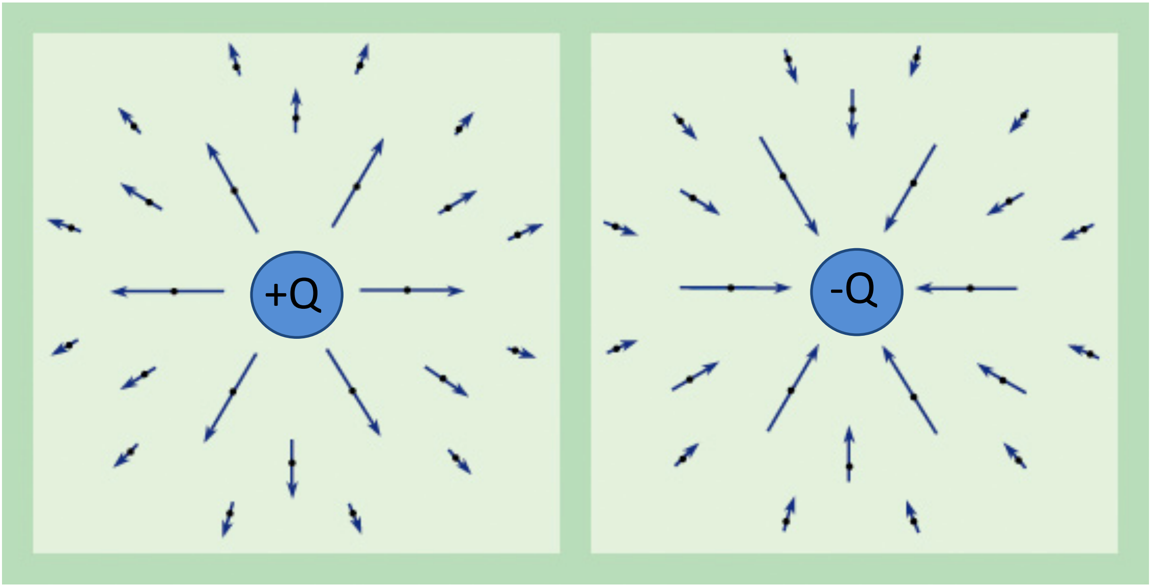
Point Charge: Summary

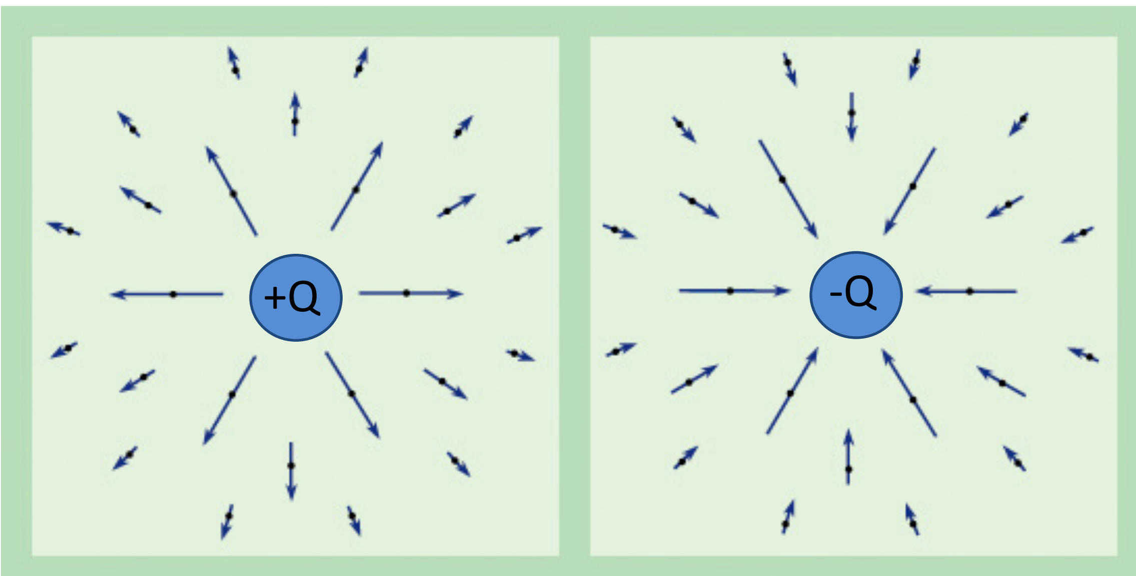
Electric Field of a Point Charge: Direction



Electric Field of a Point Charge: Direction



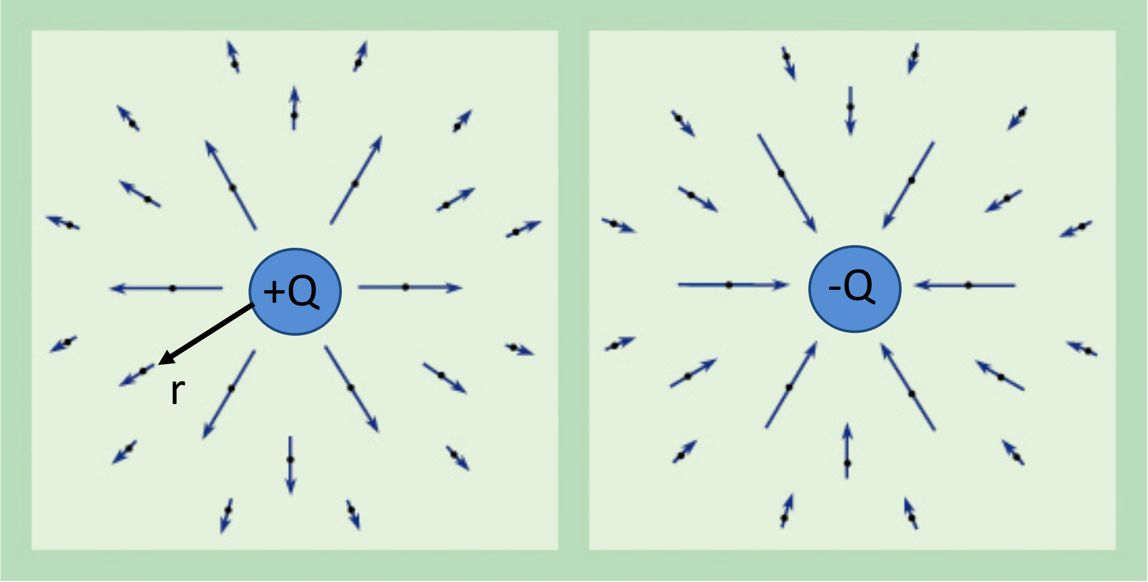
Electric Field of a Point Charge: Direction



Electric field points:

- Away from positive charges
- Towards negative charges

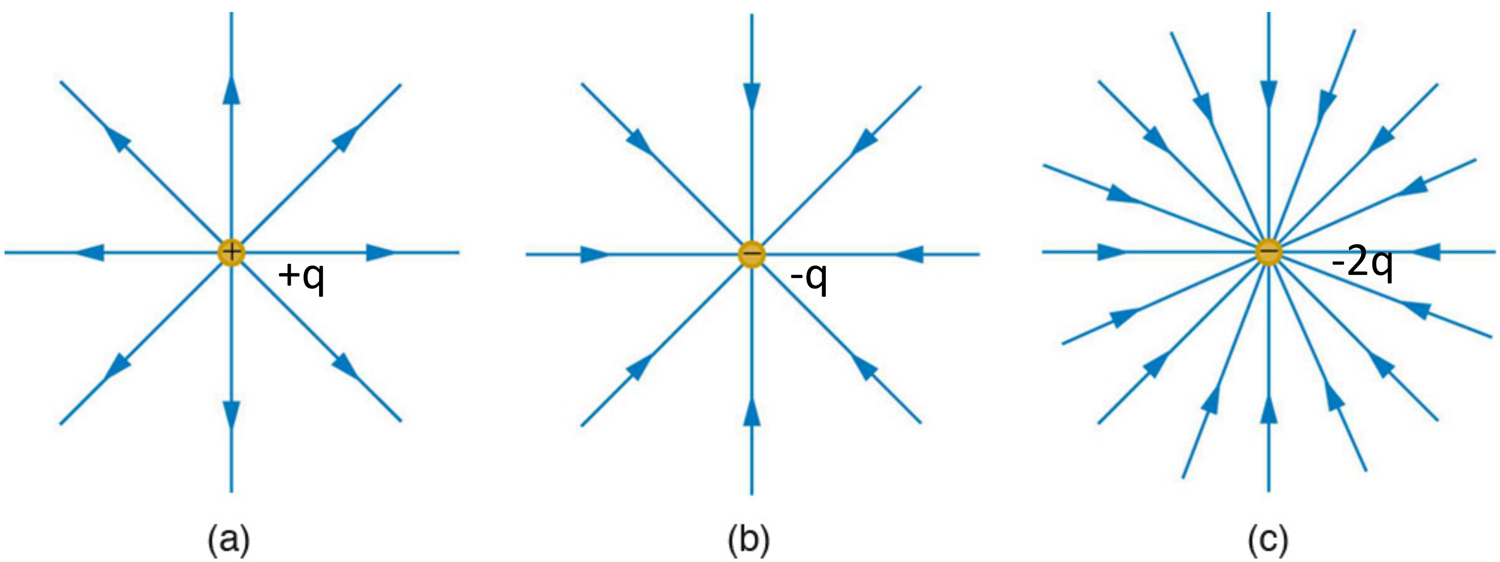
Electric Field of a Point Charge: Magnitude



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Electric Field of a Point Charge: Field Lines

- Electric field lines represent the magnitude and direction of **E**
- **Direction of E**: Direction of the line arrows
- **Magnitude of E**: Proximity of the lines
- Electron field lines depart from positive charges
- Electron field lines arrive at negative charges



Field Line Rules

- Field lines begin and end on charges or infinity
- Arrows point:

  - away from positive charges

  - toward negative charges
- No field line can cross another field line
- The density of field lines is proportional to the magnitude of the field at that point
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of that charge