

PHYS2326 Lecture #11

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Goals for this lecture

- Summary of Lecture 10
- Quantify the energy stored in capacitors
- Understand the role of dielectrics in capacitors

Chapter 24

Capacitors

- A capacitor is an electrical device capable of storing electric charge and electric potential energy.
- The quantity called capacitance (C) indicates the ability of a capacitor to store energy.

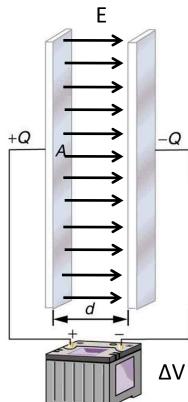
$$C = \frac{Q}{V}$$

Unit:
$$\frac{C}{V} = Farad = F$$

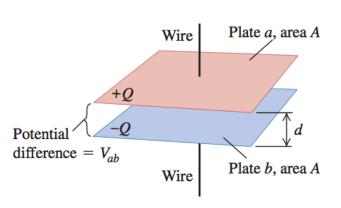
Capacitance

• The capacitance (C) tells how much charge Q (or electric field) can be stored for a given potential difference V (we start dropping the Δ).

$$Q = CV$$



Parallel-Plate Capacitor



$$C = \frac{Q}{V_{ab}} = ?$$

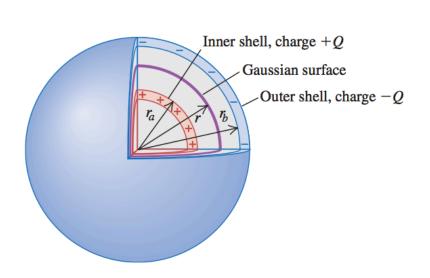
$$V_{ab} = Ed = \frac{\sigma}{\epsilon_0} d$$
$$Q = \sigma A$$

$$C = \frac{A}{d}\epsilon_0$$

Depends only on:

- Shape and size of plates
- Insulating material between them

Spherical Capacitor



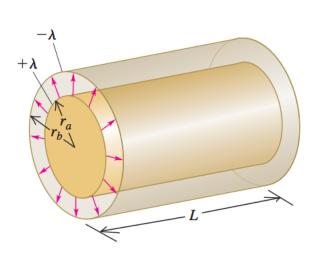
$$C = \frac{Q}{V_{ab}} = ?$$

$$Q = Q$$

$$\begin{split} V_{sphere}(r) &= \frac{Q}{4\pi\epsilon_0 r} \\ V_{ab} &= V_a - V_b = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right] \\ V_{ab} &= \frac{Q}{4\pi\epsilon_0} \left[\frac{r_b - r_a}{r_a r_b} \right] \end{split}$$

$$C = 4\pi\epsilon_0 \left[\frac{r_a r_b}{r_b - r_a} \right]$$

Cylindrical Capacitor



$$C = \frac{Q}{V_{ab}} = ?$$

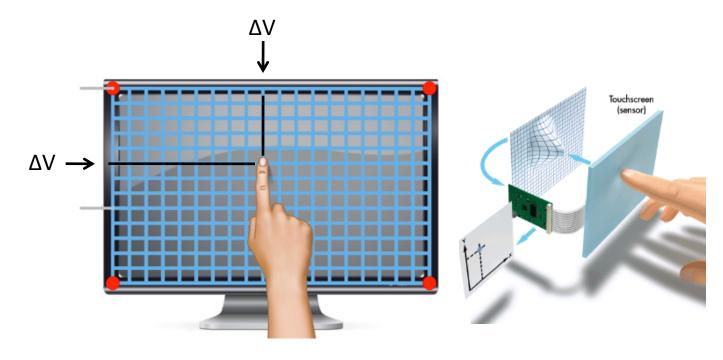
$$Q = \lambda L$$

$$V_{cylinder}(r) = \frac{\lambda}{2\pi\epsilon_0} ln \left[\frac{r_0}{r} \right]$$

$$V_{ab} = V_a - V_b = \frac{\lambda}{2\pi\epsilon_0} ln \left[\frac{r_b}{r_a} \right]$$

$$C = \frac{2\pi\epsilon_0 L}{\ln[r_b/r_a]}$$

Application: Capacitive touchscreen panel



$$C = \frac{Q}{V}$$

$$C = \frac{A}{d}\epsilon_0 \text{ Parallel plate}$$

$$V = \left(\frac{Q}{A\epsilon_0}\right)d$$



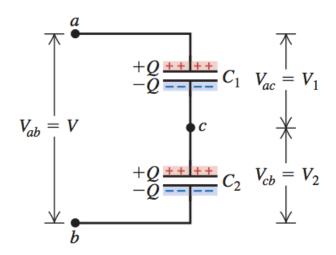
Capacitors in series and parallel

 Capacitors are made for a few specific capacitance and voltage values.

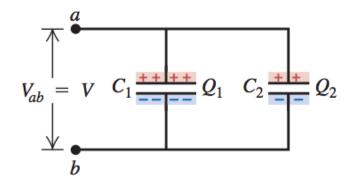
 A lot of times, these values do not meet the requirements of our projects.

 By combining capacitors in series and/or parallel connections we can obtain the capacitance values needed.

Capacitors in series and parallel

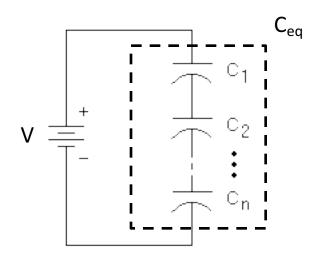


Capacitors in Series: Same Q



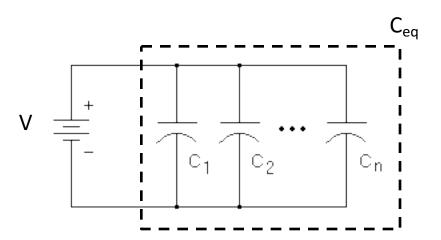
Capacitors in Parallel: Same V

Capacitors in series

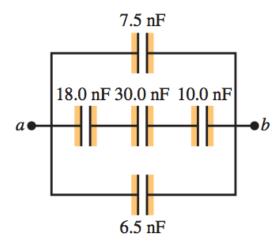


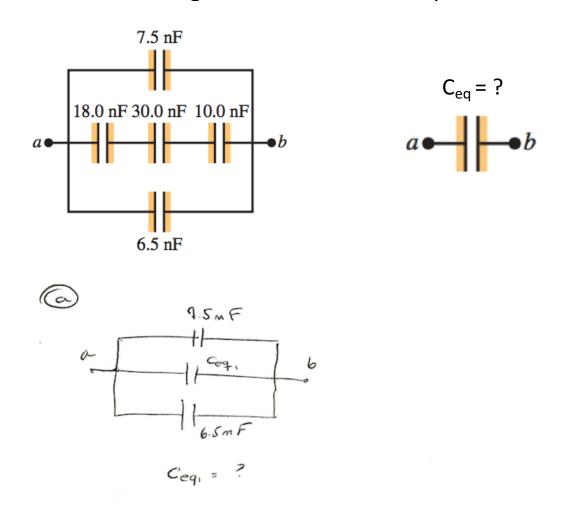
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

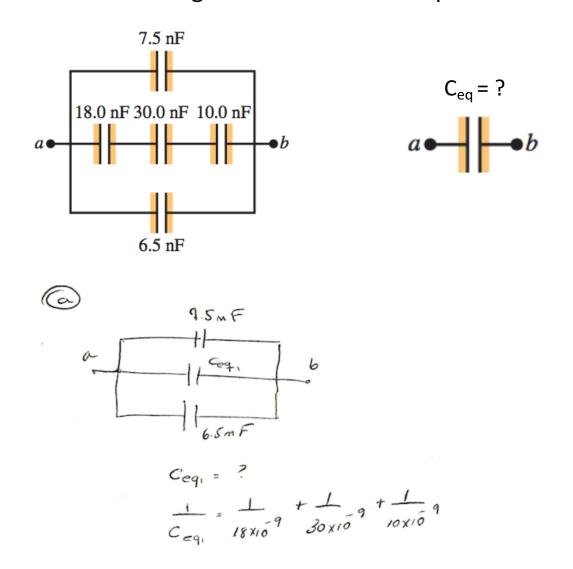
Capacitors in parallel

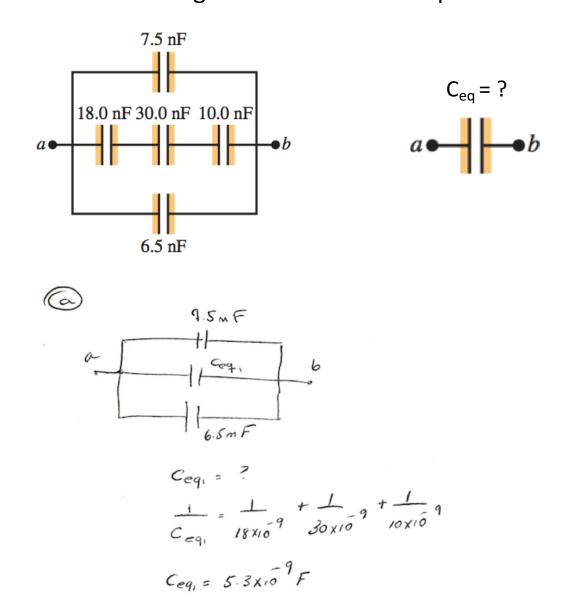


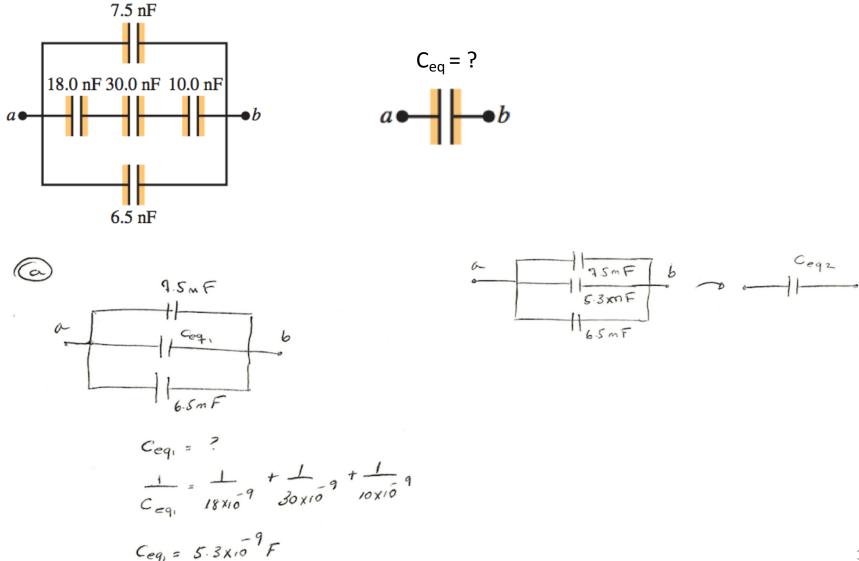
$$C_{eq} = C_1 + C_2 + C_3 + ... + C_n$$

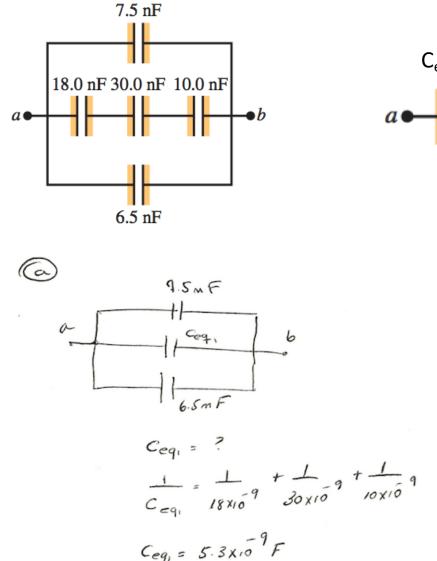


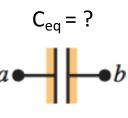


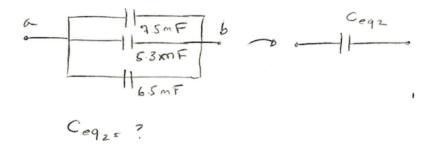


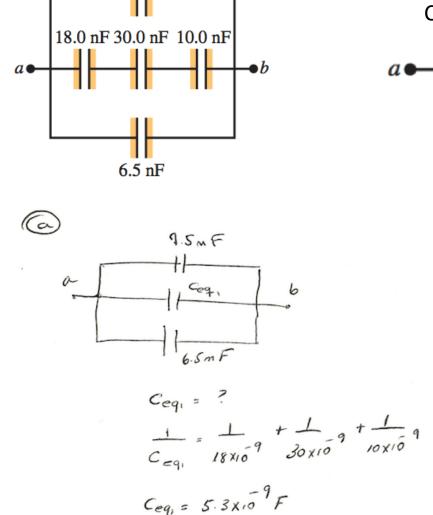




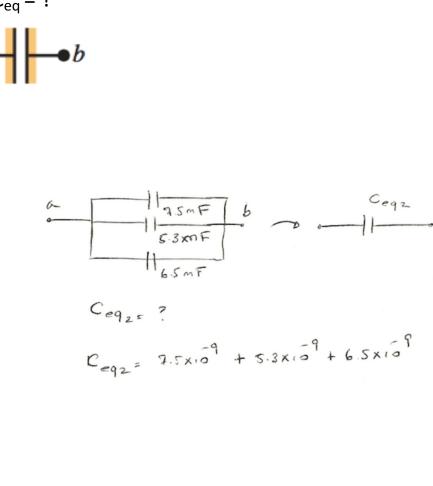


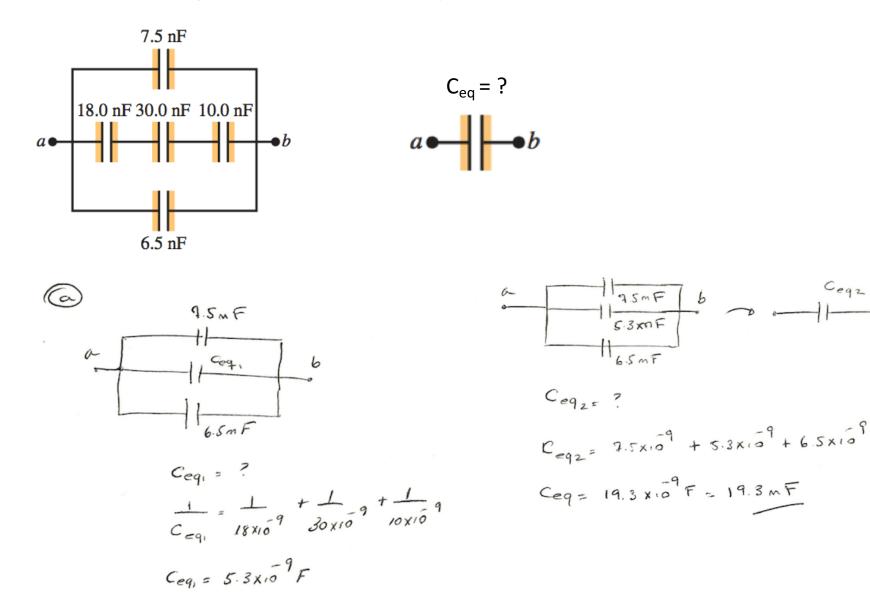


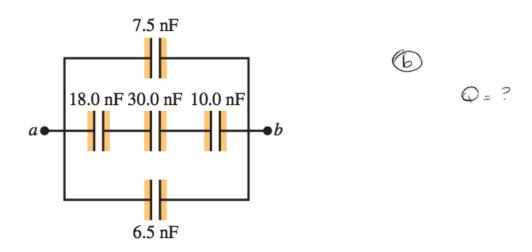




7.5 nF

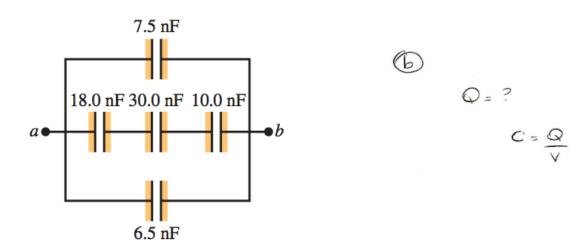




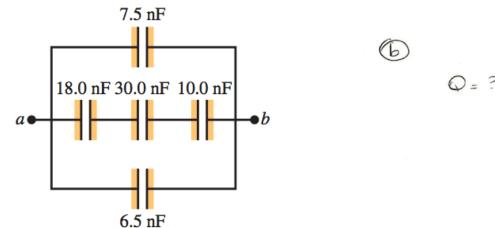


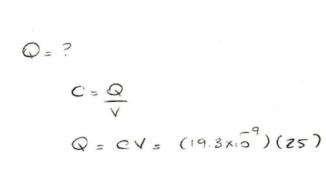
$$C_{eq} = 19.3 \text{ nF}$$





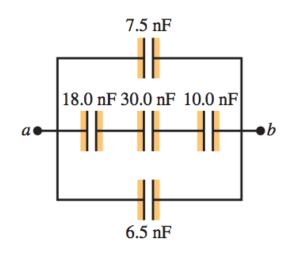
$$C_{eq} = 19.3 \text{ nF}$$

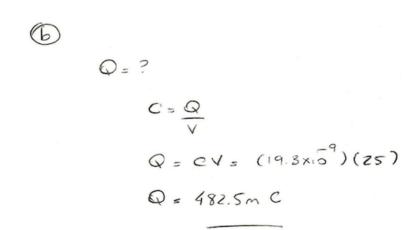




$$C_{eq} = 19.3 \text{ nF}$$

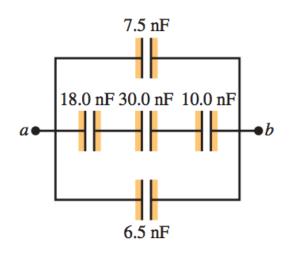
$$a \longrightarrow b$$

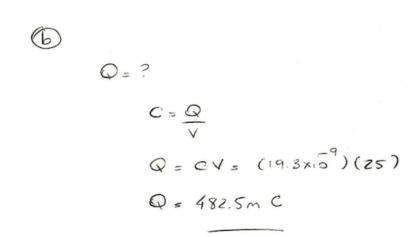




$$C_{eq} = 19.3 \text{ nF}$$

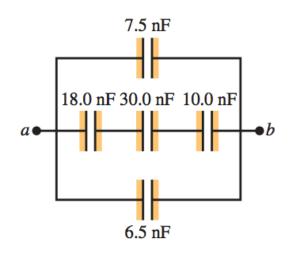
$$a \longrightarrow b$$





$$C_{eq} = 19.3 \text{ nF}$$

$$a \longrightarrow b$$



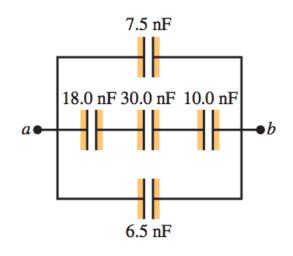
(b)

$$Q = ?$$

 $C = \frac{Q}{V}$
 $Q = CV = (19.3 \times 10^{9})(25)$
 $Q = 482.5 m C$

$$C_{eq} = 19.3 \text{ nF}$$

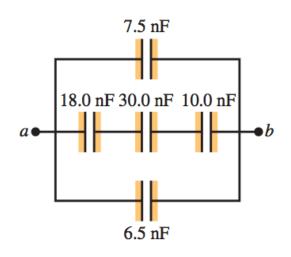
$$a \longrightarrow b$$



(b) Q = ? $Q = QV = (19.3 \times 10^{9})(25)$

$$C_{eq} = 19.3 \text{ nF}$$

© Q=?



$$Q = ?$$

$$Q = QV = (19.3 \times 10^{9})(25)$$

$$C_{eq} = 19.3 \text{ nF}$$

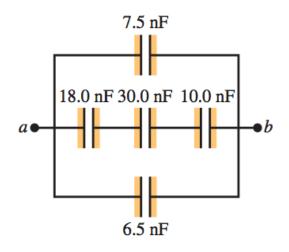
$$a \longrightarrow b$$

$$Q = ?$$

$$Q = Q$$

$$Q = CV = (6.5 \times 10^{-9})(25)$$

$$Q = 162.5 mC$$



$$C = \frac{Q}{V}$$
 $Q = QV = (19.3 \times 10^{-9})(25)$
 $Q = 482.5 m C$

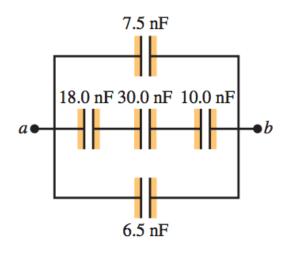
$$C_{eq}$$
 = 19.3 nF



©
$$Q = ?$$

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(b)

$$Q = ?$$

 $Q = QV = (19.3 \times 10^{9})(25)$

$$C_{eq} = 19.3 \text{ nF}$$

$$a \longrightarrow b$$

$$Q = ?$$

$$Q = Q$$

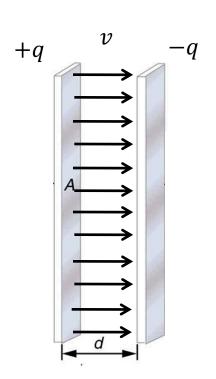
$$Q = CV = (6.5 \times 10^{-9})(25)$$

$$Q = 162.5 mC$$

How much energy does a capacitor store?

Well, it should equal the work done to charge your capacitor

At any given time during charging:



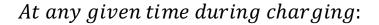
$$U = W = \int_{0}^{Q} v dq$$

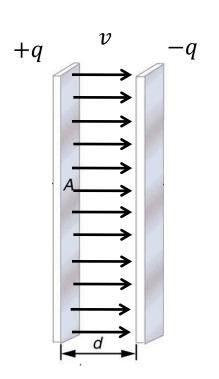
 $v = instantaneous\ potential$

q = instantaneous charge

V = final potential

 $Q = final\ charge$

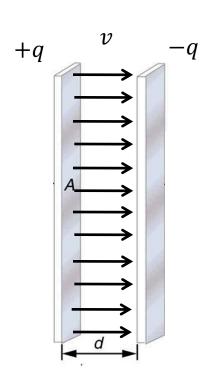




$$U = W = \int_{0}^{Q} v dq$$

$$\to C = \frac{q}{v} \longrightarrow v = \frac{q}{C}$$

At any given time during charging:

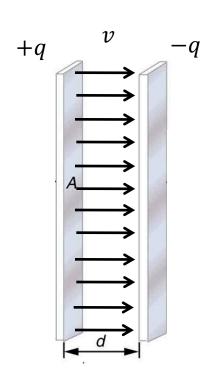


$$U = W = \int_{0}^{Q} v dq$$

$$\rightarrow C = \frac{q}{v} \longrightarrow v = \frac{q}{C}$$

$$U = W = \int_{0}^{Q} \left(\frac{q}{C}\right) dq$$

At any given time during charging:



$$U = W = \int_{0}^{Q} v dq$$

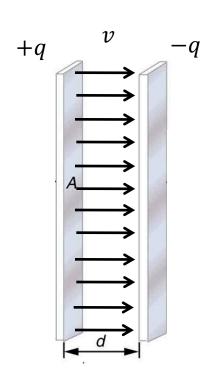
$$\rightarrow C = \frac{q}{v} \longrightarrow v = \frac{q}{C}$$

$$U = W = \int_{0}^{Q} \left(\frac{q}{C}\right) dq$$

$$U = W = \frac{Q^{2}}{2C}$$

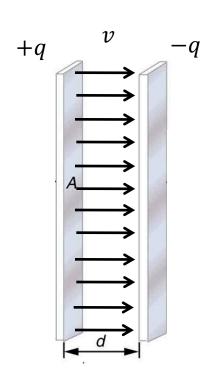
Energy stored in capacitors

At any given time during charging:



Energy stored in capacitors

At any given time during charging:



$$U = W = \int_{0}^{Q} v dq$$

$$\to C = \frac{q}{v} \longrightarrow v = \frac{q}{C}$$

$$U = W = \int_{0}^{Q} \left(\frac{q}{C}\right) dq$$

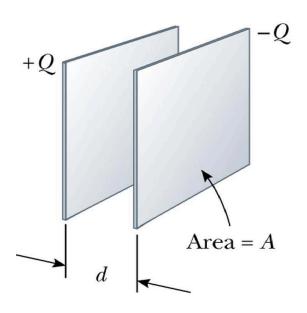
$$U = W = \frac{Q^2}{2C}$$

$$\rightarrow C = \frac{Q}{V}$$

$$U = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$$

Energy Density (u)

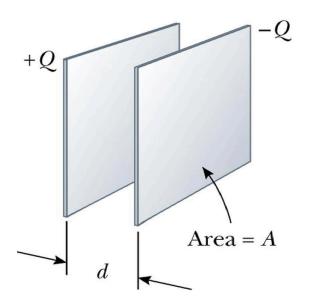
$$u = \frac{Energy}{Volume}$$



Energy Density (u)

$$u = \frac{Energy}{Volume}$$

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$



Energy Density (u)

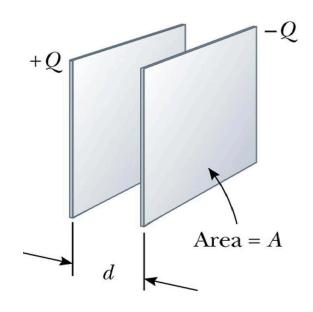
$$u = \frac{Energy}{Volume}$$

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$

But:

$$C = \frac{A}{d}\epsilon_0$$
$$V = Ed$$

$$u = \frac{1}{2}E^2\epsilon_0$$



Derived assuming E-field created by parallel plates but works for any E-field configuration

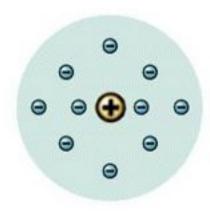
Conductors:

- Electrons are free to move (free charges)
- Electrons redistribute to cancel any E-field inside conductor

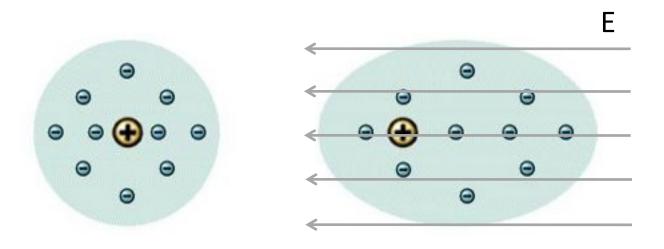
Dielectrics:

- Electrons are not free to move (bound charges)
- An external electric field will distort the electron distribution around the nucleus or align polar molecules

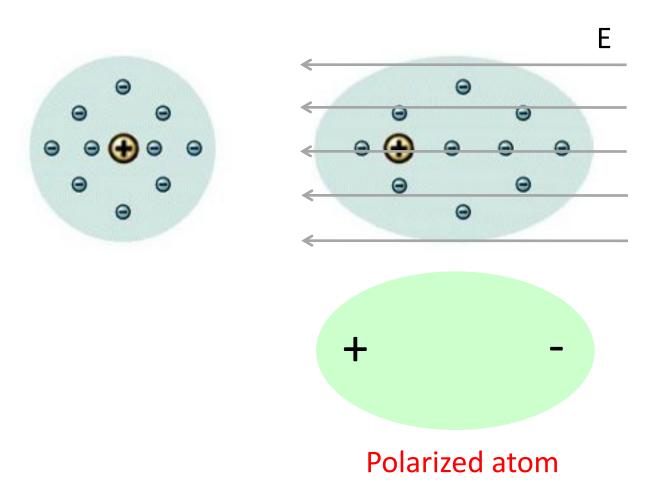
Dielectrics: Polarization



Dielectrics: Polarization



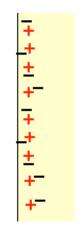
Dielectrics: Polarization



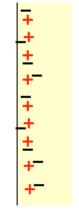




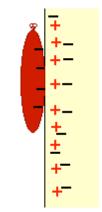




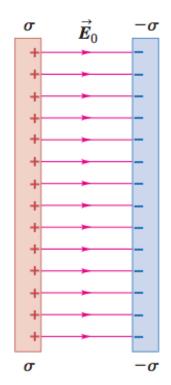




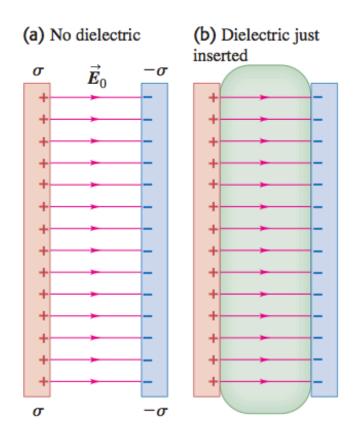




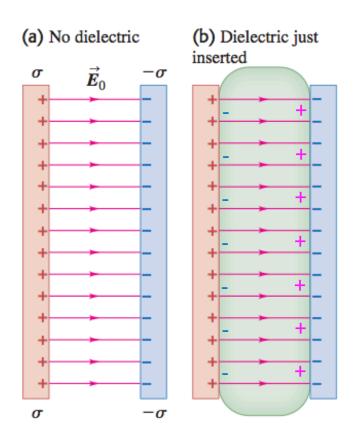
(a) No dielectric



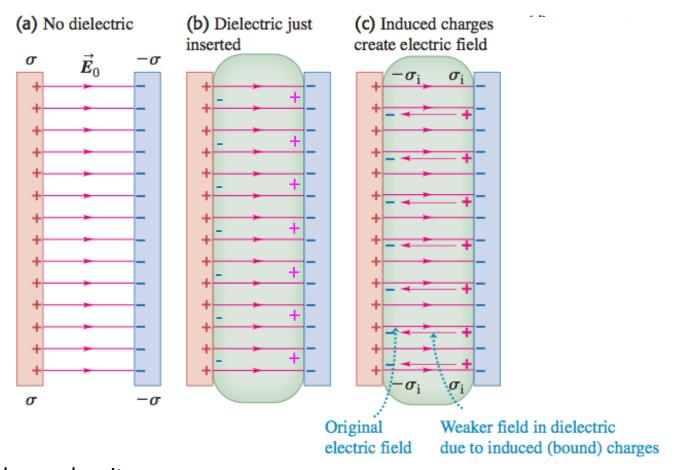
 σ = surface charge density



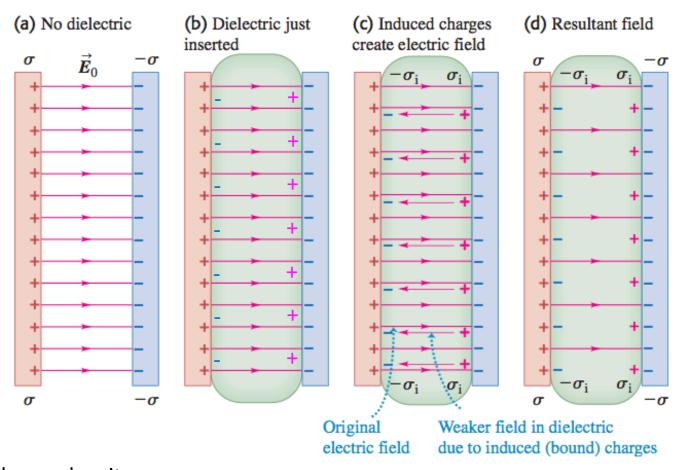
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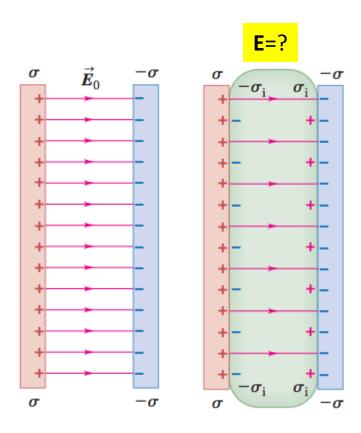
 σ = surface charge density

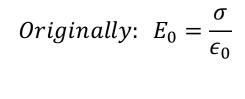


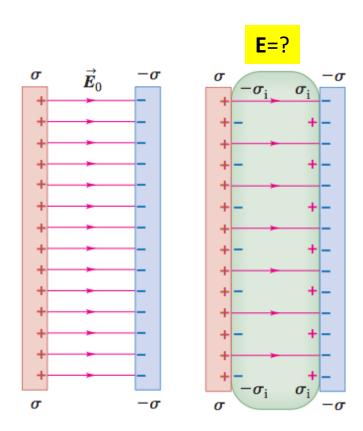
 σ = surface charge density σ_i = induced surface charge density

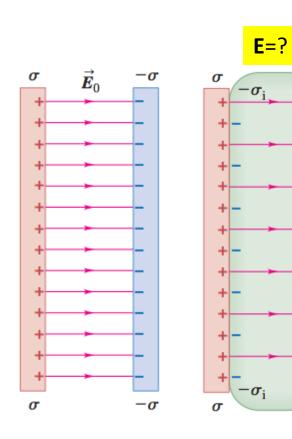


 σ = surface charge density σ_i = induced surface charge density





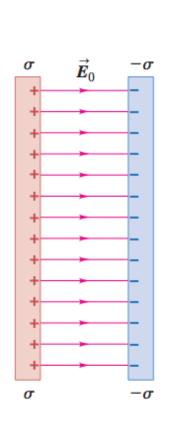


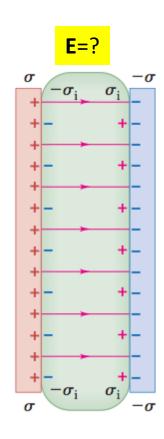


Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

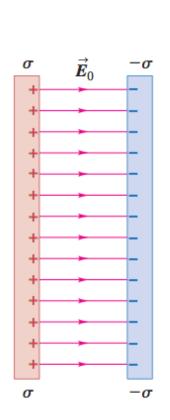


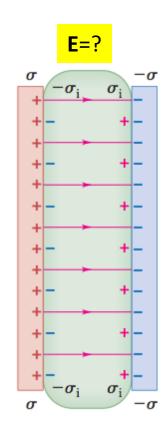


Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$
$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right]$$



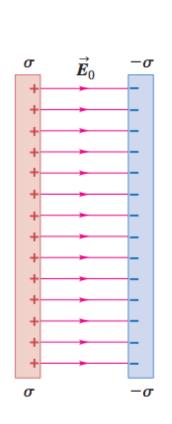


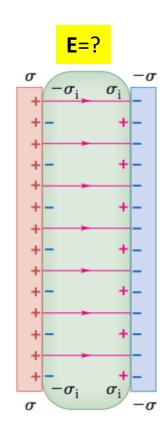
Originally:
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$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right] = \frac{\sigma}{\epsilon_0 \left[\frac{\sigma}{\sigma - \sigma_i} \right]}$$





Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

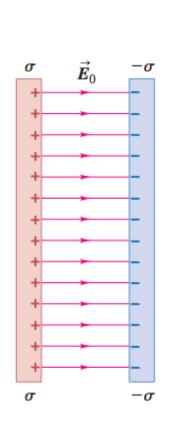
$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

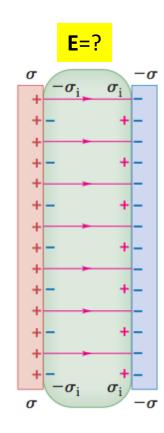
$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right] = \frac{\sigma}{\epsilon_0 \left[\frac{\sigma}{\sigma - \sigma_i} \right]}$$

$$E = \frac{\sigma}{\epsilon_0 K}$$

Where:

$$K = \frac{\sigma}{\sigma - \sigma_i} \qquad (Dieletric \ constant)$$





Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

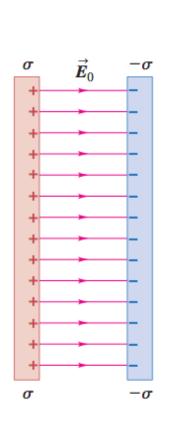
$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

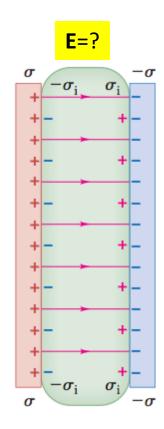
$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right] = \frac{\sigma}{\epsilon_0 \left[\frac{\sigma}{\sigma - \sigma_i} \right]}$$

$$E = \frac{\sigma}{\epsilon_0 K} = \frac{E_0}{K}$$

Where:

$$K = \frac{\sigma}{\sigma - \sigma_i} \qquad (Dieletric \ constant)$$





Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

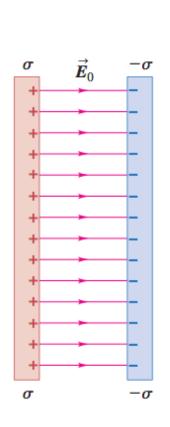
$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right] = \frac{\sigma}{\epsilon_0 \left[\frac{\sigma}{\sigma - \sigma_i} \right]}$$

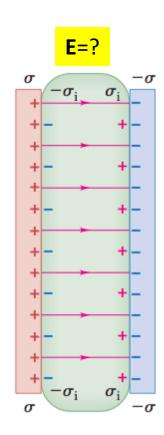
$$E = \frac{\sigma}{\epsilon_0 K} = \frac{E_0}{K} = \frac{\sigma}{\epsilon}$$

Where:

$$K = \frac{\sigma}{\sigma - \sigma_i} \qquad (Dieletric \ constant)$$

$$\epsilon = K\epsilon_0$$
 (Dieletric permittivity)





Originally:
$$E_0 = \frac{\sigma}{\epsilon_0}$$

With dielectric:

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} \left[\frac{\sigma - \sigma_i}{\sigma} \right] = \frac{\sigma}{\epsilon_0 \left[\frac{\sigma}{\sigma - \sigma_i} \right]}$$

$$E = \frac{\sigma}{\epsilon_0 K} = \frac{E_0}{K} = \frac{\sigma}{\epsilon}$$

Where:

$$K = \frac{\sigma}{\sigma - \sigma_i}$$
 (Dieletric constant)

$$\epsilon = K\epsilon_0$$

 $\epsilon = K\epsilon_0$ (Dieletric permittivity)

 How does the dielectric affect the capacitor besides making it more expensive?

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

$$V = Ed$$

$$C = \frac{Q}{V}$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

$$V = Ed = \frac{E_0}{K}d$$

$$C = \frac{Q}{V}$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V}$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V} = \frac{QK}{V_0}$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V} = \frac{QK}{V_0} = KC_0$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

Capacitor with dielectric:

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V} = \frac{QK}{V_0} = KC_0$$

The dielectric increases the capacitance by a factor of K:

$$C = KC_0$$

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

Capacitor with dielectric:

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V} = \frac{QK}{V_0} = KC_0$$

The dielectric increases the capacitance by a factor of K:

$$C = KC_0$$

$$U = \frac{CV^2}{2}$$

For a given voltage V, energy will increase with C

Capacitor without dielectric:

$$C_0 = \frac{Q}{V_0}$$
$$V_0 = E_0 d$$

Capacitor with dielectric:

$$V = Ed = \frac{E_0}{K}d = \frac{V_0}{K}$$

$$C = \frac{Q}{V} = \frac{QK}{V_0} = KC_0$$

Summary of relationships:

$$K = \frac{\sigma}{\sigma - \sigma_i} = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0}$$

The dielectric increases the capacitance by a factor of K:

$$C = KC_0$$

$$U = \frac{CV^2}{2}$$

For a given voltage V, energy stored will increase with C

$$C = \frac{Q}{V}$$

$$C = KC_0$$

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas [®]	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310