Convex Hull Data Structure

December 6th, 2023

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What is a Convex Hull?

- Convex Hull is an algorithm which reads through a set of points and solves for the points which gives the smallest polygon that encloses all the given points.

- The points outputted by convex hull connects all the outer points in which you can draw a straight line between each neighboring point and not take any right turns.

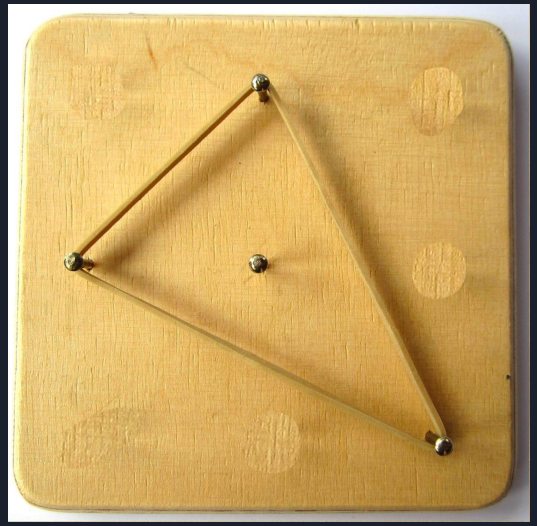
- Examples of real world applications of convex hull:
  - Computing the diameter, width and smallest polygon enclosing shape.
  - Modeling and visualization, for graphs and maps.
  - Automated detection systems such as in cars, drones, and planes,.

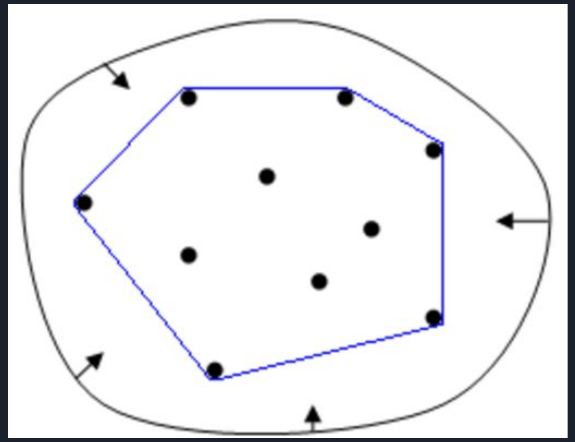
Visualizing Convex Hull

One way to better visualize convex hull can be to think of convex hull as an elastic band.

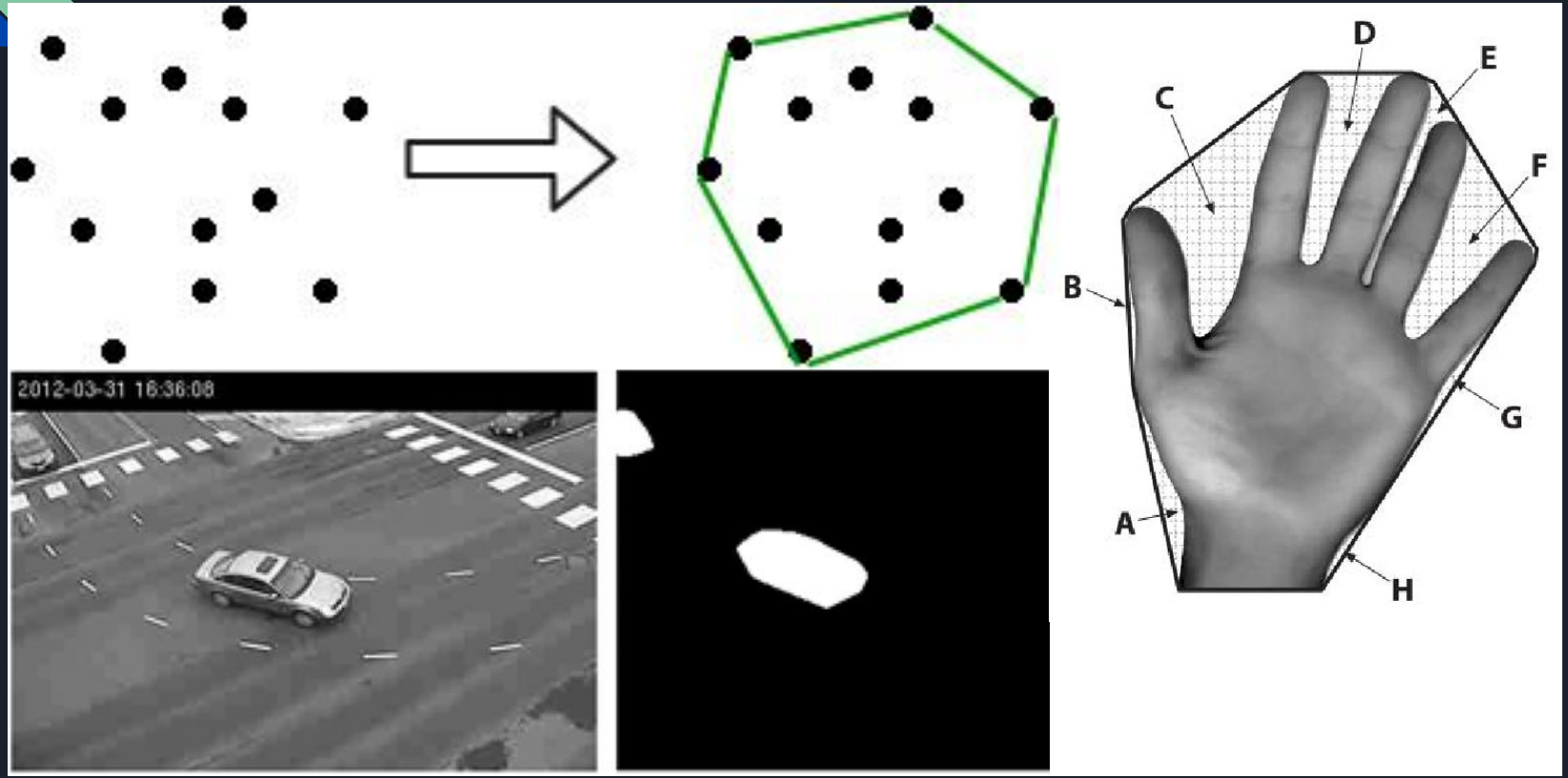
If each point was to represent a nail on a board, to solve for the convex hull, you would have to place an elastic band around all the points.

As you let go of the elastic band, the elastic band forms the smallest shape that encloses all of the given points or nails in this case.





Examples of Convex Hull



A few examples of Convex Hull Algorithms

Graham scan

Jarvis March

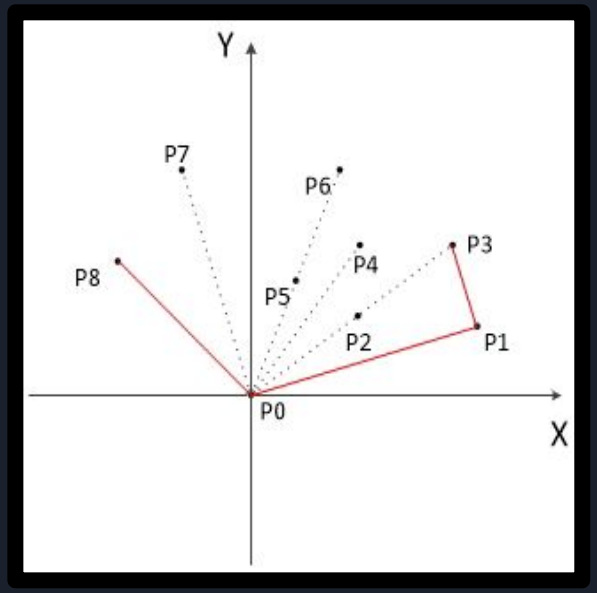
Quickhull

Monotone Chain

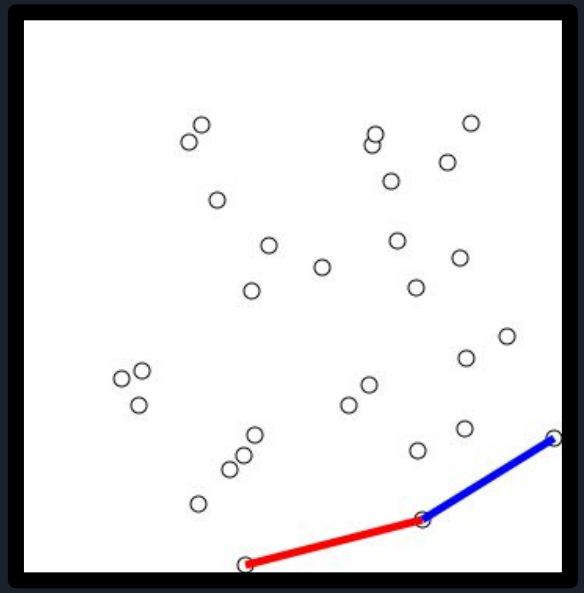
Kirkpatrick–Seidel algorithm

Chan's algorithm

How Graham Scan Algorithm Works



1. First find the lowest y coordinate out of the given points.
2. Next sort the rest of the given points by their polar angle with reference from the lowest y coordinate.
3. Then increment through the given points and push back each point that turns left and pop back each point that turns right.
4. Repeat this process until the algorithm returns to start.



Graham Scan

Advantages:

Relatively easy to implement

Efficiency with a time complexity of O(nlog(n))

Practical for sorted data with time complexity O(n) *If data is sorted based on (x,y)*

Disadvantages:

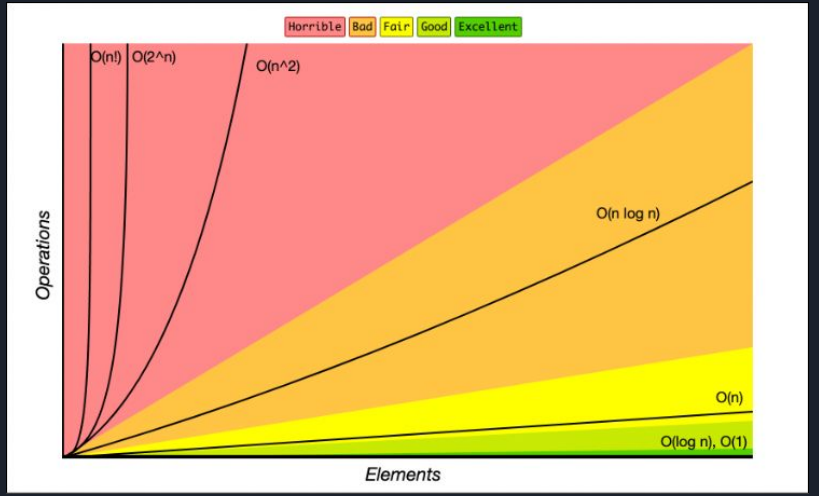
Must sort data before computing the convex

Requires use of polar points

Reference point selection dependency

Time Complexity

	Best Case	Average Case	Worst Case
Graham's Scan	O(n log n)	O(n log n)	O(n log n)
Jarvis March	O(nh)	O(nh)	O(n^2)
Quickhull	O(n log n)	O(n log n)	O(n^2)
Monotone Chain	O(n log n)	O(n log n)	O(n log n)
Kirkpatrick–Seidel algorithm	O(n log h)	O(n log h)	O(n log n)
Chan's Algorithm	O(n log h)	O(n log h)	O(n log h)



n represents the number of input points
h represents the number of convex hull points

Code/Implementation

Convex Hull Project GitHub Link

Our Report

Questions?

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