

# HW1

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## 0.1

Given a proposition, convert it to CNF.

Proposition:

$$\phi = \neg(\neg r \rightarrow \neg(p \wedge q))$$

Substitutions:

$$\begin{aligned} a_\phi &= \neg(\neg r \rightarrow \neg a_{p \wedge q}) \\ a_{p \wedge q} &= (p \wedge q) \end{aligned}$$

Conjunction:

$$a_\phi \wedge (a_\phi \leftrightarrow \neg(\neg r \rightarrow \neg a_{p \wedge q})) \wedge (a_{p \wedge q} \leftrightarrow (p \wedge q))$$

where

$$(a_\phi \leftrightarrow \neg(\neg r \rightarrow \neg a_{p \wedge q})) = (\neg a_\phi \vee \neg r) \wedge (\neg a_\phi \vee a_{p \wedge q}) \wedge (r \vee \neg a_{p \wedge q} \vee a_\phi)$$

and

$$(a_{p \wedge q} \leftrightarrow (p \wedge q)) = (\neg a_{p \wedge q} \vee p) \wedge (\neg a_{p \wedge q} \vee q) \wedge (\neg p \vee \neg q \vee a_{p \wedge q})$$

so

$$\begin{aligned} \phi &= (\neg a_\phi \vee \neg r) \wedge (\neg a_\phi \vee a_{p \wedge q}) \wedge (r \vee \neg a_{p \wedge q} \vee a_\phi) \wedge \\ &\quad (\neg a_{p \wedge q} \vee p) \wedge (\neg a_{p \wedge q} \vee q) \wedge (\neg p \vee \neg q \vee a_{p \wedge q}) \end{aligned}$$