Algorithms and Data Structures
Jacobs University Bremen
Dr. Florian Rabe

Homework 2
given: 2017-02-14

Homework 2

You have to submit your solutions as announced in the lecture.
Unless mentioned otherwise, all problems are due 2017-02-23
There will be no deadline extensions unless mentioned otherwise in the lecture.

Problem 2.1 *Fibonacci*

Points: 4+4+2

In the programming language of your choice:

1. Implement the exponential, the linear, and the inexact algorithm given in the lecture notes for the function $fib(n)$. Use arbitrary precision integer arithmetic for the exponential and the linear algorithm, and use floating point numbers for the inexact one.
2. Write a program to determine the largest n for which $fib(n)$ returns a result in at most 10 seconds. Determine the value n for the exponential and for the linear algorithm.
3. Write a program to determine the smallest n for which the inexact algorithm returns an incorrect result. Run your program to find that value.

(Of course, the answers to the last two questions will depend on your programming language and computer.)

Problem 2.2 *Asymptotic Notation: Examples*

Points: (2+1+1)+2+2+2

For each of the following functions f and g, determine whether $f \in O(g)$, $g \in O(f)$, neither, or both.

1. $f(n) = 7n$ and $g(n) = n^7$
2. $f(n) = n^2/\log_2 n$ and $g(n) = n$
3. $f(n) = 27\log_2 n$ and $g(n) = (\log_2(4n))^2$
4. $f(n) = \sqrt{n}$ and $g(n) = 5n^{0.5} + 7n^{0.3} + 11\log_2 n$

Show your work for the first problem, i.e.,

- give k and N such that the property in the definition of $\varolessthan$ holds, or
- show that no such k and N exist.

Problem 2.3 *Asymptotic Notation: Theory*

Points: 2+3

Prove that $\varolessthan$ is

1. reflexive
2. transitive

Hint: To prove $f \varolessthan g$, you have to give N and k such that $f(n) \leq k \cdot g(n)$ for all $n > N$.
For reflexivity, that is easy.
For transitivity, assume k_1 and N_1 exist for $f \varolessthan g$ and k_2 and N_2 exist for $g \varolessthan h$, then give k and N for $f \varolessthan h$.