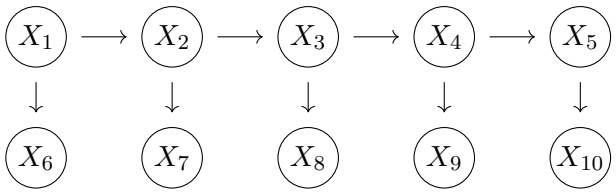
DS-GA 1018: Homework 1

Due Friday September 29th at 5:00 pm

Problem 1 (5 points): Consider the graphical model below:



i. (1 points): Write down a full factorization of $p(X_{1:10})$ implied by the graphical model. Your factorization should be as simple as possible (*simplicity is measured by the number of $X_\star$ terms that show up in your final expression*).

ii. (1 points): What is the Markov boundary of X_4?

iii. (1 points): What is the Markov boundary of X_8?

iv. (2 points): Write down a full factorization of $p(X_{1:3}, X_{5:10}|X_4)$ that is as simple as possible.

Problem 2 (13 points): Consider the two following MA(4) processes:

$$X_t = W_t + \theta_3 W_{t-3} + \theta_4 W_{t-4} + \theta_c \tag{1}$$
$$Y_t = W_t + \theta_1 W_{t-1} + \theta_4 W_{t-4}, \tag{2}$$

where W_t is drawn from $\mathcal{N}(0, \sigma_W^2)$ and all the $\theta_\star$ are constants.

i. (2 points): What is the mean, $\mu_X(t)$, of the $\{X_t\}$ process? Justify your answer.

ii. (3 points): What is the covariance, $\gamma_X(t,s)$, of the $\{X_t\}$ process?

iii. (1 points): Is $\{X_t\}$ drawn from a weakly stationary process?

iv. (5 points): What is the cross-covariance, $\gamma_{X,Y}(t,s)$, between X_t and Y_s?

v. (2 points): Is it possible for $\gamma_X(t,t) = 0$? If so, what is one value of $\theta_1, \theta_2, \theta_3, \theta_4$ that satisfied this? Limit yourself to the real numbers.

Problem 3 (10 points): Consider the following two models:

$$X_t = 2.5X_{t-1} - X_{t-2} + W_t - 2W_{t-1} \tag{3}$$
$$Y_t = 0.7Y_{t-1} + 0.3Y_{t-2} + W_t - 0.4W_{t-1}, \tag{4}$$

where W_t is drawn from $\mathcal{N}(0, \sigma_W^2)$.

i. (3 points): Identify $\{X_t\}$ as ARMA(p,q). *Watch out for parameter redundancy.*

ii. (1 points): Is $\{X_t\}$ causal? Justify your answer.

iii. (1 points): Is $\{X_t\}$ invertible? Justify your answer.

iv. (3 points): Identify $\{Y_t\}$ as ARMA(p,q).

v. (1 points): Is $\{Y_t\}$ causal? Justify your answer.

vi. (1 points): Is $\{Y_t\}$ invertible? Justify your answer.

Problem 4 (7 points): Consider an AR(2) process with the equations:

$$P(B) = (1 - 0.4B)(1 + 0.4B). \tag{5}$$

Please answer the following questions:

i. (1 points): Is the process causal?

ii. (6 points): What is the correlation function $\rho(t, t+h) = \rho(h)$? *Hint: remember that* $\rho(0) = 1$.