In [ ]:
```
import torch
import numpy as np
import matplotlib.pyplot as plt
```

Problem 6.1

In [ ]:
```
# Problem 6.1
mean = 0.0
std = 1.0
N = 50
sample_generator = torch.distributions.Normal(mean, std)
samples = sample_generator.sample((N,))

# P6.1 Compute the square of each sample and record the average
samples.square().mean()
```

Out[ ]:
```
tensor(0.7462)
```

Problem 6.2

In [ ]:
```
# P6.2 Compute the estimator T times
T = 50
estimators = [(sample_generator.sample((N,))).square().mean().item() for i i
mean = np.mean(estimators)
std = np.std(estimators)
std
```

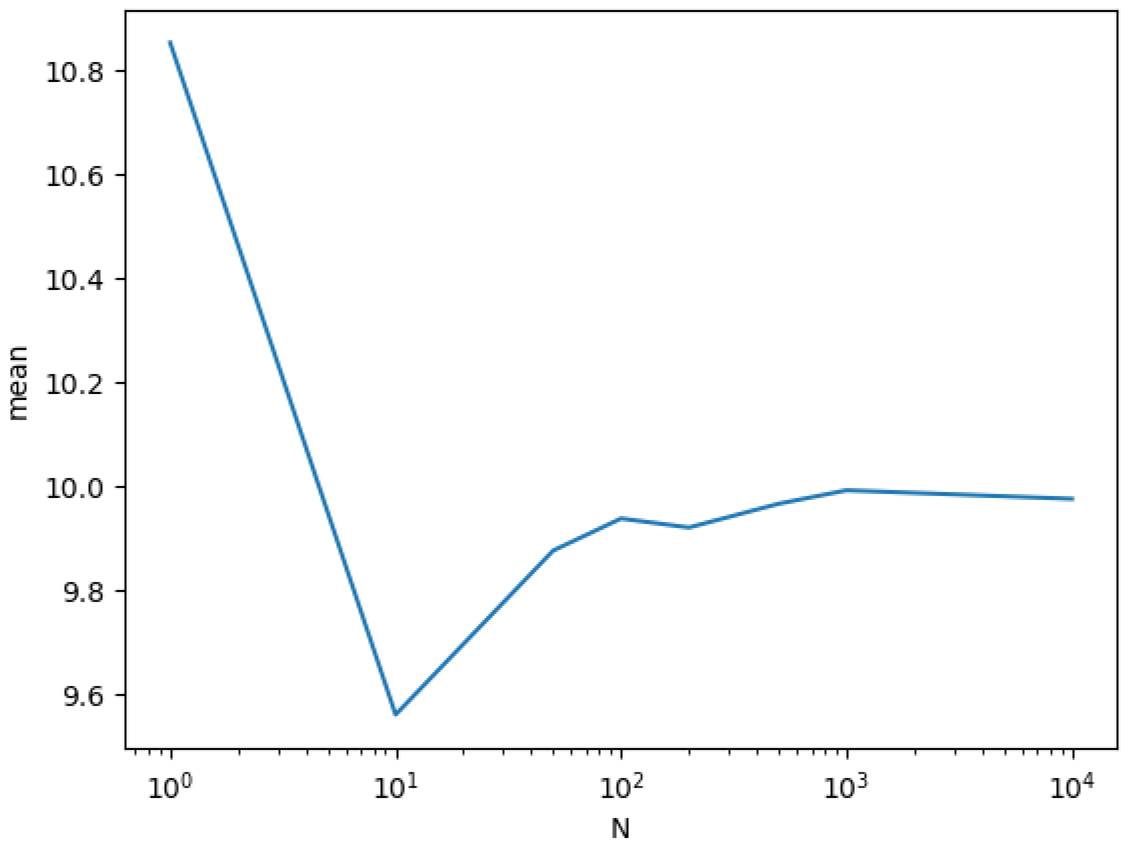
Out[ ]:
```
0.18128779864147673
```

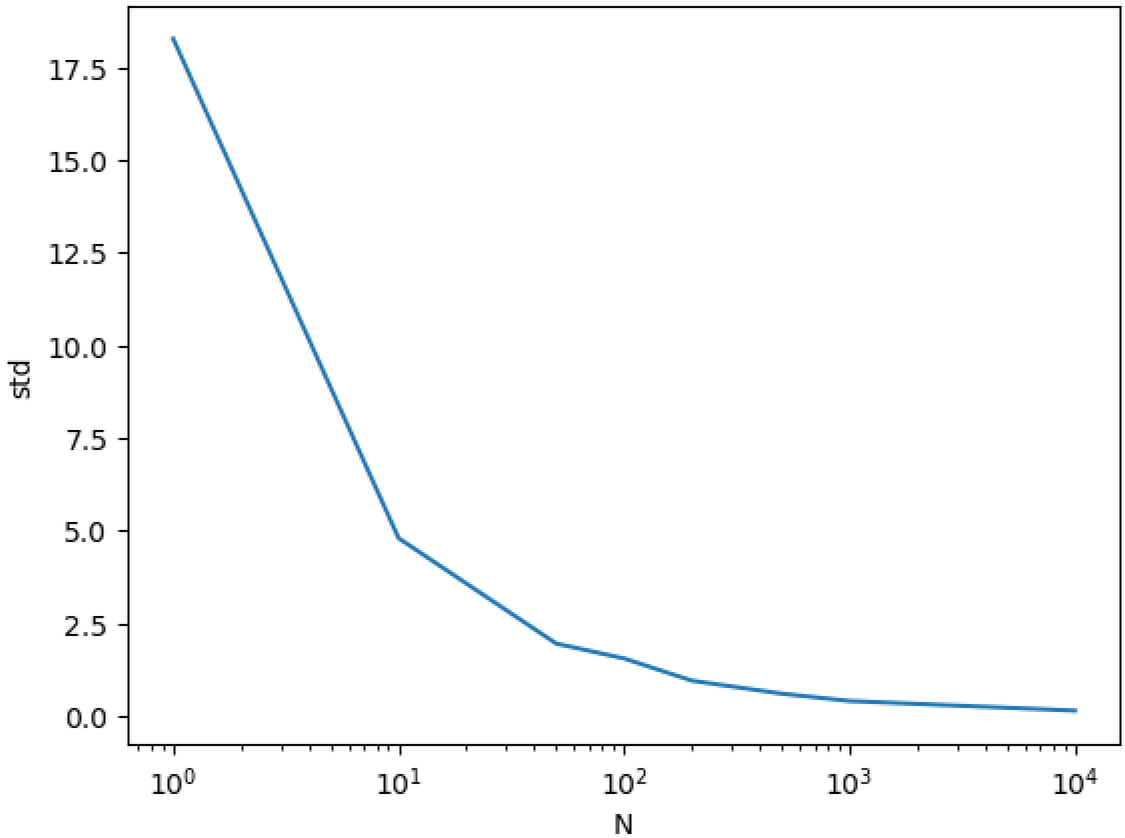
Problem 6.3(A)

In [ ]:
```
# P6.3
def generate_T_sample(mean, std, N):
    sample_generator = torch.distributions.Normal(mean, std)
    samples = sample_generator.sample((N,))
    return samples

def compute_estimator_desc(mean, std, N, T):
    T_samples = [generate_T_sample(mean , std, N).square().mean().item() for
    estimator_mean = np.mean(T_samples)
    estimator_std = np.std(T_samples)
    return [estimator_mean, estimator_std]

def collect_estimator(mean, std, N_list, T):
    return [compute_estimator_desc(mean, std, N, T) for N in N_list]

def plot_estimator(estimators_desc, N_list):
```

```
    print(estimators_desc)
    mean_list = list(map(lambda x: x[0], estimators_desc))
    std_list = list(map(lambda x: x[1], estimators_desc))
    plt.plot()
    plt.semilogx(N_list, mean_list)
    plt.xlabel("N")
    plt.ylabel("mean")
    plt.show()
    plt.semilogx(N_list, std_list)
    plt.xlabel("N")
    plt.ylabel("std")
    plt.show()

N_list = [1,10,50,100,200,500,1000, 10000]
mean = 0
std = np.sqrt(10)
T = 100
estimators_desc = collect_estimator(mean, std, N_list, T)
plot_estimator(estimators_desc, N_list)
```

```
[[10.854389454252013, 18.267171864183172], [9.561431233882905, 4.79973323344
9349], [9.877097930908203, 1.9590385411132993], [9.938786916732788, 1.557371
4609819649], [9.921476850509643, 0.9582612686629485], [9.96716851234436, 0.6
086864385648383], [9.993071165084839, 0.41086452832110637], [9.9765987205505
38, 0.15048812253635147]]
```





Problem 6.3(B)

Observation:

- $\hat{\mu_N} = \frac{1}{N}\sum_i x_i^2$ is an unbiased estimator for σ^2 since it is always close to the true variance, no matter what value N takes.
- It is also consistent since we observe that the variance of this estimator goes to zero as $N \to \infty$, and the plot verifies this.

Claim, $\hat{\mu_N}$ is an unbiased and consistent estimator of σ^2.

Proof:

- MGF of X_i: $M_X(t) = e^{\frac{\sigma^2 t^2}{2}}$.
- Second Moment: $\nabla M_X(t)\big|_{t=0} = \sigma^2$
- Fourth Moment: $\nabla^{(4)} M_X(t)\big|_{t=0} = 5\sigma^4$
- Unbiasedness: $E[\hat{\mu_N}] = \frac{1}{N}\sum_{i=1}^N E[X_i^2] = \frac{\sigma^2}{N} \cdot N = \sigma^2$
- Consistency: $\lim_{N\to\infty} P(|\widehat{\mu_N} - \sigma^2| > \epsilon) \le \frac{Var(\widehat{\mu_N})}{k^2} = \frac{5\sigma^4}{Nk^2}$(using theorem from problem 4E), by squeeze theorem we have consistency.

```
In [ ]:
```