جامعة نيويورك أبوظبي
NYU | ABU DHABI

MATH-UH-3413-001 (Spring 2022)

Numerical Methods

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Homework Set 05

Due: 11:59 pm on Sunday April 03, 2022

Notes:

1. Do all assigned problems
2. The set is worth 100 points evenly distributed among problems for the entire set. All HW sets will contribute 25 credits of the 100-credit system for the semester.
3. No late HW is accepted
4. Please be reminded of the lectured material (the shaded text is not used for composing this HW set):

Lecture03	02/01	Interpolation: Polynomial interpolation Lagrange, Newton forms. Runge phenomenon	3.1
Lecture04	02/03	Error in polynomial interpolation	3.2
Lecture05	02/08	Chebyshev interpolation	3.3
Lecture06	02/10	Cubic splines	3.4
Lecture07	02/15	Numerical differentiation	5.1
Lecture08	02/17	Trapezoid, Simpson and generic Newton-Cotes formulas for numerical quadrature. Gaussian quadrature	5.2 5.5
Lecture09 Lecture10	02/22 02/24	Linear systems of equations: simple direct methods. Gaussian elimination. LU factorization. Evaluation of numerical errors. Pivoting.	2.1 2.2 2.3 2.4
Lecture11 Lecture12	03/01 03/03	Linear systems of equations: iterative methods; methods for symmetric-positive matrices.	2.5 2.6
Lecture13 Lecture14	03/08 03/10	Trigonometric interpolation FFT	Ch10 Ch10
Lecture15 Lecture16	03/15 03/17	Spring break Spring break	
Lecture17 Lecture18	03/22 03/24	Spring break Least-square methods.	4.1 4.2

Note 1: Suggested solution reports' contents:

For this and all future HW sets, please include the following **four parts** for a self-contained report:

(1) Problem description for each problem. This could be copied from the problem assignment.
(2) Algorithm description and pseudo-code describing the main structure of your program(s).
(3) Results (numbers, figures, and tables) as requested by the assignments.
(4) Brief comments of your program performance (fast, correct, etc)

Note 2: Your HW report should be consolidated as one PDF (for each HW set) which include the 4 parts stated above. If the source code is too long, a Google Drive link is OK too. How long is too long, you are the judge.

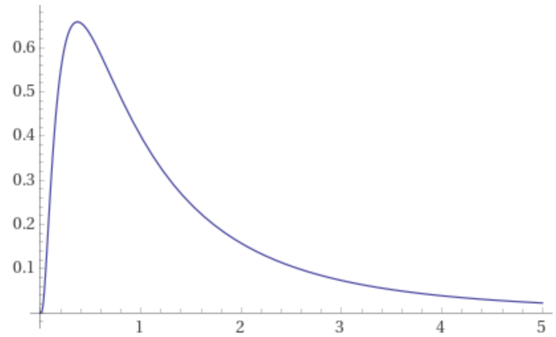
Problem 5.1 The well-known function

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}}{x}$$

where μ, σ are parameters and x is independent variable. At $\mu = 0, \sigma = 1$, the function becomes

$$f(x;0,1) = \frac{1}{\sqrt{2\pi}}\frac{e^{-\frac{1}{2}(\ln x)^2}}{x}$$

and it looks like



Select 6 points between $x \in [0, 5]$ evenly and interpolate these points in a polynomial of the appropriate order. Estimate the upper bound of the interpolation errors.

Problem 5.2 Same as Problem 5.1, select 6 points between $x \in [0, 5]$ as required by Chebyshev interpolation and interpolate these points by Chebyshev polynomials. Estimate the upper bound of the interpolation errors.

Problem 5.3 Using data from Problem 5.1, select 6 points between $x \in [0, 5]$ evenly and fit these points in the following form

$$y = c_0 + c_1x + c_2x^2 + c_3x^3$$

Also, compute the RMSE for this fit.

Problem 5.4 Using data from Problem 5.1, select 6 points between $x \in [0, 5]$ evenly and fit these points in the following form

$$y = c_1x\, e^{c_2x}$$

Also, compute the RMSE for this fit.