Yuefan Deng | yuefan.deng@nyu.edu | https://nyu.zoom.us/my/ydeng007

Midterm

2:40-3:455 pm on Thursday March 24, 2022

Notes:

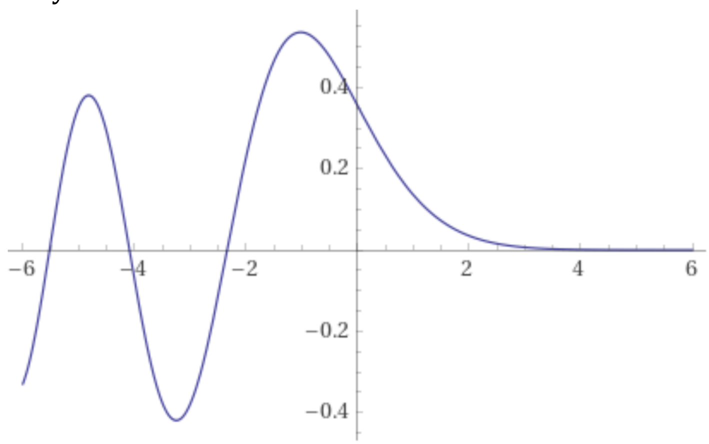
1. Do any 2 of the 3 problems. If all 3 are attempted, 2 (and only 2) best will be counted for grades.
2. The set is worth 100 points evenly distributed among problems, i.e., each problem can earn you 50 points for a max of 100. This test will contribute 30 credits to the 100-credit system.
3. Use any language, e.g., C, C++, Fortran, Java, MATLAB, Python, etc.
4. Use any sources for programs as long as you quote the source.
5. Use any computer systems as long as you can e-submit your solutions to Brightspace.
6. Compose a self-contained report for each problem, as you did for HW sets.
7. No emails/text/chat or any communication with a live person is allowed.
8. Please be reminded of the lectured material (the shaded text is not used for composing this test):

Lecture03	02/01	Interpolation: Polynomial interpolation Lagrange, Newton forms. Runge phenomenon	3.1
Lecture04	02/03	Error in polynomial interpolation	3.2
Lecture05	02/08	Chebyshev interpolation	3.3
Lecture06	02/10	Cubic splines	3.4
Lecture07	02/15	Numerical differentiation	5.1
Lecture08	02/17	Trapezoid, Simpson and generic Newton-Cotes formulas for numerical quadrature. Gaussian quadrature	5.2 5.5
Lecture09 Lecture10 Lecture11 Lecture12	02/22 02/24 03/01 03/03	Linear systems of equations: simple direct methods. Gaussian elimination. LU factorization. Evaluation of numerical errors. Pivoting. Linear systems of equations: iterative methods; methods for symmetric-positive matrices.	2.1 2.2 2.3 2.4 2.5 2.6
Lecture13	03/08	Least-square methods.	4.1 4.2
Lecture14	03/10	Trigonometric interpolation FFT	Ch10 Ch10
Lecture15 Lecture16	03/15 03/17	7-week finals Spring break	
Lecture17	03/22	Spring break	
Lecture18	03/24	Midterm	
Lecture19 Lecture20 Lecture21 Lecture22	03/29 03/31 04/05 04/07	ODEs ODEs ODEs Boundary value problems (BVP)	Ch6 Ch6 Ch6 Ch7

Problem M.1 (50 points) The very famed Airy function is defined in an integral

$$\mathrm{Ai}(x) = \frac{1}{\pi}\int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

and, for $x \in (-6, 6)$, the Airy functions looks like this:



Write a program to implement any one or more of the numerical integration methods to fill up the last row of the following table.

Since the integrand is crazy (see figure below) and so is to perform numerical integrate to ∞, please approximate the upper bound ∞ by 25 or your age in years, whichever is greater. (I respect any age!)

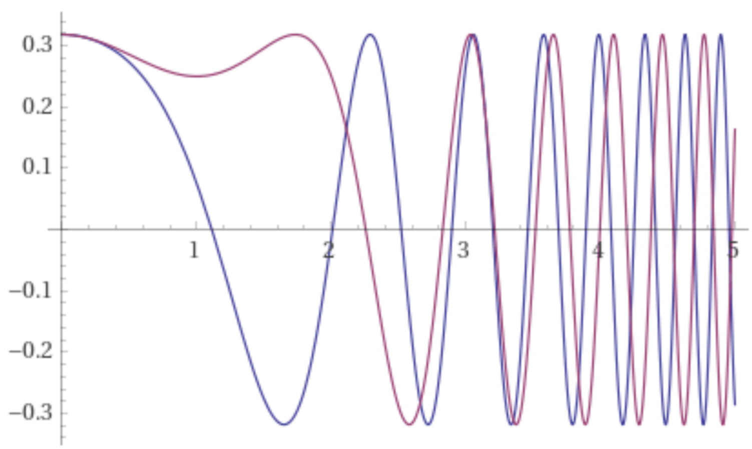
Make your results accurate till the 3rd digit after the decimal point, e.g., $\mathrm{Ai}(x = -3) \approx 0.378$ and $\mathrm{Ai}(x = 3) \approx 0.006$.

x	-3	-2	-1	0	1	2	3
Ai(x) I got	-0.378814	0.227407	0.535561	0.355028	0.135292	0.034924	0.006591
Ai(x) You get							

A picture of the integrand

$$\frac{1}{\pi}\cos\left(\frac{t^3}{3} + xt\right)$$

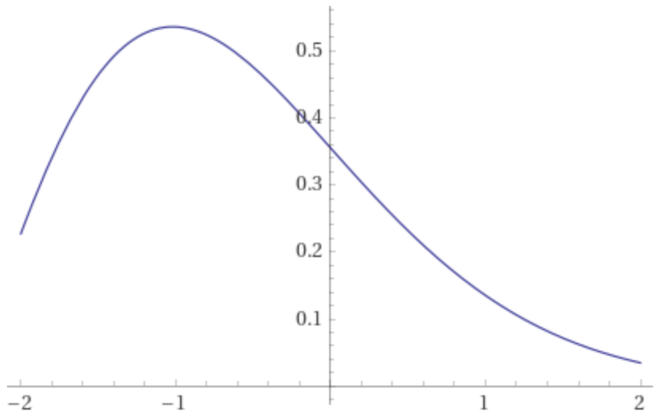
with $x = +1$ (blue) and $x = -1$ (red) for $t = 0$ to 5:



Problem M.2 (50 points) The very famed Airy function is defined in an integral

$$\mathrm{Ai}(x) = \frac{1}{\pi}\int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

and, for $x \in (-2, 2)$, it looks like



I calculated the 5 data points tabulated below:

x	-2	-1	0	1	2
$\mathrm{Ai}(x)$	0.227407	0.535561	0.355028	0.135292	0.034924

Write a program to best fit these 5 points with the following given function:

$$\mathrm{f}(x) = c_1 x\, e^{c_2 x}$$

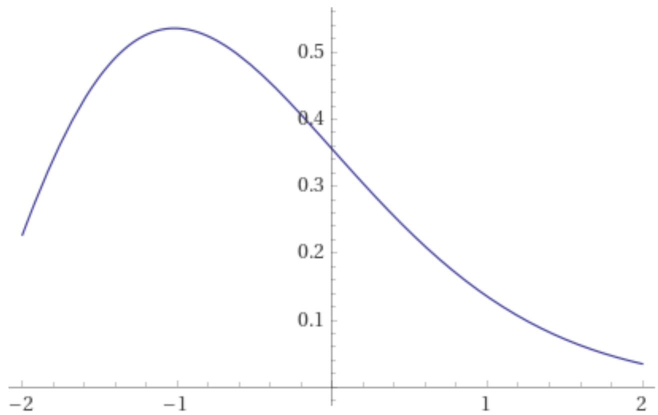
At a minimum, you need to perform the following steps:

(1) Construct the inconsistent system of equations: $AX = b$.
(2) Establish the normal equations: $A^T A\bar{x} = A^T b$.
(3) Solve for the coefficients: $\bar{x} = (c_1, c_2)^T$.
(4) Compute RMSE for your fit.

Problem M.3 (50 points) The very famed Airy function is defined in an integral

$$\mathrm{Ai}(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

and, for $x \in (-2, 2)$, it looks like



I calculated the 5 data points tabulated below:

x	-2	-1	0	1	2
$\mathrm{Ai}(x)$	0.227407	0.535561	0.355028	0.135292	0.034924

Write a program to implement any of the interpolation algorithms, including (1) Lagrange interpolation, (2) Newton's divided differences, and (3) Cubic spline, to interpolate these 5 points with

$$\mathrm{P}_4(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

At a minimum, you do:

(1) Construct the equations for the coefficients.
(2) Compute the coefficients.
(3) Plot your $\mathrm{P}_4(x)$ vs. x with the 5 given points shown.