MATH-UH-3413-001 (Spring 2022)

Numerical Methods

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Homework Set 06

Due: 11:59 pm on Sunday April 17, 2022

Notes:

1. Do all assigned problems
2. The set is worth 100 points evenly distributed among problems for the entire set. All HW sets will contribute 25 credits of the 100-credit system for the semester.
3. No late HW is accepted
4. Please be reminded of the lectured material (the shaded text is not used for composing this HW set):

Lecture15	03/15	7-week finals	
Lecture16	03/17	Spring break	
Lecture17	03/22	Spring break	
Lecture18	03/24	Midterm	
Lecture19	03/29	Trigonometric interpolation	Ch10
Lecture20	03/31	FFT	Ch10
Lecture21	04/05	ODEs	Ch6
Lecture22	04/07	ODEs	Ch6
Lecture23	04/12	ODEs	Ch6
Lecture24	04/14	Boundary value problems: Shooting method	Ch7
Lecture25	04/19	BVP: Finite difference methods	Ch7
Lecture26	04/21	PDEs: Hyperbolic Eq	Ch8
Lecture27	04/26	PDEs: Parabolic eq	Ch8
Lecture28	04/28	PDEs: Elliptic eq	Ch8
Lecture29	05/03	Using 05/06 (Friday): More PDE's	Ch8
Lecture30	05/05	No class	
Lecture31	05/10	Final presentations	
Lecture32	05/12	Final presentations	

Note 1: Suggested solution reports' contents:

For this and all future HW sets, please include the following **four parts** for a self-contained report:

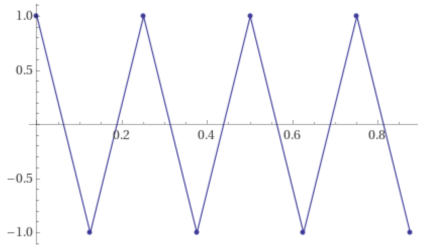
(1) Problem description for each problem. This could be copied from the problem assignment.
(2) Algorithm description and pseudo-code describing the main structure of your program(s).
(3) Results (numbers, figures, and tables) as requested by the assignments.
(4) Brief comments of your program performance (fast, correct, etc)

Note 2: Your HW report should be consolidated as one PDF (for each HW set) which include the 4 parts stated above. If the source code is too long, a Google Drive link is OK too. How long is too long, you are the judge.

Problem 6.1: Given the following 8 data points

t	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
x	1	-1	1	-1	1	-1	1	-1

which can be plotted as (although we do not know the values in-between the points):

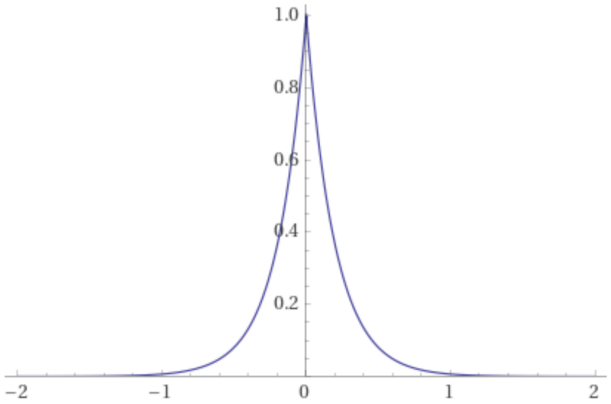


Please find the trigonometric interpolating function (and plot your function with these 8 points shown).

Problem 6.2: Given the following function

$$f(t) = e^{-\frac{|t|}{T}}$$

where T is some positive number and, for convenience, please set it as $T = 0.1984$. The function looks like



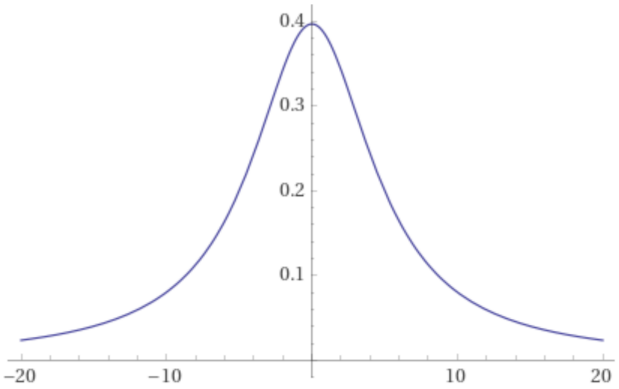
Evenly select $N = 1024 = 2^{10}$, in $t_i \in [-2, 2]$, to evaluate $f(t_i)$ where $i = 0, 1, 2 \ldots, N-1$.

(1) Perform DFT on this vector of N dimensions whose elements are $f(t_0)$, $f(t_1)$, $f(t_2)$, …
(2) Perform FFT on the same vector as in (1).
(3) Plot your vector in the Fourier space for both cases.

Hint: Analytically, we know the FT should be

$$F(\omega) = \frac{2T}{1 + (\omega T)^2}$$

and it looks like



and your FT should not been too far off.

Problem 6.3: The following is the exchange rate of EUR/USD for the past ~15 years. It has some interesting patterns and, certainly, it appears oscillatory.

At the following web site, you can find the daily rates for an entire past year:
https://finance.yahoo.com/quote/EURUSD%3DX/history?p=EURUSD%3DX

Please glean the exchanges for the last day of the months in the following table:

Time "t"	Last day of month	EUR/USD "x"	
0	04/2021		
1	05		
2	06		
3	07		
4	08		
5	09		
6	10		
7	11		
8	12		
9	01/2022		
10	02		
11	03		

Find the trigonometric interpolating function for the rates in these 12 months.