MACs, PRNGs and Diffie-Hellman Key Exchange

CS 161 Spring 2023 - Lecture 9

Announcements

- **HW 3 is released and due this Friday, February 16th at 11:59 PT**
- **The midterm is on Thursday, February 29th from 7:00-9:00 PM PT**.
  - If you would like to request an alternate exam time or remote exam, or have DSP accommodations or any special requests, please **fill out the Exam Logistics Form by Monday, February 19, 11:59 PM PT**.
  - We'll announce more information regarding review sessions, past exams, etc next week.

Last Time: Hashes

- Map arbitrary-length input to fixed-length output
- Output is deterministic and unpredictable
- Security properties
  - One way: Given an output y, it is infeasible to find any input x such that $H(x) = y$.
  - Collision resistant: It is infeasible to find another any pair of inputs $x' \neq x$ such that $H(x) = H(x')$.
- Some hashes are vulnerable to length extension attacks
- Hashes don’t provide integrity (unless you can publish the hash securely)

Message Authentication Codes (MACs)

Textbook Chapter 8.1–8.3 & 8.5–8.6

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	• One-time pads • Block ciphers with chaining modes (e.g. AES-CBC)	• RSA encryption • ElGamal encryption
Integrity, Authentication	• MACs (e.g. HMAC)	• Digital signatures (e.g. RSA signatures)

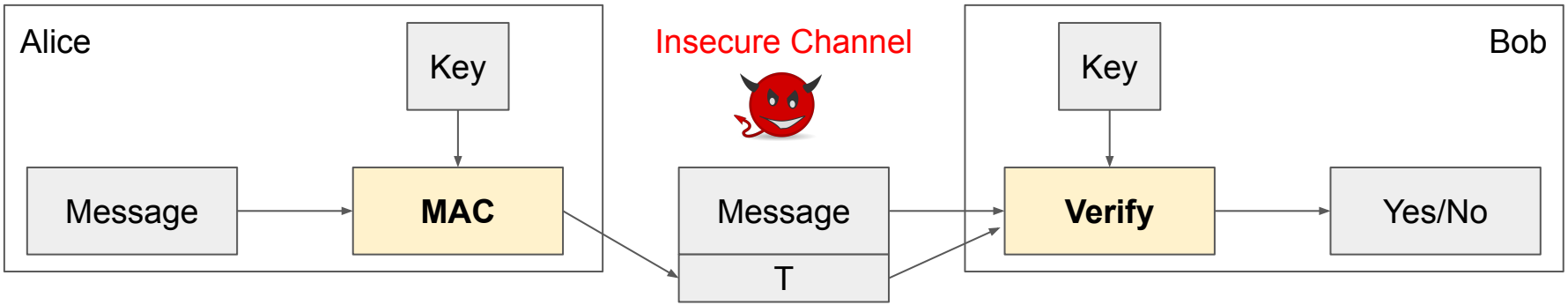
- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

How to Provide Integrity

- Reminder: We're still in the symmetric-key setting
  - Assume that Alice and Bob share a secret key, and attackers don't know the key
- We want to attach some piece of information to *convince* Bob that Alice sent this message even if Mallory is intercepting the message on the network, or to *detect* if Mallory tampered with the message
  - This piece of information can only be generated by someone with the key

MACs: Usage

- Alice wants to send *M* to Bob, but doesn't want Mallory to tamper with it
- Alice sends *M* and *T* = MAC(*K*, *M*) to Bob
- Bob recomputes MAC(*K*, *M*) and checks that it matches *T*
- If the MACs match, Bob is confident the message has not been tampered with (integrity)



MACs: Definition

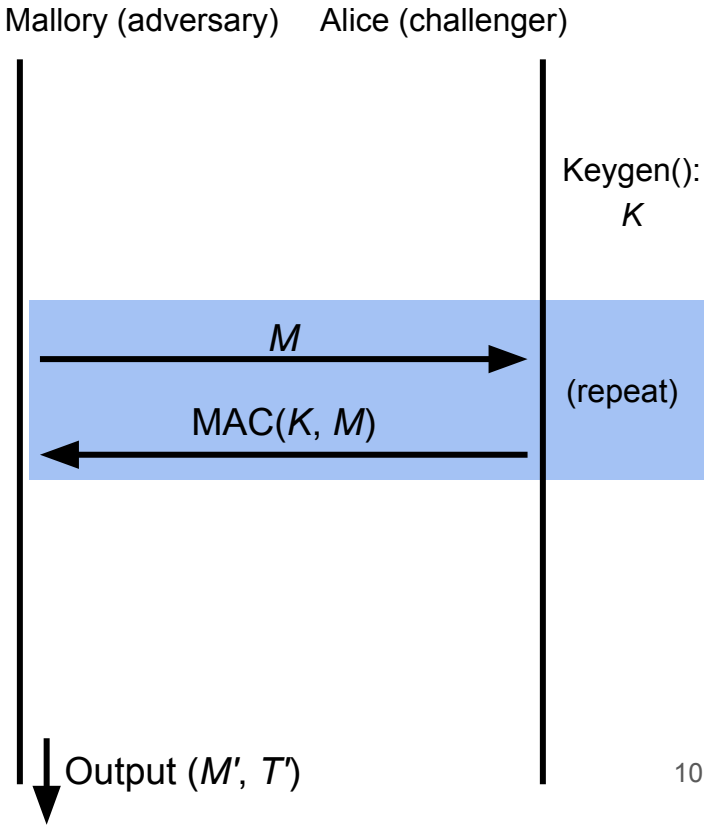
- Two parts:
  - KeyGen() → *K*: Generate a key *K*
  - MAC(*K*, *M*) → *T*: Generate a tag *T* for the message *M* using key *K*
    - Inputs: A secret key and an arbitrary-length message
    - Output: A fixed-length **tag** on the message
- Properties
  - **Correctness**: Determinism
    - Note: Some more complicated MAC schemes have an additional Verify(*K*, *M*, *T*) function that don't require determinism, but this is out of scope
  - **Efficiency**: Computing a MAC should be efficient
  - **Security**: EU-CPA (existentially unforgeable under chosen plaintext attack)

Defining Integrity: EU-CPA

- A secure MAC is **existentially unforgeable**: without the key, an attacker cannot create a valid tag on a message
  - Mallory cannot generate MAC(K, M') without K
  - Mallory cannot find any $M' \neq M$ such that MAC(K, M') = MAC(K, M)
- Formally defined by a security game: existential unforgeability under chosen-plaintext attack, or EU-CPA
- MACs should be unforgeable under chosen plaintext attack
  - Intuition: Like IND-CPA, but for integrity and authenticity
  - Even if Mallory can trick Alice into creating MACs for messages that Mallory chooses, Mallory cannot create a valid MAC on a message that she hasn't seen before

Defining Integrity: EU-CPA

1. Mallory may send messages to Alice and receive their tags
2. Eventually, Mallory creates a message-tag pair (*M′*, *T′*)
   - *M′* cannot be a message that Mallory requested earlier
   - If *T′* is a valid tag for *M′*, then Mallory wins. Otherwise, she loses.
3. A scheme is EU-CPA secure if for *all* polynomial time adversaries, the probability of winning is 0 or negligible



Example: NMAC

- Can we use secure cryptographic hashes to build a secure MAC?
  - Intuition: Hash output is unpredictable and looks random, so let's hash the key and the message together
- KeyGen():
  - Output two random, n-bit keys K_1 and K_2, where n is the length of the hash output
- NMAC(K_1, K_2, M):
  - Output $H(K_1 \,||\, H(K_2 \,||\, M))$
- NMAC is EU-CPA secure if the two keys are different
  - Provably secure if the underlying hash function is secure
- Intuition: Using two hashes prevents a length extension attack
  - Otherwise, an attacker who sees a tag for M could generate a tag for $M \,||\, M'$

Example: HMAC

- Issues with NMAC:
  - Recall: NMAC(K_1, K_2, M) = $H(K_1 \,||\, H\,(K_2 \,||\, M))$
  - We need two different keys
  - NMAC requires the keys to be the same length as the hash output (n bits)
- HMAC(K, M):
  - Compute K' as a version of K that is the length of the hash output
    - If K is too short, pad K with 0's to make it n bits (be careful with keys that are too short and lack randomness)
    - If K is too long, hash it so it's n bits
  - Output $H((K' \oplus opad) \,||\, H((K' \oplus ipad) \,||\, M))$

Example: HMAC

- HMAC(*K*, *M*):
  - Compute *K*' as a version of *K* that is the length of the hash output
    - If *K* is too short, pad *K* with 0's to make it *n* bits (be careful with keys that are too short and lack randomness)
    - If *K* is too long, hash it so it's *n* bits
  - Output $H((K' \oplus opad) \,||\, H((K' \oplus ipad) \,||\, M))$
- Use *K*' to derive two different keys
  - *opad* (outer pad) is the hard-coded byte `0x5c` repeated until it's the same length as *K*'
  - *ipad* (inner pad) is the hard-coded byte `0x36` repeated until it's the same length as *K*'
  - As long as *opad* and *ipad* are different, you'll get two different keys
  - For paranoia, the designers chose two very different bit patterns, even though they theoretically need only differ in one bit

HMAC Properties

- HMAC(K, M) = $H((K' \oplus opad) \,||\, H((K' \oplus ipad) \,||\, M))$
- HMAC is a hash function, so it has the properties of the underlying hash too
  - It is collision resistant
  - Given HMAC(K, M) and K, an attacker can't learn M
  - If the underlying hash is secure, HMAC doesn't reveal M, but it is still deterministic
- You can't verify a tag T if you don't have K
  - The attacker can't brute-force the message M without knowing K

Do MACs provide integrity?

- Do MACs provide integrity?
  - Yes. An attacker cannot tamper with the message without being detected
- Do MACs provide authenticity?
  - It depends on your threat model
  - If a message has a valid MAC, you can be sure it came from *someone with the secret key*, but you can't narrow it down to one person
  - If only two people have the secret key, MACs provide authenticity: it has a valid MAC, and it's not from me, so it must be from the other person
- Do MACs provide confidentiality?
  - MACs are deterministic ⇒ No IND-CPA security
  - MACs in general have no confidentiality guarantees; they can leak information about the message

MACs: Summary

- Inputs: a secret key and a message
- Output: a tag on the message
- A secure MAC is unforgeable: Even if Mallory can trick Alice into creating MACs for messages that Mallory chooses, Mallory cannot create a valid MAC on a message that she hasn't seen before
  - Example: HMAC(K, M) = $H((K' \oplus opad) \,||\, H((K' \oplus ipad) \,||\, M))$
- MACs do not provide confidentiality

Authenticated Encryption

Textbook Chapter 8.7 & 8.8

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	• One-time pads • Block ciphers with chaining modes (e.g. AES-CBC)	• RSA encryption • ElGamal encryption
Integrity, Authentication	• MACs (e.g. HMAC)	• Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

Authenticated Encryption: Definition

- **Authenticated encryption** (**AE**): A scheme that simultaneously guarantees confidentiality and integrity (and authenticity, depending on your threat model) on a message
- Two ways of achieving authenticated encryption:
  - Combine schemes that provide confidentiality with schemes that provide integrity
  - Use a scheme that is designed to provide confidentiality and integrity

Combining Schemes: Let's design it together

- You can use:
  - An IND-CPA encryption scheme (e.g. AES-CBC): Enc(K, M) and Dec(K, M)
  - An unforgeable MAC scheme (e.g. HMAC): MAC(K, M)
- First attempt: Alice sends Enc(K_1, M) and MAC(K_2, M)
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? No, the MAC is not IND-CPA secure
- Idea: Let's compute the MAC on the *ciphertext* instead of the plaintext: Enc(K_1, M) and MAC(k_2, Enc(K_1, M))
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? Yes, the MAC might leak info about the ciphertext, but that's okay
- Idea: Let's encrypt the MAC too: Enc(K_1, M || MAC(K_2, M))
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? Yes, everything is encrypted

MAC-then-Encrypt or Encrypt-then-MAC?

- MAC-then-encrypt
  - First compute MAC(K_2, M)
  - Then encrypt the message and the MAC together: Enc(K_1, M || MAC(K_2, M))
- Encrypt-then-MAC
  - First compute Enc(K_1, M)
  - Then MAC the ciphertext: MAC(K_2, Enc(K_1, M))
- Which is better?
  - In theory, both are IND-CPA and EU-CPA secure if applied properly
  - MAC-then-encrypt has a downside: You don't know if tampering has occurred until after decrypting
    - Attacker can supply arbitrary tampered input, and you always have to decrypt it
    - Passing attacker-chosen input through the decryption function can cause side-channel leaks
- **Always use encrypt-then-MAC** because it's more robust to mistakes

Key Reuse

- **Key reuse problem**: Using the same key in two different use cases
  - Note: Using the same key multiple times for the same use (e.g. computing HMACs on different messages in the same context with the same key) is not key reuse problem
- Reusing keys can cause the underlying algorithms to interfere with each other and affect security guarantees
  - Example: If you use a block-cipher-based MAC algorithm and a block cipher chaining mode, the underlying block ciphers may no longer be secure
  - Thinking about these attacks is hard

Key Reuse

- Simplest solution: Do not reuse keys across schemes! One key per *scheme instance*.
  - Encrypt a piece of data and MAC a piece of data?
    - Different use; different key
  - MAC one of Alice's messages to Bob and MAC one of Bob's messages to Alice?
    - Different use; different key
  - Encrypt one of Alice's files and encrypt another one of Alice's files?
    - It's *probably* fine to use the same key, but cryptographic design is tricky to get right!
  - Encrypt user metadata, encrypt file metadata, and encrypt file data?
    - You'll have to think about this in Project 2!

TLS 1.0 “Lucky 13” Attack

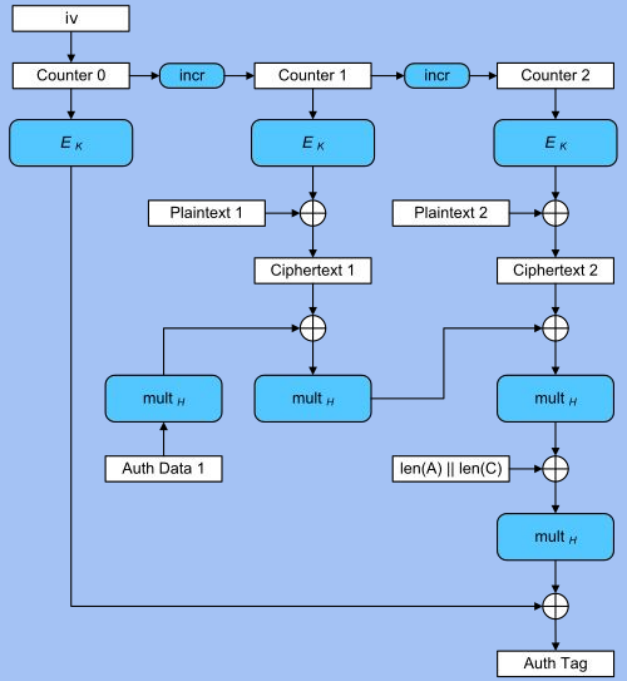
- TLS: A protocol for sending encrypted and authenticated messages over the Internet (we’ll study it more in the networking unit)
- TLS 1.0 uses MAC-then-encrypt: Enc(K_1, M || MAC(K_2, M))
  - The encryption algorithm is AES-CBC
- The Lucky 13 attack abuses MAC-then-encrypt to read encrypted messages
  - Guess a byte of plaintext and change the ciphertext accordingly
  - The MAC will error, but the time it takes to error is different depending on if the guess is correct
  - Attacker measures how long it takes to error in order to learn information about plaintext
  - TLS will send the message again if the MAC errors, so the attacker can guess repeatedly
- Takeaways
  - Side channel attack: The algorithm is proved secure, but poor implementation made it vulnerable
  - Always encrypt-then-MAC

AEAD Encryption

- Second method for authenticated encryption: Use a scheme that is designed to provide confidentiality, integrity, and authenticity
- **Authenticated encryption with additional data** (**AEAD**): An algorithm that provides both confidentiality and integrity over the plaintext and integrity over *additional data*
  - Additional data is usually context (e.g. memory address), so you can't change the context without breaking the MAC
- Great if used correctly: No more worrying about MAC-then-encrypt
  - If you use AEAD incorrectly, you lose *both* confidentiality and integrity/authentication
  - Example of correct usage: Using a crypto library with AEAD

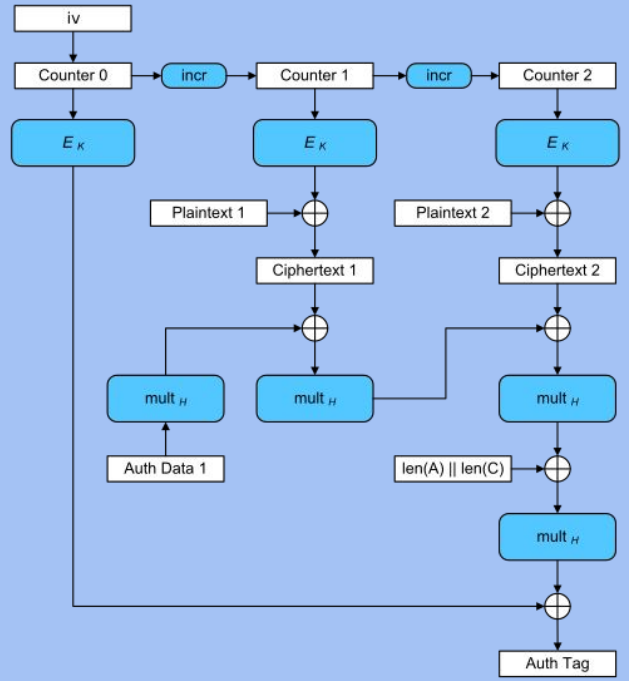
AEAD Example: Galois Counter Mode (GCM)

- **Galois Counter Mode** (**GCM**): An AEAD block cipher mode of operation
- E_K is standard block cipher encryption
- mult_H is 128-bit multiplication over a special field (Galois multiplication)
  - Don't worry about the math



AEAD Example: Galois Counter Mode (GCM)

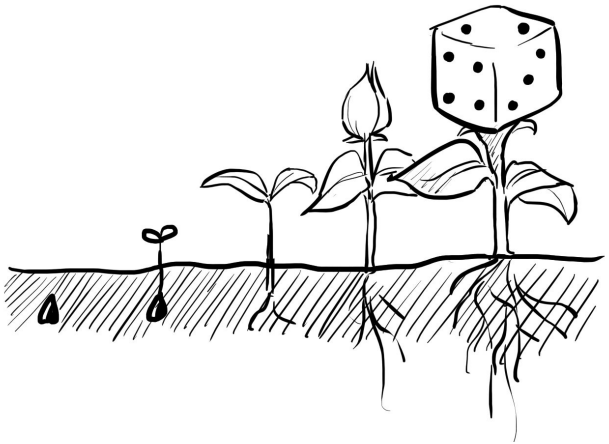
- Very fast mode of operation
  - Fully parallel encryption
  - Galois multiplication isn't parallelizable, but it's very fast
- Drawbacks
  - IV reuse leads to loss of confidentiality, integrity, and authentication
  - This wouldn't happen if you used AES-CTR and HMAC-SHA256
  - Implementing Galois is difficult and easy to screw up
- **Takeaway**: GCM provides integrity and confidentiality, but if you misuse it, it's even worse than CTR mode



Authenticated Encryption: Summary

- Authenticated encryption: A scheme that simultaneously guarantees confidentiality and integrity (and authenticity) on a message
- First approach: Combine schemes that provide confidentiality with schemes that provide integrity and authenticity
  - MAC-then-encrypt: Enc(K_1, M || MAC(K_2, M))
  - Encrypt-then-MAC: Enc(K_1, M) || MAC(K_2, Enc(K_1, M))
  - Always use Encrypt-then-MAC because it's more robust to mistakes
- Second approach: Use AEAD encryption modes designed to provide confidentiality, integrity, and authenticity
  - Drawback: Incorrectly using AEAD modes leads to losing *both* confidentiality and integrity/authentication

Pseudorandom Number Generators (PRNGs)



Textbook Chapter 9

Cryptography Roadmap

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Randomness

- Randomness is essential for symmetric-key encryption
  - A random key
  - A random IV/nonce
  - Universally unique identifiers (we'll see this shortly)
  - We'll see more applications later
- If an attacker can predict a random number, things can catastrophically fail
- How do we securely generate random numbers?

Entropy

- In cryptography, “random” usually means “random and unpredictable”
- Scenario
  - You want to generate a secret bitstring that the attacker can't guess
  - You generate random bits by tossing a fair (50-50) coin
  - The outcomes of the fair coin are harder for the attacker to guess
- **Entropy**: A measure of uncertainty
  - In other words, a measure of how unpredictable the outcomes are
  - High entropy = unpredictable outcomes = desirable in cryptography
  - The uniform distribution has the highest entropy (every outcome equally likely, e.g. fair coin toss)
  - Usually measured in bits (so 3 bits of entropy = uniform, random distribution over 8 values)

Breaking Bitcoin Wallets

- What happens if we use a poor source of entropy?
- Bitcoin users use a randomly-generated private key to access their account (and money)
  - An attacker who learns the key can access the money
  - We'll learn more about Bitcoin later
- An "improvment" [sic] to the algorithm reduced the entropy used to generate the private keys
  - Any private key created with this "improvment" could be brute-forced

Improvments to RNG

[redacted] committed on Dec 7, 2014 1 parent b0d5639

Showing 1 changed file with 26 additions and 28 deletions.

54 bitcoinjs-lib/src/jsbn/rng.js

```
@@ -8,15 +8,16 @@ var rng_state;
 8  8   var rng_pool;
 9  9   var rng_pptr;
10 10
11    - // Mix in a 32-bit integer into the pool
12    - function rng_seed_int(x) {
13    -     rng_pool[rng_pptr++] ^= x & 255;
14    -     rng_pool[rng_pptr++] ^= (x >> 8) & 255;
15    -     rng_pool[rng_pptr++] ^= (x >> 16) & 255;
16    -     rng_pool[rng_pptr++] ^= (x >> 24) & 255;
```

True Randomness

- To generate truly random numbers, we need a physical source of entropy
  - An unpredictable circuit on a CPU
  - Human activity measured at very fine time scales (e.g. the microsecond you pressed a key)
- Unbiased entropy usually requires combining multiple entropy sources
  - Goal: Total number of bits of entropy is the sum of all the input numbers of bits of entropy
    - Many poor sources + 1 good source = good entropy
- Issues with true randomness
  - It's expensive and slow to generate
  - Physical entropy sources are often biased

Exotic entropy source: Cloudflare has a wall of lava lamps that are recorded by an HD video camera that views the lamps through a rotating prism

Pseudorandom Number Generators (PRNGs)

- True randomness is expensive and biased
- **Pseudorandom number generator** (**PRNGs**): An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
  - Also called **deterministic random bit generators** (**DRBGs**)
- Usage
  - Generate some expensive true randomness (e.g. noisy circuit on your CPU)
  - Use the true randomness as input to the PRNG
  - Generate random-looking numbers quickly and cheaply with the PRNG
- PRNGs are deterministic: Output is generated according to a set algorithm
  - However, for an attacker who can't see the internal state, the output is *computationally indistinguishable* from true randomness

Stream Ciphers

Textbook Chapter 9.5

PRNG: Definition

- A PRNG has two functions:
  - PRNG.Seed(randomness): Initializes the internal state using the entropy
    - Input: Some truly random bits
  - PRNG.Generate(*m*): Generate *m* pseudorandom bits
    - Input: A number *m*
    - Output: *m* pseudorandom bits
    - Updates the internal state as needed

Properties

- **Correctness**: Deterministic
- **Efficiency**: Efficient to generate pseudorandom bits
- **Security**: Indistinguishability from random

PRNG: Security

- Can we design a PRNG that is truly random?
- A PRNG cannot be truly random
  - The output is deterministic given the initial seed
  - If the initial seed is *s* bits long, there are only 2^s possible output sequences
- A secure PRNG is computationally indistinguishable from random to an attacker
  - Game: Present an attacker with a truly random sequence and a sequence outputted from a secure PRNG
  - An attacker should not be able to determine which is which with probability > ½+negl
- Equivalence: An attacker cannot predict future output of the PRNG

Insecure PRNGs: Breaking Slot Machines

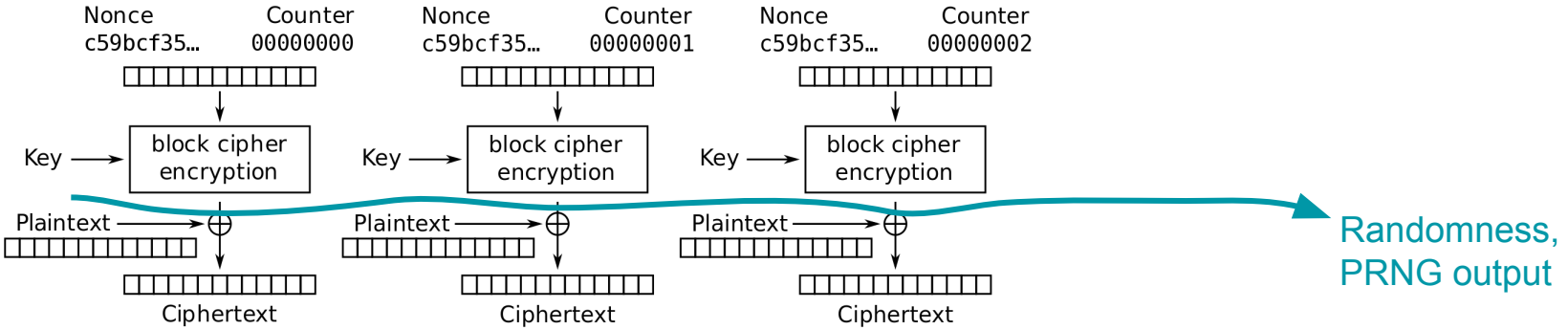
- What happens if PRNGs are used improperly?
- A casino in St. Louis experienced unusual bad “luck”
  - Suspicious players would hover over the lever and then spin at a specific time to win
- Vulnerability: Slot machines used predictable PRNGs
  - The PRNG output was based on the current time
- Strategy:
  - Use a smartphone to alert you to when to pull the lever for the best chance of winning
- Las Vegas was not affected by the vulnerability
  - Nevada slot machines must follow evaluation standards designed to address this sort of issue

Insecure PRNGs: OpenSSL PRNG bug

- What happens if we don’t use enough entropy?
- Debian OpenSSL CVE-2008-0166
  - Debian: A Linux distribution
  - OpenSSL: A cryptographic library
  - In “cleaning up” OpenSSL (Debian “bug” #363516), the author “fixed” how OpenSSL seeds random numbers
  - The existing code caused Purify and Valgrind to complain about reading uninitialized memory
  - The cleanup caused the PRNG to only be seeded with the process ID
  - There are only 2^{15} (32,768) possible process IDs, so the PRNG only has 15 bits of entropy
- Easy to deduce private keys generated with the PRNG
  - Set the PRNG to every possible starting state and generate a few private/public key pairs
  - See if the matching public key is anywhere on the Internet

Example construction of PRNG

- Using block cipher in CTR mode:
- If you want m random bits, and a block cipher with E_k has n bits, apply the block cipher m/n times and concatenate the result:
- PRNG.Seed(K | IV);
- Generate(m) = E_k(IV|1) | E_k(IV| 2) | E_k(IV|3) … E_k(IV| ceil(m/n)),
  - | is concatenation



Counter (CTR) mode encryption

Insecure PRNGs: Rust Rand_Core

- A Rust library has an interface for “secure” random number generators… but it isn’t actually secure!
- Example: ChaCha8Rng
  - A stream cipher PRNG
  - No reseed function: no way of adding extra entropy after the initial seed
  - Seed only takes 32 bits: no way to combine entropy
  - No rollback resistance
- None of the “secure” RNGs are cryptographically secure
  - None have a reseed function to add extra entropy
  - None take arbitrarily long seeds
- **Takeaway**: Always make sure you use a secure PRNG
  - Consider human factors? Use fail-safe defaults?

Insecure PRNGs: CVE-2019-16303

- Relevant if you wrote an app in JHipster before 2019
- Password reset functions
  - When you forget your password, receive an email with a special link to reset your password
  - The special link should contain a randomly-generated code (so attackers can't make their own link)
- Vulnerability: Bad PRNG
  - You can figure out the PRNG’s internal state from the reset link
  - Request password reset links for other people's accounts
  - Predict the “random” reset link and take over any account you want!

Application: Universally Unique Identifiers (UUIDs)

- Scenario
  - You have a set of objects (e.g. files)
  - You need to assign a unique name to every object
  - Every name must be unique and unpredictable
- Solution: choose a random value
  - If you use enough randomness, the probability of generating the same random value twice are astronomically small (basically 0)
- Universally Unique Identifiers (UUIDs)
  - 128-bit unique values
  - To generate a new UUID, seed a secure PRNG properly, and generate a random value
  - Often written in hexadecimal: `00112233-4455-6677-8899-aabbccddeeff`
  - You'll work with UUIDs in Project 2

PRNGs: Summary

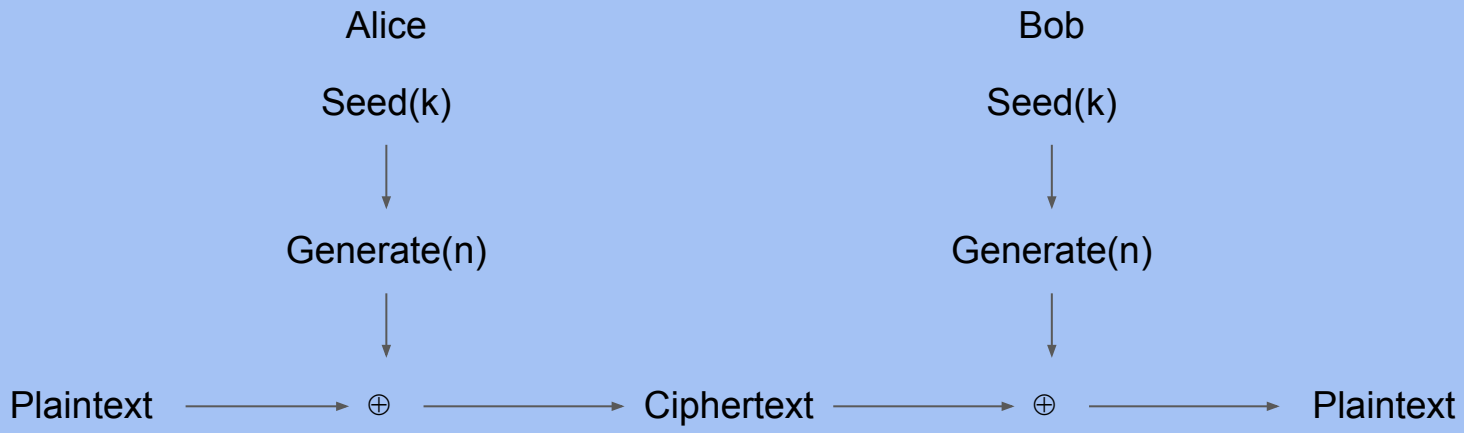
- True randomness requires sampling a physical process
  - Slow, expensive, and biased (low entropy)
- PRNG: An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
  - Seed(entropy): Initialize internal state
  - Generate(n): Generate n bits of pseudorandom output
- Security: computationally indistinguishable from truly random bits
- Example using AES in CTR mode
- Application: UUIDs

Stream Ciphers

- Another way to construct symmetric key encryption schemes
- Idea
  - A secure PRNG produces output that looks indistinguishable from random
  - An attacker who can't see the internal PRNG state can't learn any output
  - What if we used PRNG output as the key to a one-time pad?
- **Stream cipher**: A symmetric encryption algorithm that uses pseudorandom bits as the key to a one-time pad

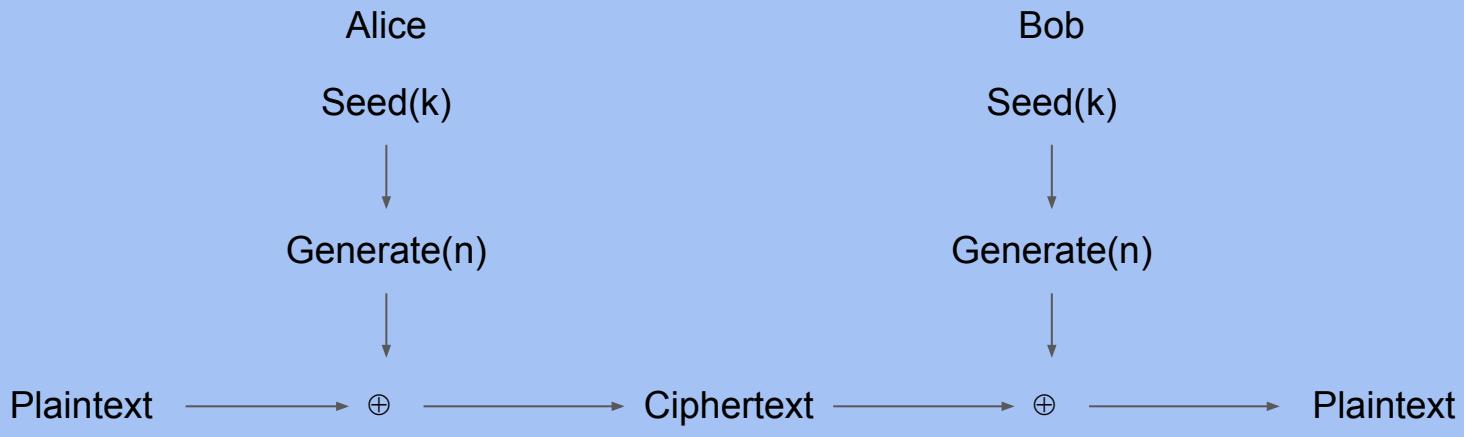
Stream Ciphers

- Protocol: Alice and Bob both seed a secure PRNG with their symmetric secret key, and then use the output as the key for a one-time pad



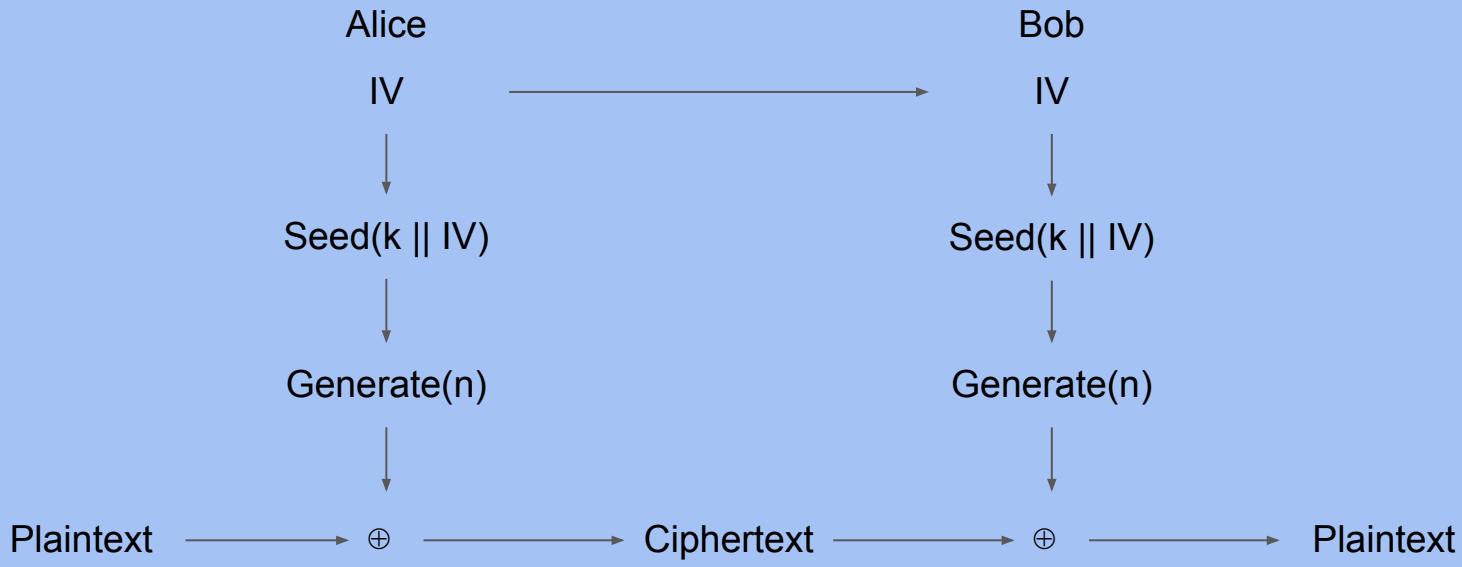
Stream Ciphers: Encrypting Multiple Messages

- Recall: One-time pads are insecure when the key is reused. How do we encrypt multiple messages without key reuse?



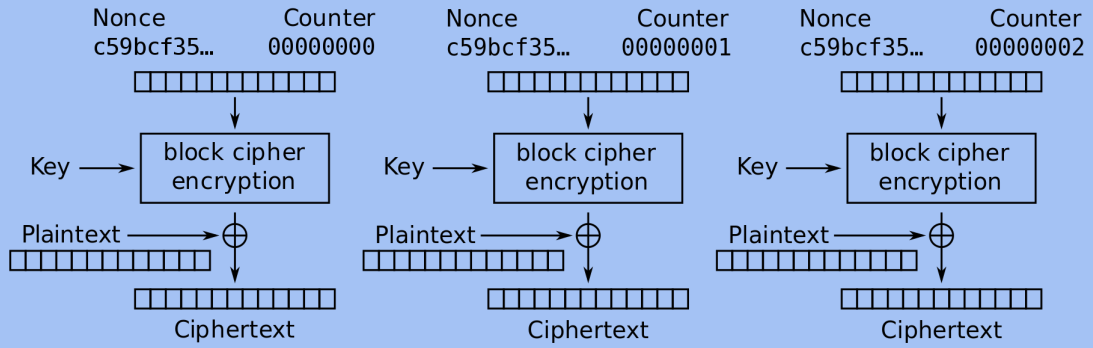
Stream Ciphers: Encrypting Multiple Messages

- Solution: For each message, seed the PRNG with the key and a random IV, concatenated. Send the IV with the ciphertext



Stream Ciphers: AES-CTR

- If you squint carefully, AES-CTR is a type of stream cipher
- Output of the block ciphers is pseudorandom and used as a one-time pad



Counter (CTR) mode encryption

Stream Ciphers: Security

- Stream ciphers are IND-CPA secure, assuming the pseudorandom output is secure
- In some stream ciphers, security is compromised if too much plaintext is encrypted
  - Example: In AES-CTR, if you encrypt so many blocks that the counter wraps around, you'll start reusing keys
  - In practice, if the key is n bits long, usually stop after $2^{n/2}$ bits of output
  - Example: In AES-CTR with 128-bit counters, stop after 2^{64} blocks of output

Stream Ciphers: Encryption Efficiency

- Stream ciphers can continually process new elements as they arrive
  - Only need to maintain internal state of the PRNG
  - Keep generating more PRNG output as more input arrives
- Compare to block ciphers: Need modes of operations to handle longer messages, and modes like AES-CBC need padding to function, so doesn't function well on streams

Stream Ciphers: Decryption Efficiency

- Suppose you received a 1 GB ciphertext (encryption of a 1 GB message) and you only wanted to decrypt the last 128 bytes
- Benefit of some stream ciphers: You can decrypt one part of the ciphertext without decrypting the entire ciphertext
  - Example: In AES-CTR, to decrypt only block *i*, compute E_K(nonce || *i*) and XOR with the *i*th block of ciphertext
  - Example: ChaCha20 (another stream cipher) lets you decrypt arbitrary parts of ciphertext
  - What about HMAC-DRBG? You have to generate all the PRNG output up until the block you want to decrypt

Diffie-Hellman Key Exchange

Textbook Chapter 10

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	• One-time pads • Block ciphers with chaining modes (e.g. AES-CBC)	• RSA encryption • ElGamal encryption
Integrity, Authentication	• MACs (e.g. HMAC)	• Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

Discrete Log Problem and Diffie-Hellman Problem

- Assume everyone knows a large prime p (e.g. 2048 bits long) and a generator g
  - Don't worry about what a generator is
- **Discrete logarithm problem** (**discrete log problem**): Given g, p, $g^a \bmod p$ for random a, it is computationally hard to find a
- **Diffie-Hellman assumption**: Given g, p, $g^a \bmod p$, and $g^b \bmod p$ for random a, b, no polynomial time attacker can distinguish between a random value R and $g^{ab} \bmod p$.
  - Intuition: The best known algorithm is to first calculate a and then compute $(g^b)^a \bmod p$, but this requires solving the discrete log problem, which is hard!
  - Note: Multiplying the values doesn't work, since you get $g^{a+b} \bmod p \neq g^{ab} \bmod p$

Discrete Log Problem and Diffie-Hellman Problem

For a random a, b, R:

$g, p, \quad g^a \bmod p, \quad g^b \bmod p, \quad g^{ab} \bmod p$

$\sim$ ← Indistinguishable from the perspective of a polynomial time attacker

$g, p, \quad g^a \bmod p, \quad g^b \bmod p, \quad R$

Diffie-Hellman Key Exchange

Alice

Public: g, p

Eve

Bob

Secret key → Generate a

Generate b

Calculate $g^a \bmod p$

g^a

g^b

Calculate $g^b \bmod p$

Public key

Receive $g^b \bmod p$

Receive $g^a \bmod p$

Calculate $(g^b)^a \bmod p$

Calculate $(g^a)^b \bmod p$

$a, g^a, g^b \Rightarrow g^{ab}$

$g^a, g^b \not\Rightarrow g^{ab}$

$b, g^a, g^b \Rightarrow g^{ab}$

Shared symmetric key is g^{ab}