# Intro to Cryptography

CS 161 Spring 2024 - Lecture 5

### Last Time: Memory Safety Mitigations

Computer Science 161

#### Memory-safe languages

- Using a memory-safe language (e.g. Python, Java) stops all memory safety vulnerabilities.
- Why use a non-memory-safe language?
  - Commonly-cited reason, but mostly a myth: Performance
  - Real reason: Legacy, existing code

#### Writing memory-safe code

- Carefully write and reason about your code to ensure memory safety in a non-memory-safe language
- Requires programmer discipline, and can be tedious sometimes

#### Building secure software

- Use tools for analyzing and patching insecure code
- Test your code for memory safety vulnerabilities
- Keep any external libraries updated for security patches

### Last Time: Memory Safety Mitigations

Computer Science 161

#### Mitigation: Non-executable pages

- Make portions of memory either executable or writable, but not both
- Defeats attacker writing shellcode to memory and executing it
- Subversions
  - **Return-to-libc**: Execute an existing function in the C library
  - Return-oriented programming (ROP): Create your own code by chaining together small gadgets in existing library code

#### Mitigation: Stack canaries

- Add a sacrificial value on the stack. If the canary has been changed, someone's probably attacking our system
- Defeats attacker overwriting the RIP with address of shellcode
- Subversions
  - An attacker can write around the canary
  - The canary can be leaked by another vulnerability (e.g. format string vulnerability)
  - The canary can be brute-forced by the attacker

### Last Time: Memory Safety Mitigations

Computer Science 161

#### Mitigation: Pointer authentication

- When storing a pointer in memory, replace the unused bits with a pointer authentication code
   (PAC). Before using the pointer in memory, check if the PAC is still valid
- Defeats attacker overwriting the RIP (or any pointer) with address of shellcode

#### Mitigation: Address space layout randomization (ASLR)

- Put each segment of memory in a different location each time the program is run
- Defeats attacker knowing the address of shellcode
- Subversions
  - Leak addresses with another vulnerability
  - Brute-force attack to guess the addresses

#### Combining mitigations

 Using multiple mitigations usually forces the attacker to find multiple vulnerabilities to exploit the program (defense-in-depth)

# What is cryptography?

#### What is cryptography?

- Older definition: The study of secure communication over insecure channels
- Newer definition: Provide rigorous guarantees about the data and computation in the presence of an attacker
  - Not just confidentiality but also integrity and authenticity (we'll see these definitions today)
- Modern cryptography involves a lot of math
  - We'll review any necessary CS 70 prerequisites as they come up

# Don't try this at home!

Computer Science 16

- We will teach you the basic building blocks of cryptography, but you should never try to write your own cryptographic algorithms
- It's very easy to make a mistake that makes your code insecure
  - Required formal rigor and tricky edge cases that we won't cover
  - One small bug could compromise the security of your code
- Instead, use existing well-vetted cryptographic libraries
  - This portion of the class is as much about making you a good consumer of cryptography



Link

February 15, 2017

Cryptography is nightmare magic math that cares what kind of pen you use.

### Don't try this at home!

- In summer 2020, CS 61A wrote a program to distribute online exams
- However, when writing cryptographic code, they used a secure algorithm in an insecure way
- Because of their mistake, it was possible to see exam questions before they were released!
  - Exam leakage was reported, but we never found out if anyone actually attacked the insecure scheme
- The TAs who wrote this code were former CS 161 students!
- Takeaway: Do not write your own crypto code!

# **Definitions**

### Meet Alice, Bob, Eve, and Mallory

- Alice and Bob: The main characters trying to send messages to each other over an insecure communication channel
  - Carol and Dave can also join the party later
- Eve: An eavesdropper who can read any data sent over the channel
  - Also called an honest-but-curious attacker
  - Does not tamper with data or code
- Mallory: A manipulator who can read and modify any data sent over the channel
  - Also called a malicious attacker



### Meet Alice, Bob, Eve, and Mallory

- We often describe cryptographic problems using a common cast of characters
- One scenario:
  - Alice wants to send a message to Bob.
  - However, Eve is going to eavesdrop on the communication channel.
  - How does Alice send the message to Bob without Eve learning about the message?
- Another scenario:
  - Bob wants to send a message to Alice.
  - However, Mallory is going to tamper with the communication channel.
  - How does Bob send the message to Alice without Mallory changing the message?

### Three Main Goals of Cryptography

- In cryptography, there are three common properties that we want on our data
- Confidentiality: An adversary cannot read our messages.
- Integrity: An adversary cannot change our messages without being detected.
- Authenticity: I can prove that this message came from the person who claims to have written it.
  - Integrity and authenticity are closely related properties...
    - Before I can prove that a message came from a certain person, I have to prove that the message wasn't changed!
  - ... but they're not identical properties
    - Later we'll see some edge cases

# Keys

- The most basic building block of any cryptographic scheme: The key
- Properly chosen and guarded keys "power" the security of our cryptographic algorithms
- Two models of keys:
  - Symmetric key model: Alice and Bob both know the value of the same secret key.
  - Asymmetric key model: A user has two keys, a secret key and a public key.
    - Example: You might remember RSA encryption from CS 70

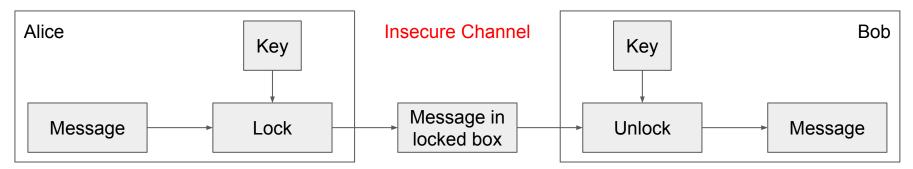


### Security Principle: Kerckhoff's Principle

- This principle is closely related to Shannon's Maxim
  - Don't use security through obscurity. Assume the attacker knows the system.
- Kerckhoff's principle says:
  - Cryptosystems should remain secure even when the attacker knows all internal details of the system
  - The key should be the only thing that must be kept secret
  - The system should be designed to make it easy to change keys that are leaked (or suspected to be leaked)
    - If your secrets are leaked, it is usually a lot easier to change the key than to replace every instance of the running software
- Our assumption: The attacker knows all the algorithms we use. The only information the attacker is missing is the secret key(s).

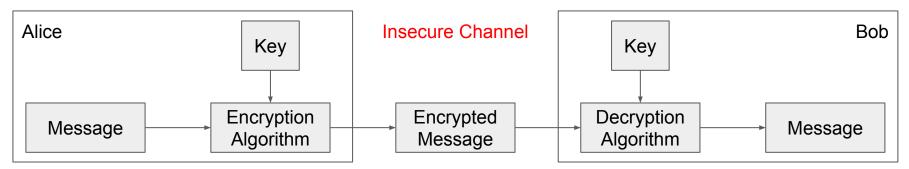
### Confidentiality

- Confidentiality: An adversary cannot read our messages.
- Analogy: Locking and unlocking the message
  - Alice uses the key to lock the message in a box
  - Alice sends the message (locked in the box) over the insecure channel
  - Eve sees the locked box, but cannot access the message without the key
  - Bob receives the message (locked in the box) and uses the key to unlock the message



#### Confidentiality

- Schemes provide confidentiality by encrypting messages
  - Alice uses the key to encrypt the message: Change the message into a scrambled form.
  - Alice sends the encrypted message over the insecure channel
  - Eve sees the encrypted message, but cannot figure out the original message without the key
  - Bob receives the encrypted message and uses the key to decrypt the message back into its original form

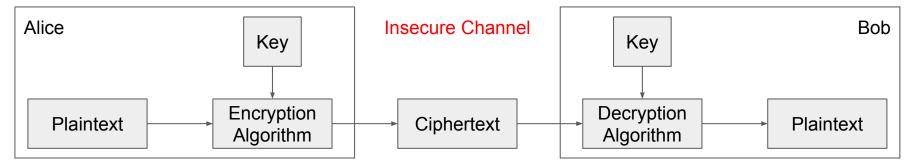


### Confidentiality

Computer Science 161

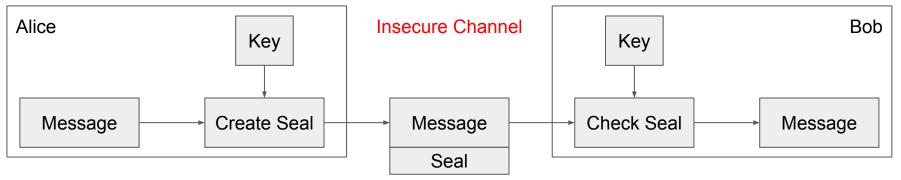
Plaintext: The original message

• **Ciphertext**: The encrypted message



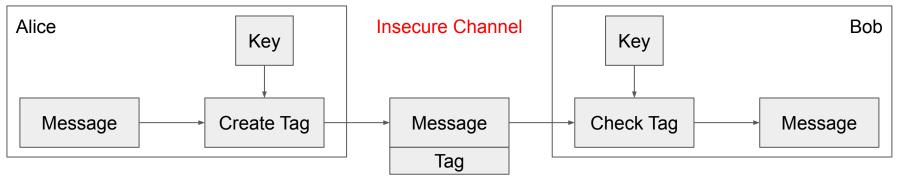
# Integrity (and Authenticity)

- Integrity: An adversary cannot change our messages without being detected.
- Analogy: Adding a seal on the message
  - Alice uses the key to add a special seal on the message (e.g. puts tape on the envelope)
  - Alice sends the message and the seal over the insecure channel
  - o If Mallory tampers with the message, she'll break the seal (e.g. break the tape on the envelope)
  - Without the key, Mallory cannot create her own seal
  - o Bob receives the message and the seal and checks that the seal has not been broken



# Integrity (and Authenticity)

- Schemes provide integrity by adding a tag or signature on messages
  - Alice uses the key to generate a special tag for the message
  - Alice sends the message and the tag over the insecure channel
  - If Mallory tampers with the message, the tag will no longer be valid
  - Bob receives the message and the tag and checks that the tag is still valid
- More on integrity in a future lecture



#### **Threat Models**

- What if Eve can do more than eavesdrop?
- Real-world schemes are often vulnerable to more sophisticated attackers, so cryptographers have created more sophisticated threat models too
- Some threat models for analyzing confidentiality:

	Can Eve trick Alice into encrypting messages of Eve's choosing?	Can Eve trick Bob into decrypting messages of Eve's choosing?	
Ciphertext-only	No	No	
Chosen-plaintext	Yes	No	
Chosen-ciphertext	No	Yes	
Chosen plaintext-ciphertext	Yes	Yes	

#### **Threat Models**

- In this class, we'll explain the chosen plaintext attack model
- In practice, cryptographers use the chosen plaintext-ciphertext model
  - It's the most powerful
  - It can actually be defended against

	Can Eve trick Alice into encrypting messages of Eve's choosing?	Can Eve trick Bob into decrypting messages of Eve's choosing?	
Ciphertext-only	No	No	
Chosen-plaintext	Yes	No	
Chosen-ciphertext	No	Yes	
Chosen plaintext-ciphertext	Yes	Yes	

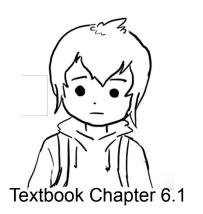
# Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	<ul> <li>One-time pads</li> <li>Block ciphers with chaining modes (e.g. AES-CBC)</li> <li>Stream ciphers</li> </ul>	<ul><li>RSA encryption</li><li>ElGamal encryption</li></ul>
Integrity, Authentication	MACs (e.g. HMAC)	Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)

- Key management (certificates)
- Password management

# Symmetric-Key Encryption



# Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	<ul> <li>One-time pads</li> <li>Block ciphers with chaining modes (e.g. AES-CBC)</li> <li>Stream ciphers</li> </ul>	<ul><li>RSA encryption</li><li>ElGamal encryption</li></ul>
Integrity, Authentication	MACs (e.g. HMAC)	<ul> <li>Digital signatures (e.g. RSA signatures)</li> </ul>

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)

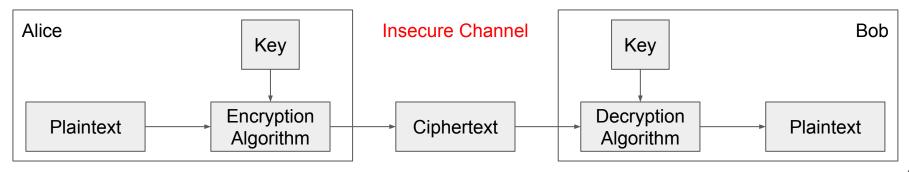
- Key management (certificates)
- Password management

### Symmetric-Key Encryption

- The next few schemes are symmetric-key encryption schemes
  - Encryption schemes aim to provide confidentiality (but not integrity or authentication)
  - Symmetric-key means Alice and Bob share the same secret key that the attacker doesn't know
    - Don't worry about how Alice and Bob share the key for now
- For modern schemes, we're going to assume that messages are bitstrings
  - o **Bitstring**: A sequence of bits (0 or 1), e.g. 11010101001001010
  - Text, images, etc. can be converted into bitstrings before encryption, so bitstrings are a useful abstraction. After all, everything in a computer is just a sequence of bits!

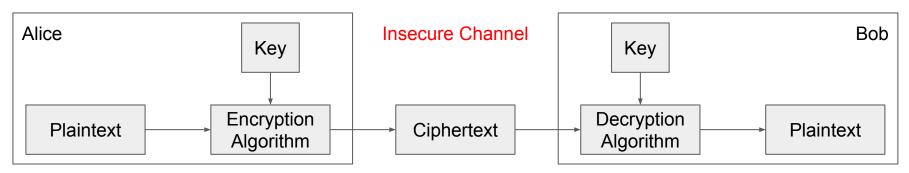
### Symmetric-Key Encryption: Definition

- A symmetric-key encryption scheme has three algorithms:
  - KeyGen() → K: Generate a key K
  - $\circ$  Enc(K, M)  $\to$  C: Encrypt a **plaintext** M using the key K to produce **ciphertext** C
  - $\circ$  Dec(K, C)  $\to M$ : Decrypt a ciphertext C using the key K



# Symmetric-Key Encryption: Definition

- What properties do we want from a symmetric encryption scheme?
  - Correctness: Decrypting a ciphertext should result in the message that was originally encrypted
    - Dec(K, Enc(K, M)) = M for all  $K \leftarrow KeyGen()$  and M
  - Efficiency: Encryption/decryption algorithms should be fast: >1 Gbps on a standard computer
  - Security: Confidentiality



# **Defining Confidentiality**

- Recall our definition of confidentiality from earlier: "An adversary cannot read our messages"
  - This definition isn't very specific
    - What if Eve can read the first half of Alice's message, but not the second half?
    - What if Eve figures out that Alice's message starts with "Dear Bob"?
  - This definition doesn't account for prior knowledge
    - What if Eve already knew that Alice's message ends in "Sincerely, Alice"?
    - What if Eve knows that Alice's message is "BUY!" or "SELL" but doesn't know which?





# **Defining Confidentiality**

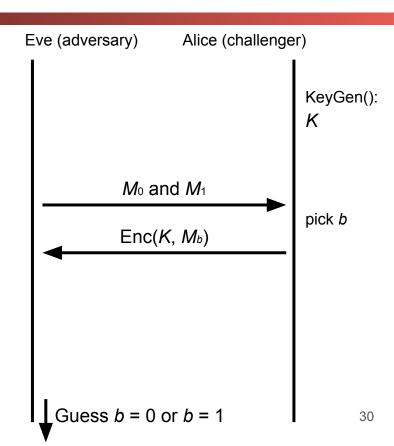
- A better definition of confidentiality: The ciphertext should not give the attacker any additional information about the plaintext.
- Let's design an experiment/security game to test our definition

# Security game: first attempt at confidentiality

**Computer Science 161** 

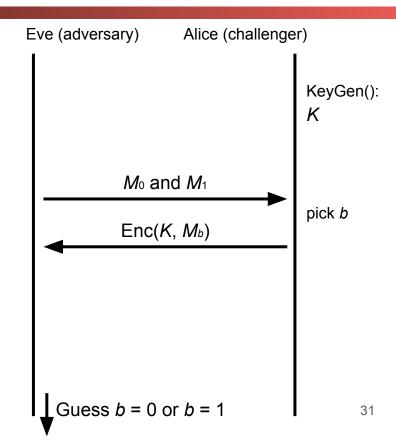
- 1. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice of the same length
- 2. Alice randomly chooses either *M*<sub>0</sub> or *M*<sub>1</sub> to encrypt and sends the encryption back
  - a. Alice does not tell Eve which one was encrypted!
- 3. Eventually, Eve outputs a guess as to whether Alice encrypted *M*<sub>0</sub> or *M*<sub>1</sub>

Q: If the scheme provides confidentiality, what chance does the attacker have to guess b?



#### **Security game**: intuition

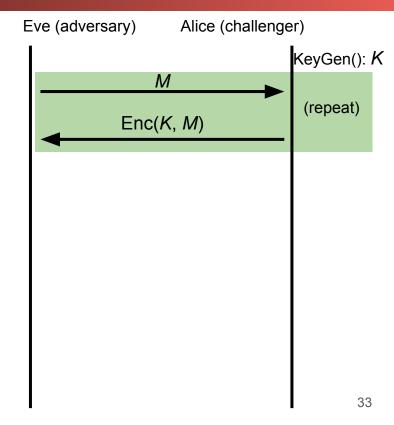
- If the scheme is secure Eve can only guess with probability 1/2, which is no different than if Eve hadn't sent the ciphertext at all
- In other words: the ciphertext gave Eve no additional information about which plaintext was sent!



- Recall our threat model: Eve can also perform a chosen plaintext attack
  - Eve can trick Alice into encrypting arbitrary messages of Eve's choice
  - We can adapt our experiment to account for this threat model
- A better definition of confidentiality: Even if Eve is able to trick Alice into encrypting messages, Eve can still only guess what message Alice sent with probability 1/2.
  - This definition is called IND-CPA (indistinguishability under chosen plaintext attack)
- Cryptographic properties are often defined in terms of "games" that an adversary can either "win" or "lose"
  - We will use one to define confidentiality precisely

Computer Science 161

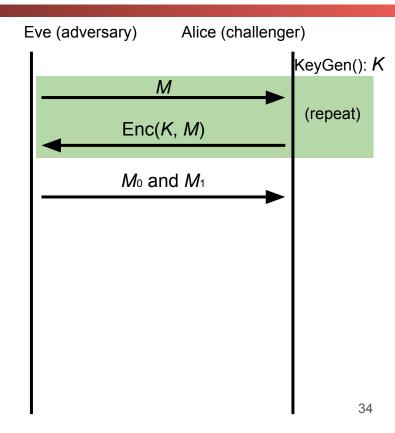
1. Eve may choose plaintexts to send to Alice and receives their ciphertexts



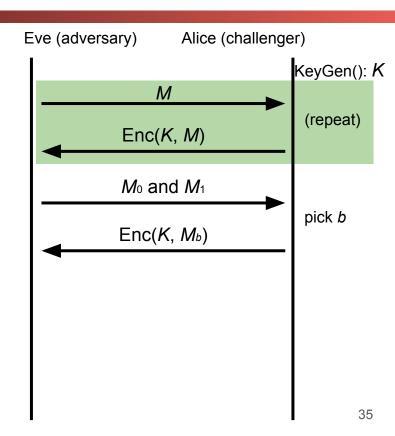
Computer Science 161

1. Eve may choose plaintexts to send to Alice and receives their ciphertexts

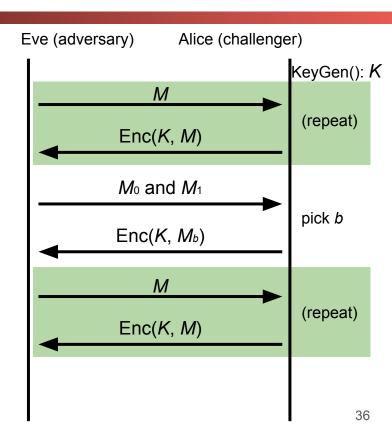
2. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice



- 1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
- 2. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice
- 3. Alice randomly chooses either  $M_0$  or  $M_1$  to encrypt and sends the encryption back
  - a. Alice does not tell Eve which one was encrypted!

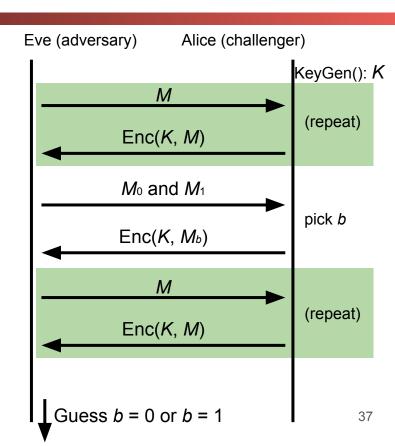


- 1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
- 2. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice
- 3. Alice randomly chooses either  $M_0$  or  $M_1$  to encrypt and sends the encryption back
  - a. Alice does not tell Eve which one was encrypted!
- 4. Eve may again choose plaintexts to send to Alice and receives their ciphertexts



#### Defining Confidentiality: IND-CPA

- 1. Eve may choose plaintexts to send to Alice and receives their ciphertexts
- 2. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice
- 3. Alice randomly chooses either  $M_0$  or  $M_1$  to encrypt and sends the encryption back
  - a. Alice does not tell Eve which one was encrypted!
- 4. Eve may again choose plaintexts to send to Alice and receives their ciphertexts
- 5. Eventually, Eve outputs a guess as to whether Alice encrypted *M*<sub>0</sub> or *M*<sub>1</sub>



#### Defining Confidentiality: IND-CPA

- If Eve correctly guesses which message Alice encrypted, then Eve wins.
   Otherwise, she loses.
- How does Eve guess whether Mo or M1 was encrypted? What strategy does she use?
  - We don't assume she uses a particular strategy; Eve represents all possible strategies
- Proving insecurity: There exists at least one strategy that can win the IND-CPA game with probability > 1/2
  - 1/2 is the probability of winning by random guessing
  - If you can be better than random, then the ciphertext has leaked information, and Eve is able to learn it and use it to gain an advantage!
- Proving security: For all (polynomial-time) attackers/Eve-s, the probability of winning the IND-CPA game is at most ½+negl

#### **Edge Cases: Length**

- Cryptographic schemes are (usually) allowed to leak the length of the message
  - o To hide length: All messages must always be the same length
    - 16-byte messages: We can't encrypt large messages (images, videos, etc.)!
    - 1-GB messages: Sending small messages (text, Tweets, etc.) needs 1 GB of bandwidth!
    - This is unpractical
  - Applications can choose to hide length by *padding* their own messages to the maximum possible length before encrypting
- In the IND-CPA game: Mo and M1 must be the same length
  - o To break IND-CPA, Eve must learn something other than message length

#### Edge Cases: Attacker Runtime

Computer Science 16

- Some schemes are theoretically vulnerable, but secure in any real-world setting
  - o If an attack takes longer than the life of the solar system to complete, it probably won't happen!
  - o Or if it would require a computer made out of a literal galaxy worth of science-fiction nanotech
- In the IND-CPA game: Eve is limited to a practical runtime

One common practical limit: Eve is limited to polynomial runtime algorithms (no exponential-time algorithms)

4

#### Edge Cases: Negligible Advantage

- Sometimes it's possible for Eve to win with probability 1/2 + 1/2<sup>128</sup>
  - This probability is greater than 1/2, but it's so close to 1/2 that it's as good as 1/2.
  - Eve's advantage is so small that she can't use it for any practical attacks
  - 2<sup>128</sup> is larger than the total number of atoms in the universe
- In the IND-CPA game: The scheme is secure even if Eve can win with probability ≤ 1/2 + E, where E is negligible
  - The actual mathematical definition of negligible is out of scope
  - Example: 1/2 + 1/2<sup>128</sup>: Negligible advantage
  - Example: 2/3: Non-negligible advantage



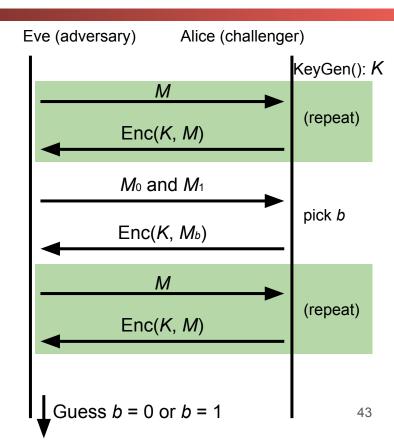
#### Edge Cases: Negligible Advantage

- Defining negligibility mathematically:
  - Advantage of the adversary should be exponentially small, based on the security parameters of the algorithm
  - $\circ$  Example: For an encryption scheme with a k-bit key, the advantage should be  $O(1/2^k)$
- Defining negligibility practically:
  - A 1/2<sup>128</sup> probability is currently unlikely
  - A 1/2<sup>20</sup> probability is fairly likely
    - "One in a million events happen every day in New York City"
  - In between these extremes, it can be messy
    - Different algorithms run faster or slower and have their own security parameters
    - Computers get more powerful over time
    - Recall: Know your threat model!
- **Takeaway**: For now, 2<sup>80</sup> is a reasonable threshold, but this will change over time!

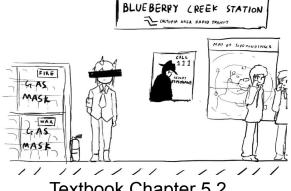
#### IND-CPA: Putting it together

- Eve may choose plaintexts to send to Alice and receives their ciphertexts
- 2. Eve issues a pair of plaintexts  $M_0$  and  $M_1$  to Alice
- 3. Alice randomly chooses either  $M_0$  or  $M_1$  to encrypt and sends the encryption back
  - Alice does not tell Eve which one was encrypted!
- Eve may again choose plaintexts to send to Alice and receives their ciphertexts
- 5. Eventually, Eve outputs a guess as to whether Alice encrypted  $M_0$  or  $M_1$

- An encryption scheme is IND-CPA secure if for all polynomial time attackers Eve:
  - Eve can win with probability  $\leq 1/2 + \mathcal{E}$ , where  $\mathcal{E}$  is *negligible*.



# A Brief History of Cryptography



Textbook Chapter 5.2

### Cryptography by Hand: Caesar Cipher

- One of the earliest cryptographic schemes was the Caesar cipher
  - Used by Julius Caesar over 2,000 years ago
- KeyGen():
  - Choose a key K randomly between 0 and 25
- Enc(*K*, *M*):
  - Replace each letter in *M* with the letter *K* positions later in the alphabet
  - $\circ$  If K = 3, plaintext DOG becomes GRJ
- Dec(*K*, *C*):
  - Replace each letter in C with the letter K positions earlier in the alphabet
  - $\circ$  If K = 3, ciphertext GRJ becomes DOG

K = 3					
M	С	М	С		
Α	D	N	Q		
В	E	0	R		
С	F	Р	S		
D	G	Q	Т		
Е	Н	R	U		
F	I	S	V		
G	J	Т	W		
Н	K	U	Χ		
l	L	V	Υ		
J	М	W	Z		
K	N	X	Α		
L	0	Υ	В		
М	Р	Z	С		

### Cryptography by Hand: Attacks on the Caesar Cipher

- Eve sees the ciphertext JCKN ECGUCT, but doesn't know the key K
- If you were Eve, how would you try to break this algorithm?
- Brute-force attack: Try all 26 possible keys!
- Use existing knowledge: Assume that the message is in English

+1	IBJM	DBFTBS	+9	ATBE	VTXLTK	+17	SLTW	NLPDLC
+2	HAIL	CAESAR	+10	ZSAD	USWKSJ	+18	RKSV	MKOCKB
+3	GZHK	BZDRZQ	+11	YRZC	TRVJRI	+19	QJRU	LJNBJA
+4	FYGJ	AYCQYP	+12	XQYB	SQUIQH	+20	PIQT	KIMAIZ
+5	EXFI	ZXBPXO	+13	WPXA	RPTHPG	+21	OHPS	JHLZHY
+6	DWEH	YWAOWN	+14	VOWZ	QOSGOF	+22	NGOR	IGKYGX
+7	CVDG	XVZNVM	+15	UNVY	PNRFNE	+23	MFNQ	HFJXFW
+8	BUCF	WUYMUL	+16	TMUX	OMQEMD	+24	LEMP	GEIWEV
						+25	KDLO	FDHVDU

### Cryptography by Hand: Attacks on the Caesar Cipher

- Eve sees the ciphertext JCKN ECGUCT, but doesn't know the key K
- Chosen-plaintext attack: Eve tricks Alice into encrypting plaintext of her choice
  - Eve sends a message M = AAA and receives C = CCC
  - $\sim$  Eve can deduce the key: C is 2 letters after A, so K = 2
  - Eve has the key, so she can decrypt the ciphertext

### Cryptography by Hand: Substitution Cipher

- A better cipher: create a mapping of each character to another character.
  - Example: A = N, B = Q, C = L, D = Z, etc.
  - Unlike the Caesar cipher, the shift is no longer constant!
- KeyGen():
  - Generate a random, one-to-one mapping of characters
- Enc(K, M):
  - Map each letter in M to the output according to the mapping K
- Dec(K, C):
  - Map each letter in C to the output according to the reverse of the mapping K

К					
М	С	М	С		
Α	N	N	G		
В	Q	0	Р		
С	L	Р	Т		
D	Z	Q	Α		
Е	K	R	J		
F	R	S	0		
G	V	Т	D		
Н	U	U	I		
I	Е	V	С		
J	S	W	F		
K	В	X	М		
L	W	Υ	Х		
М	Υ	Z	Н		

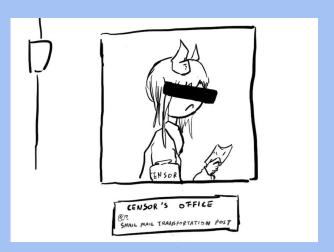
#### Cryptography by Hand: Attacks on Substitution Ciphers

- Does the brute-force attack still work?
  - There are 26! ≈ 2<sup>88</sup> possible mappings to try
    - Too much for most modern computers... for now
- How about the chosen-plaintext attack?
  - Trick Alice into encrypting ABCDEFGHIJKLMNOPQRSTUVWXYZ, and you'll get the whole mapping!
- Another strategy: cryptanalysis
  - The most common english letters in text are
     E, T, A, O, I, N

К				
М	С	М	С	
Α	N	N	G	
В	Q	0	Р	
С	L	Р	Т	
D	Z	Q	Α	
Е	K	R	J	
F	R	S	0	
G	V	Т	D	
Н	U	U	ı	
ı	Е	V	С	
J	S	W	F	
K	В	X	М	
L	W	Υ	X	
М	Υ	Z	Н	

#### **Takeaways**

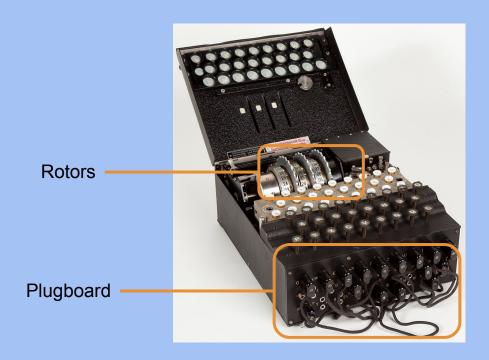
- Cryptography started with paper-and-pencil algorithms (Caesar cipher)
- Then cryptography moved to machines (Enigma)
- Finally, cryptography moved to computers (which we're about to study)
- Hopefully you gained some intuition for some of the cryptographic definitions



### Cryptography by Machines: Enigma

Computer Science 161

A mechanical encryption machine used by the Germans in WWII





#### Enigma Operating Principle: Rotor Machine

- The encryption core was composed of 3 or 4 rotors
  - Each rotor was a fixed permutation (e.g. A maps to F, B maps to Q...)
  - And the end was a "reflector", a rotor that sent things backwards
  - Plus a fixed-permutation plugboard
- A series of rotors were arranged in a sequence
  - Each keypress would generate a current from the input to one light for the output
  - Each keypress also advanced the first rotor
    - When the first rotor makes a full rotation, the second rotor advances one step
    - When the second rotor makes a full rotation, the third rotor advances once step

#### Cryptography by Machines: Enigma

Computer Science 161

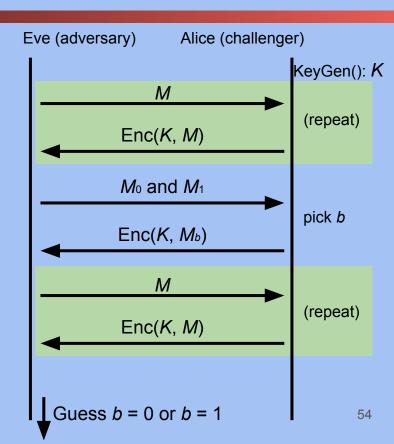
#### KeyGen():

- Choose rotors, rotor orders, rotor positions, and plugboard settings
- 158,962,555,217,826,360,000 possible keys
- Enc(K, M) and Dec(K, C):
  - Input the rotor settings *K* into the Enigma machine
  - Press each letter in the input, and the lampboard will light up the corresponding output letter
  - Encryption and decryption are the same algorithm!
- Germans believed that Enigma was an "unbreakable code"



## Cryptography by Machines: Enigma

- Enigma has a significant weakness: a letter never maps to itself!
  - No rotor maps a letter to itself
  - The reflector never maps a letter to itself
  - This property is necessary for Enigma's mechanical system to work
- What pair of messages should Eve send to Alice in the challenge phase?
  - $\circ$  Send  $M_0 = A^k$ ,  $M_1 = B^k$
  - M₀ is a string of k 'A' characters, M₁ is a string of k 'B' characters
- How can Eve probably know which message Alice encrypted?



## Cryptography by Machines: Attack on Enigma

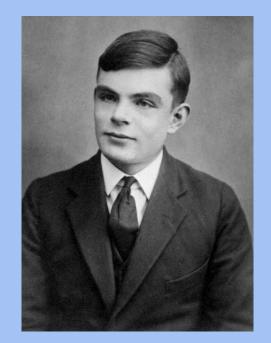
- Polish and British cryptographers built BOMBE, a machine to brute-force Enigma keys
- Why was Enigma breakable?
  - Kerckhoff's principle: The Allies stole Enigma machines, so they knew the algorithm
  - Known plaintext attacks: the Germans often sent predictable messages (e.g. the weather report every morning)
  - Chosen plaintext attacks: the Allies could trick the Germans into sending a message (e.g. "newly deployed minefield")
  - Brute-force: BOMBE would try many keys until the correct one was found
    - Plus a weakness: You'd be able to try multiple keys with the same hardware configuration



**BOMBE** machine

#### Cryptography by Machines: Legacy of Enigma

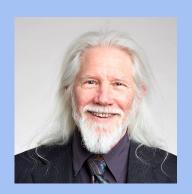
- Alan Turing, one of the cryptographers who broke Enigma, would go on to become one of the founding fathers of computer science
- Most experts agree that the Allies breaking Enigma shortened the war in Europe by about a year



Alan Turing

#### Cryptography by Computers

- The modern era of cryptography started after WWII, with the work of Claude Shannon
- "New Directions in Cryptography" (1976) showed how number theory can be used in cryptography
  - Its authors, Whitfield Diffie and Martin Hellman, won the Turing Award in 2015 for this paper
- This is the era of cryptography we'll be focusing on



One of these is Diffie, and the other one is Hellman.

