EECS 182 Deep Neural Networks
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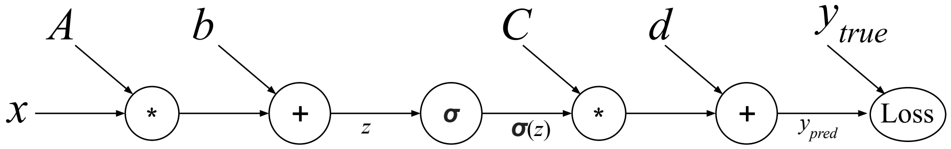
Midterm Review: Basics

Consider the simple neural network that takes a scalar real input, has 1 hidden layer with k units in it and a sigmoid nonlinearity for those units, and an output linear (affine) layer to predict a scalar output. We can algebraically write any function that it represents as

$$y_{pred} = C\sigma(Ax + \mathbf{b}) + d$$

The $\sigma(.)$ represents an arbitrary nonlinearity, with derivative $\sigma'(.)$ Where $x \in \mathbb{R}$, $A \in \mathbb{R}^{k\times 1}$, $\mathbf{b} \in \mathbb{R}^{k\times 1}$, $C \in \mathbb{R}^{1\times k}$, $d \in \mathbb{R}$, and $y_{pred} \in \mathbb{R}$ We can write it as $y_{pred} = C\sigma(\mathbf{z}) + d$, where $z = Ax + \mathbf{b}$ and the nonlinearity is applied element-wise. We have the true label y_{true} for each x, and we use the L2 Loss $L(y_{true}, y_{pred}) = (y_{true} - y_{pred})^2$.



1. (a) Consider the sigmoid nonlinearity function $\sigma(z) = \frac{1}{1+e^{-z}}$. Show that $\frac{d}{dz}\sigma(z) = \sigma(z)(1-\sigma(z))$

(b) Calculate $\frac{\partial L}{\partial d}$

(c) Calculate $\frac{\partial L}{\partial C_i}$

(d) Calculate $\frac{\partial L}{\partial b_i}$

(e) Calculate $\frac{\partial L}{\partial A_i}$

(f) Write the gradient-descent update rule for $\mathbf{b}_{t+1}$ with learning rate α.

2. Given the Regularized Objective function:

$$\underset{\mathbf{x}}{\operatorname{argmin}} \|A\mathbf{x} - \mathbf{b}\|^2 + \lambda\|\mathbf{x}\|^2$$

Use vector calculus to find the closed form solution for $\mathbf{x}$. Interpret what this means in terms of the singular values.

3. Consider a simple neural network that spits out 1-dim values after a nonlinearity. These values for a batch are $\{1, 7, 7, 9\}$. What is the output of running batchnorm with this data and $\gamma = 1$ and $\beta = 0$. In other words, standardize the data to have mean 0 and variance 1.

4. Consider a simplified batchnorm layer where we don't actually divide by standard deviation, instead we just de-mean our data before scaling it by γ, then passing it to the next layer. That is, we calculate our mini-batch mean μ, then simply let $\hat{x}_i = x_i - \mu$, and $y_i = \gamma\hat{x}_i$ is passed onto the next layer. Assume batchsize of m. If our final loss function is L, Calculate $\frac{\partial L}{\partial x_i}$ in terms of $\frac{\partial L}{\partial y_j}$ for $j = 1, ...m$, γ, and m.