UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 05
Spring 2023

1. **Order Statistics Practice**

For the random variables $X_1, \ldots, X_5 \sim_{\text{i.i.d.}} \text{Uniform}([0,1])$, let $X_{(i)}$ be the ith order statistic, i.e. the ith smallest value of $\{X_1, \ldots, X_5\}$. Recall that the pdf of $X_{(i)}$ is

$$f_{X_{(i)}}(x) = n\binom{n-1}{i-1} f(x)F(x)^{i-1}(1-F(x))^{n-i},$$

where f and F are the pdf and cdf of X_i.

a. What is the pdf of $X_{(i)}$ for $i = 1, \ldots, 5$?

b. What is $\mathbb{E}(X_{(i)})$?

c. Find the expected value of the range of $\{X_1, \ldots, X_5\}$, that is, the difference between the lowest and highest values.

2. **Exponential Bounds**

   Let $X \sim \text{Exponential}(\lambda)$. For $x > \lambda^{-1}$, find bounds on $\mathbb{P}(X \geq x)$ using Markov's inequality, Chebyshev's inequality, and the Chernoff bound.

3. **Convergence in Probability**

Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of i.i.d. random variables distributed uniformly in $[-1,1]$. Show that the following sequences $(Y_n)_{n\in\mathbb{N}}$ converge in probability to some limit.

a. $Y_n = \prod_{i=1}^{n} X_i$.

b. $Y_n = \max\{X_1,\ldots,X_n\}$.

c. $Y_n = (X_1^2+\cdots+X_n^2)/n$.