UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 09
Spring 2023

1. **Product of Rolls of a Die**

   A fair die with labels 1 through 6 is rolled until the product of the last two rolls is 12. What is the expected number of rolls?

   Hint: You can model this process as a Markov chain with 3 states, choosing your states according to the outcome of last roll. For example, assign one state if its outcome was 1 or 5, which is useless if you want the product to be 12. If the outcome was 2, 3, 4 or 6, it's useful and can be assigned to another state. Assign a third state to the case when the product of the last two outcomes was 12.

2. **Poisson Process Warmup**

Give an interpretation of the following fact in terms of a Poisson process with rate λ. If N is Geometric with parameter p and $(X_k)_{k\in\mathbb{N}}$ are i.i.d. Exponential(λ), then $X_1 + \cdots + X_N$ has an Exponential distribution with parameter λp.

3. **Customers in a Store**

Consider two independent Poisson processes with rates λ_1 and λ_2, which measure the number of customers arriving in store 1 and 2.

a. What is the probability that a customer arrives in store 1 before any arrives in store 2?

b. What is the probability that in the first hour, a total of exactly 6 customers arrive in the two stores?

c. Given that exactly 6 have arrived in total at the two stores, what is the probability that exactly 4 went to store 1?