UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 03
Spring 2023

1. **Uncorrelatedness and Independence**

   a. Show that if $X_1, \ldots, X_n$ are pairwise uncorrelated, then

   $$\mathrm{var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathrm{var}(X_i).$$

   b. Find an example where a pair of random variables are uncorrelated but not independent.

2. **Sampling Without Replacement**

   Suppose you have N items, G of which are good and B of which are bad. $B + G = N$ are all positive integers. You start to draw items without replacement, and suppose that the first good item appears on draw X. Compute the mean and variance of X.

3. **Galton–Watson Branching Process**

   Consider a population of N individuals for some positive integer N. Let ξ be a random variable taking values in $\mathbb{N}$ with $\mathbb{E}(\xi) = \mu$ and $\mathrm{var}(\xi) = \sigma^2$. At the end of each year, each individual, independently of all other individuals and generations, leaves behind a number of offspring which has the same distribution as ξ. For each $n \in \mathbb{N}$, let X_n denote the size of the population at the end of the nth year.

   a. Compute $\mathbb{E}(X_n)$.
   b. Compute $\mathrm{var}(X_n|X_{n-1})$. Then, write $\mathrm{var}(X_n)$ in terms of $\mathrm{var}(X_{n-1})$.