UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 13
Spring 2023

1. **Orthogonal LLSE**

   a. Consider zero-mean random variables X, Y, Z with Y, Z orthogonal. Show that

   $$\mathbb{L}(X \mid Y, Z) = \mathbb{L}(X \mid Y) + \mathbb{L}(X \mid Z).$$

   b. Now, for *any* zero-mean random variables X, Y, Z, explain why it holds that

   $$\mathbb{L}(X \mid Y, Z) = \mathbb{L}(X \mid Y) + \mathbb{L}[X \mid (Z - \mathbb{L}(Z \mid Y))].$$

2. **Improving Estimation Error**

   Show that "more information yields better estimation error":

   $$\mathbb{E}[(X - \mathbb{E}[X \mid Y, Z])^2] \leq \mathbb{E}[(X - \mathbb{E}[X \mid Y])^2].$$

   Hint: $\mathbb{E}[X \mid Y]$ can be interpreted as the orthogonal projection of X onto the subspace of all functions of Y.

3. **Photodetector LLSE**

Consider a photodetector in an optical communications system that counts the number of photons arriving during a certain interval. A user can convey information to the system using a photon transmitter. Suppose the transmitter is constantly on with probability p and constantly off otherwise.

- If the transmitter is on, the number of photons transmitted over the interval of interest is a Poisson random variable Θ with mean λ.
- If the transmitter is off, the number of photons transmitted is 0.

Unfortunately, regardless of whether the transmitter is on or off, photons may be detected due to "shot noise." The number N of detected shot noise photons is a Poisson(μ) random variable, independent of the number T of transmitted photons. If D is the total number of photons detected, find $\mathbb{L}(T \mid D)$.

(You do not have to simplify your final expression.)