UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Spring 2023

1. **Generating Erdős–Rényi Random Graphs**

   Let G_1 and G_2 be independent Erdős–Rényi random graphs on n vertices with probabilities p_1 and p_2 respectively. Let G be $G_1 \cup G_2$, that is, the graph generated by combining the edges in G_1 and G_2.

   a. Is G an Erdős–Rényi random graph on n vertices with probability $p_1 + p_2$?

   b. Is G an Erdős–Rényi random graph?

2. **Degree of Vertex in Random Graph**

   Consider a random undirected graph on n vertices in which each of the $\binom{n}{2}$ possible edges is present with probability p, independent of all other edges.

   a. Fix a particular vertex of the graph, and let D be the random variable equal to its degree. Find the pmf and expected value of D.

   b. Suppose that p depends on n in a way such that np tends to a fixed constant λ as $n \to \infty$. For large n, how could we model the pmf of D without using n?

3. **Maximum Likelihood Estimation for the Binary Symmetric Channel**

You are testing a digital link that corresponds to a BSC with some error probability $\varepsilon \in [0, 0.5]$.

a. Suppose that you observe an input bit X and an output bit Y. Calculate the MLE of ε, which is the value of $\varepsilon \in [0, 0.5]$ maximizing the probability you observe the given data. Here, since the observation is (X, Y), we want to find

$$\varepsilon_{\text{MLE}} = \operatorname*{argmax}_{\varepsilon \in [0,0.5]} \mathbb{P}_{\varepsilon}(X = x, Y = y).$$

b. You are now told that the inputs $X_1, \ldots, X_n$ are i.i.d. Bernoulli(0.6) bits. Suppose that you observe n outputs $Y_1, \ldots, Y_n$. Calculate the MLE of ϵ.