UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 12
Spring 2023

1. **Exponential MLE, MAP, and Hypothesis Testing**

   Let X be Exponentially distributed with rate 1. Given X, the random variable Y is Exponentially distributed with parameter X.

   a. Find $\text{MLE}(X \mid Y)$.

   b. Find $\text{MAP}(X \mid Y)$.

   c. Let $c > 1$. Suppose that

      - The null hypothesis is $X = 1$: $Y \sim \text{Exponential}(1)$, and
      - The alternative hypothesis is $X = c$: $Y \sim \text{Exponential}(c)$.

      Find the decision rule $\hat{X}$ (a function of Y) that maximizes $\mathbb{P}(\hat{X} = 1 \mid X = 1)$ subject to $\mathbb{P}(\hat{X} = 1 \mid X = c) \le 5\%$.

2. **Hypothesis Testing for Bernoulli Random Variables**

   Suppose that

   - The null hypothesis is $X = 0$: $Y \sim \text{Bernoulli}(\frac{1}{4})$, and
   - The alternative hypothesis is $X = 1$: $Y \sim \text{Bernoulli}(\frac{3}{4})$.

   Using the Neyman–Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $\hat{X}$ with respect to the criterion

$$\begin{aligned} \min \quad & \mathbb{P}(\hat{X} = 0 \mid X = 1) \\ \text{s.t.} \quad & \mathbb{P}(\hat{X} = 1 \mid X = 0) \leq \beta, \end{aligned}$$

   where $\beta \in [0, 1]$ is a given upper bound on the probability of false alarm (PFA).

3. **Hypothesis Testing for Uniform Random Variables**

Suppose that

- The null hypothesis is $X = 0$: $Y \sim \text{Uniform}([-1, 1])$, and
- The alternative hypothesis is $X = 1$: $Y \sim \text{Uniform}([0, 2])$.

Using the Neyman–Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $\hat{X}$ with respect to the criterion

$$\begin{aligned} \min \quad & \mathbb{P}(\hat{X} = 0 \mid X = 1) \\ \text{s.t.} \quad & \mathbb{P}(\hat{X} = 1 \mid X = 0) \leq \beta, \end{aligned}$$

where $\beta \in [0, 1]$ is a given upper bound on the probability of false alarm (PFA).