UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 04
Spring 2023

1. **Drawing Batteries I**

   You have an endless box of used batteries. The number of hours remaining in a battery is i.i.d. Uniform($[0, 1]$).

   a. Suppose you draw n batteries, and the ith battery you draw has X_i hours remaining. What is $\mathbb{P}(X_1 \leq X_2 \leq \cdots \leq X_n)$?

   b. Now suppose you draw batteries until you have enough batteries to last one hour in total. Let N be the number of batteries you draw. What is $\mathbb{P}(N > 2)$? $\mathbb{P}(N > 3)$?

2. **Graphical Density**

Figure 1 shows the joint density $f_{X,Y}$ of the random variables X and Y.

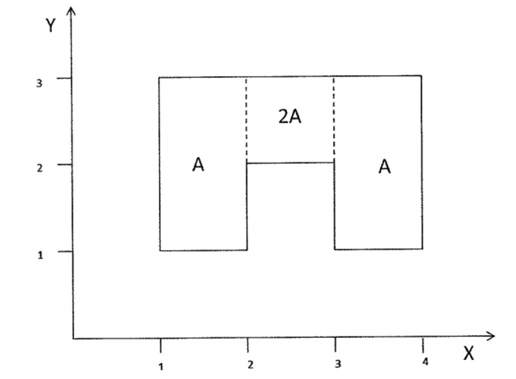

Figure 1: Joint density of X and Y.

a. Find A and sketch f_X, f_Y, and $f_{X|X+Y\leq 3}$.

b. Find $\mathbb{E}(X \mid Y = y)$ for $1 \leq y \leq 3$ and $\mathbb{E}(Y \mid X = x)$ for $1 \leq x \leq 4$.

c. Find $\text{cov}(X, Y)$.

3. **Revisiting Proofs Using Transforms**

   a. Calculate the MGF of $X \sim \text{Poisson}(\lambda)$.

   b. Let $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ be independent. Calculate the MGF of $X + Y$, and use this to show that $X + Y \sim \text{Poisson}(\lambda + \mu)$.

   c. Repeat parts a and b above, this time for $X \sim \mathcal{N}(0, \sigma_X^2)$ and $Y \sim \mathcal{N}(0, \sigma_Y^2)$.