UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

**Discussion 10**
Spring 2023

1. **Expected Squared Arrival Times**

   Let $(N_t)_{t \geq 0}$ be a Poisson process with arrival times $(T_n)_{n \geq 1}$. Find $\mathbb{E}(T_1^2 + T_2^2 + T_3^2 \mid N_1 = 3)$.

2. **CTMC Introduction**

Consider the continuous-time Markov chain defined on the state space $\{1, 2, 3, 4\}$ which has transition rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

a. Find the stationary distribution $\pi$ of this chain.

b. Find the stationary distribution $\mu$ of the jump chain, the DTMC which only keeps track of the jumps. Formally, if $(X_t)_{t\geq 0}$ transitions at times $T_1, T_2, \ldots$, then its jump chain is $(Y_n)_{n=1}^{\infty}$, where $Y_n \coloneqq X_{T_n}$.

c. Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

d. From state 1, what is the expected amount of time until the chain is in state 4?

3. **Taxi Queue**

   Empty taxis pass by a street corner according to a Poisson process of rate 2 per minute, and passengers arrive at the street corner according to a Poisson process of rate 1 per minute. Taxis always pick up a passenger if one is waiting; passengers wait for a taxi only if there are less than four people waiting, otherwise leaving and never returning. John arrives at the street corner at a given time. Find his expected waiting time, given that he joins the queue. Assume that the process is in steady state.