UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 08
Spring 2023

1. **Moving Books Around**

   You have N books labelled $1, \dots, N$ on your shelf. At each time step, you pick a book i with probability $\frac{1}{N}$, place it on the left of all others on the shelf, then repeat this process, each step independent of any other step. Construct a suitable Markov chain which takes values in the set of all $N!$ permutations of the books.

   a. Find the transition probabilities of the Markov chain.
   b. Find its stationary distribution.
      Hint: You can guess the stationary distribution before computing it.

2. **Markov Chain Practice**

Consider a Markov chain with three states 0, 1, 2, and suppose its transition probabilities are $P(0,1) = P(0,2) = \frac{1}{2}$, $P(1,0) = P(1,1) = \frac{1}{2}$, $P(2,0) = \frac{2}{3}$, and $P(2,2) = \frac{1}{3}$.

a. Classify the states in the chain. Is this chain periodic or aperiodic?

b. In the long run, what fraction of time does the chain spend in state 1?

c. Suppose that X_0 is chosen according to the steady-state or stationary distribution. What is $\mathbb{P}(X_0 = 0 \mid X_2 = 2)$?

3. **Hitting Time with Coins**

   Consider a sequence of fair coin flips.

   a. What is the expected number of flips until we first see two heads in a row?
   b. What is the expected number of flips until we see a head followed immediately by a tail?