

This homework is *optional* and consists of review problems. It will be run like a regular homework, but will not be graded, so you may also use it to familiarize yourself with the submission and self-grading system.

This homework is due at 11 PM on January 19, 2023.

Submission Format: Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned).

1. Subspaces and Dimensions

Consider the set \mathcal{S} of points $(x_1, x_2, x_3) \in \mathbb{R}^3$ such that

$$x_1 + 2x_2 + 3x_3 = 0, \quad 3x_1 + 2x_2 + x_3 = 0. \tag{1}$$

- (a) Find a 2×3 matrix A for which \mathcal{S} is exactly the null space of A .
- (b) Determine the dimension of \mathcal{S} and find a basis for it.

2. Orthogonality

Let $\vec{x}, \vec{y} \in \mathbb{R}^n$ be two linearly independent unit-norm vectors; that is, $\|\vec{x}\|_2 = \|\vec{y}\|_2 = 1$.

- (a) Show that the vectors $\vec{u} = \vec{x} - \vec{y}$ and $\vec{v} = \vec{x} + \vec{y}$ are orthogonal.
- (b) Find an orthonormal basis for $\text{span}(\vec{x}, \vec{y})$, the subspace spanned by \vec{x} and \vec{y} .

3. Homework Process

With whom did you work on this homework? List the names and SIDs of your group members.

NOTE: If you didn't work with anyone, you can put "none" as your answer.