Package 'volesti'

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Type Package
License GPL (>= 2)
Title Volume approximation of convex polytopes.
Description Package provides an R interface for VolEsti C++ package. VolEsti computes approximations of volume of polytopes given as a set of points or linear inequalities or Minkowski sum of segments (zonotopes). There are two algorithms for volume approximation as well as algorithms for sampling, rounding and rotating polytopes.
Maintainer Fisikopoulos Vissarion < vissarion.fysikopoulos@oracle.com >, Chalkis Apostolos < tolis.chal@gmail.com >
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Author Fisikopoulos Vissarion [cph, cre, aut], Chalkis Apostolos [cph, ctb, aut]
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CheBall

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Compute the Chebychev ball of a H-polytope.

Description

For a H-polytope described by a $m \times d$ matrix A and a m-dimensional vector b, s.t.: $Ax \leq b$, this function computes the largest inscribed ball (Chebychev ball) of that polytope by solving the corresponding linear program.

Usage

```
CheBall(A, b)
```

Arguments

A The matrix of the H-polytope.

b The m-dimensional vector b that containes the constants of the m facets.

Value

A (d+1)-dimensional vector that containes the Chebychev ball. The first d coordinates corresponds to the center and the last one to the radius of the Chebychev ball.

```
# compute the Chebychev ball of a 2d unit simplex A = matrix(c(-1,0,0,-1,1,1), ncol=2, nrow=3, byrow=TRUE) b = c(0,0,1) ball_vec = CheBall(A,b)
```

demoRounding 3

demoRounding	Run rounding and rotating tests.
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Description

Choose volume algorithm between CoolingGaussian and SequenceOfBalls and run rounding tests for some skinny cubes. In the first test we apply a random rotation as well before the rounding. We run 10 volume experiments for SequenceOfBalls and 20 for CoolingGaussian and we consider the mean value as the volume approximation.

Usage

```
demoRounding(algo)
```

Arguments

CG	The string "CG" to choose CoolingGaussian algorithm
SOB	The string "SOB" to choose SequenceOfBalls algorithm

Value

Print the computed volume and print a failure message if the error is larger than the expected.

Examples

```
# run tests for SOB algorithm
demoRounding("SOB")

# run tests for CV algorithm
demoRounding("CG")
```

demoSampling

Run some sampling experiments.

Description

Use uniform or spherical gaussian to sample from some convex H-polytopes, i.e. cubes, simplices, skinny cubes, cross polytopes and birkhoff polytopes. We use the default values, i.e. walklength = |10 + dimension/10|, N = 100, Cordinate Directions HnR, variance = 1.

Usage

```
demoSampling(distribution)
```

4 ExactZonoVol

Arguments

uniform The string "uniform" to choose uniform as the target distribution.

gaussian The string "gaussian" to choose spherical gaussian as the target distribution.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# choose uniform distribution
demoSampling("uniform")
# choose spherical gaussian distribution
demoSampling("gaussian")
```

ExactZonoVol

Compute the exact volume of a zonotope.

Description

Given the $m \times d$ matrix that containes the m segments that define the d-dimensional zonotope, this function computes the sum of the determinants of all the $d \times d$ submatrices.

Usage

```
ExactZonoVol(Matrix)
```

Arguments

Matrix

The $m \times d$ matrix that containes the segments that define the zonotope.

Value

The exact volume of the zonotope

```
# compute the exact volume of a 5-dimensional zonotope defined by the Minkowski sum of 10 segments
zonotope = GenZonotope(5, 10)
vol = ExactZonoVol(zonotope)
```

GenCross 5

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Gen	ı (`ı	rn	CC

Generator function for cross polytopes.

Description

This function can be used to generate a d-dimensional cross polytope in H or V representation.

Usage

```
GenCross(dimension, repr)
```

Arguments

dimension The dimension of the cross polytope.

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

Value

A cross polytope in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $2^d \times d$ matrix A and the "vector" containing a 2^d -dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $2d \times d$ matrix that containes the vertices row-wise.

Examples

```
# generate a 10-dimensional cross polytope in H-representation
PolyList = GenCross(10, 'H')

# generate a 15-dimension cross polytope in V-representation
PolyList = GenCross(15, 'V')
```

GenCube

Generator function for hypercubes.

Description

This function can be used to generate a d-dimensional Hypercube $[-1,1]^d$ in H or V representation.

Usage

```
GenCube(dimension, repr)
```

6 GenProdSimplex

Arguments

dimension The dimension of the hypercube

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

Value

A hypercube in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $2d \times d$ matrix A and the "vector" containing a 2d-dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $2^d \times d$ matrix that containes the vertices row-wise.

Examples

```
# generate a 10-dimensional hypercube in H-representation
PolyList = GenCube(10, 'H')
# generate a 15-dimension hypercube in V-representation
PolyList = GenCube(15, 'V')
```

GenProdSimplex

Generator function for product of simplices.

Description

This function can be used to generate a 2d-dimensional polytope that is defined as the product of two d-dimensional unit simplices in H-representation.

Usage

```
GenProdSimplex(dimension, repr = "H")
```

Arguments

dimension

The dimension of the simplices.

Value

A polytope defined as the product of two unit simplices in H-representation. The return value is a list with two elements: the "matrix" containing a $(2d+1) \times 2d$ matrix A and the "vector" containing a (2d+1)-dimensional vector b, s.t. $Ax \leq b$.

```
# generate a product of two 5-dimensional simplices.
PolyList = GenProdSimplex(5)
```

GenSimplex 7

Gensimplex Generator function for simplices.	GenSimplex	Generator function for simplices.
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Description

This function can be used to generate a d-dimensional unit simplex in H or V representation.

Usage

```
GenSimplex(dimension, repr)
```

Arguments

dimension The dimension of the simplex.

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

Value

A simplex in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $(d+1) \times d$ matrix A and the "vector" containing a (d+1)-dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $(d+1) \times d$ matrix that containes the vertices row-wise.

Examples

```
# generate a 10-dimensional simplex in H-representation
PolyList = GenSimplex(10, 'H')

# generate a 20-dimensional simplex in V-representation
PolyList = GenSimplex(20, 'V')
```

GenSkinnyCube

Generator function for skinny hypercubes.

Description

This function can be used to generate a *d*-dimensional skinny hypercube only in H-representation.

Usage

```
GenSkinnyCube(dimension, repr = "H")
```

Arguments

dimension

The dimension of the skinny hypercube.

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Value

A d-dimensional skinny hypercube in H-representation. The return value is a list with two elements: the "matrix" containing a $2d \times d$ matrix A and the "vector" containing a 2d-dimensional vector b, s.t. $Ax \leq b$.

Examples

```
# generate a 10-dimensional skinny hypercube.
PolyList = GenSkinnyCube(10)
```

GenZonotope

Generator function for zonotopes.

Description

This function can be used to generate a d-dimensional zonotope described by the Minkowski sum of m segments. We consider the e_1, \ldots, e_d generators and m-d random generators. Then we shift the zonotope in order to contain the origin. The origin is the center of symmetry as well. It might needs rounding before the volume computation.

Usage

```
GenZonotope(dimension, NumGen)
```

Arguments

dimension The dimension of the zonotope.

NumGen The number of segments that generate the zonotope.

Value

A $m \times d$ matrix that containes the m d-dimensional segments.

```
\# generate a 10-dimensional zonotope defined by the Minkowski sum of 20 segments zonotope = GenZonotope(10, 20)
```

HdemoVolume 9

HdemoVolume

Run some volume approximation experiments for H-polytopes.

Description

Choose between SequenceOfBalls and CoolingGaussian algorithm to approximate the volume of some cubes, simplices, skinny_cubes, cross polytopes and birkhoff polytopes in H-representation. For each polytope we run 10 volume experiments for SequenceOfBalls and 20 for CoolingGaussian and we consider the mean value as the volume approximation. We demand error=0.1 for the most of them. For all the other parameters we use the default values for both algorithms.

Usage

HdemoVolume(algo)

Arguments

CG The string "CG" to choose CoolingGaussian algorithm.

SOB The string "SOB" to choose SequenceOfBalls algorithm.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# test SequenceOfBalls
HdemoVolume("SOB")
# test CoolingGausian
HdemoVolume("CG")
```

ineToMatrix

function to get a ine file and returns a numerical matrix A.

Description

This function takes an ine file as a string (using read.csv()) and returns a numerical matrix A in ine format for function volume (see volume function examples).

Usage

```
ineToMatrix(P)
```

Arguments

Ρ

It is in format, read.cs('path/to/file.ine'). The ine file describes a H-polytope.

10 modifyMat

Value

The numerical matrix in ine format.

Examples

```
# give the path to birk4.ine
A = ineToMatrix(read.csv('path/to/data/birk4.ine'))
```

modifyMat

Takes a numerical matrix in ine format and returns the matrix A and the vector b s.t.: $Ax \leq b$.

Description

This function can be used to extract from a numerical matrix in ine format (see example), that describes a H-polytope, the $m \times d$ matrix A and the m-dimensional vector b, s.t.: $Ax \leq b$.

Usage

```
modifyMat(A)
```

Arguments

Α

The numerical matrix in ine format (see example) of the H-polytope.

Value

A list that containes elements "matrix" and "vector", i.e. the numerical $m \times d$ matrix A and the numerical m-dimensional vector b, defining H-polytope P, s.t.: $Ax \leq b$. For V polytopes the element "vector" is useless in practice.

```
# a 2d unit simplex in H-representation using numerical matrix in ine format A = matrix(c(3,3,0,0,-1,0,0,0,-1,1,1,1)), ncol=3, nrow=4, byrow=TRUE) list_of_matrix_and_vector = modifyMat(A)
```

rand_rotate 11

o. a zonoropo).	rand_rotate	Apply a random rotation to a convex polytope (H-polytope, V-polytope or a zonotope).
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Description

Given a convex H or V polytope or a zonotope as input this function applies a random rotation.

Usage

```
rand_rotate(Inputs)
```

Arguments

list("argument"=value)		
	A list that includes elements that describe the convex body that is given as input.	
path	The path to an ine (H-polytope) or ext (V-polytope, zonotope) file that describes the polytope. If path is given then "matrix" and "vector" inputs are not needed.	
matrix	The $m \times d$ matrix A of the H polytope or the $m \times d$ matrix that containes all the m d -dimensional vertices of a V-polytope row-wise or a $m \times d$ matrix that containes all the m d -dimensional segments that define a zonotope row-wise. If the matrix is in ine format, for H-polytopes only (see $volume$ function example), then the "vector" input is not needed.	
vector	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets.	
Vpoly	A boolean parameter, has to be true when a V-polytope is given as input. Default value is false.	
Zonotope	A boolean parameter, has to be true when a zonotope is given as input. Default value is false.	
verbose	Optional. A boolean parameter for printing. Default is false.	

Value

A random rotation of the polytope that is given as an input. The output for a H-polytope is a list that containes elements "matrix" and "vector". For a V-polytope the output is a $m \times d$ matrix that containes the m d-dimensional vertices of the V-polytope row-wise. For a zonotope is a $m \times d$ matrix that containes the m d-dimensional segments row-wise.

```
# rotate a H-polytope (2d unit simplex)
A = matrix(c(-1,0,0,-1,1,1), ncol=2, nrow=3, byrow=TRUE)
b = c(0,0,1)
listHpoly = rand_rotate(list("matrix"=A, "vector"=b))
# rotate a V-polytope (3d cube)
```

12 round_polytope

round_polytope Apply rounding to a convex polytope (H-polytope, V-polytope or a zonotope).

Description

Given a convex H or V polytope or a zonotope as input this function computes a rounding based on minimum volume enclosing ellipsoid of a pointset.

Usage

```
round_polytope(Inputs)
```

Arguments

verbose

list("argument"=value) A list that includes parameters for the rounding. path The path to an ine (H-polytope) or ext (V-polytope, zonotope) file that describes the polytope. If path is given then "matrix" and "vector" inputs are not needed. matrix The $m \times d$ matrix A of the H polytope or the $m \times d$ matrix that containes all the m d-dimensional vertices of a V-polytope row-wise or a $m \times d$ matrix that containes all the m d-dimensional segments that define a zonotope row-wise. If the matrix is in ine format, for H-polytopes only (see *volume* function example), then the "vector" input is not needed. vector Only for H-polytopes. The m-dimensional vector b that contains the constants of the m facets, s.t.: $Ax \leq b$. A boolean parameter, has to be true when a V-polytope is given as input. Default Vpoly value is false. Zonotope A boolean parameter, has to be true when a zonotope is given as input. Default value is false. walk_length Optional. The number of the steps for the random walk, default is |10 + d/10|. ball_walk Optional. Boolean parameter to use ball walk, only for CG algorithm. Default value is false. delta Optional. The radius for the ball walk. Optional. A boolean parameter for the hit-and-run. True for Coordinate Direccoordinate tions HnR, false for Random Directions HnR. Default value is true.

Optional. A boolean parameter for printing. Default is false.

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Value

Is a list that containes elements to describe the rounded polytope, i.e. "matrix" and "vector" for Hpolytopes and just "matrix" for V-polytopes and zonotopes, containing the verices or segments rowwise. For both representations the list containes element "round_value" which is the determinant of the square matrix of the linear transformation that was applied on the polytope that is given as input.

Examples

```
# rotate a H-polytope (2d unit simplex)
A = matrix(c(-1,0,0,-1,1,1), ncol=2, nrow=3, byrow=TRUE)
b = c(0,0,1)
listHpoly = round_polytope(list("matrix"=A, "vector"=b))
# rotate a V-polytope (3d cube) using Random Directions HnR
ListVpoly = round_polytope(list("matrix"=V, "Vpoly"=TRUE, "coordinate"=FALSE))
# rotate a 10-dimensional zonotope defined by the Minkowski sum of 20 segments
Zono = GenZonotope(10,20)
ListZono = round_polytope(list("matrix"=Zono, "Zonotope"=TRUE))
```

sample_points

Sample points from a convex Polytope (H-polytope, V-polytope or a zonotope).

Description

Sample N points from a H or a V-polytope or a zonotope with uniform or spherical gaussian centered in an internal point- target distribution.

Usage

```
sample_points(Inputs)
```

Arguments

list("argument"=value)

A list that includes parameters for the chosen target distribution and the random

walk algorithm.

path

The path to an ine (H-polytope) or ext (V-polytope, zonotope) file that describes the polytope. If path is given then "matrix" and "vector" inputs are not needed.

matrix

The $m \times d$ matrix A of the H polytope or the $m \times d$ matrix that containes all the m d-dimensional vertices of a V-polytope row-wise or a $m \times d$ matrix that containes all the m d-dimensional segments that define a zonotope row-wise. If the matrix is in ine format, for H-polytopes only (see *volume* function example),

then the "vector" input is not needed.

sample_points

vector	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets, s.t.: $Ax \leq b.$
walk_length	Optional. The number of the steps for the random walk, default is $\lfloor 10 + d/10 \rfloor$.
internal_point	Optional. A d -dimensional vector that containes the coordinates of an internal point of the polytope. If it is not given then for H-polytopes the Chebychev center is computed, for V-polytopes $d+1$ vertices are picked randomly and the Chebychev center of the defined simplex is computed. For a zonotope that is defined by the Minkowski sum of m segments we use the origin.
gaussian	Optional. A boolean parameter to sample with gaussian target distribution. Default value is false.
variance	Optional. The variance for the spherical gaussian. Default value is 1.
N	The number of points that the function is going to sample from the convex polytope. Default value is 100 .
ball_walk	Optional. Boolean parameter to use ball walk for the sampling. Default value is false.
delta	Optional. The radius for the ball walk.
verbose	Optional. A boolean parameter for printing. Default is false.
Vpoly	A boolean parameter, has to be true when a V-polytope is given as input. Default value is false.
Zonotope	A boolean parameter, has to be true when a zonotope is given as input. Default value is false.
coordinate	Optional. A boolean parameter for the hit-and-run. True for Coordinate Directions HnR, false for Random Directions HnR. Default value is true.

Value

A $d \times N$ matrix that contains, column-wise, the sampled points from the convex polytope.

VdemoVolume 15

Vdemo'	Volume	Run some volume approximation experiments for V-polytopes.

Description

Choose between SequenceOfBalls and CoolingGaussian algorithm to approximate the volume of some cubes, simplices and cross polytopes in V-representation. For each polytope we run 10 volume experiments and we consider the mean value as the volume approximation. For SOB algorithm we demand error=0.1 and for CG algorithm we demand error=0.2.

Usage

VdemoVolume(algo)

Arguments

CG The string "CG" to choose CoolingGaussian algorithm.

SOB The string "SOB" to choose SequenceOfBalls algorithm.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# test SequenceOfBalls
VdemoVolume("SOB")
# test CoolingGausian
VdemoVolume("CG")
```

volume

The main R function for volume approximation of a convex Polytope (H-polytope, V-polytope or a zonotope).

Description

For the volume approximation can be used two algorithms. Either SequenceOfBalls or Cooling-Gaussian. A H-polytope with m facets is described by a $m \times d$ matrix A and a m-dimensional vector b, s.t.: $Ax \leq b$. A V-polytope is described as a set of d-dimensional points. A zonotope is described by the Minkowski sum of d-dimensional segments.

Usage

```
volume(Inputs)
```

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Arguments

list("argument"=value)

A list that includes parameters for the chosen algorithm.

path The path to an ine (H-polytope) or ext (V-polytope, zonotope) file that describes

the polytope. If path is given then "matrix" and "vector" inputs are not needed.

matrix The $m \times d$ matrix A of the H polytope or the $m \times d$ matrix that containes all

the m d-dimensional vertices of a V-polytope row-wise or a $m \times d$ matrix that containes all the m d-dimensional segments that define a zonotope row-wise. If the matrix is in ine format, for H-polytopes only (see examples), then the

"vector" input is not needed.

vector Only for H-polytopes. The m-dimensional vector b that containes the constants

of the m facets s.t.: $Ax \leq b$.

walk_length Optional. The number of the steps for the random walk, default is $\lfloor 10 + d/10 \rfloor$.

error Optional. Declare the goal for the approximation error. Default is 1 for Se-

quenceOfBalls and 0.2 for CoolingGaussian.

InnerVec Optional. A d+1 vector that containes an inner ball. The first d coordinates

corresponds to the center and the last one to the radius of the ball. If it is not given then for H-polytopes the Chebychev ball is computed, for V-polytopes d+1 vertices are picked randomly and the Chebychev ball of the defined simplex is computed. For a zonotope that is defined as the Minkowski sum of m segments

we compute the maximal r s.t.: $re_i \in Z$ for all i = 1, ..., m.

CG Optional. A boolean parameter to use CoolingGaussian algorithm. Default

value is false.

win_len Optional. The size of the window for the ratios' approximation in CG algorithm.

Default value is $4 \ dimension^2 + 500$.

C Optional. A constant for the lower bound of $variance/mean^2$ in schedule an-

nealing of CG algorithm.

N optional. The number of points we sample in each step of schedule annealing in

CG algorithm. Default value is $500C + dimension^2/2$.

ratio Optional. Parameter of schedule annealing of CG algorithm, larger ratio means

larger steps in schedule annealing. Default value is 1 - 1/dimension.

frac Optional. The fraction of the total error to spend in the first gaussian in CG

algorithm. Default value is 0.1.

ball_walk Optional. Boolean parameter to use ball walk. Default value is false.

delta Optional. The radius for the ball walk.

verbose Optional. A boolean parameter for printing. Default is false.

Vpoly A boolean parameter, has to be true when a V-polytope is given as input. Default

value is false.

Zonotope A boolean parameter, has to be true when a zonotope is given as input. Default

value is false.

coordinate Optional. A boolean parameter for the hit-and-run. True for Coordinate Direc-

tions HnR, false for Random Directions HnR. Default value is true.

rounding Optional. A boolean parameter to activate the rounding option. Default value is

false.

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Value

The approximation of the volume of a convex H or V polytope.

References

I.Z.Emiris and V. Fisikopoulos, "Practical polytope volume approximation," ACM Trans. Math. Soft., 2014.,

B. Cousins and S. Vempala, "A practical volume algorithm," Springer-Verlag Berlin Heidelberg and The Mathematical Programming Society, 2015.

Examples

ZdemoVolume

Run some volume approximation experiments for zonotopes.

Description

Run SequenceOfBalls or CoolingGaussian algorithm to approximate the volume of some zonotopes. In each test we use GenZonotope() function to generate a random zonotope and then we apply rounding before the volume approximation. For each polytope we run 10 volume experiments and we consider the mean value as the volume approximation. For SOB algorithm we demand error=0.1 and for CG algorithm we demand error=0.2.

Usage

```
ZdemoVolume(algo)
```

Arguments

```
CG The string "CG" to choose CoolingGaussian algorithm.
```

The string "SOB" to choose SequenceOfBalls algorithm.

Zdemo Volume

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

test SequenceOfBalls
ZdemoVolume("SOB")
test CoolingGausian
ZdemoVolume("CG")

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