Package 'volesti'

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License	e GPL ($>= 2$)
Title V	Volume approximation of convex polytopes.
aj oi	ption Package provides an R interface for VolEsti C++ package. VolEsti computes pproximations of volume of polytopes given as a set of points or linear inequalities r Minkowski sum of segments (zonotopes). There are two algorithms for volume pproximation as well as algorithms for sampling, rounding and rotating polytopes.
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Imports Suggest Linking 1. Roxyge Author	ports https://github.com/vissarion/volume_approximation/issues s Rcpp (>= 0.12.17), RcppEigen (>= 0.3.3.4.0), BH (>= 1.66.0-1) ts lpSolveAPI (>= 5.5.2.0-17) gTo Rcpp (>= 0.12.17), RcppEigen (>= 0.3.3.4.0), BH (>= .66.0-1) enNote 6.0.1 Fisikopoulos Vissarion [cph, cre, aut], Chalkis Apostolos [cph, ctb, aut] Dics documented:
	CheBall demoRounding demoSampling ExactZonoVol fileToMatrix GenCross GenCube GenProdSimplex GenSimplex

2 CheBall

Index		17
	ZdemoVolume	16
	volume	
	VdemoVolume	
	sample_points	12
	round_polytope	11
	rand_rotate	10
	HdemoVolume	9
	GenZonotope	9
	GenSkinnyCube	8

CheBall

Compute the Chebychev ball of a H-polytope.

Description

For a H-polytope described by a $m \times d$ matrix A and a m-dimensional vector b, s.t.: $Ax \leq b$, this function computes the largest inscribed ball (Chebychev ball) of that polytope by solving the corresponding linear program. This function needs suggested R-package lpSolveAPI.

Usage

```
CheBall(A, b)
```

Arguments

A The matrix of the H-polytope.

b The m-dimensional vector b that containes the constants of the m facets.

Value

A (d+1)-dimensional vector that containes the Chebychev ball. The first d coordinates corresponds to the center and the last one to the radius of the Chebychev ball.

```
# compute the Chebychev ball of a 2d unit simplex A = matrix(c(-1,0,0,-1,1,1), ncol=2, nrow=3, byrow=TRUE) b = c(0,0,1) ball_vec = CheBall(A,b)
```

demoRounding 3

demoRounding	Run rounding and rotating tests.
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Description

Choose volume algorithm between CoolingGaussian and SequenceOfBalls and run rounding tests for some skinny cubes. In the first test we apply a random rotation as well before the rounding. We run 10 volume experiments for SequenceOfBalls and 20 for CoolingGaussian and we consider the mean value as the volume approximation.

Usage

```
demoRounding(algo)
```

Arguments

CG	The string "CG" to choose CoolingGaussian algorithm
SOB	The string "SOB" to choose SequenceOfBalls algorithm

Value

Print the computed volume and print a failure message if the error is larger than the expected.

Examples

```
# run tests for SOB algorithm
demoRounding("SOB")

# run tests for CV algorithm
demoRounding("CG")
```

demoSampling

Run some sampling experiments.

Description

Use uniform or spherical gaussian to sample from some convex H-polytopes, i.e. cubes, simplices, skinny cubes, cross polytopes and birkhoff polytopes. We use the default values, i.e. walklength = |10 + dimension/10|, N = 100, Cordinate Directions HnR, variance = 1.

Usage

```
demoSampling(distribution)
```

4 ExactZonoVol

Arguments

uniform The string "uniform" to choose uniform as the target distribution.

gaussian The string "gaussian" to choose spherical gaussian as the target distribution.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# choose uniform distribution
demoSampling("uniform")
# choose spherical gaussian distribution
demoSampling("gaussian")
```

ExactZonoVol

Compute the exact volume of a zonotope.

Description

Given the $m \times d$ matrix that containes the m segments that define the d-dimensional zonotope, this function computes the sum of the determinants of all the $d \times d$ submatrices.

Usage

ExactZonoVol(Matrix)

Arguments

Matrix

The $m \times d$ matrix that containes the segments that define the zonotope.

Value

The exact volume of the zonotope

```
# compute the exact volume of a 5-dimensional zonotope defined by the Minkowski sum of 10 segments
zonotope = GenZonotope(5, 10)
vol = ExactZonoVol(zonotope)
```

fileToMatrix 5

Description

This function takes the path for an ine or an ext file and returns the corresponding numerical matrix and vector that are compatible with volesti package's functions.

Usage

```
fileToMatrix(path)
```

Arguments

path

A string that containes the path to an ine or a ext file. The ine file describes a H-polytope and ext file describes a V-polytope or a zonotope.

Value

If the path corresponds to an ine file then the return value is a list that containes elements "A" and "b", i.e. the numerical $m \times d$ matrix A and the numerical m-dimensional vector b, defining H-polytope P, s.t.: $Ax \leq b$. If it corresponds to an ext file (V-polytopes or zonotopes) then the return value is a $m \times d$ matrix that containes row-wise the vertices or the segments respectively.

Examples

```
# give the path to birk4.ine
ListPoly = fileToMatrix(path/to/data/birk4.ine)
```

Generator function for cross polytopes.

Description

This function can be used to generate a d-dimensional cross polytope in H or V representation.

Usage

```
GenCross(dimension, repr)
```

Arguments

dimension The dimension of the cross polytope.

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

6 GenCube

Value

A cross polytope in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $2^d \times d$ matrix A and the "vector" containing a 2^d -dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $2d \times d$ matrix that containes the vertices row-wise.

Examples

```
# generate a 10-dimensional cross polytope in H-representation
PolyList = GenCross(10, 'H')

# generate a 15-dimension cross polytope in V-representation
PolyList = GenCross(15, 'V')
```

GenCube

Generator function for hypercubes.

Description

This function can be used to generate a d-dimensional Hypercube $[-1,1]^d$ in H or V representation.

Usage

```
GenCube(dimension, repr)
```

Arguments

dimension The dimension of the hypercube

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

Value

A hypercube in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $2d \times d$ matrix A and the "vector" containing a 2d-dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $2^d \times d$ matrix that containes the vertices row-wise.

```
# generate a 10-dimensional hypercube in H-representation
PolyList = GenCube(10, 'H')

# generate a 15-dimension hypercube in V-representation
PolyList = GenCube(15, 'V')
```

GenProdSimplex 7

GenProdSimplex Ge	nerator function for product of simplices.
-------------------	--

Description

This function can be used to generate a 2d-dimensional polytope that is defined as the product of two d-dimensional unit simplices in H-representation.

Usage

```
GenProdSimplex(dimension, repr = "H")
```

Arguments

dimension

The dimension of the simplices.

Value

A polytope defined as the product of two unit simplices in H-representation. The return value is a list with two elements: the "matrix" containing a $(2d+1) \times 2d$ matrix A and the "vector" containing a (2d+1)-dimensional vector b, s.t. $Ax \leq b$.

Examples

```
# generate a product of two 5-dimensional simplices.
PolyList = GenProdSimplex(5)
```

GenSimplex

Generator function for simplices.

Description

This function can be used to generate a d-dimensional unit simplex in H or V representation.

Usage

```
GenSimplex(dimension, repr)
```

Arguments

dimension The dimension of the simplex.

repr A string to declare the representation. It has to be 'H' for H-representation or

'V' for V-representation.

8 GenSkinnyCube

Value

A simplex in H or V-representation. For an H polytope the return value is a list with two elements: the "matrix" containing a $(d+1) \times d$ matrix A and the "vector" containing a (d+1)-dimensional vector b, s.t. $Ax \leq b$. When the V-representation is chosen the return value is a $(d+1) \times d$ matrix that containes the vertices row-wise.

Examples

```
# generate a 10-dimensional simplex in H-representation
PolyList = GenSimplex(10, 'H')

# generate a 20-dimensional simplex in V-representation
PolyList = GenSimplex(20, 'V')
```

GenSkinnyCube

Generator function for skinny hypercubes.

Description

This function can be used to generate a d-dimensional skinny hypercube only in H-representation.

Usage

```
GenSkinnyCube(dimension, repr = "H")
```

Arguments

dimension

The dimension of the skinny hypercube.

Value

A d-dimensional skinny hypercube in H-representation. The return value is a list with two elements: the "matrix" containing a $2d \times d$ matrix A and the "vector" containing a 2d-dimensional vector b, s.t. $Ax \leq b$.

```
# generate a 10-dimensional skinny hypercube.
PolyList = GenSkinnyCube(10)
```

GenZonotope 9

GenZonotope	Generator function for zonotopes.

Description

This function can be used to generate a d-dimensional zonotope described by the Minkowski sum of m segments. We consider the e_1, \ldots, e_d generators and m-d random generators. Then we shift the zonotope in order to contain the origin. The origin is the center of symmetry as well. It might needs rounding before the volume computation.

Usage

```
GenZonotope(dimension, NumGen)
```

Arguments

dimension The dimension of the zonotope.

NumGen The number of segments that generate the zonotope.

Value

A $m \times d$ matrix that containes the m d-dimensional segments.

Examples

```
# generate a 10-dimensional zonotope defined by the Minkowski sum of 20 segments
zonotope = GenZonotope(10, 20)
```

HdemoVolume

Run some volume approximation experiments for H-polytopes.

Description

Choose between SequenceOfBalls and CoolingGaussian algorithm to approximate the volume of some cubes, simplices, skinny_cubes, cross polytopes and birkhoff polytopes in H-representation. For each polytope we run 10 volume experiments for SequenceOfBalls and 20 for CoolingGaussian and we consider the mean value as the volume approximation. We demand error=0.1 for the most of them. For all the other parameters we use the default values for both algorithms.

Usage

HdemoVolume(algo)

CG	The string "CG" to choose CoolingGaussian algorithm.
SOB	The string "SOB" to choose SequenceOfBalls algorithm.

rand_rotate

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# test SequenceOfBalls
HdemoVolume("SOB")
# test CoolingGausian
HdemoVolume("CG")
```

rand_rotate	Apply a random rotation to a convex polytope (H-polytope, V-polytope or a zonotope).
rand_rotate	

Description

Given a convex H or V polytope or a zonotope as input this function applies a random rotation.

Usage

```
rand_rotate(A, b, V, G)
```

Arguments

A	Only for H-polytopes. The $m \times d$ matrix A that containes the directions of the m facets.
b	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets s.t.: $Ax \leq b$.
V	Only for V-polytopes. The $m\times d$ matrix V that containes row-wise the m d -dimensional vertices of the polytope.
G	Only for zonotopes. The $m \times d$ matrix G that containes row-wise the m d-dimensional segments that define a zonotope.

Value

A random rotation of the polytope that is given as an input. The output for a H-polytope is a list that containes elements "matrix" and "vector". For a V-polytope the output is a $m \times d$ matrix that containes the m d-dimensional vertices of the V-polytope row-wise. For a zonotope is a $m \times d$ matrix that containes the m d-dimensional segments row-wise.

round_polytope 11

Examples

round_polytope Apply rounding to a convex polytope (H-polytope, V-polytope or a zonotope).

Description

Given a convex H or V polytope or a zonotope as input this function computes a rounding based on minimum volume enclosing ellipsoid of a pointset.

Usage

```
round_polytope(A, b, V, G, walk_length, ball_walk, delta, coordinate, verbose)
```

A	Only for H-polytopes. The $m \times d$ matrix A that containes the directions of the m facets.
b	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets s.t.: $Ax \leq b$.
V	Only for V-polytopes. The $m\times d$ matrix V that containes row-wise the m d -dimensional vertices of the polytope.
G	Only for zonotopes. The $m\times d$ matrix G that containes row-wise the m d -dimensional segments that define a zonotope.
walk_length	Optional. The number of the steps for the random walk, default is $\lfloor 10 + d/10 \rfloor$.
ball_walk	Optional. Boolean parameter to use ball walk, only for CG algorithm. Default value is false.
delta	Optional. The radius for the ball walk.
coordinate	Optional. A boolean parameter for the hit-and-run. True for Coordinate Directions HnR, false for Random Directions HnR. Default value is true.
verbose	Optional. A boolean parameter for printing. Default is false.

12 sample_points

Value

Is a list that containes elements to describe the rounded polytope, i.e. "matrix" and "vector" for H-polytopes and just "matrix" for V-polytopes and zonotopes, containing the verices or segments rowwise. For both representations the list containes element "round_value" which is the determinant of the square matrix of the linear transformation that was applied on the polytope that is given as input.

Examples

sample_points	Sample points from a convex Polytope (H-polytope, V-polytope or a
	zonotope).

Description

Sample N points from a H or a V-polytope or a zonotope with uniform or spherical gaussian - centered in an internal point- target distribution.

Usage

```
sample_points(A, b, V, G, walk_length, internal_point, gaussian, variance, N,
ball_walk, delta, verbose, coordinate)
```

A	Only for H-polytopes. The $m\times d$ matrix A that containes the directions of the m facets.
b	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets s.t.: $Ax \leq b$.
V	Only for V-polytopes. The $m \times d$ matrix V that containes row-wise the m d -dimensional vertices of the polytope.
G	Only for zonotopes. The $m \times d$ matrix G that containes row-wise the m d -dimensional segments that define a zonotope.
walk_length	Optional. The number of the steps for the random walk, default is $\lfloor 10 + d/10 \rfloor$.

VdemoVolume 13

internal_point	Optional. A d -dimensional vector that containes the coordinates of an internal point of the polytope. If it is not given then for H-polytopes the Chebychev center is computed, for V-polytopes $d+1$ vertices are picked randomly and the Chebychev center of the defined simplex is computed. For a zonotope that is defined by the Minkowski sum of m segments we use the origin.
gaussian	Optional. A boolean parameter to sample with gaussian target distribution. Default value is false.
variance	Optional. The variance for the spherical gaussian. Default value is 1.
N	The number of points that the function is going to sample from the convex polytope. Default value is 100 .
ball_walk	Optional. Boolean parameter to use ball walk for the sampling. Default value is false.
delta	Optional. The radius for the ball walk.
verbose	Optional. A boolean parameter for printing. Default is false.
coordinate	Optional. A boolean parameter for the hit-and-run. True for Coordinate Directions HnR, false for Random Directions HnR. Default value is true.

Value

A $d \times N$ matrix that contains, column-wise, the sampled points from the convex polytope.

Examples

VdemoVolume

 ${\it Run some volume approximation experiments for V-polytopes.}$

Description

Choose between SequenceOfBalls and CoolingGaussian algorithm to approximate the volume of some cubes, simplices and cross polytopes in V-representation. For each polytope we run 10 volume experiments and we consider the mean value as the volume approximation. For SOB algorithm we demand error = 0.1 and for CG algorithm we demand error = 0.2.

Usage

VdemoVolume(algo)

14 volume

Arguments

CG	The string "CG" to choose CoolingGaussian algorithm.
SOB	The string "SOB" to choose SequenceOfBalls algorithm.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

```
# test SequenceOfBalls
VdemoVolume("SOB")
# test CoolingGausian
VdemoVolume("CG")
```

volume

The main R function for volume approximation of a convex Polytope (H-polytope, V-polytope or a zonotope).

Description

For the volume approximation can be used two algorithms. Either SequenceOfBalls or Cooling-Gaussian. A H-polytope with m facets is described by a $m \times d$ matrix A and a m-dimensional vector b, s.t.: $Ax \leq b$. A V-polytope is described as a set of d-dimensional points. A zonotope is described by the Minkowski sum of d-dimensional segments.

Usage

```
volume(A, b, V, G, walk_length, error, InnerVec, CG, win_len, C, N, ratio, frac,
ball_walk, delta, verbose, coordinate, rounding)
```

Α	Only for H-polytopes. The $m \times d$ matrix A that containes the directions of the m facets.
b	Only for H-polytopes. The m -dimensional vector b that containes the constants of the m facets s.t.: $Ax \leq b$.
V	Only for V-polytopes. The $m\times d$ matrix V that containes row-wise the m d -dimensional vertices of the polytope.
G	Only for zonotopes. The $m\times d$ matrix G that containes row-wise the m d -dimensional segments that define a zonotope.
walk_length	Optional. The number of the steps for the random walk, default is $\lfloor 10 + d/10 \rfloor$.
error	Optional. Declare the goal for the approximation error. Default is 1 for SequenceOfBalls and 0.2 for CoolingGaussian.

volume 15

InnerVec	Optional. A $d+1$ vector that containes an inner ball. The first d coordinates corresponds to the center and the last one to the radius of the ball. If it is not given then for H-polytopes the Chebychev ball is computed, for V-polytopes $d+1$ vertices are picked randomly and the Chebychev ball of the defined simplex is computed. For a zonotope that is defined as the Minkowski sum of m segments we compute the maximal r s.t.: $re_i \in Z$ for all $i=1,\ldots,m$.
CG	Optional. A boolean parameter to use CoolingGaussian algorithm. Default value is false.
win_len	Optional. The size of the window for the ratios' approximation in CG algorithm. Default value is $4\ dimension^2+500$.
С	Optional. A constant for the lower bound of $variance/mean^2$ in schedule annealing of CG algorithm.
N	optional. The number of points we sample in each step of schedule annealing in CG algorithm. Default value is $500C+dimension^2/2$.
ratio	Optional. Parameter of schedule annealing of CG algorithm, larger ratio means larger steps in schedule annealing. Default value is $1-1/dimension$.
frac	Optional. The fraction of the total error to spend in the first gaussian in CG algorithm. Default value is 0.1.
ball_walk	Optional. Boolean parameter to use ball walk. Default value is false.
delta	Optional. The radius for the ball walk.
verbose	Optional. A boolean parameter for printing. Default is false.
coordinate	Optional. A boolean parameter for the hit-and-run. True for Coordinate Directions HnR, false for Random Directions HnR. Default value is true.
rounding	Optional. A boolean parameter to activate the rounding option. Default value is false.

Value

The approximation of the volume of a convex H or V polytope.

References

I.Z.Emiris and V. Fisikopoulos, "Practical polytope volume approximation," ACM Trans. Math. Soft., 2014.,

B. Cousins and S. Vempala, "A practical volume algorithm," Springer-Verlag Berlin Heidelberg and The Mathematical Programming Society, 2015.

16 ZdemoVolume

```
vol = volume(V=V, CG=TRUE)

# calling Gaussian-Cooling algorithm for a 5-dimensional zonotope defined as the Minkowski sum of 10 segments
zonotope = GenZonotope(5, 10)
vol = volume(G=zonotope, rounding=TRUE, CG=TRUE)
```

ZdemoVolume

Run some volume approximation experiments for zonotopes.

Description

Run SequenceOfBalls or CoolingGaussian algorithm to approximate the volume of some zonotopes. In each test we use GenZonotope() function to generate a random zonotope and then we apply rounding before the volume approximation. For each polytope we run 10 volume experiments and we consider the mean value as the volume approximation. For SOB algorithm we demand error=0.1 and for CG algorithm we demand error=0.2.

Usage

ZdemoVolume(algo)

Arguments

CG The string "CG" to choose CoolingGaussian algorithm.

SOB The string "SOB" to choose SequenceOfBalls algorithm.

Value

Print the computed volumes and the error. If the test fails a message is printed.

Examples

test SequenceOfBalls
ZdemoVolume("SOB")
test CoolingGausian
ZdemoVolume("CG")

Index

```
{\sf CheBall}, \textcolor{red}{2}
demoRounding, 3
{\tt demoSampling}, {\tt 3}
ExactZonoVol, 4
fileToMatrix, 5
GenCross, 5
GenCube, 6
GenProdSimplex, 7
GenSimplex, 7
{\tt GenSkinnyCube}, {\color{red} 8}
GenZonotope, 9
HdemoVolume, 9
\verb"rand_rotate", \\ 10
\verb"round_polytope", \verb"11"
\verb|sample_points|, \\ 12
VdemoVolume, 13
volume, 14
```