1 Inflating sequence (from the Exam 2017) A sequence $x_1 < x_2 < \ldots < x_n$ of integers is *inflating* if $x_i - x_{i-1} < x_{i+1} - x_i$ for all $1 < i < n$. For example, the sequence $3, 5, 8, 13$ is an inflating sequence, since $5-3 < 8-5 < 13-8$, whereas $3, 4, 6, 7$ is not, since $6-4 > 7-6$.

Given a sequence $X = [x_1, \ldots, x_n]$ of integers, where $x_1 < x_2 < \ldots < x_n$, we want to find the longest inflating *subsequence* of X. For example, the sequence $6, 8, 12$ is a longest inflating subsequence of $3, 6, 8, 10, 12$.

For $1 \le i \le j \le n$, let $D(i, j)$ be the length of the longest inflating subsequence of $x_1, \ldots, x_j$, where x_i and x_j are respectively the second last and last elements in the subsequence. Let $D(i, i) = 1$ for all i.

1.1 Fill out the table below for the sequence $X = [1, 4, 5, 7, 9]$.

$D(i,j)$	1	2	3	4	5
1					
2					
3					
4					
5					

1.2 Which of the following recurrences correctly computes $D(i, j)$:

$\boxed{\text{A}}$
$$D(i,j) = \begin{cases} 1 & \text{if } i = j \\ 1 + \max\{D(i,\ell) \mid 1 \le \ell \le j \text{ and } x_i - x_\ell < x_j - x_i\} & \text{otherwise} \end{cases}$$

$\boxed{\text{B}}$
$$D(i,j) = \begin{cases} 1 & \text{if } i = j \\ 1 + \max\{D(\ell,i) \mid 1 \le \ell < i \text{ and } x_i - x_\ell < x_j - x_i\} & \text{otherwise} \end{cases}$$

$\boxed{\text{C}}$
$$D(i,j) = \begin{cases} 1 & \text{if } i = j \\ 1 + \max\{D(\ell,i) \mid 1 \le \ell \le i \text{ and } x_i - x_\ell < x_j - x_i\} & \text{otherwise} \end{cases}$$

1.3 Write pseudocode for an algorithm *based on dynamic programming and the recurrence you chose in Question 1.2* that computes the maximum total score you can achieve. The input to your algorithm is an array $X[1 \ldots n]$ of increasing integers.

Analyze the space usage and running time of your algorithm in terms of n.