Práctica 1. Análisis de eficiencia de algoritmos

Noelia Escalera Mejías — Alejandro Menor Molinero Javier Núñez Suárez — Adra Sánchez Ruiz Jesús Torres Sánchez

10 de marzo de 2019

1. Introducción

2. Eficiencia empírica

Vamos a medir el tiempo que tarda en ejecutarse cada uno de los ocho algoritmos: Quicksort, Mergesort, Heapsort, Inserción, Selección, Burbuja, Floyd y Fibonacci.

Además, los compararemos entre ellos cuando sea interesante hacerlo.

2.1. Algoritmos de ordenación

2.1.1. Ordenación rápida

Empezamos con los algoritmos de ordenación rápidos. Estos pertenecen al orden de eficiencia O(n * log(n)) es decir, "superlineales".

Tamaño del vector	Tiempo con Quicksort	Tiempo con Mergesort	Tiempo con Heapsort
500	0.000125572	0.000185121	0.000192113
1000	0.000271314	0.000389135	0.000444516
1500	0.00041891	0.000760708	0.000705254
2000	0.000594146	0.00102568	0.00097424
2500	0.000740502	0.000482617	0.000443686
3000	0.000596825	0.000494674	0.000381796
3500	0.000343663	0.000452999	0.000429375
4000	0.000373585	0.000644118	0.00053697
4500	0.00042863	0.000694048	0.000605903
5000	0.000513286	0.000788472	0.000634107
5500	0.000557894	0.000948842	0.000715611
6000	0.000605284	0.00105457	0.000794382
6500	0.000657624	0.000926247	0.000862167
7000	0.000714435	0.00100227	0.000929307
7500	0.000757684	0.0011178	0.00100083

8000 0.000831217 0.001331409 0.00118477 8500 0.00087632 0.00131409 0.00114999 9000 0.000951436 0.00139895 0.00124178 9500 0.00100672 0.00153253 0.001302 10000 0.00104054 0.00163203 0.00136841 10500 0.00111741 0.00176789 0.00144557 11500 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.00161126 12000 0.00134991 0.00229752 0.0017724 13000 0.00144951 0.0020339 0.00186281 13500 0.00144951 0.0020339 0.00193143 14000 0.00152673 0.0020339 0.00193143 14500 0.0016307 0.00232988 0.00199139 14500 0.0016856 0.00219523 0.00207509 15000 0.0016857 0.00232089 0.00216104 15500 0.0016855 0.0024091 0.00223161 16000 <				
9000 0.000951436 0.00139895 0.00124178 9500 0.00100672 0.00153253 0.001302 10000 0.00104054 0.00163203 0.00136841 10500 0.00111741 0.00176789 0.00144557 11000 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.0016126 12000 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.0020339 0.00193143 14000 0.00152673 0.00208988 0.00199139 14500 0.0016307 0.00232089 0.00216104 15500 0.0016857 0.0024091 0.00223611 16000 0.00175315 0.00251567 0.00231843 16500 0.00180967 0.00251567 0.00231843 16500 0.00189917 0.0025037 0.00240901 17000 0.00187919 0.00250362 0.00250362 18500	8000	0.000831217	0.00123515	0.00108477
9500 0.00100672 0.00153253 0.001302 10000 0.00104054 0.00163203 0.00136841 10500 0.00111741 0.00176789 0.00144557 11000 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.00161126 12000 0.00128353 0.00217296 0.00168296 12500 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.00202339 0.00193143 14000 0.00152673 0.00208988 0.00199139 14500 0.00158276 0.00219523 0.00207509 15000 0.0016807 0.00232089 0.00216104 15500 0.0016855 0.0024091 0.0022361 16500 0.00180967 0.002501567 0.00231843 16500 0.00189967 0.0026037 0.00240901 17000 0.00189967 0.0026037 0.00256264 18000	8500	0.00087632	0.00131409	0.00114999
10000 0.00104054 0.00163203 0.00136841 10500 0.00111741 0.00176789 0.00144557 11000 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.00161126 12000 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.00202339 0.00193143 14000 0.00158263 0.00208888 0.00199139 14500 0.00168367 0.00208898 0.00199139 15000 0.0016837 0.00232089 0.00216104 15500 0.0016855 0.0024091 0.00223611 16000 0.00175315 0.00251567 0.00231843 16500 0.00180967 0.0026037 0.00240901 17000 0.00187919 0.0027362 0.00250793 17500 0.00192917 0.00287752 0.00256264 18000 0.00224495 0.00310007 0.0026382 18500	9000	0.000951436	0.00139895	0.00124178
10500 0.00111741 0.00176789 0.00144557 11000 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.00161126 12000 0.00128353 0.00217296 0.00168296 12500 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.00202339 0.00193143 14000 0.00158263 0.00208988 0.00199139 14500 0.00158276 0.00219523 0.00207509 15000 0.0016307 0.0023089 0.00216104 15500 0.0016855 0.0024091 0.0023611 16000 0.00175315 0.00251567 0.00231843 16500 0.00180967 0.00262037 0.00240901 17500 0.00187919 0.0027362 0.00250793 17500 0.00192917 0.00287752 0.0025664 18500 0.0024495 0.00310153 0.00272564 19000	9500	0.00100672	0.00153253	0.001302
11000 0.0011769 0.00188453 0.00152554 11500 0.00124374 0.00208893 0.00161126 12000 0.00128353 0.00217296 0.00168296 12500 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.00202339 0.00193143 14000 0.00152673 0.00208988 0.00199139 14500 0.00158276 0.00219523 0.00207509 15000 0.0016307 0.00232089 0.00216104 15500 0.0016855 0.0024091 0.00223611 16500 0.00175315 0.00251567 0.00231843 16500 0.00180967 0.0026037 0.00240901 17000 0.00189967 0.00262037 0.00250793 17500 0.00192917 0.00287752 0.00256264 18000 0.0020248 0.00300007 0.00263882 18500 0.00218022 0.00310153 0.00272534 19500	10000	0.00104054	0.00163203	0.00136841
11500 0.00124374 0.00208893 0.00161126 12000 0.00128353 0.00217296 0.00168296 12500 0.00134991 0.00229752 0.0017724 13000 0.00142095 0.00192418 0.00186281 13500 0.00144951 0.0020339 0.00193143 14000 0.00152673 0.00208988 0.00193139 14500 0.00158276 0.00219523 0.00207509 15500 0.0016307 0.00232089 0.00216104 15500 0.0016855 0.0024091 0.00223611 16000 0.00175315 0.00251567 0.00231843 16500 0.00180967 0.00262037 0.00240901 17000 0.00187919 0.00287752 0.00250793 17500 0.00192917 0.00287752 0.00256264 18000 0.00204495 0.00310153 0.00272534 19500 0.0024495 0.00310153 0.00272534 19500 0.0021357 0.00338002 0.00280503 19500	10500	0.00111741	0.00176789	0.00144557
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	11000	0.0011769	0.00188453	0.00152554
$\begin{array}{c} 12500 & 0.00134991 & 0.00229752 & 0.0017724 \\ 13000 & 0.00142095 & 0.00192418 & 0.00186281 \\ 13500 & 0.00144951 & 0.00202339 & 0.00193143 \\ 14000 & 0.00152673 & 0.00208988 & 0.00199139 \\ 14500 & 0.00158276 & 0.00219523 & 0.00207509 \\ 15000 & 0.0016307 & 0.00232089 & 0.00216104 \\ 15500 & 0.0016855 & 0.0024091 & 0.00223611 \\ 16000 & 0.00175315 & 0.00251567 & 0.00231843 \\ 16500 & 0.00180967 & 0.00251567 & 0.00231843 \\ 16500 & 0.00189967 & 0.00262037 & 0.00240901 \\ 17000 & 0.00187919 & 0.0027362 & 0.00250793 \\ 17500 & 0.00187919 & 0.00287752 & 0.00256264 \\ 18000 & 0.00192917 & 0.00287752 & 0.00256264 \\ 18000 & 0.0020248 & 0.00300007 & 0.00263882 \\ 18500 & 0.00204495 & 0.00310153 & 0.00272534 \\ 19000 & 0.00211357 & 0.00325465 & 0.00280503 \\ 19500 & 0.00218022 & 0.0038002 & 0.00280503 \\ 19500 & 0.00223461 & 0.00350399 & 0.00304415 \\ 20500 & 0.00232654 & 0.00358945 & 0.00314781 \\ 21500 & 0.0023261 & 0.00372468 & 0.00329918 \\ 22000 & 0.00232654 & 0.00385273 & 0.00329943 \\ 22500 & 0.00244845 & 0.00385273 & 0.00330935 \\ 22000 & 0.0024441 & 0.00385273 & 0.00339943 \\ 22500 & 0.00248485 & 0.00398946 & 0.00339943 \\ 22500 & 0.00256673 & 0.00411845 & 0.00385966 \\ 24000 & 0.0027772 & 0.00445179 & 0.00366066 \\ 24000 & 0.00276691 & 0.00454967 & 0.00373309 \\ 24500 & 0.00285553 & 0.00466454 & 0.00382896 \\ \end{array}$	11500	0.00124374	0.00208893	0.00161126
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	12000	0.00128353	0.00217296	0.00168296
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	12500	0.00134991	0.00229752	0.0017724
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	13000	0.00142095	0.00192418	0.00186281
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	13500	0.00144951	0.00202339	0.00193143
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	14000	0.00152673	0.00208988	0.00199139
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	14500	0.00158276	0.00219523	0.00207509
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	15000	0.0016307	0.00232089	0.00216104
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	15500	0.0016855	0.0024091	0.00223611
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	16000	0.00175315	0.00251567	0.00231843
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	16500	0.00180967	0.00262037	0.00240901
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	17000	0.00187919	0.0027362	0.00250793
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	17500	0.00192917	0.00287752	0.00256264
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	18000	0.0020248	0.00300007	0.00263882
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	18500	0.00204495	0.00310153	0.00272534
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	19000	0.00211357	0.00325465	0.00280503
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	19500	0.00218022	0.00338002	0.00289392
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20000	0.00223461	0.00350399	0.00304415
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20500	0.00232654	0.00358945	0.00314781
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	21000	0.0023512	0.00372468	0.00322618
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	21500	0.0024141	0.00385273	0.00330935
23000 0.00264539 0.00433311 0.00357741 23500 0.00272772 0.00445179 0.00366066 24000 0.00270691 0.00454967 0.00373309 24500 0.00285553 0.00466454 0.00382896		0.00248485	0.00398946	0.00339943
23500 0.00272772 0.00445179 0.00366066 24000 0.00270691 0.00454967 0.00373309 24500 0.00285553 0.00466454 0.00382896	22500	0.00255673		0.00348261
24000 0.00270691 0.00454967 0.00373309 24500 0.00285553 0.00466454 0.00382896				
24500 0.00285553 0.00466454 0.00382896				
	24000	0.00270691	0.00454967	0.00373309
25000 0.00282962 0.0048426 0.00392208				
	25000	0.00282962	0.0048426	0.00392208

Tabla comparativa de tiempos

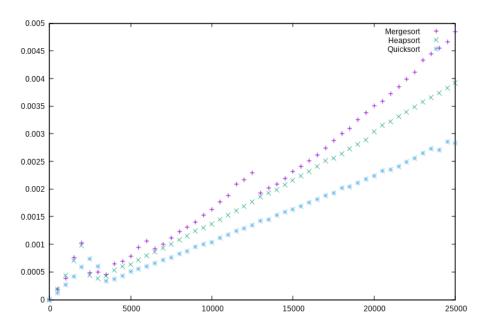


Figura 1: Comparación gráfica del rendimiento de los algoritmos de ordenación rápida $\,$

2.1.2. Ordenación lentos

Estos algoritmos de ordenación, menos sofisticados, son de orden $\mathcal{O}(n^2)$ es decir, cuadráticos.

Tamaño del vector	Tiempo con Burbuja	Tiempo con Selección	Tiempo con Inserción
500	0.00178596	0.00147628	0.00114028
1000	0.0028655	0.0022588	0.00172961
1500	0.00448784	0.00309903	0.00230721
2000	0.00786624	0.00525987	0.00405115
2500	0.0124692	0.00811555	0.00630397
3000	0.0181514	0.0116717	0.00910679
3500	0.0252785	0.0157854	0.0125022
4000	0.0337448	0.0205625	0.0158871
4500	0.0436306	0.0268227	0.0201791
5000	0.0551609	0.0331552	0.026194
5500	0.0681233	0.0401148	0.030802
6000	0.0824843	0.0467118	0.035932
6500	0.0984357	0.0540054	0.042335
7000	0.11589	0.0626111	0.0497211

7500	0.135017	0.0717969	0.0573054
8000	0.155683	0.0817153	0.0657382
8500	0.176902	0.0921947	0.0768291
9000	0.199919	0.103297	0.0861508
9500	0.225075	0.115035	0.0981397
10000	0.251881	0.127486	0.103923
10500	0.279234	0.140492	0.122772
11000	0.309941	0.154166	0.131101
11500	0.34121	0.171219	0.142071
12000	0.371406	0.183355	0.158711
12500	0.405278	0.198969	0.168258
13000	0.441736	0.215243	0.178126
13500	0.478529	0.232051	0.195711
14000	0.517851	0.249406	0.215179
14500	0.557069	0.26754	0.223471
15000	0.623507	0.286271	0.245298
15500	0.64346	0.305662	0.257939
16000	0.693738	0.325702	0.277471
16500	0.734539	0.346204	0.297803
17000	0.778796	0.367458	0.311583
17500	0.829418	0.39475	0.322414
18000	0.880487	0.412826	0.352076
18500	0.933294	0.435126	0.360694
19000	0.986121	0.460939	0.379935
19500	1.07066	0.483263	0.396013
20000	1.09964	0.515923	0.421674
20500	1.15639	0.544332	0.447574
21000	1.22045	0.5604	0.471736
21500	1.32645	0.590167	0.483069
22000	1.39171	0.618805	0.504104
22500	1.55601	0.646724	0.53811
23000	1.52041	0.671924	0.56646
23500	1.60414	0.701547	0.596336
24000	1.6872	0.745452	0.613182
24500	1.7148	0.770377	0.635088
25000	1.78348	0.79409	0.638414

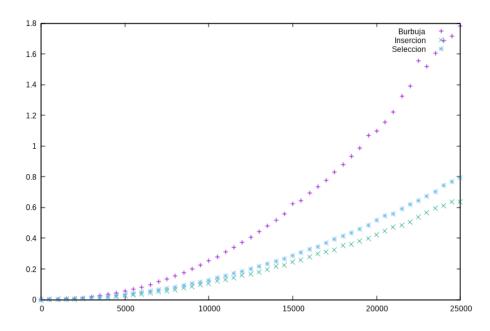


Figura 2: Comparación gráfica del rendimiento de los algoritmos de ordenación lenta

Por último, en las figuras 3 y 4, se muestra el rendimiento de todos los algoritmos de ordenación, rápidos y lentos.

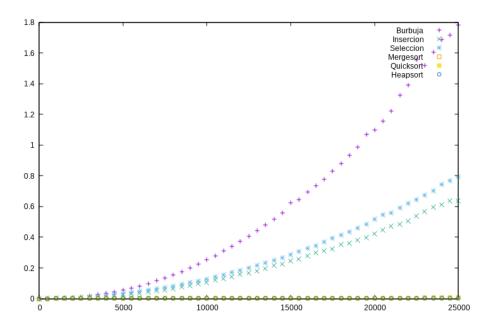


Figura 3: Comparación gráfica del rendimiento de los algoritmos de ordenación

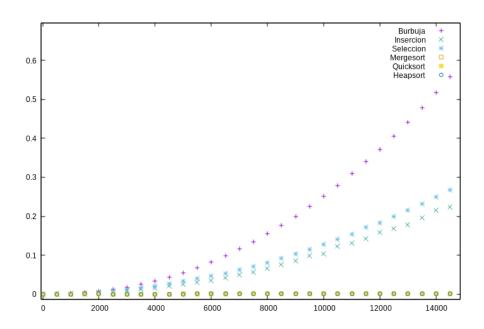


Figura 4: Zoomen el intervalo $\left[0\text{-}15000\right]$ de la figura 3

2.2. Floyd

Dado un conjunto de nodos de un grafo dirigido, el algoritmo de Floyd calcula el costo del camino mínimo entre cada par. Pertenece al orden de eficiencia $O(n^3)$, se muestra una gráfica en la figura 5.

Tamaño	Tiempo
1	1.954e-07
21	0.000155079
41	0.00103966
61	0.00231581
81	0.00315898
101	0.00603818
121	0.0102868
141	0.0163929
161	0.0241221
181	0.0351787
201	0.0473705
221	0.0620468
241	0.0834537
261	0.103527
281	0.128019
301	0.1575
321	0.194009
341	0.231567
361	0.275895
381	0.32918
401	0.367902
421	0.4349
441	0.490013
461	0.562313
481	0.631398
501	0.716334
521	0.832386
541	0.918103
561	1.03886
581	1.12256
601	1.23978
621	1.36486
641	1.5045
661	1.64389
681	1.79002
701	1.96247
721	2.16406
741	2.39603

761	2.64711
781	2.81278
801	3.1552
821	3.19514
841	3.45967
861	3.82836
881	3.88608
901	4.45056
921	4.35145
941	4.50856
961	4.88424
981	5.52174
1001	5.60464

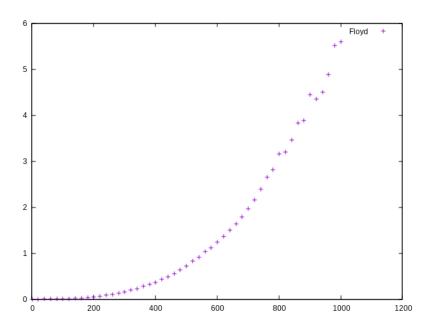


Figura 5: Tiempos de ejecución en el algoritmo de Floyd

2.3. Fibonacci

Este algoritmo calcula los números de la sucesión de Fibonacci.

Hace uso de la recursión y como hemos visto en clase, esto puede derivar muy facilmente en un algoritmo de orden exponencial, es este uno de esos casos. $fib(n)\in O((\frac{1+\sqrt{5}}{2})^n)$

$$fib(n) \in O((\frac{1+\sqrt{5}}{2})^n)$$

0 2.856e-07 1 1.512e-07 2 2.096e-07 3 2.086e-07 4 2.698e-07 5 3.788e-07 6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.00339488 30 0.00540311 31 0.0087656 32 0.0143048 33 0.022	Tamaño	Tiempo
2 2.096e-07 3 2.086e-07 4 2.698e-07 5 3.788e-07 6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 <td< td=""><td>0</td><td>2.856e-07</td></td<>	0	2.856e-07
3 2.086e-07 4 2.698e-07 5 3.788e-07 6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 <t< th=""><th>1</th><th>1.512e-07</th></t<>	1	1.512e-07
4 2.698e-07 5 3.788e-07 6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 <		2.096e-07
5 3.788e-07 6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 <	3	2.086e-07
6 5.074e-07 7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36		
7 7.382e-07 8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37	5	3.788e-07
8 1.0928e-06 9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 <td>6</td> <td>5.074e-07</td>	6	5.074e-07
9 1.599e-06 10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40	7	
10 2.4596e-06 11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	8	1.0928e-06
11 3.8526e-06 12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	9	1.599e-06
12 6.078e-06 13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	10	
13 9.6922e-06 14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		3.8526e-06
14 1.44386e-05 15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
15 2.0433e-05 16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	13	
16 3.66042e-05 17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 21 0.000291334 22 0.000485881 23 0.000766585 24 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
17 4.68404e-05 18 7.15016e-05 19 0.000113647 20 0.000291334 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	15	
18 7.15016e-05 19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
19 0.000113647 20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		4.68404e-05
20 0.000181842 21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	18	7.15016e-05
21 0.000291334 22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
22 0.000485881 23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	20	
23 0.000766585 24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
24 0.00061927 25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
25 0.000491851 26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	23	
26 0.000754886 27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
27 0.0013577 28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
28 0.00209638 29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265	26	
29 0.00339488 30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
30 0.00540311 31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
31 0.00876756 32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
32 0.0143048 33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
33 0.0226266 34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
34 0.036777 35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
35 0.0581737 36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
36 0.0948025 37 0.158607 38 0.260101 39 0.40173 40 0.654265		
37 0.158607 38 0.260101 39 0.40173 40 0.654265	35	
38 0.260101 39 0.40173 40 0.654265		
39 0.40173 40 0.654265		
40 0.654265		
41 1.06958		
	41	1.06958

42	1.70722
43	2.74853
44	4.60926
45	7.51057
46	11.6822
47	18.6488
48	30.9733
49	52.57
50	83.3349

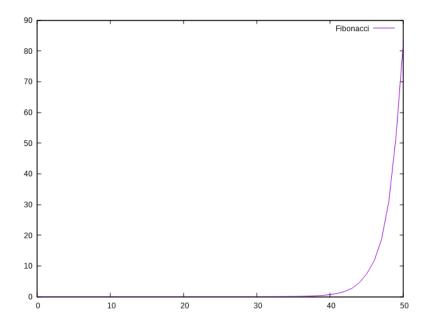


Figura 6: Tiempos de ejecución en el algoritmo de Fibonacci

3. Eficiencia híbrida

Ajuste de las diferentes funciones de acuerdo a la eficiencia teórica de los algoritmos y los datos obtenidos en la eficiencia empírica.

3.1. Algoritmos de ordenación rápidos

Para el algoritmo Mergesort:

Constante	Valor	Error estándar
a0	3.66473e-12	12.16

a1	8.67345e-08	13.28
a2	0.000308646	20.18

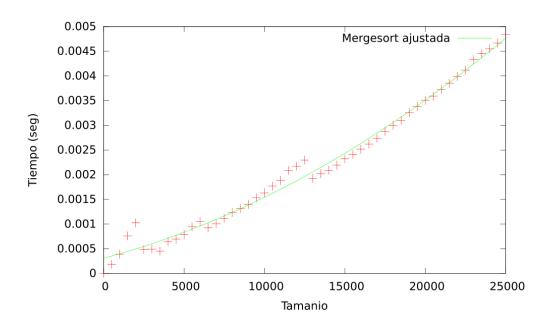


Figura 7: Ajuste función Mergesort

Para el algoritmo Heapsort:

Constante	Valor	Error estándar
a0	2.32227e-12	14.09
a1	9.24005 e-08	9.152
a2	0.000224702	20.34

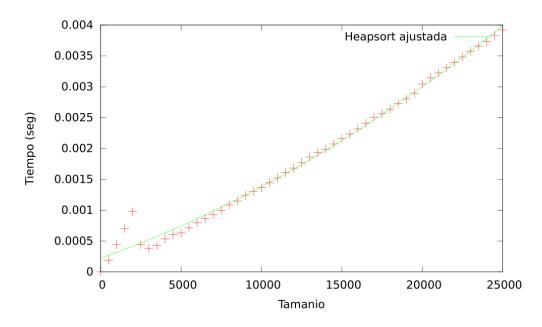


Figura 8: Ajuste función Heapsort

Para el algoritmo Quicksort:

Constante	Valor	Error estándar
a0	1.37793e-12	18.84
a1	7.47827e-08	8.974
a2	0.000181972	19.93

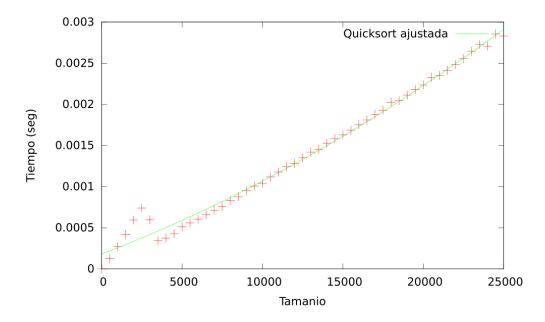


Figura 9: Ajuste función Quicksort

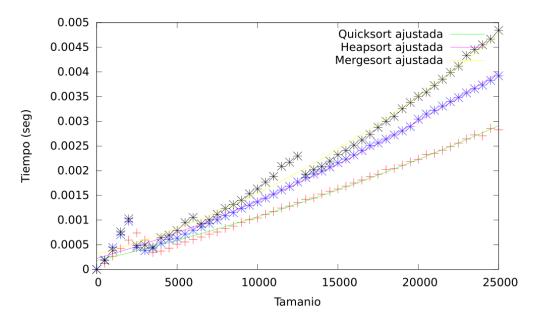


Figura 10: Comparativa ajuste para algoritmos de ordenación rápidos

3.2. Algoritmos de ordenación lentos

Para el algoritmo Burbuja:

Constante	Valor	Error estándar
a0	3.25024e-09	1.809
a1	-9.48063e-06	16.03
a2	0.0160101	51.32

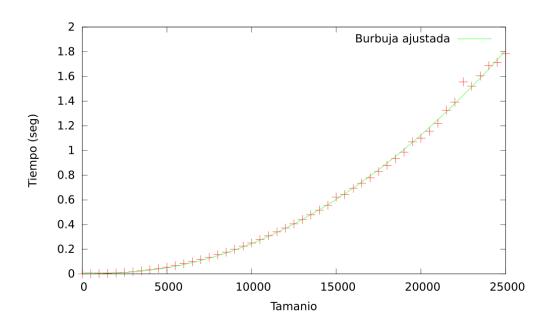


Figura 11: Ajuste función Burbuja

Para el algoritmo Insercion:

Constante	Valor	Error estándar
a0	1.02502e-09	1.459
a1	8.53837e-07	45.29
a2	-0.00285116	73.31

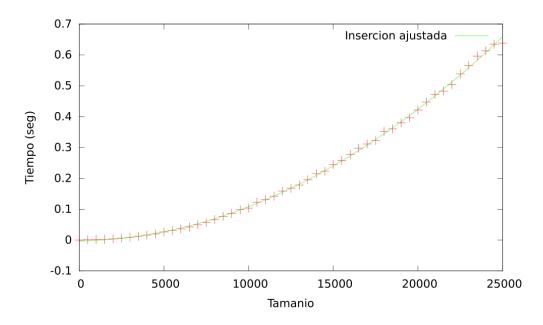


Figura 12: Ajuste función Inserción

Para el algoritmo de Seleccion:

Constante	Valor	Error estándar
a0	1.28478e-09	0.5517
a1	-1.91843e-07	95.52
a2	0.00095162	104.1

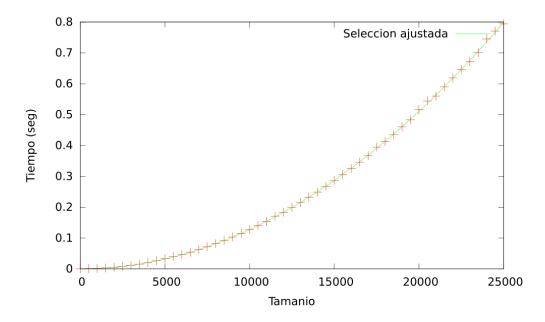


Figura 13: Ajuste función Selección

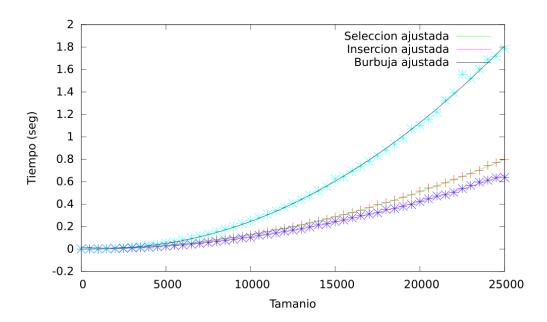


Figura 14: Comparativa ajuste para algoritmos de ordenación lentos

3.3. Algoritmo de Fibonacci

Constante	Valor	Error estándar
a0	2.9613e-09	0.2375

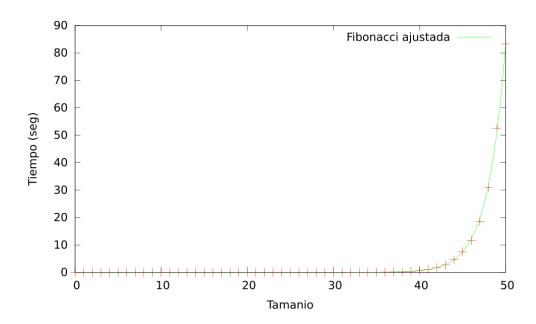


Figura 15: Ajuste función de Fibonacci

3.4. Algoritmo de Floyd

Constante	Valor	Error estándar
a0	4.5232e-09	12.19
a1	1.64e-06	51.27
a2	-0.000541551	66.61
a3	0.0340052	121.6

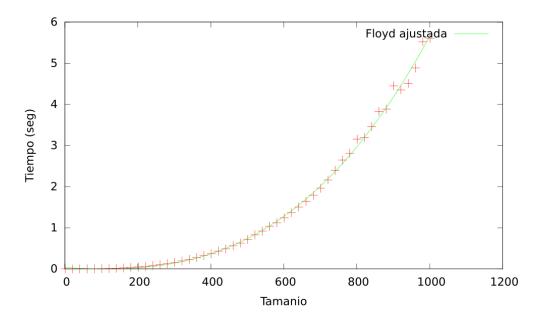


Figura 16: Ajuste función de Floyd

3.5. Ajustes de tiempos con funciones no correspondientes

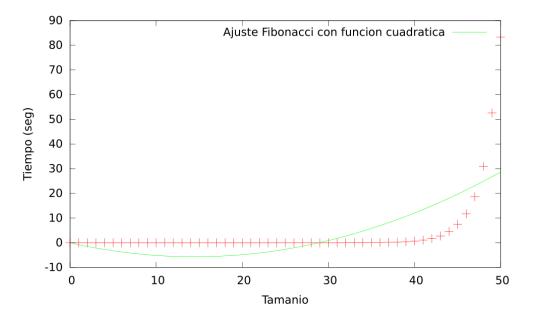


Figura 17: Ajuste Fibonacci con función cuadrática

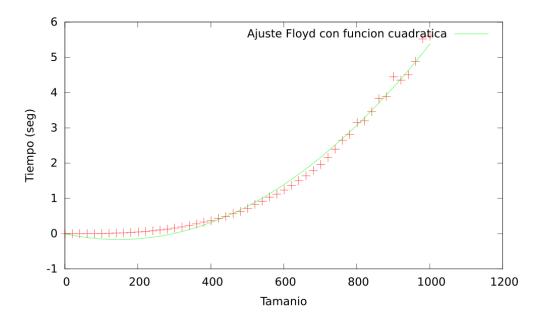


Figura 18: Ajuste Floyd con función cuadrática

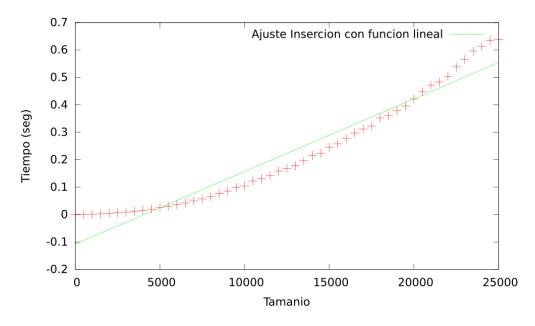


Figura 19: Ajuste Inserción con función lineal

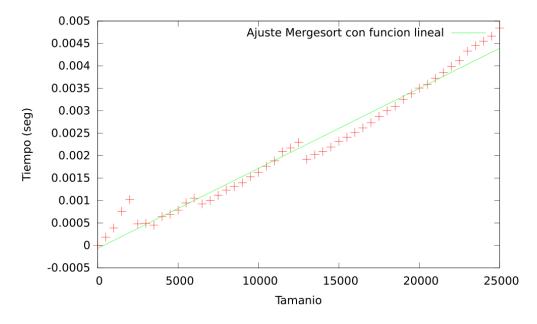


Figura 20: Ajuste Mergesort con función lineal

3.6. Probando en distintas condiciones

3.6.1. Distintos ordenadores

Hemos probado la misma implementación de un algoritmo en dos ordenadores distintos y así de paso demostrar el principio de invarianza.

En la figura 21 se muestran los tiempos de ambos ordenadores en la misma gráfica y en la figura 22, la función cociente entre los tiempos de las dos ejecuciones.

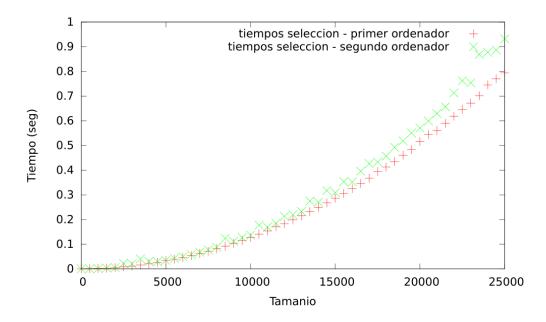


Figura 21: Equipos diferentes, tiempos distintos

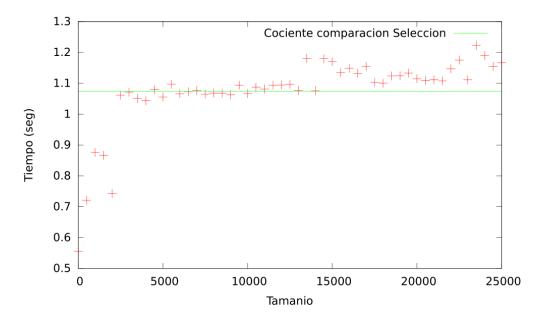


Figura 22: Demostrando el principio de invarianza

3.6.2. Distintas opciones de compilación

En este caso hemos probado el algoritmo de Floyd en el mismo equipo, pero compilando con y sin optimización. (Hemos utilizado el switch -O3 para la versión optimizada).

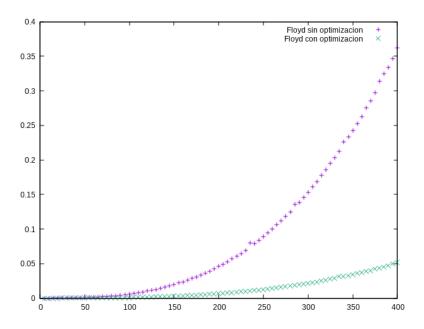


Figura 23: Tiempos de ejecución compilando con y sin optimización

A pesar de todo, como se muestra en la figura 24, sólo se diferencian en una constante $k\approx 0{,}141.$

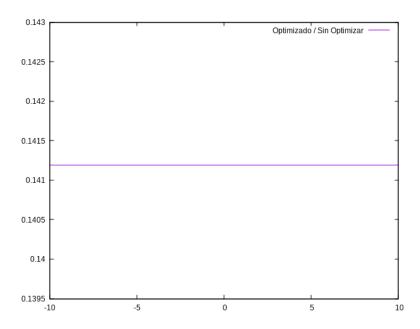


Figura 24: El principio de invarianza, esta vez aplicado a las opciones del compilador.